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Inflation Targeting Under Asymmetric Preferences

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Abstract

This paper develops and estimates a game-theoretical model of inflation targeting where the central banker's preferences are asymmetric around the targeted rate. Specifically, positive deviations from the target can be weighted more, or less, severely than negative ones in the central banker's loss function. It is shown that some of the previous results derived under the assumption of symmetry are not robust to this generalization of preferences. Estimates of the central banker's preference parameters for Canada, Sweden, and the United Kingdom are statistically different from the one implied by the commonly-used quadratic loss function.

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I. Introduction

This paper develops and estimates a game-theoretical model of monetary policy where the central banker's preferences are asymmetric around the targeted inflation rate. The preference specification permits different weights for positive and negative inflation deviations from the target, and includes as a special case the quadratic loss function employed by previous literature.[1] The symmetry of the quadratic form implies that the loss associated with an inflation deviation from the target depends solely on its magnitude. In contrast, under asymmetric preferences both the magnitude and sign of a deviation matter to the central banker. In addition, because certainty equivalence no longer holds, uncertainty can induce a prudent behavior on the part of the central banker.

Arguments in favor of the quadratic loss function include that it is tractable, yields simple analytical results, and might provide a reasonable approximation to the central banker's preferences. On the other hand, recent anecdotal and empirical evidence appears consistent with the notion of asymmetric preferences. For example, Clarida and Gertler (1999) estimate a reaction function for the Bundesbank and find that it raises the day-to-day interest rate when inflation is above its steady-state trend value but barely responds when it is below. Ruge-Murcia (2000) estimates implicit bounds for the Canadian inflation target zone using data on market-determined nominal interest rates. Results indicate that financial markets perceive the band to be of approximately the same width as announced, but asymmetrically distributed around the official target.

The functional form of the central banker's loss function is based on the linex model proposed by Varian (1974). Zellner (1986), Granger and Pesaran (1996), and Christoffersen and Diebold (1997) study this function in the context of optimal forecasting. Nobay and Peel (1998) use the linex function to study optimal commitment and discretion in monetary policy. This paper complements and extends their analysis by fully characterizing the theoretical and empirical implications of the model. In particular, the properties of the central banker's reaction function are derived, conditions for the existence and uniqueness of the Nash equilibrium are established, and the government's problem of optimal delegation is formulated and solved. Rather than using simulations (as Nobay and Peel), this paper (*i*) derives and tests the empirical predictions of the model and (*ii*) obtains estimates of the central banker's preference parameters.

The study of central bank preferences is framed here in a specific institutional setup. Publicly-announced inflation targets have been adopted recently by several countries (for example, New Zealand, Australia, Canada, the United Kingdom, and Sweden). Under this arrangement, the central bank commits itself to gear monetary policy towards keeping a measure of inflation close to an explicit target. Following Svensson (1997), Beetsma and Jensen (1998), and Muscatelli (1999), the central banker's loss function is defined around this target. Because the target is observable,

[1] See, among others, Kydland and Prescott (1977), Barro and Gordon (1983), Rogoff (1985), Walsh (1995), Svensson (1999), Clarida, Galí, and Gertler (1999), and Eijffinger, Hoeberichts, and Schaling (2000). In the inflation targeting literature, see Green (1996), Svensson (1997), Beetsma and Jensen (1998), and Muscatelli (1999).

it is possible to compare inflation realizations and the stated policy goal. This simplifies the estimation strategy, reduces the number of parameters to be estimated, and means that the model generates testable empirical implications regardless of the whether the target is the socially optimal or not.

The analysis is carried out in a standard game-theoretical framework. As the other game-theoretical models of monetary policy, the model here predicts a positive relation between inflation and unemployment (or a negative relation between inflation and output). However, in contrast to models with quadratic preferences that predict a linear relationship, asymmetric preferences predict a nonlinear and concave relation between these variables.

Game-theoretical models of inflation targeting with quadratic preferences predict that inflation should be on average above its target [see Green (1996) and Svensson (1997)]. Under asymmetric preferences, inflation can be on average above or below the target depending on the central banker's preference parameters. To understand this result, recall that relaxing the assumption of quadratic preferences means that certainty equivalence no longer holds. Then, the expected marginal cost of departing from the inflation target is nonlinear in inflation. When the central banker associates a larger loss to positive than negation inflation deviations from the target, uncertainty raises the expected marginal cost and induces a prudent behavior on the part of the monetary authority.

The asymmetric model also implies that the conditional variance of inflation is helpful in forecasting its mean. The asymmetry preference parameter is the coefficient of this conditional variance. Since the quadratic model corresponds to the special case where this parameter is zero, one can test the null hypothesis of quadratic preferences against the well-defined alternative of asymmetric preferences. In an empirical application to Canada, Sweden, and the United Kingdom, results support the notion of asymmetric preferences in the form a positive and statistically significant estimate of the asymmetry preference parameter. These results appear robust to modeling the rate of unemployment as a stationary or unit-root process and to relaxing the assumption of normality of the shocks, but not to the use of measures of inflation broader than the one targeted.

The paper is organized as follows: section 2 presents the model, derives conditions for the existence and uniqueness of the Nash equilibrium, and outlines the empirical implications; section 3 constructs a reduced-form version of the model, reports empirical estimates, and examines the robustness of the results; and section 4 concludes.

II. The Model

The model consists of (*i*) a central banker that implements monetary policy, and (*ii*) a continuum of identical individuals assumed to construct their expectations rationally. In what follows, this continuum will be referred to as the public.

A. The Economy

The economic behavior of the public is summarized by an expectations-augmented Phillips curve:[2]

$$u_t = u_t^n - \lambda(\pi_t - \pi_t^e) + \eta_t, \tag{1}$$

where λ is positive coefficient, u_t is the rate of unemployment, u_t^n is the natural rate of unemployment, π_t is the rate of inflation, π_t^e is the public's forecast of inflation at time t constructed at time $t-1$, and η_t is a supply disturbance. The choice of unemployment as measure of real economic activity has no effect on the analytical derivations below and allows the comparison of empirical results in section 3 with theoretical predictions in earlier literature.

Under the assumption of rational expectations,

$$\pi_t^e = E_{t-1}\pi_t, \tag{2}$$

where E_{t-1} is the expectation conditional on all information available at time $t-1$. The public's information set is denoted by I_t and is assumed to contain all model parameters, current and past realizations of the variables, and current and next-period inflation targets. The assumption that future inflation targets form part of I_t follows from the fact that inflation targets are preannounced.

The natural rate of unemployment is assumed to evolve over time according to:

$$\Delta u_t^n = \psi - (1-\delta)u_{t-1}^n + \sum_{i=1}^{q-1} \theta_i \Delta u_{t-i}^n + \zeta_t, \tag{3}$$

where ζ_t denotes the unpredictable component of the natural rate. This specification includes as special cases the stationary ($0 \leq \delta < 1$) and unit root ($\delta = 1$) models of the natural rate employed in earlier literature. For example, the model in Barro and Gordon (1983a) can be obtained by setting $\theta_i = 0$ for $i = 1, \ldots, q-1$, the one in Ireland (1999) by setting $\delta = 1$, $\psi = 0$, and $\theta_i = 0$ for $i = 2, \ldots, q-1$, and the one in Cukierman (2000) by setting $\delta = 0$, and $\theta_i = 0$ for $i = 1, \ldots, q-1$. The intention here is to adopt a general time series specification for the natural rate and then examine to what extent results are robust to the use of different forecasting models for u_t^n.[3] The constant term in (3) could be written as $\psi(1-\delta)$ in order to rule out the presence of a drift when unemployment is $I(1)$. However, for the estimation of the model, it is desirable to allow a nonzero intercept. Standard procedures can then be used to test whether this term is

[2]For the microfoundations of this view of aggregate supply, see Lucas (1972).

[3]In principle, one could also consider a nonlinear process for the natural rate. However, the construction of the reduced-form version of the model in section 3.3 does make use of the assumption of linearity.

significantly different from zero or not. In order to exclude the possibility of u_t^n being an $I(2)$ process, all roots of the polynomial $1 - \sum_{i=1}^{q-1} \theta_i z^i$ are assumed to lay outside the unit circle.

Modeling the natural rate of unemployment as time-varying is important for at least two reasons. *First*, it seems plausible that changes in technology, labor force demographics, unionization rates, and welfare benefits could affect the labor market and generate movements in the natural rate. For the United States, Weiner (1993), Tootell (1994), and Staiger, Stock, and Watson (1997) report evidence that the natural rate has changed during the postwar period. Shimer (1998) argues that in the absence of the baby boom, the rate of unemployment would have neither increased in the 1960's and 1970's, nor fallen afterwards.

Second, although the assumption that u_t^n is constant can be innocuous for some theoretical results, it leads to the empirical prediction that realized unemployment is white noise. However, for most countries and sample periods the rate of unemployment is serially correlated and substantially persistent. As an example, Table 1 presents the first 10 autocorrelations of the quarterly rate of unemployment in three inflation-targeting countries, namely Canada, Sweden, and the United Kingdom. In all cases, the correlation between current and previous unemployment (*i.e.*, the first autocorrelation) is above 0.90, and the correlation between current unemployment and its realization two years before (*i.e.*, the eight autocorrelation) is above 0.55.

As in Barro and Gordon (1983a) and Ireland (1999), the process of u_t^n is not affected by the monetary policy instrument (see below) or lagged unemployment rates. This assumption reflects the view that the natural rate is primarily determined by factors outside the scope of monetary policy (for example, exogenous demographic or technological variables). In this sense, this paper adopts Friedman's interpretation of the natural unemployment rate as "the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availability, the costs of mobility, and so on" [Friedman (1968, p. 8)].

An alternative process for the natural rate is proposed by Lockwood and Philippopoulos (1994). Their specification also seeks to capture the persistence observed in the unemployment rate, and consists of a strictly stationary process for the natural rate where the first unemployment lag is one of the regressors. Unfortunately, when coupled with the more general preference specification employed here, the model has no closed-form solution, the value function needs to be approximated by a finite-order polynomial, and the uniqueness of the Nash equilibrium cannot be insured. Since (3) also captures the serial correlation in unemployment but in a more tractable manner, and permits both stationary and nonstationary specifications, I did not pursue their approach further.

The policy maker is assumed to affect the rate of inflation through a policy instrument (as in Walsh (1995, 1998 ch. 8)). We can think of this instrument as a monetary aggregate or a short-term nominal interest rate. The instrument is imperfect in the sense that it cannot determine inflation completely and its effect takes place with a (one-period) lag:

$$\pi_t = f(i_t) + \epsilon_t, \tag{4}$$

where $f(\cdot)$ is a monotonic, continuous, and differentiable function, i_t is the policy instrument, and ϵ_t is a control error that represents imperfections in the conduct of monetary policy. Since i_t is chosen at time $t-1$, it follows that $i_t \in I_{t-1}$. This simple specification serves two purposes. *First*, it relaxes the usual assumption that the monetary authority chooses directly the rate of inflation after observing (before the public does) the random shocks. The policy maker here has no informational advantage over the public since neither of them observe at time $t-1$ the realization of the disturbances at time t. Because there is no private information, the central banker's information set coincides with the public's and is also given by I_t. *Second*, (4) introduces an additional structural disturbance to the model and permits the derivation of time-series predictions on the joint probability distribution function of inflation and unemployment.[4]

Finally, to complete the description of the economy, define $\boldsymbol{\xi}_t$ to be the 3×1 vector that contains all the model's structural disturbances at time t. It is assumed that $\boldsymbol{\xi}_t$ is serially uncorrelated and normally distributed with zero mean and constant conditional variance-covariance matrix:

$$\boldsymbol{\xi}_t | I_{t-1} = \left[\begin{array}{c} \eta_t \\ \zeta_t \\ \epsilon_t \end{array}\right] \Bigg| \; I_{t-1} \sim N(\mathbf{0},\Omega),$$

where Ω is a 3×3 positive-definite matrix. Evidence supporting the assumption that $\boldsymbol{\xi}_t$ is conditionally homoskedastic is presented in section 3.4. The assumption of normality is crucial for obtaining an analytical solution, but section 3.5 examines the robustness of empirical results to shocks drawn from a different distribution. For the moment, no restrictions are imposed on the off-diagonal elements of Ω.

B. THE CENTRAL BANKER

The central banker is assumed to have preferences over inflation and unemployment. Preferences are described by the function:

$$C(\pi_t, u_t) = [\exp(\alpha(\pi_t - \tilde{\pi}_t)) - \alpha(\pi_t - \tilde{\pi}_t) - 1]/\alpha^2 + (\phi/2)(u_t - \tilde{u}_t)^2, \qquad \alpha \neq 0, \tag{5}$$

[4]Previous literature frequently assumes that the only random shock is the aggregate supply disturbance. Consequently, the dynamics of π_t and u_t are driven by exactly the same random term. As an alternative to (4), one could postulate an aggregate demand relation [see, for example, Orphanides and Wilcox (1996)]. In this case, the model solution is unchanged but the structural interpretation of the reduced-form disturbances is slightly different.

where $C : \Re^2 \rightarrow \Re$, $\tilde{\pi}_t$ is the inflation target, $\tilde{u}_t$ is the unemployment target, and ϕ is a positive coefficient that measures the relative importance of unemployment stabilization. In the institutional setup considered here, the inflation target is assigned by the government to the monetary authority before time t. Since the target is preannounced, it follows that $\tilde{\pi}_t \in I_{t-1}$. A general model for $\tilde{u}_t$ is presented below.

