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Columbia or High School? Understanding the Roles of Education in Development

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Abstract

No country has achieved sustained economic development without investment in education. Thus, education policy can play a vital role in facilitating development. But which types of schooling—secondary or tertiary—should public policy promote? This paper develops an analytical framework to address this question. It shows how the composition of human capital stock determines a country's development. Hence, promoting the "wrong" type of schooling can have little effect on development. In addition to identifying some characteristics of an optimal education policy, the paper helps in understanding why empirical studies have failed to find a significant relationship between schooling and growth.

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I. Introduction

No country has achieved sustained economic development without substantial investment in human capital. Indeed, a hallmark of the development process is the increasing utilization of different types of skilled labor in the production process. Thus, education policy can play a vital role in facilitating economic development. But which types of skilled labor: secondary or tertiary should public policy promote? Many countries have experimented with different policies at different stages of development. For example, before the explosion of secondary education beginning in the 1910's,[2] the U.S. as early as 1862[3] promoted tertiary schooling.[4] However, in exploring the channels through which human capital can affect development, the theoretical literature has largely treated human capital as a homogenous concept.[5] Therefore, there is little theoretical framework for understanding how the different types of schooling affect development, and in turn, the characteristics of an optimal education policy.

In addressing these questions, this paper develops a simple analytic framework that emphasizes the role of the composition of the human capital stock in shaping the incentives for education investment. In particular, this framework relies on two key assumptions. Firstly, each skill type performs a specific but complementary function within the production process in the skilled sector. The highly educated, such as scientists and technicians, have a comparative advantage in understanding and adapting new or existing ideas into a production process.[6] Meanwhile, some minimum level of education is required to follow the production

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[2] See Goldin and Katz (1995) and Ramcharan (2001) for a discussion of this remarkable event.

[3] The Morrill Acts of 1862 and 1890 granted federal funds to existing and future states to endow universities and colleges that specialized in agriculture. The 1890 act provided funding for many institutions created by the first act.

[4] Similarly, in the last few decades India and many countries in Latin America have encouraged heavy tertiary investment; in contrast, several East Asian countries have focused on basic or secondary education.

[5] The literature is extensive. See for example, Lucas (1988), Nelson and Phelps (1966), Romer (1990).

[6] The importance of tertiary or advanced education in generating and adapting ideas is underscored by a recent study of a thousand Indian inventors. The authors (Deolalikar and Evenson (1990)) found that almost 90% had a university degree, more than half had some post graduate training, and nearly 30% held doctorate degrees.

template, and successfully execute the production steps.[7] This paper assume that the ideas developed by the highly skilled are non rival but excludable. This creates demand linkages between the education types that are external to the firm. And thus, the rate of return for either skill input depends on the educational composition of the entire workforce.[8]

Secondly, the paper studies these demand factors within the context of endogenous schooling costs. In many countries, the lack of access to schools and the limited supply of teachers[9] negatively affect the schooling investment decision. I assume that previous enrollments—the current stock of educated labor—engender improvements in the educational infrastructure: more potential teachers, more schools and more suitable curricula, which in turn diminish the current size of the sunk cost associated with human capital investment, and outwardly shifts the supply curve for skilled labor. For example, it has been observed that the expansion of the schooling infrastructure in response to previous

[7] Nelson and Phelps (1966) formalized the argument that some minimum level of education speeds the adoption process. Bartel and Lichtenberg (1987) has since found evidence of this in the U.S. manufacturing sector.

[8] The World Bank in its recent World Development Report (1998/1999) has also noted the inherent complementarity of this relationship: "Basic education increases people's capacity to learn and interpret information. But this is just the start. Higher education and technical training are also needed, to build a labor force that can keep up with a constant stream of technological advances, which compress product cycles and speed the depreciation of human capital." These linkages have also been well documented in the green revolution in Asia. While advances in biotechnology, pioneered in the developed countries, made the development of the high yielding variety (HYV) seeds in such staples as rice and wheat possible, local scientists and agronomists were necessary in order to adapt these HYV to the local climatic conditions. Once developed, the use of these seed strains are non-rival, but can be excluded. However, using HYV seeds requires a greater attention to fertilizer quantity, irrigation and soil conditions. Thus, as indicated by Foster and Rosenzweig (1996) in the case of India, educated farmers adopted the more technologically advance seed strains more rapidly than those without sufficient schooling. Furthermore, the authors found that the returns to education increased in those areas where adoption had the highest potential gains. In addition, the expansion of schooling and the many agricultural extension projects designed to facilitate adoption increased the rate of return to R&D in seed technology through the market size effect, as well as through the fact that the feedback from more educated farmers was more useful in developing better seed varieties (see Pray and Ruttan (1990)).

[9] In their investigation into the causes of inequality, Mokherjee and Ray (1998) also use the idea that the existing stock of human capital limits the availability of teachers. And empirically, Mingat and Tan (1998) found that the greater supply of potential teachers account for a significant share of the difference in educational attainment between rich and poor countries.

enrollments (the current stock of skilled workers) makes it easier for the present cohort of students to attend school. Foster and Rosenzweig (1996) make this point using Indian schooling data, where they observed that the presence of a school within a village significantly increases the probability of attendance.[10] Furthermore, the authors found that high previous enrollments within a village often led to the construction of new schools within the village, which in turn benefited later cohorts of students.

Using this framework, the paper argues that the confluence of demand and supply forces create a pattern of circularity between educational investment across the various skill categories, and demonstrates how the composition—not the level—of the human capital stock determines the long run steady state level of development. In so doing, the decentralized equilibrium illustrates the importance of the right education policy: promoting the "wrong" type of education can have no effect on an economy's potential development steady state. For instance, consider the case of an economy converging to a low steady state because of a limited number of secondary educated labor. That is, the inability of the economy to adequately use technology within the skilled sector due to the limited supply of secondary educated labor reduces the productivity of tertiary educated workers and dampens the overall incentives for education investment. The model makes clear that in this case, even large investments in tertiary schooling will have little effect on long run development. For the extra tertiary skilled labor may not sufficiently raise the return to secondary education in order to create a self-sustaining investment cycle towards a higher steady state.

The analysis is able to isolate two important characteristics of an optimal education policy. Firstly, education investment should be ongoing over time, but its rate of increase should be diminishing. Thus, the first generation should experience the biggest increase in schooling investment. Each subsequent generation should be better educated than its predecessor, but the difference in attainment across generations should be declining with time. Intuitively, the cost of education increases in the size of the enrollment levels—the flow of investment. Also, because of diminishing marginal productivity into the unskilled sector, the shadow cost of moving labor in the skilled sector increases with attainment. Therefore, it is cost minimizing to incur the largest flow of investment initially, when the shadow cost of secondary schooling investment is at its minimum. Secondly, the analysis argues that because the social marginal product of labor in the skilled sector depends on the level of the complementary input, the expansion in schooling should occur across both types of schooling simultaneously.

The decentralized model also helps in explaining the failure of many empirical studies to observe the expected strong correlation between economic growth and human

[10] In their point estimates, the building a school in a village can more than double the enrollment rate for children ages 5 through 14 years of age. Likewise, in Indonesia, Duflo (2001) finds that each primary school constructed per 1000 children led to 0.12 to 0.19 increase in the years of education.

capital accumulation.[11] Much of this research uses the average years of schooling within the population[12] as the sole measure of educational attainment. This methodology implicitly treats each year of schooling as identical, assumes that workers of each education category are perfect substitutes for workers of other education categories,[13] and assumes that the marginal productivity of an additional year of schooling is the same given every level of schooling attainment.[14] But as the model indicates, the average years of schooling can mask fundamental differences in the composition of the human capital stock. Indeed, examples in the paper show that countries with identical average years of schooling can converge to very different development steady states. This differential impact of schooling on growth is consistent with the Barro and Sala-i-Martin (1995) results. They find that while the levels of primary schooling attainment and economic growth are not significantly related, there is a strong association between initial levels of secondary and tertiary attainment and subsequent economic growth. Furthermore, the magnitude of the tertiary coefficient exceeded that found on secondary attainment, suggesting some kind convex relationship between years of schooling and growth.[15]

The rest of the paper is organized as follows. In Section II, I formalize the idea of demand linkages using a simple static model of educational investment. Section III integrates both demand and supply forces within a dynamic model of educational investment with endogenous schooling costs in order to analyze precisely how the composition of the human capital stock shapes the incentives for educational investment. This model is related to the literature on costly investment across multiple sectors (Matsuyama (1991,1988), Krugman

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[11] Islam (1995), Hoeffler (1997), Benhabib and Spiegel (1994), Spiegel (1994), Dasgupta and Weale (1992), Pritchett (1999), Barro and Sala-I-Martin (1995) and Lau, Jamison and Louat (1991) all find an insignificant or negative correlation between various measures of educational attainment and economic growth. Recently Krueger and Lindahl (1998) have questioned the accuracy of these results. They argue that measurement error in the education data negatively biases the human capital coefficient.

[12] To devise these measures, it is common to divide the total number of years of schooling attained by the size of the population. See Barro and Lee (1993).

[13] See Mulligan and Sala-I-Martin (1995).

[14] This means for example that one extra of year of primary schooling has the same effect on marginal productivity as one extra year of post secondary education.

[15] Recently, Glewwe (1991), using detailed data drawn from Ghana's 1988–1989 Ghana Living Standards Survey (GLSS) found that the private rate of return to secondary and tertiary schooling are much higher than primary schooling. See Nielsen and Westergard-Nielsen (1998), Bennel (1996) and Bigsten et. al (1997) for similar results using African data. Lachler (1999) reports similar findings using Mexican data.

(1991) and Carringtion et. al (1996)), as well as to those which explore the relationship between human capital and development: Lucas (1988), Azariadis and Drazen (1992), and Romer (1990) for example. Section IV analyzes the model's implications, and Section V discusses some characteristics of an optimal education policy.

II. External Demand Linkages

In this section, I specify the production structure of the economy, and develop the demand side of the argument, illustrating how the composition of the human capital stock influences the demand for educated labor. There are three labor categories: unskilled or unschooled (U), low skilled (L) —those with only basic education such as secondary schooling—and the high skilled (H) or tertiary educated.[16] The economy produces a single consumption good. Production of this good occurs both in the unskilled sector, where only unskilled labor is used, and in the skilled sector, where both low skilled workers and tertiary educated managers are complementary inputs. I assume, following Romer (1990), that some of the ideas developed by the high skilled agents spillover across firm boundaries and improve the productivity of all secondary educated workers within the skilled sector. This externality ensures that low skilled labor productivity is in part a function of the *total* employment of high skilled labor. Furthermore, some of the ideas generated by the tertiary educated labor within a firm become proprietary and are licensed for use by other firms within the sector. Thus, the reward to tertiary investment depends on the number of low skilled workers—this is the market size effect.[17] Using a standard Cobb Douglas framework, I describe the production structure of a representative firm in the skilled sector:

$$(1) \qquad y_i^s = A\left[(g(H)l_i)^{\alpha}(f(L)h_i)^{1-\alpha}\right], \text{ where } g(0)=f(0)=0, \frac{dg}{dH}>0, \frac{df}{dL}>0 \text{ and } \alpha \in (0,1)$$

The functions $g(H)$ and $f(L)$ denote the external effects of aggregate high and low skilled labor at the firm level respectively. To simplify the analysis further, let $g(H)=H$ and $f(L)=L$. Production at the firm level occurs using a constant returns to scale technology, but the external demand linkages between secondary and tertiary educated labor generate

[16]To reiterate, the terminology tertiary and secondary is used for convenience. These linkages can potentially exist across other types of human capital.

