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Testing for Purchasing Power Parity in Cointegrated Panels

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Abstract

This paper applies the maximum likelihood panel cointegration method of Larsson and Lyhagen (2007) to test the strong PPP hypothesis using data for the G7 countries. This method is robust in several important dimensions relative to previous methods, including the well-known issue of cross-sectional dependence of error terms. The findings using this new method are contrasted to those from the Pedroni (1995) cointegration tests and fully modified OLS and dynamic OLS esimators of the cointegrating vectors. Our overall results are the same across all approaches: The strong PPP hypothesis is rejected in favour of weak PPP with heterogenenous cointegrating vectors.

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1 Introduction

Considerable effort has been put into testing the purchasing power parity (PPP) hypothesis in the empirical international macroeconomics literature. The most recent step in this literature has been to apply panel methods to increase the power of tests and the efficiency of estimators. This paper takes an additional step in this line of research by employing the recently developed panel cointegration method of Larsson and Lyhagen (2007) that produces tests and estimates that are robust in several important dimensions relative to previous methods.

The basic idea behind the (long-run) PPP hypothesis is that since any international goods market arbitrage should be traded away over time, we should expect the real exchange rate to return to a constant equilibrium value in the long run. Moreover, since the bulk of empirical evidence indicates that nominal exchange rates and aggregate price ratios are integrated of order one – henceforth denoted I(1) – the PPP hypothesis implies that nominal exchange rates and aggregate price ratios are cointegrated. This notion leads to the strong PPP hypothesis, stating that the cointegrating vector between the nominal exchange rate and aggregate price ratios is $[1, -1]$. Or, put differently, that nominal exchange rate and aggregate price ratios move one for one. In a panel setting this amounts to the cointegrating vectors in a panel of countries being homogeneous with value $[1, -1]$.

However, various mechanisms, such as measurement errors, differences in price indices between countries, transportation costs or differential productivity shocks, could give rise to a cointegrating vector that is different from $[1, -1]$; see, for example, Taylor (1988). Under such circumstances, we instead talk about the weak PPP hypothesis; in a panel setting, the weak PPP hypothesis can then be said to hold in the data if there are (*i*) homogeneous cointegrating vectors across countries, but different from $[1, -1]$ or (*ii*) heterogeneous cointegrating vectors across countries. The latter version stems from the notion that the relative influence of the factors mentioned above may vary across countries. Note also that by definition, weak PPP implies that the real exchange rate, defined as the residual to a $[1, -1]$ vector, will not return to a time invariant equilibrium value or, in other words, be stationary.

However, given that the nominal exchange rate and aggregate price ratios are cointegrated, it is not of primary importance exactly which type of PPP hypothesis that the data prefers *per se*. What is important is whether strong PPP is a reasonable approximation since the overwhelming majority of open-economy models assume – explicitly or implicitly – that strong PPP holds.

Although the null of no cointegration between nominal exchange rates and aggregate price ratios tend to be rejected when using very long spans of data, the results for the recent float period, following the breakdown of the Bretton Woods agreement, has been mixed; see, for example, Frankel and Rose (1996), Oh

(1996), Wu (1996), Papell (1997) and O'Connell (1998) for studies on data from the recent float period. However, as pointed out by Pedroni (2001), these studies impose strong PPP under the null in the cointegration test. When allowing for heterogeneous cointegrating vectors under the null of no cointegration, the data do support the (weak) PPP hypothesis; see, for example, Pedroni (1995), Obstfeld and Taylor (1996), Taylor (1996), Chinn (1997), Canzoneri, Cumby, and Diba (1999) and Jacobson *et al.* (2002).

Besides testing the null of no cointegration in a panel setting, Pedroni (2001) uses the panel cointegration method developed in Pedroni (1996) (Fully Modified OLS [FMOLS]) and Kao and Chiang (2000) (Dynamic OLS [DOLS]) to estimate the cointegrating vectors. Pedroni's findings are in line with the weak PPP hypothesis although with panel estimates not very far from $[1, -1]$ on average.[1]

There are however weaknesses associated with both the FMOLS and the DOLS estimators. One of these is that they rely on a very strong assumption of cross-sectional independence of the error terms. Moreover, these methods are not independent of different normalizations, such as the choice of which variable to put on the left-hand side in the regression. In this paper, we take a new approach by applying the maximum-likelihood-panel-cointegration method of Larsson and Lyhagen (2007) to test the strong PPP hypothesis during the recent float period on data for the G7 countries. This method allows for a test of the null of no cointegration between nominal exchange rate and aggregate price ratios without imposing strong PPP. Moreover, if we find support for cointegration, the method allows us to test whether the cointegrating vector is homogeneous across countries and, if support for homogeneity is found, if the common vector is $[1, -1]$ or not.

