



WP/04/222

# IMF Working Paper

---

## Currency Bloc Formation as a Dynamic Process Based on Trade Network Externalities

*Etienne B. Yehoue*

**IMF Working Paper**

IMF Institute

**Currency Bloc Formation as a Dynamic Process Based on Trade Network Externalities**

Prepared by Etienne B. Yehoue<sup>1</sup>

Authorized for distribution by Roland Daumont

November 2004

**Abstract**

**This Working Paper should not be reported as representing the views of the IMF.**

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

The recent experience of the European Economic and Monetary Union (EMU) has stimulated the debate over currency union and reinforced the incentive for the emergence of currency blocs in other regions of the world. This paper builds a dynamic stochastic model—based on network externalities operating through trade channels—to explain the emergence of currency blocs, and specifically, why some countries join a currency union earlier than others. The paper develops and formalizes the intuition that currency bloc formation is path dependent, and that countries join currency blocs sooner the more they trade with the bloc member countries, with each additional member serving in a dynamic way to attract more members into the bloc. Evidence from the current pattern of EMU expansion supports the model, which is later used to elaborate on the pattern of further expansion of the union.

JEL Classification Numbers: C61, C73, F12, F33, F4

Keywords: Currency blocs, trade network externalities, stochastic and dynamic games, path dependency

Author(s) E-Mail Address: [eyehoue@imf.org](mailto:eyehoue@imf.org)

---

<sup>1</sup> This paper draws on the first chapter of my Ph.D. dissertation at Harvard University. I am especially grateful to my advisors—Alberto Alesina, Jeffrey Frankel, and Dani Rodrik—for their guidance and support. I am also very grateful to Christopher Avery, Jorg Decressin, Andrew Feltenstein, Elhanan Helpman, Marc Melitz, Nolan Miller, and Andres Velasco, and to seminar participants at Harvard University and the IMF Institute for useful comments and suggestions. I wish to thank the Weatherhead Center for International Affairs at Harvard University for financial support. The views expressed in this paper are mine; they do not necessarily represent those of the IMF or IMF policy. All errors remain mine.

Contents	Page
I. Introduction .....	3
II. The Model .....	5
A. Static Currency Bloc Formation .....	6
Trade and Benefits of Currency Unions .....	6
Monetary Policy and Costs of Currency Unions .....	10
Currency Union in Static Equilibrium .....	12
B. Currency Bloc Formation as a Dynamic Process.....	13
Bloc Configuration.....	13
Dynamic Currency Bloc Formation Game .....	14
Equilibrium of the Dynamic Game.....	18
III. Characterizing the Dynamic and Equilibrium Outcome.....	22
A. Dynamic and Equilibrium Outcome in Three-Country Case.....	22
B. A Contemporaneous Example: The Pattern of Gradual Expansion of EMU.....	24
IV. Conclusions.....	26
Tables	
Table 1a. Average Trade-to-GDP Ratio with EMU, 1980–2000 .....	27
Table 1b. Average Trade-to-GDP Ratio with EU, 1980–2000.....	28
Table 2a: Average Trade-to-GDP Ratio with EMU and Next 10, 1980–2000.....	29
Table 2b: Average Trade-to-GDP Ratio with EU and Next 10, 1980–2000 .....	29
Appendix	
Proof of Proposition 1 .....	30
Proof of Proposition 2 .....	31
References.....	34

## I. INTRODUCTION

In 1868, only four countries—the United Kingdom, Australia, Canada and Portugal—were on the gold standard in the sense that their currencies were unconditionally and exclusively convertible into gold. By 1912, virtually all of Europe had joined the gold standard; the only exception was Spain, which remained on paper. In addition, many non-European countries had also adopted gold; exceptions included China, which was on silver, and portions of Latin America, which were on silver or bimetallic standards. This raises the question: why were so many more countries on gold in 1912 than four decades earlier?

A similar story—but of different scale—may be about to be repeated. With a renewed emphasis on price stability and the need for credible commitment, currency adoption has moved to center-stage of the current policy debate. Also, the need to take advantage of reduced transaction costs and the positive impact that sharing a common currency would have on trade have further increased the incentive for countries to join currency unions. In 2002, twelve European countries abandoned their different currencies and adopted a single currency: the euro.<sup>2</sup> As a result, from 169 currencies in circulation in 2001, there are now 158. This decline in the number of currencies is apparently not over. Six oil-producing countries have expressed their willingness to form a currency union by 2010.<sup>3</sup> Many African countries are discussing entering economic and monetary unions. For example, five West African countries have agreed to create a common currency by 2005 that will eventually merge with the West African CFA zone to create a single currency for the whole of West Africa.<sup>4</sup>

Eleven members of the Southern African Development Community (SADC) are debating whether to adopt the dollar or to create an independent monetary union possibly anchored to the South African Rand.<sup>5</sup> The newly launched African Union has even discussed adopting a single continental currency. Clearly, the number of currencies in the world is likely to keep declining further in the future.

The number of currencies globally is clearly less than the number of countries in the world, which is currently 193. If the optimal number of currencies is less than the number of countries and if the number of currencies is decreasing, which groups of countries should form currency unions, and when should an individual country join a currency bloc?

---

<sup>2</sup> I refer here to the year euro began performing the three basic functions assigned to a currency: store of value, legal tender, and medium of exchange.

<sup>3</sup> The group comprises Saudi Arabia, United Arab Emirates, Bahrain, Qatar, Oman, and Kuwait.

<sup>4</sup> The five countries are: Ghana, Guinea, Nigeria, Sierra Leone, and The Gambia. Note that Liberia participated in the primary discussions on the creation of the union.

<sup>5</sup> The group comprises the members of the existing Rand zone—South Africa, Lesotho, Swaziland, Namibia—and other countries such as Botswana, Malawi, Mauritius, Mozambique, Tanzania, and Zimbabwe. Zambia is expected to confirm its membership.

The first part of this question was analyzed in the pioneering work of Mundell (1961), and by many others; and it was recently extended in Alesina and Barro (2002). These works discuss the trade-off between the costs and benefits of currency unions. While they provide a general framework for the study of currency unions, the second part of the question—that is, the timing of joining a currency bloc, or its gradual expansion—is still missing. Baldwin (1996) studies the expansion of trade blocs using a political economy argument, and Frankel (1997) analyzes regional trading blocs in the world economy. In other words, free trade blocs have received attention in the literature. Static analysis or spontaneous emergence of currency blocs has also received attention. However, the dynamic or timing of currency bloc expansion remains to be analyzed.

This paper studies the dynamic time path as well as the path dependency of currency bloc formation. In particular, it asks why some countries join currency unions earlier than others. I contend that currency bloc formation is path dependent, and that network externalities operating through trade channels help explain the gradual expansion of currency blocs. More precisely, countries join a currency bloc sooner the more they trade with the bloc member countries and each additional member serves in a dynamic way to attract further members. The dynamics are especially driven by trade induced from earlier expansions, as a currency union increases trade among member countries. In other words, this paper not only proposes a new insight—that countries' decisions to join a currency bloc are influenced by prior choices of their trading partners—but from that insight it derives the notion of path dependency of currency bloc formation.

Before proceeding further, some caveats should be highlighted. The ultimate decision to join a currency bloc goes beyond solely economic considerations. After all, a country's critical decision to surrender its monetary policy to a supranational institution is not only economic but political. Money is sometime seen as a symbol of national pride and sovereignty. For example, even though the United States, Mexico, and Canada trade a lot, they have not formed a currency union. In a companion paper (Yehoue, 2004); the political dimension is discussed in more detail. This paper does not aim to provide policy recommendations for joining a currency union. Instead, it aims to go beyond the static setting used so far in the literature on currency union, and it proposes a new theoretical framework that allows us to think about currency blocs in a dynamic setting.

In this regard, the paper proposes a rationale for the gradual expansion of currency blocs as it really happens. It proposes a model that emphasizes the trade network—a key channel through which a country can gain from joining a currency union—by first presenting a real economy where transaction costs play a key role. But joining a currency union also involves costs, especially the loss of monetary policy to counteract real shocks. Consequently, I introduce money in the economy and explicitly show how inflation can bring about real effects.

To fully formalize the intuition laid out above, I first extend the static currency union formation model in Alesina and Barro (2002) to allow for more than two countries. With this extension as the background, I then present a dynamic stochastic game, where the

intertemporal coalition or currency bloc formation is explicitly modeled. The dynamic game helps to study the convergence toward a stable bloc configuration as well as the path dependency of the process. In the model, the state variable is the current partition of countries into currency blocs. The value for a country of being in a particular state depends not only on the members of its currency bloc, but also on its discount factor.

Using the Markov perfect equilibrium concept, I first show that for a given currency bloc and a number of individual countries, there exists for each individual country a trade cut-off level with the bloc below which the country will not be willing or able to join the currency bloc. In other words, trade volume determines the sequence of joining a currency bloc. Second, I show the existence of an equilibrium process of currency bloc formation that leads to a steady state or a stable bloc configuration. That is to say, the gradual expansion of currency bloc can lead to a bloc configuration where no country belonging to a bloc would like to leave the bloc to have its own currency or to join another bloc. In addition, no country not belonging to a bloc would like to join one.

Third, using a three-country case, I characterize both the dynamic and the equilibrium outcome and show that the currency bloc formation is path dependent. Using the model as a theoretical background, I analyze the pattern of the European Economic and Monetary Union (EMU). The pattern of EMU expansion fits well the model and provides evidence supporting the intuition. Based on the trade dimension highlighted in this paper, the model is further used to elaborate on the countries that are likely to be the good candidates or frontrunners for joining EMU in the round after 2004.

The paper is organized as follows. Section II presents the model. Section III characterizes the equilibrium outcome and analyzes the pattern of the gradual expansion of EMU. Section IV concludes the paper.

## **II. THE MODEL**

Currency bloc formation is modeled in a dynamic stochastic game where currency blocs are endogenously formed in a step-by-step process of bloc expansion. The model is intended to mimic in a stylized manner the creation and expansion of currency blocs, such as the classical gold standard, the CFA zone in Africa, and the European Monetary Union. Underlying the dynamic currency bloc formation game is a static currency bloc formation model that specifies the costs and benefits of membership in a currency union. I begin with a simple one-period currency union formation model. I then present the dynamic stochastic currency bloc formation game and use the stationary Markov perfect strategy concept. In the dynamic model, I explicitly show through example the difficulty of jumping from the status quo to a steady state characterized by a big bloc, and that the emergence of currency bloc is gradual. Through example I also show that currency bloc formation is path dependent.

