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Estimation of Equilibrium Exchange Rates in the WAEMU: A Robustness Approach

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Estimation of Equilibrium Exchange Rates in the WAEMU: A Robustness Analysis

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Abstract

Using the FEER approach we investigate the long-run equilibrium paths of the real effective exchange rates (REERs) of countries in the West African Economic and Monetary Union (WAEMU). In an attempt to address econometric estimation uncertainty, we employ both single-country (Johansen and ARDL) and panel-data (FMOLS and PMG) cointegration techniques. We find that (i) much of the long-run behavior of REERs in WAEMU countries can be explained by fluctuations in terms of trade, government consumption, investment, and productivity; (ii) the use of different econometric techniques suggests that there is significant uncertainty about the path of the underlying equilibrium REERs and the degree of exchange rate misalignment, which underscores the need for robustness analyses in exchange rate modeling; and (iii) results from panel-data cointegration may sometimes be useful, but should always be complemented with single-country estimations to ensure that the results take into account country-specific characteristics.

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I. INTRODUCTION

The 1994 devaluation of the CFA franc—the only change in the peg since the zone was created over half a century ago—was instrumental in strengthening the competitiveness of the CFA franc zone and economic activity. Since the end of 2002, however, the CFA franc has appreciated—along with the euro to which it is pegged—by more than 30 percent in nominal terms against the U.S. dollar, putting upward pressure on the region's real effective exchange rate (REER) and raising questions about REER valuation and its impact on external competitiveness.[2]

These recent exchange rate developments have revived interest in the valuation of the REER in the region. In a recent paper, Abdih and Tsangarides (2006) investigate the behavior of the REERs in both the Central African Economic and Monetary Community (CEMAC) and the West African Economic and Monetary Union (WAEMU). The authors conclude that fundamentals account for most of the fluctuations of the real exchange rate and that by the end of 2005 both REERs were in line with their long-run equilibrium values. While modeling the regional exchange rate is important because the peg is determined at the regional level, concluding that the regional exchange rate is not misaligned does not shed any particular information on the behavior of the REERs of individual countries within the region. The regional exchange rate may actually mask opposing trends in individual countries. For example, in some countries in the region there may be sustained departures of the REER from equilibrium—implying that macroeconomic imbalances need to be corrected through macroeconomic adjustment; in others, where movements of the REERs are in line with equilibrium, to influence economic fundamentals that affect the EREER structural reforms to improve the competitiveness of the export sector may be warranted.

Assessing equilibrium real effective exchange rates (EREERs) and the associated misalignments involves considerable uncertainties, including the choice of a model to describe the behavior, the time frame of the analysis, and the actual estimation method. Because results may be influenced by the choice of model or the estimation method, a systematic analysis of the behavior of exchange rates would require estimation of alternative theoretical and statistical models, or a formal robustness analysis.[3]

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[2] The CFA franc zone has France on one side and two monetary unions in Central and West Africa on the other, WAEMU (Benin, Burkina Faso, Côte d'Ivoire, Guinea-Bissau, Mali, Niger, Senegal, and Togo) and CEMAC (Cameroon, the Central African Republic, Chad, the Republic of the Congo, Equatorial Guinea, and Gabon). See Hadjimichael and Galy (1997) for a thorough analysis of the CFA zone and its institutions.

[3] Ideally, in order to provide a comprehensive evaluation, the robustness analysis would include estimating all theoretical and statistical approaches across all dimensions and possible model variants. In practice, it is more reasonable to isolate one dimension and examine the uncertainty within that.

In this paper we propose to follow up the work of Abdih and Tsangarides (2006) in two ways: (i) by analyzing the sensitivity of our results to the estimation method; and (ii) by estimating the REER of each individual WAEMU country. First, we analyze the robustness of our results by comparing the results of different econometric estimation methods—but not different theoretical models—in an attempt to examine the uncertainty surrounding our estimates and the size of any real misalignments. Specifically, we restrict the model to the fundamentals equilibrium exchange rate (FEER) approach based on Edwards (1989) and investigate the behavior of the WAEMU country REERs using alternative econometric methods. Second, we model the EREER for each WAEMU country using both single-country and panel-data econometric techniques in an attempt to address problems related to the estimation of long-run cointegrated relationships that have relatively few data points.

Our approach of providing a range of estimates for each country is similar to that of Egert and Lommatzsch (2003) and Egert (2004), who investigate the EREERs of Central and Eastern Europe accession countries; Saxegaard (2007), who analyzes the REER in Senegal; and Tsangarides (2007), who analyzes the WAEMU regional exchange rate. Chudik and Mongardini (2006) also apply panel data techniques to equilibrium real exchange rates for a large number of African countries. However, unlike them, we focus our analysis on a group of countries that have experienced some economic convergence in recent years. Because WAEMU member countries conduct their national economic policies in terms of criteria for clearly defined growth and convergence horizons, they can be considered more homogeneous and therefore more suitable to panel econometrics techniques.

Our results can be summarized as follows: First, we show that the proposed fundamentals—terms of trade, government consumption, productivity, investment, and openness—account for most of the fluctuation in the REERs. Second, the use of different econometric techniques suggests that there is significant uncertainty about the path of the underlying EREER in WAEMU countries and the associated degree of exchange rate misalignment. In particular, our results indicate that (i) because the results of using panel cointegration techniques employing data on all WAEMU countries often differ significantly from the single-country results, they may not, due to important differences between countries, provide an accurate picture; (ii) subtracting time means (in FMOLS) or including an impulse dummy for the 1994 devaluation (PMG) dramatically changes the estimated EREER for many countries, but not consistently. Third, due to the uncertainty introduced by the different econometric techniques, we cannot provide conclusive evidence as to whether the REER in any of the WAEMU countries is over- or undervalued; in some cases the conclusion about whether or not it is misaligned will depend on the importance one attaches to some or all of the EREERs derived using the panel data estimators. Finally, in terms of the policy implications, our analysis underscores the need for formal robustness analysis in modeling exchange rates (and the associated misalignments) and cautions that in using single-country and panel estimation techniques one should be mindful of the applicability and limitations of each technique.

The rest of the chapter is organized as follows: Section II presents some background, including a review of the literature and an overview of the FEER estimation and the FEER empirical model. Section III presents the robustness methodology and the motivation of the variables. Section IV presents the empirical results. Section V concludes.

II. Background

A. Literature Review

The literature offers a number of different approaches to calculating the EREER,[4] among them traditional uncovered interest parity (UIP) and purchasing power parity (PPP) theories and more recent approaches, such as the fundamental equilibrium exchange rate (FEER) approach, the underlying internal-external balance approach (UIEB), and the behavioral equilibrium exchange rate (BEER) approach.

The UIP and PPP arbitrage conditions are common starting points when analyzing movements in the exchange rate. The UIP condition is more informative in explaining the rate of change (or the adjustment path back to equilibrium) and not the level of the exchange rate. UIP by itself has not been successful at predicting exchange rate movements, partly because UIP estimation does not account for possible shifts in the equilibrium exchange rate. Along the same lines, the PPP theory predicts that price levels are equalized when measured in the same currency, which suggests that the real equilibrium exchange rate should be constant and equal to unity. However, empirical work on testing PPP (see, e.g., Rogoff, 1996, and MacDonald, 2000) is not very supportive of the theory, which suggests that alternative approaches are needed. To explain the persistence in real exchange rates, it is possible to combine the UIP and PPP and estimate a cointegrating relationship between relative prices, nominal interest rate differentials, and the nominal exchange rate (see, for example, Johansen and Juselius, 1992). This approach is known as the capital enhanced equilibrium exchange rate (CHEER) approach, which has produced higher speed-of-convergence estimates than other simple PPP models.

Another popular approach used to estimate equilibrium exchange rates is the underlying internal-external balance (UIEB) approach (also known as the macroeconomic balance approach). This approach defines the equilibrium real exchange rate as the rate that satisfies both internal and external balance. For the underlying balance to hold, planned output must equal aggregate demand (the sum of domestic demand and net trade), with the real exchange rate playing the role of relative price, which must move to equilibrate demand and supply. The most popular variants of the UIEB approach are the FEER approach of Edwards (1989),

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[4] Driver and Westaway (2004) provide a complete taxonomy of the different empirical approaches on equilibrium exchange rates estimation used in the literature.

Williamson (1994), and Wren-Lewis (1992); the desired equilibrium exchange rate (DEER); and the natural real exchange rate (NATREX) approach of Stein (1994).[5]

Finally, a method with a shorter time horizon is the behavioral equilibrium exchange rate (BEER) approach associated with Clark and MacDonald (1999). BEERs aim to use a modeling technique that captures movements in real exchange rates over time, not just movements in the medium- or long-term equilibrium level. Partly reflecting this, the emphasis in the BEER approach is largely empirical, with variables used to represent long-run fundamentals in the same way they would influence FEERs.

As Driver and Westaway (2004) emphasize, there is no one single definition of equilibrium exchange rate. The choice of approach depends on the question of interest, and in particular the time horizon. In addition to methodological issues (e.g., definition and measurement of the REER), the choice between approaches must therefore be judged in terms of the question of interest. Approaches may differ, for example, in the treatment of dynamics and the time frame they concentrate on.

B. The Theory Underlying the FEER

FEER is a well-recognized approach for calculating EREERs; it is particularly appropriate for assessing whether movements of the REER represent misalignments or whether the EREER itself has shifted as a result of changes in economic fundamentals. The FEER methodology is well suited to our analysis. First, traditional PPP theories and UIP cannot be used. Testing the validity of PPP reduces to testing whether the REER series is stationary; this is easily refuted in the data as the REER series contain a unit root, thus rejecting the PPP theory and its implications. Additionally, certain characteristics of the WAEMU economies (e.g., the importance of a single commodity export, capital account restrictions, and the lack of a forward foreign exchange market) affect the plausibility of interest rate parities and UIP. Second, the Edwards model was designed to describe nominal misalignments in fixed exchange rate regimes. Third, we believe that FEER model fundamentals represent more accurately the current situation of the WAEMU economies,[6] and the FEER period of analysis (medium to longer run) is most relevant to our analysis.

The Edwards dynamic model (1989) of a three-good (exportables, importables, and nontradables) small open economy with a fixed exchange rate provides a coherent method of

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[5] In the DEER, the theoretical assumptions are as in FEER but the external balance is based on optimal policy. The NATREX has a longer time horizon than the FEER and DEER and adds the assumption of portfolio balance.

[6] The equation that underpins the BEER analysis suggests that the REER set of fundamentals includes real interest rate differentials, terms of trade, net foreign assets and government debt, and the horizon of the analysis is shorter. Some of the BEER model fundamentals may have been more relevant for our sample of countries in the pre-1994 years (given the significantly high debt levels).

identifying the fundamental variables that are associated with the EREER.[7] The equilibrium exchange rate is defined as the exchange rate that results when internal and external equilibrium are attained simultaneously in an economy. Internal equilibrium is achieved when the market for nontradable goods clears in the present and is expected to clear in the future because price and wage flexibility ensure that the condition of internal balance (demand equal to supply) is satisfied. External equilibrium is achieved when the current account balance is "sustainable." Since only real factors (the fundamentals) can influence the EREER, the model can be used to describe nominal misalignments by separating factors that can affect the long-run EREER with permanent changes and short-run misalignments of the nominal exchange rate stemming from policy variables.

The instantaneous equilibrium in the nontraded goods market for given levels of some exogenous and policy fundamentals is:

$$e = e(\mathit{Terms\ of\ trade}, \mathit{Government\ spending}, \mathit{Trade\ controls}, \mathit{Technological\ progress}, \mathit{Investment})$$

The model predictions suggest the following expected signs for the fundamentals:

- *Terms of trade for goods.* The terms of trade affect the REER through the wealth effect. A positive terms of trade shock induces an increase in domestic demand, hence an increase in the relative price of nontradable goods, which leads to an REER appreciation. Alternatively, viewed from an internal-external balance angle, an increase in the terms of trade leads to an increase in real wages in the export sector and a trade surplus. To restore external balance the REER must appreciate and the expected sign is positive.

- *Government spending.* This is a proxy for government demand for nontradables. Changes in the composition of government spending affect the long-run equilibrium in different ways, depending on whether the spending is directed toward traded or nontraded goods. If government spending is primarily directed toward nontradable goods, an increase in government consumption will result in an appreciation of the REER; if spending is directed toward tradables, an increase in government consumption will cause a depreciation. The expected sign is ambiguous when government spending is not broken down by tradable and nontradable goods.

- *Openness.* This is a proxy for trade controls/restrictions. A reduction in controls will tend to increase the total amount of trade. The equilibrium response of the REER will depend on whether this leads to a deterioration or an improvement in the current

[7] The model is discussed in detail in Edwards (1989) and Williamson (1994). Cerra and Saxena (2002), Mathisen (2003), and Abdih and Tsangarides (2006) apply the Edwards methodology.

account. If the current account deteriorates, the REER must depreciate; the reverse is true if the current account improves. Hence, the expected sign is ambiguous.[8]

- *Technological progress/productivity.* This captures the Balassa-Samuelson effect. An increase in the productivity of tradables versus nontradables of one country relative to a foreign country raises relative wages in the first country. This increases the relative price of nontradables to tradables and thus causes REER appreciation. The expected sign is positive.

- *Investment.* Edwards suggests that including investment in the theoretical model results in supply-side effects that are dependent on the relative factor intensities between sectors, so the expected sign may a priori be ambiguous.

Predictions of the FEER model also suggest that capital controls may play an important role as a determinant of real effective exchange rates, as the liberalization of capital inflows increases present consumption (through the wealth effect), increases the demand for nontradables and hence leads to an appreciation of the REER in the short-run. However, given that (i) the long-run effect of reducing capital controls is ambiguous, (ii) choosing a variable that best represents sustainable or long-run capital flows has been a controversial issue in the literature, and (iii) capital inflows are small in the WAEMU region, we excluded this variable from the list of the fundamentals.[9]

III. Methodology and Data

A. Econometric Methodology

Considering that our objective is to examine the uncertainty surrounding the EREER and the extent of exchange rate misalignments, we use alternative econometric methodologies.[10] First, we apply single-country econometric techniques to each WAEMU country. Specifically, we estimate the long-run relationships with the Johansen (1988, 1991, and 1995) maximum likelihood approach and the autoregressive distributed lag (ARDL) approach of Pesaran, Shin, and Smith (2001). Then, given the traditional problems related to the estimation of long-run relationships and the cointegration analysis using short-term data,

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[8] Openness as a measure of trade restrictions is used by Hinkle and Montiel (1999), who define it as the ratio of the sum of exports and imports to GDP. Edwards uses two alternative measures (import tariffs as a ratio of tariff revenues and the spread between the parallel and official rates) that could have a different effect on the REER compared to the measure used by Montiel.

[9] See also the discussion in Cerra and Saxena (2002).

[10] This approach is similar to those of Egert and Lommatzsch (2003), and Egert (2004), who investigate the EREERs of Central and Eastern Europe accession countries. See also Chudik and Mongardini (2006) for an application to modeling EREERs in Africa.

we also compare the results of these single-country estimates to those derived using panel-data econometric techniques. By combining time series and cross-country information, the latter compensate for the brevity of the period. The panel-data techniques we draw on are the Pedroni (2000) group-mean fully modified ordinary least squares (FMOLS) approach and the pooled mean group (PMG) method proposed by Pesaran, Shin, and Smith (1999).

The choice between using a single country or a panel technique essentially reduces to a trade-off between two competing objectives. Single country time series estimation may suffer from shorter spans of data with associated less degrees of freedom and low power. Panel cointegration allows more variation in the data that could result in increased efficiency of the estimators. It may suffer, however, because the long-term parameters are either assumed to be identical or represent averages across the different members of the panel, neither of which is necessarily appropriate for every member. Our estimation methods are described below.

Single-country cointegration techniques

Johansen

The first methodology used to estimate the EREER for each country is the Johansen (1988, 1991, and 1995) maximum likelihood procedure. The procedure initially tests for the existence of a long-run cointegrating relationship between the exchange rate and its fundamentals. Next, the equilibrium levels of the fundamentals are computed by extracting the permanent component from the series. Then, the vector of long-run parameters and the extracted permanent component of the fundamentals are combined to calculate the EREER.

Define a vector of variables Y_t assumed to be in vector autoregressive form (VAR):

$$Y_t = \pi^0 + \sum_{i=1}^{p} \pi_i Y_{t-i} + \Psi D_t + \varepsilon_t \tag{1}$$

where Y_t is a $(n \times 1)$ vector:

$$Y_t = \begin{bmatrix} Real\ effective\ exchange\ rate_t \\ Terms\ of\ trade\ of\ goods_t \\ Government\ consumption_t \\ Investment_t \\ Technological\ progress_t \\ Openness_t \end{bmatrix}$$

π^0 is a (6×1) vector of deterministic variables; π_i is a (6×6) matrix of coefficients on lags of Y_t ; D_t is a vector of impulse dummy variables; p is the lag length; and ε_t is a (6×1)

vector of independent and identically distributed errors assumed to be normal with zero mean and covariance matrix $\mathbf{\Omega}$. The VAR thus comprises six equations; the right-hand side of each equation comprises a common set of lagged and deterministic regressors.

The VAR specification in (1) provides the basis for cointegration analysis. Adding and subtracting various lags of Y_t yields an expression for the VAR in first differences:

$$\Delta Y_t = \pi^0 + \pi\ Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \Psi\ D_t + e_t\,, \tag{2}$$

where Δ denotes the difference operator, $\Gamma_i = -(\pi_{i+1} + ... + \pi_p)$ is a (6×6) coefficient matrix, and $\pi \equiv \left(\sum_{i=1}^{p} \pi_i \right) - I$.