[17]Within a different context, Acemoglu (1998) also studies the link between the potential market for a technology and the incentives to develop that technology. Crucial to his argument, in much the same way it is here, is the idea that the use of the technology is non rival but excludable.

increasing returns to scale,[18] at the sector level. This functional form specification also produces the global absence of diminishing marginal returns to all factors in the skilled sector.[19] Output in the unskilled sector relies solely on unskilled labor, and uses a standard Cobb-Douglas technology, and so is subject to diminishing marginal productivity. All factor prices are determined by the marginal productivity of the factor. The wages of the unskilled, low skilled and high skilled are given respectively by:

(2) $$w^u = B\alpha U^{\alpha-1}, w^l = A\alpha H, w^H = A(1-\alpha)L,$$

where $B > 0$.

Education investment is irreversible; for example, a low skilled agent can no longer operate in the unskilled sector.[20] The investment process is sequential, and agents incur a unique fixed cost at each step in the educational ladder. The size of this sunk cost depends on an agent's personal characteristics such as preferences, family background, and intrinsic ability, as well as policy variables such as the development of the education infrastructure: distance from home to school, the quality of instruction and the nature of the curriculum.[21] I assume that these factors are uncorrelated with future productivity. These characteristics are summarized by a cost index $\theta \in \Theta$ and $q(\theta)$ denotes the fraction of the population of type less than or equal to θ. I also assume that the population is constant and without loss of generality normalized to a constant p:

(3) $$H + L + U = p$$

The private cost of secondary schooling for an agent of type θ is given by :

(4) $$c^l(\theta), \text{ where } c^l_\theta > 0$$

The cost structure of tertiary schooling is similarly defined for an agent of type θ :

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[18] Matsuyama (1992) uses a qualitatively similar setup, albeit with a single factor of production, to generate IRS within an industrial sector.

[19] Romer (1986) also uses a similar framework to analyze knowledge spillovers at the sector level stemming from the use of capital at the firm level. There, the absence of demising marginal returns to capital, because of knowledge spillovers produces endogenous growth.

[20] This is only a simplifying assumption; in equilibrium, the skill premia are always positive (Lemma 2).

[21] In the dynamic model, I endogenize the aspect of this cost imposed by policy variables.

(5) $c^h(\theta)$, where $c^h_\theta > 0$.

I assume that for any given type, the private cost of tertiary schooling always exceeds the private cost of secondary schooling:

(6) $c^h(\theta) > c^l(\theta) \; \forall \theta \in \Theta$.

Let $\gamma^L(H,L)$ denote the premium to secondary schooling:

(7) $\gamma^L = A\alpha H - A\alpha U^{1-\alpha}$.

An agent of type θ_i invests in secondary schooling if and only if the education premium exceeds the cost of schooling:

(8) $\gamma^L(H,U) > c^l(\theta_i)$,

and all agents of type less than θ^* invest in secondary schooling:

(9) $\gamma^L(H,U) = c^l(\theta^*)$

A similar reasoning lies behind the decision to invest in tertiary schooling and the fraction of the population investing in tertiary schooling, $q(\theta^{**})$, is determined by the condition below:

(11) $\gamma^H(H,U) = c^l(\theta^{**})$,

where $\gamma^H(H,U)$ denotes the premium to tertiary education:

(12) $\gamma^H(H,U) = A(1-\alpha)L - A\alpha H$.

The absence of diminishing marginal returns in the skilled sector makes the educational investment process extremely sensitive to schooling costs and the composition of the human capital stock. To see how this works, suppose instead that all factors were subject to diminishing marginal productivity. Then, the demand for tertiary educated labor would be high in economies with a small initial stock of tertiary educated the labor. But with external demand linkages, the demand depends only on the supply of secondary educated workers. And if this is small, then investment in tertiary education would not be observed, despite its small initial stock. Indeed, the absence of diminishing marginal returns and the presence of sunk costs can combine to make education investment highly dependant on the composition of the human capital stock. Using the condition $L = p - H - U$, the lemma below states the argument more precisely.

Lemma 1: *Investment in education fails to occur if the skill composition of the economy belongs to the set* $R = \{H,U : \gamma^H(H,U) \le c^H(\theta) \text{ and } \gamma^L(H,U) \le c^L(\theta), \forall \theta \in \Theta\}$.

This simple static framework outlines the basic argument underlying the relationship between the composition of the human capital stock and educational investment incentives. External demand linkages between the various education categories break the inverse relationship between marginal productivity and the available supply of skilled factors, and the presence of investment sunk costs can lead to education traps. Therefore, in order to better understand the nature of the transition from unskilled to skilled in the production process, it is necessary to explicitly model, and endogenize how *both* these demand and supply forces interact over time to shape the pattern and level of educational investment. To this end, in Section 3 I develop a dynamic model that endogenizes schooling costs.

III. Dynamic Model of Educational Attainment with Endogenous Schooling Costs

The aim of this section is to develop a framework to analyze the long run implications of the relationship between external demand linkages and declining schooling costs. The model is related to the literature on costly investment across multiple sectors, where the externalities associated with an individual's investment decision help determine the investment incentives for the remaining agents; for example, see Matsuyama (1991,1988), Krugman (1991) and Carringtion et. al (1996). The argument developed below shows that the initial composition of the human stock, operating through both the demand and supply side plays a crucial role in determining the economy's long run pattern of educational investment, as well as its long run steady state level of educational attainment and wage inequality. Hence, economies with seemingly identical initial 'average' years of schooling—a variable oft used in the empirical literature—can converge to very different steady states. Also, the growth in the average years of schooling can be unrelated to economic growth. Additionally, the model provides a useful framework, with which to discuss some aspects of government education policy.

The production structure of the economy is identical to that discussed in Section 2. I continue to assume that the labor force consists of a continuum of infinitely lived workers whose sum is normalized to p and each agent is indexed by type θ. Let $q(\theta)$ denote the fraction of workers of type less than or equal to θ. This function is strictly increasing, continuous and differentiable. Let $c^h(H(t),\theta)$ denote the private cost of tertiary schooling for an agent of type θ, given the stock of tertiary educated workers $H(t)$ at time *t*. In keeping with the idea that the size of the sunk cost diminishes with the stock of educated agents, I assume that $c_1^H(\bullet,\bullet) < 0$ while $c_2^H(\bullet,\bullet) > 0$. The private cost of secondary schooling,

$c^L(H(t)+L(t),\theta)$ is similarly defined, and for any θ it naturally follows that $c^L(H(t)+L(t),\theta)<c^H(H(t),\theta)$. More formally, note that $c^i:[0,p]\times\Theta\to R_+$ and is assumed to be differentiable everywhere. For simplicity, I also assume that agents are endowed with perfect foresight and investment in education is irreversible. Individuals maximize the present discounted value of their income stream by choosing the optimal dates on which to invest in education.[22]

Define $V^H(L(\tau_2))$ to be the value of tertiary education at some date τ_2. Since educational investment is irreversible, the value of tertiary education is the present discounted value of its income stream from date τ_2 onwards:

$$(13)\quad V^H(L(\tau_2))=\int_{\tau_2}^{\infty}A(1-\alpha)L(t)e^{-r(t-\tau_2)}dt$$

where $r>0$ is the constant and exogenously given discount rate, and the high skilled wage at time t is $W^H(t)=A(1-\alpha)L(t)$. Let $V^L(H(\tau_1),L(\tau_1)|\theta)$ denote the value of secondary education for an agent of type θ at a date τ_1 such that $\tau_1<\tau_2$. A secondary educated agent must choose the optimal date on which to incur the sunk cost and invest in tertiary education. This problem can be written as:

(14)

$$V^L(H(\tau_1),L(\tau_1)|\theta)=\max_{\tau_2}\left\{\int_{\tau_1}^{\tau_2}A\alpha H(t)e^{-r(t-\tau_1)}dt+e^{-r(\tau_2-\tau_1)}\left[V^H(L(\tau_2))-c(H(\tau_2),\theta)\right]\right\}.$$

where the low skilled wage at time t is $w^L(H(t))=A\alpha H(t)$. The structure of the investment problem facing an unskilled agent is similar to the one described above. For some date τ_0, where $\tau_0<\tau_1<\tau_2$, let $V^U(H(\tau_0),U(\tau_0),L(\tau_0)|\theta)$ denote the value of the

[22]One interpretation of the idea that individuals are infinitely lived, and wait until the optimal date to invest in education is that generations or families pass on their existing level of education to their children. Given the cost of schooling and the demand for [skilled] labor in the current period, these children then decide whether to invest in schooling and add to their family's capital stock or delay and pass on only the existing level to future generations. In this way, if the educational infrastructure rapidly expands, then families, and by extension society quickly become educated, otherwise it takes a longer time. See Galor and Tsiddon (1997) for an overlapping generations model with some of these characteristics.

unskilled state for an individual of type θ. An unskilled individual then selects the optimal date on which to invest in secondary schooling:

(15)
$$V^U\left(H(\tau_0),U(\tau_0),L(\tau_0)|\theta\right)=\max_{\tau_1}\left\{\begin{array}{l}\int_{\tau_0}^{\tau_1} w^H\left(U(t)\right)e^{-r(\tau-\tau_0)}dt+\\ e^{-r(\tau_1-\tau_0)}\left[V^L\left(H(\tau_1),L(\tau_1)|\theta\right)-c\left(H(\tau_1)+L(\tau_1),\theta\right)\right]\end{array}\right\}.$$

Using the condition $L=p-H-U$, let $\gamma^H\left(H(t),U(t)\right)$ denote the premium induced by tertiary education relative to secondary schooling in period t. Similarly, $\gamma^L\left(H(t),U(t)\right)$ represents the premium to secondary education relative to the unskilled state. The following Lemma shows that these respective skill premia are always positive. Hence, there is no incentive to revert to a previous employment type after undertaking the educational investment decision. This ensures that the assumption of irreversibility used in simplifying the Bellman equations does not impose any dynamic inconsistency within the model, whereby along an equilibrium path an agent finds it optimal to reverse his employment decision but cannot.

Lemma 2: *Along an equilibrium path* $\gamma^H\left(H(t),U(t)\right)\geq 0$ *and* $\gamma^L\left(H(t),U(t)\right)\geq 0$ *for all* t.

