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The Forward Premium Puzzle Revisited

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Abstract

The forward premium is a notoriously poor predictor of exchange rate movements. This failure must reflect deviations from risk neutrality and/or rational expectations. In addition, a mechanism is needed that generates the appropriate correlation between the forward premium and shocks arising from risk premia or expectations errors. This paper extends McCallum (1994) to show how such a correlation can arise from the response of monetary policy to output and inflation, which are in turn affected by the exchange rate. The theoretical models considered all generate results that are consistent with the forward premium being a biased predictor of short-term exchange rate movements; the bias decreases, however, as the horizon of the exchange rate change lengthens. Another common feature of the models is that the true reduced-form equation for exchange rate changes contains variables other than the interest differential, providing a justification for "eclectic" relationships for forecasting exchange rates. The results, however, remain consistent with using uncovered interest parity as a building block for structural models.

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[1] IMF Research Department and Lingnan University, Hong Kong, respectively. This paper stems from earlier work on long-horizon uncovered interest parity undertaken with Menzie Chinn, whose contribution is gratefully acknowledged. Any errors and misinterpretations, of course, remain the authors' responsibility.

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I. Introduction

The "forward premium puzzle" refers to the well-known finding that forward rates in exchange markets are biased predictors of future spot rates. Indeed, not only are forward rates biased, but they are generally perverse: currencies that command a forward premium tend, on average, to depreciate, while those with a forward discount tend to appreciate. This puzzle has been documented in numerous studies as surveyed, for instance, in Hodrick (1987), Froot and Thaler (1990), and Lewis (1995).

Logically, the puzzle must reflect the failure of one or both legs of the joint hypothesis of efficient markets and risk neutrality. Efficient markets ensure that expectations of future variables, including the exchange rate, incorporate all information available at the time the expectations are formed. Risk neutrality implies that the forward rate is equal to the market expectation of the future spot rate. Combining these assumptions in the risk-neutral efficient-markets hypothesis (RNEMH) implies that the deviation between the forward rate and the realization of the future spot rate is a white-noise disturbance uncorrelated with past information. A regression of the change in the spot rate on the forward premium should then yield a coefficient of unity.

For the forward rate to be a biased predictor of the future spot rate, either rational expectations or risk neutrality must be violated — the failure of RNEMH is necessary for the puzzle to exist. But it is not sufficient. In addition, a mechanism is needed that generates the appropriate correlation between the forward premium and the difference between the forward rate and the rational expectation of the future spot rate; absent such a correlation, the forward premium would still be an unbiased predictor of exchange rate changes.[2] The correlation requires that shocks that cause future exchange rate appreciation must, at the same time, tend to increase the forward discount. Assuming covered interest parity, the forward discount equals the difference between domestic and foreign interest rates. Equivalently, then, shocks causing future exchange rate appreciation must raise domestic interest rates relative to those abroad.

This paper develops an explanation for such a correlation based on the reaction of monetary policy to output and prices, which in turn depend on the exchange rate. In this sense, it builds on the work of McCallum (1994) and Meredith and Chinn (1998) in establishing a model-based explanation for the puzzle. It is shown that shocks that cause future exchange rate appreciation tend to cause the current spot rate to depreciate. Such depreciation raises both output and prices, leading to higher interest rates. In these circumstances, the interest differential will be a biased predictor of exchange rate movements, particularly over short horizons. Other lagged macroeconomic information, beyond the interest differential, should be useful in predicting exchange rate movements.

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[2] See Meredith (2002) for further discussion of this issue.

Over longer horizons, the model results indicate that interest rates should be less biased as predictors of exchange rate movements.

The paper is organized as follows. The next section presents concepts and notation. Section III provides updated evidence on the forward premium puzzle, showing that it appears to be robust in the more recent data, at least for the major currencies over short horizons. Section IV analyzes deterministic solutions for the change in the exchange rate in two stylized macroeconomic models, illustrating the mechanism that leads to the forward premium puzzle at short horizons but causes it to disappear over longer horizons. Section V extends the analysis to a stochastic environment, allowing an assessment of a richer variety of issues, including the properties of "misspecified" regressions. Section VI provides concluding remarks.

II. Concepts and Notation

Under the assumption that the forward exchange rate contains all available information about the future spot rate, the following relationship must hold:

$$s_{t+1} = f_t + e_{t+1} , \tag{II.1}$$

where s_t is the log of the domestic-currency price of foreign currency, s_{t+1} is its realization at time t+1, f_t is the forward rate quoted at time t, and e_{t+1} is a white-noise error term uncorrelated with information at time t. Then, the slope coefficient in a regression of the realized change in the exchange rate on the forward discount, i.e.:

$$s_{t+1} - s_t = \alpha + \beta (f_t - s_t) , \tag{II.2}$$

will have an expected value of unity, consistent with the "unbiasedness hypothesis."

Failure of unbiasedness requires either that expectations errors be correlated with information available at time t, or that the forward rate differ from the market expectation of the future spot rate. This is apparent from the following relationships:

$$s_{t+1} = E_t (s_{t+1}) + e_{t+1} \tag{II.3}$$

$$s^e_{t+1,t} = E_t (s_{t+1}) + \eta_t \tag{II.4}$$

$$f_t = s^e_{t+1,t} + \rho_t . \tag{II.5}$$

Equation (II.3) indicates that the realization of the spot rate equals its rational expectation, $E_t (s_{t+1})$, plus a white-noise error term. Equation (II.4) allows for deviations from rational expectations, as the subjective expectation $s^e_{t+1,t}$ equals its rational counterpart plus an error term η_t . Equation (II.5) states that the forward rate equals the expected future spot rate plus a risk premium, implying the absence of expected (risk-adjusted) profits.[3]

Substitution of (II.5) into (II.4) and then into (II.3) allows the future spot rate to be expressed as the sum of the forward rate and a composite error term with three components:

$$s_{t+1} = f_t - \eta_t - \rho_t + e_{t+1} \ . \tag{II.6}$$

Subtracting the current spot rate from both sides of (II.6), and using the relationship implied by covered interest parity—i.e. that the interest rate differential equals the forward rate minus the spot rate—gives:

$$s_{t+1} - s_t = (i_t - i_t^*) - \eta_t - \rho_t + e_{t+1} \ , \tag{II.7}$$

where $(i_t - i_t^*)$ is the difference between the domestic and foreign one-period interest rates.[4]

Two components of the error term in (II.7), specifically the non-rational expectations error η_t and the risk premium ρ_t , are determined at time *t*, while the rational expectations error is determined at *t*+1. The latter is obviously uncorrelated with the time-*t* interest differential. If the components determined at time *t* are similarly uncorrelated, then the slope coefficient in the uncovered interest parity (UIP) regression is expected to be unity:[5]

$$s_{t+1} - s_t = \alpha + \beta(i_t - i_t^*) \ . \tag{II.8}$$

In this case, unbiasedness will hold, even though RNEMH is violated. For the expectation of β to be less than unity, there must be a positive correlation between $\eta_t + \rho_t$ and the interest differential. Given that short-term interest rates are generally set as the instruments of monetary policymakers, this suggests the need for a link between monetary policy and disturbances in exchange markets. We develop below such a link based on standard model dynamics, without relying on special properties of the composite error $\eta_t + \rho_t$. First, however, we review the evidence on the forward premium puzzle in light of evidence that UIP may have worked better recently than it did in the past.

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[3] This relationship is approximate in that it ignores Jensen's inequality, which is likely to be small in practice and of uncertain sign in any event.

[4] Interest rates are defined as the log of 1 plus the yield.

[5] We use common terminology in calling this a test of UIP, although a narrower view would restrict this usage to the case where there is no risk premium in equation (II.5).

III. How Robust Is The Puzzle?

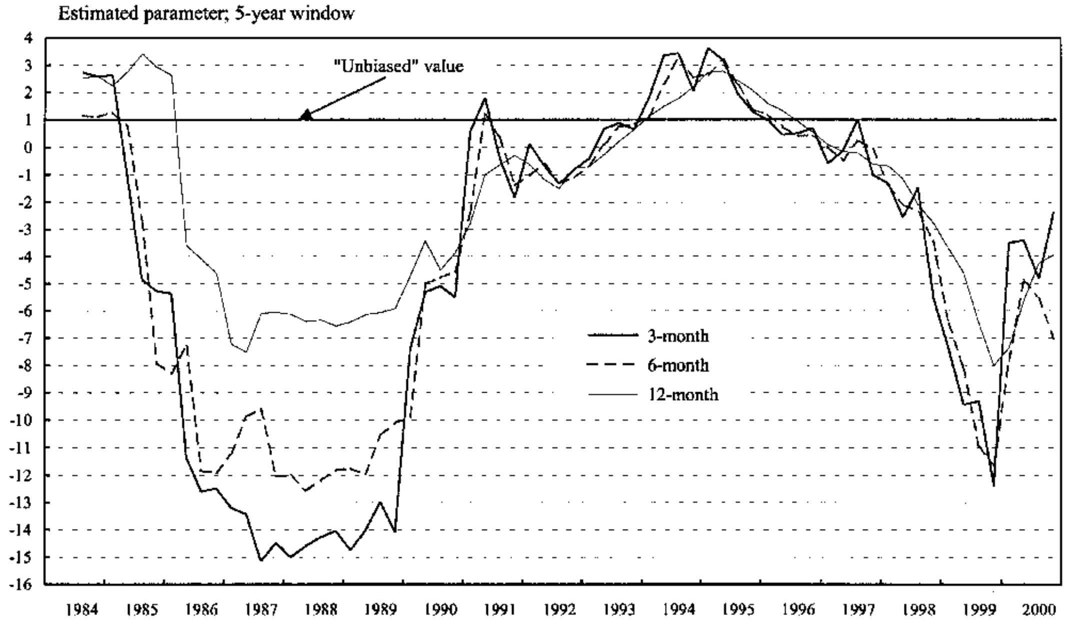
Studies from the 1970s and 1980s found overwhelming evidence for the forward premium puzzle. Of course, as with most stylized facts, such unanimity invites contradiction, or at least the identification of exceptions. In this vein, Flood and Rose (1996 and 2001) show that UIP has received more support during currency crises than non-crisis periods. Other studies indicate that the evidence is more favorable to UIP at longer horizons (Flood and Taylor (1997), Meredith and Chinn (1998), Alexius (1998), and Bleaney and Laxton (2001)). Furthermore, Bansal and Dahlquist (2000) find that UIP generally works better for emerging market than developed countries, a result also supported by Flood and Rose (2001).