### A. Static Currency Bloc Formation

Here I discuss a static one-period currency union formation. I then characterize the condition under which a period or static currency union will be formed. I do so by extending Alesina and Barro (2002)'s two country model—which in turn draws on Barro and Tenreyro (2000)—to allow for more countries.

#### Trade and Benefits of Currency Unions

Competitive firms produce output with a varieties-type production function<sup>6</sup> using labor and a composite intermediate good, which is assembled from a set of differentiated intermediate goods. The output of a representative firm  $f$  is given by:

$$Y_f = AL_f^{1-\alpha} \cdot \sum_{k=1}^N X_{kf}^\alpha \quad (1)$$

where  $A > 0$  is a productivity parameter,  $L_f$  stands for labor employed by firm  $f$ ,  $0 < \alpha < 1$ ,  $X_{kf}$  is the amount of intermediate available and used. Final output is a homogenous good that can be used for consumption or to produce intermediate goods, and I assume there is only one type of consumption good.

Suppose that our world is composed of a finite number of countries assembled in a set  $I$  with cardinality  $|I|$ , where  $|I| > 2$ , country  $i$  produces the intermediates  $k = 1, \dots, N^i$ , and country  $i+1$  produces the intermediates:

$k = N^i + 1, \dots, N^i + N^{i+1}$  with  $\sum_{i=1}^{|I|} N^i = N$ . I assume that the countries do not overlap in the

types of intermediate goods that they produce. Consequently, there is no direct competition between domestic and foreign producers in the provision of a particular type of intermediate input.

Within each country, I assume free trade and no transaction costs for shipping goods. Due to transport expenses and trade barriers, there are transaction costs for shipping an intermediate good across borders. I parameterize the transaction costs between two countries  $i$  and  $j$  by  $b_{ij}$ , where  $0 < b_{ij} < 1$ , using an iceberg technology whereby, for each unit of intermediate good shipped from any country  $i$  to another country  $j$ , or the reverse, only  $1 - b_{ij}$  units arrive. I think of trading cost  $b_{ij}$  as reflecting the using up of real resources, not a transfer from one party to another. Notice no money is in our economy so far. The final good, and hence the single type of consumer good in this real economy, is used as a numeraire. Consequently, the price of the final good is the same in both countries and is normalized to

---

<sup>6</sup> This specification was proposed by Spence (1976), Dixit and Stiglitz (1977), and Ethier (1982).

one. The labor market is competitive within each country, and the total quantity of labor for a given country  $i$  is fixed at  $L^i$ . I further assume that the country that is larger in terms of numbers of products is larger in the same proportion in terms of labor. In other words, for any two countries  $i$  and  $j$ ,  $N^i/L^i = N^j/L^j$ , and I normalize each of these ratio at unity.

Taking as given the country's wage rate and the price  $P_k$  of each intermediate good, each firm maximizes its profit. The prices are measured in units of final good and are assumed to apply uniformly at the point of origin to all purchasers (domestic and foreign alike). Taking the first order conditions for the choices of intermediate inputs by the producers in a given country  $i$ , and setting the markup price of intermediates to some value  $P_k = \mu^i$  in country  $i$ , where  $1 \leq \mu^i \leq 1/\alpha$ , and aggregating over the firms, one can show<sup>7</sup> that the level of aggregate output in country  $i$  is given by:

$$Y^i = A^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} N^i \left[ \left( \frac{1}{\mu^i} \right)^{\alpha/(1-\alpha)} N^i + \sum_{\substack{j=1 \\ j \neq i}}^{|I|} \left( \frac{(1-b_{ij})}{\mu^j} \right)^{\alpha/(1-\alpha)} N^j \right] \quad (2)$$

Note that the markup parameters reduce the aggregate output through a lower demand for intermediates. Similarly, the trading costs  $b$ , reinforce the foreign countries' markups and also reduce the aggregate output.

The second part of equation (2) captures the quantity of intermediate goods produced in the remaining  $|I|-1$  countries and used for final goods' production in country  $i$ . One can derive from the first order conditions, mentioned above, the optimal quantity of intermediates provided by each country  $j$  ( $\neq i$ ). Hence, one can determine the value of these imported goods gross of shipping costs, by adding the value obtained after multiplying the optimal quantity imported from each country  $j$  ( $\neq i$ ) by  $\mu^j/(1-b_{ij})$ . As a result, the value of imports for the intermediate goods shipped from the other countries  $j$  ( $\neq i$ ) to country  $i$ , and denoted  $VIIIG^i$ , is given by:<sup>8</sup>

$$VIIIG^i = A^{1/(1-\alpha)} N^i \sum_{\substack{j=1 \\ j \neq i}}^{|I|} \left( \frac{(1-b_{ij})}{\mu^j} \right)^{\alpha/(1-\alpha)} N^j \quad (3)$$

By analogous reasoning, the value of the exports of intermediate goods from country  $i$  to other countries  $j$  ( $\neq i$ ), denoted  $VEIG^i$ , can be computed after multiplication by  $\mu^i/(1-b_{ij})$ :

<sup>7</sup> See Alesina and Barro (2002) for details on the case of two countries.

<sup>8</sup> Ibid.



$$VEIG^i = A^{1/(1-\alpha)} N^i \left( \frac{1}{\mu^i} \right)^{\alpha/(1-\alpha)} \sum_{\substack{j=1 \\ j \neq i}}^{|J|} (1-b_{ij})^{\alpha/(1-\alpha)} N^j \quad (4)$$

I assume here that there is no international trade in financial assets so that any net surplus or deficit in the trade of intermediate goods is matched by an equal net deficit or surplus in the trade of final goods. Note that from equations (3) and (4) the balanced trade in intermediate goods will naturally result if  $\mu^i = \mu^j$ .

The only firms in country  $i$  that make profits in equilibrium are the monopolistic providers of the intermediate goods. I assume that the ownership rights in these firms are evenly distributed across the households of country  $i$ . Consequently, the net income and consumption of the country  $i$ 's representative household corresponds to gross output less the production of intermediates plus the country's net surplus in intermediate trade with the  $|J|-1$  remaining countries. Hence, the consumption per capita in country  $i$  can be expressed as follows:

$$\frac{C^i}{N^i} = A^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} \left[ \left( \frac{1}{\mu^i} \right)^{1/(1-\alpha)} \left\{ (\mu^i - \alpha) N^i + \alpha (\mu^i - 1) \sum_{\substack{j=1 \\ j \neq i}}^{|J|} (1-b_{ij})^{\alpha/(1-\alpha)} N^j \right\} + (1-\alpha) \sum_{\substack{j=1 \\ j \neq i}}^{|J|} \left( \frac{(1-b_{ij})}{\mu^j} \right)^{\alpha/(1-\alpha)} N^j \right] \quad (5)$$

The analysis highlights how the transaction costs affect the volume of foreign trade, and therefore, the level of production and consumption in each country. Specifically equation (5) shows that a decline in  $b_{ij}$  has a positive impact on consumption in country  $i$ . Assuming that utility is increasing in consumption, then the utility of the representative consumer would be maximized if the world consisted of just one country, because cross-border transactions costs would then be eliminated. Notice that the formalization in equation (5) is an intermediate step. It will be used in the subsection below to derive a loss function—which is compatible with this framework—and is due to the loss of the monetary policy.

The trading cost  $b_{ij}$  involves several components. One is shipping cost, which depends on distance and available methods of transportation. Other components include government regulations, speaking the same language, and familiarity with foreign business practices. Of particular importance, trading costs  $b_{ij}$  would positively depend upon financial transactions involving currency exchanges. Here I am interested in the reduction of the trading cost

induced by removing the financial transactions involving currency exchanges through sharing the same currency.

Acknowledging that the adoption of a common currency entails costs (for example, the loss of monetary policy to counteract real shocks), consider two countries,  $i$  and  $j$ , that use different currencies but trade a lot because of other characteristics that reduce trading costs. The question one might ask is whether both countries would be motivated to reduce the financial costs of trading by adopting a common currency and bear the cost this adoption would involve.

Assuming that the incremental cost associated with the adoption of a common currency does not depend on the trading cost  $b_{ij}$ , the sufficient condition for the two countries to be willing to adopt a common currency is that the marginal effect of  $1 - b_{ij}$  on consumption be increasing in  $1 - b_{ij}$ . Note from equation (5) that the marginal effect of  $1 - b_{ij}$  on  $C^i/N^i$  is positive. Hence, if this effect rises with  $1 - b_{ij}$ , one is assured that at some point it will outweigh the cost of adopting a common currency, since the latter is assumed not to depend on  $1 - b_{ij}$ .

From equation (5) the marginal effect mentioned above can be expressed for any two countries  $i$  and  $j$ , as follows:

$$\frac{\partial(C^i/N^i)}{\partial(1-b_{ij})} = (1-b_{ij})^{(2\alpha-1)/(1-\alpha)} HN^j \quad (6)$$

where:

$$H = \alpha A^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} \left[ \left[ \alpha/(1-\alpha) \right] \left( \frac{1}{\mu'} \right)^{1/(1-\alpha)} (\mu^i - 1) + \left( \frac{1}{\mu'} \right)^{\alpha/(1-\alpha)} \right] \text{ is a positive constant.}$$

The sufficient condition will be met if the second derivative is positive, that is, if  $\alpha > 1/2$ , which is true if the intermediate goods are relatively close substitutes. To see the underlying intuition, remember that providers of the intermediate goods have monopoly power and would like to charge monopolistic prices. Now if the intermediate goods are relatively close substitutes, this will provide incentives to governments in the concerned countries to introduce some imperfect competition to bring the price of intermediate goods down even slightly. This can be done through trade incentives, in particular by removing financial transaction costs involving currency exchange costs. That's because promoting trade allows one to boycott goods produced in one country and import close substitutes from abroad.

To summarize, reducing  $b_{ij}$ —through removing currency exchange costs—saves on the trading costs, which become more important as the volume of trade increases. Clearly if one assumes that the tradable goods are relatively close substitutes, the model suggests that countries that trade more are more likely to form currency unions. Consequently, a rise in the trade shares among countries increases their incentive to form currency unions.

If one interprets  $j$  as being a currency bloc, it is clear that the larger the entity  $j$ , the greater the marginal benefit on joining  $j$ . This highlights the network-related benefit to  $i$  from joining  $j$ , so that the consumption of the representative agent in country  $i$  rises with the number of other countries using the same currency. Intuitively, the more countries use the same currency as country  $i$ , the easier it is for the representative agent in  $i$  to understand the price these countries quote, the less the transaction costs that the representative agent in country  $i$  is likely to pay for switching currencies to trade with them, and so the greater are its output and consumption.