Using Larsson and Lyhagen's methodology, a robust test of the strong PPP hypothesis can be conducted in the empirically relevant case with cross-sectional dependence in the error terms of the panel and it is done without arbitrary assumptions regarding which variable to put on which side in the regression. The results of the maximum-likelihood method are contrasted to those obtained from employing Pedroni's cointegration tests and the FMOLS and the DOLS estimators for the cointegrating relationships. This comparison indicates that the shortcomings of the FMOLS and DOLS do matter in practice since they affect the inference in various cases. However, the overall conclusions from all methods point in the same direction: The strong PPP hypothesis is forcefully rejected in favor of the weak PPP hypothesis with heterogeneous cointegrating vectors. As a consequence, the strong PPP hypothesis does not even seem to be an acceptable approximation of observed data.

This paper is organized as follows: Section 2 discusses the basic model and

[1] It should be noted though that note that Pedroni (2001) never actually tests the null of homogeneous cointegrating vectors, that is, homogeneous weak PPP. However, the presented individual country estimates are very disparate.

data in our study. In Section 3, we present the Larsson and Lyhagen panel-cointegration framework in some detail and briefly discuss Pedroni's panel-cointegration tests and the between-dimension FMOLS and DOLS estimators. Section 4 presents our results and, finally, Section 5 concludes.

2 Basic model and data

The relationship we consider is

$$s_{i,t} = \mu_i + \beta_i p_{i,t} + \epsilon_{i,t}, \tag{1}$$

where $s_{i,t}$ is the log bilateral nominal exchange rate, $p_{i,t}$ is the log aggregate price ratio in terms of CPI between the base country and country i, and $s_{i,t}$ and $p_{i,t}$ are cointegrated with slopes β_i, which may or may not be homogenous across i. Note that since we rely on the price ratio in terms of CPI we only consider relative PPP; any base period price-level differences is then included in the constant, μ_i. Strong PPP is said to hold if $\beta_i = 1$, $\forall i$.

The countries we consider are the Canada, France, Germany, Italy, Japan, the United Kingdom and the United States, that is, the G7 countries, where the United States is used as the base for comparison. We use data on monthly frequency starting from after the transition period following the aftermath of the breakdown of the Bretton Woods agreement (January 1973) up until when the exchange rates where irrevocably fixed for the countries joining the EMU (December 1998).[2] This yields a panel with dimensions $T = 312$ and $N = 6$. Data are shown in Figures 1 and 2.

3 Tests and estimators

The econometric methodology behind testing and estimation in cointegrated panels has only recently been developed. Important contributions in this field have been made by for instance Pedroni (1995), McKoskey and Kao (1998) and Kao (1999) regarding tests and by Pedroni (1996), Phillips and Moon (1999), Kao and Chiang (2000) and Mark and Sul (2003) regarding estimation of the cointegrating vectors. These contributions do, however, take a fairly similar approach methodologically; though the null and alternative hypotheses differ between some of the cointegration tests, they are all still residual based and can thereby be seen as multivariate generalizations of the Augmented Engle-Granger (Engle and Granger, 1987) and the KPSS (Kwiatkowski *et al.*, 1992) tests. Regarding the panel estimators, these have generally taken their starting

[2] The latter cut-off of the sample is important since we would otherwise violate the assumption of no cointegration relationships across individual exchange rates, by construction.

points in standard pooled OLS or fixed-effects estimation; the more sophisticated methods then modify these estimators to correct for the endogeneity bias and serial correlation likely to be present in most empirical applications.

3.1 Maximum likelihood

Larsson and Lyhagen (2007) on the other hand introduce a new likelihood-based framework for testing and estimation in cointegrated panels by employing a panel-vector-error-correction model setting that can be seen as a generalizations of the Johansen (1988) methodology. As such, Larsson and Lyhagen's approach also has several advantages over the residual-based cointegration tests. First, it allows for more than one cointegrating vector. Most other methods, such as Pedroni (1995), assume that there is only one cointegrating vector. In contrast, this assumption can be tested in the Larsson and Lyhagen (2007) framework.[3] Secondly, using Larsson and Lyhagen's method, no choice has to be made regarding which variable(s) should be "dependent" or "explanatory". This means that we will have one result, whereas in the other methods there will be one result for every possible, and generally equally plausible, normalization. Finally, whilst the cointegrating relations are restricted to each cross-section, the rest of the model is unrestricted.[4] This allows for a substantial amount of short-run dependence between the groups, of which the most appealing perhaps is that of cross-sectional dependence of the error terms. Most of the literature regarding both testing (panel-unit-root tests as well as panel-cointegration tests) and estimation, such as Pedroni (1995), Phillips and Moon (1999), Levin *et al.* (2002), Im *et al.* (2003), relies on this assumption which is highly unlikely to be fulfilled in practice.[5]

Larsson and Lyhagen's model can be seen as a further development of the Larsson *et al.* (2001) panel-vector-error-correction model – the latter model being much more restrictive in terms of short-run dependence between the groups since everything is assumed to be block diagonal.

