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Solow Versus Harrod-Domar: Reexamining the Aid Costs of the First Millennium Development Goal

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Abstract

The First Millennium Development Goal (MDG#1) is to cut the fraction of global population living on less than one dollar per day in half, by 2015. Foreign aid financed investments may contribute to the attainment of this goal. But how much can aid be reasonably expected to accomplish? A widespread calibration approach to answering this question is to employ the so-called development planning technique, which has the Harrod-Domar growth model at its base. Two particularly problematic assumptions in this sort of analysis are the absence of diminishing returns to capital input and an infinite speed of adjustment to steady state after a shock to the economy. We remove both of these assumptions by employing a Solow model as an organizing framework for an otherwise similar analysis. We find that in order to successfully meet the MDG#1 in the context of the currently proposed aid flows, these flows will have to be accompanied by either an acceleration in the underlying productivity growth rate or a major boost to domestic savings and investment in sub-Saharan Africa. In the absence of such changes in the economic environment, the MDG#1 is unlikely to be reached.

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I. INTRODUCTION

In September of 2000, world leaders met in New York to adopt a new framework for addressing the urgent needs of people in developing countries. The Millennium Summit adopted eight Millennium Development Goals (MDGs), covering a wide range of issues such as health, education, gender equality, the environment, and more. The First Millennium Development Goal (MDG#1) sought to "eradicate extreme hunger and poverty." A quantifiable target was set to measure progress toward that goal, namely the reduction by half of the global proportion of people living in extreme poverty in 1990 by the year 2015. This goal has been endorsed by all major international financial institutions, including the IMF.[2] The purpose of this paper is to offer a basic theoretical framework with which to analyze progress toward this goal.[3]

In some respects, this project is on schedule. Sala-i-Martin (2006) conducts an empirical study of the global income distribution and world poverty, concluding that 69 percent of the MDG#1 has already been achieved. This reduction in poverty has not been evenly distributed geographically, however, with the lion's share of progress having been made in China and India. Most notably, Africa has lagged. Indeed, poverty rates in Africa increased slightly between 1990 and 2000. While justly celebrating the reduction of extreme poverty in many parts of Asia, few observers would feel that the MDG#1 can be said to have been achieved when an entire continent has been left behind. Therefore, in our calculations in this paper regarding the costs associated with the achievement of MDG#1, our focus will be on sub-Saharan Africa.

One of the fundamental justifications for development aid to poor countries is its potential for reducing poverty. Many analysts have proposed that aid can play a crucial role in fighting poverty, particularly in Africa. Of particular note has been the plan put forth by Sachs et al. (2004) and subsequently adopted by the United Nations (UN). The plan argues that a "big push" is needed to spur growth and reduce poverty in Africa, with concordant increases in aid flows from developed countries. The relationship between growth and poverty is multifaceted, but many studies have found a strong relationship between increases in per-capita GDP and the lowering of poverty (for example, Ravallion, 2001; Dollar and Kraay, 2002; and Besley and Burgess, 2003). Therefore, if a "big push" were to jump-start growth through a virtuous cycle of increasing income, investment, and productivity, one could envision the achievement of the MDG#1 in Africa.

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[2] "We are united in our desire to achieve the Millennium Development Goals and in our assessment that more aid is needed to achieve them," statement by Rodrigo de Rato, Managing Director, IMF at the Annual Meetings the Board of Governors of the IMF and World Bank, Washington, October 3, 2004.

[3] The other target contemplated in the MDG#1 was to, in the same period, reduce by half the number of people who suffer from hunger. This will not be addressed in the present paper.

However, the empirical literature on the effectiveness of aid in raising growth is mixed. Hansen and Tarp (2001) and Dalgaard et al. (2004) find modest, though significantly positive, effects of aid. The recent analysis by Clemens et al. (2004) claim a more substantial effect of aid, whereas Rajan and Subramanian (2005) fail to find a positive effect of aid on growth. Accordingly, since debate persists as to whether aid has a significant effect on growth in the first place (or as to the circumstances under which aid has a positive effect), it should be clear that no consensus exists on the *magnitude* of its effect. It therefore seems infeasible, at this stage, to assess the aid costs of MDG#1 on the basis of the econometric literature.

Consequently, this paper follows a different tack by adopting a theory-based calibration approach. Essentially, we incorporate aid into basic models of growth, set the parameters in those models to plausible values on which basis we calibrate the necessary amounts of aid so as to attain MDG#1. This exercise is in the tradition of "development planning" as practiced by Leontieff (1963), Chenery and Stout (1966), and many others, forming the backbone of theoretical support for aid flows in many development agencies, most prominently the World Bank (see Easterly, 1999). However, our approach differs from the traditional approaches in a number of key respects.

To see these differences clearly, consider Figure 1, which illustrates the principles of the development planning technique. We begin by assuming that the economy in question is proceeding along a balanced growth path, at a constant (possibly very low) growth rate. The trajectory is labelled "Old growth path" in Figure 1.

A development planner would then set either a new target growth rate directly or, alternatively, a target income level to be reached within a specified amount of time (say T years from now, $y(T)$), thus implying a required target growth rate, given the initial condition ($y(0)$). Next, assume the aggregate production technology is linear in capital input, as in a Harrod-Domar model when capital is believed to be the limiting factor, or in an endogenous growth model of the *AK*-variety (for example, Rebelo, 1991). This assumption implies that we can write the transition equation for capital, in the absence of foreign aid-financed investments, in the following way

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = A\frac{I}{Y} - \delta,$$

where a dot over a variable refers to (absolute) changes, Y is GDP, K is the capital stock, I is investments, δ is the rate of depreciation, and A is the marginal productivity of capital (under the AK interpretation) or the inverse of the "Incremental Capital Output Ratio (ICOR)," in the terminology of development planning. For a given ICOR and observed (and/or expected) domestic investment effort, the development planner could now assess how much additional aid-financed investments would be needed to fill out the "gap" = target growth rate—(1/ICOR)**I/Y*-depreciation rate. As illustrated in Figure 1, upon an infusion of a certain amount of investment financed by foreign aid, the economy would (in theory) react by shifting to a new growth path, attaining the desired income target, or fulfilling a general growth target.

A recent example of the use of this methodology is Deverajan et al. (2002). The authors determine a target GDP per capita level required to reach MDG#1 in 2015, and proceed to calibrate aid requirement under assumptions about the ICOR in different regions, and their investment capabilities. The end result is an estimated global "aid requirement" of US$40–70 billion per year.

It is clear that aid requirements, thus derived, are critically affected by the presumed production technology being "*AK*." This sort of an assumption has not gone unchallenged. Indeed, after discussing the properties of the *AK* endogenous-growth model, Solow (1994, p. 51) concluded that: "[…] this version of the endogenous-growth model is very un-robust. It can not survive without *exactly* constant returns to capital. But you would have to believe in the tooth fairy to expect that kind of luck."[4] Around the same time, Jones (1995) launched an empirical attack on the *AK* theory of endogenous growth, observing that in the member countries of the Organization for Economic Cooperation and Development (OECD), economic growth has been very persistent over the last century, whereas investment rates have increased. After performing various time-series tests, Jones rejects the implied linear association between investment rates and economic growth. More recently, Easterly (1999) extends the critique using data from the poorest countries in the world. Easterly also rejects the pure *AK* structure, but his reason is a slightly different one.[5] In practice, investment—partly fuelled by foreign aid—is fairly persistent in developing countries, whereas growth is not. This finding also contradicts a simple association between growth and investment. Moreover, Easterly performs counterfactuals under the assumptions that (aid) investments actually went into capital, and capital enabled higher income levels in the manner suggested by the linear technology. He finds that poor countries today are much too poor to be consistent with such a scenario.

In light of this criticism, we modify the underlying growth framework. Our approach recognizes that capital is likely to be subject to diminishing returns. We also take into account that convergence from one steady state to the next is unlikely to be instantaneous. So, although we perform an analysis with the same aim as development planning, the underlying theoretical framework is different. Instead of the *AK* model, we adapt the Solow (1956) model as our organizing framework. Figure 2 shows how the calibration works.

As in development planning, we assume the economy initially is in steady state, labeled "old steady state trajectory." The underlying rate of productivity growth is exogenous and can be varied in the calibration. We then calibrate an income target so that MDG#1 can be attained within in the specified amount of time; in theory at time *T*. Given the theoretically predicted path of an economy *off steady state* we can then determine the required increase in steady

[4] For an earlier critique; see the comments Allais (1963) on Leontieffs (1963) calculations.

[5] Strictly speaking Easterly attacks the "two-gap" model, rather than endogenous growth models. Since the underlying structure of the two models is the same, however, one could equally well see it as a criticism of "pure" *AK* models. That is, an endogenous growth model where the marginal product of capital is constant *at all points in time*.

state income (labeled "new steady state trajectory") for this target to be reached in transition, given the time schedule. Finally, making assumptions about how aid affects GDP per capita in the long run, which essentially are symmetrical to those adopted in development planning (that is, by enabling higher investment levels), we can calibrate how much aid would be needed to attain the target level of GDP per capita by $t = T$, and thereby the poverty target.

Our exercises should not be interpreted as forecasts in the usual sense. We are not proposing one correct way to model the effect of aid on developing economies, beyond our contention that Solow-based foundation for such models is far superior to the Harrod-Domar approach. Indeed, in our analysis we do not take a stand on either the values of key parameters nor on the specification of the growth process. More broadly still, our sole purpose is to model the potential effect of aid. Nothing in our analysis argues that the MDG#1 could not be achieved through other means. For example, a growth take-off in sub-Saharan Africa along the lines of the experienced by China, which did not receive significant amounts of aid. With our exclusive focus on aid, however, such larger issues are beyond the scope of the present paper.

Instead, we seek to answer a different set of questions. In our exercises, we take as given a set of parameters and as given a specification for the growth process and then ask how, under those assumptions, the MDG#1 could be achieved. Then we vary the parameters, the growth process, or both, and repeat. Thus, we obtain results from a range of broadly plausible parameters and also a range of model specifications, without advocating any specific formulation.

The analysis indicates that aid requirements will vary, depending on the structural characteristics of the economy in question (though perhaps not as much as one would expect). For example, aid requirements will depend on the "domestic investment rate" and the underlying productivity trend. As a result, we also perform calibrations with fixed flows of aid and ask how structural characteristics need to change, given those fixed amounts, in order to achieve the MDG#1. These exercises complement the recent debate about whether poor countries are in a "poverty trap" or not (for example Sachs et al., 2004; Kraay and Raddatz, 2006). Our analysis has no bearing on whether such traps exist or not and we make no attempt to model the mechanism by which countries could escape from such traps.

Our analysis can, however, be informative about how "deep" they would have to be in order for the amounts of aid given to be able to reach the target of cutting poverty into half within the next decade. That is, if indeed aid pushes the economy out of, say, a "savings trap," then how much of an increase in domestic investment effort would *as a minimum* be required (together with aid flows) to reach MDG#1? A similar exercise can be made with respect to a "productivity growth trap." Again, we have nothing to say regarding the plausibility of such traps, nor do we speculate about their underlying structure. Rather, our exercises focus solely on the implied magnitude of such traps, be they savings based or productivity based.