### Monetary Policy and Costs of Currency Unions

The model has considered so far a real economy and highlights the benefits of sharing the same currency through reduced transaction costs. But these benefits of currency union are not without costs. Sharing the same currency especially entails the loss of monetary policy—hence, the need to introduce money in order to explore these costs and see whether they outweigh the benefits. Here I then introduce money in the economy. This allows me to evaluate the net benefit of adopting a currency union.

To introduce a role for monetary policy, I follow Barro and Teneyro (2000) in assuming that more specialized and less competitive products tend to exhibit less flexibility in their nominal prices. Specifically, I assume that the prices of intermediate goods exhibit some stickiness, whereas the prices of final goods are flexible. In order to model the costs related to the monetary policy, one can establish a relation between the consumption of a representative agent and actual and unexpected inflations. To this end, one simply needs to extend equations (2) and (5) to account for the effects of both actual and unexpected inflations. The relation will be derived from equations (2) and (5). More precisely, these equations will be extended to account for the effects of both actual and unexpected inflations. The loss function, which is derived according to the way these inflations affect consumption, can then be approximated by a quadratic function as follows.

$$\mathcal{L}_i = a\pi_i + (\gamma/2)\pi_i^2 + (\theta/2)\left[\phi(\pi_i - \pi_i^e) - z_i - \eta_i\right]^2 \quad (7)$$

where  $a > 0$ ,  $\gamma \geq 0$ ,  $\theta > 0$ ,  $\phi > 0$ ,  $z_i (> 0)$  is an increasing function of  $\mu^i$ , and  $\eta_i$  is an error term with zero mean, serial independence, and constant variance  $\sigma_{\eta_i}^2$ .

Country  $i$  can manage its monetary policy on its own or anchor to another country. I will now examine the expected loss function in each case. If country  $i$  chooses autonomy, then inflation is determined under discretion rules to minimize  $\mathcal{L}_i$ . In particular, following Barro and Gordon (1983), inflation will be set as follows:

$$\hat{\pi}_i = -\frac{a}{\gamma} + \frac{\theta\phi z_i}{\gamma} + \frac{\theta\phi\eta_i}{\gamma + \theta\phi^2} \quad (8)$$

The corresponding estimated loss is:

$$E\mathcal{L}_i = \frac{1}{2} \left[ -\frac{a^2}{\gamma} + \theta z_i^2 + \frac{(\theta\phi z)^2}{\gamma} + \frac{\theta\gamma\sigma_{\eta_i}^2}{\gamma + \theta\phi^2} \right] \quad (9)$$

I now turn to the case of anchorage. Suppose country  $j$  is a potential anchor for country  $i$ , specifically  $j$  can commit at least one period ahead and eliminate the inflation bias  $\frac{\theta\phi z_i}{\gamma}$ . I assume as in the case of the European Monetary Union that the anchor adjusts its monetary policy to align better with the interests of its clients. In particular, the anchor minimizes the weighted average of the loss functions of the union members, assuming that the clients compensate the anchor for deviating from policies that are otherwise best for the anchor's domestic residents. Thus, for a union formed only of countries  $i$  and  $j$ , the anchor determines its policy rules to minimize the following total loss:

$$\mathcal{L} = \tau_i \mathcal{L}_i + \tau_j \mathcal{L}_j \quad (10)$$

where  $\tau_i = C_i / (C_i + C_j)$  and  $\tau_j = C_j / (C_i + C_j)$ . The weights are expressed as consumption ratios because the net loss due to inflation,  $\mathcal{L}_i$ , in equation (7) applies as a fraction of country  $i$ 's consumption  $C_i$ .

If country  $i$  gives up its autonomy and chooses country  $j$  as an anchor, then country  $i$  will import the low inflation  $\pi_j$  from  $j$ , and its inflation rate will be given by:

$$\pi_i^j = \pi_j + \varepsilon_{ij} \quad (11)$$

where  $\varepsilon_{ij}$  is an exogenous error term, which captures the rate of change of the price of a basket of final goods in country  $i$  relative to the change of the price of the same basket in country  $j$ . This error term is assumed to be serially independent with zero mean and constant variance  $\sigma_\varepsilon^2$ , and is distributed independently of  $\eta_i$  and  $\eta_j$ .

Assuming that both countries have the same underlying preference and cost parameters, but the shocks  $\eta_i$  and  $\eta_j$  can differ, then country  $j$  can use a contingent rule to minimize the prior expectation of  $\mathcal{L}$ , and set its policy rule as follows:

$$\pi_j = v + v_i \eta_i + v_j \eta_j + v_\varepsilon \varepsilon_{ij} \quad (12)$$

The optimal inflation turns out to be:

$$\hat{\pi}_j = -\frac{a}{\gamma} + \tau_i \frac{\theta\phi}{\gamma + \theta\phi^2} \eta_i + \tau_j \frac{\theta\phi}{\gamma + \theta\phi^2} \eta_j - \tau_i \varepsilon_{ij} \quad (13)$$

Note that the inflation bias  $\frac{\theta\phi z_i}{\gamma}$  is now effectively eliminated, and that the optimal inflation now reacts to the shocks  $\eta_i$  and  $\eta_j$ , in accordance with the consumption shares  $\tau_i$  and  $\tau_j$ . In

addition, country  $j$  partly offsets an increase in inflation in country  $i$  due to  $\varepsilon_{ij}$  by reducing its inflation, and the extent of the offsetting effect is given by  $\tau_i$ . Note also that, if country  $j$  were not to internalize the loss of country  $i$ , country  $j$  would simply set its inflation to minimize  $\mathcal{L}_j$ , which would be obtained by setting  $\tau_i = 0$  and  $\tau_j = 1$  in equation (13). That is:

$$\hat{\pi}_j = -\frac{a}{\gamma} + \frac{\theta\phi}{\gamma + \theta\phi^2}\eta_j \quad (14)$$

The expected loss for country  $i$  under agreement where inflation is set according to (13) is given by:

$$E\mathcal{L}_i^j = -\frac{a^2}{2\gamma} + \frac{\theta z_i^2}{2} + \frac{(\gamma + \theta\phi^2)\sigma_\varepsilon^2}{2} + \frac{\theta\sigma_\eta^2}{2} + \frac{\theta^2\phi^2\sigma_{\eta_j}^2}{2(\gamma + \theta\phi^2)} - \frac{\theta^2\phi^2\text{Cov}(\eta_i, \eta_j)}{\gamma + \theta\phi^2} + \frac{1}{2}(\tau_i^2) \left[ (\gamma + \theta\phi^2)\sigma_\varepsilon^2 + \left( \frac{\theta^2\phi^2}{\gamma + \theta\phi^2} \right) \text{Var}(\eta_i - \eta_j) \right] \quad (15)$$

Notice that the last term involving  $\tau_i$  stands for the accommodation cost that country  $i$  would have to pay to country  $j$  to compensate the latter for choosing inflation according to (13) instead of choosing inflation on its own according to (14). It is easy to see that the cost for country  $i$  to form a currency union with country  $j$ , which is expressed in equation (15), is lower, the lower are the variances of relative prices and output shocks,  $\sigma_\varepsilon^2$  and  $\text{Var}(\eta_i - \eta_j)$ .

### Currency Union in Static Equilibrium

Above, I separately established the benefit of currency union through the impact of a reduced trading cost, and the cost of currency union through the effects of monetary policy. Here I combine the trade benefits and monetary policy effects. Thus, the net benefit (per person) for country  $i$  from forming a currency union with country  $j$  can be expressed as follows:

$$\Omega_i^j = (1 - b_{ij})^{(2\alpha-1)/(1-\alpha)} HN^j - E\mathcal{L}_i^j \quad (16)$$

where  $H$  is defined in equation (6) and  $E\mathcal{L}_i^j$  by equation (15).

Similarly, the net benefit for country  $i$  from being autonomous is given by:

$$\Omega_i = -E\mathcal{L}_i \quad (17)$$

where  $E\mathcal{L}_i$  is defined by equation (9). Notice that if country  $i$  stays alone, there would be no currency union benefit as expressed in equation (6). Hence, the net benefit boils down to the expression (17).

Assuming that the utility is linear in the net benefit, one can express the one-period utility of the representative agent in country  $i$  as follows:

$$U_i^j = (1 - b_{ij})^{(2\alpha-1)/(1-\alpha)} HN^j - E\mathcal{L}_i^j, \text{ if } i \text{ is linked to } j \quad (18)$$

and:

$$U_i^i = -E\mathcal{L}_i, \text{ if } i \text{ is autonomous} \quad (19)$$

Hence country  $i$  would like to form a currency union with country  $j$  if:

$$U_i^j > U_i^i \quad (20)$$

Notice from equation (18) that high  $N^j$ , that is, more varieties and hence more trade, increases the utility for country  $i$  from linking to country  $j$ .

From equations (15), (16) and (18), one can see that in a static framework, linkage is more attractive, the lower the variances of relative prices and output shocks  $\sigma_\varepsilon^2$  and  $Var(\eta_i - \eta_j)$ .

In this framework specifically from the production function defined in equation (1), notice that international trade affects output since intermediate goods imported from abroad enter in the production of final goods at home. Therefore, all else being equal, the intensity of trade is correlated with output shock. In other words, more trade in intermediate goods leads to high correlation of output shocks. Hence, I claim that more trade is likely to reduce output variance  $Var(\eta_i - \eta_j)$ , and the variance of the relative price  $\sigma_\varepsilon^2$ . This is consistent with

Frankel and Rose's (1998) empirical finding that over longer horizons trade among countries increases the correlation between their outputs. Thus, more trade not only increases the marginal benefit from reduced trading cost as expressed in the first term of equation (18) but also reduces the monetary policy cost that would result from linkage. *This suggests that countries' utilities are increasing in trade and that countries that trade more are more likely to form a currency union.*

## B. Currency Bloc Formation as a Dynamic Process

In this section, I describe the configuration of blocs and present a dynamic stochastic currency bloc formation game. It helps to study the convergence towards a steady state or a stable bloc configuration as well as the path dependency of the process.

