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The Convergence Dynamics of a Transition Economy: The Case of the Czech Republic

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Abstract

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In this paper we develop a two-country dynamic general equilibrium model by means of which we seek to explain the long-run paths of a converging emerging market economy. We borrow a paradigm from the New Open Economy Macroeconomics literature and amend it to address specific features such as initial asymmetry in development and size of economies as well as different speed of capital accumulation. Using a calibration of productivity and deep parameters for the Czech economy we demonstrate the ability of the model to consistently replicate dynamics in key macroeconomic variables that are essential inputs for commonly used "gap models" in monetary policy routine. Based on the calibration we draw implications for future convergence of the Czech economy.

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I. INTRODUCTION

One of the most challenging tasks for policy makers in an emerging market converging open economy is to correctly judge and to predict the dynamics of the endogenously determined key policy relevant variables. The long term trajectories of these variables constitute the ‘equilibrium’ trends and hereby, anchor the monetary policy models in practice. Actual deviations from these trends represent the rationale of “gap forecast models” and that are currently used by many inflation targeting countries, among them Canada, the Czech Republic, Norway, or Romania (see Coats et al., 2003). Therefore, a coherent explanation of the trends in a theoretic framework and their simulations of the future are of utmost importance for appropriate policy implementation. To contribute to this task, the paper analyzes the potential of two-country dynamic general equilibrium modeling initiated by the so-called New International Macroeconomic (*henceforth NIM*). The paper also offers a promising extension to assess the convergence of emerging market economies.

The NIM framework has become increasingly popular in recent past. The reason is that it is able to provide a rigorous microfoundation for a bulk of observations, which are puzzling from the perspective of the standard DSGE models (such as persistent deviations from the PPP or low volatility in the relative price of nontraded goods). Thus, this type of models may be a suitable tool not only to explain certain puzzling phenomena for academic curiosity, but also for policy purposes. Typical features of the NIM framework include monopolistic competition, heterogeneity of production entities and trade self-selectivness, as in Melitz (2003). The framework is used, for example, by Ghironi and Melitz (2005) to explain international business-cycle dynamics, by Naknoi (2006) to decompose real exchange rate movements, by Bergin and Glick (2005) to study the behavior of price dispersion during episodes of international economic integration, or by Bergin and Glick (2006) to explain low degree of volatility in the relative price of nontraded goods. Since the NIM framework appears to have a better microfoundation than standard open-economy dynamic general equilibrium models, it also seems to be more promising as a tool for welfare evaluation of policy regimes. Naknoi et al. (2005) use the NIM framework to compare benefits and costs of fixed versus flexible exchange rate regimes and Baldwin and Okubo (2005) integrate the NIM approach to a New Economic Geography model and derive a set of useful normative assessments and positive political-economy predictions of economic integration.

Recently, Bayoumi et al. (2004) constructed a DSGE model with the NIM features and calibrated it for a transition economy (the Czech Republic). This is an important step, since macroeconomic dynamics of transition economies are even more puzzling from the perspective of standard DSGE models than in the case of advanced economies. Unfortunately, the model of Bayoumi et al. (2004) does not address any specific transition feature and thus its applicability for convergence projections or policy prescriptions may be limited.[1] Nevertheless, the NIM framework may still be a promising tool for explaining the pace of transition countries if the framework is coupled with structural issues relevant for transition economies. Structural stories are better suited for understanding important phenomena of external position of emerging market economies and can provide a more solid basis for understanding, explaining, and possibly forecasting the real exchange rate development.

Recently, many authors suggest that quality improvements might play a role among determinants of real exchange rate appreciation of transition economies. Also, empirical studies reflect the symptoms of quality investments in transition economy. Studies appealing to quality driven real exchange rate for tradables, such as Broeck and Slok (2006) or Égert and Lommatzsch (2004) find that quality improvements of tradable goods in catching-up economies is a source of the real exchange rate appreciation. Also, in the case of the Czech Republic, Podpiera (2005) shows that large gains in exported volumes were associated with improving terms of trade, which, in turn implies quality improvements. At the same time, quality improvements are not accounted for by the statistical offices in transition economies, such as the Czech Republic, Hungary, Poland, Slovakia, or Slovenia (see Ahnert and Kenny (2004) for a comprehensive survey). In addition, according to the assessment of the quality bias of consumer price index in the Czech Republic, the inflation overstatement could have been as high as 5 percentage points a year in the first decade of economic transformation (see for instance Hanousek and Filer, 2004). Therefore, quality-unadjusted price indexes might be well responsible for a substantial part of the pace of the real exchange rate development in a transition economy.

In order to capture the key features of emerging market economy and simulate the transition dynamics in the key macroeconomic variables in the consistent framework of general equilibrium we use a deterministic model in aggregate variables. We build our model on postulates developed by Ghironi and Melitz (2005) and extend the framework. NIM models such as by Ghironi and Melitz (2005), can give only a limited insight in understanding the external position of emerging market economies. The reason is that the production side operates with one production factor (labor) only. This feature does not address additionally important factors of production capacities. Mainly, in this paper we argue that for successful replication of the pace of relative prices of goods produced in the converging economy, it is necessary to enrich the production structure by what we call *investments to quality*. In addition, the model allows for non trivial cross-border assets ownership, i.e., modeling foreign direct and portfolio investment. Our model is solved for the transition dynamics of a transition country, which converges to its more advanced counterpart. Thus, it contrasts with the standard DSGE models, which aim at explaining deviations from exogenously given long-run trends.

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[1] Thus, it is not surprising that the model is not able to replicate the significant observed pace of the real exchange rate appreciation in Central and Eastern European countries.

In calibrating the model for the Czech economy - by involving a (continuous) drop in fixed exporting costs and the rest of parameters - we succeed in replicating the development in all endogenously modeled variables, such as real exchange rate, consumption and investment to GDP ratios, foreign direct and other investment balances, exports, imports, and trade balance to GDP ratio, as well as real return on assets. We also conclude that, based on our simulation, there will be an expected policy tightening in the Czech real interest rate compared to the EU15. This is expected to align the Czech excess real return with the trend trajectory in the future.

The rest of the paper is organized as follows: Section II describes some relevant stylized facts and Section III presents the two-country model. Section IV is on calibration and explains dynamics of some of the endogenous variables, and Section V presents the conclusions. Section VI contains an Appendix with a detailed derivation of the model, its reformulation using a recursive form and discusses numerical techniques used to solve the model.

II. Some stylized facts

Transition economies need to develop a fully functioning market economy. From the economics point of view, the prime policy interest is mostly focused on the foundation of private ownership and full liberalization, i.e., price, current account, and financial account liberalization. Undoubtedly changes in the economic environment must be complemented by building up of the political, legal, and institutional infrastructure. These measures are meant to facilitate economic convergence and foster long term sustainable growth.

The evidence on the positive effects of current account liberalization is largely documented in the literature. Fischer et al. (1996)[2] established a positive link between the cumulative liberalization and the output dynamics in a panel of twenty transition economies. Similarly, Sachs (1996) confirms the aforementioned relation by employing the reform index constructed by the EBRD. Kaminski et al. (1996) also report that among other factors, liberalization and openness to international trade were the key factors underpinning the export performance in a large sample of transition economies.

In relation to income differentials elimination among less and more developed countries, liberalization is often cited as a prominent factor. For instance Ben-David (1993) studied the income differentials within the European Economic Community and concludes that the income disparities started to diminish only after removal of the trade barriers among member countries. Similar empirical support can be found in the literature in the case of the financial account. Henry (2003) provides sample evidence on eighteen emerging markets

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[2] Fischer et al (1996) used De Melo et al. (1996) liberalization index, which comprises degree of liberalization of internal markets, of external markets, and of private sector entry

and shows that following capital (financial) account liberalization, the cost of capital declines and both, the capital stock growth as well as output growth per worker, accelerate.

The external liberalization generates in small emerging market economy various effects during its economic development, enables and promotes flows of capital, boosts capital accumulation in the home economy, affects selectivity to goods trade, and creates pressures on terms of trade and real exchange rate. These aspects remain, however, largely unaddressed in traditional models of Open Economy Macroeconomics. Most importantly, the assumption of the purchasing power parity condition in tradable goods, see for instance Edison and Pauls (1993) or Obstfeld and Rogoff (1995), renders these models inapplicable for explaining of the transition economy dynamics. The empirical evidence for emerging market economies documents significant violation of the PPP assumption. For evidence of the trend development of the real exchange rate for tradables in the Central and Eastern European transition economies, see Cincibuch and Podpiera (2006) and for evidence of small Harrod-Balassa-Samuelson (HBS) type of convergence, see for instance Mihaljek and Klau (2006).

The trend real exchange rate appreciation (also in tradables) observed in the majority of CEET economies constitutes a puzzle and renders the standard models incomplete for explanation of the transition economy dynamics (see Cincibuch and Podpiera (2006) for recent empirical evidence). Indeed, the observed inconstancy of the real exchange rate for tradables seems to be in contradiction with the view of the traditional models of Open Economy Macroeconomics, where the purchasing power parity condition in tradable goods is a standard assumption (Edison and Pauls, 1993; Obstfeld and Rogoff, 1995).

The New Open Economy Macroeconomics of two-country models, such as by Ghironi and Melitz (2005), provides a solid base for tackling some of the issues, for instance that of inconstancy of the real exchange rate for tradables and endogenously determined foreign trade. It only allows for an endogenous short-run deviation from purchasing power parity, i.e. for an endogenously generated HBS effect. Indeed, the empirical evidence of small HBS type of convergence dominates the recent literature (Mihaljek and Klau, 2006; Flek et al., 2003). However, the permanent, equilibrium trend in real exchange rate remains unaddressed. As already noted, the trend equilibrium in the real exchange rate is also a puzzle for the alternative stream of two-country modeling in recent literature. A standard DSGE model, even if applied to a transition country with various real and nominal rigidities (Bayoumi et al., 2004), does not predict a long-run appreciation of the real exchange rate, despite its relatively rich structure. In this regard, our approach offers a promising amendment.

III. THE TWO-COUNTRY MODEL[3]

The two countries are modeled in a discrete time that runs from zero to infinity. The home country is populated by a representative competitive household who has recursive preferences over discounted streams of period utilities. The period utility is derived from consumption. A similar household inhabits the foreign country. Production takes place in heterogeneous production entities called firms.[4]

A. Firms

There is a continuum of firms in the domestic country. In each period there is an unbounded mass of new, ex-ante identical, entrants. Firms ex-post differ by the total factor productivity: upon entry, it draws a shock z from a distribution $G(z)$, which has the support on $[z_L, z_U)$ with $0 \leq z_L < z_U < \infty$. This shock determines the idiosyncratic part of the firm productivity. At the end of each period, there is an exogenous probability that a firm is hit by an exit shock δ, which is assumed to be independent on aggregate as well as individual states. Hit firms shut down.

The production function maps two inputs into two outputs. The one of the input is fixed and we label it as 'capital', the second of the input is variable and is labeled as 'labor'. The variable input – labor – is available in inelastic supply in each country and is immobile between countries.

One of the output is quality h and if the firm j uses k_j units of capital, then the quality of its product is given simply as $h_j = k_j$. Capital investment can be thus considered as an improvement in quality. The second output is the physical quantity of produced goods x. The production function is given as follows: $x_{jt}=z_jA_t\,\ell(l_{jt},k_j)$. The production function ℓ is strictly increasing in the first argument (labor), but strictly decreasing in the second argument.[5] This implies that investments into quality increase the needed labor inputs to produce physical quantities. One may think that the production of a better good requires more labor or more skilled labor. Thus, quality investment is costly for two reasons: first, it requires fixed input k_j, second more labor is required to produce better goods.

[3] This section presents the core of the two-country model. A more detailed discussion is provided by Brůha and Podpiera (2007a).

[4] The production entities are called firms, however, since we aim at understanding equilibrium convergence of a transition economy, which is likely to experience a significant change in production structure, it would be appropriate to associate production entities with *production projects*.

[5] We require that the function ℓ is strictly decreasing in the capital. If the function ℓ does not depend on the capital, the linearity of h_j in k_j will imply endogenous growth, as in Young (1998) or Baldwin, Forslid (2000). Although it may be interesting to investigate the model under the endogenous growth paradigm, this draft avoids this issue to concentrate on the potential of the *NIM* framework to explain convergence experience of some emerging economies.

The production of the physical quantities is increasing in the level of firm total factor productivity $A_t z_j$, which has two components: (a) idiosyncratic component z_j, which is i.i.d. across firms and which follows distribution $G(z)$ introduced above, and (b) the common component A_t. The total factor productivity A_t pertains to the ownerships: firms owned by the domestic household enjoy at time t the productivity A_t^H, while firms owned by the foreign household enjoy the productivity A_t^F. The productivity does not depend on the location of production or on the time of entry (the time of entry is henceforth called *vintage*) of firms.

We assume that the final output of the firm is given by the product of quality and quantity: $q_{jt}=h_j x_{jt}$ and that this final quality-quantity bundle is what is sold at the market. This assumption reflects the nowadays standard approach of growth theoreticians, for example Young (1998). Thus, the production of the final bundle can be described as $q_{jt}=z_j A_t f(k_j,l_{jt})$, where f is given as $f(k_j,l_{jt})\equiv k_j \ell(l_{jt},k_j)$. We assume that the final bundle production function is increasing in both arguments and is homogenous of degree one. This places some restrictions on the quantity production function ℓ; the most important restriction is that ℓ should be homogenous of degree zero.