The present paper is directly related to the literature on development planning, indeed can be viewed as an extension to this literature. Early contributions include that of Leontieff (1963) and Chenery and Strout (1966), whereas a recent example is Deverajan et al. (2002). Easterly (1999) contains a discussion of how often this approach has been invoked over the intervening years, and some additional references. Another related contribution is that of Mourmouras and Ranzagas (2006), which studies a transition model, in the tradition of the pioneering contribution by Galor and Weil (2000). They allow for a utility-maximizing government, and go on to use the model to examine aid costs of reforms which might spur growth. If or when aid is given to a national government, the constraint faced by it will matter for the extent to which aid is actually translated into investment effort. The circumstances under which this happens can be studied in detail in a fully micro founded framework, such as that studied by Mourmouras and Ranzagas (2006). We view this contribution as an interesting complement to the present analysis. Our analysis is in some sense more limited in scope, in that we are interested in examining the implications of aid for the attainment of MDG#1. For this purpose, we simply assume that aid inflows are, in fact, able to spur capital accumulation, so as to assess its likely quantitative impact on growth within a coherent theoretical framework. In contrast, Mourmouras and Ranzagas (2006) examine the impact of aid in the broader context of ensuring a transition to a modern growth regime. Whereas the time dimension is very important for our analysis, it does not play a role in the Mourmouras and Ranzagas (2006) analysis—another marked difference.[6]

The plan for the paper is as follows. Section II lays out the basic theoretical framework in detail and derives the formulae needed for the calibrations that follow. Section III has two parts. We begin by using the basic model to examine the effectiveness of *past* aid donations, in the context of sub-Saharan Africa. We conclude that the Solow framework provides a much better organizing framework than the *AK* model. In the second part, we present our baseline calibrations of future aid requirements for sub-Saharan Africa, using the standard Solow assumptions with a range of parameter values for key structural characteristics. In each simulation we answer the following question: if the basic Solow model accurately describes African economies and if the specific parameters used are, in fact, the correct ones, then how much extra aid would be necessary to meet the MDG#1?

Suppose, however, that sub-Saharan Africa is caught in some form of poverty trap, as has been suggested by some observers. To address this issue, in Section IV we conduct calibrations examining under which circumstances changing domestic fundamentals along with aid inflows together may allow for a halving of poverty by 2015. We begin by modeling savings-based poverty traps. After deriving the corresponding formulae to incorporate a potential jump in savings, we consider how big that jump would have to be for a given aid flow and given parameters to achieve the MDG#1. We then turn to productivity-based poverty traps, deriving the formulae to incorporate a jump in the growth rate of productivity and then examining how big the magnitude of that jump would have to be, again using a given flow of aid and parameters. Finally, we model increases in *both* savings and productivity.

Section V discusses various extensions of the baseline model. In particular, we show that our basic approach to calibration is robust to the inclusion of externalities from capital

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[6] Mourmouras and Ranzagas also study policies that aim to affect fertility, which is an issue not examined in the present paper.

accumulation and if the number of accumulated factors is extended. We also point out how our results would be affected if domestic savings were endogenous, rather than exogenous. Section VI contains concluding remarks.

II. Basic Framework

The MDG#1 is formulated in terms of poverty reduction: the cutting in half of the headcount ratio (that is, the fraction of population living under the US$1 a day threshold).[7] The neoclassical growth model, which we will use as underlying framework for the analysis, only allows us to study the impact of aid on mean income. So we need to introduce a link between GDP per capita and poverty. Following the empirical literature on the topic of poverty reduction (for example, Ravallion, 2001; Bourguignon, 2002), we assume that

$$p \propto y^{-\pi}, \tag{1}$$

where p is the poverty headcount ratio, y is GDP per capita, and π is a parameter which specifies how much poverty declines, as average living standards rise. For example, Ravallion (2001) find across a sample of developing countries that $\pi \approx 2$. But estimates of the "poverty elasticity" (or "growth elasticity") vary across countries and time, so we will invoke a range of them in the calculations below.[8]

The virtue of adopting this simple association between poverty and mean income is that it is easy to back out the increase in GDP per capita which ensures that a given poverty objective can be reached. Specifically, if the goal is to cut poverty rates by half, GDP per capita will have to increase by $(1/2)^{-1/\pi}$. Hence if the poverty elasticity is, say, 2, then mean income would have to rise by 41 percent.

If we were indifferent as to exactly when this increase were to materialize, one could simply think of 41 percent as the required increase in long-run (steady state) income per capita and proceed to back out the needed aid requirement to make this happen, using a suitable growth model while making additional assumptions regarding how aid affects capital accumulation and long run GDP per capita. However, the MDG#1 involves a timetable: poverty should be

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[7] Technically the poverty line is US$1.08 a day per person, calculated at 1993 prices (Besley and Burgess, 2003).

[8] As pointed out by Bourguignon (2002), the elasticity will in general depend on the characteristics of the underlying income distribution and the level of GDP per capita. Assuming a lognormal distribution of income, Bourguignon shows that the elasticity is increasing in the standard deviation of log income, and in the ratio between the poverty line and GDP per capita. Hence, whether the elasticity rises or declines in the medium run (the focus of our analysis) would depend on the changes in both the mean and variance of the distribution.

cut in half by 2015. Hence, the above considerations give us a required increase in mean income, which needs to be reached within *T* periods, or years:

$$\frac{y(T)}{y(0)}=\left(\frac{1}{2}\right)^{-1/\pi}. \tag{2}$$

Therefore, the question which we now seek to answer is what increase in long-run (steady state) GDP per capita is required for the economy to be able to reach the target level of mean income, within the designated timeframe? Intuitively, if we can characterize the trajectory of income per capita, from an initial condition to its future steady state, we can calibrate the necessary increase in steady state mean income (and thereby the required aid inflow) which allows us to reach target income in *T* years (c.f., Figure 2). Fortunately, the neoclassical growth model allows us to trace the path of GDP per capita, thus enabling us to answer this question.

Consider an economy which can be described by a Solow (1956) model, formulated in continuous time.[9] The economy is in a vicinity of its future steady state, and it utilizes an aggregate production function of the Cobb-Douglas variety. Under these circumstances, it is well known (see for example, Mankiw et al., 1992) that the evolution of GDP per *efficiency* units of labor, $\tilde{y}$, follows:

$$\log\tilde{y}(T)=\left(1-e^{-\lambda t}\right)\log\tilde{y}^{*}+e^{-\lambda t}\log\tilde{y}(0), \tag{3}$$

where λ is the rate of convergence to steady state and $\tilde{y}^{*}$ is steady state GDP per efficiency units of labor. We can rearrange this equation so as to yield:

$$\frac{\log\left(\frac{y(T)}{y(0)}\right)-gT}{1-e^{-\lambda T}}=\log\left(\frac{\tilde{y}^{*}}{\tilde{y}(0)}\right). \tag{4}$$

To obtain equation (4) we have assumed that technological progress expands at a constant rate $g=\dot{A}(t)/A(t)$, and that $\tilde{y}(t)\equiv y(t)/A(t)$. Equations (2) and (4) together give us the answer to the aforementioned question, provided we have reasonable priors about trend growth, the rate of convergence, *and* assume that the economy, at *t=0*, is in steady state. In

[9] In the analysis which follows we invoke the positive representative agent assumption. This assumption, of course, does not imply that the underlying distribution of income needs to be perfectly egalitarian. Instead we implicitly assume that the underlying distribution of income is such that the elasticity rule (1) is a meaningful description of how individuals move above a certain threshold level of income. See Stiglitz (1969) for an analysis of the dynamics of wealth (and income) inequality within a representative agent Solow model, augmented by subsistence consumption. Caselli and Ventura (2000) extend the analysis to the Ramsey-Cass-Koopmans framework.

this case, $\tilde{y}^* / \tilde{y}(0)$ can be interpreted as the required increase in steady state income (per efficiency units of labor), which ensures that the income target, and therefore poverty target, is reached within *T* years.

The initial condition that the economy is in steady state may seem restrictive, which of course it is. But it is worth observing that previous work also imposed this initial condition. In fact, if the aid-calibration framework invokes an *AK*-type production function (which is what development planners have traditionally used, when calculating aid requirements) the economy is *always* in steady state (or on a balanced growth path). Hence our initial condition simply mirror the assumption made in previous calibrations. Instead, the fundamental difference to previous work lies in how the economy is supposed to react when aid is infused. In a development planning calibration, changes in investment rates and aid flows induce the economy to instantaneously "jump" to a new steady state trajectory, without any transitional dynamics. In other words, the assumption of past calibrations is that of an *infinite* rate of convergence to steady state. The present analysis merely relaxes the assumption of immediate adjustment and assumes that convergence from the initial steady state to the new one is gradual.

The general lesson from equation (4) is that for the economy to reach the target level of mean income in *T* years, aid needs to push the long-run level of income, y^*, even further. As an illustration; suppose mean income has to increase by 41 percent within 10 years, the rate of convergence is 10 percent per year, and that $g=0$, then aid inflows need to increase the steady state level of GDP per capita by 72 percent.

The required increase in steady state GDP per efficiency units of labor is clearly sensitive to assumptions about g and λ. Obviously, if the trend growth rate (g) is "high" the level of income will rise (and poverty fall) even without outside stimulus. The requirements from aid will therefore be lower, *ceteris paribus*. As is also clear from equation (4), the higher the rate of convergence, the lower the required increase in long-run GDP per capita. Accordingly, an economy which converges slowly to the steady state will need to be "pushed" harder, that is, requires a larger inflow of aid, than an economy which exhibits rapid convergence, for a given poverty reduction objective to be attained within a pre-specified amount of time.

It is important to realize that the trend growth rate and the rate of convergence are not independent of one another—in theory at least. Within the neoclassical growth model the rate of convergence depends on the structural characteristics of the economy (again, see for example, Mankiw et al., 1992). Specifically:

$$\lambda = (1-\alpha)(n+\delta+g), \qquad (5)$$

where α is the capital-output elasticity from the (Cobb-Douglas) production function, n is the rate of population growth and δ is the rate of capital depreciation. Accordingly, faster trend growth will be doubly useful in reaching poverty targets. It reduces the need for outside stimulus, and it increases the rate of convergence implying that less of an increase in steady state income is needed for a given objective to be reached in time. We take this link into account in the calculations below. However, we also present calculations where equation (5) is violated, so as to study the impact from convergence in isolation.

Next, we need to specify how aid matters for the process of capital accumulation and work out its impact on steady state prosperity. In keeping with past calibrations of "aid requirements," we assume the transfer of aid comes in the shape of investment. That is, the transition equation for the capital stock, *K(t)* can be written

$$\dot{K}(t) = I(t) + \varpi F(t) - \delta K(t), \varpi \leq 1, \tag{6}$$

where *I(t)* is domestically generated investments and *F(t)* is the aid inflow. The parameter ϖ is introduced to capture the potential for "waste." Accordingly ϖ=1 means that all aid flows are turned into investments. If instead ϖ<1, then some part of aid does not go towards capital formation; it could be dead-weight loss, it could go towards socially undesirable consumption (for example, corruption by government officials) or to socially desirable consumption (for example, disaster relief which doesn't increase the capital stock). In the calculations below we discuss how different assumptions about ϖ matter for the aid costs of MDG#1.

