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Debt Stabilization Bias and the Taylor Principle: Optimal Policy in a New Keynesian Model with Government Debt and Inflation Persistence

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Debt Stabilization Bias and the Taylor Principle: Optimal Policy in a New Keynesian Model with Government Debt and Inflation Persistence

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Abstract

We analyse optimal monetary and fiscal policy in a New-Keynesian model with public debt and inflation persistence. Leith and Wren-Lewis (2007) have shown that optimal discretionary policy is subject to a 'debt stabilization bias' which requires debt to be returned to its pre-shock level. This finding has two important implications for optimal discretionary policy. Firstly, as Leith and Wren-Lewis have shown, optimal monetary policy in an economy with high steady-state debt cuts the interest rate in response to a cost-push shock - and therefore violates the Taylor principle. We show that this striking result is not true with high degrees of inflation persistence. Secondly, we show that optimal fiscal policy is more active under discretion than commitment at all degrees of inflation persistence and all levels of debt.

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I. Introduction

"*Central banks are often accused of being obsessed with inflation. This is untrue. If they are obsessed with anything, it is with fiscal policy.*" (Mervyn King, 1995)

It is conventional wisdom that well-designed monetary policy can, on its own, do a good job of stabilising an economy in the face of cost-push shocks (Svensson, 1997, Bean, 1998, Allsopp and Vines, 2000, Woodford, 2003). Such well-designed monetary policy satisfies the Taylor principle, i.e. it raises the real interest rate in response to a rise in inflation. It is also widely believed that the role for fiscal policy in the macroeconomic stabilisation of an economy can be limited to that of ensuring that the fiscal position is solvent (Allsopp and Vines, 2005). Kirsanova and Wren-Lewis (2007), for example, show that the fully optimal policy under commitment has this form. Many countries have established policymaking institutions whose purpose is to ensure that macroeconomic policy is conducted in this manner. For example, in the UK, the Bank of England is given the task of achieving an inflation target, and, subject to that, of stabilising demand. But fiscal policy has been circumscribed by rules which, in effect, tightly constrain discretionary fiscal policy and ensure that fiscal policy is only used, gradually, so as to ensure the sustainability of public debt[1].

Leeper (1991), for example, has shown that monetary policy cannot be conducted in this way if fiscal policy fails to ensure debt sustainability[2]. With such 'irresponsible' fiscal policy, the optimal policy regime becomes one in which monetary policy lowers the interest rate in response to a cost-push shock to stabilise debt[3]. That is, optimal monetary policy becomes 'passive', and violates the Taylor principle - essentially because the actions which are possible for monetary policy are tightly constrained by the need to stabilise debt. This setup has become known as the fiscal theory of the price level.

In this paper we show that, under discretionary policy, the conventional wisdom will be inappropriate, even although both monetary and fiscal policy are set optimally. Our argument makes use of the fact that, in this case both the control of inflation and the control of debt are subject to a dynamic bias which has become known as 'stabilisation bias'.

Stabilisation Bias

Stabilisation bias results from the inability to commit to a time-inconsistent policy path. The problem caused by time inconsistency has been well understood since Kydland and

[1] Fiscal rules in the UK consist of the 'Golden Rule', which restricts fiscal policy from borrowing over the cycle other than to finance investment, and the 'Sustainable Investment Rule' which requires public debt to remain below the 'prudent' level of 40% of GDP.

[2] See also Woodford (2000) and Kirsanova and Wren-Lewis (2007).

[3] Recently Sims (2005) and Benigno and Woodford (2006) have shown that the standard inflation targeting regime becomes inappropriate when fiscal policy is exogenous.

Prescott (1977). The resulting inefficiencies can take two forms. Firstly, there is a level bias in the system if the policymaker attempts to attain an excess value for one or more of his target variables (Barro and Gordon 1983). Secondly, following from the work of Currie and Levine (1987, 1993), it has been realised that the dynamic control of a system under optimal commitment can be time-inconsistent, even if the policymaker does not attempt to attain an excess target level. This is because a policymaker may have the incentive to promise to follow a policy path that he will subsequently not find optimal to follow. Under commitment the policymaker is able to commit to such a time-inconsistent path, thereby manipulating private sector behaviour by means of time-inconsistent promises. Under discretionary policy, by contrast, the policymaker cannot commit to such promises and is hence unable to manipulate private sector expectations in the same way.

The effects of stabilisation bias in the control of inflation by means of monetary policy have been widely explored in New Keynesian models (Clarida et al 1999, Woodford 2003b). Following a cost-push shock, optimal commitment policy reduces inflation in the current period partly by promising tight monetary policy in the future. But such policy is time inconsistent. This is because, once inflation has been reduced in the current period, the policymaker faces the incentive to renege on his announced plan and to not keep interest rates high in subsequent periods, even although he promised to do so. Under optimal discretionary policy, by contrast, the policymaker re-optimises every period and is not able to make time-inconsistent promises about the future. Without the ability to reduce current inflation by manipulating inflation expectations optimal discretionary policy leads to a suboptimally slow rate of disinflation. Such a policymaker is forced to strongly raise interest rates in response to a cost-push shock. Then, once inflation is controlled, interest rates are returned to zero. This costly bias in dynamic inflation control under optimal discretionary policy has been labelled 'inflation stabilisation bias' by Woodford (2003b).

Recently, these ideas about stabilisation bias have also been applied to the optimal control of public debt in New Keynesian models. Previously, a number of studies had analysed responses to shocks in such models with optimal monetary and fiscal policy under the commitment. These papers show that government debt under optimal commitment policy follows a random walk (Benigno and Woodford 2003, Schmitt-Grohe and Uribe 2004, Leith and Wren-Lewis 2007). With permanently higher debt, there will be permanently higher interest payments, and thus there will need to be a permanently lower level of public expenditure. But this is optimal because the current-period costs, to both inflation and public spending, of reducing the debt stock back to its original level outweigh the discounted costs of the permanently lower public expenditure. Recently, Leith and Wren-Lewis (2007), have shown that such behaviour for debt is time inconsistent[4]. This is because, when debt is above its original level, the policymaker faces an incentive to reduce debt slightly in the first period. That, in turn, is because, up until the first period, inflation expectations have already been set, so that cutting debt in the first period does not induce higher expected inflation, and thus higher actual inflation, in the periods before the cut in

[4]Notice that the idea that the government can use inflation surprises to reduce the real value of debt, and hence behave in a time-inconsistent manner, goes back to Lucas and Stokey (1982), Persson et al (1987) and Calvo and Obstfeld (1990).

debt, whereas it would do this in subsequent periods. As a result, as long as debt remains at all above its pre-shock value, there is an incentive in each period to re-optimise and to cut debt. This means that, following a cost-push shock, time-consistent optimal discretionary policy is required to return debt to its initial value - rather than following a random walk. We will label this costly bias in dynamic debt control under optimal discretionary policy 'debt stabilisation bias'.

Following on from this, Leith and Wren-Lewis (2007) show that the means of adjustment of debt to its initial value depends crucially on the steady-state ratio of debt to output (because this determines the relative effectiveness of monetary and fiscal policy in controlling debt). With a low steady-state value of debt, the burden of adjustment is shared by fiscal and monetary policy. With a high steady-state level of debt, however, monetary policy becomes highly effective in controlling debt through its large leverage over interest payments. Leith and Wren-Lewis (2007) show that it becomes optimal for interest rates to *fall* in the first period in response to a cost-push shock. Such violation of the Taylor principle in the first period is reminiscent of optimal monetary policy in the fiscal theory of the price level, which we have described above. That latter result relies on the assumption that there is no fiscal feedback on debt, which is implausible for most countries. But the violation of the Taylor principle, displayed here for a high debt economy, is an outcome when both fiscal and monetary policy are set fully optimally under discretion. This seems to us to be a highly counter-intuitive result and it does not correspond to what is observed in practice[5].

The Contribution of this Paper

The present paper investigates these implications of stabilisation bias for optimal monetary policy and optimal fiscal policy. We do this by generalising the work of Benigno and Woodford (2003) and Leith and Wren-Lewis (2007) in two ways. First, we add inflation persistence to the model. Second, we present, for the first time, an analysis of optimal policy under discretion when, under commitment, the control of *both* inflation *and* debt would be time inconsistent. It is important to note that Leith and Wren-Lewis (2007) did not do this. They treated the distortionary income tax rate as an additional fiscal instrument which enters the Phillips curve directly and hence allows exact control of inflation in each period. As a result, there can be no inflation stabilisation bias.

We show that the outcomes under optimal discretionary policy depend on the steady-state ratio of debt to output and the degree of inflation persistence. With a low steady-state value of debt, the burden of adjustment is shared by fiscal and monetary policy, as in the work of Leith and Wren-Lewis (2007). We show that monetary policy fulfils the Taylor Principle and fiscal policy is more active under discretion than commitment to assist the stabilisation of inflation and debt in the presence of stabilisation bias. This is true for all levels of inflation persistence. But with a high steady-state level of debt, the behaviour of monetary policy is different, and highly striking. We will consider two different cases.

[5] Clarida et al (1998) is one study amongst many, which finds that the Taylor principle has been fulfilled by, for example, the US, Germany, Japan and the UK over the last two decades.

First, with low inflation persistence, optimal discretionary monetary policy replicates the findings of Leith and Wren-Lewis (2007): interest rates are cut in the first period in response to a cost-push shock. This is true even although the control of inflation is subject to inflation stabilisation bias. This finding is important because inflation stabilisation bias on its own would cause interest rates to be raised strongly initially, compared to optimal commitment policy. Debt stabilisation bias, in contrast, tends to pull interest rates down initially, when the level of debt is high. The results here show that this debt stabilisation bias effect dominates, when the initial level of debt is high and inflation persistence is low.

Second, we also show that cutting the interest rate in the first period is only optimal when inflation persistence is low. The reason for this is simple. If inflation is to be controlled despite a first-period cut in interest rates, then there must be enough forward-looking expectations of future tight policy to have a strong enough effect. With more inflation persistence this becomes increasingly difficult. Beyond a certain threshold value for inflation persistence it becomes impossible to carry out a monetary policy which violates the Taylor principle but is nevertheless consistent with the control of inflation. Simultaneously, of course, there is less opportunity to do this as the magnitude of inflation stabilisation bias becomes smaller, the more inflation persistence there is. Beyond this threshold value interest rates must rise in response to the cost-push shock. We show that, as would be expected, this inflation-persistence threshold, after which falling first period interest rates cease to be optimal, is higher for larger steady-state values of debt, as the constraint imposed on policies by debt stabilisation bias becomes more and more powerful. Fiscal policy, in such a high debt economy, cuts spending very strongly in response to the cost-push shock to assist the debt-constrained monetary authority in the stabilisation of both inflation and debt.

Summary

The contribution of this paper can be summarised as follows. Leith and Wren-Lewis (2007) have shown that public debt under optimal discretionary policy does not follow a random walk but has to be returned to its pre-shock level to ensure time consistency. This finding has two important implications for optimal monetary and optimal fiscal policy under discretion. Firstly, as Leith and Wren-Lewis (2007) show, optimal monetary policy in an economy with high steady-state debt cuts the interest rate in response to a cost-push shock - and therefore violates the Taylor principle. This is a striking and unintuitive result. We show that this is not true with high degrees of inflation persistence. Secondly, because debt does not follow a random walk under discretionary policy, we show that optimal fiscal policy is more active under discretion than commitment - at all levels of inflation persistence and all levels of debt - to assist the constrained monetary authority. We conclude that monetary policy should fulfil the Taylor principle and fiscal policy should play an active role in the stabilisation of cost-push shocks in an economy with public debt and inflation persistence.

The remainder of the paper is structured as follows. Section II. introduces the New Keynesian model. Section III. solves for optimal policy and shows that fully optimal commitment policy is time inconsistent. Section IV. presents simulations for optimal policy under commitment and discretion. Section V. concludes.

II. The Model

We use a microfounded model which extends the standard closed-economy New Keynesian model of, for example, Woodford (2003) in two important ways. Firstly, following Steinsson (2003) it contains rule-of-thumb price setters which induce inflation persistence. Secondly, the setup includes fiscal policy with government spending and dynamic debt accumulation. The model we use in this paper is based on Kirsanova and Wren-Lewis (2005)[6].

A. Consumers

The economy is populated by a continuum of infinitely lived individuals, who specialise in the production of a differentiated good (indexed by z), and who spend $h(z)$ of effort in its production. They consume a basket of goods C, and derive utility from per capita government consumption G. The individual's maximisation problem is:

$$\max_{\{C_s,h_s\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[u(C_s) + f(G_s) - v(h_s(z))\right] \tag{1}$$

The price of the differentiated good z is given by $p(z)$ and the corresponding aggregate price level is given by P. Each individual chooses his optimal consumption and work effort to maximise his utility function (1) subject to the demand system and the intertemporal budget constraint:

$$P_t C_t + E_t R_{t,t+1} \bar{A}_{t+1} \leq \bar{A}_t + (1-\tau)(w_t(z) h_t(z) + \Omega_t(z)) + T_t$$

where $P_t C_t = \int_0^1 p(z) c(z) dz$ is nominal consumption, $\bar{A}_t$ are nominal financial assets of a household, Ω_t is profit and T_t is a lump sum subsidy. The nominal wage rate is given by ω_t and τ is an exogenous labour income tax rate. $R_{t,t+1}$ is the stochastic discount factor which denotes the price in period t of carrying the state-contingent asset $\bar{A}_{t+1}$ into period $t+1$. We can express the stochastic discount factor in terms of the riskless one period nominal interest rate i_t:

$$E_t(R_{t,t+1}) = \frac{1}{1+i_t}$$

Individuals consume identical baskets of goods which are aggregated into a Dixit and Stiglitz (1977) consumption index. The elasticity of substitution between any pair of goods

[6] See, for example, Benigno and Woodford (2003), Schmitt-Grohe and Uribe (2004), Beetsma and Jensen (2004) and Blake and Kirsanova (2006) for similar setups.

is assumed to be stochastic to allow for shocks to the mark-up of firms and is given by $\varepsilon_t > 1$ with mean ε. The consumption index is given by $C_t = \left[\int_0^1 c_t^{\frac{\varepsilon_t - 1}{\varepsilon_t}}(z)\,dz\right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}}$.

We assume no Ponzi schemes, that the net present value of individual's income and wealth is bounded[7] and that the nominal interest rate is always positive. By ruling out infinite consumption, this allows us to summarise the infinite sequence of budget constraints as a single intertemporal constraint:

$$E_t \sum_{s=t}^{\infty} R_{t,s} C_s P_s \leq \bar{A}_t + E_t \sum_{s=t}^{\infty} R_{t,s}\left[(1-\tau)\left(w_s(z)\, h_s(z) + \Omega_s(z)\right) + T_s\right]$$

We assume that the utility functions for both private and government consumption are iso-elastic with intertemporal elasticity of substitution σ (that is $u(C_s) = \frac{C_s^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$ and $f(G_s) = \frac{G_s^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$). Household optimisation leads to the following dynamic evolution of consumption that dictates how consumption is optimally allocated between periods:

$$\beta E_t \left(\left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\sigma}} \frac{P_t}{P_{t+1}} \right) = \frac{1}{1+i_t} \tag{2}$$

Aggregate nominal assets accumulate according to:

$$\bar{A}_{t+1} = (1+i_t)\left(\bar{A}_t + (1-\tau)\, P_t Y_t - P_t C_t\right) \tag{3}$$

We define real assets as $A_t = \bar{A}_t / P_{t-1}$ and linearise (2) and (3) around the steady state. For each variable X_t we denote its steady-state value as X and its logarithmic deviation from this steady state as $\hat{X}_t = \ln(X_t/X)$. Linearising equation (2) leads to the well-known Euler equation:

$$\hat{C}_t = E_t \hat{C}_{t+1} - \sigma\left(\hat{\imath}_t - E_t \pi_{t+1}\right) \tag{4}$$

Where we define inflation as $\pi_t = P_t/P_{t-1} - 1$ and assume that inflation is zero in equilibrium. Linearising (3) gives:

$$\hat{A}_{t+1} = \hat{\imath}_t + \frac{1}{\beta}\left(\hat{A}_t - \pi_t + \frac{(1-\tau)}{A}\hat{Y}_t - \frac{\theta}{A}\hat{C}_t \right) \tag{5}$$

Where $\theta = C/Y$ is the steady-state share of private consumption in output and A is the steady-state level of real assets as a share of Y.

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[7] The requirement that the household's wealth accumulation satisfies the transversality condition is given by $\lim_{s\to\infty} E_t\left(R_{t,s}\bar{A}_s\right) = 0$.

B. Price Setting

Following Steinsson (2003), we model price setting as a mix of Calvo contracting and rule-of-thumb behaviour. As in Woodford (2003), agents re-calculate their prices with fixed probability $(1-\gamma)$. If prices are re-calculated then a proportion ω of the price re-setting agents use a rule of thumb to set their price and proportion $(1-\omega)$ calculate the optimum price. With probability γ prices are not re-calculated and are assumed to rise at the average rate of inflation.

