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Rational Speculation, Financial Crises, and Optimal Policy Responses

Jay Surti

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Rational Speculation, Financial Crises, and Optimal Policy Responses[1]

Prepared by Jay Surti

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Abstract

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The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

This paper develops a theory of the onset of financial crises by solving for the optimal trading strategies of speculators in financial markets, in a model where each speculator tries to coordinate her trades with the market's by observing the decisions of other speculators, while simultaneously trying to preempt the market. The interaction and resolution of these two conflicting incentives are analyzed under alternate central bank policy regimes. Our model explains how imperfect information structures prevent traders from exploiting profitable opportunities and suggests how large traders help alleviate this problem by undertaking risky arbitrage early in the investment process, in return for higher profits, if successful. The central bank's defense strategy is a parameter of this model. We compare the likelihood of a crisis under alternate defense strategies and show that credible monetary authorities can provide a better defense of exchange rate regimes against adverse shocks by not disclosing their commitment value to the market.

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Author's E-Mail Address: jsurti@imf.org

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Contents Page

I. Rational Speculation and Financial Crises

Economists often explain financial market crises as arising from a particular coordination of actions by traders or speculators, which is rationalized ex post by the market outcome. For example, it is considered optimal to have sold pounds sterling for dollars prior to September 16, 1992 because the pound was devalued that day. Similarly, it is considered optimal to withdraw deposits from an illiquid bank prior to a run, or to short a bubble asset prior to a (price) crash. The common link in these examples is the observation that the outcome (crisis) at the aggregate level that rationalizes individual actions is, in fact, generated by the latter, thereby presenting the possibility of another equally rationalizable outcome: if there are no speculative attacks or runs, there is no crisis, thereby rendering as rational, the decision not to attack. [2]

The attraction of such models lies in their apparent ability to provide an avenue through which a sudden shift in market sentiment appears rationalizable, without any visible change in the fundamentals. Unfortunately, however, these models fail to explain why sentiments shift without introducing extraneous elements that are usually impossible to identify empirically.[3] Specifically, there is no dynamic process of endogenous information revelation or transmission, either through trading or price changes, that can convey the rapid changes in expectations that seem to precede most financial crises.

In this paper, we argue that a dynamic model that computes the optimal trading strategies in these asset markets is required to provide such explanatory power. We show that analyzing optimal trading strategies in such markets reduces to solving for the optimal moment for a trader to *time* the market (e.g., when to short a bubble asset, when to sell currency futures, when to withdraw from a bank), in an environment where all other traders are doing the same thing simultaneously. Formally, the optimal trading strategies turn out to be solutions to an optimal stopping problem in a strategic environment.

In several economic applications, the resolution of optimal stopping problems is driven by a well-defined preference over positions in the decision making queue. In patent races, for example, there is an obvious advantage to being first with a new innovation, which generates strong incentives for firms to preempt each other, thereby yielding overinvestment in research and development, relative to the optimum. In contrast, coordination problems such as investing in capacity expansion during a recession, generate an opposite preference: an individual firm

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[2] Classic examples of macro-finance models with multiple equilibria include the papers of Bryant (1980) and Diamond-Dybvig (1983) for bank runs, Obstfeld (1994, 1996) for currency attacks of the 1992 ERM variant, and Tornell and Sachs (1999) for the 1997 Asian debt crises. Most of these papers utilize the canonical static coordination game model of Cooper and John (1988).

[3] Such as sunspots and behavioral rules related to sunspot variables that follow cycles.

finds it optimal to invest after a sufficient amount of investment has already been undertaken by other firms in the economy. This late-mover advantage generates inefficient *waiting games* among firms, thereby often lengthening a recession.[4] Problems where payoffs to the agents can be ordered in a one-to-one correspondence with the order of their moves have received exhaustive analytical attention, and consequently, the nature and resolution of the optimal stopping problem is reasonably well understood.

In many financial markets, however, agents who act earlier also receive a higher payoff (conditional on successful coordination of asset trades). Here, a first-mover advantage coexists with a coordination problem. In this case, solving the optimal stopping problem within the strategic setting of a market may be quite cumbersome, owing to the presence of these two conflicting incentives.

As an example, consider the perspective of a large speculator (like George Soros) in mid-1992 when the pound sterling was weak (and overvalued) against the U.S. dollar. He knows that if—by selling a sufficiently large quantity of pound futures—he can move the market against the overvalued pound, he could net a sizable profit. If, however, the market does not move (quickly) behind him, he will lose the margin he must put up to speculate. While Soros is a large trader in the sense that his speculative trade changes the net trade in pound futures in a manner *clearly visible to the market*, his speculative trade, alone, is generally insufficient to imply a collapse of the pound out of the exchange rate mechanism (ERM) band; that is, he needs the market to replicate his actions. Hence, financial market arbitrage possesses elements of a standard coordination game.

In a dynamic market setting, however, there are other payoff-relevant features that move us far beyond the standard coordination game analysis. To see this, consider an institutional investor holding pounds because of an interest rate differential. When the pound is overvalued, holding on to it is akin to a gamble: if the market moves, a devaluation could occur before the investor can sell. The result of this *market shutdown* (devaluation) is a substantial loss of asset value. It is this fear of being preempted by a devaluation that generates an incentive to preempt the market instead. On the other hand, short-sale constraints of individual traders simultaneously generate incentives to wait and observe the way the market is moving. It is an interesting and unresolved question how these incentives play themselves out, and, in particular, whether, when aggregated, they imply anything interesting for speculative behavior and sentiment swings in asset markets.

While the motivating example is speculation in currency futures, similar intermeshed incentives coexist in any financial market where large swings in demand can induce a crash in the asset price. In bank runs, for example, depositors prefer not to withdraw beyond liquidity needs in the absence of a panic, in order to continue earning interest. In the event of a run, however, there is

[4] See Chamley and Gale (1994) and Gale (1995) for the canonical dynamic models.

much greater incentive to withdraw at the earliest possible opportunity, so as to minimize the likelihood of losing one's deposits if the bank is left insolvent.

Formally, the analysis of these problems falls outside the scope of the existing set of coordination game models owing to the coexistence of strategic complementarities and substitutabilities in agent-payoff functions. In the investment-in-a-recession problem, the optimal time to invest is with (or just after) most other agents, and the greater the aggregate investment, the higher is the firm-specific payoff. Financial market arbitrage, on the other hand, takes place in a setting where the optimal time to speculate is before the rest of the traders. Moreover, once speculation exceeds a critical level, each agent prefers strictly lower aggregate volumes of transactions. This first section of the paper presents a general framework that simultaneously captures both these incentives within a single model, designed to analyze and solve for the strategic equilibrium behavior of traders.

A major weakness shared by most coordination (game) models of economic crises is their inability to link equilibrium outcomes to economic fundamentals or policy regimes.[5] In our leading example, we can ask whether the presence of large traders accentuates the likelihood of a speculative attack. Another interesting question is what defense strategies a central bank may utilize to defend an existing regime such as a currency peg. For example, should the central bank maintain complete transparency regarding its commitment value to the peg (e.g., as measured in terms of the amount of foreign exchange it is willing to sacrifice), or would it be better off by keeping this information private? Specifically, in what way may fundamentals and policy transparency combine to (de)stabilize the market?

A major strength of our approach is that it provides a way to compare the likelihood of crises under alternative policy regimes and distributions of liquidity. Specifically, we characterize optimal strategies of traders under alternate parametric environments, where the parameters are policy variables and liquidity distributions. This approach provides valuable insights about the desirability of transparency in central bank operations in the foreign exchange market.

In a market where all payoff-relevant parameters (including, for example, market liquidity and central bank reserves) are publicly known, all agents attempt to preempt the market. Depending on the market in question, this would entail immediate withdrawal of deposits (bank run), or shorting a currency at the first available opportunity (foreign exchange speculation). This means that in the resulting unique equilibrium, a crisis will occur immediately if fundamentals permit a price crash.