The rank of π determines the number of cointegrating vectors:

(i) If $rank(\pi) = 6$ or $rank(\pi) = 0$, then there is no cointegration among the variables. In this case, it is appropriate to estimate the model in levels (for $rank(\pi) = n$), and first differences (for $rank(\pi) = 0$).

(ii) If $0 < rank(\pi) \equiv r < 6$, then there are r cointegrating vectors and relationships. In this case, matrix π can be expressed as the outer product of two full-column rank ($6 \times r$) matrices α and β, where $\pi = \alpha\beta'$.

If the condition in (ii) is met, the VAR can be expressed as a vector error correction model (VECM):

$$\Delta Y_t = \pi^0 + \alpha\beta' Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + \Psi\ D_t + \varepsilon_t\ . \tag{3}$$

The matrix β' contains cointegrating vectors, and the matrix α has the weighting elements for the r^{th} cointegrating relation in each equation of the VAR. The matrix rows of $\beta' Y_{t-1}$ are normalized on the variables of interest in the cointegrating relations and interpreted as deviations from long-run equilibrium conditions. The α columns thus represent the speed of adjustment to long-run equilibrium.[11] The estimated vector β can be used to provide a measure of the EREER and also quantify the misalignment between the prevailing real exchange rate and its equilibrium level. The estimated α associated with the REER captures the speed at which the real exchange rate converges to equilibrium.

[11] If the coefficient is zero in a particular equation, that variable is considered to be weakly exogenous and the VAR can be conditioned on that variable. Variables found to be non-weakly exogenous jointly respond to move the system back to equilibrium.

ARDL

The second methodology used to estimate the EREER for WAEMU countries is the ARDL approach advanced by Pesaran and Shin (1999) and Pesaran et al. (2001). As Fosu and Magnus (2006) point out, the benefits of the ARDL methodology advanced by Pesaran and Shin (1999) and Pesaran, Shin, and Smith (2001) include: (i) the ARDL approach is relatively simple compared to the Johansen methodology: Once the existence of a cointegrating relationship has been established and the appropriate number of lags has been determined, the model can be estimated by OLS; (ii) unlike most other approaches to cointegration, the ARDL approach does not require pretesting for the existence of a unit root in the variables, a requirement that introduces additional uncertainly into the analysis. As Pesaran, Shin, and Smith (2001) point out, the ARDL method of testing for a relationship between the variables can be applied whether or not the underlying regressors are purely I(0), purely I(1), or cointegrated; and (iii) the underlying model is a single-equation ARDL, rather than a VAR as in the Johansen methodology, thus reducing the number of parameters to be estimated and improving efficiency in small samples (as in this study).

Consider the following ARDL$\left(p, q_1, \ldots, q_K\right)$ model:

$$\phi\left(L, p\right) y_t = c^0 + \sum_{i=1}^{K} \delta_i \left(L, q_i\right) x_{it} + \Psi\, D_t + \varepsilon_t^y \tag{4}$$

where L is the lag operator and we have partitioned the $(n \times 1)$ vector Y_t in equation 1 into the dependent variable y and fundamentals x_{it}.

The ARDL procedure has two stages. In the first, we set out to ascertain whether there is a long-run relationship between the variables by testing for the significance of the lagged levels of the variables in the error-correction form of the underlying ARDL model in equation (4):

$$\Delta y_t = c^0 - \phi_1 y_{t-1} + \sum_{i=1}^{K} \theta_i x_{it} - \sum_{j=1}^{p-1} \psi_j \Delta y_{t-j} + \sum_{i=1}^{K} \sum_{j=0}^{q-1} \varpi_{ij} \Delta x_{it-j} + \Psi\, D_t + \varepsilon_t^y \tag{5}$$

The test for the absence of a level relationship in the error-correction model (ECM) is a test for the joint hypothesis that $\phi_1 = 0$ and $\theta_i = 0$ for all i in equation (5).[12] However, this is complicated by the fact the asymptotic distribution of the *F*-statistic is nonstandard and will

[12] Note that we assume that contemporaneous effect Δx_{it} is not correlated with the disturbance term ε_{yt} so that instrumental variable estimation is not necessary and the ECM can be consistently estimated by OLS. For details see Pesaran, Shin, and Smith (2001).

depend on whether the regressors are I(0) or I(1). Pesaran, Shin, and Smith (2001) have tabulated two sets of critical values. The first assumes that all the variables in the ARDL model are I(1) and the second that they are all I(0). This gives a band of critical values that covers all possible classifications of the variables into I(0), I(1), or even fractionally integrated. If the computed *F*-statistic falls outside this band, a decision about the existence of a level relationship can be made without knowing whether the underlying variables are I(0) or I(1). If the *F*-statistic falls within this band, the results are inconclusive and will depend on whether or not the variables are stationary.

Once the existence of a relationship in levels between the variables has been confirmed, the second stage of the procedure involves estimating the ARDL model in equation 4 using OLS. However, whereas in the first stage it is important that the lagged changes of the ECM are unrestricted to avoid any pretesting problem, the use of a more parsimonious specification is advisable in the second stage. We therefore follow the approach discussed in Pesaran and Shin (1999) of searching across different combinations of lag-lengths for each of the underlying repressors in equation 4 using the Schwarz Bayesian criterion (SBC). The vector of long-run parameters can then be retrieved by rewriting the estimated ARDL model as an ECM.

Multicountry panel techniques

Over the past decade, the literature on nonstationary panels has expanded rapidly (see Banerjee, 1999, and Breitung and Pesaran, 2005). Contrary to earlier nonstationary panel techniques, which have traditionally been applied to micropanels—characterized by short periods of time and large cross-sections—the recent literature focuses on techniques that can be applied to macroeconomic data for numerous countries—characterized by substantial time series and moderate cross-sections. These techniques have been appealing relative to single-country cointegration techniques of the kind discussed because the cross-sectional variation in the data provides more freedom and more efficiency when estimating the models, and therefore helps overcome the low power of time series unit root tests and cointegration analysis.

The attractiveness of these nonstationary panel techniques has also increased with the ability of more recent estimators to model cross-sectional heterogeneity. This relates to the issue of imposing (or not imposing) slope homogeneity. Group-mean estimators imply estimating the parameters country by country and then averaging them—essentially taking the long-run cointegrating vectors (in addition to the short-run dynamics) to be heterogeneous. In contrast, pooled approaches treat the parameters of the cointegrating vector as common across countries, thus maximizing freedom. In this paper, we use approaches that allow for flexibility related to cross-country heterogeneity. In particular we employ the pooled mean group (PMG) estimator, which allows for heterogeneity in the short-run dynamics but assumes that the cointegrating vector is homogenous across countries, and the group-mean fully modified OLS (GM-FMOLS) estimator, which allows for heterogeneity of both the long-run cointegrating vector and short-run dynamics. The next two sections outline these two estimators.

The Group-Mean Panel Fully Modified OLS (GM-FMOLS)

The starting point for GM-FMOLS is the static equation

$$y_{it} = a_i + \sum_{j=1}^{K} \beta_j x_{jit} + \varepsilon_{it} \tag{6}$$

for $i = 1,...,N$ countries where y_{it} is the dependent variable, a_i is a country specific intercept, and x_{it} is a vector of K fundamentals. This corresponds to an Engle-Granger-type approach, which for a single country can be estimated consistently using OLS if there is cointegration between the dependent variable and the K fundamentals. In a nonstationary panel, however, Pedroni (2000) demonstrates that OLS is asymptotically biased while the distribution of the parameters is data-dependent (due to the presence of nuisance parameters associated with serial correlation of the data) unless the regressors are assumed to be exogenous and the dynamics homogenous across countries.

The GM-FMOLS panel technique was proposed by Pedroni (1996, 2000) to deal with these issues in a way that allows for heterogeneity in the cointegrating vectors. As discussed in Maeso-Fernandez, Osbat, and Schnatz (2004), the FMOLS takes into account the presence of the constant term and the possible correlation between the error term and the differences of the regressors. Adjustments are made to the dependent variable and then to the estimated long-run parameters obtained from regressing the adjusted dependent variable on the regressors. In the panel setting, the GM-FMOLS long-run coefficients are obtained by averaging the group estimates over N.

The form of the GM-FMOLS estimator is outlined in Pedroni (1996, 2000);

$\hat{\beta}^{GM-FMOLS} = N^{-1} \sum_{i=1}^{N} \left[\left(\sum_{t=1}^{T} x_{it}' x_{it} \right)^{-1} \left(\sum_{t=1}^{T} x_{it}' y_{it}^{*} - T\hat{\lambda}_i \right) \right]$, where y_{it}^{*} are the regressands adjusted for the covariance between the error term and the x_{it}, and $T\hat{\lambda}_i$ is the adjustment for the presence of the constant. The term in the brackets is the individual FMOLS estimator for the K fundamentals.

The advantages of this method include robustness to endogenous regressors and to many forms of omitted variables and measurement errors. It also allows for heterogeneity of the long-run parameters between countries, in which case the estimated parameters can be interpreted as the mean values of the cointegrating vector. Using this approach, the existence of a cointegration relationship can be tested formally.

The Pooled-Mean Group (PMG) Estimator

The pooled-mean group estimator (PMG) proposed by Pesaran, Shin, and Smith (1999) constrains the long-run coefficients to be identical in an error correction framework but allows the short-run coefficients and error variances to differ.

The basic PMG estimator involves estimating an ARDL model of order (p_i, q_i):

$$\Delta y_{it} = \phi_i y_{it-1} + \beta_i x_{it} + \sum_{j=1}^{p_i - 1} \psi_{ij} \Delta y_{it-j} + \sum_{j=0}^{q_i - 1} \delta_{ij} \Delta x_{it-j} + a_i + \varepsilon_{it} \tag{7}$$

where y_{it} is the dependent variable, x_{it} is a vector of explanatory variables, a_i represents country-specific intercepts, ψ_{ij} and δ_{ij} are the country-specific coefficients of the short-term dynamics, and ε_{it} is an error term. The long-run coefficients are assumed to be the same for all countries.[13] If ϕ_i is significantly negative, there is a long-run relationship between y_{it} and x_{it}.

The PMG approach is essentially a panel version of the ARDL procedure and consists of a maximum likelihood estimation of an ARDL model, which can be rewritten as an error correction model (ECM). Estimation of this model involves both pooling (within-dimension) and averaging (between-dimension). Pesaran, Shin, and Smith (1999) do not propose a formal cointegration test but derive asymptotic properties of the estimator for both stationary and nonstationary regressors.

Relative to GM-FMOLS the PMG estimator is a maximum likelihood estimator while GM-FMOLS, as its name suggests, is a modified OLS estimator. In principle, FMOLS requires fewer assumptions and tends to be more robust.[14] In particular, Pedroni (2000) finds that GM-FMOLS estimators have satisfactory size and power properties even for small panels, as long as T is larger than N. Pedroni also notes that even when the cointegrating vector is homogeneous, mean-group estimators have better small sample performance than within-group estimators. Because the PMG estimator imposes long-run homogeneity, it can also produce inconsistent estimates of the average values of the parameters if the assumption of homogeneity is violated in practice—see, for instance, Pesaran and Smith (1995). However,

[13] Equation (7) can also be estimated with individual specific long–run coefficients which can be then averaged to obtain an Group Mean estimator. The two estimators can then be used to test for slope homogeneity using the Hausman test.

[14] For more details on the choice of panel methods see Maeso-Fernandez, Osbat, and Schnatz (2004).

if the long-run cointegrating vector is homogenous across countries, GM-FMOLS estimates are inefficient relative to PMG.

B. Data

As noted, we employ the following variables in our analysis of the EREER: the natural logarithm of the real effective exchange rate (LREER), the natural logarithm of terms-of-trade (LTTT), the natural logarithm of government consumption as a share of GDP (LCGR), the natural logarithm of real GDP per capita relative to trading partners (LPROD) to capture the Balassa-Samuelson effect, the natural logarithm of openness to GDP (LOPEN), and the natural logarithm of investment to GDP (LNIR). Dummy variables were used to capture the effect of the 1994 devaluation and the presence of outliers.[15] The dataset consists of annual observations for 1970 through 2005.[16] The REER and the fundamentals employed in the empirical analysis are plotted in Figures C1 to C7 (Rename to B1-B7).

Some interesting patterns in the REER and the fundamentals are worth highlighting. First, member country REERs vary significantly around the WAEMU regional average. Benin's REER had appreciated the most (97 percent of the pre-devaluation level) at the end of 2006 and Mali's the least (67 percent). In addition, there is continued divergence in WAEMU member REERs, particularly in the latter period.[17] This could stem from the high degree of market separation, or, said differently, the lack of sufficient economic integration in the region.[18] Further, our analysis suggests that large differences in country specific REERs have been the driven primarily by different domestic CPI trends rather than NEER differences (e.g. different partner-country weights), suggesting a market driven real exchange rate adjustment. Nevertheless, it appears that there is important country-specific information about the individual countries' REERs that simply a regional average does not capture.

As expected, terms of trade patterns have been quite volatile in the region: in general, since devaluation, there have been net declines for Burkina Faso, Mali, Niger, and Togo, but terms of trade remained roughly constant for Benin, Côte d'Ivoire, and Senegal. Also, since about

[15] Impulse dummies are included as required by the data but are constrained to lie outside the cointegrating space. For the single-country estimates the dummy variables employed in the univariate ARDL method are, with a few exceptions, a subset of those used in the multivariate Johansen method. In the panel estimation, we employ an impulse dummy to capture the devaluation in 1994 in one version of the PMG model. Further details are available from the authors upon request.

[16] There are no missing observations and the panels are balanced. More details on the variable definitions and sources are presented in Appendix A.

[17] See Tsangarides (2007) for more details on the estimation of the measure of the dispersion.

[18] By comparison, our analysis suggests that EU countries' REERs have been converging. For example, looking at the REERs of the group of the 15 EU countries (i.e. those that joined prior to 2004), there has been a trend reduction in the standard deviation of their REER changes since 1980, suggesting convergence.

the mid-1990s the ratio of government consumption to GDP has increased for Benin, Côte d'Ivoire, Mali, and Senegal; remained roughly constant for Burkina Faso; and declined for Niger and Togo. For all WAEMU countries (and the regional average), there was a persistent decline in real GDP per capita with respect to trading partners from the mid-1970s to the end of the sample period. Moreover, for the majority of the countries in the region—Benin, Mali, Niger, Senegal, and Togo—there have been some increases in the investment to GDP ratio since about the mid-1980s, but the ratio remained roughly constant in Burkina Faso and Côte d'Ivoire. Finally, there have been improvements in all countries in openness since the mid- to late 1990s, particularly for Côte d'Ivoire, Mali, Niger, Senegal, and Togo.

It is worth noting that while the movement in each countries' REER is clearly dominated by the 1994 devaluation, we do feel that an econometric analysis of the EREER is possible. In particular, studies (see inter alia Johansen, Mosconi, and Nielsen (2000)) suggest that the cointegrating framework employed in this paper is robust to structural breaks as long as these are captured by dummy variables. Moreover, we do not believe that the 1994 devaluation affected the underlying equilibrium relationship between the REER and its fundamental determinants. Hence, while the 1994 devaluation clearly had a large impact on the REER, we do not believe it directly affected the EREER. This is likely to facilitate the modeling of the EREER. Finally, we draw confidence from the fact that numerous studies have successfully analyzed the REER in WAEMU countries, obtaining results that are both statistically and economically meaningful.

It should be noted, however, that all econometric studies of real exchange rates in SSA are likely to suffer from data limitations. This is particularly true with respect to the pre-1980 data and the data on the REER. It is well-known that the pre-1980 data has in some instances been constructed using extrapolation techniques. The REER data also suffers from a number of limitations. In particular, the REER relies on data for the consumer price index (CPI), which for many countries reflects largely import prices and developments in the agricultural sector, and may therefore not provide a good measure of price developments in the economy as a whole. In addition, it is well known that the REER for several countries does not include data on some of its most important trading partners.[19] Finally, it is important to note that the CPI based REER may not provide an accurate picture of competitiveness. In particular, Saxegaard (2007) notes that the CPI may understate movements in competitiveness as consumer prices include the price of imported final goods, which tend to reflect movements in the nominal exchange rate.[20] While we recognize these data limitations, their importance is difficult to judge ex-ante. In addition, we do not consider them to invalidate our analysis given that our objective is primarily to investigate the sensitivity of estimates of the EREER

[19] This is the case, for example with Niger and Benin where the REER does not include data on Nigeria.

[20] Alternative measures of the real exchange rate are typically not available for the countries in our sample, however.

to different estimation techniques. Nevertheless, it is clear that interpretation of the country results should take into account the data limitations referred to above.

Unit root tests

Before estimating the long-run real exchange rate equations, we test for the presence of unit roots on all the series used in the analysis. Nelson and Plosser (1982) find that many macroeconomic and aggregate level series are shown to be well modeled as stochastic trends, i.e., integrated of order one, or *I(1)*. To mirror the different steps of our econometric analysis—single-country followed by a panel approach—country-by-country augmented Dickey-Fuller (ADF) tests and panel unit root tests are systematically conducted.

