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Reserve Requirements, the Maturity Structure of Debt, and Bank Runs

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Abstract

The paper looks at the relationship between reserve requirements and the choice of the maturity structure of external debt in a general equilibrium setup, by incorporating the role of international lenders. A date- and maturity-specific reserve requirement is a fraction of the debt to be deposited in a non-interest bearing account at the central bank. At maturity, the central bank returns the reserves. There exist some specific combinations of date- and maturity-specific reserve requirements that reduce the vulnerability to bank runs. In such setup, lenders may still want to provide new short-term lending to the bank after a bank run.

JEL Classification Numbers: E43, E44, F34

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I. Introduction

> […] the right starting point for thinking about capital controls must be on very focused, temporary measures aimed at stemming massive temporary inflows or outflows of debt. (Rogoff, 2002, ¶ 6)

After the financial crises of the 1990s, preventing large speculative and volatile inflows of short-term debt became a major concern. Following the spread of liberalization policies in the 1990s, many developing countries have seen their private and financial sectors accumulate high short-term borrowing. The increase in private short-term borrowing resulted in a situation where the private sector became unable to fully service the debt without new borrowing. With high inherited short-term debt, any new borrowing becomes costlier. A debt overhang arises where the expected present value of output is lower than the accumulated debt. The private sector with a debt overhang signals its inability to pay back its debt and therefore may not be able to get any new borrowing. This in turn leads to default crises, bank runs, or speculative attacks.

In this paper, we explore the relationship between capital controls and the choice of the maturity structure of external debt in a general equilibrium setup while explicitly incorporating the role of international lenders. We look at specific types of capital controls which take the form of date-specific and maturity-specific reserve requirements on external borrowing. A date- and maturity-specific reserve requirement is a fraction of the amount borrowed by private agents that has to be deposited in a non-interest bearing account at the central bank. When the bond matures, the central bank returns the reserves. Despite the extensive literature analyzing capital controls, collateral and external debt, the issue of reserve requirements on external borrowing has received little attention. What are the effects of date- and maturity-specific reserve requirements on the maturity structure of external debt? Can they prevent a bank run?

II. Motivation and Literature

The success of the Chilean experience with reserve requirements on external debt brought increasing attention to the use of these types of capital controls. De Gregorio and others (2000) show that such policies have tilted the maturity composition of capital inflows toward a longer maturity structure in Chile. The Chilean financial liberalization dates from 1974. It had a banking crisis in September 1981 that peaked in March 1983. The closest balance of payment crisis was in August 1982 (Reinhart and Kaminsky, 1999). In the 1990s, Chile did not experience any financial crisis. The literature explaining the success of the Chilean case refers to the use of reserve requirements as capital controls among many sound monetary policies. In January 1992, 20-percent reserve requirements were imposed on deposits and loans in foreign currency held by commercial banks. The reserves had to be maintained for one year. In May 1992, the rate was increased to 30 percent and it was set such that the reserve requirement rate was lower for longer maturities. In September 1998, a year marked by financial crisis in other Latin American countries, the rate of reserve requirements was set to zero. The share of private debt in Chile in the 1990s has increased. In addition, short-term debt share has decreased significantly from 19.41 percent in 1990 to 5.08 percent in 1998 (Reinhart and Reinhart, 1999, and De Gregorio and others, 2000).

Theoretically, the answer on the benefit of capital controls is not really straight forward: as much as it prevents excessive short-term debt ex-ante, it puts more constraint on the provision of liquidity ex-post. Diamond and Rajan (2000) argue that once illiquid investment has been made, a ban on short-term debt may precipitate a crisis. Reinhart and Smith (2002) study the effect of temporary controls on capital inflows and find that temporary capital controls need to be very high to be effective. The welfare benefits of such taxes are estimated to be very low. Aizenman and Turnovsky (2002) analyze the effect of reserve requirements in a model with only short-term debt and show that they reduce the probability of default and thus increase welfare. Forbes (2007) shows that "during the period of the encaje (reserve requirements), smaller traded firms experienced significant financial constraints."

Reserve requirements could play the role of liquidity guarantee for any issuance of debt ex-ante, and therefore may prevent a crisis ex-ante. But in the event of a liquidity shock, short-term borrowing becomes more difficult when reserve requirements are imposed and therefore might constrain the role of short-term borrowing as a liquidity provider. These two effects working in opposite directions are consistent with results related to the benefits or losses associated with capital controls. The scarce literature that explored the role of these reserve requirements has identified their tax equivalent role. In this paper we try to capture the liquidity role in addition to the tax role.

The above discussion suggests that a rigorous analysis of reserve requirements should consider not only the maturity structure of external private debt, but also the behavior of banks and international lenders in the external debt market. At least three features of the environment would be desirable to analyze an endogenous maturity structure: First, liquidity risk must be present to endogenize alternative scenarios with different maturity structures. Second, for the choice between short-term and long-term debt to be explicit, the possibility of issuing long-term and short-term debt simultaneously should be considered. Third, the behavior of international lenders has to be considered in a world general equilibrium.

Diamond and Dybvig (1983) provide a natural framework to think about reserve requirements. Chang and Velasco (2000) extend their model to a small open economy in which they show that bank runs are associated with high levels of short-term debt. We extend the basic Chang and Velasco (2000) framework to a general equilibrium model in which international lending is endogenous and then analyze the effects of capital controls that take the form of reserve requirements on external borrowing. More specifically, our model consists of a simple Diamond-Dybvig-type model with three dates and two large open economies: one with high income and one with low income. Banks arise endogenously in the low income countries. There are two short-term bonds and one long-term bond offered by the domestic banks to international lenders. We consider international lending behavior explicitly. The high income countries lend to low income countries. In the world general equilibrium, the interest rates will be such that markets clear. Accordingly, the equilibrium term structure of the debt will be determined.

If the bank holds a liquid position, there is never a bank run, independently of whether or not there is a bad dream. If the bank holds an illiquid position, domestic depositors will run to the bank if and only if they see a bad dream. In such a setup, reserve requirements not only play the role of a tax but also provide liquidity for each bond at different dates. We show that they reduce the scope of indeterminacy. Some specific combinations of date- and maturity-specific reserve requirements reduce the vulnerability to bank runs.

This paper deals with the cause of a banking crisis, the assessment of an ex-ante preventive policy, and an ex-post bailout strategy by international lenders. In section III, we develop the model with and without reserve requirements without considering bank runs. In section IV, we look into how bank runs emerge. We identify illiquidity conditions that are necessary for bank runs to emerge. We explore whether date- and maturity- specific reserve requirements can prevent the occurrence of a bank run. We analyze the re-optimizing behavior of international lenders in deciding whether or not to lend when a bank run occurs. In section V, we discuss the justification of some of the key assumptions in this setup.

III. The Model

A. The Domestic Economy

The economy faces a two-period planning horizon with three dates: t=0, 1, and 2. In the domestic country, there is a continuum of agents with unit-mass. These agents are born at t=0. There is one type of goods each period, which is homogeneous across countries. This is an endowment economy. At t=0 each depositor gets an endowment of e_0 units of the good. At t=1 and t=2, they do not receive any endowments of goods[2]. At $t = 0$, depositors do not consume. The domestic agents are ex-ante identical but they may be ex-post different owing to a preferences shock that is realized at t=1. The distribution of this shock is known at t=0, and it is i.i.d. across agents. A t=0 with probability $\lambda \in (0,1)$, the depositors could be impatient and derive utility only from consuming goods at t=1. With probability $(1-\lambda)$, they could be patient and would want to consume goods at t=2 only. Thus domestic depositors will consume in either period. The distribution is known ex-ante and is public information. However when the event is realized at t=1, each depositor's type realization is private information. Unlike Aizenman and Turnovsky (2002), there is no aggregate uncertainty. The only uncertainty in the borrowing country is private.