In contrast to most of the previous literature, where both components of the policy maker's loss function are quadratic, the inflation component in (5) is given by the linex function $g(x) = [\exp(\alpha x) - \alpha x - 1]/\alpha^2$ [Varian (1974)].[5] Zellner (1986), Granger and Pesaran (1996), and Christoffersen and Diebold (1997) study this function in the context of optimal forecasting. In order to develop some intuition, the linex function is plotted in Figure 1(a) for the special case where $\alpha > 0$. For rates of inflation above the target, the exponential term eventually dominates and the loss associated with a positive deviation rises exponentially. For rates of inflation below the target, it is the linear term that becomes progressively more important as inflation decreases and, consequently, the loss rises linearly. This asymmetry can be easily seen by considering, for example, the loss associated with a ± 1 inflation deviation from $\tilde{\pi}_t$. It is apparent that even though their magnitudes are the same, the -1 deviation delivers a smaller loss than the $+1$ deviation. Hence, positive inflation deviations are weighted more severely than negative ones in the central banker's loss function. The converse holds when $\alpha < 0$.

It is useful to compare the linex function with the very-familiar quadratic loss function in Figure 1(b). The quadratic function is symmetric around zero. This means that positive and negative deviations of the same size yield exactly the same loss. Hence, only the magnitude, and not the sign, of the deviation is important for the policy maker. Applying L'Hôpital's rule, it is possible to show that the quadratic function can be obtained as a special case of the linex function when the asymmetry parameter (α) tends to zero.[6] This result is important because it proves that the preference specification employed here nests the one assumed in earlier literature as a special case. It also suggests that the hypothesis that the central banker's preferences are quadratic over inflation could be tested econometrically by evaluating whether α is significantly different from zero.

The functional form in (5) is attractive because it is analytically tractable, yields a closed-form solution, and generates clear empirical predictions. In principle, one could extend it to allow asymmetries regarding unemployment. This possibility is discussed in section 3.5, where it is shown that the basic predictions of the model are unchanged by this extension. In related research, Ruge-Murcia (2001) estimates a model with asymmetric unemployment preferences for the G7

[5] Papers that also relax the assumption of quadratic preferences include Barro and Gordon (1983b), where the unemployment component is linear; Orphanides and Wilcox (1996), where the unemployment component has a kink; and Nobay and Peel (1998), where preferences are characterized by the same linex function employed here.

[6] Formally,

$$\underset{\alpha \rightarrow 0}{\text{Lim}} \frac{\exp(\alpha x) - \alpha x - 1}{\alpha^2} = \underset{\alpha \rightarrow 0}{\text{Lim}} \frac{x\exp(\alpha x) - x}{2\alpha} = \underset{\alpha \rightarrow 0}{\text{Lim}} \frac{x^2 \exp(\alpha x)}{2} = \frac{x^2}{2}.$$

countries. Reduced-form estimates do not support the hypothesis of asymmetric unemployment preferences for Canada and the United Kingdom.

The targeted rate of unemployment is assumed proportional to the expected natural rate:

$$\tilde{u}_t = kE_{t-1}u_t^n, \qquad 0 < k \leq 1. \tag{6}$$

Previous literature usually assumes that $\tilde{u}_t$ is strictly less than the natural rate (*i.e.*, $0 < k < 1$). This assumption is based on the notion that distortions in goods and labor markets render the natural rate of unemployment higher than socially optimal. Persson and Tabellini (2000) note that this assumption is crucial in generating an inflation bias in the linear-quadratic framework of Kydland and Prescott (1977) and Barro and Gordon (1983a). On the other hand, King (1996) and Blinder (1998) suggest on the basis of institutional evidence, that central bankers target the expected natural rate of unemployment (*i.e.*, $k = 1$). Both views are accommodated here by allowing $0 < k \leq 1$. Analytical results obtained under both assumptions are compared below.

C. Nash Equilibrium

Consider the problem of a central banker who must choose the sequence of instruments that minimizes the present discounted value of her loss function:

$$\underset{\{i_{t+s}\}_{s=0}^{\infty}}{Min} \quad E_{t-1}\sum_{s=0}^{\infty}\beta^s C\left(\pi_{t+s}, u_{t+s}\right),$$

where $\beta \in (0, 1)$ is the discount rate. The optimization is made subject to the expectations-augmented Phillips curve [eq. (1)], and takes as given the public's inflation forecasts and the inflation targets. Recall that the natural rate of unemployment is assumed to be determined primarily by factors outside the scope of monetary policy. Hence, the policy instrument does not affect the path of u_t^n and the central banker's objective function can be break down into a sequence of one-period problems. This decomposition simplifies the solution of the model and, as it will be shown below, delivers a unique Nash equilibrium.[7]

The first-order necessary condition is

$$E_{t-1}[(\exp(\alpha(\pi_t - \tilde{\pi}_t)) - 1)/\alpha - \lambda\phi(u_t - kE_{t-1}u_t^n)] = 0, \tag{7}$$

[7] In contrast, Lockwood and Philippopoulos (1994) show that in a model where the natural rate depends on lagged realizations of the unemployment rate, two stable equilibria are possible with different comparative-statics properties. Only one of the equilibria is intuitive in its economic predictions and corresponds to the one obtained in the static case. However, it is not possible to rule out the second, nonintuitive equilibrium based on theoretical considerations alone.

where $kE_{t-1}u_t^n$ substitutes $\tilde{u}_t$. Since the loss function is globally convex, the solution to (7) delivers a unique minimum. In order to find the conditional expectation in (7), two intermediate results are useful. *First*, as shown below, the assumption that shocks are normal implies that, conditional on the information set, inflation is normally distributed. Then, $\exp[\alpha(\pi_t - \tilde{\pi}_t)]$ is log normal with mean $\exp[\alpha(E_{t-1}\pi_t - \tilde{\pi}_t + (\alpha/2)\sigma_\pi^2)]$, where σ_π^2 denotes the conditional variance of the inflation rate. (See below for the derivation of σ_π^2 in terms of the elements of ξ_t.) *Second*, the conditional expectation of unemployment can be found by taking E_{t-1} in both sides of (1) to obtain $E_{t-1}u_t = E_{t-1}u_t^n - \lambda(E_{t-1}\pi_t - \pi_t^e)$. With these results, the first-order condition can be rewritten as

$$[\exp(\alpha(E_{t-1}\pi_t - \tilde{\pi}_t) + \alpha^2\sigma_\pi^2/2) - 1]/\alpha + \lambda^2\phi(E_{t-1}\pi_t - \pi_t^e) - \lambda\phi(1-k)E_{t-1}u_t^n = 0. \quad (8)$$

In the quadratic model, the first-order condition of the central banker's minimization problem is linear and can be solved explicitly to obtain her reaction function in terms of the public's inflation forecast, π_t^e.[8] To see this, take the limit of (8) as $\alpha \to 0$ and rearrange to obtain

$$E_{t-1}\pi_t = [\tilde{\pi}_t + \lambda^2\phi\pi_t^e + \lambda\phi(1-k)E_{t-1}u_t^n]/(1+\lambda^2\phi). \quad (9)$$

The central banker's reaction is linear and monotonically increasing on the public's inflation forecast (note that $0 < \lambda^2\phi/(1+\lambda^2\phi) < 1$). It is trivial to show that in this case, the Nash equilibrium always exists and is unique.

In contrast, under asymmetric preferences, the first-order condition only defines the reaction function implicitly:

$$\begin{aligned} &h(E_{t-1}\pi_t, \pi_t^e) \\ &= [\exp(\alpha(E_{t-1}\pi_t - \tilde{\pi}_t) + \alpha^2\sigma_\pi^2/2) - 1]/\alpha + \lambda^2\phi(E_{t-1}\pi_t - \pi_t^e) - \lambda\phi(1-k)E_{t-1}u_t^n, \quad (10) \\ &= 0. \end{aligned}$$

Using the implicit function theorem, it is possible to show that

$$\partial E_{t-1}\pi_t/\partial\pi_t^e = \lambda^2\phi/[\lambda^2\phi + \exp(\alpha(E_{t-1}\pi_t - \tilde{\pi}_t) + \alpha^2\sigma_\pi^2/2)] \in (0,1),$$

for all values of α. Hence, as in the quadratic model, the central banker's reaction is a monotonically increasing function of the public's inflation forecast. Also

$$\begin{aligned} &\partial^2 E_{t-1}\pi_t/\partial(\pi_t^e)^2 = \\ &\qquad -\alpha\lambda^4\phi^2\exp[\alpha(E_{t-1}\pi_t - \tilde{\pi}_t) + \alpha^2\sigma_\pi^2/2]/[\lambda^2\phi + \exp(\alpha(E_{t-1}\pi_t - \tilde{\pi}_t) + \alpha^2\sigma_\pi^2/2)]^3, \end{aligned}$$

[8] Strictly speaking, the reaction function relates the policy instrument, i_t, to π_t^e, both of which are determined in the previous period. However, in what follows it will be convenient to work with $E_{t-1}\pi_t$ rather than i_t. Since these two variables are monotonically related by the function $f(\cdot)$, this approach entails no loss of generality.

that is less than zero for $\alpha > 0$, equal to zero for $\alpha \to 0$, and larger than zero for $\alpha < 0$. In other words, for $\alpha > 0$ ($\alpha < 0$) the central banker's reaction is a concave (convex) function of π_t^e.

In order to develop further the reader's intuition and to illustrate future theoretical results, it is useful to plot the central banker's reaction function for different values of the preference parameter α. This is done in Figure 2 under the assumption that the inflation target is $\tilde{\pi} = 0$, and remaining parameters are $\lambda = 2$, $\phi = 0.5$, $k = 0.8$, $u^n = 5$, and $\sigma_\pi^2 = 2.5^2$. Treating all parameters as fixed, the central banker's reaction was computed by solving numerically the implicit function (10) for given values of π^e. The figure also includes the reaction function of the quadratic central banker, and the public's reaction function that is summarized by the rational expectations relation (2). Graphically, the Nash equilibrium is the point where (10) and (2) intersect.

Although in all cases the central banker's reaction is an increasing function of π^e, her willingness to accommodate the public's inflation forecast depends on the preference parameter α. Consider the case where $\alpha < 0$. The central banker responds to π_t^e at an increasing rate and the inflation rate will be larger than under quadratic preferences. For values of $\alpha \leq -1/(\lambda\phi(1-k)E_{t-1}u_t^n)$, there is no finite rate of inflation at which (10) and (2) intersect, and the Nash equilibrium will not exist [see proposition 1 below]. Consider now the case where $\alpha > 0$. The central banker accommodates the public's inflation forecast at a decreasing rate and inflation will be always smaller than under quadratic preferences. For large-enough values of α, a deflationary bias, whereby inflation systematically undershoots its targeted value, can arise in equilibrium. This result is important because it shows that asymmetric preferences can provide a theoretical foundation for Stanley Fischer's observation that a deflationary bias is a possible outcome in the practice of monetary policy [Fischer (1994)].[9]

Conditions for the existence and uniqueness of the Nash equilibrium are presented in the following proposition:

Proposition 1. *Provided* $1 + \alpha\lambda\phi(1-k)E_{t-1}u_t^n > 0$, *there exists a unique* $\pi_t^e = E_{t-1}\pi_t$, *such that* $h(E_{t-1}\pi_t, \pi_t^e) = 0$.

Proof. To prove existence, construct a

$$\pi_t^e = E_{t-1}\pi_t = \tilde{\pi}_t - (\alpha/2)\sigma_\pi^2 + (1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)E_{t-1}u_t^n). \tag{11}$$

Plugging (11) into (10) and using $\pi_t^e = E_{t-1}\pi_t$ delivers $h(E_{t-1}\pi_t, \pi^e) = 0$. To show uniqueness, assume there exists a second inflation forecast, say $\hat{\pi}^e = \tilde{\pi}_t - (\alpha/2)\sigma_\pi^2 + (1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)E_{t-1}u_t^n) + x$, that also lies on the 45° line on the plane $(\pi_t^e, E_{t-1}\pi_t)$ and satisfies $h(E_{t-1}\pi_t, \pi_t^e) = 0$. Replace $\hat{\pi}_t^e$ in (10) and simplify to obtain $[1 + \alpha\lambda\phi(1-k)E_{t-1}u^n][\exp(\alpha x) - 1]/\alpha = 0$. Since $1 + \alpha\lambda\phi(1-k)E_{t-1}u_t^n > 0$ and $\alpha \neq 0$, then it must be the case that $x = 0$.¶

[9] To my knowledge, this point was first made by Nobay and Peel (1998).

From this proposition follows:

Corollary 1. *In the special case where the central banker targets the expected natural rate of unemployment, there always exists a unique* $\pi_t^e = E_{t-1}\pi_t$, *such that* $h(E_{t-1}\pi_t, \pi_t^e) = 0$.

Proof. The result follows from noting that when $k = 1$, the condition $1 + \alpha\lambda\phi(1-k)E_{t-1}u_t^n > 0$ is always satisfied.¶

Notice in (11) that depending on the sign of α, inflation is an increasing or decreasing function of its conditional variance. Recall that when the loss function is quadratic, certainty equivalence holds and the model solution is the same regardless of whether there is uncertainty or not. In contrast, when the loss function is asymmetric on inflation, the marginal cost of departing from $\tilde{\pi}_t$ is not linear in inflation, but convex (when $\alpha > 0$) or concave (when $\alpha < 0$). When $\alpha > 0$, an increase in uncertainty rises the expected marginal cost of deviating from $\tilde{\pi}_t$. Then, uncertainty induces a prudent behavior on the part of the central banker. A comparable result can be found in the literature on precautionary savings. When the assumption of quadratic utility is relaxed and labor-income risk in nondiversifiable, uncertainty increases the expected marginal utility of future consumption. To satisfy the Euler condition, prudent households decrease current consumption compared to future consumption and increase their savings.

I now derive the stochastic processes of the rates of inflation and unemployment and show that, conditional on the information set, both are normally distributed. Taking conditional expectations of both sides of (4), noting that i_t forms part of the public's information set, substituting back into (4), and using (11) yields

$$\pi_t(\alpha) = \tilde{\pi}_t - (\alpha/2)\sigma_\pi^2 + (1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)E_{t-1}u_t^n) + \epsilon_t. \tag{12}$$

The notation $\pi_t(\alpha)$ makes explicit the dependence of the inflation process on the parameter that measures the asymmetry in the central banker's preferences. Since $\tilde{\pi}_t$ and $E_{t-1}u_t^n$ are included in the public's information set at time $t-1$: $E(\pi_t|I_{t-1}) = E_{t-1}\pi_t$ and $Var(\pi_t|I_{t-1}) = \sigma_\pi^2 = \sigma_\epsilon^2 = \mathbf{A}\Omega\mathbf{A}'$, where $\mathbf{A} = (0,0,1)$. Since the linear combination $\mathbf{A}\boldsymbol{\xi}_t$ is normally distributed, then

$$\pi_t|I_{t-1} \sim N(E_{t-1}\pi_t, \mathbf{A}\Omega\mathbf{A}'),$$

as claimed above.