Using superscript $*$ to denote firms that re-set their price we see that the average price is a weighted average between forward (P_t^F) and backward-looking prices (P_t^B):

$$P_t^* = \left(P_t^F\right)^{1-\omega}\left(P_t^B\right)^{\omega}$$

Backward-looking agents set their prices P_t^B using the rule of thumb:

$$P_t^B = P_{t-1}^* \Pi_{t-1}\left(\frac{Y_{t-1}}{Y_{t-1}^n}\right)^{\delta} \tag{6}$$

where $\Pi_t = P_t/P_{t-1}$ and Y_t^n is the flexible-price equilibrium of output which we define later. The coefficient δ defines the relative weight of output considerations in the rule of thumb. The forward-looking price setters solve the first order conditions for profit maximisation and obtain the optimal solution as in Rotemberg and Woodford (1997). The rest of the prices will rise at the steady-state rate of inflation $\bar{\Pi}$, defined as $P_t = \bar{\Pi}P_{t-1}$, with probability γ. We can write the price equation for the economy as a whole as:

$$P_t = \left[\gamma\left(\bar{\Pi}P_{t-1}\right)^{1-\varepsilon_t} + (1-\gamma)(1-\omega)\left(P_t^F\right)^{1-\varepsilon_t} + (1-\gamma)\,\omega\left(P_t^B\right)^{1-\varepsilon_t}\right]^{\frac{1}{1-\varepsilon_t}}$$

Following Steinsson (2003) we obtain a hybrid Phillips curve which is amended to include government spending in the utility function and mark-up shocks[8]:

$$\hat{\pi}_t = \chi^f \beta E_t \hat{\pi}_{t+1} + \chi^b \hat{\pi}_{t-1} + \kappa_c \hat{C}_t + \kappa_{y0}\hat{Y}_t + \kappa_{y1}\hat{Y}_{t-1} + \hat{\mu}_t \tag{7}$$

where $\hat{\mu}_t$ is a mark-up shock. The coefficients are defined as:

$$\chi^f = \frac{\gamma}{\gamma + \omega(1-\gamma+\gamma\beta)}, \quad \chi^b = \frac{\omega}{\gamma + \omega(1-\gamma+\gamma\beta)}$$

$$\kappa_c = \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)\psi}{(\gamma+\omega(1-\gamma+\gamma\beta))(\psi+\varepsilon)\sigma}, \quad \kappa_{y1} = \frac{(1-\gamma)\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta$$

$$\kappa_{y0} = \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)}{(\gamma+\omega(1-\gamma+\gamma\beta))(\psi+\varepsilon)} - \frac{(1-\gamma)\gamma\beta\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta, \quad \delta = \frac{(1-\gamma\beta)(\psi+\sigma)}{\gamma\sigma(\psi+\varepsilon)}$$

where the elasticity of disutility of labour is defined as $\psi = \frac{v_y}{v_{yy}Y}$.

[8] A detailed derivation is provided in Appendix 1..

C. Aggregate Demand

Aggregate demand is given by the national income identity:

$$Y_t = C_t + G_t \tag{8}$$

In steady state we assume $G = (1-\theta)\,Y$ where θ is the share of private consumption in GDP. Linearising the income identity:

$$\hat{Y}_t = (1-\theta)\,\hat{C}_t + \theta\hat{G}_t$$

D. Fiscal Policy

The government buys goods (G), taxes income with a constant income tax rate τ and issues nominal debt $\bar{B}$. The evolution of nominal debt is given by:

$$\bar{B}_{t+1} = (1+i_t)\left(\bar{B}_t + P_tG_t - \tau P_tY_t\right) \tag{9}$$

Linearising the debt evolution equation:

$$\hat{B}_{t+1} = \hat{\imath}_t + \frac{1}{\beta}\left(\hat{B}_t - \pi_t + \frac{(1-\theta)}{B}\hat{G}_t - \frac{\tau}{B}\hat{Y}_t\right) \tag{10}$$

where we define the real debt stock as $B_t = \bar{B}_t/P_{t-1}$ and B is the steady-state ratio of debt to output.

E. The System

Finally, we obtain the system of equations that describes the evolution of the out-of-equilibrium economy. We follow convention in denoting lower case letters to denote 'gap' variables, where the gap is the difference between actual and natural levels (that is we define $x_t = \hat{X}_t - \hat{X}_t^n$). As government debt is the only asset in the economy we have $\hat{A}_t = \hat{B}_t$. We obtain the following system:

$$c_t = E_tc_{t+1} - \sigma\left(i_t - E_t\pi_{t+1}\right) \tag{11}$$

$$\pi_t = \chi^f\beta E_t\pi_{t+1} + \chi^b\pi_{t-1} + \kappa_c c_t + \kappa_{y0}y_t + \kappa_{y1}y_{t-1} + \mu_t \tag{12}$$

$$y_t = (1-\theta)\,g_t + \theta c_t \tag{13}$$

$$b_{t+1} = i_t + \frac{1}{\beta}\left(b_t - \pi_t + \frac{(1-\theta)}{B}g_t - \frac{\tau}{B}y_t\right) \tag{14}$$

The complete model consists of four equations. Equation (11) is a standard intertemporal Euler equation in which current consumption depends on its future expected value, because consumers smooth consumption, and negatively on the intertemporal price of consumption, the real interest rate. Secondly, (12) describes a hybrid Phillips curve in which current inflation depends on both forward- and backward-looking components due to firms that set their prices optimally and using the rule of thumb respectively. Equation (13) describes a simple linearised aggregate demand relationship. Finally, (14) describes public debt accumulation in which debt at the beginning of period $t+1$ depends on existing debt, real interest payments, government spending and tax revenues through the constant income tax rate.

F. Social Welfare Function

Kirsanova and Wren-Lewis (2005) follow Steinsson (2003) in using a second-order approximation of the aggregate utility function to show that the model-consistent social welfare function can be expressed as:

$$\frac{1}{2}E_t\sum_{s=t}^{\infty}\beta^{s-t}\left[u\left(C_s\right)+f\left(G_s\right)-\int_0^1 v\left(h_s\left(z\right)\right)dz\right]=\frac{1}{2}E_t\sum_{s=t}^{\infty}\beta^{s-t}W_s$$

where the period loss function W_s given by:

$$W_s=\lambda_c c_s^2+\lambda_g g_s^2+\lambda_y y_s^2+\pi_s^2+\lambda_2\left(\triangle\pi_s\right)^2+\lambda_3 y_{s-1}^2+\lambda_4 y_{s-1}\triangle\pi_s+O\left(3\right) \qquad (15)$$

where $O\left(3\right)$ denotes terms of higher than second order and terms independent of policy. The coefficients are determined by the parameters of the model and are given by:

$$\lambda_c=\frac{\theta}{\sigma}\frac{\psi\left(1-\gamma\beta\right)\left(1-\gamma\right)}{\varepsilon\left(\varepsilon+\psi\right)\gamma},\quad \lambda_g=\frac{\left(1-\theta\right)}{\sigma}\frac{\psi\left(1-\gamma\beta\right)\left(1-\gamma\right)}{\varepsilon\left(\varepsilon+\psi\right)\gamma},\quad \lambda_y=\frac{1}{\psi}\frac{\psi\left(1-\gamma\beta\right)\left(1-\gamma\right)}{\varepsilon\left(\varepsilon+\psi\right)\gamma}$$

$$\lambda_2=\frac{\omega}{\left(1-\omega\right)\gamma},\quad \lambda_3=\frac{\omega\left(1-\gamma\right)^2\delta^2}{\left(1-\omega\right)\gamma},\quad \lambda_4=-2\frac{\omega\left(1-\gamma\right)\delta}{\left(1-\omega\right)\gamma}$$

Appendix 2. provides the details of this derivation. We see that the social welfare function consists of three terms in a model with fiscal policy and inflation persistence. Firstly, the 'demand terms' (c, g and y) arise because the representative consumer has an incentive to smooth both private and public consumption and dislikes fluctuations in hours worked. Secondly, the 'inflation level' term (π) captures the cost of inflation with sticky prices. With nominal rigidities, a higher level of inflation induces greater price dispersion across industries, which is costly. Thirdly, social welfare contains 'smoothing' terms ($\triangle\pi$ and y_{-1}) which arise with inflation persistence, because current inflation depends on past output and inflation with rule-of-thumb price setters. With rule-of-thumb price setters, who base their current pricing decisions on past period's output and prices using (6), those past values will affect the dispersion of prices across industries which is costly in a similar vein to current inflation levels.

To attach an economic meaning to values of the social loss, we will express the loss in terms of compensating consumption. That is, the required steady-state fall in consumption that would balance the welfare gain from eliminating the variability of consumption, government spending and leisure. Appendix 3. explains how to derive this measure.

G. Calibration

We follow the recent literature in assuming a time period to be a quarter and set $\beta = 0.99$, $\sigma = 0.5$, $\psi = 2$, $\varepsilon = 5$ and $\gamma = 0.75$ (e.g. Rotemberg and Woodford 1997). The Calvo parameter γ implies that prices are on average set once a year. Following Kirsanova and Wren-Lewis (2005), we set the steady-state share of private consumption in output to $\theta = 0.75$.

Whilst the above calibration is standard, there is little consensus on how to calibrate the proportion of rule-of-thumb price setters ω and the steady-state ratio of debt to output B[9]. Estimates of the persistence of inflation (χ^b) vary widely. Gali and Gertler (1999), for example, find a predominantly forward-looking Phillips curve (with $\chi^b = 0.3$) whilst Mehra (2004) and Rudebusch (2002) find a predominantly backward-looking inflation process (with $\chi^b = 0.7$). The steady-state ratio of debt to output evidently varies strongly between countries, even within OECD. Given this disagreement, we will vary these two key parameters throughout the paper.

1. The Proportion of Rule-of-Thumb Price Setters

Raising the proportion of rule-of-thumb price setters has important effects on the model. Firstly, higher ω reduces the degree of forward relative to backward-lookingness in the Phillips curve (reduces χ^f and raises χ^b). This in turn determines the extent to which current inflation is determined by past inflation relative to future expected policy. At the extreme without persistence ($\omega = 0$) we obtain the standard New Keynesian Phillips curve without a backward-looking component (with $\chi^f = 1$ and $\chi^b = \kappa_{y1} = 0$).

Secondly, the proportion of rule-of-thumb price setters is important for the relative effectiveness of monetary and fiscal policy in controlling inflation. We can write the inflation rate as the sum of a forward-looking component ($\pi_t^F = \chi^f \beta E_t \pi_{t+1} + \kappa_c c_t + \kappa_{y0} y_t$), a backward-looking component ($\pi_t^B = \chi^b \pi_{t-1} + \kappa_{y1} y_{t-1}$) and the mark-up shock. For a given level of inflation expectations, forward-looking inflation depends on both consumption and the output gap because Calvo price setters base their decisions on real marginal cost (which in turn depend on consumption and output via the real wage). For a

[9] The choice of B in turn determines the steady-state tax rate τ.

given level of past prices, backward-looking price setters are assumed to base their decisions on past output, and not marginal cost. Holding expectations of future inflation and consumption constant and dropping time subscripts for simplicity, we to obtain a simple expression for the relative effectiveness of monetary and fiscal policy in controlling the forward-looking and backward-looking elements of inflation:

$$\frac{\delta\pi^F}{\delta c}/\frac{\delta\pi^B}{\delta c} = \frac{\kappa_c}{\theta\kappa_{y1}} + \frac{\kappa_{y0}}{\kappa_{y1}}, \qquad \frac{\delta\pi^F}{\delta g}/\frac{\delta\pi^B}{\delta g} = \frac{\kappa_{y0}}{\kappa_{y1}}$$

As long as we have some forward-looking price setters ($\kappa_c > 0$) we see that monetary policy is relatively more effective in controlling the forward-looking component of inflation than the backward-looking component as compared to fiscal policy. That is, monetary policy has a 'comparative advantage' in controlling forward-looking inflation and fiscal policy has a comparative advantage in controlling backward-looking inflation. Intuitively this is because monetary policy has a relatively stronger effect on real marginal cost than on output, because the real wage, and hence real marginal cost, depend on both the marginal disutility of working and the marginal utility of consumption (i.e. real marginal cost depends on consumption directly and also indirectly through output)[10]. Unlike in simple backward-looking models, such as Kirsanova et al (2005), monetary and fiscal policy are therefore not perfect substitutes in their control of inflation.

Finally, we see from (15) that a larger proportion of rule-of-thumb price setters raises the weight of terms on $\Delta\pi_s$ and y_{s-1} in the microfounded loss function. The weights λ_2, λ_3 and λ_4 dominate the loss function for high ω because more firms base their pricing decisions on past output and inflation (see Steinsson 2003). We notice, however, that there is no solution to the model in the polar case of $\omega = 1$. Whilst the Phillips curve converges to an accelerationist form, the social welfare function is not well defined in this limit[11]. This means that we cannot nest simple backward-looking models, such as those of Bean (1998) or Kirsanova et al (2005), in this New Keynesian model.

2. The Steady-State Value of Debt

By changing the leverage of monetary policy over interest payments, the steady-state ratio of debt to output (B) plays a crucial role in determining the relative effectiveness of monetary and fiscal policy in affecting the debt stock. For an economy with high B, monetary policy becomes relatively more effective in controlling debt as it gains greater leverage over interest payments. We will hence define two regimes: firstly, a 'low' debt economy with a debt to output ratio of 0.1 and, secondly, a 'high' debt economy with a debt ratio of 0.4. These calibrations imply annual debt to GDP ratios of 2.5% and 10% respectively. We will see that the choice of the steady-state debt ratio plays a crucial role in this paper.

[10] The forward-looking elements of the system will leave this finding unchanged as monetary policy will become more effective in both consumption and output control.

[11] To see this, we take the limit of (15) for $\omega \to 1$. We substitute out using the Phillips curve in this limit, which Steinsson (2003) has shown equals $\pi_s = \pi_{s-1} + \delta(1-\gamma) y_{t-1}$, and obtain $\lim_{\omega\to 1}(1-\omega) W_s = 0$.

III. Solving for Optimal Policy

The policymaker minimises the social loss by choosing the interest rate and spending subject to the evolution of the economy (11) to (14). Optimal policy differs considerably under commitment and discretion. Whilst the policymaker can credibly commit to future policies under commitment and hence affect expectations, under discretion he is forced to re-optimise every period and treats non-predetermined variables parametrically. We will outline a canonical representation that we will subsequently use to solve for optimal commitment and discretionary policy.

A. Canonical Form

Following Currie and Levine (1993) a linear quadratic optimisation problem for the policymaker can be written as:

$$\min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} E_t \sum_{s=t}^{\infty} \beta^{s-t} L_s$$

subject to the constraints

$$\begin{bmatrix} X_{1,t+1} \\ E_t X_{2,t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} U_t + \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \varepsilon_{t+1} \tag{16}$$

where $X_{1,t}$ is a $n_1 \times 1$ vector of predetermined ('state') variables with initial conditions $X_{1,0}$ given, $X_{2,t}$ is a $n_2 \times 1$ vector of forward-looking ('jump) variables, U_t is the instrument vector with dimension k and ε_{t+1} is a white noise process. We can define the $n = n_1 + n_2$ vector $X_t = \left(X'_{1,t}, X'_{2,t}\right)'$ and write the model in canonical form:

$$X_{t+1} = AX_t + BU_t + E\varepsilon_{t+1}$$

For our model we have $X_{1,t} = (\mu_t, \pi_{t-1}, y_{t-1}, b_t)'$, $X_{2,t} = (\pi_t, c_t)'$, $U_t = (i_t, g_t)'$ and $\varepsilon_{t+1} = \left(\eta_{t+1}, 0, 0, 0\right)'$, where η_t is an i.i.d process. Appendix B. defines the matrices A, B and E.

The quadratic loss function L_t has target variables G_t, such that $L_t = G'_t Q G_t$, where the target variables are functions of the state variables and the instruments of the system, $G_t = CZ_t$ where $Z_t = \left(X'_{1,t}, X'_{2,t}, U'_t\right)'$. The period loss function L_t can hence be re-written as:

$$L_t = Z'_t \Omega Z_t \tag{17}$$

with $\Omega = C'QC$ which is given by

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} & 0 \\ \Omega_{21} & \Omega_{22} & \Omega_{23} \\ 0 & \Omega_{32} & \Omega_{33} \end{pmatrix}$$

The diagonal elements Ω_{ii} constitute the squared terms in the loss function, whilst the off-diagonal elements Ω_{12} and Ω_{21} define the weights on the smoothing terms $\Delta\pi_s y_{s-1}$[12]. Appendix B. defines these weight matrices for our model in terms of the structural parameters.

B. Optimal Policy under Commitment

Following Currie and Levine (1993) we can write the objective function of the policymaker under commitment (C) as a constrained loss function:

$$H^C = \min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} E_t \sum_{s=t}^{\infty} H_s^C \tag{18}$$

with

$$\begin{aligned} H_s^C = \frac{1}{2}\beta^{s-t}\{L_s + \hat{\mu}'_{s+1}\left(A_{11}X_{1,s} + A_{12}X_{2,s} + B_1 U_s - X_{1,s+1}\right) \\ + \hat{\rho}'_{s+1}\left(A_{21}X_{1,s} + A_{22}X_{2,s} + B_2 U_s - X_{2,s+1}\right)\} \end{aligned}$$

where L_s is defined in (17), $\hat{\mu}_{t+1}$ is a n_1-dimensional non-predetermined Lagrange multiplier associated with the predetermined variables $X_{1,t}$ and $\hat{\rho}_{t+1}$ is a n_2-dimensional predetermined Lagrange multiplier associated with the non-predetermined variables $X_{2,t}$. The Lagrange multipliers have the usual interpretation as shadow prices of the system constraints.