Country-by-country unit root tests are conducted using the ADF tests in both levels and first differences of the variables of interest. The results for each country are provided in Table B1. The t-ADF statistics are reported in the second column for each country, and the p-values are reported in the third.[21] We cannot reject the null hypothesis of a unit root for any of the variables in levels, but we strongly reject the null of a unit root in first differences. Hence, we conclude that all our variables are *I(1)* in levels or, equivalently, stationary in first differences.[22] Further, because the presence of structural breaks may bias traditional unit root tests towards finding evidence of nonstationarity, we supplement the ADF tests with the Zivot and Andrews (1992), and the Clemente, Montanes, and Reyes (1998) tests. These two tests allow for a grid search over possible structural break points—the latter test even allows for the (realistic) possibility of more than on break in the series—and then conduct a Dickey-Fuller type test conditional on the series inclusive of the estimated breaks(s).[23] Broadly speaking, both tests confirm that the LREER series for each country are nonstationary.

As noted by Maeso-Fernandez, Osbat, and Schnatz (2004), relative to single-country tests panel unit root tests have the advantage of increasing the power of the test by exploiting the cross-sectional as well as time-series information in the data. In this paper we carry out the ρ and $t-\rho$ tests proposed by Levin, Lin and Chub (2002) ADF test; all three tests are based on pooled within-dimension estimators and hence assume that the parameters of interest are homogeneous across countries. We also employ the ADF-t panel unit root test proposed by

[21] The appropriate lag-length for the dependent variable in each test, chosen using the Schwarz Information Criterion.

[22] The ADF test results in Table B2 are based on a specification with a constant term included (see notes for Table B2). We experimented with specifications that include both a constant and a deterministic time trend. The results are virtually unchanged from those reported in Table B2.

[23] In particular, the Zivot and Andrews (1992) test allows for a one-time break in the intercept and/or the trend of the series; the Clemente, Montanes, and Reyes (1998) test allows for one and/or two breaks in the series and sudden or gradual changes in the means of the series. Results from these tests are available from the authors.

Im, Pesaran and Shin (2003), which is based on a group mean estimator and thus treats the parameters of interest as heterogeneous between members. The results are presented in Table B2. Except for LNIR and LOPEN, which appear to be stationary in levels, the majority of the panel unit root tests employed suggest that all series are nonstationary in levels and stationary in first differences.[24]

Decomposition of the fundamentals

Measuring the degree of misalignment requires constructing an unobserved variable, the EREER, which requires decomposing the fundamentals into permanent and transitory components. In particular, the EREER is defined as that value of the REER which is consistent in the long-run with the *equilibrium* value of the fundamentals. As is common in the literature, we construct a measure of the equilibrium by extracting the permanent component of each fundamental.[25] In this paper we apply the Hodrick-Prescott (HP) (1997) filter, which has become a popular choice among business cycle analysts. Given that our analysis requires comparing three different estimation techniques, to ensure consistent comparison across the estimation methods we apply the HP filter to each of the fundamentals.[26]

IV. Empirical Results

A. Integration Analysis

The results in the previous section suggested that most of the fundamentals were integrated of order one and should therefore be modeled in a suitable econometric framework in order to avoid drawing conclusions based on spurious results. The unit root tests showed that all of the variables were stationary in first differences so that simple first differencing of the data would remove the nonstationarity problem. However, this would entail discarding information about any long-run "equilibrium" relationship among the variables, in particular the relationship between the REER and its fundamentals. Before turning to our estimation results, therefore, we first test for the presence of cointegration between the REER and the fundamentals in both a single-country and a panel-data environment.

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[24] The fact that one or more fundamentals are stationary in levels is not of immediate concern as long as there is a cointegrating relationship between the REER and the nonstationary fundamentals.

[25] Simply applying HP smoothing to the REER to get an estimate of the EREER is a fundamentally flawed approach, not least because it does not take advantage of the information resulting from the cointegration relationship and the interaction of the fundamentals in determining the EREER.

[26] The Gonzalo-Granger (GG) (1995) decomposition may be an alternative (and perhaps more theoretically appealing) approach; however, it can only be used with the Johansen methodology (and hence, precluded its use for our analysis). For applications of GG, see for example, Cerra and Saxena (2002), Maeso-Fernandez, Osbat, and Schnatz (2002), and Abdih and Tsangarides (2006).

The country-by-country cointegration tests are reported in Table B3. The top part of each panel presents the results of the Johansen test. The null hypothesis that there are zero cointegrating vectors versus the alternative that there are more than zero is soundly rejected for all countries. Furthermore, the null that there is at most one cointegrating vector versus the alternative that there are more than one is, with a few exceptions, not rejected at the 5 percent significance level.[27] The ARDL bounds test for cointegration for each country is reported in the bottom part of each panel. For WAEMU and four of the seven countries the results confirm cointegration between the variables at the 5 percent significance level. For the other three countries the results are ambiguous at the 5 percent level, but at the 10 percent level the bounds tests do suggest cointegration. We proceed on this basis.[28]

Several different tests for cointegration in nonstationary panels have been proposed in the literature. In this paper, we apply the seven tests proposed by Pedroni (1999) based on the null hypothesis of no cointegration; they have several benefits relative to other popular alternatives.[29] In particular, Maeso-Fernandez, Osbat, and Schnatz. (2004) note that the Pedroni tests allow for heterogeneous variances and some dependence between countries. Three of the tests (the "group" tests) are based on pooling along the between-dimension and thus allow for heterogeneous slope coefficients. The other four are based on pooling along the within-dimension.

The results are presented in Table B4 for raw and time-demeaned data. The null of no cointegration is rejected in three out of seven tests when raw data are used and in four out of seven cases when time-demeaned data are used. It is worth noting, however, that two of the group tests that allow for some degree of heterogeneity indicate cointegration among the variables. The one that does not suggest cointegration (the group ρ statistic) is well known to be grossly undersized (see, e.g., Maeso-Fernandez, Osbat, and Schnatz, 2004). We proceed therefore with estimation of the long-run parameters using the nonstationary panel techniques described.

B. Single-Country Estimation

The results using the ARDL and Johansen single-country approaches are consistent with predictions from economic theory and with earlier analysis. The estimated elasticities of the cointegrating relation for the ARDL and Johansen techniques and their t-statistics (in

[27] We report results of two cointegration tests, trace and max-eigenvalue. Both tests fail to reject "at most one vector" except in two cases: (i) Senegal where the trace (max-eigenvalue) test suggests that there may be two (one) cointegrating vector(s), and (ii) Niger where the trace (max-eigenvalue) test suggests that there may be one (zero) cointegrating vector.

[28] The 10 percent critical values are not reported in Table B3. The relevant lower bound is 2.65 and the upper bound 3.35.

[29] See inter alia Kao (1999) and McCoskey and Kao (1998).

brackets) are presented in Table 1. The top part of the table presents the estimated long-run relationship for each country; the bottom part of the table shows the estimate of the quantitative importance of the error correction term, which determines the speed at which the system moves back to equilibrium after deviations from long-run equilibrium. The bottom part of the table also reports the implied half-life of deviations away from equilibrium. While the diagnostic tests reported in Table B5 identify a few isolated issues, the estimated models generally appear to be well-specified.

Benin

The single-country estimation results for Benin are relatively consistent across methodologies and with the predictions from economic theory. They suggest that (i) the terms of trade have a significant positive effect on the REER using both the ARDL and the Johansen methodologies, indicating that an improvement in the terms of trade would result in an appreciation of the long-run EREER through a possible wealth effect; (ii) government consumption has a significant positive (appreciating) impact on the REER, suggesting that most government spending is directed toward nontradables—the effect is particularly strong and significant in the model estimated using the Johansen technique; (iii) technological progress, proxied by relative real GDP per capita, has a significant positive long-term impact on the REER, thus confirming the Ballasa-Samuelson effect; (iv) investment has a significant positive effect on the REER in both the ARDL and the Johansen model, suggesting that investment in Benin moves spending toward nontradable goods; and (v) openness does not seem to have a significant effect on the REER.

Both estimation methodologies suggest that the error correction term is negative and significantly different from zero, suggesting that the error correction mechanism is stable. Surprisingly, however, the implied half-life of deviation is more than twice as long in the model estimated using the Johansen method relative to that using the ARDL model. The difference between the implied half-lives calculated using the ARDL and the Johansen approaches is replicated across countries and is consistent with previous findings by Saxegaard (2006) and Tsangarides (2007).[30] In addition, within the multivariate Johansen

[30] In comparison to other studies, the overall adjustment speeds estimated in our paper appear on the high side. Mathisen (2003), and Cashin, Cespedes and Sahay (2003) estimate an adjustment speed with a half-life of less than a year for Malawi; MacDonald and Ricci (2003) estimate a half-life of 2 to 2.5 years for South Africa; Rogoff (1996) estimates the longer half-life of 3 to 5 years. While an analysis of the causes underlying the differing speeds of adjustment across our sample is clearly of interest, it is beyond the scope of this paper.

Table 1. Results of Cointegration Estimation[1,2]

	Benin		Burkina Faso		Côte d'Ivoire		Mali		Niger		Senegal		Togo		WAEMU	
	Johansen	ARDL	Johansen	ARDL	Johansen	ARDL	Johansen	ARDL	Johansen	ARDL	Johansen	ARDL	Johansen	ARDL	Johansen	ARDL
Estimates of the Long Run Coefficients																
ln(terms of trade)	1.03 ***	1.01 ***	1.96 ***	0.82 **	0.47 ***	0.40 *	0.56 **	-0.93	0.48 ***	0.36 **	0.38 ***	0.53 ***	-0.08	-0.15	0.25 *	0.31 ***
	[8.71]	[3.97]	[6.56]	[2.60]	[5.18]	[1.93]	[2.45]	[-1.20]	[4.37]	[2.29]	[2.94]	[3.15]	[-1.54]	[-1.43]	[1.92]	[3.76]
ln(government consumption)	1.59 ***	0.59 *	1.08 ***	0.51 ***	0.33 ***	0.01	0.60 ***	0.09	-0.20	-0.23	0.37 **	0.51 **	-0.46 **	-0.86 **	0.78 ***	0.59 ***
	[7.12]	[1.87]	[8.42]	[2.89]	[4.48]	[0.08]	[4.31]	[0.27]	[1.57]	[-1.44]	[2.58]	[2.50]	[2.87]	[-2.20]	[9.31]	[15.32]
ln(technological progress)	1.27 ***	0.85 ***	3.08 ***	0.56	0.78 ***	1.21 **	1.22 ***	2.23 *	0.60 ***	0.74 ***	1.25 ***	0.78 ***	0.39 ***	0.86 **	0.17 **	0.22 ***
	[11.36]	[3.48]	[4.52]	[0.69]	[4.09]	[2.07]	[5.50]	[1.92]	[6.38]	[4.40]	[8.29]	[3.11]	[2.76]	[2.54]	[2.23]	[5.30]
ln(investment)	0.56 ***	0.34 **	-2.16 ***	-1.06 ***	-0.82 ***	-1.00 **	-0.55 ***	-0.55 ***	0.08	-0.02	-0.54 ***	-0.21 *	0.29 ***	0.23 **	-0.43 ***	-0.58 ***
	[3.74]	[2.10]	[-5.78]	[-3.0]	[6.79]	[-2.39]	[-5.65]	[-3.22]	[1.02]	[-0.37]	[-6.59]	[-1.73]	[5.74]	[2.34]	[-4.40]	[-8.74]
ln(openness)	-0.23	-0.10	2.54 ***	0.62	0.82 ***	0.96 *	0.27	-1.68 *	-0.34 **	-0.36 *	0.24 **	-0.04	-0.33 ***	-0.14	0.16	0.23 ***
	[-1.48]	[-0.55]	[4.22]	[1.19]	[4.36]	[2.04]	[0.99]	[-1.79]	[-2.36]	[-1.79]	[2.04]	[-0.26]	[-4.52]	[-1.04]	[1.47]	[3.70]
Constant	-4.77	-2.04	-9.09	0.70	0.35	1.37	1.04	16.63	3.95	4.85	2.70	1.78	6.75	7.30	2.24	2.61
Estimates of the ECM coefficients																
D[ln(real effective exchange rate)]	-0.09 **	-0.23 ***	-0.07 **	-0.33 ***	-0.18 ***	-0.29 **	-0.12 **	-0.15 *	-0.29 ***	-0.41 ***	-0.19 **	-0.33 ***	-0.23 *	-0.55 ***	-0.13 **	-1.05 ***
	[-2.27]	[-3.22]	[-2.04]	[-4.04]	[-3.06]	[-2.55]	[-2.26]	[-1.84]	[-3.88]	[-6.44]	[-2.08]	[-4.68]	[-1.94]	[-4.00]	[-2.01]	[-6.20]
Half-life of deviation	7.4	3.1	10.6	2.1	3.9	2.4	5.9	4.6	2.4	1.7	3.6	2.1	3.0	1.3	5.3	0.7

Notes:
[1] Three, two, and one asterisks,denote statistical significance at the 0.01, 0.05, and 0.10 levels, respectively; t-statistics in brackets.
[2] The speed of adjustment is derived from the error correction model.

framework the adjustment coefficient of the cointegrating vector is estimated to be significantly different from zero in more than one equation. This suggests that the fundamentals associated with these equations are not weakly exogenous with respect to the parameters of the cointegrating relationship, implying that in the face of a deviation from the long-run equilibrium these variables jointly respond to move the system back to equilibrium

Burkina Faso

The results for Burkina Faso suggest that (i) terms of trade and government consumption have a significant positive effect on the REER, although these two effects are much stronger when the Johansen methodology is used; (ii) technological progress and openness have a positive effect on the REER in the Johansen model but are both insignificant in the ARDL model; and (iii) investment has a negative effect on the REER, suggesting that, unlike our findings for Benin, an increasing share of investment in Burkina Faso shifts spending toward tradable goods in the long run. Although significant across both methodologies, this effect is much stronger in the Johansen model. Finally, while the estimates of the error correction term confirm that the mechanism is stable in both models, the implied half-life differs significantly, as was the case for Benin.

Côte d'Ivoire

In Côte D'Ivoire, the estimated effect of fundamentals on the REER in the long run is consistent across econometric techniques and with the results for Burkina Faso. However, increased parameter uncertainty implies that the effect of the fundamentals is not as significant statistically in the ARDL model as in the Johansen model. This is particularly true for government consumption, which has a significant positive effect on the REER in the model estimated using the Johansen method but is insignificant in the ARDL model. The estimated error correction terms confirm the stability of the mechanism and imply that half of any deviation away from equilibrium is corrected in two and a half to four years, depending on methodology.

Mali

The econometric methods give significantly different results for Mali. In particular, while terms of trade and government consumption have a significant positive effect on the REER in the Johansen model, we are unable to detect any significant effect of these two fundamentals in the ARDL model. The opposite is the case with openness, which has a negative and significant effect on the REER in the ARDL model, suggesting that increased openness leads to a deterioration of the current account, but it is insignificant in the Johansen model. As was the case for Burkina Faso and Côte D'Ivoire, investment has a significant negative effect on the REER and the estimated parameter for technological change confirms the Balassa-

Samuelson effect. The error correction terms confirm that both models are stable and suggest a half-life of deviation of four and a half to six years.

Niger

Our estimation results from Niger are consistent for both methodologies. As was the case for Mali, in the ARDL model we find evidence that increasing openness puts pressure on the REER to depreciate in the long run. Otherwise, we are unable to find any significant effect of investment or government consumption on the REER in the long run, though terms of trade and technological progress have the expected significant positive effect. The results indicate that the model is stable with a half-life of deviation of one and a half to two and a half years—relatively short compared to the previous results.

Senegal

As with Niger, the single-country results for Senegal are relatively similar for both econometric techniques. With the exception of openness in the ARDL model, all the fundamentals have significant effects on the REER in the long run; this is consistent with economic theory. Quantitatively, results for Senegal are similar to those for Côte d'Ivoire: They suggest that the model is stable and the half-life of deviation is comparable to that of Côte d'Ivoire.

Togo

The results for Togo differ from those for the previous countries in that we are unable to find any significant effect of terms of trade on the REER in either of the two single-country models. Moreover, an increase in government consumption leads to a depreciation of the REER in both models, suggesting that, unlike in previous countries, an increase in government consumption moves spending toward tradable goods. Otherwise, productivity and investment have a significant positive effect on the REER and increasing openness is associated with a depreciation of the REER. However, the latter is not significant in the ARDL model. The error correction terms suggest that both models are stable with half-lives comparable to those of Niger.

WAEMU

Our final set of single-country estimation results is based on data aggregated for WAEMU as a whole. As one would expect, the results are consistent with the results for individual countries. In particular, (i) the terms of trade have a positive and significant effect on the REER, as was true for every country except Mali; (ii) an increase in the share of government consumption shifts spending toward the nontradable sector, putting pressure on the REER to appreciate, as was true for all countries except Niger; (iii) our results for WAEMU, as was true for every country confirm the positive long-term impact of technological progress on the REER, which is consistent with the Balassa-Samuelson hypothesis; (iv) an increased share of investment expenditure shifts spending toward tradables, as was true for the majority of its members; and (v) increasing openness has a positive effect on the REER in the long run, as in Burkina Faso and Côte D'Ivoire (although this effect is only significant in the ARDL model). In addition to being stable, the results suggest a half-life of deviation between six months and five years, depending on methodology.

C. Multicountry Panel Estimation

The results from the multicountry panel estimation are reported in Table 2. The first two columns report the results from estimating the nonstationary panel using GM-FMOLS and the final two columns report the results from estimating the panel using PMG. The two columns for each estimator differ in the extent to which they correct for time-effects that are common to countries (e.g., the 1994 devaluation). The first set of results for each estimator uses the uncorrected data and the second set uses either common time dummies and time-demeaned data (FMOLS) or an impulse dummy for 1994 (PMG).