Turning now to the rate of unemployment, plug (12) into (1) and use the assumption of rational expectations to obtain: $u_t = u_t^n - \lambda\epsilon_t + \eta_t$. As usual, the rate of unemployment realized in equilibrium does not differ systematically from the natural rate. Since u_t^n is serially correlated, u_t will be serially correlated too. Decompose the process of the natural rate as $u_t^n = E_{t-1}u_t^n + \zeta_t$ to write

$$u_t = E_{t-1}u_t^n + \zeta_t - \lambda\epsilon_t + \eta_t. \tag{13}$$

Since $E_{t-1}u_t^n$ forms part of I_{t-1} : $E(u_t|I_{t-1}) = E_{t-1}u_t^n$ and $Var(u_t|I_{t-1}) = \sigma_u^2 = \mathbf{B\Omega B'}$, where $B= (1, 1, -\lambda)$. Since the linear combination $\mathbf{B}\boldsymbol{\xi}_t$ is normally distributed, then

$$u_t|I_{t-1} \sim N(E_{t-1}u_t^n, \sigma_u^2).$$

In contrast to models where the supply disturbance is the only shock, the error term here is a linear combination of the structural disturbances, rather than only a scaled version of the supply shock.

Under current inflation targeting arrangements, the targets are publicly announced and, consequently, are observable by the econometrician. This means that (12) generates testable empirical implications regardless of whether $\tilde{\pi}_t$ is socially optimal or not.[10] Readers interested on these predictions are invited to go directly to section 2.5. However, from the normative point of view, it is still important to derive the optimal inflation target in the more general setup where preferences are asymmetric and compare results with the ones obtained using a quadratic loss function. This task is undertaken in the following section.

D. Digression: The Optimal Inflation Target

In order to make results comparable with the ones is earlier literature , assume that society's preferences are described by the quadratic loss function:

$$G(\pi_t, u_t) = (1/2)(\pi_t - \pi_t^*)^2 + (\phi/2)(u_t - u_t^*)^2, \tag{14}$$

where π_t^* and u_t^* denote the socially-optimal rates of inflation and unemployment, and remaining notation is as previously defined. Consider the problem of a government that chooses the sequence $\{\tilde{\pi}_t\}_{t=0}^{\infty}$ that minimizes the present discounted value of the social loss function. It is important to note that in practice, targets are sometimes jointly agreed by the government and the monetary authority. For example, section 9 of the Reserve Bank of New Zealand Act requires the Finance Minister *and* the Governor of the Reserve Bank to negotiate and make public the Policy Target Agreements (PTA's) that state the inflation targets [Bernanke *et al.* (1999, p. 88)]. Similarly, in Canada the targets are jointly determined and announced by the government and central bank. However, since in this model there is no private information and preferences are observable prior to delegation, the simplifying assumption that the choice of targets rests with the government is innocuous.[11]

[10] Although the natural rate of unemployment is not directly observed, it is possible to write the model in terms of inflation and unemployment alone, for which data is available. This is done below in section 3.3.

[11] Beestma and Jensen (1998) and Muscatelli (1999) develop static models where the central banker's preferences are unobserved by the government prior to delegation. The uncertainty concerns the weight of inflation (or unemployment) in the central banker loss function. This weight is stochastic in an otherwise standard quadratic loss function. The rigorous modeling of preference uncertainty in a dynamic setup would also require the specification of a learning mechanism on the part of the public/government. Thus, in order to keep the scope of this project manageable, I have adopted the simplifying assumption that preferences are public information. To motivate this assumption, it could be argued that the track record and the previously-expressed opinions of individuals might provide information about their preferences when in office.

The government's problem is

$$\underset{\{\tilde{\pi}_{t+s}\}_{s=0}^{\infty}}{Min} \quad E_{t-1}\sum_{s=0}^{\infty}\beta^s G\left(\pi_{t+s}(\tilde{\pi}_{t+s}), u_{t+s}\right),$$

where the minimization is made subject to (12). Because the choice of the inflation target for time t does not affect the inflation process beyond time t, the government's objective function can also be decomposed into a sequence of one-period problems. Noting that the unemployment rate does not depend on $\tilde{\pi}_t$ and that $\partial\pi_t/\partial\tilde{\pi}_t = 1$, the first-order condition is $E_{t-1}[(\partial G/\partial\pi_t)(\partial\pi_t/\partial\tilde{\pi}_t)] = E_{t-1}\pi_t - \pi_t^* = 0$. Using (11) and solving for the optimal inflation target yields

$$\tilde{\pi}_t(\alpha) = \pi_t^* + (\alpha/2)\sigma_\pi^2 - (1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)E_{t-1}u_t^n). \tag{15}$$

The notation $\tilde{\pi}_t(\alpha)$ makes explicit the dependence of the optimal inflation target on the parameter that measures the asymmetry in the central banker's preferences. As in Svensson (1997), the optimal target is time-varying, rather than the constant value typically observed in practice. However, the optimal target here is nonlinearly related to unemployment, depends on the conditional variance of inflation, and can be smaller or larger than the optimal inflation rate.

The optimal inflation target under quadratic preferences can be obtained by taking the limit of (15) as $\alpha \to 0$ to obtain

$$\tilde{\pi}_t(0) = \pi_t^* - \lambda\phi(1-k)E_{t-1}u_t^n < \pi_t^*.$$

Notice that when $0 < k < 1$, the optimal target is strictly lower than optimal inflation. The result $\tilde{\pi}_t(0) < \pi_t^*$ means that the government entrusts the quadratic central banker with a target that is low enough *vis a vis* π_t^* that, even if the latter is subject to an inflation bias, the marginal costs and benefits of inflation to the central banker are equalized at the socially-optimal inflation rate. This result was first derived by Svensson (1997). In the special case where the central banker targets the natural rate of unemployment ($k = 1$), the optimal inflation target corresponds exactly to the socially-optimal inflation rate.

The optimal inflation target can deliver the optimal monetary policy. To see this, replace (15) into (12) to obtain

$$\pi_t = \pi_t^* + \epsilon_t. \tag{16}$$

Inflation realizations differ from π_t^* only by a mean-zero and serially uncorrelated random term. The unconditional mean of inflation is the socially-optimal rate. Since the disturbance ϵ_t is outside the control of the monetary authority, the model highlights the fact that inflation is not perfectly controllable by the central banker. Consequently, announced inflation targets are likely to be specified in terms of a tolerance range rather than as a sole numerical value. This implication of the model appears consistent with empirical evidence: of the eight inflation targeting countries surveyed by Bernanke and Mishkin (1997), only Finland defines its target as a number alone. Israel has defined its inflation target as range in 1993 and from 1996 onwards, and a as number in 1994 and 1995. Since June 1995, the United Kingdom defines its target as a point rather than as a range. However, after June 1997 there is a 1% threshold around the target rate that, if breached by the inflation rate, prompts and explanatory letter from the Monetary Policy Committee to the Chancellor of the Exchequer.

E. IMPLICATIONS

As noted above, the fact that inflation targets are observable means that (12) generates testable predictions regardless of whether the targets are optimal or not. These predictions can be formulated in terms of either the rate of inflation or its deviation from the announced target. For the general case where the loss function is asymmetric, it is trivial to rewrite (12) as:

$$\pi_t(\alpha) - \tilde{\pi}_t = -(\alpha/2)\sigma_\pi^2 + (1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)E_{t-1}u_t^n) + \epsilon_t. \tag{17}$$

Notice that the term $(1/\alpha)\ln(1+\alpha\lambda\phi(1-k)E_{t-1}u_t^n)$ is nonnegative but $-(\alpha/2)\sigma_\pi^2$ can be positive or negative depending on the sign of the preference parameter α.

Consider first the case $\alpha < 0$, meaning that the central banker weights positive deviations from the target *less* severely than negative ones. Since $-(\alpha/2)\sigma_\pi^2$ is larger than zero, the average inflation deviation from the target is unambiguously positive. Consider now the case $\alpha > 0$, meaning that the central banker weights positive deviations from the target *more* severely than negative ones. Then, $(\alpha/2)\sigma_\pi^2 > 0$ and for certain parameter values, it is possible that its magnitude be large enough that $\pi_t(\alpha) - \tilde{\pi}_t < 0$. Hence, inflation could be on average below the announced target. In summary, under asymmetric preferences, inflation could be systematically above or below its target depending on the central banker's preference parameter α. It is shown below that the quadratic model predicts that average inflation is no smaller than the announced target. In the special case examined by Green (1996) and Svensson (1997) where $0 < k < 1$, average inflation is always larger than the target.

The asymmetric model predicts that the conditional variance of inflation is helpful in predicting its mean. In a cross section, this means, for example, that if $\alpha > 0$, countries with more volatile inflation should have a lower average inflation deviation from target. Some limited evidence to this effect is presented in section 3.4. In a time-series, the preference parameter is statistically identified from the coefficient of the conditional variance of inflation and from the nonlinear term on unemployment.

As a comparison, it is useful to derive the implications of the model with quadratic preferences. Take the limit of (17) as $\alpha \to 0$ to obtain

$$\pi_t(0) - \tilde{\pi}_t = \lambda\phi(1-k)E_{t-1}u_t^n + \epsilon_t.$$

Since the term $\lambda\phi(1-k)E_{t-1}u_t^n$ is nonnegative, the average realization of $\pi_t(0) - \tilde{\pi}_t$ is also nonnegative. When $0 < k < 1$, this model predicts that inflation is systematically above the announced target. Because this overshooting is rationally anticipated by the public, it follows that the targeting policy can only be imperfectly credible [see Svensson (1997)]. Quadratic preferences also imply that the relation between inflation and unemployment is linear and that the conditional variance has no explanatory power on the current deviation from the target. If one generalizes Svensson's model to allow the case where the central banker targets the natural rate of unemployment ($k = 1$), the model predicts that the average inflation deviation from the target is zero.

A more direct comparison of both models can be obtained by examining the variable

$$\begin{aligned}[\pi_t(\alpha) - \tilde{\pi}_t] - [\pi_t(0) - \tilde{\pi}_t] &= -(\alpha/2)\sigma_\pi^2 + (1/\alpha)\ln(1 + \alpha\lambda\phi(1-k)E_{t-1}u_t^n) \\ &\quad -\lambda\phi(1-k)E_{t-1}u_t^n.\end{aligned}$$

This variable compares explicitly the deviations predicted by the quadratic and asymmetric models. It is plotted in Figure 3 as a function of the preference parameter α and for three different values of $E_{t-1}u_t^n$, namely $2\%, 4\%$, and 6%. In constructing this graph, I have set $\lambda = 2$, $\sigma_\pi^2 = 1$, $\phi = 0.5$, and $k = 0.8$. Notice that when $\alpha \to 0$ both models predict exactly the same deviation from the target and, consequently, their difference is zero. For negative values of α, the asymmetric model predicts positive deviations that are larger than the (also positive) deviations predicted by the quadratic model. The nonlinear relation between unemployment and inflation can be quite relevant for certain negative values of α. For positive values of α, the asymmetric model predicts negative deviations, or positive deviations that are smaller than the ones predicted by the quadratic model. Consequently, their difference is negative. Note that for $\alpha > 0$, the relationship between the variables is approximately linear.

As previous game-theoretical models of monetary policy [see, among others, Barro and Gordon (1983a, b), Rogoff (1985), and Svensson (1997)], the rates of inflation and unemployment are predicted to be positively correlated when $0 < k < 1$. However, under asymmetric preferences, the relationship is nonlinear. The analysis of the data in section 3.2 suggests that the nonlinear model yields a more accurate fit of the inflation/unemployment observations than the linear specification.

In the special case where the central banker targets the natural rate of unemployment ($k = 1$), the model predicts no systematic relationship between inflation and unemployment. However, an inflation bias can still arise in the case where $\alpha < 0$. This suggests that the result that the inflation bias is zero when $k = 1$ [see McCallum (1995, 1997) and Blinder (1998)] is not robust to the generalization of the policy maker's loss function.[12] In the more plausible case where $\alpha > 0$, a pure deflationary bias arises and inflation is on average below its target for any positive α.

Finally, consider the effect on the inflation deviation from target of an innovation that increases the public's forecast of the natural rate. In the quadratic case:

$$\partial[\pi_t(0) - \tilde{\pi}_t]/\partial E_{t-1}u_t^n = \lambda\phi(1-k),$$

that is positive, constant and independent of the expected rate of unemployment when the innovation takes place. In the asymmetric case:

$$\partial[\pi_t(\alpha) - \tilde{\pi}_t]/\partial E_{t-1}u_t^n = \lambda\phi(1-k)/[1+\alpha\lambda\phi(1-k)E_{t-1}u_t^n],$$

that is also positive but depends on the expected unemployment rate. For example, when $\alpha > 0$, the innovation always has a smaller effect than in the quadratic model and decreases with the rate of unemployment. The empirical predictions of the models with quadratic and asymmetric preferences are summarized in Table 2.

