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Counterfeit Goods and Income Inequality

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Abstract

This paper examines the effect of counterfeit goods in a world where consumers are differentiated by level of income and innovation is quality enhancing. Counterfeit goods are defined as products with the same characteristics as "originals", but of lower quality. The effect of imitation on firms' profits and consumer welfare depends on the distribution of income within the country. In particular, the greater the level of income inequality the larger the increase in consumer welfare due to the imitation, and the smaller the effect on profits of the state-of-the-art firm.

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I. Introduction

The standard argument for intellectual property rights (IPR) protection is that there is a trade-off between the need to concede monopoly power as an incentive for innovation, and the loss in consumer surplus such a concession entails. From strictly the consumers' point of view, imitation is a good thing mainly because it reduces such monopoly power.

Society wishes to encourage innovation to promote the introduction of new products and more efficient technologies, and conceding a certain amount of monopoly power to successful firms is necessary to induce them to take on the heavy R&D investments that are often required. Allowing imitation, however, increases the dissemination of knowledge, and makes available to consumers the cheaper, lower cost varieties. IPR protection regimes are an attempt to achieve an equilibrium between these contrasting objectives.

In an open economy the issue of IPR protection becomes more complex, in that the efficient level of global protection seldom maximizes the interests of the single governments that must actually apply the protection. Innovative activity in the global economy entails large international profits, but how different countries choose to capture these profits depends on their level of technological development. A more developed country will try to maximize the probability of being a winner in the innovation race, encouraging its firms to engage in R&D through strong IPR protection and strategic intervention policies. This approach is impractical for less developed countries with little or nonexistent innovative activity: promoting imitation, so as to capture at least a part of the international profits associated with others' research, is often the strategy of choice.

A common opinion in the international arena is that although strong IPR protection is not in the short term interest of developing countries, one must take into consideration the dynamic benefits associated with innovation. The introduction of new products and the reduction of production costs entail an increase in consumer surplus worldwide, independently from where the innovation takes place and which country is receiving the monopoly profits. Consequently, given that IPR protection serves as an incentive to R&D, developing countries must realize that they are faced with the following trade-off: weak IPR protection allows them to appropriate part of the innovation profits, however it reduces R&D incentives in developed countries, with negative effects on global welfare. A short-term vision tends to highlight only the benefits of imitation, at the expense of the future international benefits of innovation.

Exporting firms in advanced economies have been extremely vocal regarding profit losses in foreign markets with inadequate protection. The U.S. Trade Representative, for example, each year comes out with a "hit list" of countries with inadequate IPR protection, and with estimates of industry losses in different markets. In the international arena, and within the WTO framework in particular, the achievement of international harmonization of IPR protection is being hotly debated. Developing countries are concerned that they will reap none of the benefits of such increased protection, and experience only a loss in the benefits associated with imitation.[2] This paper seeks to shed light on this issue, by examining the effect of the introduction of counterfeit goods in economies with unequal income distribution.

Counterfeit goods, characterized by infringement of IPRs and usually of inferior quality with respect to the original good, have three main effects: 1) they reduce incentives for innovation, by reducing the payoff to successful innovation; 2) they "cheat" consumers into buying lower quality goods; 3) they lower the status of high quality, original goods. The later two effects have been examined extensively by Grossman and Shapiro (1988a and 1988b). This paper will concentrate on the case of counterfeit goods that are recognized as such by consumers (i.e., perfect information) in markets where status plays no role in consumer preference.

Consider a world in which consumers differ in income, and therefore in their consumption choices.[3] If all qualities were offered at identical cost, all consumers would prefer the highest quality good. Given instead differing prices (in particular, higher quality goods offered at higher prices), consumers will split into different groups, with the lowest income brackets choosing lower quality goods, and higher income bracket consumers choosing higher quality. In this scenario, competition from imitation goods will force firms producing state-of-the-art products to lower price. Consumer gain is twofold: poorer consumers obtain a higher quality good, and richer consumers obtain the high quality product at a lower price.

Technological development takes place by quality increments (this will allow for vertical product differentiation). Unit costs are the same for all qualities of the good, given they have been discovered. The discoverer of a new product is granted a patent for the duration of a period, and therefore has monopoly power. The follower good, for which the patent has expired, will be freely produced by whoever wishes to, and therefore its market will operate under conditions of perfect competition and price will be equal to marginal cost.

Define *counterfeit goods* as imitated goods that drive a wedge between the state-of-the-art variety and the older generation varieties, i.e. a counterfeit good provides strictly more services than the older generation good, but strictly less than the state-of-the-art. Depending on the actual quality of this good (i.e., whether it is closer to that of the state-of-the-art or of that of the follower good) and the price at which it is sold (marginal cost or higher) it will

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[2]See for example Diwan and Rodrik (1991) and Lanjouw (1998) for differing views on this issue.

