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Currency Mismatches and Corporate Default Risk: Modeling, Measurement, and Surveillance Applications

Jorge A. Chan-Lau and Andre O. Santos

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Currency Mismatches and Corporate Default Risk: Modeling, Measurement, and Surveillance Applications[1]

Prepared by Jorge A. Chan-Lau and Andre O. Santos

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Abstract

Currency mismatches in corporate balance sheets have been singled out as an important factor underlying the severity of recent financial crises. We propose several structural models for measuring default risk for firms with currency mismatches in their asset/liability structure. The proposed models can be adapted to different exchange rate regimes, are analytically tractable, and can be estimated using available equity price and balance sheet data. The paper provides a detailed explanation on how to calibrate the models and discusses two applications to financial surveillance: the measurement of systematic risk in the corporate sector and the estimation of prudential leverage ratios consistent with regulatory capital ratios in the banking sector.

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Authors' E-Mail Addresses: jchanlau@imf.org, asantos2@imf.org

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I. INTRODUCTION

A currency mismatch exists when a borrower funds its operations in one currency while the earnings derived from these operations accrue in another currency. In emerging market countries, and especially Latin America, currency mismatches in the corporate sector arise from balance sheets heavily tilted towards foreign-currency-denominated debt and local-currency-denominated assets and/or earnings.

This paper presents models for measuring default risk when there are currency mismatches in a firm's balance sheet. Specifically, we propose a number of tractable models for evaluating the credit risk or default probability of a corporation when the assets and liabilities in the balance sheet are denominated in different currencies.[2]

The models are firmly grounded on the structural approach to modeling credit risk as first proposed by Merton (1974). The structural approach is naturally suited to analyze how currency mismatches affect credit risk, since the approach factors in how the capital structure affects the probability of default. Under certain assumptions commonly used in commercial implementations of structural models such as Moody's KMV, it is relatively easy to calibrate the models using equity price and balance sheet data information and standard maximum likelihood procedures.

The paper contributes to the literature in several dimensions. First, it complements previous empirical studies such as Bleakley and Cowan (2005) and Claessens, Djankov, and Xu (2000). These studies attempted to measure the impact of currency mismtaches on the soundness and competitiveness of the corporate sector. Such studies, however, cannot answer the key question of how much currency mismatches contribute to the credit risk of an individual firm. The models presented here answer this question. Second, because the models map equity prices into default probabilities, they provide a forward-looking measure of the default risk of a corporation provided local equity markets are reasonably efficient.[3] Even if equity prices are not available, the models could be extended by using balance sheet proxies of equity prices. Finally, the models can be used as basic building blocks to construct system-wide measures of vulnerability as explained in detail below.

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[2] Therefore, the paper does not address currency mismatches in the cash flow of the firm. However, the modeling techniques described here could be extended to this case since the cash flow can be represented as a sequence of zero-coupon bonds.

[3] It is important to note that model calibration requires using balance sheet data. The models still apply if firms use off-balance sheet derivatives contracts to manage their foreign-currency-denominated liabilities provided the level of liabilities is adjusted accordingly to reflect the unhedged share of the liabilities. See Chan-Lau and Santos (2006b) for a description of the conceptual issues and suggested modeling approaches to factor in off-balance sheet exposure in the measurement of default risk.

The rest of the paper is structured as follows. Section II explains briefly why currency mismatches matter. Section III explains the structural approach to default risk adopted in this paper. Section IV to VI present the benchmark models and describe in detail how the models can be calibrated using equity price return and balance sheet data. Section VII shows how the models can be integrated into surveillance frameworks for analyzing and monitoring financial stability. Section VIII concludes. A companion paper, Chan-Lau and Santos (2006a), illustrates the application of the methodology described in this paper in detail.

II. Why do Currency Mismatches Matter?

Currency mismatches in the corporate sector pose a latent risk to financial stability in the event of a sharp currency depreciation, as experienced during the financial crises of the late 1990s. In the aftermath of the currency depreciations, corporate borrowers, especially those in the nontradable sector, struggled to cope with the sudden increase of their debt service. As bankruptcies in the corporate sector mounted, nonperforming loans accumulated rapidly, contributing to the demise of the banking and financial systems of the affected countries.[4]

Even if firms do not default, depreciations may have substantial welfare effects through the balance sheet effect. The deterioration of the borrowing capacity in the corporate sector results in lower output and investment, as noted by Aghion, Bacchetta, and Banerjee (2001), Krugman (1999), and Chang, Céspedes, and Velasco (2001) among others.

Currency mismatches tend to be very persistent, as in the case of Latin America (Table 1). The only exception is Argentina, where mandatory pesoization reduced currency mismatches from 80 percent to less than 7 percent in 2001. By 2004, however, there was a slight increase in foreign-currency-denominated loans.

What drives currency mismatches?[5] One main driver of currency mismatches is the uncertainty associated with high inflation periods. A casual glance at Table 1 suggests that currency mismatches are more prevalent in countries that have experienced high inflation rates. Despite rapidly falling and low inflation rates, currency mismatches persist arguably due to the low credibility of government policies and/or expectations that current economic policies that have kept inflation in check may be discontinued. For instance, the election of governments associated with populist policies in Latin America could weaken fiscal discipline and trigger an increase in inflation.

[4] See, for instance, Calvo, Izquierdo, and Mejía (2003), and DeNicolo, Honohan, and Ize (2005).

[5] See Cowan (2004) and Rennhack and Nozaki (2006) and references therein for a comprehensive analysis of the causes of financial dollarization and currency mismatches.

Table 1. Latin America: Foreign-Currency-Denominated Loans

(In percent of total loans)

	2001	2002	2003	2004
Argentina	80.0	7.2	7.1	14.1
Bolivia	97.0	97.3	97.7	97.7
Brazil	18.0	19.4	…	12.0
Chile	13.8	13.0	10.3	10.3
Colombia	11.0	11.6	8.8	6.1
Costa Rica	67.2	53.0	55.5	53.3
Dominican Republic	27.6	30.9	37.0	27.3
Ecuador	100.0	100.0	100.0	100.0
El Salvador	100.0	100.0	100.0	100.0
Guatemala	…	15.3	16.7	17.7
Honduras	22.2	22.8	26.4	30.9
Mexico	20.5	12.9	12.3	9.8
Nicaragua	83.6	83.1	84.3	85.0
Paraguay	52.8	58.2	55.7	51.7
Peru	80.5	79.7	77.9	75.9
Uruguay	66.0	81.0	76.0	70.0
Venezuela	0.7	0.8	0.7	0.6

Source: Rennhack and Nozaki (2006).

While the focus of the literature has been on emerging market countries, currency mismatches in the balance sheet are not limited only to corporations in emerging market countries. The continued popularity enjoyed by the eurobond and samurai bond markets as a source of corporate funding as well as investors' appetite for emerging market currency-denominated bonds have led corporations in developed countries to issue foreign-currency denominated bonds.