The good is perishable if not invested or consumed. There is an investment technology that works as follows: one unit of the good invested at $t=0$ yields a real gross rate of return $R > 1$

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[2] With this assumption we avoid the complexity that may arise with different types of deposits while we are focusing on different types of debts. For instance, a depositor receiving a new endowment at t=1 would face the choice of whether or not to deposit her new endowment at the bank at t=1. We rule out such a possibility. The assumption that the domestic depositors have no endowments at t=2 is just a normalization that is standard in the literature.

at t=2, but produces only r units of the good if liquidated earlier at t=1, where $r^2 < R$.[3] Let k denote the amount of goods invested in this technology at $t=0$.[4]

Because of the uncertainty and the ex-post heterogeneity, a bank or a coalition of depositors arises endogenously. Henceforth we will use the terms "domestic bank" and "borrower" interchangeably. As is standard in the literature, owing to the preference shock that depositors face, they may gain from acting jointly. In fact, each depositor faces a probability λ of being of the impatient type at t=1. Liquidating the investment at t=1 with a lower return becomes unavoidable if she was to choose autarky and learns that she is impatient. Therefore, depositors find it optimal to form a bank that can provide some insurance. In this setup, banks arise endogenously.[5] Banks will offer contracts that maximize the domestic depositors expected utility inducing truth-telling self-selection. We assume a logarithmic form of utility.[6] Thus a domestic depositor's expected utility is given by

$$U(c_1, c_2) = \lambda \ln(c_1) + (1-\lambda)\ln(c_2), \tag{1}$$

where c_1 and c_2 are consumption quantities at t=1 and t=2 respectively. $\ln(c)$ is twice continuously differentiable, strictly increasing in its arguments and satisfies the Inada conditions.

All domestic agents deposit their endowments at the bank at t=0. In addition to receiving deposits in the amount e_0 at t=0, only banks can borrow from international lenders. At t=0, two types of international debt are available to banks: short term and long term. At t=1, banks can only borrow short term. d_{01} is the short-term bond issued by the domestic bank at t=0 that matures at t=1, paying a gross real interest rate ρ_{01}. d_{12} is the short-term bond issued by the domestic bank at t=1 that matures at t=2, paying a gross real interest rate ρ_{12}. Finally, d_{02} is the long-term bond issued by the domestic bank at t=0 that matures at t=2 paying a gross real interest rate ρ_{02}. The bank takes ρ_{01}, ρ_{02}, and ρ_{12} as given.

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[3] Note that a one unit of the good can yield r^2 units at t=2, if invested at t=0, and then liquidated and reinvested at t=1 and withdrawn at t=2.

[4] In Chang and Velasco (2000), the borrower has the possibility to invest in an additional technology: "international reserves" b_t with gross return equal to one. Here we abstract from such a possibility. It is important to note that in this paper model when $r > 1$, the rate of return on the investment technology would be higher than "international reserves" technology. The investment in k would therefore dominate the saving in "international reserves." In fact Chang and Velasco (2000) assume $r = 0$.

[5] One could also think of the bank as a possible contracting scheme, in which agents exchange contracts to hedge against uncertainty.

[6] Note that with the logarithmic function, the substitution effect dominates the income effect, improving greatly the tractability of the model.

Banks have access to the same investment technology that depositors have. The bank will decide how much to invest long-term k at *t*=0 and how much to liquidate l at *t*=1, given R and r as explained above.

B. Date-Specific and Maturity-Specific Reserve Requirements

Now, suppose that the social planner for the domestic economy imposes the reserve-requirement rates exogenously. She allows them to be both date-specific and maturity-specific. For instance, when the bank borrows from abroad, it must deposit in the central bank a fraction θ_{01}, θ_{12}, and θ_{02} on d_{01}, d_{12}, and d_{02}, respectively. In other words at *t*=0, a fraction θ_{01} of the first short-term debt is placed as reserves at the central bank and will be returned to the domestic bank at *t*=1, i.e., when the debt matures. At *t*=0, a fraction θ_{02} of the long-term debt is placed as reserves at the central bank and will be returned to the bank at *t*=2. At *t*=1, a fraction θ_{12} of the second short-term debt is placed at the central bank as reserves and will be returned to the bank at *t*=2.[7] Clearly, $\theta_{01} \in [0,1)$, $\theta_{12} \in [0,1)$ and $\theta_{02} \in [0,1)$.

Then the general forms of the budget constraints of the bank become:

$$k \leq e_0 + (1-\theta_{01})d_{01} + (1-\theta_{02})d_{02}, \tag{2}$$

$$\lambda c_1 + \rho_{01}d_{01} \leq (1-\theta_{12})d_{12} + rl + \theta_{01}d_{01}, \tag{3}$$

$$(1-\lambda)c_2 + \rho_{12}d_{12} + \rho_{02}d_{02} \leq R(k-l) + \theta_{12}d_{12} + \theta_{02}d_{02} \; . \tag{4}$$

In addition, the following incentive compatibility constraint needs to hold:[8]

$$c_2 \geq rc_1 \; . \tag{5}$$

[7] Note that in this setup the focus is on the case in which reserve requirements are returned to the bank when the debt matures, neither before nor after. It is also assumed, for simplicity, that the reserve requirement does not earn any interest when held at the central bank. This is consistent with reality in which interest rates on reserve requirements are relatively lower than on other assets. One can think of the reserve requirement as a risk-less technology which gives back the good when the debt matures at a gross rate of return equal to one.

[8] We allow domestic depositors to have access to investment technology even after the bank is formed. A patient depositor has therefore the option of concealing her type and withdrawing c_1 units of *t*=1 goods from the bank. After investing this withdrawal for one period, at *t*=2, she could consume rc_1 units of the goods at *t*=2. If the patient depositor reveals her true type and waits until date *t*=2, she would consume c_2 units of *t*=2 goods. Therefore the patient depositor will have an incentive to reveal her type if and only if $c_2 \geq rc_1$. In Chang and Velasco (2000), once the bank is formed, depositors are not allowed to have access to the investment technology. Since they have a storage technology, there the incentive compatibility constraint takes the form: $c_2 \geq c_1$.

By looking at the budget constraints in equations (2), (3) and (4), it is possible to distinguish two roles that reserve requirements may play if a bank run at $t = 1$ occurs. First, at t=0, a fraction θ_{01} and a fraction θ_{02} are placed as reserves for every unit of d_{01} and d_{02}; therefore investment in k is penalized. Thus at t=0, the reserve requirement rate θ_{01} and θ_{02} operates like a tax. Similarly at t=1, θ_{12} plays the role of a "tax." However at t=1, a fraction θ_{01} of the maturing short-term debt is returned as reserves and guarantees at least a partial payment of the debt to the international lenders, reducing the need for early liquidation of the long-term asset. This will be called a "liquidity" role. Note that when θ_{12} is applied to any new issuance of debt at t=1, the new borrowing which is financing any liquidity shortage would need to be higher. This could precipitate a crisis if the demand for short-term borrowing is not satisfied by the supply side. At t=2, fractions θ_{12} and θ_{02} of the respective short-term and long-term maturing bonds are returned to the domestic bank. This is again the "liquidity" role of θ_{12} and θ_{02}. Notice that for a single date- and maturity- specific reserve requirement rate, the "liquidity" role and the "tax" role work in opposite directions and at two different dates. It is therefore an interesting policy question to ask which of the two effects dominates and under which circumstances. The "liquidity" role is an ex-ante preventive mechanism of a crisis. But in the event of a crisis, the "tax" role might create a worse outcome because it could make borrowing more difficult.

