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The Role of Stock Markets in Current Account Dynamics: a Time-Series Approach

Benoît Mercereau

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Asia and Pacific Department

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Prepared by Benoît Mercereau¹

Authorized for distribution by Alessandro Zanello

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Abstract

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This paper develops a simple model to study the impact of stock markets on the current account. A closed-form solution for the current account is derived from the optimal portfolio and consumption/saving choices of a representative agent. Formally, the model can be seen as a stock market-augmented version of the “fundamental equation of the current account” popularized by Jeffrey Sachs. It appears to shed light on recent developments in the U.S. current account deficit. The model also shows how the current account may help predict future stock market performance and/or endowment streams.

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Author's E-Mail Address: Bmercereau@imf.org

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	Contents	Page
I.	Introduction.....	3
II.	The Model: A Stock Market-Augmented Fundamental Equation of the Current Account.....	4
	A. The Model.....	4
	B. Expression of Current Account.....	7
	C. Interpretation.....	9
III.	Some Implications of Model.....	11
	A. United States' Current Account Deficit: Irrational Exuberance?	11
	B. Current Account as a Potential Predictor of Future Stock Market Performance.....	13
	C. Other Implications of Model.....	15
IV.	Conclusion.....	17
Appendix		
I.	Proof of Propositions.....	18
II.	Summary of Main Notation.....	27
	References.....	28

I. INTRODUCTION

How important is the performance of the stock market to the current account? Given the extraordinary development of financial markets, one would expect stock market events to play an important role in the dynamics of the current account. For example, it is widely believed that the large and unprecedented U.S. current account deficit in the second half of the 1990s was at least partially caused by the dramatic stock market boom. Yet there are surprisingly few models studying how stock markets can affect current account dynamics.² The aim of this paper is to explore this issue theoretically. In a companion paper, Mercereau (2003), I also study the subject empirically.

This paper develops a simple model to study the impact of stock markets on the current account. The basic mechanism is a stock market-augmented version of a consumption-smoothing story. A country's representative agent receives a stochastic endowment at each period. The agent will then use all the available financial instruments to maximize her expected intertemporal utility. These financial instruments include an arbitrary number of risky assets (both foreign and domestic), which form an incomplete market, as well as a risk-free bond. A closed-form solution for the current account is then derived from the optimal portfolio and consumption/saving choices of the agent. This solution relates the current account to the present and expected future performance of the stock markets, as well as to the evolution of the structure of risk across markets and assets. Formally, the model can be seen as a stock market-augmented version of the so-called fundamental equation of the current account popularized by Jeffrey Sachs (Sachs, 1982).

This fundamental equation of the current account is without doubt the most popular model of the last twenty years. For example, Obstfeld and Rogoff (1995) devote most of their paper to a survey of this line of literature. Their further work in their graduate textbook (Obstfeld and Rogoff, 1997) also summarizes the literature. This fundamental equation model is based on consumption smoothing. One of its main limitations, though, is that it features a risk-free bond as the unique financial instrument. It is, therefore, poorly suited to study the impact of stock markets on the dynamics of the current account.

In order to make the main points of the model clear, I first solve it taking prices as given. In Mercereau (2002), I develop a general-equilibrium version of the model in which risky assets prices are derived endogenously. One can then use the equation of the current account found in this paper to gain insights on the role of stock markets in the dynamics of the current account. For example, as will be seen, the model sheds light on recent developments in the U.S. current account deficit.³ Some claim that this current account deficit reflected over-

² Important exceptions include the work by Kraay and Ventura (2000), and Ventura (2001).

³ The most direct interpretation of a partial-equilibrium model in which asset prices are exogenous is a small-country model. Nevertheless, I think that the model can help one to analyze the U.S. situation. Given the amount of U.S. assets owned by foreigners, U.S. stock prices should be substantially influenced by foreign shifts in demand for U.S. assets that are not strongly related to fundamentals in the U.S. economy itself. As shown in the general-equilibrium version of the model, in Mercereau (2002), the possible causes of such shifts
(continued...)

optimistic, even “irrationally exuberant” expectations of future stock market performance. The model, on the other hand, suggests that it is optimal for a country to run a current account deficit even if people do not expect a stock market boom to last. (Expectations of a continuing boom would only result in a deficit of a larger magnitude.)

Another insight afforded by the model is that the current account may help predict future stock market performance and/or future endowment streams. The reason is that the current account is derived from the optimal portfolio and consumption/savings choices of the agents. As a consequence, the current account should both incorporate and reflect all the relevant information agents have about future stock market performance and future endowments, including the pieces of information which are not observed by econometricians. This forecasting property can be formally expressed by a set of Granger causality and Granger causal priority propositions. Since stock market performance is very difficult to predict, one should regard the above proposition with caution. This paper will nevertheless discuss why this property may be less surprising than it seems. Other implications of the model are also briefly analyzed.

In a companion paper, Mercereau (2003), the model is put to the test using U.S. data. The traditional Sachs model (Sachs, 1982) had been tested for large majority of countries using a methodology developed by Campbell and Shiller (1987) in a different context (see Obstfeld and Rogoff, 1995 and 1997 for a survey).⁴ The model developed in this paper performed better than the same model without stock markets. The forecasting property of the current account as a predictor of future stock market performance also received preliminary empirical confirmation.

The remainder of the paper is organized as follows: Section II presents the model; Section III discusses its main implications; and Section IV concludes the analysis.

II. THE MODEL: A STOCK MARKET-AUGMENTED FUNDAMENTAL EQUATION OF THE CURRENT ACCOUNT

A. The Model

The basic mechanism of the model is a stock-market-augmented version of a consumption-smoothing story. A country’s representative agent receives a stochastic endowment at each period. The agent will use all the financial instruments at her disposal to maximize her expected intertemporal utility. These financial instruments include an arbitrary number of risky assets (both foreign and domestic), which form an incomplete market, as well as a risk-free bond. A closed-form solution for the current account is then derived from the optimal

include demographic changes, changes in risk in foreign economies, and changes in risk aversion.

⁴ Some of these studies include Ghosh (1995), Sheffrin and Woo (1990), Otto (1992), Ghosh and Ostry (1995), Ostry (1997), and Cashin and McDermott (1998), Agénor and others (1999), Callen and Cashin (1999), Adedeji (2001), and Milo (2001).

portfolio and consumption/saving choices of the agent⁵. The framework used for the model was developed in a different context by Davis and Willen (2000) and Davis, Nalewaik, and Willen (2000).⁶

There is a single consumption good, which serves as a *numéraire*. All variables are expressed in units of this consumption good. Writing C as the vector of consumption levels and δ as the discount factor, the program of the agent is:

$$\text{Max}_{\{c_t; \omega_{0,t}; \omega_t\}_{t=0}^{+\infty}} U(C) = E_0 \left\{ \sum_{t=0}^{+\infty} \delta^t u(c_t) \right\},$$

under the budget constraints BC_t :

$$c_t + \omega_{0,t} + \sum_{j=1}^J \omega_{j,t} = NI_t + R_0 \omega_{0,t-1} + \sum_{j=1}^J R_{j,t} \omega_{j,t-1} \quad (1)$$

(and we have the initial conditions: $\omega_{0,-1} = \omega_{j,-1} = 0$)

A sufficient condition of transversality is:⁷

$$\lim_{s \rightarrow +\infty} E_t \left(\frac{1}{1+r} \right)^s \left[\omega_{0,t+s} + \sum_{j=1}^J \omega_{j,t+s} \right] \leq 0. \quad (2)$$

Let us briefly define the variables used above (Appendix II summarizes the notation):

- NI_t (“net income”) is a stochastic endowment received in each period by the representative agent. This is all the income she receives in period t , with the exception of the revenues from her past financial investments.