Table 2. Results of Panel Estimations[1,2]

	FMOLS	FMOLS[3]	PMG	PMG[4]
Estimates of the Long Run Coefficients				
ln(terms of trade)	0.17 ***	0.16 ***	0.59 ***	0.31 ***
	[4.25]	[6.22]	[4.46]	[3.38]
ln(government consumption)	0.26 ***	-0.06	0.11	0.20 *
	[5.26]	[-0.53]	[1.09]	[1.86]
ln(technological progress)	0.23 ***	0.34	1.03 ***	0.80 ***
	[8.63]	[0.53]	[4.71]	[7.64]
ln(investment)	-0.06 **	-0.04 ***	-0.78 ***	-0.01
	[-2.18]	[-2.16]	[-5.18]	[-0.2]
ln(openness)	-0.25 ***	-0.04	0.97 ***	-0.07
	[-3.82]	[-0.94]	[4.26]	[-3.82]

Notes:

[1] Three, two, and one asterisks,denote statistical significance at the 0.01, 0.05, and 0.10 levels, respectively; t-statistics in brackets.

[2] For PMG estimates, ARDL lags selected wih SBC criteria (max. 3).

[3] With time demeaned variables.

[4] With 1994 dummy included.

Both sets of GM-FMOLS results suggest that productivity has had a significant positive effect on the REER and investment a significant negative impact, suggesting that, on average, an increase in the investment-to-GDP ratio shifts spending toward tradable goods. Similarly, both government consumption and openness have significant effects on the REER in the GM-FMOLS model with uncorrected data but not in the model with time-demeaned data.

The PMG results are similar to the GM-FMOLS results. Terms of trade and productivity are positive and significant in both the model with a dummy for 1994 and the model without; government spending has a positive significant effect in the former but not the latter. The opposite is true for investment and openness, which has a significant effect on the REER in the model with uncorrected data but not in the model corrected for the 1994 devaluation. However, contrary to the results from the GM-FMOLS model, the PMG model using uncorrected data suggests that increasing openness has a significant positive effect on the REER. The error correction term in the PMG model suggests a half-life of two and a half to five years, which is comparable to our single-country results.

One of the strengths of the PMG framework is the fact that it allows formal testing, using a Hausmann test, of whether the assumption of long-run homogeneity across countries is valid. Conducting this test parameter by parameter produces mixed results for this hypothesis, suggesting that while the long-run effect of some fundamentals on the REER, such as productivity and openness, seems to be similar across countries, the long-run effect of parameters such as terms of trade is not.[31] This puts into question the validity of using the PMG framework, which assumes that the cointegrating vector is homogeneous for all countries. GM-FMOLS, however, does not suffer from this weakness. It allows us to relax the assumption of long-run homogeneity, but because our GM-FMOLS results are an average of the individual cointegrating vectors, the fact that we reject homogeneity in the PMG frameworks suggests that for any given country the actual GM-FMOLS cointegrating vector may be significantly different from the average. This in turn may lead to difficulties when we employ these results to analyze country-by-country REER misalignment.

D. Real Misalignments

The long-run relationships between the REER and the fundamentals estimated permit the calculation of the EREER. Based on the results of each of the five methods, EREERs were computed using the estimated long-term parameters listed in Table 1 and the permanent component of the fundamentals derived using the HP methodology. Figures 1 to 8 illustrate our results for each country. The two panels at the top display the evolution of the actual REER and the estimated EREER and the two bottom panels display the implied misalignment as a percent. The two left panels illustrate the results derived using the single-

[31] The results from the Hausmann tests are not reported but are available from the authors upon request.

country estimation techniques, and the two right-hand side panels display the results from the panel cointegration estimates.[32] In general the single-country estimates and to a lesser extent the panel estimates provide plausible estimates of the degree of REER misalignment. However, while the single-country estimates are typically fairly consistent, they are often relatively different from the panel estimates, suggesting that the results from the panel estimation are not representative of every country in the sample. Finally, the range of estimates underlines the uncertainty involved in estimating REER misalignment and the importance of treating the results of any single estimation with caution.

Benin

The single-country results for Benin indicate that the REER went through a period of overvaluation before 1994, which suggests that for Benin the 1994 CFA devaluation was warranted. The number of years of overvaluation differs significantly between methods, however. The devaluation in 1994 led to an undervaluation, which in the model estimated using the ARDL approach has gradually disappeared. However, the model estimated using the Johansen approach suggests that the EREER has been appreciating in line with the actual REER, so the degree of undervaluation in the Johansen model has remained relatively stable since the devaluation. In 2006, therefore, the degree of undervaluation implied by the single-country estimators ranges from about 3 to 17 percent.

The panel estimators differ sufficiently from the single-country results to cast doubt on the validity of the results for Benin. Except for the PMG estimates, the results from the panel estimates are relatively consistent, suggesting that in the case of Benin, to replicate the data it is necessary to include an impulse dummy in the PMG model to capture the effect of the 1994 devaluation. Including an impulse dummy for 1994 in the PMG model, however, results in a model that does not show any overvaluation before 1994. This is also the case with the FMOLS model estimated on demeaned data. Instead, the two models suggest that the REER in Benin had been undervalued since approximately 1982. In 2006, however, the results of these two models and the FMOLS model estimated on the raw data are consistent with the single-country estimates, with the degree of undervaluation ranging from about 0 to 12 percent.

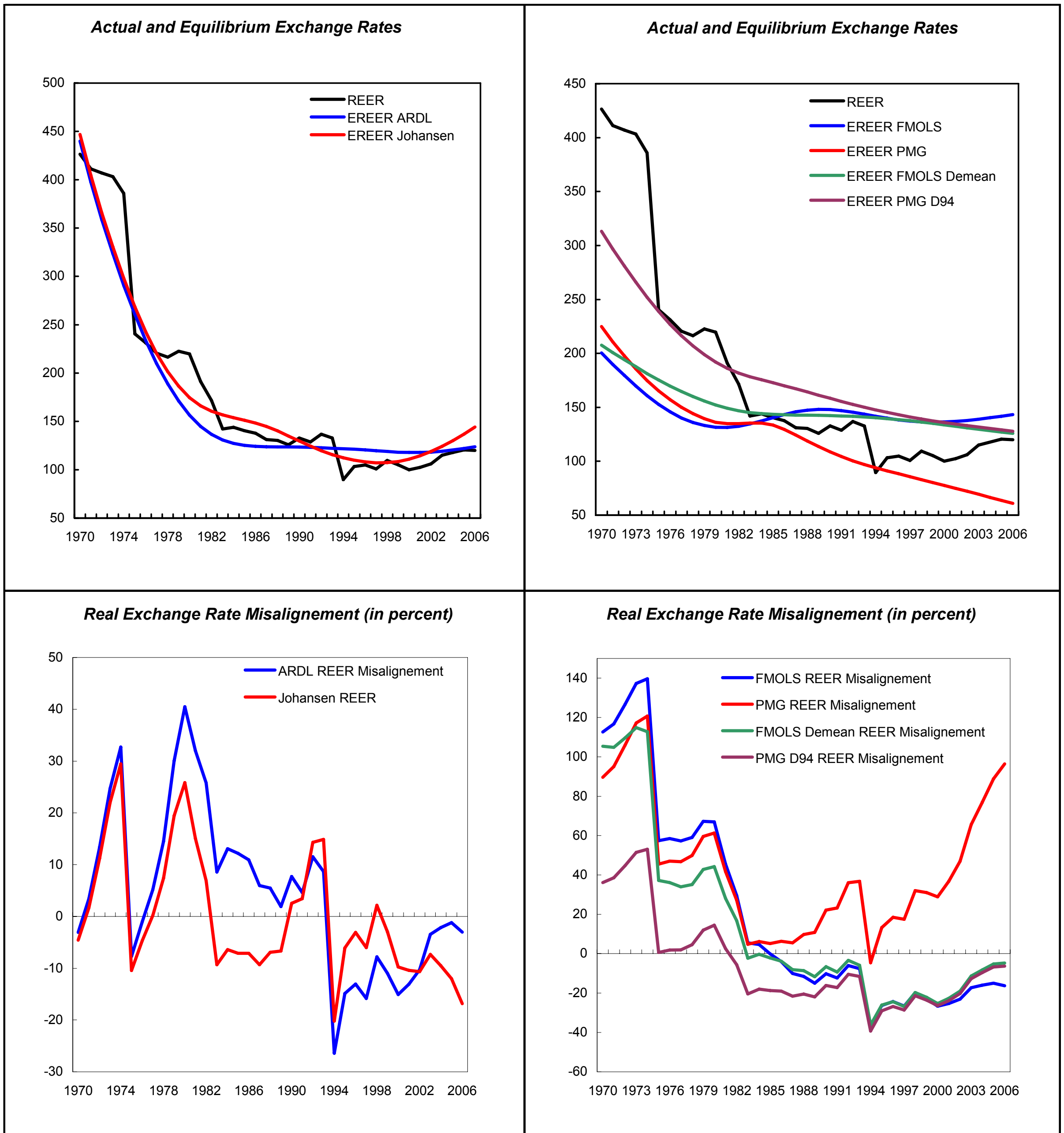
Burkina Faso

The single-country estimators and the panel estimators also give different results for Burkina Faso. The single-country results, which are more or less consistent, find evidence of an

[32] Because the focus of this paper is on the uncertainty inherent in the choice of econometric estimator rather than the statistical uncertainty surrounding our results, we do not include statistical error bands.

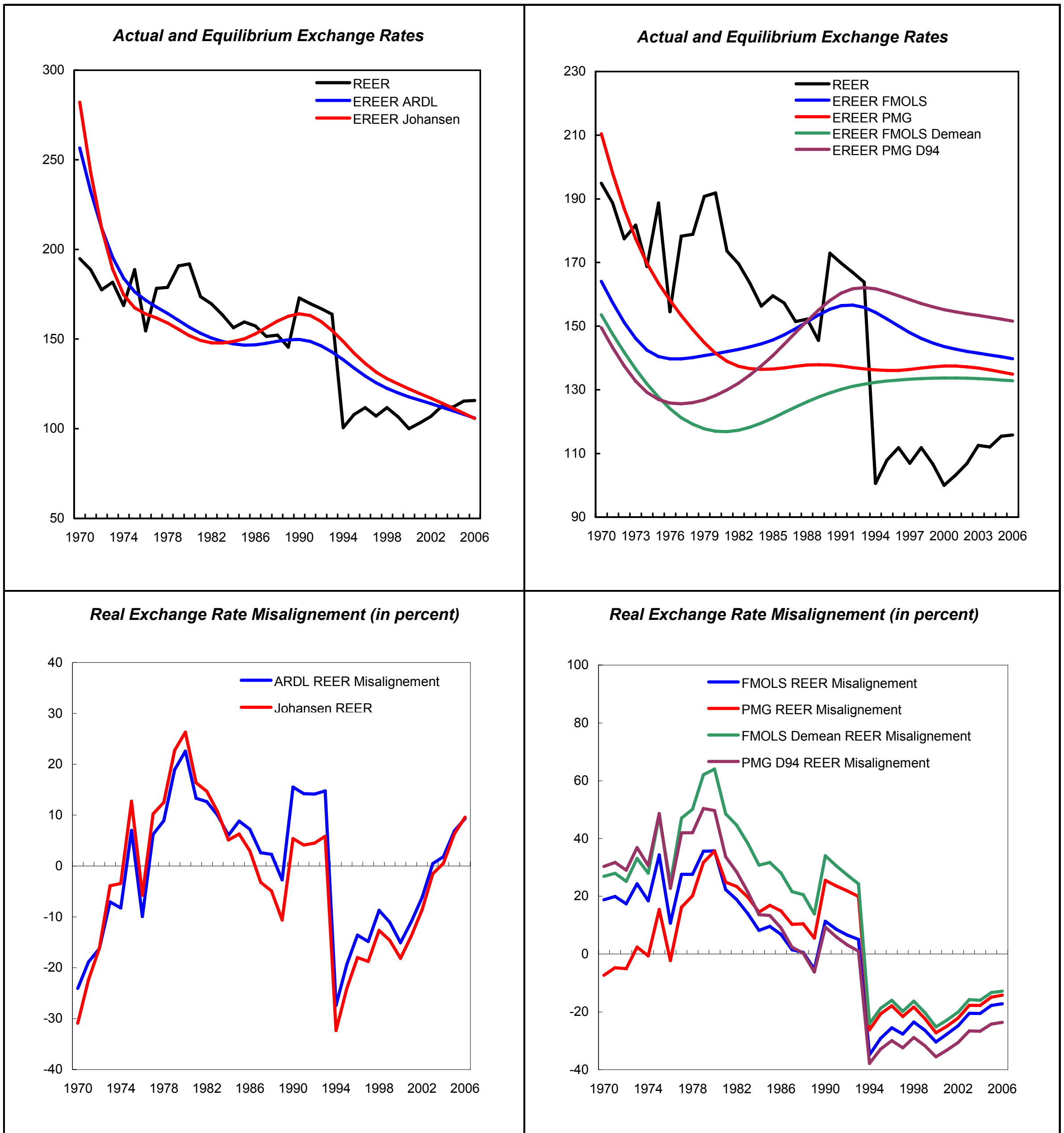
overvaluation ranging from 6 to 15 percent before the 1994 devaluation. Moreover, both the Johansen and the ARDL model indicate that the EREER continued to depreciate later, to the extent that the undervaluation caused by devaluation is no longer a problem. Indeed in 2006 the results consistently suggest that the REER may be overvalued by about 9 percent. This is in sharp contrast with the results obtained using the panel estimators, which suggest an undervaluation ranging from about 1 to 24 percent. Unlike our results for Benin, the PMG estimator without an impulse dummy for the devaluation provides results that are consistent with those of the other panel-data estimators.

Figure 1. Benin: Actual and Equilibium Exchange Rates, 1970–2006



Source: Authors Calculations

Figure 2. Burkina Faso: Actual and Equilibium Exchange Rates, 1970–2006



Source: Authors Calculations

Côte d'Ivoire

The single-country results for Côte d'Ivoire provide conflicting evidence about whether the exchange is over- or undervalued. The Johansen model suggests that the REER is currently about 3 percent overvalued; the ARDL model suggests a 6 percent undervaluation. The panel data results similarly conflict, ranging from a 14 percent undervaluation to a 53 percent overvaluation, depending on the estimator. All estimators, including the panel estimators, find evidence of REER overvaluation in the years before 1994, providing strong evidence that devaluation was warranted for Côte d'Ivoire.

Mali

As with Côte d'Ivoire, results of the ARDL and the Johansen model conflict, with estimates ranging from a 6 percent REER undervaluation to a 16 percent overvaluation in 2006. The panel data results for 2006 also conflict, ranging from a 24 percent undervaluation to a 6 percent overvaluation. As with Côte d'Ivoire, all estimators suggest that the REER was overvalued before 1994 and thus that the devaluation was warranted.

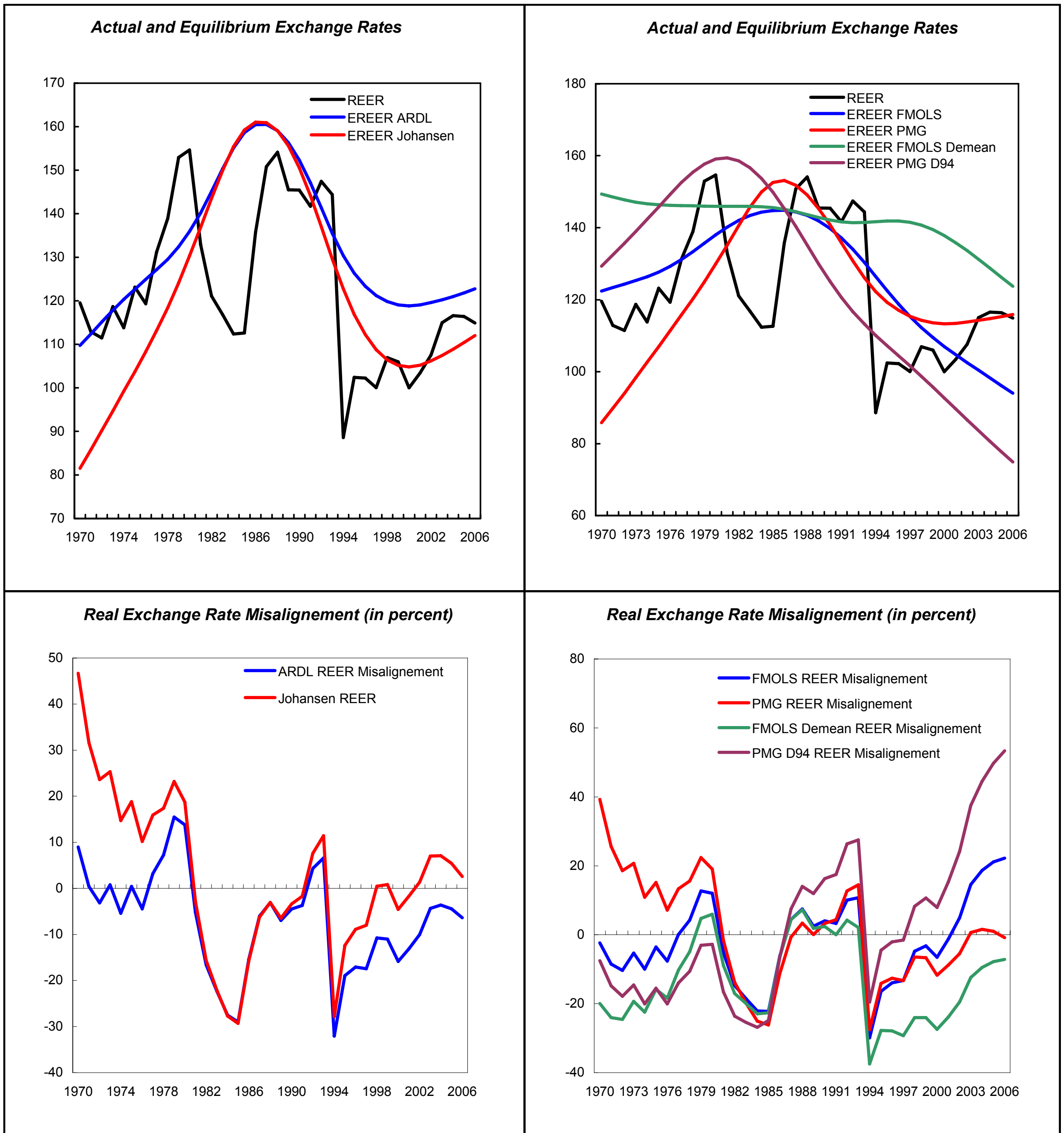
Niger

The single-country results for Niger in 2006 suggest an overvaluation of from 4 to 10 percent. Both single-country estimators suggest that the REER was overvalued before 1994, with estimates ranging from 6 to 9 percent, depending on the estimator. As with Benin, the estimates derived using the PMG model qualitatively and quantitatively are completely different from the other three panel estimators, suggesting that accounting for the 1994 devaluation in the estimation is necessary for Niger. Doing so in the PMG estimator leads to results that are remarkably similar to those obtained using the ARDL single-country model, with an implied overvaluation of 11 percent in 2006; the results for the two FMOLS estimators range from a 22 percent undervaluation to a 1 percent overvaluation.

