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# How to Fight Deflation in a Liquidity Trap: Committing to Being Irresponsible

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# How to Fight Deflation in a Liquidity Trap: Committing to Being Irresponsible

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## Abstract



I model deflation, at zero nominal interest rate, in a microfounded general equilibrium model. I show that deflation can be analyzed as a credibility problem if the government has only one policy instrument, money supply carried out by means of open market operations in short-term bonds, and cannot commit to future policies. I propose several policies to solve the credibility problem. They involve printing money or nominal debt and either (1) cutting taxes, (2) buying real assets such as stocks, or (3) purchasing foreign exchange. The government credibly "commits to being irresponsible" by using these policy instruments. It commits to higher money supply in the future so that the private sector expects inflation instead of deflation. This is optimal, since it curbs deflation and increases output by lowering the real rate of return.

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## I. INTRODUCTION

Can the government lose its control over the price level and economic activity so that no matter how much money it prints, it has no effect on inflation or output? Ever since Keynes' *General Theory,* this question has been hotly debated. Keynes answered yes, Friedman and the monetarists no. At low nominal interest rates, Keynes argued, increasing money supply has no effect. This is what is referred to as the liquidity trap. The zero short-term nominal interest rate in Japan today, together with the lowest short-term interest rate in the United States since the Great Depression, make this old question urgent again. The Bank of Japan (BOJ) has nearly doubled the monetary base over the past five years, yet the economy still suffers deflation, and growth is stagnant or negative. Was Keynes right? Is money supply irrelevant when the interest rate is zero? In this paper I revisit this question using a microfounded intertemporal general equilibrium model and assuming rational expectations. For Keynes to be right, two extreme assumptions are needed: first, Ricardian equivalence holds, and, second, the government cannot commit to future policy. If these assumptions fail, however, Friedman's position is not vindicated. The role of money supply is much more subtle—and interesting—than is indicated by the quantity theory of money.

### A. The Deflation Bias: Deflation as a Credibility Problem

The paper's first contribution is to show that deflation, at a zero short-term nominal interest rate, can be modeled as a credibility problem. This theory of deflation is in sharp contrast to conventional wisdom about the deflation in Japan today (or, for that matter, United States during the Great Depression). The conventional wisdom blames deflation on policy mistakes by the central bank or bad policy rules (see, for example, Friedman and Schwartz (1963), Krugman (1998), Buiter (1999), Bernanke (2000) and Benabib and others (2002)).[2] In this paper, however, deflation is not attributed to an inept central bank or bad policy rules. It is a direct consequence of the central bank's policy constraints and inability to commit when faced with large negative demand shocks. As Krugman (1998) shows, increasing money supply has no effect in a liquidity trap if the private sector expects the increase to be reversed in the future. Krugman's analysis thus can explain why the BOJ has nearly doubled the monetary base without affecting inflation expectations. Monetary policy is still effective, however, if the private sector expects the increase in the money supply to be *permanent.* I show that, under certain conditions, large demand shocks make the zero bound binding and the private sector *always* expects current monetary expansion to be reversed in the future.[3] The central assumption behind this result is that the government is *discretionary,* so that it is unable to commit to future policy. This result indicates that Krugman's (1998) proposal, that the BOJ should announce an inflation target, is ineffective absent other

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[2] There is a large literture that discusses optimal monetray policy rules when the zero bound is binding. Contributions include Summers (1991), Fuhrer and Madigan (1997), Woodford and Rotemberg (1997), Wolman (1998), Reifschneider and Williams (1999) and references there in. Since monetary policy rules arguably become credible over time these contributions can be viewed as illustration of how to *avoid* a liquidity trap rather than a prescription of how to escape them which is the focus here.

[3] In contrast Krugman (1998) shows that this is only true for a "bad" policy rule, i.e. if the government has committed to fixed money supply in the future.

policy actions. It is simply not credible.[4] This unexplored *deflation bias* of discretionary policy can be viewed as the inverse of the *inflation bias* that Kydland and Prescott (1977) and Barro and Gordon (1983) analyze. A critical assumption driving the deflation bias is that the government has only one policy instrument. This policy instrument is the money supply, which is influenced by open market operations in short-term government bonds. The key idea behind the deflation bias is simple: aggregate demand depends on the level of current and future real interest rates. Even if the zero bound is binding, monetary policy can still lower the real rate of interest, and thus increase aggregate demand, by increasing inflation expectations. When the zero bound is binding due to large deflationary shocks, therefore, the government has an incentive to promise future inflation to lower the real rate of return. When those deflationary shocks have subsided, however, the ex post optimal inflation is below the government's previous promises. Then the government has an incentive to renege. It follows that if the government is unable to commit to future policy, and agents are rational, the optimal inflation target is not credible. The result is excessive deflation when the zero bound is binding. The next two sections show how deflation can be eliminated, even if the government is unable to commit to future policy, by introducing additional policy tools.

### B. Escaping a Liquidity Trap: Deficit Spending

The paper's second contribution is to show how the government can eliminate deflation by deficit spending. The reason why deficit spending achieves this is as follows: If the government cuts taxes and increases nominal debt, and taxation is costly, inflation expectations increase (i.e. the private sector expects higher money supply in the future). That is because higher nominal debt gives the government an incentive to inflate to reduce the real value of the debt. To eliminate deflation, then, the government simply cuts taxes until the private sector expects inflation instead of deflation. Higher inflation expectations reduce the real rate of return, and thereby raises aggregate demand and the price level. The central assumption behind this result is that there is some cost of taxation which makes this policy credible.[5]

Deficit spending has exactly the same effect as if the government followed Friedman's famous suggestion to "drop money from helicopters" to increase inflation. At zero nominal interest rates money and bonds are perfect substitutes. They are one and the same thing: A government issued

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[4] This has been a common objection of BOJ officials as well. Responding to Krugman's policy proposal, Kunio Okina, director of the Institute for Monetary and Economic Studies at BOJ, said in Dow Jones News (08/11/1999): "Because short-term interest rates are already at zero setting an inflation target of, say, 2 percent, wouldn't carry much credibility." Similar objections have also been raised by economists such as for example Dominguez (1998), Svensson (1999, 2001), and Woodford (1999)

[5] The fiscal theory of the price level (FTPL) popularized by Leeper (1992), Sims (1994) and Woodford (1994,1996) also stresses that fiscal policy can influence the price level. What separates this analysis from the FTPL (and the seminal contribution of Sargent and Wallace (1982) is that in my setting fiscal policy only affects the price level because it changes the *inflation incentive* of the government. In contrast, according to the FTPL, fiscal policy affects the price level because it is *assumed* that the monetary authority commits to a (possibly suboptimal) interest rate rule and fiscal policy is modelled as a (possibly suboptimal) exogenous path of real government surpluses. Under these assumptions, innovations in real government surpluses can influence the price level, since the prices may have to move for the government budget constraint to be satisfied. In my setting, however, the government budget constraint is a *constraint* on the policy choices of the government.

piece of paper that carries no interest but has nominal value. It does not matter, therefore, if the government drops money from helicopters or government bonds. Friedman's proposal thus increases the price level through the same mechanism as deficit spending. This result, however, is not a vindication of the quantity theory of money. Dropping money from helicopters does *not* increase prices because it increases money supply. It creates inflation by increasing government debt which is defined as the sum of money and bonds. *At zero nominal interest rates it is government debt that determines the price level* because it determines expectations about future money supply.

### C. Escaping a Liquidity Trap: Buying Real Assets or Foreign Exchange

The paper's third contribution is to show that open market operations in any asset other than government bonds eliminates deflation through the same channel as deficit spending. Cutting taxes, or dropping money from helicopters, are only two ways of increasing government debt. Government debt can also be increased by printing money (or issuing nominal bonds) and buying real assets, such as stocks, or foreign exchange. I show that these operations increase prices and output *because they change the inflation incentive of the government by increasing government debt* (money+bonds). This formalizes an insight by Bernanke (2000):

> Despite the apparent liquidity trap – monetary policymakers retain the power to increase nominal aggregate demand and the price level [...] One can make what amounts to an arbitrage argument – the most convincing type of argument in an economic context – that it must be true [...] The monetary authorities can issue as much money as they like. Hence, if the price level were truly independent of money issuance, then the monetary authorities could use the money they create to acquire indefinite quantities of goods and assets. This is manifestly impossible in equilibrium. Therefore money issuance must ultimately raise the price level, even if nominal interest rates are bounded at zero.

Bernanke's argument against the "manifestly impossible equilibrium" can be interpreted as an argument against Ricardian equivalence. If Ricardian equivalence holds, increasing debt has no effect. Since money and bonds are perfect substitutes at zero nominal interest rate, this applies to money supply as well. This paper excludes the "manifestly impossible equilibrium" Bernanke refers to by assuming cost of taxation. If the government prints money and buys assets it increases the price level because the private sector expect inflation, i.e. expects higher money supply in the future. The failure of Ricardian equivalence (that is, cost of taxation) eliminates the liquidity trap.[6] This channel of monetary policy does not rely on the portfolio effect of buying real assets or foreign exchange. In this paper these policies are effective because they increase inflation expectations even if all assets are perfect substitutes. This paper thus offers a complimentary arguments to Meltzer's (1999) and McCallum (1999) proposals for foreign exchange interventions

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[6] Costly taxation is also a natural interpretation of Bernanke's "arbitrage argument". To make an "arbitrage argument", the arbitrager needs to care about capital gains and losses. Why should the government care about capital losses? If there is no cost of taxation it would not be concerned since it could make up for any capital losses by lump sum taxes. Costly taxation can thus be thought of as a parable that explains the government's aversion to capital losses.

which that rely on the portfolio channel.[7]

A surprising and radical result is immediate under plausible institutional arrangement that Eggertsson (2001) calls a "goal independent central bank": *When the central bank is goal independent, deficit spending has no effect on either output or prices.* A "goal independent" central bank, according to this definition, does not maximize social welfare. The results of this paper, therefore, are not merely a roundabout way of reaching Keynes' famous conclusion that the government should use deficit spending to get out of a liquidity trap. *Deficit spending increases output and prices only if the central bank and the treasury coordinate policy to maximize social welfare.* As pointed out in Eggertsson (2001), and discussed in the conclusion of this paper, the goal independence of the BOJ can explain why the high nominal debt in Japan today has failed to increase inflation expectations.

Deflationary pressures in this paper are due to temporary exogenous real shocks that shift aggregate demand.[8] The paper, therefore, does not answer questions such as what was the origin of the deflationary shocks during the Great Depression in the United States or in Japan today. These deflationary shocks are most likely due to a host of factors, such as for example the stock market crash and banking problems. Here I take these deflationary pressures as given and ask: How can the government eliminate deflation by monetary and fiscal policy even if the zero bound is binding?

The outline of the rest of the paper is as follows: Section II presents the model, Section III illustrates the deflation bias, Section IV how deficit spending is used to eliminate deflation, and Section V shows how non-standard open market operations can achieve the same aim, Section VI concludes and discusses to what extent the model can be applied to the case of Japan. Appendices I-V contain discussions of technical issues.

## II. A Simple Model

In this section I derive a simple rational expectations model from micro foundations.

### A. The Private Sector

The representative household maximizes expected utility:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [u(C_t, \xi_t) + g(G_t) + q(\frac{M_t}{P_t}) - v(h_t)] \right\} \tag{1}$$

where $C_t$ is the consumption, $\xi_t$ is a vector of exogenous shocks, $\frac{M_t}{P_t}$ real money balances held at the end of period $t$, $G_t$ is real government consumption, and $h_t$ are hours worked. $E_0$ denotes

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[7] The argument in the paper is also complimentary to Svensson's (2000) "foolproof" way of escaping the liquidity trap by foreing exchange intervention. I show explicitly how foreign exchange rate intervention increase inflation expectation even if the government cannot commit to future policy and maximizes social welfare.

[8] In contrast to Benhabib et al (2002) where deflation is due to selffulfilling deflationary spirals.

mathematical expectation conditional on information available in period 0. The utility function is concave and satisfies standard assumptions. The household maximizes utility subject to a standard intertemporal budget constraint shown in Appendix I. The household optimal consumption decisions imply an Euler or "IS" equation:

$$1 + i_t = \frac{u_c(C_t, \xi_t)}{\beta f_t^e} \tag{2}$$

where $\pi_t$ is inflation, $i_t$ is the nominal interest rate on a one—period riskless bond, and $f_t^e \equiv E_t \frac{u_c(C_{t+1}, \xi_{t+1})}{1+\pi_{t+1}}$. The households optimal money holdings implies:

$$\frac{q_{\frac{M}{P}}(\frac{M_t}{P_t})}{u_c(C_t, \xi_t)} = \frac{i_t}{1 + i_t} \tag{3}$$

This equation defines money demand or the "LM" equation. Utility is increasing in real money balances up to a satiation point (at some finite level of real money balances) as in Friedman (1969).[9] The left-hand side of (3) is therefore weakly positive. Thus there is a zero bound on the interest rate:

$$i_t \geq 0 \tag{4}$$

The production function of the representative firm is:

$$y_t = F(l_t) \tag{5}$$

where $F$ is a concave and $l_t$ is labor. I abstract from capital dynamics. I assume competitive firms that maximize profits. Wages are set one period in advance in a competitive labor market detailed in Appendix I. As Rotemberg (1983), I assume that firms face a cost of price changes given by the function $d(\pi_t)$.[10] Price variations have a welfare cost that is separate from the cost of expected inflation due to real money balances in utility.[11] (The results in this paper do not depend on that the cost of price changes being large). Equating the firm labor demand and the household labor supply I show in Appendix I that aggregate output can be expressed as:[12]

$$Y_t = S(\pi_t, u_{t-1}^e, v_{t-1}^e) \tag{6}$$

[9] A satiation level in real money balances is also implied by several cash-in-advance models such as Lucas and Stokey (1987) or Woodford (2001a).