We next assume *I(t)* (and savings) are given as a constant fraction *s* of total income, that the population grows at a constant rate *n*, and that the economy is closed (except to foreign assistance, of course). We also assume that the amount of foreign assistance is kept constant as a fraction of GDP,

$$F(t) = f \cdot Y(t). \tag{7}$$

so that we can treat *f* as the "aid investment rate." Restated in efficiency units of labor the transition equation for capital reads

$$\dot{\tilde{k}}(t) = (s + \varpi f)\tilde{y}(t) - (n + g + \delta)\tilde{k}(t), \tag{8}$$

where $\tilde{k}(t) \equiv K(t)/A(t)L(t)$. To complete the model, we assume that the production function is Cobb-Douglas, that is

$$Y(t) = K(t)^{\alpha}\left(A(t)L(t)\right)^{1-\alpha} \Leftrightarrow \tilde{y}(t) = \tilde{k}^{\alpha}(t). \tag{9}$$

It is now straight forward to derive the steady state level of GDP per efficiency units of labor. Imposing $\dot{\tilde{k}} = 0$, solving for the steady state level of capital in efficiency units, and substituting the result into the production function yields

$$\tilde{y}^* = \left(\frac{\varpi f + s}{n + g + \delta}\right)^{\frac{\alpha}{1-\alpha}}, \tag{10}$$

where $\frac{\varpi f + s}{n + g + \delta}$ is the steady state *K/Y* ratio. The sum $\varpi f + s$ is total investment to GDP. Hence, in this sense there is absolutely nothing new here. The only difference to the standard Solow model is that we assume that some of the total investments made derive from aid inflows, while the rest are the result of domestic resource mobilization (*s*).

We can now examine the impact of increasing aid investments on long run prosperity. Log differentiation of equation (10) with respect to investment effort yields:

$$\frac{d\tilde{y}^*}{\tilde{y}} = \frac{\alpha}{1-\alpha}\left(\frac{\varpi}{\varpi f + s} df + \frac{1}{\varpi f + s} ds\right). \qquad (11)$$

Now, suppose domestic resource mobilization is kept constant, so that *ds =0*. We can then restate equation (11) in the following manner:

$$df = \frac{dF}{Y} = \frac{d\tilde{y}^*}{\tilde{y}^*}\left[\frac{1-\alpha}{\alpha}\frac{\varpi f + s}{\varpi}\right] \qquad (12)$$

which relates changes in the aid to GDP ratio to changes in long-run GDP per efficiency units of labor, and parameters of the model. Noting that $d\tilde{y}^* / \tilde{y}^* \approx \left[\tilde{y}^* - \tilde{y}(0)\right] / \tilde{y}(0)$, given our assumption that $\tilde{y}(0)$ is a steady state, we could combine equation (4) and (12), and proceed to calibrate aid requirements.

A more useful formula, however, is obtained if we insert the marginal product of capital, evaluated in the initial steady state. In the steady state we can write the marginal product of capital as:

$$MP_K = \alpha\left(\frac{Y}{K}\right)^* = \alpha\left(\frac{n+\delta+g}{\varpi f + s}\right) \Leftrightarrow \varpi f + s = \frac{\alpha(n+\delta+g)}{MP_K} \qquad (13)$$

If we substitute the last expression into equation (12) we get

$$\frac{dF}{Y} = \frac{d\tilde{y}^*}{\tilde{y}^*}\left[(1-\alpha)\frac{(n+\delta+g)}{MP_K^{\ *}\varpi}\right] \qquad (14)$$

The point of this is that we now can use the marginal product of capital as an input in the calibrations below, extracting estimates of its value from the literature. Empirical estimations of *s*, the domestic contribution to gross capital formation, present more of a challenge. Of course, once we make assumptions about MP_K and other variables, a size for *s* is implied by equation (13). We report these numbers when we do our calibrations.

In sum, the fruit of our labor is a simple formula, which gives required aid inflows as a function of the structural characteristics of an economy. Using equations (2), (4), the approximation $d\tilde{y}^* / \tilde{y}^* \approx \left[\tilde{y}^* - \tilde{y}(0)\right] / \tilde{y}(0)$, and (14) we have

$$\frac{dF}{Y} = \left[e^{\frac{-\frac{1}{\pi}\log\left(\frac{1}{2}\right) - gT}{1-e^{-\lambda T}}} - 1 \right] \left[(1-\alpha) \frac{n+\delta+g}{MP_K^{\ *} \varpi} \right], \tag{15}$$

where λ is given by equation (5). Notice that this formula, of course, does not confine itself to the case where poverty rates are to be cut in half. Any other target could be accommodated, by substituting $-(1/\pi)\log(1/2)$ for the general expression $-(1/\pi)\log\left(p(T)/p(0)\right)$.

Compared with the formula used in development planning, equation (15) is more demanding on the input side. However, it is not much more demanding. Consider a development planning exercise which strives to reduce poverty by way of an influx of aid. The development planner would then need to know $-(1/\pi)\log\left(p(T)/p(0)\right)$, since it provides the income target. Given initial income, a target growth rate (of GDP) would be implied. Thus n and g would implicitly be "input" as well. Since aid in such exercises funds capital (as it does here), the depreciation rate of capital would also be an input. Finally, the planner would need to have a sense of how productive aid investments are supposed to be. In the realm of development planning this is what a "reasonable" ICOR provides. Strictly speaking, $MP_K^{\ *}\varpi$ is therefore also required input for development planning. For our calibrations we need a bit more information, which reflect the dimensions in which we abandon the theoretical framework of development planning. First, we assume that the economy only gradually returns to its steady state—hence the presence of the rate of convergence, λ. As we pointed out above, the development planning approach would assume instantaneous adjustment to a new steady state trajectory, $\lambda = \infty$. The second difference is that we allow capital to be subject to diminishing returns. This is why we need to know α. In traditional planning exercises, $\alpha = 1$ would be implicit. Hence, as should be clear, the fundamental source of differences in estimated "price tag" on MDG#1 will be due to these two "new" assumptions: convergence and diminishing returns.

III. Baseline Calibrations

A. Evaluating Impact of Past Aid Flows

Before we start calibrating aid requirements for the future, it seems like a prudent "check" of the framework to do a little "back casting." As demonstrated by Easterly (1999), the AK-based approach over predicts actual GDP per capita of aid-recipients to a rather extreme extent. Is the same true for the present Solow-based framework?

To answer this question we focus on the sub-Saharan region, where growth over the last 3 decades has been dismal, in spite of continuous infusions of aid. Figure 3 illustrates these facts. The figure comprises 30 sub-Saharan African countries for the period 1970–2000, and shows the evolution of GDP per capita (constant 2000 US$) and the aid to GDP ratio. The countries are chosen based on the criterion that data is available for all years.[10] GDP per capita in any year is defined at the sum of GDP in the 30 countries, divided by the sum of populations. Total aid in any given year is similarly calculated as the inflow to all the 30 countries. In effect, therefore, we treat this group of countries as "one big country." As seen, the period in question can be described as one of stagnation in living standards; GDP per capita actually fell slightly by roughly 2 percent. Simultaneously, aid inflows rose from about 2 percent of total GDP in 1970 to about 10 percent in the mid-90s after which it fell to around 5 percent of GDP as the millennium came to a close—roughly the average for the period.

To assess the growth implications of aid for this "country," we begin by calculating the impact on steady state GDP per capita (using equation (10)) from a permanent increase in "aid investments" to 5 percent, starting at 2 percent of GDP. We thereby match the initial "aid investment rate" and the average aid/GDP ratio for the period. Assuming the economy initially is in steady state, we can then project GDP per capita in 2000, using the predicted time path for GDP per capita, under the Solow model (equation (3)). Based on available data for the sample of countries under consideration, we employ a 3 percent rate of population growth (the average for 1970–2000), put g=0, δ=0.05 and impose s=0.12. The latter assumption implies, when ω=1, that the GDP share of gross capital formation at the end of the period is 0.17, in accordance with the evidence. Finally, suppose capital's share is fairly large: ½ say.

These assumptions imply a rate of convergence of 4 percent, which matches the finding of Hoeffler (2002) who fit the (augmented) Solow model to data pertaining to Africa in isolation. The "predicted" gain in GDP per worker from observed aid flows over the last 30 ears is only slightly more than 7 percent. If we reduce capital's share to 1/3, the gain in GDP per capita falls to a mere 4 percent, or, what amounts of an acceleration in (average) GDP per capita growth of roughly 0.1 percent. Obviously, if parts of the aid inflow is not invested (so that ω is smaller than 1), the predicted gain is further reduced.

It is disturbing that GDP per capita has stagnated in sub-Saharan Africa. But the above calculations suggest that under the neoclassical growth model this stagnation does not necessarily lead us to believe we are faced by an "aid effectiveness" puzzle. In the end, the

[10] The data source is the 2005 edition of World Development Indicators. "Aid" refers to ODA. The 30 countries are: Benin, Botswana, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Democratic Republic of Congo, Republic of Congo, Cote d'Ivoire, Gabon, The Gambia, Ghana, Kenya, Lesotho, Madagascar, Malawi, Mauritania, Niger, Nigeria, Rwanda, Senegal, Seychelles, Sierra Leone, Sudan, Swaziland, Togo, Uganda, Zambia, and Zimbabwe.

amounts of aid given should not have been expected to make a dramatic difference, seen through the lenses of the neoclassical growth model.

We believe these calculations illustrate that the Solow model is a plausible tool for forming priors about the impact from aid in an African context. Looking forward, we can ask how the model can inform us about the effects of future aid. Specifically, what combination of aid flows and parameters values that will enable the MDG#1 to be reached in sub-Saharan Africa.

B. Calibrating Aid Requirements for the Future

Of the set of parameters to be chosen, in order to use equation (15), the expected population growth rate is probably the easiest to pin down. Throughout, we will use UN's population growth projections for the period 2005–15 for Africa.[11] Regardless of which "variant" we choose (low, medium, high), the population growth rate is about 1 percent per year on average.

A second key input is the trend growth rate, g. As should be clear from Figure 3, there is little evidence of persistent growth in total factor productivity (TFP) in sub-Saharan Africa, over the past 30 years. This would suggest that assuming a constant level of TFP would be a reasonable baseline assumption, which we therefore adopt.[12] However, in the next section we will explore the consequences of an increase in the underlying productivity trend.

A third input parameter to be chosen is the *aggregate* marginal product of capital (MPK) in sub-Sahara Africa. In this regard, we are not faced with an abundance of evidence on which to rely. However, a recent paper by Caselli and Feyrer (2006) calibrate marginal products for a cross section of countries, including 6 sub-Saharan African countries: Botswana, Burundi, Congo, Cote d'Ivoire, South Africa and Zambia. Without adjustments for natural capital, the MPKs fall in range from 8 percent (Burundi) to 24 percent (Botswana). Taking natural resources into account the MPKs are reduced to a range from 1 percent (Burundi) to 14 percent (Botswana).[13] These estimates presume competitive markets and price taking

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[11] The projections are found at http://esa.un.org/unpp/ .