Unlike Chang and Velasco (2000), we do not impose credit limits, since quantities and prices will be determined in equilibrium once the lenders' problem is considered explicitly. The domestic bank's problem is to choose $\{c_1, c_2, d_{01}, d_{02}, d_{12}, k, l\}$ to maximize equation (1) subject to equations (2), (3), (4), and (5), given the endowment e_0 at t=0, the investment technology rate of return R, the costly liquidation return r, and the world real gross interest rates $\rho_{01}{}^{d}, \rho_{02}{}^{d}, \rho_{12}{}^{d}$.[9]

C. The Lenders' Problem

There is a continuum of international lenders with unit-mass. Unlike agents in the borrowing country, the agents in the lending block are ex-ante and ex-post homogeneous. They derive utility from consuming goods at both t=1 and t=2. Let c_1^* denote a representative lender's consumption of goods at t=1 and c_2^* be her consumption at t=2. Again for simplicity, we assume a logarithmic utility function. The representative lender's utility is [10]

$$U^* = \ln(c_1^*) + \beta^* \ln(c_2^*) \ , \tag{6}$$

[9] Refer to Appendix (section A).

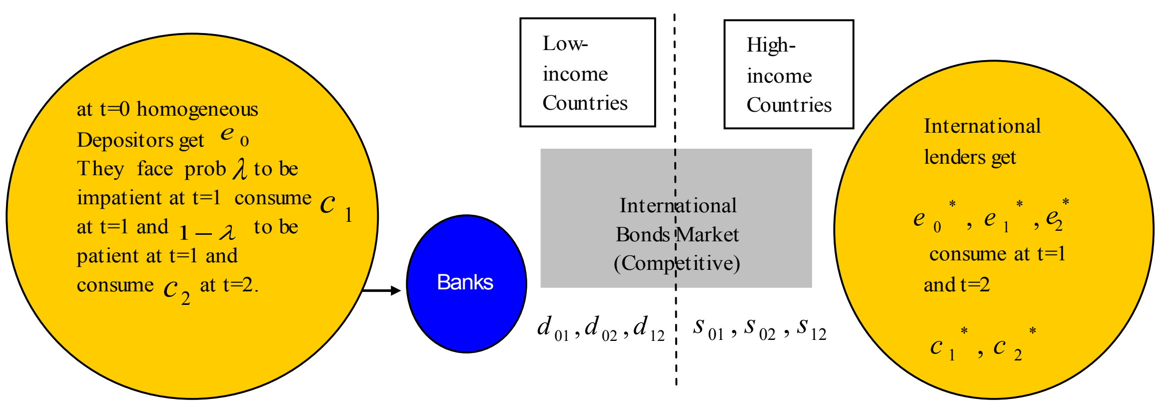
[10] For simplicity, U^* is assumed to be additively separable. Also note that the lender maximizes her lifetime utility, whereas the borrower maximizes its expected lifetime utility across two contingent states (patience and impatience.)

where β^*, the discount factor is identical across lenders. Each international lender receives endowments of goods in the amounts e_0^*, e_1^* and e_2^* at t=0, t=1, and t=2 respectively. For simplicity, we assume that her only investment opportunity is lending to the low-income country through the domestic banks in the amounts of s_{01}, s_{12}, and s_{02}. s_{01} is the short-term loan to the domestic bank at t=0 and maturing at t=1, with a gross real interest rate of $\rho_{01}{}^s$. s_{12} is the short-term loan to the domestic bank at t=1 and maturing at t=2 with a gross real interest rate of $\rho_{12}{}^s$. s_{02} is the long-term loan to the domestic bank at t=0 and maturing at t=2 with a gross real interest rate of $\rho_{02}{}^s$.The budget constraints of a typical international lender are:

$$s_{01} + s_{02} \le e_0^* \,, \tag{7}$$
$$c_1^* + s_{12} \le e_1^* + \rho_{01}{}^s s_{01} \,, \tag{8}$$
$$c_2^* \le e_2^* + \rho_{02}{}^s s_{02} + \rho_{12}{}^s s_{12} \;. \tag{9}$$

The international lender's problem is therefore to choose c_1^*, c_2^*, s_{01}, s_{02}, s_{12} to maximize equation (6) subject to equations (7), (8), and (9) taking as given the endowments e_0^*, e_1^*, e_2^* and the world interest rates $\rho_{01}{}^s, \rho_{02}{}^s, \rho_{12}{}^s$. In appendix (section A), we derive the relevant first-order conditions. A summary of the players and market clearance is illustrated in Figure 1.

Figure 1. Structure of the Model



D. Defining the Equilibrium

In this economy, a competitive equilibrium is a set of interest rates $\{\hat{\rho}_{01}, \hat{\rho}_{12}, \hat{\rho}_{02}\}$, a set of international bonds' allocations $\{\hat{q}_{01}, \hat{q}_{12}, \hat{q}_{02}\}$, a set of allocations for the typical domestic bank $\{\hat{k}, \hat{c}_1, \hat{c}_2, d_{01}, d_{12}, d_{02}\}$, and a set of allocations for the typical international lender $\{\hat{c}_1^*, \hat{c}_2, s_{01}, s_{12}, s_{02}\}$, given $\{e_0, e_0^*, e_1^*, e_2^*, \lambda, \beta^*\}$ and the reserve requirements rates $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ such that:

i. $\{\hat{k}, \hat{c}_1, \hat{c}_2, d_{01}, d_{12}, d_{02}\}$ solve the domestic bank problem of maximizing (1) subject to equations (2), (3), (4), and (5).
ii. $\{\hat{c}_1^*, \hat{c}_2, s_{01}, s_{12}, s_{02}\}$ solve the international lender problem of maximizing equation (6) subject to equations (7), (8) and (9).
iii. Markets clear: $d_{01} = s_{01} = \hat{q}_{01}, d_{12} = s_{12} = \hat{q}_{12}$, $d_{02} = s_{02} = \hat{q}_{02}$.

If perfect arbitrage is assumed for the international lenders, this implies that at equilibrium the following equation should hold at all times.[11]

$$\rho_{01}\rho_{12} = \rho_{02}. \tag{10}$$

On the borrower side, if perfect arbitrage is assumed, then the following equation would hold.

$$\frac{(\rho_{01} - \theta_{01})}{(1-\theta_{01})} \cdot \frac{(\rho_{12} - \theta_{12})}{(1-\theta_{12})} = \frac{(\rho_{02} - \theta_{02})}{(1-\theta_{02})}. \tag{11}$$

All the derived conditions for the existence of any equilibrium should be consistent with equations (10) and (11), under the assumption of perfect arbitrage. The derivation of the borrower and lender problems is identified in the Appendix. In this paper, the focus will be at the interior solution. Two propositions are stated with regard to the solutions with and without reserve requirements, respectively.

Proposition 1:
In the case without reserve requirements and under perfect arbitrage, the equilibrium at the interior solution is characterized by determinate and locally unique interest rates but the quantities of each bond are not determinate.

See proof in Appendix (section C).