III. The Structural Approach to Default Risk

There are two main approaches for modeling default risk. In reverse chronological order, they are the reduced form or intensity-based approach and the structural approach. The intensity-based approach assumes that the time of default is determined by an exogenous stochastic process. The default event is not linked to any observable characteristic of the firm analyzed, which raises questions on what drives default risk in these models. In contrast, the structural approach starts from the observation that default or bankruptcy occurs when a firm is unable to continue servicing its debt, which can be traced to economic reasons such as the stage of the business cycle and so on. The structural approach, hence, relies on the modeling of the capital structure of the firm and is the approach followed henceafter.[6]

Structural models were born from the insight of Black and Scholes (1973) and Merton (1974), who linked the analysis of the capital structure to option pricing theory. Under absolute priority rules, equity shareholders are residual claimants on the assets of the firm since bondholders are paid first. Thus, equity shareholders are holding a call option on the assets of the firm, where the strike price is equal to the debt owed to bondholders. Similarly, the value of the debt owed by the firm is equivalent to a default-free bond plus a short position on a put option on the assets of the firm. The option pricing analogy described above is useful for calibrating structural models of default risk with market prices and balance sheet data as explained later in sections IV to VI.

The main conceptual insight for modeling default risk in structural models is that default occurs if the asset value of the firm is less than what the firm owes to its debtors. Structural models differ in their assumptions regarding the timing of default. In the benchmark model of Merton (1974), the firm issues a zero-coupon bond.[7] Default only occurs at maturity since this is the only period in which creditors verify the asset value of the firm. Hence, the asset value of the firm can be less than the face value of debt in any period prior to default, i.e. the firm is technically insolvent but it is not declared bankrupt. This type of structural models can be grouped together under the category of ***default-at-maturity*** models.

In contrast, another type of structural models define bankruptcy as the first time the asset value of the firm falls below the face value of its liabilities. The problem of default, in mathematical language, is equivalent to a ***first passage time*** problem, also known as a first stopping or exit time problem.[8] First passage time models include, among others, those of Kim, Ramaswamy, and Sundaresan (1993), Nielsen, Saá-Requejo, and Santa-Clara (1993), Longstaff and Schwartz (1995) and Saá-Requejo and Santa Clara (1999). First-passage time models offer an answer to the following question, which summarizes the default risk problem: when is the first time that the asset value of the firm, V, falls below the value of the liabilities of the firm, L? The time of this event is defined as the ***default time*** τ. Once the problem has been mapped into the language of first passage time problems, it is also possible to answer the question of how likely the default of the firm is during a certain period of time. In mathematical language, we want to know the probability that the default time occurs during a certain period of time.

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[6] See Duffie and Singleton (2003), among others, for a comprehensive description of the intensity-based and structural approaches to credit risk.

[7] See Geske (1977) for an extension to coupon bonds.

[8] For a comprehensive discussion of stopping times see Karatzas and Shreve (1991) or Protter (1992).

First passage time models could be better suited than default-at-maturity models for analyzing default risk when currency mismatches exist. If default only occurs at maturity it is likely that the risk of default will be underestimated. The experience of East Asia in 1997 and Argentina in 2001 indicates that in the event of a drastic currency depreciation, highly leveraged corporations, especially those with earnings mainly denominated in local currency, will likely default well in advance of the maturity of their foreign-currency denominated debt. First passage time models, however, are complex and more difficult to estimate than default-at-maturity models.

Below, we introduce several benchmark models under both categories, default-at-maturity and first passage time models, that can accommodate a number of different assumptions about the behavior of the exchange rate. The first model is a first passage time model that assumes that the exchange rate follows a diffusion process. The use of a diffusion process is indirectly validated by the empirical success of simple implementations of the Merton model in capturing default risk in the corporate and banking sector.[9] The model is easy to calibrate since it yields simple closed form solutions.

The second model is a default-at-maturity model that assumes implicitly that the exchange rate follows a jump-diffusion process. Empirical studies such as Jorion (1988), Dumas, Jennergren, and Naslund (1995) and Bates (1996) have found that jump-diffusion processes are better suited to capture the behavior of exchange rates better than alternative models such as diffusion processes and stochastic volatility models.

The third model is a first passage time model based on a double exponential jump-diffusion process (Kou, 2002). In contrast to jump-diffusion processes, the double exponential jump-diffusion process allows capturing the stylized fact that the distribution of returns is asymmetric by specifying different probability distributions for positive and negative jumps. This model is well suited for analyzing situations under which the exchange rate is more prone to move in one direction rather than other, i.e. undervalued or overvalued exchange rate pegs.

IV. THE DIFUSSION MODEL

The diffusion model presented in this section is appropriate for relatively stable exchange rate regimes in which sudden large exchange rate movements are seldom experienced, or at least not expected during the time horizon considered by the analysts. Hence, the characterization of the asset and liability values of the firm by diffusion processes is justified when the exchange rate regime could be vaguely characterized as "normal." That is, day-to-

[9] See Crosbie and Bohn (2003) for corporates, and Gropp, Vessala, and Vulpes (2006) and Chan-Lau, Jobert, and Kong (2004) for banks in mature and emerging market countries respectively.

day changes of the exchange rate are relatively "small." The assumptions of the diffusion model are presented below.

A. Assumptions

In order to ensure model tractability and for expositional convenience, we take as given the following assumptions made by Merton (1974) which are also shared by most structural models:

a) There are no transaction costs nor taxes and assets are indivisible.

b) Markets are perfectly elastic, i.e., investors buy and sell orders do not affect the market price.

c) Short-sales are allowed, and the lending and borrowing rates are the same.

d) There is continuous trading in time and the firm's assets are a tradeable security.

e) The firm is financed by equity and a zero-coupon bond.

Before presenting the diffusion model in detail, some useful assumptions are needed.

Assumption A1. The firm issues only one type of debt, a zero-coupon bond that matures at time *T*. The bond is denominated in foreign currency and has a face value *D*. The face value of the bond in local currency is *L=DX*, where *X* is the exchange rate expressed in units of local currency per foreign currency.

Assumption 1 simplifies the complex liability structure of a firm. Despite the simplification, structural models of corporate debt have proved useful for forecasting default (Crosbie and Bohn, 2003). Assumption 1 may also appear unduly restrictive since firms may carry both local currency-denominated and foreign-currency denominated liabilities in their balance sheet. We will show below that the benchmark model can be easily extended to deal with this situation.

Assumption A2. There is an underlying probability space (Ω, F, P) endowed with a reference filtration $\mathbf{F}=(F_t)_{0\le t\le T}$ that captures all the information available in the economy.