On the other hand, if some of these parameters are not known ex ante to the traders, then the distribution of liquidity within the market becomes a key factor in determining the likelihood of a crisis. Specifically, it is shown that in the unique equilibrium, given any history of trades, only agents with liquidity above a certain threshold are willing to risk speculation; the *largest traders*

[5] Chamley (2003) is an exception and is discussed in Section IV.B.

at any point in time, have the greatest incentive to speculate immediately rather than wait for more information. This information, in turn, is generated through trading itself, and, hence, it is also the large traders who generate the information required to push the speculation process forward. The absence of large traders (such as in a homogeneous market) thwarts further speculation in equilibrium because if no one speculates, the market infers that the likelihood of successful speculation is low because of insufficient liquidity. In a similar vein, if the central bank's commitment to an exchange parity is not perfectly known by the market, large traders can signal their private information about a weak commitment more effectively by taking larger positions. Conversely, lack of speculation by these traders signals to the rest of the market that fundamentals are likely to be sound and, therefore, that the commitment of the central bank to the regime is strong. [6, 7]

This result has important implications for informational aspects of a central bank's defense strategy regarding an exchange rate regime. In markets where payoff-relevant information is close to perfect, central bank vulnerability to a speculative attack will be exploited by the market very quickly. On the other hand, secrecy clauses or partial information obfuscation of payoff-relevant parameters (e.g., the reserves commitment to the peg) can, by generating aggregate uncertainty, infuse caution into speculator behavior, which may postpone the speculative attack. Indeed, sometimes the mere uncertainty about how a central bank responds to an attack may stave off runs on reserves. France thwarted a run on the franc in 1992 by widening the band, thus indicating to the arbitrageurs both its commitment to the mechanism as well as its willingness to accommodate further depreciation within the existing exchange rate regime.

The paper is organized as follows: Section II describes a model of a foreign exchange market composed of a finite number of risk-neutral traders who have perfect public information regarding market fundamentals and relevant policy variables. Section III introduces uncertainty into this model by making the liquidity held by each trader private information, which implies that the traders can no longer evaluate with certainty, the strength of the central bank's commitment to the exchange rate regime relative to market liquidity. Section IV.A applies the results to provide an explnation for the dynamics of some recent financial market crises by

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[6] In this paper, we assume that a trader's liquidity constraint is private information. We could equivalently assume that the amount of reserves the central bank is willing to commit to defend an exchange rate regime is information that is not disclosed to the market or, more generally, that the market is uncertain about the manner and agility with which the central bank will respond to a speculative attack on the regime. Both models yield qualitatively identical outcomes.

[7] An important implication of the analysis of this paper is that this intuition does not depend upon traders' preferences exhibiting risk aversion. In fact, risk-neutral traders will also exhibit this type of behavior in the asset market equilibrium. Hence, it is structural uncertainty that drives this positional sorting of traders in asset markets.

endogenizing the self-fulfilling expectations that generate multiple rational-expectations equilibria in static models of these phenomena. Consequently, we are able to provide a satisfactory explanation for foreign exchange market dynamics of the type exhibited in some of the European economies in 1992. Section IV.B relates the analysis of this paper to the existing literature on optimal dynamic trading/investment in strategic environments. Section V concludes and discusses some future directions. All proofs are given in an appendix.

II. The Model

A finite number (*N*) of risk-neutral traders have the opportunity to exercise an option, the scale of which is specific to each trader (depending on her net wealth and borrowing constraints). To be concrete, we will refer to the scale of trader *I*'s option as her *liquidity constraint*. The option could be a put or call order on a currency, a forward position in that currency, or simply a spot transaction at the (central) bank. Trader *I*'s liquidity constraint will be denoted ω_I.[8] Investing one's liquidity in the option yields positive returns as soon as the market's net investment in the option reaches a critical amount ω^*. The profitability of the option disappears once the total amount invested by the market reaches this critical amount.

A trader is free to choose her time to invest; at any time, she can choose to invest her liquidity immediately or to postpone this decision to some future date. We assume that time is discrete and that there is no exogenous deadline constraining the traders' decisions. Once trader *I* has exercised her option, she cannot reverse her decision in the future; the investment is irreversible. To exercise her option, *I* must pay up-front an amount $c\omega_I$, that can only be recovered if the investment pays off, that is, only if total investment ever reaches ω^*.

Consider a central bank defending a fixed exchange parity. Let the parity be *e* dollars per unit of domestic currency, and assume that this pegged value is greater than the floating rate of θ dollars per unit that is determined by the economic fundamentals. The bank is willing to borrow or drain up to ω^* dollars of foreign exchange to defend the parity. Speculators can short the domestic currency at anytime. Standard methods of doing so are costly, however; selling domestic currency futures for example, requires putting up a margin that can be recovered only if the profit is ultimately realized. Speculator *I* can short up to a dollar equivalent of ω_I units of domestic currency at the fixed exchange rate, but must put up a cost of *c* dollars per unit.[9] The

[8] Capital letters will be used to index traders and lower-case letters to index time periods.

[9] In this paper, we do not model the demand curve for the asset that generates the price *c*. An alternate approach to the results in this paper lies in a model where the earlier one purchases the right to sell domestic currency futures, the lower is the margin price one pays. Notice that this is an alternate way to generate a payoff function for traders that implies higher returns to earlier speculation. Chamley (2003) builds a model of speculative attacks by interacting the incentives to coordinate and preempt in this fashion. However, his model does not deal with the case of

(continued…)

central bank devalues to the floating rate once speculative short-sales exceed ω^*. Then speculator I's profit if she manages to short ω_I dollars worth of domestic currency is $(e-\theta)\omega_I = \alpha\omega_I$. If I shorts the domestic currency at a time the central bank has sufficient dollar reserves to transact her order, then her profit in the event of a devaluation is $\alpha\omega_I$. But if she attempts to short sell (along with other traders) at a time when the bank's reserves fall short, then with positive probability, the parity will fall before she can purchase ω_I dollars at the pegged exchange rate. In this event, her profit will be strictly less than $\alpha\omega_I$. So the time at which I shorts, relative to the time of the devaluation has an important bearing upon her eventual profit.

Consequently, it is best to describe I's payoffs by separating them in accordance with the relative positions of the period in which they exercise the option and the period in which the critical amount of investment ω^*, is reached.

Suppose I invests in some period and the devaluation occurs t periods thereafter. In this event, I earns a profit of α on each dollar speculated. Given her liquidity constraint of ω_I dollars, her total discounted profits are $\delta^t\alpha\omega_I$.

If, on the other hand, I shorted the domestic currency in the same period in which the devaluation occurs (say period t), then her profits depend upon the total amount of speculation attempted in that period. Let $\mathbf{I}_{t-1}$ denote the subset of N traders who have sold short at the end of period $t-1$. Then the amount of additional speculation that will induce a devaluation of the currency is $\omega^* - \sum_{J\in \mathrm{I}_{t-1}} \omega_J$. Suppose that total amount of speculation in period t is $\omega_t \geq \max\left\{\omega_I, \omega^* - \sum_{J\in \mathrm{I}_{t-1}} \omega_J\right\}$. Then I's expected profits are $\alpha\omega_I\left(\dfrac{\omega^* - \sum_{J\in \mathrm{I}_{t-1}} \omega_J}{\omega_t}\right)$. Since (generically), the ratio in the parenthesis is strictly less than one, the profit accruing to I is also strictly less than the amount he could have earned had he invested slightly earlier, had he the opportunity to do so.

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large traders who can influence the direction of trade in an asset market by anchoring expectations around their trading decisions, an effect that our model is able to fully capture. Moreover, Chamley's model does not deliver a comparison of the stability of exchange rate regimes under alternate degrees of transparency in central bank policy.

If *I* shorts the currency in the case where the central bank successfully staves off a devaluation, she loses the margin on the speculation $c\omega_I$. If she never speculates, her net return is zero.

All traders discount the future using a common per-period discount factor δ. This discount factor reflects the length of a time period. So if $\tau > 0$ is period length and $\rho > 0$ is the rate of interest, then the corresponding per period discount factor, $\delta(\tau, \rho) = e^{-\rho\tau}$. It follows that for any positive rate of interest, as period length converges to zero, the per-period discount factor converges to unity. In this paper, we will be interested in situations where the period length is very short.[10] In modern financial markets, these lags are typically negligible, and hence, we will assume in this paper that period length is very short, or equivalently, that $\delta \sim 1$.

The dynamic coordination game is first solved for the case where all payoff-relevant variables (the distribution of options $(\omega_A, \bullet, \bullet, \bullet, \omega_I)$ and the critical mass ω^*) are common knowledge. The following result describes the set of market equilibria.

Proposition 1. *There exists a unique (sub-game perfect) equilibrium. In this equilirium, if* $\sum_{I=A}^{N} \omega_I > \omega^*$, *then at any trading history, any trader who is yet to invest will immediately exercises her option. If* $\sum_{I=A}^{N} \omega_I < \omega^*$, *then no trader ever exercises her option.*

Proof: See the appendix. ‖

Discussion

The second part of the result is easily explained: if it is publicly known that the central bank is capable of withstanding the maximum attack, then shorting the currency cannot yield positive returns. Given the positive costs associated with speculation, no trader would be willing to pick up a sure loss. The first part of the result is non-trivial; in particular, it provides a striking contrast with corresponding results in the coordination game literature, where there is a multiplicity of equilibria, with most equilibria implying slower coordination of actions than in this model. Gale (1995) shows that getting out of a recession may be difficult because the incentive to coordinate the timing of one's investment with other investors introduces an incentive to delay investment. In that model, there are multiple equilibria with most of them involving investment after a delay of a number of periods.