- $R_{j,t}$ is the gross rate of return of asset j at time t . There are J stocks available on the world stock markets. They are exogenously given, and they form an incomplete market⁸. They can be either domestic or foreign stocks. Each risky asset $j=1, \dots, J$ pays a

⁵ The framework of the Sachs model is the same, except that the model featured a unique financial instrument: a risk-free bond.

⁶ The focus of these papers is the estimation of risk-sharing benefits provided by financial assets in the cases of labor income and international trade.

⁷ A discussion of a “no Ponzi game” condition is presented in Appendix I.

⁸ In the model, to have incomplete markets simply means that the agent’s endowment stream cannot be duplicated, and thus cannot be perfectly hedged, with any combination of the available assets.

stochastic dividend $d_{j,t}$ at time t , and has a market price $P_{j,t}$. $R_{j,t}$ is thus formally defined by:

$$R_{j,t} = \frac{d_{j,t} + P_{j,t}}{P_{j,t-1}}.$$

- $\omega_{j,t}$ is the holdings of risky asset j at time t (like all other variables, it is expressed in units of the *numéraire* good).
- $R_0 = 1 + r$ is the constant international risk-free rate, at which all agents can lend or borrow. This international interest rate is assumed to be constant over time and exogenously given⁹.
- $\omega_{0,t}$ is the holdings of risk-free asset at time t .

In order to facilitate the derivation of the results, I need to make a few assumptions:

- The agent has an exponential utility function: $u(c) = \frac{-1}{A} \exp(-Ac)$, where A is the coefficient of absolute risk aversion.¹⁰
- Net income NI_t and the gross returns have a joint normal distribution (its moments can be time varying).^{11,12}

⁹ Consequently, I assume a perfectly elastic supply (or demand) from foreigners for bonds. The risk-free rate can be made time-varying at the cost of additional notational complexity.

¹⁰ Similar results can be derived with a quadratic utility function. With a quadratic utility function, though, one could not have a closed-form solution for the portfolio ω_t . The main intuition would not be altered by alternative forms of utility functions. What would be different is the presence of a “wealth effect.” With an exponential utility function, portfolio holdings do not depend on wealth, which is not realistic. So the main implication of this exponential utility framework is the absence of this wealth effect. A realistic model should include wealth effects in the analysis. The advantage of using an exponential utility function is that one is able to derive closed-form solutions for all the variables in the model, which one could not do with traditional utility functions.

¹¹ The normality assumption makes the model much easier to solve with the exponential utility function. It can nevertheless be relaxed, but at a very high technical cost in the case of exponential utility (see Gron, Jorgenson, and Polson, 2000). This normality assumption is not needed with a quadratic utility function.

¹² The model is thus a partial equilibrium one, in which economic developments in other countries do not explicitly affect the current account. In another paper (Mercereau, 2002), I develop a general-equilibrium version of the model, in which risky asset prices are determined endogenously.

B. Expression of Current Account

The full solution of the model, as well as the proofs, are given in Appendix II. I use this solution to derive a closed-form solution for the current account. In order to do this, I first recall that, by definition, the current account is the change in the net foreign position of a country¹³. I then use the expression found for the optimal portfolio of the representative agent. This gives us the following expression for the current account (I call this solution the global current account (GCA) for reasons that will become clear momentarily):

Proposition 1¹⁴. Stock-market-augmented fundamental equation of the current account.

$$\begin{aligned}
 GCA_t = & \underbrace{\left(NI_t - E_t(NI_t^*) \right)}_{(1) \text{ endowment income effect}} + \underbrace{\left(X_t' \omega_{t-1} - E_t \left[\left(X_t' \omega_{t-1} \right)^* \right] \right)}_{(2) \text{ stock market effect}} \\
 & + \underbrace{\frac{1}{rA} \text{Ln}[\delta(1+r)]}_{(3) \text{ consumption tilting}} + \underbrace{\frac{A}{2} \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} \text{var}_t c_{t+i}}_{(4) \text{ precautionary savings}} - \underbrace{\frac{(e_t - e_{t-1})}{e_t}}_{(5) \text{ assets stock change}}, \quad (3)
 \end{aligned}$$

where

- X_t is the $J \times 1$ vector of excess returns: $X_t = [R_{j,t} - R_0]_{j=1}^J$.
- $\omega_{t-1} = (\omega_{j,t-1})_{j=1}^J$ is the $J \times 1$ portfolio of risky assets of the representative agent at time $t-1$.
- e_t is the total per capita valuation of all financial assets located in the home country of the representative agent (note that this is independent from the citizenship of the shareholders: an asset located in a given country can be entirely owned by foreigners). Risky assets are indeed in positive supply. There are ϕ_j shares of asset j . The total market valuation of asset j is $S_{j,t} = P_{j,t} \phi_j$. The total per capita valuation of all financial assets located in the home country of the representative agent is then defined by: $e_t = \sum_{j \in \left\{ \begin{smallmatrix} \text{home country's} \\ \text{stocks} \end{smallmatrix} \right\}} S_{j,t}$.
- For any variable Z_t , Z_t^* is the “(future) **permanent level**” of the variable,

which is defined so as to satisfy the following equation:

$$\sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} Z_{t+i} = Z_t^* \sum_{i=0}^{+\infty} \frac{1}{(1+r)^i}$$

¹³ An equivalent way to define the current account is as the sum of the trade balance plus all returns on net foreign assets (interest payments, capital gains, and dividends).

¹⁴ The proof is given in Appendix I.

(it follows that $Z_t^* = \frac{r}{1+r} \sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} Z_{t+i}$).

- Primed variables denote the transpose of the corresponding vector (e.g., Z' is the transpose of vector Z).
- $E_t(Z)$ denotes the expected value of variable Z as of time t .

The above proposition stresses the three components of the current account. Terms (1) and (2) constitute the consumption-smoothing component of the current account. Term (3) is the consumption-tilting component; term (4) the precautionary savings one; and term (5) represents the change in the stock of assets.

The traditional (i.e., stock-market-free) fundamental equation of the current account first derived by Sachs (1982) did not include terms (2) and (5).

The fundamental equation of the current account has been very popular (for a survey of the literature on the topic, see Obstfeld and Rogoff (1995) or Obstfeld and Rogoff (1997, Chap. 2)). Its analysis, both theoretical and empirical, has focused mainly on the main current account component, the consumption-smoothing one. While terms (3) and (4) are also interesting by themselves, I will also follow this literature and focus on the consumption-smoothing component of the current account. For accounting reasons, the change in the domestic stock market valuation term must also be included in the analysis. This term is also necessary for a comprehensive study of the role of stock markets. But before doing so, brief mention must be made of the other two components.

Term (3) reflects the consumption tilting components of consumption. *Ceteris paribus*, the more patient people tend to save more (and, therefore, consume less).

Term (4) is a precautionary saving term. It reflects the fact that the agent wants to protect against future consumption *variability*,¹⁵ thus saving more in order to reach this goal.¹⁶

¹⁵ Formally, this comes from the fact that $u''' > 0$. Consequently, no precautionary savings term appears when one uses the quadratic utility function instead of the exponential one (with a quadratic utility function $u''' = 0$).

¹⁶ A more complete analysis of this term is as follows. Term (4) is due to traditional precautionary saving behavior. The agent cares not only about expected utility, but also about the variability of consumption: greater variability of the agent's consumption induces a loss of utility. This means that the agent will save in order to protect against the variability of future income (precautionary saving). The precautionary saving motive—and hence term (4)—would exist in any stochastic model with $u''' > 0$, whether it includes stock markets or not. Stock markets have an impact on term (4), though the variability of future generalized wealth (and therefore consumption) depends on how well one can hedge labor income risk using the stock market, as well as on the exploitation of the risk premium. Willen (1997) focuses on the impact of financial sophistication on the trade balance as channeled through precautionary savings in a simpler framework.