Assumption A3. Under the objective probability measure *P* the dynamics of the value of the firm in local currency, *V*, and the face value of the firm's debt, *L*, are given by the stochastic differential equations:

(3) $$dV_t = V_t\left(\mu_V(t)dt + \sigma_V(t)dW_t^V\right),$$

(4) $$dL_t = L_t\left(\mu_L(t)dt + \sigma_L(t)dW_t^L\right),$$

where W_t^V and W_t^L are independent Wiener process (i.e. standard Brownian motions) with respect to the filtration **F.**[10] By Assumption 2, it follows that the filtration generated by (*V*, *L)* is contained in **F**. Notice that a credible fixed exchange rate regime implies that the coefficients μ_L and σ_L in equation (4) are equal to zero.

Assumption A4. $\mu_i(t)$ and $\sigma_i(t)$, *i=V,L* are **F**-predictable real functions satisfying:

(5) $$|\mu_i(t)| + |\sigma_i(t)| \leq C,\ i = V, L$$

for some real finite *C*.

This assumption is required to ensure the existence and uniqueness of the asset value function, *V*, and the liability value function, *L*, specified in Assumption A3.

Lemma 1. Under Assumptions A2 and A4, there exists unique solutions to the stochastic differential equations (3) and (4).

Proof. Since the drift and diffusion coefficients $\mu_i(t)$ and $\sigma_i(t)$, *i=V,L* depend neither on *V* nor *L*, the global Lipschitz conditions are trivially satisfied. Under Assumption 4 the linear growth conditions are satisfied, and the lemma follows (see Karatzas and Shreve, 1991, chapter 5).□

B. Main Results

Intuitively, the firm defaults if the asset value, *V*, is less than the value of the liabilities, *L*, sometime during the interval $(0, T]$. Let the normalized asset value of the firm, or asset-debt ratio, be denoted by $Y_t = \ln\left(V_t / L_t\right)$. An straightforward application of Ito's Lemma yields the dynamics of the asset-debt ratio, which is stated without proof in the next lemma.

Lemma 2. Properties of the asset-debt ratio. The asset-debt ratio Y_t satisfies the stochastic differential equation

(6) $$dY_t = \mu_Y(t)dt + \sigma_V(t)dW_t^V - \sigma_L(t)dW_t^L,$$

with

[10] Similar results will hold if the assumption of independence is relaxed.

(7) $$\mu_Y(t) = \mu_V(t) - \mu_L(t) + \frac{1}{2}\left(\sigma_V^2(t) - \sigma_L^2(t)\right).$$

Declines in the asset-debt ratio drive up the default risk of the firm. An inspection of equation (7) indicates that there are two factors that could contribute to a decline of the asset-debt ratio. The first factor is an exchange rate that depreciates at a rate faster than the firm's asset growth rate, i.e. $\mu_V(t) - \mu_L(t) < 0$. The second factor is an exchange rate that is more volatile than the firm's asset volatility, i.e. $\sigma_V^2(t) - \sigma_L^2(t) < 0$. Thus, a rapidly depreciating and volatile exchange rate has a negative impact on the firm's solvency.

We now provide precise definitions of the default event and the default time in terms of the asset-debt ratio Y_t.

Definitions: Default event and Default Time in the Diffusion Model. Given a positive asset-debt ratio at time *t*, $Y_t > 0$, the default event is defined as the first time the asset-debt ratio becomes negative. The corresponding default time, τ, is defined as:

(8) $$\tau = \inf\left\{s \geq t : Y_s \leq 0\right\},$$

Notice that the definition of default event corresponds intuitively to the event that $V_t \leq L_t$. A direct application of results in Harrison (1985) yields a closed-form solution for the probability that the default time occurs sometime in the time interval $(t, s]$. The closed-form solution is stated in Lemma 3.

Lemma 3. The probability of default. Assume that the coefficients $\mu_V(t) = \mu_V$, $\mu_L(t) = \mu_L$, $\sigma_V(t) = \sigma_V$, and $\sigma_L(t) = \sigma_L$ for all t so Assumption 4 is satisfied. Then, the probability that the default time occurs before time *s* conditional on the information known at time *t* is:

(9) $$P\left\{\tau \leq s \mid F_t\right\} = \Phi\left(\frac{-Y_t - \mu_Y(s-t)}{\sigma_Y\sqrt{s-t}}\right) + \exp\left(-2\mu_Y\sigma^{-2}Y_t\right)\Phi\left(\frac{-Y_t + \mu_Y(s-t)}{\sigma_Y\sqrt{s-t}}\right),$$

where $\Phi(.)$ is the cumulative normal distribution, and with μ_Y given by equation (5) and

(10) $$\sigma_Y^2 = \sigma_V^2 + \sigma_L^2.$$

Proof. Under the assumption of constant drift and diffusion coefficients, equation (4) can be rewritten as a brownian motion with drift, i.e. $Y_s = Y_t + \mu_Y(s-t) + \sigma_Y W_S^*$, with μ_Y

and σ_Y given by equations (7) and (10) respectively. Define a survival event during the period $(t,s]$ as $\tau_2 = \inf\left\{t \le u \le s : \mu_Y(u-t) + \sigma_Y(W_u^* - W_t^*) \ge -Y_t\right\}$. Notice that τ_2 is indistinguishable from the survival event of the firm during the period $(t,s]$, $\tau_1 = \inf\left\{t \le u \le s : Y_s \ge 0\right\}$. From Harrison (1985) it follows that the probability that the firm survives during the period $(t,s]$, conditional on the information available at time *t* in the sigma-algebra $F_t \in \mathbf{F}$ is given by:

$$P\left\{\tau_2 \mid F_t\right\} = N\left(\frac{-Y_t + \mu_Y(s-t)}{\sigma_Y(s-t)}\right) - \exp(-2\mu_Y\sigma_Y^{-2}Y_t)N\left(\frac{-Y_t + \mu_Y(s-t)}{\sigma_Y(s-t)}\right).$$

Equation (9) follows from the fact that $P\left\{\tau \mid F_t\right\} = 1 - P\left\{\tau_2 \mid F_t\right\}$.□

Notice that Lemma 3 encompasses also the case of a credible pegged or fixed exchange rate regime. In this case, it suffices to set the drift and diffusion coefficients in equation (4) to zero.

C. Calibration Methodology

The examples presented above were based on ad-hoc parameter values for illustration purposes. In practice, measuring corporate default risk implies estimating the parameters using real world price and balance-sheet data. In this section, we explain how the diffusion model can be implemented using observable equity prices, market capitalization, and balance-sheet information.

The basis for the implementation of the diffusion model is the observation that the market value of equity or market capitalization, *E*, is equivalent to a call option on the assets value of the firm, *A*:

(11) $$E = \max\left(A - D, 0\right),$$

where *D* denotes the face value of the debt owed by the firm. The calibration of the model with real data requires specifying the appropriate call option function corresponding to the diffusion model and constructing the appropriate maximum likelihood function.[11]

In the case of the diffusion model, the calculation of the default probability of the firm using equation (9) in Lemma 2 requires the estimation of the set of parameters θ:

$$\theta = \left\{\theta_L, \theta_V\right\} = \left\{\left(\mu_L, \sigma_L\right), \left(\mu_V, \sigma_V\right)\right\}.$$

[11] See Duan, Gauthier, and Simonato (2004) for a similar application for estimating the plain vanilla Merton model.