[12] A recent paper by Young (2005), which aims to assess the welfare cost of the AIDS epidemic, also assumes g=0. In a footnote Young presents some evidence in favor of the assumption deriving from growth accounting, while recognizing that the quality of input data for such exercises in Africa is rather doubtful.

[13] The basic approach taken by Caselli and Feyrer consists of using the fact that capital's share in theory equals (User cost of capital) *(K/Y). With data on K , Y, capital's share, and the relative price of investment to output, one can calibrate MPK= [(Price of output)/(Price of investment)]*(Y/K)*Capitals share. A key element in their analysis relates to which measure of capitals share to invoke, which is where natural wealth adjustments comes into the picture.

behavior on the part of produces. If externalities are important, the private MPK and the social MPK can be different. In order to detect such effects, direct estimation is necessary. Such an analysis is conducted in Dalgaard and Hansen (2005), where the *average* aggregate marginal products across time and countries is estimated for a group of aid receiving countries, based on an observable (modified) growth accounting equation. According to this analysis, the average marginal product falls in a range from 20 to 30 percent. There is no evidence that "aid financed investments" are less productive than those financed by other sources (domestic resource mobilization, FDI).[14] As a result, for the calibrations below we assume MPKs in a range from 10 to 30 percent, capturing the findings of these two studies in broad strokes.[15]

The fourth important input variable is capital's share, which in theory parameterizes the curvature of the aggregate production function. The standard assumption in macroeconomics is to allow α to fall in a 1/3 to 0.4 range. However, in a recent study Kraay and Raddatz (2006) argue that somewhat higher values are appropriate for poor countries; they explore values of α in a 0.5 to 0.6 range. The studies by Gollin (2002) and Caselli and Feyrer (2006) detect substantial cross-country variation in capital's share, though not variation that seems to be systematic to levels of development. Accordingly we will allow α to fall in the range 1/3 to 0.6.

Fifth, for our main calibrations we need to choose a poverty-elasticity. A recent paper by Besley and Burgess (2003) is the most appropriate for present purposes, as it estimates equation (1) using poverty rates and GDP per capita. They find $\pi = 0.75$, with a standard deviation of 0.25, using data for the entire less developed world. When they confine attention to sub-Saharan Africa the elasticity falls to 0.49 (with a standard deviation of 0.23). Hence, using the "full sample estimate" Besley and Burgess' study would suggest allowing π to fall in a range from 0.25 to 1.25, which engulfs the point estimate for sub-Saharan Africa taken in isolation. Larger poverty elasticity's are found in the literature, but typically when expenditures, rather than GDP per capita, are used as right hand side variable in the regression. As this does not correspond exactly to our specification, it is not obvious that using such estimates would be appropriate. Nevertheless, for completeness we will allow π to go as high as 3, which is the upper limit to the estimate found by Ravallion (2001) on data including both income and expenditures.

Finally, we also provide calibrations where the rate of convergence is exogenous, implying the need for a prior. To ensure consistency with the underlying theoretical framework we only look to studies estimating the (augmented) Solow model. Even narrowing attention to

[14] MPKs likely vary across countries and time. Hence Dalgaard and Hansen develop a correlated random coefficient approach to estimating average returns. A range of estimators are employed to take endogeniety into account. The issue of weak instrumentation is addressed also.

[15] Is equation (1.15) still valid if externalities are present? The answer is a qualified "yes" and the details are shown in Section V.

this group of contributions, estimates of the rate of convergence vary. Mankiw et al. (1992) estimate that countries tend close about 2 percent of the gap to steady state each year, using the OLS estimator on a cross-section data set. Subsequent studies, invoking panel data, initially found evidence in favour of more rapid convergence. In particular the study by Caselli et al. (1996), who invoke the first-difference GMM estimator, estimate λ to be 10 percent per annum. However in the last few years the rate of convergence has tended to "converge" back to the original cross-section estimate. Bond et al. (2001) argue that the first-difference GMM estimator may be poorly behaved since lagged levels of GDP per capita provide poor instruments for subsequent growth rates. Instead they invoke SYS-GMM and proceed to produce a 2.4 percent estimate for λ. Arellano (2003) discuss optimal selection of instruments in a GMM setting, and provides an illustration for the Solow model. Arellano finds a rate of convergence of about 4 percent. Accordingly, we examine rates of convergence in the range 2–10 percent. Still, for present purposes the finding of Hoeffler (2002) seems particular relevant (that is, 4 percent, as mentioned above), in that it speaks to Africa in isolation.

Table 1 shows the results of the aid calibrations. In all cells we maintain a set of assumptions, which are reported at the bottom of the table. Notice that we maintain ϖ=1 and *MPK*=0.2 in all reported calibrations. The reason is simply that an interested reader quickly can assess the consequences of changing these assumptions, as evident from equation (15). Both parameters enter the denominator of the expression, which means that simple multiplication of the reported aid requirements in the table is sufficient to get new results. For example, if ϖ=0.5 were to be considered more appropriate, all numbers in the *dF/Y* column should be multiplied by 2.[16] Likewise, if *MPK*=0.3 is thought to be more relevant, all aid requirements should be multiplied by 2/3.

In the first 18 rows the rate of convergence is endogenous. Accordingly if capitals share is changed the rate of convergence also responds (c.f. equation (5)). The implied rates of convergence are broadly consistent with Hoeffler's (2002) estimates for Africa. Nevertheless, in the last 6 rows we decouple this link to see the effects of varying assumptions about λ.

Turning to the results it is clear that aid requirements are rather steep, especially if we use the poverty elasticity estimated by Besley and Burgess (0.75). If the poverty elasticity rises to its perceived upper limit, aid requirements are dramatically lowered, though remain large. For example, with a poverty elasticity of 2, dF/Y falls in a 37–49 percent range depending on the assumption about capital's share. This amounts to a required increase in aid inflows to what Sachs et al. (2004) label "Tropical sub-Saharan Africa" of between US$ 76 and 100 billion in

[16] Setting ω=1 is almost certainly an overstatement of the capital accumulation content of aid donations. For example, about 6 percent of aid flows are emergency relief (White, 2002, Table 15). However, as noted, it is comparatively easy to assess the consequences of changing the size of ω for aid requirements, for which reason we will continue to assume that all aid propels capital accumulation.

the first year; total flows will subsequently have to increase over time so as to keep pace with GDP and maintain a constant aid to GDP ratio.[17]

Comparing the results for varying assumptions about α, it might at first seem odd that as capital's share is increased, aid requirements *increase*. The explanation is, however, simple. A larger capital share will, on the one hand, imply less diminishing return to capital, which tends to make additional investment more able to expand long run income. On the other hand, however, less diminishing returns lowers the rate of convergence, which implies that the economy need a bigger "push" to reach a given income target within 10 years time. Accordingly, with slower convergence more aid is needed for the fulfilment of MDG#1. As it turns out, the latter effect dominates, which explains why assuming a larger share of capital does not bring down the calibrated costs of halving poverty.

The last 6 columns show the "pure" influence from the rate of convergence on the aid costs of MDG#1, in a setting where capital's share is favourably chosen (from the perspective of aid effectiveness). The rate of convergence is an important variable, in that fast convergence at 10 percent lowers aid requirements considerably. The last row provides an example where λ is "huge," mimicking one aspect of traditional development planning; immediate convergence.[18] This assumption would, in its own right, lower aid requirements significantly. This shows the importance of taking convergence into account when calibrations such as these is performed, and illustrates how the traditional assumption of infinitely fast adjustment have lead researchers and practitioners to overestimate the impact of aid on growth.

The calibrations are useful in highlighting which structural characteristics are important for aid effectiveness in the context of poverty reduction. For example, our calculations are not very sensitive to assumptions about the extent of diminishing returns (α). Instead, the poverty elasticity is a key input. Accordingly, getting an accurate estimate for this parameter is of practical importance when forming reasonable priors about the impact from aid on poverty reduction in a specific context.

The conclusion from these exercises is that aid inflows of realistic magnitudes are unlikely to ensure a halving of poverty in sub-Saharan Africa, over the course of 10 years. As explained in the next section, currently contemplated aid flows to tropical sub-Saharan Africa amounts to an increase of about 12 percent of GDP, or US$ 25 billion, well short of the 20–23 percent requirement calibrated above as a "best-case" scenario involving a poverty elasticity as high

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[17] This range for aid requirements is based on pooled GDP data for 2002. Specifically, the countries included are: Angola, Benin, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Democratic Republic of Congo, Republic of Congo, Cote d'Ivoire, Eritrea, Ethiopia, Ghana, Guinea, Kenya, Liberia, Madagascar, Malawi, Mali, Mauritania, Mozambique, Niger, Nigeria, Rwanda, Senegal, Sierra Leone, Sudan, Tanzania, Togo, Uganda, Zambia, and Zimbabwe.

[18] When λ=0.5, the term $[1-e^{-\lambda T}]$ in equation (15) is 0.99; associating this case with an "infinite rate of convergence" (where the term is 1 exactly) is therefore fairly reasonable.

as 3. To get a sense of the difference between these two numbers, in terms of poverty reduction, we can calibrate the poverty reduction a 12 percent increase in the aid to GDP ratio might "buy." In order to do so, we begin by simulating the expected increase in GDP per capita, from 2005 to 2015, using the approach from Section 3A. In 2002 Tropical sub-Saharan Africa received about US$ 18 billion in aid, which amounts to roughly 9 percent of total GDP. Accordingly, suppose f increases from 9 percent to 21 percent, and let s=0.1, n=0.01, g=0, α=0.5, ϖ=1 and δ=0.05 (the implied initial *MPK* is 20 percent). Under this set of assumptions we find an increase in GDP per worker, within the 10 year window, of roughly 14 percent. Accordingly, given a poverty elasticity of 3, the reduction in poverty is 32 percent, rather than the 50 percent target. However, if the poverty rate is 0.75, the 12 percent increase in the aid/GDP ratio will only be associated with a 9 percent reduction in the headcount ratio, *ceteris paribus*.

IV. Domestic Additional Effort, Poverty Traps, and "Takeoff"

In the calibrations above we assume that nothing changes in the aid receiving nations when aid flows into the country. That is, domestic savings, productivity growth, etc., remained unaffected by the outside stimulus. The assumption of "business as usual" is the standard in calibration exercises such as these. But there are circumstances under which one, in theory, would expect aid inflows to induce change in key structural characteristics.

Increased aid donations could be associated with some form of conditionality. As an example, one could imagine donors requesting reforms, aimed at stimulating private investment efforts; these could perhaps involve institutional reform aimed at providing more secure property rights, a crack-down on corruption and so on. Increased domestic investment effort would naturally reduce the aid requirements for the attainment of MDG#1. Conversely, the amount of additional domestic effort needed (on top of managing aid inflows of course), would be a function of how much foreign assistance will be forthcoming. Hence we can do a simple calibration to assess this "burden." Figure 4 illustrates it.