The starting point of Larsson and Lyhagen is the following panel vector error correction model. Initially, let $i = 1, ..., N$ denote the different groups in the panel, $t = 1, ..., T$ the sample period and $j = 1, ..., p$ the variables in each group. The system can then be represented as

[3] Although it would be very surprising to find more than one cointegrating vector in this application.

[4] The assumption of no cross-member cointegration is also made by, for example, Pedroni (1995).

[5] For example, Maddala and Wu (1999) shows that there will be size distortions in panel-unit-root tests when cross-sectional dependence of the error terms is not accounted for properly.

$$\begin{bmatrix} \Delta\mathbf{y}_{1,t} \\ \Delta\mathbf{y}_{2,t} \\ \vdots \\ \Delta\mathbf{y}_{N,t} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \\ \vdots \\ \boldsymbol{\mu}_N \end{bmatrix} + \begin{bmatrix} \mathbf{\Pi}_{11} & \mathbf{\Pi}_{12} & \cdots & \mathbf{\Pi}_{1N} \\ \mathbf{\Pi}_{21} & \mathbf{\Pi}_{22} & & \mathbf{\Pi}_{2N} \\ \vdots & & \ddots & \vdots \\ \mathbf{\Pi}_{N1} & \mathbf{\Pi}_{N2} & \cdots & \mathbf{\Pi}_{NN} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{1,t-1} \\ \mathbf{y}_{2,t-1} \\ \vdots \\ \mathbf{y}_{N,t-1} \end{bmatrix} + \sum_{k=1}^{m-1} \begin{bmatrix} \mathbf{\Gamma}_{11,k} & \mathbf{\Gamma}_{12,k} & \cdots & \mathbf{\Gamma}_{1N,k} \\ \mathbf{\Gamma}_{21,k} & \mathbf{\Gamma}_{22,k} & & \mathbf{\Gamma}_{2N,k} \\ \vdots & & \ddots & \vdots \\ \mathbf{\Gamma}_{N1,k} & \mathbf{\Gamma}_{N2,k} & \cdots & \mathbf{\Gamma}_{NN,k} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{y}_{1,t-k} \\ \Delta\mathbf{y}_{2,t-k} \\ \vdots \\ \Delta\mathbf{y}_{N,t-k} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{N,t} \end{bmatrix}, \quad (2)$$

where $\mathbf{y}_{i,t}$ is the $p \times 1$ vector of variables for country i., in the present application given by $\mathbf{y}_{i,t} = [s_{i,t}, p_{i,t}]'$. With more compact notation, (2) can be written as

$$\Delta\mathbf{Y}_t = \boldsymbol{\mu} + \mathbf{\Pi}\mathbf{Y}_{t-1} + \sum_{k=1}^{m-1} \mathbf{\Gamma}_k \Delta\mathbf{Y}_{t-k} + \boldsymbol{\varepsilon}_t, \quad (3)$$

where $\mathbf{Y}_t = \left[\mathbf{y}'_{1,t}, \mathbf{y}'_{2,t}, \ldots, \mathbf{y}'_{N,t}\right]'$ and $\boldsymbol{\varepsilon}_t = \left[\varepsilon'_{1,t}, \varepsilon'_{2,t}, \ldots, \varepsilon'_{N,t}\right]'$ are $Np \times 1$ vectors and $\boldsymbol{\varepsilon}_t$ is multivariate normally distributed as $\boldsymbol{\varepsilon} \sim \mathbf{N}(\mathbf{0}, \mathbf{\Omega})$, where

$$\mathbf{\Omega} = \begin{bmatrix} \mathbf{\Omega}_{11} & \mathbf{\Omega}_{12} & \cdots & \mathbf{\Omega}_{1N} \\ \mathbf{\Omega}_{21} & \mathbf{\Omega}_{22} & & \mathbf{\Omega}_{2N} \\ \vdots & & \ddots & \vdots \\ \mathbf{\Omega}_{N1} & \mathbf{\Omega}_{N2} & \cdots & \mathbf{\Omega}_{NN} \end{bmatrix}. \quad (4)$$

Imposing some more structure on the model, the matrix $\mathbf{\Pi}$ – which is assumed to have reduced rank – is defined as $\mathbf{\Pi} = \mathbf{A}\mathbf{B}'$ where

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{\alpha}_{11} & \boldsymbol{\alpha}_{12} & \cdots & \boldsymbol{\alpha}_{1N} \\ \boldsymbol{\alpha}_{21} & \boldsymbol{\alpha}_{22} & & \boldsymbol{\alpha}_{2N} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{\alpha}_{N1} & \boldsymbol{\alpha}_{N2} & \cdots & \boldsymbol{\alpha}_{NN} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} \boldsymbol{\beta}_{11} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\beta}_{22} & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\beta}_{NN} \end{bmatrix}.$$