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[10] In a discrete-time economy, the period length represents both the lag with which information about actions taken by other traders get transmitted to any given trader as well as the lag with which traders can respond to actions taken elsewhere in the market.

From a policy perspective, this result has the interesting implication that if a central bank were to operate in an environment where all payoff relevant information was publicly available, then it cannot defend an exchange rate parity when it is vulnerable to a speculative attack, or equivalently, transparency destroys any hope of salvaging exchange rate regimes when fundamentals are weak.

The intuition for this unique *immediate attack* equilibrium can best be highlighted by focusing on a market where two traders (A and B) can sell short sufficient amount of domestic currency to induce a devaluation. In this case, a devaluation must occur in any equilibrium. This is because if there were an equilibrium with no devaluation, then no trader would sell short.[11] But then all traders (including A and B) would obtain zero profits, in which event both of them have an incentive to deviate and short. For example, if A were to short in the first period, B would follow suit in the second period (as B knows that she can induce a devaluation by doing so) and both would make positive profits. In a similar vein, we can argue that in any equilibrium, A and B must invest in the same period. For if B were to short in a period before A (say period $t-1$), then A (and all the other traders) would short in period t, yielding A strictly lower expected profit than if he had shorted with B.[12] The reason for this is that A must share a strictly smaller pool of central bank dollars in period t with the others than he could have shared with B in period $t-1$. Finally, why can't A and B short together after some delay, say in period $t > 1$? If this were possible in equilibrium, then all other traders would also sell in period t, because they know that A and B will induce a devaluation at that time. But then, A (also B) has an incentive to deviate and short one period earlier so that he does not have to share the profits from the devaluation with the rest of the traders. It follows from this argument that in this example, A and B (and all the other traders) will short the domestic currency at the first opportunity (in period 1) and induce a devaluation.

III. TRADING UNDER UNCERTAINTY AND COORDINATION FAILURE

We now move to a market composed of N risk-neutral traders whose liquidity constraints are distributed *i.i.d.* $F\left(\left[\underline{\omega},\overline{\omega}\right]\right)$, where $\left[\underline{\omega},\overline{\omega}\right];\ 0 \le \underline{\omega} < \overline{\omega} < \infty$. A trader's liquidity constraint is now private information. The common cumulative distribution function of the traders' liquidity, F, is continuously differentiable, has full support, and no mass points. There is no other change to the structure of the economy described in Section II.

The following assumption will be made to avoid trivial cases:

$$\omega^* \in \left(N\underline{\omega}, N\overline{\omega}\right)$$

[11] Because if a trader shorts ω, she pays $c\omega$ and receives zero return (no devaluation), so she would in fact prefer not to short at all.

[12] This is easily verified by comparing the payoffs under the two strategies. See the appendix for details.

If the critical mass, $\omega^* \geq N\overline{\omega}$, then exercising one's option implies negative returns with probability one. On the other hand, if $\omega^* \leq N\underline{\omega}$, then the analysis is equally trivial as the traders know ex ante that there is sufficient liquidity in the market to overcome the reserves commitment of the central bank, and the analysis of the previous section implies that there will be an immediate attack. If however, ω^* lies somewhere in between these two values, a trader is not certain ex-ante, whether speculation is profitable even if all other traders short the currency.

Given the independence assumption, the larger is the liquidity constraint of trader *I*, the larger is the probability assigned by her to the event that $\omega_I > \omega^* - \sum_{J \neq I} \omega_J \Leftrightarrow \sum_J \omega_J > \omega^*$. This implies that, *ceterius paribus*, larger traders are more optimistic about the possibility of a successful speculative attack. Moreover, the payoff structure guarantees that the amount of profit that may lost by postponing speculative trade to a future period is also greater for a larger trader. On the other hand, since the cost of speculative trade is an increasing function of the size of the position taken, a large trader will lose more in the event of unsuccessful speculation, as happens if there is no subsequent devaluation.

This situation is vastly different from the previous section. Uncertainty regarding the sufficiency of market liquidity to induce a devaluation makes traders more reluctant in paying the upfront margin cost of speculative trade. Traders with tight liquidity constraints, may be quite pessimistic about the possibility of successful speculation ex ante; moreover, early speculative trades emanating from these traders also send negative signals to the rest of the market about the total volume of liquidity. As a consequence, these traders will strategically wait for positive information from the market. Large traders—knowing this—will need to guide expectations by adopting the riskier strategy of speculating in the absence of positive public information, but also earn a higher (per dollar) return because they invest earlier. Consequently, the distribution of liquidity becomes an important structural variable in determining the likelihood of a devaluation: the presence of large traders is critical to initiating and carrying forward the speculative attack. In the remainder of this section, we formalize this intuition.

Strategies and Equilibrium

The state of the market at the beginning of period *t* (which includes information on trading activity up to period *t* – 1) is an ordered pair $\left(\omega^*_t, \mathrm{I}_{t-1}\right)$. ω^*_t is the investment that is still required to generate the positive returns and I_{t-1} denotes the set of traders who have invested by the end of period *t* – 1. Hence, $\omega^*_t = \omega^* - \sum_{J \in \mathrm{I}_{t-1}} \omega_J$. A history of trade up to period *t* is any set of the form $\left\{\left(\omega^*_1, \mathrm{I}_0\right), \bullet, \bullet, \bullet, \left(\omega^*_t, \mathrm{I}_{t-1}\right)\right\}$.. Letting h be a generic trading history, we may now define a **trading strategy** as a probability of investment defined as a function of both the history of market activity as well as a trader's own liquidity constraint. Formally, a trading strategy is a function, $\sigma : \{h\} \times [\underline{\omega}, \overline{\omega}] \rightarrow [0,1]$.

In subsequent analysis, a particular type of trading strategy will be of considerable interest. An action at a history *h* is said to possess the **trigger property** if there is a level of liquidity, $\hat{\omega}_t$, such that

$$\sigma(h,\omega) = \begin{cases} 1 \\ 0 \end{cases} \text{as } \omega \begin{Bmatrix} \geq \\ < \end{Bmatrix} \hat{\omega}_t$$

A **trigger strategy** is a strategy where for each history $h > 0$, the action at *h* possesses the trigger property. A **market equilibrium** is a perfect Bayesian equilibrium of this trading game. The following result characterizes the symmetric market equilibria of the trading game.

Proposition 2. *In asset markets where information flows are sufficiently fast and reaction lags are sufficiently short* ($\delta \sim 1$)*, there is a unique symmetric market equilibrium. In this equilibrium, the traders' common strategy is a trigger strategy. If after any history of trade in the market, no trader invests in a single period, then there is no further investment thereafter.*

Proof: See the appendix. ‖

Discussion
This result strikingly differentiates equilibrium market dynamics when there is trading uncertainty from markets where there is none: in the latter, there can be no coordination failure in equilibrium (Proposition 1). If a devaluation is possible, then it will happen and immediately. With private information, however, this need not be that case as the following simple example illustrates: Consider a market where two traders *A* and *B* have (stochastically) independent liquidity constraints, distributed uniformly on $[0,75]$. Let $\omega^* = 100; \alpha = 1; \delta = 1-\varepsilon;\ \varepsilon \sim 0;\ c =$ 0.75 dollars. Then the unique first period equilibrium trigger is approximately 61.5 dollars.[13] So even if both traders had sufficient liquidity to induce a devaluation through speculative trade, this need not imply a devaluation; for example, if both traders could demand 60 dollars each from the central bank, neither would take the risk of shorting in the first period, whereupon further speculation would not occur thereafter. But if one had 70 dollars and the other 50 dollars, then the first would trade in period 1 and the second would follow in period 2. So with private information, it is not just the total amount but also the distribution of liquidity that matters. A large trader is pivotal in transmitting positive information to the rest of the market.