### Bloc Configuration

I consider above a finite set of countries ( $I$ ). Denote by  $\chi$  the finite set of all possible partitions of  $I$ . A bloc configuration is a particular partition  $x$  of  $I$ ; that is, any element belonging to  $\chi$  is a bloc configuration. The bloc configuration is also referred to as state. A bloc configuration is a partition of countries into currency blocs, where a bloc can consist of a single or many countries. I consider an infinite horizon, where the current state or bloc configuration is denoted by  $x^t$ . Let  $|x^t|$  be the cardinality of  $x^t$ , that is the number of currency blocs in the bloc configuration  $x^t$ . Within the bloc structure  $x$ , denote by  $x_i$  the currency

bloc to which country  $i$  belongs. Notice that for a given bloc configuration  $x$ , countries  $i$  and  $j$  belong to the same currency bloc if and only if  $x_i = x_j$ .

### **Dynamic Currency Bloc Formation Game**

Before starting the process of currency bloc formation, it is worth mentioning some prerequisites for the analysis. Trade affects the utility of joining a currency union, as highlighted above in subsection (A); and the pattern of trade flows changes over time. So a world in which countries were initially using their own currencies or where there is a large number of small blocs will not be able to jump to a steady state. By steady state I mean a bloc configuration in which no country belonging to a currency bloc would want to leave in order to have its own currency or to join another bloc, and no country not belonging to a bloc would want to join one.

In order to convince us that a one-step jump from a status quo—to a steady state where a big currency bloc emerges—is likely to be the exception but not the rule, I propose the following example, which highlights that the emergence of currency blocs is likely to be gradual and that is driven by trade. Consider a world constituted at time  $t$  of a currency bloc ( $CB$ ) and four individual countries  $C_1, C_2, C_3, C_4$ . The analysis in subsection (A) highlights that for a country to form a currency union with another country, the volume of trade needs to be sufficiently large so that the benefits coming from a reduced transaction cost outweigh the loss coming from the monetary side. In other words, trade needs to reach some threshold, which I explicitly show later in the paper.

Suppose that the pattern of trade flows between the different entities (currency bloc or countries) is as follows:  $Tr(CB, C_1) = \Lambda_1$ ,  $Tr(CB, C_2) = Tr(CB, C_3) = Tr(CB, C_4) = \varepsilon$ ,  $Tr(C_1, C_2) = \Lambda_2$ ,  $Tr(C_2, C_3) = \Lambda_3$ ,  $Tr(C_3, C_4) = \Lambda_4$ ; trade flows in any other direction or between any other entities are zero, and  $\Lambda_1 > \Lambda_2 > \Lambda_3 > \Lambda_4 > \varepsilon$ . In addition,  $\varepsilon$  is assumed to be small and  $\Lambda_1$  relatively high, so that the bloc  $CB$  trades substantially with  $C_1$  but very little with the remaining countries. Suppose that  $\Lambda_1$  is high enough for  $C_1$  to be willing and able to join  $CB$ , and that  $\varepsilon$  is not high enough for  $C_2$  to join. However, if  $C_1$  joins  $CB$ , trade between the new emerging bloc is high enough for  $C_2$  to join, that is  $\Lambda_2 + \varepsilon$  is high enough and reaches the threshold of joining.

Suppose that the trade flows  $\Lambda_3 + \varepsilon$  and  $\Lambda_4 + \varepsilon$  are not high enough and do not reach the threshold. Based on the trade pattern just described, by rational expectation, countries  $C_1$  and  $C_2$  would be able to join at time  $t$ , because  $CB$ ,  $C_1$ , and  $C_2$  can anticipate that if  $C_1$  joins,  $\Lambda_2 + \varepsilon$  will be high enough and make it optimal for  $C_2$  to join as well. However contrary to the case between  $C_1$  and  $C_2$ , country  $C_3$  (though a trade partner for  $C_2$ ), will not be able to join immediately with  $C_2$  because  $\Lambda_3 + \varepsilon$  is not enough. But, as the new bloc

$(CB + C_1 + C_2)$  emerges or is realized, the pattern of trade flows between  $CB$  and  $C_1$ ;  $CB$  and  $C_2$ ;  $C_1$  and  $C_2$  will change because of the positive impact of currency union on trade. The resulting income effect, will lead to additional trade between the bloc  $(CB + C_1 + C_2)$  and  $C_3$ , and between  $(CB + C_1 + C_2)$  and  $C_4$ , as high income is likely to lead to high demand and hence high imports. Denoting these additional trades respectively by  $\Gamma_3$  and  $\Gamma_4$ , trade flows between the new bloc and countries  $C_1$  and  $C_2$  will respectively be  $\Lambda_3 + \varepsilon + \Gamma_3$  and  $\Lambda_4 + \varepsilon + \Gamma_4$ . Assume that  $\Gamma_3$  is sufficiently high so that  $\Lambda_3 + \varepsilon + \Gamma_3$  reach the threshold of joining. This will make it optimal for country  $C_3$  to join the bloc  $(CB + C_1 + C_2)$ —which would find it optimal to accept—as  $C_3$  would have become a key trading partner. This would become possible however, only after the bloc  $(CB + C_1 + C_2)$  were realized, that is, in sequence and not simultaneously. Assuming that  $\Gamma_4$  is not high enough for  $\Lambda_4 + \varepsilon + \Gamma_4$  to reach the threshold,  $C_4$  will not even be able to join with  $C_3$  but would have to await another round.

Notice that international trade naturally increases over time because of innovation and reduced transportation costs. This trade dynamic can alone induce some dynamics in the process of the emergence of a currency bloc. But the focus of this paper is more on the externalities coming from other countries that had previously joined. Hence the emphasis on trade induced by an earlier expansion of a currency bloc. In the dynamic model below, the trade parameter  $N^x$  is considered to change with the state or bloc configuration  $x$ .

The example described above clearly shows that jumping to the steady state is not always possible. Countries often join in sequence or gradually. This conforms to the reality of currency bloc formation. For instance, the CFA zones in Africa were set up in 1948, but only recently (1997) did Guinea Bissau join. The EMU began in 1998 with eleven countries, but later Greece joined in 2001 and the bloc will enlarge further after 2004 with new entrants. This highlights that the emergence of big currency blocs is gradual. The modest goal of this paper is to propose a model to capture in a stylized manner this reality of gradual emergence of currency blocs.

In this model, for simplicity of tractability I will assume that from one period to another only two blocs/countries will be allowed to merge in order to form a bigger bloc. That is, the number of currencies in the world will either be the same (status quo) or decrease by unity from one period to another.

I now turn to the dynamic model. Each period begins with nature choosing a proposer  $j \in I$ . Once chosen, given the current state  $x^t$ , country  $j$  can propose a bloc structure  $y$  from the set of compatible bloc configurations, where compatibility is defined as follows. A bloc



configuration  $y$  is compatible with proposer  $j$ , given the current state  $x^t$  if the following conditions are satisfied:<sup>9</sup>

$$\begin{aligned}
 (a) \quad & |y| = |x^t| - 1 \text{ or } y = x^t \\
 (b) \quad & \text{For } i \in I, \text{ if } i \notin y_j \text{ then } y_i = x_i^t \\
 (c) \quad & \text{If } y \neq x^t, \text{ then } y_j \neq x_j^t
 \end{aligned} \tag{21}$$

Denote by  $S(x^t, j)$  the set of bloc configurations compatible with proposer  $j$ , given the current state  $x^t$ .  $S(x^t, j)$  represents the set of possible actions for  $j$ , given the current state  $x^t$ . Condition (a) restricts the proposal to a new bloc configuration where either two currency blocs are merged, reducing the cardinality of the existing bloc configuration by unity, or the status quo is maintained. Condition (b) states that within the newly proposed bloc configuration, countries that are not in the proposer's currency bloc remain in their previous currency blocs. Finally, condition (c) states that if the newly proposed bloc structure is different from the previous one, the proposer's new currency bloc will not be the same as the one it was in before; it will have merged with another currency bloc.

After a proposal has been made by a country  $j$ , each country that would belong to  $j$ 's currency bloc  $y_j$  in the new proposed bloc configuration has the opportunity to veto the proposal and maintain the status quo. Thus, once a proposal  $y$  is made by  $j$ , the response of each country  $i$  belonging to  $y_j$ , is to choose an action  $r_i(y)$  in the set  $\{Accept, Reject\}$ .

Notice that each period of the game has three stages: nature moves first, then the proposer, and finally the relevant countries accept or reject the proposal. Note however that the strategy space for each country at the beginning of the period is the same, but the action set for each possible proposer evolves as the state changes. Given a state  $x$ , a pure strategy for each country  $j$  is (1) a proposal  $y \in S(x, j)$  and (2) for each proposal  $y \in S(x, j)$  a list of  $r_i(y) \in \{Accept, Reject\}$ , where  $i \neq j$ . If  $i \notin y_j$ , then  $r_i(y) = Accept$ , since in this case the country has no veto power.

The period utility of the representative agent in each country  $i$  for each state  $x$  is  $U_i(x)$  and defined by equations (18) and (19), depending on whether or not the country is linked to another. Notice that the linkage is now captured by the state  $x$  and that  $j$  in equation (18) will be thought as grouping all the other countries in the same currency bloc as  $i$ . Also notice that the utility in country  $i$  only depends on the currency bloc  $x_i$  to which country  $i$  belongs, and should be written as  $U_i(x_i)$ , but for convenience I use the notation  $U_i(x)$ . I assume that the representative agent in country  $i$  has a discount factor  $\delta_i \in (0,1)$ . I also assume that for

---

<sup>9</sup> Tjornhom (2000) uses similar characterizations in the context of trade bloc formation.

each period, each country has the same probability  $q$  to be chosen by the nature as the proposer, that is,  $q = 1/|I|$ .

To summarize, given a state  $x$ , a country  $j$  is chosen by the nature with probability  $1/|I|$  to propose a new bloc configuration  $y \in S(x, j)$  then each country in the proposer's existing bloc and the merging bloc responds either by accepting or rejecting the proposal. The game is played in a manner that voting to accept or reject is sequential with the voting order chosen randomly. Given that each country in the two merging blocs has the right to veto the proposal, two blocs will successfully merge if each of the concerning countries accepts the proposal. If the proposal is accepted by all the relevant countries, the new state is realized and the process is repeated in the next period. If the proposal is rejected or the proposer proposes the status quo, then the state does not change; the status quo is maintained; and the game moves to the next period.