More generally, Baillie and Bolerslev (2000) have questioned the apparent failure of UIP on statistical grounds. They observe that the slope parameter in 5-year rolling regressions using the deutschemark/dollar exchange rate exhibits substantial variation during 1978-1995, ranging from –13.0 to +3.5. They also find that the coefficients become consistently positive towards the end of their sample, suggesting that the puzzle may have disappeared in the 1990s. On econometric grounds, they argue that conventional standard errors overstate the precision of parameter estimates in UIP regressions, leading to unwarranted confidence in rejecting unbiasedness. Working with a model where the true value of the slope coefficient is unity, they show that the estimated standard errors of the coefficients are biased down due to autocorrelation in the forward premium, overstating the power of the usual tests.

Here, we present evidence that the forward discount bias remains robust in the post-1980s data, at least for the major currencies. While the distribution of the slope parameters in standard regressions may be wider than conventional statistics suggest, it seems unlikely that the consistent pattern of negative estimates simply reflects sampling error. To show this, we start with an update of the estimation results reported in Baillie and Bollerslev, with the sample period extended through end-2000 as opposed to end-1995. Figure 1 shows the evolution of the slope parameter from 5-year rolling regressions of equation (II.8) using the deutschemark/dollar rate as the dependent variable. These estimates confirm the volatility of the slope parameter observed by Baillie and Bollerslev. The parameters also become positive during 1993-96, and exceed the unbiased value of unity during 1994-95. Their general pattern is not particularly sensitive to the choice of a 3-, 6-, or 12-month regression horizon, although the shorter-horizon estimates do exhibit somewhat greater volatility. With the extended sample, though, it becomes apparent that the 1994-96 experience was short-lived. In the second half of the 1990s, the slope parameter returns to deep negative territory, typical of earlier estimates of the failure of unbiasedness.

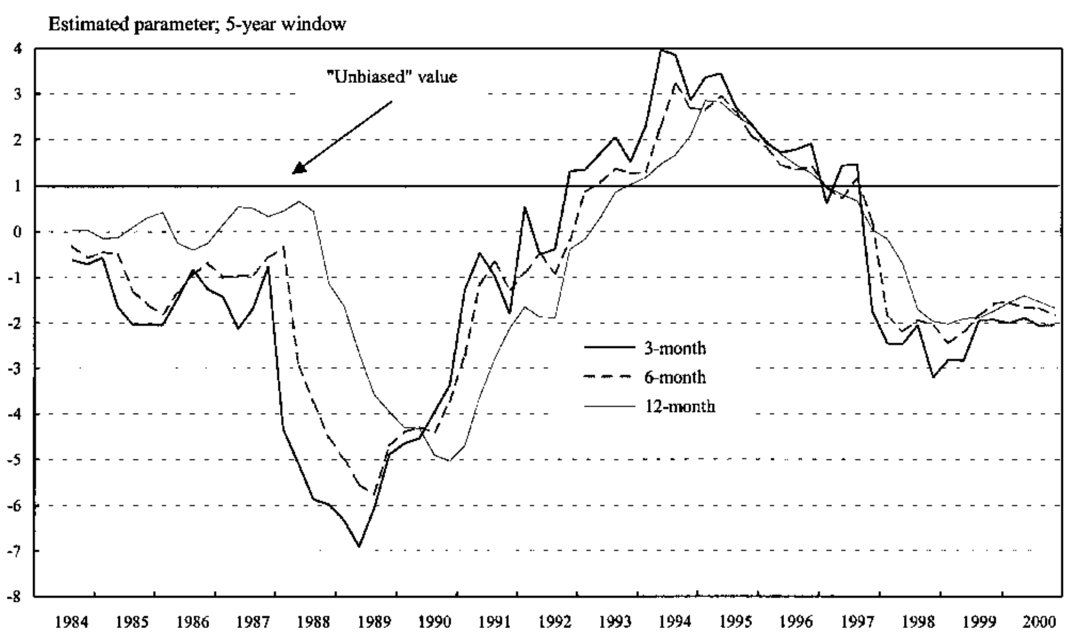
Extending this analysis to other major currencies, Figure 2 shows the coefficients from rolling regressions for the other five G-7 exchange rates against the dollar over the same sample period. The coefficients are obtained using the pooled estimator described in Meredith and Chinn (1998). They still exhibit significant volatility, with values for the 3-month regressions ranging from about –7 to +4, and also become positive in the mid-1990s. Again, however, the more recent data show that this episode was short-lived, and

Figure 1. Slope Parameters from Rolling UIP Regressions: US Dollar Numeraire; Deutschmark Only



Source: Staff calculation.

Figure 2. Slope Parameters from Rolling UIP Regressions: US Dollar Numeraire; Deutschmark Excluded



Source: Staff calculation.

from 1997-2000 the coefficients become negative, stabilizing at about –2 from 1997 on for all three regression horizons.

To examine the sensitivity of these results to the choice of the U.S. dollar as numeraire, Figure 3 shows the results of rolling regressions for the G-7 currencies (excluding the dollar) against the deutschemark. The pattern is now more stable, with the slope parameter never reaching unity and rarely exceeding zero. The range of roughly zero to –4 is typical of the results of the studies from the 1970s and 1980s. Figure 4 shows the same results but with the French franc and Italian lira excluded, as the introduction of the euro in 1999 eliminated these cross-rates with the deutschemark. The parameters are generally more volatile, and jump above unity at the end of the sample at the 3- and 6-month horizons. But the picture of the parameters being almost uniformly less than unity, and predominantly below zero, remains.

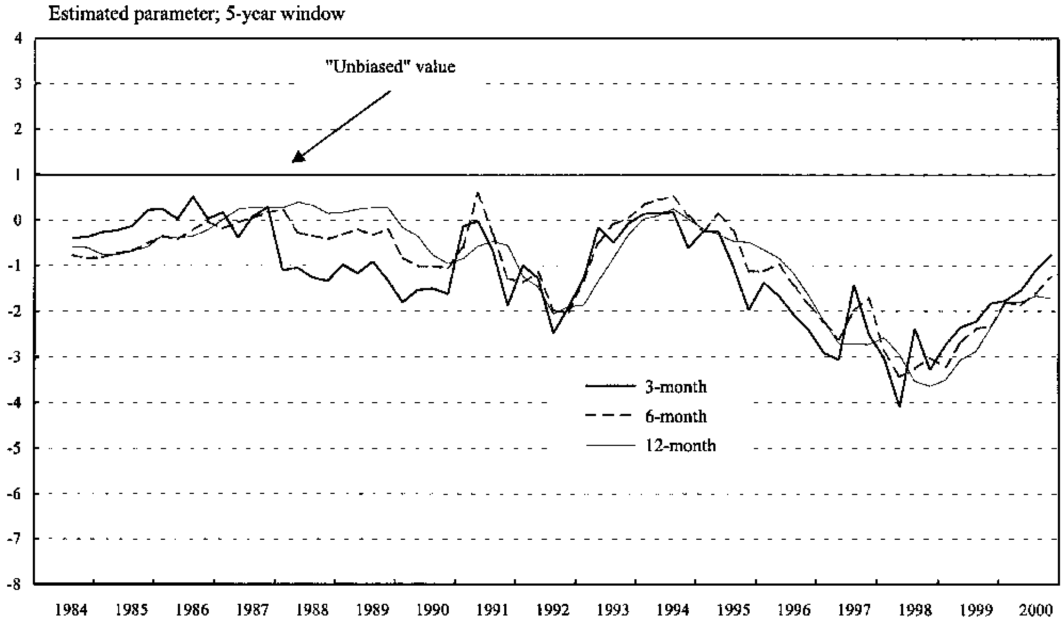
Taken together, Figures 1 to 4 provide little support for the view that the forward discount puzzle has faded in the more recent data, at least for the major currencies. This is consistent with the evidence in Meredith and Chinn (1998), and suggests that the findings from studies using data from the 1970s and 1980s did not simply reflect a transitional learning process in the initial stages of generalized floating exchange rate regimes.

Of course, these estimates only describe regularities in the data. Whether or not they point to a true underlying value of the slope parameter that differs from unity depends on whether the negative values result from sampling bias. As Baillie and Bollerslev point out, standard tests probably overstate the precision of the estimates. Absent a robust statistical means of determining the sampling distribution of the slope parameter, the plausibility that the results are due to sampling bias is inherently subjective. In our view, the likelihood that the negative estimates found both here and in many other studies are simply an accident of the data is implausible. This is especially true given that the models considered below generate a negative underlying relationship between exchange rate changes and lagged interest rates through conventional macro channels.

Before turning to the model-based results, though, we identify another interesting aspect of the parameter estimates—that they tend to be more negative in shorter than longer samples. This regularity is apparent in Table 1, which compares the average values of the coefficients from the rolling regressions using a five-year window with the values for the 20-year period as a whole. In all cases, the parameters based on 20-years of observations are less negative than the average of those from the rolling regressions, even though they are obtained using the same data. Indeed, in the case of the deutschemark regressions, the coefficients for the overall sample are slightly positive, while for the average of 5-year samples they are clustered around minus one.

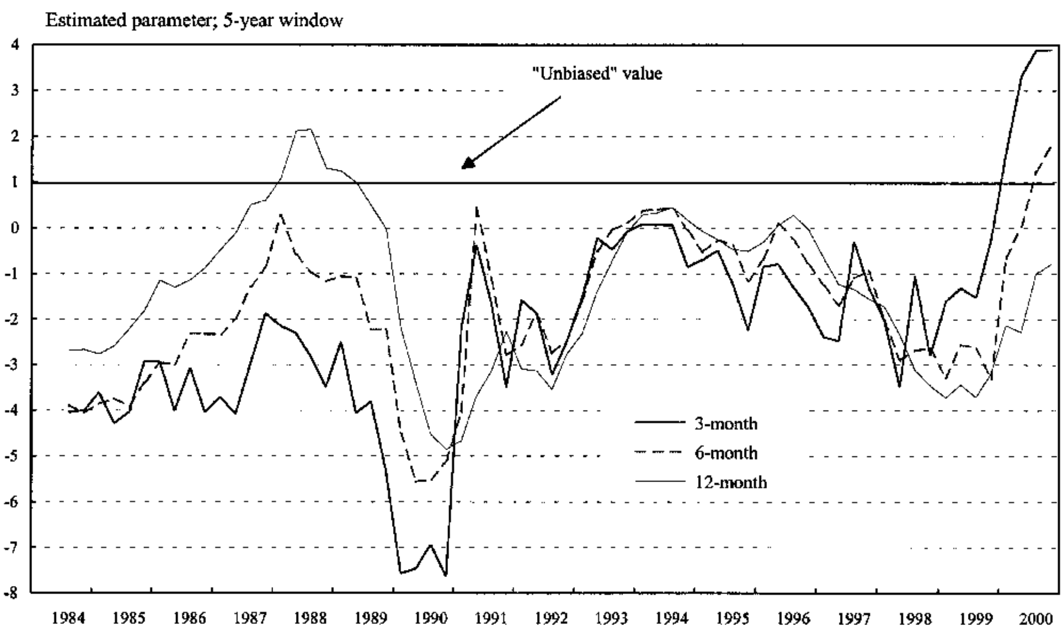
As will be discussed in Section V, this dependence of the parameter estimate on the sample length is also present in data generated by model simulations. It can be explained by the bias induced in OLS estimates using small samples by serial correlation between the lagged interest differential and the error term in the regression. Thus, the slope parameters

Figure 3. Slope Parameters from Rolling UIP Regressions: Deutschmark Numeraire; US Dollar Excluded



Source: Staff calculation.