When there are no reserve requirements, equations (10) and (11) become identical. There, both lenders and borrowers are indifferent between holding any combinations of the three bonds as long as their budget constraints are satisfied. In each period the overall borrowing is determined. However, the absolute quantities of bonds by maturity cannot be determined. Gross interest rates are uniquely determined by the exogenous parameters (return on investment, endowment, and the fraction of impatient depositors). The gross interest rate on the long-term bond is equal to the gross return on long-term investment. The higher the

[11] This is a version that can be comparable to "pure expectations theory (that) hypothesizes that $R_{2t}^{-1} = R_{1t}^{-1} E_t R^{-1}{}_{1t+1}$, which results in Lucas-Tree type Model, when utility is linear in consumption or there is no uncertainty" (Sargent, 1987, pp. 105).

aggregate fraction of impatient depositors is, the lower the short-term interest rate on the first period bond and the higher the short-term interest rate on the second-period bond. To anticipate what is coming in section IV, the following intuition can be made: The first-period bond is cheaper when the fraction of patient depositors ($1-\lambda$) who could unexpectedly run to the bank after one period is lower.

Proposition 2:
In the case with reserve requirements and under perfect arbitrage, there exist either one or two equilibria depending on the combinations of date- and maturity- specific reserve requirements. The quantities of bonds are determinate for any set of equilibrium interest rate.

See proof in Appendix (section C).

There is an indeterminate set of the quantities at the interior solution without reserve requirements. There, a unique set for interest rates exists. The number of equilibria is infinite because of indeterminate quantities. Thus, a combination of non-zero reserve requirements causes bifurcations toward two equilibria or a unique equilibrium, depending whether the vector $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ is arbitrary or satisfies uniqueness (equation (C.8) in Appendix (sectionC)). In this setup, which is not dynamic, a bifurcation is defined as a change in number of equilibria when the values of $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ change.[12]

To check whether date- and maturity-specific reserve requirements matter, the question becomes: Would a policy of equal reserve requirements $\{\theta_{01} = \theta_{12} = \theta_{02} = \theta\}$ reduce the scope of indeterminacy and result in a unique equilibrium?

Proposition 3:
If all reserve requirements are equal, there is no unique equilibrium.

Proof.

(a) There does not exist a unique equilibrium with a $\hat{\rho}_{12}$ that solves the quadratic equation (C.8) with equal $\{\theta_{01}, \theta_{12}, \theta_{02}\}$, where $0 < \theta < 1$.
With a unique $\{\theta_{01} = \theta_{12} = \theta_{02}\}$, $\Delta = \varsigma_2{}^2 - 4\varsigma_1\varsigma_3 = \theta^2(1-\theta)^2(R-1)^2$. Since by assumption, $R > 1$, then only (i) $\theta = 0$ and (ii) $\theta = 1$ could generate $\Delta = 0$.

(b) With $\Delta = \varsigma_2{}^2 - 4\varsigma_1\varsigma_3 = \theta^2(1-\theta)^2(R-1)^2 > 0$, there are two possible equilibria
If $r > 1$, there is no equilibrium at the interior solution.
If $r \le 1$, there are two equilibria:

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[12] Bifurcations of equilibria are of three types "when structural parameters change, that is, in changes in the number of steady states, their stability type, and the nature of orbits near a given equilibrium"(Azariadis 1993, pp. 91). In this setup, which is not dynamic, we are interested in the first type: the change in number of equilibria.

(i) $\hat{\rho}_{02} = R(1-\theta)+\theta = \hat{\rho}_{01}$, $\hat{\rho}_{12} = 1$.
(ii) $\hat{\rho}_{02} = R(1-\theta)+\theta = \hat{\rho}_{12}$, $\hat{\rho}_{01} = 1$.

Note that from the first order conditions and the incentive compatibility we should have $\frac{\hat{\rho}_{12}-\theta}{1-\theta} \geq r$ and $\frac{\hat{\rho}_{01}-\theta}{1-\theta} \geq r$. Therefore both types of equilibria do not exist for $r > 1$.

The equilibrium with low $\hat{\rho}_{01} = 1$ has a lower welfare for the domestic economy than the equilibrium with $\hat{\rho}_{01} = R(1-\theta)+\theta \geq 1$. Proposition 3 shows that it matters whether the reserve requirements are date- and maturity- specific or not.

In the following section, the scope of analysis shifts to the more interesting question on how bank runs could emerge.

IV. The Emergence of Bank Runs

To introduce bank runs in the simplest possible way, we assume that a bad dream will unexpectedly occur at $t = 1$. The bank signals that it is illiquid when it cannot pay the withdrawals of all deposits at *t*=1. A bank run will occur if an unexpected bad dream happens and the bank is illiquid. We focus on cases where the rule of the domestic banking system is such that, in the case of a bank run, domestic depositors are paid prior to international lenders. At *t*=0, international lenders and the domestic banks agree on $\hat{q}_{01}$, $\hat{q}_{12}$ and $\hat{q}_{02}$. However, only $\hat{q}_{01}$ and $\hat{q}_{02}$ are actually traded at *t*=0. Only at *t*=1 would $\hat{q}_{12}$ be actually traded. $\hat{q}_{12}$ indicates an optimal anticipated amount at *t*=0 of new lending at $t = 1$. If new events happen such as a bank run, international lenders may find it optimal to deviate from the original decision of lending $\hat{q}_{12}$.

A. The Emergence of Bank Runs in the Setup Without Reserve Requirements

We will first look at how a bank run emerges. What is the illiquidity condition under which a bad dream creates the bank run? Second, we will see whether date- and maturity- specific reserve requirements can prevent the occurrence of a bank run.

Defining the Illiquidity Condition

Given the decisions of $\hat{q}_{01}, \hat{q}_{12}$ and $\hat{q}_{02}$ at *t*=0, international lenders and domestic depositors can observe whether the bank would be liquid or not in the case of a bad dream at *t*=1. There are two possibilities: either the bank holds a liquid position or it holds an illiquid position. If the bank holds a liquid position, there is never a bank run, independently of whether or not there is a bad dream. If the bank holds an illiquid position, domestic depositors will run to the bank if and only if they see a bad dream. Under the rules assumed for the domestic financial system, the domestic depositors perceive the bank position as strongly illiquid when the bank cannot pay for all their withdrawals, even if it liquidated at *t*=1 all its investment *k*. Thus the illiquidity condition is as follows

$$\hat{c}_1 + \hat{\rho}_{01}\hat{q}_{01} > r\hat{k}\ . \qquad (12)$$

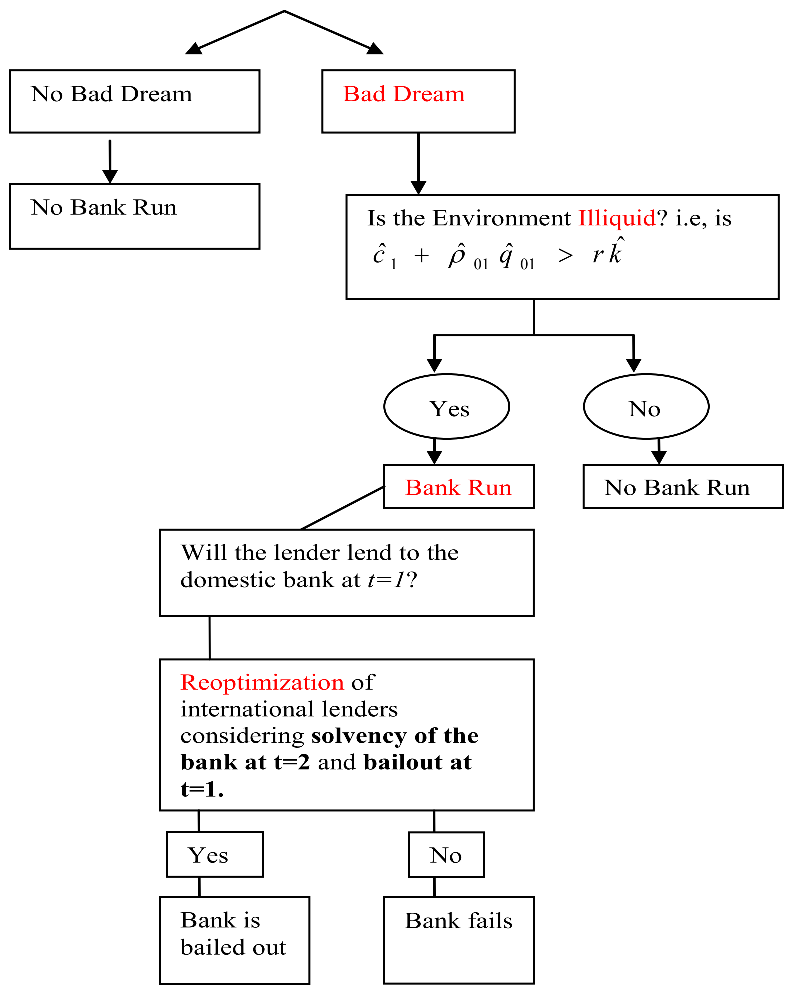
A stronger illiquidity condition would therefore be $\hat{c}_1 > r\hat{k}$. Note that if $\hat{c}_1 > r\hat{k}$ holds, the illiquidity condition in equation (12) must hold.