The second aspect of the result which is of considerable empirical interest is that there is a considerable bunching of speculation against an exchange rate regime just prior to the devaluation, rather than a gradual, slow burn-down of reserves: in each period, there must be

[13] The example can be numerically solved in the following manner: following the proof of Proposition 2 in the appendix (see particularly lemma 3), the solution to the two-trader case reduces to solving a two-period arbitrage equation. In this case, we first show that the trigger must be greater than 50 dollars, which simplifies the arbitrage equation, enabling a numerical solution.

some speculation against the currency, otherwise expectations of a quick devaluation disappear, thwarting further speculation. Since periods are very short (when measured in *real time* units), a devaluation in N periods corresponds to an intense speculative frenzy in a short period of time that results in a quick devaluation. In this manner, financial crises that appear to be generated purely by mood or sentiment swings and appear as unpleasant surprises are fully rationalizable as the outcome of optimal strategic arbitrage undertaken by traders in these markets.

Finally, as mentioned in the introduction, trading uncertainty should be broadly interpreted to mean uncertainty regarding the ability of the market to put the sufficient amount of pressure on an exchange rate regime to destabilize it in a relatively short period of time. In the current framework, this comes about because liquidity constraints are private information. Alternately, this situation could also be the outcome of central bank secrecy regarding its commitment value to a peg or an exchange rate band regime, or even from uncertainty regarding the manner in which the bank may respond to a speculative attack.[14] Section IV.A provides a more detailed discussion of these issues in the context of the ERM band crisis of 1992.

IV. Applications and Connections

A. Financial Crises

Static models of self-fulfilling financial crises, such as Obstfeld (1996) of the currency market or Bryant (1980) and Diamond and Dybvig (1983) of bank runs, have stressed the key role played by market expectations in determining the equilibrium outcome that is selected. These models lack the dynamic structure, however, that clarify the formation of—and change in—these expectations over time. Hence, they are agnostic about the dynamic market process that leads to the selection of the crisis as an equilibrium outcome and cannot throw light upon how a crisis may arise and what factors make a crisis more likely.

In contrast, the model developed in this paper takes an explicitly dynamic path to analyzing the onset of financial market crises by studying the optimal timing decisions of speculators in such markets. In particular, we have stressed the role of two key factors that impinge upon the likelihood of a crisis given the optimal strategic decisions of the traders. First, the quality of information regarding the payoff relevant parameters and second, the distribution of wealth in the market. An example of an attack on a fixed exchange parity to illustrates the advantages of using this approach.

Regarding the quality of information available to the traders, a comparison of Propositions 1 and 2 indicates that in a setting where speculators have precise and complete information on all

[14] Indeed the speculation game of this paper is isomorphic to a speculation game where the bank's reserves are private information and where traders receive private signals, whose accuracy may be (positively) correlated to the wealth of the trader.

payoff relevant parameters, then it is very unlikely that a weak central bank will be able to preclude a successful attack. In fact, we can expect such a weakness to be exploited quickly. However, once there is genuine uncertainty about the weakness of a central bank, as for example, in the case where agents are uncertain about whether the reserves position is high enough to match the speculative capacity of the market, then the presence of large traders who can coordinate expectations in the market by large early short sales becomes essential to uncovering this weakness, and exploiting it.

A prominent example of the latter phenomenon is the exit of the pound from the ERM in September 1992. It is rather far-fetched to say that George Soros forced the U.K. authorities to such a decision. It is however, likely that Soros engineered the exit by shifting market expectations. He did this by (a) undertaking a large initial forward position against the pound; (b) thereby, inducing other large players, in this case the British banks to follow suit by selling pounds in the spot market so as to make necessary portfolio adjustments; (c) which was sufficient to force the U.K. authorities to choose the exit option over further increases in domestic interest rates and foreign exchange reserve losses.[15]

There are other examples of how a change in the market's perceptions can be brought about by a combination of the information released through trading and visible exogenous shocks to the commitment value. One is the case of the Swedish Riksbank's unsuccessful defense of the Swedish krona in the period September through November 1992. The Riksbank demonstrated its considerable commitment to the band in September by raising interest rates to unprecedented levels and taking massive reserve losses as result of sterilized interventions. The overnight lending rate for example, was increased to over 500 percent per annum by September 16, when the first wave of the attack was well along. With its resources thus depleted, the collapse of the pound sterling, the Italian lira, and the Danish krone out of the band weakened the Riksbank's commitment value to the peg rather visibly (as it led to considerable doubts about the future of the EMS itself, the primary reason for which Sweden joined the band in May 1991). This lowered expectations about the likelihood that the Riksbank would put up a further fight, induced a second attack in November, forcing the krona into a float on November 19, 1992.[16]

The mysterious transition of speculators' expectations that seems to occur at the time of a speculative outburst can thus be rationalized as an outcome of dynamic speculative trade, where the uncertainty about overcoming the central bank's reserves is overcome due to the initial transactions of a few large agents. Perceptions about the weakness of a central bank are only

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[15] Soros reportedly borrowed heavily in the U.S. treasury bills market, used the bills to sell a large value in three-month pound futures, thereby forcing large British banks to adjust their own asset portfolios through dynamic hedges (which included large spot sales of pound sterling). Once the pound crashed out of the ERM, Soros pocketed the profits on the pound trades, which were more than sufficient to make good the payments due on his treasury bills transaction.

[16] See Obstfeld (1994) for an excellent exegesis of the Swedish devaluation of 1992.

meaningful when viewed in comparison to the pressure that can be brought on it through speculation. Agents who hold a large quantity of liquid domestic assets are more optimistic about the likelihood of generating the required amount of pressure, than are agents who have a lower quantity of domestic assets. The predictions of the model developed in Section IV, therefore, translate into a very intuitive picture: large speculators take the lead by bailing out of the currency first, thereby increasing the market's expectations of a collapse of the exchange parity, which induces further pressure in subsequent periods.

The major policy implication emerging from the results of the previous sections is the possibility that central banks may reap considerable benefit from non-transparent modes of operation in foreign exchange markets; these include not disclosing its reserves commitment to an existing exchange rate arrangement and also, the manner in which they will adjust pegs or bands to speculative attacks.

An example is the manner in which the Banque de France responded to speculation against the franc in 1992; instead of hiking interest rates and sacrificing reserves (as in the case of the United Kingdom and Sweden), its response was to widen the band, sending a mixed signal more difficult to interpret than the responses of the other two countries. It signaled that it was willing to accommodate more speculation, while remaining committed to the band regime, thereby raising the likelihood of its value being pushed back up (provided it did not crash out of the wider band). This was sufficient to thwart further speculation against the franc.

B. Related Literature

The model presented in this paper attempts to provide a coherent theory of the resolution of coordination problems in markets where agents have dual and conflicting incentives in determining when to act. A number of recent papers have examined these incentives in specific financial market applications.

Chamley (2003) analyzes a dynamic speculative attack upon an exchange rate band mechanism and draws similar conclusions about the impact of various degrees of transparency upon the likelihood of a speculative attack. In his model, speculation initially raises the domestic price of foreign currency (within the band), implying larger profits for the early speculators, if a devaluation occurs (as when a currency exits the band). A critical distinction between our models lies in the strategic influence that individual traders exercise in the game. The model developed here allows for players whose actions change the strategic nature of the game and who take this effect into account in computing their optimal strategy. Arbitrage in Chamley's model is non-strategic since individual agents have negligible investment size relative to the market and hence, an individual’s action has no influence on market outcomes or parameters and does not affect other traders’ expectations (and hence, their actions).

Corsetti and others (2003, Section 5) present a two-period currency attack model of a market with two types of traders. There is one large trader and a continuum of small agents. While in principle they allow for an endogenous selection of the time to make the irreversible decision to short the domestic currency, in fact, the presence of this option is redundant given the structure of their model. In particular, the absence of strategic substitutability in their model implies that there is no cost to waiting for information unlike in the model presented here. Specifically, their central bank accommodates all orders demanding U.S. dollars at the overvalued exchange rate despite devaluating the currency. Given zero waiting costs and the fact that a negligible agent's action has zero informational content, he has no incentive to take an action first. Knowing this, the single large agent will act in the first period to rid the market of any uncertainty concerning his intentions; that is, to lessen the strategic uncertainty in the market. Effectively, therefore, Corsetti and others impose an exogenous order on the decision-making process, and hence, their paper has little to say about optimal timing in this kind of market.

Abreu and Brunnermeier (2003) study a dynamic game between arbitrageurs seeking to optimally time their short sale of a *bubble* asset. In their paper, a coordination problem is generated by the temporary inability of agents to coordinate their selling strategies owing to lack of common knowledge of the time at which the bubble will be corrected exogenously. Since the bubble widens over time before ultimately being corrected, this generates incentives to wait for a while before shorting the asset. This explains the resilience of bubbles in their model. Likewise, in the model here, lack of common knowledge of agents' wealth levels generates incentives to wait and observe the market and thereby, possibly delaying price crashes. Abreu and Brunnermeier, however, preclude strategic arbitrage since they model a market composed of a continuum of agents. Finally, the existence of an exogenous deadline on the bubble exerts a considerable impact on their analysis which is absent here, owing to the infinite horizon.