The basic idea is to infuse a fixed aid-investment rate into the economy. This means that we need priors about how much aid is likely to be forthcoming over the next years, and we return to this issue below. For now, simply think about aid flows as exogenously given. Moreover, suppose this aid inflow is insufficient so as to ensure the economy reaches the target at time T, as illustrated in Figure 4. Then we can back out the needed increase in *domestic* investments which, together with aid, would allow the economy to reach $y(T)$.

Technically we do the following. Instead of assuming ds=0, we re-write equation (11) so as to yield

$$ds = \frac{d\tilde{y}^*}{\tilde{y}^*}\left[\frac{1-\alpha}{\alpha}\left(\varpi f + s\right)\right] - \varpi df, \qquad (16)$$

where df now is treated as an input in the calibration, so as to back out ds. As explained in Section 2, we can insert the marginal product of capital in the steady state, as well as the formula for the required increase in steady state income. The final equation used to back out the investment increase is therefore

$$\frac{dS}{Y} = \left[e^{\frac{-\frac{1}{\pi}\log\left(\frac{1}{2}\right) - gT}{1 - e^{-\lambda T}}} - 1 \right] \left[(1-\alpha) \frac{n+\delta+g}{MP_K^{\ *}} \right] - \varpi \frac{dF}{Y}. \tag{17}$$

The rate of convergence, λ, is still given by equation (5).

Observe that this calibrated increase will be a *minimum* requirement. The reason is that the calibration assumes that the domestic investment rate *immediately* rises to its new level. If reforms are gradual, and only take effect over time, the required increase in the domestic investment rate would be greater for MDG#1 to be achieved within a fixed number of years.

Aside from policy reform, there could be another cause for a sudden change in domestic investment effort. The most frequently cited reason is the existence of poverty traps. A plausible example would be the "savings trap." The idea is that, due to the presence of subsistence consumption, poor people save nothing, or next to nothing. As this implies a stagnating capital stock (at best) and therefore stagnating standards of living, the initial situation of low income and savings is perpetuated. This vicious circle can be broken, however, if income rises sufficiently (even if temporary). A higher level of income, perhaps attained through foreign aid, leads to savings, capital accumulation and rising income: the beginning of a virtuous circle. This idea has recently been advanced as a key element in explaining Africa's dismal growth performance over the last half century and as a reason why aid inflows could have a large impact on prosperity (Sachs et al., 2004).[19] This diagnosis, however, is currently being questioned (see Easterly, 2005; Kraay and Raddatz, 2006).

In this paper, we do not wish to participate in the on-going debate about whether or not Africa is in a poverty trap. To be sure, the methodology employed in our analysis has nothing to say regarding the existence of poverty traps in Africa, or the lack thereof. Nonetheless, the present framework can address a related question. Supposing that Africa *is* in a poverty trap, how "deep" must that trap be in order for a given increase in aid inflows to be sufficient to cut poverty in half on the continent?

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[19] As pointed out by these authors (pp. 121–22): "Our explanation is that tropical Africa, even the well-governed parts, is stuck in a poverty trap, too poor to achieve robust, high levels of economic growth and, in many places, simply too poor to grow at all… We argue that what is needed is a 'big push' in public investments to produce a rapid 'step' increase in Africa's underlying productivity… In particular, we argue that well-governed African countries should be offered a substantial increase in official development assistance (ODA) to enable them to achieve the Millennium Development Goals (MDG)… by 2015."

From the perspective of the calibration discussed above, there is fundamentally *no* difference between an assumed increase in *s* due to policy reform, and an assumed increase in *s* because a country exits a poverty trap.[20] Hence, another way of interpreting the calibration depicted in Figure 4 is that the additional domestic investment effort appears as the result of an escape from the poverty trap. The calibration therefore tells us how big of an increase would be necessary for aid, along with an escape-of-the-trap induced investment spurt, to allow the MDG#1 to be reached. Again, note that the calibrated "jump" in investments is an immediate one. So the experiments amount to asking the following question. Suppose aid is given and the economy immediately shifts into a high savings regime; how much of an increase in domestic savings and investments would be needed for the economy to achieve MDG#1, for given aid flows?

To do these calibrations we need to specify an inflow of aid. This choice is unavoidably somewhat arbitrary. Given that many different levels of aid have been proposed, we will rely on aid flows called for by the most prominent plan for development of Africa, namely that of Sachs and associates, mentioned above. That is, we increase aid by US$25 billion in Tropical sub-Saharan Africa.[21] Sachs et al. (2004) argue forcefully that sub-Sahara Africa is in a savings poverty trap, and that this infusion of aid should allow the region to reach (among other goals) the MDG#1. Accordingly, our exercise can be viewed as delivering the required size of the poverty trap, so as to make this argument internally consistent—under the assumptions of the basic neoclassical growth model.[22]

It should be emphasized that under the plan laid out in Sachs et al (2004) this US$25 billion would not be used exclusively for direct capital accumulation. The plan contemplates myriad expenditures on education, health etc, which we do not take into account here. Therefore, in

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[20] In practice, of course, there is a major difference: In the former case some actual effort on the part of the government is required in the sense of reforms, in the latter case the investment increase will appear by itself.

[21] As Sachs et al. (2004, p. 185) puts it, "The current level of ODA is surely a limiting factor for achieving the MDGs in the well-governed African countries. It is not only necessary but also possible to remove this constraint. We have found that well-governed African countries need an additional $40 or so per capita per year in development assistance. If we supposed that 620 million Africans were to receive that amount, it would add about $25 billion a year to the $18 billon a year provided in 2002, or more than a doubling of aid flows to the continent."

[22] Alternatively, one could have invoked potential flows stemming from the over-all pledge by world leaders to raise aid flows to 0.7 percent of GNI, from its 2004 level of about 0.3 percent. In this case, however, we would have to speculate as to what fraction of this additional aid would be allocated to Africa. Yet another approach would be to implement an increase in aid, which was pledged at the 2006 G8 summit at Gleneagles, i.e. a doubling of aid to sub-Saharan Africa. Under the proposal of Sachs et al., used below, aid is assumed to more than double in the years to come.

using the US$25 billion figure in this exercise we are effectively modeling an upper bar for the direct effect of the "Sachs plan" in capital accumulation. It is conceivable that some of the alternative uses of the US$25 billion could have important indirect effects on capital accumulation. Accordingly, the calibrated increase in the domestic investment rate may be interpreted as resulting from these indirect effects, from exiting the poverty trap, or a combination of the two.[23]

We use data for 32 countries situated in what Sachs et al. (2004) refer to as Tropical sub-Saharan Africa; this set excludes South Africa. Following the same procedure as in Section 3A, we pool all countries with respect to GDP and aid flows. Upon doing so we find that these countries received aid in what amounts to about 9 percent of total GDP. This corresponds to total aid flows of US$ 18 billion. Accordingly, we assume this number is raised to US$43 billion and is increased thereafter to maintain a constant ratio of aid to GDP of roughly 21 percent.

Finally, we need to choose parameters from the ranges discussed in Section 3B. Accordingly, we set the marginal product at the highest value we consider (30 percent) and capital's share to the lowest value in its range, 1/3. Thus, we are choosing values for these parameters which would generate the greatest reduction in poverty in response to aid flows, *ceteris paribus*. The other variables are set at the level assumed in Section III, as summarized at the bottom of Table 2. The implied initial domestic savings rate and the implicit rate of convergence are also reported.

Unsurprisingly, in light of our calibrations from Section III, we generally find that the required increase in domestic savings/investment intensity is large. Staying within the range of poverty elasticity's consistent with Besley and Burgess (2003) estimate, the smallest (minimum) increase in domestic investment effort is around 46 percent.[24] But if the poverty elasticity is considerably larger, reaching the upper limit of Ravallion's (2001) findings, the required increase shrinks to 2 percent. It is important to recall, however, that we assume aid flows are turned into savings/investment on a 1:1 basis. Hence the last case would require a *total* increase in the savings/investment rate of 14 percent (aid flows plus additional domestic effort).

How big of an increase in savings is *a priori* plausible? Rodrik (2000) examines the contours of what he labels "saving transitions." That is, periods during which the savings rate of an economy rises to a sustained higher level. Rodrik define a saving transition as a scenario

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[23] Alternatively one could view expenditures on health and schooling as accumulation of another capital good, human capital. In Section V we discuss how our calibrations are affected if multiple capital goods are introduced.

[24] The calibrations are unrestricted, as can be seen from row 1 of the table; it goes without saying that an increase in s of anything close to (or in excess of) 100 percent is meaningless for practical purposes.

where (a three year moving average of) the savings rate increases by at least 5 percentage points over a 10 year period. Using this filter, Rodrik detects 20 such transitions (when excluding natural resource abundant economies from the sample), for the 1965–87 period. In this sample of countries, the median savings rate increases from 14 to 23 percent within a 5 year period, and further to 25 percent within a 10 year window. The most spectacular case, however, would be that of Lesotho in the 1970s, with an increase from 9 to 22 percent. Hence, there are no cases of savings transitions involving increases of more than 12–13 percentage points and taking place over relatively short periods of time during the second half of the 20th century.

As a result, we are inclined to believe that increases in domestic savings, of the magnitudes reported in Table 2, are unlikely to occur, poverty traps or no, since it would require a savings transition in the entire sub-Saharan African region of historically unseen proportions.[25]

A more dramatic poverty traps story could involve the trend growth rate itself (g). That is, perhaps sub-Saharan Africa is caught in a zero productivity growth trap, but upon a sufficient infusion of aid, productivity growth is ignited. One can imagine many possible channels by which this could occur: strengthening governance and the rule of law, greater technology transfer from developed countries or improvements in health or education.[26] Returning to the example of Sachs et al. (2004), many of the policies advocated are explicitly or implicitly justified by appealing to their possible effects of productivity. As with savings-based poverty traps, Kraay and Raddatz (2006) find scant evidence for the existence of productivity-based poverty traps in Africa. Once again, whether or not such traps exist is well beyond the methodological scope of the present paper. However, we can examine the magnitudes of productivity increases which would be necessary to achieve the MDG#1.

Table 3 models different scenarios of a productivity "takeoff." As in the previous exercises, we start out with in steady-state and no productivity growth at all, corresponding loosely to the productivity stagnation in Africa. At t=0, aid flows permanently increase and, through some unspecified channels, cause an immediate and permanent increase in productivity growth. Under varying assumptions regarding the poverty elasticity, we examine the necessary increases productivity growth g which would have to accompany the increase in aid in order to achieve a reduction of poverty by one half, thereby achieving the MDG#1. Obviously, if the increase in aid is large, a smaller increase in g will be necessary. It is

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[25] Rodrik (2000) finds evidence that aid inflows stimulate savings. According to his estimates roughly half the inflows are turned into savings. This could be taken to suggest that ϖ=0.5 would be roughly appropriate, rather than ϖ=1 as we assume in Tables 1 and 2.

[26] Here, we would only be considering improvements in health or education which would operate through increases in productivity. Alternatively, such improvements could be modeled as improving the stock of health or human capital. These will be discussed in Section V.

important to note that the idea here is not that higher aid would *lead* to a lower increase in *g*. Rather, a higher level of aid inflows means that a smaller increase in *g* is *needed* in order to make up the difference in GDP growth necessary to achieve the MDG#1. Additionally, a lower elasticity of poverty reduction will imply greater necessary increases in productivity, aid flows, or both.