Based on the structure of the game, given the current period  $t$  (hence the state  $x^t$ ), and a proposal  $y \in S(x^t, j(t))$ , the next state is defined as follows:

$$x^{t+1} = \begin{cases} y & \text{if for all } i \in y_{j(t)}, r_i(y) = \text{Accept} \\ x^t & \text{if there exists } i \in y_{j(t)} / r_i(y) = \text{Reject, or } y = x^t \end{cases} \quad (22)$$

In this infinite horizon game, the decision to join a currency bloc will not be based on the period utility but on the intertemporal or discounted utility. I now turn to the discounted utility. Each period, a state will emerge according to some probabilistic path. Denote by  $\Delta(\chi)$  the space of all probabilities  $\sigma$  on  $\chi$ . Then for any sequence  $\sigma \equiv \{\sigma_t\}$  in  $\Delta(\chi)$ , the discounted utility of the representative agent in country  $i$  is given by:

$$V_i(\sigma) = \sum_{t=0}^{\infty} \delta_i^t \left( \sum_{x \in \chi} \sigma_t(x) \right) U_i(x) \quad (23)$$

$\sigma_t(x)$  captures the probability that the state or bloc configuration  $x$  occurs at  $t$ .

Equation (27) can be rewritten as follows:

$$V_i(\sigma) = \sum_{t=0}^{\infty} \delta_i^t \left( \sum_{x \in \chi} \sigma_t(x) \right) \left[ (1 - b_{ix})^{(2\alpha-1)/(1-\alpha)} HN^{x_i} - E\mathcal{L}_i^x \right] \quad (24)$$

where  $b_{ix}$  captures the transaction costs between country  $i$  and other countries of the bloc  $x_i$ , and  $N^{x_i}$  captures the size of intermediate goods and hence the size of trade between country  $i$  and others member of the  $x_i$ . Finally  $E\mathcal{L}_i^x$  is defined as in equation (15) where country  $j$  is now replaced by currency bloc  $x_i$ , and abuse of notation  $E\mathcal{L}_i^x$  is used instead of  $E\mathcal{L}_i^{x_i}$ . Note from (24) that the greater  $N^{x_i}$ , that is, the higher the trade size between country  $i$  and other

members of bloc  $x_i$ , the greater the intertemporal utility country  $i$  derives by belonging the bloc  $x_i$ , as it increases the first term in bracket and lowers the second.

A process of currency bloc formation (*PCBF*) is a transition probability  $p: \mathcal{X} \times \mathcal{X} \rightarrow [0,1]$ , such that  $\sum_{y \in \mathcal{Z}} p(x, y) = 1$  for each  $x \in \mathcal{X}$ .

The probability  $p$  captures the transitions from one state to another, the transitions being induced by some coalitions of countries that stand to benefit. Each coalition consists of the two merging groups after a proposal is made.

### Equilibrium of the Dynamic Game

I will use the stationary Markov perfect equilibrium concept so that the history of currency bloc formation will not matter when countries make their decisions at a given state  $x$ . Strategies are stationary in the sense that countries play the same strategies whenever the state is  $x$ . As countries play Markov, they induce a transition probability that follows a Markov process. Starting from any given state  $x$ , and noticing that nature will first choose a proposer  $j$  with a probability  $1/|I|$ , the value function under a Markov process  $p$  is given by the following Bellman-type equation:

$$V_i(x, p) = \sum_{j=1}^{|I|} \frac{1}{|I|} \left[ U_i(x) + \delta_i \cdot \sum_{y \in S(x, j)} p(x, y) V_i(y, p) \right] \quad (25)$$

Given a proposal  $y$ , I am now ready to define the conditions of its acceptance by a country  $i \in y_j$ . These will be used to define profitable moves and impose restrictions on the transition process. Given a state  $x$  and a proposer  $j$ , a bloc configuration  $y$  is strictly acceptable for country  $i$  if:

$$U_i(y) + \delta_i \cdot \sum_{z \in S(y, j)} p(y, z) V_i(z, p) > U_i(x) + \delta_i \cdot \sum_{y \in S(x, j)} p(x, y) V_i(y, p) \quad (26)$$

It will be weakly acceptable if the inequality in (26) is replaced by  $\geq$ . Equation (26) can be written explicitly as follows:

$$\begin{aligned} & \left[ (1 - b_{iy})^{(2\alpha-1)/(1-\alpha)} HN^{y_i} - E\mathcal{L}_i^y \right] + \delta_i \cdot \sum_{z \in S(y, j)} p(y, z) V_i(z, p) \\ & > \\ & \left[ (1 - b_{ix})^{(2\alpha-1)/(1-\alpha)} HN^{x_i} - E\mathcal{L}_i^x \right] + \delta_i \cdot \sum_{y \in S(x, j)} p(x, y) V_i(y, p) \end{aligned} \quad (27)$$

Since the expression in brackets is increasing in the size of trade, which is captured by  $N$ , a country  $i$  would accept a proposal  $y$  if it trades a lot with the members of country  $j$ 's currency bloc.

For a given transition process  $p$ , and a state  $x$ , there will be a strictly profitable move from  $x$  under  $p$ , if there exists  $j \in I$  with a proposal  $y \neq x$  such that (26) or (27) holds for all  $i \in y_j$ . The move will be weakly profitable if the inequality in (26) or (27) holds with  $\geq$  for all  $i \in y_j$ . Finally, a move  $y$  is efficient for  $y_j$  if there is no other move, say  $y'$ , such that the following holds for all  $i \in y_j$ :

$$U_i(y') + \delta_i \cdot \sum_{z \in S(y', j)} p(y', z) V_i(z, p) > U_i(y) + \delta_i \cdot \sum_{z \in S(y, j)} p(y, z) V_i(z, p) \quad (28)$$

In other words, the problem of the proposer is to propose a bloc structure  $y^* \in S(x, j)$  according to:

$$y^* = \arg \max_{y \in S(x, j)} \left\{ \left[ (1 - b_{iy})^{(2\alpha-1)/(1-\alpha)} HN^{y_i} - E\mathcal{L}_i^y \right] + \delta_i \cdot \sum_{z \in S(y, j)} p(y, z) V_i(z, p) \right\} \quad (29)$$

subject to acceptability conditions, and where  $V_i$  is defined in the same spirit as expression (24) and satisfied the Bellman-type equation (25). Subject to the acceptability condition, it is clear that the proposer will propose the bloc configuration that provides the highest utility, that is a bloc configuration with high  $N^y$  or substantial trade level. In other words, the proposer will propose a configuration where it will join the bloc with which it trades the most, with, again subject to acceptability. Of course the relevant countries will only accept a bloc configuration to the extent that they trade substantially with the bloc to which they will belong. Note that, as I have argued earlier, more trade reduces  $Var(\eta_i - \eta_j)$  and  $\sigma_\varepsilon^2$ , and increases country utilities. I now claim that the following is true.

**Proposition 1.** *Suppose there is a currency bloc with  $k$  countries  $x_k = \{1, 2, \dots, k\}$ , and  $n - k$  individual countries  $k + 1, \dots, n$ . For any individual country  $i \in (k + 1, \dots, n)$ , there exists a trade cut-off level  $N_i^{x_k^*}$  between country  $i$  and the bloc  $x_k$  such that country  $i$  is willing and able to join the bloc  $x_k$  if and only if the level of trade between country  $i$  and the bloc  $x_k$  is greater or equal to  $N_i^{x_k^*}$ , that is,  $N_i^{x_k} \geq N_i^{x_k^*}$ .*

A key implication of this result is that trade volume determines the sequence of joining a currency bloc. In other words, given a currency bloc and a number of individual countries characterized by substantial differences in trade level, some countries will join the bloc earlier than others. A proof of this result appears in appendix.

It might appear that the emphasis is only on trade, with less focus on the symmetry of output shocks. However, I provide here an additional argument to back my claim on the timing of joining a currency bloc. Consider a currency bloc with  $n$  countries, where each country  $i$ 's

output shock is  $\eta_i$ . The common central bank sets its monetary policy to counteract the aggregate shock  $\sum_{i=1}^n \alpha_i \eta_i$ , where  $\alpha_i$  stands for the weight put on country  $i$ 's output shock. Suppose there are two other countries  $(n+1)$  and  $(n+2)$ , which substantially trade with one another. Suppose further that the  $(n+1)$ th country substantially trades with the bloc so that the benefits from reduced transaction cost are high enough to make it optimal for country  $(n+1)$  to join the bloc, which also finds it optimal and accepts that  $(n+1)$  join at time  $t$ . However, the  $(n+2)$ th country trades less with the bloc and cannot join at time  $t$ . As country  $(n+1)$  joins, the new emerging bloc will now have to set its monetary policy to counteract the aggregate shock  $\sum_{i=1}^{n+1} \alpha_i \eta_i$ . Suppose that country  $(n+1)$  is large and that the weight  $\alpha_{n+1}$  is high enough. Since countries  $(n+1)$  and  $(n+2)$  trade substantially with one another, the variance  $Var(\eta_{n+1} - \eta_{n+2})$  is low as trade affects output (see equation (1)). This implies that the variance  $Var\left(\sum_{i=1}^{n+1} \alpha_i \eta_i - \eta_{n+2}\right)$  is lower than  $Var\left(\sum_{i=1}^n \alpha_i \eta_i - \eta_{n+2}\right)$ , especially because  $\alpha_{n+1}$  is high. Thus, not only country  $(n+2)$  now trades more with the new bloc including country  $(n+1)$  and hence has higher benefits from reduced transaction costs, but also its shock is now more correlated with the new bloc's aggregate shock. From equation (18), where  $j$  is replaced by the new bloc, this leads to a net increase in country  $(n+2)$  utility from joining the bloc. It becomes optimal for country  $(n+2)$  to join the new bloc at time  $t+1$ . Since country  $(n+2)$ 's shock is now more correlated with the bloc's shock and that  $(n+2)$ 's admission will increase the intra-bloc trade volume and hence the aggregate benefits from reduced transaction costs, it is optimal for the bloc to accept country  $(n+2)$ . I implicitly assume an equitable distribution scheme of the benefits of the bloc. This shock correlation argument further confirms the gradual expansion of the currency bloc.

The implication drawn from the result above induces an ordering of states in the currency bloc formation process. States must move down the ordering from finer to coarser partitions or the status quo is maintained and the current bloc configuration remains unchanged.

**Definition 1.** A *stationary Markov perfect equilibrium* is a profile of strategies (proposals)  $\{y(x^t, j(t))\}_{t=0}^{\infty}$  and value function  $\{V_i[y(x^t, j(t)), p]\}_{t=0}^{\infty}$  such that  $y(x^t, j(t))$  is weakly profitable and efficient move for  $y_{j(t)}$  from  $x$ . This equilibrium profiles induces an equilibrium process of currency bloc formation (*EPCBF*)  $p^*$ , which is characterized by the following:

(1) Whenever  $p^*(x, y) > 0$  for some  $y \neq x$ , then there is a country  $j$  with proposal  $y$ , such that  $y$  is weakly profitable and efficient move for  $y_j$  from  $x$ .