III. Econometric Analysis

A. The Data

The empirical predictions of the model are examined using monthly observations of the rates of inflation and unemployment for Canada, Sweden, and the United Kingdom. These inflation-targeting countries feature time series that are sufficiently long to allow the meaningful estimation and testing of the model. In addition, their monetary policy objectives appear not to involve (at least explicitly) exchange rate considerations. Thus, despite very occasional interventions in foreign exchange markets, their exchange rate regime could be characterized as floating [see Bernanke *et al.* (1999)]. This is important because modeling the objectives of a central banker explicitly concerned about the nominal exchange rate, would require a more general model with the loss function appropriately augmented to include this institutional feature. For example, Finland had an inflation target of 2% per year between February 1993 and December 1998, and was simultaneously a member of the Exchange Rate Mechanism (ERM). Under the ERM, the nominal exchange rate was to be kept in a $\pm 15\%$ range around a target value and later fixed at an "irrevocable" rate prior to monetary union. Because of this, some authors argue that Finland's monetary policy might not correspond to one of pure inflation targeting [see Gerlach (1999, p. 1263) for a discussion].

Australia, New Zealand, and Israel also employ inflation targets in the conduct of their monetary policy. Unfortunately, their statistical offices publish data on inflation and/or unemployment

[12]This point has also been made by Nobay and Peel (1998) and Cukierman (2000).

only on a quarterly basis. The Australian Bureau of Statistics collects price data every quarter, pricing most articles in the first two months of each quarter. Only certain items (for example, milk, bread, fresh meat, fresh fruit and vegetables, holiday travel and accommodation) are surveyed each month. Similarly, Statistics New Zealand prices most items at the mid-point of each quarter. Food is the only commodity group of the Consumer Price Index (CPI) for which an index is prepared each month. Although monthly CPI data is available for Israel, the labor force survey of the Central Bureau of Statistics takes place only once per quarter. All this means that, since inflation targeting is a relatively recent policy, the number of observations available to estimate and test the model for these countries is too small to yield reliable results.[13]

The Canadian inflation targets were announced in February 1991 and originally envisaged a reduction in the annual rate of inflation to 3% by the end of 1992, 2.5% by mid-1994, and 2% by the end of 1995. The inflation target of 2% per year has been renewed in December 1993 (to the end of 1998), in February 1998 (to December 2001) and in May 2001 (to December 2006). I follow the literature [for example, Lafrance (1997)], in interpreting the Canadian targets during the period December 1992 to December 1995 as the linear interpolation of the announced values. The targets are regarded as mid-points in a band of plus or minus 1 percentage point and apply to a measure of "core" inflation that excludes the volatile food and energy components and the first-round effect of changes in indirect taxes. The annual percentage change of the CPI and Core CPI were obtained from the *Weekly Financial Statistics* published by the Bank of Canada (http://www.bankofcanada.ca).

In Sweden, the Governing Board of the Riksbank announced in January 1993 an inflation target of 2% per year from January 1995 onwards. The target applies to the annual change in the CPI with no exclusions and involves a tolerance range of $\pm 1\%$ around the target.[14] The raw CPI series was taken from the Web Site of the Riksbank (http://www.riksbank.se).

In the United Kingdom, the Chancellor of the Exchequer announced an inflation target of between 1% and 4% per year in October 1992. Initially, targets were meant to apply "through the end of the present parliament" alone. In June 1995, the government reinterpreted the inflation target as a numerical value of 2.5% per year. Since June 1997 there has been a 1% threshold around the target rate that, if breached by the inflation rate, prompts an explanatory letter from the Monetary Policy Committee to the Chancellor of the Exchequer. Throughout, the targeted measure of inflation is the annual change in the Retail Price Index excluding mortgage interest payments (or RPIX). Both the RPI and the RPIX were taken from the *OECD Main Economic Indicators*. Plots of the rates of inflation and announced targets for Canada, Sweden, and the United Kingdom are presented in Figure 4.

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[13] At the time of collecting the sample (fall of 2000), there are only 23, 39, and 34 observations of both inflation and unemployment for Australia, New Zealand, and Israel, respectively.

[14] The Riksbank issued a clarification on February 1999 stating that in setting interest rates it would also take into account underlying inflation as calculated from the UND1X index.

Note that by design, inflation targets are intended to apply to annual, rather than to monthly, inflation. However, because (*i*) annual inflation is the sum of the twelve most recent observations of monthly inflation,[15] (*ii*) the policy holds continuously, and (*iii*) past rates of monthly inflation are predetermined at time t, it is possible estimate the model using data observed at a higher frequency than the time horizon for which the policy is defined. See Ruge-Murcia (2000) for a discussion.

Unemployment is measured by the survey-based, seasonally-adjusted rate of unemployment published by the *OECD Main Economic Indicators*.[16] The sample periods are as follows: Canada from 1992:12 to 2000:6, Sweden from 1995:1 to 2000:6, and the United Kingdom from 1992:10 to 2000:6. The different sample periods start at the time when the inflation targeting policy took effect in each of country and end with the latest available observation of the variables at the time the data was collected.

B. A Rough Look at the Data

Prior to estimation, this section uses simple summary statistics and plots to examine to what extend the model predictions are (or are not) borne by the data. Consider first the predictions of the quadratic and asymmetric models regarding the average inflation deviation from the target. Under quadratic preferences, inflation should be on average above or at the target. Under asymmetric preferences, inflation can be on average above or below the target depending on the sign and magnitude of the preference parameter α. A negative average inflation deviation from the announced target is suggestive of a positive value of α.

Table 3 contains the sample mean of $\pi_t - \tilde{\pi}_t$ and the percentage of inflation observations above and below the target. For Canada and Sweden, the average deviation from target is negative and quantitatively important (-0.61 and -1.23, respectively), and the proportion of observations below target is high (91.2% and 81.2%, respectively). In the case of Sweden, the average deviation is below the lower limit of its inflation target zone. For the United Kingdom, the average inflation deviation from the target is small and positive (0.15) and 69.9% of the observations are above $\tilde{\pi}_t$.[17]

Although these statistics are provocative and appear to challenge the assumption of quadratic preferences, they must be interpreted with caution for two reasons. *First*, since the series correspond to monthly observations of annual inflation, the data points are serially correlated and do not constitute independent evidence against quadratic preferences. *Second*, these statistics make no

[15] This result relies on the definition of the rate of inflation as the change in the log of the price level. While, strictly speaking, this is an approximation to its percentage change, the numerical difference between the two measures is negligible for small rates of inflation.

[16] In preliminary work, I also used the seasonally unadjusted unemployment series. Results are similar to the ones reported and are available from the author upon request.

[17] These statistics are robust to using broader aggregate price measures. For Canada, the average CPI inflation deviation from the target is -0.69, and 76.9% of its observations are below target. For the United Kingdom the average RPI inflation deviation from target is 0.06, and 55.9% of its observations are above target.

use of the restrictions embodied in the economic model constructed in section 2. A more formal analysis of the data is performed below in sections 3.4 and 3.5.

A second prediction common to game-theoretical models of monetary policy is that the rates of inflation and unemployment are positively correlated.[18] Analytical results in section 2 show that this prediction is robust to generalizing the functional form of the central banker's loss function. However, under asymmetric preferences the relation between inflation and unemployment is non-linear and concave. In the special case where $k = 1$, both models predict no relationship between inflation and unemployment.

Because the inflation target is predetermined and publicly known, the prediction that π_t and u_t are positively related means that the inflation deviation from the target and unemployment should be positively related as well. Figure 5 presents scatter plots of $\pi_t - \tilde{\pi}_t$ with the rate of unemployment in the horizontal axis. As a very rough evaluation of this model prediction, the figures include the fitted line of a Least Squares projection of $\pi_t - \tilde{\pi}_t$ on a constant and the unemployment rate. Because u_t is endogenous, this projection was carried out by Two-Stage Least Squares (TSLS) using lagged unemployment as an instrument for current unemployment. The estimates of this regression are reported in Table 4. Notice that for Canada, the coefficient on unemployment is negative and significantly different from zero. On other hand, for Sweden and the United Kingdom the coefficients are positive, but significantly different from zero only in the latter case. Hence, the prediction that the inflation deviation from target and the unemployment rate are positively related is supported by UK and (to some extent by) Swedish data, but not by Canadian data.

Since the prediction of the model with asymmetric preferences is that the relation is positive but nonlinear, Table 4 also reports the results of a TSLS regression of $\pi_t - \tilde{\pi}_t$ on a constant, u_t, and u_t^2. Notice that the coefficients on u_t are positive and statistically different from zero at the 5% level in all cases. The coefficients on u_t^2 are negative and statistically different from zero at the 5% level in all cases. This result supports the notion of a nonlinear, concave relation between the inflation deviation from target and unemployment. Note that the R^2's of the nonlinear model are considerably larger than the ones of the linear model.

C. Reduced-Form of the Model

The simple game-theoretical model developed in section 2 consists of three variables, namely the natural rate of unemployment and the rates of inflation and unemployment. Although data

[18] When the models are written in terms of output, rather than unemployment, the prediction is that output and inflation are negatively related. This result is independent of assuming a neoclassical expectations-augmented Phillips curve (as done here) or a New Keynesian Phillips curve. For example, the first-order condition in Clarida Galí and Gertler (1999, p. 1672) predicts a negative relation between output gap and inflation.

on inflation and unemployment is readily available, the natural rate is not directly observable.[19] In order to allow the estimation of the model using observations on inflation and unemployment alone, a reduced-form version is now constructed. Taking conditional expectations in both sides of (3) and substituting into (13) yields

$$u_t = \psi + \delta u^n_{t-1} + \sum_{i=1}^{q-1} \theta_i \Delta u^n_{t-i} + \zeta_t - \lambda\epsilon_t + \eta_t.$$

Adding and substracting $\psi + \delta u_{t-1} + \sum_{i=1}^{q-1} \theta_i \Delta u_{t-i}$, noting that $u^n_t - u_t = \lambda\epsilon_t - \eta_t$, and substracting u_{t-1} in both sides:

$$\begin{aligned} \Delta u_t = \; & \psi - (1-\delta)u_{t-1} + \sum_{i=1}^{q-1} \theta_i \Delta u_{t-i} \\ & + \zeta_t - \lambda\epsilon_t + \eta_t + \delta(\lambda\epsilon_{t-1} - \eta_{t-1}) + \sum_{i=1}^{q-1} \theta_i(\lambda\Delta\epsilon_{t-i} - \Delta\eta_{t-i}). \end{aligned} \tag{18}$$

Equation (18) describes the process of the rate of unemployment as a function of its lagged values and a linear combination of current and past structural disturbances. An advantage of this representation is that it does not include the unobserved natural rate as one of the regressors. However, with only data on π_t and u_t, one cannot disentangle the residuals of each structural disturbance to construct empirical counterparts for the lagged ϵ_t and η_t that enter (18). Hence, the unemployment process cannot be estimated without additional statistical restrictions.

Consider a strategy that involves (*i*) assuming that ζ_t, ϵ_t and η_t are mutually uncorrelated with each other at all leads and lags,[20] and (*ii*) using time series results on the aggregation of $ARMA$ processes [see Harvey (1981, p. 43)]. Two of these results are relevant here. *First*, adding two moving average (MA) processes that are uncorrelated with each other at all leads and lags yields another MA process with order equal to the maximum order of its two components. *Second*, adding a white noise series to a moving average with which it is uncorrelated at all leads and lags produces a new MA process of the same order. Since the sequence $\zeta_t - \lambda\epsilon_t + \eta_t + \delta(\lambda\epsilon_{t-1} - \eta_{t-1}) + \sum_{i=1}^{q-1} \theta_i(\lambda\Delta\epsilon_{t-i} - \Delta\eta_{t-i})$ aggregates a white noise and two moving averages of order q, the two results above imply that there exists a $MA(q)$ process, say $w_t + \sum_{i=1}^{q} \tau_i w_{t-i}$, with *exactly*

[19] A number of authors [for example, Staiger, Stock, and Watson (1997)] construct estimates of the natural rate using data on inflation and unemployment. However, in the context of this model, it is clear that such estimates provide no additional information beyond that already contained in π_t and u_t.

[20] Note that this entails the restriction that the off-diagonal elements of Ω are zero.

the same statistical properties as the original series. Then, the process for Δu_t can be written in reduced-form as the unrestricted $ARMA(q-1, q)$:

$$\Delta u_t = \psi - (1-\delta)u_{t-1} + \sum_{i=1}^{q-1} \theta_i \Delta u_{t-i} + w_t + \sum_{i=1}^{q} \tau_i w_{t-i}. \tag{19}$$

Equation (19) makes apparent a number of advantages of the strategy outlined above. *First,* it involves weaker assumptions than alternative identification schemes [for example, Ireland (1999)]. *Second,* although restrictions are imposed on the variance-covariance matrix of the structural disturbances, no restrictions arise on the variance-covariance matrix of the reduced-form disturbances (see below). *Third,* estimation is straightforward and can be carried out using standard procedures. For example, lagged values of w_t could be proxied empirically by lagged residuals of (19). *Finally,* since the $ARMA$ process is unrestricted, it could be well approximated by a finite-order autoregressive or moving average processes.

Regarding the rate of inflation, take E_{t-1} in both sides of (13) to obtain

$$E_{t-1}u_t^n = E_{t-1}u_t.$$

This result follows from the fact that unemployment differs from the natural rate only by a mean-zero and serially-uncorrelated random term. Hence, the forecast of u_t^n is numerically equivalent to the forecast of u_t, when both are based on the same information set, I_{t-1}. By eq. (19), the latter can be constructed on the basis of past observations of unemployment alone. With this result, equation (12) can be rewritten as

$$\pi_t(\alpha) = \tilde{\pi}_t - (\alpha/2)\sigma_\pi^2 + (1/\alpha)\ln(1 + \alpha\gamma E_{t-1}u_t) + \epsilon_t, \tag{20}$$

where $\gamma = \lambda\phi(1-k) \geqslant 0$ is a constant coefficient.

Finally, from the assumptions about the structural shocks, it follows that the reduced-form disturbance, w_t, and ϵ_t are serially uncorrelated and jointly normally distributed with zero mean:

$$\left[\begin{array}{c} w_t \\ \epsilon_t \end{array}\right] \Bigg| \; I_{t-1} \sim N(\mathbf{0}, \boldsymbol{\Psi}),$$

where Ψ represents a 2×2 positive-definite variance-covariance matrix. Since w_t is an aggregation of structural shocks, w_t and ϵ_t are contemporaneously correlated and the off-diagonal element in Ψ is nonzero.

