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# A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to The Czech Republic

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Asia and Pacific Department

## A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to the Czech Republic

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### Abstract



This paper presents a multivariate (MV) methodology for obtaining measures of excess demand that can facilitate discussion of monetary policy issues and improve policy decisions. Using data for the Czech Republic, a growing economy undergoing major structural change, it shows how the use of more information to condition the paths of potential output and the non accelerating inflation rate of unemployment (NAIRU) improves on univariate methods as the Hodrick-Prescott (HP) filter.

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[1] Czech National Bank and IMF Research Department (at the time this paper was written), respectively. We are grateful to Peter Isard, Douglas Laxton, and David Rose for helpful comments and suggestions. We are entirely responsible for any errors. The opinions expressed in the paper are solely those of the authors and do not represent the views of the International Monetary Fund.

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## I. Introduction

Since an important part of the influence of monetary policy on inflation comes from an adjustment of the monetary instrument to excess demand and from the credibility maintained by the monetary authorities, it is crucial that the measures of excess demand be clearly understood and accepted by both those involved in the policy process and others. Unfortunately, because it is not possible for policymakers and modelers to observe the state of excess demand directly, they must infer it. To this end, policymakers and modelers have developed various techniques to construct measures of potential output –the level of output that can be produced and sold without creating pressures for the rate of inflation to rise or fall— and the NAIRU (the non accelerating inflation rate of unemployment) –the level of unemployment at which there is no pressure for inflation to rise or fall.

This paper presents a multivariate (MV) methodology for obtaining measures of excess demand that can facilitate discussion of monetary policy issues and improve policy decisions. Using data for the Czech Republic, a growing economy undergoing major structural change, it shows how the use of more information to condition the paths of potential output and the NAIRU improves on univariate methods such as the Hodrick-Prescott (HP) filter.

The remainder of the paper is organized as follows. Section II provides an overview of the different methods for measuring potential output and the NAIRU. Section III presents the MV methodology that we use to measure these variables and discusses the results. Section IV compares the results of the MV filter to those of the HP filter. Section V gives conclusions. An appendix provides details on the MV filter used.

## II. Measures of Potential Output and the NAIRU

Over the years, many methods have been proposed for measuring potential output and the NAIRU.[2] In the case of potential output, one idea that always lies close to the surface is that there is some production function that links output to available inputs of labor, capital and raw materials, given the current technology, and that we can think of the current level of potential output as what would emerge from the production function, given the current levels of fixed inputs and sustainable levels of variable inputs. Although this idea is useful in a general sense, and indeed motivates the idea that there is some link between conditions in labor markets and conditions in product markets, it has been found that, in practice, not much is added to the precision of measures of excess demand by the structure of the production function. The uncertainty in pinning down potential output is simply transferred into

[2] See, for example OECD (2000).

uncertainty about total factor productivity.[3] Moreover, for an economy which is experiencing major structural change, a production function approach may not be that reliable because it may not fully account for an increase in the share of capital stock that has become obsolete.

The modern standard methodology for measuring potential output and the NAIRU is to use some variant of filtering. What this means is that time-series techniques are used to fit trend lines through the data, and these trends provide the measures of the underlying "equilibrium" values.[4] It is important to stress that in referring to these values as "equilibrium" values, we use the perspective of the effect on inflation. The trend lines are used to define "gaps"--deviations of actual observed values from these trends-- that are, in turn, used to describe the dynamics of inflation and the policy control process. The measures are determined, at least in part, by their ability to represent these processes. It is not the case, for example, that our measure of "potential" output means that this is the best that could be done with the best possible use of all resources without constraints. For example, capital put in place under different conditions may not be malleable or useful at all in new industries. Labor may need new skills or to be relocated to achieve the longer-term production function possibilities frontier. What we need for monetary policy is a measure that represents what can be produced today, given all the constraints, without generating inflationary pressures.

Filtering methodologies are many and varied. One economist summarized the early methodology as using "a long and flexible ruler" to draw a bendy line through the data on a graph.[5] In modern methodology, the long and flexible ruler has been replaced by numerical methods that do the same thing on a computer, with more or less complexity.

In the simplest variants, which are called univariate filters, only the data for the series itself are used to fit the trends. A popular example is the HP filter.[6] In the HP filter, and all other similar filters, the user must supply some judgment as to how smooth the trend should be. In other words, just how flexible should that ruler be? Should it be very stiff so that the trend does not move much with actual cycles in the data, or should it be more flexible and follow the data more closely? The key point is that the methodology itself cannot provide this information; the user must impose it or infer it from other information or criteria (such as embedding it in a broader estimation problem, where some other criterion will effectively determine the degree of smoothing).

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[3] This does not mean that production functions are not useful in other ways. In more complex models with stocks, it is essential to have an explicit link between investment spending and the creation of productive capacity.

[4] We use the phrase "trend lines" to describe the series we identify as potential output, the NAIRU, and so on. They are not necessarily "straight" lines.

[5] See Laxton and Tetlow (1992) for a survey that includes this observation.

[6] See Hodrick and Prescott (1980).

The issue of the degree of smoothing to use in a filter has a direct link to the issue of the nature of the shocks to the economy. If the shocks to the economy are primarily shocks to aggregate demand, with supply conditions largely unaffected, then potential output does not move closely with the data, and it is appropriate to use a high level of smoothing in the filter. If, on the other hand, there is a high proportion of supply shocks, then potential output is indeed moving with the data, and a lower degree of smoothing is appropriate. Thus, it is important that the judgment of knowledgeable specialists be used to condition what is otherwise a purely mechanical exercise.

One example of a univariate methodology that makes a small step in formalizing the use of judgment is the simple Prior-Consistent (PC) filter, which allows some weight to be given to priors on the evolution of the trend through time or its variability relative to the observed data, in the fitting of the trend.[7] Univariate methodologies all suffer from a number of problems. An important one is that estimates become relatively imprecise at the end of the sample. In effect, trends are estimated as two-sided moving averages of the data, with future outcomes used to condition estimates of the current trend value. At the end of sample, where future values are not available, the filter does not have the benefit of hindsight to infer the current trend value. This means that the precision of the trend estimates deteriorates markedly right when those estimates are needed most to prepare a forecast or make judgments as to the appropriate settings of the policy instrument.

The methodology we use improves on univariate methods by using more information to condition the estimates of potential output and the NAIRU. Our approach is a version of a MV filter. The essential idea behind the MV filter is that we can profit from considering more than just the data on output. In particular, since we know that there is a link between labor input and output, it may be useful to exploit information about the degree of excess demand in the labor market. Similarly, if we observe inflation accelerating, it is more likely that there is excess demand in the product market.[8] Our methodology treats the filtering problem as a small system, where the estimates of potential output, the NAIRU, and some of the parameters of the dynamic model are determined simultaneously, allowing us to account for interactions among unemployment, output and inflation.

[7] We document this procedure later in the paper and in the appendix. See also Laxton and others (1998).

[8] This may not be true. It could be that special factors are driving inflation up, factors that have nothing to do directly with the state of domestic excess demand, such as an external energy price shock. But, all else equal, an observation that inflation is rising should lead us to give more weight to the idea that there is excess demand.

## III. The Multivariate System

In this section we describe how potential output and the NAIRU have been estimated using a MV filter which takes into account the paths of output, unemployment and inflation[9].

Table 1 presents the equations of the MV filter.[10] In equation (1), $y_t$ is the log of observed output, $\bar{y}_t$ the potential output, and $ygap_t$ is the output gap. The scaling in equation 1 is to convert the gap units to units that are approximately measured as percent of potential output. Equation 1 is an identity; it simply defines what we mean by $ygap_t$, or more precisely the relationship between $y_t$, $\bar{y}_t$, and $ygap_t$. The concept of potential output that we use here is tailored to the purpose of the model—to facilitate forecasts and policy decisions in a central bank attempting to respect an inflation target. It represents the amount of output that can be produced under current conditions without generating pressures for inflation to rise or fall.

In equation (2), $u_t$ is the unemployment rate, $\bar{u}_t$ is the level of the NAIRU and $ugap_t$ is the unemployment gap. Again this is an identity defining a relationship among these variables. Note that we have defined the unemployment gap such that positive values mean excess demand for labor, implying an expected positive correlation with the output gap measure and positive coefficients in equations linking these measures. It should be clear from the term NAIRU, that we are thinking of "equilibrium" in the labor market using the perspective of the effect on inflation.[11]

[9] The program that implements the MV filter has been written in GAUSS and can be obtained by contacting one of the authors.

[10] The equations that are presented here are those used for the estimation of the potential output and the NAIRU in a macroeconomic model of the Czech Republic. See Chapter III of Coats, Laxton, and Rose (2003).

[11] For more discussion on this point see Coats, Laxton, and Rose (2003).