Senegal

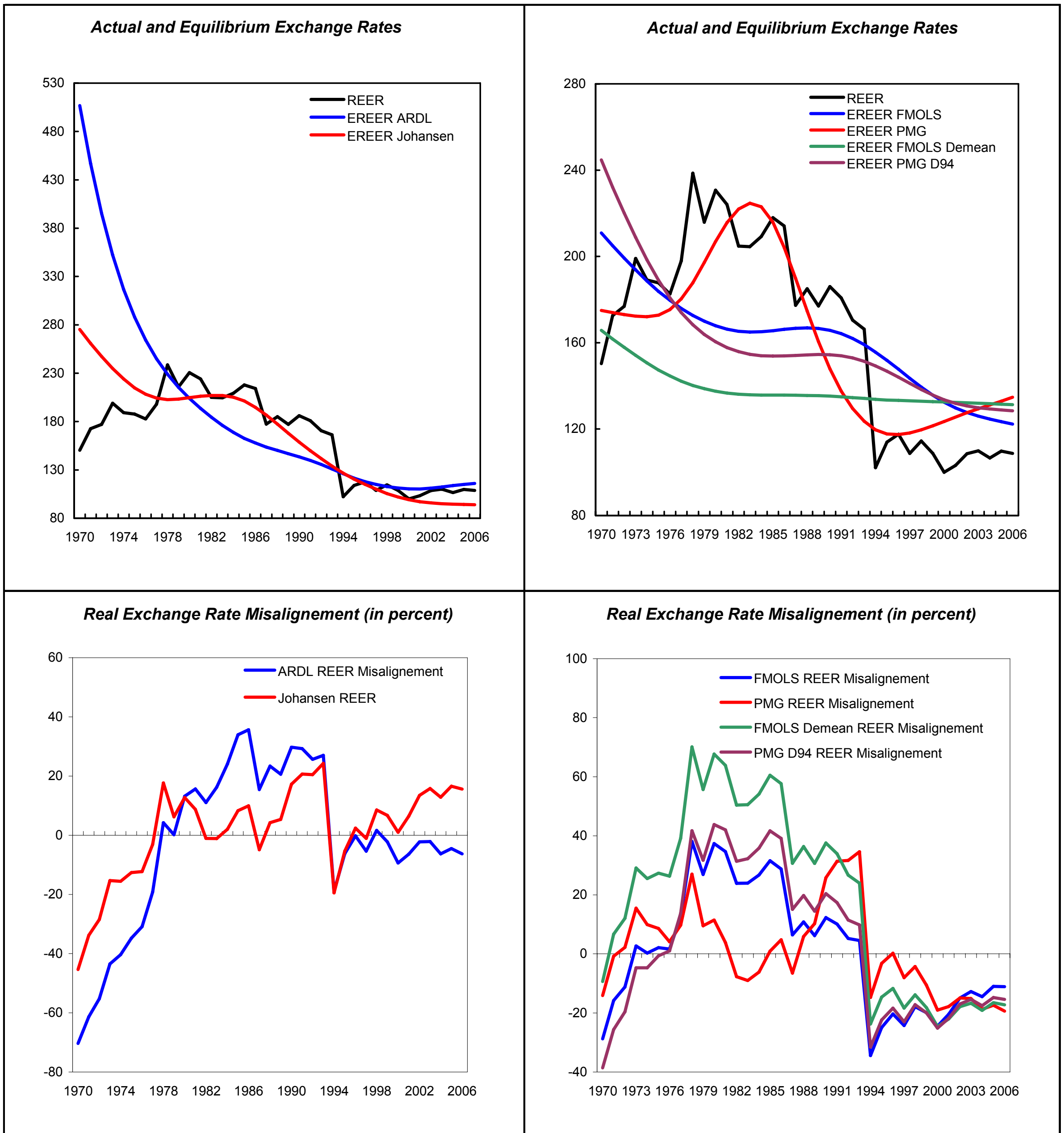
As with Côte d'Ivoire and Mali, the single-country results for Mali do not provide conclusive evidence about whether or not the REER is under- or overvalued; estimates range from a 4 percent undervaluation to a 9 percent overvaluation. Otherwise, however, the two single-country estimators provide consistent results, which suggests that the REER had been more or less overvalued in the 10 years before 1994. Similarly, the panel estimators all indicate that the REER was at least somewhat overvalued in the two years before the devaluation, thus validating that it was appropriate. The panel estimators also mirror the single-country estimators in that they do not provide conclusive evidence of either over- or undervaluation; estimates range from a 6 percent overvaluation to a 23 percent undervaluation.

Figure 3. Côte D'Ivoire: Actual and Equilibium Exchange Rates, 1970–2006



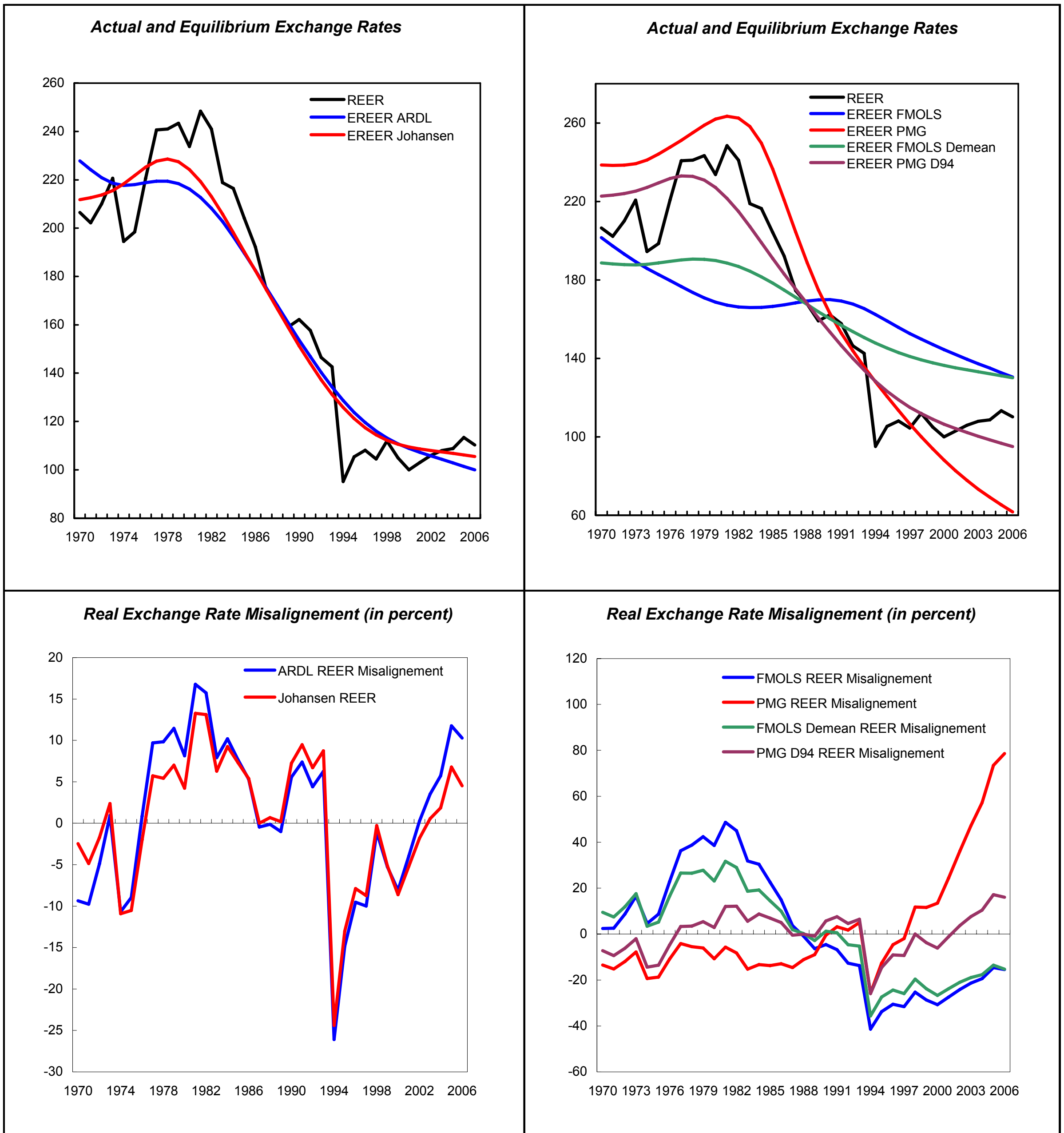
Source: Authors Calculations

Figure 4. Mali: Actual and Equilibium Exchange Rates, 1970–2006



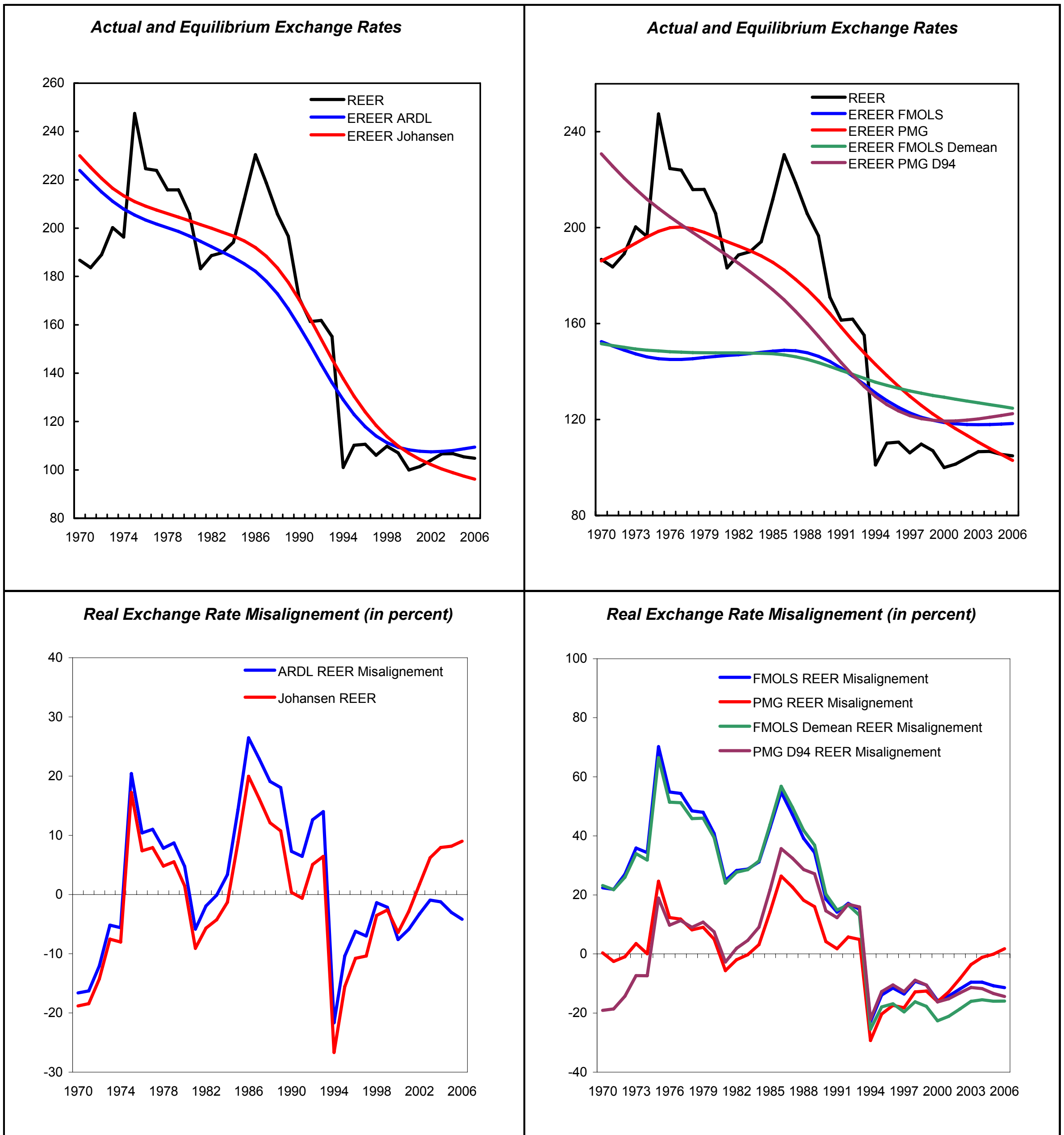
Source: Authors Calculations

Figure 5. Niger: Actual and Equilibium Exchange Rates, 1970–2006



Source: Authors Calculations

Figure 6. Senegal: Actual and Equilibium Exchange Rates, 1970–2006



Source: Authors Calculations

Togo

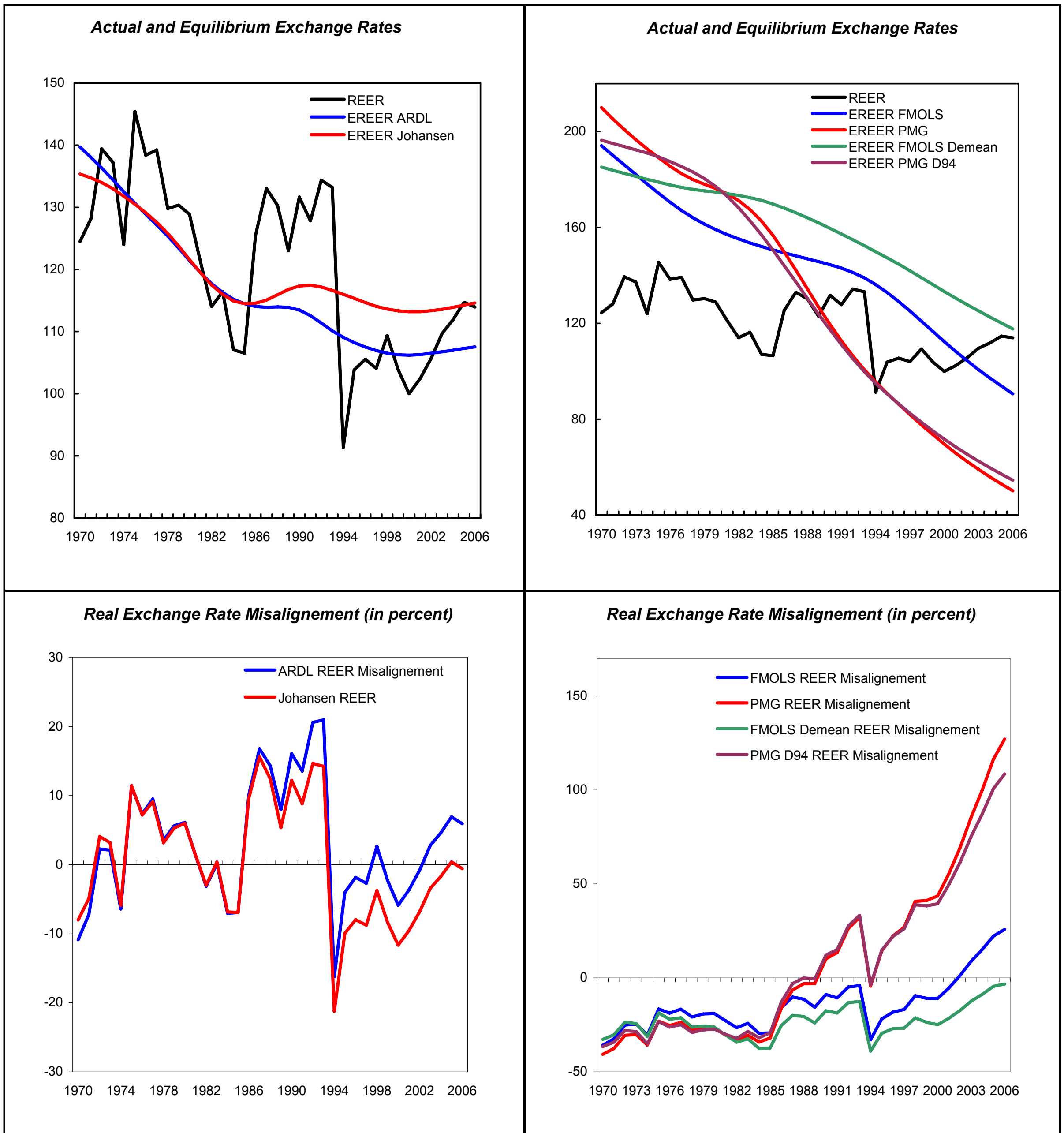
As with Senegal and other countries, the single-country results for Togo do not provide conclusive answers about whether the REER is over- or undervalued. The Johansen approach suggests that in 2006 the REER was more or less aligned with its underlying equilibrium whereas the ARDL model suggests a 6 percent overvaluation. The latter is consistent with findings from the panel cointegration models, which except for the FMOLS model estimated on demeaned data all indicate that the REER is overvalued. However, the discrepancy between the single-country and the panel results casts doubt on whether the latter are valid for Togo. Both single-country models and three of the four panel data models suggest, however, that the REER was overvalued before devaluation, lending support to the idea that in Togo's case the devaluation was justified.