As with the savings-based poverty traps our results are very sensitive to the value of the poverty elasticity. For plausibly high poverty elasticities, the MDG#1 could be achieved with feasible levels of aid flows. However, the corresponding increases in productivity growth would be substantial. For example, with a poverty elasticity of 2—well above the range posited by Besley and Burgess (2004)—and an increase in aid flows of 8 percent the MDG#1 could be achieved; such an increase would correspond to the increase in aid flows to sub-Saharan Africa proposed in Sachs et al. (2004). However, productivity growth would have to increase from zero to 1.6 percent. This would constitute a dramatic change in the evolution of productivity growth. By way of comparison, during the boom years of 1966–90, productivity grew in Taiwan by 1.8 percent per year, while in South Korea during the same period it grew by 1.2 percent. (Rodrik, 1994).[27] Thus, even under these relatively optimistic assumptions about the poverty elasticity and aid flows, Africa would have to immediately attain East Asian levels of productivity growth in order to meet the MDG#1.

Of course, savings-based poverty traps and productivity-based poverty traps could occur simultaneously. Thus, some convex combination of the two types of poverty traps could imply lower needs for increased aid flows. To model such cases, we again turn to the 12 percent of GDP increase in aid flows to sub-Saharan Africa suggested in Sachs et al. (2004). We continue to assume the future underlying growth trend in sub-Saharan Africa rises immediately when aid inflows are increased. We can then calculate how varying assumptions about this acceleration influences the required (immediate) increase in domestic investment effort. Accordingly, the present calculation can be seen as either a scenario where aid increases both the growth trend and the savings rate as a result of an emergence from a "generalized" poverty trap, or as the result of some deliberate policy choices in recipient countries.

Results are reported in Table 4. As in Tables 2 and 3, we allow the poverty elasticity to vary. If π is about 0.75, one would need very large growth accelerations. As seen, increasing the underlying growth trend by 2 percentage points is not sufficient for the (simultaneously) required investment hike to fall to a "reasonable" size. That is, saving transitions of a magnitude which has been observed in poor countries during the last half of the 20th century. However, if π is 2 a growth acceleration of about 1.0 percent, combined with a total investment increase of about 12 percentage points would be enough to ensure MDG#1, under

[27] It should be noted, however, that there are relatively recent cases of countries achieving even higher productivity growth. Estimates for Total Factor Productivity (TFP) growth in China during 1978–95 range up to 3.7 percent (Yeh, 2005), while high estimates for TFP in India reach to 2.4 percent (Rodrik and Subramanian, 2004).

the model. This, at least, would correspond to a savings/investment boom of magnitudes observed in recent history.

It is of course an open question whether the growth trend can be lifted in sub-Saharan Africa as a result of aid inflows. The above calculations show that such acceleration would be critically important for the attainment of the stipulated goal of halving poverty in sub-Saharan Africa within the next decade. If the trend does not move, the required saving transition becomes implausible for the contemplated increases in aid inflows. Moreover, reaching MDG#1 in sub-Saharan Africa also critically depends on a strong relationship between growth and poverty reduction. A poverty-elasticity around 2 is required.

It is once again worth pointing out that theses scenarios represent lower-bounds for the implied necessary increases in domestic investment rates. Just as above where new aid flows triggered as increase in domestic resource mobilization immediately, here the increase in domestic investments *and* productivity growth likewise happens immediately. Thus, if there are any lags in achieving the productivity increases, there will be a corresponding need for investment rates.

V. EXTENSIONS

A. Externalities and Multiple Capital Goods

The calibrations in Section III and IV assumed that there is only one accumulated factor of production: physical capital. This raises the issue of how the results would change, within a Solow-type framework, if we were to allow for additional sources of growth. That is to say, if we allow for external effects from capital accumulation or multiple capital goods.

To begin, consider externalities from capital accumulation. Specifically suppose we assume that productivity rises in response to Learning-by-Doing. That is, investment leads to productive knowledge, which raises productivity, that is, "*A*." Let productivity be given by:

$$A = \tilde{A}\tilde{k}^{\vartheta}, \tag{18}$$

where $\tilde{A}$ grows at the rate *g*. Accordingly, the analysis conducted in Section II represents the situation where $\vartheta = 0$. If we continue to assume production is Cobb-Douglas, we have the following reduced form aggregate production function (where the external effect is taken into account):

$$\tilde{y} = \tilde{k}^{\psi} \tag{19}$$

where $\psi = \alpha + (1-\alpha)\vartheta$. We can plug this expression into the law of motion for capital per unit of labor (equation (8)) and derive a new expression for the steady state level of GDP per efficiency units of labor:

$$\tilde{y}^{*} = \left(\frac{s + \varpi f}{n + \delta + g} \right)^{\frac{\psi}{1-\psi}}. \tag{20}$$

The new version of equation (12)

$$df = \frac{dF}{Y} = \frac{d\tilde{y}^{*}}{\tilde{y}^{*}} \left[\frac{1-\psi}{\psi} \frac{\varpi f + s}{\varpi} \right] \tag{21}$$

Now, the step involving substituting the marginal product of capital is slightly different from that in Section II. The externality from capital accumulation is not internalized by producers, so the marginal product derived in Section II would correspond to the *private* marginal product in the present context. The *social* marginal product of capital in steady state is obtained by differentiating the reduced form production function (equation (19)) with respect to k:

$$MP_{K}^{Soc} = \psi \left(\frac{Y}{K} \right)^{*} = \psi \frac{n + \delta + g}{s + \varpi f}, \tag{22}$$

As a result, the final formula for the model extended by external effects from capital accumulation would read

$$\frac{dF}{Y} = \left[e^{\frac{-\frac{1}{\pi}\log\frac{p(T)}{p(0)} - gT}{1-e^{-\lambda T}}} - 1 \right] \left[(1-\psi) \frac{n + \delta + g}{MP_{K}^{Soc} \varpi} \right], \tag{23}$$

where it is noted that the rate of convergence, reflecting the new curvature of the production function, is given by $\lambda = (1-\psi)(n + \delta + g)$.

Accordingly, from a quantitative perspective the model from Section II can match the externality extension fully, provided ψ <1. But we would need to change: (*i*) the assumed share for capital (it would be defensible to raise it, compared with the baseline case) and (*ii*) the marginal product of capital, which also should be raised, if private returns are used in baseline calibrations.

Such changes are, however, unlikely to lead to substantially different results than those reported in Section III and IV. As demonstrated in Section III, the results are not very sensitive to the assumed share of capital, which we had varying from 1/3 to 0.6. In fact, larger output-capital elasticity works to *increase* aid requirements. As explained in the context of the simulations, this is because of two counteracting effects. First, a higher capital share means that investments become more productive, which in isolation would lower aid requirements. Second, however, a higher capital share lowers the rate of convergence, which means that the economy needs to be "pushed harder" to reach a given poverty objective within a given amount of time—10 years, say.

Now, in theory the tendency for aid requirements to go up (due to the change in output-capital elasticity) could be off set by a higher marginal return, as evaluated at the aggregate level. The evidence in favour of large returns from capital investments is, however, somewhat limited. In a cross-section context the mentioned study by Dalgaard and Hansen (2005) estimate that the average gross return on capital investments (funded by aid or otherwise) is 20–30 percent. In principle this is the (average) social return on investments in aid-receiving countries, which we applied above. Therefore, increasing the estimate for the marginal product of capital much above this level seems hard to justify. At least we are not aware of evidence which could support it. If an increase can be justified after all, however, the ramifications of a change in *MPK* for the results are easy to assess. A doubling of the marginal product of capital (from 20 to 40 percent, say), compared to the baseline number, will simply reduce calculated aid requirements by half.

What about multiple capital goods? This is a relevant case if some aid investments go into durables, other parts into equipment. Alternatively, one could distinguish between public and private capital, or between physical capital and human capital. In all these cases one may write the production function (stated in efficiency units) as

$$\frac{dF}{Y}=\left[e^{\frac{-\frac{1}{\pi}\log\frac{p(T)}{p(0)}-gT}{1-e^{-\lambda T}}}-1\right]\left[\left(1-\psi\right)\frac{n+\delta+g}{MP_K^{Soc}\varpi}\right], \tag{24}$$

where x is a generic capital good, which may capture any of the interpretations mentioned above. In "the augmented Solow model" (Mankiw et al. (1992)) x is human capital, for example. To proceed, we assume that accumulation of physical capital and x is structurally identical. That is

$$\dot{\tilde{x}}=\left[s_x+\varpi f_x\right]\tilde{y}-\left(n+g+\delta\right)\tilde{x} \tag{25}$$

$$\dot{\tilde{k}}=\left[s_k+\varpi f_k\right]\tilde{y}-\left(n+g+\delta\right)\tilde{k}, \tag{26}$$

where total aid flows $F=f_x Y+f_k Y$.

We can now solve the model for steady state GDP per capita:

$$\tilde{y}^*=\left(\frac{s_x+\varpi f_x}{n+g+\delta}\right)^{\frac{\beta}{1-\alpha-\beta}}\left(\frac{s_k+\varpi f_k}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha-\beta}}, \tag{27}$$

and proceed to derive the impact from a *balanced* increase in aid investments in both activities, that is to say we assume $df_x=df_k=df$. We also exploit the "trick" of substituting

$s_x + \varpi f_x$ and $s_k + \varpi f_k$ for the respective private marginal products; precisely as in Section II. The end result is the following formula

$$df = \left[e^{\frac{-\frac{1}{\pi}\log\left(\frac{1}{2}\right)-gT}{1-e^{-\lambda T}}} - 1 \right] \left[\frac{(1-\alpha-\beta)(n+g+\delta)}{\varpi\left[MP_K + MP_X\right]} \right] \qquad (28)$$

which distinguishes itself from equation (15) in two ways: $1\text{-}\alpha\text{–}\beta$ enters instead of just $1\text{-}\alpha$, and in addition the *sum* of the marginal products, evaluated in the initial steady state, enters the denominator, rather than only MP_K.

The first difference is due to dynamic complementarily between *k* and *x* in the growth process. If the investment rate in physical capital goes up, this will spur capital accumulation and thereby output in the next "period." But this increase in output will not only lead to more physical capital accumulation in the next instant (as in the standard Solow model), but also more accumulation of *x*. This "propagation mechanism" causes the elasticity of long-run output, with respect to "broad" physical capital input, to go up. In a sense, investments are more productive in the present framework. But again, this will not lead required aid flows to shrink, since the rate of convergence in this model is $\lambda = (1-\alpha-\beta)(n+g+\delta)$. Thus, when we increase total capital's share ("broad capital''), the rate of convergence will be lowered.

The second difference to equation (14) lies in the fact that two marginal products enter. This effect stems from the opportunity to divide a given aid transfer across different capital goods. Since each are subject to diminishing returns, it is clear that a "balanced increase" will yield a bigger effect on productivity than if all investments went into, say, *k*. As a result, just as were the case under externalities, multiple capital goods may lead to a lowering of the aid costs, via an increasing marginal product effect.