Table 1. Model Equations

| | |
|---|---|
| $y_t = \bar{y}_t + ygap_t / 100$ | **( 1 )** |
| $u_t = \bar{u}_t - ugap_t$ | **( 2 )** |
| $\pi_t = a_0\left[\pi 4_t^{MexE} + 100 * \Delta_4 lz_t^{eq}\right] + a_1 E_t(\pi 4_t) + (1 - a_0 - a_1)\pi_{t-1} + a_2 ygap_{t-1} + \varepsilon_t^{\pi}$ | **( 3 )** |
| $\bar{y}_t = \bar{y}_{t-1} + \mu_{t-1} - b_0 \Delta \bar{u}_t + \varepsilon_t^{\bar{y}}$ | **( 4 )** |
| $\mu_t = c_0 \mu_{t-1} + (1 - c_0)\bar{\mu} + \varepsilon_t^{\mu}$ | **( 5 )** |
| $ygap_t = d_0 ygap_{t-1} - d_1 rr12gap_t - d_2 rr4gap_t - d_3 gr\_rrgap_t - d_4 lzgap_t + \varepsilon_t^{ygap}$ | **( 6 )** |
| $\bar{u}_t = \bar{u}_{t-1} + \varepsilon_t^{\bar{u}}$ | **( 7 )** |
| $ugap_t = f_0 ugap_{t-1} + f_1 ygap_t + \varepsilon_t^{ugap}$ | **( 8 )** |

Equation (3) is the model's (reduced-form) equation for the dynamics of inflation. Variable $\pi_t$ is the measure of inflation we choose to model, based on the "core CPI" (CPI excluding energy prices, essentially petrol, and all regulated prices).[12] It is measured as a percentage change, quarter-over-quarter, at annual rates. We choose to model this core measure of inflation because the logic of the link between excess demand and inflation does not apply in the same way to the parts of the CPI that change for exogenous reasons. It is important to note that the choice of an inflation measure to explain in the model's Phillips curve does not limit, in any way, the inflation concept that the policymaker will choose to try to control.

The equation describes inflation using an expectations-augmented Phillips curve, with some special features to reflect the local economy. The influence of excess demand is captured through the one-quarter-lagged value of the output gap. The lag reflects our judgment that there is delay in the response of prices to economic conditions.

Expected inflation enters through the term $E(\cdot)$, which is defined in the model as a weighted combination of a backward-looking component (the one-quarter lag of the four-quarter rate of change of the overall CPI) and a forward-looking component (the predicted

[12] The data were constructed by the Czech National Bank (CNB) based on information provided by the Statistical Office.

value of overall CPI inflation over the next four quarters). Using the overall CPI in the expectations model provides an explicit link between changes in regulated and energy prices and pressures on the rate of inflation for market prices. One can think of this as capturing, at least in part, the effect of pressures on costs coming from the influence on wage bargaining.

Import price effects enter through the first term. The variable $\pi 4_t^{MexE}$ is the four-quarter rate of change in import prices, excluding energy, and $lz_t^{eq}$ is a proxy for the log of the equilibrium real exchange rate, defined such that an increase is an appreciation.[13] The operator $\Delta_4$ denotes a four-quarter difference. The real exchange rate term removes the effect of a trend in the equilibrium real exchange rate on imported goods prices.[14]

Finally, in addition to the lags operating through expectations, we have a direct effect of lagged core inflation. This represents intrinsic dynamics—the effects of things like contractual lags or other costs of adjustment that lead to stickiness in prices, even in the absence of expectations lags.

Equation (6) describes the dynamics of the output gap. The gap is formulated to evolve according to a first-order autoregressive process, reverting to zero in a steady state, but allowing for effects from interest rates and exchange rates. There are four additional gap terms used for this: $rr12gap$ is the deviation of the three-year (12-quarter) real interest rate from its equilibrium trend level, $rr4gap$ is the deviation of the one-year (4-quarter) real interest rate from its trend level, $gr\_rrgap$ is the deviation of the German real interest rate from its equilibrium trend level, $lz\_gap$ is the deviation of the real exchange rate from its trend level, where this is defined such that a positive value means that the real exchange rate is appreciated relative to its equilibrium level.

The German real interest rate gap is included to capture the effect of foreign financing of domestic investment. In the interest rate part, we allowed for two channels, reflecting a judgment about what rates are important in influencing aggregate demand.

The other term in equation (6) allows for the influence of the exchange rate on aggregate demand. We refer to the combined effects of the interest rate and the exchange rate terms in this equation as our index of real monetary conditions.

Equation (8) is an Okun equation that links the movements in unemployment to those in output gap. Some degree of persistence in the dynamics of the unemployment gap is captured by the presence of the lagged values of unemployment gap.

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[13] The measure of import prices excluding energy was constructed by the CNB based on information supplied by the Statistical Office.

[14] This makes the term in square brackets equal to the domestic inflation rate in the steady state, even if the real exchange rate has a trend in it.

Equations (4), (5), and (7) describe the properties of the trends we assume for the MV filter.

Equation (4) describes the dynamics of potential output. Variable $\mu_t$ is the growth rate of potential output. In equation (5), this growth rate is specified to evolve according to a first-order, stationary autoregressive process, reverting in the long run to a fixed steady state level, $\overline{\mu}$. Our judgment is that a reasonable value for a sustainable steady-state real growth rate is 3.5 percent per annum. In the quarterly equation, this is divided by 4 to express it at a quarterly rate, giving a value for $\overline{\mu}$ in equation (5) of 0.875. In an economy experiencing large structural change, there is good reason to think that the trend growth rate will not converge quickly to the assumed steady-state rate of 3.5 percent per annum. We have set the parameter $c_0$ to 0.9, which means that in the absence of shocks, output growth would converge to within 1 percent of the steady-state rate in just over 10 years.

In the absence of changes in the NAIRU, equation (4) describes the evolution of potential output as a random walk, driven by disturbances, $\varepsilon_t^{\bar{y}}$, which are interpreted as supply shocks—shocks to total factor productivity and so on. When the NAIRU is changing, however, there is an additional dynamic effect. The operator $\Delta$ is a first (quarter) difference operator; a rising NAIRU implies a falling level of potential in this specification. The parameter $b_0$ is set at 0.6, based on the approximate share of labor income in total income, which would be the right magnitude if the production function technology were approximately Cobb-Douglas in form.

The evolution of the NAIRU is specified in equation (7) as a pure random walk driven by shocks $\varepsilon_t^{\bar{u}}$. Despite the fact that the NAIRU cannot literally follow a random walk, this represents a useful empirical assumption when the NAIRU has a tendency to drift over time in ways that are difficult to explain sensibly on the basis of variation in conventional "structural determinants."[15]

The variables $\varepsilon_t^{\pi}, \varepsilon_t^{\bar{y}}, \varepsilon_t^{\mu}, \varepsilon_t^{ygap}, \varepsilon_t^{\bar{u}}, \varepsilon_t^{ugap}$ are random variables that are assumed to be identically, independently normally distributed and to be uncorrelated.

The system of equation (1)-(8) is processed using an application of Kalman filtering; see the appendix for a formal description of the methodology. Before completing the discussion of the application of this methodology to derive measures of potential output and the NAIRU, we need to establish the methodology and results for certain input variables, and in particular the measures for the components of monetary conditions.

[15] See Boone and others (forthcoming) for further discussion of this point.

### A. Methodology for Prefiltering: The PC Filter

Because a number of variables are treated as exogenous in estimating potential output and the NAIRU, we need to specify values for the four contributors to real monetary conditions. The methodology used to establish these values is described in detail in the appendix. We describe, below, a simplified version. The model is called the PC filter, because it permits the imposition of certain "priors" on the properties of the measures.[16]

Consider, as an example, the problem of inferring a measure of trend equilibrium real interest rates. Calling $r_t$ the real interest rate, $\bar{r}_t$ its trend equilibrium value, and $rgap_t$ the deviation of $r_t$ from its equilibrium values, the measurement equation which links $r_t$ to the two state variables $\{\bar{r}_t, rgap_t\}$ is given by:

$$r_t = \bar{r}_t + rgap_t \tag{9}$$

The transition equations which summarize the dynamics of the state variables are:

$$\bar{r}_t = \bar{r}_{t-1} + \varepsilon_t^{\bar{r}} \tag{10}$$

$$rgap_t = \varepsilon_t^{rgap} \tag{11}$$

The covariance matrix of the error terms in equations (10) and (11) is:

[16] See Laxton and others (1998), in particular Box 7, pages 30-31. It can be shown that the equations we present can be derived from minimizing the value of the following objective function:

$$\sum_{t=1}^{T}(r_t - \bar{r}_t)^2 + \lambda\sum_{t=2}^{T}\left[(\bar{r}_t - \bar{r}_{t-1}) - \Delta\bar{r}^*\right],$$

where $\Delta\bar{r}_t^*$ is the steady-state change in the equilibrium value, which is set to zero here. Thus, we trade off fitting the data (first term) against penalizing the change in the trend estimate, with a relative weight, $\lambda$, on the latter. The higher is $\lambda$, the smoother will be the estimates of $\bar{r}_t$.

$$Q = \begin{pmatrix} \sigma^2_{\varepsilon^{\bar{r}}} & 0 \\ 0 & \sigma^2_{\varepsilon^{\text{rgap}}} \end{pmatrix}\text{, or equivalently}$$

$$Q = \begin{pmatrix} \dfrac{\sigma^2_{\varepsilon^{\bar{r}}}}{\sigma^2_{\varepsilon^{\text{rgap}}}} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\lambda} & 0 \\ 0 & 1 \end{pmatrix} \tag{12}$$

The measurement equation is an identity that states that the variable $r_t$ is the sum of an equilibrium value and a gap. The first transition equation says that the equilibrium values of $r_t$ follow a random walk.[17] The second transition equation says that $r_t$ deviates from its equilibrium level by a random disturbance.[18] The error terms $\{\varepsilon_t^{\bar{r}}, \varepsilon_t^{rgap}\}$ are assumed to be identically, independently and normally distributed.