Caplin and Leahy (1993) and Chamley and Gale (1994) explore the macroeconomic implications of individual incentives to delay costly actions in order to make more informed decisions, after observing the actions of others. In both these papers, strategic delay results in waiting games and can give rise to equilibrium behavior similar to that seen in this paper. In Chamley and Gale, at least one agent must invest in each period to keep the investment process from stopping altogether. Failure to invest generates too much pessimism and ends the game in spite of an infinite horizon. These papers are characterized by a lack of pecuniary externalities unlike the model presented in this paper. This significantly affects the analysis of equilibrium behavior. In the absence of pecuniary externalities, an agent's payoff is determined by the realization of some exogenous random variable and not by the number of agents who exercise their investment option. This implies that while agents delay in order to obtain better information (as in our model), *at the time they invest*, they are not concerned about the impact of their investment upon future paths of play, because (unlike in our model), their own payoff is unaffected by this.

V. CONCLUSIONS

This paper outlines a theory of optimal trading in strategic environments characterized by an interaction of two conflicting incentives: the need to coordinate and the desire to preempt. My results clearly explicate the nature of the optimal decision rules in equilibrium. Moreover, optimal arbitrage implies very different likelihoods of a financial crisis under different degrees of trading uncertainty. This fact implies, in particular, that central banks which value an existing exchange rate arrangement may have greater success in avoiding speculative attacks on the exchange regime, designed to exploit short-term adverse shocks, by not revealing its commitment value to the arrangement ex ante.

While the paper discusses optimal trading in the context of a currency market, the results are also relevant to runs on banks by rational depositors seeking to avoid asset losses in the face of liquidity shocks to the bank's balance sheet, and drawing conclusions about the benefits and costs of imposing balance-sheet disclosure to depositors in a banking system.

Proof of Proposition 1: The case where $\sum_{I=A}^{N} \omega_I < \omega^*$ is trivial. So consider the case where $\sum_{I=A}^{N} \omega_I > \omega^*$. If one of the N traders in the market has sufficient liquidity to beat the bank: $\exists I : \omega_I \geq \omega^*$, then, given $\delta < 1$, this trader will prefer to short in period 1 itself. Knowing this, all other agents will do the same. This fact establishes the basis for an induction argument, whose hypothesis is as follows: suppose that the proposition is true for all $k \leq n$; for some $1 \leq n \leq N-1$; if $\sum_{I=A}^{k} \omega_I > \omega^*$. We will prove under this hypothesis that the result is also true when $k = n+1$. We prove by contradiction. Take a market where $\neg\left(\exists k \leq n : \exists\left(I_1, \bullet, \bullet, \bullet, I_k\right) \wedge \sum_{i=1}^{k} \omega_{I_i} > \omega^* \right)$, but where $\exists\left(\tilde{I}_1, \bullet, \bullet, \bullet, \tilde{I}_{n+1}\right) : \sum_{i=1}^{n+1} \omega_{\tilde{I}_i} > \omega^*$. Suppose $\left(\sigma^*_I\right)_{I=A,\bullet,\bullet,\bullet,N}$ represents an equilibrium mixed strategy profile for this market game, where $\neg\left(\forall I; \sigma^*_{I,1} = 1\right)$. Given the induction hypothesis, it follows that there is a finite t and a trader $\tilde{I}_i$ such that – in the event that no trader has shorted the currency by the beginning of period t, then $\sigma^*_{I,t} > 0$. Let this trader be called B. For trader A $\neq$ B, let us compare the expected payoff at the beginning of period t, from two alternate trading strategies: immediate short-selling and waiting for at least one period more. Denote by the set S(A), those collections of traders who – together with A – have sufficient liquidity to induce a devaluation. For example, $\left\{I_1, \bullet, \bullet, \bullet, I_r\right\} \in S(A) \Rightarrow \sum_{1}^{r} \omega_{I_i} + \omega_A \geq \omega^*$. Let NS(A) denote all collections of traders other than A for which this condition is violated. Finally, let the set of traders other than A who short the currency in period t be denoted $\mathbf{I}_t$. Then A's expected payoff from investing in period t is:

$$\alpha\omega_A \sum_{\mathrm{I_t} \in S(A)} \Pr\left(\mathrm{I_t} \;\middle|\; \sigma^*_{-A}\right) \frac{\omega^*}{\omega_A + \sum_{I \in \mathrm{I_t}} \omega_I} + \delta\alpha\omega_A \sum_{\mathrm{I_t} \in NS(A)} \Pr\left(\mathrm{I_t} \;\middle|\; \sigma^*_{-A}\right)$$

where σ^*_{-A} denotes the strategy profile of all traders excluding trader A. If instead, A decided to not invest in period t, his payoff under the event that $\mathrm{I_t} \in S(A)$ would have been strictly lower than if he had. In the event that $\mathrm{I_t} \in NS(A)$, it would be at most $\delta\alpha\omega_A$.[17] So trader A is

[17] For example, in the event that trader B shorted the currency in period t, the induction hypothesis implies that all other traders – including A – will short the currency in period t+1, yielding A at most the expected payoff $\delta\alpha\omega_A \frac{\omega^* - \omega_B}{\sum_{I \neq B} \omega_I} < \delta\alpha\omega_A$.

not indifferent between shorting in period *t* and waiting beyond period *t*. As A was arbitrarily chosen among the set of traders, it follows that each trader (other than B) will strictly prefer to short in period *t* itself. This argument establishes that ${\sigma^*}_{B,t} > 0$ implies that $\forall I; {\sigma^*}_{I,t} = 1$. If so, then in period $t - 1$, each trader will strictly prefer to short over postponing to period *t*. But then each trader will strictly prefer to short in period $t - 2$ instead. This argument leads in a finite number of steps to establishing that $\forall I; {\sigma^*}_{I,1} = 1$, a contradiction. This completes the proof of Proposition 1. ‖

Proof of Proposition 2: The proof of this result is quite involved. Here we will provide details for the case of two traders A and B, and then sketch the argument for the general case of *N* traders.[18] The proof for the case of two traders rests upon proving three lemmatta.

Lemma 1. *A's best response to any trading strategy,* σ_B*, is a trigger strategy.*

Proof. If $\omega_A \geq \omega^*$, then the optimal strategy for trader A is to short the currency in the first period itself. So let me assume henceforward, that $\omega_A < \omega^*$. I will constrain the strategy σ_B to satisfy $\left(\forall \omega_B \geq \omega^*\right); \sigma_B\left(\omega_B\right) = 1$, but otherwise let it be completely arbitrary.[19] The strategy of proof is as follows: in any period, a best response to B's trading strategy is found by comparing two expected payoffs. The first is the expected payoff to shorting the currency in that period, the second is the expected payoff from not shorting this period, and reoptimizing the next period (given B's decision in this period and her continuation strategy, given by σ_B. The computation of the former value is straightforward, given that B will (a) either short in the current period; (b) not short this period, and will respond optimally to A by – (i) shorting the next period if $\omega_B \geq \omega^* - \omega_A$ or (ii) never shorting if $\omega_B < \omega^* - \omega_A$. In order to compute the second value, I will first compute the expected payoff to A (in the current period) of postponing investment for *t* more periods and then shorting (in the event that B has not invested by then). If B invests in the interim, then A will respond in the same manner as described above in (a) – (b-(i),(ii)). The next step is to compute the supremum of these payoffs. It is this latter value which clearly represents the expected payoff to postponing the decision to speculate to the next period and reoptimizing

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[18] Details are available upon request.

[19] *Technical Note*: The space of admissible trading strategies must satisfy a condition of measurability with respect to the product space $\mathbf{H} \times \mathbf{B}\left(\left[\underline{\omega}, \overline{\omega}\right]\right)$, where **H** is the sigma-algebra formed by power sets of all finite t – histories, $\mathbf{H_t}$, (the Kolmogorov construction), and **B** is the Borel-sigma algebra of open sets in $\left[\underline{\omega}, \overline{\omega}\right]$.

at that time.[20] Once the two values are computed, it will be shown that their difference is (strictly) increasing in ω_A, whence the lemma.