The domestic depositors will run to the bank if two conditions exist:

(i) The domestic depositors see a bad dream, in this context, the random variable takes at t=1 the value 1.

(ii) The illiquidity condition $\hat{c}_1 + \hat{\rho}_{01}\hat{q}_{01} > r\hat{k}$ holds.

Figure 2. Decision Tree at t=1 Summarizes How a Bank Run Would Occur



For the interior solution, when there are no reserve requirements, the quantities of $\hat{q}_{01}$ are not determined. There the illiquidity condition is the following:

$$\hat{q}_{01} > r\frac{[\lambda R\, e_0(1+\beta^*) + e_2{}^* + Re_0{}^*]}{R\beta^* e_1{}^*}(e_0 + e_0{}^*) - e_0 \ . \tag{13}$$

Result 1: Since $\hat{q}_{01} \in [0, e_0^*]$, if $\hat{q}_{01} \to 0$, this equilibrium is less likely to be vulnerable to illiquidity. If $\hat{q}_{01} \to e_0^*$, it will be more likely to be vulnerable.

B. Can Reserve Requirements Prevent the Occurrence of a Bank Run?

Illiquidity Conditions with Reserve Requirements

In the setup with reserve requirements, at $t = 1$, the reserves—$\theta_{01}\hat{q}_{01}$ goods—are returned to the domestic bank by the domestic authority. With reserve requirements, the illiquidity condition takes the following form:

$$(\hat{\rho}_{01} - \theta_{01})\hat{q}_{01} + \hat{c}_1 > r\hat{k} . \tag{14}$$

At the interior solution we compare the vulnerabilities to a bank run of the cases with and without reserve requirements. At t=0, investment becomes:

$$\hat{k} = e_0 + e_0^*(1 - \theta_{02}) + (\theta_{02} - \theta_{01})\hat{q}_{01} . \tag{15}$$

The reserve requirements θ_{01} and θ_{02} play a tax role: for $\theta_{01} > 0$ and $\theta_{02} > 0$, one concludes that $\hat{k} < e_0 + e_0^*$. Thus, with these reserve requirements, investment in k is lower than investment in the case without reserve requirements. The illiquidity condition becomes

$$\hat{q}_{01}(\theta_{01}, \theta_{12}, \theta_{02}) > \{r - \frac{\hat{\rho}_{01} - \theta_{01}}{1 - \theta_{01}}\}e_0 + re_0^*(1 - \theta_{02})\} \frac{1}{(\hat{\rho}_{01} - r\theta_{02} + (r-1)\theta_{01})} . \tag{16}$$

Result 2: If $\theta_{01}, \theta_{12}, \theta_{02}$ are such that they can guarantee

(i) Uniqueness, i.e., the reserve requirements are set to have C.8.

(ii) $\hat{q}_{01}(\theta_{01}, \theta_{12}, \theta_{02}) \le \{r - \frac{\hat{\rho}_{01} - \theta_{01}}{1 - \theta_{01}}\}e_0 + re_0^*(1 - \theta_{02})\} \frac{1}{\hat{\rho}_{01} - r\theta_{02} + (r-1)\theta_{01})}$.

There exists an equilibrium which is invulnerable to a bank run but with lower investment where:

$$\widehat{\rho}_{12} = \frac{[R[(1-\theta_{02}) - (1-\theta_{01})(1-\theta_{12})] + (\theta_{01}\theta_{12} + \theta_{02})}{2\theta_{01}},$$

$$\widehat{\rho}_{02} = R(1-\theta_{02}) + \theta_{02},$$

$$\widehat{\rho}_{01} = \frac{2(\theta_{01}R(1-\theta_{02}) + \theta_{02}\theta_{01})}{[R[(1-\theta_{02}) - (1-\theta_{01})(1-\theta_{12})] + (\theta_{01}\theta_{12} + \theta_{02})]}.$$

The liquidity role of θ_{01} and θ_{02} will be captured at *t=1* and *t=2,* respectively.

From the above analysis one can conclude two points: On one hand, the effectiveness of reserve requirements depends on how rich the policy mix is. A preventive policy of bank runs should consider date-specific and maturity-specific reserve requirements. On the other hand, only specific combinations of these types of capital controls could prevent a bank run.[13]

Note that for $\widehat{\rho}_{12} > 0$, we should have that

$$\theta_{01} \geq \frac{(R-1)}{R(1-\theta_{12})+\theta_{12}}\theta_{02}. \tag{17}$$

Since $\theta_{01} \leq 1$, it automatically follows that:

$$\theta_{12} + \theta_{02} \leq \frac{R}{R-1}. \tag{18}$$

The inequality in equation (17) shows that the reserve requirements on the first short-term bond should be higher than a linear function of the reserve requirements on the long-term bond, given θ_{12}. This feature reminds us of the Chilean reserve requirements being higher on short-term borrowing in comparison with long-term borrowing. However, in this case it follows directly from guaranteeing uniqueness.

Reserve Requirements and Market Failure

We have assumed that only the bank can borrow from international lenders. Since the bank is pooling the risk across the type of depositors, it is indifferent between (i) borrowing at date 0 for the short term and then renewing its borrowing at date 1 to date 2, and (ii) long-term borrowing at date 0. In the setup without reserve requirements, interest rates and each-period overall borrowing are determined; however the composition of the debt structure is not determined. Since the bank solves the aggregate problem and thus is indifferent to its type of borrowing, two types of equilibria might result: one with low and one with high short-term borrowing. If the accumulated short-term debt is very high ex-post, the system becomes vulnerable and a bank run becomes possible.

The bank is an intermediary that pools the depositors’ problem. International lenders offer a unique contract for a pool of depositors, who could be either patient or impatient. If the international lender lends to the depositors directly, she can offer a contingent contract based on the type turnout of the depositor. For example, the lender could directly lend with the contingency that the maturity structure of the debt depends on the borrower’s declaring herself patient or impatient. In other words, the lender would like to retrieve all his lending at date 1 from those who declare themselves impatient, but at date 2 from those who declare themselves patient. It is the lack of such type-contingent contracts that creates the sort of market failure leading to indeterminacy of debt structure.

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[13] For future analysis, it would be interesting to find which combination is optimal and leads to higher welfare. We abstract from this problem at this stage.