***Assumptions on the Initial State Vector***

To apply the Kalman filter to the system (9)-(12) we need to make some assumptions on the initial values of the state variables, their covariance matrix, and the value of the parameter $\lambda$. The parameter $\lambda$ has been fixed to 25 in all applications. It is easiest to think of the intuition for this in terms of the standard deviations. We judge that if a "large"

[17] In this case, we do not allow for any permanent trend change in the equilibrium value. The real interest rate is presumed to be constant in a steady state, and movements in the sample are interpreted, statistically, as the result of a sequence of random shocks. For the real exchange rate, we do allow for a trend change in a steady state. The model for this case is more complicated. See the appendix for details.

[18] It may seem odd that we assume, statistically, that the gap measure has no persistence, when our economic stories always feature persistence in macro cycles. It would be interesting to investigate the sensitivity of the results to this particular assumption. However, for now we stick to the simplest possible specification, for two reasons. First, the two equations interact to give reasonable persistence properties in gap measures, so we do not have to introduce a more complicated statistical assumption to get reasonable output. Second, it has been found (see Boone and others, forthcoming) that the system we use has reasonable updating properties, that is, as new data arrive, the estimates from the filter change in a sensible manner. We do not know what would happen, in this regard, in a more complex model.

deviation for the trend is 1, say, then the corresponding measure in gap terms would have a large value at 5. This assumption has been found useful in applications elsewhere.[19]

The initial value of $\bar{r}_t$ is set to the value of the first observation of $r_t$ (the initial gap is set to zero). The initial covariance matrix of the state variables is diagonal with each variance set at 10. This high value denotes the degree of uncertainty on the initial values of the state vector; assuming a diffuse prior is a standard procedure.[20]

The results are shown in Figures 1 – 5. In Figure 1, we show the results for the German 90-day real interest rate. There is not much movement, and the value at the end of the sample is just over 2.5 percent per annum. In Figure 2, we have results for the equilibrium real exchange rate, in log form. Note that there is a clear trend in the equilibrium rate. For this application of the PC filter, we set the trend real appreciation to the historical mean, which is 1.26 percent per annum over this sample. The main story of the cyclical variation around the trend line is that there was what appears, ex post, to have been an unsustainable appreciation in the period leading up to the exchange crisis in 1997. The abrupt depreciation at that time removed the disequilibrium, and according to these measures the real exchange rate remained systematically below its equilibrium level. However, more recently our measure shows that by 2000, the actual rate was not so far from its equilibrium.

Figures 3, 4, and 5 show the results across the term structure of the three domestic measures of real interest rates. Figure 3 shows the results for the real 90-day rate. According to our results, the trend increase in domestic real rates from the first part of the sample has been reversed, and the risk premium is beginning to fall. Indeed, the estimated real equilibrium rate has come down from a peak of about 5 percent to about 3 percent, at annual rates, by the end of the sample. The picture is dominated by the spike from the period of the exchange crisis. The monetary response to the developing recession is evident, as the rate passes below its equilibrium by the second quarter of 1998 and moves increasingly below for the next year.

Figure 4 shows the results for the one-year rate. Again, our results show the pattern of a rise in the equilibrium in the first part of the sample, which is then reversed. The initial rise is not as dramatic as for the 90-day rate, and the end point is about the same, at about 3 percent. Figure 5 shows a flatter profile for the three-year rate, and no strong evidence of a decline in the risk premium over recent periods.

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[19] See Laxton and others (1998), especially Box 7, pages 30-31, for a discussion of this point, and an application to measuring the NAIRU in a number of countries.

[20] The assumptions made for the case of the real exchange rate are slightly different. The main point is that we set the initial equilibrium real exchange rate variance term to zero, effectively constraining the first observation of the equilibrium real exchange rate to be very close to the actual measure in the first period.

Figures 4 and 5 show that the effects of the emergency hike in the 90-day rate in 1997 lingered even longer in the one-year and three-year rates. Thereafter, the movements in the three-year rate, which is measured from the rate for newly issued long-term loans to businesses, are similar, though larger in magnitude than those for the one-year rates.

### B. Estimates of Potential Output and the NAIRU

The estimates of the four contributors to monetary conditions are used to estimate potential output and the NAIRU by applying the Kalman filter to the system (1)-(8). The results are reported in Figures (6) and (7).

In Figure 6, we show the estimates of potential output and the NAIRU. The pattern of the potential output indicates two major phases in the sample. A short period of excess demand, 1994Q2-1996Q4, is followed by a period of excess supply, which attained its peak during the first quarter of 1999. Clearly, the increase in the domestic interest rates has contributed to the deepening recession. Another notable feature of the pattern of potential output is that between 1996Q3 and 1997Q1 major negative supply shocks have occurred, leading to a reduction in its actual growth rate.

Comparing the two panels of Figure 6 indicates that the output gap leads the unemployment gap by approximately three quarters revealing the presence of some rigidities in the labor market.

Figure 7 displays the series for the output gap, unemployment gap and year-on-year inflation. Clearly, the excess supply period that started in early 1997 following the tightening of the monetary policy drove, with some lags, inflation down to its lowest level in 1999Q3.

Figure 1: Actual and Trend Equilibrium German 90-Day Real Interest Rates

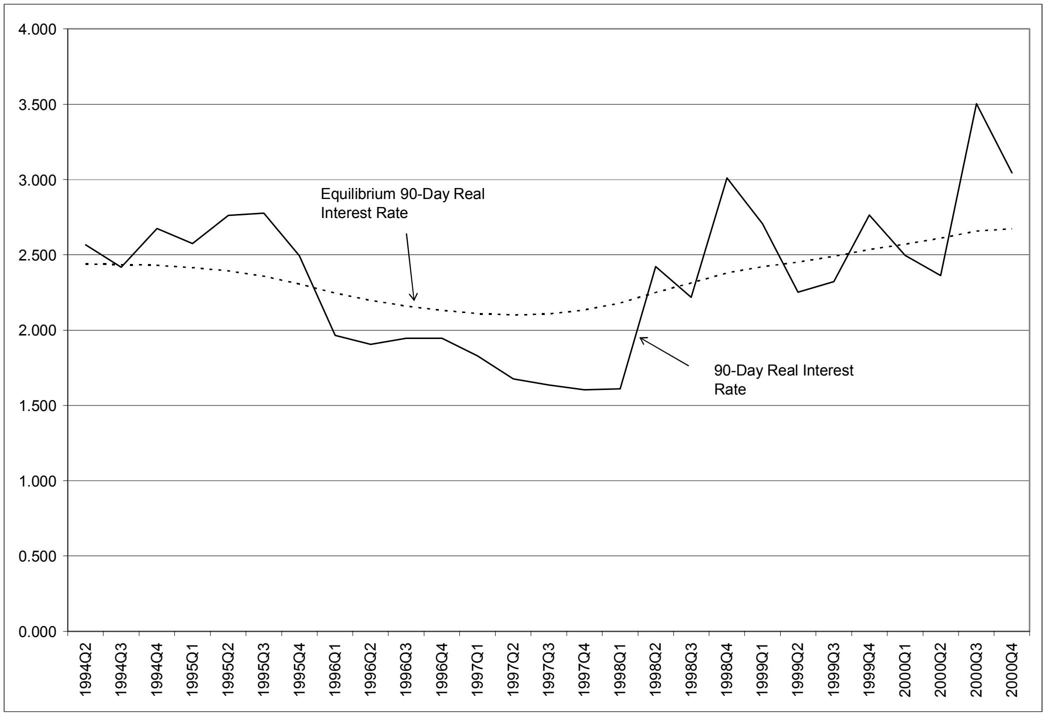



Figure 2: Actual and Trend Equilibrium Real Exchange Rates (DM/CZK)