[10] $d'(\pi) > 0$ if $\pi > 0$ and $d'(\pi) < 0$ if $\pi < 0$. Thus both inflation and deflation are costly. $d(0) = 0$ so that the optimal inflation rate is zero (consistent with the interepretation that this represent a cost of changing prices). Finally, $d'(0) = 0$ so that in the neighborhood of the optimal inflation rate the cost of price changes is of second order. The assumption of perfect competition implies that firms always set prices equal to the aggregate price level.

[11] This cost can be more explicitly modelled by assuming price setting a la Calvo (1982). In that case the cost of inflation is due to staggered price setting. All the results shown in this paper can be derived in that framework and are available upon request. The advantage of the framework presented here is that it allows for simple closed form solution (the Calvo pricing framework requires numerical simulations) and the structure of the game analysed is somewhat simpler.

[12] To simplify notation we have replaced the function $v(L_t)$ with a simple transformation $\tilde{v}(Y_t) \equiv v(F^{-1}(Y_t))$.

where $u_{t-1}^e \equiv E_{t-1}u_c(Y_t - d(\pi_t) - F, \xi_t)\frac{F^{-1}(Y_t)}{1+\pi_t}$ and $v_{t-1}^e = E_{t-1}\tilde{v}_y(Y_t)F'(F^{-1}(Y_t))F^{-1}(Y_t)$. Equation (6) is what I refer to as the AS equation. It is, to a linear approximation, equivalent to the Phillips curve used by Kydland and Prescott (1977) and Barro and Gordon (1983) in deriving the inflation bias. All the results presented here can also be obtained in the "New Keynesian" framework that is popular in the literature (although that model does not allow closed form solutions – see footnote 11).

### B. The Government Policy Instruments

The government consists of the central bank and the treasury. Government debt is the sum of the monetary base and government bonds held by the public, $B_t \equiv D_t + M_t$. The treasury determines government debt, $B_t$, by taxation and the issuance of debt $D_t$. The central bank, on the other hand, determines how government debt is split between money supply, $M_t$, and bonds, $D_t$, by open market operations in bonds. By its choice of money supply the central bank determines the nominal interest rate, $i_t$, through money demand (3).

I assume that the treasury can only issue one period nominal bonds and that there is an output cost of taxation (e.g. due to tax collection costs as in Barro (1979)) captured by the function $s(\tau_t)$. For every dollar collected in taxes $s(\tau_t)$ units of output are waisted without contributing anything to utility. Government real spending is then given by:

$$F_t = G_t + s(\tau_t)$$

Real government spending, $F_t$, is an exogenous process that is for simplicity is a constant $F$. I make this assumption to focus the analysis on deficit spending, $F_t - \tau_t$, as opposed to real spending $F_t$.[13] It is useful to write the government budget constraint in terms of $\tilde{b}_t \equiv \frac{B_t}{P_t}u_c(C_t, \xi_t)$ (see Appendix I):

$$\frac{\tilde{b}_t}{u_c(C_t, \xi_t)} = \frac{1}{\beta f_{t-1}^e}\frac{\tilde{b}_{t-1}}{1+\pi_t} + F - \tau_t \tag{7}$$

I impose a borrowing limit on the government that rules out Ponzi schemes:

$$\tilde{b}_t \leq \bar{b} < \infty \tag{8}$$

where $\bar{b}$ is an arbitrarily high finite number.[14]

## III. The Deflation Bias

In this section, I show how deflation can be modeled as a credibility problem. The government maximizes social welfare, which is given by the utility of the representative household.[15] In

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13 The latter is treated in a companion paper Eggertsson (2002) where $F_t$ is a choice variable of the government.

14 This condition guarantees that the transversality condition of the houshold is satisfied at all times.

15 In this paper, I abstract from the effects of real money balances on the government objectives. The analysis can thus be interpreted as referring to the "cashless limit" often studied in the literature. This abstaction is not important

the absence of shocks the optimal inflation rate is zero according to this objective. Government spending and taxes are assumed to be constant. Money supply, by open market operations in short-term government bonds, is assumed to be the only policy instrument. This is equivalent to assuming that the nominal interest rate is the only policy instrument. An appealing interpretation of the results is that they apply if the central bank does not coordinate its action with the treasury, i.e. if the central bank is "goal independent". This interpretation is discussed in the conclusion of the paper and illustrated in Eggertsson (2001).

I analyze two equilibria, the commitment and the discretionary equilibrium. The commitment equilibrium is the first best solution, i.e. the solution if the government can commit to future policy (this is often called the Ramsey or Planner solution). The discretionary equilibrium, by contrast, is the equilibrium if the government cannot commit to future policy (this is often called the Markov solution). Discretion is identical to commitment if the zero bound is not binding. For sufficiently large demand shocks, however, the zero bound is binding, and there is a deflation bias of discretionary policy. The inability of the government to commit results in excessive deflation.

### A. The Commitment Equilibrium

The optimal commitment can be found as the solution to a Lagrangian maximization problem. Under commitment the government optimally chooses the expectation variables $f_t^e, v_t^e$ and $u_t^e$ in addition to the endogenous variables $\pi_t, Y_t, i_t$. This choice is subject to "implementability constraints", i.e. the private sector equilibrium conditions and the condition that expectations must be rational. This maximization problem can be solved by a Lagrangian method shown in Appendix II.

### B. The Discretion Equilibrium

In a discretionary equilibrium the strategies of the government and the private sector depend on a well defined minimum set of variables that are directly relevant to current market conditions. I do not consider equilibria built on reputation and rule out trigger strategies. Consider a repeated game between the government (i.e. the central bank) and the private sector. The sequence of actions in each round of the game is:

(1) Each round starts with initial values for $\xi_{t-1}$. The private sector uses $\xi_{t-1}$ to form expectations about $\xi_t$.

(2) The private sector forms expectations $f_{t-1}^e$, $u_{t-1}^e$ and $v_{t-1}^e$.[16]

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to derive deflationary bias of discretionary policy which is at the center of this study. Furthermore, given the low level of seignorage in industrialized countries, including real money balances does not yield quantitatively much different results for a realistic calibration. Including real money balances in the government objectives would yield tractable solutions but make the algebra considerably more tedious. It would not change the nature of the deflation bias described in this section (which is a result of inefficient response to shocks) but would change the steady state rate of inflation (some deflation would be optimal, its size would depend on the cost of changing prices).

[16] The variables $u_{t-1}^e$ and $v_{t-1}^e$ influence the equilibrium through the AS equation since

(3) The vector of shocks $\xi_t$ is realized.

(4) The government chooses $i_t$ to maximize social welfare.[17]

Defining the game in this way reduces the number of state variables. This is particularly helpful in the following sections when other policy instruments are introduced. $\xi_t$ is assumed to follow a Markov process. When $i_t$ is the only policy instrument there is only one state variable in the game given by $\xi_{t-1}$ so that $f^e_{t-1} = f^e(\xi_{t-1})$, $u^e_{t-1} = u^e(\xi_{t-1})$ and $v^e_{t-1} = v^e(\xi_{t-1})$. Since the actions of the government have no effect on the state of the economy at time $t+1$, the problem is a one period maximization problem:

$$\max_{i_t}[u(Y_t - d(\pi_t) - F, \xi_t) - \tilde{v}(Y_t) + g(G)] \tag{9}$$

s.t. (2),(4),(6) and the strategy functions of the private sector. Appendix II illustrates the first order conditions for this problem.

### C. Approximate Solution

In this subsection I derive approximate solutions by a first-order Taylor expansion shown in Appendix II. Due to the inequality constraint stemming from the zero bound the approximate equations cannot be solved with standard methods. Appendix III proposes a solution method for this non-standard class of models. In the approximate solution the shocks can be summarized by a single disturbance $g_t = -\frac{\bar{u}_{c\xi}}{\bar{u}_{cc}Y}\xi_t$.[18] I first derive a closed form solution for the most simple process for $g_t$ (Case 1). I then show a numerical solution for a more general stochastic process (Case 2).

**Case 1** *In period zero there is an unexpected shock so that $g_0 \neq 0$. In period $t > 0$ $g_t = 0$. There is perfect foresight from period 0 onward.*

Movements in $g_0$ can be interpreted as exogenous shifts in spending (e.g. an exogenous collapse in spending if $g_0$ is negative) or exogenous shifts in preferences (e.g. a temporary increase in the propensity to save if $g_0$ is negative). I first consider the commitment and discretion solution under in two different cases. Case 1a (*CNoTr*) applies when the zero bound is not binding, and Case 1b (*CTr*) applies when it is binding.

**Case 1a (CNoTr)** $g_0 \leq g_{Tr}$

**Case 1b (CTr) as Krugman (1998)** $g_0 \leq g_{Tr}$

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it is the ratio $\frac{v^e_{t-1}}{u^e_{t-1}}$ that determines the nominal wage rate on the labor market. $f^e_{t-1}$ is the part of nominal interest rate at time $t-1$ that is determined by expectations about future marginal utility and inflation.

[17] More precisely the treasury determines $B^s_t$ and the central bank $M^s_t$ by open market operations in government bonds. In equilibrium $M^s_t$ will be equal to money demand. This in turn, determines the nominal interest rate for any given price level and output. Since there are no shocks between the actions of the government and market clearing one can think of the central bank as determining a correspondance between $i_t$ and $Y_t$ and $\pi_t$,

[18] Here $u_{c\xi}$ and $u_{cc}$ are evaluated in steady state.

**Proposition 1** *Suppose that $i_t$ is the only policy instrument. Then if condition CNoTr holds ($g_0 > g_{Tr}$) the solution for optimal monetary policy under discretion and commitment is identical and the zero bound is never binding. If condition CTr holds ($g_0 \leq g_{Tr}$) the zero bound is always binding. The value of $g_{Tr}$ is determined by $g_{Tr} = -\sigma\frac{\kappa^2+(\sigma^{-1}+\omega)\lambda_\pi^{-1}}{\kappa^2+\omega\lambda_\pi^{-1}}(1-\beta)$ where the parameters $\lambda_\pi = d''$, $\omega = \frac{v_{yy}Y}{v_y}$, $\sigma \equiv -\frac{u_c}{u_{cc}Y}$, $\kappa \equiv -\frac{F''\bar{Y}}{F'^2} > 0$ are all a function of preferences and technology evaluated in the constant (steady-state) solution shown in Appendix II.*

Proof: See Appendix II.

If condition *CTr* is satisfied the discretion solution is different from the commitment solution. The commitment solutions yields:[19]

$$\pi_1^{com} = -\frac{\kappa^2+\omega\lambda_\pi^{-1}}{\kappa^2+(\sigma^{-1}+\omega)\lambda_\pi^{-1}+\beta\sigma^{-2}}\sigma^{-1}(g_0-g_{Tr}) > 0,\ \pi_t^{com} = 0\ \forall\ t > 1 \qquad (10)$$

where the superscript *com* refers to the commitment solution. This establishes that it is optimal to commit to a future inflation target that is above zero in a liquidity trap.

**Proposition 2** ***Optimal Inflation Target in a Liquidity Trap.*** *Suppose CTr and $i_t$ is the only policy instrument. Optimal monetary policy under commitment results in expected inflation*

Proof: See equation (10).

This result is suggested by Krugman (1998) (although he does not derive the optimal solution if the government maximizes social welfare). To gain some insight into to logic of this proposition consider the second order expansion of the objectives of the government:[20]

$$\sum_{t=0}^{\infty}\beta^t U_t \approx -\sum_{t=0}^{\infty}\beta^t(\lambda_\pi\pi_t^2+\lambda_x x_t^2) \qquad (11)$$

where $\lambda_x = \sigma^{-1}+\omega$, $\lambda_\pi = d''$ and $x_t$ is the output gap, i.e. the percentage difference between actual output and the output that would be produced under flexible wages. This illustrates that if the government maximizes the utility of the representative household (using the nominal interest rate as its only policy instrument) it suffers losses from both price variations (captured by the coefficient $\lambda_\pi$) and output variations (captured by the coefficient $\lambda_x$).[21] Consider the linearized

[19] This can be seen as a special case of the solution method presented in Appendix III for a more general stochastic process.

[20] This objective is derived only for illustration since I use the exact utility of the representative household to find the optimal policy in Appendix II. Derivation of this loss function is available from the author upon request.