The first order conditions, of which there are $2n_1 + 2n_2 + k$, are obtained by differentiating with respect to X_1, X_2, U, $\hat{\mu}$ and $\hat{\rho}$. Appendix 1. presents the general first order conditions of the system. Here we summarise the first order conditions for $s > 0$ before turning to $s = 0$.

1. For Periods after the Initial $(s > 0)$

We start with periods after the initial ($s > 0$) and obtain eight first order conditions. For notational simplicity we will define $\mu_s = \beta^{-s}\hat{\mu}_s$ and $\rho_s = \beta_s^{-s}\hat{\rho}_s$ and drop the rational expectations operator E_s (that is, denote $E_s X_{s+1} = X_{s+1}$). The first block of optimality conditions are for the state variables in vector $X_{1,s}$:

$$\frac{\partial H^C}{\partial \mu_s} = -\frac{1}{\chi^f}\rho_{s+1}^{\pi} + \frac{\sigma}{\chi^f}\rho_{s+1}^{c} - \mu_s^{\mu} = 0 \tag{19}$$

[12]The weights Ω_{23} and Ω_{32} are non-zero because they constitute the weight on the squared output gap which we substituted out for in terms of consumption and government spending using (13).

$$\frac{\partial H^C}{\partial \pi_{s-1}} = \lambda_2 \pi_{s-1} - \frac{1}{2}\lambda_4 y_{s-1} - \lambda_2 \pi_s - \frac{\chi^b}{\chi^f}\rho^\pi_{s+1} + \frac{\sigma\chi^b}{\chi^f}\rho^c_{s+1} - \mu^\pi_s = 0 \tag{20}$$

$$\frac{\partial H^C}{\partial y_{s-1}} = -\frac{1}{2}\lambda_4 \pi_{s-1} + \lambda_3 y_{s-1} + \frac{1}{2}\lambda_4 \pi_s - \frac{\kappa_{y1}}{\chi^f}\rho^\pi_{s+1} + \frac{\sigma\kappa_{y1}}{\chi^f}\rho^c_{s+1} - \mu^y_s = 0 \tag{21}$$

$$\frac{\partial H^C}{\partial b_s} = \mu^b_{s+1} - \mu^b_s = 0 \tag{22}$$

Whilst (19) to (21) offer little analytical insight, we see from (22) that the Lagrange multiplier for debt follows a random walk:

$$\mu^b_{s+1} = \mu^b_s \tag{23}$$

We will discuss below how this behaviour of the debt Lagrange multiplier is important in understanding the result that debt under optimal commitment policy follows a random walk. The second block of optimality conditions are for the jump variables in vector $X_{2,s}$:

$$\tfrac{\partial H^C}{\partial \pi_s} = -\lambda_2 \pi_{s-1} + \tfrac{1}{2}\lambda_4 y_{s-1} + (\lambda_\pi + \lambda_2)\,\pi_s + \mu^b_{s+1} + \sigma\rho^c_{s+1} - \rho^\pi_s = 0 \tag{24}$$

$$\begin{aligned} \tfrac{\partial H^C}{\partial c_s} &= \left(\theta^2\lambda_y + \lambda_c\right) c_s + 2\theta\lambda_y\,(1-\theta)\, g_s + \beta\,(1-\theta)\,\mu^y_{s+1} \\ &+ \tfrac{(1-\tau)(1-\theta)}{B}\mu^b_{s+1} + \tfrac{\sigma\kappa_{y0}(1-\theta)}{\chi^f}\rho^c_{s+1} - \tfrac{\kappa_{y0}(1-\theta)}{\chi^f}\rho^\pi_{s+1} - \rho^c_s = 0 \end{aligned} \tag{25}$$

The third block for the instruments in vector U_s is:

$$\tfrac{\partial H^C}{\partial i_s} = \mu^b_{s+1} + \sigma\rho^c_{s+1} = 0 \tag{26}$$

$$\begin{aligned} \tfrac{\partial H^C}{\partial g_s} &= 2\theta\lambda_y\,(1-\theta)\, c_s + \left((1-\theta)^2\lambda_y + \lambda_g\right) g_s + (1-\theta)\,\mu^y_{s+1} \\ &+ \left(\tfrac{(1-\tau)(1-\theta)}{\beta B}\right)\mu^b_{s+1} - \tfrac{\kappa_{y0}(1-\theta)}{\chi^f\beta}\rho^\pi_{s+1} + \left(\tfrac{\sigma\kappa_{y0}(1-\theta)}{\chi^f\beta}\right)\rho^c_{s+1} = 0 \end{aligned} \tag{27}$$

The first order condition for the interest rate, (26), offers an important insight into optimal commitment policy. It describes how optimal interest rate setting at time s equates the marginal cost of raising debt through higher interest payments (μ^b_{s+1}) with the marginal cost of lower consumption through a higher intertemporal price ($-\sigma\rho^c_{s+1}$):

$$\mu^b_{s+1} = -\sigma\rho^c_{s+1} \tag{28}$$

Along the optimal commitment path, the non-predetermined Lagrange multiplier on debt is therefore proportional to the predetermined Lagrange multiplier of the jump variable, c_s. A non-zero value for ρ^c_{s+1} will imply a non-zero value for μ^b_{s+1}. This will be important in our analysis of the time inconsistency of such policy to which we return below.

The final block of first order conditions is the evolution of the system (16) which, for brevity, we do not replicate here.

2. For the Initial Period ($s = 0$)

Following Currie and Levine (1993) we set the first period Lagrange multipliers corresponding to the jump variables of inflation and consumption to zero ($\rho_0 = 0$) which ensures that the policymaker starts from an optimal position. We see that all first order conditions are unchanged, except for those of the jump variables $X_{2,0}$. Using (26) we can write (24) and (25) for the first period as:

$$\frac{\partial H^C}{\partial \pi_0} = -\lambda_2 \pi_{-1} + \tfrac{1}{2}\lambda_4 y_{-1} + (\lambda_\pi + \lambda_2)\, \pi_0 = 0 \tag{29}$$

$$\begin{aligned} \frac{\partial H^C}{\partial c_0} = & \left(\theta^2 \lambda_y + \lambda_c\right) c_0 + 2\left(1 + \theta \lambda_y \left(1 - \theta\right)\right) g_0 + \beta \left(1 - \theta\right) \mu_1^y \\ & + \tfrac{\sigma(1-\theta)(B\kappa_{y0} - (1-\tau))}{B} \rho_1^c - \tfrac{\kappa_{y0}(1-\theta)}{\chi^f} \rho_1^\pi = 0 \end{aligned} \tag{30}$$

We see clearly see that the first order conditions differ for $s > 0$ and $s = 0$ differ to the extent that ρ_0^π and ρ_0^c are zero (which are the initial values)[13]. We will discuss the importance of this difference below.

3. Solution

Following Currie and Levine (1993) we abstract from stochastic terms and obtain a certainty-equivalent solution. They show that the evolution of the economy under optimal commitment policy can be written as:

$$\begin{bmatrix} U_s \\ X_{2,s} \\ \mu_s \end{bmatrix} = \Phi \begin{bmatrix} X_{1,s} \\ \rho_s \end{bmatrix} \tag{31}$$

$$\begin{bmatrix} X_{1,s+1} \\ \rho_{s+1} \end{bmatrix} = \Psi \begin{bmatrix} X_{1,s} \\ \rho_s \end{bmatrix} + E\varepsilon_{s+1} \tag{32}$$

where Φ and Ψ are found by solving the above system using the initial conditions for all predetermined variables ($X_{1,0}$ and ρ_0) and terminal conditions for all non-predetermined variables (X_2, μ and U). The solution can be obtained using the algorithm of Soderlind (1999). For future reference, let us define:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \\ \Phi_{31} & \Phi_{32} \end{bmatrix} \text{ with } \Phi_{11} = \begin{bmatrix} \theta_\mu^C & \theta_\pi^C & \theta_y^C & \theta_b^C \\ \phi_\mu^C & \phi_\pi^C & \phi_y^C & \phi_b^C \end{bmatrix}$$

where Φ_{11} defines the feedback coefficients on the state variables of the system (exclusive of the predetermined Lagrange multipliers) under optimal commitment. For example, θ_μ^C constitutes the optimal feedback of the interest rate onto the cost-push shock under optimal commitment policy. Equations (31) and (32) together with the initial conditions $X_{1,0}$ and $\rho_0 = 0$ provide a complete description of the evolution of the economy.

[13] We also notice from (28) that ρ_1^c only pins down μ_1^b but not μ_0^b (i.e. ρ_0^c is zero but (28) does not allow us to pin down μ_0^b which instead is determined by its own initial condition).

4. Time Inconsistency

The first order conditions for the non-predetermined variables $X_{2,s}$, (29) and (30), highlight the problem of time inconsistency in the optimal commitment solution[14]. Once optimal policy has been found at time $t = 0$, and ρ_0 is set to zero, such optimal policy implies a time path for ρ_s such that ρ_s is not necessarily equal to zero anymore for $s > 0$. That is, given a chance to re-optimise at $s > 0$ the policymaker will choose to set ρ_s equal to zero, reneging on the previously optimal plan. The magnitude of ρ_s therefore captures the extent of the time inconsistency problem. With two jump variables we have two sources of time inconsistency: (29) and (30) show that the control of both inflation and consumption is time inconsistent. Further we recall from (28) that a non-zero value for ρ^c_{s+1} implies a non-zero value for μ^b_{s+1} which allows us to connect the time-inconsistent control of consumption to the time-inconsistent control of debt. Given the structure of the problem in (18), we see that negative values of ρ^π_s and ρ^c_s indicate that the social loss under optimal commitment could be reduced by raising inflation and consumption, or equivalently by raising inflation and lowering debt.

Currie and Levine (1993) quantify this incentive to renege in terms of the social welfare gain by showing that the cost-to-go at time s under optimal commitment policy can be written as a function of the predetermined variables of the model:

$$L^{OPC}_s = -\frac{1}{2}\left[tr\left(\Phi_{21}X_{1,s}X'_{1,s}\right) + tr\left(\Phi_{22}\rho_s\rho'_s\right)\right] \tag{33}$$

where Φ_{21} and Φ_{22} are defined in (31). At any time $s > 0$ there exists a gain, from reneging by re-setting $\rho_s = 0$, given by the last term in (33)[15]. A non-zero value of ρ_s therefore identifies a potential welfare gain equal to $-tr\left(\Phi_{22}\rho_s\rho'_s\right)$ from reneging on the optimal commitment path.

C. Optimal Policy under Discretion

Optimal policy under discretion, in contrast, must be time consistent. Currie and Levine (1993) show that the first step in finding the discretionary solution is to postulate how the private agents determine their expectations of non-predetermined variables. Given the linear-quadratic setup of the model, we guess that the reaction function of the public takes the following linear form:

$$X_{2,t} = -GX_{1,t} - KU_t \tag{34}$$

[14] Woodford (2003b) has suggested an alternative solution approach in which the system, in all periods including the initial, follows the first order conditions for $s > 0$. This ‘timeless perspective’ policy is not time inconsistent as it involves ignoring the conditions that prevail at the regime’s inception ($s = 0$). We want to analyse the time inconsistency inherent in commitment policy and will therefore consider fully optimal commitment policy.

[15] Levine (1988) has shown that the diagonal entries of Φ_{22} are negative and hence that the incentive to renege exists at all points along the trajectory path of optimal commitment policy.

where the matrices G and K are unknown and will be found later. We substitute for (34) and form the Lagrangean:

$$H^D = \min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} E_t \sum_{s=t}^{\infty} H_s^D \tag{35}$$

with

$$H_s^D = \frac{1}{2}\beta^{s-t}\{L_s + \hat{\eta}'_{s+1}\left(\left(A_{11} - GA_{12}\right)X_{1,s} + A_{12}X_{2,s} + \left(B_1 - A_{12}K\right)U_s - X_{1,s+1}\right)\}$$

where L_s is defined in (17) and $\hat{\eta}_{t+1}$ is a vector of non-predetermined Lagrange multipliers associated with the predetermined variables $X_{1,t}$. To ensure time consistency, the objective function is only constrained by predetermined variables, as the policymaker takes non-predetermined ones as given (i.e. time consistency requires $\rho_s = 0$ for all s). The first order conditions for this general linear quadratic problem under discretion are outlined in Appendix 2..

Due to the complexity of the model, the first order conditions for optimal discretionary policy are complex and unrevealing. Currie and Levine (1993) show that the certainty equivalent solution of the first order conditions converges to:

$$U_t = FX_{1,t} \tag{36}$$

$$X_{2,t} = CX_{1,t} \tag{37}$$

where F and C are found by means of a numerical algorithm. The dynamics of $X_{1,t}$ are then found by substituting these expressions into (16). The solution can be obtained using the method of Soderlind (1999). For future reference we define the instrument feedback coefficients under optimal discretionary policy as:

$$F = \begin{bmatrix} \theta_\mu^D & \theta_\pi^D & \theta_y^D & \theta_b^D \\ \phi_\mu^D & \phi_\pi^D & \phi_y^D & \phi_b^D \end{bmatrix}$$

Together with the initial conditions $X_{1,0}$, these expressions give a complete description of the evolution of the economy.

IV. Simulating Optimal Policy

To analyse the behaviour of optimal monetary and optimal fiscal policy, we simulate the impulse responses of the system to a unit cost-push under optimal policy. We will analyse optimal monetary and optimal fiscal policy under commitment in Section A. and under discretion in Section B.. We will look at both a low debt ($B = 0.1$) and high debt economy ($B = 0.4$) for three versions of the Phillips curve: we will consider a 'New Keynesian'

Phillips curve ($\omega = 0$), a 'hybrid' Phillips curve ($\omega = 0.75$)[16] and a predominantly backward-looking Phillips curve ($\omega = 0.99$). In Section C. we will extend the analysis to more general values of debt and inflation persistence.

Table 1 summarises the simulations of optimal policy for these Phillips curve specifications for the low and high debt economies in the top and bottom parts of the Table respectively. We report the absolute welfare loss ('Loss') and the excess loss over the commitment solution. This excess loss is expressed both in terms of percentage loss above that of commitment (% W) and in terms of percentage of steady-state consumption foregone (% C). We also present the maximum eigenvalue of the system of predetermined variables, which is indicative of the speed of adjustment of debt in our model[17]. Finally, Table 1 reports the optimal feedback coefficients under commitment and discretion. (These coefficients are the optimised values for the reaction functions in (31) and (36) respectively[18]). Notice that the instrument feedback coefficients onto the cost-push shock are identical to the first period movement of the instrument. That is, with a unit cost-push shock, the first period interest rate and spending movements are respectively given by θ_μ^C and ϕ_μ^C under commitment and θ_μ^D and ϕ_μ^D under discretion[19].

A. Optimal Policy under Commitment

We will consider optimal policy for our three values of inflation persistence in Figures 1, 2 and 3. The solid line in these Figures (labelled C) plots the dynamic responses of the model under optimal commitment policy. (We will turn to discretionary policy, labelled D, in Section B. below).

1. The Low Debt Economy

Let us start by characterising optimal commitment policy for a low debt economy and turn to Figure 1 which plots the impulse responses to a unit cost-push shock under optimal policy for a purely forward-looking Phillips curve. The policymaker raises the nominal interest rate to control inflation. We see from column (1) in Table 1 that nominal interest rates rise sufficiently strongly to increase the real interest rate ($\theta_\mu^C > 1$). That is, the

[16] This calibration is chosen to correspond approximately to $\chi^f = \chi^b = 0.5$, as in Fuhrer and Moore (1995).

[17] The maximum eigenvalue of a system describes the speed of adjustment of the system and hence that of its most persistent process.

[18] These optimal coefficients under commitment have to be interpreted with care as the fully optimal rule includes feedback onto the pre-determined Lagrange multipliers (see (31)).

[19] From (31) we see that under commitment the first period value of the instrument is given by $(i_1, g_1)' = (\Phi_{11}, \Phi_{12})(X_{1,0}, \lambda_0)' = \left(\theta_\mu^C, \phi_\mu^C\right)'$, where we have substituted for the unit shock $\mu_1 = 1$, the initial conditions $\pi_0 = y_0 = b_1 = 0$ because we start from equilibrium, and $\rho^\pi = \rho^c = 0$. Under discretion it follows from (36) that we have $(i_1, g_1)' = \left(\theta_\mu^D, \phi_\mu^D\right)'$.

Table 1: Optimal Monetary and Optimal Fiscal Policy Summary for Commitment ($i = C$) and Discretion ($i = D$).