(2) If there is a strictly profitable move from  $x$ , then  $p^*(x, x) = 0$  and there is a strictly profitable and efficient move  $y$  with  $p^*(x, y) > 0$ .

I can now define an equilibrium outcome.

**Definition 2.** An *equilibrium outcome* of the currency bloc formation game is a bloc configuration  $x^*$  that can be supported by an *EPCBF*  $p$ . That is, starting from a state  $x$ , there exists an integer  $k \geq 1$  such that  $p^{(k)}(x, x^*) > 0$  and  $p(x^*, x^*) = 1$ , where notation  $p^{(k)}$  describes the  $k$ -step transition probability derived from  $p$  in the usual way. In this case, I say that  $p$  is an *absorbing process* and  $x^*$  an *absorbing state*.

In other words, an equilibrium outcome is a configuration of currency blocs in which no country belonging to a bloc would like to leave the bloc to have its own currency or to join another bloc. In addition, no country not belonging to a bloc would like to join one. The following establishes that currency bloc formation is path dependent and can lead to a stable outcome.

**Proposition 2.** *In a region with a finite number of countries, and hence a finite number of partition of countries, there exists a currency bloc formation process which has a steady state, that is, a stable equilibrium bloc configuration.*

To characterize an equilibrium outcome, one needs a specific case. But before turning to that, I summarize the key insight that the framework highlights: Countries' decision to join a bloc (by proposing or accepting a new bloc configuration) is driven according to equations (24) and (25) by their intertemporal utilities, which depend on the within-period utilities and the discount factors. Suppose that the discount factors are the same for countries. For a given discount factor, then the key driving force will be the within-period utilities, which turn out to be increasing in trade volume as discussed above.

From the perspective of the current period  $t$ , a country may reject the option to join a currency bloc today but accept it in the future. This is because its trade volume with the bloc may increase in the future, leading to a higher intertemporal utility for the country. Note that the value function defined in equation (25) depends on the state or bloc configuration. Thus, a country may have rejected joining a bloc given a current state  $x$ , but would accept it given a different current state  $x'$ . In other words, it might not be optimal for a given country  $i$  to join a given bloc in period  $t$ . But if in period  $t+1$  one of its key trade partner, say  $j$ , joins the bloc, country  $i$  may find it optimal to join the bloc in period  $t+2$ , because its trade share with the new bloc, which now includes  $j$ , would have increased. Therefore, this model helps explain the gradual emergence of currency blocs.

### III. CHARACTERIZING THE DYNAMIC AND EQUILIBRIUM OUTCOME

For the characterization of an equilibrium outcome, Alesina and Barro (2002) propose in a static framework the equilibrium configuration of currency unions for any finite number of countries. I will not repeat that here. Rather, this paper complements their work and studies the dynamic that governs the transition toward this equilibrium outcome. Here I simply propose what the dynamic and equilibrium outcome look like in the case of three countries. This also illustrates how the currency bloc formation is path dependent. Finally, I elaborate on the expansion of EMU in light of the model.

#### A. Dynamic and Equilibrium Outcome in Three-Country Case

The following example is inspired from other examples related to the coalition formation games studied in Konishi and Ray (2003), and Bogomolnaia and Jackson (2002). Consider a region with three countries 1, 2, and 3, that is,  $|I|=3$  and  $I = \{1,2,3\}$ . For a bloc configuration  $x$ ,  $U_1(x)$ ,  $U_2(x)$ ,  $U_3(x)$ —defined as in equation (24)—represent the payoffs that countries 1, 2, and 3 derive respectively from the blocs to which they belong in the bloc configuration  $x$ . Denote by  $U^x$  the vector of these payoffs that is  $U^x = (U_1(x), U_2(x), U_3(x))$ . Suppose that for  $x = (\{1,2\}, \{3\})$ ,  $U^{(\{1,2\}, \{3\})} = (3, 3, 0)$ . Similarly, suppose  $U^{(\{1\}, \{2,3\})} = (0, 4, 1)$ ,  $U^{(\{1,2,3\})} = (2, 2, 2)$ , and that for any other bloc structure  $x$ ,  $U^x = (0, 0, 0)$ .

In this three-country case, there are five possible bloc structures:  $x_1 = (\{1\}, \{2\}, \{3\})$ ,  $x_2 = (\{1,2\}, \{3\})$ ,  $x_3 = (\{1,3\}, \{2\})$ ,  $x_4 = (\{1\}, \{2,3\})$ ,  $x_5 = (\{1,2,3\})$ . Suppose that the current state is  $x_1 = (\{1\}, \{2\}, \{3\})$ , then each country receives a payoff of zero. As a consequence, countries 2 and 3 can easily convince each other to form a bloc, where country 2 can get 4 and country 3 can get 1. Note that country 3 would like country 1 to join, as this would increase country 3's payoff to 2; but country 2 will veto such a move as it would decrease country 2's payoff to 2. Country 3, which would be worse off by forming an independent bloc with country 1, will simply accept country 2's veto and stay with country 2. Intuitively, country 1 might be seen as a bad anchor with a bad inflation record, but it trades and has a more correlated shock with country 3. As a result, country 3 will not be able to import low inflation from country 1 and will not be better off being only with country 1. However, consider if country 2—which can be seen as a very good anchor and with whom country 3 is better off—were to accept country 1 and take into account its shock in setting the common monetary policy for the bloc. This would provide further benefits to country 3, whose shock is correlated with country 1's shock. However, this would be too demanding for country 2, which would reject country 1. Therefore, the outcome would be  $x_4 = (\{1\}, \{2,3\})$ .

Let us now assume other current states. If the current state is  $x_2 = (\{1,2\}, \{3\})$ , then the bloc  $\{1,2\}$  would be worse off accepting country 3. They will not do so since their payoffs would decrease from 3 to 2. Though country 2 would like to cut its link with country 1 and can easily convince and form a bloc with country 3, given the irreversibility of action in this framework, this will not be allowed. As a result, the status quo that is  $x_2 = (\{1,2\}, \{3\})$  will prevail in this case. However, if reversibility were allowed, country 2 would break up its tie with country 1 and would form a bloc with country 3, leading to the bloc structure  $x_4 = (\{1\}, \{2,3\})$  as outcome.

If the current state is  $x_3 = (\{1,3\}, \{2\})$ , then by an argument similar to those developed above, the bloc  $\{1,3\}$  and  $\{2\}$  have an incentive to convince each other to form a larger bloc. The equilibrium outcome would be  $x_5 = (\{1,2,3\})$  and each country would increase its payoff from 0 to 2.

If the current state is  $x_4 = (\{1\}, \{2,3\})$ , then there would be no move, and the status quo would prevail. Finally, if the current state is  $x_5 = (\{1,2,3\})$ , in this framework no country has incentive to move, so the status quo prevails. However, should reversibility be allowed, country 2 would in a first step break up its link with countries 1 and 3, thus leading to the intermediate bloc structure  $x_3 = (\{1,3\}, \{2\})$ . In a second step, country 2 can now easily convince country 3, which now will be better off accepting. This would lead to the bloc structure  $x_4 = (\{1\}, \{2,3\})$  as outcome. The macroeconomic intuition is similar to the one provided earlier in this example.

Consistent with irreversibility of actions, the scheme described above and is indeed an *EPCBF*, regardless the discount factor, as all the moves are incentive compatible. It also illustrates that currency bloc formation is path dependent as it leads to different outcomes depending on the starting point.

However, the scheme described with reversibility of actions is an *EPCBF* only if country 2's discount factor is greater than 1/2. To see why, consider the case in which the current state is  $x_5 = (\{1,2,3\})$ . Normally the only bloc structure that can defeat  $x_5$  is a bloc structure with a coalition formed by countries 1 and 2, that is  $x_2 = (\{1,2\}, \{3\})$ . However, if country 1 is sufficiently forward looking, country 1 would not accept such a proposal since it would expect to be abandoned because country 2 would be highly better off to form a bloc with country 3 to achieve the bloc structure  $x_4$ . Thus, country 1 would be better off by not deviating from  $x_5$  in the first place. The reason why the bloc structure  $x_5$  is not stable is that country 2 deviates alone, expecting to create a further subsequent move with country 3. Country 2 with a discount factor exceeding 1/2 is willing to suffer from a low payoff for one period



right after the unilateral deviation, and enjoys higher payoffs forever from the next period on. Note that with a discount factor exceeding  $1/2$ , country 2 has an incentive to give up a payoff of 2 today to get a payoff of 4 tomorrow, highlighting the importance of looking forward.

While it provides a complete picture of both the dynamic and equilibrium outcome, this example is hypothetical. I now wish to turn to a real case: the gradual expansion of EMU. This real case also serves as contemporaneous evidence supporting the intuition that I formalize in this paper.

### **B. A Contemporaneous Example: The Pattern of Gradual Expansion of EMU**

I have developed and formalized in this paper the intuition that currency bloc formation is path dependent and that countries join a currency bloc sooner the more they trade with the bloc member countries, and each additional member attracts in a dynamic way other members into the bloc. In this subsection I investigate whether this model can explain the pattern of the gradual expansion of EMU. As it will become clear below, the model performs quite well in explaining the pattern of EMU expansion. I then elaborate on further expansion of EMU in light of the model.

The exercise consists of investigating whether the choice of the early joiners—that is, the countries that join the European Union (EU) in 2004, with the option of adopting the euro—can be explained by the model developed in this paper. The model is also used to elaborate on the next joiners, that is, the likely frontrunners for joining after the 2004 round. Since EU countries—that is, Sweden, Denmark and the United Kingdom—that initially opted not to adopt the euro may do so at any time in the future, I also conduct the same exercise using the entire EU as the reference bloc.

In order to do all of this, I extract data on bilateral trade from the International Monetary Fund's *Direction of Trade Statistics*. These data are expressed in real U.S. dollars. In fact, I follow Glick and Rose (2002), and Alesina, Barro, and Tenreyro (2002), by deflating the original nominal values of trade by the U.S. consumer price index and express trade values in 1995 U.S. dollars. Data on trade are available for the period 1980–2000. I use these data and data on real GDP, which come from the World Bank's *World Development Indicators (WDI)*, to compute the trade shares between the different pairs of European countries. The share is computed as the ratio of real bilateral trade to real GDP. Trade is computed as exports plus imports. In order to correct the discrepancies due to differences in countries' reporting, I compute the bilateral trade between two countries  $i$  and  $j$  as being the average of trades reported by countries  $i$  and  $j$ .