[21] Note that there is no inflation bias in the objective of the government. The deflation bias would still be obtained in an economy with an inflation bias in steady state but the shocks that give rise to it would need to be correspondingly larger.

version of the IS equation and solve it forward:

$$x_0 = -\sigma E_0 \sum_{t=0}^{\infty}(r_t - r_t^n) \tag{12}$$

where $r_t = i_t - E_t\pi_{t+1}$ and $r_t^n = \bar{r} + \frac{\sigma^{-1}\omega}{\sigma^{-1}-\omega}(g_t - E_t g_{t+1})$ is the natural rate of interest that depends only on the exogenous disturbance. This illustrates that the output gap depends on the difference between expectations of future and current real short rates and the natural rate of interest.[22] The central bank can then keep the output gap close to zero at all times (which raises social welfare) if it keeps the real rate close to the natural rate of interest. In case *CTr* the natural rate is negative for large enough shocks. Then a negative real rate of return is required to prevent an excessive negative output gap and deflation. This cannot be achieved through the nominal interest rates because of the zero bound. The real rate of return, however, can still be lowered by expected inflation which in turn increases aggregate demand by (12). To increase inflation expectations the central bank commits to higher money supply in the future. This is the logic behind Proposition 2. The commitment solution under *CTr* is shown as the solid line in Figure (1) from period -1 to period 2 for a calibrated version of the model.[23] The government commits to inflation in period 1 to reduce the real rate of return in period 0. To achieve this commitment in period 0 the government announces that money supply in period 1 will be higher relative to its initial value. Note that in period 0 money supply is indeterminate. This is because in period 0 the nominal interest rate is zero so that increasing money supply by buying government bonds has no effect.

If the government is discretionary it is easy to show by solving (A-28)–(A-32) that $\pi_1^{com} > \pi_1^{dis} = 0$ where the superscript $dis$ refers to the discretionary equilibrium. Although optimal policy under commitment requires inflation in period 1, a discretionary central bank cannot commit to positive inflation. The result is excessive deflation and output gap in period zero. This is the deflation bias of discretionary policy. The deflation bias can be shown by solving (A-11)–(A-18) and (A-28)–(A-32) in Appendix II:[24]

$$\pi_0^{dis} - \pi_0^{com} = \kappa \frac{\omega\lambda_\pi^{-1} + \kappa^2}{\kappa^2 + (\sigma^{-1}+\omega)\lambda_\pi^{-1} + \beta\sigma^{-2}}(g_0 - g_{Tr}) < 0 \tag{13}$$

$$x_0^{dis} - x_0^{com} = \frac{\omega\lambda_\pi^{-1} + \kappa^2}{\kappa^2 + (\sigma^{-1}+\omega)\lambda_\pi^{-1} + \beta\sigma^{-2}}(g_0 - g_{Tr}) < 0 \tag{14}$$

Equation (13) and (14) reveal that the deflation bias of discretionary policy does not depend on the cost of price changes, $\lambda_\pi$, being large. Indeed the contrary is the case. The deflation bias is most severe as the costs of changing prices approaches zero!

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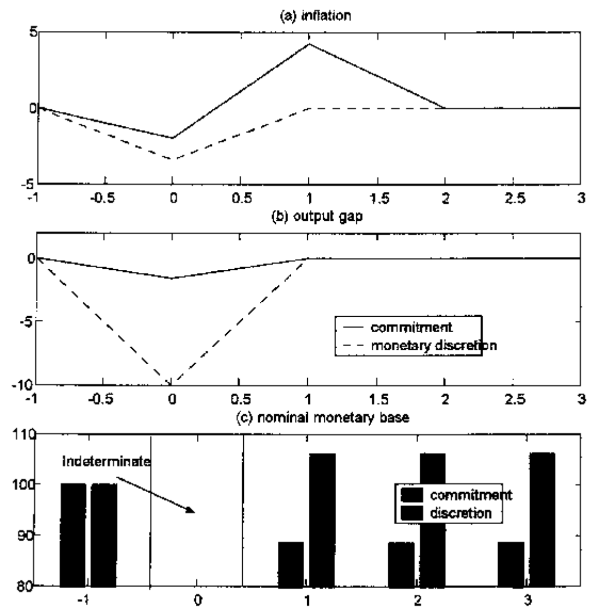
22 The natural rate of interest is the real rate of interest if actual output is equal to the flexible wage output.

23 As should be clear from the discussion above, the result do not in any way depend on these particular values of the parameters or the special functional form chosen. See Appendix D for the calibration parameters assumed.

24 This can be seen as a special case of the solution method presented in Appendix C for a more general stochastic process.

Figure 1: The Deflation Bias




**Proposition 3** ***The Deflation Bias.*** *Suppose CTr. If the only instrument of the government is the nominal interest rate, a discretionary central bank suffers excessive deflation and output gap compared to a central bank that can commit. The deflation bias of discretionary policy is decreasing with the cost of inflation* $\lambda_\pi$.

Proof: See equations (13) and (14), and take the derivative with respect to $\lambda_\pi$.

The discretion solution is shown by the dotted line in Figure (1) for a calibrated version of the model. The inability of the central bank to commit to future inflation results in excessive deflation. Why has the central bank an incentive to renege on an inflation promise? Since the government is trying to minimize (approximately) $\pi_0^2 + \lambda_x x_0^2$ (and future losses), it is indeed beneficial in period 0 to promise higher money supply in period 1 to increase expected $\pi_1$ and lower the real rate of return (thereby increasing demand to accommodate a negative $r_t^n$). This is what the commitment solution illustrates. In period 1, however, the central bank is minimizing $\pi_1^2 + \lambda_x x_1^2$ (and future losses) regardless of promises made in period 0. It thus has an incentive to renege on its inflation promise. The private sector understands this so in rational expectation a credible inflation target cannot be established. The central bank cannot credibly promise higher future money supply! Since the nominal interest is zero, open market operations in short-term government bonds have no effect, household (or banks) simply replace the government bonds in their vaults with money. This is why the money supply in Figure (1) in period 0 is indeterminate. No matter how much the central bank increases money supply in period 0, it is irrelevant since the private sector expects it to be contracted again in period 1. This set of expectations, consistent with rational behavior, can explain why the dramatic increase in the monetary base in Japan has failed to curb deflation – the private sector expects the BOJ to contract as soon as there is any sign of inflation! It is a credibility problem of a *rational* central bank that *cannot commit to future policy*.[25]

Two aspects of a liquidity trap render the deflation bias a particularly acute problem, and possibly a more serious one for policy makers than the inflation bias analyzed by Kydland and Prescott (1977) and Barro and Gordon (1983). First, if the central bank announces a higher inflation target in a liquidity trap it involves no direct policy action - since the short term nominal interest rate is at zero it cannot lower them any further. The central bank has therefore no means to manifest its desire for inflation. Thus announcing an inflation target in a liquidity trap may be less credible than under normal circumstances when the central bank can take direct actions to show its commitment. Second, unfavorable shocks create the deflation bias. If these shocks are infrequent (which is presumably the case given the few examples of a binding zero bound in economic history) it is hard for the central bank to acquire any reputation for dealing with them. To make matters worse, optimal policy in a liquidity trap involves committing to inflation. In an era of price stability the optimal policy under commitment is fundamentally different from what has been observed in the past.

The deflation bias is not unique to the simple process we assumed for $g_t$. The result can be generalized by considering a stochastic process for $g_t$.

**Case 2 (C2)** *In period zero there is an unexpected shock to* $g_0$. *Conditional on that* $g_{t-1} \neq 0$ *in every period* $t > 0$ *there is a probability* $\alpha_t > 0$ *that the vector of shocks* $\xi_t$ *return back to zero so that* $g_t = 0$. *Let us call the stochastic date* $g_t$ *returns to zero* $T$. *In periods* $t > T$ *there are no further shocks to the economy.*

Appendix C illustrates a general solution method for the stochastic process in C2. Figure (2) illustrates several different contingencies for a simple stochastic process that is a special case of C2 which allow for the possibility that $g_t$ does not return to zero with certainty in period 1 but only reverses back with a certain probability.[26]

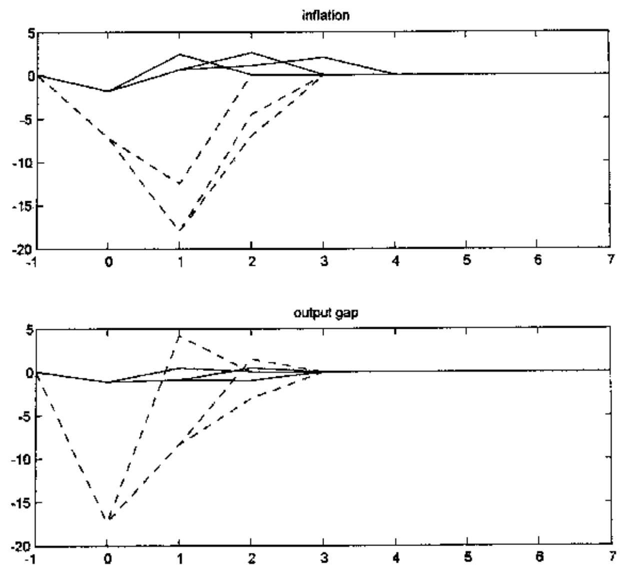
## IV. Committing to Being Irresponsible

In the last section I showed that a discretionary government is unable to commit to higher money

[25] Note that although $r_t^n$ is only temporarily negative (so that output will automatically go back to normal in the future) there is nothing about this dynamic inconsistency problem that requires this to be the case. It is possible to write a model in which the natural rate of interest is endogenous (so that the liquidity trap can be maintained for arbitary large number of periods) and still obtain the same solution.

[26] I suppose that in period 0 there is a $2/3$ probability that the shocks return back to their zero in period 1. Similarly, conditional on being in the trap in period 1, there is a $2/3$ probability that the shocks return back to their zero in period 2. Finally we suppose that the shocks are back to zero in period 3 with probability 1. Once the shocks return back to zero there are no further shocks to the economy. In the figure I suppose that $g_t$ follows a path so that in each of the periods the economy is trapped, the natural rate of interest is $-3/\beta\%$ (so that 3% expected inflation would be required to close the output gap). The figure shows all the different contingencies given this simple stochastic process. If the liquidity trap lasts over several periods, the optimal commitment policy does not only involve expected inflation, there will also be inflation during the trap. This can for example be seen by the line that illustrates the contingency that $r_t^n = -3/\beta\%$ in periods 0,1, and 2. Again, although the optimal commitment solution involves expected inflation in the trap, a discretionary policy maker cannot commit to positive inflation.

Figure 2: The Deflation Bias in Stochastic Setting




Note: The figure shows the commitment solution (solid line) and discretionary solution (dashed lines) when the nominal interest rate is the only policy instrument

supply in the future by open market operations in short-term bonds. It cannot commit to the optimal inflation target with only one policy instrument. The challenge for policy makers is to take actions that renders a higher inflation target credible. The government must "commit to being irresponsible" in the words of Krugman (1998).[27] Here I illustrate one simple solution to this credibility problem. Suppose the government coordinates fiscal and monetary policy to maximize social welfare. In this case, the government can use fiscal policy to effectively commit the central bank to future inflation by cutting taxes and issuing nominal debt.

### A. The Discretion Equilibrium Under Coordination

The structure of the game is exactly the same as described as described in previous section except for that in this case the treasury and the central bank coordinate their policy instruments, $i_t$ and $\tau_t$, to maximize social welfare. Introducing fiscal policy adds two new constraints on the policy choices of the government. The taxing decisions of the treasury must satisfy the government budget constraint (7) and the borrowing constraint (8). There are now two state variables of the game, i.e. $\tilde{b}_{t-1}$ and $\xi_{t-1}$ so that the strategy functions of the players depend on both these variables i.e. $f_t^e = f^e(\xi_t, \tilde{b}_t)$,. $u_t^e = u^e(\xi_t, \tilde{b}_t)$ and $v_t^e = v^e(\xi_t, \tilde{b}_t)$. Equation (7) and the IS and the AS equations indicate that government policy depends on $u_{t-1}^e, f_{t-1}^e, v_{t-1}^e$ and $\xi_t$. The maximization problem of the government is:

$$\max_{i_t, \tau_t}[u(Y_t - d(\pi_t) - F, \xi_t) + g(F - s(\tau_t), \xi_t) - \tilde{v}(Y_t) + \beta V(\tilde{b}_t, \xi_t)] \tag{15}$$

---

[27] Although Krugman does not model how this commitment migh be achieved.

s.t. (2),(4),(6),(7),(8) and the strategy function of the players. Here $V(\tilde{b}_t, \xi_t)$ is the value function of the government, i.e. the expected value at time $t$ of utility in period $t+1$ onwards. This value is calculated under the expectations that the government will maximize under discretion from period $t+1$ onwards. The value function satisfies the Bellman equation:

$$V(\tilde{b}_{t-1}, \xi_{t-1}) = E_{t-1} \max_{i_t, \tau_t} \{u(Y_t - d(\pi_t) - F, \xi_t) + g(F - s(\tau_t), \xi_t) - \tilde{v}(Y_t) + \beta V(\tilde{b}_t, \xi_t)\} \quad (16)$$

s.t. (2),(4),(6),(7),(8) and the strategy functions of the players. To characterize the strategy functions of the households and the government I write a Lagrangian for the maximization problem defined in Appendix B.