		$\omega = 0$		$\omega = 0.75$		$\omega = 0.99$	
		C	D	C	D	C	D
		(1)	(2)	(3)	(4)	(5)	(6)
Low Debt ($B = 0.1$)							
Welfare	Loss	0.18	0.28	4.03	4.31	134.90	135.10
	% W	-	55.6	-	6.95	-	0.22
	% C	-	0.09	-	0.28	-	0.29
Eigenvalue	ν^i_{Max}	1.00	0.73	1.00	0.98	1.00	0.99
Optimal Monetary Feedback Coefficients							
Shock	θ^i_μ	2.99	6.27	13.95	24.63	17.69	28.48
Inflation	θ^i_π	0.00	0.00	3.68	4.26	6.10	6.16
Output	θ^i_y	0.00	0.00	0.23	0.26	0.37	0.38
Debt	θ^i_b	-0.003	-0.012	-0.003	-0.001	-0.002	-0.001
Optimal Fiscal Feedback Coefficients							
Shock	ϕ^i_μ	-0.11	-4.10	-4.48	-5.73	-8.02	-8.40
Inflation	ϕ^i_π	0.00	0.00	-3.47	-3.62	-6.69	-6.70
Output	ϕ^i_y	0.00	0.00	-0.21	-0.22	-0.41	-0.41
Debt	ϕ^i_b	-0.005	-0.107	-0.005	-0.011	-0.005	-0.006
High Debt ($B = 0.4$)							
Welfare	Loss	0.19	0.28	4.07	4.87	134.95	135.99
	% W	-	47.4	-	19.70	-	0.76
	% C	-	0.09	-	0.79	-	1.02
Eigenvalue	ν^i_{Max}	1.00	0.11	1.00	0.44	1.00	0.57
Optimal Monetary Feedback Coefficients							
Shock	θ^i_μ	2.59	-3.96	12.66	-2.82	16.08	1.04
Inflation	θ^i_π	0.00	0.00	3.39	0.19	5.72	2.41
Output	θ^i_y	0.00	0.00	0.21	0.01	0.35	0.15
Debt	θ^i_b	-0.045	-0.568	-0.038	-0.410	-0.033	-0.301
Optimal Fiscal Feedback Coefficients							
Shock	ϕ^i_μ	-0.24	-4.17	-5.12	-20.19	-8.86	-24.56
Inflation	ϕ^i_π	0.00	0.00	-3.62	-6.74	-6.93	-10.50
Output	ϕ^i_y	0.00	0.00	-0.22	-0.41	-0.43	-0.64
Debt	ϕ^i_b	-0.023	-0.346	-0.027	-0.390	-0.035	-0.314

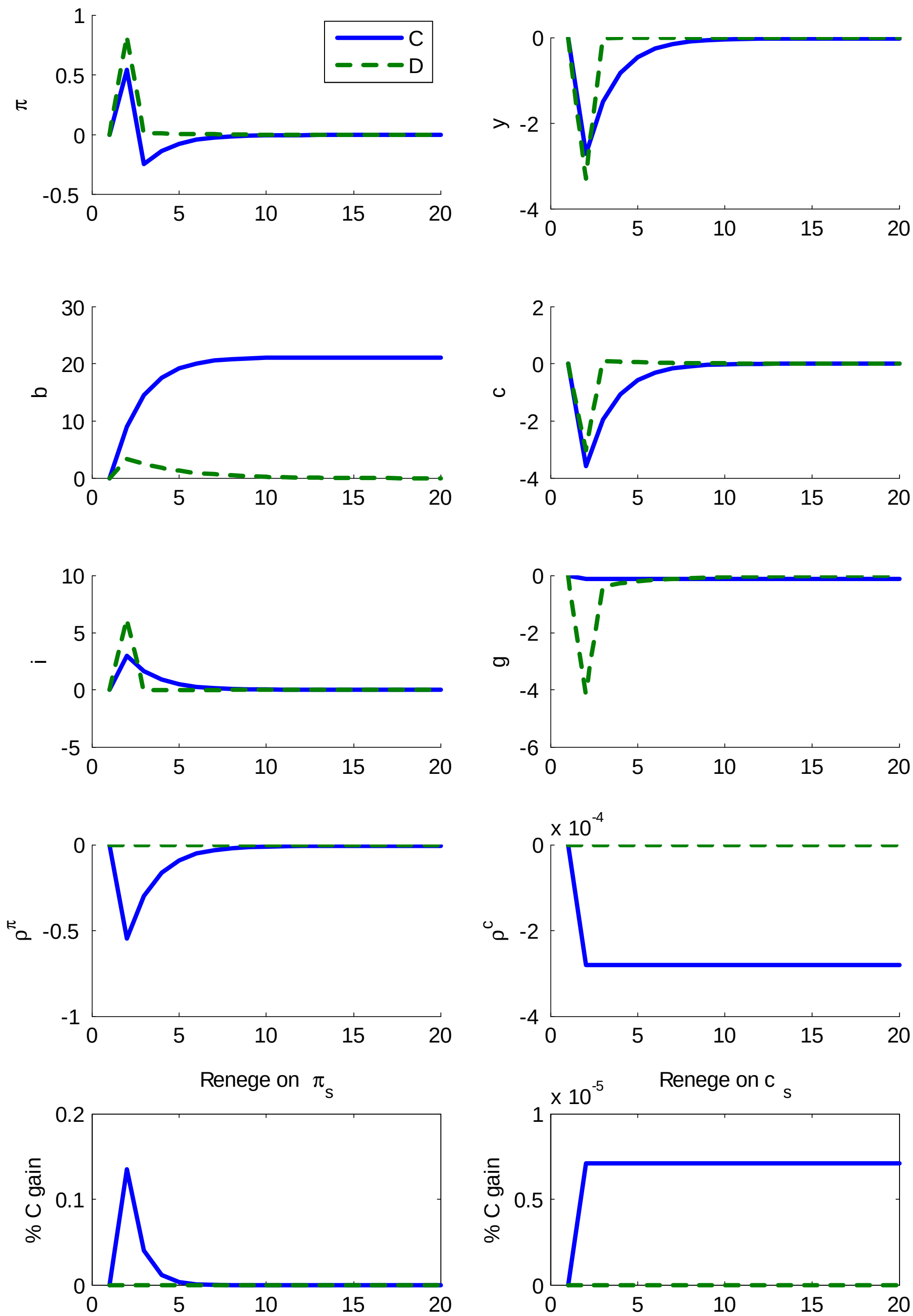
policymaker fulfils the Taylor principle to ensure inflation stability[20]. This rise in real interest rates induces a fall in consumption, and hence the output gap, which reduce real marginal cost and therefore inflation. Optimal policy under commitment is highly effective in achieving such a disinflation by steering inflation expectations through committing to and delivering contractionary policy in the future. This is achieved through a gradual response to the cost-push shock in which interest rates are slowly smoothed back to zero. We further see that, in the benchmark New Keynesian model without inflation persistence, optimal fiscal policy is almost inactive in response to the cost-push shock (ϕ_μ^C is negative but small at -0.11). The fiscal authority therefore leaves the stabilisation of the cost-push shock almost entirely to monetary policy. This is because movements in the fiscal instrument, in contrast to the monetary instrument, are costly as they induce a suboptimal quantity of public goods provision. This policy mix for a New Keynesian Phillips curve underpins the widely held view that monetary policy performs almost the entire stabilisation of cost-push shocks. Fiscal policy with a New Keynesian Phillips curve, as we are about to see, simply ensures the sustainability of debt (Allsopp and Vines 2005).

We see from Figure 1 that debt accumulates strongly through persistently higher interest rates and the corresponding fall in income tax revenues. Following the shock, debt remains permanently higher: debt under optimal commitment policy follows a random walk (as in Benigno and Woodford 2003, Schmitt-Grohe and Uribe 2004 and Leith and Wren-Lewis 2007). Column (1) in Table 1 shows that the maximum eigenvalue for the simulated system is exactly equal to one ($\nu_{Max}^C = 1$). This random walk result is related to the random walk of the debt Lagrange multiplier in (23). (We will discuss this in more detail below). The intuitive reason for the random walk of debt is as follows. We saw above that the cost-push shock induces contractionary monetary policy which in turn creates debt. When determining to what extent to reduce such debt, the policymaker will weigh benefits against costs. The benefits of reducing debt are that permanently higher debt leads to permanently higher interest payments, which will require a permanently lower level of government spending as the government needs to be solvent at given rates of tax. Lower government spending is costly both because the level of public spending appears in the welfare function directly and also because lower government spending leads to permanently higher consumption. The costs of reducing debt are that doing so would be inflationary: this is both because higher inflation helps to reduce real interest payments directly and because any optimal mix of lower interest rates and spending will, on balance, raise inflation[21]. As the benefits from permanently reducing government debt are discounted, there will only be finite gains. This means that the commitment solution will be a point such that, at the margin, these gains are balanced with the costs of debt reduction which hence involves a permanent increase in the ratio of debt to output. Consequently, in the face of random cost-push shocks, it is optimal to allow debt to become a random walk.

[20] Given the setup of the model, the condition $\theta_\mu^i > 1$ amounts to the Taylor principle in the first period. However, we notice that there is no simple expression available for the Taylor principle for subsequent periods.

[21] Reducing debt is necessarily inflationary in this setup because it will be done, to a large extent, by lowering interest rates. That is both because inflation helps to reduce debt directly and because reducing debt only by lowering government spending would be costly, because the level of government expenditure features in the utility function.

Figure 1: Optimal Policy with a New Keynesian Phillips curve ($\omega = 0$) in a 'low' debt economy ($B = 0.1$).



Such a permanent increase in debt requires permanently lower government spending, so that the new, permanently higher, level of debt can be serviced. Table 1 shows that debt sustainability is ensured through permanently lower spending via negative fiscal feedback on debt ($\phi_b^C < 0$ and $\theta_b^C < 0$)[22]. This implies that in response to the shock not only public debt, but also government spending, and hence consumption and output, will converge to a new steady state[23].

Inflation Persistence

The introduction of inflation persistence through rule-of-thumb price setters has important consequences for this optimal policy mix. With a hybrid Phillips curve Figure 2 shows that, as inflation becomes harder to control through expectations of future policies, monetary policy has to raise interest rates significantly more in the first period and induce a larger fall in consumption. With persistent inflation, we also see in Column (3) of Table 1 that monetary policy furthermore feeds back onto past inflation and output in a stabilising manner along the disinflation path ($\theta_\pi^C, \theta_y^C > 0$). The combination of positive monetary responses to the cost-push shock and the resulting output and inflation dynamics ensures the stability of inflation.

Fiscal policy now plays an active role with inflation persistence: government spending falls in response to the cost-push shock. The explanation follows from our 'comparative advantage' discussion in Section 1.. During a disinflation, the Calvo component of inflation falls strongly due to low current marginal cost and expectations of low future marginal cost (as we saw in Figure 1). The rule-of-thumb component of inflation, in contrast, falls less rapidly as it depends on past output and prices. This difference in price adjustment speed contributes to price dispersion in the economy and is costly. (This is of course why the 'smoothing' terms appear in the social loss function). As long as there are some forward-looking price setters, we showed above that fiscal policy has a 'comparative advantage' in controlling the rule-of-thumb component of inflation whilst monetary policy is relatively more effective in affecting the Calvo part of inflation. Fiscal policy therefore becomes helpful in raising the speed of disinflation of rule-of-thumb price setters towards that of the Calvo price setters through cuts in government spending. The gains in terms of better inflation control outweigh the costs of moving the fiscal instrument and Column (3) in Table 1 shows that spending optimally falls on impact of the shock ($\phi_\mu^C < 0$) [24].

Despite this fall in spending, the strong rise in interest rates leads to a more rapid accumulation of public debt than without inflation persistence. For the same reasons as with the New Keynesian Phillips curve, debt remains permanently higher and therefore follows a random walk. The optimal behaviour of monetary and fiscal policy remains

[22] We notice that as in Kirsanova and Wren-Lewis (2007) the optimal fiscal feedback on debt is negative and small.

[23] The initial linearisation remains valid despite this shift in steady state if this change is small in magnitude.

[24] We further notice that inflation falls below zero and then rises back to zero. For the same reasons as just discussed, it becomes optimal for fiscal policy to raise spending to align the rule-of-thumb price setters with the Calvo price setters when inflation is negative.

Figure 2: Optimal Policy with a hybrid Phillips curve ($\omega = 0.75$) in a 'low' debt economy ($B = 0.1$).

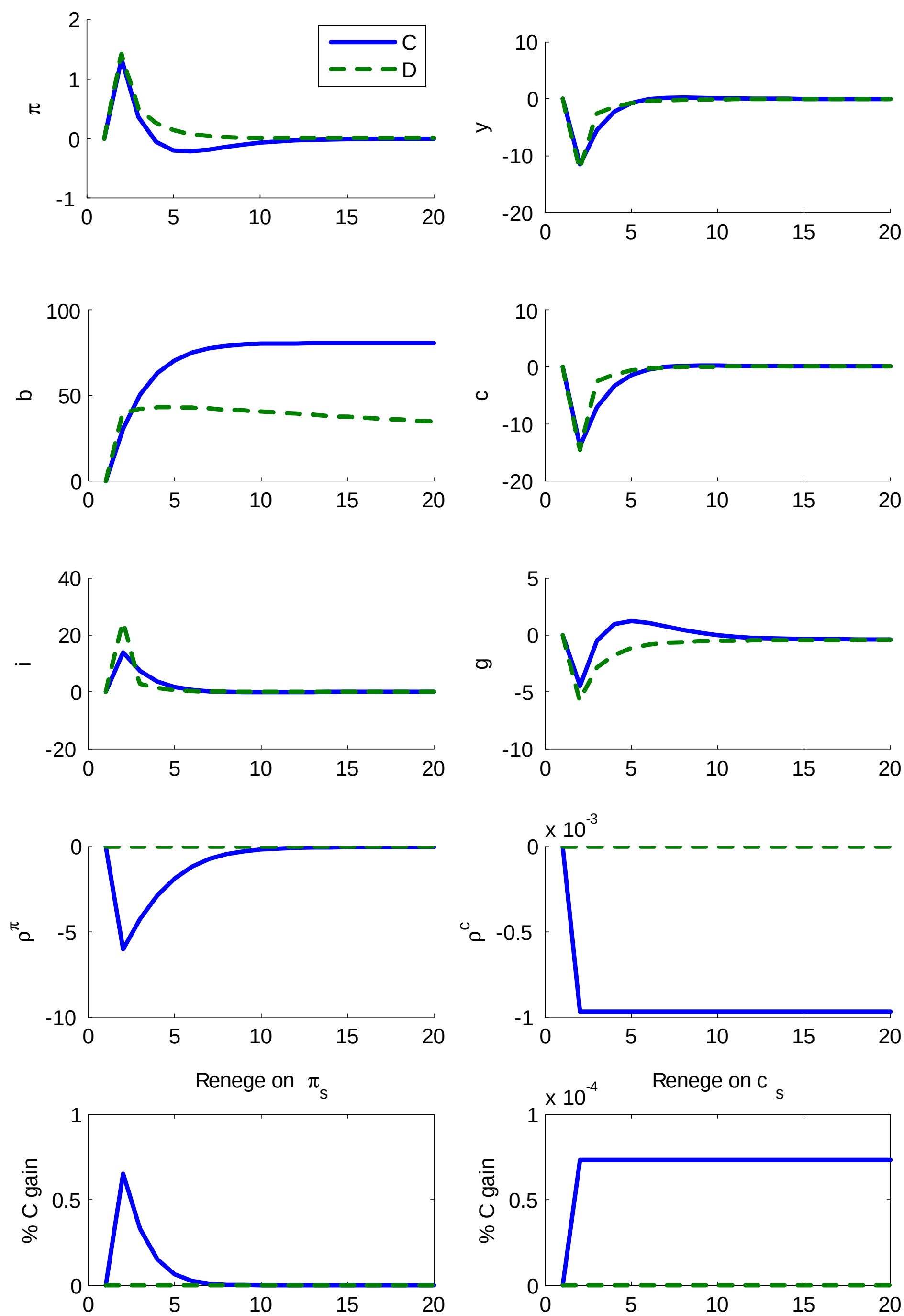
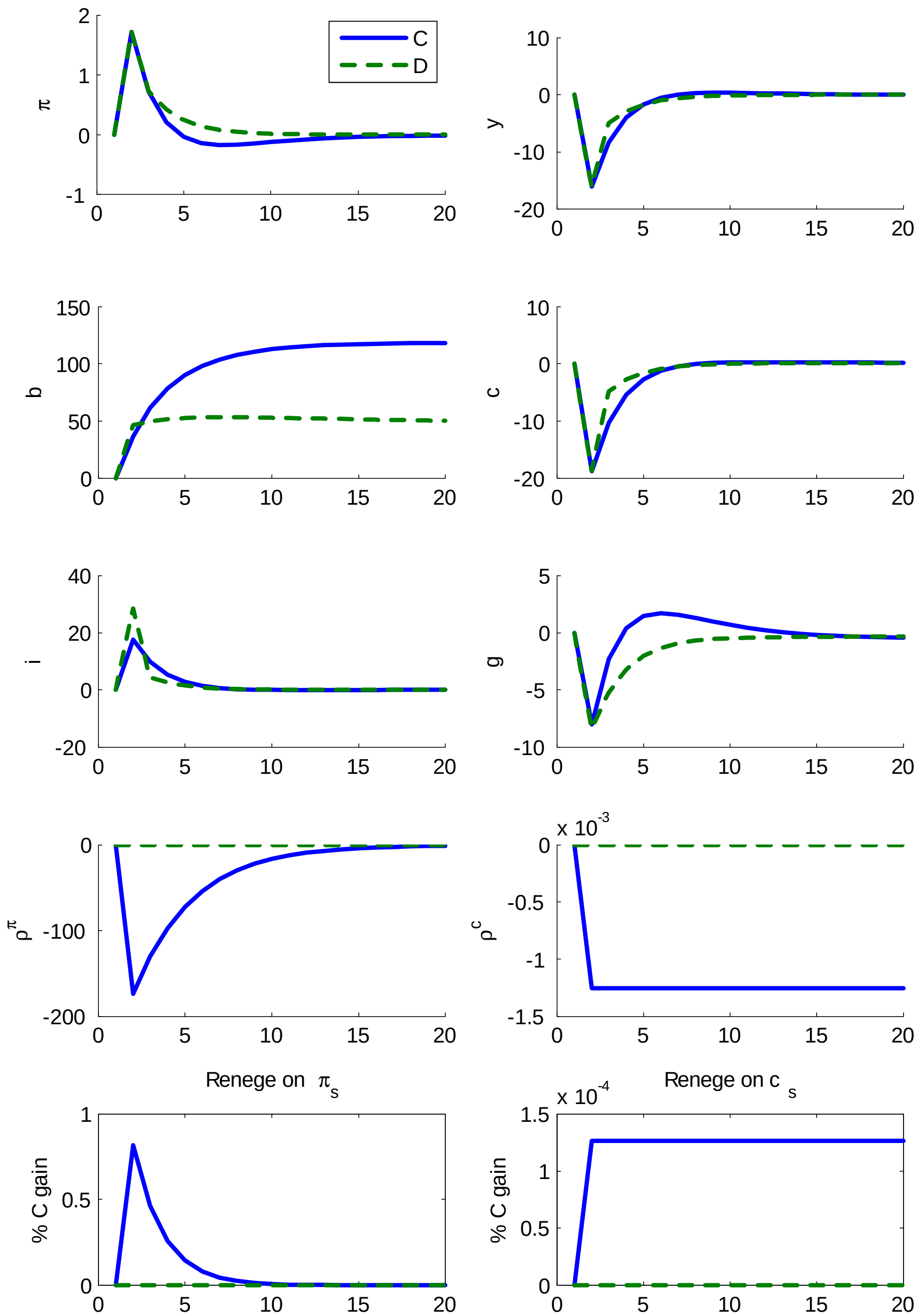