The set of parameters θ_L can be estimated from the time series of the exchange rate. Under Assumptions 1 to 3 in section IV.A, and those in Lemma 2, it follows that the exchange rate, X, is governed by the process:

$$dX_t = X_t\left(\mu_L dt + \sigma_L dW_t^L\right),$$

so the log of exchange rate returns, $r_t^X = \ln(X_t / X_{t-1})$, is normally distributed with mean $\mu = \mu_L - \sigma_L^2/2$, and variance σ_L^2. With N independent observations, equally spaced by Δ units of time, the set of parameters θ_L maximizes the log-likelihood function:

(12) $$\arg\max_{\theta_L} l_N\left(\theta_L;\left\{r_t^X\right\}_{t=1}^T\right)$$

where

$$l_N\left(\theta_L;\left\{r_t^X\right\}_{t=1}^T\right) = -\frac{T}{2}\ln(2\pi) + \sum_{t=1}^T \ln\left[\frac{1}{\sqrt{\sigma_L^2\Delta}}\exp\left(\frac{-(r_t^X\Delta - \left(\mu_L - \sigma_L^2\Delta/2\right))}{2\sigma_L^2\Delta}\right)\right].$$

Once the set of parameters θ_L has been estimated, the set of parameters θ_V can be estimated from the time series of the firm's market capitalization. The market capitalization of the firm in period t is given by:

(13) $$E_t = \max\left(V_t - L_t, 0\right).$$

Equation (13) is equivalent to an American option to exchange one asset for another. Since there are not closed form solutions for this option, it has to be valued either by numerical methods (Rubinstein, 1991) or analytical approximations (Bjerksund and Stensland, 1993). Instead, we choose to approximate the value of the option by an European option to exchange, which has a closed form solution (Margrabe, 1978). The expression for the European option is derived in Lemma 4.

Lemma 4. Under Assumptions A1 to A4 in section IV.A., the assumptions in Lemma 3, and constant domestic and foreign risk-free rates r and r_f respectively, the European option to exchange approximation to equation (13) is given by:

(14) $$E_t = V_t\Phi\left(d_1\right) - L_t\exp\left(r - r_f\right)\Phi(d_2),$$

where

$$d_1 = \frac{\ln(V_t / L_t) - (r - r_f + \sigma_Y^2/2)\left(T - t\right)}{\sigma_Y\sqrt{T - t}},$$

$$d_2 = d_1 - \sigma_Y \sqrt{T-t}\ ,$$

and σ_Y is given by equation (10), $\sigma_Y^2 = \sigma_V^2 + \sigma_L^2$.
Proof. The proof follows directly from Margrabe (1978) after noting that $r - r_f$ is the dividend yield of the exchange rate in a risk-neutral world.□

Equation (14) determines an implicit function *g* such that

(15) $$E_t = g(V_t, \sigma_V; r, r_f, \sigma_L, T-t),$$

where the unknown variables are the asset value of the firm, V_t, and the asset volatility, σ_V. The market capitalization of the firm, *E*, and aall the other parameters are either observable, or estimated from equation (12). From equation (15), the value of the firm can be expressed in terms of the unknown parameter σ_V as:

(16) $$V_t = g^{-1}(E_t, r, r_f, \sigma_L, T-t; \sigma_V).$$

The log-asset return of the firm, $r_t^V = \ln(V_t / V_{t-1})$, can be obtained from the time series of asset values given by equation (16). Under Assumption A3, section V.A., the log-asset return is normally distributed with mean $\mu = \mu_V - \sigma_V^2/2$, and variance σ_V^2. It follows that the set of parameters θ_V is the solution to the following maximization problem:

(17) $$\underset{\theta_V}{\arg\max}\, l_N\left(\theta_V; \left\{r_t^V\right\}_{t=1}^T\right)$$

where

$$l_N\left(\theta_L; \left\{r_t^V\right\}_{t=1}^T\right) = -\frac{T}{2}\ln(2\pi) + \sum_{t=1}^T \ln\left[\frac{1}{\sqrt{\sigma_V^2 \Delta}} \exp\left(\frac{-(r_t^V \Delta - \left(\mu_V - \sigma_V^2 \Delta / 2\right))}{2\sigma_V^2 \Delta}\right)\right].$$

Summarizing, given the time series of market capitalization of the firm, $\left\{E_t\right\}_{t=1}^N$, the exchange rate, $\left\{X_t\right\}_{t=1}^N$, the set of parameters θ is estimated in a two-step sequence. First, the set of parameters θ_L is estimated by maximizing the log-likelihood function (12). Second, the set of parameters θ_V is estimated by maximizing the log-likelihood function (17). Finally, the default probability of the firm is obtained from equation (9).

There are three important remarks regarding the calibration method described in this section. First, equation (9) yields objective or real-world default probabilities since the log-likelihood functions (12) and (17) are estimated under the physical probability measure rather than the risk-neutral probability measure. The latter probability measure is used only for obtaining the pricing equation (14).

Second, in contrast to the standard calibration method used by Moody's KMV (Crosbie and Bohn, 2003), the maximum likelihood estimation is consistent with the prior assumption on the dynamics of the value of the firm. The point-in-time estimation of the Moody's KMV calibration methodology implies that asset volatility estimates are time-varying although the underlying Merton structural model requires asset volatility to be constant.

Finally, the diffusion model and the calibration procedure are still valid if the firm carries both local currency-denominated and foreign currency-denominated liabilities. After replacing Assumptions A1 and A3 in section IV.A by the Assumptions A1′ and A3′ below respectively, the results in section IV.B and the calibration method in section IV.C still apply:

Assumption A1′. The firm issues two types of debt that mature at time *T*. The first type of debt is a zero-coupon bond denominated in foreign currency with a face value of *D*. The face value of the bond in local currency is $L=DX$, where *X* is the exchange rate expressed in units of local currency per foreign currency. The second type of debt is a zero-coupon bond denominated in local currency with a face value of C.

Assumption A3′. Denote the value of the firm by *Z* and the value of the firm net of the local currency debt by $V=Z-C$. Under the objective (physical) probability measure *P* the dynamics of the value of the firm in local currency net of the local currency debt, *V*, and the face value of the firm's foreign denominated debt, *L*, are given by the stochastic differential equations:

$$dV_t = V_t\left(\mu_V(t)dt + \sigma_V(t)dW_t^V\right),$$
$$dL_t = L_t\left(\mu_L(t)dt + \sigma_L(t)dW_t^L\right),$$

where W_t^V and W_t^L are independent Wiener process (i.e. standard Brownian motions) with respect to the filtration **F.**

To see why the Assumptions A1' and A3' are sufficient to adapt the diffusion model to the case of a firm with local currency-denominated and foreign currency-denominated liabilities, notice that the firm defaults when $Z \le L + C$, or equivalently, when $V \equiv Z - C \le L$.