## B. Approximate Solution

In this subsection I derive explicit solutions by a first-order Taylor expansion shown in Appendix B. I use the same assumption about the stochastic process for $g_t$ as in last section in Cases 1 and 2 and apply the solution method proposed in Appendix III. In Appendix II I show that in the absence of shocks the solution takes the form:

$$b_t = \rho b_{t-1} \quad (17)$$

$$\pi_t = \Pi b_{t-1} \quad (18)$$

I show in Appendix II that $\rho$ is a real number between 0 and 1. The coefficient $\Pi$ is shown to be a positive number and is given by

$$\Pi = \frac{\lambda_\tau}{\lambda_\pi}\beta^{-1} + \frac{\lambda_\tau}{\lambda_\pi}\rho\kappa^{-1}\sigma^{-1} \quad (19)$$

where $\lambda_\tau = g_G s'$ is the marginal cost of taxes in utility units. This solution shows that nominal debt effectively commits the government to inflation even if it is discretionary. To gain further understanding of this result it is useful to consider a second order expansion of the representative household utility[28]:

$$\sum_{t=0}^{\infty} \beta^t U_t \approx -\sum_{t=0}^{\infty} \beta^t (\lambda_\pi \pi_t^2 + \lambda_x x_t^2 + \lambda_\tau \tau_t^2)$$

In any given period $t$ the government has outstanding nominal debt $B_{t-1}$. The government can reduce the real value of this debt (and future interest payments) by either increasing taxes or increasing inflation. Since both inflation and taxes are costly it will choose a combination of the two. The presence of debt creates inflation through two channels in our model: (1) If the government has outstanding *nominal* debt it has incentives to create inflation to reduce the real value of the debt. This effect is captured by $\frac{\lambda_\tau}{\lambda_\pi}\beta^{-1}$ in the expression for $\Pi$. (2) If the government issues debt at time $t$ it has incentives to lower the real rate of return its pays on the debt it rolls

[28] The expansion is around zero government spending. Derivation is available upon request from the author. Note that I use the exact utility of the representative household when deriving the solution in Appendix B- the second order expansion is just meant to clarify the logic of the results.

over to time $t+1$. This incentive also translates into higher inflation and is captured by the term $\frac{\lambda_\tau}{\lambda_\pi}\rho\kappa^{-1}\sigma^{-1}$ in $\Pi$.[29] The next proposition is immediate from the expressions above.

**Proposition 4** ***Committing to Being Irresponsible.*** *If the central bank and the treasury coordinate policy to maximize social welfare a government can commit to future inflation in a liquidity trap by cutting taxes and issuing nominal debt. Inflation is highest as the economy emerges from the liquidity trap. It declines with public debt over the infinite horizon and converges to zero in absence of other shocks.*

Proof: see equations (17) and (18)

I now show the optimal debt issued when the government is discretionary. Suppose *CTr* so that $g_0 < g_{Tr}^{Cdis}$ where $g_{Tr}^{Cdis}$ is the critical value of the shock that makes the zero bound binding. The superscript *Cdis* refers to the discretion equilibrium when the government coordinates fiscal policy and monetary policy. Then solving the equations (A-17), (A-18), and (A-28)–(A-39) in Appendix II yields:

$$b_0^{Cdis} = -\tau_0 = -\phi(g_0 - g_{Tr}^{Cdis}) + b_{Tr}^{Cdis} > 0 \tag{20}$$

where $\phi > 0$ and $b_{Tr}^{Ddis} > 0$ is a constant equal to the debt issued if $g_0 = g_{Tr}^{Cdis}$. I obtain closed form solutions for these constants given in Appendix III. Equation (20) illustrates that deficit spending is optimal in a liquidity trap. The government cuts taxes and issues debt to effectively commit to future inflation. The solution for inflation in period 1 onward is given by (17)–(20):

$$\pi_t = \Pi\rho^t b_0 > 0 \text{ for } t \geq 1 \tag{21}$$

By committing to inflation the government curbs deflation and reduces output losses in period zero relative to the discretionary solution when it is unable to coordinate monetary and fiscal policy. This can be shown by solving (A-17), (A-18), and(A-28)–(A-39) in Appendix II:

$$x_0^D - x_0^{Ddis} = \sigma\Pi[\phi(g_0 - g_{Tr}^{Ddis}) - b_{Tr}^{Ddis}] < 0 \tag{22}$$

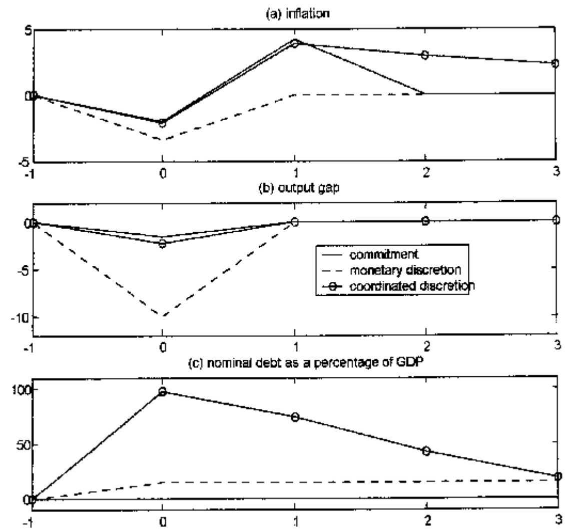
$$\pi_0^D - \pi_0^{Ddis} = \sigma k\Pi[\phi(g_0 - g_{Tr}^{Ddis}) - b_{Tr}^{Ddis}] < 0 \tag{23}$$

**Proposition 5** ***The Optimality of Committing to Being Irresponsible.*** *Suppose CTr and that the government has two policy instruments, fiscal policy and the nominal interest rate. Then the government cuts taxes and issues positive amount of public debt in a liquidity trap. In doing so it will reduce the output gap and commits to inflation.*

---

[29] Obstfeld (1991,1997) analyses a flexible price model with real debt (as opposed to nominal as in our model) but seignorage revenues due to money creation. He obtains a solution similar to (17) and (18) (i.e. debt in his model creates inflation but is paid down over time). Calvo and Guidotti (1990) similarly illustrate a flexible price model that has a similar solution. The influence of debt on inflation these authors illustrate is closely related to the first channel we discuss above. The second channel we show, however, is not present in these papers since they assume flexible prices.

Figure 3: Committing to being Irresponsible.




Proof: See equations (20)–(23).

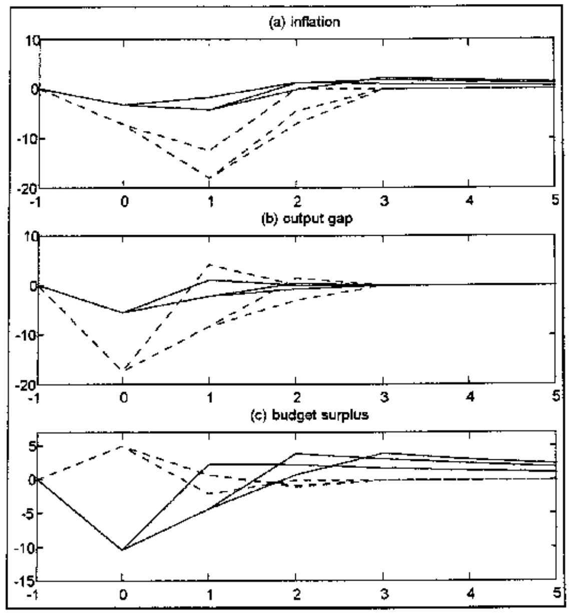
The evolution of each of the endogenous variables is shown in Figure (3) and are labelled coordinated discretion for the same calibration parameters as in the last section.[30] This figure shows the evolution under coordinated discretion (described above) and contrasts it to the solution paths when fiscal policy is not coordinated with monetary policy which I illustrated in the last section (this case is labelled monetary discretion). By cutting taxes and issuing debt in a liquidity trap the government curbs deflation and increases output almost to the same level as obtained under commitment. The discretion solution is still inferior to the commitment solution since the price that has to be paid for this is higher inflation in period 2 onward (when no inflation is desirable) and higher future taxes. The assumed calibration parameters for this figure is shown in Appendix D along with a sensitivity analysis. Figure (5) illustrates that the same result holds true in the simple stochastic example we considered in the last section. Again a discretionary government can effectively commit to an inflation target in a liquidity trap by deficit spending, thereby curbing deflation and increasing the output gap.[31]

Propositions 5 and 6 summarize the central results of this paper. Even if the government cannot make commitment about future policy, it can control the price level at zero nominal interest rates. A simple way to increase inflation expectations is to coordinate fiscal and monetary policy and run budget deficits. This increases output and prices. The channel is simple. Budget deficits generate nominal debt. Nominal debt in turn makes a higher inflation target in the future credible

[30] Again as the analytical results indicate our results do not in any way depend on the numerical values assumed. See Appendix D for the calibration parameters and some sensitivity analysis.

[31] It is worth noting that in both the numerical examples discussed the social welfare is higher under cooperation than non-cooperation when the government is discretionary as one would expect.

Figure 4: Committing to being Irresponsible in a Stochastic Setting.




Note: The figure shows optimal cooperation under discretion (solid line) and discretion when the Central Bank is independent (dashed lines).

because the real value of the debt increases if the government reneges on the target. Higher inflation expectations lower the real rate of interest and thus stimulate aggregate demand. This policy involves direct actions by the government as opposed to only announcements about future policies. The government can announce an inflation target and then increase budget deficits until the target is reached.

## V. NON-STANDARD OPEN MARKET OPERATIONS

In this section I show how the government can increase prices by printing money (or debt) and buying real assets. I introduce a real asset that has an exogenous real rate of return given by $1 + q_t$. I *assume* that the government can buy this asset in unlimited amounts without affecting $q_t$. This asset can be a parable for, say, foreign exchange, stocks or any real asset that is in fairly elastic supply since I assume that the government cannot influence its return $q_t$. Now the government budget constraint is:

$$\frac{\tilde{b}_t}{u_c(C,\xi_t)} + a_t = \frac{1}{\beta f^e_{t-1}} \frac{\tilde{b}_{t-1}}{1+\pi_t} + (1+q_{t-1})a_{t-1} + F - \tau_t$$

where $a_t$ is the quantity of the asset held measured in terms of the consumption good. As shown in Appendix B the presence of this asset implies a first order condition approximated by:

$$\hat{\tau}_t = E_t\hat{\tau}_{t+1} - \chi^{-1}\hat{q}_t \qquad (24)$$

where the system has been linearized around a constant solution in which $1 + q = 1/\beta$ and

$\chi = \frac{\tau s'}{G}\sigma_g^{-1} + \frac{s''\tau}{s'}$. Condition (24) is Barro's (1979) famous tax—smoothing result.[32] Furthermore, it can be verified that nominal debt is zero if the nominal interest rates is positive:

$$\text{if } i_t > 0 \; b_t = 0 \tag{25}$$

The result in (25) indicates that the government always eliminates any outstanding nominal debt when the nominal interest rate is positive. The logic behind is simple: Nominal debt creates inflation expectations. If the nominal interest rate is positive the government has no incentive to create inflation expectations. It then sells real assets (or issue real bonds) to collect all outstanding nominal debt to eliminate the inflation bias it creates. For any initial value of nominal debt, $b_{t-1}$, the inflation in period $t$ is given by:

$$\pi_t = \Pi^{Adis} b_{t-1}$$

where

$$\Pi^{Adis} = \frac{\lambda_\tau}{\lambda_\pi}\beta^{-1}$$

here the superscript *Adis* refers to the discretion equilibrium if the government can trade in the real asset. Note that $\Pi^{Adis} < \Pi$ so that nominal debt creates less inflation incentive when real assets are traded. The logic is simple: According to (25) the government does not roll over any nominal debt when the nominal interest rate is positive. The government has then no incentive to reduce the real rate of return of any debt rolled over to the next period with surprise inflation (recall that I assume that the government cannot influence $q_t$). This reduces its inflation incentive.

I now consider the optimal solution under discretion under condition *CTr*, i.e. the case when there is a large negative demand shock in period 0 so that the zero bound is binding. If the zero bound is binding, nominal debt (money + bonds) is still a desirable commitment device. Solving the system of equations given in Appendix B, the optimal level of debt in period 0 is:

$$b_0^{Adis} = -a_0 = -\phi^a(g_0 - g_{Tr}^{Adis}) > 0 \tag{26}$$

According to this equation the government prints money (or debt) and buys asset, $a_0$, in period 0. Under condition *CTr* it sells all its assets in period 1 by (25), thereby, to a first order approximation, collecting almost all of its outstanding nominal debt.