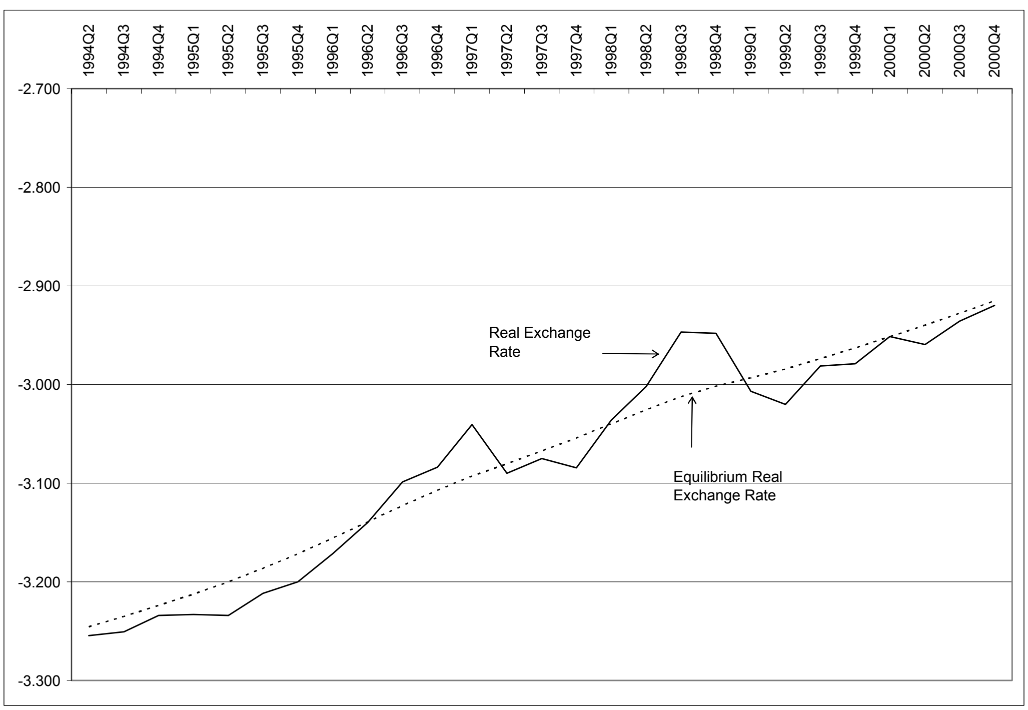


Figure 3: Actual and Trend Equilibrium Czech 90-Day Real Interest Rates

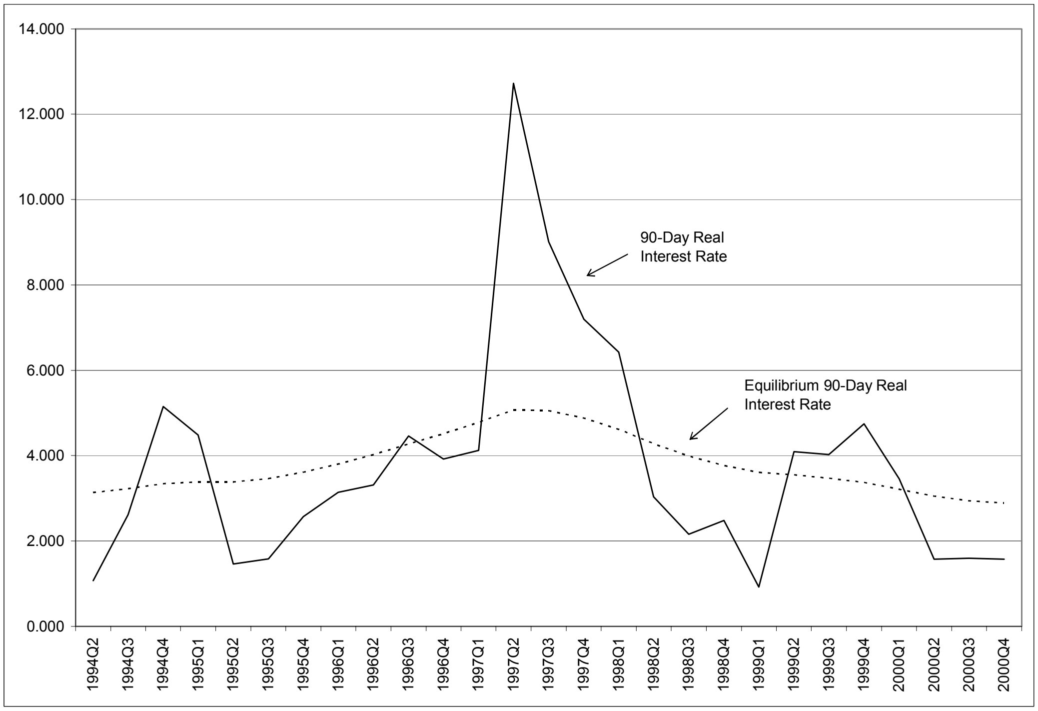



Figure 4: Actual Trend and Trend Equilibrium Czech One-Year Real Interest Rates

Figure 5: Actual and Trend Equilibrium Czech Three-Year Real Interest Rates

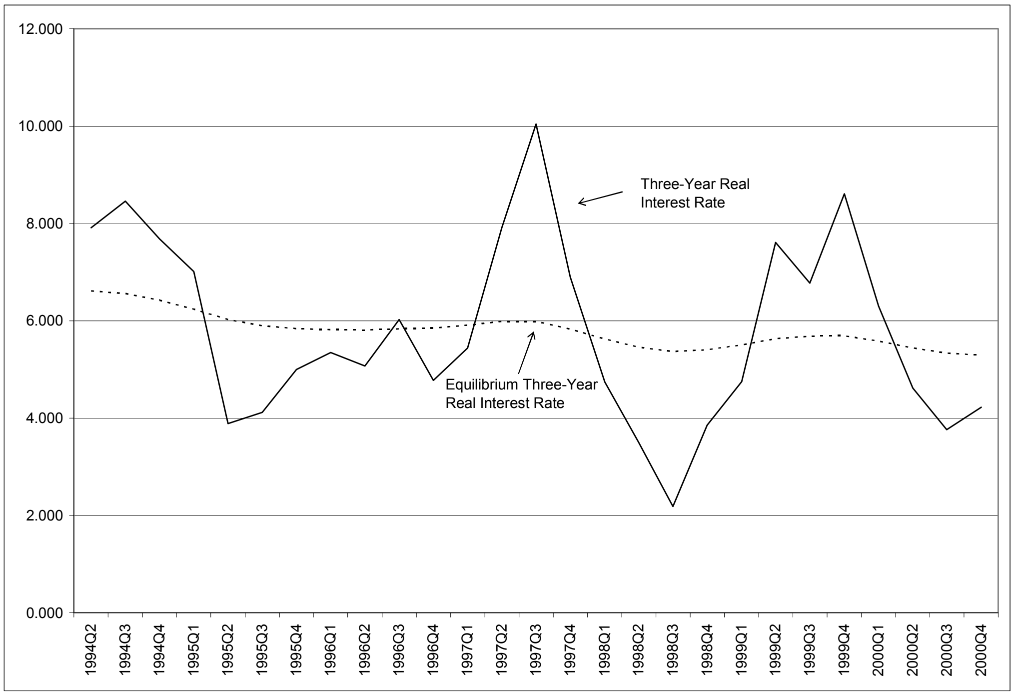
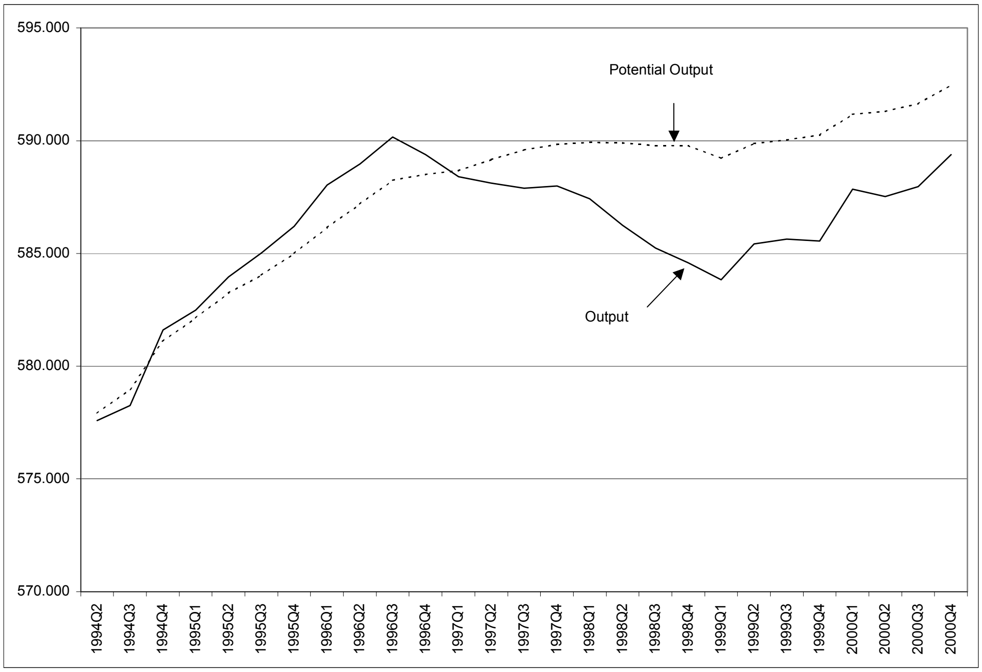