V. The Jump-Diffusion Model

One common shortcoming of default risk models based on diffusion processes is that the model estimates of default risk for very short horizons are too low compared to what is observed in reality or priced in the markets. In addition, the distribution properties of time series of exchange rates appear to be consistent with fat-tail processes rather than with

diffusion processes, as found by Jorion (1988) and Bates (1996) among others. Arguably, the presence of jumps in the exchange rate process is more likely among emerging market countries than in advanced economies.
The jump-diffusion model in this section addresses this shortcoming.[12] Of particular interest is that the model, as it will be shown below, accommodates the special case of fixed or pegged exchange rate regimes that could come under attack, as experienced by Argentina in 2001.

A. Assumptions

In addition to the standard assumptions (a) – (d) stated in section IV.A., a number of assumptions are needed to use some well known results from the literature on jump-diffusion processes.

Assumption B1. The firm issues only one type of debt, a zero-coupon bond that matures at time *T*. The bond is denominated in foreign currency and has a face value *D*.

Assumption B2. There is an underlying probability space (Ω, F, P) endowed with a reference filtration $\mathbf{F}=(F_t)_{0\leq t\leq T}$ that captures all the information available in the economy.

Assumption B3. (Merton, 1976) Under the objective probability measure *P* the dynamics of the value of the firm in foreign currency, *V*, is given by the following jump-diffusion process (or Generalised Ito process):

(18)
$$dV_t = V_t\left(\mu_V dt + \sigma_V dW_t^V + (J-1)\, dN_t\right),$$

where μ_V and σ_V are constants, W_t^V is a standard Brownian Motion, N_t is a Poisson process with constant arrival rate λ, and the log of the jump size, $\log(J)$, is an i.i.d. normal variable with mean μ_J and variance σ_J^2.

Assumption B3 indicates that the percentage change of the asset value in *foreign currency* is driven by two different processes. The first process is a geometric brownian motion described by the first two elements in the right-hand side of equation (18). The second process is a jump process that occurs with probability λdt during a short interval *dt*. If a jump occurs in time *t*, the asset value changes by $J-1$. The asset value is given in Lemma 5.

Lemma 5. Under Assumptions B1 to B3, there is a unique solution to equation (18):

[12] See Bichteler (2002) and Protter (2002) for a comprehensive discussion of jump-diffusion processes, and Zhou (2003) for a semi-analytical model of default risk where the asset value of the firm follows a jump-diffusion process.

(19) $$V_t = V_0 \exp\left(\left(\mu_V - 1/2\sigma_V^2\right)t + \sigma_V W_t^V\right) \prod_{1 \le j \le N(t)} \left(J_j\right)$$

where $N(t)$ is the number of realized jumps in $(0,t)$.

Proof. The four conditions of Lemma 1 in Yu (2005) are satisfied under Assumptions B1 to B3 ensuring the existence and uniqueness of the solution.[13] The derivation of (19) follows directly from Merton (1976). □

B. Main Results

Definitions: Default event and Default Time in the Merton Jump-Diffusion Model. Given an initial asset value, V_0, in excess of the face value of the debt, *D*, i.e. $V_0 > D$, the default event is defined as the first time the asset value falls below the face value of debt. The corresponding default time, τ, is defined as:

(20) $$\tau = \inf\left\{s \ge t : V_s \le D\right\},$$

The firm is in default the first time the asset value, *V*, is less than the value of the liabilities, *D*, sometime during the interval $(0,T]$. The probability of default before period *s*>*t*, conditional on the information available at time *t*, $P(\tau \le s \mid F_t)$, needs to be computed using numerical methods since no closed-form solution exists for the jump-diffusion process specified in equation (18).[14] In the special case when default can only occur at maturity, *T*, it is possible to obtain a closed-form solution for the default probability, as shown in Lemma 6.

Lemma 6. Probability of default at maturity. The probability of default at maturity conditional on the information available at time *t*, $P(\tau = T \mid F_t)$, is given by:

(21) $$P(\tau = T \mid F_t) = \sum_{j=0}^{\infty} \frac{e^{-\lambda(T-t)}\left(\lambda\left(T-t\right)\right)^j}{j!} \Phi\left(\frac{\ln D - \ln V_t - \left(\mu_V - \frac{1}{2}\sigma_V^2\right)\left(T-t\right) - j\mu_J}{\sqrt{\sigma_V^2(T-t) + j\sigma_J^2}}\right).$$

Proof. Since the diffusion and jump processes are independent, the result follows immediately after noticing that

$$P\left(\tau = T \mid F_t\right) = P(\ln V_t \le \ln D \mid F_t) = \sum\nolimits_{j=0}^{\infty} P(n = j) P(\ln V_T \le \ln D \mid F_t, n = j).$$ □

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[13] See Lo (1988) for more general results on existence and uniqueness.

[14] See Atiya and Metwally (2002).

C. Calibration Methodology

As in the case of the diffusion model, the calibration of the Merton jump-diffusion model rests on the observation that the equity value of the firm in foreign currency is a call option on the asset value of the firm in foreign currency with a strike price equal to the face value of the firm's liability in foreign currency:

(22) $$E_t = \max\left(V_t - D, 0\right).$$

Equation (21) corresponds to the payoff of an American option on an underlying asset governed by a jump-diffusion process. While there are analytical solutions for this option (Gukhal, 2001), the fact that there is a closed form solution for the default probability at maturity (Lemma 6) suggests approximating the equity value of the firm in foreign currency with an European option to simplify the analysis.

Lemma 7. Equity value of the firm in foreign currency. Under Assumptions B1 to B3, the equity value of the firm is equal to:

(23) $$E_t = \sum_{n=0}^{\infty} \frac{e^{-\lambda\mu_J}\left(\lambda\mu_J T\right)^n}{n!} BS\left(V_t, \sqrt{\sigma_V^2 + n\sigma_J^2/(T-t)}, r_F - \lambda(\mu_J - 1) + \frac{n\log(\mu_J)}{T-t}, D\right),$$

where r_F is the risk-free rate in foreign currency and $BS(V, \sigma, r, T, D)$ is the Black and Scholes price of a call option with spot price *V*, volatility σ, risk-free rate *r*, maturity *T*, and strike price *D*.