The quality investment is a fixed factor, set at the time of entry, while labor can be freely adjusted. Given a realization of the productivity shock z_j, the probability of the exit shock δ, and a chosen production plan, the value of a firm is determined by the stream of discounted profits.

Since the presented model involves several kinds of goods and firms, we will use indexes to distinguish among them. To make reading of the paper easier, we introduce the following convention. Firms differ by location, ownerships, and vintage. Location of firms is distinguished by superscripts d and f, where the former stands for the *domestic* and the latter for the *foreign* country. Firms owned by household from the foreign country are denoted by the superscript *, while the ownership of domestic household is given no special superscript. The vintage is denoted by Greek letters τ, σ, while the real time is denoted by the Latin character t, υ.

Firms produce differentiated goods, which are labeled as follows: the good produced by the firm located in the country in which the good is also sold is denoted by the superscript d, while goods imported (produced in the non-resident country) are denoted by the superscript m. The sale market is denoted by the superscript *. Namely, goods consumed by the domestic household are without superscript, while goods consumed by the foreign household do have it.

Similarly, p_{jt}^d will denote the price of a good produced by a firm j located in the domestic country at time t sold to the domestic market, p_{jt}^m is the price of a good j imported to the domestic market from the foreign country, while p_{jt}^{m*} would be the price of a good from the domestic country to the foreign household. We further assume that prices are denominated in the currency of the market of sale.

According to the introduced convention, $\Pi_{j\tau t}{}^{d}$ denotes a t-period profit of the firm located in the domestic country of vintage τ and owned by the domestic household. The nominal profit $\Pi_{j\tau t}{}^{d}$ is given as follows:

$$\Pi^{d}_{j\tau t} = \left[\kappa_{jt} p^{d}_{jt} + (1-\kappa_{jt}) \frac{s_t}{1+\mathbf{t}} p^{m*}_{jt} \right] A^{H}_{t} z_j f(k_j, l_{jt}) - \omega_t l_{jt},$$

where $0 \leq \kappa_{jt} \leq 1$ is a share of product q_{jt} sold in the domestic market, s_t being a nominal *foreign exchange rate*, and $\mathbf{t} \geq 0$ represents unit iceberg exporting costs. Firms of different vintages and different ownership have different levels of investment into quality; that is why $\Pi_{j\tau t}{}^{d}$ will be naturally different along these dimensions. Similar definitions apply to the remaining types of firms as well.

Firms may export only if special fixed costs are sunk. If a firm at the time of entry decides to sink the fixed export costs, then it becomes eligible to export in all subsequent periods, otherwise it is for all periods not eligible to export. The export decisions of the *eligible* firms are taken on a period-by-period basis. Thus an eligible firm may decide not to export in a given period.

Unit iceberg exporting costs $\mathbf{t}$ represents transportation costs and policy barriers such as tariffs, while the fixed export eligibility costs may represent expenditures associated with acquiring necessary expertise such as legal, business, or accounting standards of the foreign market. It is worth to note that the unit iceberg costs $\mathbf{t}$ is related to the degree of trade frictions, while the ratio c^e/c^n speaks for the trade openness. Obviously, non-eligible firms have $\kappa_{jt} \equiv 1$ regardless of the state of the world.

We assume that nominal investment costs take the following form: $P_t(k+c^{\xi})$, $\xi \in \{e, n\}$, where P_t represents the 'ideal' price index, which is the price of both consumption and investment goods. We assume that:

$$c^e > c^n > 0,$$

where the superscript refers to eligibility, i.e. *e – eligible* or *n – noneligible*: eligible firms pay larger fixed costs. This implies – as in Melitz (2003) – that in equilibrium there is an endogenous cut-off productivity value $\bar{z}$, such that firms with lower idiosyncratic productivity $z_j < \bar{z}$ will not invest to become eligible, while firms with a sufficiently high productivity level $z_j \geq \bar{z}$ will do.

We assume that firm's manager maximizes the expected stream of discounted profits. The discounting respects the ownerships. Thus the value of the profit stream of the firm of vintage τ, enjoying the idiosyncratic productivity level z_j and owned by the domestic household in real terms is:

$$V_\tau^d(z_j) = \max_{\xi, k, \{l_\tau\}} \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu_\tau^t \frac{\Pi_{j\pi}^d}{P_t} - (c^\xi + k) \quad (1)$$

where $\frac{\Pi_{j\pi}^d}{P_t}$ is the t-time real profit of a firm of vintage τ, enjoying the productivity level j under the optimal production plan (derived later in Subsubsection III.A), and the effective discount factor is given as $(1-\delta)^{\tau-t}\mu_\tau^t$, where μ_τ^t is the marginal rate of intertemporal substitution between dates τ and t. The rate of the intertemporal substitution is defined in Subsection III.B.

The value of the firm owned by the foreign household is defined analogously with the exception that the marginal rate of the intertemporal substitution is taken from the perspective of the foreign household.

To summarize the sequencing, the timing proceeds first with the domestic and foreign households' decision about a number of new entrants in both countries. Then, each new entrant draws a productivity level from the distribution G and the owner decides the amount of investment into quality and whether to invest for export eligibility. Then labor demand and production (of both entrants and incumbents) take place.[6] At the end of the period, some firms experience the exit shock and shut down.

Even firms located in the same country and owned by the same household differ along two dimensions: idiosyncratic productivity variance z_j and vintage τ. The ownership within each country affects the amount of investment into quality, since both households have different rates of the intertemporal substitution along the transition path. Likewise the vintage affects incentives to invest. This implies that firms of different vintages and ownership will invest different amounts into quality, even if they experience the same idiosyncratic productivity level. Therefore we shall define the time-varying distribution measure over firms: $\Gamma_t^d(j,\tau)$ for the firms in the home country owned by the domestic household and the star version $\Gamma_t^{d*}(j,\tau)$ will denote the analogous measure for the firms owned by the foreign household. The counterparts of firms located in the foreign country are denoted by $\Gamma_t^f(j,\tau)$, and $\Gamma_t^{f*}(j,\tau)$. The superscript convention applied to the distributions follows the one applied to firms.

[6] The capital is firm specific and the model lacks the usual one-lag time-to-build assumption. The time-to-build is not needed in our model since we aim at long-run dynamics, not at short-run fluctuations.

Market structure

The final good Q in home country[7] is composed of a continuum of intermediate goods, some of them are produced in the home country and some are imported. There is an imperfect substitution among these goods. The parameter $\theta > 1$ measures substitution among goods. The limit case $\theta \to \infty$ implies perfect substitution and hence perfect competition. The aggregate good in the domestic country is defined as:

$$Q_t = \left(\sum_{\xi \in \{d, d^*\}} \int_{\Omega^\xi} (q_{jt}^d)^{\frac{\theta-1}{\theta}} d\Gamma_t^\xi(j,\tau) + \sum_{\xi \in \{f, f^*\}} \int_{\Omega_e^\xi} (q_{jt}^m)^{\frac{\theta-1}{\theta}} d\Gamma_t^\xi(j,\tau) \right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

where, q_j is the output of the firm j, Ω^d denotes the set of products of firms located in the domestic country and owned by the domestic household and Ω^{d^*} denotes the set of products of firms located in the domestic country and owned by the foreign household. Analogously, for sets of firms located in the foreign country we have: Ω^f , Ω^{f^*} . If a set is labeled by the subscript e, it reads as a subset of eligible firms: thus $\Omega_e^{f^*} \subset \Omega^{f^*}$ is the subset of goods produced by *eligible* firms owned by the foreign household located in the foreign country.[8] The final good in the foreign country is defined similarly. The market structure implies the following definition of the aggregate price index:

$$P_t = \left(\sum_{\xi \in \{d, d^*\}} \int_{\Omega^\xi} (p_{jt}^d)^{1-\theta} d\Gamma_t^\xi(j,\tau) + \sum_{\xi \in \{f, f^*\}} \int_{\Omega_e^\xi} (p_{jt}^m)^{1-\theta} d\Gamma_t^\xi(j,\tau) \right)^{\frac{1}{1-\theta}},$$

where p_{jt} is the price of products of firm j at time t.

The CES market structure implies that the demand for individual producer's products in the domestic market satisfies:

$$q_{jt}^d = \left(\frac{p_{jt}^d}{P_t} \right)^{-\theta} Q_t,$$

[7] The final good is consumption as well as investment good, so that Q can be interpreted as domestic absorption.

[8] It holds that $q_j^d \in \Omega^d$ or $p_j^d \in \Omega^{d^*}$ and $q_j^{m^*} \in \Omega_e^d$, $q_j^{m^*} \in \Omega_e^{d^*}$, but $q_j^{m^*} \notin \Omega^d \setminus \Omega_e^d$ nor $q_j^{m^*} \notin \Omega^{d^{*}} \setminus \Omega_e^{d^*}$.

$$q_{jt}^{m} = \left(\frac{p_{jt}^{m}}{P_t} \right)^{-\theta} Q_t .$$

Analogous formulae apply to the demand for the products in the foreign market as well.

Optimal plans

The optimal production and investment plans are derived using backward induction. We present the derivation for a firm located in the domestic country and owned by the domestic household. The reader can then similarly derive optimal plans for other types of firms.

Thus, let us assume the problem of maximizing the value of a firm, under given location, ownership, and sunk investments. Since there are no labor adjustment costs, labor decisions are made on a period-by-period basis. Standard results of monopolistically competitive pricing under the CES market structure suggest that prices are set as a mark-up over marginal costs. Nevertheless, an important issue here is that the standard assumption of symmetric equilibrium is given up: firms enjoying identical productivity levels z_j and identical capital levels k_j are supposed to price identically, but firms with different characteristics charge different prices $\{p_{jt}^{d}, p_{jt}^{m*}\}$, and obviously produce different output q_{jt}.

Simultaneously with prices, firms also decide κ_j. Brůha and Podpiera (2007a) show that - for a general neoclassical production function *f* - eligible firms would produce goods for both markets, i.e., $0 < \kappa < 1$ for an eligible firm. This part of the paper derives the optimal production plan for such a general production function. See Appendix A.1 for the derivation of the model for the specific parameterization used in calibration and policy scenario. We denote *real* quantities by the *Monotype Corsiva* scripts: $\mathcal{P}_{j\tau t}^{d} \equiv \prod_{j\tau t}^{d} / P_t$ is the real profit of a domestic firm and $\mathcal{W}_t \equiv w_t / P_t$ is the real domestic wage.

Now, let us take the perspective of a non-eligible firm of vintage τ and productivity level A_t^H. Its real profit $\mathcal{P}_{j\tau t}^{d}$ in a period *t* is given - conditional on non-eligibility status, aggregate productivity, idiosyncratic productivity z_j, - as a solution to the following:

$$\mathcal{P}_{j\tau t}^{d} = \max_{l_{jt}} \{ \tfrac{p_{jt}}{P_t} A_t^H z_j f(k_j, l_{jt}) - \mathcal{W}_t l_{jt} \} = \tag{3}$$

$$= \max_{l_{jt}} \{ [A_t^H z_j f(k_j, l_{jt})]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathcal{W}_t l_{jt} \} .$$

The second row of Expression in (3) and in the subsequent expression follows from the CES market structure. Similarly, the real profit of an eligible firm $\mathcal{P}^{d}_{j\tau t}$ of vintage τ in a period *t* is given by:

$$\mathcal{P}^{d}_{j\tau t} = \max_{l_{jt}} \{[\kappa_{jt} \tfrac{p_{jt}}{P_t} + (1-\kappa_{jt}) \tfrac{\eta_t}{1+\mathrm{t}} \tfrac{p^*_{jt}}{P^*_t}] A^H_t z_j f(k_j, l_{jt}) - \mathcal{W}_t l_{jt}\} = \qquad (4)$$

$$= \max_{l_{jt}} \{[\kappa_{jt} Q_t^{\frac{1}{\theta}} + (1-\kappa_{jt}) \tfrac{\eta_t}{1+t} Q_t^{*\frac{1}{\theta}}][A^H_t z_j f(k_j, l_{jt})]^{\frac{\theta-1}{\theta}} - \mathcal{W}_t l_{jt}\}.$$

The expected present value of profit streams is as follows:

$$\mathcal{P}^{d\xi}_{j\tau} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \mathcal{P}^d_{j\tau t},$$

where $\xi \in \{n, e\}$. The expected present values depend on idiosyncratic productivity z_j, invested capital k_j, and the future path of productivities, real wages, and demands.