The secondary educated agent indifferent between investing in tertiary education at time t is implicitly defined by the condition:

$$\text{(16)}\qquad \int_t^{\infty}\gamma^H\left(H(s),U(s)\right)e^{-r(s-t)}ds=c^H\left(H(t),\theta^{**}\right)$$

Since $q(\theta)$, the fraction of the population of type less than or equal to θ, is monotonic, it can be inverted:

$$\text{(17)}\qquad \theta^{**}=q^{-1}\left(H(t)\right)=M\left(H(t)\right)$$

For notational simplicity, I express the cost of tertiary education as $c^H\left(H(t)\right)$, suppressing $M\left(H(t)\right)$, and I assume that $c_H^H\left(H(t)\right)<0$. Similarly, the unskilled agent indifferent between investing in secondary schooling and continuing at his current skill level at some time t is defined by:

$$(18) \quad \int_t^{\infty} \gamma^L\left(H(s),U(s)\right)e^{-r(s-t)}ds = c^L\left(p-U(t),\theta^*\right)$$

where I have made use of $H+L=p-U$. Using the monotonicity of $q(\theta)$, I express the cost of secondary education solely as a function of the level of unskilled labor in the population: $c^L\left(p-U(t)\right)$, and $c_U^L(\bullet)>0$. The following lemma specifies some characteristics of the optimal investment date τ_2 and τ_1 for tertiary and secondary education respectively.

Lemma 3: *The optimal investment dates,* τ_2 *and* τ_1*, satisfy the following conditions:*

$$(19) \quad c^H\left(H(\tau_2)\right) \le \int_{\tau_2}^{\infty} \gamma^H\left(H(t),U(t)\right)e^{-r(t-\tau_2)}dt$$

$$(20) \quad c^L\left(p-U(\tau_1)\right) \le \int_{\tau_1}^{\tau_2} \gamma^L\left(H(t),U(t)\right)e^{-r(t-\tau_1)}dt$$

there does not exists a $\tau' > \tau_2$ *such that*

$$(21) \quad c^H\left(H(\tau_2)\right) - c^H\left(H(\tau')\right)e^{-r\tau'} > \int_{\tau_2}^{\tau'} \gamma^H\left(H(t),U(t)\right)e^{-r(t-\tau_2)}dt$$

there does not exists a $\tau_1 < \tau' < \tau_2$ *such that*

$$(22) \quad c^L\left(p-U(\tau_1)\right) - c^L\left(p-U(\tau')\right)e^{-r\tau'} > \int_{\tau_1}^{\tau'} \gamma^L\left(H(t),U(t)\right)e^{-r(t-\tau_1)}dt$$

From conditions (19) and (20), on the optimal investment date, the benefit from investing in education—the present discounted value of the skill premium—is greater than or equal to the cost of schooling. Conditions (21) and (22) state that after choosing the optimal time, the marginal benefit from waiting beyond the optimal time—lower schooling costs—cannot be greater than or equal to the forgone skill premium—the opportunity cost of waiting. Therefore, conditions (21) and (22) imply that for any date $\tau_2' = \tau_2 + \Delta t$

$$(23) \quad c^H\left(H(\tau_2)\right) - \frac{c^H\left(H(\tau_2+\Delta t)\right)}{1+\Delta tr} \le \frac{\Delta t\gamma^H\left(H(t),U(t)\right)}{1+\Delta tr}, \text{ for } \Delta t > 0$$

and

$$(24) \quad c^H\left(H(\tau_2)\right) - \frac{c^H\left(H(\tau_2+\Delta t)\right)}{1+\Delta tr} \ge \frac{\Delta t\gamma^H\left(H(t),U(t)\right)}{1+\Delta tr}, \text{ for } \Delta t < 0$$

Similarly for any date $\tau_1' = \tau_1 + \Delta t$

$$(25)\quad c^{L}\left(p-U(\tau_1)\right)-\frac{c^{L}\left(p-U(\tau_1+\Delta t)\right)}{1+\Delta tr}\leq\frac{\Delta t\gamma^{L}\left(H(t),U(t)\right)}{1+\Delta tr},\text{ for }\Delta t>0$$

And

$$(26)\quad c^{L}\left(p-U(\tau_1)\right)-\frac{c^{L}\left(p-U(\tau_1+\Delta t)\right)}{1+\Delta tr}\geq\frac{\Delta t\gamma^{L}\left(H(t),U(t)\right)}{1+\Delta tr},\text{ for }\Delta t<0$$

Rearranging the above expressions and taking the limit as $\Delta t\to 0$ leads to Proposition 1.

Proposition 1: *The behavior of educational attainment along a perfect foresight equilibrium path is described by*

$$(27)\quad \dot{U}=\frac{w^{U}\left(U(t)\right)-\left[w^{L}\left(H(t)\right)-rc^{L}\left(p-U(t)\right)\right]}{c_{U}^{L}\left(p-U(t)\right)}$$

$$(28)\quad \dot{H}=\frac{w^{H}\left(L(t)\right)-w^{L}\left(H(t),U(t)\right)-rc^{H}\left(H(t)\right)}{-c_{H}^{H}\left(H(t)\right)}.$$

There is no investment in tertiary education if:

$$(29)\quad rc^{H}\left(H(t)\right)\geq\gamma^{H}\left(H(t),U(t)\right).$$

There is no investment in secondary education if:

$$(30)\quad rc^{L}\left(p-U(t)\right)\geq\gamma^{L}\left(H(t),U(t)\right).$$

Equations (27) and (28) define a planar dynamical system in (H,U) space that describes the aggregate behavior of educational attainment. Investment in education is ongoing if and only if its rate of return is positive. The size of the externality, $c_1(\cdot)$, determines the sensitivity of aggregate behavior towards the capital gains rate. And unless the externality is zero, convergence towards the equilibrium stock of education is gradual, implying that educational attainment occurs slowly over time. Intuitively, some agents find it optimal to wait for the cost of schooling to diminish rather than invest in education given the current incentives. And the bigger the reduction in schooling costs over time, then the more attractive waiting becomes. A stationary state occurs when the present discounted value of the skill premium is less than or equal to the cost of schooling. It should be reiterated that whenever the cost of schooling exceeds the skill premium there is no *new* investment in schooling. But this does not imply that those already educated wish to reverse their educational decision. As Lemma 2 indicates, the skill premia are always positive along an equilibrium path. The phase diagrams in the next section consider how the equilibrium dynamics of the model respond to various assumptions about the supply structure of the model.

A. Equilibrium and Dynamics

In this subsection, I use phase diagrams to characterize the equilibrium behavior of the educational attainment paths generated by the dynamical system in Proposition 1. I compare and contrast the implications of a linear schooling externality to the non linear case. Using this methodology, I am able to isolate how assumptions about the supply of educated labor coupled with the maintained production structure affect the economic development process. Previewing these results, I find that a linear schooling externality produces a single stable steady state. Notwithstanding this, the presence of the sunk cost produces education traps, where depending on the initial conditions, new educational investment is unprofitable for all types. A non linear externality captures the idea that over some range of attainment, the educational infrastructure maybe slow to develop, which leads to continued limited access and a slow decline in schooling costs. However, at a higher cumulative stock of educated labor, infrastructure development proceeds at a more rapid pace, leading to a more substantial decline in schooling costs. Early on for example, the stock of teachers may be too limited to significantly improve the quality of education.[23] As educated labor becomes more plentiful, education quality and quantity improve and schooling costs fall. Incorporating a non-linear externality in the analysis produces multiple stable steady states. That is, the composition of the human capital stock determines both the pattern of educational attainment and the long run steady state level of educational attainment. To illustrate the idea that average years of schooling can mask important differences in the composition of education stock, I construct examples which show that the growth in 'average' years of schooling can be quite uncorrelated with economic growth.

A.1 Linear Schooling Externality

In this the simplest case, I assume that the private cost of schooling diminishes linearly with attainment. For example, in the case of tertiary schooling:

(31) $$c^H\left(H(t)\right)=k-aH(t),$$

[23] This idea bears some resemblance to Azariadis and Drazen's (1992) concept of threshold externalities.

where the parameter: a captures the size of the externality. Likewise, secondary schooling can be written as[24]:

$$(32) \quad c^{L}\left(U(t)\right)=k-ap+aU(t)$$

Since the schooling externality is constant, the decline in schooling costs are independent of the level of attainment. The dynamical system for the linear case can be expressed as:

$$(33) \quad \dot{H}=\frac{H(t)\left[ra-A\right]-AU(t)\left[1-\alpha\right]+A(1-\alpha)p-rk}{a}=\frac{M\left(U(t),H(t)\right)}{a}$$

$$(34) \quad \dot{U}=\frac{\alpha BU(t)^{\alpha-1}-\alpha AH(t)+rk+raU(t)-par}{a}=\frac{N\left(U(t),H(t)\right)}{a}$$

The curves labeled $\mu=M^{-1}(0)$ and $\upsilon=N^{-1}(0)$ in Figure 3 represent the loci of points (U,H) such that the rate of return to tertiary and secondary schooling are respectively zero.

Steady states occur when these level curves intersect and both $\dot{H}=0$ and $\dot{U}=0$. These intersections divide the space into four regions, and the arrows in Figure 3 denote the behavior of the investment trajectories over time. The line $H+U\leq p$ represents the population constraint. By limiting the size of the externality relative to the production technologies in Assumption 1, I ensure that at least one stable steady state exists:

Assumption 1: $ra<A$ and $ra<\alpha(\alpha-1)BU(t)^{\alpha-2}<p$

From the above assumption, the level curves are always downward sloping:

$$(35) \quad \left.\frac{dH}{dU}\right|_{\dot{H}=0}=\frac{A(1-\alpha)}{ra-A}<0$$

$$(36) \quad \left.\frac{dH}{dU}\right|_{\dot{U}=0}=\frac{\alpha(\alpha-1)U^{\alpha-2}+ar}{\alpha A}<0$$

Intuitively, the rate of return to both tertiary investment and remaining in the unskilled state negatively depend on the stock of H and U. Therefore, holding the rate of return constant

[24] To conserve notation, I assume that size of the externality, a, is the same in both categories of schooling. Relaxing this assumption does not substantially alter the results.

along a level curve, an increase in U must be accompanied by a decline in H.[25] The curvature of the $\dot{U}=0$ stems from the assumption of diminishing marginal productivity in the unskilled sector. In drawing μ and ν, I have made the additional assumptions that $\mu \cap \nu$ is non-empty, and at each intersection point, μ and ν cross transversely, that is, they have distinct tangent lines. The first assumption ensures that a steady state exists, while the second assumption combined with the curvature of the level curves obtains two steady states.[26] The next Lemma discusses the behavior of investment trajectories depicted in Figure 3.

Lemma 4: *Above the $\dot{H}=0$ level curve, the rate of return to tertiary education, $R^H(H,U)=(w^H-w^L-rc^H)$, is negative, and there is no tertiary attainment. Below the $\dot{H}=0$ level curve $R^H(H,U)>0$, and tertiary educated labor is accumulated. Above the $\dot{U}=0$ level curve the rate of return to the unskilled state, $R^U(H,U)=(w^U-w^L+rc^L)$, is negative and there is investment in secondary schooling. Below this curve, $R^U(H,U)>0$, and there is no decline in the stock of unskilled.*

Proposition 2 then follows from the preceding Lemma.