Proof. Merton (1976). □

Equation (23) can be used to estimate the set of parameters $\theta_J = \left(\mu_V, \sigma_V, \lambda, \mu_J, \sigma_J\right)$ needed to estimate the probability of default at maturity using maximum likelihood. Equation (23) implies that the value of the firm can be expressed in terms of the unknown set of parameters, θ_J, the market value of the firm, E_t, and the known variables *T-t* and r_F:

(24) $$V_t = g^{-1}(E_t, r_f, T-t; \theta_J).$$

Equation (24) can be used to obtain the log-asset return of the firm, $r_t^V = \ln(V_t / V_{t-1})$, as a function of the unknown parameters. Since the log-asset return is normally distributed conditionally on the number of jumps, the density function of the log-asset return is given by:

(25) $$p(r_{t+1}^V; \theta_J) = \sum_{j=0}^{\infty} \frac{e^{-\lambda\Delta}\left(\lambda\Delta\right)^j}{j!} \phi\left(r_{t+1}^V; \left(\mu_V - \frac{1}{2}\sigma_V^2\right)\Delta + j\mu_J, \sigma_V^2\Delta + j\sigma_J^2\right),$$

where $\phi(x;m,v)$ is the density function of a normally distributed variable with mean m and variance v, and Δ is the sampling interval of the market capitalization of the firm. Hence, the set of parameters θ_J should solve the following maximum likelihood problem:

$$l_J\left(\theta_J;\left\{r_t^V\right\}_{t=1}^T\right)=-\frac{T}{2}\ln(2\pi)+ \sum_{t=1}^{T}\ln\left[\sum_{j=0}^{\infty}\frac{e^{-\lambda\Delta}\lambda^j}{j!}\frac{1}{\sqrt{\sigma_V^2\Delta+j\sigma_J^2}}\exp\left(\frac{-\left(r_t^V\Delta-\left(\mu_V-\sigma_V^2/2\right)\Delta-j\mu_J\right)}{2\left(\sigma_V^2\Delta+j\sigma_J^2\right)}\right)\right]. \quad (26)$$

Maximization of the log-likelihood function (26) requires truncating the number of terms in the internal sum. Ball and Torous (1985) derived a formula for an upper truncation bound, and previous empirical estimations suggest that an upper bound of 10 is enough to provide satisfactory results (Jorion, 1988). Another alternative to the use of an upper bound truncation is to use closed-form likelihood approximations, as proposed by Yu (2005).

Once the parameters are estimated, computation of the default probability at maturity using Lemma 6 is trivial. As in the case of the diffusion process, the estimation procedure yields real-world default probabilities and is consistent with the dynamics of the value of the firm. Furthermore, the framework can accommodate the case of a fixed exchange rate by simply setting μ_V and σ_V equal to zero.

VI. The Double Exponential Jump-Diffusion Model

The jump-diffusion model still implies the assumption that negative and positive jumps are equally likely. But financial returns data, in general, suggests that the distribution of returns should be leptokurtic and asymmetric with respect to zero. To address this characteristic of the data Kou (2002) introduced the double-exponential jump-diffusion model which is adapted below to analyze credit risk when there are currency mismatches.

A. Assumptions

As in the previous models, the standard assumptions (a) – (d) stated in section IV.A. hold in this model. In addition, we add the following assumptions:

Assumption C1. The firm issues only one type of debt, a zero-coupon bond that matures at time *T*. The bond is denominated in foreign currency and has a face value *D*.

Assumption C2. There is an underlying probability space (Ω, F, P) endowed with a reference filtration $\mathbf{F}=(F_t)_{0\le t\le T}$ that captures all the information available in the economy.

Assumption C3. (Kou, 2002) Under the objective probability measure *P* the dynamics of the value of the firm in foreign currency, *V*, is given by the following jump-diffusion process (or Generalised Ito process):

(27) $$dV_t = V_t\left(\mu_V dt + \sigma_V dW_t^V + d\left(\sum_{i=1}^{N_t}(J_i - 1)\right)_t\right),$$

where μ_V and σ_V are constants, W_t^V is a standard Brownian Motion, N_t is a Poisson process with constant arrival rate λ, and $\{J_i\}$ is a sequence of i.i.d. nonnegative random variables such that $X = \ln(J)$ has a double exponential distribution with density function given by:

(28) $$f_X(x) = p\eta_1 e^{-\eta_1 x} 1_{\{x \geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{x<0\}},$$

where $\eta_1 > 1$ and $\eta_2 > 0$, *p* and *q* represent the probability of upward and downward jumps respectively, and $p + q = 1$. That is, *X* is distributed as an exponential random variable ξ^+ with mean $1/\eta_1$ with probability *p*, and as a random variable $-\xi^-$ with probability *q*, where ξ^- is an exponential random variable with mean $1/\eta_2$. W_t^V, N_t, and $X = \ln(J)$ are independent.

If $\eta_1 \neq \eta_2$, the asset value of the firm is asymmetrically leptokurtic, that is, the tails of the distribution are asymmetric. Assumption C3, thus, is appropriate for modeling situations in which movements of the exchange rate in a given direction are more frequent than in the other direction. Equation (27) admits the solution below, which we state without proof.

Lemma 8. (Kou, 2002) Under Assumptions C1 to C3,

(29) $$V_t = V_0 \exp\left(\left(\mu_V - 1/2\sigma_V^2\right)t + \sigma_V W_t^V\right) \prod_{1 \leq i \leq N(t)} J_i$$

is a solution to equation (27), where V_0 is asset value of the firm at time 0 and $N(t)$ is the number of realized jumps in $(0, t)$.

B. Main Results

Definitions: Default Event and Default Time in Kou's Double Exponential Jump-Diffusion Model. As in the Merton Jump-Diffusion model, the default event is defined as the first time the asset value falls below the face value of debt. The corresponding default time, τ, is defined similary as:

(30) $$\tau = \inf\{s \geq t : V_s \leq D\},$$

where V_0 is the initial asset value in excess of the face value of the debt, *D*, with $V_0 > D$.

There are no closed form solutions for the probability distribution of the first passage time, $P(\tau \le s),\ s \in\in [0,T],$ in Kou's double exponential jump diffusion model. However, Kou and Wang (2003) derived a closed form solution for the Laplace transform of the first passage time. If the Laplace transform is known, the probability distribution of the first passage time can be obtained by numerical inversion of the Laplace transform, which is presented in Lemma 9.