The optimal investment decision of an eligible firm located in the domestic country and owned by the domestic household, which enjoys a productivity level z_j, maximizes the value of the firm, which is given as

$$V^{de}_\tau(k_j \mid z_j) = \mathcal{P}^{de}_{j\tau}(z_j, k_j, \{\mathcal{W}_{t+\tau}, Q_{\tau+t}, Q^*_{\tau+t}, A^H_{\tau+t}, \eta_{\tau+t}\}^\infty_{t=0}) - (c^e + k_j) \qquad (5)$$

and similarly for a non-eligible firm:

$$V^{dn}_\tau(k_j \mid z_j) = \mathcal{P}^{dn}_{j\tau}(z_j, k_j, \{\mathcal{W}_{t+\tau}, Q_{\tau+t}, A^H_{\tau+t}\}^\infty_{t=0}) - (c^n + k_j). \qquad (6)$$

Maximization of $V^{de}_\tau(k_j \mid z_j)$ (resp. $V^{dn}_\tau(k_j \mid z_j)$) yields the optimal demand for quality investment (capital) for eligible (resp. non-eligible) firms, and the value of a firm is:[9]

$$\mathrm{V}^{d\xi}_\tau(z_j) = \max_{k_j \geq 0} V^{d\xi}_\tau(k_j \mid z_j),$$

where $\xi \in \{n, e\}$. The value functions $\mathrm{V}^{dn}_\tau(z_j)$, $\mathrm{V}^{de}_\tau(z_j)$ implicitly define the cut-off value $\bar{z}$, which is the least idiosyncratic shock, which makes the export-eligibility investment profitable.[10]

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[9] There are two distinct values functions, the one with the arguments V(k_j | z_j) and the other with z_j. The first function denotes the expected value of a firm, which enjoys the productivity level z_s and invest k_j, the second function is the value under the optimal investment and therefore depends on the productivity z_j only.
The functions are distinguished by fonts: the second function is typed using the bold font.

Thus it is defined as:

$$\bar{z}_{\tau}^{d} = \min_{z_j}(\mathrm{V}_{\tau}^{de}(z_j) \geq \mathrm{V}_{\tau}^{dn}(z_j)).$$

The value of a firm is given by:

$$\mathrm{V}_{\tau}^{d}(z_j) = \max_{\xi \in \{n,e\}} \mathrm{V}_{\tau}^{d\xi}(z_j) = \begin{cases} \mathrm{V}_{\tau}^{de}(z_j) \ \ if \ \ z_j \geq \bar{z}_{\tau}^{d} \\ \mathrm{V}_{\tau}^{dn}(z_j) \ \ if \ \ z_j < \bar{z}_{\tau}^{d} \end{cases},$$

and the expected value of a new entrant, owned by the domestic household, of vintage τ, $\mathbf{V}_{\tau}^{d}$ is:

$$\mathbf{V}_{\tau}^{d} = \int_{z_L}^{z_U} \mathrm{V}_{\tau}^{d}(z) G(dz). \tag{7}$$

This completes the backward induction.

The, just derived, optimal production plan naturally induces a measure over firms. We denote $\tilde{P}^{d}_{\tau,t}$ as the *t*-time expected profit of a domestically-owned firm, which enters in time τ, expectation being taken with respect to that measure $\tilde{\mathcal{P}}_{\tau t}^{d} = \int_{z_L}^{z_U} \mathcal{P}_{\tau t j}^{d} G(dz_j)$ and $\tilde{c}_{\tau}^{d}$ the expected investment costs under such measure. Then:

$$\mathbf{V}_{\tau}^{d} = \sum_{\sigma \geq 0} \mu_{\tau}^{\tau+\sigma} (1-\delta)^{\sigma} \tilde{\mathcal{P}}_{\tau,\tau+\sigma}^{d} - \tilde{c}_{\tau}^{d}.$$

Similarly, one can express the expected real investment costs as:

$$\tilde{c}_{\tau}^{d} = G(\bar{z}_{\tau}^{d}) c^{n} + (1 - G(\bar{z}_{\tau}^{d})) c^{e} + \int_{z_L}^{\bar{z}_{\tau}^{d}} k_j^{opt,n} G(dz) + \int_{\bar{z}_{\tau}^{d}}^{z_U} k_j^{opt,e} G(dz).$$

The first two terms correspond to the expected fixed costs, while the last two terms correspond to the expected costs of capital investment. The expected investment costs differ across locations, vintages and ownerships and this is because (i) the cut-off values differ across these dimensions too (as was already described) and (ii) these dimensions also vary the optimal amount of invested capital $k_j^{opt,e}$ and $k_j^{opt,n}$. Therefore – in accordance to the convention introduced above – we will denote expected investment costs in the domestic

[10] It is worth to mention that the cut-off value differs across locations and vintages (since firms located in different location or firms appeared in different times face different relative prices) and across ownership (because the marginal rate of substitution is - in general - different).

country from the perspective of the domestic household $\tilde{c}_t^d$ and from the perspective of the foreign household $\tilde{c}_t^{d*}$. The counterpart of these costs in the foreign country will be denoted as $\tilde{c}_t^f$ (from the perspective of the domestic household) and as $\tilde{c}_t^{f*}$ (when foreign household's perspective is taken).

B. Household behavior

The home country is populated by a representative competitive household who has recursive preferences over discounted streams of period utilities. The period utilities are derived from consumption of the aggregate good. Leisure does not enter the utility and so labor is supplied inelastically. The aggregate labor supply in the domestic country is *L*, while L^* is the aggregate labor supply in the foreign country. Households can trade bonds denominated in the foreign currency.

The domestic household maximizes

$$\max U = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

subject to

$$\begin{aligned} B_t &= (1+r_{t-1}^*)B_{t-1} + \frac{1}{\eta_t}\left(-C_t + \frac{w_t}{P_t}L\right) + \frac{1}{\eta_t}\left(\sum_{\sigma\le t}(1-\delta)^{t-\sigma} n_\sigma^d \frac{\tilde{\Pi}_{\sigma,t}^d}{P_t} - \tilde{\chi}(n_t^d)\right) + \\ &+ \left(\sum_{\sigma\le t}(1-\delta)^{t-\sigma} n_\sigma^f \frac{\tilde{\Pi}_{\sigma,t}^f}{P_t^*} - \hat{\chi}(n_t^f)\right) - \frac{\Psi_B}{2}B_t^2 + \mathcal{T}_t \end{aligned} \tag{8}$$

where B_t is the real bond holding of the domestic household. Bonds are denominated in the foreign currency by our convention; however, since the model is deterministic, this assumption is completely innocent. C_t denotes consumption and r_{t-1}^* is the real interest rate of the internationally traded bond. Ψ_B represents adjustment portfolio costs as in Schmitt-Grohe and Uribe (2003) to stabilize the model[11] and $\mathcal{T}_t$ is the rebate of these costs in a lump-sum fashion to the household.

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[11] In a strict sense, the model is stable even without adjustment costs (i.e. under $\Psi_B = 0$). The model is deterministic and therefore it would not exhibit the unit-root behavior even under $\Psi_B = 0$. On the other hand, if $\Psi_B = 0$, then the model would exhibit the steady state dependence on the initial asset holding and we do not like such a model property. Therefore we use the nontrivial adjustment costs $\Psi_B > 0$ to give up the dependence of the steady state on the initial asset holding.

The momentary utility function *u(C)* is assumed to take the conventional constant-relative-risk-aversion form: $u(C)=\frac{C^{1-\varepsilon}}{1-\varepsilon}$, with the parameter of intertemporal substitution *ε*. As usually, the case of *ε =1* is interpreted as *log(C)*.

The number of new domestically located entrants owned by the domestic household in time *t* is n_t^d, while $\tilde{\chi}(n_t^d)$ presents the investment cost associated with entry of n_t^d entrants. These costs are given as follows:

$$\tilde{\chi}(n_t^d)=\tilde{c}_t^d n_t^d+\frac{\Psi_d}{2}(n_t^d)^2.$$

The first term is obvious – it is the expected[12] investment cost (where the expectation is taken with respect to the measure induced by the optimal production plan). The second term may be interpreted as adjustment costs (e.g. due to limited supply of skills needed to run firms, such as legal experts), and its purpose is to mitigate knife-edge conditions on household investments. These adjustment costs are assumed to be rebated by the lump-sum fashion to households (they are included in $\mathcal{T}_t$).

Similarly, n_t^f denotes number of new entrants in the foreign country owned by the domestic household. The associated costs are given as:

$$\hat{\chi}(n_t^f)=\tilde{c}_t^f n_t^f+\frac{\Psi_f}{2}(n_t^f).$$

The two functions $\tilde{\chi},\hat{\chi}$ differ by the terms Ψ_d,Ψ_f only. The parameter Ψ_d is the adjustment cost of investing in the resident country (i.e., in the domestic country for the domestic household and in the foreign country for the foreign household), while the parameter Ψ_f is the adjustment cost of investing in the non-resident country. While the parameter **t** and the ratio c^e/c^n (conditionally on the values of the remaining parameters) model trade friction and the degree of trade openness, respectively, the ratio Ψ_f/Ψ_d and the parameter Ψ_B are used to model the degree of financial openness.

The first order conditions for the domestic household are as follows:

[12] Because of the law of large numbers and of perfect foresight, the *ex-ante* expected values of the key variables for household decisions (such as investment costs or profit flows) coincide with *ex-post* realizations.

$$u'(C_t)(1+\Psi_B B_t) = \frac{\eta_{t+1}}{\eta_t}(1+r_t^*)\beta u'(C_{t+1}), \tag{9}$$

$$\lim_{t\to\infty} B_{t+1} = 0, \tag{10}$$

$$\widetilde{\chi}'(n_t^d)u'(C_t) = \sum_{v\geq 0}(1-\delta)^v \beta^v u'(C_{t+v})\frac{\widetilde{\Pi}^d_{t,t+v}}{P_{t+v}},$$

In a strict sense, the equation (10) should read as $\lim_{t\to\infty} B_t u'(C_t)\beta^t = 0$, as a combination of the transversality condition and the non-Ponzi game conditions. However, because of nontrivial bond adjustment costs $\Psi_B > 0$, such a condition reduces to a simpler form of (10). The last two optimality conditions read as:

$$\widetilde{c}_t^d + \Psi_d n_t^d = \sum_{v\geq 0}(1-\delta)^v \mu_t^{t+v}\frac{\widetilde{\Pi}^d_{t,t+v}}{P_{t+v}}, \tag{11}$$

$$\eta_t(\widetilde{c}_t^f + \Psi_f n_t^f) = \sum_{v\geq 0}(1-\delta)^v \eta_{t+v}\mu_t^{t+v}\frac{\widetilde{\Pi}^f_{t,t+v}}{P^*_{t+v}}. \tag{12}$$

The marginal rate of substitution between times t_1 and t_2 is defined as:

$$\mu_{t_1}^{t_2} \equiv \beta^{t_2-t_1}\frac{u'(C_{t_2})}{u'(C_{t_1})}.$$

Although there is an idiosyncratic variance at the firm level, the model is deterministic at the aggregate level, thus the dynasty problem is deterministic too. Therefore the marginal rate of substitution does not involve the expectation operator.

The part of the model related to the foreign household is defined analogously and details of the derivations are given in Brůha and Podpiera (2007a).

C. General equilibrium

The general equilibrium is defined as a time profile of prices and quantities such that all households optimize and all markets clear. Since there are no price stickiness, nominal prices are indeterminate. Therefore, only the relative prices matter. The general equilibrium requires that the market-clearing conditions hold.

The aggregate resources constraint is given as follows:

$$C_t + n_t^d \tilde{c}_t^d + n_t^{d*} \tilde{c}_t^{d*} = Q_t, \tag{13}$$

$$C_t^* + n_t^f \tilde{c}_t^f + n_t^{f*} \tilde{c}_t^{f*} = Q_t^*, \tag{14}$$

Similarly, the labor market equilibrium requires:

$$\int l_{jt} d\Gamma_t^d (j,k) + \int l_{jt} d\Gamma_t^{d*} (j,k) = L, \tag{15}$$

where L is the aggregate, inelastic, domestic labor supply.

Analogous market clearing conditions hold in the foreign country. The international bond market equilibrium requires that:

$$B_t + B_t^* = 0. \tag{16}$$

The last equilibrium condition is the balance-of-payment equilibrium, which requires that:

$$B_{t+1} = (1 + r_t^*) B_t + X_t + (\Xi_t - \hat{\chi}(n_t^f)) - \frac{1}{\eta_t} (\Xi_t^* - \hat{\chi}(n_t^{d*})), \tag{17}$$

where X_t is the value of *net* real exports of the domestic country expressed in the foreign currency, and real profit flows are given as:

$$\Xi_t = \sum_{\sigma \leq t} (1-\delta)^{t-\sigma} n_\sigma^f \frac{\tilde{\Pi}_{\sigma,t}^f}{P_t^*},$$

$$\Xi_t^* = \sum_{\sigma \leq t} (1-\delta)^{t-\sigma} n_\sigma^{d*} \frac{\tilde{\Pi}_{\sigma,t}^{d*}}{P_t}.$$

The definition of the general equilibrium is standard. A more complicated task is to simulate the dynamic path, because the model is effectively a vintage type model. However, the model can be rewritten in the recursive (first-order) form, and the recursive form makes it convenient for application of a variety of efficient numerical methods. It turns out that the notorious domain-truncation approach seems to be the most efficient approach. The full set of equations of the model in the recursive form and a detailed discussion on methods are available in Appendix A.