Proposition 2: *Vertex b is the only asymptotically stable steady state.*

The above proposition states that if the size of the schooling externality is not too big, and education investment commences, then the economy converges to a unique asymptotically stable equilibrium point. Because the size of the cost externality is invariant to the initial skill level in the economy, it is not too surprsising that there is a unique asymptotically stable steady state. However, convergence towards this equilibrium is remarkably varied. And as in the static argument, there exists a set of initial conditions—Region 4 in Figure 3—for which the private rates of return to both kinds of educational investment are negative: $R^H(H,U)<0$ and $R^U(H,U)>0$.[27] In this case, there are no

[25] If the rate of return to tertiary schooling increased with the stock of tertiary educated workers, then a stable steady state would not exist. Any perturbation around a steady state, say an increase in H would raise the rate of return to tertiary investment and lead to new round of investment until the population constraint was reached.

[26] Without the latter assumption, a single steady state occurs where the two curves share the same tangent line. The dynamics associated with this case is not very interesting.

[27] Note that $-R^u(\cdot)$ is the rate of return to secondary schooling.

private incentives for the educational and economic development process to begin. The following series of remarks discuss the intuition behind Lemma 4.

Remark 1: *In region 3, the rate of return to tertiary schooling is negative,* $R^H(H,U)<0$ *and only secondary investment is ongoing* $R^U(H,U)<0$..

Since the number of secondary educated workers is small, the market size effect implies that the wage for tertiary educated workers does not exceed the total cost of tertiary investment: the forgone wages plus the actual schooling costs. In contrast, the number of tertiary educated workers is large enough to induce investment in secondary education. As the secondary educated workforce expands, the rate of return to tertiary education increases, and becomes positive after the trajectory crosses the level curve labeled u.

Remark 2: *In region 1, the rate of return to tertiary investment is positive,* $R^H(H,U)>0$ *while there is no investment in secondary education* $R^U(H,U)>0$.

In this case, the relatively large number of secondary educated workers means that the wage of the tertiary educated offset the actual and opportunity cost of tertiary investment. However, because the initial stock of tertiary educated workers is small, there is a negative rate of return to secondary education. As tertiary investment proceeds, rising productivity amongst the low skilled raises the rate of return to secondary education, which eventually becomes positive when the trajectory crosses the level curve v.

Remark 3: *In region 2 the rate of return to both secondary and tertiary education are positive.*

The dynamic process is self reinforcing. An increase in secondary investment increases the flow of tertiary educated labor:

(37) $$\frac{d\dot{H}}{dU}=-A(1-\alpha)<0$$

In turn, rising tertiary investment increases the flow of secondary investment:

(38) $$\frac{d\dot{U}}{dH}=-A\alpha<0$$

This process converges to the steady state level of attainment defined by *b*, where the rate of return to both types of schooling is exactly zero.

A.2 Non-linear Externality

The presence of a non-linear externality alters the results of the previous section. In particular, I consider the case of an externality that is weak at first, becomes stronger over an intermediate range of attainment, and then attenuates as attainment rises beyond some level. As mentioned previously, this scenario seems to be a better approximation of reality, for it is likely that early in the development process the educational infrastructure may be slow to respond to increased enrollment; but beyond some critical level, schooling costs fall rapidly as schools are built, the curriculum modified, and teachers are recruited. Eventually, the impact of additional educational infrastructure ceases to have a substantial effect on the private cost of schooling. For example, after building the first few schools within in a village, additional infrastructure only minimally reduces the private cost of schooling. Figure 4 qualitatively depicts this idea for the case of tertiary schooling; the slope of the cost function indicates the size of the externality.

The dynamical system corresponding to this setup is:

$$(39)\quad \dot{H} = \frac{H(t)[ra-A]-AU(t)[1-\alpha]-rc(H(t))}{\frac{-dc}{dH}} = \frac{M(U(t),H(t))}{\frac{-dc}{dH}}$$

$$(40)\quad \dot{U} = \frac{\alpha BU(t)^{\alpha-1}-\alpha AH(t)+rc^{L}(p-U(t))}{\frac{dc}{dU}} = \frac{N(U(t),H(t))}{\frac{dc}{dU}}$$

Assumption 2: $A > \left|\frac{dc}{dH}\right| \forall H < p$

Assumption 3: $\left|w^{U}{}_{UU}\right| > c_{U}^{L} \forall U < p$

Fundamental to the existence of stability is the idea that the rate of return to both tertiary investment and remaining in the unskilled state negatively depend on the stock of H and U. Thus, the level curves are downward sloping, because holding the rate of return constant along a level curve, an increase in U must be accompanied by a decline in H:

$$(41)\quad \left.\frac{dH}{dU}\right|_{\dot{H}=0} = \frac{-A(1-\alpha)}{A+r\frac{dc^{H}}{dH}} < 0$$

$$(42) \qquad \left.\frac{d\dot{H}}{dU}\right|_{\dot{U}=0} = \frac{\alpha(\alpha-1)U^{\alpha-2} - rc_U^L}{\alpha A} < 0$$

By limiting the size of the externality relative to the respective production technologies, Assumptions 2 and 3 produce dynamics similar to that described in Lemma 4, and ensure that stable steady states exist.

However unlike the previous section, the non-linearity in the behavior of the schooling externality can produce multiple stable steady states. I use equations (34) and (35) to draw the level curves, $\mu = M^{-1}(0), \nu = N^{-1}(0)$ in Figure 5, for the case where the externality resembles that depicted in Figure 4. That is, schooling costs decline slowly when the initial stock of attainment is low, but accelerate beyond the range of attainment. I assume that μ and ν intersect at three locations.[28] Hence, the dynamics described in Lemma 4 combined with the "threshold" nature of the externality can produce multiple stable steady states. Intuitively, in an economy with a small stock of educational attainment, the high schooling costs and its slow decline are unable to offset the initially weak demand for skilled labor. Therefore, the economy converges to a low steady state level of attainment. The proposition below and the discussion that follows make this idea more precise.

Proposition 3: *Vertices a and c are the only asymptotically stable steady states.*

The composition of the human capital stock determines not only the dynamic pattern of educational investment, but also the steady state level of educational attainment. In Figure 5, there exist stable steady states such as 'd', where production is predominantly undertaken in the skilled sector, as well as the case of steady state '*c*': a backward economy with a largely uneducated workforce. Given the crucial role of educational attainment in influencing the distribution of income,[29] the next proposition discusses how wage and income inequality differ across the two stable steady states.

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[28] The level curves in Figure 5 qualitatively depict the case where the cost of schooling takes the functional form: $c(\cdot) = -\tanh(\cdot)$. See the Appendix for the parameter values that produce curves similar to the level curves in Figure 5. Alternatively, a similar result can be obtained by assuming a discontinuous schooling externality. The size of the externality is constant, but discontinuously jumps after some threshold level of attainment. This would produce multiple stable steady states.

[29] For example, The Inter-American Development Bank's 1998–1999 Annual Report argues that differences in education is the most significant factor behind wage inequality in Latin America.

Proposition 4: *The skill premia are lower in the higher attainment steady state, a, but the share of national income accruing to the highly skilled,* $\frac{Hw^H}{Hw^H + Lw^L + Uw^U}$*, is greater in steady state a than c.*

At an asymptotically stable steady state, the discounted skill premia equal the cost of schooling $\frac{\gamma^L(H,U)}{r} = c^i(\bullet)$. Therefore, the high attainment steady state, with its better developed infrastructure has a smaller skill premia. However, since educational development shifts the bulk of production into the skilled sector, the highly skilled garner a larger share of national income. This result consistent with some of the differences between Latin America and East Asia. The lower wage inequality in East Asia might stem from its better developed educational infrastructure. However, since production mainly occurs in the skilled sector, the share of national income earned by the skilled is higher than in Latin America. The following series of remarks describe the intuition behind Lemma 4 for the non-linear case, and discuss how the eventual denouement is shaped by both demand and supply forces. Note that the dashed lines in Figure 5 define the basin of attraction for the different equilibria.

Remark 4: *In the set defined by region 4, the rate of return to both types of schooling are negative:* $R^H(H,U) < 0, R^U(H,U) > 0$. *Therefore, there's no investment in schooling.*

Although educational attainment maybe initially ongoing, trajectories that enter region 4 never leave and educational investment ceases. Consider an initial point '*d*' in region 6. Investment in secondary schooling is ongoing $\dot{U} < 0$, while the rate of return to tertiary schooling is negative but increasing as the size of the low skilled labor force grows: $\frac{\partial R^H}{\partial U} < 0$. When the trajectory reaches the boundary of region 4, the rate of return to tertiary schooling is still negative. But rising marginal productivity in the unskilled sector means that the rate of return to secondary schooling is now zero and educational investment ceases. Economies with endowments in the interior of region 4 never experience economic growth.

Remark 5: *Economies with initial conditions located in regions 1, 2 and 3 converge to equilibrium point 'a'.*

Continuing with the example above, consider an economy defined by point '*e*' in region 2. It has more tertiary educated labor $\left(H^e > H^d\right)$ but the same level of unskilled labor as economy '*d*': $\left(U^e = U^d\right)$. Starting from both economies, the rate of return to tertiary educated labor is negative and only secondary investment is ongoing. However, since $H^e > H^d$, then $\left|R^U\left(H^e,U^e\right)\right| > \left|R^U\left(H^d,U^d\right)\right|$, which leads to greater investment in secondary education in economy '*e*'. Because of the external demand linkages, the growing secondary educated labor force coupled with the lower initial cost of tertiary schooling spark investment in tertiary schooling in economy *'e'*. This occurs when the trajectory crosses the level curve *u*. Therefore, while economy *'d'* converges to a steady state on the boundary of region 4, with no increase in its initial stock of tertiary educated labor, economy *'e'* converges to the high attainment steady state *'a'*.

Divergent behavior can emerge when economies differ critically in their endowment of secondary educated labor. Consider two economies beginning at points *'f'* and *'g'* in regions 5 and 3 respectively. Both economies have identical levels of tertiary educated labor $\left(H^f = H^g\right)$ but differ in their endowment of secondary educated labor $U^g < U^f$. Initially, in both economies tertiary investment is ongoing, while there is no new investment in secondary schooling. Since its initial endowment of secondary educated labor is bigger, tertiary investment is greater in economy *'g'*: $R^H\left(H^g, U^g\right) > R^H\left(H^f, U^f\right)$, and when the investment trajectory crosses the v level curve, the rate of return to secondary schooling becomes positive and both types of labor are accumulated as the economy converges to point *'a'*. In contrast, because its endowment of secondary educated labor was too small, economy *'f'* never accumulates enough tertiary schooling in order to make secondary investment profitable, and educational investment ceases once the trajectory reaches the boundary of Region 4.

Remark 6: *Economies with initial conditions located in regions 7, 8 and 9 converge to equilibrium point 'c'.*

In this case, the relatively small initial stocks of secondary and tertiary educated labor mean that schooling costs are high and the demand for each type of skilled labor is low. These twin effects diminish the private incentive to invest in education, and the economy converges to a low steady state level of development.