[3]Analogously, one could consider a world in which consumers differ in their preference for quality. However, the assumption of differing income is most in line with existing literature on the economic theory of product differentiation, and furthermore will permit considerations particular to income inequality.

"steal" part of the market share of either the lesser quality good or of the state-of-the-art. Either way, consumer surplus in the economy increases: either poorer individuals consume a better product at a low enough price, or richer consumers choose a slightly poorer quality good for a much lower price. In every case, the monopolist loses.

In general, the paper finds that counterfeit goods have the following effects on society. Firstly, the price charged for state-of-the-art goods will decrease. Secondly, consumer welfare will increase while firms' profits decrease. Lastly, the magnitude of these effects will depend on the income distribution of the economy. In particular, the more skewed the distribution is towards the poor, i.e., the greater the degree of income inequality, the greater the welfare effect and the smaller the profit effect.

This last result is the most interesting from a policy point of view. The income distribution within the economy will determine the extent of the effect of the presence of counterfeit goods, in particular the effect on firms' profits and on consumers' welfare. This will affect the trade-off faced by different countries in deciding their IPR regime: the greater the income inequality, the greater the gains to consumers, and the smaller the losses to producers. It follows that the degree of income inequality should be an important component in deciding the optimal level of IPR protection for any country, and should be considered when negotiating internationally acceptable IPR standards.

This paper builds on three different strands of literature. Firstly, it is closely tied to the literature on markets for vertically differentiated goods, as in Gabszewicz and Thisse (1979) and various papers by Shaked and Sutton (1982, 1983, 1987). Beath and Katsoulacos (1991) give a useful review of the literature in the area. Markets for vertically differentiated products in an international trading environment have been examined by Flam and Helpman (1987) and Glass (1997), among others.

Secondly, although this paper focuses on the demand side of the market for vertically differentiated products, it is nonetheless closely related to the literature on patent races, as in Loury (1979), Lee and Wilde (1980), Reinganum (1982) and Dixit (1988). The seminal paper in North-South trade in innovation goods, also based on the patent race literature, is Grossman and Helpman (1991).

Lastly, this paper makes use of a vast literature on income distribution and inequality representation. Van der Linden and Manders (1999) give an idea of the controversy in the field of the economics of income distribution. Basman, Hayes and Slottje (1993) present traditional and modern methods of measuring and describing economic inequality. Merkies and Steyn (1993) present new insights on Pareto Laws, a traditional way of representing income inequality.

The paper is organized as follows. Firstly, the supply side of the model is introduced, presenting a patent race for goods such that technological development is quality enhancing. The market structure that results is then considered, given differing taste for quality among consumers. Lastly, the effects of the introduction of counterfeit goods are examined in different cases of income distribution.

II. The Model

A. Innovation

Technological development takes place by fixed steps along quality ladders, i.e., by quality increments. When a new generation good is introduced, it provides λ times as many services as the previous state-of-the-art good:

$$q_k = \lambda q_{k-1} \tag{1}$$

where q_k is the quality of variety k of the good.[4] Assume λ to be exogenous, constant and greater than 1. Technological development is uncertain, and entails a fixed cost investment in research and development.

B. The Patent Race

This paper is less interested in the patent race per se, as in the resulting market structure. The usual patent race a' la Dixit (1988) will be used, including a risk of imitation (due to imperfect IPR protection) that reduces the expected payoff from the patent race. Assume that innovation is sufficiently difficult that it is extremely unlikely that a discovery of a new generation good will be made while a patent is still valid on the last generation good. This will simplify the analysis in that there will always be only two varieties on offer in the economy.

Assume that there are X firms in the economy, and each must decide between investing in innovation, or not investing and being an imitator. X can be treated as a continuous variable. Each firm that participates in innovative activity has a probability of success μdt in any small interval of time dt. The hazard rate μ is constant over time and the same for all firms. The chances of success are independent across all firms. The number of innovating firms will be $x^* \in [0, X]$.

Once one firm succeeds, the R&D race ends. The prize of the successful firm is a patent for the state-of-the-art good.

Assume θ is the probability of imitation by a single firm. The resulting imitated (i.e., counterfeit) good is an imperfect copy of the state-of-the-art: superior to the last generation good, but inferior to the state-of-the-art. Given X firms operating in the industry and x^* innovators, $(X-x^*)$ firms will be imitators. Once imitation occurs, all imitators acquire the capability of producing the counterfeit good.

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[4]The terms "generation" and "variety" are interchangeable in this model given that goods are vertically differentiated and innovation is quality enhancing.