Figure 4. Slope Parameters from Rolling UIP Regressions: Deutschmark Numeraire; US, FR and IT Excluded



Source: Staff calculation.

found in studies using relatively short sample periods may be more negative than the underlying population parameter for statistical reasons. While this would exaggerate the forward premium puzzle, it by no means explains it, as the asymptotic parameter is still well below unity.

Table 1. Regression Results Using Different Sample Lengths, 1984-2000

	5-year rolling regressions: average	Whole sample: 20 years
Versus U.S. dollar		
3-month	-1.09	-0.78
6-month	-0.91	-0.65
12-month	-0.58	-0.37
Versus deutschemark		
3-month	-1.14	0.14
6-month	-0.98	0.09
12-month	-0.99	0.09

Source: Authors' estimates.

IV. Model Solutions with Monetary Policy Endogeneity

We argued above that the perverse correlation between exchange rate changes and interest differentials remains in the more recent data, at least for the major currencies. Theoretically, however, the reason for the failure of unbiasedness remains controversial. While relaxing the joint assumption of risk neutrality and efficient markets allows time-*t* dated shocks to be introduced into equation (II.7), a framework is also required that generates the required positive correlation between these shocks and the interest differential.

The explanation pursued here involves the response of monetary policy to exchange market shocks. The theoretical framework for such a mechanism was first developed by McCallum (1994).[6] Meredith and Chinn (1998) extended this approach in a more general

[6] A similar mechanism is present in the model of Obstfeld and Rogoff (1998), although in the context of a different monetary policy framework. In their model, the nominal money supply is exogenous. Assuming the law of one price, nominal exchange rate shocks are immediately transmitted to the domestic price level, leading to an automatic interest-rate response via the money demand relationship.

setting to rationalize, not only the failure of UIP at short horizons, but also the evidence favoring UIP at longer horizons. Their results, however, were based on stochastic simulations that did not reveal the theoretical mechanism that switches the sign of the coefficient on interest differentials at short versus long horizons.

We extend this analysis by deriving closed-form solutions for the change in the exchange rate in two representative models. The first is simple enough to allow the derivation of an analytical solution, while the second is solved by numerical methods. In both cases, the parameter on the lagged interest rate is negative in the short run, but approaches unity at longer horizons. The explanation is the same: at short horizons, UIP is violated because monetary authorities respond to innovations in inflation and output, which in turn depend on exchange market shocks. This generalizes the result obtained by McCallum with an exchange-rate targeting rule. Over longer horizons, UIP is restored in both models, as monetary responses to exchange market shocks fade in relation to fundamentals that are "UIP-consistent."

A. Generalization of McCallum's (1994) Model

The first model is a generalization of McCallum (1994), where his exchange-rate targeting rule is replaced by one where interest rates respond to innovations in inflation and output. This brings the monetary reaction function in line with those conventionally used in macro models, addressing the critique that McCallum's specification was unrealistic (Mark and Wu (1996)). This extension requires explicitly modeling inflation and output. We use specifications that embody key elements of the linkages between financial variables, output, and prices, but leave the model simple enough to derive an analytic solution.

The first equation is a stochastic UIP condition, identical to McCallum (1994):

$$\Delta s^e_{t+1,t} = i_t - \omega_t \;, \tag{IV.1}$$

where $\Delta s^e_{t+1,t} = E(\Delta s_{t+1} \mid \Theta_t) = E(s_{t+1} - s_t \mid \Theta_t)$ is the conditional (rational) expectation of the change in the exchange rate given the information set Θ_t. The one-period interest rate is represented by i_t, and ω_t is a random shock, which we refer to generically as an "exchange market shock." It represents the sum of the risk premium, ρ_t, and the non-rational expectations error, η_t, defined above. There is no way to distinguish between these two shocks in the models considered in this section—the results are the same whichever interpretation is given.

The second equation is a Taylor-rule type of monetary reaction function (Taylor (1993)) with interest rate inertia:

$$i_t = \alpha_{ii}\, i_{t-1} + \alpha_{ip}(\pi_t + y_t) \;, \tag{IV.2}$$

where π_t is the deviation of inflation from target and y_t is the log deviation of output from a potential or trend level, and α_{ii} and α_{ip} are parameters. The same parameter is imposed on inflation and output for analytic convenience, although the specification could be generalized without altering the central result.

The following equations for inflation and output complete the model:

$$\pi_t = \alpha_{py} y_t + \alpha_{ps}(\Delta s_t - \pi_t) + \alpha_{pp}\pi_{t-1} + \nu_t \ , \tag{IV.3}$$

and

$$y_t = -\alpha_{yi}(i_t - \pi_t) + \varepsilon_t \ , \tag{IV.4}$$

where ν_t and ε_t are random shocks. Inflation depends on the output gap, the change in the real exchange rate (reflecting the role of imported prices), and lagged inflation. Setting α_{pp} to one implies a traditional "accelerationist" model of inflation. A more general specification would also include expectations of future inflation—the implications of this change are explored below; for the moment, we assume that expectations are anchored by the credibility of monetary policy. Output depends negatively on the real interest rate. One could also add the real exchange rate, but doing so would significantly complicate the analytical solution. Again, solutions to more general models are considered later. For the moment, it can be observed that having output respond to the real exchange rate would strengthen the link from exchange market shocks to interest rates, reinforcing the "anti-UIP" channels in the model.

Equations (IV.1) to (IV.4) represent a linear rational expectations model. A reduced-form solution exists in which the endogenous variables are expressed as linear functions of lagged endogenous variables and contemporaneous disturbances. To obtain the bubble-free minimal-state-variable (MSV) solution, we apply the undetermined-coefficients (UC) approach of McCallum (1983, 1998).[7] The procedure is described in the Annex; the solution can be expressed as:

$$\Delta s_t = \beta_{sp}\pi_{t-1} + \beta_{si} i_{t-1} + \Gamma_s u_t \tag{IV.5}$$

$$\pi_t = \beta_{pi} i_{t-1} + \Gamma_p u_t \tag{IV.6}$$

$$i_t = \beta_{ii} i_{t-1} + \Gamma_i u_t \tag{IV.7}$$

$$y_t = \beta_{yi} i_{t-1} + \Gamma_y u_t \ , \tag{IV.8}$$

[7] For alternative solution methods of linear models with rational expectations, see, for example, Blanchard and Kahn (1980) and Ma (1992).

where the β s are reduced-form parameters, and u_t is a vector of contemporaneous disturbances with the Γs as vectors of associated weights. As described in the Annex, the reduced-form parameters map into the structural parameters as follows:

$$\beta_{sp} = -\alpha_{pp}/\alpha_{ps}$$

$$\beta_{si} = \frac{z\_\{(1+\alpha_{ps})(1+\alpha_{ip}\alpha_{yi})+\alpha_{py}\alpha_{yi}(\alpha_{ip}-1)\}-\alpha_{ii}(1+\alpha_{ps}-\alpha_{py}\alpha_{yi})}{\alpha_{ps}\alpha_{ip}(1+\alpha_{yi})}$$

$$\beta_{pi} = \frac{z\_(1+\alpha_{ip}\alpha_{yi})-\alpha_{ii}}{\alpha_{ip}(1+\alpha_{yl})}$$

$$\beta_{ii} = z\_$$

$$\beta_{yi} = -\alpha_{yi}(\beta_{ii}+\beta_{pi})$$

where:

$$z\_ = \frac{-b-\sqrt{b^2-4ac}}{2a},$$

$$a = (1+\alpha_{ps})(1+\alpha_{ip}\alpha_{yi})+\alpha_{py}\alpha_{yi}(\alpha_{ip}-1)$$

$$b = -[\alpha_{ii}(1+\alpha_{ps}-\alpha_{py}\alpha_{yi})+\alpha_{ps}\alpha_{ip}(1+\alpha_{yi})+\alpha_{pp}(1+\alpha_{ip}\alpha_{iy})]$$

$$c = \alpha_{pp}\alpha_{ii} \; .$$

To be consistent with standard tests of unbiasedness, it would be necessary that $\beta_{sp}=0$ and $\beta_{si}=1$ in equation (IV.5). Yet plausible values for the model parameters yield quite different results. For example, setting $\alpha_{ii}=0.5$, $\alpha_{ip}=0.5$, $\alpha_{py}=0.25$, $\alpha_{ps}=0.1$, $\alpha_{pp}=0.6$, and $\alpha_{yi}=0.5$ yields $\beta_{si}=-0.31$ and $\beta_{sp}=-6.0$. The negative value for β_{si} is consistent with the "perverse" response of the exchange rate to the lagged interest differential, while the (large) negative parameter on the lagged inflation rate violates the hypothesis that no other information available at time t should explain future exchange rate changes.

In general, the derived values of the reduced-form parameters are sensitive to the choice of model parameters. Suppose, for instance, that inflation persistence is less that assumed above by changing α_{pp} to 0.1 from 0.6. Holding the other parameters unchanged implies values for β_{si} of –4.97 and β_{sp} of –1.0, reflecting a downward jump of an order of magnitude in the reduced-form coefficient on the interest differential, and a sharp drop in that on inflation. It would not be surprising, then, to find that the relationship between exchange rate changes and lagged interest differentials is not stable in actual data, but varies according to the economic environment.