D. Note on real exchange rate

The prices p_{jt} and the corresponding price indexes P_t and P_t^* are quality-adjusted prices. Therefore, the real exchange rate η_t is measured in the terms of qualities. These measures correspond to real-world price indexes only if the latter are quality-adjusted perhaps using a hedonic approach, which is rarely the case for transition countries, see Ahnert and Kenny (2004) for a survey of quality adjustments in prices. It is a fact that price indexes in transition economies are not adjusted for quality changes.

Thus, in order to obtain indexes closer to real-world measures, we have to define aggregate indexes over prices pertaining to physical quantities. Let us denote such indexes as $\wp_t$ and $\wp_t^*$. Ideally, one can compute these indexes based on theoretical-consistent aggregation. We use a simpler approximation instead and set:

$$\wp_t = \mathcal{K}_t P_t ,$$

where $\mathcal{K}_t$ is the total amount of quality investment by firms selling its products in the domestic country:

$$\mathcal{K}_t = \sum_{\xi \in \{d, d^*\}} \int_{\Omega^\xi} k_{j\tau} d\Gamma_t^\xi (j, \tau) + \sum_{\xi \in \{f, f^*\}} \int_{\Omega_e^\xi} k_{j\tau} d\Gamma_t^\xi (j, \tau).$$

Nevertheless, $\wp_t$ might differ from the CPI-based real-world indexes by one more term. The market structure based on the CES aggregation implies the *love-for-variety* effect. This means that the welfare-theoretical price indexes differ from the 'average' price by the term $n^{\frac{1}{\theta-1}}$, where n is the number of available varieties and θ is the parameter of substitution in the CES function (see Melitz, 2003 for rigorous definition and derivation of the average price).

Hence, quality-unadjusted CPI-based real exchange rate (empirical real exchange rate) is the correct model counterpart of the *measured real exchange rate in reality* and is defined as:

$$\eta_t^e = \left(\frac{n_t^*}{n_t} \right)^{\frac{1}{\theta-1}} \frac{\mathcal{K}_t^*}{\mathcal{K}_t} \eta_t .$$

The reader is referred to Brůha and Podpiera (2007a) for a more detailed discussion on real exchange rate measurements.

IV. Calibration and Projections

This section discusses model's calibration for the Czech economy. The investigated time span runs from 1995 to 2005. The choice of the start date is motivated by the fact that by 1995 the full external (trade and financial) and price liberalization has been completed – see Roland (2004) for a comparison of transition EBRD indexes of liberalization and reforms.

The Czech economy is considered to be roughly 6 times smaller than the EU15, nevertheless the exact size is not the crucial parameter, and it only says that already at this ratio the Czech economy does not significantly influence the large developed economy. When we calibrate the model, we use the iso-elastic production function

$$\ell(l,k) \equiv \left(\frac{l}{k}\right)^{1-\alpha}$$

for production of physical quantities. This formulation implies the Cobb-Douglas production function $f(k,l) = k^{\alpha} l^{1-\alpha}$ for the production of the quality-quantity bundle. The momentary utility function is parameterized using the common constant-relative-risk-aversion form $u(C) = (1-\varepsilon)^{-1} C^{1-\varepsilon}$, with the parameter of intertemporal substitution ε and was calibrated at standard vale of 2.09. The distribution G is calibrated to be uniform[13] on the interval [0,1].

The model is calibrated to replicate the observed trends in the Czech data during 1995-2005 in a set of major variables. The data for the Czech and EU15 economies has been taken from various sources: the Czech Statistical Office, the Czech National Bank, Bloomberg, and the Eurostat. The values of parameters are chosen such that the variables in the model are jointly as close as possible to the empirical counterparts. In practice, the model requires knowledge of constant parameters as well one moving (transitory) parameter. The summary of the parameters can be found in Table 1.

The fixed parameters take the respective value as can be seen in Table 1. In the case of the one transitory parameter (c^e/c^n), a path during the convergence is allowed (initial and terminal value is specified). The direction its change appears intuitive. While the productivity of the domestic firms increases, the export eligibility decreases along the convergence path

[13] Microeconomists usually use other distributions than uniform for modeling the distribution of productivities across firms. The usual choice is the Pareto distribution. However, since we aim at calibrating the long-run trajectories, the uniform distribution is sufficient for that purpose.

(effects of integration). In the steady state, the transitory parameter reaches the *SS value* (terminal value).

One of the important factors for convergence is the exit rate, where the firms do exit and new firms enter. This is a typical feature of a transition economy, where the closure and start-ups of firms is relatively high. Therefore, we have chosen the value of $\delta = 0.46$. And the discount factor β takes the conventional vale of 0.95.

A. Czech policy relevant variables

Key monetary policy variables for a small open economy such as the Czech Republic are: output, real exchange rate, and interest rate. Deriving their 'equilibrium' paths plays an instrumental role for 'gap models' which drive actual monetary policy actions. Also, understanding the long-term interaction of these variables is essential for the country's monetary integration into the European Monetary Union (participation in the ERM II and adoption of euro). Since our model is designed to simultaneously deliver all three convergence trajectories endogenously and interdependently, it can naturally be of use.

First, we interpret the convergence of the *output per capita* to the average of the EU 15. Starting with the Czech GDP per capita at the 60 % of the EU15 average in mid-1990s, and remaining at that level for the rest of the 1990s, in the early-2000s, the Czech economy started to converge more apparently, standing at roughly 70 % in 2005. The model's outcome along with the data (and forecast of the Ministry of Finance, which predicts this ratio) is displayed in the Figure 1.1. The calibration (as summarized in Table 1) of the logistic curve assumes an average growth (1995-2005) in the total factor productivity of 3 % p.a. This is roughly in congruence with the other empirically found values.[14]

Second, we aim at replicating *the real exchange rate* appreciation, which has reflected the economic convergence. The real exchange rate has been appreciating and stood approximately 30 % stronger in 2005 compared to base of 1997. Figure 1.2 compares the actual real exchange rate and the model's trajectory. The series are rebased such that the value of the average of the years 1997 and 1998 of the original data equals to the model's outcome. Although this is an arbitrary normalization, the reason behind is that in order to facilitate comparison of price indexes, we need to choose a benchmark equilibrium year. Since all available estimates of the equilibrium or parity of the real exchange rate falls into these two years – a summary of the evidence is provided by Babetskii and Égert (2005) - we choose it as a benchmark equilibrium year.

The model can explain the real exchange rate appreciation due to the presence of two factors. First is the fact that the CES aggregation implies the *love for variety*, which means

[14] The Czech Ministry of Finance (2006) for instance found the growth in TFP between 1-3 % during the period 1995-2005.

that the expansion of the number of domestic production entities can be considered a *quality improvement* of the domestic goods basket. However, the expansion of the domestic production has two effects. On one hand, it diminishes the price of the domestic basket, since domestic products become less scarce (and this effect causes fall in the international price of the domestic basket, i.e. it pushes the exchange rate to depreciation). On the other hand, the perceived quality of the domestic basket increases, and therefore this effect causes the real exchange rate to appreciate. The net effect depends on the relative importance of these two effects.

The effect of the increased variety is only part of the explanation for the real exchange rate appreciation. The second, and more important part, is the explicit investment into quality (the quality input is quite intensive in production, $\alpha = 0.32$) that causes increase in the portion of the quality in the produced quality-quantity bundle. The accumulation of the quality brings about the empirically observed exchange rate appreciation (see section III.D for theoretical note).

In a strict sense, the quality of goods basket increases in both countries. However, this effect is much stronger in the converging country and it is amplified by trade and financial openness,[15] therefore the perceived quality of domestic goods increases relatively more.

The quality improvements of the domestic composite basket is the very explanation why the converging country is able to sell more and - at the same time - for relatively higher price as its total factor productivity increases. The value of the parameter $\theta = 4.7$ that was chosen for calibration falls into the range of parameters in the literature to replicate the empirically observed mark-ups, for instance see Ghironi and Melitz (2005) who used the value 3.8 and claimed that this value implies reasonable mark-up over average costs. Indeed, the value around 4.7 delivers the mark-up over average costs close to the observed mark-up in the Czech manufacturing industry (20-25 % on average over 1995-2005, see Podpiera and Raková, 2006).

It is worth noting that the pace of real exchange rate appreciation in the model is obtained without any explicit assumption of exogenous productivity differential in tradable and non-tradable sectors (although the model displays endogenous productivity differential between traded and non-traded goods). In fact, the reason for the appreciation comes from the improvement of the domestic composite good through the variety expansion and explicit investment into quality. Moreover, hypotheses explaining real exchange rate appreciation based on exogenous productivity differential (Harrod-Balassa-Samuleson hypothesis) are empirically flawed (Mihaljek and Klau 2006 and Flek et al. 2003). Indeed, models with

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[15] That is why our model implies welfare gains from trade and financial liberalization in both countries. It also implies that these welfare gains will be more significant in a smaller (less developed) country, since such a country will benefit more from the variety expansion.

exogenous productivity differential imply that the terms-of-trade will not move so strongly as the real exchange rate.

The third crucial information for the monetary policy implementation is the *implicit "equilibrium" trajectory of return on domestic relative to foreign assets*. Since a small open emerging market economy exhibits convergence in the output, the corresponding (neutral) level of the real interest rate is hard to judge based on the historical averages of output growth (a standard approach in steady state economies). This stands in contrast to the developed foreign country in the model, where the neutral interest rate is easily set to the long run average of the output growth.

Nevertheless, deriving the neutral interest rate level from the output growth seems intuitive as any economy can pay return on assets equal to the growth in value added. Therefore, the interest rate trajectory can be derived from the excess growth of the domestic long-term output growth over the long-term growth in the foreign country. It is apparent that as the domestic economy develops and converges to the foreign one, the real output per capita increases and the domestic country gets richer. The convergence-implied neutral, cumulative return on investment made at the beginning of the convergence process is derived from the speed of convergence.

In a small open economy, there are two channels through which the foreign investor can gain from the economic convergence. The first is the traditional channel, i.e., the real interest rate differential *vis à vis* the developed foreign country, while the second is the domestic currency real appreciation channel. The mechanics of the former channel is very standard and apparent. Since the economy growth is higher than that of the developed country, the interest received on the investment (portfolio or direct) in converging country is higher accordingly. The latter channel is mainly due to improvement in quality of the products (variety or explicit quality investment) of the converging country.

In the calibration exercise we compare model's outcome of the excess return received by the foreign investor with the actual data on excess return from the 1Y governmental bonds. The relative 1Y return in both countries (Czech Republic vs. Euro area, prior 1999 German governmental Bonds) is tightly linked to the monetary policy settings. The cumulative actual and modeled yield differential is shown in Figure 1.4.

In summary, our analysis suggests that while the exchange rate followed the convergence trajectory with limited deviations, somewhat more pronounced deviations can be observed in the case of the remaining two variables. The output has recorded substantial deviation from its convergence trajectory starting 1998 and has never returned fully on the trajectory, albeit getting closer to it at the end of 2005. Similarly, the cumulative excess return departed from the implied convergence trajectory starting 1999 and the distance from the trajectory even increased after 2002. These concurrent observations are in our opinion intuitive. The return on 1Y bonds has decreased as a reaction to lowering monetary policy rates (since 1999). This stimulated the economic activity and stopped the economy from departing from its convergence trajectory. Further decrease in the real policy rate from

2002 on has stimulated economic activity even more and helped to close the gap between output and its convergence trajectory. Nevertheless, as the closure of this gap is underway and the discrepancy in excess real return is high, the policy is expected to gradually react by tightening of the real policy rate.

B. Czech GDP components

Consumption and investment have been steadily proportional to gross domestic product over the entire period 1995-2005. The calibrated parameters of the production function and the investment costs are chosen such that to replicate the observed shares of consumption and investment on the domestic absorption. For the Czech Republic, these shares are 72 % and 28 % (these numbers add up to 100 % since we divided government spending into consumption and investments), which complies with the hypothesis of consumption smoothing and low frictions in financing investment in the Czech economy. The actual data as well as model's outcome are presented in Figure 1.3.

Exports as well as *imports* have been gaining on importance over the studied period. The model[16] implies an increasing involvement in trade and thus replicates the tendency observed in the data. Figures 2.1 and 2.2 present the actual data and model's outcome. Slight excess in imports share of GDP over exports at the beginning on 1990s implies a negative trade balance with gradual improvement towards positive numbers at the end of the sample period, which is also observed in the data. This is elaborated more in the next subsection.

C. Czech balance of payments

The *financial account* recorded a net inflow of investment. We set portfolio adjustment costs Ψ_B and Ψ_f so as to replicate the net investment in 1995-2005. The development of financial account is shown in Figure 2.4 (positive values denote Czech net debit). As a consequence of increasing net inflow of investment, the foreign owned companies have increased their share quite rapidly. Based on the financial survey of the Czech Statistical Office among non-financial companies, in 1998, the foreign owned companies represented only one tenth of the total number of firms, while in 2004 it exceeded one forth by large margin (28 %).

Also, the real wage paid by the foreign-owned sector attained 112 % of the average wage in the economy in 2004, thus concentrating a higher productivity then the rest of the economy. The excess productivity is also apparent from the share of the value added produced in this sector, which reached roughly half of the produced total value added

[16] The iceberg transportation cost was calibrated at 4%.