Figure 6: Estimates of Potential Output and the NAIRU



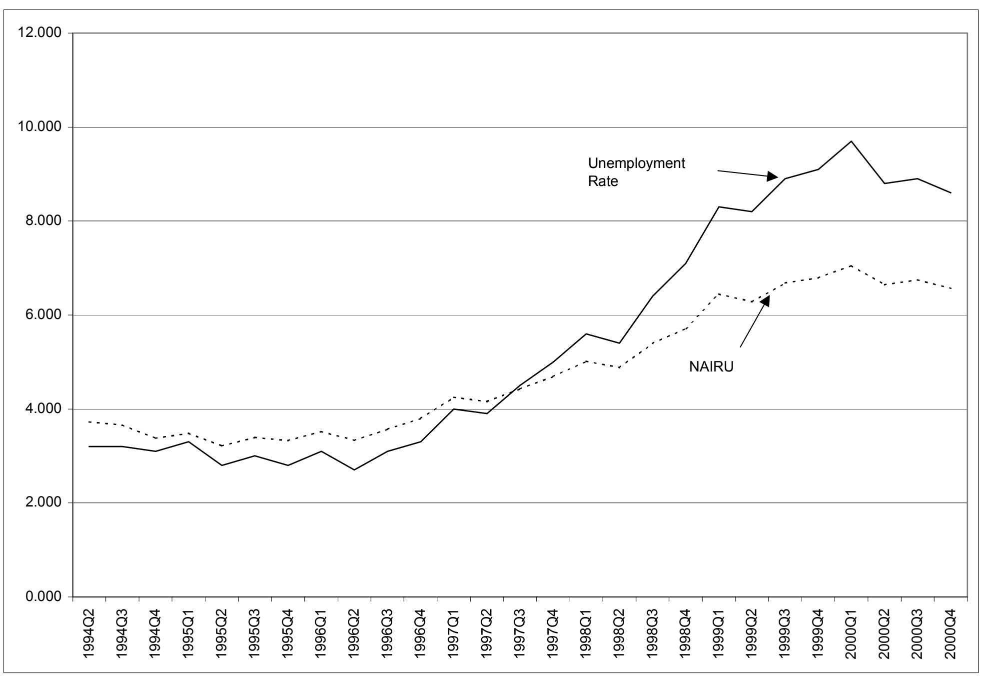



Figure 7: Output Gap, Unemployment Gap, and Inflation

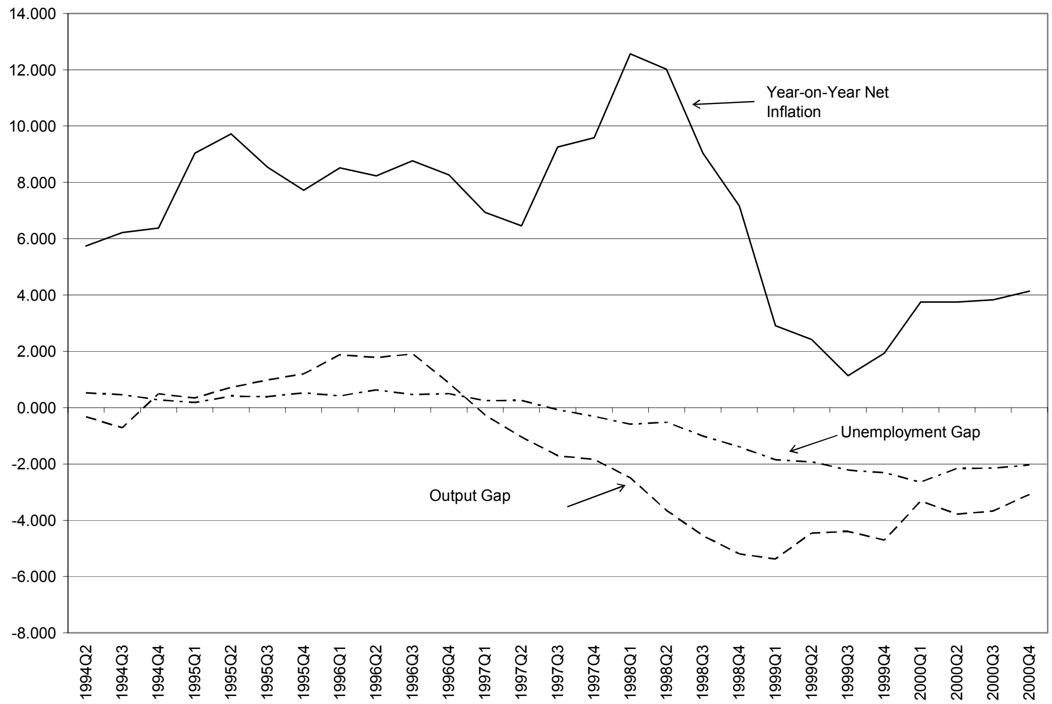
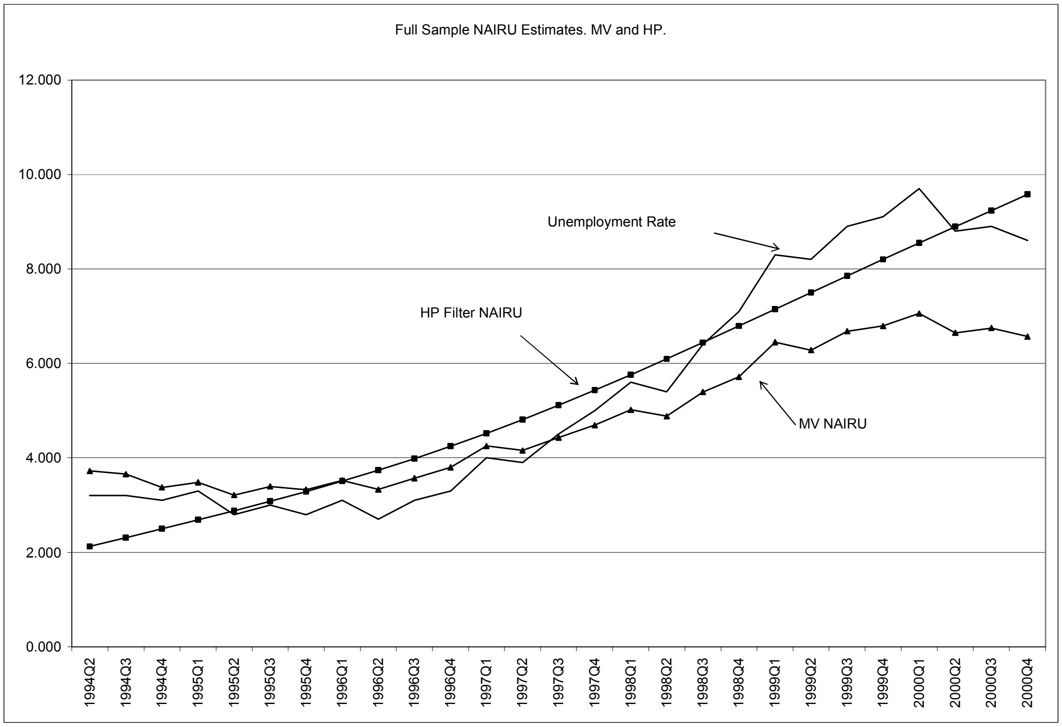