Figure (5) shows the evolution of inflation and the output gap. In our numerical example, the discretionary government comes very close to replicating the commitment solution. Figure (6) shows the asset and debt side of the balance sheet of the government in period 0 to 4. To increase inflation expectations the government buys real assets in period 0 by printing money (or bonds) and thereby increasing government debt (which is defined as money+bonds). In period 1 the government reverses this transaction by selling the bulk of its real assets. The nominal debt issued in period 0 effectively commits it to higher inflation in period 1. The logic behind our result is

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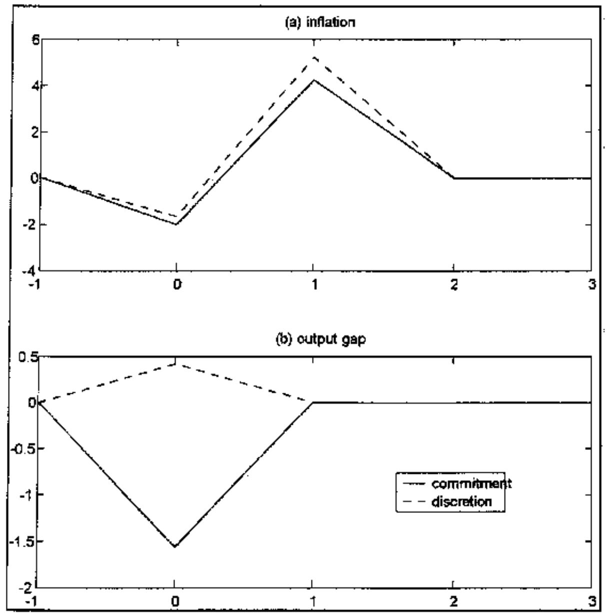
[32] This condition is in sharp contrast to the solution derived in last section when the government only traded in nominal bonds. In that case the government was unable to eliminate nominal debt by going short on real assets. Instead it violated the Barro's tax smooting principle by raising current taxes relative to future taxes.

Figure 5: Bying Real Assets to Commit to Inflation




along the same lines as before. In period 1 the government has outstanding $b_0^{defD}$ nominal debt. Thus in period 1 the government budget constraint is given by:

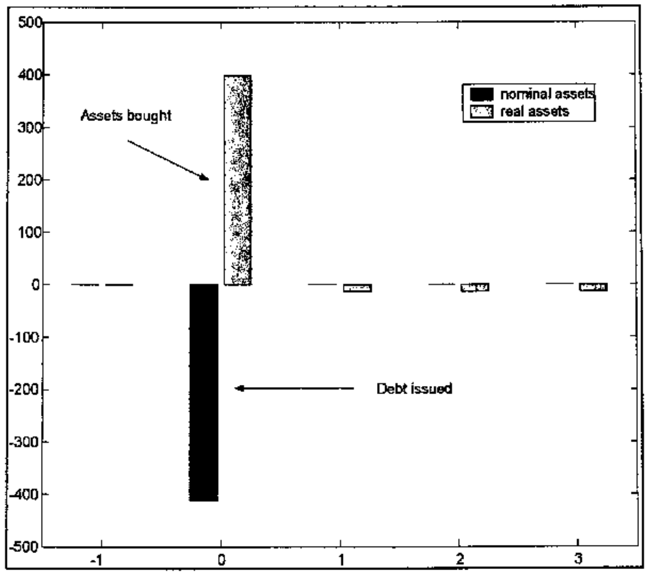
$$a_1 + b_1 = \frac{1}{\beta f_0^e} \frac{\tilde{b}_0}{1 + \pi_1} + (1 + q_0) a_0 + F - \tau_1$$

In period 1 $\tilde{b}_0, f_0, q_0$ and $a_0$ are all predetermined. If the government provides less inflation, $\pi_1$, than was expected when the expectation variable $f_0^e$ was formed, the real value of its assets, $-(1 + q_0)a_0$, decreases relative to the real value of its nominal debt $\frac{1}{\beta f_0^e}\frac{\tilde{b}_0}{1+\pi_1}$, i.e. the government incurs *capital losses*. Thus if the government sets the inflation rate below the optimal target announced in period 0, it has to increase taxes today and in the future to pay for these losses. If the government cares about tax distortions this is not in its interest. Thus the government effectively commits to the optimal inflation target by printing money and buying real assets. *This illustrates that in this model the cost of taxation can be thought of as a parable for the central bank being concerned about capital losses.* This is a topic I will discuss in better detail in the conclusion.

To summarize:

**Proposition 6** ***The Optimality of Buying Real Assets.*** *Suppose CTr. If the government buys real assets by issuing nominal debt or printing money, this effectively commits it to higher money supply in the future and inflation. If it has access to an elastic supply of real assets, a discretionary government is able to commit to future inflation and comes close to replicating the commitment solution.*

Figure 6: The Government Balance Sheet when Buying Real Assets.




As can be seen by Figure (6) the discretion solution when the government buys real assets is superior to the solution when deficit spending is the only tool available to the government. The reason is straight forward: When the government buys real assets, rather than cutting taxes, it can sell these assets in period 1 and thereby eliminate the inflationary bias in period 2 onwards when inflation is not desirable. A critical assumption, however, is that I *assume* that the rate of return of the asset, $q_t$, is exogenously given and is not influenced by the actions of the government. Thus it is implicitly assumed that the asset exists in perfectly elastic supply and that it is a perfect a substitute for nominal bonds or money, so that there are no portfolio balance effects of these transactions. This assumption may be questionable, even as a first order approximation, if the open market operation required to achieve the inflation target are very large. Our calibration indicates that this may indeed be the case. Figure (6) shows that in that the government would need to print money (or bonds) and buy assets corresponding to over 400 percent of annual GDP! This number however depends on the cost of taxes assumed. Appendix IV shows sensitivity analysis with respect to this number. Even when tax costs are as high as ten percent of government spending, the size of the open market operation in real assets are around 70 percent of GDP. Given the size of this transaction it is likely, therefore, to have large effects on the relative return of assets (which I abstract from here by assuming constant $q_t$). This may, or may not, be desirable. Some combination of deficit spending and purchases of real assets may, therefore, be the best solution.

## VI. Conclusion

What are the implications of this model for Japan today? During the last few years, the short-term nominal interest rate has been zero in Japan. Furthermore, there has been deflation by most indicators and substantial unemployment, and the government has run large budget deficits.

Although the model does not explain the source of the deflationary shocks in Japan (which may be difficulties in the banking sector or the collapse in the asset market in the early 1990s), it provides some ideas on how to eliminate deflation in a liquidity trap by increasing inflation expectations, thereby lowering real interest rates and stimulating aggregate activity. Two other questions arise when this model is applied to Japan. First, why has deficit spending failed to increase inflation expectations in Japan? Second, why has the BOJ not engaged in open market operations in real assets, since our model suggests that it could be a way to curb deflation? I address these questions in turn.

Over the last 10 years gross public debt as a fraction of GDP in Japan has more than doubled, from roughly 64.5 percent in 1990 to 130 percent in 2001, largely owing to deficit spending. Why has this failed to increase inflation expectations? Eggertsson (2001) suggests an answer: The central bank is goal independent. The assumption behind our results is that *monetary and fiscal policy are coordinated to maximize social welfare*. This does not need to be the case under all circumstances. Consider the most simple example illustrated in Eggertsson (2001): Imagine that the BOJ can commit to a gold standard as it did in the 1920's. In this case $M_t = k * gold_t$ where $k$ is some constant. As long as deficit spending does not influence $gold_t$ (which one has no reason to expect in our model), it has no effect on either output or prices. Deficit spending only increases demand if *it influences the central bank choice of the money supply in the future*. A central bank on a gold-standard is only one example of a "goal independent" central bank. It can, in principle, have any objectives different from social welfare. One example of a goal-independent bank that is of particular interest is a bank that maximizes the utility of the representative household but ignores the welfare consequences of government spending. The loss criterion of a bank with this objective is to a second order equal to $\pi_t^2 + \lambda_x x_t^2$ , which is commonly assumed in the literature. In this case, as shown in Eggertsson (2001), deficit spending has no effect on either prices or output. Then the equilibrium is the same as was illustrated in Section III, when the nominal interest rate was the only policy instrument of the government. The channel for fiscal policy illustrated here, therefore, works only if fiscal and monetary policies are coordinated to maximize social welfare.

Turning to the second question, it is useful to review first whether or not a goal-independent central bank (for example, one that has a loss function $\pi_t^2 + \lambda_x x_t^2$) can can curb deflation. Suppose, for example, that it prints money and buys foreign exchange or real assets. Will this increase inflation expectations? Costly taxation can be thought of as a parable for having the government, or a goal-independent central bank, being concerned about its balance sheet—that is caring about capital gains and losses. The central bank, therefore, has to care only about *its own capital gains or losses* for open market operations in foreign exchange or real assets to be effective. Do goal- independent central banks care about capital losses? If a central bank incurs capital losses (for example, by buying real assets that lose value), it can compensate for them by one of two ways: printing money or obtaining tax revenues collected by the treasury. The first may imply excessive inflation, the other a bailout from the treasury associated with a loss of independence. For most central banks, this choice is like choosing between death by fire or by drowning. There is, therefore, every reason to expect a goal-independent central bank to care about capital losses, and this concern enables the bank to use foreign exchange intervention or buying real assets as a commitment device.[33]

If the central bank is risk averse, however, there is a catch. It may be reluctant to buy any assets that have uncertain returns; and, apart from short-term government bonds, any asset has an uncertain return. If the bank prints money and buys foreign exchange, to take one example, there is always a positive probability that at some future date the bank will have to choose between high inflation or high capital losses. Risk aversion, therefore, may limit the bank from taking *any actions that carry risk* when the zero bound is binding. This remains true even if these actions enable the bank to achieve some of its goals, such as output and price stabilization. One implication is that even though a central bank has several policy instruments with which to escape a liquidity trap, in its arsenal the bank's concern about its independence may limit its use of them. The cause of the deflation bias was that the only instrument of policy was open market operations in short-term bonds. This assumption, therefore, can be justified as a description of central bank behavior because extensive use of other instruments implies balance-sheet risks. In this case, cooperation between the treasury and the central bank may be required to persuade a goal-independent central bank that is "too risk averse" to effectively commit to end deflation.

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33 See e.g. discussion by the BOJ about option that may put its balance sheet at risk.

## I. APPENDIX - MICROFOUNDATIONS

### A. Aggregate supply equation

The budget constraint of the representative household is:

$$\frac{D_t}{P_t} + \frac{M_t}{P_t} = \frac{A_t}{P_t} + \frac{M_{t-1}}{P_t} + h_t\frac{W_t}{P_t} - \tau_t - C_t + \int_0^{N_t} Z_t(j)dj \qquad \text{(A-1)}$$

where $D_t$ is the nominal value of the end of period bond portfolio, $M_t$ is money balances held at the end of period $t$, $W_t$ is the nominal wage rate, $A_t$ is the beginning of period nominal wealth, $\tau_t$ is net real tax collections by the government, $Z_t(j)$ is the real profit from firm $j$ and $N_t$ is the number of firms. The consumption plan of the representative household must satisfy a transversality condition out Ponzi schemes:[34]

$$\lim_{T\to\infty} \beta^T E_t(u_c(C_T, \xi_T)\frac{B_T}{P_T}) = 0 \qquad \text{(A-2)}$$

where $B_T$ is total nominal debt.

At time $t$ there is a fixed number $N_{t+1}$ of labor contracts offered by firms for the next period.[35] The household chooses how many contracts, $n_{t+1}$, to accept facing the market wage $W_{t+1}$ that it takes as exogenous. We assume that $W_{t+1}$ is determined at time $t$ so as to clear the labor market. In period $t+1$ the firms are free to choose the hours worked at the given wage rate. Thus at time $t+1$ the representative household supplies the labor $h_{t+1} = \frac{n_{t+1}}{N_{t+1}} L_{t-1}$ where $L_{t+1}$ is aggregate labor demand of firms. The optimal labor supply condition for the household is:

$$E_{t-1}\frac{u_c(C_t, \xi_t)}{P_t} W_t L_t - E_{t-1}\tilde{v}_y(Y_t)F'(L_t)L_t = 0$$

To derive this condition, substitute the budget constraint into the household utility, take a derivative with respect to $n_t$, and normalize the number of jobs to 1. In every period $t$ each firm offers one labor contract for the next period and the chooses labor to maximize profits:

$$\frac{W_t}{P_t} = F'(L_t)$$

Equating these two first order condition yields

$$F'(F^{-1}(Y_t)) = \frac{1}{1+\pi_t} \frac{E_{t-1}(\tilde{v}_y(Y_t)F'(F^{-1}(Y_t))F^{-1}(Y_t))}{E_{t-1}(\frac{u_c(Y_t - d(\pi_t), \xi_t)}{1+\pi_t} F^{-1}(Y_t))} \qquad \text{(A-3)}$$

To obtain this condition note that $W_t$ is determined at time $t-1$ and can thus be taken outside the expectation in the first order condition of the household. Equation (A-3) gives (6) in the text.

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[34] For a detailed discussion of this borrowing limit and its interpretation see Woodford (2001a).

[35] This contracting structure is proposed by Levin (1990).