For the innovating firm that makes the discovery, it will obtain a monopoly profit Π_M until imitation occurs, and then the lower profit Π_I. Therefore an innovator's expected return, discounted back to the date of discovery, is

$$F = \left[\Pi_M + (X - x^*) \frac{\theta}{r} \Pi_I \right] \Big/ \left(r + (X - x^*)\theta \right) \tag{2}$$

Unsuccessful innovating firms receive zero.

Once innovation has taken place, firms $(X\text{-}x^*)$ begin the imitation phase. Imitation is costless, but since there are so many imitators profits are pushed to zero.[5]

To participate in innovation, firms must pay a fixed cost $K_R(x)$. For simplicity, assume that all costs are sunk at the outset. Firms differ in their ability to do R&D (that is why some choose to innovate and others not): firms are ordered such that $K_R(x)$ is continuous and increasing in x.

Firms will choose to innovate as long as the payoff to doing so is not negative. The payoff to an innovating firm will be

$$P(x) = \frac{\mu F}{r + x^* \mu} - K_R(x) \tag{3}$$

where r is the rate of time preference. The equilibrium number of innovating firms will be x^* such that $P(x^*) = 0$.

C. Production

For simplicity, assume identical production technologies across all qualities: this is to stress that the burden of quality improvements falls on fixed rather than variable costs. Assuming labor is the only factor of production, units are chosen so that one unit of each producible good requires one unit of labor input, this way the marginal cost of every good is equal to the wage rate w, which we set equal to 1. Firms engage in Bertrand price competition.

D. Consumer Preferences

Consumers either buy a single unit of one of the qualities on offer or else buy nothing at all. Consumers are not identical but instead differ in their level of income y, which

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[5]Once could consider costly imitation, with imitators entering one at a time, but these aspects are ignored in the interest of simplicity and given that the focus is on market structure *after* discovery takes place.

translates into a difference in their willingness to pay for quality improvements. Consumers' preferences are identical and are given by the utility function

$$U(y,k) = q_k(y - p_k),\ k = 1, 2, \ldots, n \tag{4}$$

where p_k is the price of generation k and $q_1 < q_2 < \ldots < q_k$.

Once a consumer decides which quality of the good to buy, $(y - p_k)$ is the amount of income left over to buy other goods. One also needs to allow for the possibility that a consumer decides to not buy the good at all: in this case all her income is available for spending on other goods

$$U(y,0) = q_0 y \tag{5}$$

and assume $q_0 = 1$. Furthermore, the utility derived from having no income is equal to zero:

$$U(0,k) = 0 \tag{6}$$

For any two products k and $k-1$, of corresponding quality q_k and q_{k-1}, that sell at prices p_k and p_{k-1}, define the indifference income y_k as the income of the consumer who is indifferent between consuming thc two goods. This entails

$$U(y_k, k) = U(y_k, k-1) \tag{7}$$

which is equivalent to

$$q_k(y_k - p_k) = q_{k-1}(y_k - p_{k-1}) \tag{8}$$

y_k defines the lower limit of the market share of the firm/s selling variety k. Another interpretation is that given p_{k-1}, p_k is the maximum price that firm can charge and just attract the consumer with income y_k.

Define r_k as an index of the qualitative superiority of variety k over the next best variety being offered:

$$r_k = \frac{q_k}{q_k - q_{k-1}} \tag{9}$$

Solving for y_k

$$y_k = r_k p_k - (r_k - 1)p_{k-1},\quad k = 1, 2, \ldots, n \tag{10}$$

The number of qualities consumed will depend on the distribution of income of consumers: in particular, if income doesn't vary much between consumers, only one quality (the highest) will be consumed. With a greater distribution of income, more qualities will be consumed, but always a finite number. This *finiteness property*, labeled such by Shaked and Sutton, is a particular characteristic of vertically differentiated markets. (In fact it does not apply to horizontally differentiated products.) The necessary condition for this property to hold is that unit variable costs of production increase "not too steeply" with the level of quality, in particular, more slowly than consumers' willingness to pay for increased quality. (See Shaked and Sutton 1987.) This type of relationship between technology and tastes reflects the fact that the main burden of quality improvement falls on fixed rather than variable costs (as is the case in patent races).[6]

III. Income Distribution

A. The General Case

Consumers' incomes are described by a generic density function $f(y)$ and its cumulative $F(y)$. Minimum income will be given by a and maximum income by b. Before introducing specific income distribution function, this section will present some general results.