(46.2 %) in 2004. In addition, the share of exported value added by this sector to the total value added exported was at a considerable 45.3 % in 2004 as well.

The model responds to an improving productivity (investment driven) and decreasing export costs of domestic and foreign firms in domestic country (c^E, see Table 1), which leads to an increasing number of firms that are exporting and thus it is able to explain the increase in export dynamics that exceeded the import dynamics and in 2005 led to *trade balance* surplus. As a logical consequence of the development on the *current account*, the direct investment has produced deepening deficit in *income balance*.

Thus, the initial smoothing of consumption represented by an excess in imports of goods and services (goods for final consumption in early stages, later moderated by increasing share of investment goods import) over exports, was replaced with stronger exporting performance and excess of export over import. This is in line with the intuition about the phases of convergence in an open transition economy as represented by the model's projection; see Figure 2.3 for the trade balance actual and simulated values.

D. Long-run convergence projection

We carried out projection of the Czech economy convergence using the calibrated model. We present scenario for the two policy relevant variables, i.e., the output convergence and the real exchange rate path. The scenario, showed in Figures 3.1 and 3.2, assumes that the Czech GDP per capita will reach the EU15 average in 2020. The path of the 'equilibrium' output suggests its fast growth in an upcoming decade (in excess of the EU15 long-term growth). Around 2015 it is anticipated to moderate towards the EU15 growth. As for the real exchange rate, the projected trend appreciation by the model is slowly moderating and stabilizing around 2010 at a level, which is roughly 45 % more appreciated than the exchange rate in 1997.

V. Conclusion

In this paper, we aim at providing an essential input for the Czech monetary policy makers - the long-run trend in the key policy relevant variables. Unlike a developed economy, which exhibits standard and settled characteristics for sufficiently long period of time and long run values (sometimes called *equilibrium*) can be obtained by averaging past observations, every emerging market economy falls short in this respect. In order to find and assess these variables for an emerging market economy, one needs a specific model that would deliver simultaneously determined their long-term trajectories.

We present a two-country model, in which there is an underdeveloped economy converging to its large and developed counterpart. We built on the New International Macroeconomic literature to capture main stylized facts of transition dynamics in key variables. The model calibrated for the Czech economy and EU15 shows that the symptoms of the convergence in selected policy relevant variables can be explained by decreasing export costs (direct investment enhanced) and by growing productivity in the converging country. The development of the economy is described by the endogenously determined trajectories for a large set of variables starting with gross domestic product, consumption, investment, exports and imports, direct foreign investment, and ending up with real exchange rate, and excess real return on domestic assets.

In particular, following the external liberalization, which triggered the inflow of portfolio and direct investment and excess of imports over exports, the economy started to benefit from foreign ownership and exhibited productivity gains. Increased productivity fostered the export performance and caused improvement in trade balance. Real exchange rate appreciated and profitable companies started to pay out dividends (also to foreign owners). As result, the income balance deteriorated. Subsequently, the excess return paid on domestic assets starts to moderate and the economy stabilizes in all variables as it approaches the steady state after completing the convergence move. The initial few years of simulated convergence path convincingly match the observed development in the Czech economy.

The presented modeling framework can be used to answer a number of policy questions, since the derived trends can be used for assessing the size of the medium term deviations of the output gap, real exchange rate gap, and the gap in the excess return on Czech assets. In particular, the real monetary policy conditions (excess return on assets in the converging economy) speak directly to the monetary policy.

In addition, the long-run trajectories might be of high importance when considering the timing of the monetary integration of the Czech Republic and other new EU member states[17].

VI. Appendix: Detailed derivation of the Model

A. Model equations under particular functional form

In this part of the paper, we derive the main model equation for particular functional forms of the production function, utility function and investment cost functions. In particular, as a benchmark calibration, we use the iso-elastic production function $\ell(l,k) \equiv \left(\frac{l}{k}\right)^{1-\alpha}$ for production of physical quantities. This formulation implies the Cobb-Douglas production function $f(k,l) = k^{\alpha} l^{1-\alpha}$ for the production of the quality-quantity bundle. The momentary

[17] For such an application, see Brůha and Podpiera (2007b).

utility function is parameterized using the common constant-relative-risk-aversion form $u(C) = (1-\varepsilon)^{-1} C^{1-\varepsilon}$, with the parameter of intertemporal substitution ε. As usually, the case of $\varepsilon = 1$ is interpreted as log (*C*). The distribution *G* of idiosyncratic shocks is uniform on the interval [0,1].

The real cost function associated with the Cobb-Douglas production function is given as follows:[18]

$$\mathbb{C}\,(q,\ \mathbb{W}_t\ ,A_t^H,\ z_j,\ k_j) = \mathbb{W}_t \left[\frac{q}{A_t^H z_j k_j^{\alpha}} \right]^{\frac{1}{1-\alpha}} .$$

First, we derive the optimal investment decision, and the present value of profit flows for a non-eligible firm.[19] Such a firm will supply the following quantity-quality bundle q_{jt}^d to the domestic market (at time *t*):

$$q_{jt}^d = \left([\tfrac{\theta-1}{\theta}(1-\alpha)\, \mathbb{W}_t^{\,-1}\, [A_t^H z_j k_j^{\alpha}]^{\frac{1}{1-\alpha}}]^{\theta} Q_t \right)^{\frac{(1-\alpha)}{\alpha\theta+(1-\alpha)}},$$

the real turnover is:

$$\frac{p_{jt}^d}{P_t} q_{jt}^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} [\frac{\theta-1}{\theta}(1-\alpha)\, \mathbb{W}_t^{\,-1}\, [A_t^H]^{\frac{1}{1-\alpha}}]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

And the real profit is given by:

$$\mathbb{P}_{j\pi t}^{\,d} = \frac{p_{jt}^d}{P_t} q_{jt}^d - \mathbb{C}\,(q_{jt}^d,\ \mathbb{W}_t\ ,A_t^H,\ z_j,\ k_j) =$$

$$= z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} [A_t^H]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}}\ \mathbb{W}_1,$$

where we define

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[18] Recall that blackboard fonts, such as $\mathbb{W}_t$ and $\mathbb{P}_t$ denote real variables such as real wage and real profits. Following that convention, the blackboard $\mathbb{C}$ denotes real cost function.

[19] Also, in this part of the paper, we derive expression only for firms located in the domestic country and owned by the domestic agent. The expressions for other types of firms are easily derived then.

$$\mathcal{W} \equiv \left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} - \left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{(1-\alpha)+\alpha\theta}} =$$

$$\frac{\alpha(\theta-1)+1}{(\theta-1)(1-\alpha)}\left[\frac{\theta-1}{\theta}(1-\alpha)\right]^{\frac{\theta}{(1-\alpha)+\alpha\theta}},$$

which is obviously positive.

Second, we derive optimal production decisions of eligible firms. The optimal production decision implies that $q_{jt}^{d} = \left[\frac{\theta-1}{\theta}\left(\frac{MC_{jt}}{P_t}\right)^{-1}\right]^{\theta} Q_t$, and $q_{jt}^{m*} = \left[\frac{\theta-1}{\theta}\frac{\eta_t}{1+t}\left(\frac{MC_{jt}}{P_t}\right)^{-1}\right]^{\theta} Q_t^{*}$. Some simple, but tedious, algebraic manipulations yield:

$$\kappa_{jt} q_{jt} \equiv q_{jt}^{d} = \left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathcal{W}_t^{-1}\left(A_t^H z_j k_j^{\alpha}\right)^{\frac{1}{1-\alpha}}\right]^{\theta} \frac{Q_t}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}},$$

$$(1-\kappa_{jt}) q_{jt} \equiv q_{jt}^{m*} = \left[\frac{\theta-1}{\theta}(1-\alpha)\frac{\eta_t}{1+t}\mathcal{W}_t^{-1}\left(A_t^H z_j k_j^{\alpha}\right)^{\frac{1}{1-\alpha}}\right]^{\theta} \frac{Q_t^{*}}{q_{jt}^{\frac{\alpha\theta}{1-\alpha}}}.$$

This implies that

$$\kappa_{jt} = \frac{Q_t}{Q_t + Q_t^{*}\left(\frac{\eta_t}{1+t}\right)^{\theta}},$$

observe that κ_{jt} does not depend on individual characteristics of firms: z_j and k_j; it depends only on relative tightness of both markets and on the real exchange rate corrected for transport costs ***t***. Therefore, all eligible firms will sell the same share of its products to the domestic, respectively foreign markets. Thus henceforth we will simply write κ_t for κ_{jt}. Define

$$\xi_t \equiv Q_t + Q_t^{*}\left(\frac{\eta_t}{1+t}\right)^{\theta} = \frac{Q_t}{\kappa_t}.$$

The total production of eligible firms can be written as follows:

$$q_{jt} = \left(z_j^{\theta} k_j^{\alpha\theta}\right)^{\frac{1}{(1-\alpha)+\alpha\theta}} \left\{ \left[\frac{\theta-1}{\theta}(1-\alpha)\, \mathcal{W}_t^{-1} \left[A_t^H\right]^{\frac{1}{1-\alpha}} \right]^{\theta} \xi_t \right\}^{\frac{(1-\alpha)}{(1-\alpha)+\alpha\theta}},$$

and the real turnover on the domestic and the foreign markets, respectively are given by:

$$\frac{p_{jt}^d}{P_t} q_{jt}^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \kappa_t^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[\frac{\theta-1}{\theta}(1-\alpha)\, \mathcal{W}_t^{-1} \left[A_t^H\right]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

$$\frac{\eta_t}{1+t} \frac{p_{jt}^{m*}}{P_t^*} q_{jt}^{m*} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} (1-\kappa_t)^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left(\frac{\eta_t}{1+t}\right)^{\frac{\theta}{(1-\alpha)+\alpha\theta}} x$$
$$x \left[\frac{\theta-1}{\theta}(1-\alpha)\, \mathcal{W}_t^{-1} \left[A_t^H\right]^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q_t^{*\frac{1}{(1-\alpha)+\alpha\theta}}.$$

Real production costs of eligible firms read as follows:

$$C_{jt} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[A_t^H\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \left\{ \left[\frac{\theta-1}{\theta}(1-\alpha) \right]^{\theta} \xi_t \right\}^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

thus, the real profit in a period t is given as:

$$\mathcal{P}_{j\tau}^{d} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[A_t^H\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \mathsf{W} \xi_t^{\frac{1}{(1-\alpha)+\alpha\theta}}.$$

Now, we are able to derive the expected present value of profit stream. We start with an eligible firm $P_{j\tau}^{de}$, the expected present value satisfies:

$$\mathcal{P}_{j\tau}^{de} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathsf{W} \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu_\tau^t \left[A_t^H\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_t^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \xi_t^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

and denote

$$\varpi_{\tau}^{e} = \mathrm{W}\sum_{t=\tau}^{\infty}(1-\delta)^{t-\tau}\,\mu_{\tau}^{t}\left[A_{t}^{H}\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}}\,\mathcal{W}_{t}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}}\,\xi_{t}^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

while the expected present value $\mathcal{P}_{j\tau}^{dn}$ of a non-eligible firm satisfies:

$$\mathcal{P}_{j\tau}^{dn} = z_{j}^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}}\,k_{j}^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}}\mathrm{W}\sum_{t=\tau}^{\infty}(1-\delta)^{t-\tau}\,\mu_{\tau}^{t}\left[A_{t}^{H}\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}}\,\mathcal{W}_{t}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}}\,Q_{t}^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

and similarly denote

$$\varpi_{\tau}^{n} = \mathrm{W}\sum_{t=\tau}^{\infty}(1-\delta)^{t-\tau}\,\mu_{\tau}^{t}\left[A_{t}^{H}\right]^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}}\,\mathcal{W}_{t}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}}\,Q_{t}^{\frac{1}{(1-\alpha)+\alpha\theta}}.$$

The value of an eligible firm located in the domestic country and owned by the domestic household – which enjoys a productivity level z_j – is determined by capital investment:

$$V_{\tau}^{de}(k_{j}\mid z_{j}) = \mathcal{P}_{j\tau}^{de} - (c^{e}+k_{j}) \equiv z_{j}^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}}\,k_{j}^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}}\,\varpi_{\tau}^{e} - (c^{e}+k_{j});$$

and similarly for a non-eligible firm

$$V_{\tau}^{dn}(k_{j}\mid z_{j}) = \mathcal{P}_{j\tau}^{dn} - (c^{n}+k_{j}) \equiv z_{j}^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}}\,k_{j}^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}}\,\varpi_{\tau}^{n} - (c^{n}+k_{j}).$$

If firms' managers maximize the value of firms, they choose the following capital level:

$$k_{j}^{opt,e} = z_{j}^{\theta-1}\left[\frac{\alpha(\theta-1)\varpi_{\tau}^{e}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1},$$

and the value of an eligible firm is

$$\mathrm{V}_{\tau}^{de}(z_{j}) = \max_{k_{j}\geq 0} V_{\tau}^{de}(k_{j}\mid z_{j}) = z_{j}^{(\theta-1)}\left[\varpi_{\tau}^{e}\right]^{\alpha(\theta-1)+1}\Im - c^{e},$$

where

$$\Im \equiv \left[\left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)} - \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)+1} \right] =$$

$$= \frac{1}{\alpha(\theta-1)+1} \left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1} \right)^{\alpha(\theta-1)} .$$

Similarly, the value of a non-eligible firm is

$$\mathrm{V}_{\tau}^{dn}(z_j) = \max_{k_j \geq 0} V_{\tau}^{dn}(k_j \mid z_j) = z_j^{(\theta-1)} \left[\varpi_{\tau}^{n} \right]^{\alpha(\theta-1)+1} \Im - c^{n},$$

and the optimal capital investment into quality is

$$k_j^{opt,n} = z_j^{\theta-1} \left[\frac{\alpha(\theta-1)\varpi_{\tau}^{n}}{\alpha(\theta-1)+1} \right]^{\alpha(\theta-1)+1} . \qquad \text{(A.1)}$$

The value functions $\mathrm{V}_{\tau}^{dn}(z_j)$, $\mathrm{V}_{\tau}^{de}(z_j)$ implicitly define the cut-off value $\bar{z}$, which is the least idiosyncratic shock, which makes the export-eligibility investment profitable. Thus it is defined as

$$\bar{z}_{\tau}^{d} = \min_{z_j} (\mathrm{V}_{\tau}^{de}(z_j) \geq \mathrm{V}_{\tau}^{dn}(z_j)).$$

Also for the chosen parametrization of the production function, one can derive the labor demand. The formula is complicated and is given in the next section, since it involves integration over labor demands of firms of various vintages, see (A.8), (A.9), and (A.10) below.