## B. Government Budget Constraint

The consolidated government budget constraint is:

$$D_t + M_t = (1 + i_{t-1})D_{t-1} + M_{t-1} + FP_t - \tau_t P_t \tag{A-4}$$

where $D_t$ now refers to one period nominal government debt. Let total government liabilities be $B_t \equiv D_t + M_t$. Rewrite the government budget constraint as:

$$B_t = (1 + i_{t-1})B_{t-1} + FP_t - \tau_t P_t - S_t \tag{A-5}$$

where $S_t \equiv i_{t-1}M_{t-1}$ is seigniorage revenues. For simplicity we abstract from the effects of seigniorage revenues on the government budget constraint. Thus $S_t$ is dropped in equation (A-5). By abstracting from seigniorage revenues we are in effect stacking the cards against the result we obtain. The central conclusion of Section IV is that the government can commit to future inflation by issuing debt because it creates inflation incentives in the future. The presence of seigniorage revenues would make these inflation incentives even stronger. Abstracting from seigniorage revenues has little quantitative impact on the results if they are low relative to other items in the government budget.[36] This is indeed the case in most industrial countries.[37] Seigniorage revenues in Japan, measured as $S_t = i_{t-1}M_{t-1}$ [38], were 0.34 percent of GDP in 1980–1990 and only 0.15 percent in 1990–2000 as the nominal interest rate approached zero.[39] Ignoring $S_t$ , the budget constrain can now be written in terms of $\tilde{b}_t$ , yielding equation (7) in the text.

[36] In our model seignorage revenues are low if monetary friction are small. Let us write the utlitity of real money balances as $q(\frac{M_t}{P_t}, \xi_t) \equiv \gamma\tilde{q}(\frac{M_t}{P_t}, \xi_t)$ where the parameter $\gamma$ is greater than zero. Then small monetary frictions simply referes to a low value of $\gamma$ in the representative houshold utility function. As shown by Woodford (2001a) the Central Banks control over the nominal interest rate is still well defined even in the "cashless limit" when there are no monetary frictions, i.e. when $\gamma \to 0$. In this case open market operations needed to move the nominal interest by a given amount become arbitarily small. This of course has no effect on the nature of the zero bound since the sole presence of money as an asset (even though little or none of it is actually held in equilibrium) will prevent consumers from ever accepting a negative nominal interest rate on bonds. One interpretation of our results is that they refer to an economy where monetary frictions are close to zero.

[37] see e.g. King and Plosser (1985) who report that seignorage revenues for the United States were 0.2% of GDP in 1929–52 and 0.47% in 1952–82.

[38] Thus in calculating this number we multiply the montetary base over GDP by some measure of short term nominal interest rate. In the caluculation above we used the official discount rate but using short term yield on Treasury bond would result in similar numbers.

[39] At zero interest rate the seignorage revenues as defined above are zero.

## II. APPENDIX - FIRST ORDER CONDITIONS

### A. Commitment

For compactness I only write the first order condition for the case when the government coordinates fiscal and monetary policy. The same conditions apply if the nominal interest rate is the only policy instrument but in this case $\mu_t = 0$ at all times and condition (A-4), (A-5), and (A-10) do not apply:[40]

$$\begin{aligned} L_s &= E_s \sum_{t=s}^{\infty} \beta^t [u(Y_t - d(\pi_t) - F, \xi_t) + g(F - s(\tau_t)) - \tilde{v}(Y_t) \\ &+\mu_t(\frac{\tilde{b}_t}{u_c(Y_t - d(\pi_t), \xi_t)} - \frac{1}{\beta f_{t-1}^e}\frac{\tilde{b}_{t-1}}{1+\pi_t} + F - \tau_t) \\ &+\eta_t(Y_t - S(\pi_t, u_{t-1}^e, v_{t-1}^e)) + \psi_t(1 - \frac{u_c(Y_t - d(\pi_t) - F, \xi_t)}{\beta f_t^e}) \\ &+\phi_t^1(u_t^e - E_t\frac{u_c(Y_{t+1} - d(\pi_{t+1}) - F, \xi_{t+1})}{1+\pi_{t+1}}F^{-1}(Y_{t+1})) \\ &+\phi_t^2(v_t^e - E_t\tilde{v}_y(Y_{t+1})F'(F^{-1}(Y_{t+1}))F^{-1}(Y_{t+1})) + \phi_t^3(f_t^e - \frac{u_c(Y_{t+1} - d(\pi_{t+1}) - F, \xi_{t+1})}{1+\pi_{t-1}}) \\ &+\gamma_t(\tilde{b}_t - b)] \end{aligned} \quad \text{(A-1)}$$

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$$\begin{aligned} \frac{\delta L_s}{\delta \pi_t} &= -u_c d' \pi_t + \mu_t(\frac{\tilde{b}_t u_{cc} d'}{u_c^2} + \frac{1}{\beta f_{t-1}^e}\frac{\tilde{b}_{t-1}}{(1+\pi_t)^2}) - \eta_t S_\pi + \psi_t \frac{u_{cc}}{\beta f_t^e} d' \\ &+\frac{1}{\beta}\phi_{t-1}^1(\frac{u_c F^{-1}}{(1+\pi_t)^2} + \frac{u_{cc}F^{-1}d'}{1+\pi_t}) + \frac{1}{\beta}\phi_{t-1}^3(\frac{u_c(Y_t, \xi_t)}{(1+\pi_t)^2} + \frac{u_{cc}d'}{1+\pi_t}) = 0 \end{aligned} \quad \text{(A-2)}$$

$$\begin{aligned} \frac{\delta L_s}{\delta Y_t} &= u_c(Y_t - d(\pi_t), \xi_t) - \tilde{v}_y(Y_t) - \mu_t \frac{u_{cc}}{u_c{}^2}\tilde{b}_t + \eta_t - \frac{\psi_t}{\beta}\frac{u_{cc}}{f_t^e} \\ &-\frac{\phi_{t-1}^1}{\beta}(\frac{u_{cc}}{1+\pi_t} + \frac{u_c}{1+\pi_t}\frac{1}{F'}) \\ &-\frac{\phi_{t-1}^2}{\beta}(\tilde{v}_{yy}F^{-1} + \tilde{v}_y + v_y F'' F^{-1}) - \frac{\phi_{t-1}^3}{\beta}\frac{u_{cc}}{1+\pi_t} \end{aligned} \quad \text{(A-3)}$$

$$\frac{\delta L_s}{\delta \tau_t} = -g_G(F - s(\tau_t))s'(\tau_t) + \mu_t = 0 \quad \text{(A-4)}$$

$$\frac{\delta L_s}{\delta \tilde{b}_t} = \frac{\mu_t}{u_c(Y_t, \xi_t)} - \frac{1}{f_t^e}E_t\frac{\mu_{t-1}}{1+\pi_{t+1}} + \gamma_t = 0 \quad \text{(A-5)}$$

[40] I simplify the Lagrangian by combining the IS equation and the zero bound to yield the inequality $1 - \frac{u_c(Y_t, \xi_t)}{\beta f_t^e} \leq 0$. This eliminates the nominal interest rate from the system reducing the number of first order conditions by one

$$\frac{\delta L_s}{\delta u_t^e} = -\beta E_t \eta_{t+1} S_{u^e}(\pi_{t+1}, u_t^e, v_t^e) + \phi_t^1 = 0 \tag{A-6}$$

$$\frac{\delta L_s}{\delta v_t^e} = -\beta E_t \eta_{t+1} S_{v^e}(\pi_{t-1}, u_t^e, v_t^e) + \phi_t^2 = 0 \tag{A-7}$$

$$\frac{\delta L_s}{\delta f_t^e} = \frac{u_c(Y_t, \xi_t)}{\beta f_t^{e2}}\psi_t + \frac{\tilde{b}_t}{f_t^{e2}} E_t \frac{\mu_{t+1}}{1+\pi_{t+1}} + \phi_t^3 = 0 \tag{A-8}$$

Complementary slackness condition:

$$\frac{u_c(Y_t, \xi_t)}{\beta f_t^e} - 1 \geq 0, \psi_t \leq 0, (\frac{u_c(Y_t, \xi_t)}{\beta f_t^e} - 1),\ \psi_t = 0\, \forall\, t \tag{A-9}$$

$$\gamma_t \geq 0,\ \ \bar{b} - \tilde{b}_t \leq 0,\ \ \gamma_t(\bar{b} - \tilde{b}_t) = 0 \tag{A-10}$$

Note that a solution for the commitment program has to be accompanied by initial conditions for $\phi_t^1, \phi_t^2, \phi_t^3$ and $b_{t-1}$. We assume the initial conditions: $\phi_{-1}^1 = \phi_{-1}^2 = \phi_{-1}^3 = \psi_{-1} = 0$ and $\tilde{b}_{-1} = 0$.

**Approximate Solution**

To obtain an approximate solution I do a first order Taylor expansion. The constant solution I expand around and solves the equation above is $\pi = b = \phi = \eta = \phi = 0$, $\mu = g's'$and $\bar{\imath} = \frac{1}{\beta} - 1$. This constant solution would result for the initial conditions $\phi_{-1}^1 = \phi_{-1}^2 = \phi_{-1}^3 = \psi_{-1} = b_{-1} = 0$ at all times if there are no shocks to the economy so that $\xi_t = 0$ at all times. If the shocks are small enough the approximate equations will be arbitrarily close to the exact equations. Substitution for $\phi_t^1$, $\phi_t^2$ and $\phi_t^3$ the linearized first order are:[41]

$$-u_c d'' \pi_t - \kappa^{-1}\eta_t + \kappa^{-1}E_{t-1}\eta_t - \frac{1}{\beta^2}\psi_{t-1} = 0 \tag{A-11}$$

$$-u_c(\sigma^{-1}+\omega)\hat{Y}_t + u_c\sigma^{-1}g_t + \frac{\sigma^{-1}}{\beta}\psi_t - \frac{\sigma^{-1}}{\beta^2}\psi_{t-1} + \eta_t + \kappa^{-1}(\omega+\sigma^{-1})E_{t-1}\eta_t + g's'\sigma^{-1}b_t - \frac{\sigma^{-1}g's'}{\beta}b_{t-1} = 0 \tag{A-12}$$

$$-(\frac{\tau}{G}s'\sigma_G^{-1} + \frac{s''\tau}{s'})\hat{\tau}_t + \hat{\mu}_t = 0 \tag{A-13}$$

$$\hat{\mu}_t - E_t\hat{\mu}_{t+1} + \sigma^{-1}E_t(Y_t - Y_{t+1} - g_t + g_{t+1}) + \frac{1}{g_G s'}\gamma_t = 0 \tag{A-14}$$

[41] It is useful to note that in the constant solution $S_{u^e} = \frac{\kappa^{-1}}{u^e}$, $S_{v^e} = -\frac{\kappa^{-1}}{v^e}$ and $S_\pi = \kappa^{-1}$.

The complementary slackness conditions are:

$$E_t\hat{Y}_{t+1}-\hat{Y}_t-E_tg_{t+1}+g_t+\sigma E_t\pi_{t+1}+\beta\sigma\bar{\imath} \geq 0,\ \psi_t \leq 0,\ \psi_t(E_t\hat{Y}_{t+1}-\hat{Y}_t-E_tg_{t-1}-g_t+\sigma E_t\pi_{t+1}+\beta\sigma\bar{\imath}) = 0 \tag{A-15}$$

$$\gamma_t \leq 0,\ \ \bar{b}-u_cb_t \geq 0,\ \ \gamma_t(\bar{b}-u_cb_t)=0 \tag{A-16}$$

The linearized AS equation and the budget constraint are:

$$\pi_t = \kappa\hat{Y}_t + E_{t-1}\pi_t + \epsilon_t \tag{A-17}$$

$$b_t = \frac{1}{\beta}b_{t-1} - \tau\hat{\tau}_t \tag{A-18}$$

Here the exogenous term $\epsilon_t$ is defined as $\epsilon_t = -\kappa\frac{\sigma^{-1}}{\sigma^{-1}+\omega}E_{t-1}g_t$. Finally the linearized money demand equation can be written as:

$$\hat{m}_t = \eta_y\hat{Y}_t - \eta_i\hat{\imath}_t \tag{A-19}$$

where $\hat{m}_t$ is the deviation of real money balances from steady state. All the hatted variables above are defined as a percentage deviation from the constant solution. By making assumptions about the stochastic process of $g_t$ one can find an approximate solution using the linearized equations. .