Figure 3: Optimal Policy with a predominantly backward looking Phillips curve ($\omega = 0.99$) in a 'low' debt economy ($B = 0.1$).



qualitatively unchanged for an economy with a strongly backward-looking Phillips curve. Column (5) in Table 1 and Figure 3 show that the interest rate has to rise by even more and that spending has to fall more strongly to assist the stabilisation of inflation. Together these imply that more debt is accumulated than with a hybrid Phillips curve.

Time Inconsistency

We have seen already that such optimal commitment policy is time inconsistent in its control of *both* inflation *and* debt. The fourth row in Figures 1, 2 and 3 plots the evolution of the predetermined Lagrange multipliers ρ_s^π and ρ_s^c for the optimal policy scenarios discussed above. We see that both Lagrange multipliers are different from zero during the disinflation process for all levels of inflation persistence. As discussed in Section 4., such non-zero values of predetermined Lagrange multipliers indicate that optimal commitment policy is time inconsistent.

Firstly, we see that the control of inflation along the optimal policy path is time inconsistent. We observe that ρ_s^π is negative for $s > 0$, which indicates that the social loss could be reduced by setting a higher inflation rate in periods after the initial than was optimal at time $s = 0$, where we had $\rho_0^\pi = 0$. It is therefore optimal for the policymaker to announce at time $s = 0$ a rapid disinflation through higher interest rates in that and future periods. Expectations of future tight policy help to reduce current inflation without a large fall in current period consumption or output gap through the forward-looking part of the Phillips curve. However, at $s > 0$, once inflation has fallen substantially, it becomes optimal for the policymaker not to implement tight policy to lower inflation as this would depress demand. This incentive to renege induces less contractionary policy at time $s > 0$ than announced at time $s = 0$. As inflation converges back to zero, the incentive to renege disappears gradually.

Secondly, as in Leith and Wren-Lewis (2007), the control of debt under optimal commitment policy is time inconsistent. Figure 1, for example, shows that ρ_s^c is different from zero for $s > 0$, which from (28) implies a non-zero value of μ_s^b. A negative value of ρ_s^c is equivalent to a positive value of μ_s^b and hence indicates the incentive to reduce debt under optimal commitment policy. This incentive to cut debt does not vanish over time because μ_s^b and ρ_s^c follow a random walk. The intuition for this result is as follows. In any period, there is a benefit from reducing debt through cutting government expenditure and/or interest rates so as to cut debt service costs. We have explained above that doing so entails a cost because it will be inflationary[25]. The key insight is that, whilst the gain of cutting debt is constant over time, the cost of reducing debt in the first period is smaller than in subsequent periods. This is because, in the first period, the effect on inflation will, of course, be confined to that and subsequent periods; there will be no effects on inflation in previous periods (since they do not exist). But in all subsequent periods any attempt to change policy so as to reduce debt which was expected would, because it was expected in

[25] It follows that this incentive to reduce debt through inflation is higher if debt is denominated in nominal terms, rather than in real terms as in the present model. See Leith and Wren-Lewis (2007).

the periods before the period in which it occurred, lead to an increase in inflation not only in the period in which it happened (and in subsequent periods) but also in periods before it was implemented, because it was already expected. It would thus be more costly to cut debt in future periods, as compared with cutting debt in the first period. But this means that a policymaker who re-optimises every period would face an incentive to unexpectedly lower debt in every period in the future because he had not been expected to do such lowering of debt. The random walk in debt under optimal commitment is therefore time inconsistent[26]. This discussion suggests that the random walk of μ_s^b in (23) serves as a sufficient condition for the random walk of debt under commitment: if optimal policy is described by a permanent incentive to cut debt after starting from a an initial equilibrium position, then the debt stock must be permanently different from its initial level.

Using (33) we can quantify the incentive to renege on the optimal inflation and debt paths in terms of the gain in social welfare. (We denote this in % of steady-state consumption gained). For the New Keynesian Phillips curve, the last row of Figure 1 plots the period-to-period incentive to renege on the optimal inflation and consumption path respectively. We see that the incentive to deviate from the optimal path of inflation is largest in the first period for which expectations have already been set and then fall over time as inflation returns to zero. The incentive to renege on the optimal consumption path, and therefore on the debt path, follows a random walk. This is because, as discussed above, the gain from reducing debt is constant over time as it stems from steady-state gains resulting from higher government spending and lower consumption. Figure 1 further suggests that the welfare gains from reneging on the optimal inflation path are significantly larger than those on debt. Given the importance of inflation in the social welfare function, the smaller welfare consequences of reneging on debt control is not surprising[27].

As we see in the bottom rows of Figures 2 and 3, the incentive to renege on the optimal inflation path disappears more slowly for economies with higher inflation persistence. We observe that the welfare incentive to renege on inflation path becomes stronger in magnitude with more inflation persistence because, with higher ω, the weights on inflation related terms in the social welfare function increase strongly (see Section 1.).

2. The High Debt Economy

Let us now turn to optimal commitment policy in an economy with higher steady-state debt. The solid line in Figures 4, 5 and 6 plot optimal commitment policy for such a high debt economy with a New Keynesian Phillips curve, a hybrid Phillips curve and a predominantly backward-looking Phillips curve respectively. The bottom part of Table 1 reports the corresponding welfare losses and optimal feedback coefficients.

[26] This discussion implies that policy under fully optimal commitment policy will induce slightly higher inflation and lower debt in the first period as compared with ‘timeless’ commitment policy, see Leith and Wren-Lewis (2007).

[27] A smaller incentive to renege under commitment does not mean, however, that the requirement to conduct time-consistent control of debt will imposes a smaller distortion onto the system. We will return to this issue below.

Figure 4: Optimal Policy with a New Keynesian Phillips curve ($\omega = 0$) in a 'high' debt economy ($B = 0.4$).

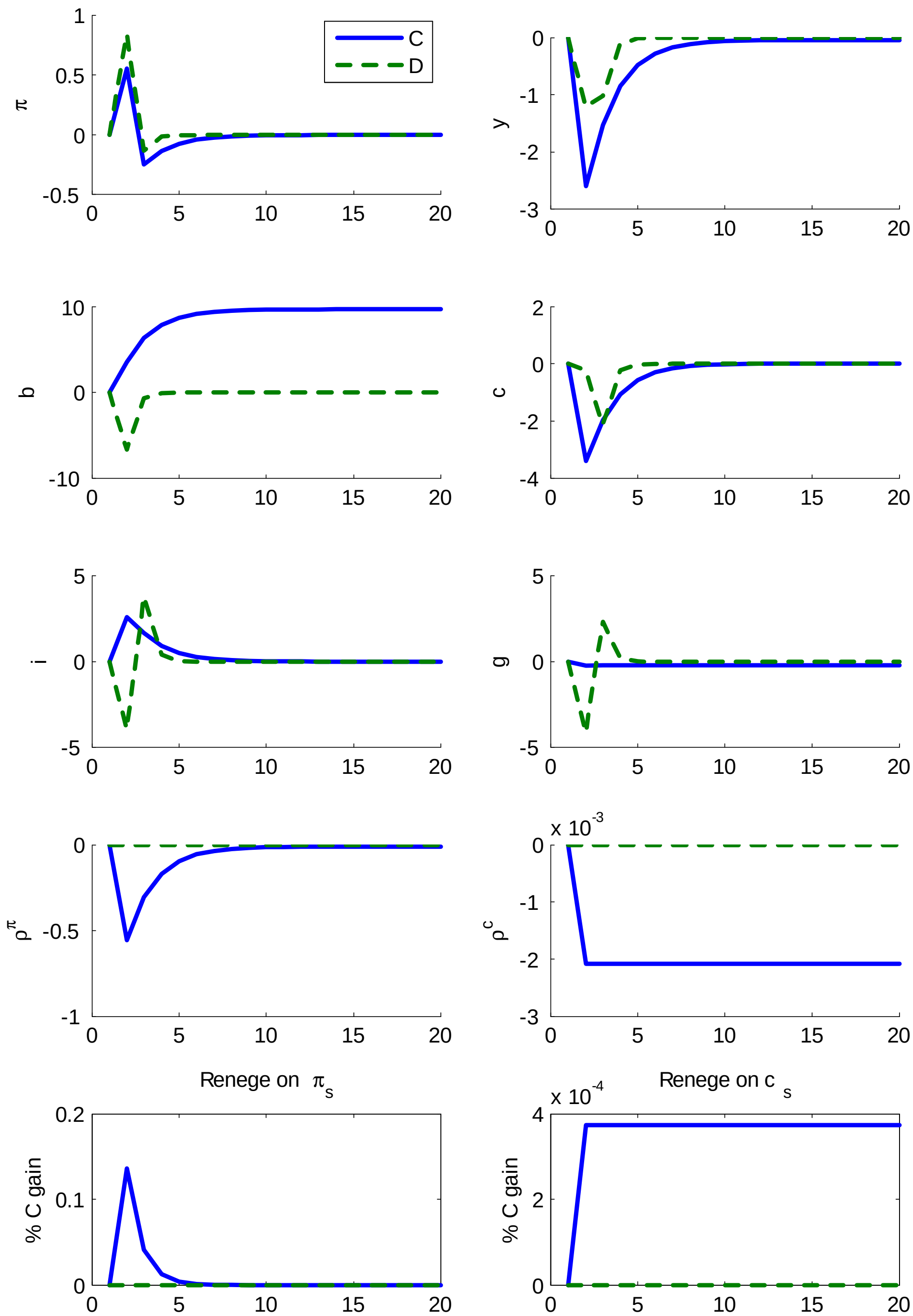


Figure 5: Optimal Policy with a hybrid Phillips curve ($\omega = 0.75$) in a 'high' debt economy ($B = 0.4$).

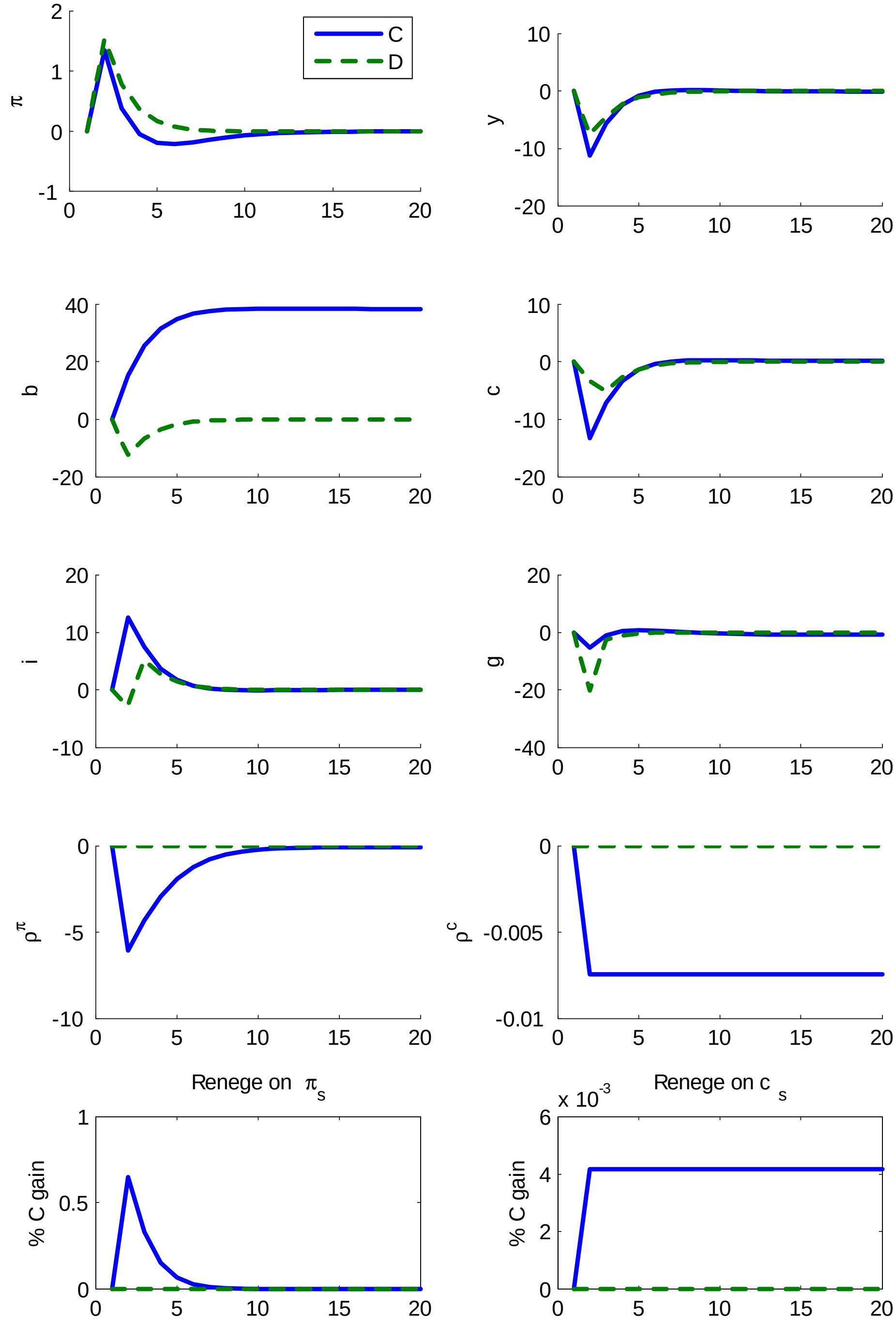
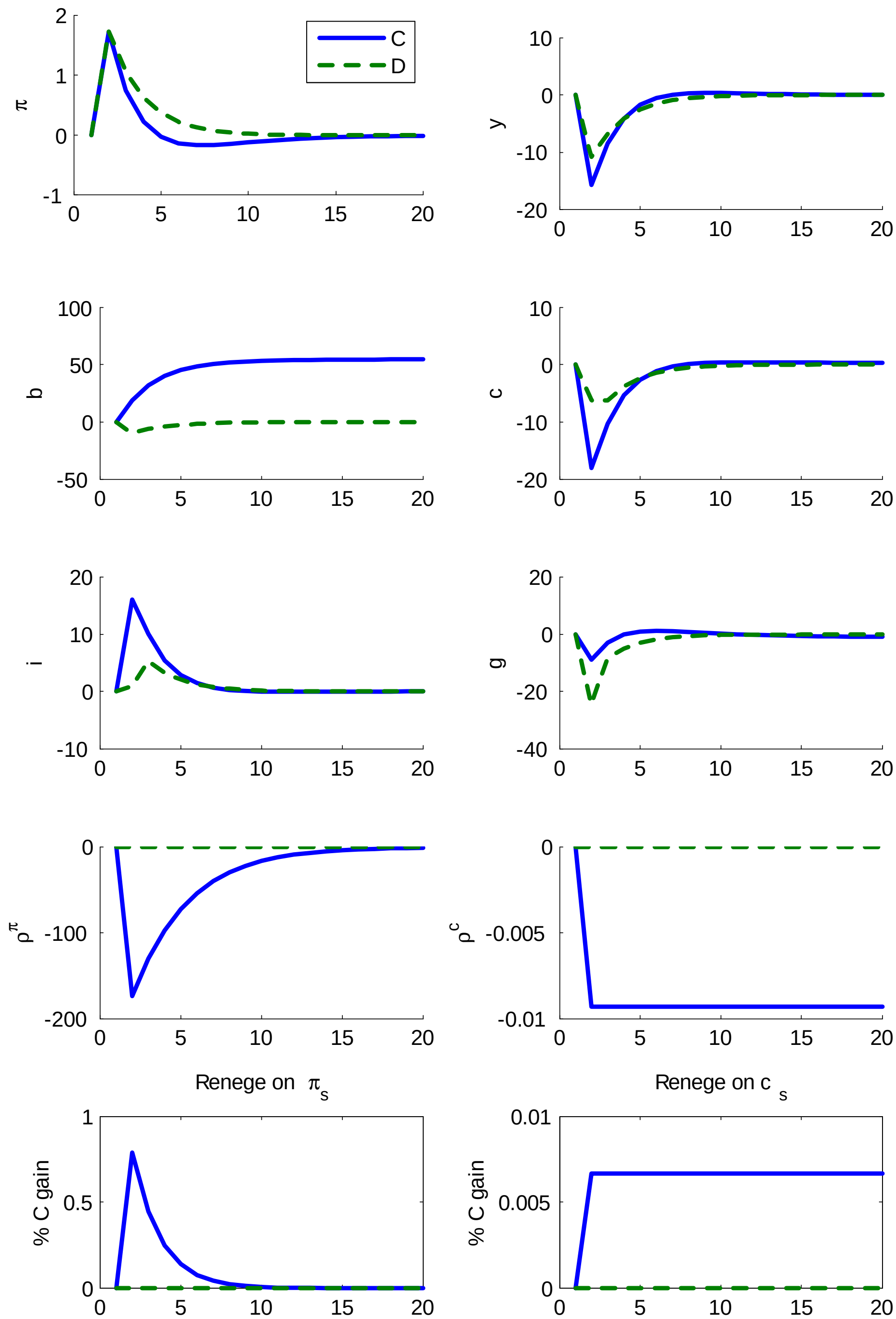