B. Model in the recursive form

In this part of the paper, we show how to transform the model into the recursive (first-order) form, which is suitable for numerical evaluation. We do it for parametrization used in Section VI.A. Although it is in principle possible to apply selected (but not all) numerical techniques directly to the vintage-formulation of the model, such a strategy would be very inefficient: numerical experiments suggest that the computation time is substantially reduced when the numerical techniques are applied to the recursive formulation of the model.

The first-order form consists of dynamic and static equations. These are listed below.

Dynamic equations

Intertemporal marginal rate of substitution:

$$\mu_t^{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{\varepsilon}, \tag{A.2}$$

$$\mu^{*t+1}_{\ t} = \beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{\varepsilon}.$$

Profit flows:

$$\varpi_t^{ndd} = W_1\left(\left[A_t^H\right]^{(\theta-1)} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{ndd}, \tag{A.3}$$

$$\varpi_t^{edd} = W_1\left(\left[A_t^H\right]^{(\theta-1)} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} \xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{edd},$$

$$\varpi_t^{nfd} = W_1\left(\left[A_t^F\right]^{(\theta-1)} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} Q_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu^{*t+1}_{\ t}\varpi_{t+1}^{nfd},$$

$$\varpi_t^{efd} = W_1\left(\left[A_t^F\right]^{(\theta-1)} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} \xi_t\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu^{*t+1}_{\ t}\varpi_{t+1}^{efd},$$

$$\varpi_t^{nff} = W_1\left(\left[A_t^F\right]^{(\theta-1)} \mathcal{W}_t^{*\,-(\theta-1)(1-\alpha)} Q_t^*\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu^{*t+1}_{\ t}\varpi_{t+1}^{nff},$$

$$\varpi_t^{eff} = W_1\left(\left[A_t^F\right]^{(\theta-1)} \mathcal{W}_t^{*\,-(\theta-1)(1-\alpha)} \xi_t^*\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu^{*t+1}_{\ t}\varpi_{t+1}^{eff},$$

$$\varpi_t^{ndf} = W_1\left(\left[A_t^H\right]^{(\theta-1)} \mathcal{W}_t^{*\,-(\theta-1)(1-\alpha)} Q_t^*\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{ndf},$$

$$\varpi_t^{edf} = W_1\left(\left[A_t^H\right]^{(\theta-1)} \mathcal{W}_t^{*\,-(\theta-1)(1-\alpha)} \xi_t^*\right)^{\frac{1}{\alpha(\theta-1)+1}} + (1-\delta)\mu_t^{t+1}\varpi_{t+1}^{edf};$$

where $\xi_t = Q_t + Q_t^*\left(\frac{\eta_t}{1+t}\right)^{\theta}$, and $\xi_t^* = Q_t^* + Q_t\left(\frac{\eta_t^{-1}}{1+t}\right)^{\theta}$.

Expected value of the stream of future profits (from the unit investment now) Ω_t^o are given as the sum of weighted expected values from eligible and non-eligible profits:[20] $\Omega_t^{x_1x_2} = \Omega_t^{nx_1x_2} + \Omega_t^{ex_1x_2}$, (with $x_i \in \{d, f\}$). Also to make the notation as transparent as possible, henceforth the superscript *dd* denotes domestically-owned firms located in the domestic country, *fd* denotes foreignly-owned firms located in the domestic country, *ff* denotes foreignly-owned firms located in the foreign country, and *df* denotes domestically-owned firms located in the foreign country, and where:

[20] Henceforth, in order to diminish the notational burden, we use A^o in lieu of $\{A^{ndd}, \ldots, A^{eff}\}$.

$$\Omega_t^{edd} = \widetilde{\mathcal{P}}_t^{edd} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{edd}\left(\frac{\varpi_t^{edd}}{\varpi_{t+1}^{edd}}\right)\frac{\int_{\bar{z}_t^{dd}}^{z_U} z^{\theta-1}G(z)}{\int_{\bar{z}_{t+1}^{dd}}^{z_U} z^{\theta-1}G(z)}, \tag{A.4}$$

$$\Omega_t^{ndd} = \widetilde{\mathcal{P}}_t^{ndd} + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{ndd}\left(\frac{\varpi_t^{ndd}}{\varpi_{t+1}^{ndd}}\right)\frac{\int_{z_L}^{\bar{z}_t^{dd}} z^{\theta-1}G(z)}{\int_{z_L}^{\bar{z}_{t+1}^{dd}} z^{\theta-1}G(z)},$$

$$\Omega_t^{efd} = \frac{\widetilde{\mathcal{P}}_t^{efd}}{\eta_t} + \mu^{*t+1}_t(1-\delta)\Omega_{t+1}^{efd}\left(\frac{\varpi_t^{efd}}{\varpi_{t+1}^{efd}}\right)\frac{\int_{\bar{z}_t^{fd}}^{z_U} z^{\theta-1}G(z)}{\int_{\bar{z}_{t+1}^{fd}}^{z_U} z^{\theta-1}G(z)},$$

$$\Omega_t^{nfd} = \frac{\widetilde{\mathcal{P}}_t^{nfd}}{\eta_t} + \mu^{*t+1}_t(1-\delta)\Omega_{t+1}^{nfd}\left(\frac{\varpi_t^{nfd}}{\varpi_{t+1}^{nfd}}\right)\frac{\int_{z_L}^{\bar{z}_t^{fd}} z^{\theta-1}G(z)}{\int_{z_L}^{\bar{z}_{t+1}^{fd}} z^{\theta-1}G(z)},$$

$$\Omega_t^{eff} = \widetilde{\mathcal{P}}_t^{eff} + \mu^{*t+1}_t(1-\delta)\Omega_{t+1}^{eff}\left(\frac{\varpi_t^{eff}}{\varpi_{t+1}^{eff}}\right)\frac{\int_{\bar{z}_t^{ff}}^{z_U} z^{\theta-1}G(z)}{\int_{\bar{z}_{t+1}^{ff}}^{z_U} z^{\theta-1}G(z)},$$

$$\Omega_t^{nff} = \widetilde{\mathcal{P}}_t^{nff} + \mu^{*t+1}_t(1-\delta)\Omega_{t+1}^{nff}\left(\frac{\varpi_t^{nff}}{\varpi_{t+1}^{nff}}\right)\frac{\int_{z_L}^{\bar{z}_t^{ff}} z^{\theta-1}G(z)}{\int_{z_L}^{\bar{z}_{t+1}^{ff}} z^{\theta-1}G(z)},$$

$$\Omega_t^{edf} = \widetilde{\mathcal{P}}_t^{edd}\eta_t + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{edf}\left(\frac{\varpi_t^{edf}}{\varpi_{t+1}^{edf}}\right)\frac{\int_{\bar{z}_t^{df}}^{z_U} z^{\theta-1}G(z)}{\int_{\bar{z}_{t+1}^{df}}^{z_U} z^{\theta-1}G(z)},$$

$$\Omega_t^{ndf} = \widetilde{\mathcal{P}}_t^{edd}\eta_t + \mu_t^{t+1}(1-\delta)\Omega_{t+1}^{ndf}\left(\frac{\varpi_t^{ndf}}{\varpi_{t+1}^{ndf}}\right)\frac{\int_{z_L}^{\bar{z}_t^{df}} z^{\theta-1}G(z)}{\int_{z_L}^{\bar{z}_{t+1}^{df}} z^{\theta-1}G(z)};$$

where definitions of expectations of profits Π_t^{xxx} and cut-off values will be given in the next subsection.

To get equations for actual realized profits $\Xi_t^{x_1x_2}$, $x_i \in \{d, f\}$, we have to split into two parts (according to eligibility): $\Xi_t^{x_1x_2} = \Xi_t^{ex_1x_2} + \Xi_t^{nx_1x_2}$. The first-order equations are then:

$$\Xi_{t+1}^{edd} = (1-\delta)\left(\frac{\left[A_{t+1}^{H}\right]^{\theta-1}\xi_{t+1}}{\left[A_{t}^{H}\right]^{\theta-1}\xi_{t}}\right)^{\frac{1}{\alpha(\theta-1)+1}} (\mathcal{W}_{t+1}/\mathcal{W}_{t})^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}} \Xi_{t}^{edd} + n_{t+1}^{edd}\widetilde{\mathcal{P}}_{t}^{edd}, \qquad \text{(A.5)}$$

$$\Xi_{t+1}^{ndd} = (1-\delta)\left(\frac{\left[A_{t+1}^{H}\right]^{\theta-1}Q_{t+1}}{\left[A_{t}^{H}\right]^{\theta-1}Q_{t}}\right)^{\frac{1}{\alpha(\theta-1)+1}} (\mathcal{W}_{t+1}/\mathcal{W}_{t})^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}} \Xi_{t}^{ndd} + n_{t+1}^{ndd}\widetilde{\mathcal{P}}_{t}^{ndd},$$

$$\Xi_{t+1}^{efd} = (1-\delta)\left(\frac{\left[A_{t+1}^{F}\right]^{\theta-1}\xi_{t+1}}{\left[A_{t}^{F}\right]^{\theta-1}\xi_{t}}\right)^{\frac{1}{\alpha(\theta-1)+1}} (\mathcal{W}_{t+1}/\mathcal{W}_{t})^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}} \Xi_{t}^{efd} + n_{t+1}^{efd}\widetilde{\mathcal{P}}_{t}^{efd},$$

$$\Xi_{t+1}^{nfd} = (1-\delta)\left(\frac{\left[A_{t+1}^{F}\right]^{\theta-1}Q_{t+1}}{\left[A_{t}^{F}\right]^{\theta-1}Q_{t}}\right)^{\frac{1}{\alpha(\theta-1)+1}} (\mathcal{W}_{t+1}/\mathcal{W}_{t})^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}} \Xi_{t}^{nfd} + n_{t+1}^{nfd}\widetilde{\mathcal{P}}_{t}^{nfd},$$

$$\Xi_{t+1}^{eff} = (1-\delta)\left(\frac{\left[A_{t+1}^{F}\right]^{\theta-1}\xi^{*}{}_{t+1}}{\left[A_{t}^{F}\right]^{\theta-1}\xi^{*}{}_{t}}\right)^{\frac{1}{\alpha(\theta-1)+1}} (\mathcal{W}^{*}_{t+1}/\mathcal{W}^{*}_{t})^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}} \Xi_{t}^{eff} + n_{t+1}^{eff}\widetilde{\mathcal{P}}_{t}^{eff},$$

$$\Xi_{t+1}^{nff} = (1-\delta)\left(\frac{\left[A_{t+1}^{F}\right]^{\theta-1}Q^{*}{}_{t+1}}{\left[A_{t}^{F}\right]^{\theta-1}Q^{*}{}_{t}}\right)^{\frac{1}{\alpha(\theta-1)+1}} (\mathcal{W}^{*}_{t+1}/\mathcal{W}^{*}_{t})^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}} \Xi_{t}^{nff} + n_{t+1}^{nff}\widetilde{\mathcal{P}}_{t}^{nff},$$

$$\Xi_{t+1}^{edf} = (1-\delta)\left(\frac{\left[A_{t+1}^{H}\right]^{\theta-1}\xi^{*}{}_{t+1}}{\left[A_{t}^{H}\right]^{\theta-1}\xi^{*}{}_{t}}\right)^{\frac{1}{\alpha(\theta-1)+1}} (\mathcal{W}^{*}_{t+1}/\mathcal{W}^{*}_{t})^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}} \Xi_{t}^{edf} + n_{t+1}^{edf}\widetilde{\mathcal{P}}_{t}^{edf},$$

$$\Xi_{t+1}^{ndf} = (1-\delta)\left(\frac{\left[A_{t+1}^{H}\right]^{\theta-1}Q^{*}{}_{t+1}}{\left[A_{t}^{H}\right]^{\theta-1}Q^{*}{}_{t}}\right)^{\frac{1}{\alpha(\theta-1)+1}} (\mathcal{W}^{*}_{t+1}/\mathcal{W}^{*}_{t})^{\frac{-(\theta-1)}{\alpha(\theta-1)+1}} \Xi_{t}^{ndf} + n_{t+1}^{ndf}\widetilde{\mathcal{P}}_{t}^{ndf},$$

where the numbers of eligible and non-eligible firms distinguished by location and ownerships (i.e. n_t^o) is given in the next subsection.