$\mathbf{A}$ and $\mathbf{B}$ are both $Np \times Nr$ matrices of full rank. The constant term in (3) is furthermore restricted such that a constant term is allowed for in the cointegrating relations, but no drift in the system. This is accomplished by imposing the restriction $\mathbf{A}'_{\perp}\boldsymbol{\mu} = \mathbf{0}$, where $\mathbf{A}'_{\perp}$ is given by $\mathbf{A}'_{\perp}\mathbf{A} = \mathbf{0}$ and $[\mathbf{A}, \mathbf{A}_{\perp}]$ has full rank, that is, Np.[6]

A number of tests will be performed using this framework. Initially, we are interested whether the $\mathbf{\Pi}$ matrix is of reduced rank and whether all N groups can each be characterized by r cointegrating vectors.[7] Next, if cointegration is supported, we are interested in whether the cointegrating vectors span the

[6] Note that we also have estimated with an unrestricted constant yielding the same empirical results as with the restricted constant.

[7] This in turn implies that the matrix Π has rank Nr.

same space for all groups. Put differently, we want to investigate whether the cointegrating vectors are homogeneous or not. Should this be the case, we are then interested in if the common cointegrating vector has the value $[1, -1]$, thereby providing evidence in favour of strong PPP.

Having presented the Larsson and Lyhagen methodology, we next turn to the more frequently used methods that also will be applied to our data: the cointegration tests of Pedroni (1995) and the between dimension panel FMOLS and DOLS estimators.

3.2 Pedroni's cointegration tests, FMOLS and DOLS

The cointegration tests based on Pedroni's methodology take their starting point in the system of equations given by

$$y_{1,i,t} = \mu_i + \boldsymbol{\beta}_i' \mathbf{y}_{2,i,t} + u_{i,t}, \tag{5}$$

where the scalar $y_{1,i,t}$ and the $(p-1) \times 1$ vector $\mathbf{y}_{2,i,t}$ are country specific variables, $i = 1, ..., N$ and $t = 1, ..., T$.[8] The potential cointegrating relationships are estimated individually for each i by OLS and the residuals from these regressions are then tested for unit roots where the parameter of interest is ρ_i in equation (6) below:

$$\hat{u}_{i,t} = \rho_i \hat{u}_{i,t-1} + \sum_{j=1}^{j_i} \theta_{i,j} \Delta \hat{u}_{i,t-j} + \psi_{i,t}. \tag{6}$$

Given this heterogeneous cointegration framework proposed by Pedroni, the results using this method are then straightforward to compare to the results from the first stage of the cointegration test using Larsson and Lyhagen's framework.

To estimate the cointegration vector(s) the between dimension FMOLS and DOLS estimators are used. The starting point for both these estimators is equation (5).[9] The basic idea behind both the FMOLS and DOLS estimators is to correct for endogeneity bias and serial correlation and thereby allow for standard normal inference.[10] The FMOLS estimator accomplishes this by employing a non-parametric correction using $\hat{u}_{i,t}$ and $\Delta\mathbf{y}_{2,i,t}$, whereas the DOLS estimator

[8] For a detailed description of Pedroni's methodology, see Pedroni (1995, 1999).

[9] For a detailed description of these estimators, see Pedroni (2001).

[10] But whilst asymptotic results show that both the FMOLS and DOLS estimators should have t-statistics that allow for standard normal inference (Kao and Chiang, 2000) this need not be the case in small samples. In a Monte Carlo study by Kao and Chiang (2000), in which the behavior of within-dimension FMOLS and DOLS estimators was examined, the FMOLS estimator was shown to have serious size distortions in most cases. The DOLS estimator on the other hand was shown to perform fairly well and was favoured due to both smaller bias and more correct inference. There is less Monte-Carlo evidence available for the between-dimension estimators, but Pedroni (2001) argues that between-dimension estimators have smaller size distortions than the within-dimension estimators for both FMOLS and DOLS.

employs a parametric correction for the endogeneity achieved by augmenting (5) with leads and lags of $\Delta\mathbf{y}_{2,i,t}$ as below:

$$y_{1,i,t} = \mu_i + \boldsymbol{\beta}_i'\mathbf{y}_{2,i,t} + \sum_{s=-s_i}^{s_i} \boldsymbol{\tau}_{i,s}'\Delta\mathbf{y}_{2,i,t-s} + v_{i,t}. \tag{7}$$

Information about the cointegrating vectors is then pooled in order to generate both more precise estimation as well as more powerful tests compared to single equation methods. The hypothesis $H_0 : \beta_i = 1, \forall i$ is then tested versus $H_1 : \beta_i \neq 1$ using the t-statistics.

The alternative hypothesis implies that the cointegrating vectors are not restricted to be the same under the alternative hypothesis which, as pointed out by Pedroni (2001), is an advantage over the between-dimension estimators over the within-dimension estimators in the presence of heterogeneity of the cointegrating vectors. Through their construction, the point estimates from between-dimension estimators can also be interpreted as the mean value for the cointegrating vectors.