Figure 6: Optimal Policy with a predominantly backward looking Phillips curve ($\omega = 0.99$) in a 'high' debt economy ($B = 0.4$).



We see from these impulse responses, and from the feedback coefficients, that optimal commitment policy is similar in low and high debt economies. For a hybrid Phillips curve, Figure 4 shows that, as before, the interest rate rises and spending falls in response to the cost-push shock. Monetary policy fulfils the Taylor principle and debt remains a random walk. The only important change, as one might expect, occurs with respect to debt control. We see from any of the Figures that less debt is accumulated in economies with higher steady-state debt[28]. This is because with higher steady-state debt, the interest payments for an additional percentage of debt are larger and therefore even lower government spending is needed in steady-state to service them. It follows that less debt is accumulated. The tighter control of debt is reflected in stronger fiscal debt feedback coefficients[29].

As one might expect, the steady-state ratio of debt strengthens the time inconsistency problem with respect to debt control. Taking the hybrid Phillips curve as an example, the comparison of the bottom rows of Figures 2 and 5 shows that the welfare incentive of reneging on the announced optimal path of debt rises strongly in the high debt economy. This is because the gains from cutting debt - in terms of higher government spending - are higher with more debt as the cost-push shock leads to a larger shift in steady state. The incentive to renege on the inflation path, in contrast, remains roughly unchanged[30].

B. Optimal Policy under Discretion

We next turn to characterising optimal discretionary policy which, as discussed above, has to be time consistent. We will see that this time consistency requirement will have important consequences for optimal policy behaviour. As both the control of inflation and debt are time inconsistent under optimal commitment policy, it follows that a time consistency constraint will impose two distortions onto optimal discretionary policy.

1. The Low Debt Economy

The dashed line (denoted by D) in Figures 1, 2 and 3 plots optimal discretionary policy alongside optimal commitment policy in a low debt economy. Columns (2), (4) and (6) in the top part of Table 1 display the welfare losses and the corresponding optimal feedback coefficients.

Starting again with optimal policy with a New Keynesian Phillips curve we see in Figure 1 that inflation is controlled much less effectively under discretion than commitment. The inability to control inflation tightly by steering inflation expectations under discretion results in the classic inflation stabilisation bias of Currie and Levine (1987, 1993) and

[28] That is, we observe a smaller percentage deviation of debt from its steady-state level.

[29] We see in Column (3) of Table 1 that the fiscal debt feedback rises in absolute value from -0.005 to -0.027.

[30] We also see this by observing that B does not feature in (29), which is the source of the time inconsistency problem in inflation control.

Woodford (2003b). Unable to promise high interest rates in the future, the policymaker raises interest rates very strongly in the first period. This first period hike in interest rates induces a large recession but then interest rates are much more quickly returned to zero than under commitment which leads to slower inflation control.

We further see from Figure 1 that debt does not follow a random walk under optimal discretionary policy but returns to its initial value. This result was first discovered by Leith and Wren-Lewis (2007). This 'debt stabilisation bias' is a direct consequence of the incentive to cut debt that we found under commitment. Under discretion the policymaker cannot commit to 'not cutting debt tomorrow'. The only time-consistent solution is one in which there is no incentive, at any stage, to reduce debt in an unexpected way through unexpected changes in spending or interest rates. As inflation, the interest rate, and spending fall back to towards their zero steady-state values, the only time-consistent solution is one in which debt returns to its pre-shock level (i.e. equals its steady-state value)[31]. Otherwise, as described above, there would always be an incentive to carry out an unexpected reduction in debt. Debt under optimal discretionary policy does therefore not follow a random walk. For a New Keynesian Phillips curve, Column (2) in Table 1 shows that the maximum eigenvalue for the simulated system is considerably below unity at $\nu^D_{\max} = 0.73$.

Next we consider how the adjustment of debt takes place. The key difference between the control of inflation and debt is that inflation is a partly forward-looking process, whilst debt is an entirely backward-looking process. This implies that inflation in the first period may be reduced through expectations of future contractionary policy. Debt, in contrast, can only be reduced in the first period through lower interest rates and/or lower spending *in that period*. As discussed above, the policymaker will choose to do the bulk of the debt adjustment in the first period when the inflationary costs of doing so are smallest. Leith and Wren-Lewis (2007) show that whether to lower the interest rate or spending, or both, to do this adjustment in the first period depends critically on the steady-state ratio of debt to output. This is because the steady-state value of debt determines the relative effectiveness of monetary and fiscal policy in affecting the debt stock. For the low debt economy, we see from Figure 1 that the adjustment of debt in the first period is done mostly through lower spending[32]. Column (2) in Table 1 shows that spending under discretionary policy falls much more strongly in response to the cost-push shock than under commitment. Strongly negative fiscal and monetary feedback onto the debt stock subsequently ensures a fast convergence of debt back to its initial level. The debt stabilisation bias therefore necessitates a much more active role for fiscal policy under discretion than commitment for a New Keynesian Phillips curve.

[31] An argument by reductio ad absurdum makes this point clear. Any candidate for a discretionary outcome which had a positive outcome for debt would be vulnerable, at any point after this supposed equilibrium had been reached to re-optimisation by the policymaker to reduce debt, taking inflation expectations as given. But this vulnerability would cause the candidate equilibrium to unravel, by backwards induction.

[32] Notice how interest rates help to accumulate less debt. Whilst interest rates rise more strongly under discretion they are returned much more quickly to zero than under commitment. This lower cumulative effect of interest rates helps to limit the accumulation of debt.

As one would expect, the inability to commit to a time-inconsistent policy plan is very costly with government debt. Column (2) in Table 1 shows that the combination of the inflation and debt stabilisation bias induces a loss that is equivalent to a 0.09% fall in steady-state consumption.

Inflation Persistence

The introduction of inflation persistence again has important consequences for the optimal policy mix under discretion. We see in Figures 2 and 3 that monetary policy, just as under commitment, raises interest rates more strongly. We also observe that fiscal policy cuts spending more actively. This is because with rule-of-thumb price setters fiscal policy needs to assist monetary policy *both* in its control of both inflation - as it did under commitment - *and* debt.

As the degree of inflation persistence rises further, the dynamics of inflation and debt under discretion become more similar to that of commitment. We see that the disinflation under discretion is less slow compared to commitment than it was in a more forward-looking regime. This is because the link from expected inflation to current inflation weakens and time-inconsistent promises about future policy become less effective in controlling inflation. An implication of a weaker inflation stabilisation bias is that debt can be controlled more slowly, because for less forward-looking Phillips curves there is weaker pressure to reduce debt through inflationary policies. Table 1 confirms that the maximum eigenvalue rises from 0.73 to 0.98 and 0.99 as we raise the degree of inflation persistence from $\omega = 0$ to $\omega = 0.75$ and $\omega = 0.99$ respectively.

However, Table 1 also indicates that the combined effect of the inflation and debt stabilisation biases continues to be very costly for higher ω. In fact, the cost of discretionary policy rises to 0.28% and 0.29% of steady-state consumption for a hybrid and backward-looking Phillips curve respectively. These higher welfare costs are again driven by the fact that the weight on price dispersion in the social welfare function rises strongly with inflation persistence.

2. The High Debt Economy

We saw above that in the low debt economy, a combination of lower interest rates and spending than under commitment delivered the required adjustment of debt. As the steady-state ratio of debt to output in the economy rises, monetary policy becomes more powerful in controlling the debt stock relative to fiscal policy. This is because the leverage of monetary policy over interest payments rises. For a New Keynesian Phillips curve in a high debt economy we see in Figure 4 that it turns out for the interest rate to *fall* in the first period (we see that $\theta_{\mu}^{D} < 0$ in Column (2) of Table 1). Cutting interest rates under optimal policy in response to a cost-push shock seems deeply counter-intuitive. As found in Leith and Wren-Lewis (2007), monetary policy is forced to lower interest rates in the first period - and hence violate the Taylor Principle - because debt has to be returned to its initial level to ensure time consistency. Lower interest rates also serve to fuel inflation and hence additionally reduce debt through lower real interest payments.

Notice that we have shown that interest rates optimally fall in the first period, even although the control of inflation is subject to the inflation stabilisation bias. With such a bias, the effect of promises about the effects of future monetary policy is weakened because these promises cannot be time inconsistent. Nevertheless the effects of debt stabilisation bias are so severe that it remains optimal to cut interest rates, despite the weak link of expected future monetary policy to current inflation in a regime of discretionary policy. This finding is interesting because it relates to circumstances in which the effects of inflation stabilisation bias and debt stabilisation bias point in opposite directions as to the initial movement of monetary policy. We saw above that inflation stabilisation bias would, if operating on its own, cause interest rates to be raised strongly initially, compared with optimal commitment policy. And we have noted that debt stabilisation bias, operating on its own, pulls interest rates down initially, when the level of debt is high. The results here show that this debt stabilisation bias effect dominates, when the initial level of debt is high and inflation persistence is low. This interaction of the inflation and debt stabilisation biases makes the inability to commit particularly costly for a high debt economy, as we will see below.

Once interest rates have fallen in the first period to reduce debt, they rise strongly in the second period. Even under discretion rational agents anticipate in the first period that interest rates will have to rise in subsequent periods to control inflation. Expectations of future contractionary policy ensure the stability of inflation in the first period, despite the cut in interest rates[33]. In other words, optimal policy incurs the 'damage' necessary for debt control in the first period and postpones the control of inflation to subsequent periods.

Given that monetary policy is constrained by having to cut debt, it is not surprising that fiscal policy reduces spending very aggressively in the first period to assist monetary policy[34]. In fact, we see in Figure 4 that on impact of the shock, interest rates and spending are cut so strongly that debt actually falls below its steady-state value and then returns to its initial level from below. This is, as we discussed above, because interest rates have to rise in future periods to ensure inflation stability. The only way optimal policy can deliver contractionary policy in future periods, but still fulfil the time consistency requirement, is by cutting debt below its pre-shock value in the first period. Debt is then rather quickly returned to its initial level. This fast adjustment of debt is reflected in a maximum eigenvalue of 0.11 in Table 1, which indicates much faster convergence than in the low debt economy.

[33] That is, the policymaker under discretion can still make promises about future policy but these promises have to be time consistent.

[34] Notice, however, that this cut in spending is not necessary to control inflation in the first period. Stehn (2007) shows that interest rates under optimal discretionary policy continue to be cut initially even if government spending is unable to fall because fiscal policy is constrained to a simple feedback on debt (and hence cannot respond to the cost-push shock in the first period).

Inflation Persistence

As we raise the persistence of inflation it becomes more and more difficult to control inflation despite cutting interest rates in the first period. For a hybrid Phillips curve Figure 5 shows that interest rates continue to fall but that this is only possible through very large cuts in spending: fiscal policy not only helps to reduce debt but also assists monetary policy in controlling first period inflation as it becomes harder to do so through expectations of future tight monetary policy. As the Phillips curve becomes strongly backward looking, however, the effect of future inflation expectations on current inflation is not strong enough and monetary policy cannot cut interest rates in the first period. Figure 6 shows that the social planner under discretion *raises* the interest rate in the first period in response to the cost-push shock with very strong inflation persistence (this is confirmed by $\theta_{\mu}^{C} > 0$ in Table 1). We conclude that the debt stabilisation bias ceases to force monetary policy into the perverse cut in interest rates in the first period when the Phillips curve becomes strongly backward looking. This conclusion is consistent with the description of optimal policy in a fully backward-looking system, in which interest rates rise and spending falls in response to an inflation shock (see Kirsanova et al 2005).

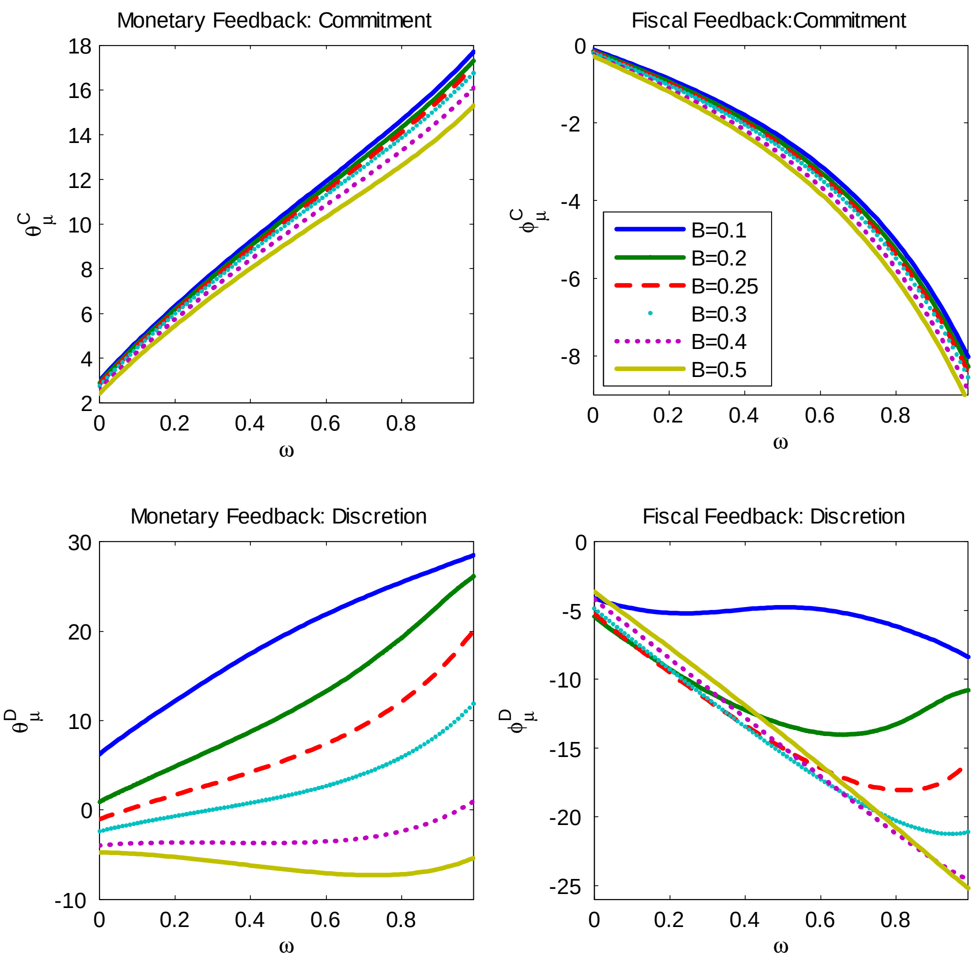
It is not surprising that the system implications of the stabilisation bias are more severe in the high debt economy. In contrast to the low debt economy, where the dynamics of the system under discretion approached that under commitment for $\omega = 0.99$, we see from Figure 6 that this is not true with a high debt economy. Debt continues to converge much faster to its pre-shock level than in a low debt economy. (The maximum eigenvalue of the system remains considerably below unity at 0.57). It follows that the welfare costs are considerably higher for the high debt economy even for high values of inflation persistence. For a hybrid Phillips curve the excess loss of discretionary policy over commitment is equivalent to a 0.79% fall in steady-state consumption. For the predominantly backward-looking Phillips curve this cost rises further to 1.02%. We conclude that, in contrast to the low debt economy, the distortion of the stabilisation bias does not fall and that the welfare cost of the inability to commit continues to rise with the degree of inflation persistence.