D. Estimation

Recall that unemployment can be written in reduced-form as an unrestricted $ARMA(q-1,q)$. Since (*i*) any stationary $ARMA$ process can be approximated arbitrarily well by a finite autoregression, and (*ii*) the estimation of $ARMA$ processes is frequently complicated by common factors, the unemployment process is estimated here in autoregressive form. Results using a low-order $ARMA$ process yielded virtually the same results as reported below and are available from the author upon request.

Under asymmetric preferences, inflation depends nonlinearly on the public's unemployment forecast. In order to examine the robustness of the results to the use of different forecasting models of unemployment, this paper considers two processes for u_t. The first one is a stationary model that corresponds to the case where $0 \leq \delta < 1$ in (3). The second one is a unit root model that corresponds to the case where $\delta = 1$ in (3). For both models, the lag length of the AR representation was determined using Akaike's Information Criterion (AIC). After estimating processes with lag length 1 to 9, results indicated that the most appropriate stationary specifications for the rates of unemployment in Canada, Sweden, and the United Kingdom are $AR(3)$, $AR(4)$, and $AR(4)$, respectively. When modeling unemployment as nonstationary, the most appropriate specifications are $AR(2)$, $AR(3)$, and $AR(3)$, respectively.

Specification tests for both forecasting models of unemployment are reported in Table 5. Durbin-Watson and Breusch-Godfrey tests for serial correlation of the residuals are reported in rows 1 and 2. The Breusch-Godfrey test statistics were calculated as the product of the number of observations and the uncentered R^2 of the Ordinary Least Square (OLS) regression of the unemployment residuals on a constant, lagged unemployment rates, and two lagged residuals. Under the null hypothesis of no serial correlation, the statistic is distributed chi-square with as many degrees of freedom as lagged residuals are included in the regression. For all countries and models, the null of hypothesis of no serial correlation cannot be rejected at the 5% level.

Lagrange Multiplier (LM) tests for neglected Autoregressive Conditional Heteroskedasticity (ARCH) are reported in row 3. The statistics were calculated as the product of the number of observations and the uncentered R^2 of the OLS regression of the squared unemployment residual on a constant and two of its lags. Under the null hypothesis of no conditional heteroskedasticity, the statistic is distributed chi-square with as many degrees of freedom as squared residuals are included in the regression. The null of hypothesis of no conditional heteroskedasticity cannot be rejected at the 5% level in any country.[21]

The bivariate process of inflation and unemployment was estimated by the numerical maximization of their joint log likelihood function. Since this function imposes the cross-equation restrictions that arise from the dependence of inflation on $E_{t-1}u_t$, its maximization yields consistent

[21] Note, however, that for Sweden the null hypothesis is rejected at the 10% significance level.

and efficient Full Information Maximum Likelihood (FIML) estimates of the parameters. Asymptotic standard errors were computed using as estimate of the variance-covariance matrix the inverse of the Hessian of the log likelihood function at the maximum. In order to assess the robustness of the results to deviations from the assumption of normality, Quasi-Maximum Likelihood standard errors [White (1982)] were also computed and used in statistical inference. A well-known feature of nonlinear models is that their log likelihood function might present numerous local maxima. In order to address this issue, the robustness of the global maximum was examined using the method of simulated annealing. This approach is very efficient computationally and by linking different points in the domain through a Markov chain insures that ultimately any point on the surface will be visited. For more details the reader is referred to Judd (1998, ch. 8.3).

Parameter estimates for Canada, Sweden and the United Kingdom are reported in Tables 6, 7 and 8, respectively. Note that this results are based on the targeted measure of inflation, that in the case of Canada and the United Kingdom is calculated using a price index that excludes certain CPI components. (Section 3.5 reports estimates based on broader inflation measures). Several results are apparent from these tables. *First*, estimates are robust to assuming that the public forecasts unemployment using a stationary or a nonstationary model. This result is not surprising because the one-step-ahead predictions of a persistent variable computed using a nonstationary or a persistent stationary process are likely to be very similar. The fact that estimates of the θ-coefficients are different simply reflects the fact that their interpretation is different in each model. For example, in the stationary specification, θ_1 is the coefficient of lagged unemployment, while in the nonstationary specification, it is the coefficient of the lagged first-difference of unemployment.

Second, estimates of γ vary substantially across countries. As expected from the empirical analysis in section 3.2 and Figure 5, the estimate of γ is negative and statistically different from zero for Canada but positive for Sweden and the United Kingdom. Consider first the results for Canada. The conclusion that γ is negative and statistically different from zero does not depend on the estimate of the standard error employed to construct the t-statistic. This result indicates that unemployment is helpful in forecasting the rate of inflation (as expected) but in an opposite direction as predicted by the model. Thus, it would appear that this simple game-theoretical model of monetary policy might not completely capture the statistical relation between inflation and unemployment in the case of Canada.

Consider now the results for Sweden. In this case, γ is positive but whether it is statistically different from zero or not depends on the estimate of the standard error employed to construct the t-statistic. A Likelihood Ratio test of the restriction $\gamma = 0$ yields a statistic of 0.62 that is smaller than the 5% critical value of chi-square variable with 1 degree of freedom.[22] Hence, the restriction cannot be rejected at the 5% significance level. The finding that unemployment is not helpful in forecasting inflation is still consistent with a version of the model where the central banker targets the natural rate of unemployment. Since the sample size for Sweden is the smallest among

[22]This statistic, and the one reported for the United Kingdom below, were obtained using the stationary model for the rate of unemployment. Conclusions using the unit-root model are the same.

the countries considered, it is also possible that this result simply reflects the larger uncertainty regarding the model parameters that is associated with small samples.

Finally, consider the results for the United Kingdom. As in the case of Sweden, the statistical significance γ depends on the estimate of the standard error employed to construct the t-statistic. However, a Likelihood Ratio test of the restriction $\gamma = 0$ yields a statistic of 15.26 that is larger than the 5% critical value of a chi-square variable with 1 degree of freedom. Hence, the restriction is rejected at the 5% significance level. The result that $\gamma > 0$ is not meant to suggest that the Bank of England targets a rate of unemployment below the natural rate, but simply that unemployment is helpful in forecasting the inflation rate in a manner consistent with the simple game-theoretical model. LM tests of the overidentifying restrictions are reported in section 3.5.

Third, estimates of the preference parameter α are positive and quantitatively large: 4.54 for Canada, 3.42 for Sweden, and 2.64 for the United Kingdom. This means that for the countries under consideration, positive inflation deviations from the target appear to be weighted more severely than negative ones in the central banker's loss function, even if they are of the same magnitude. In the case of Canada, the null hypothesis that the true preference parameter is zero is rejected at the 1% significance level against the two-sided alternative that it is different from zero.[23] In the case of the United Kingdom, the hypothesis is rejected at the 1% level when one computes the t-statistic using the robust standard error, and at the 10% when one employs the asymptotic standard error. In the case of Sweden, the hypothesis is rejected at the 1% level when the t-statistic is computed using the robust standard error but cannot be rejected when one uses the asymptotic standard error. When one considers the test of the same hypothesis ($\alpha = 0$) against the one-sided alternative that $\alpha > 0$, the null hypothesis is rejected in all cases at the 10% level (1% level in most cases). However, given the limited sample sizes currently available to estimate inflation target models, these results should be interpreted with caution.

The result that $\alpha > 0$ for Canada and Sweden was to some extent anticipated by the sample statistics reported in Table 3. However, recall that for the United Kingdom, these statistics revealed a small, but positive, average inflation deviation from target (0.15) and a substantial proportion of observations above target (69.9%). The result that α is positive and significantly different from zero for the UK means that the average inflation deviation from target is too low to be consistent with a model with quadratic preferences.

Figure 6 plots the central banker's loss functions implied by the estimates of the preference parameter α and compares them with the usual quadratic loss function (dotted line). Although there are some differences in the numerical estimates of α for the three countries, their loss function are similar in the range of interest. In all cases, negative deviations from the target are penalized much less than under a quadratic loss function. Small positive deviations from the target (say between 0 and 0.5) are penalized roughly in the same manner in all loss functions, including the

[23]Even though the limit of the log-likelihood function as $\alpha \to 0$ exists, strictly speaking the function is not continuous at the point $\alpha = 0$. To circumvent this problem, I have used Wald-type t-tests to assess the significance of $\hat{\alpha}$.

quadratic. Large positive deviations from the target are penalized much more severely in the estimated asymmetric loss functions than in the quadratic.

Under asymmetric preferences the marginal cost of departing from the target is convex when $\alpha > 0$. Hence, uncertainty raises the expected marginal cost of deviating from $\tilde{\pi}_t$ and induces a prudent behavior on the part of the central banker. The analytical counterpart of this prudence motive is given by the term $-(\alpha/2)\sigma_\pi^2$ in (12). This term increases (in absolute value) with the conditional variance of inflation and the asymmetry in preferences. Prudence moderates the central banker's inflation bias (if there is any) and for large enough values of either α or σ_π^2, can override it. Then, a prudent central banker would pursue a more conservative monetary policy than her quadratic counterpart, in the sense that average inflation is lower. Depending on the preference parameters, it might be possible to observe realizations of inflation that are below target for seemingly long periods of time.

From the above discussion it follows that, for a given preference parameter $\alpha > 0$, countries with a larger conditional variance of inflation, should have a lower (that is, more negative) average inflation deviation from the target. Although the number of countries in the sample is too small to allow a fully-fledged cross-section analysis, one can still verify graphically to what extent this prediction is supported by the data. Figure 7 plots the relation between the conditional variance of inflation and the average deviation from target. Notice that Sweden has the largest conditional variance of inflation (1.14) and the smallest average deviation from the target (-1.23). On the other hand, Canada and the United Kingdom have much smaller conditional variances of inflation (0.129 and 0.130, respectively) and larger (that is, less negative) average inflation deviations from the target (-0.63 and 0.15, respectively). The larger conditional variance of inflation in Sweden and the prudence motive that arises under asymmetric preferences, could explain why average $\pi_t - \tilde{\pi}_t$ is much lower in Sweden than in Canada and the United Kingdom, even though the estimate of their preference parameter are similar.

E. Specification Tests and Robustness Analysis

This section reports specification tests of the model, examines the robustness of the results to using broader measures of inflation and a different estimation procedure, and discusses other theories that could also account for the empirical results.

Because inflation depends on the unemployment forecast, the model imposes overidentifying restrictions on the joint process of inflation and unemployment. These restrictions were tested by means of a Lagrange Multiplier test. The alternative specification was constructed to nest the model as a special case and differed from the null in that the unemployment coefficients in (20) were unrestricted. Results for both unemployment models and all countries are reported in the first row of Table 9. Under the null hypothesis, the LM test statistics is asymptotically distributed chi-square with as many degrees of freedom as restrictions are tested. Comparing the statistics with the 5% critical value of the appropriate distribution indicates that the restrictions cannot be rejected at the 5% significance level. However, since the small-sample distribution of the statistic might differ from the asymptotic one, these results are best regarded as indicative.[24]

The inflation equation (20) allows for the intercept term $-\alpha\sigma_\pi^2/2$. Since α is also a coefficient of the unemployment forecast, and σ_π^2 appears in the variance-covariance matrix of the residuals, this intercept is restricted. The restriction is tested by means of a LM test where the alternative allows for a free intercept. Results are reported in the second row of Table 9. Under the null hypothesis, the LM statistic is asymptotically distributed chi-square with 1 degree of freedom. Comparing the statistics with the 5% critical value leads to the conclusion that the restrictions imposed by the model on the intercept cannot be rejected at the 5% significance level.

Less favorable to the model are the tests for serial correlation of the inflation residuals reported in rows 3 and 4 of Table 9. Comparing the Durbin-Watson test statistics with the upper and lower bounds of 5% critical value of its tabulated distribution leads to the rejection of the null hypothesis of no serial correlation in favor of the alternative of positive serial correlation. Similarly, Portmanteau tests for the first order autocorrelation of the residuals yield statistics that are well above their 5% critical value. (Under the null hypothesis of no autocorrelation, the test statistic is distributed chi-square with as many degrees of freedom as autocorrelations are tested for.) However, note that since (20) does not include lagged inflation rates in the right-hand side, serial correlation reduces the efficiency, but does not affect the consistency of the FIML estimates reported above.

Consider the case where the conditional variance of inflation is assumed to be a function of lagged squared residuals, as in the ARCH model proposed by Engle (1982). In this case, lagged squared rates of inflation are implicitly included among the explanatory variables. Since residuals are serially correlated, estimates are likely to be biased and inconsistent.[25] In preliminary work, I parameterized the conditional variance of inflation as function of lagged squared changes in the oil price, that could be plausibly assumed to be exogenous [see Hamilton (1983)]. However, its coefficient is not statistically different from zero and results are similar to the ones obtained under the assumption of conditional homoskedasticity.[26]

One could also consider a more general model with asymmetric preferences over both inflation and unemployment. The model solution differs from (20) in two ways. *First*, although the relationship between π_t and u_t is still positive and nonlinear, the functional form is slightly different. *Second*, the conditional variance of unemployment becomes one of the arguments of the nonlinear function along with $E_{t-1}u_t$. Since the tests for neglected ARCH reported in Table 5, indicate that unemployment is conditionally homoskedastic, this amounts to including a second constant term in the nonlinear function. The restricted intercept $-(\alpha/2)\sigma_\pi^2$ remain unchanged. Hence,

[24]For the case of Sweden, results might also reflect low power because the coefficient of unemployment is very imprecisely estimated.

[25]Monte-Carlo experiments (not reported) indicate that when the residuals are positively autocorrelated, the estimate of the preference parameter α is biased downwards. This explains the results in a previous version of this paper, where only mild evidence of preference asymmetry was reported.

[26]This results are not reported to save space, but are available from the author upon request.

this generalization of the model, does not appear to fundamentally alter the model predictions. In related research, Ruge-Murcia (2001) estimates a model with asymmetric unemployment preferences for the G7 countries. Reduced-form estimates do not support the hypothesis as asymmetric unemployment preferences for Canada and the United Kingdom.