A.3 Implications for Empirical Research

The preceding discussion has argued that differences in the composition of the human capital stock can lead to very different levels of economic development. Therefore, average years of schooling is potentially of little use in identifying an economy's steady state level of development. Furthermore, the growth of average years of schooling can be unrelated to economic growth. The lines labeled **x, y** and **z** in Figure 6 help to illustrate these ideas. These are iso-average lines, where the stocks of *H* and *U* vary so that the average years of schooling is held constant. That is, suppose that the years of schooling required for performing high skilled, low skilled and unskilled tasks are: n^H, n^L, n^U, where $n^H > n^L > n^U$, then the average years of schooling in the population is:

$$(43) \qquad \bar{n} = \frac{n^H H + n^L L + n^U U}{p}.$$

Using $L = p - H - U$, the slope of this line:

$$(44) \qquad \frac{dH}{dU} = \frac{n^L - n^U}{n^H - n^L}$$

is always positive. An increase in the number of unskilled reduces the total years of schooling in the population. In order to offset this decline and hold the average years constant, there must be a rise in the number of high skilled. The magnitude of this change depends on the number of years of schooling required for each type of education. For example, consider two economies with identical population levels, and with endowments that lie on the same iso average line. If the years of schooling required for secondary education is much bigger than that required for being unskilled, then the economy with the greater number of unskilled must also have a significantly larger high skilled labor force. Thus, the composition of the human capital stock can vary quite dramatically along an iso-average line. In Figure 4, I draw three such iso-average lines: x, **y** and **z**, where $\overline{n}^x > \overline{n}^y > \overline{n}^z$. All endowments located along the **x** iso- average line converge to the high steady state, while all endowments along the **z** iso-average line converge to the low steady state. In the former case, the average years of schooling imply a skilled sector large enough to be self-sustaining. In the latter case, the skilled sector is too small to produce a dramatic shift in the production process, and the economy converges to a low steady state. This indicates the threshold nature of development *across* iso-average lines: small changes in the average years of schooling can lead to dramatic differences in educational investment. A similar point has been made by Azariadis and Drazen (1991). But the intermediate case makes a novel argument. From the dynamics described in Lemma 4, the endowments '*d*', '*e*' and '*f*' located on the iso-average line **y** lead to very dissimilar development experiences. Because average years of schooling can mask fundamental differences in the composition of the human capital stock, *along* an iso-average line sharp differences in development can still be observed. Although the average years of schooling remain constant, as we move from point '*d*' to point '*f*' on the iso-average line **y**, the size of the skilled sector shrinks and so do the private incentives for educational investment.

The following numerical examples not only illustrate this bifurcation behavior, but also the potentially weak relationship between the growth in average years of schooling and economic growth. In the first case, the two economies critically differ in their initial stock of tertiary educated labor and as a result, their economies diverge. In the second example, an economy converging to its steady state mainly accumulates high skilled labor. The second economy, converging to a much higher steady state mainly accumulates secondary educated labor. Since secondary education requires less years of schooling than tertiary training, the growth in the average years of schooling is higher in the first economy than in the second. Yet, economic growth is higher in the economy that primarily experiences investment in secondary educated labor.

Table 5.1. Bifurcation Dynamics

Initial Composition	Initial Average Years of Schooling	Equilibrium Composition	Growth in Output
$U=0.75, H=0.05, L=0.2$	4.2 years	$U=0.65, H=0.07, L=0.27$	89.8%
$U=0.64, H=0.017, L=0.3$	4.2 years	$U=0.2, H=0.3, L=0.48$	2487%

Table 5.2. Change in Average Years of Schooling Versus Growth in Output

Initial Composition	Equilibrium Composition	Growth in Average Years of Schooling	Growth in Output
$U=0.75, H=0.05, L=0.2$	$U=0.65, H=0.07, L=0.27$	8.2%	35.1%
$U=0.5, H=0.2, L=0.3$	$U=0.2, H=0.31, L=0.48$	6.5%	153.4%

See Appendix

B. Policy Implications

The previous section has demonstrated how large and slowly decreasing private education costs coupled with demand linkages can lead to multiple equilibria. Within this context, unless government policy is carefully chosen, it may ultimately have little or no impact on the economy'*s* long run steady state. For example as Figure 5 indicates, consider an economy with a composition of educated labor defined by point *d.* Suppose that education policy is focused on heavily subsidizing tertiary schooling, but the behavior of private education costs is the same as that studied in the previous section: large and slowly decreasing. To this end, the government affects an increase in the number of tertiary educated from point *d* to point *d'*. From Figure 7, it is easy see that in the long run this educational policy regime will have no impact on the economy's long run level of educational attainment. In contrast, a policy stance which led to greater investment in secondary schooling would have had a greater impact on long run educational development. In this instance, the effect on long run development would have been more dramatic had policy transformed the composition from point *d* to point *e.*

For economies with a large initial base of unskilled workers, such as point *f,* education policy geared to reducing the number of unskilled is a necessary condition for development. Otherwise, it is impossible to change the long run steady state by only changing the endowment of tertiary educated labor. From the diagram, if the government converted all of the available low skilled workers to high skilled workers, it still would be insufficient, given private incentives, to escape steady state *c*. Policy that either subsidizes secondary education (*g*), or some combination of secondary and tertiary (*h*) is necessary.

While the above arguments are suggestive, they assume that the behavior of infrastructure development is predetermined, and that government policy is only focused on once and for all changes in the level of the human capital stock. To address this shortcoming and provide a more complete description of an optimal education policy, I consider the case of a social planner who chooses the level of tertiary and secondary enrollments in order to maximize the present discounted value of output net of education costs:

$$\max_{\dot{U},\dot{H}} \int_0^\infty e^{-rt}\left[F(H,U)-G\left(H,\dot{H}\right)-J\left(U,\dot{U}\right)\right]dt$$

subject to

$$H(0)=H_0, U(0)=U_0, H+U\leq p, \dot{H}\geq 0, \dot{U}\leq 0$$

where the government internalizes the demand linkages:

(45) $\quad F(H,U)=AHL+BU^{\alpha}$

and the total cost of tertiary schooling is given by:

(46) $\quad G=G\left(H,\dot{H}\right), G_H<0, G_{\dot{H}}>0$ and $G_{\dot{H}\dot{H}}>0$.

The first argument in the cost function reflects the idea that the existing level of tertiary attainment lowers the private cost of tertiary investment. However, a rise in the level of schooling enrollments requires increased expenditures on infrastructure and other education inputs. Thus, I assume that at any instant the total cost of tertiary schooling is an increasing and convex function of the flow of current investment in tertiary schooling: the enrollment level. The cost of secondary schooling is similarly defined:

(47) $\quad J=J\left(U,\dot{U}\right), J_U>0, J_{\dot{U}}<0$ and $J_{\dot{U}\dot{U}}>0$

By making the simplifying assumptions that the marginal impact of attainment on the private cost of schooling is independent of the current enrollment levels:

(48) $\quad \dfrac{\partial^2 G}{\partial\dot{H}\,\partial H}=\dfrac{\partial^2 J}{\partial U\,\partial\dot{U}}=0,$

and that the marginal impact of attainment decreases in the level of attainment:

$$\frac{\partial^2 J}{\partial U^2}<0, \frac{\partial^2 G}{\partial H^2}<0$$

The first order conditions for an optimal policy are both necessary and sufficient:

(49) $$e^{-rt}\left[F_H - G_H\right] = -\frac{d\left(e^{-rt}G_H\right)}{dt}$$

(50) $$e^{-rt}\left[F_U - G_U\right] = -\frac{d\left(e^{-rt}J_U\right)}{dt}$$

Equation (51) implies that along an optimal path, the social planner chooses the level of tertiary investment such that the cost difference of endowing the marginal agent with tertiary education at time t rather than at $t+\Delta t$ is just offsetted by the net social marginal product contributed by that agent over the interval $\left[t, t+\Delta t\right]$:

(51) $$\int_t^{t+\Delta t} e^{-rs}\left[F_H - G_H\right]ds = e^{-rt}G_{\dot{H}(t)} - e^{-r(t+\Delta t)}G_{\dot{H}(t+\Delta t)}.$$

The intuition for secondary investment is similar, and the first order conditions imply the following lemma.

Lemma 4: *If* $\dot{H} \geq 0, \dot{U} \leq 0, G_{\dot{H}\dot{H}} > 0$ *and* $J_{\dot{U}\dot{U}}$ *then* $\ddot{H} < 0, \ddot{U} > 0$.

From Lemma 4, along an optimal path, the change in the flow of human capital investment diminishes over time. Therefore, while attainment increase over time, the initial change in the level of enrollment in both secondary and tertiary education should be the greatest. That is, the first generations should experience the biggest increase in schooling investment. The simultaneous expansion of both kinds of schooling follows from the fact that the social marginal product of labor in the skilled sector depends on the level of the complementary input. Thus, by investing in both types of education, the social planner increases the social marginal product of each unit of labor in the skilled sector.

A combination of two factors leads to the result that the change in enrollment levels should diminish over time. An optimal education policy postpones investment in secondary education for the marginal agent from the current to a later date if the net present value of the marginal agent's contribution in the current instant is less than the difference in the marginal costs over the interval:

(52) $$\int_t^{t+\Delta t} e^{-rs}\left[F_U - F_U\right]ds < e^{-rt}J_{\dot{U}(t)} - e^{-r(t+\Delta t)}J_{\dot{U}(t+\Delta t)}$$

Because the marginal cost of investment in secondary schooling increases in the flow of investment $\left(J_{\dot{H}\dot{H}} < 0\right)$, at each instant policy makers face an upward sloping supply curve for new skilled labor. Secondly over time diminishing marginal productivity in the unskilled sector reduces the net marginal benefit of adding to the secondary skilled capital stock and expanding the skilled sector. Therefore, along an optimal path, the net marginal benefit of skilled labor is at its greatest initially. Hence, it is optimal for the policy maker to increase enrollments over time with the biggest increases occurring early in the development process. Smaller increases occur later on as the shadow cost of skilled labor—the marginal product of unskilled labor—increases, making it profitable to slow the rate of educational investment. In contrast, if the cost of educating the marginal agent was constant (a flat supply curve), then there is no incentive to postpone investments in order to lower current marginal costs, and the social planner solves a simple static problem. Note that since the social planner internalizes the demand linkages, as well as the private cost of schooling, the steady state level of attainment exceeds the decentralized equilibrium.

The educational experience of economies such as the U.S.[30] and some East Asian countries share certain characteristics with the optimal education policy discussed above. By facilitating heavy initial investments in both types of schooling early in the development process, these economies may have made the development process self-sustaining. The private incentives to invest in education became stronger, and the economies were able to shift more and more production into the skilled sector. In contrast, the analysis implies that although many sub-Saharan African nations invested heavily in primary schooling, this may have had little impact in absorbing or using knowledge and did not fundamentally alter their economies.