Tables 1a and 1b present the average trade-to-GDP ratios for European countries with EMU and EU. The trade shares are ranked from the highest to the lowest. The results are very

appealing. Overall, the early joiners, as predicted by the model, are effectively the countries accepted to join in 2004.<sup>10</sup> From Tables 1a and 1b, one sees that among the ten first countries that should join according to the model, seven are among the 2004 candidates for joining. Going further down in Tables 1a and 1b, one can see that nine out of the thirteen early joiners predicted by the model are among the ten countries accepted for 2004, with a trade share varying from 75 percent to 20 percent. If one disregards countries like Iceland, Norway, and Switzerland, which have not decided to join the EU, then with the exception of Poland the nine others invited to join in 2004 are the top nine that should join according to the model. Among the ten 2004 candidates for joining, only Poland seems not to fit well, occupying the ranks of nineteenth with EMU and twentieth with EU.

Some caveats need to be highlighted about Poland. Notice that during 1980–1989, Poland had a centrally planned economy and traded mostly with the Council for Mutual Economic Assistance (CMEA) countries. This negatively affected Poland’s trade with EU countries. But since 1989 when Poland opted for a free market system, the trade pattern has substantially changed in favor of EU countries. Nowadays, Poland’s trade with EU countries is about 70 percent. Clearly the low trade share with EU over 1980–2000 is driven by Poland’s trade structure over 1980–1989, which I however consider in order to have the same study period for all countries analyzed in the paper. Hence, overall, the ten EMU/EU joiners fit the model and provide contemporaneous evidence supporting the model.

Based on the model’s good performance, I take a further step to elaborate on the countries likely to become the frontrunners for joining after the 2004 round. Here I compute the trade shares for the remaining European countries with the bloc that will emerge following the 2004 round, that is, EMU/EU plus the ten new members. Tables 2a and 2b present the results. This “model-based” forecast suggests that countries like Croatia, Macedonia, Bulgaria, Bosnia and Herzegovina, Romania, and Albania are likely to become strong candidates for joining in the round that will follow the 2004 round. Their trade shares with the new bloc to emerge after the 2004 round have risen relative to their shares with the current bloc, increasing the benefits from reduced transaction costs for both the bloc members and the new candidates for joining. In other words, joining might have not been optimal for both parties in 2004, but it becomes more attractive post-2004 with other countries having joined. The next country in line—Turkey, with a trade share of 12 percent with EMU and 15 percent with the EU—seems an interesting candidate as well. The analysis also finds little evidence in support of other countries, especially Ukraine, Belarus, and Moldova, joining soon because, on average, their trade with EU is low. Although the possible evolution of EU membership highlighted in this paper finds little evidence supporting the joining of the later countries any time soon, their EU membership is not necessarily precluded because it entails—as pointed out in the introduction—much more than high trade.

---

<sup>10</sup> The ten countries that joined the EU on May 1st 2004 are Cyprus, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia, and Slovenia.

#### IV. CONCLUSIONS

This paper argues that currency bloc formation is best viewed as a dynamic process based on network externalities operating through trade channels. The paper presents an approach to bloc formation based on the notion that currency blocs dynamically emerge out of individual decisions that not only trade off the costs of forming links against the potential rewards, but equally important, are also influenced by prior choices of their trading partners.

The basic message of this paper is two-fold. First, the paper argues that currency bloc formation is path dependent and that trade network externalities help explain the gradual expansion of currency blocs. Countries join a currency union sooner the more they trade with the bloc member countries, and each additional member serves in a dynamic way to attract additional members into the bloc.

Second, using the model as a theoretical background, the paper analyzes the pattern of EMU expansion. The paper finds that the current pattern of EMU expansion fits the model well and provides evidence supporting the model. The model is further used to elaborate on which countries are most likely to be frontrunners in the next round of EU membership with the option of adopting the euro. The “model-based” forecasts suggest that countries such as Croatia, Macedonia, Bulgaria, Bosnia and Herzegovina, Romania, and Albania are likely to become strong candidates in the next round of EU accession talks. The analysis also finds little evidence supporting other countries, especially Ukraine, Belarus, and Moldova to become frontrunners any time soon because, on average, they trade less with the EU than the countries mentioned earlier do. The paper also makes clear that a nation’s decision to surrender some policy decisions, especially the monetary one, to a supranational institution transcends purely economic considerations, as political will is sometime crucial. As a consequence, although the “model-based” forecasts in this paper find little evidence for some countries to join, their EU membership is not necessarily precluded.

In the model, I have imposed the restriction of sequential unanimity for currency bloc formation. This restriction is empirically consistent since the politics of currency bloc formation are similar to those of trade bloc formation, and the voting rule used by the major trade blocs require consensus on the admission of new members. This requirement could conceivably change as currency blocs become larger and the agreement among all members becomes harder to obtain.

Table 1a. Average Trade-to-GDP Ratio with EMU 1/, 1980–2000  
(In percent)

---

High Trade-Ratio European Countries	
Malta	75
Slovenia	55
Czech Republic	49
Slovak Republic	41
Estonia	35
Hungary	31
Croatia	29
Switzerland	27
Lithuania	26
Macedonia, FYR	26
Bulgaria	24
Cyprus	21
Latvia	20
Norway	19
Bosnia and Herzegovina	18
Albania	18
Iceland	18
Romania	17
Poland	15
Turkey	12
Russia	08
Ukraine	06
Belarus	05
Moldova	04

---

1/ EMU: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain.

EU: EMU, Denmark, Sweden, and United Kingdom.

Next 10: Ten countries invited to join the EU on May, 2004: Cyprus (Greek part), the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia, and Slovenia.

Table 1b. Average Trade-to-GDP Ratio with EU, 1980–2000  
(In percent)

---

High Trade-Ratio European Countries	
Malta	92
Slovenia	58
Czech Republic	54
Estonia	51
Slovak Republic	44
Norway	37
Lithuania	35
Hungary	33
Switzerland	32
Iceland	31
Latvia	31
Croatia	31
Cyprus	30
Macedonia, FYR	27
Bulgaria	27
Bosnia and Herzegovina	20
Albania	19
Romania	19
Poland	18
Turkey	14
Russia	10
Ukraine	06
Belarus	06
Moldova	04

---

Table 2a. Average Trade-to-GDP Ratio with EMU and Next 10, 1980–2000  
(In percent)

---

High Trade-Ratio European Countries	
Croatia	37
Macedonia, FYR	32
Bulgaria	30
Bosnia and Herzegovina	29
Switzerland	28
Romania	21
Albania	21
Norway	20
Iceland	19
Turkey	12
Russia	11
Ukraine	09
Belarus	09
Moldova	05

---

Table 2b: Average Trade-to-GDP Ratio with EU and Next 10, 1980–2000  
(In percent)

---

High Trade-Ratio European Countries	
Croatia	40
Norway	38
Macedonia, FYR	34
Bulgaria	33
Switzerland	33
Iceland	32
Bosnia & Herzegovina	30
Romania	23
Albania	21
Turkey	15
Russia	13
Ukraine	10
Belarus	10
Moldova	05

---

### Proof of Proposition 1

Consider a country  $i \in (k+1, \dots, n)$ . Suppose that the new currency bloc that would emerge if country  $i$  were to join the bloc  $x_k$  is simply  $x_k \cup \{i\}$ . Denote by  $\hat{y}$  the corresponding proposal or bloc configuration. Note that given the current state  $x$  there can be some proposals other than  $\hat{y}$ . Assume without loss of generality that there are distribution mechanisms within the bloc  $x_k$  that make decisions within the bloc unanimous, so that the bloc can be considered as an entity. Given the current state or bloc configuration  $x$ , country  $i$  would be able to join  $x_k$ , that is, the proposal  $\hat{y}$  would be accepted if and only if for  $l = x_k, i$ :

$$\begin{aligned} & \left[ (1 - b_{\hat{y}})^{(2\alpha-1)/(1-\alpha)} HN^{\hat{y}_l} - E\mathcal{F}_l^{\hat{y}} \right] + \delta_l \cdot \sum_{z \in S(\hat{y}, j)} p(\hat{y}, z) V_l(z, p) \\ & \geq \\ & (1 - b_x)^{(2\alpha-1)/(1-\alpha)} HN^{x_l} - E\mathcal{F}_l^{x_l} + \delta_l \cdot \sum_{y \in S(x, j)} p(x, y) V_l(y, p) \end{aligned} \quad (30)$$

and:

$$\hat{y} = \arg \max_{y \in S(x, j)} \left\{ \left[ (1 - b_y)^{(2\alpha-1)/(1-\alpha)} HN^{y_l} - E\mathcal{F}_l^y \right] + \delta_l \cdot \sum_{z \in S(y, j)} p(y, z) V_l(z, p) \right\} \quad (31)$$

Notice that country  $i$  can be willing to join but can be rejected.

Condition (30) is the acceptability condition. Condition (31) is the efficiency condition capturing the idea that among acceptable proposals, country or bloc  $l$  would only accept the efficient one, that is, the one that cannot be beaten by any other proposal.

Set:

$$\hat{N}_l^{y_l} = \arg \max_{y \in S(x, j)} \left\{ \left[ (1 - b_y)^{(2\alpha-1)/(1-\alpha)} HN^{y_l} - E\mathcal{F}_l^y \right] + \delta_l \cdot \sum_{z \in S(y, j)} p(y, z) V_l(z, p) \right\} \forall l = x_k, i \quad (32)$$

where  $N_l^{y_l}$  denote here the level of trade between the entity  $l$  and the currency bloc  $y_l$  to which entity  $l$  will belong in any new proposal  $y$ .  $\hat{N}_l^{y_l}$  represents the level of trade between entity  $l$  and the currency bloc to which entity  $l$  would belong to in the efficient bloc configuration or proposal that cannot be beaten.

Now set:

$$\tilde{N}_l^{\hat{y}^i} = \min \left\{ \begin{array}{l} \left[ (1-b_{\hat{y}})^{(2\alpha-1)/(1-\alpha)} HN^{\hat{y}^i} - E\mathcal{L}_l^{\hat{y}} \right] + \delta_l \cdot \sum_{z \in S(\hat{y}, j)} p(\hat{y}, z) V_l(z, p) \\ \geq \\ (1-b_{x_k})^{(2\alpha-1)/(1-\alpha)} HN^{x_k} - E\mathcal{L}_l^{x_k} + \delta_l \cdot \sum_{y \in S(x, j)} p(x, y) V_l(y, p) \end{array} \right\} \forall l = x_k, i \quad (33)$$

$\tilde{N}_l^{\hat{y}^i}$  captures the minimum level of trade that would make the proposal or bloc configuration  $\hat{y}$  acceptable for entity  $l$ .