This paper relaxes the usual linear-quadratic framework in a particular dimension. That is, it relaxes the assumption of a quadratic objective function but preserves the linear constraint (the expectations-augmented Phillips curve). Alternatively, one could consider a model where the objective function is quadratic but the supply function is nonlinear. This is the strategy followed by Nobay and Peel (2000). These authors show analytically that the nonlinearity of the supply schedule yields ambiguous implications for average inflation. Numerical simulations indicate that a convex supply function produces upward, rather than downward, inflation deviations from the target. Only a less-plausible, concave supply function yields deviations consistent with observed inflation.

In order to assess the robustness of the results with respect to the measure of inflation and examine if central banker's preferences might be different when cast in terms of broader inflation measures, the models were also estimated for Canada and the United Kingdom using CPI and RPI inflation, respectively. Results are reported in Tables 10 and 11. In contrast to previous results, in this case it is not possible to reject the null hypothesis of quadratic preferences. It appears that although the central banker treats asymmetrically the targeted inflation deviation from its goal, the loss associated with a headline inflation deviation depends primarily on its magnitude and not on its sign.

The assumption of normality is useful in deriving closed-form analytical results. However, it is possible that in reality the distribution of the shocks deviates in an important manner from normality. Whether misspecification in the distribution of the shocks could affect the results is examined in three ways. *First,* Quasi-Maximum Likelihood (QML) standard errors robust to deviations from normality were also computed and used in statistical inference. As reported in the preceding section, t-tests based on QML standard errors tend to reject the null hypothesis of quadratic preferences more strongly than those based on asymptotic standard errors.

Second, the model is estimated by the Generalized Method of Moments (GMM), that does not require explicit assumptions regarding the model disturbances.[27] To be precise, I estimate a version of the first-order condition of the central banker's problem [eq. (7)] that does not exploit the assumption of normality, and use $E_{t-1}u_t^n = E_{t-1}u_t$ to substitute out the unobserved natural unemployment rate:

$$E_{t-1}[(\exp(\alpha(\pi_t - \tilde{\pi}_t)) - 1)/\alpha - \gamma u_t] = 0,$$

[27] The idea of using GMM to estimate the model was suggested to me by René Garcia.

where, as before, $\gamma = \lambda\phi(1-k)$. The instruments are a constant and two unemployment lags. GMM results are reported in Table 12. Note that in all cases, $\hat{\alpha}$ is positive and statistically different from zero. In the case of Canada, the GMM estimate of this preference parameter is numerically similar to the FIML estimate. In the case of Sweden and the United Kingdom, the GMM estimate of α is smaller than, but still consistent with, the FIML estimate. In all cases, $\hat{\gamma}$ is positive and significantly different from zero. The last row of Table 12 reports the chi-square statistic of the test of the overidentifying restriction. Under the null hypothesis this statistic is distributed chi-square with 1 degree of freedom. The overidentifying restriction is not rejected at the 5% level for Canada or Sweden. In the case of the United Kingdom, it is rejected at the 5% level but not at the 1% level. In summary, GMM delivers qualitatively the same results as those obtained using FIML.

Finally, a Monte-Carlo study is used to assess whether the finding $\hat{\alpha} > 0$ can be the result of asymmetric shocks rather than asymmetric preferences. The strategy is to generate artificial data using the quadratic model but with shocks drawn for an asymmetric distribution. Then, an asymmetric-preference model is estimated under the (incorrect) assumption that shocks are normally distributed. The question asked is whether a researcher would be more likely to find $\alpha \neq 0$ in these circumstances. The data is generated by the following quadratic specification:

$$u_t = 0.5 + 0.9u_{t-1} + w_t,$$

and

$$\pi_t - \tilde{\pi}_t = 0.8E_{t-1}u_t + \epsilon_t,$$

where $w_t \sim N(0, 0.2^2)$. The disturbance ϵ_t is drawn from an asymmetric chi-square distribution with 1 degree of freedom and is independent of w_t.[28] Since the mean of a chi-square distribution equals the number of degrees of freedom, the distribution of ϵ_t is centered around zero by substracting -1 from each draw. Recall that the variance of a chi-square distribution is twice the number of degrees of freedom. Thus, $\sigma_\pi = \sigma_\epsilon = \sqrt{2}$. The values of the parameters where chosen to be roughly on line with the estimates reported above. For example, the unconditional mean of unemployment is $0.5/(1-0.9) = 5\%$, and its process is highly persistent as suggested by the autocorrelations presented in Table 1. The R^2 associated with the unemployment process is calculated to be $R_u^2 = 0.9^2 = 0.81$. Experiments are based on 100 replications using sample sizes of 2000 and 200 observations. For the unemployment process, 100 extra observations were generated in every replication. Then, for the estimation of the model, the initial 100 observations were discarded in order to limit the effect of starting values used to generate the observations of u_t.

Monte-Carlo results are reported in Table 13. In both the small- and large-sample experiments, all parameter estimates are close to their true value. The type I error of the test that the parameters

[28]Recall that for all estimated models, the correlation coefficient between ϵ_t and w_t, namely $\rho_{\epsilon w}$, was found to be insignificantly different from zero.

take their true values are well within their 95% confidence interval around the nominal size of 5%. The only exception is the estimate of σ_π ($= \sigma_\epsilon$) where there is considerable overrejection regardless of the sample size. Focusing more precisely on the preference parameter, note that if the true model were quadratic ($\alpha = 0$), estimates using a misspecified normal distribution when the true distribution is asymmetric would be unlikely to lead to the conclusion that the central banker's preferences are asymmetric.

IV. Conclusions

This paper constructs a tractable game-theoretical model of monetary policy that permits asymmetries in the central banker's preferences. In particular, the central banker is concerned about both the sign and magnitude of inflation deviations from the desired rate. The preference specification is general in that it nests the standard quadratic function as a special case. It is shown that some of the conclusions derived under the assumption of symmetry are not robust to the functional form of the central banker's loss function. Quadratic preferences predict that the average inflation deviation from the target should be nonnegative and linearly related to unemployment. Since this overshooting is rationally anticipated by the public, the targeting policy can only be imperfectly credible. Asymmetric preferences predict that the average inflation deviation from the target depends on the conditional mean of inflation, is related nonlinearly to unemployment, and can be on positive or negative depending on the central banker's preference parameters. Theoretical results are based on the idea that under asymmetric preferences, certainty equivalence no longer holds and uncertainty can induce a prudent behavior on the part of the central banker.

The empirical predictions of the model are examined using data from three inflation-targeting countries, namely Canada, Sweden, and the United Kingdom. Results support the notion of asymmetric preferences in the form of a positive and statistically significant estimate of the asymmetry preference parameter. This suggests that departures from the linear-quadratic framework could be relevant in actual policy making and that inflation targeting might be a credible framework for the conduct of monetary policy. Empirical results basically reflect the fact that for these countries, inflation has been generally below target. This observation is inconsistent with quadratic preferences and with standard formulations of the Phillips curve (linear or convex).

However, in interpreting the empirical results it is very important to keep in mind other explanations that might contribute to the finding of asymmetry in preferences. Asymmetric shocks (of a form not considered above) coupled with long lags in monetary policy might yield persistent deviations from the target. The strategic interaction of the central banker and the government could affect the selection of the inflation targets themselves. Although this paper allows for the strategic interaction of the central banker and the public, equilibrium concepts other than Nash might be empirically important. For example, it could be argued that the persistent undershooting of inflation targets might be part of an effort by the central banker to show its commitment to the policy. Current and future research by the author seeks to address these observations. Still, given our limited understanding of central bankers' behavior and preferences, it is probably too early to dismiss the notion that prudence can play a role in modern monetary policy making.

Table 1. Autocorrelations of the Unemployment Rate

Autocorrelation	Canada	Sweden	United Kingdom
1	0.966	0.935	0.987
2	0.930	0.875	0.971
3	0.892	0.823	0.954
4	0.849	0.773	0.937
5	0.803	0.724	0.917
6	0.751	0.685	0.896
7	0.702	0.639	0.875
8	0.657	0.598	0.851
9	0.614	0.556	0.827
10	0.577	0.512	0.802

Notes: The statistics were calculated using quarterly, seasonally-adjusted data for the periods 1990:01 to 2000:06 (Canada), 1993:01 to 2000:06 (Sweden), and 1991:01 to 2000:06 (United Kingdom).

Table 2. Summary of Predictions

Prediction	Model	
	Quadratic	Asymmetric
Positive relation between $\pi_t - \tilde{\pi}_t$ and u_t	Yes	Yes
Nonlinear relation between $\pi_t - \tilde{\pi}_t$ and u_t	No	Yes
Average deviation from target is nonnegative	Yes	Not necessarily
Conditional variance helps forecast $\pi_t - \tilde{\pi}_t$	No	Yes

Table 3. Inflation Deviation from Target

	Canada	Sweden	United Kingdom
Average Deviation from Target	−0.61	−1.23	0.15
Observations Above Target (in %)	8.79	18.18	69.89
Observations Below Target (in %)	91.21	81.82	30.11

Notes: The statistics were calculated using monthly observations for the periods 1992:12 to 2000:06 (Canada), 1995:01 to 2000:06 (Sweden), and 1992:10 to 2000:06 (United Kingdom).

Table 4. Results of Two-Stage Least Squares Regressions

	Canada		Sweden		United Kingdom	
	Linear	Nonlinear	Linear	Nonlinear	Linear	Nonlinear
Intercept	0.23	-8.20^{**}	-1.99^{*}	-12.87^{*}	-0.40^{**}	-1.31^{**}
	(0.32)	(1.76)	(0.81)	(5.46)	(0.11)	(0.45)
u_t	-0.09^{**}	1.79^{**}	0.11	3.42^{*}	0.08^{**}	0.36^{**}
	(0.03)	(0.39)	(0.11)	(1.65)	(0.01)	(0.14)
u_t^2		-0.10^{**}		-0.24^{*}		-0.02^{*}
		(0.02)		(0.12)		(0.01)
R^2	0.07	0.27	0.02	0.07	0.23	0.25

Notes: The figures in parenthesis are standard errors. The superscripts ** and * denote the rejection of the null hypothesis that the coefficient is zero at the 1% and 5% significance levels, respectively.

Table 5. Tests of Unemployment Residuals

Test	Canada		Sweden		United Kingdom	
	(3, 0, 0)	(2, 1, 0)	(4, 0, 0)	(3, 1, 0)	(4, 0, 0)	(3, 1, 0)
DW	1.91	1.96	2.01	2.00	2.00	1.99
BG	1.43	1.25	2.91	2.84	2.47	1.93
ARCH	0.48	0.08	5.60	5.47	2.44	2.81

Notes: DW and BG stand for Durbin-Watson and Breusch-Godfrey, respectively. Under the null hypothesis of no serial correlation up to order 2, the Breusch-Godfrey test statistic is distributed χ^2 with 2 degrees of freedom. Under the null hypothesis of no conditional heteroskedasticity, the LM statistic for neglected ARCH is distributed χ^2 with 2 degrees of freedom. The superscripts ** and * denote the rejection of the null hypothesis at the 1% and 5% significance levels, respectively.

Table 6. FIML Estimates

Canada

Core Inflation

Parameter	Unemployment Model					
		(3,0,0)			(2,1,0)	
	Estimate	S.E.	Robust S.E.	Estimate	S.E.	Robust S.E.
α	4.54	(0.87)	[0.32]	4.53	(0.87)	[0.32]
γ	−0.02	(0.003)	[0.001]	−0.02	(0.003)	[0.001]
σ_π	0.36	(0.02)	[0.02]	0.36	(0.02)	[0.02]
ψ	−0.005	(0.13)	[0.10]	−0.08	(0.02)	[0.02]
θ_1	0.82	(0.10)	[0.10]	−0.18	(0.10)	[0.10]
θ_2	−0.11	(0.13)	[0.14]	−0.28	(0.10)	[0.10]
θ_3	0.28	(0.10)	[0.10]			
σ_w	0.21	(0.02)	[0.02]	0.21	(0.02)	[0.02]
$\rho_{\epsilon w}$	−0.04	(0.10)	[0.10]	−0.04	(0.10)	[0.11]
L		−133.39			−133.28	

Notes: S.E. and L are, respectively, the standard error and the value of the joint log-likelihood function at the maximum.

Table 7. FIML Estimates

Sweden

CPI Inflation

Parameter	Unemployment Model					
	(4, 0, 0)			(3, 1, 0)		
	Estimate	S.E.	Robust S.E.	Estimate	S.E.	Robust S.E.
α	3.42	(2.58)	[0.47]	3.42	(2.59)	[0.48]
γ	0.45	(2.97)	[0.08]	0.45	(2.98)	[0.09]
σ_π	1.07	(0.09)	[0.07]	1.07	(0.09)	[0.07]
ψ	−0.06	(0.22)	[0.21]	−0.04	(0.03)	[0.03]
θ_1	0.80	(0.12)	[0.15]	−0.20	(0.12)	[0.15]
θ_2	0.16	(0.15)	[0.18]	−0.04	(0.12)	[0.12]
θ_3	0.32	(0.15)	[0.17]	0.27	(0.12)	[0.12]
θ_4	−0.27	(0.13)	[0.13]			
σ_w	0.26	(0.02)	[0.02]	0.26	(0.02)	[0.02]
$\rho_{\epsilon w}$	0.05	(0.12)	[0.13]	0.05	(0.12)	[0.13]
L		−17.75			−17.75	

Notes: See notes in Table 6.