WAEMU

Finally, we investigate the implications of our estimation of REER misalignment in WAEMU as a whole, using aggregated data. The single-country estimators provide remarkably consistent results that mirror those documented in Abdih and Tsangarides (2006) and in Tsangarides (2007). In particular, they provide evidence for the fact that the REER may be somewhat overvalued. This is consistent with the results obtained using both the PMG estimators, which range from a 15 to a 26 percent overvaluation. The results from the FMOLS panel data models are not as clear-cut, however, ranging from a 15 percent undervaluation to a 21 percent overvaluation. Except for the FMOLS model estimated on demeaned data, all estimators provide evidence that the REER was overvalued up to 1994, suggesting that the devaluation was warranted for the region as a whole.[33]

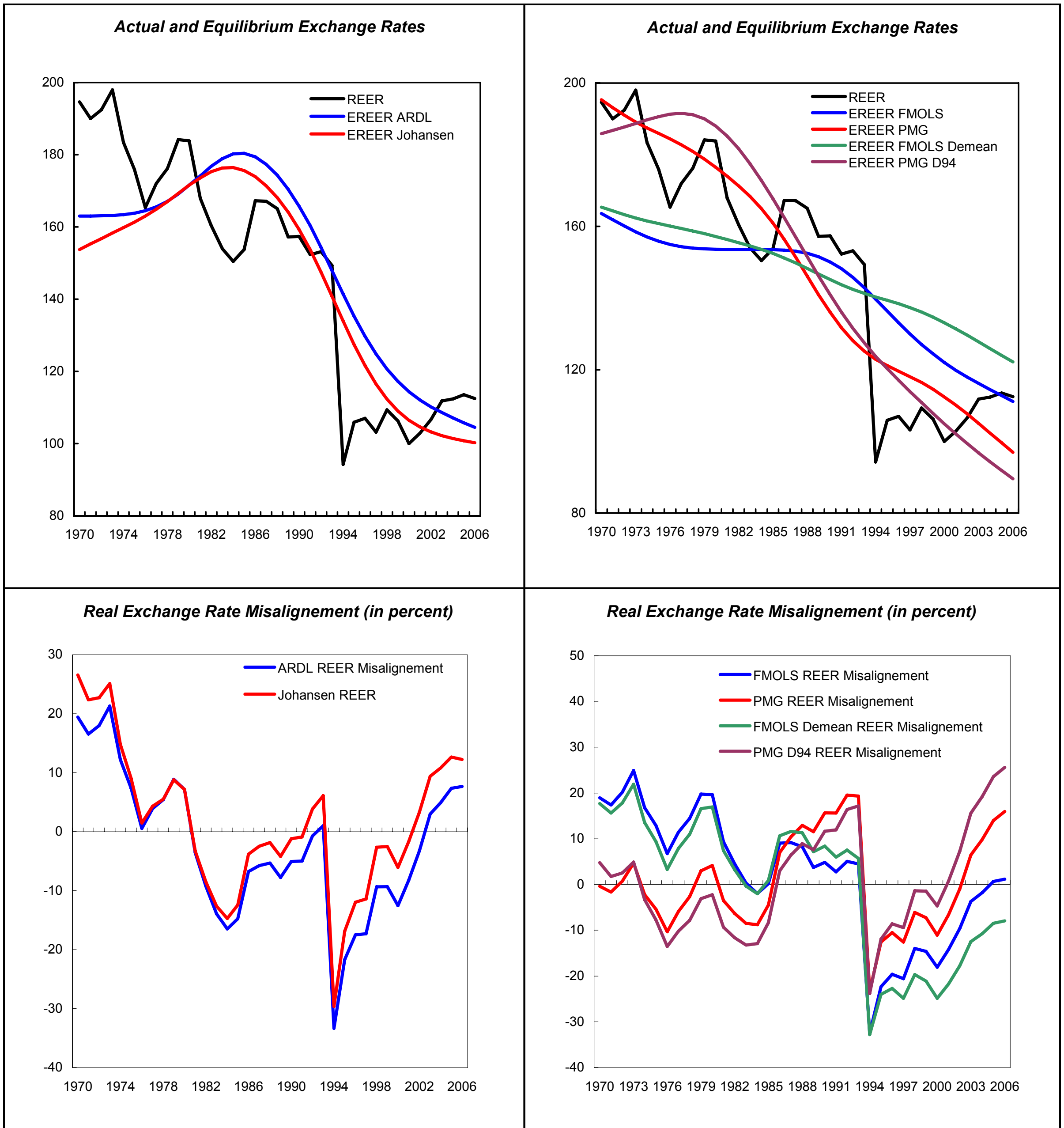
[33] These results are consistent with the findings (derived using the HP filter) in Abdih and Tsangarides (2006).

Figure 7. Togo: Actual and Equilibium Exchange Rates, 1970–2006



Source: Authors Calculations

Figure 8. WAEMU: Actual and Equilibium Exchange Rates, 1970–2006



Source: Authors Calculations

To conclude the discussion in this section we attempt to summarize the information about the degree of REER misalignment in Table 3 which shows the share of estimators suggesting a overvaluation or an undervaluation of the REER and the share of estimators suggesting that the REER is broadly aligned with its underlying equilibrium. The results show that for no country do all the estimators provide unambiguous results regarding whether the exchange rate is misaligned or not. This underlines the lack of robustness in many cases of estimates of the degree of misalignment to the choice of estimators. This is true even when we consider only the two single-country estimators. In particular, only in three out of eight country cases do the single-country estimators provide unambiguous results.

Table 3. Evidence of Exchange Rate Misalignement in 2006[1,2]
(In Percent of Number of Estimators)

	Single Country			Panel Estimation			All Estimators		
Country	Over Valuation	Under Valuation	Aligned	Over Valuation	Under Valuation	Aligned	Over Valuation	Under Valuation	Aligned
Benin	0%	50%	50%	25%	50%	25%	17%	50%	33%
Burkina Faso	100%	0%	0%	0%	75%	25%	33%	50%	17%
Côte d'Ivoire	0%	50%	50%	50%	25%	25%	33%	33%	33%
Mali	50%	0%	50%	25%	75%	0%	33%	50%	17%
Niger	100%	0%	0%	50%	25%	25%	67%	17%	17%
Senegal	50%	50%	0%	25%	50%	25%	33%	50%	17%
Togo	50%	0%	50%	75%	25%	0%	67%	17%	17%
WAEMU	100%	0%	0%	75%	25%	0%	83%	17%	0%

Notes:
[1] The table shows the percentage of estimators that indicate overvaluation, undervaluation, or no misalignement in 2006. For example, a 100% overvaluation for a single country (panel) estimation suggests that 2 out of 2 (4 out of 4) estimators indicate overvaluation.
[2] An aligned exchange rate is defined as a REER which is less than 5 percent misaligned.

V. Conclusion

Using a dynamic model of a small open economy, this paper has analyzed the EREER of the each WAEMU member country and of the WAEMU as a whole in order to assess whether movements in the REERs were consistent with macroeconomic fundamentals. In an attempt to address estimation uncertainty, we estimated EREER series based on both single-country (Johansen and ARDL) and panel-data cointegration techniques (FMOLS and PMG). These estimation methods have the benefit of incorporating country-specific information that might otherwise be lost in single-country estimations of the regional exchange rate.

We find that much of the long-run behavior of the REER can be explained by fluctuations in terms of trade, government consumption, investment, and productivity. The use of different econometric techniques suggests, however, that there is significant uncertainty about the path of the EREER in WAEMU countries and thus about the degree of exchange rate misalignment. In particular, (i) the results of panel cointegration techniques employing data on all WAEMU countries often differ significantly from the single-country results and therefore, due to important differences across countries, may not provide an accurate picture of the EREER in some WAEMU countries, particularly since the hypothesis of homogeneity is rejected in the PMG framework for at least some of the long-run parameters; (ii) demeaning (in FMOLS) or including an impulse dummy for the 1994 devaluation (PMG) dramatically changes the estimated EREER for many countries but does not do so consistently, thus increasing uncertainty about the correct approach; (iii) due to the uncertainty introduced by the different econometric techniques, it is impossible to conclude that the REER in any WAEMU country is over- or undervalued. This is particularly true for Côte d'Ivoire, Mali, Senegal, and Togo, where even the single-country estimation results are not conclusive. In the other countries, a conclusion about whether or not the REER is misaligned will depend on the importance that is attached to some or all of the EREERs derived using panel cointegration estimators. For these countries, close evaluation and monitoring of exchange rate developments seems prudent.

Our analysis has a number of policy implications which can be applied more widely: First, the differences observed in the estimated EREER across econometric techniques suggests a robust analysis of exchange rate misalignment needs to take seriously the issue of estimation uncertainty by considering results from different estimators. Second, while results from panel-data cointegration may be useful in some cases, they should always be compared with single-country estimators to ensure that the panel results are representative of a particular country. Third, the use of nonstationary panel techniques like FMOLS that allow the long-run parameters to be heterogeneous across countries may be less useful for macroeconomic analysis than previously thought because the average cointegrating vector may not be a good representation of any particular country in the panel due to heterogeneity. In particular, the average cointegrating vector may unduly dampen (or strengthen) the effect of a particular fundamental on a country's EREER.

While we estimate that short-term fluctuations or deviations around the REERs are temporary and revert to equilibrium, close evaluation and monitoring of exchange rate developments is required if a deviation remains one-sided and builds up to a longer-term significant and persistent misalignment. In addition, a complete analysis of the environment that impacts the short term sustainability of the arrangement requires examining possible pressures on balance of payments flows, adequacy of reserve levels, deterioration of other measures of competitiveness (including structural), unfavorable financial market perceptions, and sustained deviations from the EREER. For the latter, fixed exchange rate regimes can be sustainable in theory, as long as actual deviations from long-term equilibrium rates are small and mean reverting. In contrast, if deviations are one-sided and build up to longer-term significant misalignments, it is generally argued that, in addition to demand side management policies, real exchange rate action may be needed to restore balance.

Finally, this paper has sought to emphasize the uncertainty inherent in estimates of the EREER due to different estimation techniques. We have been candid, however, in acknowledging that we do not consider (i) statistical uncertainty surrounding our estimates of the EREER; and (ii) the uncertainty associated with the fact that there exists more than one model of the EREER. For WAEMU countries, the former has been extensively analyzed by Abdih and Tsangarides (2006); the latter is being considered for future research.

Appendix A

Variable Definitions and Sources

The dataset used in this paper consists of annual data series (1970–2006) for Togo, Senegal, Mali, Niger, Burkina Faso, Côte d'Ivoire, and Benin.[34] Acronyms, definitions, and sources for the variables are as follows:

LREER Natural logarithm of the real effective exchange rate (Index 2000=100)
Source: Information Notice System (INS) and IMF staff calculations.

LNCGR Natural logarithm of public consumption expenditure to GDP
Source: IMF, World Economic Outlook (WEO).

LTTT Natural logarithm of terms of trade (Index 2000=100)
Source: IMF, World Economic Outlook (WEO).

LNIR Natural logarithm of gross capital formation to GDP
Source: IMF, World Economic Outlook (WEO).

LPROD Natural logarithm of real per capita GDP relative to main trade partners, normalized to 1 in 2000 with weights as discussed above
Source: IMF, World Economic Outlook (WEO).

LOPEN Sum of exports and imports to GDP
Source: IMF, World Economic Outlook (WEO).

The regional aggregate variables for WAEMU were constructed using member country annual observations and nominal GDP weights.

Country REERs before 1980 were unavailable in the IMF Information Notice System (INS) database and were constructed based on CPI indices from the *World Economic Outlook* with partner weights renormalized. The "foreign" variable (used in calculating the productivity proxy) was calculated as the renormalized weighted average of the five trading partners based on the INS weights for the REER. The WAEMU partner country weights were: France, 0.35; United States, 0.13; Germany, 0.11; China, 0.09; Japan, 0.08; and Italy, 0.08.

[34] Guinea Bissau was excluded from the sample because it joined WAEMU only in 1997.