Wlog, let the current period, $t = 1$. Let A adopt the following strategy: wait for $t-1$ periods for B to move first (and respond optimally if she does). If B does not short in the first $t-1$ periods, then short the currency in period t. Under this strategy, A's expected payoff in period 1 is:

$$\begin{aligned}
&\alpha\sum_{s=1}^{t-1}\delta^s\int_{\omega^*-\omega_A}^{\omega^*}\prod_{\tau=0}^{s-1}\Big[\big(1-\sigma_{B,\tau}(\omega_B)\big)\sigma_{B,s}(\omega_B)\Big]\big(\omega^*-\omega_B\big)dG_B(\omega_B)\\
&+\alpha\delta^{t-1}\int_{\omega^*-\omega_A}^{\omega^*}\prod_{\tau=0}^{t-1}\big(1-\sigma_{B,\tau}(\omega_B)\big)\sigma_{B,t}(\omega_B)\,\omega_A\frac{\omega^*}{\omega_A+\omega_B}dG(\omega_B)\\
&+\alpha\delta^{t}\int_{\omega^*-\omega_A}^{\omega^*}\prod_{\tau=0}^{t}\big(1-\sigma_{B,\tau}(\omega_B)\big)\omega_A dG(\omega_B)-\delta^{t-1}c\omega_A\int_{\underline{\omega}}^{\omega^*-\omega_A}\prod_{\tau=0}^{t-1}\big(1-\sigma_{B,\tau}(\omega_B)\big)dG(\omega_B)
\end{aligned}\tag{1}$$

where in (1), the first term is the expected payoff under the event that B shorts in some period prior to period t, and A and B have sufficient liquidity to beat the central bank; the second term corresponds to the expected payoff under the event that B shorts in period t together with A, and they have sufficient liquidity to beat the bank; the third term corresponds to the expected payoff under the event that B fails to short in the first t periods, and A and B have sufficient liquidity to beat the bank; while the fourth term is the expected payoff under the event that B does not short the currency in the first $t-1$ periods and that A and B do not have enough liquidity to beat the bank. We denote the expected payoff under this strategy as $V_t(\omega_A \mid \sigma_B)$. We wish to compare this payoff to the expected payoff from shorting the currency in the first period itself, denoted $V_1(\omega_A \mid \sigma_B)$:

$$\begin{aligned}
&\alpha\int_{\omega^*-\omega_A}^{\omega^*}\omega_A\frac{\omega^*}{\omega_A+\omega_B}\sigma_{B,1}(\omega_B)dG(\omega_B)\\
&+\alpha\delta\int_{\omega^*-\omega_A}^{\omega^*}\big(1-\sigma_{B,1}(\omega_B)\big)\omega_A dG(\omega_B)-c\omega_A G\big(\omega^*-\omega_B\big)
\end{aligned}\tag{2}$$

Claim 1: $V_\infty(\omega_A \mid \sigma_B) := \lim_{t\to\infty} V_t(\omega_A \mid \sigma_B)$ *exists.*

Proof of Claim 1: Observe that the integrands in the second, third, and fourth terms of (1) are uniformly bounded by $\alpha\bar{\omega}, \alpha\bar{\omega}$, and $c\bar{\omega}$ respectively. Denoting these integrands respectively by S_t, T_t, and F_t and assuming that $\alpha\bar{\omega} > c$, we have – independent of t –

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[20] The alert reader may worry about mixed strategies here. Observe that in the event that A's best response in the next period, following postponement of speculation in the current period, entails a mixed strategy at some future period t, it is interpreted as A being indifferent in period t between shorting in period t and postponing beyond t. In this event, his maximized expected payoff at period t is by definition that obtainable by shorting immediately.

$$0 \le \delta^{t-1} \begin{Bmatrix} |S_t| \\ |T_t| \\ |F_t| \end{Bmatrix} < \delta^{t-1} \begin{Bmatrix} 1 \\ \delta \\ 1 \end{Bmatrix} \alpha\bar{\omega} \tag{3}$$

and

$$\lim_{t\to\infty} \delta^{t-1}\alpha\bar{\omega} = \lim_{t\to\infty} \delta^{t}\alpha\bar{\omega} = 0 \tag{4}$$

(3) and (4) jointly imply that S_t, T_t, and F_t converge to zero, and hence all save the first term in (1) also converge to zero as $t \to \infty$. So

$$\begin{aligned} &V_\infty\left(\omega_A \;\middle|\sigma_B\right) \\ &= \alpha \lim_{t\to\infty} \sum_{s=1}^{t-1} \delta^s \int_{\omega^*-\omega_A}^{\omega^*} \prod_{\tau=0}^{s-1} \left[\left(1-\sigma_{B,\tau}\left(\omega_B\right)\right)\sigma_{B,\tau}\left(\omega_B\right)\right]\left(\omega^* - \omega_B\right) dG\left(\omega_B\right) \\ &\ge 0 \end{aligned} \tag{5}$$

In (5), we have $\prod_{\tau=0}^{s-1}\left[\left(1-\sigma_{B,\tau}\left(\omega_B\right)\right)\sigma_{B,\tau}\left(\omega_B\right)\right] \le 1$, $\left(\omega^* - \omega_B\right) \le \omega_A$, and $\int_{\omega^*-\omega_A}^{\omega^*} dG\left(\omega_B\right) \le 1$.

Hence, replacing each of these terms by the larger constants, we have

$$\begin{aligned} 0 &\le V_\infty\left(\omega_A \;\middle|\sigma_B\right) \\ &\le \alpha\omega_A \lim_{t\to\infty} \sum_{s=1}^{t-1} \delta^s \\ &= \alpha\omega_A \delta\left(1-\delta\right)^{-1} < \infty \end{aligned} \tag{6}$$

(5) implies that $V_\infty\left(\omega_A \;\middle|\sigma_B\right)$ is the limit of a sequence of non-negative, non-decreasing partial sums, and (6) implies – through Abel's limit criterion for absolutely summable series – that it is well-defined. This completes the proof of the claim. ◊

It follows immediately that

$$V^D\left(\omega_A \;\middle|\sigma_B\right) := \max_{t>1} V_t\left(\omega_A \;\middle|\sigma_B\right) = \sup_{t>1} V_t\left(\omega_A \;\middle|\sigma_B\right) \tag{7}$$

exists.

Claim 2: *If for each t > 1,* $V_t\left(\omega_A \;\middle|\sigma_B\right)$ *is continuous, then so is* $V^D\left(\omega_A \;\middle|\sigma_B\right)$.

Proof of Claim 2: We prove a slightly more general claim which implies the result. Let X be a compact metric space, and let $g, h: X \to \mathbf{R}$. Let $f = \max\{g, h\}$. Suppose there exists V open in $\mathbf{R}$ and $f^1(V) \notin O(X)$, the class of open sets in X. This implies the existence of $\left(x \in X\right);\left(f\left(x\right)\in V\right)\wedge\left(\left(\forall\delta>0\right);\exists x_\delta \in B_\delta\left(x\right), f\left(x_\delta\right)\notin V\right)$. Suppose $f(x) = g(x) > h(x)$. Then there is some $\delta_g > 0$ such that $\delta \in \left(0,\delta_g\right) \Rightarrow f\left(x_\delta\right) \ne g\left(x_\delta\right) \Leftrightarrow h\left(x_\delta\right) - g\left(x_\delta\right) > 0$ for any such δ, which contradicts $g(x) > h(x)$ assumed as the hypothesis. Hence, the initial hypothesis is inconsistent and f is a continuous function. By finite induction, if we now define for every finite $n \in \mathbf{N}$, $f_{(n)} = \max\left\{f_1, \bullet, \bullet, \bullet, f_n\right\}$, where the f_i are continuous on a compact metric space X, then $f_{(n)}$ is also continuous on X. Next, consider a sequence of continuous functions $\left(f_n\right)$, where for every $x \in X, f_n$ coverges pointwise to some function *f*. Then the function, $\bar{f}(x) = \max\{f_n(x)\}$ where the *max* operates over the set **N**, is well-defined. We wish to prove that $\bar{f}$ is continuous. One way to do so would be to prove that $f_{(n)} \to \bar{f}$, where the convergence is uniform. To do this, it is sufficient to show that $M_n \to 0$, where $M_n := \sup_{x \in X}\left|f_{(n)}(x) - \bar{f}(x)\right|$. Take any *x* satisfying $\bar{f}(x) = f_n(x)$ for some $n < \infty$. For such *x*, $\left|f_{(n)}(x) - \bar{f}(x)\right| \to 0$. Conversely, consider *x* such that $(\forall n),(\exists m > n); f_m(x) > f_n(x)$. This implies that we can pick a subsequence $\left(f_{n_k}\right)$ s.t. $n_{k+1} := \min_{\{n_k+1,\bullet,\bullet,\bullet\}}\left\{n \ \left| f_n(x) > f_{n_k}(x)\right.\right\}$. As f_n converges pointwise to *f*, we must have $\bar{f}(x) = f(x)$, whereupon it follows immediately that $\left|f_{(n)}(x) - \bar{f}(x)\right| \to 0$. This establishes that $M_n \to 0$, whence continuity of $\bar{f}$ follows. To apply this to the problem at hand, replace f_n by $V^D{}_n\left(\omega_A \ \middle|\sigma_B\right)$, then $V^D\left(\omega_A \ \middle|\sigma_B\right) = \bar{f}$ and the claim follows. ◊