Finally, set:

$$N_i^{x_k^*} = \max \{ \hat{N}_l^{y^i}, \tilde{N}_l^{\hat{y}^i}, \text{ with } l = x_k, i \} \quad (34)$$

It is easy to see that for  $N_i^{x_k} \geq N_i^{x_k^*}$ , conditions (30) and (31) are satisfied. Hence country  $i$  will be willing and able to join if and only if  $N_i^{x_k} \geq N_i^{x_k^*}$ , that is, the trade level between country  $i$  and the bloc  $x_k$  is greater or equal to the cut off level  $N_i^{x_k^*}$ .

### Proof of Proposition 2

This proof follows the same route as the ones that show the existence of an equilibrium in the process of coalition formation and rely on the well known Kakutani fixed point theorem. Let  $P$  denote the set of all process of currency bloc formation. I appropriately construct a mapping  $\Psi : P \rightarrow P$  and observe that a fixed point exists and is an *EPCBF*. Consider a process  $p \in P$ , for any country  $i$ , there exists, a unique value function  $V_i(x, p)$  satisfying the Bellman-type equation (25). Note that equation (25) is successively equivalent to:

$$V_i(x, p) = U_i(x) + \delta_i \cdot \sum_{j=1}^{|I|} \frac{1}{|I|} \sum_{y \in S(x, j)} p(x, y) V_i(y, p) \quad (35)$$

$$V_i(x, p) = U_i(x) + \delta_i \cdot \sum_{j=1}^{|I|} \frac{1}{|I|} \left( \sum_{y \in X} p(x, y) V_i(y, p) \right), \quad (36)$$

where  $p(x, y) = 0$  for any  $y \in X \setminus \bigcup_{j=i}^{|I|} S(x, j)$ .

$$V_i(x, p) = U_i(x) + \delta_i \cdot \sum_{y \in X} p(x, y) V_i(y, p), \quad (37)$$

where  $p(x, y) = 0$  for any  $y \in X \setminus \bigcup_{j=i}^{|I|} S(x, j)$ .



Denote by  $V_i(p)$  the vector of value functions  $\{V_i(x, p)\}_{x \in X}$ , by  $U_i$  the vector of current utilities  $\{U_i(x)\}_{x \in X}$ , and by  $P$  the matrix of transition probabilities (under  $p$ ). I can now rewrite this last equation as:

$$(I - \delta_i P)V_i(p) = U_i \quad (38)$$

Since  $\delta_i \in (0, 1)$ , setting  $I - \delta_i P = (a_{lk})$ , one notes that  $a_{ll} > 0$  and  $a_{lk} \leq 0$  for  $l \neq k$ , so that the matrix  $I - \delta_i P$  has a dominant diagonal and  $\det(I - \delta_i P) \neq 0$ . This guarantees the unique solvability and continuity of  $V_i(p)$  in  $p$ .

Now let us consider  $(x, p)$  such that strictly profitable moves exist, and let denote the set of all strictly profitable and efficient moves by  $Y(x, p)$ . For each  $y \in Y(x, p)$ , there is a proposer  $j$  with proposal  $y$  such that  $y$  is a strictly profitable and efficient move for  $y_j$  from  $x$  under  $p$ . Denote by  $J$  the set of such potential proposers.

Define  $\sigma(y, x, p) = \min_{i \in y_j} [V_i(y, p) - V_i(x, p)]$ . Let us now define a probability measure  $q(x, p)$  over  $Y(x, p)$  as follows:

$$q(x, p)[y] = \frac{\sigma(y, x, p)}{\sum_{y' \in Y(x, p)} \sigma(y', x, p)} \quad (39)$$

Now consider the correspondence  $\Gamma(x, p)$  defined as follows: when strictly profitable moves exist,  $\Gamma(x, p) = \{q(x, p)\}$ , otherwise  $\Gamma(x, p)$  is the collection of all probability measures with support contained in the union of  $\{x\}$  and the collection of weakly profitable and efficient moves from  $x$  under  $p$ . Note that  $\Gamma(x, p)$  is nonempty and convex-valued for each  $(x, p)$ . One can show that for given  $x$ ,  $\Gamma(x, p)$  is upper semi-continuous in  $p$ . To show this, let  $p^n$  represent some sequence in  $P$  converging to  $p$ . Now consider the corresponding sequence  $q^n \in \Gamma(x, p^n)$  and extract a subsequence converging to some  $q$ . I claim that  $q \in \Gamma(x, p)$ . To see this, consider first the case where no strictly profitable move exists at  $(x, p)$ . Observing that if  $y$  is strictly profitable for the sequence  $(x, p^n)$ , then it must be weakly profitable for  $(x, p)$ ; the claim becomes obvious in this case. Now consider the case in which a strictly profitable move exists at  $(x, p)$ . Making use of the continuity of  $V_i(x, p)$  in  $p$  for every  $i$  and  $x$ , it is easy to verify that for any  $y \in Y(x, p)$ ,  $\sigma(y, x, p^n)$  converges to  $\sigma(y, x, p)$ . As a consequence for  $n$  large enough,  $\Gamma(x, p^n)$  is a singleton containing the probability  $q(x, p^n)$  defined by expression (39). This implies that  $q(x, p^n) \longrightarrow q(x, p)$ .

Therefore  $\Gamma(x, p)$  is nonempty, convex-valued and upper semi-continuous in  $p$  for each  $x$ . Now since  $|X| < \infty$ , define the correspondence  $\Psi : P \longrightarrow P$  by  $\Psi(p) = \prod_{x \in X} \Gamma(x, p)$  for every  $p \in P$ . The fact that  $X$  is finite is crucial and guarantees that the correspondence is well defined. In addition, the arguments above show that all the conditions for the Kakutani fixed point theorem are satisfied for the correspondence  $\Psi$ . Hence there exists  $p^* \in P$  such that  $p^* \in \Psi(p^*)$ . It is easy to see that  $p^*$  satisfies all the conditions of an equilibrium process of currency bloc formation (*EPCBF*). Based on the existence of an *EPCBF*  $p^*$  and the irreversibility of actions, there exists an equilibrium outcome  $x^*$  that can be reached after a number of step-transition say  $k$ , that is  $p^{*(k)}(x, x^*) > 0$ , and the stability condition  $p^*(x^*, x^*) = 1$  is satisfied.

## REFERENCES

- Alesina, Alberto, and Robert J. Barro, 2002, "Currency Unions," *Quarterly Journal of Economics*, Vol. 117, pp. 409–36.
- \_\_\_\_\_, and Silvana Teneyro, 2002, "Optimal Currency Areas," in *NBER Macroeconomic Annual*, ed. by Mark Gertler and Kenneth Rogoff (Cambridge, Massachusetts: MIT Press).
- Baldwin, Richard, 1996, "A Domino Theory of Regionalism," in *Expanding Membership of the European Union*, ed. by Richard Baldwin, Pentti Haaparanta, and Jaako Kiander (New York: Cambridge University Press).
- Barro, J. Robert, and David B. Gordon, 1983, "Rules, Discretion, and Reputation in a Model of Monetary Policy," *Journal of Monetary Economics*, Vol. 12, No. 1, pp. 101–21.
- \_\_\_\_\_, and Silvana Teneyro, 2000, "Closed and Open Economy Models of Business Cycles with Marked-Up and Sticky Prices," NBER Working Paper No. 8043 (Cambridge, Massachusetts: National Bureau of Economic Research).
- Bayoumi, Tamin, 1994, "A Formal Model of Optimal Currency Areas," *IMF Staff Papers* International Monetary Fund, Vol. 41, No. 4, pp. 537–54.
- Bogomolnaia, Anna, and Matthew O. Jackson, 2002, "The Stability of Hedonic Coalition Structures," *Games and Economic Behavior*, Vol. 38, pp. 201–30.
- Dixit, Avinash, 2000, "A Repeated Game Model of Monetary Union," *Economic Journal*, Vol. 110, No. 466, pp. 759–80.
- \_\_\_\_\_, and Joseph Stiglitz, 1977, "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, Vol. 67, No. 3, pp. 297–308.
- Ethier, Wilfred J., 1982, "National and International Returns to Scale in the Modern Theory of International Trade," *American Economic Review*, Vol. 72, No. 3, pp. 389–405.
- Flandreau, Marc, 1996, "The French Crime of 1873: An Essay on the Emergence of the International Gold Standard, 1870–1880," *Journal of Economic History*, Vol. 56, No. 4, pp. 862–97.
- Frankel, Jeffrey, 1997, *Regional Trading Blocs in the World Economy* (Washington: Institute for International Economics).
- \_\_\_\_\_, and Andrew K. Rose, 2002, "An Estimate of the Effect of Currency Unions on Trade and Growth," *Quarterly Journal of Economics*, Vol. 117, pp. 437–66.

- \_\_\_\_\_, 1998, "The Endogeneity of the Optimum Currency Area Criteria," *Economic Journal*, Vol. 108, pp. 1009–25.
- Glick, Reuven, and Andrew K. Rose, 2002, "Does a Currency Union Affect Trade? The Time Series Evidence," *European Economic Review*, Vol. 46, No. 6, pp. 1125–51.
- Konishi, Hideo, and Debraj Ray, 2003, "Coalition Formation as a Dynamic Process," *Journal of Economic Theory*, Vol. 110, pp. 1–41.
- Mundell, Robert, 1961, "A Theory of Optimum Currency Areas," *American Economic Review*, Vol. 51, No. 4, pp. 657–65.
- Rose, Andrew K., 2000, "One Money One Market: Estimating the Effect of Common Currencies on Trade," *Economic Policy*, Vol. 15, No. 30, pp. 9–45.
- Spence, A. Michael, 1976, "Product Selection, Fixed Costs, and Monopolistic Competition," *Review of Economic Studies*, Vol. 43, No. 2, pp. 217–35.
- Tjornhom, Jessica, 2000, "Dynamic Trade Bloc Formation: Building Blocs or Stumbling Blocs?" (unpublished; Minneapolis: University of Minnesota).
- Yehoue, Etienne, 2004, "On the Pattern of Currency Blocs in Africa" (Unpublished; Washington: International Monetary Fund).