IV. Conclusion

This paper has argued that the composition of the educational stock plays an important role in shaping the incentives for investment in education. The specificity of the various tasks within the production process and the observation that some ideas tend to be non-rival but excludable generate external linkages between the skill categories. Moreover, the access to education often depends on the existing level of attainment. Together, these demand and supply factors produce a pattern of circularity in educational investment. A critical deficiency in either skill type can weaken the demand for the complementary input, and coupled with limited access to education, can lead to a low level of overall investment in education and economic development. Conversely, a generous supply of either skill input can strengthen the demand for the complementary education type. Declining schooling costs reinforce this process and help propagate this advantage through time, eventually leading the economy to high level of educational investment.

[30]See Ramcharan (2001) and Goldin and Katz (1998) for a discussion of American educational investments.

Using this framework, the paper demonstrates that unless carefully chosen, education policy can prove wasteful, leaving the potential long run development steady state unchanged. To avoid this outcome, the paper argues that the initial investments in both types of schooling should be the heaviest, and that it should be coordinated so that investments occur in both education types. In addition, the model is helpful in interpreting the empirical literature. The many empirical studies that have failed to detect a positive correlation between the growth in average years of schooling and economic growth is unsurprising. The average years of schooling can mask potentially important differences in the composition. Examples in the text highlighted this empirical difficulty.

That said, the paper has not addressed many important questions. For example, openness to trade and the flow of ideas maybe critical factors in the determination of educational investment. Hence, it may well be that developing economies need only invest in secondary schooling, importing high skilled education embodied in the foreign goods. Also, questions of administrative capacity arise when discussing optimal education. The poorest developing countries may only be able to administer basic schooling, although higher education itself maybe more profitable. In addition, the analysis does not consider the impact of wage uncertainty on the education investment decision. It is hoped that future research will incorporate these ideas.

A. Figures

Figure 1
Decentralized Dynamics
(Linear Externality)