C. Summary

Having analysed optimal policy for selected calibrations of inflation persistence and steady-state debt, let us now turn to a summary of optimal policy with a wider range of calibrations. This allows us to identify the conditions under which falling interest rates in the first period cease to be optimal.

Figure 7 plots the optimal feedback coefficient on the cost-push shock under optimal commitment and discretionary policy respectively (which we recall is equivalent to the first period instrument movement). We plot the optimal monetary (θ_{μ}^{i}) and fiscal feedback coefficients (ϕ_{μ}^{i}) against different proportions of rule-of-thumb price setters (ω) for different steady-state debt levels in the economy. The top row of Figure 7 confirms that under

Figure 7: Monetary (θ_μ^i) and fiscal (ϕ_μ^i) feedback coefficients on the cost-push shock under optimal commitment ($i = C$) and discretion ($i = D$) policy.



optimal commitment policy the social planner always raises the interest rate and cuts spending (that is $\theta_\mu^C > 0, \phi_\mu^C < 0$ for all ω and B). As the degree of inflation persistence rises, we see that the social planner becomes more active in using both monetary and fiscal policy to stabilise the cost-push shock.

Turning to the bottom two panels in Figure 7 we see the implications of stabilisation bias for optimal monetary and fiscal policy behaviour under discretion. We see that fiscal policy is more active under discretion than commitment and that the degree of activism rises with the steady-state ratio of debt to output. For low values of steady-state debt ($B \leq 0.2$) we see that the social planner under discretion raises the interest rate in response to the cost-push shock and fulfils the Taylor principle for all levels of inflation persistence. For economies with $B > 0.2$, in contrast, we see how monetary policy becomes forced to stabilise debt through cutting the interest rate in response to the cost-push shock.

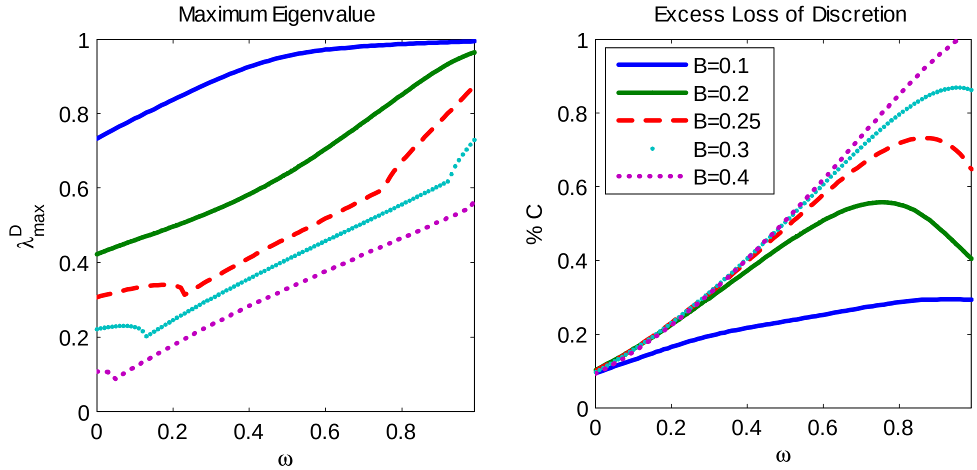
However, Figure 7 shows that this is *only* true if the Phillips curve is predominantly forward-looking and it is possible to control inflation in the first period by expectations of future tight policy. For $B = 0.25$, for example, the perverse monetary response vanishes at about $\omega = 0.1$. For higher levels of steady-state debt this threshold is larger (e.g. for $B = 0.3$ the point at which monetary policy returns to fulfilling the Taylor principle in the first period rises to about $\omega = 0.5$). This is because the higher the steady-state value of debt, the stronger is the debt stabilisation bias and the more powerful monetary policy is in cutting debt and hence the more optimal policy trades off slower control of inflation for lowering debt in the first period. However, we also observe that the violation of the Taylor principle in the first period remains optimal at all levels of inflation persistence for very high debt calibrations ($B \geqslant 0.5$). In those cases monetary policy is so powerful in affecting debt that it remains optimal to lower interest rates in the first period, even for $\omega = 0.99$[35].

Figure 8 reports simulation results on the strength of the stabilisation bias both by reporting the speed of adjustment of the system under discretion and the welfare consequences of the inability to commit. The left hand panel summarises the severity of the stabilisation bias with respect to debt control by plotting the maximum eigenvalue under discretion ($\nu^D_{\max}$). Under commitment, we recall that the maximum eigenvalue equals one. For low values of steady-state debt the eigenvalue under discretion rises towards unity as the system becomes increasingly backward looking because the stabilisation bias imposes less tight debt control onto the policymaker. However, even as ω approaches unity we see that debt does not follow a random walk because consumption remains forward looking. The speed of adjustment of the system falls for higher steady-state debt as the debt stabilisation bias becomes more severe. For those high debt economies we see that the maximum eigenvalue remains significantly below unity even as ω approaches unity. This underpins our earlier finding that the stabilisation bias continues to impose tight debt control and the violation of the Taylor principle onto optimal discretionary policy.

The right hand panel of Figure 8 evaluates the welfare consequences of these policies by plotting the excess loss of discretionary policy over commitment policy (in % steady-state consumption foregone). We see that this excess loss is large and higher for bigger values of steady-state debt. As the persistence of the system rises, the excess loss of discretionary policy over commitment policy increases as the cost of delivering less tight inflation control rises as the welfare function places more weight on price dispersion. For an empirically plausible hybrid Phillips curve the inability to commit imposes a large welfare cost which is equivalent to a fall in steady-state consumption of around 0.6%. As the persistence of the Phillips curve rises further, however, the excess loss of discretionary policy over commitment policy starts to fall for economies with intermediate debt ratios because with fewer forward-looking agents the stabilisation bias becomes less severe. For very high debt economies, which always violate the Taylor principle, we observe that the inability to commit becomes more and more costly.

[35] Notice that we would expect this violation of the Taylor principle to disappear for an entirely backward-looking inflation process, as cutting the interest rate would lead to an explosive inflation process. However, as the micro-founded social loss function (15) is not defined in this limit, we cannot compute optimal policy. Kirsanova et al (2005), for a non-microfounded model, show that monetary policy fulfils the Taylor principle with an accelerationist Phillips curve, regardless of the level of steady-state debt.

Figure 8: The excess loss of discretion over commitment (in % of steady-state consumption foregone) and the maximum eigenvalue ($v^D_{\max}$) under discretion.



Discussion

We have shown that the violation of the Taylor principle in a high debt economy ceases to be optimal with strong degrees of inflation persistence. However, Figures 7 and 8, however, suggest that optimal discretionary policy will violate the Taylor principle for reasonable debt calibrations (e.g. for a hybrid Phillips curve with an annual debt to GDP ratio of 10%). The quantitative results depend on the model setup in two important ways. Firstly, the thresholds depend critically on how we define debt. Public debt in this model has a one period maturity. This means that the entire debt stock is rolled over each period which gives monetary policy large leverage over interest payments. In practice, the fraction of debt which is refinanced every period is considerably lower[36] (i.e. a one year maturity debt to GDP ratio of 10% is therefore rather large). Denoting debt in nominal terms, in contrast to our analysis, would raise the effect of inflation on debt and increase the severity of the debt stabilisation bias.

Secondly, the threshold of inflation persistence at which it ceases to be optimal to violate the Taylor principle for a given steady-state debt ratio falls with the intertemporal elasticity of substitution. For higher elasticities of substitution, changes in interest rates have stronger effects on consumption and hence inflation. In response to a cost-push shock monetary policy can dis-inflate by accumulating less debt which renders the debt stabilisation bias less severe and makes the violation of the Taylor principle less likely.

[36] For example, whilst Euroland has a debt to GDP ratio about 60%, the amount of debt re-financed per year is much lower at around 11% of GDP (ECB Monthly Bulletin 2005).

V. Conclusion

Leith and Wren-Lewis (2007) have shown that public debt under optimal discretionary policy does not follow a random walk but has to be returned to its pre-shock level to ensure time consistency. This finding has two important implications for optimal monetary and optimal fiscal policy under discretion. Firstly, as Leith and Wren-Lewis (2007) show, optimal monetary policy in a high debt economy cuts the interest rate in response to a cost-push shock - and therefore violates the Taylor principle. This is a striking and unintuitive result. We have shown that this is not true with high degrees of inflation persistence. Secondly, because debt does not follow a random walk under discretionary policy, we have shown that optimal fiscal policy is more active under discretion than commitment - at all levels of inflation persistence and all levels of debt - to assist the constrained monetary authority. We conclude that monetary policy should fulfil the Taylor principle in an economy with inflation persistence, but that the widely held view, in which monetary policy performs the bulk of the stabilisation of cost-push shocks and fiscal policy merely ensures the sustainability of debt, is inappropriate under optimal discretionary policy.

These results suggest that the gains from commitment are much larger in an economy with public debt, especially if it is high, than the traditional monetary policy analysis identified[37]. Institutions which promote the commitment of both monetary policy and fiscal policy, such as the announcement of targets and long tenure of policymakers, are therefore highly desirable. In low-debt countries with effective institutions, such as Britain, monetary policy is unlikely to be tightly constrained by public debt. In very high debt countries with weak commitment mechanisms, in contrast, the central bank might well be hindered in its control of inflation[38].

[37] For example, Steinsson (2003) shows that the absolute value of the welfare loss under discretionary policy is 26% higher than under commitment for a New Keynesian Phillips curve in a monetary policy model. In our model with public debt, this loss rises to over 50%.

[38] Mitra (2007) and Baig et al (2006), for example, find evidence that monetary policy has been constrained by the debt stock for a panel of high debt countries.

Appendix

A. Derivations of the Model

The derivations in this section are taken from Kirsanova and Wren-Lewis (2005).

1. The Phillips Curve

From the consumer's first order conditions it follows that we can write the nominal wage w as

$$w_t(z) = \frac{v_y(y_t(z))}{(1-\tau)\, u_C(C_t)} P_t \tag{38}$$

The production function is assumed to be just a linear function of labour supplied

$$y_t(z) = h_t(z)$$

As we assume that labour is the only production input and we assume that there are no taxes, the total cost of supplying good z is given by

$$\frac{1}{\mu_w} w_t(z)\, h_t(z) = \frac{1}{\mu_w} w_t(z)\, y_t(z)$$

where μ_w is a labour subsidy (see below). The demand for good z follows from intra-temporal consumption optimisation and is given by

$$y_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\varepsilon_t} Y_t \tag{39}$$

A profit maximising firm will choose a price $p_t(z)$ that maximises

$$\begin{aligned} &\max_{p_t(z)} E_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} \left[p_s(z)\, y_s(z) - \frac{1}{\mu_w} w_s(z)\, y_s(z) \right] \\ &= \max_{p_t(z)} E_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} P_s^{\varepsilon_s} Y_s \left[p_s^{1-\varepsilon_s}(z) - \frac{1}{\mu_w} w_s(z)\, p_s^{-\varepsilon_s}(z) \right] \end{aligned}$$

Leading to the following first order condition for setting the optimal price $p_{f,t}(z)$

$$E_t \sum_{s=t}^{\infty} \gamma^{s-t} R_{t,s} P_s^{\varepsilon_s} Y_s \left[p_{f,s}^{-\varepsilon_s}(z)\,(1-\varepsilon_s) + \varepsilon_s \frac{1}{\mu_w} w_s(z)\, p_{f,s}^{-\varepsilon_s - 1}(z) \right] = 0$$

Substituting for the demand equation (39) and the nominal wage (38) we can write

$$0 = E_t \sum_{s=t}^{\infty} \gamma^{s-t} p_{f,s}^{-\varepsilon_s - 1}(z) R_{t,s} Y_s \left(\frac{p_s(z)}{P_s}\right)^{-\varepsilon_s} \left[p_{f,s}(z) - \frac{\mu_s}{\mu_w} P_s \frac{v_y\left(\left(\frac{p_{f,s}(z)}{P_s}\right)^{-\varepsilon_s} Y_s\right)}{(1-\tau) u_C(C_s)} \right] \tag{40}$$

where we denote the stochastic mark-up of firms as $\mu_s = -\frac{\varepsilon_s}{(1-\varepsilon_s)}$. This equation defines the optimal forward-looking price $p_t(z) = p_{f,t}(z)$. In flexible price equilibrium we have

$$E_t \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} \left[(1-\tau) - \frac{\mu}{\mu_w} \frac{v_y(y)}{U_C(C)}\right] = \frac{1}{1-\gamma\beta} \left[(1-\tau) - \frac{\mu}{\mu_w} \frac{v_y(y)}{U_C(C)}\right] = 0$$

where we used $\sum_{s=t}^{\infty} (\gamma\beta)^{s-t} = \frac{1}{1-\gamma\beta}$. For this to hold the term in square brackets must be zero

$$\frac{v_y(y)}{(1-\tau) U_C(C)} = \frac{\mu_w}{\mu} = \frac{w}{P}$$

This implies that the term in square brackets in (40) is zero in equilibrium. Therefore when we now linearise the equation we only need to linearise this term and take everything that multiplies this term at its steady-state level. Let us start by linearising the second part of the expression in the square brackets in (40) using $Z_t = Z\left(1 + \hat{Z}_t\right)$

$$\begin{aligned} \frac{v_y\left(\left(\frac{p_{f,s}(z)}{P_s}\right)^{-\varepsilon_s} Y_s\right)}{u_C(C_s)} &= \frac{v_y}{u_C} - \varepsilon \frac{v_{yy}}{u_C}\left(\hat{p}_{f,s}(z) - \hat{P}_s\right) + \frac{v_{yy} Y}{u_C} \hat{Y}_s - \frac{v_y u_{CC} C}{u_C^2} \hat{C}_s \\ &= \frac{v_y}{u_C}\left(1 - \varepsilon \frac{v_{yy}}{v_y}\left(\hat{p}_{f,s}(z) - \hat{P}_s\right) + \frac{v_{yy} Y}{v_y} \hat{Y}_s - \frac{u_{CC} C}{u_C} \hat{C}_s\right) \end{aligned}$$

Where the term with $\hat{\varepsilon}_s$ is zero as it is multiplied by $\ln \frac{p_{f,t}(z)}{P}$. Turning to linearise the entire expression

$$
\begin{aligned}
&(1-\tau)\,p_s(z) - \frac{\mu_s}{\mu_w} P_s \frac{v_y\left(\left(\frac{p_{f,s}(z)}{P_s}\right)^{-\varepsilon_s} Y_s\right)}{u_C(C_s)} \\
&= (1-\tau)\left(1+\hat{p}_s(z)\right) - \frac{\mu}{\mu_w}\left(1+\hat{\mu}_s\right)\left(1+\hat{P}_s\right)\frac{v_y}{u_C}[1-\varepsilon\frac{v_{yy}}{v_y}\left(\hat{p}_{f,s}(z)-\hat{P}_s\right) \\
&\qquad + \frac{v_{yy}Y}{v_y}\hat{Y}_s - \frac{u_{CC}C}{u_C}\hat{C}_s] \\
&= (1-\tau) - \frac{v_y}{u_C}\frac{\mu}{\mu_w} + (1-\tau)\,\hat{p}_s(z) - \frac{v_y}{u_C}\frac{\mu}{\mu_w}[\hat{P}_s - \varepsilon\frac{v_{yy}}{v_y}\left(\hat{p}_{f,s}(z)-\hat{P}_s\right) \\
&\qquad + \frac{v_{yy}Y}{v_y}\hat{Y}_s - \frac{u_{CC}C}{u_C}\hat{C}_s - \hat{\mu}_s] \\
&= (1-\tau)\left(\hat{p}_s(z) - \hat{P}_s + \frac{\varepsilon}{\psi}\left(\hat{p}_{f,s}(z)-\hat{P}_s\right) - \frac{1}{\psi}\hat{Y}_s - \frac{1}{\sigma}\hat{C}_s - \hat{\mu}_s\right)
\end{aligned}
$$

where we have used $\frac{v_y(y)}{U_C(C)} = \frac{\mu_w}{\mu}(1-\tau), \frac{1}{\sigma} = -\frac{u_{CC}C}{u_C}$ and $\frac{1}{\psi} = \frac{v_{yy}Y}{v_y}$. Using the fact that $\hat{P}_s = \sum_{k=1}^{s-t}\pi_{t+k}$ we can solve this equation with respect to $\hat{p}_{f,s}(z)$

$$
\hat{p}_{f,t}(z) = \frac{(1-\gamma\beta)}{\left(1+\frac{\varepsilon}{\psi}\right)} E_t \sum_{s=t}^{\infty}(\gamma\beta)^{s-t}\left[\left(1+\frac{\varepsilon}{\psi}\right)\sum_{k=1}^{s-t}\pi_{t+k} + \frac{1}{\psi}\hat{Y}_s + \frac{1}{\sigma}\hat{C}_s + \hat{\mu}_s\right]
$$

Using $\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}\sum_{k=1}^{s-t}\pi_{t+k} = \frac{1}{1-\gamma\beta}\sum_{k=1}^{s-t}(\gamma\beta)^k\pi_{t+k}$ we can simplify this to

$$
\hat{p}_{f,t}(z) = E_t\sum_{k=1}^{s-t}(\gamma\beta)^k\pi_{t+k} + \frac{(1-\gamma\beta)}{\left(1+\frac{\varepsilon}{\psi}\right)} E_t \sum_{s=t}^{\infty}(\gamma\beta)^{s-t}\left[\frac{1}{\psi}\hat{Y}_s + \frac{1}{\sigma}\hat{C}_s + \hat{\mu}_s\right]
$$

Steinsson (2003) shows how this equation can be used to derive the final specification of the hybrid Phillips curve (7) in the main text. We see how the mark-up shocks $\hat{\mu}_s$ affect the Phillips curve and we notice that the constant wage income tax rate τ does not affect the dynamic Phillips curve.