Figure 8: Comparison of MV-NAIRU and HP-NAIRU



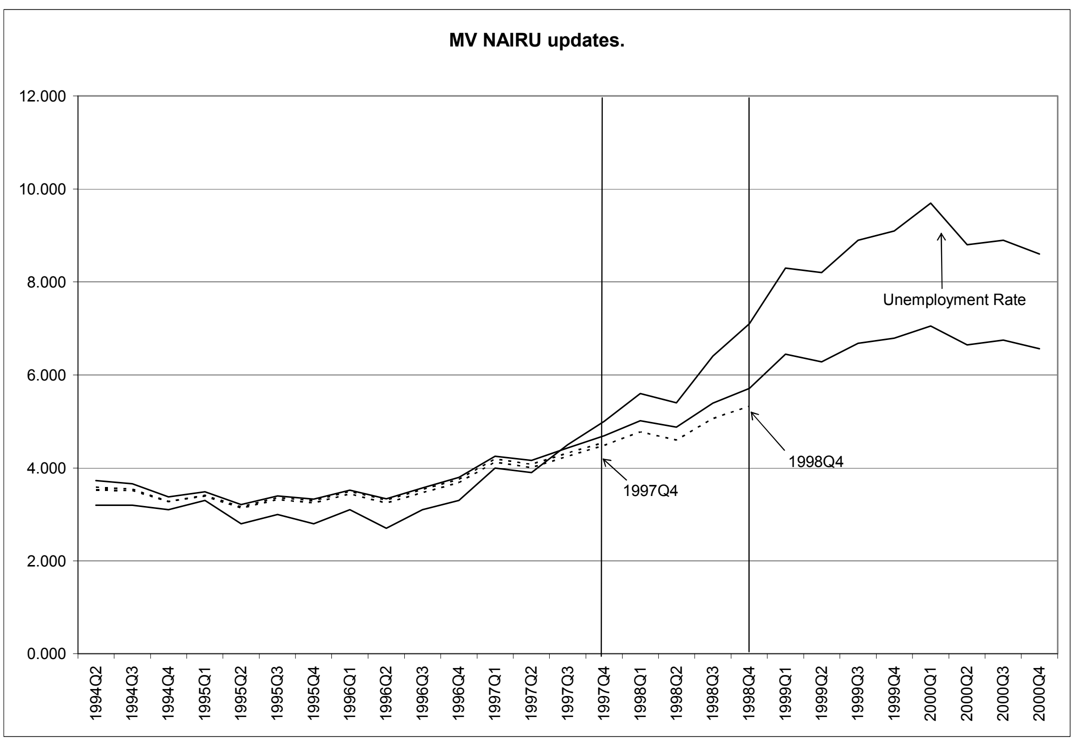



Figure 8 (Concluded). Comparison of MV-NAIRU and HP-NAIRU

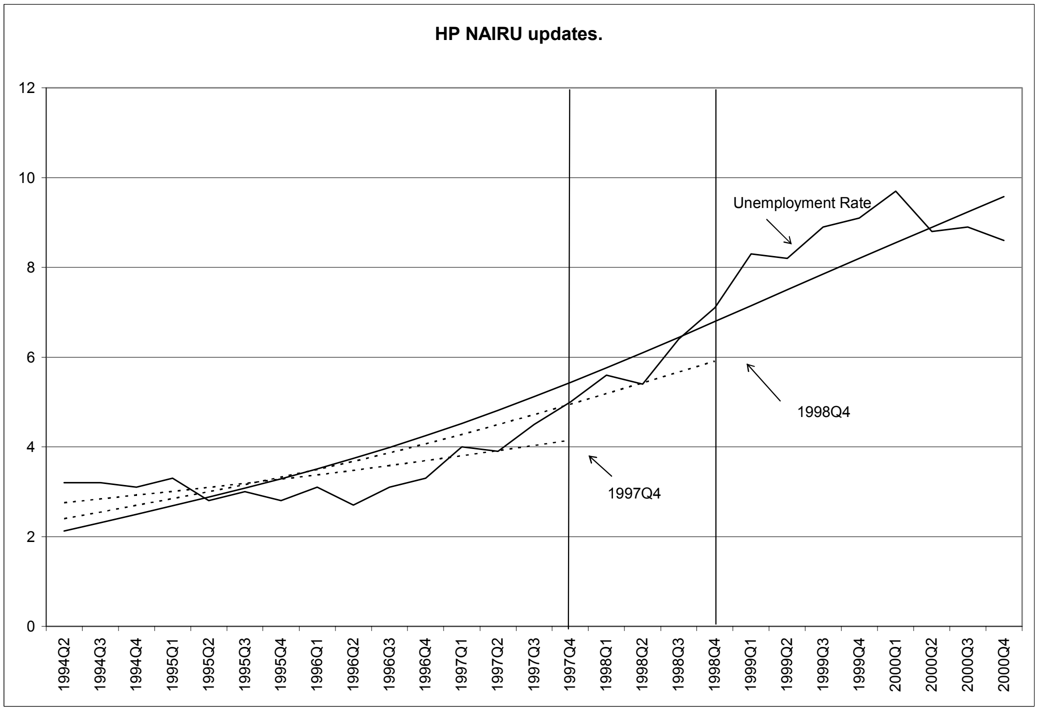



Figure 9: Comparison of MV-Potential Output and HP-Potential Output

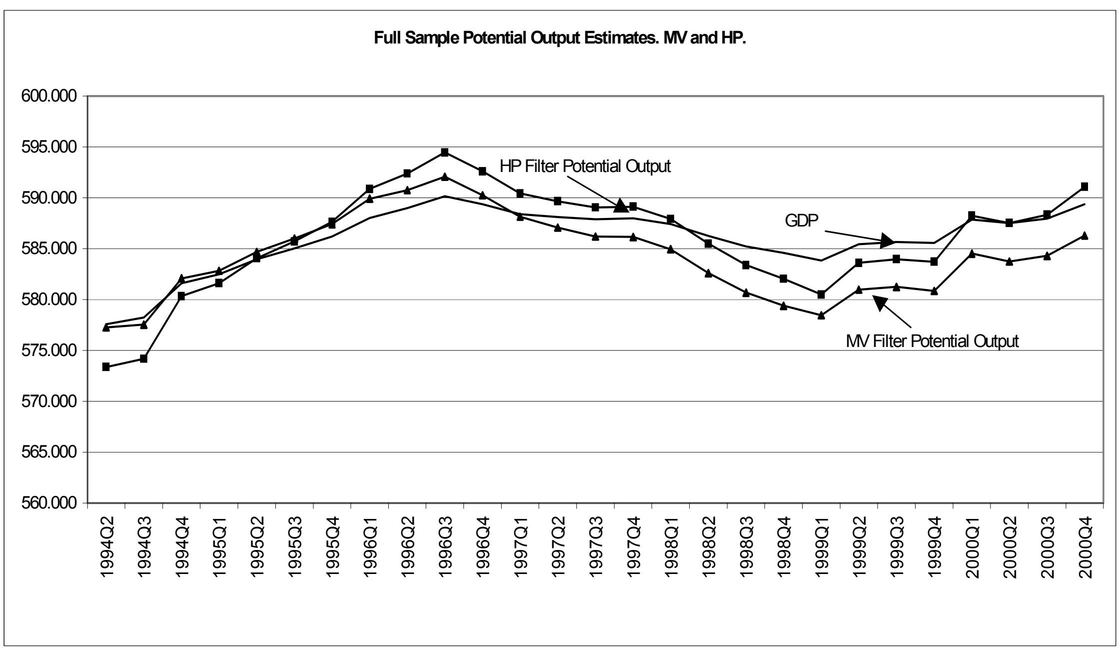
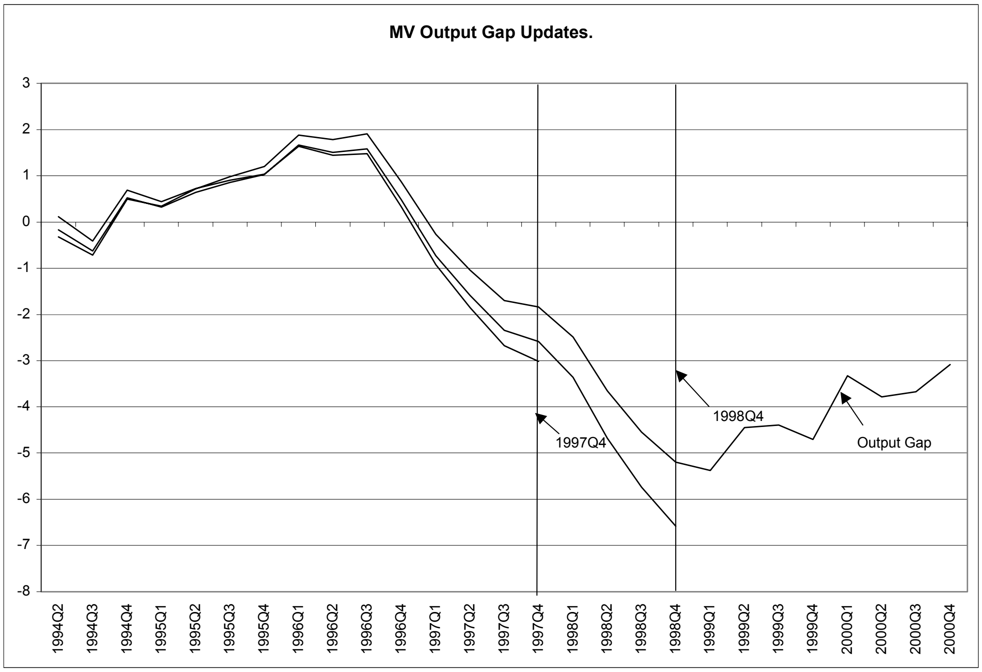
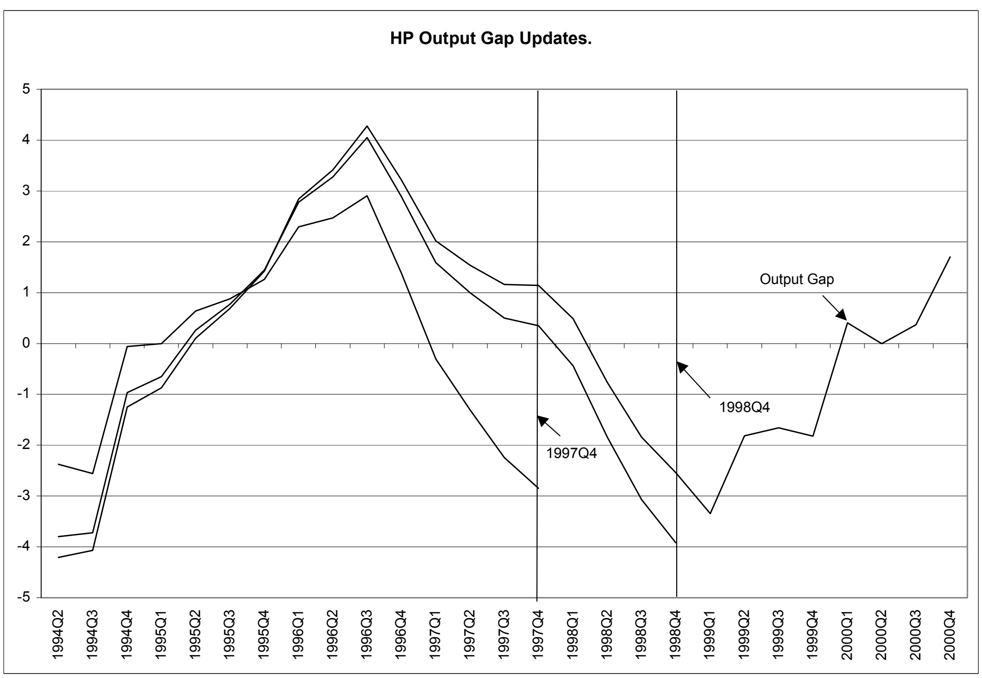


Figure 9 (Concluded). Comparison of MV-Potential Output and HP-Potential Output



## IV. Comparison with Gaps Derived from the HP Filter

In this section we compare our full-sample estimates with the results of the HP filter and perform an experiment where we look at the updating properties of the two methods during a critical period of history (1997-98).