The issue of potential heterogeneity of cointegrating vectors does, however, highlight one of the benefits of Larsson and Lyhagen's framework. Consider Pedroni's cointegration test above and compare it to Kao's (1999) tests for homogenous cointegration. Let us first assume that the null hypothesis of Pedroni's tests is rejected. This implies that there is cointegration, but we do not know if this is heterogeneous or homogenous. To find this out, we should employ the test of Kao which imposes a homogenous cointegration vector. If we, on the other hand, assume that we first apply Kao's tests to the data and are unable to reject the null of no cointegration, we must ask ourselves if this is due to heterogeneous cointegrating vectors or complete absence of cointegration. To find this, out we should instead use Pedroni's tests. Clearly, it is more appealing to be able to conduct all testing and inference in the same framework.

It should finally be noted that as the method above is residual based, it is not possible to test for more than one cointegrating relation.

4 Results

First, employing Larsson and Lyhagen's method we test for the number of cointegrating relations in the data. It should be noted though that for this type of test the asymptotic distribution does not provide a good approximation to the small sample distribution. However, for example, Swensen (2006) shows that the parametric bootstrap works well. Here, we present the results when we resampled the residuals but the results were very similar when a parametric approach were used. The number of bootstrap replicates is 399. Looking at Table 1, we see that the test – as expected – supports one cointegrating vector at the five-percent level. Note that there is a fairly large difference between the asymptotic critical values and the bootstrapped critical values. If the asymptotic critical values would be used, two cointegrating vectors would be the conclusion, which clearly would have been both surprising and unsatisfying.

Having rejected the null of no cointegration in favour of panel cointegration with one cointegrating vector, we next turn to the issue of whether these vectors are homogenous. In Table 1 we see that the null of homogenous cointegrating vectors is forcefully rejected; the observed value of 73.66 is extremely large compared to the critical value of 11.07 and the bootstrapped critical value of 39.46.

Table 1: Results from Larsson and Lyhagen's cointegration and homogeneity tests.

Cointegrating rank			
Null hypothesis	Likelihood ratio	5% critical values	
		Asymptotic	Bootstrapped
$H_0 : r = 0$	823.94*	343.24	601.33
$H_0 : r = 1$	173.03	137.27	208.78
Homogeneous cointegration vectors		5% critical value	
Null hypothesis	Likelihood ratio	Asymptotic	Bootstrapped
$H_0 : \beta_1 = \beta_2 = ... = \beta_N$	73.66*	11.07	39.46
$H_0 : B$ Block diagonal	75.06	43.77	99.86

Notes: Sample is January 1973 to December 1998. * Denotes significance at the 5 percent level.

Table 2: Parameter estimates of normalized cointegrating vectors.

	$\hat{\beta}_i$	$s.e.$
UK	-3.66	0.22
France	-2.39	0.15
Italy	-1.36	0.05
Germany	2.43	0.18
Japan	5.44	0.32
Canada	-0.22	0.27

Notes: Sample is January 1973 to December 1998. Cointegrating vector normalized such that $\beta_{ii} = [1, \beta_i]'$. The standard errors (s.e.) are bootraped using 399 replicates. All of the estimates are significantly different from -1 at the 5 percent level.

Since we could not find any support for a common cointegrating vector using Larsson and Lyhagen's methodology, it is needless to say not interesting to investigate if there is a common vector which has the value $[1, -1]$. We can thus conclude that we have found support for weak PPP with heterogeneous cointegrating vectors in the data.

In Table 2 the parameter estimates with corresponding bootstrapped standard error are displayed. As can be seen the individual estimates are quite far from the theoretical -1. For the United Kingdom, France and Italy the parameter estimates and standard errors are roughly the same, while for Germany and Japan they have the wrong sign, and significantly so. The parameter value for Canada is close to zero and not significant. One possible reason for the sligthly divergent parameter estimates could be that the assumption of block diagonality does not hold.[11] Testing this using a likelihood-ratio test and bootstrapped critical values yields the result that we cannot reject the block diagonality assumption though; see the bottom row of Table 1. Hence, we conclude that the block diagonality of the cointegrating vectors appears to be a reasonable approximation after all.

Jacobson *et al.* (2002) also use the Larsson and Lyhagen (2007) method, with a slightly different setup of their model, to test the strong PPP hypothesis on a subsample of the countries used in this study.[12] They, find support for weak PPP with homogenous cointegrating vectors not far from what is expected from the strong PPP hypothesis. The results above, however, show that this finding is not very robust.