Appendix B

Cointegration, Unit Root Tests and Model Diagnostics

Figure B1. WAEMU: Cointegration Variables

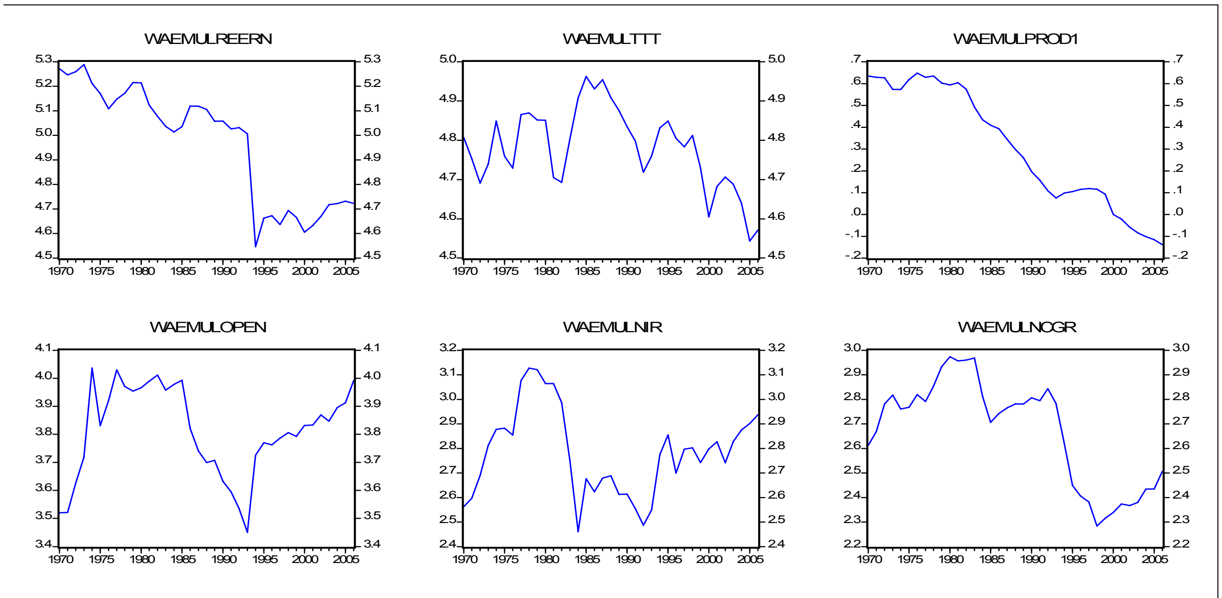


Figure B2. Benin: Cointegration Variables

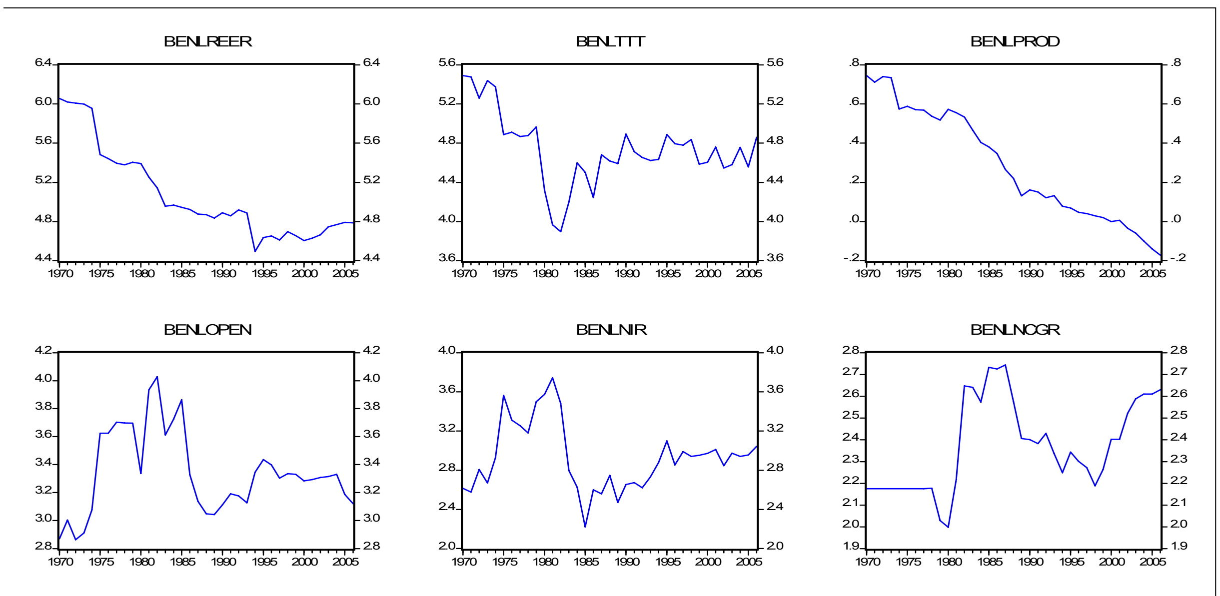


Figure B3. Burkina Faso: Cointegration Variables

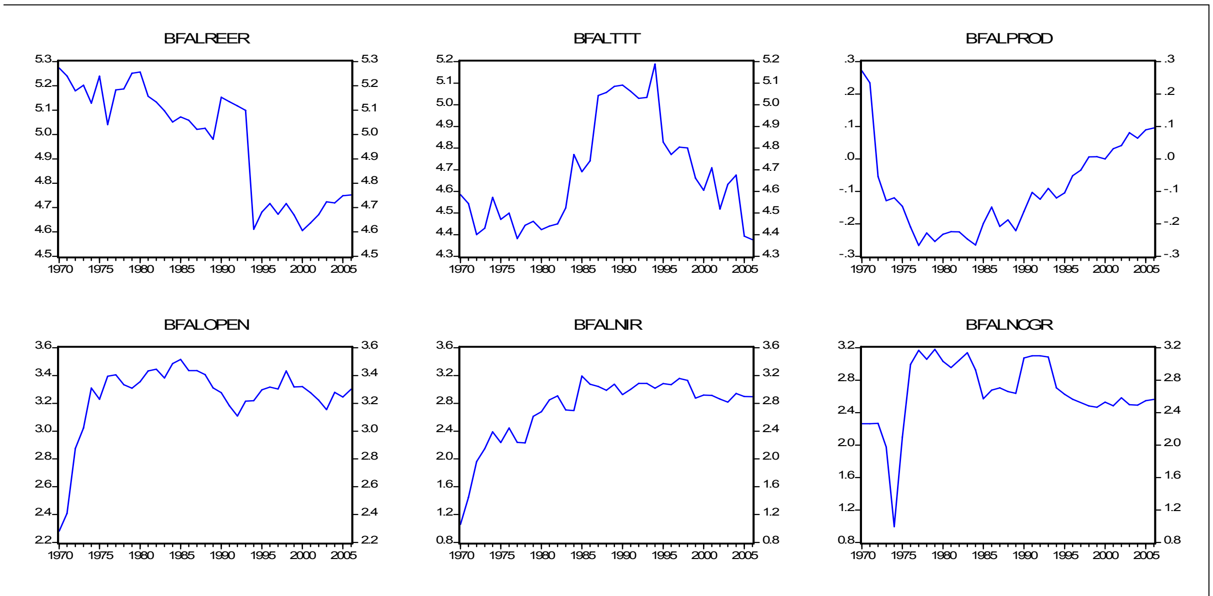


Figure B4. Cote d'Ivoire: Cointegration Variables

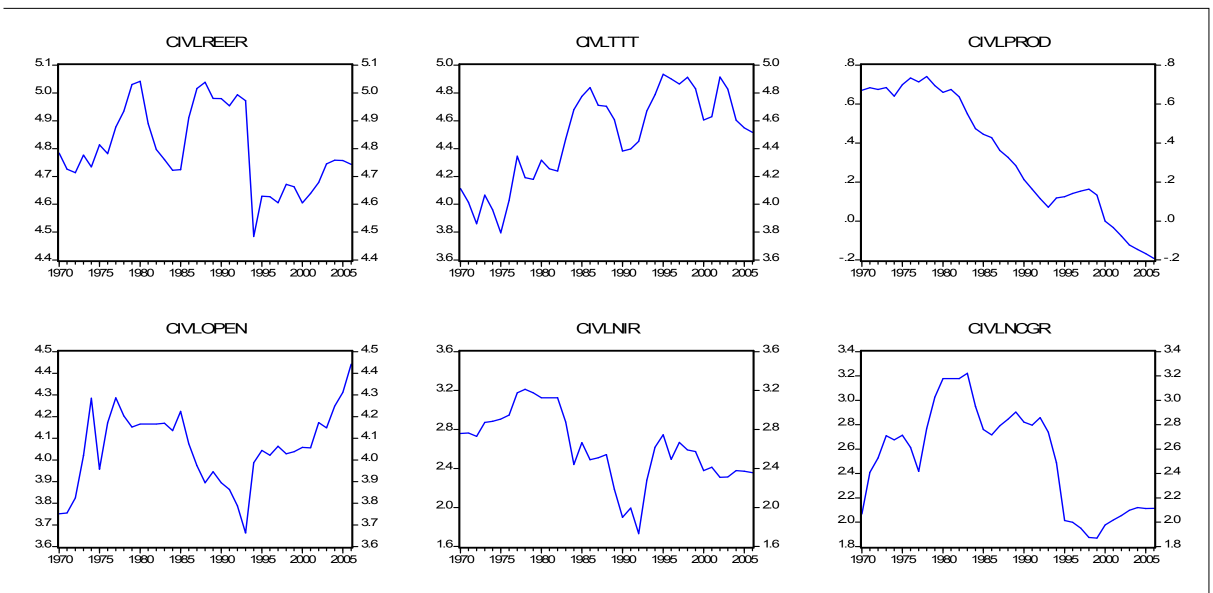


Figure B5. Mali: Cointegration Variables

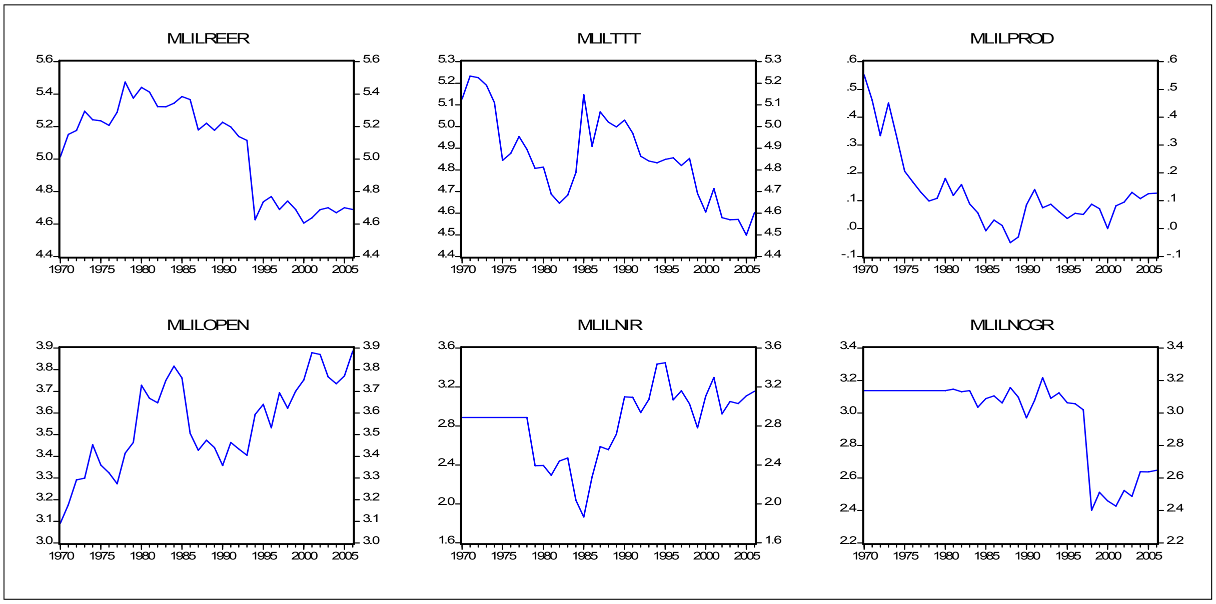


Figure B6. Niger: Cointegration Variables

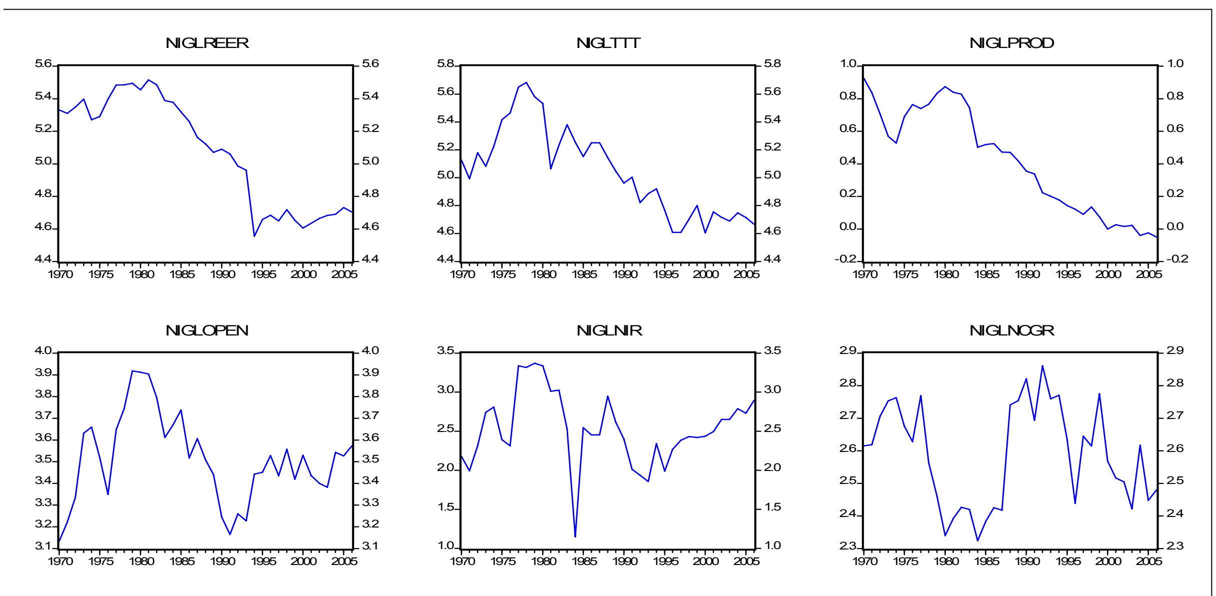


Figure B7. Senegal: Cointegration Variables

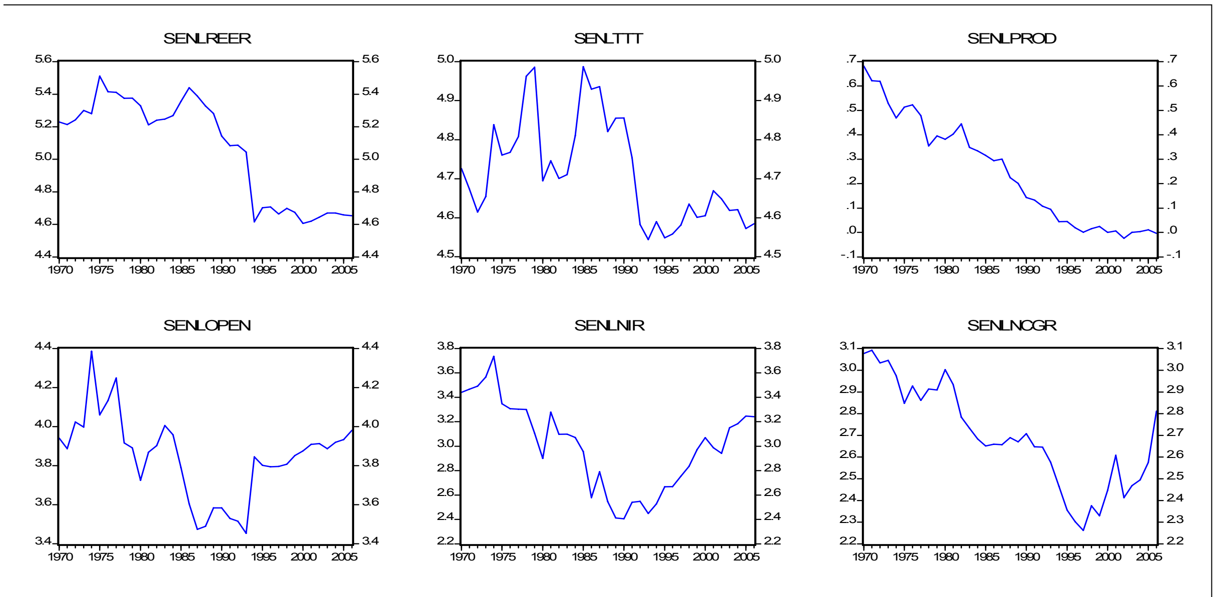


Figure B8. TGO: Cointegration Variables

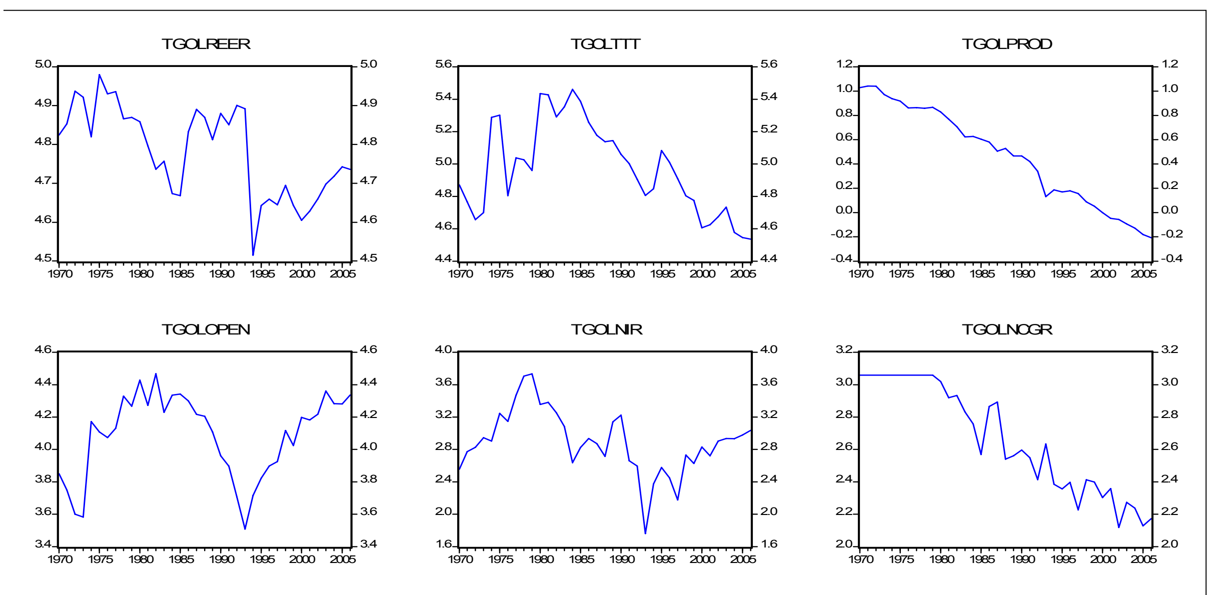


Table B1. Unit Root Tests (Series)[1,2,3]