## B. Discretion

Below I show the first order condition for the maximization problem under discretion. For compactness I only write the first order condition for the case when fiscal and monetary policy are coordinated. The same conditions apply if the nominal interest rate is the only policy instrument but in this case $\mu_t = 0$ at all times and condition (A-23), (A-24), (A-26), and (A-27) do not apply:

$$\begin{aligned} L_t \;=\; & u(Y_t - d(\pi_t) - F, \xi_t) + g(F - s(\tau_t), \xi_t) - \tilde{v}(Y_t) + \beta V(\tilde{b}_t, \xi_t) \\ & +\mu_t\left(\frac{\tilde{b}_t}{u_c(Y_t - d(\pi_t) - F, \xi_t)} - \frac{1}{\beta f^e(\tilde{b}_{t-1}, \xi_{t-1})}\frac{\tilde{b}_{t-1}}{1+\pi_t} + F - \tau_t\right) + \eta_t(Y_t - S(\pi_t, \tilde{b}_{t-1}, \xi_{t-1})) \\ & +\psi_t\left(1 - \frac{u_c(Y_t - d(\pi_t) - F, \xi_t)}{\beta f^e(\tilde{b}_t, \xi_t)}\right) + \gamma_t(\tilde{b}_t - \bar{b}) \end{aligned} \tag{A-20}$$

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$$\frac{\delta L_s}{\delta \pi_t} = -u_cd'(\pi_t) + \mu_t\frac{1}{\beta}f^e(\tilde{b}_{t-1}, \xi_{t-1})^{-1}\frac{\tilde{b}_{t-1}}{(1+\pi_t)^2} - \eta_tS_\pi + \psi_t\frac{u_cd'}{\beta f^e} = 0 \tag{A-21}$$

$$\frac{\delta L_s}{\delta Y_t} = u_c(Y_t - d(\pi_t), \xi_t) - \tilde{v}_y(Y_t) - \mu_t\frac{\tilde{b}_tu_{cc}}{{u_c}^2} + \eta_t - \psi_t\frac{u_{cc}}{\beta f^e} = 0 \tag{A-22}$$

$$\frac{\delta L_s}{\delta \tau_t} = -g_G(F - s(\tau_t))s'(\tau_t) + \mu_t = 0 \tag{A-23}$$

$$\frac{\delta L_s}{\delta \tilde{b}_t} = \beta V_{\tilde{b}}(\tilde{b}_t, \xi_t) + \frac{\mu_t}{u_c(Y_t, \xi_t)} + \psi_t \frac{u_c f^e_{\tilde{b}}}{\beta f^{e2}} + \gamma_t = 0 \qquad \text{(A-24)}$$

Complementary slackness conditions:

$$\psi_t \leq 0, \ \frac{u_c(Y_t, \xi_t)}{\beta f^e_t} - 1 \geq 0, \ \ \psi_t(\frac{u_c(Y_t, \xi_t)}{\beta f^e_t} - 1) = 0 \qquad \text{(A-25)}$$

$$\gamma_t \leq 0, \ \ \bar{b} - \tilde{b}_t \geq 0, \ \ \gamma_t(\ \bar{b} - \tilde{b}_t) = 0 \qquad \text{(A-26)}$$

The optimal plan under discretion also satisfies an envelope condition:

$$V_{\tilde{b}}(\tilde{b}_{t-1}, \xi_{t-1}) = E_{t-1}[-\frac{\mu_t}{\beta(1+\pi_t)}(f^e(\tilde{b}_{t-1}, \xi_{t-1})^{-1} - f^e(\tilde{b}_{t-1}, \xi_{t-1})^{-2} f^e_{\tilde{b}}(\tilde{b}_{t-1}, \xi_{t-1})\tilde{b}_{t-1}) - \eta_t S_{\tilde{b}}(\pi_t, \tilde{b}_{t-1}, \xi_{t-1})] \qquad \text{(A-27)}$$

Using the envelope condition we can substitute the derivative of the value function into (A-24) eliminating the value function from the first order conditions. To obtain a solution to the system we must specify initial condition for both state variables: $\tilde{b}_{-1} = 0$ and $\xi_{-1} = 0$

**Approximate Solution**

To obtain an approximate solution we once again do a first order Taylor expansion around a constant solution. The constant solution we expand around and solves the equation above is $\pi = b = \eta = \gamma = 0$, $\mu = g's'$ and $\bar{\imath} = \frac{1}{\beta} - 1$. This solution would result for the initial conditions $\phi^1_{-1} = \phi^2_{-1} = \phi^3_{-1} = \psi_{-1} = b_{-1} = 0$ if there are no shocks to the economy so that $\xi_t = 0$ at all times. If the shocks are small enough the approximate equations will be arbitrarily close to the exact equations. The linearized first order conditions are:[42]

$$-u_c d'' \pi_t + \frac{g_G s'}{\beta} b_{t-1} - \kappa^{-1} \eta_t = 0 \qquad \text{(A-28)}$$

$$-u_c(\sigma^{-1} + \omega)\hat{Y}_t + u_c \sigma^{-1} g_t + g_G s' \sigma^{-1} b_t + \eta_t + \frac{\sigma^{-1}}{\beta}\psi_t = 0 \qquad \text{(A-29)}$$

$$-(\frac{\tau s'}{G}\sigma_G^{-1} + \frac{s'' \tau}{s'})\hat{\tau}_t + \hat{\mu}_t = 0 \qquad \text{(A-30)}$$

$$\hat{\mu}_t - E_t \hat{\mu}_{t+1} + \sigma^{-1} E_t(\hat{Y}_t - \hat{Y}_{t+1} - g_t + g_{t-1}) + f^e_b b_t + \frac{f^e_b}{\beta g_G s'}\psi_t + \frac{1}{g_G s'}\gamma_t = 0 \qquad \text{(A-31)}$$

The complementary slackness conditions are:

$$E_t\hat{Y}_{t+1} - \hat{Y}_t - E_t g_{t-1} - g_t + \sigma E_t \pi_{t+1} + \beta\sigma\bar{\imath} \geq 0, \ \psi_t \leq 0, \ \psi_t(E_t\hat{Y}_{t+1} - \hat{Y}_t - E_t g_{t+1} - g_t + \sigma E_t \pi_{t+1} + \beta\sigma\bar{\imath}) = 0 \qquad \text{(A-32)}$$

[42]Note that $S_b = 0$ in the constant solution which elimintes the term $\eta_{t+1}$ in (A-31).

$$\gamma_t \leq 0,\ \ \bar{b} - u_c b_t \geq 0,\ \ \gamma_t(\bar{b} - u_c b_t) = 0 \tag{A-33}$$

A solution can now be found to the linearized equation above in addition to linearized AS equation and the budget constraint shown in last section. This solution involves the derivative of an unknown function $f^e(\tilde{b}_t, \xi_t)$ in the constant solution i.e the term $f^e_{\tilde{b}}$. As we seek to find the derivative of $f$ it is not enough to solve for the value of $f$ in the constant solution. Rather we need to solve for the value of $f$ away from the constant solution. It will not be necessary to solve for the transition dynamics of $f$ for all type of shocks since we are only interested in evaluating the derivative of $f$ with respect to $\tilde{b}$. This derivative can be found by assuming initial condition for $\tilde{b}$ that are different from the constant solution. In the absence of shocks $\tilde{b}_t$ is the only state variable of the game. Consider the value of the function $f^e(\tilde{b}_t, \xi_t)$ at time $t$ when there are no shocks to the economy and perfect foresight (given some initial value of $\tilde{b}_t$):

$$u_c f^e(\tilde{b}_t)^{-1} = 1 + \pi_{t+1} \tag{A-34}$$

A first order approximation to this equation yields

$$-f^e_{\tilde{b}} b_t = \pi_{t+1} \tag{A-35}$$

Then:

$$\pi_t = -f^e_{\tilde{b}} b_{t-1} = \Pi b_{t-1} \tag{A-36}$$

Equation (A-28)-(A-30) and (A-36) imply equation (19) in the text. The value of $\rho$ can be found considering (A-18),(A-31), (A-30) and our solution for $f^e_{\tilde{b}}$. In the absence of shocks those equations combined can be written as:

$$\beta E_t b_{t+1} - (1 + \beta + \theta\Pi)b_t + b_{t-1} + g_G s' \gamma_t = 0 \tag{A-37}$$

where $\theta = \frac{\beta\tau}{\frac{\tau s'}{G}\sigma_G^{-1} + \frac{s'' \tau}{s'}} = \frac{\beta\tau}{\chi} > 0$. Suppose the debt limit is not binding so that $\gamma_t = 0$. Then $\rho$ solves the characteristic equation:

$$\Gamma(\rho) = \beta(1 - \frac{\lambda_\tau}{\lambda_\pi}\theta k^{-1}\sigma^{-1})\rho^2 - (1 + \beta + \frac{\lambda_\tau}{\lambda_\pi}\theta\beta^{-1})\rho + 1 = 0 \tag{A-38}$$

Then the solution for $b_t$ when the debt ceiling is not binding is of the form

$$b_t = c_1\rho_1^t + c_2\rho_2^t \tag{A-39}$$

It can be shown that the characteristic equation (A-38) has one root $0 < \rho_1 < 1$ and one root $\rho_2 > \left|\frac{1}{\beta}\right|$ The borrowing limit imposes that $b_t \leq \bar{b}$. If the debt dynamic involved an explosive root the debt limit would be reached in finite time. This however cannot be an equilibrium because if the debt limit is reached in the absence of any shocks then (A-31) is violated. To see this consider the period in which the debt limit is reached.[43] Then $b_t = \bar{b} \geq b_{t+1}$ and $b_t > b_{t-1}$. Then by (A-37)

[43] The argument only considers the case when $\rho_2 > 1/\beta$ which is the case of economic interest.

$\gamma_t = -\frac{1}{g_{GS'}}(\beta E_t b_{t+1} - (1+\beta+\theta\Pi)b_t + b_{t-1}) > 0$ that violates (A-33). Thus $c_2 = 0$ in (A-39) and

$$b_t = \rho b_{t-1} \quad 0 < \rho < 1$$

## C. Discretion with Real Assets

When the government can buy real assets in unlimited supply under discretion all the same conditions apply as before. In addition there is a first order condition that determines optimal asset holding.

$$\frac{\delta L_s}{\delta a_t} = \beta V_a(\tilde{b}_t, a_t, \xi_t) + \mu_t + \psi_t \frac{u_c f_a^e}{\beta f^{e2}} + \delta_t = 0 \tag{A-40}$$

There are now three state variables $b_t$, $a_t$ and $\xi_t$. Thus there is an envelope condition associated with $a_t$:

$$\begin{aligned} & V_a(\tilde{b}_{t-1}, a_{t-1}, \xi_{t-1}) \\ = \; & E_{t-1}[\frac{\mu_t}{\beta(1+\pi_t)}(f^e(\tilde{b}_{t-1}, a_{t-1}, \xi_{t-1})^{-2} f_a^e(\tilde{b}_{t-1}, a_{t-1}, \xi_{t-1})\tilde{b}_{t-1} - (1+q_{t-1})) \\ & -\eta_t S_a(\pi_t, \tilde{b}_{t-1}, a_{t-1}, \xi_{t-1})] \end{aligned} \tag{A-41}$$

It is easy to see that $f_a$ must be zero since the government cannot influence $q_t$. Then combining these two conditions yields:

$$\mu_t - \beta E_t(1+q_t)\mu_{t+1} + \psi_t \frac{u_c f_a^e}{\beta f^{e2}} = 0 \tag{A-42}$$

A linear approximation of this condition yields

$$\hat{\mu}_t - E_t \hat{\mu}_{t+1} - \hat{q}_t = 0 \tag{A-43}$$

To eliminate arbitrage opportunities I assume in equilibrium that $\hat{q}_t = \hat{\imath}_t - E_t \pi_{t+1}$. Note that although this holds in equilibrium the government assumes that it cannot influence $\hat{q}_t$ by its decisions.[44]

## D. Proof of Proposition 1

The first-order conditions (A-2)–(A-10) and (A-21)–(A-27) are identical when $\psi_t = \mu_t = 0$. The linearized condition (A-11)–(A-18) in Appendix B can be solved for each of the endogenous

[44] A more elaborate model where this will hold is one that has an open economy with a traded good. Then the real rate of return in terms of the traded good is exogenously given and the government can buy foreign exchange (that has a fixed rate of return) to achieve this commitment. Although there are some subtle differences between the present model and an open economy version, crudely speaking one can think of the equation above as corresponding to the interest rate parity.

variables when $\psi_t = 0$ under C1A. The implied value for the nominal interest rate is:

$$i_0 = \frac{1-\beta}{\beta} + \frac{\kappa^2 + \omega\lambda_\pi^{-1}}{\kappa^2 + (\sigma^{-1} + \omega)\lambda_\pi^{-1}} \sigma^{-1}\beta^{-1} g_0 \tag{A-44}$$

Then the zero bound is binding when:

$$g_0 \leq g_{Tr} = -\sigma \frac{\kappa^2 + (\sigma^{-1} + \omega)\lambda_\pi^{-1}}{\kappa^2 + \omega\lambda_\pi^{-1}} (1-\beta) \tag{A-45}$$