Let $f : [\underline{\omega}, \bar{\omega}] \times \Sigma \to \mathrm{R}$ be an arbitrary, fixed continuous function, where Σ is the space of all (measurable) strategies. Define by

$$g_t(\omega_A;\sigma_B) := f(\omega_A;\sigma_B) - V_t\left(\omega_A \ \middle|\sigma_B\right);\ t \in \mathbf{N} \tag{11}$$

and

$$\begin{aligned} g^D(\omega_A;\sigma_B) &:= \min_{t>1} g_t(\omega_A;\sigma_B) \\ &= f(\omega_A;\sigma_B) - V^D\left(\omega_A \ \middle|\sigma_B\right) \end{aligned} \tag{12}$$

By construction and hypothesis (on *f*), the functions ($g_t; t \in \mathbf{N}$) are uniform continuous and totally bounded on $[\underline{\omega}, \bar{\omega}]$.

Claim 3: *If for each t > 1, g_t is a strictly increasing function, then so is g^D.*

Proof of Claim 3: By contradiction, suppose not; i.e., let $\omega' < \omega''$ be two points in $[\underline{\omega}, \bar{\omega}]$ for which $g^D(\omega') \geq g^D(\omega'')$. If so, then by continuity of g^D, we may choose $\hat{\omega} \in [\omega', \omega'')$ and $\varepsilon(\hat{\omega}) > 0$, sufficiently small that $\left(\forall \omega \in \left(\hat{\omega}, \hat{\omega} + \varepsilon(\hat{\omega})\right)\right); g^D(\omega) \leq g^D(\hat{\omega})$. For $\omega \in [\underline{\omega}, \bar{\omega}]$, define

$$T(\omega) := \left\{N \cup \{\infty\} \setminus \{1\} : g_t(\omega;\sigma_B) = g^D(\omega;\sigma_B)\right\} \tag{13}$$

If for some $\omega \in \left(\hat{\omega}, \hat{\omega} + \varepsilon(\hat{\omega})\right)$, $t \in T(\omega) \cap T(\hat{\omega})$, then $g_t(\hat{\omega}) > g_t(\omega)$, implying that g_t is not strictly increasing, a contradiction. So assume that for all $\omega, T(\omega) \cap T(\hat{\omega}) = \phi$, and let $t \in T(\omega), \hat{t} \in T(\hat{\omega})$. Then $g^D(\omega) = g_t(\omega) > g_{\hat{t}}(\omega) > g_{\hat{t}}(\hat{\omega}) = g^D(\hat{\omega}) > g^D(\omega)$, a contradiction. Hence, $g^D(\omega') \geq g^D(\omega'') \Rightarrow g^D(\omega') = g^D(\omega'')$. Moreover, $g^D(\bullet)$ is a constant function over the interval $[\omega', \omega'']$ as otherwise, one can find two points, $\omega_1 < \omega_2$ in $[\omega', \omega'']$

where $g^D(\omega_1) > g^D(\omega_2)$, a contradiction. Let $t', t'' \in T(\omega'), T(\omega'')$ respectively. By preceding arguments, to avoid contradiction, we must have $t' \notin T(\omega'')$. But then,

$$g^D(\omega') = g^D(\omega'') = g_{t''}(\omega'') > g_{t'}(\omega'') > g_{t'}(\omega') = g^D(\omega')$$

a contradiction. It follows that $g^D(\bullet;\sigma_B)$ is strictly increasing, hence the claim. ◊

We are now in a position to prove the lemma. From (2), $V_1(\bullet \mid \sigma_B)$ is a (uniform) continuous, bounded function on a compact domain. Define this function to be the function *f* in the definition of g_t and g^D. From claim 3, the lemma is proven if it can be established that for each $t > 1$,

$$g_t(\omega_A;\sigma_B) = V_1(\omega_A \mid \sigma_B) - V_t(\omega_A \mid \sigma_B)$$

is strictly increasing. To do so, it is convenient to decompose this difference into the sum of the following six terms:

$$[1]\ \alpha\int_{\omega^*}^{\bar{\omega}} \omega_A \frac{\omega^*}{\omega_A + \omega_B} dG(\omega_B);$$

$$[2]\ \alpha\int_{\omega^*-\omega_A}^{\omega^*} \left(\omega_A \frac{\omega^*}{\omega_A + \omega_B} - \delta(\omega^* - \omega_B)\right)\sigma_{B,1}(\omega_B)\, dG(\omega_B);$$

$$[3]\ \alpha\sum_{s=2}^{t-1}\int_{\omega^*-\omega_A}^{\omega^*} \left(\delta\omega_A - \delta^s(\omega^* - \omega_B)\right)\prod_{\tau=0}^{s-1}\left(1-\sigma_{B,\tau}(\omega_B)\right)\sigma_{B,s}(\omega_B)\, dG(\omega_B);$$

$$[4]\ \alpha\delta\int_{\omega^*-\omega_A}^{\omega^*} \omega_A\left(1-\delta^{t-2}\frac{\omega^*}{\omega_A + \omega_B}\right)\prod_{\tau=0}^{t-1}\left(1-\sigma_{B,\tau}(\omega_B)\right)\sigma_{B,t}(\omega_B)\, dG(\omega_B);$$

$$[5]\ \alpha\delta\omega_A\left(1-\delta^{t-1}\right)\int_{\omega^*-\omega_A}^{\omega^*}\prod_{\tau=0}^{t}\left(1-\sigma_{B,\tau}(\omega_B)\right)dG(\omega_B);$$

$$[6]\ -c\omega_A\left(G(\omega^* - \omega_A) - \delta^{t-1}\int_{\underline{\omega}}^{\omega^*-\omega_A}\prod_{\tau=0}^{t-1}\left(1-\sigma_{B,\tau}(\omega_B)\right)dG(\omega_B)\right)$$

Partially differentiating each of these terms with respect to ω_A, it is verified by direct computation that for the first 5 terms:[21]

$$\frac{\partial}{\partial\omega_A}[i] > 0;\ i = 1, \bullet, \bullet, \bullet, 5. \tag{14}$$

and if *c* is sufficiently small, then (14) implies that the sum of these partials is positive also. So for sufficiently small values of *c*, we have for each $t > 1$,

[21] By Leibniz's rule, differentiating the integral with respect to the *parameter* ω_A, is equivalent to summing the integral of the derivative of the integrand (w.r.t. ω_A) with the value of the integrand evaluated at $\omega_B = \omega_A$.

$$\frac{\partial}{\partial \omega_A}\left(V_1\left(\omega_A \mid \sigma_B\right)-V_t\left(\omega_A \mid \sigma_B\right)\right)>0 \tag{15}$$

Combining (15) with claim 3 implies that

$$\frac{\partial}{\partial \omega_A}\left(V_1\left(\omega_A \mid \sigma_B\right)-V^D\left(\omega_A \mid \sigma_B\right)\right)>0 \tag{16}$$

which completes the proof of the lemma. ◊

Lemma 2. *Let $\left(\sigma_A,\sigma_B\right)$ denote a symmetric market equilibrium strategy profile. Let $\hat{\omega}_{I,t}$ denote the equilibrium period t speculative trigger for trader I under this profile. Then for t > 1, $\hat{\omega}_{I,t} \geq \hat{\omega}_{I,1}$.*