Lemma 9. Laplace transform of the first passage time (Kou and Wang, 2003). The Laplace transform of the first passage time of Kou's double exponential jump diffusion model is given by:

(31) $$E[e^{-\alpha\tau}] = \frac{\eta_2 - \beta_{1,\alpha}}{\eta_2}\frac{\beta_{2,\alpha}}{\beta_{2,\alpha} - \beta_{1,\alpha}} e^{D\beta_{1,\alpha}} + \frac{\beta_{2,\alpha} - \eta_2}{\eta_2}\frac{\beta_{1,\alpha}}{\beta_{2,\alpha} - \beta_{1,\alpha}} e^{D\beta_{2,\alpha}},$$

where $\beta_{1,\alpha}$ and $\beta_{2,\alpha}$ are the only two positive roots of the equation

(32) $$\alpha = G(\beta),$$

with $G(.)$ given by:

(33) $$G(x) = -x\mu_V + \frac{1}{2}x^2\sigma_V^2 + \lambda\left(\frac{q\eta_2}{\eta_2 - x} + \frac{p\eta_1}{\eta_1 + x} - 1\right).$$

Proof. It follows from an immediate application of Theorem 3.1. in Kou and Wang (2003) using the process

$$dZ_t = Z_t\left(-\mu_V dt + \sigma_V dW_t^V - \left(\sum_{i=1}^{N_t}(J_i - 1)\right)_t\right),$$

a default barrier equal to –*D*, and noticing that the upward jumps in *Z* correspond to the downward jumps of the asset value process of the firm, *V*, and vice versa. □

C. Calibration Methodology

Equity price return data can be used to calibrate the double exponential jump-diffusion model and obtain the set of parameters $\theta_K = (\mu_V, \sigma_V, \lambda, \eta_1, \eta_2, p)$. As in the case of the jump-diffusion model, the calibration exploits the fact that the equity value of the firm in foreign currency is a call option on the asset value of the firm with a strike price equal to the face value of the firm's liability in foreign currency. The equity value of the firm in foreign currency can be obtained approximately as the value of an European call option, which is given in Lemma 10 without proof.

Lemma 10. Equity value of the firm in foreign currency. Under Assumptions C1 to C3, the equity value of the firm is equal to:

(34)
$$E_t = V_t \Upsilon\left(r_F + \frac{1}{2}\sigma_V^2 - \lambda\zeta, \sigma_V, \overline{\lambda}, \overline{p}, \overline{\eta}_1, \overline{\eta}_2; \ln\left(D/V_t\right), T-t \right) - De^{-rT}.\Upsilon\left(r_F - \frac{1}{2}\sigma_V^2 - \lambda\zeta, \sigma_V, \lambda, p, \eta_1, \eta_2; \ln\left(D/V_t\right), T-t \right),$$

where r_F is the risk-free rate in foreign currency, $\overline{p} = \frac{p}{1+\zeta}.\frac{\eta_1}{\eta_1 - 1}$, $\overline{\eta}_1 = \eta_1 - 1$, $\overline{\eta}_2 = \eta_2 + 1$, $\overline{\lambda} = \lambda(\zeta + 1)$, $\zeta = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1$, and Υ is a closed form equation described in Theorem B.1. in Kou (2002).[15]

Equation (34) implies that:

(35)
$$V_t = g^{-1}(E_t, r_f, T-t; \theta_K),$$

and the asset returns of the firm, $r_t^V = V_t / V_{t-1} - 1$, can be expressed as a function of the parameter vector θ_K. If the equity price observations are equally spaced by Δ units of time, Kou (2002) shows that the probability density of the asset returns is given by:

(36)
$$h(x) = \frac{1-\lambda\Delta}{\sigma_V\sqrt{\Delta}}\phi\left(\frac{x-\mu_V\Delta}{\sigma_V\sqrt{\Delta}}\right) + \lambda\Delta\left\{ p\eta_1 e^{\sigma_V^2\eta_1^2\Delta/2} e^{-(x-\mu_V\Delta)\eta_1}\Phi\left(\frac{x-\mu_V\Delta-\sigma_V^2\eta_1\Delta}{\sigma_V\sqrt{\Delta}}\right) + q\eta_2 e^{\sigma_V^2\eta_2^2\Delta/2} e^{-(x-\mu_V\Delta)\eta_2} \times \Phi\left(\frac{x-\mu_V\Delta-\sigma_V^2\eta_2\Delta}{\sigma_V\sqrt{\Delta}}\right)\right\}$$

where ϕ is the standard normal density function and Φ is the cumulative standard normal density function. The set of parameters θ_K maximizes the following likelihood function:

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[15] Since the expression for Υ is rather lengthy we refer the reader to the original source, Kou (2002).

(37) $$l_K\left(\theta_J;\left\{r_t^V\right\}_{t=1}^T\right)=\sum_{t=1}^T \ln\left[h(r_t^V;\theta_J)\right].$$

Once the parameters are estimated, the Laplace transforms of the default time can be computed using the results of Lemma 9, and the default probabilities can be obtained by inverting the Laplace transform.

VII. Surveillance Applications

The models described above are useful for assessing the default risk of an individual firm when there are currency mismatches in the balance sheet. The usefulness of the models does not stop there though since they could be used as the basic building blocks for constructing financial surveillance frameworks and/or assessing system-wide vulnerabilities in the corporate sector. This section offers two possible applications of the model. The first application focuses on the construction of aggregate measures of default risk in the corporate sector. The second application explains how to determine prudential leverage levels in the corporate sector consistent with regulatory capital ratios in the banking system.

A. Measuring Systemic Risk in the Corporate Sector[16]

The first step is to use any of the three models presented above to estimate the default probability of individual firms for a given time horizon. The second step is to assess the probability that a subset of the firms analyzed default during the specified time horizon. It seems reasonable to assume that, during crisis periods, a large number of defaults occur simultaneously as they are driven by a common negative shock, i.e. a large exchange rate depreciation. For supervisors, regulator, and market participants, it is of interest to know how likely joint defaults are.

Assessing the probability of default among a subset of firms requires computing the distribution of the number of defaults. We assume that the distribution is given by a one-factor normal Gaussian copula, as suggested by Vacisek (1987) and Li (2000). In the Gaussian copula, the normalized asset value of firm i, x_i, depends on a single common factor, M, and an idiosyncratic shock, Z_i:

(38) $$x_i = a_i M + \sqrt{1-a_i^2}\,Z_i,$$

where x_i, M, and Z_i are standard normally distributed variables. The coefficient a_i, or factor loading, is restricted to values between 0 and 1 and measures the dependence of the asset value on the common factor. For instance, a common factor could be the exchage rate or some economy-wide index.

[16] This section is based in Chan-Lau and Gravelle (2005).

Firm *i* defaults when the asset value x_i falls below a threshold value $\bar{x}_i$. The threshold value can be determined if the probability of default $q_i(t)$ for firm *i* in period *t* is known since $\bar{x}_i = \Phi^{-1}(q_i)$, where Φ is the cumulative standard normal distribution function. Once the threshold value is known, it follows that the conditional default probability is equal to:

$$\text{Prob}\{x_i < \bar{x}_i \mid M\} = q_i(t \mid M) = \Phi\left(\frac{\bar{x}_i - a_i M}{\sqrt{1-a_i^2}}\right). \tag{39}$$

The distribution of the number of defaults can be obtained using the recursive procedure proposed by Andersen, Sidenius, and Basu (2003). We follow Gibson (2004) closely in the remainder of the section. Let $p^K(l,t \mid M)$ be the probability of experiencing *l* defaults during a time horizon *t* conditional on the common factor *M* for a set of *K* firms. If the default distribution is known for *K* firms, the default distribution if an additional firm is added to the set can be obtained from the following recursion:

$$p^{K+1}(0,t \mid M) = p^K(0,t \mid M)\left(1 - q_{K+1}(t \mid M)\right) \tag{40}$$

$$p^{K+1}(l,t \mid M) = p^K(l,t \mid M)\left(1 - q_{K+1}(t \mid M)\right) + p^K(l-1,t \mid M)q_{K+1}(t \mid M), \; l=1, ...,K \tag{41}$$

$$p^{K+1}(K+1,t \mid M) = p^K(K,t \mid M)q_{K+1}(t \mid M) \tag{42}$$

The recursion in equations (40) to (42) can be started using the degenerate default distribution $p^0(0,t \mid M) = 1$ for $K=0$ to obtain the default distribution for a set of *N* firms, $p^N(l,t \mid M)$, $l = 0,..., N$. The unconditional default distribution $p(l,t)$ is obtained by integration:

$$p(l,t) = \int_{-\infty}^{\infty} p^N(l,t \mid M)\phi(M)dM \tag{43}$$

where ϕ is the standard normal distribution function.