Exports are given recursively as follows: $X_t^d = X_t^{dd} + X_t^{fd}$, $X_t^f = X_t^{df} + X_t^{ff}$, where X_t^{dd} is the export of firms located in the domestic country and owned by the domestic household to the foreign country (and similarly for X_t^{fd}, X_t^{df}, X_t^{ff}). We use the convention that exports are denominated in the currency of the original market (thus X_t^{dd}, X_t^{fd} are in the domestic currency). Thus, it holds that:

$$X_t^{dd} = \hat{n}_t^{edd}(1-\kappa_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}}\left(\frac{\eta_t}{1+t}\right)^{\frac{\theta}{\alpha(\theta-1)+1}}\left[\frac{\theta-1}{\theta}(1-\alpha)\,\mathcal{W}_t^{\;-1}\left[A_t^H\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{*\frac{1}{\alpha(\theta-1)+1}},$$

$$X_t^{df} = \hat{n}_t^{edf} (1-\kappa^*{}_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left(\frac{\eta_t}{1+t}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\, \mathcal{W}_t^{\,-1} \left[A_t^H\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

$$X_t^{ff} = \hat{n}_t^{eff} (1-\kappa^*{}_t)^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}} \left(\frac{\eta_t}{1+t}\right)^{\frac{\theta}{\alpha(\theta-1)+1}} \left[\frac{\theta-1}{\theta}(1-\alpha)\, \mathcal{W}_t^{\,-1} \left[A_t^F\right]^{\frac{1}{1-\alpha}}\right]^{\frac{(\theta-1)(1-\alpha)}{\alpha(\theta-1)+1}} Q_t^{\frac{1}{\alpha(\theta-1)+1}},$$

where $\hat{n}_t^{ex_1x_2}$ are weighted numbers of eligible firms, which obeys the following recursive relation:

$$\hat{n}_{t+1}^{ex_1x_2} = (1-\delta)\hat{n}_t^{ex_1x_2} + n_{t+1}^{ex_1x_2} \left[\frac{\alpha(\theta-1)\varpi_{t+1}^{ex_1x_2}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{\bar{z}_{t+1}^{x_1x_2}}^{z_U} z^{\theta-1} G(dz) \ .$$

A similar recursive equation holds for non-eligible firms:

$$\hat{n}_{t+1}^{ex_1x_2} = (1-\delta)\hat{n}_t^{ex_1x_2} + n_{t+1}^{ex_1x_2} \left[\frac{\alpha(\theta-1)\varpi_{t+1}^{nx_1x_2}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{\bar{z}_L}^{\bar{z}_{t+1}^{x_1x_2}} z^{\theta-1} G(dz) \,.$$

These recursive schemes are used in the next subsubsection too (when deriving the labor demand).

The rest of model dynamic equations are *balance-of-payment equation* (17), households' *budget constraint* (8), households' *Euler equations* (9), households' *equations*, which determines the *asset holdings*: (11), (12), plus the corresponding equations for the foreign household. Equation describing optimal asset holding are not in the recursive first-order form, but we can easily convert them into it (for sake of clarity, we put the equations for both agents):

$$\begin{aligned} &\tilde{c}_t^{dd} + \Psi_d n_t^{dd} = \Omega_t^{dd}, \\ &\eta_t \tilde{c}_t^{df} + \Psi_f n_t^{df} = \Omega_t^{df}, \\ &\tilde{c}_t^{ff} + \Psi_d n_t^{ff} = \Omega_t^{ff}, \\ &\eta_t^{-1} \tilde{c}_t^{fd} + \Psi_f n_t^{fd} = \Omega_t^{fd}; \end{aligned} \qquad \text{(A.6)}$$

where expected investment costs obey:

$$\begin{aligned} \tilde{c}_t^{x_1x_2} &= G(\bar{z}_t^{x_1x_2})c^n + (1-G(\bar{z}_t^{x_1x_2}))c^e + \ldots \\ &+ \left[\frac{\alpha(\theta-1)\varpi_{t+1}^{nx_1x_2}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{z_L}^{\bar{z}_{t+1}^{x_1x_2}} z^{\theta-1} G(dz) + \ldots \\ &+ \left[\frac{\alpha(\theta-1)\varpi_{t+1}^{ex_1x_2}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \int_{\bar{z}_{t+1}^{x_1x_2}}^{z_{tU}} z^{\theta-1} G(dz). \end{aligned} \qquad \text{(A.7)}$$

Static equations

The model has static equations too. These are mainly market clearing conditions and definitions. The market clearing conditions include the clearing of the goods markets (13), (14), international bond market clearing (16), and labor market clearing conditions. We now show how the labor market conditions look like: define $\hbar_t^o$ as

$$\hbar_t^{ndd} = \left(\frac{\theta - 1}{\theta} (1-\alpha) \left[A_t^H \right]^{\theta-1} \mathcal{W}_t^{-(\theta-1)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}}, \tag{A.8}$$

$$\hbar_t^{edd} = \left(\frac{\theta - 1}{\theta} (1-\alpha) \left[A_t^H \right]^{\theta-1} \mathcal{W}_t^{-(\theta-1)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\hbar_t^{nfd} = \left(\frac{\theta - 1}{\theta} (1-\alpha) \left[A_t^F \right]^{\theta-1} \mathcal{W}_t^{-(\theta-1)} Q_t \right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\hbar_t^{efd} = \left(\frac{\theta - 1}{\theta} (1-\alpha) \left[A_t^F \right]^{\theta-1} \mathcal{W}_t^{-(\theta-1)} \xi_t \right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\hbar_t^{ndf} = \left(\frac{\theta - 1}{\theta} (1-\alpha) \left[A_t^H \right]^{\theta-1} \mathcal{W}_t^{*\,-(\theta-1)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\hbar_t^{edf} = \left(\frac{\theta - 1}{\theta} (1-\alpha) \left[A_t^H \right]^{\theta-1} \mathcal{W}_t^{*\,-(\theta-1)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\hbar_t^{nff} = \left(\frac{\theta - 1}{\theta} (1-\alpha) \left[A_t^F \right]^{\theta-1} \mathcal{W}_t^{*\,-(\theta-1)} Q_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}},$$

$$\hbar_t^{eff} = \left(\frac{\theta - 1}{\theta} (1-\alpha) \left[A_t^F \right]^{\theta-1} \mathcal{W}_t^{*\,-(\theta-1)} \xi_t^* \right)^{\frac{1}{\alpha(\theta-1)+1}}.$$

Then the domestic labor demand is given as

$$L_t = \sum_{\xi \in \{e,n\}} \sum_{x_1 \in \{d,f\}} \hbar_t^{\xi x_1 d} \hat{n}_t^{\xi x_1 d}, \tag{A.9}$$

and the foreign labor demand is given by

$$L_t^* = \sum_{\xi \in \{e,n\}} \sum_{x_1 \in \{d,f\}} \hbar_t^{\xi x_1 f} \hat{n}_t^{\xi x_1 f}. \tag{A.10}$$

The labor demands should be equal to inelastic labor supply.

The only remaining definitions are those of average profits and expected cut-offs. They follow:

$$\bar{z}_t^{x_1 x_2} = \left(\frac{c^e - c^n}{\Im\left[\left[\varpi_t^{ex_1x_2}\right]^{\alpha(\theta-1)+1} - \left[\varpi_t^{nx_1x_2}\right]^{\alpha(\theta-1)+1}\right]} \right)^{\frac{1}{\theta-1}} , \qquad \text{(A.11)}$$

for $x_i = \{d, f\}$, and

$$\widetilde{\mathcal{P}}_t^{ndd} = \mathrm{W}\left(\int_{z_L}^{\bar{z}_{t+1}^{dd}} z^{\theta-1} G(dz)\left(\left[A_t^H\right]^{\theta-1} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} Q_t\right)^{\frac{1}{\alpha(\theta-1]+1}}\left[\frac{\alpha(\theta-1)\varpi_t^{ndd}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} ,$$

$$\widetilde{\mathcal{P}}_t^{nfd} = \mathrm{W}\left(\int_{z_L}^{\bar{z}_{t+1}^{fd}} z^{\theta-1} G(dz)\left(\left[A_t^F\right]^{\theta-1} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} Q_t\right)^{\frac{1}{\alpha(\theta-1]+1}}\left[\frac{\alpha(\theta-1)\varpi_t^{nfd}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} ,$$

$$\widetilde{\mathcal{P}}_t^{efd} = \mathrm{W}\left(\int_{\bar{z}_{t+1}^{fd}}^{z_U} z^{\theta-1} G(dz)\left(\left[A_t^F\right]^{\theta-1} \mathcal{W}_t^{-(\theta-1)(1-\alpha)} \xi_t\right)^{\frac{1}{\alpha(\theta-1]+1}}\left[\frac{\alpha(\theta-1)\varpi_t^{efd}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} ,$$

$$\widetilde{\mathcal{P}}_t^{ndf} = \mathrm{W}\left(\int_{z_L}^{\bar{z}_{t+1}^{df}} z^{\theta-1} G(dz)\left(\left[A_t^H\right]^{\theta-1} \mathcal{W}_t^{*\,-(\theta-1)(1-\alpha)} Q_t^*\right)^{\frac{1}{\alpha(\theta-1]+1}}\left[\frac{\alpha(\theta-1)\varpi_t^{ndf}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} ,$$

$$\widetilde{\mathcal{P}}_t^{efd} = \mathrm{W}\left(\int_{\bar{z}_{t+1}^{df}}^{z_U} z^{\theta-1} G(dz)\left(\left[A_t^H\right]^{\theta-1} \mathcal{W}_t^{*\,-(\theta-1)(1-\alpha)} \xi_t^*\right)^{\frac{1}{\alpha(\theta-1]+1}}\left[\frac{\alpha(\theta-1)\varpi_t^{edf}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} ,$$

$$\widetilde{\mathcal{P}}_t^{nff} = \mathrm{W}\left(\int_{z_L}^{\bar{z}_{t+1}^{ff}} z^{\theta-1} G(dz)\left(\left[A_t^F\right]^{\theta-1} \mathcal{W}_t^{*\,-(\theta-1)(1-\alpha)} Q_t^*\right)^{\frac{1}{\alpha(\theta-1]+1}}\left[\frac{\alpha(\theta-1)\varpi_t^{nff}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} ,$$

$$\widetilde{\mathcal{P}}_t^{eff} = \mathrm{W}\left(\int_{\bar{z}_{t+1}^{dd}}^{z_U} z^{\theta-1} G(dz)\left(\left[A_t^F\right]^{\theta-1} \mathcal{W}_t^{*\,-(\theta-1)(1-\alpha)} \xi_t^*\right)^{\frac{1}{\alpha(\theta-1]+1}}\left[\frac{\alpha(\theta-1)\varpi_t^{eff}}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} .$$

C. Numerical methods

This part of the appendix discusses numerical methods used to simulate the model. Basically, we have experimented with two classes of methods: (i) projection-based methods and (ii) domain-truncation methods.

Before discussing these methods, it is worth to realize a fact, which we use when applying both methods: If one can guess the time profile of the following six variables: domestic output $\{Q_t\}_{t=0}^{\infty}$, domestic real wage $\{\mathcal{W}_t\}_{t=0}^{\infty}$, domestic consumption $\{C_t\}_{t=0}^{\infty}$, their foreign counterparts: $\{Q_t^*\}_{t=0}^{\infty}$, $\{\mathcal{W}_t^*\}_{t=0}^{\infty}$, and $\{C_t^*\}_{t=0}^{\infty}$ the real exchange rate $\{\eta_t\}_{t=0}^{\infty}$, one can easily compute the time profile of all other endogenous variables (given exogenous and policy variables). Indeed, the algorithm is the following:

1. Given $\{C_t\}_{t=0}^{\infty}$, $\{C_t^*\}_{t=0}^{\infty}$ compute the marginal rate of substitutions $\{\mu_t^{t+1}\}_{t=0}^{\infty}$, $\{\mu^{*t+1}_{\ t}\}_{t=0}^{\infty}$ using (A.2).

2. Given $\{Q_t\}_{t=0}^{\infty}$, $\{\mathcal{W}_t\}_{t=0}^{\infty}$, $\{Q_t^*\}_{t=0}^{\infty}$, $\{\mathcal{W}^*_t\}_{t=0}^{\infty}$, and $\{\mu_t^{t+1}\}_{t=0}^{\infty}$, $\{\mu^{*t+1}_{\ t}\}_{t=0}^{\infty}$, it is possible to solve for $\{\varpi_t^o\}_{t=0}^{\infty}$, and therefore for $\{\bar{z}_t^o\}_{t=0}^{\infty}$; use (A.3) and (A.11).