Proof: The proof is for trader A. An identical argument works for the other trader. Let σ_B be a fixed, but arbitrary trigger strategy satisfying $\hat{\omega}_{B,1}<\omega^*$. Let me represent σ_B by the corresponding sequence of triggers: $\left(\hat{\omega}_{B,1},\bullet,\bullet,\bullet,\hat{\omega}_{B,t},\bullet,\bullet,\bullet\right)$. Observe that this yields a sequence of posterior cdf's, G_{Bt}, on ω_B satisfying for $t > 1$, and $\omega\in\left[\underline{\omega},\min\left\{\hat{\omega}_{B,s}\right\}_{s=1,\dots,t-1}\right]$, $G_{B,t}\left(\omega\right)=\dfrac{F\left(\omega\right)}{F\left(\min\left\{\hat{\omega}_{B,s}\right\}_{s=1,\dots,t-1}\right)}$. From lemma 1, it follows that A's best response (which by hypothesis is σ_A) is a trigger strategy, and also representible by a sequence of triggers, denoted $\left(\hat{\omega}_{A,1},\bullet,\bullet,\bullet,\hat{\omega}_{A,t},\bullet,\bullet,\bullet\right)$. Proving the lemma is clearly equivalent to establishing that for $t > 1$, $\hat{\omega}_{A,t}\geq\hat{\omega}_{A,1}$. By contradiction, suppose that $t > 1$ is the first period after period 1 for which A's period trigger violates this inequality; i.e., $t=\left\{\min\left\{s\in \mathrm{N}\right\}:\hat{\omega}_{A,1}>\omega_{A,s}\right\}$. Take any $\omega\in[\hat{\omega}_{A,t},\hat{\omega}_{A,1})$; for σ_A to be a best response to σ_B, it follows that $\min\left\{\hat{\omega}_{B,s}\right\}_{s=1,\dots,t-1}>\omega^*-\omega.$ For if not, then come period *t*, it is a best response for trader A with liquidity ω to not speculate before B does, which contradicts $\hat{\omega}_{A,t}\leq\omega.$ It follows that – evaluated in period 1 – A's expected payoff under $\left(\sigma_A,\sigma_B\right)$ when his liquidity constraint is ω is:

$$\begin{aligned}
&V_t\left(\omega \mid \sigma_B\right)\\
&\qquad \alpha\sum_{s=1}^{t-1}\delta^s\int_{\hat{\omega}_{B,s}}^{\min\left\{\hat{\omega}_{B,r}\right\}_{1\leq r\leq s-1}}\chi\left(\hat{\omega}_{B,s};\min\left\{\hat{\omega}_{B,r}\right\}_{1\leq r\leq s-1}\right)\left(\omega^*-\omega_B\right)dF\left(\omega_B\right)\\
&= \;+\int_{\max\left\{\hat{\omega}_{B,t},\omega^*-\omega\right\}}^{\min\left\{\hat{\omega}_{B,r}\right\}_{1\leq r\leq t-1}}\chi\left(\hat{\omega}_{B,t};\min\left\{\hat{\omega}_{B,r}\right\}_{1\leq r\leq t-1}\right)\delta^{t-1}\omega\frac{\omega^*}{\omega+\omega_B}dF\left(\omega_B\right)\\
&\qquad +\alpha\delta^t\omega\left[F\left(\max\left\{\hat{\omega}_{B,t},\omega^*-\omega\right\}\right)-F\left(\omega^*-\omega\right)\right]-\delta^{t-1}c\omega F\left(\omega^*-\omega\right)
\end{aligned} \tag{17}$$

where by convention, $\hat{\omega}_{B,0}=\omega^*$, and the characteristic function,

$$\chi(x;y)=\begin{cases}1; \text{ if } x\le y\\ 0; \text{ if } x>y\end{cases}$$

If A were to speculate in period 1 instead, his expected payoff would have been:

$$\begin{aligned}&V_1(\omega \mid \sigma_B)\\ &=\alpha\int_{\hat{\omega}_{B,1}}^{\overline{\omega}}\omega\frac{\omega^*}{\omega+\omega_B}dF(\omega_B)+\alpha\delta\omega\left[F(\hat{\omega}_{B,1})-F(\omega^*-\omega)\right]-c\omega F(\omega^*-\omega)\end{aligned} \tag{18}$$

As speculating in period t is – by assumption – a best response for trader A in period t, it must be the case that his expected payoff to immediate speculation in period t, evaluated at the beginning of that period (and given B's trading strategy) must be non-negative:

$$\begin{aligned}&\int_{\max\{\hat{\omega}_{B,t},\omega^*-\omega\}}^{\min\{\hat{\omega}_{B,r}\}_{1\le r\le t-1}}\chi\left(\hat{\omega}_{B,t};\min\{\hat{\omega}_{B,r}\}_{1\le r\le t-1}\right)\omega\frac{\omega^*}{\omega+\omega_B}dF(\omega_B)\\ &\quad+\alpha\delta\omega\left[F\left(\max\{\hat{\omega}_{B,t},\omega^*-\omega\}\right)-F(\omega^*-\omega)\right]-c\omega F(\omega^*-\omega)\\ &\ge 0\end{aligned} \tag{19}$$

A comparison of (19) with (18) establishes that $V_1(\omega \mid \sigma_B)>V_t(\omega \mid \sigma_B)$ if $V_t(\omega \mid \sigma_B)$ is non-negative. This proves the lemma. ◊

Lemma 3. *There exists a unique first period trigger that satisfies fact (ii).*
Proof: Using fact *(ii)*, let us restrict attention to $\omega\in[\underline{\omega},\overline{\omega}]$ that satisfy

$$\alpha\delta\omega\frac{\max\{F(\omega)-F(\omega^*-\omega),0\}}{F(\omega)}-c\omega\min\left\{\frac{F(\omega^*-\omega)}{F(\omega)},1\right\}<0 \tag{20}$$

If the left-hand side of (20) is positive for all values of ω, then from lemma 2, the unique symmetric equilibrium trigger is $\underline{\omega}$. However, there exists $\varepsilon>0:\forall\omega\in[\underline{\omega},\underline{\omega}+\varepsilon)$ for which the inequality (20) is valid.[22] When the proposed trigger is ω', let $\Delta(\omega;\omega')$ denote the difference in expected payoff to speculating in period 1 and delaying for one period, when one's liquidity constraint is ω. For ω to be a symmetric equilibrium trigger in the first period, it must satisfy an arbitrage equation of the following form:

[22] In particular, it is valid for all $\omega\le\frac{\omega^*}{2}$.

$$
\begin{aligned}
&\Delta(\omega;\omega) \\
&= \int_{\omega^*}^{\bar{\omega}} \alpha\omega \frac{\omega^*}{\omega+\omega_B} dF(\omega_B) + \int_{\max\{\omega,\omega^*-\omega\}}^{\omega^*} \alpha\left(\omega\frac{\omega^*}{\omega+\omega_B} - \delta(\omega^*-\omega_B)\right) dF(\omega_B) \\
&+\alpha\delta\omega\Big[F(\omega) - F\big(\min\{\omega,\omega^*-\omega\}\big)\Big] - c\omega F(\omega^*-\omega) \\
&= 0
\end{aligned} \tag{21}
$$

To proving the lemma, it is sufficient to demonstrate that along the 45^0 line; i.e., the locus of points (ω,ω), the function is strictly increasing. I will divide the analysis into two cases: first let us consider $\omega \geq \frac{\omega^*}{2}$; where the left-hand side of (21) may be written as:

$$
\begin{aligned}
&\int_{\omega^*}^{\bar{\omega}} \alpha\omega \frac{\omega^*}{\omega+\omega_B} dF(\omega_B) + \alpha\delta\omega\Big[F(\omega) - F(\omega^*-\omega)\Big] \\
&+\int_{\omega}^{\omega^*} \alpha\left(\omega\frac{\omega^*}{\omega+\omega_B} - \delta(\omega^*-\omega_B)\right) dF(\omega_B) - cF(\omega^*-\omega)
\end{aligned} \tag{22}
$$

For $\delta \square 1$, direct computation shows that the derivative of (22) with respect to ω is positive. Conversely, consider $\omega < \frac{\omega^*}{2}$. The arbitrage equation for this case reduces to:

$$
\int_{\omega^*}^{\bar{\omega}} \alpha\omega \frac{\omega^*}{\omega+\omega_B} dF(\omega_B) + \int_{\omega^*-\omega}^{\omega^*} \alpha\left(\omega\frac{\omega^*}{\omega+\omega_B} - \delta(\omega^*-\omega_B)\right) dF(\omega_B) - cF(\omega^*-\omega) \tag{23}
$$

whose derivative with respect to ω is clearly positive. The lemma is, therefore, proven. ◊

From lemma 1, it follows that an optimal trading strategy is necessarily a trigger strategy. Lemma 2 implies that in any symmetric equilibrium, the first period trigger liquidity level is chosen in a manner such that if no one speculates in period 1, then it becomes common knowledge that no trader has significantly high liquidity in the market, and this pessimism is sufficient to thwart further speculation. As a consequence, any equilibrium period 1 trigger must be the solution to a two period arbitrage equation that expresses the trade-off between the larger speculative profits emanating from shorting early against the risk of speculating with lesser information. Finally, lemma 3 proves that this arbitrage equation can have only one zero in $[\underline{\omega},\bar{\omega}]$, establishing that there is a unique first period trigger. Hence proposition 2 is proven for the case of two traders. ‖

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