Of course, β_{si} cannot be interpreted as the expected value of the slope coefficient in a traditional UIP regression, as such regressions do not include lagged inflation. The OLS estimator for the slope parameter in a traditional regression, $\hat{\beta}$, would be:[8]

$$\text{plim}(\hat{\beta}) = \beta_{si} + \beta_{sp}\,\text{cov}(i,\pi)/\text{var}(\pi) .$$

Since $\beta_{sp} < 0$, $\hat{\beta}$ could be greater or less than β_{si} depending on the sign of cov(i,π). The value cannot be pinned down more precisely without describing the stochastic properties of the model. We pursue this approach in section V. For the moment, the important result is that UIP does not hold in the short run, and the "correct" regression of the change in the exchange rate on the lagged interest differential and inflation would yield negative parameters on both variables.

These findings confirm that McCallum's explanation for the failure of UIP in the short run is robust to the choice of monetary reaction function. What does the model imply about the longer-horizon relationship between the exchange rate and lagged interest differentials? To see this, we solve for the average n-period change in the exchange rate, $\Delta_n s_t$, as a function of the interest rate on an n-period bond observed at time t–n, $i_{n,t-n}$:

$$\Delta_n s_t = \beta_{n,si}\, i_{n,t-n} , \tag{IV.9}$$

where:

$$\Delta_n s_t = \left(\sum_{j=t-n+1}^{t} \Delta s_j \right) / n , \tag{IV.10}$$

$$i_{n,t-n} = \left(i_{t-n} + \sum_{j=t-n+1}^{t-1} i^e_{j,t-n} \right) / n , \tag{IV.11}$$

and $i^e_{j,t-n}$ is the one-period interest rate at time j expected at time t-n. Thus, the n-period interest rate is defined in terms of current and expected future one-period interest rates. From the model solution, we have:

$$\begin{aligned} \Delta s_j &= \beta_{sp}\pi_{j-1} + \beta_{si}\, i_{j-1} + \xi_j \\ &= (\beta_{sp}\beta_{pi} + \beta_{si}\beta_{ii})\, i_{j-2} + \xi_{j-1,j} \\ &= \beta_{ii} i_{j-2} + \xi_{j-1,j} , \end{aligned} \tag{IV.12}$$

[8] See, for example, Greene (2000), pp. 334-5.

where the last equality uses the identity $\beta_{sp}\,\beta_{pi} + \beta_{si}\,\beta_{ii} = \beta_{ii}$ (see Annex), and $\xi_{j-k,j}$ represents a linear combination of the disturbances between periods *j-k* and *j*, the structure of which is not relevant to the following analysis. Equation (IV.12) implies:

$$\begin{aligned}\Delta s_j &= \beta_{sp}\pi_{t-n} + \beta_{si}\, i_{t-n} + \xi_j && \text{for } j = t-n+1\,, \\ &= \beta_{ii}^{j-t+n-1}\, i_{t-n} + \xi_{t-n+1,j} && \text{for } j = t-n+2 \text{ to } t\ .\end{aligned}$$

Substituting these expressions into (IV.10) gives:

$$\Delta_n s_t = (\beta_{sp}/n)\pi_{t-n} + \left(\beta_{si}/n + \sum_{j=t-n+1}^{t-1} \beta_{ii}^{j-t+n}/n\right) i_{t-n} + \xi_{t-n+1,t}\ . \tag{IV.13}$$

The n-period interest rate can also be expressed in terms of i_{t-n}:

$$i_{n,t-n} = \left(\sum_{j=t-n}^{t-1} \beta_{ii}^{j-t+n}/n\right) i_{t-n}\ .$$

Substituting this expression into (IV.13) gives:

$$\Delta_n s_t = (\beta_{sp}/n)\pi_{t-n} + ((\beta_{si}-1)/n)\, i_{t-n} + i_{n,t-n} + \xi_{t-n+1,t}\ . \tag{IV.14}$$

As n rises and the horizon of the exchange rate change lengthens, the parameters on both inflation and the one-period interest rate in (IV.14) approach zero. In contrast, that on the n-period bond yield is unity at all horizons. Consider the implications for a UIP regression of the n-period change in the exchange rate on the bond yield. At shorter horizons, the estimated coefficient will reflect the influence of the covariances between the bond yield, inflation, and the one-period interest rate along with their associated parameters in equation (IV.14)—this value could, in principle, be either greater or less than unity. As n increases, however, the coefficient on the bond yield will approach unity regardless of the parameters on inflation and the one-period interest rate and their covariances with the bond yield, so UIP holds in the long run.

B. A Numerical Model

Here we develop a closed-form solution to a model similar to that used in Meredith and Chinn (1998). It generalizes the model described above to incorporate richer dynamics and forward-looking behavior. In particular, inflation expectations enter the price equation, the real exchange rate enters the output equation, and a more general version of the Taylor rule is used. These extensions require that the solution be derived by numerical as opposed to analytical methods.

The exchange rate equation is identical to the UIP relationship (IV.1). The Taylor rule, however, is expressed in terms of the real as opposed to nominal short-term interest rate:

$$i_t - \pi_t = \alpha_{ip}\pi_t + \alpha_{iy}y_t + \alpha_{ii}(i_{t-1} - \pi_{t-1}) \ , \tag{IV.15}$$

where α_{ip} and α_{iy} have the standard values of 0.5. The interest rate response is smoothed via the presence of the lagged real interest rate, with a value for α_{ii} of 0.5. The inflation equation is similar to that used above, except that expected inflation now enters with a parameter equal to one minus that on lagged inflation:

$$\pi_t = \alpha_{py}y_t + \alpha_{ps}(\Delta s_t - \pi_t) + \alpha_{pp}\pi_{t-1} + (1-\alpha_{pp})\pi^e_{t+1,t} + \nu_t \ , \tag{IV.16}$$

where α_{py} is 0.25, α_{ps} is 0.10, and α_{pp} is 0.60 (the choice of parameter values is discussed in Meredith and Chinn). The output equation contains the real exchange rate and lagged output, in addition to the real interest rate; the latter is also redefined in terms of the long-term as opposed to short-term expected real rate:

$$y_t = -\alpha_{yi}(i^{l,e}_t - \pi^{l,e}_t) + \alpha_{ys}(s_t - p_t) + \alpha_{yy}y_{t-1} + \varepsilon_t \ , \tag{IV.17}$$

where α_{yi} is 0.5, α_{ys} is 0.1, and α_{yy} is 0.5.

The model is closed with identities that determine the long-term interest rate, expected inflation, the price level, and the spot exchange rate:

$$i^{l,e}_t = (i_t + i^e_{t+1,t} + i^e_{t+2,t} + i^e_{t+3,t} + i^e_{t+4,t})/5 \tag{IV.18}$$

$$\pi^{l,e}_t = (\pi_t + \pi^e_{t+1,t} + \pi^e_{t+2,t} + \pi^e_{t+3,t} + \pi^e_{t+4,t})/5 \tag{IV.19}$$

$$p_t = p_{t-1} + \pi_t \tag{IV.20}$$

$$s_t = s_{t-1} + \Delta s_t \ . \tag{IV.21}$$

A numerical solution for the reduced-form parameters can be obtained using the generalized Schur decomposition (McCallum (1998)), as described in the Annex. Given the values for the structural parameters, the reduced-form relationship for the change in the exchange rate is as follows:

$$\Delta s_t = -0.53i_{t-1} - 0.72y_{t-1} - 0.28\pi_{t-1} - 0.87(s_{t-1} - p_{t-1}) + \xi_t \tag{IV.22}$$

As in the case of the previous model, this equation is clearly not consistent with UIP: the parameter on the lagged interest rate is negative, and the change in the exchange rate depends on other variables known at time t-1, in addition to the orthogonal disturbance ξ_t .

Again, one cannot determine the estimated slope coefficient from a UIP regression without specifying the stochastic properties to tie down the covariances between the other variables and i_{t-1}. It could be larger or smaller than the value of –0.53 in (IV.22), but would only by coincidence be in the neighborhood of the unbiased value of unity.

The longer-horizon relationship between exchange rates and lagged variables in this model can also be evaluated. Using a procedure analogous to that in the previous section (see Annex), the n-period average change in the exchange rate can be expressed as:

$$\Delta_n s_t = (\lambda_{sy,n}/n)y_{t-n} + (\lambda_{sp,n}/n)\pi_{t-n} + (\lambda_{ss,n}/n)(s_{t-n} - p_{t-n}) + (\lambda_{si,n}/n)i_{t-n} + \xi_{t-n+1,t} \quad . \tag{IV.23}$$

Similarly, the n-period bond yield at time t-n can be expressed as:

$$i_{n,t-n} = (\lambda_{iy,n}/n)y_{t-n} + (\lambda_{ip,n}/n)\pi_{t-n} + (\lambda_{is,n}/n)(s_{t-n} - p_{t-n}) + (\lambda_{ii,n}/n)i_{t-n} \quad . \tag{IV.24}$$

Subtracting (IV.24) from (IV.23) and taking $i_{n,t-n}$ to the right-hand side gives:

$$\begin{aligned} \Delta_n s_t = {} & ((\lambda_{sy,n} - \lambda_{iy,n})/n)y_{t-n} + ((\lambda_{sp,n} - \lambda_{ip,n})/n)\pi_{t-n} + ((\lambda_{ss,n} - \lambda_{is,n})/n)(s_{t-n} - p_{t-n}) \\ & + ((\lambda_{si,n} - \lambda_{ii,n})/n)i_{t-n} + i_{n,t-n} + \xi_{t-n+1,t} \quad . \end{aligned} \tag{IV.25}$$

As in the case of the previous model, the parameters on the n-period lags on the endogenous variables asymptote to zero as n rises, while that on the n-period bond yield is unity at all horizons. Again, long-horizon UIP will tend to hold regardless of the structural parameters.

To illustrate this property, Table 2 provides numerical values for the parameters at different horizons ranging from 1 to 100 periods. In general, the parameters quickly decay from their one-period values in inverse proportion to the length of the horizon. The only exception is that on the one-period interest rate, which temporarily rises when the bond yield is introduced but then decays in the same way. It is apparent that after several periods the parameters on the other variables become insignificant compared with that on the bond yield. While, again, tying down the predicted coefficient in a short-horizon UIP regression requires a stochastic framework, these results indicate that UIP would hold at longer horizons regardless of the structure of the model disturbances.