3. Then, use backward difference equations (A.4) to compute $\{\Omega_t^o\}_{t=0}^{\infty}$, (A.7) to compute expected investment costs $\{\tilde{c}_t^o\}_{t=0}^{\infty}$ and first-order conditions (A.6) to compute the numbers of new entrants.

4. Then use the forward difference equation (A.5) to solve for profit flows $\{\Xi_{t+1}^o\}_{t=0}^{\infty}$ and (A.8), (A.9) and (A.10) to find labor demand in both countries.

5. One can use households' Euler equations to derive the optimal bond holding and from the international-bond market clearing condition (16) to derive the equilibrium interest rate $\{r_t\}_{t=0}^{\infty}$;

Now, one guesses the time profile and verifies the guess. The guess should be verified as follows:

1. Budget constraints for both households have to be satisfied: (8) and similarly for the foreign household.
2. Labor markets in both countries have to be cleared: (15) and similarly for the foreign country.
3. The balance of payment condition has to be satisfied: (17).
4. Goods markets have to be cleared as well: (13), (14).

Denote the guess of the seven variables as

$$\vec{\Re} = \{\{Q_t\}_{t=0}^{\infty}, \{\mathcal{W}_t\}_{t=0}^{\infty}, \{C_t\}_{t=0}^{\infty}, \{Q_t^*\}_{t=0}^{\infty}, \{\mathcal{W}^*_t\}_{t=0}^{\infty}, \{C_t^*\}_{t=0}^{\infty}, \{\eta\}_{t=0}^{\infty}\},$$

and the seven equilibrium conditions as $\left\{\lambda_t(\vec{\Re})\right\}_{t=0}^{\infty}$, where we interpret $\lambda_t(\vec{\Re}^o) = 0$ as the fulfillment of these conditions at time *t* for a guess $\vec{\Re}^o$. Note that the fulfillment of equilibrium condition at time *t*, $\lambda_t = 0$ does not depend on the value of the seven variables at time *t* only: it depends on their entire time profiles. It depends on future values because of expectations of profits, e.g. today's investment decisions depend on future streams of profits, cf. (11), (12), and it depends on past values because of predetermined variables in budget constraints.

In any case, the equilibrium candidate $\vec{\Re}$ is an infinite-dimensional object and for practical simulations, we have to approximate it by a finite-dimensional representation. The projection and domain-truncation methods do that in different ways.

The strategy of the projection method is the following: approximate the time profiles using an object parameterized by a low number of parameters (such as polynomials, splines, neutral networks, or wavelets). Thus approximate

$$\vec{\Re} \approx \widetilde{\Re}(\Theta),$$

where Θ is a finite vector of parameters. Then the problem is to find such a vector of parameters $\vec{\Theta}$, such that the equilibrium conditions $\lambda_t(\widetilde{\Re}(\vec{\Theta})) = 0$ nearly holds for all *t*. Judd (2002) discusses applications of the projection methods in the context of perfect foresight discrete-time models.

Another approach (called domain truncation approach) to reduce dimensionality of $\vec{\Re}$ is to set $\{Q_t\}_{t=0}^{\infty} \approx \hat{Q} = \{Q_1,...,Q_N,Q_+,Q_+,...,Q_+\}$, where Q_+ is the steady state of the variable Q_t (and similarly for other variables too) and to set

$$\hat{\Re} = \{\hat{Q}, \hat{W}, \hat{C}, \hat{Q}^*, \hat{W}^*, \hat{C}^*, \hat{\eta}\},$$

and solve the system

$$\begin{aligned} &\lambda_1(\hat{\Re}) = 0 \\ &\lambda_2(\hat{\Re}) = 0 \\ &\quad . \\ &\quad . \\ &\quad . \\ &\lambda_M(\hat{\Re}) = 0 . \end{aligned} \tag{A.12}$$

for $M > N$. This is a system of *M* unknowns. Lafargue (1990) proposed this approach, and Boucekkine (1995) and Juillard et al. (1998) exploited the sparseness of the system to apply an efficient algorithm. Hence, the approach uses to be called as L-B-J approach (see also Gilli and Pauletto, 1998 or Armstrong et al., 1998 for further discussions about efficient implementation). The stacked system (A.12) is usually solved using Newton-based iterations. When applied to the model presented in this paper, we cannot use efficient algorithms for sparse systems unless $\delta = 1$. The case of $\delta = 1$ is the only case, when the Jacobian of (A.12) is sparse.

We experimented with both approaches: as projections we chose splines and RBF neural networks. To solve the system (A.12), we apply the quasi-Newton iteration, with the Hessian update via the BFGS method suggested by Broyden (1970), Fletcher (1970), Goldfarb (1970), and Shanno (1970). Our numerical experiments suggest that for our problem the BFGS formula outperforms the Hessian update formula of Davidson (1959) and Fletcher and Powell (1963) and the steepest-descent approach.[21] Likewise, numerical experiments suggest that quasi-Newton iterations outperform the Nelder-Mead simplex algorithm by Lagarias et al. (1998) implemented in MATLAB function **fminsearch**.

Surprisingly, the L-B-J approach seems to perform better than the projection methods. Therefore, simulation results reported in this paper are based on quasi-Newton iterations on (A.12) with the BFGS Hessian update formula.

[21] These methods are implemented in the MATLAB function **fminunc**, which is used.

Figure 1: Czech policy relevant variables

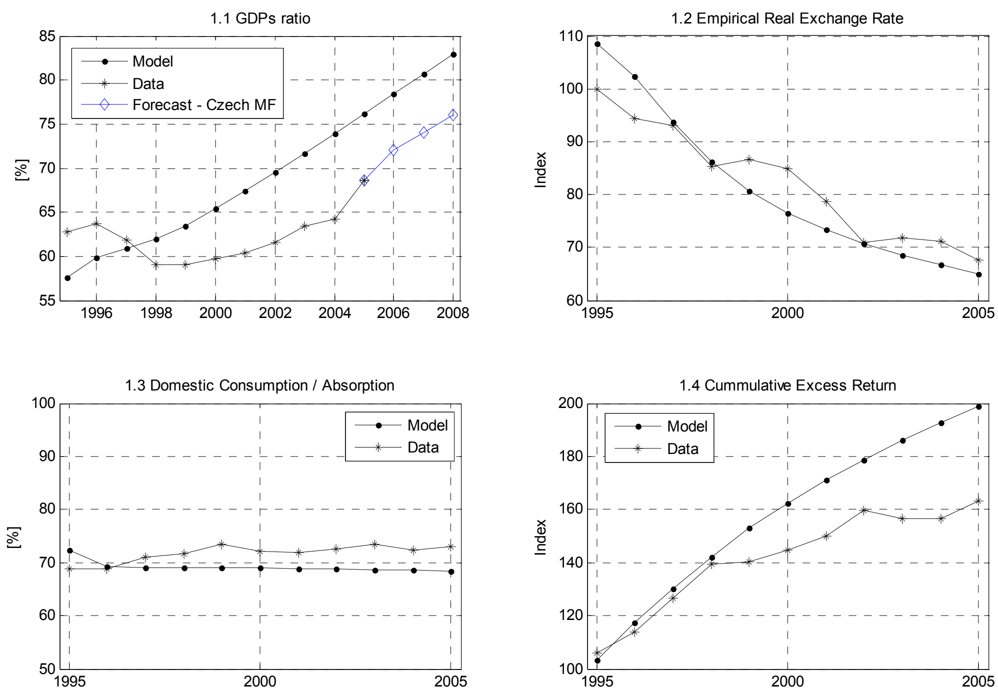

Figure 2: Czech balance of payments

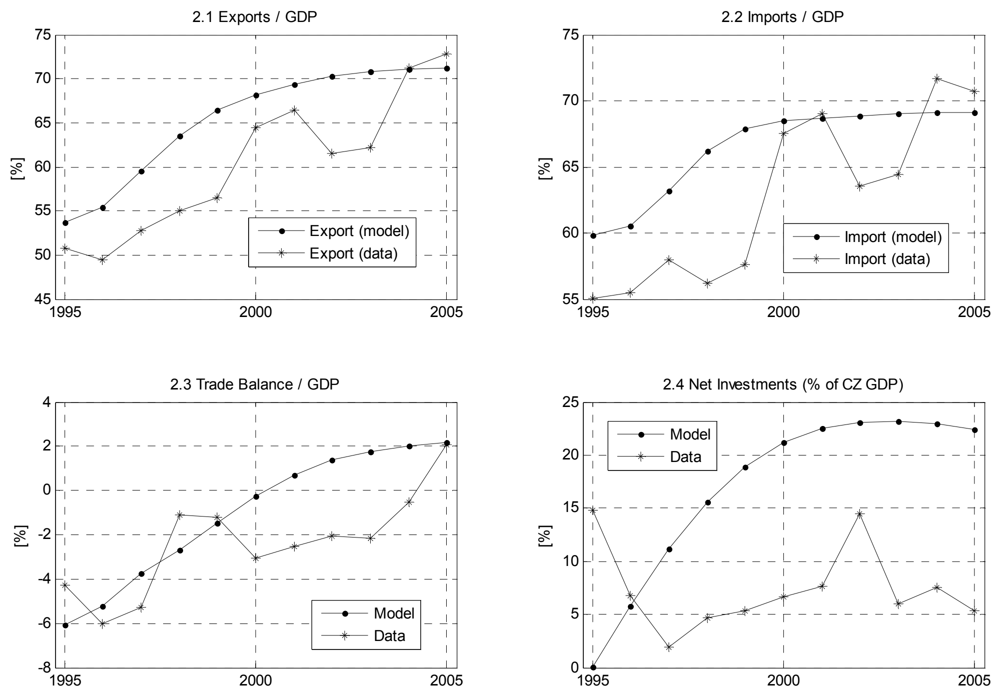

Figure 3: Long-run convergence projection

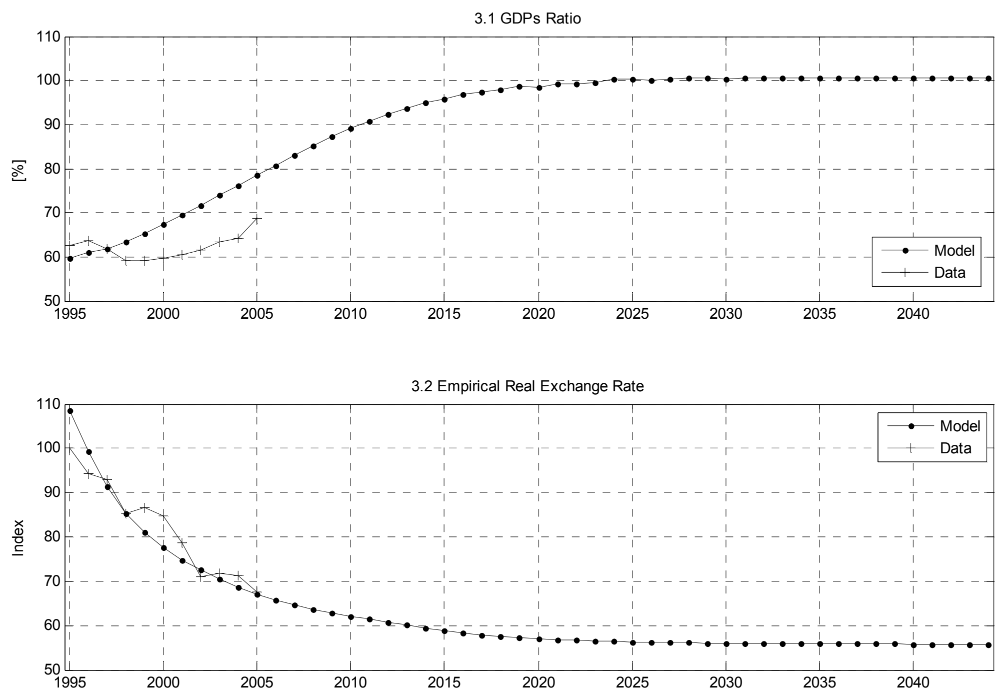

Table 1: Summary of model parameters

Parameter	Description	Value
θ	elasticity of substitution	4.71
β	discount factor	0.95
α	capital share	0.32
δ	exit rate	0.46
t	iceberg transportation cost	0.04
ε	elasticity of intertemporal substitution	2.09
m	auxiliary parameter for A^H_t	9.19
n	auxiliary parameter for A^H_t	12.03
τ	auxiliary parameter for A^H_t	4.89
A^H_{SS}	terminal value of domestic productivity	10.00
A^F	foreign productivity	10.00
c^N	fixed cost (non-eligible firms)	4.56
c^E_{SS}/c^N_{SS}	terminal ratio of fixed cost	2.10
c^E_{ini}/c^N_{ini}	initial ratio of fixed cost	3.48
$1/\psi_d$	adjustment cost parameter (domestic investment)	0.22
$1/\psi_f$	adjustment cost parameter (cross-country investment)	0.01
ψ_B	adjustment costs parameter (bond holding)	0.01
L^*/L	relative size of labor force	6.00

Note: A^H_t evolves according to the logistic curve: $A^H_t = A^H_{ss}*(1+m*\exp(-t/\tau))/(1+n*\exp(-t/\tau))$

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