Turning next to Pedroni's cointegration tests, we find in Table 3 that ignoring time specific effects yields the results that the null of no cointegration is not rejected for any of the tests regardless of which variable we choose as dependent. The plausible reason for this result is that cross sectional dependencies deflate the size, and hence, reduce the power of the tests; see, for example, Jönsson (2005). Incorporating time specific dummies in the analysis will reduce, but is not likely to completely remove, the cross-sectional dependencies and therefore makes the test more powerful. This view is supported by the results in Table 3 and the cases where time dummies are used. Out of fourteen tests, seven for each choice of dependent variable, ten reject the null hypothesis and find support for some form of PPP relationship. There is only a minor difference whether the panel or group version of a test is used.

The above findings make it apparent that the issue of cross-sectional dependence is of practical importance. Moreover, looking at the results from Pedroni's cointegration test but instead focusing on normalization, we see that this question does not appear to be without relevance either. Whilst the results are completely consistent between the two normalizations when no time dummies are included (the null of no cointegration is not rejected using any test), we find

[11] Another potential problem that could be the source of unexpected parameter estimates is structural breaks. Neither Larsson and Lyhagen's nor Pedroni's method addresses this issue but judging by Figures 1 and 2, we believe that there is no reason to believe that structural breaks would be a cause of concern in this application.

[12] Jacobson *et al.* (2002) include the price levels separately and not as a ratio, as done here, when applying the Larsson and Lyhagen (2007) method to data for France, Germany, Italy and the United Kingdom. They also restrict the constant in (3) to the cointegrating space alone, but when testing for common parameters they also assume that the constant is the same.

Table 3: Results from Pedroni's panel cointegration tests.

	Normalization			
	$s_{i,t} = \alpha_i + \beta p_{i,t} + \epsilon_{i,t}$		$p_{i,t} = \alpha_i + \beta s_{i,t} + \epsilon_{i,t}$	
Time dummies	Yes	No	Yes	No
Panel v	3.84*	1.02	-1.73	-2.45
Panel rho	-2.30*	0.18	-2.23*	0.33
Panel PP	-1.77*	0.34	-1.83*	-0.07
Panel ADF	-1.55	0.48	-1.78*	-0.07
Group rho	-1.90*	0.61	-1.34	1.40
Group PP	-1.82*	0.53	-2.07*	0.79
Group ADF	-1.60	0.67	-1.83*	0.75

Notes: Sample is January 1973 to December 1998. For the within dimension tests, the null and alternative hypotheses are $H_0 : \rho_i = 1$ for all i and $H_1 : \rho_1 = \rho_2 = ... = \rho_N < 1$. For the between dimension tests, the null and alternative hypotheses are $H_0 : \rho_i = 1$ for all i versus $H_1 : \rho_i < 1$ for all i. All tests statistics follow the standard normal distribution under the null. Under the alternative hypothesis of cointegration, the test statistics diverge to negative infinity for all tests, except the Panel v test, and the null is therefore rejected for observed values far in the left tail of the distribution. For the Panel v test, the test statistic diverges to infinity under the alternative and the null is accordingly rejected in the right tail of the distribution. * Denotes significance at the 5 percent level.

that when time dummies are included only the Panel rho, the Panel PP and the Group PP tests reach the same conclusion under the two normalizations. Such a finding is of course not completely surprising; relying on a single equation framework, it has been well-known for a long time that the choice of dependent variable, as well as the choice of cointegration test, can generate different conclusions. This finding, is nevertheless disturbing.

The evidence from Pedroni's panel cointegration tests is not unanimous. However, a majority of the tests reject the null of no cointegration when time dummies are used, regardless of normalization. We therefore argue that the most likely conclusion is that some form of PPP has support in the data, just as we concluded above when applying the Larsson and Lyhagen (2007) method to the data.

In Table 4 the results from the FMOLS and DOLS estimation are presented. Looking first at the results from when we have normalized on the exchange rate, as is done by, for example, Pedroni (2001), we find that strong PPP is rejected in all cases. The point estimate of the average of the cointegrating vectors is, however, close to one for both FMOLS and DOLS when time dummies are used. It must be made very clear though that this fact by no means is favouring strong PPP. Instead, what appears to be a value close to one is just a result from the fact that we are averaging over cointegration vectors. The individual vectors from the FMOLS and DOLS estimations, upon which the pooled estimators are based, are presented in the lower panel of Table 4. These show that it is in fact only for one country that strong PPP appears to be a good approximation, namely Italy.[13,14]

Table 4: Results from the between dimension DOLS and FMOLS estimation.