Calibration of the one-factor Vacisek model requires first estimating the correlation of each firm's asset value with the common shock or factor. This correlation can be obtained using principal component analysis. The method assumes that a limited number of unobserved variables (or factors) explain the total variation of the larger set of variables. That is, the higher is the degree of co-movement across all individual firm default probability time series, the fewer the number of principal components (factors) needed to explain a large portion of the variance of the original series.

In the case where the original variables are identical (perfectly collinear), the first principal component would explain 100 percent of the variation in the original series. Alternatively, if the series are orthogonal to one another (i.e., uncorrelated), it would take as many principal components as there are series to explain all the variance in the original series. In that case, no advantage would be gained by looking at common factors, as none exist. Results obtained by Chan-Lau and Gravelle (2005) suggest that the first principal component accounts for around 70 to 80 percent of the variance.

B. Prudential Leverage Levels and Regulatory Capital in a Dollarized Economy

In most countries, banks are required to satisfy minimum regulatory capital ratios to provide a cushion against unexpected losses. In this section we show how to assess whether foreign currency-denominated borrowing and leverage in the corporate sector are too risky relative to regulatory capital ratios. The assessment is based on the determination of a maximum leverage ratio, or prudential leverage level, that is consistent with the aggregate capital in the banking system. For illustration purposes a relatively simple analytical framework is used. The framework, however, could be extended in order to include more realistic features such as the discrete granularity of the loan portfolio in the banking system, the non-trivial dependence of default events, etc.

Assumptions

The framework makes a number of assumptions. First, it is assumed that there is only one bank in the country. This assumption implicitly states that the banking system has enough assets to take over individual banks that fail. Second, the probability of default of corporate loans are independent. Third, default probabilities, exposure-at-default, and loss-given-default are independent. Fourth, the loss distribution can be approximated by a normal distribution. Finally, the probability of default is the same for every corporate loan extended by the bank. This is not a crucial assumption and is made only to facilitate the analysis.

Analysis and results

The loss from loan *i*, x_i, is given by:

(44)
$$x_i = \begin{cases} EAD_i \times LGD_i & \text{with probability } PD_i \\ 0 & \text{with probability } 1 - PD_i \end{cases},$$

where PD_i is the probability of default, EAD_i is the exposure-at-default, and LGD_i is the loss-given-default of loan *i*. The Value-at-risk for a confidence level α, VAR_α, under the normal approximation is given by:

(45)
$$VAR_\alpha = \sum_{i=1}^{N_L} PD_i \times EAD_i \times LGD_i + z_\alpha \sqrt{\sum_{i=1}^{N_L} PD_i \left(1 - PD_i\right) EAD_i^2 \times LGD_i^2},$$

where the value of z_α depends on the confidence level. Under the assumption of a constant default probability, $PD_i = p$, the equation above reduces to:

(46)
$$VAR_\alpha = p \sum_{i=1}^{N_L} EAD_i \times LGD_i + z_\alpha \sqrt{p(1-p) \sum_{i=1}^{N_L} EAD_i^2 \times LGD_i^2}.$$

Regulatory capital requirements for individual banks imply that in the aggregate banking sector, the aggregate *VAR* should be less or equal to the aggregate minimum capital requirement. This capital requirement is simply the statutory minimum capital ratio, *MCR*, times the aggregate risk-weighted assets in the banking sector, *RWA*. From equation (46), we obtain the following inequality:

$$p\sum_{i=1}^{N_L} EAD_i \times LGD_i + z_\alpha \sqrt{p(1-p)\sum_{i=1}^{N_L} EAD_i^2 \times LGD_i^2} \leq MCR \times RWA + PROV \,, \tag{47}$$

where *PROV* is the level of aggregate provisions in the banking system. Given the values of the exposure-at-default, EAD_i, and the loss-given-default, LGD_i, when the inequality (47) holds with equality it determines an upper-bound for the probability of default, $p_{\max}$. The upper-bound for the probability of default is consistent with the minimum capital requirements and the level of risk-weighted assets in the banking sector. The upper-bound $p_{\max}$, in turn, determines upper-bound for the leverage ratio of the firm, Lev_{max}, or prudential leverage ratio such that potential losses are expected to be covered by the aggregate capital in the banking system. Depending on the model being used in the analysis, the upper-bounds for the maximum leverage ratio of the firm consistent with the existent capital buffer in the banking system can be obtained directly from either Lemma 3, Lemma 6, or the inversion of the Laplace Transform in Lemma 9.

VIII. CONCLUSIONS

Currency mismatches in the corporate sector can contribute to the occurrence of financial crisis and/or increase the severity of a crisis once it has occurred due to the balance sheet channel. It is important, hence, for practitioners and policymakers to measure the credit risk of firms with currency mismatches in their balance sheets.

This paper has proposed a number of analytically tractable credit risk models for firms with currency mismatches. Given equity price and balance sheet data, the models can be estimated using standard maximum likelihood techniques. Besides being useful for evaluating the credit risk of individual firms, the models are basic building blocks for constructing financial surveillance frameworks.

In particular, we illustrate the usefulness of these models for financial surveillance by describing two applications. The first application is the construction of a measure of systemic risk in the corporate sector based on distribution of defaults in the corporate sector. The second application explains how to use information about the aggregate loan portfolio in the banking sector to assess whether leverage ratios and foreign-denominated borrowing levels are adequately covered by the aggregate capital in the banking system.

While the models were developed with nonfinancial corporations in mind, they could be extended to banks with a proper adjustment of the default barrier. For instance, the prompt corrective action framework governing bank intervention may require increasing the level of liabilities by a multiplicative factor (Chan-Lau and Sy, 2006). More importantly, assessing and modeling the default risk of a financial institutions needs to account for the institution's off-balance sheet exposure (Chan-Lau and Santos, 2006b). In many instances, credit risk arising from loan commitments, letters of credit, and derivatives contracts could be one order of magnitude larger than the credit risk arising from the banking and trading books.

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