The bottom line of this section is, however, that the extensions involving multiple capital goods can be captured by the simpler framework of Section II. Multiple capital goods (as well as the effect of external effects) can be "captured" by increasing capitals share and the marginal product of capital appropriately, in the calibrations.

B. Endogenous Savings

The calibrations in Section III and IV assumed a standard Solow model as organizing framework. A critique of this approach is that savings behaviour thereby is taken to be exogenously given, and thus unaffected by (for example) an inflow of foreign aid. In this section we discuss how results would change, if we allowed savings to be endogenous. In particular we discuss the implications of using two alternative growth models as underlying framework: *the Ramsey-Cass-Koopmans* (*RCK*) model, and the two period *Diamond model*.

Consider a social planner's problem in a RCK environment, where time indices are omitted for brevity.[28]

$$\max_{\{c\}_0^\infty} U(0) = \int_0^\infty u[c] e^{-(\rho-n)t} dt$$

$$c \geq 0$$

$$\dot{\tilde{k}} = \tilde{k}^\alpha + \tilde{f} - \tilde{c} - (n+\delta+g)\tilde{k}, \tilde{k}(0)\, given$$

$$k \geq 0.$$

Hence, the planner strives to maximize discounted utility over an infinite horizon (*u*[*c*] is per period utility from consumption, *c*), subject to the control region (non negative consumption), the resource constraint of the economy, and a terminal condition which concerns the path of capital; here it is formulated as a non-negativity constraint on capital, since it makes little sense to work with negative capital stocks. Aid inflows, expressed in efficiency units, enter the economy's resource constraint.

The evolution of the economy over time is, as is well known, characterized by two differential equations. The first is the consumption Euler (or "Keynes-Ramsey rule"), which comes from solving the above problem:

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{1}{\varepsilon}\left(f'\left(\tilde{k}\right) - \rho - \varepsilon g\right),$$

where $(1/\varepsilon)$ is the intertemporal elasticity of substitution in consumption. For a steady state to exist, with constant growth in consumption, capital and output per capita, ε will have to be constant. That is, *u[c]* needs to be *CES*. The second differential equation is the resource constraint above.

As a matter of steady state analysis, the consumption Euler pins down $\tilde{k}$ and thereby long-run GDP per capita: $f'\left(\tilde{k}^*\right) = \rho + \varepsilon g$. As pointed out in Dalgaard et al. (2004), this means that a powerful result holds in the RCK model with respect to the impact of aid: *if foreign aid comes in the shape of a transfer of income or capital, which enters the budget of the consumer (or planner), the inflow will increase long-run consumption per capita 1:1, but leave steady state output unaffected.* Indeed, from the resource constraint it follows immediately, that in steady state: $\tilde{c}^* = \left(\tilde{k}^*\right)^\alpha + \tilde{f}^* - (n+\delta+g)k^*$, from which the 1:1 association between steady state consumption and aid inflows is apparent. Hence, a staunch believer in the standard RCK model would have to reject the notion that aid can affect long-run poverty, since it should not affect long-run GDP per capita.[29]

[28] The decentralized equilibrium in the *RCK* model is Pareto Optimal, so we can save some space by focusing on the planner's solution.
[29] Arellano et al. (2005) reach similar conclusions using a general equilibrium two-sector model. They find that permanent aid flows will be directed towards consumption.

Of course, one could modify the standard *RCK* model in ways in which aid could have an impact on growth. For example, Kraay and Raddatz (2006) discuss the implications of aid in a *RCK* model where consumers face a minimum consumption constraint (that is, subsistence consumption is introduced). In this case, if the economy initially is near subsistence, an infusion of aid can stimulate growth. More capital leads to higher income, and thereby a higher savings rate due to the presence of the minimum consumption requirement. This is, however, only a transitional phenomenon. Even in the model with subsistence consumption, aid will *not* affect steady state income. Aid can speed up the transition to steady state, but in theory the country would have undertaken the same "journey," in the absence of any foreign assistance. Moreover, Kraay and Raddatz find little evidence to suggest that aid have had a profound stimulating effect on savings in recipient nations over the last several decades, as suggested by the subsistence augmented *RCK* model.

As a result, if the *RCK* model represents "the true model" of the growth process, the calibrations from the last section are assuming much too big an effect from aid inflows on poverty reduction. The infinitely lived consumer approach is not, however, the only way to make savings endogenous.

Consider a two period OLG model (Diamond, 1965). People live for two periods. During period 1 (youth) individuals work (for which they receive a wage), consume and save. People only consume in period 2 (old age). Suppose utility is logarithmic, and that the aggregate production function is Cobb-Douglas. Then it is easy to show that the law of motion for capital per efficiency units of labor—without aid—fulfils (see for example, Romer, 2001, Chapter 2):

$$\tilde{k}_{t+1} = \frac{\sigma}{(1+g)(1+n)}(1-\alpha)\,\tilde{y}_t \tag{29}$$

where the only new notation introduced is σ, which represents the propensity to consume of the young in turn determined by the underlying rate of time preference. Capital in the next period depends on GDP per capita in this period, in a very similar way to that imposed by the Solow model. The mechanics is a little different though, since in the OLG model this association is caused by the fact that the young save only out of wage income; and wages (in efficiency units) are simply $(1-\alpha)\,\tilde{y}$, given the Cobb-Douglas assumption. Nevertheless it follows that, in this framework, aid can have an impact on long run GDP per capita and thereby poverty. We could assume, for example, that each period a transfer of capital arrives, which enters the right hand side of equation (16). In this case the resulting law of motion for capital, implied by the Diamond model, is virtually indistinguishable from the one derived on the basis of the Solow model. Alternatively, one could assume that aid enters the budget of the young, who then take it into account when they choose how much to save. In this case we would find that only the fraction σ of the aid inflow is saved; the rest is consumed. In this framework then, we could view our "waste" parameter (ϖ) from the analysis above, as reflecting the marginal propensity to save, by young individuals.[30] However, as should be clear, from a qualitative perspective one could "micro found" the law of motion for capital, employed in the above calculations, using the Diamond model.

The bottom line is that whether the calibrations in the previous sections can be viewed as sensible or not, once endogenous savings are allowed, depends on what sort of micro foundations one believes is the best approximation to reality. If one prefers the RCK model, the calibrations should be rejected as overly optimistic. If, on the other hand, one is partial to the Diamond framework, they become more persuasive.

VI. Concluding Remarks

This paper has proposed a simple framework for examining the link between aid, growth, and poverty reduction. The calibration approach is based on a Solow model. Using this framework to "back-cast" the effect of past aid flows to sub-Saharan Africa fits the historically observed growth outcomes much better than the alternative approach, which draws on the Harrod-Domar growth model.

Under a range of reasonable parameters the aid costs of ensuring the achievement of MDG#1 by 2015 are very high. Even after augmenting the Solow model to consider savings-based poverty traps and productivity accelerations, we still find the prospect for achieving a reduction of poverty following the prescribed deadline and aid donations difficult. To reach the target income, it is necessary for trend growth to rise substantially combined with a large increase in domestic investment effort. Even this scenario requires that all aid be invested without "waste" and that the marginal product of capital be high. We demonstrate that these results are robust to different microfoundational assumptions and to the inclusion of externalities and multiple capital goods.

These results are not encouraging; the burden of poverty in African and other low-income countries is immense, and our findings that aid may not be as effective in reducing that burden as other analyses have suggested are certainly not a cause for celebration. It would be a mistake, however, to interpret our results as showing that aid is simply ineffective. Rather, our analysis suggests that aid does have the potential to both increase growth and reduce poverty. While these impacts may be modest in absolute terms, for very poor people small improvements in their material conditions may have much larger impacts on welfare. Looking beyond our sample period, the experience in sub-Saharan Africa has been more positive in recent years, with growth in approaching 6 percent in 2005 (IMF, 2006). It is too early to tell, however, if this is as a result of new aid flows, some other factor, or merely cyclical variation.

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[30] A more general treatment is found in Dalgaard et al. (2004), involving a distribution of aid flows among both young and old. The paper also contains a discussion of why the two work horse models (*RCK* and Diamond) lead to so different conclusions regarding aid effectiveness.

Moreover, the present paper, by design, does not consider potential effects of aid that do not lead to higher levels of growth but which nonetheless might have substantial welfare-improving outcomes. As an illustration, even if eliminating malaria in a country had no effect on growth or poverty, surely an aid project that accomplished such a feat should not be judged a failure.

The analysis offers, we believe, an improved framework for the debate on the effectiveness of aid and the prospects for achieving the MDG#1. By examining the combinations of models and parameter values that generate different levels of poverty reduction, we can illustrate the background assumptions implicit in varying claims for the effectiveness of aid. We believe that making those assumptions explicit, and therefore subject to evaluation, is important in its own right. In terms of direct policy implications, our methodology offers no definitive prescriptions, but does point toward some possible avenues for further investigation. Given reasonable parameters across a range of models, the direct effect of aid on capital accumulation and then on growth, with growth, in turn, leading to poverty reduction, does not, by itself, seem sufficient to reach the MDG#1 in sub-Saharan Africa. A more promising avenue for investigation would be to determine if aid enhances productivity growth, though whether such an effect could be achieved is open to doubt. Our results showed a great sensitivity to varying the elasticity of poverty reduction to growth. Thus, perhaps further attention should be paid to focusing aid on projects that directly reduce poverty, either through targeting aid flows on poor individuals or on decreasing inequality.[31]

Possible extensions of this paper would include a richer modeling of the key relationship between inequality, growth and poverty reduction. Besley and Burgess (2003) find large effects of inequality in reducing poverty. If aid could have a effect on inequality (or if inequality falls as a country develops), the scope for poverty reduction could be strengthened. Thus, a potential extension to our approach would be to consider the effect of aid not just on mean income but also on the dispersion of income, following, for example, Stiglitz (1969).

Finally, it may be worthwhile to revisit the empirical studies of aid discussed in this paper in light of its findings. Our results indicate that the potential overall effect of aid on growth is likely to be modest. In terms of how aid is apportioned by donors, there is a clear and understandable tendency to allocate aid to countries that are most in need of it: poor, slow-growing countries. Of course, the empirical studies try to address this selection-bias in aid disbursements, but there are no easy solutions to the identification problem. Thus, under the assumption that this selection bias can not be completely controlled for and that the actual effect of aid on growth is relatively small, empirical estimations that aid has no effect on growth should come as no surprise. Indeed, they would be consistent with a positive, yet modest, effect of aid on growth.

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[31] For example, Paes de Barros, et al., (2002), find that forecasts of poverty reduction in Latin America depends greatly on assumptions regarding income distribution.