Normalization: $s_{i,t} = \alpha_i + \beta p_{i,t} + \epsilon_{i,t}$				
	DOLS		FMOLS	
Time dummies	Yes	No	Yes	No
		Panel		
β	0.97*	1.37*	0.92*	1.28*
	(5.76)	(6.39)	(3.48)	(3.48)
		Individual		
β_{UK}	0.73*	0.68*	0.78*	0.68*
	(-3.00)	(-3.15)	(-2.19)	(-2.23)
β_{France}	2.10*	1.87*	2.11*	2.12*
	(10.09)	(4.26)	(10.18)	(7.16)
β_{Italy}	1.01	1.02	1.03	1.16*
	(0.22)	(0.37)	(0.92)	(2.20)
β_{Japan}	1.68*	1.87*	1.69*	1.89*
	(10.98)	(6.49)	(11.22)	(7.12)
$\beta_{Germany}$	0.74*	0.94	0.65*	1.05
	(-7.20)	(-0.43)	(-4.35)	(0.33)
β_{Canada}	-0.77*	1.31	-0.44*	1.32
	(-2.56)	(0.99)	(-1.69)	(1.07)

Notes: Sample is January 1973 to December 1998. * Denotes significance at the 5 percent level. t-statistics, from testing $H_0 : \beta = 1$, in parentheses.

Finally, turning to the case where we have normalized on the ratio of price indices – shown in Table 5 – we find that the null hypothesis of strong PPP is forcefully rejected for all specifications. Worth noting is that strong PPP appears to be an even worse approximation under this normalization as the coefficient on the exchange rate is very far from one. The importance of which variable to normalize on is further demonstrated in the FMOLS and DOLS estimation where the estimates of β, when the exchange rate is the dependent variable, are often far from the inverse of that from when the price ratio is the dependent variable.[15]

Summing up our results, we conclude that the employed tests reject strong PPP in the investigated sample. Weak PPP with heterogeneous cointegrating vectors is, however, supported.

[13] Intuitively, one would perhaps expect PPP to hold best between Canada and the United States. It is therefore an interesting finding that the slope coefficient in the Candian equation has the wrong sign and also is significancly different from 1.

[14] When common time dummies have not been included, the results in Table 4 indicate that when the individual cointegrating vectors are evaluated, strong PPP is supported for Italy, Germany and Canada.

[15] Only asymptotically do we, however, expect the cointegrating vector under this normalization to be the inverse of that under the first normalization.

Table 5: Results from the between dimension DOLS and FMOLS estimation.

Normalization: $p_{i,t} = \alpha_i + \beta s_{i,t} + \epsilon_{i,t}$				
	DOLS		FMOLS	
Time dummies	Yes	No	Yes	No
Panel				
β	0.65*	0.51*	0.64*	0.51*
	(-40.60)	(-27.72)	(-36.48)	(-26.52)
Individual				
β_{UK}	0.78*	0.70*	0.77*	0.68*
	(-2.36)	(-3.05)	(-2.67)	(-3.20)
β_{France}	0.42*	0.34*	0.44*	0.35*
	(-26.82)	(-15.68)	(-31.20)	(-16.65)
β_{Italy}	0.97	0.88*	0.98	0.88*
	(-0.99)	(-2.16)	(-0.71)	(-2.09)
β_{Japan}	0.56*	0.42*	0.56*	0.42*
	(-21.69)	(-20.42)	(-24.51)	(-22.37)
$\beta_{Germany}$	1.18*	0.54*	1.18*	0.54*
	(3.18)	(-5.72)	(3.19)	(-5.91)
β_{Canada}	-0.04*	0.20*	-0.03*	0.21*
	(-40.70)	(-17.92)	(-43.54)	(-17.67)

Notes: Sample is January 1973 to December 1998. * Denotes significance at the 5 percent level. t-statistics, from testing $H_0 : \beta = 1$, in parentheses.

5 Conclusions

This paper has tested the relevance of the PPP hypothesis for the G7 countries versus the U.S. by employing the recently developed maximum likelihood based approach to panel cointegration of Larsson and Lyhagen (2007) and contrasted the results to those relying on Pedroni's (1995) cointegration tests and between dimension panel FMOLS and DOLS. Our empirical findings support that the additional steps, in terms of robustness, taken by Larsson and Lyhagen (2007) is important in this type of application. Most importantly, it is robust to cross-sectional dependence of error terms and does not rely on any particular normalization in terms of which variable to put on which side of the equation to be estimated. As has been shown in this paper, the residual based cointegration test of Pedroni is sensitive both to the usage of time dummies to account for potential cross-sectional dependence and to which variable we choose to normalize on. This makes the empirical results difficult to interpret since, from economic theory, it should not matter which variable is on the left hand side. The importance of which variable to normalize on is further demonstrated in the FMOLS and DOLS estimation where the estimate of β when the exchange rate is the dependent variable is far from the inverse of that from when the price ratio is the dependent variable. When the length of the time series and the dimension of the panel allows it, we therefore argue that Larsson and Lyhagen's methodology is to be preferred.