Table 2. Parameters in the Long-Horizon Exchange Rate Equation

Time horizon	Parameters on variables:				
n	y_{t-n}	π_{t-n}	$s_{t-n} - p_{t-n}$	i_{t-n}	$i_{n,t-n}$
1	-0.720	-0.280	-0.435	-0.530	n.a.
2	-0.362	-0.138	-0.217	-0.767	1
3	-0.241	-0.092	-0.144	-0.511	1
4	-0.181	-0.069	-0.108	-0.384	1
5	-0.145	-0.055	-0.087	-0.307	1
10	-0.072	-0.028	-0.044	-0.153	1
20	-0.036	-0.014	-0.022	-0.077	1
50	-0.015	-0.006	-0.017	-0.031	1
100	-0.007	-0.003	-0.009	-0.015	1

Source: Authors' calculations.

V. A Stochastic Model

The previous section analyzed the deterministic relationship between exchange rate changes and lagged interest rates in two representative models. These results indicated that, at short horizons, the correct relationship for future exchange rate changes differs from that implied by the standard UIP regression. They cannot, however, tie down the parameter on the interest differential in such incorrectly specified regressions, which will depend on the stochastic properties of the model and shocks. Here, we generalize the analysis in this direction. This has two benefits: the hypothetical UIP parameter can be calculated based on the assumed disturbance structure for the model; and models can be considered where the lagged interest rate does not enter the reduced-form solution. It is shown that standard UIP regressions based on data generated by such models yield negative coefficients. Furthermore, the absolute value of the negative parameter increases as the sample size of the regression diminishes, reinforcing the bias. Finally, as found earlier, long-horizon regressions are supportive of UIP.

A. Model Structure

The model used here is similar to McCallum and Nelson's (1999) open-economy framework, the main difference being that it is parameterized at an annual as opposed to quarterly frequency.[9] The inflation, output, and interest-rate equations for a single country are:

$$\pi_t = \alpha_{pp}\pi_{t-1} + (1-\alpha_{pp})\pi^e_{t+1,t} + \alpha_{py}y_t + \alpha_{ps}(\Delta s_t - \pi_t) + \nu_t \tag{V.1}$$

$$y_t = \alpha_{ys}(\Delta s^e_{t+1,t} - \pi^e_{t+1,t}) + \alpha_{yi}(i_t - \pi^e_{t+1,t} - \gamma(\Delta s^e_{t+1,t} - \pi^e_{t+1,t})) + y^e_{t+1,t} + \varepsilon_t \tag{V.2}$$

$$i_t = \alpha_{ip}\pi_t + \alpha_{iy}y_t + \psi_t \tag{V.3}$$

where the notation follows that used in the previous section. The inflation equation has forward- and backward-looking elements, with relative weights determined by α_{pp} ; this is set to 0.6 in the baseline parameterization. The parameter on the output gap, α_{py}, is set to 0.2. The change in the real exchange rate is included to capture the transmission of higher imported prices to broader cost measures, such as wages, and thus output prices. The associated parameter, α_{ps} reflects three factors: the passthrough of exchange rate changes to import prices, the share of imports in domestic spending, and the responsiveness of other costs to changes in the deflator for total spending. We initially assume a passthrough coefficient of 0.5, an import share of 0.20, and an elasticity of other costs with respect to the spending deflator of 0.5. This implies a value for α_{ps} of 0.05.

The output equation is derived from the intertemporal Euler relationship, where α_{yi} is the intertemporal elasticity of substitution, initially set to 0.25; γ is the share of imports in spending, or 0.20.[10] The parameter on the real exchange rate, α_{ys}, represents the passthrough coefficient multiplied by the share of imports in spending and the price elasticity of demand of imports; the first two are 0.5 and 0.20 respectively, while the price elasticity is assumed to be -1, giving a value for α_{ys} of -0.10. The parameters in the monetary reaction function are set to the standard Taylor-rule values of $\alpha_{ip} = 1.5$ and $\alpha_{iy} = 0.5$.

Equations (V.1) to (V.3) apply to a single "home" country. Similar relationships apply to the foreign country. Assuming that the structure of the two economies is the same, the only difference will be that the signs of the parameters on the exchange rate will be reversed (keeping the definition of the exchange rate as the domestic price of foreign currency). Subtracting equations (V.1) to (V.3) for the foreign country from those for the home country allows the model to be expressed in terms of differences between domestic and foreign values for y_t, i_t, and π_t. The resulting model is identical to equations (V.1) to (V.3), except

[9] The model is also similar to that used by Clarida, Gali, and Gertler (2001), with the principal difference that this model incorporates inflation inertia and an effect of the exchange rate on output prices.

[10] The nominal interest rate is deflated by the change in the deflator for domestic spending, where the latter is defined as $\pi_t + \gamma(\Delta s_t - \pi_t)$.

that α_{ys} and α_{ps} are twice as large as their original values, because changes in the exchange rate drive a symmetric wedge between home and foreign output and prices.

The exchange rate equation is again based on UIP with a random shock, although we now make the foreign interest rate explicit:

$$\Delta s^{e}_{t+1,t} = (i_t - i^{f}_t) - \omega_t \tag{V.4}$$

In the models discussed in section IV, ω_t could equivalently be interpreted as a risk premium shock or a non-rational expectations error. Because the output equation in this model contains the expected exchange rate, it becomes necessary to distinguish between the two. If ω_t represents a risk premium, then the exchange rate change in (V.4) is consistent with that in (V.2), and no modifications are needed to the model. If it instead represents a non-rational expectations error, ω_t must be added to $\Delta s^{e}_{t+1,t}$ in (V.2) so that output depends on the non-rational expectation instead of the model-consistent value. The implications of both interpretations are considered below.

The terms ν_t , ε_t , ψ_t ,and ω_t are disturbances to prices, output, the interest rate, and the exchange rate respectively. With the model expressed in differences between the two countries, the first three disturbances represent the difference between the shocks to the home and foreign country. All four shocks are assumed to be independent, normally-distributed, white-noise processes with variances σ^2_ν, σ^2_ε, σ^2_ψ ,and σ^2_ω respectively. The variances are calibrated so that the model replicates the volatility of actual data for the G-7 countries, including that in the exchange rate, as discussed below.

B. Solution Technique

To derive the reduced-form solution for the model, we use the method of Bullard and Mitra (2001), which also yields McCallum's minimum-state-variable solution. To illustrate the technique, it is convenient to express the model in a general form as:

$$Y_t = BY_t + CY^{e}_{t+1,t} + DY_{t-1} + EZ_t \ , \tag{V.5}$$

where Y is a vector of endogenous variables, Z is a vector of disturbances, and B, C, D, and E are matrices of parameters (many of which are zero). Eliminating Y_t from the right-hand side, the model can be simplified to:

$$\begin{aligned} Y_t &= (I-B)^{-1}CY^{e}_{t+1,t} + (I-B)^{-1}DY_{t-1} + (I-B)^{-1}EZ_t \ , \\ &= C'Y^{e}_{t+1,t} + D'Y_{t-1} + E'Z_t \end{aligned} \tag{V.6}$$

Conjecture that a solution for Y_t can be found in terms of Y_{t-1} and Z_t, where the coefficient matrices F and G are (as yet) unknown:

$$Y_t = FY_{t-1} + GZ_t \; . \tag{V.7}$$

Based on this conjectural solution, $Y^e_{t+1,t}$ can be expressed as:

$$\begin{aligned} Y^e_{t+1,t} &= FY_t \\ &= F(FY_{t-1} + GZ_t) \end{aligned} \tag{V.8}$$

Substituting this expression into (V.6) and collecting terms gives:

$$Y_t = (C'FF + D')Y_{t-1} + (FG + E')Z_t \tag{V.9}$$

The coefficient matrices in this equation must equal those in (V.7) to be consistent with the conjectural solution, implying the following relationships:

$$\begin{aligned} F &= C'FF + D' \\ G &= FG + E' \end{aligned} \tag{V.10}$$

Equations (V.10) are a nonlinear system that can be solved numerically for the elements of F and G as functions of C', D', and E'. A unique solution can be found for well-behaved models using standard algorithms, yielding a model in form (V.7)[11] This model will generate identical simulation results to (V.5) for given values of the disturbances.

C. Moments of the Simulated Data

Using the reduced-form solution, it is straightforward to derive the asymptotic moments of the model data. The variance-covariance matrix of the endogenous variables is:

$$plim(Y_t\, Y_t') = F\, plim(Y_{t-1} Y_{t-1}')F' + G\, plim(Z_t\, Z_t')G' \, , \tag{V.11}$$

as the orthogonality of Y_{t-1} and Z_t implies that the *plim* of their product is zero. Expressed in more concise notation, and observing that:

$$plim(Y_t\, Y_t') = plim(Y_{t-1} Y_{t-1}') \, ,$$

(V.11) becomes:

$$\Sigma^Y = F\Sigma^Y F' + G\Sigma^Z G' \; , \tag{V.12}$$

[11] The nonlinear solutions are obtained using the FindRoot routine in Mathematica 4.1.

where $\Sigma^Y = plim(Y_t Y_t')$ and $\Sigma^Z = plim(Z_t Z_t')$. This nonlinear system can be solved for the elements of Σ^Y in terms of the values of F, G, and Σ^Z (the latter being given by the assumed disturbance variances).

The standard deviations of the disturbances were selected to replicate the volatility in the endogenous variables using historical data for the G-7 countries (Table 3). In practice, this was achieved by endogenizing the associated elements on the diagonal of Σ^Y, while the corresponding elements in Σ^Z were endogenized. With the disturbance variances tied down in this way, initial simulations indicated that autocorrelations in the interest rate differential and the exchange rate differed from those observed in the actual data. As shown in Table 3, the interest rate differential exhibits strong serial correlation, while exchange rate changes are roughly uncorrelated. With a white-noise disturbance in the UIP relationship, however, the model-generated data exhibit only moderate serial correlation in the interest differential, and negative autocorrelation in exchange rate changes.

Table 3. Moments of G-7 Data[12]
(Annual data; 1980-2000 averages)

Standard deviations (percentage points)	
GDP gap	2.6
CPI inflation	2.3
Short-term interest rate	3.7
Exchange rate change	11.5
First-order autocorrelation	
Short-term interest rate	0.60
Exchange rate change	0.13

Source: IMF *World Economic Outlook* database.