### Comparing the Gaps

Recall that the HP filter uses only the data of the series itself to identify a trend line. The nature of the results depends a lot on the choice of the smoothing parameter. A low value will produce trend estimates that follow the data closely; a very high value will produce trend estimates that are straight lines trends. We use the standard assumption, coming from the original Hodrick-Prescott paper, setting the smoothing parameter at 1600. Harvey and Jaeger (1993) argue that this is an optimal choice for deriving estimates of potential output for the United States using the HP filter.[21]

The charts in Figure 8 compare the resulting HP estimates of the NAIRU with our MV-Filter estimates. The solid lines extending to the end of the sample are the two full sample estimates of the NAIRU. The difference between the two is dramatic. The HP estimates put a line through the data, and, in particular, the actual unemployment rate towards the end of the sample. Most of the rise in actual unemployment is identified as rise in the NAIRU, with an end-of-sample estimate of the NAIRU at well above 9 percent. Indeed, the HP estimates show the labor market in excess demand in 2000. This contrasts markedly with our results, which allocate only roughly half of the increase in unemployment to the NAIRU, and show a large measure of excess supply in 2000.

Figure 9 repeats the comparison for potential output; the results are shown in terms of the output gap that emerges from the two approaches. The same sharp contrast emerges from the estimates in the last part of the sample. The HP results show a much smaller recession, starting significantly later, and a return to excess demand by 2000. Our estimates show a deeper trough, and one that continues through the end of the sample.

We believe that the multivariate results characterize much more accurately the situation at the end of the sample and provide a better base for a forecast of inflation.

### Comparing the updating properties of the MV filter and HP filter

Figures 8 and 9 also contain the results for our "real time" illustration of the updating properties of the two methods. For both methods, we estimate the NAIRU and potential output using data up to 1997Q4; we then repeat this, adding another year of data and estimating up to 1998Q4.

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[21] The Harvey-Jaeger "optimal" argument does not necessarily carry over to an application to data for the Czech Republic, but we think that our choice is reasonable. It has been used in many application of the HP filter in many countries.

In Figure 8, the lowest dashed lines, which ends in 1997Q3, show the results for the two methods on the first sample. Note, first that the difference between the HP estimates on the short sample and the HP estimates for that same period from the full sample results are much farther apart than the two sets of estimates from the MV filter. For 1997Q4, the end-point of the short sample, the difference for the MV filter is less than 0.15 percentage points, while for the HP filter it is close to 1.30 percentage points. Now compare what happens when we add another year of data, estimating to 1998Q4. The dashed line showing the HP results has moved up sharply, about half way towards the final estimates. The MV filter estimates also rise, but by much less. The HP results are both more volatile, and from the perspective of final estimates, much less accurate than the MV results through this period.

Figure 9 shows that the same basic messages emerge from the application to output. The HP results for the short samples are indeed volatile, but their levels and the stories they tell, especially form the shortest sample, are much more like those from the MV filter during this period than they will end up being in the full sample. It may be comforting to know that had the HP approach been used in 1997, the results would have been reasonable, from an ex post perspective. However, it is hard to take too much comfort from this result, when the story is virtually revised away within a year, and totally reversed, eventually.

## V. Conclusions

This paper has developed a methodology for obtaining measures of excess demand that can facilitate discussion of monetary policy and help policy decision making. Our methodology uses a small system that accounts for the interactions among unemployment, output, and inflation to obtain, through the Kalman filter procedure, estimates of potential output, the NAIRU, and some other parameters. Our multivariate approach is less data-demanding than the "production function" approach, and more accurate and sensible in economic terms than a univariate approach based on the HP filter.

When applied to the Czech economy data, our methodology provides estimates of excess demand that, we believe, characterize much more realistically the situation at the end of the sample and provide a better base for a forecast of inflation than those of the HP filter. We have also shown by performing an experiment where we look at the updating properties of our method and the HP filter during a critical period of history, that our estimates are more accurate than the HP results because they are less subject to changes as new information arrives.

## APPENDIX

### A. The Multivariate Kalman Filter

In this appendix, we present the details of the MV filter used to provide our measures of potential output and the NAIRU. In the first section, we explain how the system of equations described in the text is transformed in order to obtain a state-space representation that allows us to apply the Kalman Filter procedure.[22] Then, we show how the same procedure can be used to obtain results for an HP filter.

#### State Space Representation of the System

The system of equations (1)-(8) from Section III (Table 1) can be represented by three measurement equations that link the current values of output, unemployment rate, and inflation rate to seven state variables $\left[\bar{y}_t, ygap_t, \mu_t, u_t, ugap_t, \pi_t, \bar{\mu}\right]$.

$$\begin{bmatrix} y_t \\ u_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \bar{y}_t \\ ygap_t \\ \mu_t \\ u_t \\ ugap_t \\ \pi_t \\ \bar{\mu} \end{bmatrix} \qquad \text{(A1)}$$

Note that owing to the presence of lagged endogenous variables, the third measurement equation has been written as an identity that states that the sixth variable is equal to the current observed values of inflation. When forecasting the next *n*-step-ahead values of inflation this allows us to take into account the errors arising from the use of predicted values.

The dynamics of the state variables are summarized by the following transition equations.

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[22] For further details on this methodology see, for example, Hamilton (1994) or Harvey (1989).

$$\begin{bmatrix} \bar{y}_t \\ ygap_t \\ \mu_t \\ \bar{u}_t \\ ugap_t \\ \pi_t \\ \bar{\mu} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & d_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_0 & 0 & 0 & 0 & (1-c_0) \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & f_1 d_0 & 0 & 0 & f_0 & 0 & 0 \\ 0 & a_2 & 0 & 0 & 0 & 0 & (1-a_0-a_1) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{y}_{t-1} \\ ygap_{t-1} \\ \mu_{t-1} \\ \bar{u}_{t-1} \\ ugap_{t-1} \\ \pi_{-1} \\ \bar{\mu} \end{bmatrix} + \begin{bmatrix} 0 \\ \kappa_{2,t} \\ 0 \\ 0 \\ \kappa_{5,t} \\ \kappa_{6,t} \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\bar{y}} - b_0 \varepsilon_t^{\bar{u}} \\ \varepsilon_t^{ygap} \\ \varepsilon_t^{\mu} \\ \varepsilon_t^{\bar{u}} \\ \varepsilon_t^{ugap} + f_1 \varepsilon_t^{ygap} \\ \varepsilon_t^{\pi} \\ 0 \end{bmatrix} \quad \text{(A2 )}$$

where

$$\kappa_{2,t} = -d_1 [e_0 rr12gap + e_1 rr4gap + e_2 gr\_rrgap] - d_2 lzgap_t$$

$$k_{5,t} = f_1 \kappa_{2,t}$$

$$\kappa_{6,t} = a_0 \left[\pi 4_t^M + 100 * \Delta_4 lz_t^{eq}\right] + a_1 E(\pi 4_t)$$

The covariance matrix of the residuals of the transition equations is as follows:

$$Q = \begin{bmatrix} \sigma^2_{\varepsilon^{\bar{y}}} + b_0^2 \sigma^2_{\varepsilon^{\bar{u}}} & 0 & 0 & -b_0 \sigma^2_{\varepsilon^{\bar{u}}} & 0 & 0 & 0 \\ 0 & \sigma^2_{\varepsilon^{ygap}} & 0 & 0 & f_1 \sigma^2_{\varepsilon^{ygap}} & 0 & 0 \\ 0 & 0 & \sigma^2_{\varepsilon^{\mu}} & 0 & 0 & 0 & 0 \\ -b_0 \sigma^2_{\varepsilon^{\bar{u}}} & 0 & 0 & \sigma^2_{\varepsilon^{\bar{u}}} & 0 & 0 & 0 \\ 0 & f_1 \sigma^2_{\varepsilon^{ygap}} & 0 & 0 & f_1^2 \sigma^2_{\varepsilon^{ygap}} + \sigma^2_{\varepsilon^{ugap}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\varepsilon^{\pi}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(A3)}$$

Once the values of the parameters have been set, and given initial values of the state variables and their corresponding covariance matrix, optimal estimates of the potential output, output gap, NAIRU and unemployment gap based on the information available at time *t* (referred to as filtered estimates) and on information available from the full sample of observations to time *T* (referred to as smoothed estimates) are obtained from the Kalman filter. The calculations are done in GAUSS. The values of the parameters are reported in table A1.[23]

[23] The parameters that are reported in table A1 have been calibrated. For more discussion, see Coats, Laxton, and Rose (2003).