(continued...)

Now, let us turn to the **consumption-smoothing and change-in-domestic-stock-market-valuation components of the current account**. They will be written as CA . In the remainder of the paper, “current account” will refer to these two components of the current account:

$$CA_t = \underbrace{\left(NI_t - E_t \left(NI_t^* \right) \right)}_{(1) \text{ endowment income effect}} + \underbrace{\left(X_t' \omega_{t-1} - E_t \left[\left(X_t' \omega_{t-1} \right)^* \right] \right)}_{(2) \text{ stock market effect}} - \underbrace{\left(e_t - e_{t-1} \right)}_{(3) \text{ assets stock change}} \quad (4)$$

C. Interpretation

Sachs’s traditional fundamental equation of the current account consists mostly of term (1). Its interpretation is as follows: when the consumers’ endowment income is higher than its expected future permanent level, the representative agent will save more in order to smooth her consumption. *Ceteris paribus*, the country’s net stock of foreign assets will, therefore, increase, and the country will run a current account surplus. Following this line of reasoning, a current account deficit is nothing to be concerned about as long as it reflects expectations of future rises in the country’s net output.

But term (2) suggests that a policymaker using such reasoning could well miss the point and reach inappropriate conclusions about the optimality of the current account level. Indeed, one also has to take into account the role of future stock market performance. The intuition behind this second effect is fairly simple: if the agents expect the stock market to do better in the future than it does today, they will borrow money in order to smooth consumption --and the country will run a current account deficit. In other (more precise) terms, if today’s excess financial gains are smaller than their expected future permanent level, consumption smoothing will lead the country to run a current account deficit. Note that what matters is not the total amount of financial gains but only the share of it in excess of what the same investment made in a risk-free bond would have yielded. The reason for this is that all the welfare gains one can realize by using a risk-free bond to smooth intertemporal consumption are already incorporated in term (1). The extra welfare gains achieved through the stock markets should, therefore, include only the gains one could not have achieved using a risk-free bond. This is what is expressed by term (2).

It is interesting to understand why this precautionary savings behavior induces a positive change in the *current account*. Indeed, the current account is the net change in foreign asset holdings. If the domestic stock market valuation does not change, then the current account is equal to the change in all assets held by the representative agents. So the question is to know why should an agent hold more assets at date t than at date $t-1$ if the individual expects the same degree of income variability in the future. The answer is that because of the agent’s past precautionary saving, the agent’s wealth grows over time, thus creating extra wealth in the next period. The agent’s total asset holding will grow, and, all things being equal, this will translate into an increase in the country’s net asset holding (i.e., the country runs a current account surplus).

The third term in the equation is the change of stocks in domestic risky assets (i.e., the change in the valuation of the domestic stock market). This term is unrelated to the behavior of the agent. It arises only because of the accounting definition of the current account. Indeed, terms (1) and (2) described the change in desired total asset holding by the representative agent. But the current account is not the change in desired total asset holding by the representative agent: the current account is, by definition, the change in net foreign assets of the country. To go from the former to the latter, one has to subtract the change in the total amount of assets located in the country.¹⁷ An example should help explain this point. Let us suppose that the representative agent finds it optimal to raise her total holding of assets by \$10 billion, but that at the same time the valuation of the stock market increases by \$15 billion. Let us first assume that the domestic stock market was entirely owned by domestic agents before the stock market boom. It is true that the total holding of assets of the agent will rise by \$10 billion. But the representative agent will nevertheless sell \$5 billion worth of shares (the difference between the \$15 billion increase in the value of her portfolio and the \$10 extra billion she decides to save). By construction, these shares have to be bought by foreigners. The net effect for the country will, therefore, be a \$5 billion transfer of domestic assets to foreigners, which is to say that the country will run a \$5 billion current account deficit.

If, alternatively, the assets located in the home country were entirely owned by foreigners before the boom, then the \$15 billion rise in domestic stock market valuation corresponds to a \$15 billion decrease in the net foreign position of the home country. But the domestic agents also want to increase their holding of assets by \$10 billion. This translates into a change of +\$10 billion in the net foreign position of the country. So finally, the total change in the net foreign position of the country (and thus, the current account) will be $+10-15=-5$ billion dollars. This example makes clear how the initial degree of foreign ownership of stocks affects the *CA* impact of asset value changes.

We can give another full numerical example to further illustrate the accounting. Let us suppose that at time $t-1$, the valuation of the financial assets located in country h is $e_{t-1} = \$30$ billion. Of these \$30 billion, \$20 billion are owned by domestic agents, and \$10 billion by foreigners. At the same time, country h owns \$5 billion of foreign assets. We thus

have: $\omega_{t-1}^h = \begin{pmatrix} 20 \\ 5 \end{pmatrix}$.

At time t , the following happens:

¹⁷ While the current account should, conceptually, be defined as the change in the net foreign position of a country, traditional empirical measures of it still do not usually include capital gains and losses. However, the IMF has started gathering data, which includes capital gains and losses (see *IMF Balance of Payments Manual*, 5th ed.).

- The domestic stock market increases by 10 percent: $\frac{P_t}{P_{t-1}} = 1.1$. Domestic dividend payments correspond to 5 percent of capital: $\frac{d_t}{P_{t-1}} = 0.05$. The gross returns on domestic stocks are, therefore, 15 percent. Moreover, $e_t = \frac{P_t}{P_{t-1}} e_{t-1} = 1.1 * 30 = \33 billion .
- The gross returns on foreign stocks happen to be 10 percent. With a 5 percent risk-free rate, this leads to the following excess financial returns vector:

$$X_t = \begin{pmatrix} 0.1 \\ 0.05 \end{pmatrix}$$
. Country h 's excess financial gains at time t are thus:

$$X_t' \omega_{t-1}^h = (0.1 \quad 0.05) \begin{pmatrix} 20 \\ 5 \end{pmatrix} = 2.25$$
.
- The agents happen to believe that their future present discounted excess financial gains will be zero: $E_t [X_t' \omega_{t-1}^h] = 0$.
- The agents also happen to expect that an annuity of their present discounted NI (or net income) endowments is exactly equal to their current NI endowment, so that $NI_t - E_t (NI_t^*) = 0$.

In such a case, country h 's current account would be:

$$CA_t = 0 + (2.25 - 0) - (3.3 - 3) = +\$1.95 \text{ billion .}$$

To conclude, term (3) underscores the importance of domestic stocks as a saving instrument: when domestic stock markets rise, the total amount of domestic assets available for savings purposes also rises. As a consequence, an increase in the savings rate does not necessarily lead to a current account surplus.

Equation (4) allows us to address a large range of issues related to the role of stock markets in the current account dynamics. Some of them are presented in the next section.

III. SOME IMPLICATIONS OF MODEL

A. United States' Current Account Deficit: Irrational Exuberance?

The model can be used to shed light on the recent and unprecedented U.S. current account deficit. It is sometimes said that the U.S. current account deficit in the late 1990s was due to the unusually high performance of the stock markets and to the (possibly irrational) belief

that the stock market would continue to increase as it had during the decade. Can my model help analyze this occurrence?¹⁸

It is the case that expectations of ever-rising stock market performance would cause a current account deficit (if one expects higher excess financial gains in the future, one will reduce savings, and the country will run a current account deficit). But such continually rising expectations are not necessary to create a current account deficit. The deficit would have appeared even if people believed that the recent stock market performances were a one time positive shock (expectations of a continuing boom would only result in a deficit of a larger magnitude). To illustrate this point, let us consider what happens to a U.S. agent whose shares value have risen by, say, \$10,000 following a rise in the stock market. If the agent thinks that these gains are a one-time windfall because of the belief that the stock market will not perform as well again in the future, the agent will consume only an annuity of this \$10,000, say \$2,500, and save the rest. In other words, what will occur is that the agent will sell \$2,500 in shares in order to finance consumption (or keep all or part of the shares and borrow the rest at a risk-free rate). But since the agent is a representative agent, all Americans will do the same. Therefore, the only individuals who can buy the \$2,500 in shares Americans want to sell to finance their extra consumption are, by construction, foreigners. This transaction is a transfer of wealth from home to foreign agents: it is a current account deficit for the home country. Prior to this surge in the market, a number of foreigners owned some U.S. assets. The capital gains they made during the boom also corresponds to a worsening of the U.S. net foreign position and, thus to a current account deficit.¹⁹

¹⁸ For an alternative portfolio-based analysis of the recent U.S. current account deficits, see Ventura (2001).