To reduce this discrepancy, a first-order moving average process was assumed for the exchange-rate disturbance:

[12] The U.S. is used as the base country. The GDP gap represents the logarithmic ratio of actual to potential GDP in each of the other G-7 countries less that in the U.S.; CPI inflation is the difference between the log change in the CPI in the other countries and that in the U.S.; the short-term interest rate is the difference between 12-month euro-market interest rates; and the exchange rate is the log change in the bilateral rate versus the U.S. dollar. The data for actual and potential GDP and CPI inflation are obtained from the IMF's *World Economic Outlook* database.

$$\omega_t = \vartheta_t + \vartheta_{t-1} \tag{V.13}$$

where ϑ_t is white noise. With this change, the model generated an autocorrelation coefficient for the interest differential of 0.41, and for the change in the exchange rate of -0.11. While the former is still somewhat below the 0.60 value in the actual data, the difference is substantially reduced compared with the results without a moving-average disturbance. Perhaps more importantly, the difference between the serial correlation coefficients for the interest rate and the change in the exchange rate is very close to that in the actual data. While the assumption of a moving average exchange market disturbance is, of course, ad hoc, there does not appear to be a strong reason for constraining this process, given that its nature remains unresolved in the theoretical literature. Similar results can be obtained using other processes, such as an autoregressive process—indeed, the results for UIP regressions are little affected even if the original white-noise specification is retained, as shown below

Based on the MA(1) process for the exchange-rate disturbance and the standard deviations of the endogenous variables shown in Table 3, the standard deviations of the disturbances were calculated as following (in percentage points):

σ_ε (output)	2.87
σ_v (inflation)	0.95
σ_φ (interest rate)	1.56
σ_ϑ (exchange rate)	5.26

It is apparent that (by far) the largest shock is to the exchange rate equation, with an implied value for σ_ω of 7.44 (i.e. $\sqrt{2}\,\sigma_\vartheta$). This is consistent with the earlier results in Meredith and Chinn (1998), and highlights the fact that the observed volatility in exchange rates is associated with large shocks in the exchange market.

D. UIP Regressions

It is straightforward to calculate moments for transformations of the model data using the above approach. For instance, the matrix of covariances between Y_t and its first lag is:

$$plim(Y_t\ Y'_{t-1}) = F\Sigma^Y \tag{V.14}$$

The element of the matrix on the right-hand side of (V.14) corresponding to the covariance between the change in the exchange rate and the lagged level of the nominal interest rate is of particular interest, as it represents the numerator of the (asymptotic) slope coefficient in the standard UIP regression. The denominator is the element of Σ^Y corresponding to the interest rate variance. Similar expressions can be derived for the coefficients in UIP regressions for longer horizons.

Under the baseline parameters, and with ω_t interpreted as a risk premium shock, the asymptotic value of the slope parameter in the standard UIP regression is -0.17, consistent with the failure of the unbiasedness hypothesis; the associated R^2 is 0.002, similar to the low values found in actual studies. Nevertheless, the slope parameter is closer to zero than the value of about -0.8 found in typical regressions, such as those described above, and the simulation value of -0.5 obtained by Meredith and Chinn (1998). This difference can be mostly attributed to differences between the asymptotic and finite-sample moments of the data. In particular, estimates of β are biased in repeated draws of the disturbances depending on the length of the assumed sample, as shown in Table 4.[13]

Table 4. Value of Slope Parameter for Different Sample Sizes

Sample size (years)	$\hat{\beta}$
10	-0.52
20	-0.39
50	-0.25
Asymptotic	-0.17

Source: Authors' estimates.

It is apparent that the parameter rises in absolute value as the sample size decreases, approaching levels typical of those found in actual studies. At the same time, it is difficult to make a direct comparison in terms of sample sizes, because most studies have used relatively high-frequency data over short samples of a few years, as opposed to the annual observations used here. To precisely assess the bias due to finite sample sizes for such studies would require parameterizing our model at a higher frequency.

The small-sample bias in the OLS estimator is worth further discussion. It arises from the properties of the joint distribution of the lagged interest rate and the disturbance term. The issue can be seen by rewriting the reduced-form model in equation (V.7) in a final form where the lagged values of the endogenous variables are replaced by the lagged disturbances:

$$Y_t = GZ_t + FGZ_{t-1} + FFGZ_{t-2} + \dots F^n GZ_{t-n} \dots \qquad \text{(V.15)}$$

[13] Table 4 is based on a population of 50,000 observations generated by stochastic simulation of the model in form (V.7), using a random number generator to draw the disturbances. The population was divided into different sample lengths, and the mean value of the coefficients obtained from the resulting OLS regressions are shown here.

The endogenous variables can thus be expressed as infinite moving averages of the structural disturbances. Furthermore, the disturbance term in a UIP regression is a transformation of the Y vector that follows a process analogous to (V.15). Because both the lagged interest rate and the disturbance are moving averages of the same innovations represented by the Z vector, they tend to be highly correlated—both contemporaneously and across periods. The contemporaneous correlation, of course, means that the OLS estimate of β is not consistent; the inter-temporal correlations also imply that the bias is dependent on the sample size.

Regarding the implications for long-horizon UIP, Table 4 shows the asymptotic parameters from regressions out to a five-year horizon. The parameter rises to 0.66 at this horizon, consistent with the findings in recent studies that unbiasedness holds better using longer-horizon data. The rise in the parameter is not monotonic as the horizon lengthens, as there is a slight decline at the two year horizon due to the effect of the two-period moving average disturbance in the exchange rate equation.

Table 5. Asymptotic Slope Parameter in UIP Regressions at Different Horizons

Horizon in years	Asymptotic $\hat{\beta}$
1	-0.17
2	-0.19
3	0.28
4	0.52
5	0.66

Source: Authors' estimates.

E. "Correct" Exchange Rate Regression

The standard UIP regression is misspecified in the context of this model, as it was for those in Section IV, because the reduced-form relationship for the change in the exchange rate involves variables other than the interest differential. Here, the rational expectation for the change in the exchange rate is given by the elements in the row of the F matrix corresponding to the change in the exchange rate times the corresponding lagged endogenous variables. The actual change in the exchange rate will equal this expectation plus a white-noise error term consisting of a weighted sum of the time-t disturbances.

Under the baseline model parameters, the reduced-form relationship is:

$$\Delta s_t = -0.93(s_{t-1} - p_{t-1}) - 0.42\pi_{t-1} + 0.93\omega_{t-1} + \varepsilon_t \ . \qquad \text{(V.16)}$$

The change in the exchange rate depends negatively on the lagged real exchange rate and the inflation rate, and positively on the lagged exchange-market disturbance. Unlike the models described earlier, this solution does not contain the lagged interest rate. The standard deviation of ε_t is 9.62, implying that the R-squared of a regression based on (V.16)would be 0.30. In other words, 30 percent of the variance of the change in the exchange rate would be explained by lagged information, and 70 percent by contemporaneous shocks.

Of course, equation (V.16) is not operational as a regression, as it includes the unobserved value of the lagged exchange-market disturbance. Using the known correlation between this disturbance and the observable variables, however, it is possible to calculate the asymptotic parameters for an OLS regression of the change in the exchange rate on all information observable at *t-1*. This is:

$$\Delta s_t = -0.59(s_{t-1} - p_{t-1}) + 0.38 i_{t-1} - 0.05 y_{t-1} - 0.12 \pi_{t-1} + \varepsilon_t' . \qquad \text{(V.17)}$$

Compared with the "true" reduced-form expression (V.16), the lagged interest rate now enters with a positive parameter; the output gap also enters with a (small) negative parameter. The parameters on the lagged real exchange rate and inflation are somewhat lower in absolute value than in (V.16).The standard deviation of the error term in this equation is 9.89, implying an R-squared of 0.26. This close to the R^2 of the true reduced-form relationship of 0.30, and much higher than that in the standard UIP regression of 0.002.

The implication of (V.17) — which also holds for the models considered in section IV — is that there is a conceptual case of including a variety of lagged macroeconomic variables, in addition to interest rates, in exchange rate regressions. Doing so, in the context of this model, results in a substantial improvement in goodness-of-fit, with the R^2 rising from about zero to one quarter. Nevertheless, about three quarters of the variance in exchange rate changes remains inherently unpredictable. Thus, while regressions of this type have often been regarded with suspicion by practitioners, there is a conceptual case for adopting an eclectic approach for forecasting purposes. Of course, there is also a danger of over-fitting such regressions when the true structural model is unknown. In the context of a more general model than the one considered here, it is likely that a wide range of variables would enter (V.17). Given limited sample lengths, and changes in economic behavior over time, it may well be difficult to confidently estimate generalized versions of (V.17) for forecasting purposes.

F. Alternative Model Parameters

The above results have been obtained under the baseline model calibration, assuming that the exchange-market disturbance is interpreted as a risk premium. It is useful to explore the implications for UIP regressions of varying some of the key parameters, and interpreting the disturbance as an expectations error. Table 6 summarizes the implications of alternative cases for the implied coefficients on interest differentials in 1-year and 5-year regressions.[14]

Table 6. Alternative Model Parameters

Case	Model parameters	UIP slope coefficients 1-year	5-year
Baseline		-0.17	0.66
High substitution elasticities	$\sigma = 0.5,\ \varepsilon = 2.0$	-0.58	-0.07
Low substitution elasticities	$\sigma = 0.1,\ \varepsilon = 0.5$	0.07	0.92
High passthrough	$\alpha_{ps} = 0.075,\ \alpha_{ys} = 0.15$	-1.02	-0.01
Low passthrough	$\alpha_{ps} = 0.025,\ \alpha_{ys} = 0.05$	0.48	0.90
Greater trade openness	trade share = 0.25	-0.56	0.42
Less trade openness	trade share = 0.15	0.18	0.80
High inflation inertia	$\alpha_{pp} = 1$	-0.26	0.44
No inflation inertia	$\alpha_{pp} = 0$	0.73	1.32
White-noise ω_t	$\omega_t = \vartheta_t$	-0.09	0.90
ω_t as expectations error		0.21	0.89

In each case, the 1-year slope parameter is below unity. Furthermore, the parameter in the long-horizon regressions is always well above that in the 1-year regressions. Increasing the elasticities of substitution tends to increase the bias in the coefficient, because it amplifies the effect of exchange rate changes on output and prices, and thus the feedback on interest rates. Similarly, raising the passthrough of exchange rates to prices or the share of trade in the economy magnifies the bias. Increasing inflation inertia raises the bias because the impact of exchange rate changes on inflation is propagated over a longer time, raising the effect on interest rates. Assuming a white-noise exchange-market disturbance as opposed to an MA(1) process has little effect on the 1-year coefficient, but does bring that in the long-horizon regression closer to unity. When the disturbance is interpreted as an expectations error as

[14] The standard deviations of the endogenous variables described in Table 3 were held constant in these exercises, while those of the associated disturbances adjusted to remain consistent with the observed data.

opposed to a risk premium, the short-horizon bias is reduced somewhat, because the disturbance no longer affects the expected exchange rate change in the output equation.