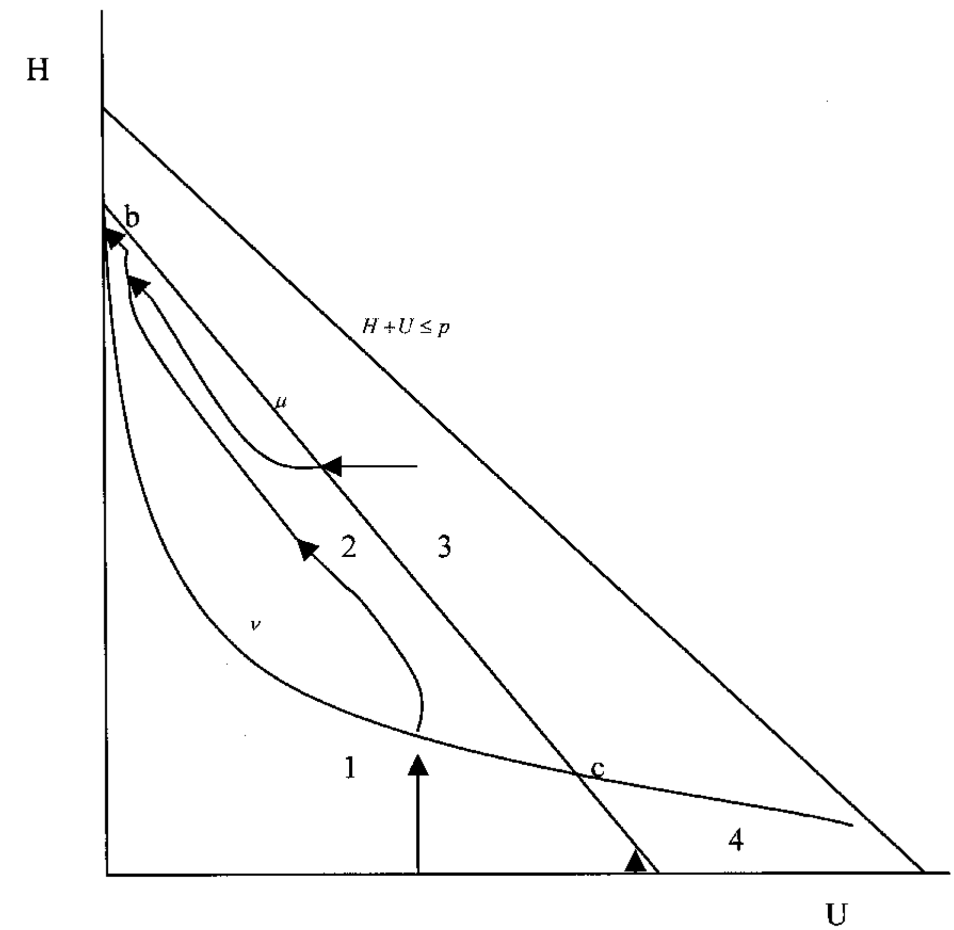


Figure 2
Non-Linear Externality

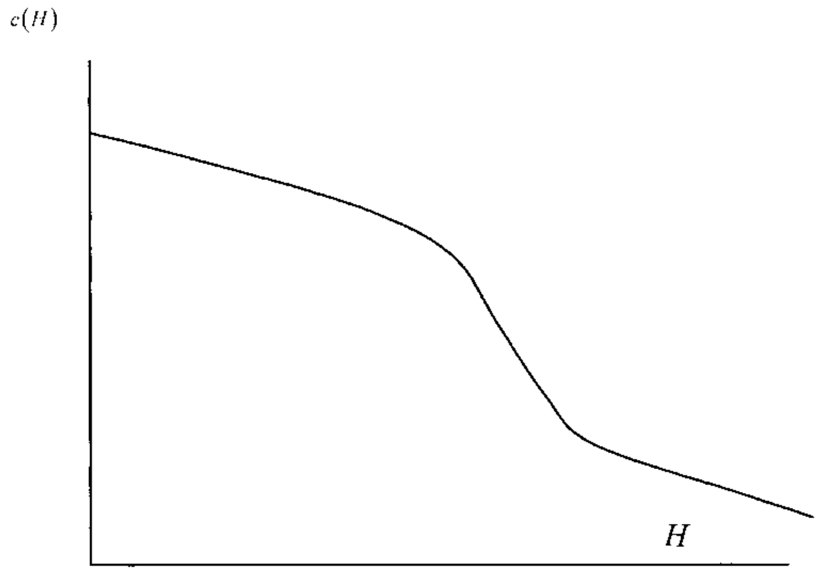


Figure 3
Decentralized Dynamics
Non-Linear Externalaity

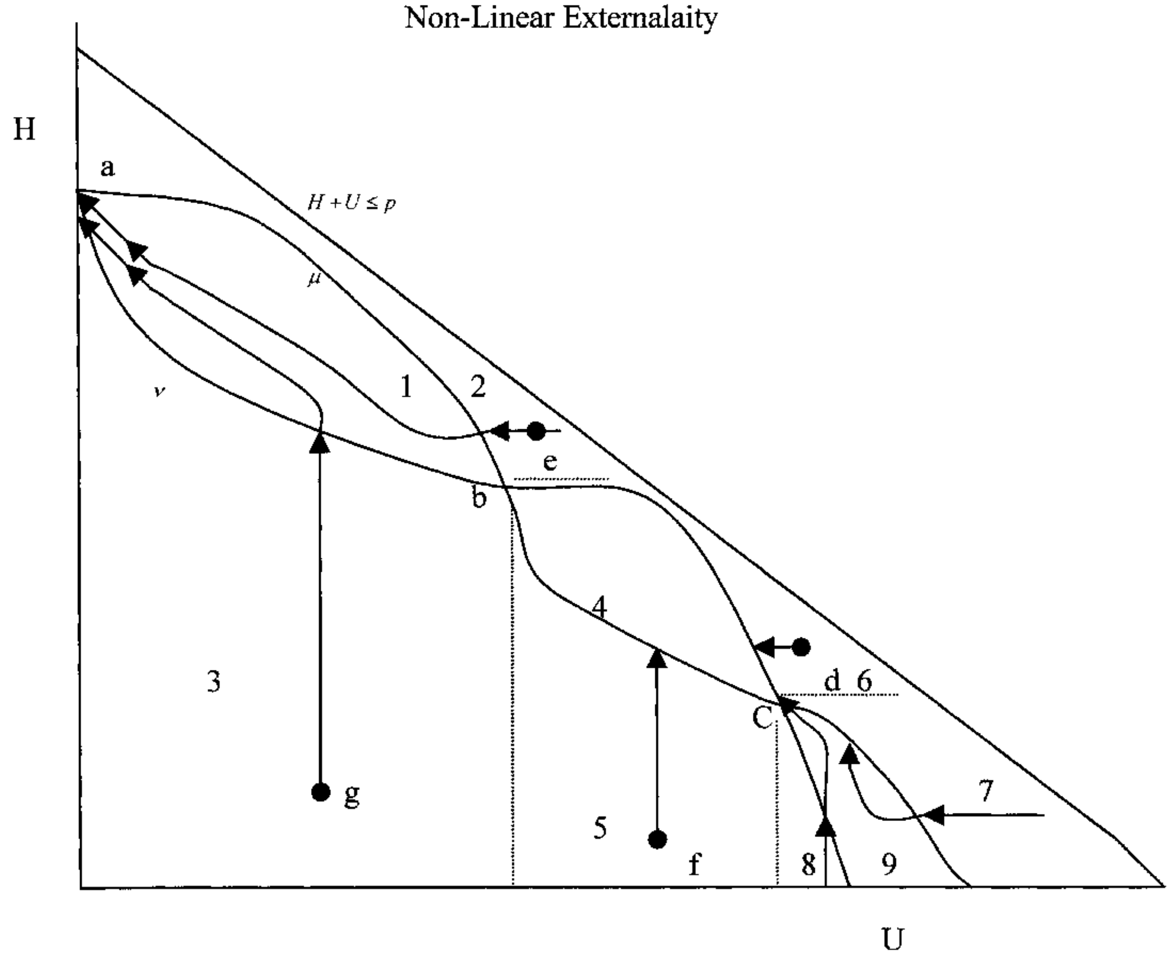


Figure 4
Decentralized Dynamics
Empirical Implications

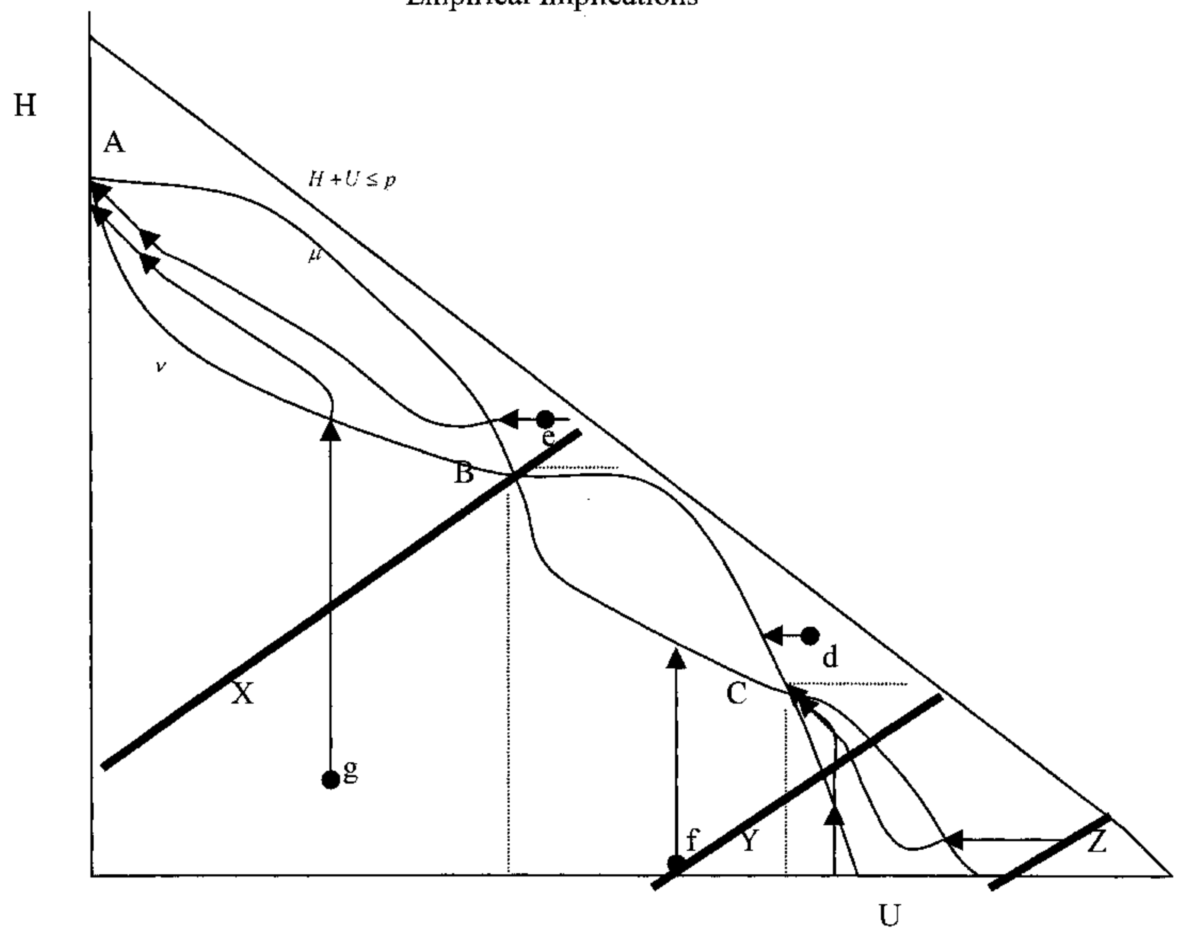
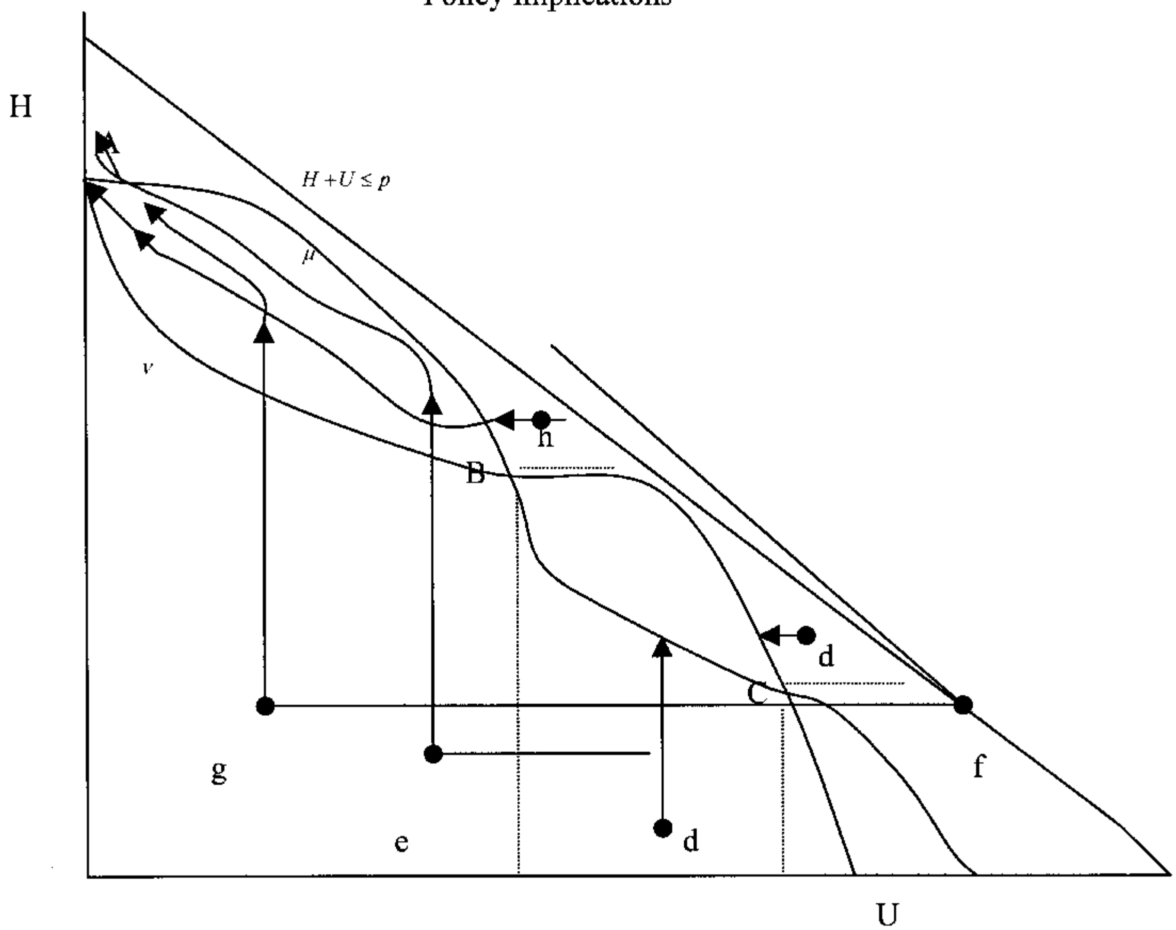


Figure 5
Decentralized Dynamics
Policy Implications



B. Mathematical Details

Lemma 1: **Investment in education fails to occur if the skill composition of the economy belongs to the set** $R = \{H, U : \gamma^h(H, U) \leq c^h(\theta^{**}) \text{ and } \gamma^l(H, U) \leq c^l(\theta^*)\}$.

Proof: Consider an initial allocation (H_0, U_0). If $\gamma^l(H_0, U_0) < c^l(\theta^*)$, then new secondary investment is privately unprofitable. Similarly, $\gamma^h(H_0, U_0) < c^h(\theta^{**})$ then new investment in tertiary education is privately unprofitable for all agents.

Lemma 2: Along an equilibrium path $\gamma^h(H(t), L(t)) \geq 0$ and $\gamma^l(H(t), L(t)) \geq 0$ for all t.

Proof: I first show that $\gamma^h(H(t), L(t)) \geq 0$. The income stream of a high skilled agent who earns the high skilled wage from date s onwards is:

$$V^h(L(t)) = \int_s^\infty L(t) e^{r(s-t)} dt, \quad \text{where } w^h = L(t). \tag{A.1}$$

I now show that any single deviation from this income stream by a high skilled agent is suboptimal. Suppose a high skilled individual finds it optimal to switch to the low skilled income stream in period t^*, and later re-enters the high skilled sector at some later date $t^* + \Delta t$. The income profile for such a strategy beginning on any date is:

$$\int_s^{t^*} L(t) e^{r(s-t)} dt + \int_{t^*}^{t^*+\Delta t} H(t^*) e^{r(t^*-t)} dt + \int_{t^*+\Delta t}^{\infty} L(t^* + \Delta t) e^{r(t^*+\Delta t - t)} dt \tag{A.2}$$

If I chose s to be arbitrarily close to t^*, then I can approximate the first two integrals by:

$$\frac{(t^* - s)L(s)}{1 + (t^* - s)r} + \frac{\Delta t H(t^*)}{1 + \Delta t r} \tag{A.3}$$

If this deviation is optimal then $L(s) > H(t)$ and $H(t^*) > L(t^*)$.

However, since the individual operated as a high skilled in instant , but switched to the low skilled sector in period $*$: $H(s) > H(t^*)$ and $L(s) < L(t^*)$. This implies $L(s) > H(s) > H(t^*) > L(t^*) > L(s)$: a contradiction. Since the wage differential stemming from tertiary schooling is always positive, this implies that

A similar argument shows that $\gamma^l(H(t), L(t)) \geq 0$. The income profile of a secondary educated agent beginning on date s who enters the unskilled sector on some instant t^*, but thereafter re-enters the low skilled sector is:

$$\int_s^{t^*} H(t) e^{r(s-t)} dt + \int_{t^*}^{t^*+\Delta t} w(u(t^*)) e^{r(t^*-t)} dt + V^l(H(t^* + \Delta t), L(t^* + \Delta t)) \tag{A.4}$$

Assuming the difference between s and t^* is arbitrarily small, the first two terms of the above expression can be approximated by:

$$\text{(A.5)}\quad \frac{(t^*-s)H(s)}{1+(t^*-s)r}+\frac{\Delta t w(u(t^*))}{1+\Delta t r}$$

If this is optimal, then $H(s) > w(u(s))$ and $w(u(t^*)) > H(t^*)$. Since the individual switched to the unskilled sector on $t^* : u(s) < u(t^*) \to w(u(s)) > w(u(t^*))$, but the condition $\gamma^h(H(t), L(t)) \geq 0$ implies that $\dot{H}(t) \geq 0 \to H(t^*) \geq H(s)$. Therefore, a deviation implies: $H(s) > w(u(s)) > w(u(t^*)) > H(t^*)$: a contradiction.

Lemma 3: *The optimal investment dates,* τ_2 *and* τ_1, *satisfy the following conditions:*

$$\text{(A.6)}\quad c^h(H(\tau_2)) \leq \int_{\tau_2}^{\infty} \gamma^h(H(t), U(t))e^{r(\tau_2-t)}dt$$

$$\text{(A.7)}\quad c^l(p-U(\tau_1)) \leq \int_{\tau_1}^{\tau_2} \gamma^l(H(t), U(t))e^{r(\tau_1-t)}dt$$

there does not exists a $\tau' > \tau_2$ *such that*

$$\text{(A.8)}\quad c^h(H(\tau_2)) - c^h(H(\tau'))e^{-r\tau'} > \int_{\tau_2}^{\tau'} \gamma^h(H(t), U(t))e^{r(\tau_2-t)}dt$$

there does not exists a $\tau_1 < \tau' < \tau_2$ *such that*

$$\text{(A.9)}\quad c^l(p-U(\tau_1)) - c^l(p-U(\tau'))e^{-r\tau'} > \int_{\tau_1}^{\tau'} \gamma^l(H(t), U(t))e^{r(\tau_1-t)}dt$$

Proof: The first two conditions are obvious. For example, suppose the optimal investment date did not satisfy condition (i), then

$$\text{(A.10)}\quad c^h(H(\tau_2)) > \int_{\tau_2}^{\infty} \gamma^h(H(t), U(t))e^{r(\tau_2-t)}dt$$

the cost of investing in tertiary schooling exceeds the present discounted value of the earnings stream.