¹⁹ A full numerical example can help further understand this point. Let us suppose that at time $t-1$ the stock market valuation is $e_{t-1} = 60,000$. Of these, 50,000 belong to domestic agents, and 10,000 to foreigners. At time t , the gross return on the domestic stock market is $R_t = 1.2$. Let us assume that there were no dividends paid at time t , so that the gross returns correspond exclusively to capital gains. The new stock market valuation will thus be: $e_t = 60,000 * 1.2 = 72,000$. Therefore, $\Delta e_t = 12,000$. If the risk-free rate is 5 percent, the excess return on domestic asset will be $X_t = 20\% - 5\% = 15\%$. As a consequence the excess financial gains on domestic assets held by domestic agents are: $0.15 * 50,000 = 7,500$.

Let us finally assume that agents expect that the present discounted value of their future excess financial gains will be zero; that they expect their current endowment to be equal to an annuity of the discounted value of their future endowments; and that they expect their current excess financial gains on foreign assets to be equal to an annuity of the present discounted value of their future financial gains on foreign assets. We then have: $CA_t = 0 + (7,500 - 0) - 12,000 = -4,500$. The country thus runs a current account deficit of \$4,500. Of this amount, \$4,500, \$2,500 are due to the utility-maximization behavior described above. The remaining \$2,000 are due to capital gains by foreigners.

In conclusion, it is not necessary that people have high or irrational expectations about future stock market performances for a rise in the stock market to create a current account deficit. Quantitative studies would be needed to know whether the phenomenon is enough to explain the magnitude of the U.S. current account deficit or whether “over-optimism” is required.²⁰

B. Current Account as a Potential Predictor of Future Stock Market Performance

Another insight afforded by the model is that the current account may help predict future stock market performance and/or future endowment streams. The reason is that the current account is derived from the optimal portfolio and consumption/savings choices of all the agents. As a consequence, the current account should both incorporate and reflect all the relevant information agents have about future stock market performance and future endowments. This forecasting property can be formally expressed by a set of Granger causality and Granger causal priority propositions (see below).

Let us now turn to the formal propositions. In order to derive them, it is useful to first rewrite the stock market-augmented equation of the current account. But before doing so, let us rewrite, for the sake of notational simplicity, $f_t = X'_t \cdot \omega_{t-1}$, the excess financial gains at time t .

Proposition 2. The stock-market-augmented equation of the current account can be rewritten

$$\text{as } CA_t + \Delta e_t = -\sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i [E_t(\Delta NI_{t+i})] - \sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i [E_t(\Delta f_{t+i})] . \quad (5)$$

(In this equation Δ denotes the first-difference operator.)

Proposition 3 (Multivariate Granger causality). If equation (5) holds, then CA_t Granger-causes *at least one of* $(\Delta f_t, \Delta NI_t)$, except in the very special case where CA_t is a linear combination of present and past ΔNI_t , Δf_t and Δe_t .

Proposition 3B (multivariate Granger causality). If equation (5) holds, then $(CA_t + \Delta e_t)$ Granger-causes *at least one of* $(\Delta f_t, \Delta NI_t)$, except in the very special case where $(CA_t + \Delta e_t)$ is a linear combination of present and past ΔNI_t and Δf_t .

Proposition 4 (Granger Causal Priority).²¹ If equation (5) holds, then $\{\Delta f_t, \Delta NI_t\}$ is not Granger Causally Prior to $(CA_t + \Delta e_t)$, except in the very special case where $(CA_t + \Delta e_t)$ is a linear combination of present and past ΔNI_t and Δf_t .

²⁰ Of course, the current account deficit can also have been created by other factors, such as expectations of higher labor income (higher NI).

²¹ For a brief and very clear presentation of the concepts of Granger Causal Priority and multivariate Granger causality, see Sims (1999).

Proposition 5 (bivariate Granger causality). If I have equation (5), then $(CA_t + \Delta e_t)$ Granger-causes $\Delta(NI_t + f_t)$, except in the very special case where $(CA_t + \Delta e_t)$ is a linear combination of present and past $\Delta(NI_t + f_t)$. For proofs: see Appendix I.

Proposition 3 suggests a new insight about the current account. It implies that the current account may help predict future stock market performance and/or future endowment streams.²² What the proposition intuitively means is that the current account should help forecast at least one of the changes in net income and in financial gains. This comes from the fact that the current account reflects the expectations of the agents about future changes in net income and in financial gains. It is interesting to note that these expectations incorporate all the information available to the agents (including the information which econometricians do not observe). Therefore, the current account should reflect this information. As a consequence, the current account may help predict future stock-market performance. The potential predictive power of future endowments is not new. It was already present in the “save for a rainy day” argument of all life cycle models. Campbell (1987) derived the corresponding Granger proposition on the predictive power of consumption for future income. But the potential predictive power of the current account with respect to future stock market performance is new.

Since future stock market performance is difficult to predict, one should take this implication of the model with caution. A few points are in order to illustrate why this implication may be more reasonable than it can seem.²³

First, in any asset-pricing model where agents are risk-averse, agents have to expect that stocks will yield higher returns than the risk-free rate (i.e., “excess returns”) in order to decide to hold stocks. If there is no such risk premium to compensate people for the risk they take by holding risky securities, agents would never purchase risky assets. Expectations of excess financial gains is, therefore, a necessary condition for holdings of risky assets to exist.²⁴ This, of course, is not easy to reconcile with a random walk view of the stock market. Two points can nevertheless be made on this subject. The first one is that the information used by consumers to make their decisions may not be available to traders on Wall Street (something which seems reasonable). If so, then it would be possible that the current account is informative about future stock market performance without implying that some arbitrage opportunity has not been exploited by traders. The second point is that while arbitrage may lead to a random walk behavior of the stock markets in the short run, there may nevertheless

²²These propositions would not be altered by non-CARA utility functions. A CARA utility function implies a wealth-independent portfolio holding. However, I do not use the particular portfolio predicted by the CARA utility function in this section’s analysis.

²³ Surveys on stock market predictability can be found in Cochrane (1999), and Lettau *et al.* (2001).

²⁴ Determinants of these expected excess returns are discussed in the general equilibrium version of the model in Mercereau (2002). They include demographic variables, variance and covariance of dividend processes, and risk aversion, among others. Since these parameters are non-stochastic but time-varying, expected excess-returns will fluctuate in a predictable way at lower frequencies.

be predictable medium-run or long-run trends in the stock market movements. The model is precisely about these longer-term trends.

Finally, it is interesting to notice that this property of the model that the current account may help forecast future changes in equity premium gains is in the same spirit of an argument recently made by Lettau and Ludvigson (2001). In this paper, they argue that the ratio of consumption to wealth should help forecast stock returns because it incorporates people's expectations about them. They make their point in a fairly general formal model. They then show that empirically their consumption to wealth ratio is the best single predictor of future stock returns. The insight given by our model is sort of a generalization of their argument to the national open economy.

The other Granger propositions are in the same spirit as the former, but are somewhat weaker or less interesting. These propositions are tested formally in Mercereau (2003).