Reserve requirements that induce determinacy in the maturity structure of bonds can correct for this market incompleteness. The reserve requirements make one type of bond costlier relative to the other. Such capital controls are not equal across different types of maturity. As Result 2 indicates, only a specific combination can resolve such incompleteness in the market contract without creating a multiplicity of equilibria.[14]

C. International Lending After the Bank Runs: Are International Lenders "Throwing Good Money After Bad Money"?

In this section, we look at the role of international lending in bailing out the bank after a bank run. In the earlier section, we have discussed the role of reserve requirements in preventing a bank run when a specific combination of $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ is implemented. Given such a result, we analyze the possibility of the post bank-run bailout of the bank by international lenders. Would the bank fail if all depositors attempt to withdraw c_1? Chang and Velasco (2000) answer the latter question in the positive. With endogenous lending, creditors may decide to bail out or not the bank by providing or not $\hat{q}_{12}$ in the event of a bank run.

International Re-Optimization Problem

At $t=1$ a bank run will occur if the bank has high levels of short-term debt and a bad dream is seen. Once a bank run has occurred, international lenders would like to reconsider their decision on the anticipated $\hat{q}_{12}$. They will re-optimize given the bank run. They will decide how much would be the net new lending x_{12} at $t=1$.[15] Thus at $t=1$ their new budget constraint is given by

$$c_1^* + x_{12} \leq e_1^* . \tag{19}$$

In equilibria where the long-term lending is zero ($\hat{q}_{02}=0$), the foreign lenders have nothing at stake to bail out the bank, therefore they will not provide any new funding ($\hat{x}_{12}=0$), in the event of a bank run. However, in equilibria where $\hat{q}_{02}>0$, it might be the case that the international lenders will provide some new lending $x_{12} \geq 0$. They would be willing to reconsider the value of the long-term debt $\hat{q}_{02}$: two possibilities may arise. First, given that their outside option is not to lend but also retrieve nothing of their loans, they may be willing to give up some of $\hat{q}_{02}$. They may accept partial payments of $\hat{q}_{02}$. Second, at $t=1$ patient and impatient depositors withdraw c_1 goods: thus no domestic depositors will be able to withdraw again at $t=2$. International lenders know that they will be the only ones paid back at $t=2$. They may be willing to provide x_{12} only by increasing the value of their long-term loan. Therefore they will choose new value of long-term debt x_{02}. While re-optimizing they

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[14] The indeterminacy due to the multiple possible interest rates associated with random combinations of reserve requirements is distinctive from that due to the indeterminacy of quantities without reserve requirements.

[15] Note that $\hat{x}_{12}$ would be the actual equilibrium new lending, net of the partial or full payment of $\hat{\rho}_{01}\hat{q}_{01}$.

take the interest rates $\hat{\rho}_{02}, \hat{\rho}_{12}$ found at *t=0* as given. Their new budget constraint at $t = 2$ is given by equation (20):

$$c_2^* \le e_2^* + \hat{\rho}_{12} x_{12} + \hat{\rho}_{02} x_{02} . \tag{20}$$

International lenders will not provide any new funding if they know that despite the new lending the bank would still fail or if the bank liquidates all $\hat{k}$ to pay back all depositors. The bank has the option of liquidating early at $t = 1$, some or all $\hat{k}$ if there is a need of liquidity. At *t*=1, the bank will choose $l \ge 0$. The magnitude of l needed to pay the domestic depositors will depend on the new lending provided by international lenders. Thus the bailout condition of the domestic bank by international lenders is given by equation (21):

$$\hat{c}_1 \le x_{12} + rl . \tag{21}$$

In addition the international lenders will only give new lending if the domestic bank will be able to pay back $\hat{\rho}_{12} x_{12}$ and $\hat{\rho}_{02} x_{02}$. The new solvency condition at $t = 2$ for the domestic bank is given by equation (22):

$$\hat{\rho}_{12} x_{12} + \hat{\rho}_{02} x_{02} \le R(\hat{k} - l) . \tag{22}$$

The domestic bank does not solve any re-optimization problem since depositors, in the event of a bank run, will withdraw $\hat{c}_1$ contracted at $t = 0$.[16] The typical international lender will choose $c_1^*, c_2^*, x_{12}, x_{02}, l$ to maximize equation (6) subject to the new budget constraints, equations (19) and (20), and the bailout and solvency conditions, equations (21) and (22). An equilibrium is defined as a set of allocations $\{\hat{c}_1^*, \hat{c}_2^*, \hat{l}, \hat{x}_{12}, \hat{x}_{02}\}$ that maximize the international lenders problem given the parameters and the *t=0* allocations $\{e_1^*, e_2^*, \hat{\rho}_{01}, \hat{\rho}_{02}, \hat{\rho}_{12}, r, R, \hat{k}, \hat{c}_1\}$. The interior solution for the optimization problem:

$$\hat{l} = \frac{1}{1+\beta^*}\left[\frac{e_2^*}{R} + e_0^* + \frac{(r + \beta^* \hat{\rho}_{01})e_0 - \beta^* e_1^*}{r}\right], \tag{23}$$

$$\hat{x}_{12} = \frac{1}{1+\beta^*}\{(\hat{\rho}_{01} - r)e_0 - r\frac{e_2^*}{R} - re_0^* + \beta^* e_1^*\} , \tag{24}$$

$$\hat{x}_{02} = e_0^* - \frac{1}{1+\beta^*}(1 - \frac{r}{\hat{\rho}_{01}})(\frac{e_2^*}{R} + e_0^* + \frac{(r + \beta^* \hat{\rho}_{01})e_0 - \beta^* e_1^*}{r}) . \tag{25}$$

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[16] Note that at *t*=1 when types are revealed to each individual, the role of the bank as an insurance mechanism disappears. The concerned reader may question for what reason the bank would still exist. Although this could be an interesting research question in the future, we would like to abstract from it in the setup to avoid complexity.

Under certain parameter configurations, it is possible that international lenders could bail out the bank after a bank run. All depositors will stand in line declaring themselves impatient independently of their true type in a bank run. There we have assumed that the bank does not adopt any suspension of convertibility on deposits. Why will international lenders behave differently from depositors if both deposits and bank foreign borrowing are on the liability side of a bank balance sheet? We need to recall some of the assumptions to answer this question. International markets are competitive. All lenders are homogeneous and they will solve the same re-optimization problem. We suppose for illustration that date $t=1$ is divided into two sub-dates. By construction the international lender problem assumes the following: Once a bad dream occurs and the environment is illiquid, the bank knows that a bank run is going to occur. Thus it suspends all payments to international lenders and serves the depositors' withdrawals in the first sub-date. In the second sub-date, international lenders solve their re-optimization problem and decide whether or not to bailout the bank. The construction of the international lender problem explained above is presupposing a suspension of payment and a request for bailout. This is similar to the concept of suspension of convertibility on deposits that Diamond and Dybvig (1983) consider. This result should only be interpreted as an encouragement to renegotiate to attract a bailout rather than an encouragement for suspension. Finally, the bailout only emerges if the bank is illiquid not insolvent.

V. DISCUSSION

The following two sub-sections discuss the treatment of the probability of a crisis in this literature and how it is linked to forming a bank. This is to justify that the bank run may be modeled as unexpected.

Sunspot and Bank Run Probability

We have assumed that the probability of a bank run is zero in the ex-ante decision making of the bank.

> "The problem is that once they have deposited, anything that causes them to anticipate a run will lead to a run. This implies that banks with pure demand deposit contracts will be very concerned about maintaining confidence because they realize that the good equilibrium is very fragile." (Diamond and Dybvig, 1984, pp. 409)

Once the bank is formed, any anticipation of a bank run could generate a bank run. It is in this spirit that the assumption of zero or negligible probability of a bank run becomes consistent with rational behavior.[17] In this literature and in our paper, the bank run will not only occur because of a sunspot, but also because of the illiquidity condition, which is an indicator of fundamentals.