Table 8. FIML Estimates

United Kingdom

RPIX Inflation

Parameter	Unemployment Model					
		(4, 0, 0)			(3, 1, 0)	
	Estimate	S.E.	Robust S.E.	Estimate	S.E.	Robust S.E.
α	2.64	(1.52)	[0.57]	2.62	(1.51)	[0.56]
γ	0.08	(0.06)	[0.01]	0.08	(0.06)	[0.01]
σ_π	0.36	(0.03)	[0.03]	0.36	(0.03)	[0.03]
ψ	0.005	(0.03)	[0.03]	−0.03	(0.01)	[0.01]
θ_1	1.05	(0.10)	[0.11]	0.06	(0.10)	[0.11]
θ_2	0.35	(0.15)	[0.14]	0.40	(0.09)	[0.10]
θ_3	−0.23	(0.15)	[0.16]	0.17	(0.10)	[0.12]
θ_4	−0.18	(0.10)	[0.11]			
σ_w	0.08	(0.006)	[0.007]	0.07	(0.006)	[0.006]
$\rho_{\epsilon w}$	0.14	(0.11)	[0.15]	0.10	(0.11)	[0.15]
L		−244.42			−243.76	

Notes: See notes in Table 6.

Table 9. Tests of Model Restrictions and

Serial Correlation of Inflation Residuals

Test	Canada		Sweden		United Kingdom	
	(3, 0, 0)	(2, 1, 0)	(4, 0, 0)	(3, 1, 0)	(4, 0, 0)	(3, 1, 0)
LM (Overidentifying)	1.00	0.73	0.06	0.05	6.13	5.80
LM (Intercept)	1.83	1.67	0.0002	0.0001	0.20	0.22
DW	0.33**	0.33**	0.09**	0.09**	0.31**	0.31**
Portmanteau	61.37**	62.34**	57.04**	57.04**	58.36**	58.36**

Notes: The number of overidentifying restrictions tested for Canada, Sweden, and the United Kingdom are 3, 4, and 4, respectively, for the stationary unemployment model, and 2, 3, and 3, respectively, for the unit-root model. The superscripts ** and * denote the rejection of the null hypothesis at the 1% and 5% significance levels, respectively.

Table 10. FIML Estimates

Canada

CPI Inflation

Parameter	Unemployment Model					
	(3, 0, 0)			(2, 1, 0)		
	Estimate	S.E.	Robust S.E.	Estimate	S.E.	Robust S.E.
α	-0.25	(0.75)	[0.19]	-0.25	(0.75)	[0.20]
γ	-0.10	(0.06)	[0.01]	-0.10	(0.06)	[0.01]
σ_π	0.81	(0.06)	[0.06]	0.81	(0.06)	[0.06]
ψ	-0.11	(0.17)	[0.17]	-0.08	(0.02)	[0.02]
θ_1	0.81	(0.10)	[0.11]	-0.19	(0.10)	[0.11]
θ_2	-0.11	(0.12)	[0.13]	-0.30	(0.10)	[0.10]
θ_3	0.30	(0.10)	[0.10]			
σ_w	0.21	(0.02)	[0.02]	0.21	(0.02)	[0.02]
$\rho_{\epsilon w}$	0.17	(0.11)	[0.10]	0.16	(0.10)	[0.10]
L		-71.40			-71.39	

Notes: See notes in Table 6.

Table 11. FIML Estimates

United Kingdom

RPI Inflation

Parameter	Unemployment Model					
		$(4,0,0)$			$(3,1,0)$	
	Estimate	S.E.	Robust S.E.	Estimate	S.E.	Robust S.E.
α	-1.46	(1.05)	[0.74]	-1.38	(1.01)	[0.77]
γ	-0.07	(0.09)	[0.05]	-0.06	(0.08)	[0.05]
σ_π	0.76	(0.06)	[0.04]	0.76	(0.06)	[0.04]
ψ	-0.003	(0.03)	[0.03]	-0.02	(0.01)	[0.01]
θ_1	1.06	(0.10)	[0.10]	0.06	(0.10)	[0.10]
θ_2	0.37	(0.15)	[0.14]	0.43	(0.09)	[0.11]
θ_3	-0.22	(0.15)	[0.15]	0.21	(0.10)	[0.12]
θ_4	-0.22	(0.10)	[0.11]			
σ_w	0.08	(0.006)	[0.007]	0.08	(0.006)	[0.006]
$\rho_{\epsilon w}$	0.16	(0.10)	[0.09]	0.16	(0.10)	[0.09]
L		-174.15			-173.80	

Notes: See notes in Table 6.

Table 12. GMM Estimates

Parameter	Canada		Sweden		United Kingdom	
	Estimate	S.E.	Estimate	S.E.	Estimate	S.E.
α	4.13	(2.01)	0.74	(0.23)	1.57	(0.34)
γ	0.006	(0.002)	0.037	(0.018)	0.050	(0.007)
Chi-square statistic	0.55		0.02		6.61	

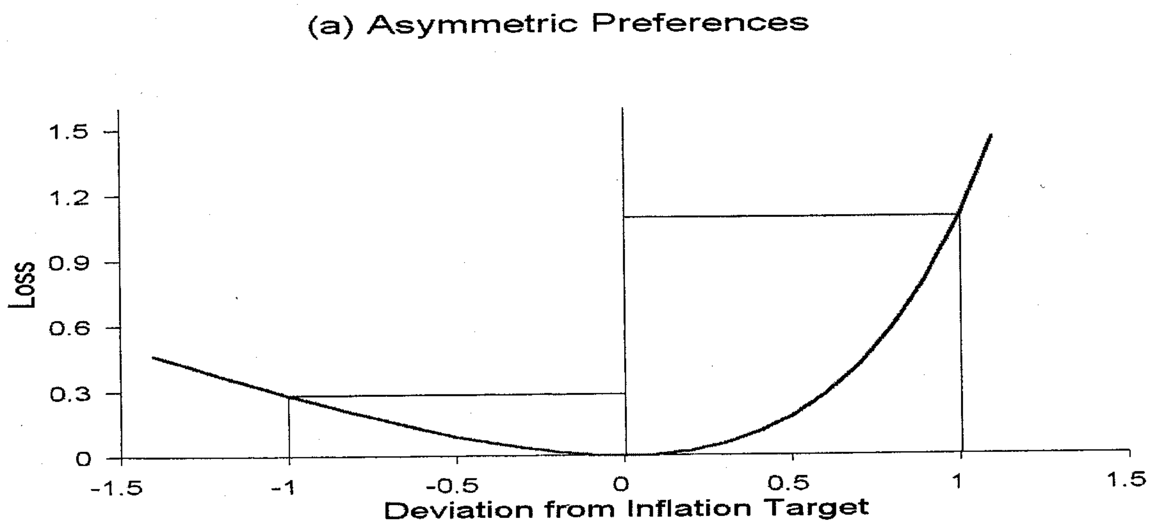
Notes: The chi-square statistic is distributed chi-square with 1 degree of freedom.

Table 13. Monte Carlo Results

Parameter	True Value	Estimate	S.E.	Type I Error	S.E.
			$n = 200$		
α	0	0.011	(0.209)	0.03	(0.017)
γ	0.8	0.953	(0.679)	0.08	(0.027)
σ_π	1.41	1.388	(0.070)	0.45	(0.050)
ψ	0.5	0.586	(0.166)	0.02	(0.014)
θ_1	0.9	0.883	(0.033)	0.02	(0.014)
σ_w	0.20	0.199	(0.010)	0.06	(0.024)
			$n = 2000$		
α	0	0.0005	(0.059)	0.03	(0.017)
γ	0.8	0.811	(0.113)	0.04	(0.020)
σ_π	1.41	1.414	(0.022)	0.04	(0.049)
ψ	0.5	0.507	(0.049)	0.03	(0.017)
θ_1	0.9	0.899	(0.010)	0.03	(0.017)
σ_w	0.20	0.199	(0.003)	0.05	(0.022)

Notes: n is the sample size. The experiment was based on 100 replications. The nominal size of the test was taken to be 0.05.