To summarize, the current account may help forecast the changes in net output and/or of excess financial gains because it reflects the agents' information and expectations about these two variables' future levels.

C. Other Implications of Model

Implication of Holding Foreign Stocks: International Transmission of Shocks

It is noteworthy that in this framework "stock markets" does not necessarily mean "domestic stock markets." The model does not differentiate between domestic and foreign shares: an agent is free to use all the international stock markets. The consequence of this is that expected performances of foreign stock markets can also have a meaningful impact for a country which has invested (or simply plans to invest) abroad. For example, a meaningful portion of the financial income of a country, which, like Canada, invests heavily in U.S. shares, comes from its revenue from U.S. shares. Therefore, their expectations about the future performance and structure of the U.S. stock market will have a meaningful impact on the Canadian current account. To give a simple example, if the Canadians believe that the U.S. stock market is going to perform exceptionally well in the future, Canada will tend to run a current account deficit today, *independently of the expected performance of the Canadian economy or stock markets.*²⁵

Oil producing countries are also good examples of countries which hold large stocks of foreign assets.

²⁵ In an extreme situation for a country with a sizable stock of foreign assets, a country's net output and financial gains on *domestic* stock markets could theoretically both be above their respective expected future permanent levels, and have the country run a current account deficit, provided that they expect high enough returns on foreign assets.

Impact of Endowment Income Shocks

Our model can also be used to discuss the impact of endowment shocks on the current account, an issue that has drawn attention in the literature. Our model stresses that endowment and stock market returns shocks should be jointly analyzed. Intuitively, buying stocks will allow one to mitigate the impact of endowment shocks. In contrast with the rest of the literature, which deals exclusively with the two extreme cases of an economy with only a risk-free bond or of complete markets, financial markets are incomplete in our framework.

To make the discussion somewhat more substantive, let us examine the case where endowment shocks are serially auto-correlated (let us write Ψ_t as the present value multiplier of a shock η_t on NI_t). In the Sachs' bond-only small open economy, a positive shock η_t on endowment income creates a current account surplus of magnitude $\frac{1}{1+r}\Psi_t\eta_t$. Indeed, if a shock has positive present discounted value, the representative consumer will consume an annuity of it ($\frac{r}{1+r}\Psi_t\eta_t$) and save the rest, creating a current account surplus.

In a complete market world, it is even simpler: the country purchases in the first period the portfolio which will insure full risk sharing among countries, and it will keep it every subsequent period. The current account, which is the change of net foreign assets, will, therefore, only reflect the capital gains on the net foreign portfolio.

Our model, with *incomplete* financial markets, is an intermediate (and more realistic) case between these two extremes. What is the implication of market incompleteness? Intuitively, the answer is that international stock markets allow the country to partially hedge its endowment shocks. It will help it hedge against shocks, because one of the determinants of the country's portfolio was risk hedging. In our model, the desired risky assets portfolio is indeed given by:

$$\omega_t = \underbrace{\frac{1+r}{rA}\Sigma_{t+1}^{-1}EX_{t+1}}_{\text{risk premium exploitation}} - \underbrace{\Psi_{t+1}\Sigma_{t+1}^{-1}\beta_{t+1}}_{\text{risk hedging}},$$

where Σ_{t+1}^{-1} is the variance-covariance matrix of the assets returns, β_{t+1} the vector of covariance of the asset returns with NI_t and Ψ_{t+1} the present value multiplier of a shock on NI_t . The risk-hedging component reflects the desire of the country to hold stocks which perform well when the endowment income performs poorly.

One way to illustrate how this (imperfect) risk hedging provided by the financial market influences the impact of an endowment shock on the current account is to consider the expected shock on the current account conditional on the realization of an endowment shock. Proposition 6 below illustrates this point.

Proposition 6. The expected shock on the current account conditional on a net income shock is:

$$E_{t-1}(CA_t / \eta_t) = \frac{1}{1+r} \left[\underbrace{\Psi_t \eta_t}_{\text{PDV of shock}} + \underbrace{\Psi_t \frac{1}{\text{var}(\eta_t)} \beta_t' \omega_{t-1} \eta_t}_{\text{PDV of expected shock hedging}} \right] - E_{t-1}(\Delta e_t / \eta_t). \quad (6)$$

For proof, see Appendix I.

We see that the effect on the current account of a negative present discounted value shock on endowment income is no longer clear. This shock is indeed three-sided. It has a direct negative effect on wealth. But it is also possible that our portfolio of risky assets provides us with more than enough hedging against endowment income risk and that the corresponding risky asset shocks [term (2)] more than offsets this negative shock. Therefore, the combined impact is uncertain, and the impact of the shock on desired holding of the agents could go either way (this is what the first two terms reflect).²⁶ Finally, to see which way the current account will move, one also has to take into account the change in domestic stocks available. This discussion illustrates that it is essential to take the stock markets into account when one wants to assess the impact of endowment shocks on a country's current account. It also shows that the phenomenon is more complex than previous models have suggested.

IV. CONCLUSION

To summarize, this paper develops a model to study the impact of stock markets on the current account. The model includes an arbitrary number of risky assets, which form an incomplete market, as well as a risk-free bond. A closed-form solution for the current account is derived from the optimal portfolio and consumption/saving choices of a representative agent. Formally, the model can be seen as a stock market-augmented version of the fundamental equation of the current account popularized by Sachs.

The model can help explain the recent and exceptional U.S. current account deficits. The model suggests that the current account may help predict future stock market performance and/or future endowments. A general-equilibrium version of the model is developed in Mercereau (2002).

In a companion paper, Mercereau (2003), the model is put to the test using U.S. data. The model performed better than the same model without stock markets. The forecasting property of the current account as a potential predictor of future stock market performance also received preliminary empirical confirmation.

To conclude, this paper developed a model to study the role of stock markets in the dynamics of the current account. To further explore this issue would be a worthy role for future research.

²⁶ Although, in practice, it is probably more often the case that financial investments of a country will only partially offset the endowment income shocks rather than overshooting them.

APPENDIXES

I. Proof of Propositions

Solution of model

The program of the agent is:

$$\text{Max}_{\{c_t; \omega_{0,t}; \omega_{j,t}\}_{t=0}^{+\infty}} U(C) = E_0 \left\{ \sum_{t=0}^T \delta^t u(c_t) \right\},$$

under the budget constraints BC_t :

$$c_t + \omega_{0,t} + \sum_{j=1}^J \omega_{j,t} = NI_t + R_0 \omega_{0,t-1} + \sum_{j=1}^J R_{j,t} \omega_{j,t-1}$$

(and we have the initial conditions: $\omega_{0,-1} = \omega_{j,-1} = 0$).