Chang and Velasco (1998) assume an unexpected crisis. Chang and Velasco (2000) assume an arbitrary probability of a sunspot. There, if the probability of the sunspot is large enough,

[17] For further justification of this assumption see footnote 10 in Chang and Velasco (1998).

then the optimal bank behavior is to borrow long term only.[18] In Chang and Velasco (2000) if the probability of a crisis is sufficiently small then the bank optimal behavior is to borrow only short term.[19] Thus if p is assumed not to be negligible, short-term borrowing may not be an equilibrium and only long-term borrowing would be possible. The focus of our paper is on how capital controls shift the maturity structure towards a longer term structure and reduce vulnerability to bank runs. The assumption of an unexpected bank run allows equilibria where both short-term and long-term borrowing could emerge.

Incentive to Form a Bank

One other issue that the concerned reader might raise in relation to the possibility of a crisis is the incentive to form a bank if a probability of a bank run is high. When the environment is illiquid, any arbitrary assumption on the probability of the occurrence of the bad dream would imply the same assumption on the probability of a bank run. Depositors would engage in forming the bank if this probability is low enough. In fact the typical depositor will face the problem of choosing whether it will actually be part of the bank coalition or not. If there is no bank, she will invest at $t = 0$ her endowment e_0 in the investment technology and will receive Re_0 at $t = 2$ if she is patient or will liquidate it at $t = 1$ receiving re_0 if she is impatient. Thus her utility without depositing at the bank is:

$$U_{NB} = \lambda re_0 + (1-\lambda)\,\mathrm{Re}_0 .$$

Let us assume that the environment is illiquid. If a bank is formed and if it faces the probability p that there is a bad dream, the depositor faces the following problem: With no bad dream, if she is patient she will consume $\hat{c}_2 = \mathrm{Re}_0$ and if she is impatient, she will consume $\hat{c}_1 = \hat{\rho}_{01}e_0$ given the market equilibrium interest rate—derived earlier—$\hat{\rho}_{01}$. If there is a bad dream, she will have to consume $\hat{c}_1 = \hat{\rho}_{01}e_0$. Thus its expected utility from depositing its endowment e_0 at the bank would be:

$$\begin{aligned} U_B &= (1-p)(\lambda\hat{\rho}_{01}e_0 + (1-\lambda)\,\mathrm{Re}_0) + p\hat{\rho}_{01}e_0 \\ &= [\lambda\hat{\rho}_{01} + (1-\lambda)R]e_0 + (1-\lambda)p[\hat{\rho}_{01} - R]. \end{aligned}$$

For the bank to exist, the depositors should face $U_{NB} \le U_B$: this implies that p needs to be sufficiently low: $p \le \frac{\lambda}{1-\lambda}\frac{\hat{\rho}_{01} - r}{R - \hat{\rho}_{01}}e_0$.[20] Note that this thought process has been included

[18] See proposition 7 pp. 186 in Chang and Velasco (2000).

[19] See proposition 3 pp. 181 in Chang and Velasco (2000).

[20] When $\lim \lambda \to 0$, for the bank to exist $p \to 0$. The above analysis assumes that $p = 0$ is consistent with very low number of λ. Since the potential runners are $1-\lambda$ is high, this is consistent with our interest in vulnerable environment ex-post despite the low probability ex-ante.

only for discussion purpose and to show that p has to be sufficiently low. However it is not entirely consistent with the multiplicity of equilibria that we have. Although there is determinacy in prices and consumption schemes, there are multiple equilibria due to indeterminacy in quantities of bonds. The probability of an environment to be illiquid cannot be predetermined. This in turn implies that a probability of a bank run is unknown in this setup and justifies assuming it to be zero. A modified Diamond and Dybvig environment in Goldstein and Pauzner (2005) has a unique equilibrium where the probability of a panic based bank run can be derived.[21]

VI. Conclusion

In this paper, we explored two related issues: the choice of the maturity structure of external debt in an environment with liquidation cost and the role of specific types of capital controls. First we construct an environment with endogenous lending where one can capture endogenous term structure and endogenous credit rationing. In such an environment the interest rate captures both the "reward for parting with liquidity" and the "impatience factor." We show that date- and maturity-specific reserve requirements reduce the scope of indeterminacy. In this model reserve requirements play the role of a tax and the role of liquidity providers at different dates for each bond. The scarce literature that explored the role of these reserve requirements has identified their tax equivalent role. The model presented in this paper captures the role of tax along with the role of liquidity. For reserve requirements to be effective, they need to guarantee liquidity and get a locally unique equilibrium.

With regard to the post bank-run role of international lenders, one can show that international lenders may still want to provide new short-term lending to the bank after the occurrence of a bank run, in order to retrieve their long-term debt.

[21] For further discussion on this issue see the thorough literature survey on financial intermediation by Gorton and Winton (2002).

APPENDIX : MODEL SOLUTION

A. International Lenders' Problem

The lenders' problem is to maximize equation (6) subject to the constraints in equations (7), (8), and (9) in the text.

A Lagrangean is formed where

$$L^{*} = \ln c_1^{*} + \beta^{*} \ln c_2^{*}, \qquad \text{(A.1)}$$

where

$$c_1^{*} = e_1^{*} + \rho_{01} s_{01} - s_{12} \ , \qquad \text{(A.2)}$$

$$c_2^{*} = e_2^{*} + \rho_{02} s_{02} + \rho_{12} s_{12}, \qquad \text{(A.3)}$$

and

$$s_{02} = e_0^{*} - s_{01}. \qquad \text{(A.4)}$$

The international lender problem becomes to choose s_{01} and s_{12} to maximize L^{*}.

$$\frac{\partial L^{*}}{\partial s_{01}} s_{01} = 0 \ , \qquad \text{(A.5)}$$

$$\frac{\partial L^{*}}{\partial s_{12}} s_{12} = 0 \ , \qquad \text{(A.6)}$$

where

$$\frac{\partial L^{*}}{\partial s_{01}} = \frac{\rho_{01}{}^{s}}{c_1^{*}} - \frac{\beta^{*} \rho_{02}{}^{s}}{c_2^{*}}, \qquad \text{(A.7)}$$

$$\frac{\partial L^{*}}{\partial s_{12}} = \frac{-1}{c_1^{*}} + \frac{\beta^{*} \rho_{12}{}^{s}}{c_2^{*}} \quad . \qquad \text{(A.8)}$$

B. Domestic Banks' Problem

The banks' problem is to maximize equation (1) subject to the constraints in equations (2), (3), (4), and (5).