The backward-looking price setters use the rule of thumb to set prices. The linearisation of this gives

$$
\hat{P}^B_{Hk,t} = (1-\omega)\ln\left(\frac{P^F_{Hk,t-1}}{P_{Hk,t-1}}\right) + \omega\ln\left(\frac{P^B_{Hk,t-1}}{P_{Hk,t-1}}\right) - \ln\Pi_{Hk,t} + \ln\Pi_{Hk,t-1} + \delta\ln\left(\frac{Y_{k,t-1}}{Y^n_{k,t-1}}\right)
$$

Doing manipulations similar to Steinsson (2003), Kirsanova and Wren-Lewis (2005) show how to obtain the Phillips curve in the main text.

2. The Social Welfare Function

The social welfare function can be written as

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} W_s = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[u\left(C_s\right) + f\left(G_s\right) - \int_0^1 v\left(h_s\left(z\right)\right) dz \right]$$

Following Woodford (2003) we can derive a second order approximation of social welfare based on the representative agent's utility in four steps. Firstly, we linearise the intra-temporal utility W_s around its equilibrium using $\hat{X}_t = \ln\left(X_t/X\right)$:

$$\begin{aligned} W_s = {} & C u_C\left(C\right)\left(\hat{C}_s + \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\hat{C}_s^2\right) + G f_G\left(G\right)\left(\hat{G}_s + \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\hat{G}_s^2\right) \\ & - Y v_y\left(Y\right)\left(\hat{Y}_s + \frac{1}{2}\left(1 + \frac{1}{\psi}\right)\hat{Y}_s^2 + \frac{1}{2}\left(\frac{1}{\psi} + \frac{1}{\varepsilon}\right) var_z \hat{y}_s\left(z\right)\right) + O\left(3\right) \end{aligned} \quad (41)$$

where $O\left(3\right)$ denotes terms of higher than second order and terms independent of policy. Secondly, we linearise the aggregate demand equation (8)

$$\hat{C}_s = \frac{1}{\theta}\left(\hat{Y}_s - \left(1-\theta\right)\hat{G}_s - \theta\frac{1}{2}\hat{C}_s^2 - \frac{1}{2}\left(1-\theta\right)\hat{G}_s^2 + \frac{1}{2}\hat{Y}_s^2\right) + O\left(3\right)$$

Substituting this expression into (41) we obtain

$$\begin{aligned} W_s = {} & \theta u_C[\left(1 - \frac{v_y}{u_C}\right)\hat{Y}_s - \left(1-\theta\right)\left(1 - \frac{f_G}{u_C}\right)\hat{G}_s - \frac{\theta}{2\sigma}\hat{C}_s^2 \\ & + \frac{1}{2}\left(1 - \frac{v_y}{u_C}\left(1 + \frac{1}{\psi}\right)\right)\hat{Y}_s^2 + \frac{\left(1-\theta\right)}{2}\left(\frac{f_G}{u_C}\left(1 - \frac{1}{\sigma}\right) - 1\right)\hat{G}_s^2 \\ & - \frac{v_y}{2u_C}\left(\frac{1}{\psi} + \frac{1}{\varepsilon}\right) var_z \hat{y}_s\left(z\right)] + O\left(3\right) \end{aligned}$$

The third step is to eliminate the linear terms in output and government spending. We can always choose a steady-state such that $\theta = 1 - \frac{G}{Y}$ such that $\frac{f_G}{u_C} = \frac{v_y}{u_C}$. The government is assumed to eliminate both the distortions resulting from monopolistic competition and the distortions resulting from income taxation with a lump sum of $\mu_w = \frac{\mu}{1-\tau}$ in steady state. Then $\frac{f_G}{u_C} = \frac{v_y}{u_C} = 1$ and hence the welfare function does not contain any linear terms. We can hence re-write it in 'gap' form in deviations from its natural levels:

$$\begin{aligned} W_s = {} & -\theta u_C[\frac{\theta}{2\sigma}\left(\hat{C}_s - \hat{C}_s^n\right)^2 + \frac{\left(1-\theta\right)}{2\sigma}\left(\hat{G}_s - \hat{G}_s^n\right)^2 \\ & + \frac{1}{2\psi}\left(\hat{Y}_s - \hat{Y}_s^n\right)^2 + \frac{1}{2}\left(\frac{1}{\psi} + \frac{1}{\varepsilon}\right) var_z \hat{y}_s\left(z\right)] + O\left(3\right) \end{aligned}$$

Finally, Steinsson (2003) has shown that we can write[39]

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} var_z \hat{y}_s(z) = E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{\varepsilon^2}{(1-\gamma\beta)} [\frac{\gamma}{(1-\gamma)} \pi_t^2 + \frac{\omega}{(1-\omega)} \frac{1}{(1-\gamma)} (\Delta\pi_t)^2 + \frac{(1-\gamma)\omega\delta^2}{(1-\omega)} y_{t-1}^2 - \frac{2\omega\delta}{(1-\omega)} y_{t-1} \Delta\pi_t]$$

Normalising on the inflation term and denoting 'gaps' by $x_t = \hat{X}_t - \hat{X}_t^n$ we obtain the period loss function (15) in the main text

$$W_s = \frac{\psi(1-\gamma\beta)(1-\gamma)}{\varepsilon\gamma(\varepsilon+\psi)} \left(\frac{\theta}{\sigma} c_s^2 + \frac{(1-\theta)}{\sigma} g_s^2 + \frac{1}{\psi} y_s^2 \right) + \pi_s^2 + \frac{\omega}{\gamma(1-\omega)} (\Delta\pi_s)^2 + \frac{(1-\gamma)^2 \omega\delta^2}{\gamma(1-\omega)} y_{s-1}^2 - \frac{2\omega\delta(1-\gamma)}{\gamma(1-\omega)} y_{s-1}\Delta\pi_s + O(3)$$

3. Compensating Consumption

Having computed a welfare outcome $W_{1,s}$ we can express this loss in the percentage reduction in steady-state consumption, Ω, that makes the household equally well off under this regime and a regime without any volatility. If the first regime results in a steady-state consumption outcome C, the regime without volatility therefore results in a consumption level of $C + \Omega C$. Using the second order approximation of the utility function derived above, the level of welfare for a utility stream U_{1s} can be written as

$$W_1 = \frac{1}{1-\beta}(u(C) + f(G) - v(Y)) - Cu_C(C) E_t \sum_{s=t}^{\infty} \beta^{s-t} U_{1s}$$

Under a benchmark policy with no volatility we have $U_{0,s} = 0$. Kirsanova et al (2007) show that this can be written as follows

$$\begin{aligned} W_0 &= \frac{1}{1-\beta}(u(C+\Omega C) + f(G) - v(Y)) \\ &= \frac{1}{1-\beta}\left(u(C)\Omega C \left(1 - \frac{\Omega}{2\sigma}\right) + u(C) + f(G) - v(Y) \right) + O(\Omega C)^3 \end{aligned}$$

The representative agent will be indifferent between W_1 and W_0 when

$$\Omega \left(1 - \frac{\Omega}{2\sigma}\right) + (1-\beta) E_t \sum_{s=t}^{\infty} \beta^{s-t} U_{1s} = 0$$

[39] Notice that we make use of the erratum to Steinsson (2003).

The solution for Ω is given by

$$\Omega = \sigma\left(1-\sqrt{1+\frac{2(1-\beta)}{\sigma}E_t\sum_{s=t}^{\infty}\beta^{s-t}U_{1s}}\right)$$

This expression can then be used to find the change in steady-state consumption required to make the individual indifferent between two regimes W_2 and W_1 that both induce volatility.

B. Canonical Form

For our model we have

$$A_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\beta} \end{pmatrix},\ A_{12} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & \theta \\ -\frac{1}{\beta} & -\frac{\tau\theta}{\beta B} \end{pmatrix},$$

$$A_{21} = \begin{pmatrix} -\frac{1}{\chi^f\beta} & -\frac{\chi^b}{\chi^f\beta} & -\frac{\kappa_{y1}}{\chi^f\beta} & 0 \\ \frac{\sigma}{\chi^f\beta} & \frac{\sigma\chi^b}{\chi^f\beta} & \frac{\sigma\kappa_{y1}}{\chi^f\beta} & 0 \end{pmatrix},\ A_{22} = \begin{pmatrix} \frac{1}{\chi^f\beta} & \frac{(\kappa_c-\theta\kappa_{y1})}{\chi^f\beta} \\ -\frac{\sigma}{\chi^f\beta} & \frac{\left(\chi^f\beta+\sigma\kappa_c+\sigma\theta\kappa_{y1}\right)}{\chi^f\beta} \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1-\theta \\ 1 & \frac{(1-\tau)(1-\theta)}{\beta B} \end{pmatrix},\ B_2 = \begin{pmatrix} 0 & \frac{-\kappa_{y0}(1-\theta)}{\chi^f\beta} \\ \sigma & \frac{\sigma\kappa_{y0}(1-\theta)}{\chi^f\beta} \end{pmatrix},\ E_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},\ E_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

And for the weight matrix

$$\Omega_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & -\frac{1}{2}\lambda_4 & 0 \\ 0 & -\frac{1}{2}\lambda_4 & \lambda_3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},\ \Omega_{12} = \Omega_{21}' = \begin{pmatrix} 0 & 0 \\ -\lambda_2 & 0 \\ \frac{1}{2}\lambda_4 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Omega_{22} = \begin{pmatrix} 1+\lambda_2 & 0 \\ 0 & \lambda_c+\theta^2\lambda_y \end{pmatrix},\ \Omega_{23} = \Omega_{32}' = \begin{pmatrix} 0 & 0 \\ 0 & \theta(1-\theta)\lambda_y \end{pmatrix}$$

$$\Omega_{33} = \begin{pmatrix} 0 & 0 \\ 0 & \lambda_g+(1-\theta)^2\lambda_y \end{pmatrix}$$

C. Optimal Policy

In this second Appendix we provide the detailed first order conditions for optimal policy under commitment (1.) and discretion (2.).

1. Commitment

We can write the objective function of the policymaker under commitment as a constrained loss function (see Currie and Levine 1993):

$$H^C = \min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} E_t \sum_{s=t}^{\infty} H_s^C$$

with

$$\begin{aligned} H_s^C = {} & \frac{1}{2}\beta^{s-t}[X_{1,s}'\Omega_{11}X_{1,s} + X_{1,s}'\Omega_{12}X_{2,s} + X_{2,s}'\Omega_{21}X_{1,s} + X_{2,s}'\Omega_{22}X_{2,s} \\ & + X_{2,s}'\Omega_{23}U_s + U_s'\Omega_{32}X_{2,s} + U_s'\Omega_{33}U_s] \\ & + \hat{\mu}'_{s+1}\left(A_{11}X_{1,s} + A_{12}X_{2,s} + B_1U_s - X_{1,s+1}\right) \\ & + \hat{\rho}'_{s+1}\left(A_{21}X_{1,s} + A_{22}X_{2,s} + B_2U_s - X_{2,s+1}\right) \end{aligned}$$

where $\hat{\mu}_{t+1}$ is a n_1-dimensional non-predetermined Lagrange multiplier associated with the predetermined variables $X_{1,t}$ and $\hat{\rho}_{t+1}$ is a n_2-dimensional predetermined Lagrange multiplier associated with the non-predetermined variables $X_{2,t}$. The first order conditions are obtained by differentiating with respect to X_1, X_2, U, $\hat{\mu}$ and $\hat{\rho}$. We simplify notation and define $\mu_s = \beta^{-s}\hat{\mu}_s$ and $\rho_s = \beta^{-s}\hat{\rho}_s$.

For $s > 0$ the first order conditions are given by:

$$\begin{aligned} \frac{\partial H^C}{\partial X_{1,s}} &= \Omega_{11}X_{1,s} + \Omega_{12}X_{2,s} + \beta A_{11}'\mu_{s+1} + \beta A_{21}'\rho_{s+1} - \mu_s = 0 \\ \frac{\partial H^C}{\partial X_{2,s}} &= \Omega_{21}X_{1,s} + \Omega_{22}X_{2,s} + \Omega_{23}U_s + \Omega_{32}'U_s + \beta A_{12}'\mu_{s+1} + \beta A_{22}'\rho_{s+1} - \rho_s = 0 \\ \frac{\partial H^C}{\partial U_s} &= X_{2,s}'\Omega_{23} + \Omega_{32}X_{2,s} + \Omega_{33}U_s + B_1'\mu_{s+1} + B_2'\rho_{s+1} = 0 \\ \frac{\partial H^C}{\partial \hat{\mu}_{s+1}'} &= A_{11}X_{1,s} + A_{12}X_{2,s} + B_1U_s - X_{1,s+1} = 0 \\ \frac{\partial H^C}{\partial \hat{\rho}_{s+1}'} &= A_{21}X_{1,s} + A_{22}X_{2,s} + B_2U_s - X_{2,s+1} = 0 \end{aligned}$$

For $s = 0$ the first order condition for $X_{2,s}$ is given by:

$$\frac{\partial H^C}{\partial X_{2,0}} = \Omega_{21}X_{1,0} + \Omega_{22}X_{2,0} + \beta A_{12}'\mu_1 + \beta A_{22}'\rho_1 = 0$$

because of the initial condition $\rho_0 = 0$.

2. Discretion

Following Currie and Levine (1993), the first step in finding the discretionary solution is to guess how the private agents determine their expectations of non-predetermined variables. That is, we guess a solution for the reaction function of the public (who can be seen as the ultimate follower in this game). Given the linear quadratic setup of the model, we know the reaction function of the public must take the following linear form:

$$X_{2,t} = -GX_{1,t} - KU_t \tag{42}$$

where the matrices G and K are unknown. We substitute for (34) and form the Lagrangean

$$H^D = \min_{\{U_s\}_{s=t}^{\infty}} \frac{1}{2} E_t \sum_{s=t}^{\infty} H_s^D$$

with

$$\begin{aligned} H_s^D = &\frac{1}{2}\beta^{s-t}[X'_{1,s}\Omega_{11}X_{1,s} + X'_{1,s}\Omega_{12}X_{2,s} + X'_{2,s}\Omega_{21}X_{1,s} + X'_{2,s}\Omega_{22}X_{2,s} \\ &+ X'_{2,s}\Omega_{32}U_s + U'_s\Omega_{23}X_{2,s} + U'_s\Omega_{33}U_s] \\ &+ \hat{\eta}'_{s+1}\left(\left(A_{11} - GA_{12}\right)X_{1,s} + A_{12}X_{2,s} + \left(B_1 - A_{12}K\right)U_s - X_{1,s+1}\right) \end{aligned}$$

where $\hat{\eta}_{t+1}$ is a vector of non-predetermined Lagrange multipliers, associated with the predetermined variables $X_{1,t}$. Notice that under discretion, the objective function is only constrained by predetermined variables, as the policymaker takes non-predetermined ones as given. We simplify notation and define $\eta_s = \beta^{-s}\hat{\eta}_s$.

The first order conditions with respect to $X_{1,s}, U_s$ and $\hat{\eta}_{s+1}$ are given by:

$$\begin{aligned} \frac{\partial H^D}{\partial X_{1,s}} &= \Omega_{11}X_{1,s} + \Omega_{12}X_{2,s} + \beta\left(A_{11} - GA_{12}\right)'\eta_{s+1} - \eta_s = 0 \\ \frac{\partial H^D}{\partial U_s} &= X'_{2,s}\Omega_{23} + \Omega_{32}X_{2,s} + \Omega_{33}U_s + \left(B_1 - A_{12}K\right)'\eta_{s+1} = 0 \\ \frac{\partial H^D}{\partial \hat{\eta}'_{s+1}} &= \left(A_{11} - GA_{12}\right)X_{1,s} + A_{12}X_{2,s} + \left(B_1 - A_{12}K\right)U_s - X_{1,s+1} = 0 \end{aligned}$$

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