Figure 1: Preferences



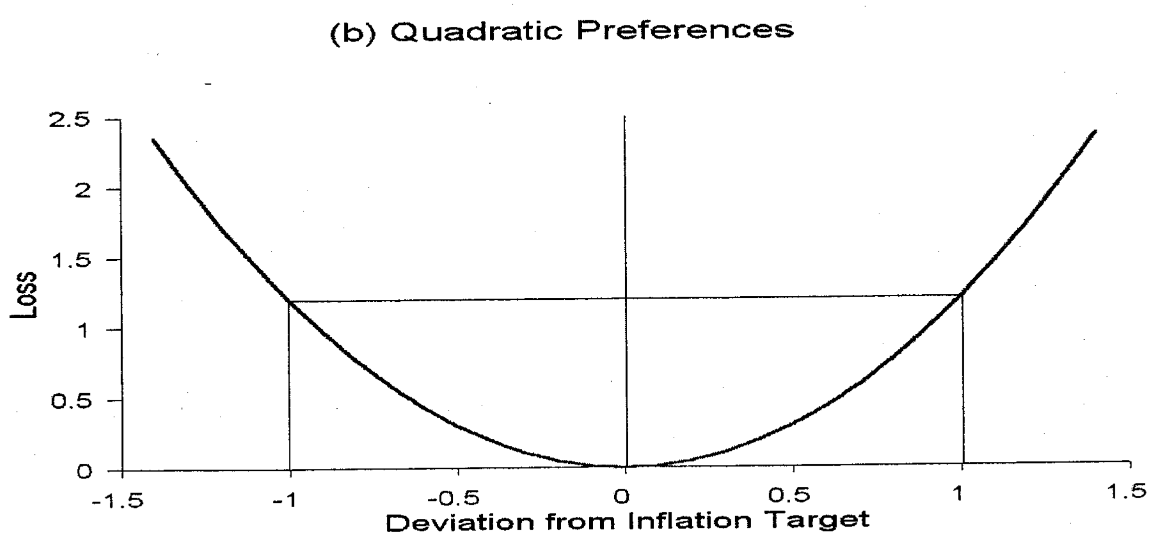


Figure 2: Reaction Functions

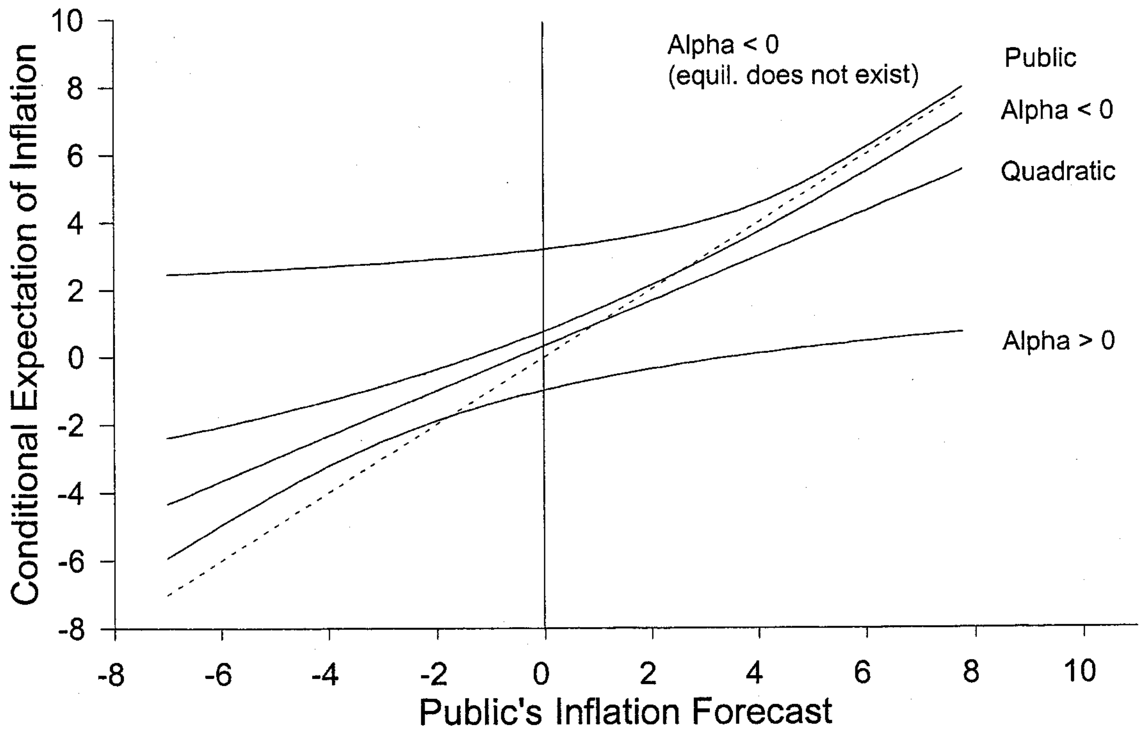


Figure 3: Difference in Predicted Inflation Deviation from Target

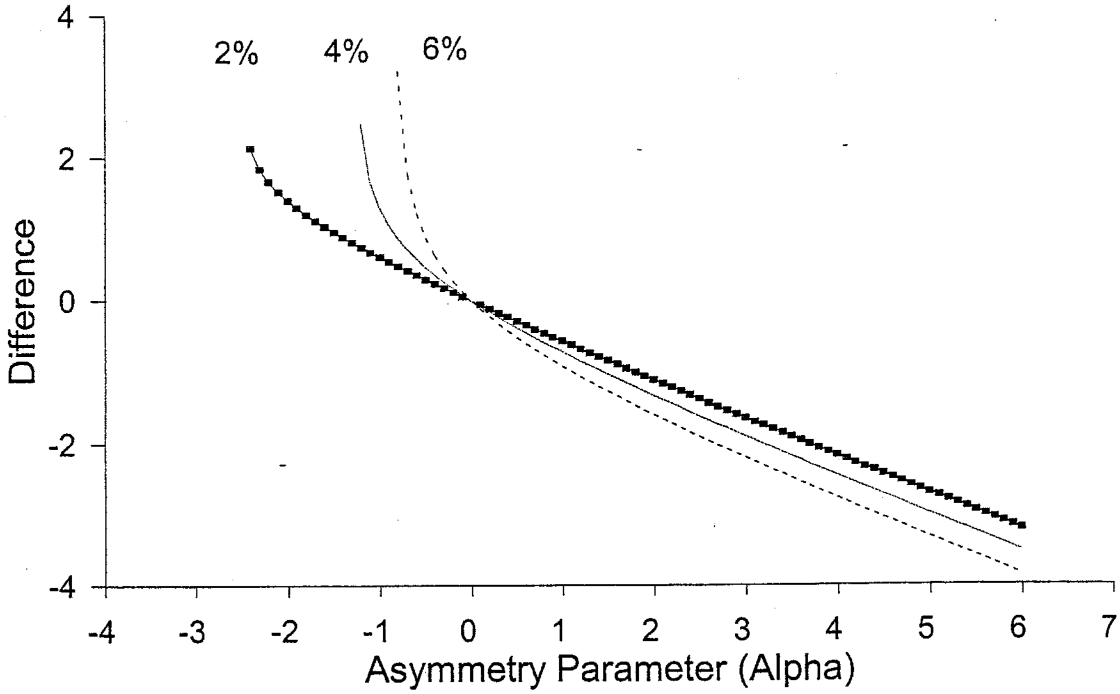


Figure 4: Inflation Rates

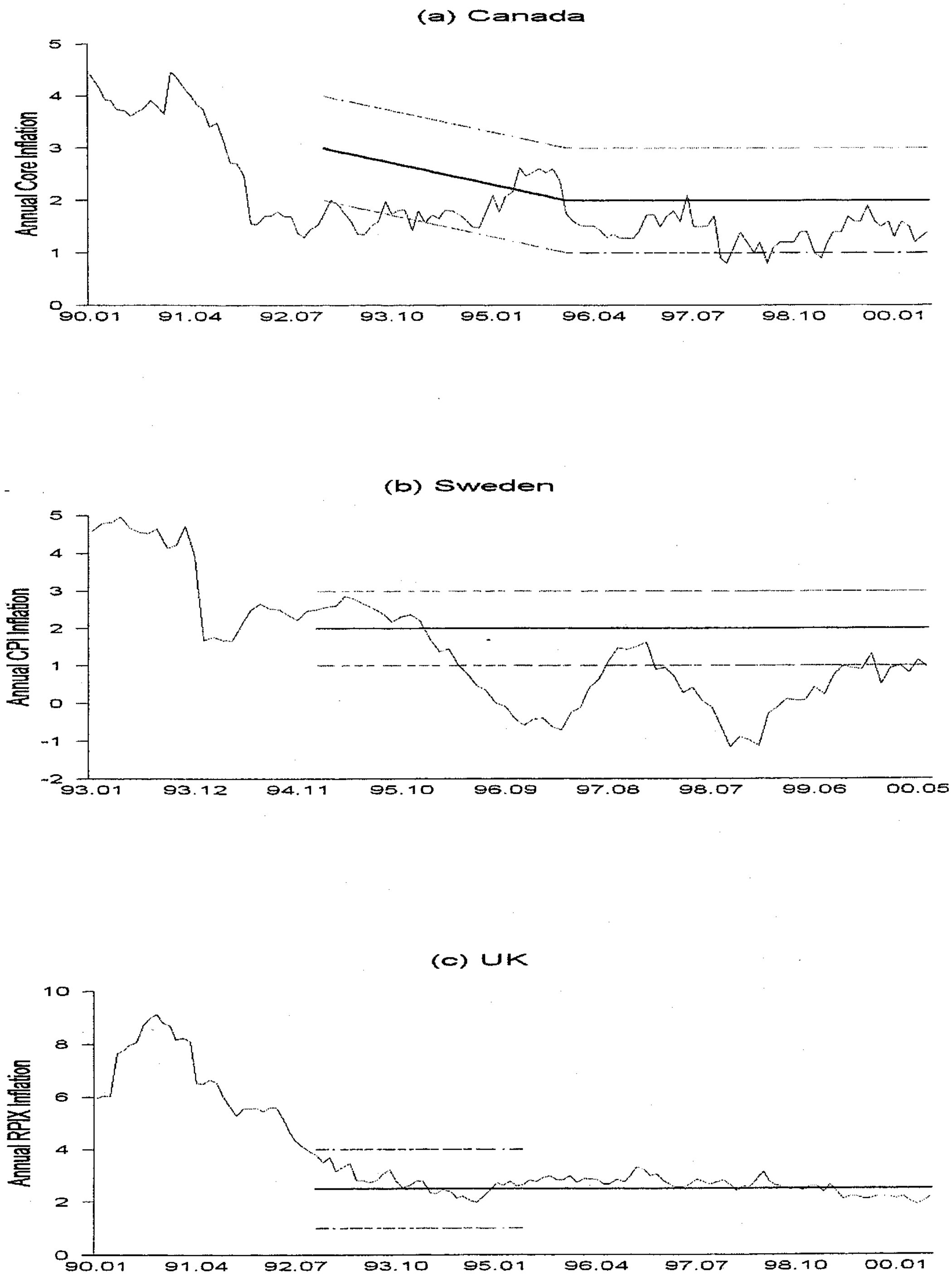


Figure 5: Unemployment and the Inflation Deviation from Target

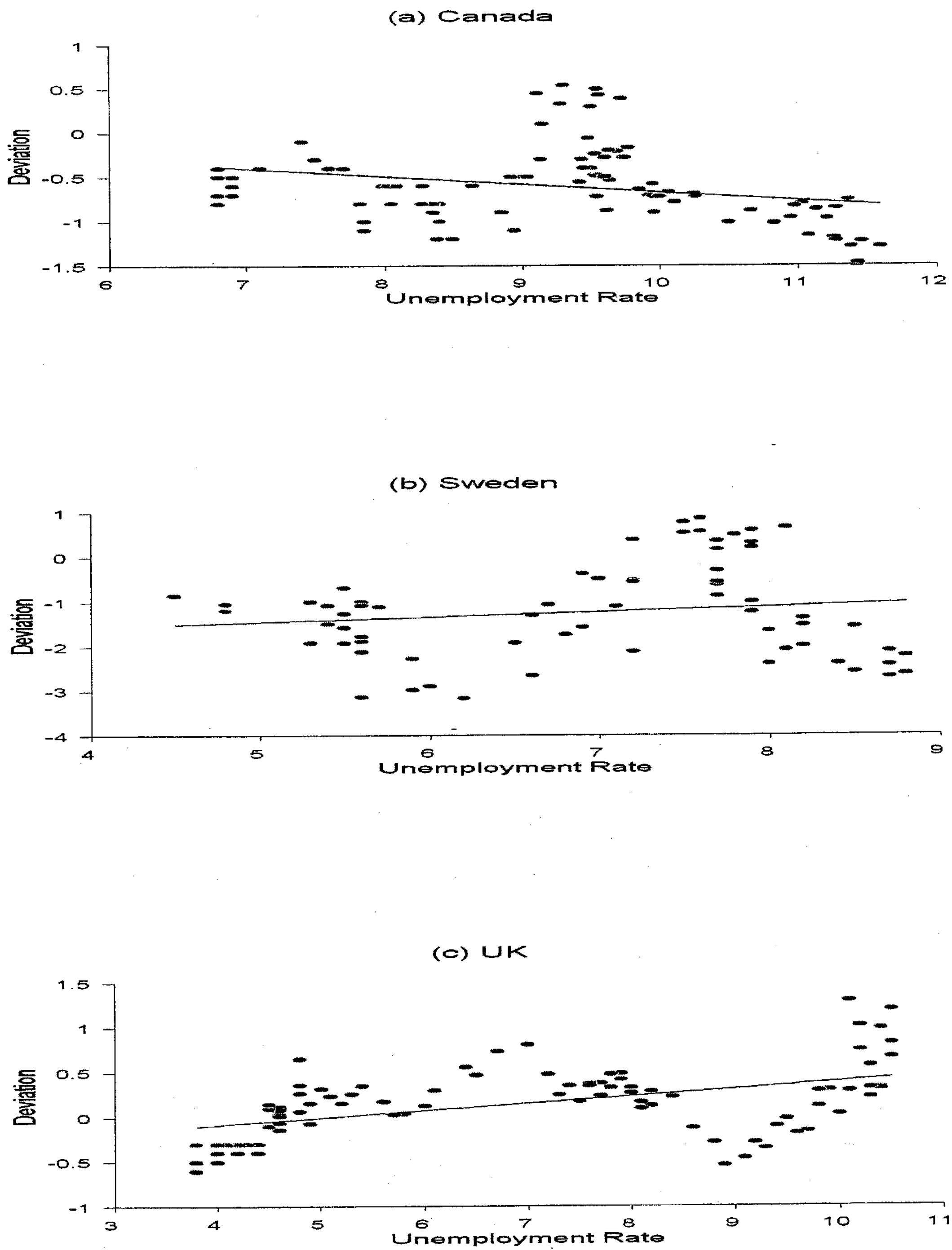


Figure 6: Estimated Preferences

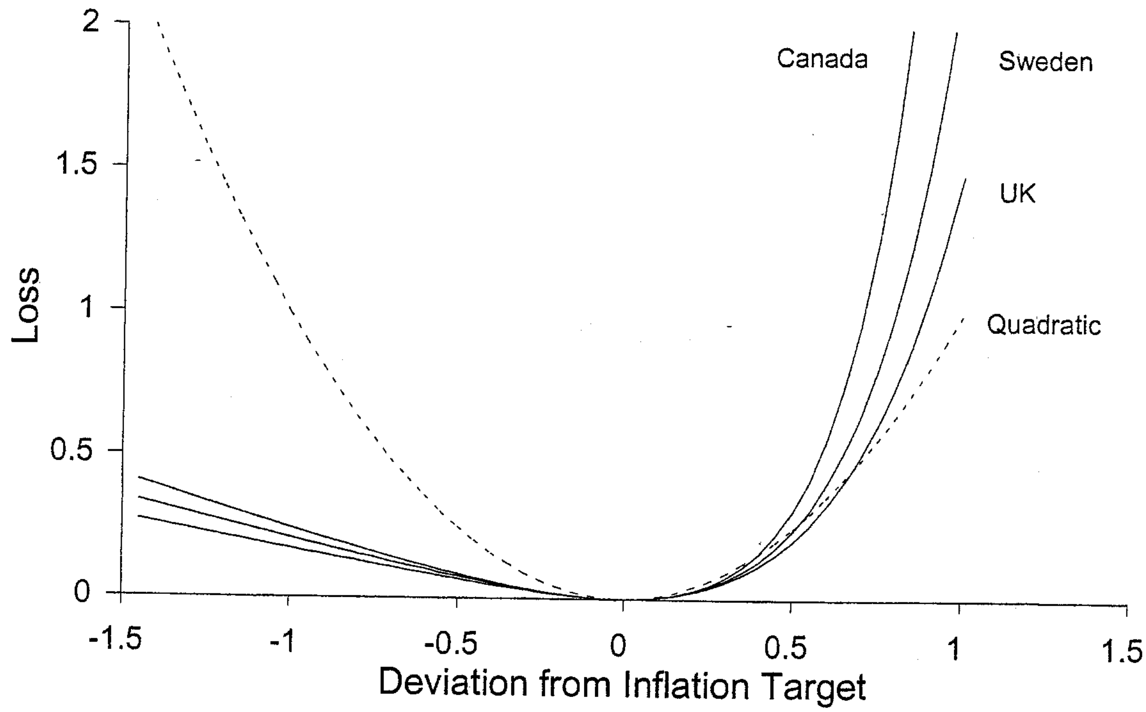
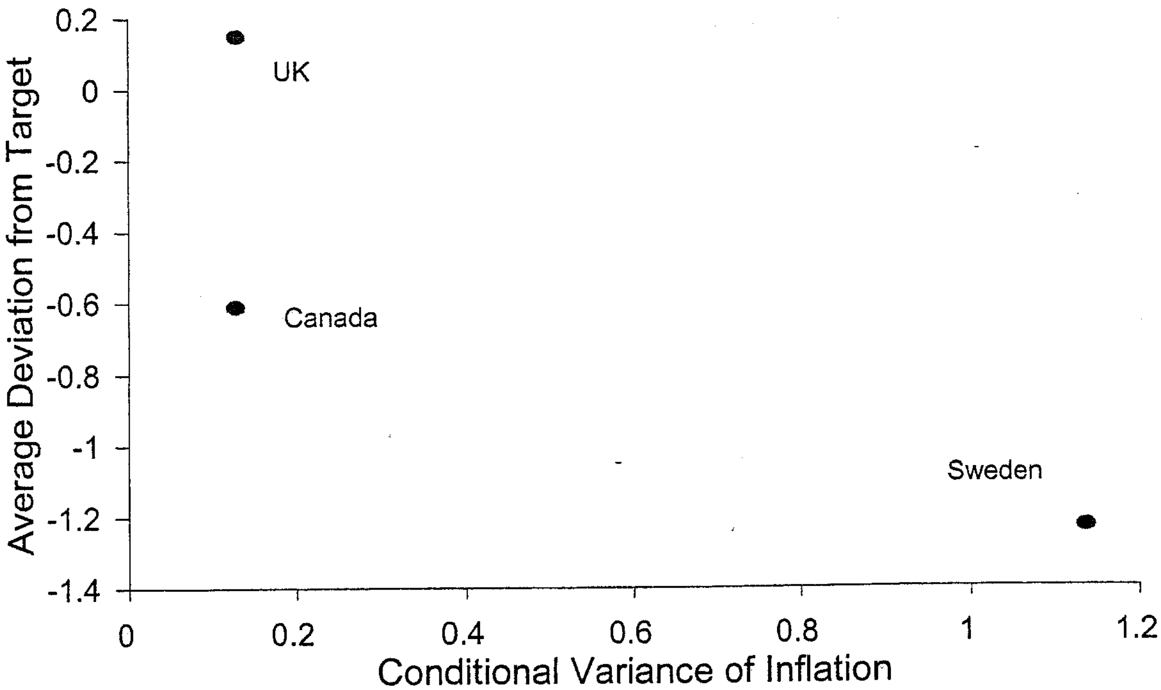


Figure 7: Conditional Variance and the Inflation Deviation from Target



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