To prove condition (A.7), define the net value of investing on date τ_2 as:

$$\text{(A.11)}\quad V^h(\tau_2) = \int_{\tau_2}^{\infty} \gamma^h(H(t), U(t))e^{r(\tau_2-t)}dt - c^h(H(\tau_2))$$

This is the present discounted value of the skill premium minus the cost of investing. If date is the optimal investment date, then $V^h(\tau_2) \geq V(t)\forall t$. If there exists which satisfies condition (A.7), then

$$\text{(A.12)}\quad c(H(\tau_2)) - c(H(\tau'))e^{-r\tau'} > \int_{\tau_2}^{\tau'} \gamma^h(H(t), U(t))e^{r(\tau_2-t)}dt$$

$$\text{(A.13)}\quad c(H(\tau_2)) - c(H(\tau'))e^{-r\tau'} > \int_{\tau_2}^{\infty} \gamma^h(H(t), U(t))e^{r(\tau_2-t)}dt - \int_{\tau'}^{\infty} \gamma^h(H(t), U(t))e^{r(\tau'-t)}dt$$

$$\text{(A.14)}\quad \int_{\tau'}^{\infty} \gamma^h(H(t), U(t))e^{r(\tau'-t)}dt - c^h(H(\tau'))e^{-r\tau'} > \int_{\tau_2}^{\infty} \gamma^h(H(t), U(t))e^{r(\tau_2-t)}dt - c^h(H(\tau_2))$$

$V(\tau') > V(\tau_2)$. A contradiction. The argument is similar for condition (A.9).

Proposition 1: *The behavior of educational attainment along a perfect foresight equilibrium path is described by*

(A.15) $$\dot{U}=\frac{w^u(U(t))-[w^l(H(t))-rc^l(p-U(t))]}{c_U^l(p-U(t))}$$

(A.16) $$\dot{H}=\frac{w^h(H(t),U(t))-w^l(H(t),U(t))-rc^h(H(t))}{-c_H^h(H(t))}.$$

There is no investment in tertiary education if:

(A.17) $rc^h(H(t))\geq \gamma^H(H(t),U(t))$.

There is no investment in secondary education if:

(A.18) $rc^l(p-U(t))\geq \gamma^L(H(t),U(t))$.

Proof:

From Lemma 3 we know that for any $\tau_2^{'}=\tau_2+\Delta t$:

(A.19) $$c^h(H(\tau_2))-\frac{c^h(H(\tau_2+\Delta t))}{1+\Delta tr}\leq\frac{\Delta t\gamma^h(H(t),U(t))}{1+\Delta tr}\text{, for } \Delta t>0$$

and

(A.20) $$c^h(H(\tau_2))-\frac{c^h(H(\tau_2+\Delta t))}{1+\Delta tr}\geq\frac{\Delta t\gamma^h(H(t),U(t))}{1+\Delta tr}\text{, for } \Delta t<0$$

Similarly for any date $\tau_1^{'}=\tau_1+\Delta t$

(A.21) $$c^l(p-U(\tau_1))-\frac{c^l(p-U(\tau_1+\Delta t))}{1+\Delta tr}\leq\frac{\Delta t\gamma^h(H(t),U(t))}{1+\Delta tr}\text{, for } \Delta t>0$$

And

(A.22) $$c^l(p-U(\tau_1))-\frac{c^l(p-U(\tau_1+\Delta t))}{1+\Delta tr}\geq\frac{\Delta t\gamma^h(H(t),U(t))}{1+\Delta tr}\text{, for } \Delta t<0$$

Rearranging the above expression:

(A.23) $$\frac{c^h(H(\tau_2))-c(H(\tau_2+\Delta t))}{\Delta t}\leq\gamma^h(H(t),U(t))-rc^h(H(\tau_2))$$

(A.24) $$\frac{c^h(H(\tau_2))-c(H(\tau_2+\Delta t))}{\Delta t}\geq\gamma^h(H(t),U(t))-rc^h(H(\tau_2))$$

Taking the limit as completes the derivation. A similar argument is constructed for $\dot{U}$.

Lemma 4: *Above the* $\dot{H}=0$ *level curve, the rate of return to tertiary education,* $(w^h - w^l - rc^h)$, *is negative, and there is no tertiary attainment. Below the* $\dot{H}=0$ *level curve* $(w^h-w^l-rc^h)>0$, *and tertiary educated labor is accumulated.*

Above the $\dot{U}=0$ *level curve the rate of return to the unskilled state,* $(w^u-w^l+rc^l)$, *is negative and there is investment in secondary schooling. Below this curve,* $(w^u-w^l+rc^l)>0$, *and there is no decline in the stock of unskilled.*

Proof:

The argument rests on the assumptions that

Assumption A.1: $A>|rc_H^h|\forall H<p$

Assumption A.2: $|w_{uu}^u|>c_u^l\forall U<p$

From Assumption A.1, the rate of return to tertiary education:

(A.25) $R^H(H,U) = A(1-a)p - AH - A(1-a)U - rc^h(H)$

negatively depends on the stock of H:

(A.26) $\frac{\partial R^H}{\partial H} = -A - rc^h_H < 0$.

Therefore, suppose $R(H^1, U^1) = 0$, then for any $H > H^1$, $R(H, U^1) < 0$. Since investment in tertiary schooling is irreversible, this implies that $\dot{H} = 0$. Likewise, for any $H < H^1$, $R(H, U^1) > 0$ and $\dot{H} > 0$. The argument for the dynamic behavior of U can be similarly constructed.

Proposition 2: *If $A > 2a$, then vertex a is the only asymptotically stable equilibrium.*
Draw a rectangle with one corner fixed at vertex *a*, while the other three are located in regions (1,2,3) and sides parallel to the axes. The rectangle is positively invariant, and since it can be made arbitrarily small, the equilibrium is asymptotically stable.
Analytically, this can be demonstrated by constructing the Jacobean matrix:

$$J(U,H) = \frac{1}{a} \begin{matrix} a(a-1)Bu^{a-2} + ra & -A \\ -A & ra - 2A \end{matrix}$$

An equilibrium is asymptotically stable iff and the absolute value of the slope of $\dot{U} = 0$ is greater than the slope of the $\dot{H} = 0$ level curve. Vertex *a* satisfies this condition.

Proposition 3: *Vertices a and c are the only asymptotically stable equilibria.*
The dynamical system corresponding to this setup is:

$$\dot{H} = \frac{A(1-a)p - H(t)A - A(1-a)U(t) - rc(H(t))}{-\frac{dc}{dH}} = -M(U(t),H(t))\left(\frac{dc}{dH}\right)^{-1} \quad \text{(A.25)}$$

$$\dot{U} = \frac{aBU(t)^{a-1} - AH(t) + rk + rc(U(t))}{\frac{dc}{dU}} = N(U(t),H(t))\left(\frac{dc}{dU}\right)^{-1} \quad \text{(A.26)}$$

Assumption 1: $A > \left|\frac{dc}{dH}\right| \forall H$

Assumption 2: $|w^u_{uu}| > c^l_u \forall U$

The Implicit Function Theorem, the above assumptions and the behavior of $c^h(H(t))$ are used to depict the level curves $\dot{H} = 0$ and $\dot{U} = 0$ in Figure 5. An examination of the Jacobean matrix at a candidate equilibrium point reveals that asymptotic stability is obtained iff the absolute value of the slope of $\dot{U} = 0$ is greater than the slope of the $\dot{H} = 0$ level curve:

$$J(U,H) = \begin{matrix} Ba(a-1)U^{a-2} + r\frac{dc^l}{dU} & -A \\ -A & \frac{2A + r\frac{dc^h}{dH}}{\frac{dc^h(H)}{dH}} + \left(\frac{d^2c^h(H)}{dH^2}\right)(A(1-2H-U) - rc^h(H)) \end{matrix}$$

Alternatively, qualitatively similar level curves can be generated from the following parameter values:
$A = 10, B = 1.2, a = 0.5, p = 6, r = 0.1, c^l(p-U) = 100\tanh(5-U) + 27, c^h(H) = 250\tanh(H-5) + 250 - 95H$.
As mentioned in the text, the non-linear nature of the schooling externality is the key to generating multiple steady states.

Proposition 4: *The skill premia are lower in the higher attainment steady state, a, but the share of national income accruing to the highly skilled,* $\frac{Hw^h}{Hw^h+Lw^l+Uw^u}$, *is greater in steady state a than c.*

In equilibrium the skill premia equals the discounted value of schooling costs $\frac{\gamma(H,U)}{r} = c(\cdot)$. Skill premia are higher in the lower attainment steady state, because schooling costs are higher. To prove the second part of the Proposition, let Y_j^i denote level of production (and income) in the sector j in steady state i. From the equilibrium conditions, we know $Y_u^a < Y_u^c$, and $Y_s^a > Y_s^c$. These conditions imply $\frac{Y_u^a}{H^aL^a} < \frac{Y_u^c}{H^cL^c}$ or $\frac{2H^aL^a+Y_u^a}{H^aL^a} < \frac{2H^cL^c+Y_u^c}{H^cL^c}$, which can be inverted to yield the proposition.

Lemma 4: *If* $\dot{H} \geq 0,\ \dot{U} \leq 0,\ G_{\dot{H}\dot{H}} > 0$ *and* $J_{\dot{U}\dot{U}} > 0$, *then* $\ddot{H} < 0, \ddot{U} > 0$.
Proof: Using techniques from the calculus of variations, the first order conditions imply:
$\frac{F_H - rG_{\dot{H}}}{-G_{\dot{H}\dot{H}}} = \ddot{H} < 0$. A similar argument holds for $\ddot{U} = 0$

A.2 Numerical Examples

In order to generate tractable numerical examples, I consider a piece-wise linear threshold as an approximation to the non-linear externality discussed in Section 5.3b. That is, the size of the externality changes as the economy's level of educational attainment exceeds a particular threshold level of attainment. Specifically,

$$a^l = \begin{cases} 1 & U \geq 0.65 \\ 4 & otherwise \end{cases}$$

The dynamical system below:

(A.14) $\dot{H} = \frac{H(ra - 2A + A(1-U) - rk)}{a}$

(A.15) $\dot{U} = \frac{aBU^{a-1} - AH + rk + rUa - 2ar}{a}$

Is linearized around the steady states: $(U = 0.65, H = 0.07, L = 0.27) and (U = 0.2, H = 0.31, L = 0.48)$, as well as for the parameters: $A = 3, B = 1, a = 2, a = 0.1, k = 0.5, r = 0.2$.

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