The overall picture is, however, the same across all approaches. The strong PPP hypothesis is forcefully rejected in favor of the weak PPP hypothesis with heterogeneous cointegrating vectors. As a consequence, the strong PPP hypothesis does not even seem to be an acceptable approximation of observed data. There are several potential mechanisms that can cause a failure of the strong PPP hypothesis, such as measurement errors, differences in price indices between countries, transportation costs or differential productivity shocks. The implications for macroeconomic theory is very much dependent on the underlying cause(s) for the failure of the strong PPP hypothesis. If it, for example, only is measurement errors, we should not be mislead by a model that imposes strong PPP, although it might be problematic to take the model to the data. A worse scenario is that the failure of strong PPP is due to some omitted fundamental mechanism of the economy. More research on the causes to the failure of the strong PPP hypothesis is clearly warranted.

6 Appendix

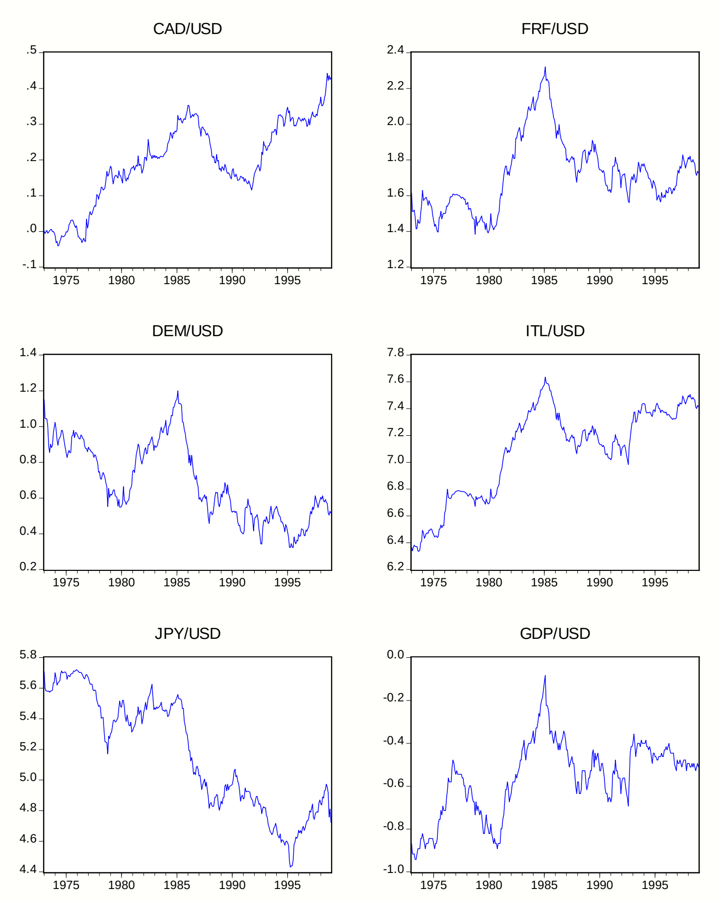

Figure 1: Logarithm of exchange rates.

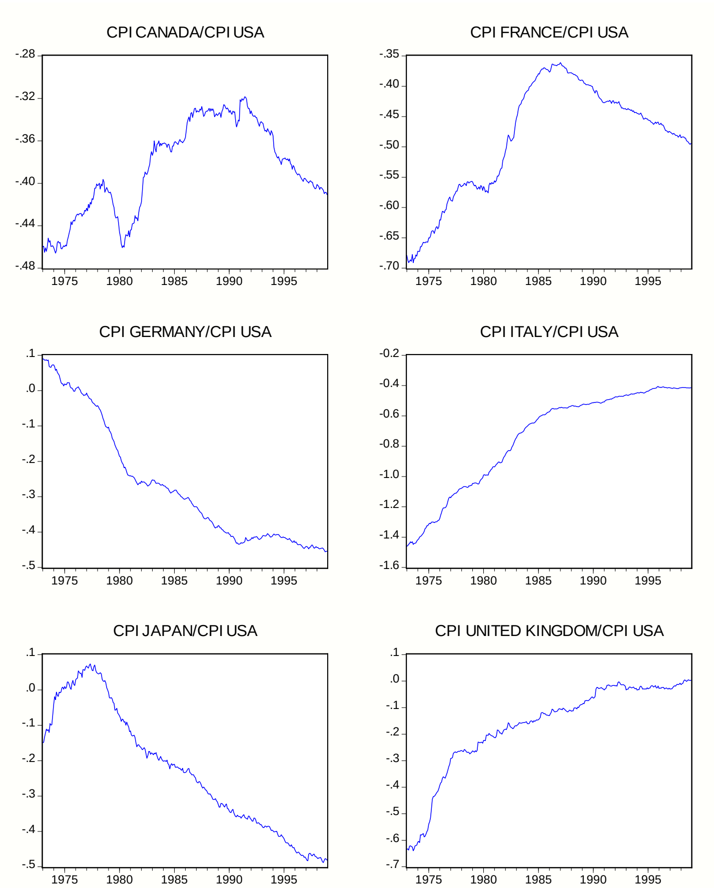

Figure 2: Logarithm of consumer price index ratios.

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