We can rewrite this program as:

$$\text{Max}_{\{\omega_{0,t}; \omega_{j,t}\}_{t=0}^{+\infty}} U(C) = E_0 \left\{ \sum_{t=0}^T \delta^t u(NI_t + R_0 \omega_{0,t-1} + \sum_{j=1}^J R_{j,t} \omega_{j,t-1} - \omega_{0,t} - \sum_{j=1}^J \omega_{j,t}) \right\}.$$

Let us write the Euler equations:

$$\frac{\partial}{\partial \omega_{0,t}} : E_t \left\{ -u'(c_t) + \delta(1+r)u'(c_{t+1}) \right\} = 0 \quad (\text{A1})$$

$$\frac{\partial}{\partial \omega_{j,t}} : E_t \left\{ -u'(c_t) + \delta R_{j,t+1} u'(c_{t+1}) \right\} = 0 \quad (\text{A2})$$

As a beginning, let us rewrite this system of equations. Equation (A1) becomes:

$\exp(-Ac_t) = \delta(1+r)E_t \left[\exp(-Ac_{t+1}) \right]$. Because all shocks are normally distributed by assumption, the budget constraint implies that consumption is normally distributed as well. Therefore, the equation now reads:

$$\exp(-Ac_t) = \delta(1+r) \exp \left[-AE_t(c_{t+1}) + \frac{A^2}{2} \text{var}_t(c_{t+1}) \right],$$

$$\text{which leads to: } E_t(c_{t+1}) = c_t + \frac{A}{2} \text{var}_t(c_{t+1}) + \ln \left[\delta(1+r) \right]. \quad (\text{A3})$$

On the other hand, differencing (A1) and (A2) leads to:

$$E_t[(1+r)u'(c_{t+1})] = E_t[R_{j,t+1}u'(c_{t+1})], \text{ which is equivalent to:}$$

$$E_t(X_{j,t+1})E_t[u'(c_{t+1})] + \text{cov}_t[X_{j,t+1};u'(c_{t+1})] = 0.$$

Since the variables are normally distributed, one can use Stein's lemma:

$$\text{cov}_t[X_{j,t+1};u'(c_{t+1})] = E_t[u''(c_{t+1})]\text{cov}_t(X_{j,t+1};c_{t+1}).$$

Using the fact that I have an exponential utility function, this can be rearranged in:

$$E_t(X_{j,t+1}) = A\text{cov}_t[R_{j,t+1};c_{t+1}] \tag{A4}$$

We will now use (A3) and (A4) to solve the model. The strategy is as follows: equation (A4) will give the risky portfolio holding expression, while the other equations will give the other variables as function of the portfolio holding. I will use a “guess and verify” method on the portfolio holding: I will “guess” its expression, then solve for all other variables as a function of portfolio holding. And finally, I will verify that equation (A4) and the expressions found for the other variables indeed result in the guessed risky portfolio allocation.

$$\text{The guess for the portfolio of risky assets is: } \omega_t = \underbrace{\frac{1+r}{rA}\Sigma_{t+1}^{-1}E_t X_{t+1}}_{\text{risk premium exploitation}} - \underbrace{\Sigma_{t+1}^{-1}\beta_{t+1}}_{\text{risk hedging}},$$

where $\Sigma_t = [\text{cov}_{t-1}(R_{i,t};R_{j,t})]_{i,j=1,\dots,J}$ is the $J \times J$ variance-covariance matrix of asset returns,

and $\beta_t = [\text{cov}_{t-1}(NI_t, R_{j,t})]_{j=1,\dots,J}$ is the $J \times 1$ matrix of covariance between net income and

asset returns. Both β_t and Σ_t are exogenous in the model. They can vary over time, however.

Let us first find the expression of consumption. The consumption is found by solving the budget constraint (“BC”) forward and using equation (A3): I take the sum

$$BC_t + E_t \sum_{i=1}^T \frac{1}{(1+r)^i} BC_{t+i} \text{ and use the fact that } E_t[c_{t+i+1}] = E_t[E_{t+i}(c_{t+i+1})].$$

Since I have assumed that the first and second moments of our exogenous random variables are bounded, each element of ω_t will be bounded, and so will be $X'_{t+i}\omega_{t+i-1}$ for all i . $\text{var}_t(c_{t+1})$ will also be bounded.

As a consequence, the series $\sum_{i=1}^T \frac{1}{(1+r)^i} E_t(X'_{t+i}\omega_{t+i-1})$ and $\sum_{i=1}^T \frac{1}{(1+r)^i} \text{var}_t c_{t+i}$, respectively, converge when $T \rightarrow +\infty$.

Solving forward the budget constraint will, after some straightforward algebra, lead to:²⁷

$$c_t = \frac{r}{1+r} \left\{ \begin{array}{l} \underbrace{\sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} E_t(NI_{t+i})}_{(1) \text{ labor income wealth}} + \underbrace{R_0 \omega_{0,t-1} + R'_t \omega_{t-1}}_{(2) \text{ current financial income}} + \underbrace{\sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} E_t(X'_{t+i} \omega_{t+i-1})}_{(3) \text{ future financial excess gains}} \\ - \underbrace{\frac{1}{rA} \text{Ln}[\delta(1+r)]}_{(4) \text{ consumption tilting}} - \underbrace{\frac{A}{2} \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} \text{var}_t c_{t+i}}_{(5) \text{ precautionary savings}} \end{array} \right\}.$$

Let us now turn to the expression of risk-free asset holding $\omega_{0,t}$. Plugging the budget constraint at time t into equation (A3) leads to the following recursive relation:

$$\omega_{0,t} = \omega_{0,t-1} + z_t,$$

$$\text{with } z_t = NI_t + R'_t \omega_{t-1} - \sum_{j=1}^J \omega_{j,t} - \frac{r}{1+r} \left\{ \sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} E_t(NI_{t+i}) + R'_t \omega_{t-1} + \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} E_t(X'_{t+i} \omega_{t+i-1}) \right\} \\ + \frac{1}{rA} \text{Ln}[\delta(1+r)] + \frac{A}{2} \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} \text{var}_t c_{t+i}$$

I can now verify our guess. Using the expression I found for consumption in equation (A4) leads to:

$$E_t(X_{j,t+1}) = \frac{Ar}{1+r} \text{cov}_t \left[R_{j,t+1}; NI_{t+1} + \sum_{i=1}^J R_{i,t+1} \omega_{i,t} \right] \quad \forall j = 1, \dots, J.$$

This can be rewritten in matrix form as: $\frac{Ar}{1+r} [\Sigma_{t+1} \omega_t + \beta_{t+1}] = E_t X_{t+1}$,

$$\text{or } \omega_t = \underbrace{\frac{1+r}{rA} \Sigma_{t+1}^{-1} E_t X_{t+1}}_{\text{risk premium exploitation}} - \underbrace{\Sigma_{t+1}^{-1} \beta_{t+1}}_{\text{risk hedging}}, \text{ which was our initial guess.}$$

²⁷ We also assume that $\sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} E_t(NI_{t+i})$ exists, which is a natural assumption. If the sum

did not converge, it would mean that the present discounted value of the agent's labor income is infinite. She would then have no intertemporal budget constraint and she would be free to consume as much as she wishes.

Transversality condition

We also have to check that the TVC is satisfied. A sufficient condition for the TVC to be

satisfied is: $\lim_{s \rightarrow +\infty} E_t \left(\frac{1}{1+r} \right)^s \left[\omega_{0,t+s} + \sum_{j=1}^J \omega_{j,t+s} \right] \leq 0$.

We saw that because the first and second moments of the exogenous stochastic variables are

bounded $\sum_{j=1}^J \omega_{j,t+s}$ is bounded as well, and, therefore, $\lim_{s \rightarrow +\infty} E_t \left[\left(\frac{1}{1+r} \right)^s \sum_{j=1}^J \omega_{j,t+s} \right] = 0$.

We also have $\omega_{0,t+s} = \sum_{i=0}^{t+s} z_i$,

with $z_t = NI_t + R'_t \omega_{t-1} - \sum_{j=1}^J \omega_{j,t} - \frac{r}{1+r} \left\{ \sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} E_t(NI_{t+i}) + R'_t \omega_{t-1} + \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} E_t(X'_{t+i} \omega_{t+i-1}) \right\}$

$$+ \frac{1}{rA} \text{Ln}[\delta(1+r)] + \frac{A}{2} \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} \text{var}_t c_{t+i}.$$

From the boundedness of the variables, which has been previously discussed, it is possible to derive that there exists a constant K such that:

$$\left| R'_t \omega_{t-1} - \sum_{j=1}^J \omega_{j,t} - \frac{r}{1+r} \left\{ R'_t \omega_{t-1} + \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} E_t(X'_{t+i} \omega_{t+i-1}) \right\} + \frac{1}{rA} \text{Ln}[\delta(1+r)] + \frac{A}{2} \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} \text{var}_t c_{t+i} \right| \leq K$$

Writing x_t the variable between the absolute value sign above, I then have:

$$\lim_{s \rightarrow +\infty} E_t \left(\frac{1}{1+r} \right)^s \sum_{i=1}^{t+s} x_i = 0.$$

So all I have left to show in order to verify that the TVC is satisfied is that :

$$\lim_{s \rightarrow +\infty} E_t \left[\left(\frac{1}{1+r} \right)^s \sum_{k=0}^{t+s} \left(NI_k - \frac{r}{1+r} \left\{ \sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} E_k(NI_{k+i}) \right\} \right) \right] \leq 0. \quad \text{inequality (A5)}$$

Let us define $Z_{t+s} \equiv \left(\frac{1}{1+r} \right)^{t+s} \sum_{k=0}^{t+s} NI_k$. I will show that $\lim_{s \rightarrow +\infty} E_t Z_{t+s} = 0$.