Figure 1. Development Planning

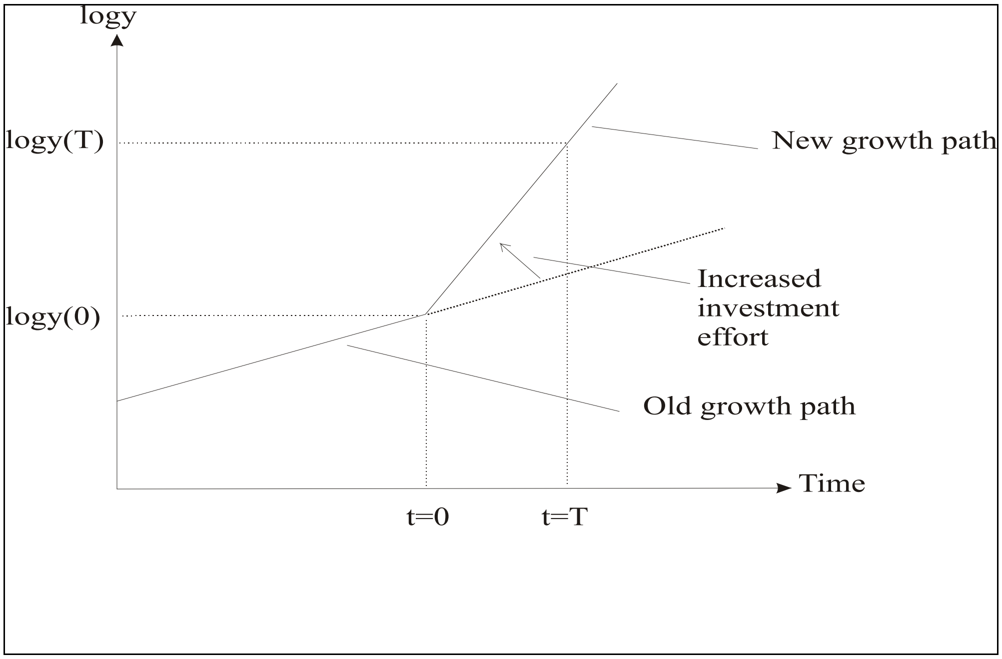

Figure 2. An Alternative Approach to Calibrating Aid Requirements

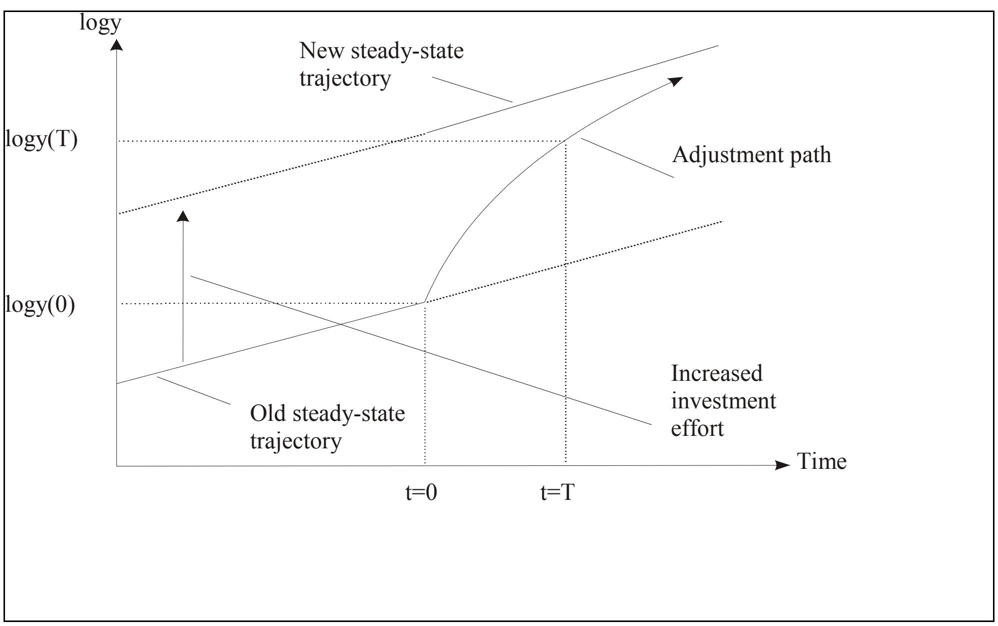

Figure 3. Aid Inflows and Growth in Sub-Saharan Africa

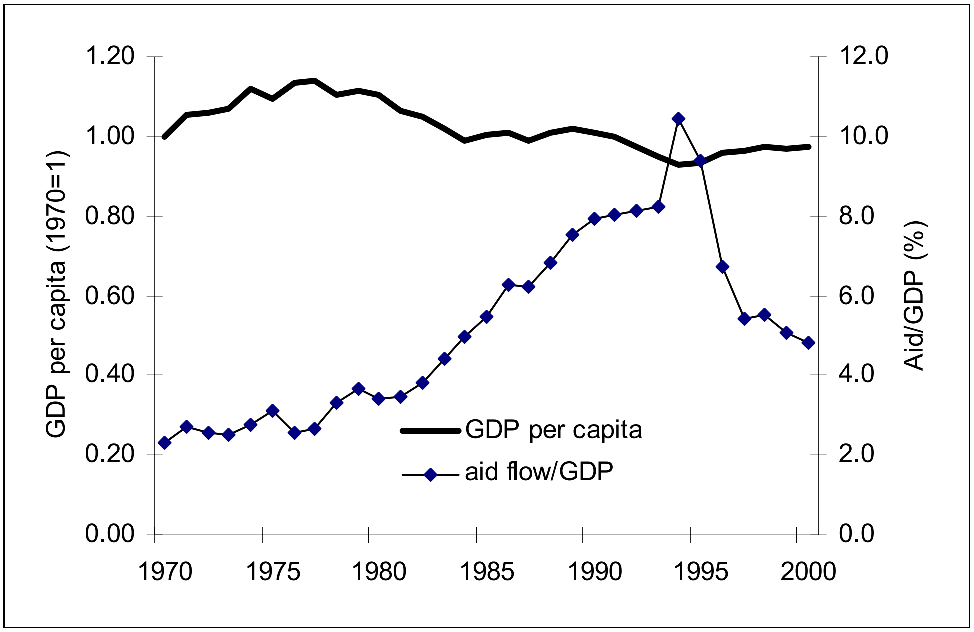

Figure 4. Division of Labor: Aid and Domestic Additional Investment Effort as Partners in Achieving MDG#1

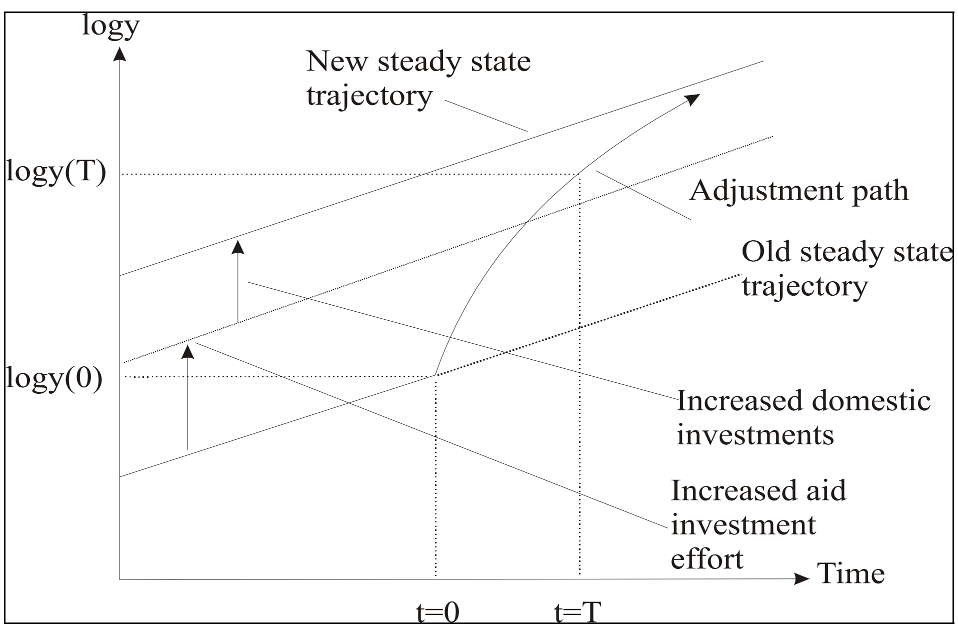

Table 1. Aid Requirements for MDG#1

Poverty Elasticity (π)	**Capital Share (α)**	***dF/Y* (Percentage of GDP)**	***s***	**λ**
0.75	0.6	901	0.18	0.024
1.00	0.6	297		
1.25	0.6	149		
1.50	0.6	93		
2.00	0.6	49		
3.00	0.6	23		
0.75	0.4	365	0.12	0.036
1.00	0.4	160		
1.25	0.4	95		
1.50	0.4	65		
2.00	0.4	39		
3.00	0.4	21		
0.75	0.33	310	0.10	0.04
1.00	0.33	144		
1.25	0.33	88		
1.50	0.33	61		
2.00	0.33	37		
3.00	0.33	20		
1.25	0.6	244	0.18	0.02
1.25	0.6	53		0.04
1.25	0.6	29		0.06
1.25	0.6	21		0.08
1.25	0.6	17		0.10
1.25	0.6	9		0.50

Assumptions: $g = 0$, $n = 0.01$, $\delta = 0.05$, $\varpi = 1$.
MPK = 0.2, T = 10.

Table 2. Required Increase in Domestic Investment Rate

Poverty Elasticity (π)	***dS/Y* (Percentage of GDP)**
0.75	195
1.00	84
1.25	46
1.50	29
2.00	13
3.00	2

Assumptions: $g = 0$, $n = 0.01$, $df = 0.12$, $\varpi = 1$, $\alpha = 1/3$.
MPK = 0.3, T = 10.
Implicit initial s = 0.07.
Implicit rate of convergence = 0.04.

Table 3. Productivity Takeoff and MDG#1

dF/Y (Percentage)	**Poverty Elasticity (π)**	**Necessary New Growth Rate (g, %)**
50	0.5	5.5
25	0.5	7.6
15	0.5	9.1
8	0.5	10.7
4	0.5	12.0
50	1	1.1
25	1	2.4
15	1	3.4
8	1	4.6
4	1	5.5
50	2	*
25	2	*
15	2	1.0
8	2	1.6
4	2	2.3
50	3	*
25	3	*
15	3	*
8	3	*
4	3	0.7

Assumptions: initial g = 0, n= 0.01, $\varpi = 1$, $\alpha = 1/3$.
MPK = 0.3, T = 10, s = 0.07.
Implicit rate of convergence = 0.04.
* Indicates that the target poverty reduction would be achieved for the corresponding increase in g without any inflow of aid.

Table 4. Takeoff, Savings Poverty Trap, and MDG#1

Poverty elasticity (π)	**Growth Take-Off (Percentage)**	**dS/Y (Percentage)**
0.75	0.00	195
0.75	0.50	135
0.75	0.75	114
0.75	1.00	96
0.75	1.50	70
0.75	2.00	52
1	0.00	84
1	0.50	58
1	0.75	48
1	1.00	40
1	1.50	28
1	2.00	19
2	0.00	13
2	0.50	6
2	0.75	3
2	1.00	*
2	1.50	*
2	2.00	*

Assumptions: $n = 0.01$, $df = 0.12$, $\varpi = 1$, $\alpha = 1/3$.
MPK = 0.3, T = 10, s = 0.07.
* Indicates that MDG#1 would be achieved without any change in S.

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