Variables/Groups Levels and Differences

WAEMU				*Mali*			
Variable	ADF	p-value	Conclusion	Variable	ADF	p-value	Conclusion
ln(REER)	-2.55	0.30	I(1)	ln(REER)	-0.74	0.82	I(1)
ln(TTT)	-2.65	0.26	I(1)	ln(TTT)	-1.77	0.39	I(1)
ln(CGR)	-2.55	0.31	I(1)	ln(CGR)	-1.20	0.67	I(1)
ln(NIR)	-2.02	0.28	I(1)	ln(NIR)	-1.69	0.43	I(1)
ln(PROD)	-2.52	0.32	I(1)	ln(PROD)	-2.82	0.20	I(1)
ln(OPEN)	-2.21	0.21	I(1)	ln(OPEN)	-2.02	0.28	I(1)
Dln(REER)	-6.51	0.00	I(0)	Dln(REER)	-6.92	0.00	I(0)
Dln(TTT)	-5.20	0.00	I(0)	Dln(TTT)	-7.21	0.00	I(0)
Dln(CGR)	-3.59	0.00	I(0)	Dln(CGR)	-6.95	0.00	I(0)
Dln(NIR)	-5.33	0.00	I(0)	Dln(NIR)	-5.83	0.00	I(0)
Dln(PROD)	-3.73	0.01	I(0)	Dln(PROD)	-6.87	0.00	I(0)
Dln(OPEN)	-6.31	0.00	I(0)	Dln(OPEN)	-5.88	0.00	I(0)
Benin				*Niger*			
Variable	ADF	p-value	Conclusion	Variable	ADF	p-value	Conclusion
ln(REER)	-2.31	0.17	I(1)	ln(REER)	-0.56	0.87	I(1)
ln(TTT)	-2.62	0.10	I(1)	ln(TTT)	-1.04	0.73	I(1)
ln(CGR)	-2.10	0.25	I(1)	ln(CGR)	-2.49	0.13	I(1)
ln(NIR)	-2.25	0.19	I(1)	ln(NIR)	-3.15	0.11	I(1)
ln(PROD)	-0.43	0.89	I(1)	ln(PROD)	-0.82	0.80	I(1)
ln(OPEN)	-2.48	0.13	I(1)	ln(OPEN)	-2.54	0.11	I(1)
Dln(REER)	-5.93	0.00	I(0)	Dln(REER)	-6.26	0.00	I(0)
Dln(TTT)	-5.97	0.00	I(0)	Dln(TTT)	-4.69	0.00	I(0)
Dln(CGR)	-4.44	0.00	I(0)	Dln(CGR)	-7.64	0.00	I(0)
Dln(NIR)	-5.97	0.00	I(0)	Dln(NIR)	-7.23	0.00	I(0)
Dln(PROD)	-6.42	0.00	I(0)	Dln(PROD)	-4.52	0.00	I(0)
Dln(OPEN)	-5.17	0.00	I(0)	Dln(OPEN)	-5.85	0.00	I(0)
Burkina Faso				*Senegal*			
Variable	ADF	p-value	Conclusion	Variable	ADF	p-value	Conclusion
ln(REER)	-1.51	0.52	I(1)	ln(REER)	-0.50	0.88	I(1)
ln(TTT)	-1.31	0.61	I(1)	ln(TTT)	-1.93	0.32	I(1)
ln(CGR)	-2.50	0.12	I(1)	ln(CGR)	-1.69	0.43	I(1)
ln(NIR)	0.97	0.91	I(1)	ln(NIR)	-1.54	0.50	I(1)
ln(PROD)	0.20	0.97	I(1)	ln(PROD)	-1.90	0.33	I(1)
ln(OPEN)	1.12	0.93	I(1)	ln(OPEN)	-2.09	0.25	I(1)
Dln(REER)	-7.44	0.00	I(0)	Dln(REER)	-6.25	0.00	I(0)
Dln(TTT)	-6.69	0.00	I(0)	Dln(TTT)	-5.76	0.00	I(0)
Dln(CGR)	-5.22	0.00	I(0)	Dln(CGR)	-4.92	0.00	I(0)
Dln(NIR)	-5.46	0.00	I(0)	Dln(NIR)	-6.97	0.00	I(0)
Dln(PROD)	-5.17	0.00	I(0)	Dln(PROD)	-1.98	0.00	I(0)
Dln(OPEN)	-4.67	0.00	I(0)	Dln(OPEN)	-7.44	0.00	I(0)
Côte d'Ivoire				*Togo*			
Variable	ADF	p-value	Conclusion	Variable	ADF	p-value	Conclusion
ln(REER)	-2.25	0.19	I(1)	ln(REER)	-2.46	0.13	I(1)
ln(TTT)	-1.99	0.29	I(1)	ln(TTT)	-1.68	0.43	I(1)
ln(CGR)	-1.39	0.58	I(1)	ln(CGR)	-0.01	0.95	I(1)
ln(NIR)	-1.52	0.51	I(1)	ln(NIR)	-2.36	0.16	I(1)
ln(PROD)	0.83	0.99	I(1)	ln(PROD)	0.31	0.98	I(1)
ln(OPEN)	-2.02	0.28	I(1)	ln(OPEN)	-1.88	0.34	I(1)
Dln(REER)	-6.10	0.00	I(0)	Dln(REER)	-7.28	0.00	I(0)
Dln(TTT)	-5.02	0.00	I(0)	Dln(TTT)	-6.59	0.00	I(0)
Dln(CGR)	-4.18	0.00	I(0)	Dln(CGR)	-8.00	0.00	I(0)
Dln(NIR)	-5.72	0.00	I(0)	Dln(NIR)	-7.13	0.00	I(0)
Dln(PROD)	-4.21	0.00	I(0)	Dln(PROD)	-6.63	0.00	I(0)
Dln(OPEN)	-6.76	0.00	I(0)	Dln(OPEN)	-6.83	0.00	I(0)

Notes:

[1] The symbol "D" denotes the difference operator.

[2] The null hypothesis is unit root (or I(1)).

[3] Unit root tests are based on the Augmented Dickey-Fuller test, Schartz criterion, maximum lags 9.

Table B2. Unit Root Tests (Panel)[1,2,3]

Variables/Groups Levels and Differences

ln(REER)

Test	Statistic	p-value	Conclusion
Levels			
Levin & Lin rho	-0.87		I(1)
Levin & Lin t-rho	-0.33		I(1)
Levin, Lin & Chu ADF	-0.23		I(1)
Im, Pesaran and Shin ADF-t	-1.22		I(1)
Differences			
Levin & Lin rho	-24.50	**	I(0)
Levin & Lin t-rho	-10.17	**	I(0)
Levin, Lin & Chu ADF	-10.30	**	I(0)
Im, Pesaran and Shin ADF-t	-13.86	**	I(0)

ln(TTT)

Test	Statistic	p-value	Conclusion
Levels			
Levin & Lin rho	-1.98	**	I(0)
Levin & Lin t-rho	-0.61		I(1)
Levin, Lin & Chu ADF	-0.68		I(1)
Im, Pesaran and Shin ADF-t	-1.30	*	I(0)
Differences			
Levin & Lin rho	-27.59	**	I(0)
Levin & Lin t-rho	-12.25	**	I(0)
Levin, Lin & Chu ADF	-10.36	**	I(0)
Im, Pesaran and Shin ADF-t	-13.40	**	I(0)

ln(CGR)

Test	Statistic	p-value	Conclusion
Levels			
Levin & Lin rho	-2.82	**	I(0)
Levin & Lin t-rho	-0.89		I(1)
Levin, Lin & Chu ADF	-0.85		I(1)
Im, Pesaran and Shin ADF-t	-1.51	*	I(0)
Differences			
Levin & Lin rho	-25.90	**	I(0)
Levin & Lin t-rho	-11.51	**	I(0)
Levin, Lin & Chu ADF	-10.35	**	I(0)
Im, Pesaran and Shin ADF-t	-13.20	**	I(0)

ln(NIR)

Test	Statistic	p-value	Conclusion
Levels			
Levin & Lin rho	-4.81	**	I(0)
Levin & Lin t-rho	-2.41	**	I(0)
Levin, Lin & Chu ADF	-2.26	**	I(0)
Im, Pesaran and Shin ADF-t	-3.84	**	I(0)
Differences			
Levin & Lin rho	-30.36	**	I(0)
Levin & Lin t-rho	-13.95	**	I(0)
Levin, Lin & Chu ADF	-10.89	**	I(0)
Im, Pesaran and Shin ADF-t	-14.72	**	I(0)

ln(PROD)

Test	Statistic	p-value	Conclusion
Levels			
Levin & Lin rho	1.79		I(1)
Levin & Lin t-rho	2.73		I(1)
Levin, Lin & Chu ADF	2.38		I(1)
Im, Pesaran and Shin ADF-t	1.97		I(1)
Differences			
Levin & Lin rho	-23.53	**	I(0)
Levin & Lin t-rho	-10.17	**	I(0)
Levin, Lin & Chu ADF	-9.87	**	I(0)
Im, Pesaran and Shin ADF-t	-11.12	**	I(0)

ln(OPEN)

Test	Statistic	p-value	Conclusion
Levels			
Levin & Lin rho	-6.38	**	I(0)
Levin & Lin t-rho	-2.60	**	I(0)
Levin, Lin & Chu ADF	-1.79	**	I(0)
Im, Pesaran and Shin ADF-t	-2.84	**	I(0)
Differences			
Levin & Lin rho	-29.84	**	I(0)
Levin & Lin t-rho	-13.66	**	I(0)
Levin, Lin & Chu ADF	-12.64	**	I(0)
Im, Pesaran and Shin ADF-t	-17.02	**	I(0)

Notes:

[1] The tests based on the Levin and Lin (1992, 1993) and Levin, Lin and Chu (2002) rho, t-rho and ADF statistics, are based on pooled, within-dimension, estimators and hence consider the parameters of interest as homogeneous across countries. The test based on the Im, Pesaran, and Shin (2003) ADF-t statistic, is based on group mean estimator and treats the parameters of interest as heterogeneous among members.

[2] The tests are one sided and the statistics are normally distributed N(0,1) under the null hypothesis of no cointegration. Under the alternative hypothesis, the statistics diverge to negative infinity (and therefore the left tail is used to reject the null).

[3] One and two asterisks indicate rejection at 10 and 5 percent, or better, respectively.

Table B3. Cointegration Tests (Series)

Johansen and ARDL Cointegration Tests

WAEMU

Johansen Cointegration Tests

Number of Hypothesized Cointegrating Equations[1]	Trace Statistic	p-value	Max-Eigen Statistic	p-value
None *	115.45	0.00	55.54	0.00
At most 1	59.91	0.24	26.16	0.31
At most 2	33.75	0.52	19.13	0.40
At most 3	14.62	0.80	9.86	0.76
At most 4	4.75	0.83	4.49	0.80
At most 5	0.27	0.61	0.27	0.61

ARDL Cointegration Test[2]

Test	F-statistic	Lower Bound	Upper Bound
Bounds test	6.42	2.62	3.79

Mali

Johansen Cointegration Tests

Number of Hypothesized Cointegrating Equations[1]	Trace Statistic	p-value	Max-Eigen Statistic	p-value
None *	115.91	0.00	46.98	0.01
At most 1	68.93	0.06	33.68	0.05
At most 2	35.25	0.43	19.60	0.37
At most 3	15.65	0.74	15.31	0.27
At most 4	0.34	1.00	0.24	1.00
At most 5	0.10	0.76	0.10	0.76

ARDL Cointegration Test[2]

Test	F-statistic	Lower Bound	Upper Bound
Bounds test	17.93	2.62	3.79

Benin

Johansen Cointegration Tests

Number of Hypothesized Cointegrating Equations[1]	Trace Statistic	p-value	Max-Eigen Statistic	p-value
None *	107.96	0.01	47.37	0.01
At most 1	60.59	0.22	29.07	0.17
At most 2	31.52	0.64	13.75	0.84
At most 3	17.77	0.58	9.86	0.76
At most 4	7.92	0.47	6.11	0.60
At most 5	1.81	0.18	1.81	0.18

ARDL Cointegration Test[2]

Test	F-statistic	Lower Bound	Upper Bound
Bounds test	4.41	2.62	3.79

Niger

Johansen Cointegration Tests

Number of Hypothesized Cointegrating Equations[1]	Trace Statistic	p-value	Max-Eigen Statistic	p-value
None *	98.64	0.03	37.38	0.10
At most 1	61.26	0.20	27.34	0.25
At most 2	33.92	0.51	20.24	0.32
At most 3	13.68	0.86	10.53	0.69
At most 4	3.15	0.96	3.04	0.94
At most 5	0.11	0.74	0.11	0.74

ARDL Cointegration Test[2]

Test	F-statistic	Lower Bound	Upper Bound
Bounds test	4.99	2.62	3.79

Burkina Faso

Johansen Cointegration Tests

Number of Hypothesized Cointegrating Equations[1]	Trace Statistic	p-value	Max-Eigen Statistic	p-value
None *	95.75	0.00	40.08	0.00
At most 1	69.82	0.08	33.88	0.26
At most 2	47.86	0.22	27.58	0.19
At most 3	29.80	0.62	21.13	0.59
At most 4	15.49	0.72	14.26	0.67
At most 5	3.84	0.64	3.84	0.64

ARDL Cointegration Test[2]

Test	F-statistic	Lower Bound	Upper Bound
Bounds test	3.54	2.62	3.79

Senegal

Johansen Cointegration Tests

Number of Hypothesized Cointegrating Equations[1]	Trace Statistic	p-value	Max-Eigen Statistic	p-value
None *	121.01	0.00	45.68	0.01
At most 1	75.33	0.02	31.64	0.09
At most 2	43.69	0.12	21.86	0.23
At most 3	21.82	0.31	13.99	0.37
At most 4	7.83	0.48	5.87	0.63
At most 5	1.96	0.16	1.96	0.16

ARDL Cointegration Test[2]

Test	F-statistic	Lower Bound	Upper Bound
Bounds test	3.45	2.62	3.79

Cote d'Ivoire

Johansen Cointegration Tests

Number of Hypothesized Cointegrating Equations[1]	Trace Statistic	p-value	Max-Eigen Statistic	p-value
None *	114.95	0.00	47.57	0.01
At most 1	67.37	0.08	32.31	0.08
At most 2	35.06	0.44	23.60	0.15
At most 3	11.46	0.95	7.58	0.93
At most 4	3.88	0.91	3.86	0.87
At most 5	0.02	0.89	0.02	0.89

ARDL Cointegration Test[2]

Test	F-statistic	Lower Bound	Upper Bound
Bounds test	14.69	2.62	3.79

Togo

Johansen Cointegration Tests

Number of Hypothesized Cointegrating Equations[1]	Trace Statistic	p-value	Max-Eigen Statistic	p-value
None *	110.59	0.00	46.67	0.01
At most 1	63.92	0.14	29.40	0.16
At most 2	34.52	0.47	22.33	0.20
At most 3	12.19	0.93	9.79	0.76
At most 4	2.39	0.99	2.37	0.98
At most 5	0.02	0.89	0.02	0.89

ARDL Cointegration Test[2]

Test	F-statistic	Lower Bound	Upper Bound
Bounds test	3.59	2.62	3.79

Notes:

[1] The small sample cointegration tests also indicate the presence of one cointegrating vector for each estimation.

[2] Asymptotic critical value bounds obtained from Pesaran and Pesaran (1997), under the specification of intercept and no trend for k=5, at the 5 % significance level. The tests suggests presence of cointegration.

Table B4. Panel Cointegration Tests[1,2]

Standard

Test	Statistic
Panel v-stat (non-parametric)	-0.21
Panel rho-stat (non-parametric)	0.47
Panel PP-stat (non-parametric)	-1.86 **
Panel ADF-stat (parametric)	-1.01
Group rho-stat (non-parametric)	0.77
Group PP-stat (non-parametric)	-2.39 **
Group ADF-stat (parametric)	-1.60 *

Time Demeaned Variables

Test	Statistic
Panel v-stat (non-parametric)	0.72
Panel rho-stat (non-parametric)	-0.63
Panel PP-stat (non-parametric)	-3.03 **
Panel ADF-stat (parametric)	-2.14 **
Group rho-stat (non-parametric)	0.38
Group PP-stat (non-parametric)	-2.87 **
Group ADF-stat (parametric)	-2.09 **

Notes:

[1] All tests are one sided and the statistics are normally distributed N(0,1) under the null hypothesis of no cointegration. Under the alternative hypothesis, the panel variance statistic diverges to positive infinity (and therefore the right tail is used to reject the null hypothesis) and the other statistics diverge to negative infinity (left tail used to reject the null).

[2] One and two asterisks indicate rejection at 10 and 5 percent respectively.

Table B5. Diagnostic Tests for the Residuals[1]

Test	*Benin*		*Burkina Faso*		*Côte d'Ivoire*		*Mali*		*Niger*		*Senegal*		*Togo*		*WAEMU*	
							Johansen									
Vector AR 1-2 test	F(72,43)	1.16	F(72,11)	1.34	F(72,43)	1.32	F(72,38)	1.65	F(72,49)	0.81	F(72,38)	1.33	F(72,38)	1.59	F(72,49)	0.93
		[0.30]		[0.31]		[0.16]		[0.05]		[0.79]		[0.17]		[0.06]		[0.62]
Vector Normality test	Chi-square(12)	21.22	Chi-square(12)	37.71	Chi-square(12)	19.48	Chi-square(12)	18.45	Chi-square(12)	18.48	Chi-square(12)	18.66	Chi-square(12)	10.91	Chi-square(12)	20.14
		[0.05]		[0.00]		[0.08]		[0.10]		[0.10]		[0.10]		[0.54]		[0.62]
Vector hetero test	Chi-square(252)	236.33	Chi-square(252)	494.87	Chi-square(252)	265.82	Chi-square(252)	240.74	Chi-square(252)	248.97	Chi-square(252)	269.79	Chi-square(252)	255.42	Chi-square(252)	257.13
		[0.75]		[0.61]		[0.26]		[0.68]		[0.54]		[0.21]		[0.43]		[0.62]
							ARDL Model Diagnostics									
Serial correlation	Chi-square(1)	0.35	Chi-square(1)	4.38	Chi-square(1)	0.01	Chi-square(1)	1.47	Chi-square(1)	0.07	Chi-square(1)	5.82	Chi-square(1)	1.99	Chi-square(1)	0.08
		[0.55]		[0.04]		[0.92]		[0.70]		[0.80]		[0.02]		[0.16]		[0.78]
Functional form	Chi-square(1)	0.04	Chi-square(1)	7.51	Chi-square(1)	0.27	Chi-square(1)	3.40	Chi-square(1)	0.07	Chi-square(1)	1.36	Chi-square(1)	1.59	Chi-square(1)	5.33
		[0.84]		[0.01]		[0.60]		[0.07]		[0.80]		[0.24]		[0.21]		[0.02]
Normality	Chi-square(2)	0.29	Chi-square(2)	1.68	Chi-square(2)	1.07	Chi-square(2)	1.69	Chi-square(2)	0.39	Chi-square(2)	0.11	Chi-square(2)	1.34	Chi-square(2)	1.53
		[0.89]		[0.43]		[0.59]		[0.43]		[0.83]		[0.94]		[0.51]		[0.47]
Heteroskedasticity	Chi-square(1)	3.77	Chi-square(1)	0.86	Chi-square(1)	1.87	Chi-square(1)	1.78	Chi-square(1)	0.12	Chi-square(1)	0.02	Chi-square(1)	0.86	Chi-square(1)	0.73
		[0.05]		[0.35]		[0.17]		[0.18]		[0.73]		[0.89]		[0.35]		[0.39]

Notes:
[1] P-values are indicated in brackets.

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