It is apparent that there is substantial heterogeneity in the UIP coefficients — particularly at short horizons — for different choices of structural parameters and disturbance processes. A plausible conclusion might be that it is unlikely that the estimated parameter in actual tests of UIP would be highly stable in practice, given changes in economic structure over time and differences across countries.

VI. Concluding Remarks

This paper has explored model-based explanations for the bias in the forward premium as a predictor of exchange rate movements, supporting explanation based on monetary policy endogeneity. Shocks that cause the current exchange rate to depreciate also cause output and prices to rise, leading to higher interest rates. The subsequent reversal of these shocks is associated with exchange rate appreciation, explaining the perverse relationship between lagged interest rates and ex post exchange rate movements.

Over longer horizons, however, the bias fades, as the short-run correlation due to policy endogeneity declines relative to longer-term model dynamics that are consistent with UIP. The empirical finding that UIP holds better over longer horizons could, of course, simply reflect the fact that inflation differentials between countries dominate nominal exchange rate movements over time. Yet this channel is not important in the theoretical models considered above, as inflation rates are tied down to a common level across countries via the same monetary reaction function. Interestingly, and consistent with this result, Bleaney and Laxton (2002) find empirical support for "real" UIP over longer horizons, suggesting that it is not simply a nominal phenomenon caused by the cumulative effect of inflation differentials.

The finding that monetary policy endogeneity can explain the forward premium puzzle should not, in a sense, be surprising. The puzzle requires a correlation between interest rates and exchange market shocks. Short-term interest rates are the operating instrument of monetary policy. If monetary policy is not influenced, directly or indirectly, by exchange market shocks, then interest rates would be exogenous to such shocks. The absence of a correlation between the two suggests that interest rates should be unbiased predictors of exchange rate movements. Of course, there is also the logical possibility that interest rates are exogenous to exchange market shocks, but that the causation runs in the opposite direction—from interest rates to either risk premia or non-rational expectations errors. While theoretical stories could no doubt be built to support such a channel of influence, consistency with the observed data would require that the exchange-market shocks induced by interest rate movements be much larger than the interest rate movements themselves. More generally, parsimony argues against models that require additional theoretical superstructure to explain the forward premium puzzle, especially when the underlying theory is likely to be difficult to test given data limitations.

Another implication of the model results is that "eclectic" exchange rate regressions have a theoretical basis, and can substantially improve predictive power relative to simple UIP specifications. An important caveat to the use of such regression, though, is that the parameters are likely to be sensitive to changes in the structure and coefficients of the underlying structural model. Analysis using different coefficients in the theoretical models suggests that this can have a large impact on estimates of UIP regressions. For this reason, it is not surprising that empirical tests do not point to a well-defined, stable value for the parameter in traditional UIP regressions.

The analysis also sheds light on the important question of whether UIP is a useful characterization of exchange rate movements in theoretical models. As Flood and Rose (2001) note, UIP is both " . . . a critical building block of most theoretical models and a dismal empirical failure." Our results indicate that UIP may be more appropriate theoretically than the empirical failure would suggest. Standard unbiasedness regressions are misspecified in the context of structural models that incorporate UIP. Indeed, all three of the models we examine embody UIP as a structural relationship with an exogenous, white-noise error term, but generate results that imply biased coefficients in standard regressions. Thus, running such regressions is not an appropriate test of the validity of the structural model—it confuses the properties of the structural specification with its reduced-form stochastic behavior.

Finally, our analysis assumes exogenous shocks in exchange markets. It does not address the question of what could generate sufficient volatility in risk premia and/or non-rational expectations errors to explain the stylized facts. Theoretical progress in this area has been limited, although recent work in the area of market dynamics and multiple equilibria such as Jeanne and Rose (2002) suggests mechanisms that could generate more volatility than earlier models.[15] In any event, the observed behavior of floating exchange rates requires larger shocks than traditional models can easily explain. We view the question of what generates these shocks as being separate from that of what explains the forward premium puzzle itself in the presence of such shocks.

[15] See Macklem (1991), for a discussion of the limited role of the risk premium in a stylized theoretical environment.

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Derivation of Solutions to the Models in Section IV

1. Analytical solution to the extended McCallum (1994) model

Define $k_t = (\pi_{t-1}, i_{t-1})'$ as a 2x1 vector of predetermined variables, and $u_t = (\omega_t, \nu_t, \varepsilon_t)'$ as a 3x1 vector of random variables; Δs_t is a non-predetermined endogenous variable. Rewrite (IV.1) to (IV.4) in the main text as follows:

$$\Delta s^e_{t+1} = M_1 k_{t+1} + M_4 u_t \tag{A.1}$$

$$k_{t+1} = P_0 \Delta s_t + P_1 k_{t+1} + P_2 k_t + P_3 y_t + P_4 u_t \tag{A.2}$$

$$y_t = N_1 k_{t+1} + N_4 u_t \tag{A.3}$$

where $M_1 = (0,1)$, $M_4 = (-1,0,0)'$, $P_0 = (\alpha_{ps}, 0)'$,

$$P_1 = \begin{bmatrix} -\alpha_{ps} & 0 \\ \alpha_{ip} & 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} \alpha_{pp} & 0 \\ 0 & \alpha_{ii} \end{bmatrix}, \quad P_3 = \begin{bmatrix} \alpha_{py} \\ \alpha_{ip} \end{bmatrix}, \quad P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$N_1 = (\alpha_{yi}, -\alpha_{yi})$, and $N_4 = (0,0,1)$. Equations (A.1) and (A.3) correspond to (IV.1) and (IV.4) in the main text, respectively, while (A.2) is the compact form of (IV.2) and (IV.3).

Substituting (A.3) into (A.2) gives:

$$k_{t+1} = B_{21}\Delta s_t + B_{22}k_t + C_2 u_t\,, \tag{A.4}$$

where:

$$\begin{aligned} B_{21} &= (I - P_1 - P_3 N_1)^{-1} P_0 \\ B_{22} &= (I - P_1 - P_3 N_1)^{-1} P_2 \\ C_2 &= (I - P_1 - P_3 N_1)^{-1} (P_4 + P_3 N_4)\ . \end{aligned}$$

Substituting (A.4) into (A.1) gives:

$$\Delta s^e_{t+1} = B_{11}\Delta s_t + B_{12} k_t + C_1 u_t\ , \tag{A.5}$$

where:

$$\begin{aligned} B_{11} &= M_1 B_{21} \\ B_{12} &= M_1 B_{22} \\ C_1 &= M_4 + M_1 C_2\ . \end{aligned}$$

Equations (A.4) and (A.5) are presented in the standard format of the linear rational expectations model in McCallum (1998, equations (1) and (3)). A undetermined-coefficients solution will be of the form:

$$\begin{aligned}\Delta s_t^e &= \Omega k_t + \Gamma u_t \\ &= (\beta_{sp}, \beta_{si})(\pi_{t-1}, i_{t-1})' + \Gamma u_t \ ,\end{aligned} \tag{A.6}$$

$$k_{t+1} = \Pi_1 k_t + \Pi_2 u_t \ , \tag{A.7}$$

where Ω, Γ , Π_1 and Π_2 are coefficient matrices to be determined later. It is apparent that β_{si} is the coefficient of one-period, short-run, UIP from equation (A.6).

Following McCallum (1998, equation (8)), we solve for the unknown matrices Ω and Π_1 using the following matrix equation:

$$\begin{bmatrix} 1 & 0 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} \Omega\Pi_1 \\ \Pi_1 \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \Omega \\ I_2 \end{bmatrix}, \tag{A.8}$$

where I_2 is a 2x2 identity matrix.

There are three solutions that satisfy the matrix equation (A.8):

Solution (i): $\Omega = (\beta_{sp}, \beta_{si}) = (0,1)$

Solution (ii): $\Omega = (\beta_{sp}, \beta_{si})$

where:

$$\begin{aligned}\beta_{sp} &= -\alpha_{pp}/\alpha_{ps} \\ \beta_{si} &= \frac{z\_[(1+\alpha_{ps})(1+\alpha_{ip}\alpha_{yi}) + \alpha_{py}\alpha_{yi}(\alpha_{ip}-1)] - \alpha_{ii}(1+\alpha_{ps}-\alpha_{pi}\alpha_{yi})}{\alpha_{ps}\alpha_{ip}(1+\alpha_{yi})} \\ \beta_{pi} &= \frac{z\_(1+\alpha_{ip}\alpha_{yi})-\alpha_{ii}}{\alpha_{ip}(1+\alpha_{yi})} \\ \beta_{ii} &= z\_ \\ \beta_{yi} &= -\alpha_{yi}(\beta_{ii}+\beta_{pi})\end{aligned}$$

and:

$$
\begin{aligned}
z_{-} &= \frac{-b-\sqrt{b^2-4ac}}{2a} \\
a &= (1+\alpha_{ps})(1+\alpha_{ip}\alpha_{yi}) + \alpha_{py}\alpha_{yi}(\alpha_{ip}-1) \\
b &= -[\alpha_{ii}(1+\alpha_{ps}-\alpha_{py}\alpha_{yi}) + \alpha_{ps}\alpha_{ip}(1+\alpha_{yi}) + \alpha_{pp}(1+\alpha_{ip}\alpha_{yi})] \\
c &= \alpha_{pp}\alpha_{ii}
\end{aligned}
$$

Solution (iii):

The same as solution (ii), except that z_{-} is replaced by:

$$z_{+} = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

Applying the minimal-state-variable (MSV) criterion of McCallum (1983, 1998), we choose the bubble-free solution from the three alternatives. The MSV criterion requires that a bubble-free solution be valid for *all* admissible values of the structural parameters. Consider the special case in which $\alpha_{ii}=0$ in equation (IV.2) in the main text. Then i_{t-1} would not appear in the model, and so would not be included in the minimal set of state variables. As a result, $\beta_{si}=0$ in equation (IV.5). This immediately eliminates solution *(i)*, in which $\beta_{si}=1$. It also eliminates solution *(iii)*: if $\alpha_{ii}=0$, then $b<0$ and $c=0$ (recall that all model parameters are defined as positive values). It follows that $z_{+} = -b/(2a) \neq 0$, and β_{si} in solution *(iii)* becomes:

$$
\begin{aligned}
\beta_{si} &= \frac{z_{+}[(1+\alpha_{ps})(1+\alpha_{ip}\alpha_{yi}) + \alpha_{py}\alpha_{yi}(\alpha_{ip}-1)] - \alpha_{ii}(1+\alpha_{ps}-\alpha_{pi}\alpha_{yi})}{\alpha_{ps}\alpha_{ip}(1+\alpha_{yi})} \\
&\neq 0
\end{aligned}
$$

This implies that *(iii)* cannot be bubble-free under the MSV criterion. For solution *(ii)*, however, we have $z_{-}=c=0$ if $\alpha_{ii}=0$, implying that $\beta_{si}=0$, indicating that i_{t-1} indeed is *not* included in the minimal set of state variables under solution *(ii)*. Thus, it is the only bubble-free solution.