Table A1. Values of the Parameters

| $a_0$ | $a_1$ | $a_2$ | $b_0$ | $c_0$ | $d_0$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ | $f_0$ | $f_1$ | $\overline{\mu}$ | $\sigma^2_{\varepsilon^{\bar{y}}}$ | $\sigma^2_{\varepsilon^{ygap}}$ | $\sigma^2_{\varepsilon^{\mu}}$ | $\sigma^2_{\varepsilon^{\bar{u}}}$ | $\sigma^2_{\varepsilon^{ugap}}$ | $\sigma^2_{\varepsilon^{\pi}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| .25 | .33 | .50 | .60 | .90 | .90 | .06 | .13 | .06 | .15 | .85 | .10 | .88 | .70 | 1 | .07 | .74 | .53 | 3 |

## B. A Special Case: The HP filter

In the past, HP filter has been widely used by policymaking institutions to measure the potential output and the NAIRU. The popularity of this univariate filter resides in its simplicity and its ability to fit quite well, at least for some countries, the historical variations of inflation when the estimated unemployment gap or output gap is included in a Phillips curve.[24]

HP filter estimates of the potential output and the NAIRU can be obtained from the Kalman filter when the state space representation is as follows. Calling $x_t$ the output or the unemployment rate, $\overline{x}_t$ the potential output or the NAIRU, and $xgap_t$ the output gap or the unemployment gap, the measurement equation is given by,

$$x_t = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \overline{x}_t \\ \overline{x}_{t-1} \\ xgap_t \end{bmatrix} \tag{A4}$$

with the transition equations, which summarize the dynamics of the state variables,

$$\begin{bmatrix} \overline{x}_t \\ \overline{x}_{t-1} \\ xgap_t \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \overline{x}_{t-1} \\ \overline{x}_{t-2} \\ xgap_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\overline{x}} \\ 0 \\ \varepsilon_t^{xgap} \end{bmatrix} \tag{A5}$$

The variance covariance matrix of the two shocks is:

[24] See Boone and others (forthcoming) for a discussion on this point. See Coe and McDermott 1997), Bank of England (1999), and Cozier and Wilkinson (1990) for examples.

$$Q = \begin{pmatrix} \sigma^2_{\varepsilon^{\bar{x}}} & 0 \\ 0 & \sigma^2_{\varepsilon^{xgap}} \end{pmatrix}, \text{ or equivalently}$$

$$Q = \begin{pmatrix} \dfrac{\sigma^2_{\varepsilon^{\bar{x}}}}{\sigma^2_{\varepsilon^{xgap}}} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\lambda} & 0 \\ 0 & 1 \end{pmatrix} \quad \text{( A6 )}$$

The smoothed estimates obtained from the Kalman filter correspond to the HP filter estimates. The degree of volatility of the estimates depends on the value of the smoothness parameter $\lambda$. The higher $\lambda$ the less volatile the trend, as $\lambda$ tends to infinity (zero) the trend tends to be deterministic (highly volatile). The value of this parameter determines how much the trend should fit the data. More specifically it determines the weight given to the past and future observations relative to the current observations. Small values (high values) of $\lambda$ correspond to small (high) weight on the past and future observations.

### C. A Methodology for Prefiltering: The PC filter

We do not use the HP filter in our work, except for illustrative purposes. For applications where a univariate approach is judged appropriate, we use a filter called the Prior-Consistent (PC) filter.

**State-Space Representation**

Calling $x_t$ the variable we wish to filter, $\bar{x}_t$ the equilibrium values of $x_t$, $\bar{\bar{x}}_t$ the growth rate of $x_t$, and $xgap_t$ the deviation of $x_t$ to its equilibrium values, the measurement equation which links $x_t$ to the three state variables $\{\bar{x}_t, \bar{\bar{x}}_t, xgap_t\}$ is given by:

$$x_t = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{x}_t \\ \bar{\bar{x}}_t \\ xgap_t \end{bmatrix}. \quad \text{(A7)}$$

The transition equations which summarize the dynamics of the state variables are,

$$\begin{bmatrix} \bar{x}_t \\ \bar{\bar{x}}_t \\ xgap_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_{t-1} \\ \bar{\bar{x}}_{t-1} \\ xgap_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\bar{x}} \\ 0 \\ \varepsilon_t^{xgap} \end{bmatrix} \quad \text{(A8)}$$

The Matrix of variance covariance of the error term in (A8) is

$$Q = \begin{pmatrix} \sigma^2_{\varepsilon^{\bar{x}}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\varepsilon^{xgap}} \end{pmatrix}, \text{ or equivalently}$$

$$Q = \begin{pmatrix} \dfrac{\sigma^2_{\varepsilon^{\bar{x}}}}{\sigma^2_{\varepsilon^{xgap}}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\lambda} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{A9}$$

The measurement equation is an identity that states that, the variable $x_t$ is the sum of an equilibrium value and a gap. The first transition equation states that the equilibrium values of $x_t$ follow a random walk plus drift. The drift term, $\bar{\bar{x}}_t$, is assumed to be constant as described in the second equation of (A9). When this constant term is assumed to be equal to zero, as it is the case for most applications, the main exception being the real exchange rate, the number of state variables reduces to two. The last equation of (A9) states that $x_t$ deviates from its equilibrium level according to a white noise. The error terms $\left\{\varepsilon_t^{\bar{x}}, \varepsilon_t^{xgap}\right\}$ are assumed to be identically, independently and normally distributed.

**Assumptions on the Initial State Vector**

In order to apply the Kalman filter to the system (A8)-(A9) we need to make some assumptions on the initial values of the state variables, their matrix of variance covariance, and the value of the parameter $\lambda$.

The parameter $\lambda$ has been fixed to 25 for all the values of $x$. It is easiest to think of the intuition for this prior in terms of the standard deviations. We judge that if a “large” deviation for the trend is 1, say, then the corresponding measure in gap terms would have a large value at 5. This assumption has been found useful in applications elsewhere.[25]

[25] See Laxton and others (1998), especially Box 7, pages 30-31.

The initial value of $\bar{x}_t$ is set to the value of the first observation of $x_t$ (the initial gap is set to zero). In applications to reduced systems (two state variables), the initial covariance matrix of the state variables is diagonal with each element of the diagonal set at 10. This high value denotes the degree of uncertainty on the initial values of the state vector. Assuming a diffuse prior is a standard procedure. In expanded (three state variables) systems, the mean of the growth rate is set at a calculated historical average, or some number set by judgment in the light of the historical value. The initial variance of this variable is fixed at zero. In this case, we also set the initial variance of $\bar{x}_t$ at zero. The zero initial variance of the growth rate, taken with the other assumptions on its dynamics, allow us to treat $\bar{\bar{x}}_t$ as a constant term during the prediction-updating process of the Kalman filter. The zero initial variance on $\bar{x}_t$ reduces the uncertainty due to its initial value in the filtering process. The filtered and smoothed initial equilibrium levels will be close to the first observation of $x_t$ in this case.

## References


Bank of England, 1999, Economic Models at the Bank of England, pp. 77-156.

Boone, Laurence, Douglas M. Laxton, Michel Juillard and Papa M'B. P. N'Diaye, "How Well Do Alternative Time Varying Parameter Models of the NAIRU Help Policymakers Forecast Unemployment and Inflation in the OECD Countries?" Forthcoming IMF Working Paper.

Coats, Warren, Douglas M. Laxton, and David Rose, 2003, "The Czech National Bank. Forecasting and Policy Analysis System," Czech National Bank.

Coe, David T. and C. John McDermott, 1997, "Does the Gap Model Work in Asia?" IMF Staff Papers, vol. 44, No. 1.

Cozier, Barry and Gordon Wilkinson, 1990, "How Large Are the Costs of Disinflation in Canada?" Bank of Canada Working Paper No. 90-6.

Hamilton, James D., 1994, *Time Series Analysis*, New Jersey, Princeton University Press.

Harvey, Andrew C., 1989, *Forecasting and Structural Time Series Models and the Kalman Filter*, Cambridge University Press.

Harvey, Andrew C. and A. Jaeger, 1993, "Detrending, Stylized Facts, and The Business Cycle," Journal of Applied Econometrics, Vol. 8, pp. 231-247.

Hodrick, Robert J., and Edward C. Prescott, 1980, "Post-War U.S. Business Cycles: An Empirical Investigation," Discussion Paper No. 451, Evanston, Center for Mathematical Studies in Economics and Management Science, Northwestern University.

Laxton, Douglas M., Peter Isard, Hamid Faruquee, Eswar Prasad and Bart Turtelboom, 1998, *Multimod Mark III. The Core Dynamic and Steady-State Models*, IMF Occasional Paper No. 164, pp. 30.

Laxton, Douglas and Robert Tetlow, 1992, "A Simple Multivariate Filter for the Measurement of Potential Output," Technical Report No. 59, Ottawa, Bank of Canada.

OECD, 2000, "The Concept, Policy Use and Measurement of Structural Unemployment: Estimating a Time Varying NAIRU Across 21 OECD Countries," Economics Department Working Paper No. 250, 2000.