A Lagrangean is formed:

$$L = \lambda \ln c_1 + (1-\lambda)\ln c_2 + \varepsilon[c_2 - rc_1]. \tag{B.1}$$

From the budget constraint (2), (3) and (4), one finds that:

$$c_1 = \frac{(1-\theta_{12})d_{12} + rl + \theta_{01}d_{01} - \rho_{01}d_{01}}{\lambda}, \tag{B.2}$$

$$c_2 = \frac{R(e_0 + (1-\theta_{01})d_{01} + (1-\theta_{02})d_{02} - l) + (\theta_{12} - \rho_{12})d_{12} + (\theta_{02} - \rho_{02})d_{02}}{1-\lambda}. \tag{B.3}$$

After replacing (B.2) and (B.3) in (B.1), the banks' problem becomes to choose d_{01}, d_{12}, d_{02} and l to maximize L. Thus the Kuhn-Tucker conditions are the following:

$$\frac{\partial L}{\partial d_{01}} d_{01} = 0, \tag{B.4}$$

$$\frac{\partial L}{\partial d_{12}} d_{12} = 0, \tag{B.5}$$

$$\frac{\partial L}{\partial d_{02}} d_{02} = 0, \tag{B.6}$$

$$\frac{\partial L}{\partial l} l = 0, \tag{B.7}$$

where

$$\frac{\partial L}{\partial d_{01}} = \left[\frac{\lambda}{c_1} - r\varepsilon\right]\left[-\frac{\rho_{01}{}^{d} - \theta_{01}}{\lambda}\right] + \left[\frac{(1-\lambda)}{c_2} + \varepsilon\right]\left[\frac{R(1-\theta_{01})}{1-\lambda}\right], \tag{B.8}$$

$$\frac{\partial L}{\partial d_{02}} = \left[\frac{1-\lambda}{c_2} + \varepsilon\right]\left[\frac{R(1-\theta_{02}) + \theta_{02} - \rho_{02}{}^{d}}{1-\lambda}\right], \tag{B.9}$$

$$\frac{\partial L}{\partial d_{12}} = \left[\frac{\lambda}{c_1} - r\varepsilon\right]\left[\frac{1-\theta_{12}}{\lambda}\right] + \left[\frac{1-\lambda}{c_2} + \varepsilon\right]\left[\frac{\theta_{12} - \rho_{12}{}^{d}}{1-\lambda}\right], \tag{B.10}$$

$$\frac{\partial L}{\partial l} = \left[\frac{\lambda}{c_1} - r\varepsilon\right]\left[\frac{r}{\lambda}\right] + \left[\frac{1-\lambda}{c_2} + \varepsilon\right]\left[\frac{-R}{1-\lambda}\right]. \tag{B.11}$$

C. Solving for Equilibria

C.1. Case Without Reserve Requirements:

Both the demands of the borrowers and supply of lenders are interior. Thus the first order conditions for both the borrower and the lender are equal to zero. Here the arbitrage condition in equation (10) is derived from both the lender and borrower problems. This is a locally unique equilibrium point: The interest rates are functions of endowments and other parameters in the world economy:

The interest rates:

$$\hat{\rho}_{01} = \frac{R\beta^* e_1^*}{\lambda R\, e_0(1+\beta^*) + e_2^* + R\, e_0^*}, \tag{C.1}$$

$$\hat{\rho}_{12} = \frac{\lambda\, \mathrm{Re}_0(1+\beta^*) + e_2^* + \mathrm{Re}_0^*}{\beta^* e_1^*}, \tag{C.2}$$

$$\hat{\rho}_{02} = R\ . \tag{C.3}$$

The above equilibrium has determinate and locally unique interest rates. Because of the perfect arbitrage condition, the quantities of debt are not determined: there are two equations with three unknowns.

$$\hat{q}_{01} + \hat{q}_{02} = e_0^*\ , \tag{C.4}$$

$$\hat{q}_{12} = \hat{\rho}_{01}\hat{q}_{01} + \frac{\lambda R\, e_0}{\hat{\rho}_{12}}\ . \tag{C.5}$$

Note that the net new lending at t=1 $\{\hat{q}_{12} - \hat{\rho}_{01}\hat{q}_{01}\}$ and the total debt at t=0 are determined. The ranges for the first short-term debt, the second short-term and the long-term debt are $\hat{q}_{01} \in [0, e_0^*]$, $\hat{q}_{12} \in [\lambda\hat{\rho}_{01}e_0, (\hat{\rho}_{01}e_0^* + \lambda\hat{\rho}_{01}e_0)]$, and $\hat{q}_{02} \in [0, e_0^*]$ respectively. In this case all $\hat{c}_1, \hat{c}_2, \hat{c}_1^*, \hat{c}_2^*, \hat{k}$ have a unique solution.[22] Note that considering the incentive compatibility constraint in (5), a sufficient condition for the interior solution not to exist is

$$R < \frac{re_2^*}{\beta^* e_1^* - r[\lambda(1+\beta^*)e_0 + e_0^*]}.$$

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[22] The depositors consumptions are: $\hat{c}_1 = \dfrac{R\, e_0 \beta^* e_1^*}{R\, e_0^* + e_2^* + \lambda R\, e_0(1+\beta^*)}, \hat{c}_2 = R\, e_0$

The international lenders consumptions are: $\hat{c}_1^* = \dfrac{1}{1+\beta^*}[e_1^* + \dfrac{\beta^* e_1^*(R\, e_0^* + e_2^*)}{R\, e_0^* + e_2^* + \lambda R\, e_0(1+\beta^*)}]$

$\hat{c}_2^* = \dfrac{\beta^*}{1+\beta^*}[e_2^* + R\, e_0^* + \dfrac{R\, e_0^* + e_2^* + \lambda R\, e_0(1+\beta^*)}{\beta^* e_1^*}]$

C.2. Case with Reserve Requirements

Both lenders and borrowers are at their interior solutions: $s_{01} = d_{01} = \hat{q}_{01} \geq 0$, $s_{02} = d_{02} = \hat{q}_{01} \geq 0$ and $s_{12} = d_{12} = \hat{q}_{12} > 0$. There exist at most two possible equilibria depending on the combination of reserve requirements. The interest rates satisfy three equations: equations (10), (11), and the following:

$$\hat{\rho}_{02} = R(1-\theta_{02}) + \theta_{02} \quad . \tag{C.6}$$

Equation (C.6) is derived from the first order condition with respect to d_{02}. Note that without reserve requirements, $\hat{\rho}_{02} = R$, a higher long-term interest rate than with reserve requirements. Thus the interest rate $\hat{\rho}_{12}$ solves the quadratic equation

$$\varsigma_1 \hat{\rho}_{12}{}^2 + \varsigma_2 \hat{\rho}_{12} + \varsigma_3 = 0 , \tag{C.7}$$

where

$\varsigma_1 = \theta_{01}$, $\varsigma_2 = R[(1-\theta_{12})(1-\theta_{01}) - (1-\theta_{02})] - (\theta_{02} + \theta_{01}\theta_{12})$ and $\varsigma_3 = [R(1-\theta_{02}) + \theta_{02}]\theta_{12}$.

In order to guarantee that a unique set of interest rates arises rather than two sets of interest rate with the imposition of reserve requirements, the policymaker would implement a combination of $\{\theta_{01}, \theta_{12}, \theta_{02}\}$ such that a unique interest rate $\hat{\rho}_{12}$ would solve equation (C.7), following the rule:

$$\varsigma_2{}^2 = 4\varsigma_1\varsigma_3 \quad . \tag{C.8}$$

With reserve requirements, unlike the case without reserve requirements, the bonds are determined by the following equations at the interior solution for the equilibrium interest rates:

$$\hat{q}_{01} = \frac{(1-\theta_{12})}{\theta_{12}\rho_{01} - \theta_{01}} \{ \frac{1}{1+\beta^*} \left[\beta^* e_1{}^* - \hat{\rho}_{01} e_0{}^* - \frac{e_2{}^*}{\hat{\rho}_{12}} \right] - \frac{\lambda \operatorname{Re}_0}{(\hat{\rho}_{12} - \theta_{12})} \} \ , \tag{C.9}$$

$$\hat{q}_{02} = e_0{}^* - \hat{q}_{01} \ , \tag{C.10}$$

$$\hat{q}_{12} = \frac{\hat{\rho}_{01} - \theta_{01}}{1-\theta_{12}} \hat{q}_{01} + \frac{\lambda R e_0}{\hat{\rho}_{12} - \theta_{12}} \quad . \tag{C.11}$$

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