2. Numerical solution to the Meredith-Chinn (1998) model

We first define:

$$Y_t = (\Delta s^e_t, i_t, i^e_{t+1}, i^e_{t+2}, i^e_{t+3}, i^{L,e}_t, \pi^{L,e}_t, \pi_t, \pi^e_{t+1}, \pi^e_{t+2}, \pi^e_{t+3})'$$

as an 11x1 vector of non-predetermined endogenous variables. Divide Y_t into two components:

$$Y_t = (\Delta s^e_t, \quad {}_t')',$$

where $\;_t$ excludes the first element of Y_t, i.e. Δs^e_t. Then define:

$$k_t = (y_{t-1}, \pi_{t-1}, p_{t-1}, s_{t-1}, i_{t-1})'$$

as a 5x1 vector of predetermined variables. Also divide k_t into two components:

$$k_t = (\hat{k}_t', i_{t-1})',$$

where $\hat{k}_t$ excludes the last element of k_t, i.e. i_t. Finally, let u_t be a 3x1 vector of structural disturbances.

Rewriting the model in matrix form, we have:

$$M_1 Y_{t+1} + M_2 k_{t+1} + M_3 Y_t \qquad + M_5 u_t = 0, \tag{A.9}$$

$$N_2 k_{t+1} + N_3 Y_t + N_4 k_t + N_5 u_t = 0, \tag{A.10}$$

where M and N are parameter matrices.

To express the model in the standard form of McCallum (1998), pre-multiply (A.10) by N_2^{-1} and rearrange as follows:

$$k_{t+1} = -N_2^{-1} N_3 Y_t - N_2^{-1} N_4 k_t - N_2^{-1} N_5 u_t. \tag{A.11}$$

Substituting (A.11) into (A.9) and rearranging gives:

$$M_1 Y_{t+1} = (M_2 N_2^{-1} N_3 - M_3) Y_t + (M_2 N_2^{-1} N_4 - M_4) k_t + (M_2 N_2^{-1} N_5 - M_5) u_t. \tag{A.12}$$

Equations (A.11) and (A.12) form McCallum's standard setup:

$$A \begin{bmatrix} Y_{t+1} \\ k_{t+1} \end{bmatrix} = B \begin{bmatrix} Y_t \\ k_t \end{bmatrix} + Cu_t, \tag{A.13}$$

where:

$$A = \begin{bmatrix} M_1 & 0 \\ 0 & I_5 \end{bmatrix},$$

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} M_2 N_2^{-1} N_3 - M_3 & M_2 N_2^{-1} N_4 - M_4 \\ -N_2^{-1} N_3 & -N_2^{-1} N_4 \end{bmatrix},$$

$$C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} M_2 N_2^{-1} N_5 - M_5 \\ -N_2^{-1} N_5 \end{bmatrix}.$$

Applying the undetermined-coefficients approach, the solution has the following form:

$$Y_t = \Omega k_t + \Gamma u_t, \tag{A.14}$$

$$k_{t+1} = \Pi_1 k_t + \Pi_2 u_t, \tag{A.15}$$

where Ω, Γ, Π_1 and Π_2 are coefficient matrices. Rewrite (A.14) in detail:

$$\begin{bmatrix} \Delta S_t \\ \hat{Y}_t \end{bmatrix} = \Omega \begin{bmatrix} \hat{k}_t \\ i_{t-1} \end{bmatrix} + \Gamma u_t = \begin{bmatrix} \Omega_{11} & \beta_1 \\ \Omega_{21} & \Omega_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ i_{t-1} \end{bmatrix} + \Gamma u_t \tag{A.16}$$

where $\Omega = \begin{bmatrix} \Omega_{11} & \beta_1 \\ \Omega_{21} & \Omega_{22} \end{bmatrix}$.

From (A.16), we have:

$$\Delta s^e_t = \beta_1 i_{t-1} + \Omega_{11} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ p_{t-1} \\ s_{t-1} \end{bmatrix} + \Gamma u_t \tag{A.17}$$

As above, we solve for Ω and Π_1 using the matrix equation:

$$\begin{bmatrix} 1 & 0 \\ 0 & I_5 \end{bmatrix} \begin{bmatrix} \Omega\Pi_1 \\ \Pi_1 \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \Omega \\ I_5 \end{bmatrix}, \tag{A.18}$$

where I_5 is a 5x5 identity matrix.

The generalized Schur decomposition yields the numerical solution for Ω, Γ, Π_1 and Π_2 (for details, see McCallum (1998) p.145)[16]. In particular:

$\beta_1 = -0.5341$,

[16] We are grateful to Paul Soderlind for generously providing his Gauss program to perform the generalized Schur decomposition.

$$\Omega_{11} = [-0.7235\ -0.2750\ \ 0.8651\ -0.8651]\ .$$

To derive the long-horizon parameters, we have:

$$Y_j = \begin{bmatrix} \Delta S_j \\ \hat{Y}_j \end{bmatrix} = \Omega\, k_j + \Gamma\, u_j$$

$$= \Omega(\Pi_1 k_{j-1} + \Pi_2 u_{j-1}) + \Gamma\, u_j$$

$$= \ldots\ldots$$

$$= \Omega \Pi_1^{j-t+n-1} k_{j-n+1} + \xi^{Y}_{t-n+1,j}, \qquad \text{for j=t-n+1 to t} \qquad \text{(A.19)}$$

Summing up (A.19) from period t-n+1 to t gives:

$$\sum_{j=t-n+1}^{t} Y_j = \begin{bmatrix} \sum_{j=t-n+1}^{t} \Delta s_j \\ \sum_{j=t-n+1}^{t} \hat{Y}_j \end{bmatrix} = \Omega \sum_{j=t-n+1}^{t} \Pi_1^{j-t+n-1}\, k_{t-n+1} + \xi^{Y}_{t-n+1,t}.$$

$$= \begin{bmatrix} \lambda_{sy,n} & \lambda_{sp,n} & -\lambda_{ss,n} & \lambda_{ss,n} & \lambda_{si,n} \\ \lambda_{1n} & \lambda_{2n} & \lambda_{3n} & \lambda_{4n} & \lambda_{5n} \end{bmatrix} \begin{bmatrix} y_{t-n} \\ \pi_{t-n} \\ p_{t-n} \\ s_{t-n} \\ i_{t-n} \end{bmatrix} + \begin{bmatrix} \zeta_{t-n+1,t} \\ \psi_{t-n+1,t} \end{bmatrix} \qquad \text{(A.20)}$$

where it is found that the coefficients on p_{t-n} and s_{t-n} have opposite signs in the first row.

Substituting (A.20) into (IV.10) in the main text gives:

$$\Delta_n s_t = \left(\sum_{j=t-n+1}^{t} \Delta s_j \right) / n$$

$$= (\lambda_{sy,n}/n) y_{t-n} + (\lambda_{sp,n}/n)\pi_{t-n} + (\lambda_{ss,n}/n)(s_{t-n} - p_{t-n}) + (\lambda_{si,n}/n) i_{t-n} + \xi_{t-n+1,t} \qquad \text{(A.21)}$$

Similarly we have:

$$k^{e}_{j+1} = \begin{bmatrix} \hat{k}^{e}_{j+1,\ t-n} \\ i^{e}_{j,\ t-n} \end{bmatrix} = \Pi_1\, k_j$$

$$= \ldots\ldots$$

$$= \Pi_1^{j-t+n} k_{t-n+1} \tag{A.22}$$

Summing up (A.22):

$$k_{t-n+1} + \sum_{j=t-n+1}^{t-1} k^e_{j+1} = \begin{bmatrix} \hat{k}_{t-n+1} + \sum\limits_{j=t-n+1}^{t-1} \hat{k}^e_{j+1,t-n} \\ i_{t-n} + \sum\limits_{j=t-n+1}^{t-1} i^e_{j,\, t-n} \end{bmatrix} = \sum_{j=t-n}^{t-1} \Pi_1^{j-t+n} k_{t-n+1}$$

$$= \begin{bmatrix} \alpha_{1n} & \alpha_{2n} & \alpha_{3n} & \alpha_{4n} & \alpha_{5n} \\ \lambda_{iy,n} & \lambda_{ip,n} & -\lambda_{is,n} & \lambda_{is,n} & \lambda_{ii,n} \end{bmatrix} \begin{pmatrix} y_{t-n} \\ \pi_{t-n} \\ p_{t-n} \\ s_{t-n} \\ i_{t-n} \end{pmatrix} \tag{A.23}$$

where it was found that coefficients on p_{t-n} and s_{t-n} have opposite signs in the last row. Substituting (A.23) into (IV.11) gives:

$$i_{n,t-n} = \left(i_{t-n} + \sum_{j=t-n+1}^{t-1} i^e_{j,t-n} \right) / n = (\lambda_{iy,n}/n) y_{t-n} + (\lambda_{ip,n}/n) \pi_{t-n} + (\lambda_{is,n}/n)(s_{t-n} - p_{t-n}) + (\lambda_{ii,n}/n)\, i_{t-n} \tag{A.24}$$

Subtracting (A.24) from (A.21) gives:

$$\Delta_n s_t = [(\lambda_{sy,n} - \lambda_{iy,n})/n]\, y_{t-n} + [(\lambda_{sp,n} - \lambda_{ip,n})/n]\, \pi_{t-n} + [(\lambda_{ss,n} - \lambda_{is,n})/n]\, (s_{t-n} - p_{t-n}) + [(\lambda_{si,n} - \lambda_{ii,n})/n]\, i_{t-n} + i_{n,t-n} + \xi_{t-n+1,t} \tag{A.25}$$

As n increases, the parameters on y_{t-n}, π_{t-n}, $s_{t-n} - p_{t-n}$ and i_{t-n} in the long-horizon UIP equation asymptote to zero (see Table 2 in the main text), while that on the bond yield asymptotes to unity. Hence, (A.25) implies that UIP is restored over the longer horizon.