I will do the proof in the sub case where $t+s$ is an even number. The other sub case where $t+s$ is odd is then straightforward. Posit $t+s \equiv 2n$, with $n \in \mathbb{N}$.

$$E_t Z_{t+s} = \left(\frac{1}{1+r} \right)^{2n} E_t \left(\sum_{k=0}^n NI_k + \sum_{k=n}^{2n} NI_k \right).$$

$$E_t Z_{t+s} \leq \left(\frac{1}{1+r} \right)^n \sum_{k=0}^n \left(\frac{1}{1+r} \right)^n E_t NI_k + \sum_{k=n}^{2n} \left(\frac{1}{1+r} \right)^{2n} E_t NI_k, \text{ which leads to:}$$

$$E_t Z_{t+s} \leq \left(\frac{1}{1+r} \right)^n \sum_{k=0}^n \left(\frac{1}{1+r} \right)^k E_t NI_k + \sum_{k=n}^{2n} \left(\frac{1}{1+r} \right)^k E_t NI_k.$$

By assumption, $\sum_{k=0}^n \left(\frac{1}{1+r} \right)^k E_t NI_k$ converges. Therefore,

$$\left(\frac{1}{1+r} \right)^n \left[\sum_{k=0}^n \left(\frac{1}{1+r} \right)^k E_t NI_k \right] \rightarrow 0 \text{ when } n \rightarrow +\infty.$$

Moreover, $\sum_{k=n}^{2n} \left(\frac{1}{1+r} \right)^k E_t NI_k \rightarrow 0$ when $n \rightarrow +\infty$ as part of the residual of a converging positive series. Hence, $\lim_{s \rightarrow +\infty} E_t Z_{t+s} = 0$.

This result implies that inequality (A5) is necessarily verified. Indeed, all the other terms on the LHS are negative. I, therefore, have finally proved that the TVC is verified.

No-Ponzi-game condition

What would be a no-Ponzi-game condition for this problem? One way to think about it would be to say that the total amount of money foreigners are willing to lend to the domestic economy should not grow faster than the total stock of risky assets in the economy:

$$\lim_{s \rightarrow +\infty} E_t \left(\frac{1}{1+r} \right)^s \left[\omega_{0,t+s} + \sum_{j=1}^J \omega_{j,t+s} \right] \geq \lim_{s \rightarrow +\infty} E_t \left[\left(\frac{1}{1+r} \right)^s e_{t+s} \right].$$

More assumptions should be made on the model to formally check that such a condition is verified, but these new conditions would be very weak. Indeed, it is sufficient to assume, for example, that the Net Income follows an exponential growth path whose rate is below the

risk-free rate²⁸, then it is possible to show that $\lim_{s \rightarrow +\infty} E_t \left(\frac{1}{1+r} \right)^s \left[\omega_{0,t+s} + \sum_{j=1}^J \omega_{j,t+s} \right] = 0$, so that

the No Ponzi game condition is satisfied.

²⁸ Again, if this rate were higher, the present discounted value of the country's endowment income would be infinite. The country's representative agent would then be able to consume
(continued...)

Proof of proposition 1 (stock market-augmented fundamental equation of the current account).

By definition, the current account is the change in net foreign assets:

$GCA_t = \omega_{0,t} + 1' \omega_t^{h,f} - 1' \omega_t^{f,h} - (\omega_{0,t-1} + 1' \omega_{t-1}^{h,f} - 1' \omega_{t-1}^{f,h})$, where $\omega_t^{h,f}$ is the vector of foreign assets owned by domestic (“home”) agents, and $\omega_t^{f,h}$ is the vector of domestic assets owned by foreign agents.

Using the fact that $\sum_{j=1}^J \omega_{j,t} = e_t - 1' \omega_t^{f,h} + 1' \omega_t^{h,f}$, the GCA can be rewritten as:

$$GCA_t = \omega_{0,t} + \sum_{j=1}^J \omega_{j,t} - \left(\omega_{0,t-1} + \sum_{j=1}^J \omega_{j,t-1} \right) - (e_t - e_{t-1}).$$

Using the recursive relation found above for the risk-free asset holding yields:

$$\begin{aligned} GCA_t = & \sum_{j=1}^J \omega_{j,t} - \sum_{j=1}^J \omega_{j,t-1} + NI_t + R'_t \omega_{t-1} - \sum_{j=1}^J \omega_{j,t} \\ & - \frac{r}{1+r} \left\{ \sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} E_t(NI_{t+i}) + R'_t \omega_{t-1} + \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} E_t(X'_{t+i} \omega_{t+i-1}) \right\} \\ & + \frac{1}{rA} Ln[\delta(1+r)] + \frac{A}{2} \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} \text{var}_t c_{t+i} - (e_t - e_{t-1}) \end{aligned}$$

Using the definition of the “future permanent level” operator

$(\sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} Z_{t+i} = \sum_{i=0}^{+\infty} \frac{1}{(1+r)^i} Z_t^*)$ and rearranging the terms gives:

$$\begin{aligned} GCA_t = & \underbrace{(NI_t - E_t NI_t^*)}_{(1) \text{ labor income effect}} + \underbrace{(X'_t \omega_{t-1} - E_t [(X'_t \omega_{t-1})^*])}_{(2) \text{ stock market effect}} \\ & + \underbrace{\frac{1}{rA} Ln[\delta(1+r)]}_{(3) \text{ consumption tilting}} + \underbrace{\frac{A}{2} \sum_{i=1}^{+\infty} \frac{1}{(1+r)^i} \text{var}_t c_{t+i}}_{(4) \text{ precautionary savings}} - (e_t - e_{t-1}) \end{aligned}$$

as much as she wants at each period, for she would not face any intertemporal budget constraint. So the required conditions are weak indeed.

Finally, note that $\text{var}(c)$ can also be expressed as a function of the exogenous parameters:

$$\text{var}(c_t) = \left(\frac{r}{1+r} \right)^2 \underbrace{(\Psi_t)^2 [\text{var}(\eta_t) - \beta_t' \Sigma_t^{-1} \beta_t]}_{\text{undiversifiable part of idiosyncratic risk}} + \underbrace{\frac{1}{A^2} EX_t' \Sigma_t^{-1} EX_t}_{\text{premium exploitation risk}} \quad \text{where } \eta_t \text{ is the}$$

innovation of net output, Ψ_t is the present value multiplier of the innovation on net output, Σ_t^{-1} is the variance-covariance matrix of the assets returns, β_t the vector of covariance of the asset returns with NI_t .

Proof of proposition 2

To prove proposition 2, one should start with writing down the RHS of the equation I want to

$$\text{prove: } RHS \equiv - \sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i [E_t(\Delta NI_{t+i})] - \sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i [E_t(\Delta f_{t+i})].$$

Then all one has to do is to split each sum into two sums (recall that both series

$\sum_{i=1}^T \frac{1}{(1+r)^i} E_t(f_{t+i-1})$ and $\sum_{i=1}^T \frac{1}{(1+r)^i} NI_{t+i}$ converge when $T \rightarrow +\infty$). Simplifying term by term and rearranging then yields the stock market-augmented equation of the current account.