## III. APPENDIX - RECURSIVE SOLUTION METHOD

### A. Commitment

Here I outline a recursive solution method for discretion and commitment for Case 2. A more detailed description is available from the author on request. The solution for Case 1 can be seen as a special case of the solution illustrated. In Case 2 we assume that there is an unexpected shock to $g_0$ and that in every period $t > 0$ there is a probability $\alpha_t > 0$ that it reverses back to 0. Once it reverses back to zero it stay's there forever after. We call the stochastic date it reverses back to zero $T$. We assume that there is some date $S$ (that can be arbitrarily far into the future) at which $g_t$ must have returned back to zero with probability 1 (in the Case 1 $S = 1$). Let us denote the any variable $\hat{q}_t^T$ as the value of the variable conditional on that the economy is in the trap and $\hat{q}_t^N$ as the value of the variable conditional on that the economy is out of the trap. Let the vectors $Z_t$ and $P_t$ be defined as:

$$Z_t = \begin{bmatrix} \pi_t \\ \hat{Y}_t \\ \eta_t \\ b_t \end{bmatrix}, \; P_t = \begin{bmatrix} \psi_t \\ b_t \end{bmatrix}$$

t>T then $\psi_t^N = \hat{Y}_t^N = g_t^N = 0$ Then by (A-11) $\pi_t^N = 0$ and by (A-14) $b_t^N = b_T^N$

t=T. The system of equations can be written on the form

$$Z_T^N = K_T Z_T^T + F_T P_{T-1}^T + W_T \tag{A-1}$$

t<T. Using the solution from t=T and t>T the system can be written as:

$$\begin{bmatrix} P_t^T \\ Z_t^T \end{bmatrix} = \begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix} \begin{bmatrix} P_{t-1}^T \\ Z_{t+1}^T \end{bmatrix} + \begin{bmatrix} M_t \\ V_t \end{bmatrix} \tag{A-2}$$

The matrixes $A_t$, $B_t$, $C_t$,and $D_t$ are known. They only vary with time because of $\alpha_t$ . $M_t$ and $V_t$ are also known since they are only a function of the exogenous shocks and the parameters of the model. Note that $B_{S-1} = D_{S-1} = 0$. By recursive substitution we can find a solution of the form:

$$P_t^T = \Omega_t P_{t-1}^T + \Phi_t \tag{A-3}$$

$$Z_t^T = \Lambda_t P_{t-1}^T + \Theta_t \tag{A-4}$$

To find the solution for the coefficients $\Lambda_t, \Omega_t, \Theta_t$, and $\Phi_t$ consider the solution of the system in period $S-1$ when $B_{S-1} = D_{S-1} = 0$ . Consider the solution of the system in period $S-1$. By (A-2)

$$P_{S-1}^T = A_{S-1} P_{S-1}^T + M_{S-1}$$

$$Z_{S-1}^T = C_{S-1} P_{S-1}^T + V_{S-1}$$

Then we have $\Omega_{S-1} = A_{S-1}$, $\Phi_{S-1} = M_{S-1}$, $\Lambda_{S-1} = C_{S-1}$ and $\Theta_{S-1} = V_{S-1}$. The numbers $\Lambda_t, \Omega_t, \Theta_t$ and $\Phi_t$ for period 0 to $S-2$ can be found by solving

$$\Omega_t = [I - B_t \Lambda_{t+1}]^{-1} A_t \tag{A-5}$$

$$\Lambda_t = C_t + D_t \Lambda_{t+1} \Omega_t \tag{A-6}$$

$$\Phi_t = (I - B_t \Lambda_{t+1})^{-1} [B_t \Theta_{t+1} + M_t] \tag{A-7}$$

$$\Theta_t = D_t \Lambda_{t+1} \Phi_t + D_t \Theta_{t+1} + V_t \tag{A-8}$$

Given the solution for $\Lambda_t, \Omega_t, \Theta_t$ and $\Phi_t$ we can find the solution for each of the endogenous variables in (A-3) and (A-4) using the initial condition for $P^T_{-1} = 0$. This defines the solution under the contingency that the economy stays in the trap for the maximum period times given by $S$ (which can be made arbitrarily large). The solution for all of the other contingencies is then given by (A-1) where we now know the values for $Z^T_t$ and $P^T_{t-1}$ from the solution derived above.

### B. Discretion

t>T then $\psi^N_t = \hat{Y}^N_t = \hat{r}^{nN}_t = 0$. Then $\pi^N_t = \Pi b^N_{t-1}$ and $b^N_t = \rho_1 b^N_{t-1}$. Using this solution the discretion case can be solved in the same fashion as commitment illustrated above.

### C. Closed-Form Solutions for Case 1b

Using the solution method described above closed form solutions can be obtained for Case 1b. For compactness I only report the solution for the case when monetary and fiscal policy are coordinated. The case when $i_t$ is the only policy instrument can be seen as a special case of this solution for $\lambda_\tau \to \infty$, $\chi \to \infty$ and $\frac{\lambda_\tau}{\chi} \to \infty$. In this case the government will keep taxes unchanged and $i_t$ is the only policy instrument:

**Commitment**

$$g^{Ccom}_{Tr} = -\frac{\sigma^{-1} + \omega + d''\kappa^2 - \lambda_\tau \beta \sigma^{-2} \chi^{-1} \tau}{\omega + d''\kappa^2} \sigma(1-\beta) \tag{A-9}$$

$$\pi_1 = -\frac{\sigma^{-1}(\omega + \lambda_\pi \kappa^2)}{\sigma^{-1} + \omega + \lambda_\pi(\kappa^2 + \beta\sigma^{-2}) - \beta\sigma^{-2}\lambda_\tau \chi^{-1}\tau}(g_0 - g_{Tr}) = -\phi(g_0 - g_{Tr}) \tag{A-10}$$

$$b_{Tr} = \frac{\beta}{1-\beta} \chi^{-1} \tau \bar{i} \tag{A-11}$$

$Y_0$, $\tau_0$, $\pi_0$ and $x_0$ can now be found by equations (A-13)-(A-19).

**Discretion**

$$g_{Tr}^{Cdis} = -\frac{\sigma^{-1} + \omega + \kappa\lambda_\pi + \frac{\tau\Pi(\sigma^{-1} + \omega + d''\kappa^2 - \frac{g's'}{u_c}\Pi^{-1}\sigma^{-2})}{\chi(1+\frac{1}{\beta}-\rho)}}{\omega + \kappa^2\lambda_\pi}\sigma(1-\beta) \tag{A-12}$$

$$b_{Tr}^{Cdis} = \frac{\tau}{\chi(1+\frac{1}{\beta}-\rho)}(1-\beta) > 0 \tag{A-13}$$

$$\begin{aligned} b_0 &= -\frac{\frac{\Pi\sigma u_c}{\lambda_\tau}(\omega + \lambda_\pi\kappa^2)}{[\frac{\chi}{\tau}(1+\frac{1}{\beta}-\rho) + \frac{\sigma\Pi}{\lambda_\tau}((\sigma^{-1}+\omega)\sigma\Pi + d''\kappa^2\sigma\Pi - \frac{g's'}{u_c}\sigma^{-1})]}(g_0 - g_{Tr}) + b_{Tr} \qquad \text{(A-14)} \\ &= -\phi(g_0 - g_{Tr}) + b_{Tr} \end{aligned}$$

$Y_0$, $x_0$ and $\pi_0$ can now be found by equations (A-17), (A-19) and (A-30)–(A-33).

**Discretion with Real Assets**

$$g_{Tr}^{Adis} = -\frac{\sigma^{-1} + \omega + d''\kappa^2}{\omega + d''\kappa^2}\sigma(1-\beta)$$

$$b_{Tr}^{Adis} = 0$$

$$b_0 = -\frac{1}{\sigma\Pi^{Adis}}\frac{\omega + \lambda_\pi\kappa^2}{\sigma^{-1} + \omega + \lambda_\pi\kappa^2 - \lambda_\pi(1-\beta)\beta\sigma^{-2}}(g_0 - g_{Tr}) == -\phi^{Adis}(g_0 - g_{Tr}) + b_{Tr}$$

$Y_0$, $x_0$ and $\pi_0$ can now be found by equations (A-17), (A-19), (A-30)–(A-33), and (A-43).

Table 1. Varying the Cost of Taxes

| | Full commitment | | | Discretion when real assets are trade | | |
|---|---|---|---|---|---|---|
| $\gamma$ | $\pi_1$ | $x_1$ | $B_0$/GDP | $\pi_1$ | $x_1$ | $B_0$/GDP |
| 0.25% | 4.19% | -1.62% | 17.27% | 5.21% | 0.42% | 3416.18% |
| 0.50% | 4.19% | -1.61% | 17.20% | 5.21% | 0.42% | 1699.51% |
| 0.75% | 4.20% | -1.60% | 17.13% | 5.21% | 0.42% | 1127.28% |
| 1.00% | 4.20% | -1.59% | 17.06% | 5.21% | 0.42% | 841.17% |
| 1.50% | 4.21% | -1.57% | 16.92% | 5.21% | 0.42% | 555.06% |
| 2.00% | 4.22% | -1.55% | 16.77% | 5.21% | 0.42% | 412.00% |
| 3.00% | 4.24% | -1.51% | 16.49% | 5.21% | 0.42% | 268.95% |
| 5.00% | 4.28% | -1.43% | 15.95% | 5.21% | 0.42% | 154.50% |
| 7.50% | 4.34% | -1.33% | 15.30% | 5.21% | 0.42% | 97.28% |
| 10.00% | 4.39% | -1.22% | 14.67% | 5.21% | 0.42% | 68.67% |

| | Discretion with fiscal policy | | | Monetary discretion | | |
|---|---|---|---|---|---|---|
| $\gamma$ | $\pi_1$ | $x_1$ | $B_0$/GDP | $\pi_1$ | $x_1$ | $B_0$/GDP |
| 0.25% | 1.06% | -7.89% | 194.23% | 0.00% | -10.00% | 0.00% |
| 0.50% | 1.79% | -6.43% | 169.09% | 0.00% | -10.00% | 0.00% |
| 0.75% | 2.36% | -5.28% | 151.75% | 0.00% | -10.00% | 0.00% |
| 1.00% | 2.83% | -4.34% | 138.56% | 0.00% | -10.00% | 0.00% |
| 1.50% | 3.57% | -2.86% | 119.22% | 0.00% | -10.00% | 0.00% |
| 2.00% | 4.14% | -1.73% | 105.38% | 0.00% | -10.00% | 0.00% |
| 3.00% | 4.98% | -0.05% | 86.34% | 0.00% | -10.00% | 0.00% |
| 5.00% | 6.04% | 2.08% | 64.14% | 0.00% | -10.00% | 0.00% |
| 7.50% | 6.85% | 3.71% | 48.55% | 0.00% | -10.00% | 0.00% |
| 10.00% | 7.40% | 4.80% | 38.73% | 0.00% | -10.00% | 0.00% |

## IV. APPENDIX - CALIBRATION

### A. Baseline Calibration

I assume that each period is three years.$\beta = 0.98^3$. I assume log utility. $\sigma = -\frac{u_c}{u_{cc}C}\frac{C}{Y} = 1 * 2/3$, $\sigma_G = -\frac{g_G}{u_{GG}G} = -\frac{u_c}{u_{cc}C} = 1$, $\tau = F = 1/3$, $w = 2$, $\lambda_x = \omega + \sigma^{-1} = 3$. A second order approximation to the loss function yields: $L_t = \lambda_\pi \pi_t^2 + \lambda_x x_t^2 + ...$ In our model the $\pi_t$ refers to inflation in one period which is three years. Our calibration is $\lambda_\pi^a = \lambda_x$ so that the weight on annual inflation is the same as output gap. Then $\lambda_\pi^a = \lambda_x = \lambda_\pi 9$. $\lambda_\pi = \frac{\lambda_x}{9} = \frac{\omega+\sigma^{-1}}{9}$.I assume a particular functional form for tax collections: $s(\tau) = \frac{1}{2}a\tau_t^2$. This implies that in steady state: $\frac{s''\tau}{s'} = 1$. I assume tax collection is $\gamma$ fraction of government consumption. $\gamma = 0.02$ (see sensitivity analysis below). $s(\tau) = \gamma\tau = \frac{1}{2}a\tau^2 \rightarrow a = \frac{2\gamma}{\tau}$. $dF = dG + s'd\tau = 0$. $\frac{dG}{d\tau}\frac{\tau}{G} = -\frac{s'\tau}{G}$. $\frac{s'\tau}{G} = \frac{a\tau^2}{G} = \frac{2\gamma}{1-\gamma}$. $\chi = \frac{\tau s'}{G}\sigma_G^{-1} + \frac{s''\tau}{s'} = \frac{a\tau}{\gamma}\sigma_G^{-1} + 1 = \frac{2\gamma}{1-\gamma}\sigma_G^{-1} + 1$. In the constant (steady state) solution I linearize around the government spending that is optimal. This implies the first order condition: $-u_c + g_G - g_G s' = -u_c + g_G(1 - s') = 0$. $\frac{g_G}{u_c} = \frac{1}{1-s'} = \frac{1}{1-a\tau}$ Hence: $\lambda_\tau = \frac{s' g_G}{u_c} = \frac{s'}{1-s'} = \frac{2\gamma}{1-2\gamma}$. In Case 1b I assume that the natural rate of interest is $-5/\beta percent$ in period 0 so that 5 percent expected inflation is required to eliminate the output gap.

## B. Sensitivity Analysis

This table shows the equilibrium outcome for various values for $\gamma$, which is the fraction of real government spending that is spent in tax collection (or any other output cost of taxes). In the baseline calibration, I assumed that $\gamma = 0.02$, i.e. the cost of taxation is 2 percent of real government spending.

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