Proof of propositions 3 to 5 (Granger propositions)

I will do the proof for propositions 3B and 4 (I will give two alternative proofs in each case). The other proofs are similar.

Let us first recall the equation I started with:

$$CA_t + \Delta e_t = - \sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i [E_t(\Delta NI_{t+i})] - \sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i [E_t(\Delta f_t)] \quad (A6)$$

(where f_t denotes the equity premium gains previously written as $X'_{t+i} \omega_{t+i-1}$).

Let us also write I_t the information set containing all the present and past values of $(CA_t + \Delta e_t)$, ΔNI_t , and Δf_t .

Then I have the following proposition:

Proposition 3B (multivariate Granger causality). If equation (A6) holds, then $(CA_t + \Delta e_t)$ Granger-causes *at least one of* $(\Delta f_t, \Delta NI_t)$, except in the very special case where $(CA_t + \Delta e_t)$ is a linear combination of present and past ΔNI_t and Δf_t .

Proof:

In the entire analysis, I treat conditional expectations as equivalent to linear projections on information.

Let us proceed ad absurdum. Let us assume that $(CA_t + \Delta e_t)$ Granger-causes neither Δf_t nor ΔNI_t , and that $(CA_t + \Delta e_t)$ is *not* a linear combination of present and past ΔNI_t and Δf_t . Then the fact that $(CA_t + \Delta e_t)$ does not Granger-cause Δf_t implies that:

$$E(\Delta f_{t+i} / I_t) = E(\Delta f_{t+i} / \Delta f_t, \Delta f_{t-1}, \Delta f_{t-2}, \dots, \Delta NI_t, \Delta NI_{t-1}, \Delta NI_{t-2}, \dots) \text{ for all } i.$$

In the same way, the fact that $(CA_t + \Delta e_t)$ does not Granger-cause ΔNI_t implies that $E(\Delta NI_{t+i} / I_t) = E(\Delta NI_{t+i} / \Delta f_t, \Delta f_{t-1}, \Delta f_{t-2}, \dots, \Delta NI_t, \Delta NI_{t-1}, \Delta NI_{t-2}, \dots) \text{ for all } i.$

Using equation (1), I have:

$$E(CA_t + \Delta e_t / I_t) = -\sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i \left[E(\Delta NI_{t+i} / I_t) \right] - \sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i \left[E_t(\Delta f_t / I_t) \right], \text{ and therefore:}$$

$$E(CA_t + \Delta e_t / I_t) = -\sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i \left[E(\Delta NI_{t+i} / \Delta f_t, \Delta f_{t-1}, \dots, \Delta NI_t, \Delta NI_{t-1}, \dots) \right]$$

$$- \sum_{i=1}^{+\infty} \left(\frac{1}{1+r} \right)^i \left[E_t(\Delta f_t / \Delta f_t, \Delta f_{t-1}, \dots, \Delta NI_t, \Delta NI_{t-1}, \dots) \right],$$

which is to say that $E(CA_t + \Delta e_t / I_t)$ is an exact linear combination of the present and past ΔNI_t and Δf_t . But $(CA_t + \Delta e_t)$ belongs to I_t . Hence, $E(CA_t + \Delta e_t / I_t) = CA_t + \Delta e_t$. Therefore $(CA_t + \Delta e_t)$ is an exact linear combination of the present and past ΔNI_t and Δf_t , which contradicts our initial assumption.

Note: An alternative proof could be done in the special of a VAR model.

$$\text{Let us write } \left(I - \begin{bmatrix} A_{11}(L) & A_{12}(L) & A_{13}(L) \\ A_{21}(L) & A_{22}(L) & A_{23}(L) \\ A_{31}(L) & A_{32}(L) & A_{33}(L) \end{bmatrix} \right) \begin{bmatrix} \Delta NI_t \\ \Delta f_t \\ CA_t + \Delta e_t \end{bmatrix} = \varepsilon_t. \text{ Stacking this equation to form}$$

a first order system, it is then easy to show that if $A_{13}=A_{23}=0$ then $E(\Delta f_{t+i} / I_t)$ and $E(\Delta NI_{t+i} / I_t)$ are linear combinations of present and past ΔNI_t and Δf_t (all one has to do is to write the VAR forecast of each of this terms). Therefore, $(CA_t + \Delta e_t)$ itself is an exact linear combination of the present and past ΔNI_t and Δf_t .

Proposition 4 (Granger Causal Priority). If equation (A6) holds, then $\{\Delta f_t, \Delta NI_t\}$ is not Granger Causally Prior to $(CA_t + \Delta e_t)$, except in the very special case where $(CA_t + \Delta e_t)$ is a linear combination of present and past ΔNI_t and Δf_t .

Proof: This is an immediate consequence of Proposition 1. Indeed, let's suppose that $\{\Delta f_t, \Delta NI_t\}$ is Granger Causally Prior to $(CA_t + \Delta e_t)$. Then, by definition of Granger-causal priority, $(CA_t + \Delta e_t)$ does not Granger-cause $\{\Delta f_t, \Delta NI_t\}$. But this contradicts Proposition 1 (except in the very special case where $(CA_t + \Delta e_t)$ is a linear combination of present and past ΔNI_t and Δf_t).

Note that it is also straightforward to do a proof in the VAR case (in a 3 variable-system, $\{\Delta f_t, \Delta NI_t\}$ is Granger Causally Prior to $(CA_t + \Delta e_t)$ necessarily implies $A_{13}=A_{23}=0$, which contradicts Proposition 1).

Proof of proposition 6

Proposition 6 is a simple application of the fact that if two variables x_1 and x_2 have a joint normal distribution:

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N \left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}; \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{pmatrix} \right], \text{ then the distribution of } x_1 \text{ conditional on } x_2 \text{ is:}$$

$$(x_1 / x_2) \sim N \left[\mu_1 + \Sigma_{12} \Sigma_{22}^{-1} (x_2 - \mu_2); \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{12} \right]. \text{ In our case } x_1 \text{ is } CA_t \text{ and } x_2 \text{ } NI_t.$$

II. Summary of Main Notation

Description	Notation	Dimension
Net income	$NI_t = Y_t - I_t - G_t - D_t$	1x1
Total amount of dividends distributed by domestic companies	D_t	1x1
Risk free interest rate	$R_0 = 1 + r$	1x1
Dividends paid by company j	$d_{j,t}$	1x1
Price of stock j	$P_{j,t}$	1x1
Gross returns	R_t	Jx1
Gross return of stock j	$R_{j,t} = \frac{d_{j,t} + P_{j,t}}{P_{j,t-1}}$	1x1
Excess returns	X_t	Jx1
Excess return of stock j	$X_{j,t} = (R_{j,t} - R_0)$	1x1
Risk free asset holding	$\omega_{0,t}$	1x1
Risky asset holding by domestic agent	$\omega_t = (\omega_{j,t})_{j=1}^J$	Jx1
Total endowment of risky assets in the domestic economy (=stock market valuation)	e_t	1x1
Excess financial gains	$f_t = X'_t \omega_{t-1}$	1x1
Trade balance	$TB_t \equiv Y_t - C_t - I_t - G_t$	1x1
Rate of time preference	δ	1x1
Coefficient of absolute risk aversion	A	1x1
Variance-covariance matrix of asset returns	$\Sigma_t = [\text{cov}_{t-1}(R_{i,t}, R_{j,t})]_{i,j=1 \dots J}$	JxJ
Covariance between net income and asset returns	$\beta_t = [\text{cov}_{t-1}(NI_t, R_{j,t})]_{j=1, \dots, J}$	Jx1

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