Since production costs are the same for all firms and the last generation good is sold at cost, in the absence of imitation only two qualities of the good will be sold, the last generation good and the state-of-the-art. The unit cost of production of all varieties of the good is the same (as long as the variety in question has been discovered). If the patent on the last generation good has expired, Bertrand competition will drive the price of such good down to cost and profits will be zero. The discoverer of the state-of-the-art good holds a patent and therefore is a monopolist. There will be no other qualities offered: qualities higher than the state-of-the-art have not yet been discovered, and qualities less than the last generation will charge the same price but are of lower quality, and therefore will have zero demand.

The state-of-the-art firm will set price to maximize profits:[7]

$$\Pi_s = (p_s - 1)\left(F(b) - F(y_s)\right) \tag{11}$$

where $F(b) = 1$.

$$\frac{d\Pi_s}{dp_s} = \left(1 - F(y_s)\right) - r_s f(y_s)(p_s - 1) \tag{12}$$

[6]An extreme case in which the necessary condition holds is that in which the unit variable cost is the same for all qualities: which is in fact the situation in this model. In this case, if consumer preferences were homogeneous, all consumers would prefer to consume the high quality good. Given differing taste for quality, a finite number of qualities will be consumed, with poorer individuals choosing the lower quality varieties.

[7]From hereon, the subscript s will be used to refer to state-of-the-art variables, the subscript l to refer to last generation variables, and the subscript c to refer to counterfeit variables.

$$p_s = \frac{(1 - F(y_s))}{r_s f(y_s)} + 1 \tag{13}$$

where y_s is given by (from equation (10))

$$y_s = r_s p_s^{*} - (r_s - 1) \tag{14}$$

Consider now the effect of an introduction of a counterfeit good. By definition, the quality of a counterfeit good is higher than that of the old generation, and lower than that of the state-of-the-art. If there is more than one counterfeiter, as assumed, Bertrand competition will push price down to cost, i.e., $p_c = w = 1$. This will immediately push the old generation good out of the market, in as it offers an inferior product at the same price. The market will now be divided between the state-of-the-art good and the counterfeit.

The following will be used in the rest of the paper:

Lemma 1 : The introduction of a counterfeit good increases r_s.

Proof. The introduction of a counterfeit good, by driving a wedge between the state-of-the-art and the follower good, implies a reduction in the quality step between the state-of-the-art and the next best good. Given the assumptions on the technology of innovation and $q_0 = 1$, the quality levels of the last generation good, of the state-of-the-art and of the counterfeit good are respectively:

$$q_l = \lambda^{k-1}$$
$$q_s = \lambda^{k}$$
$$q_c = \lambda^{k-\delta}$$

where k is the generation of the good and $\delta \in (0,1)$ is the (exogenously determined) qualitative distance between the state-of-the-art good and the counterfeit good. In the absence of imitation

$$r_s = \frac{q_s}{q_s - q_l} = \frac{\lambda^k}{\lambda^k - \lambda^{k-1}} = \frac{\lambda}{\lambda - 1} \tag{15}$$

With the introduction of a counterfeit good of quality greater than the follower good but inferior to the state-of-the-art

$$r_{s'} = \frac{q_s}{q_s - q_c} = \frac{\lambda^k}{\lambda^k - \lambda^{k-\delta}} = \frac{\lambda^\delta}{\lambda^\delta - 1} \tag{16}$$

And $r_{s'} > r_s$ since $\frac{\lambda^\delta}{\lambda^\delta - 1} > \frac{\lambda}{\lambda - 1}$ given that $\lambda > 1$ and $\delta \in (0,1)$.

Lemma 2: The higher the quality of the counterfeit good, i.e., the closer to the quality of the state-of-the-art, the greater r_s.

Proof. δ represents the quality gap between the state-of-the-art and the counterfeit good. When $\delta = 1$ there is no counterfeiting (or the counterfeit is of very low quality, equal to that of the last generation good). When $\delta = 0$ the counterfeit is perfect, and can be substituted for the state-of-the-art.

$$\frac{dr_s}{d\delta} = -\frac{\lambda^{\delta} \log \delta}{(\lambda^{\delta} - 1)^2} < 0$$

The smaller the quality gap, the greater r_s.

Proposition 1: The introduction of a counterfeit good will always decrease the price of the state-of-the-art good, and market share will remain unchanged.

Proof. Substituting (13) into (14), one finds that y_s is independent of r_s. Substituting instead (14) into (13), and taking the derivative of p_s with respect to r_s to determine the effect of the introduction of a counterfeit good on the price of the state-of-the-art firm.

$$\frac{dp_s}{dr_s} = -\frac{(p_s - 1)}{r_s} < 0 \tag{17}$$

Consumer welfare in the economy, W_C, is given by the sum of the utility of consumers that choose the lower quality good and of the utility of consumers who choose the high quality good:

$$W_C = \int_a^{y_s} \lambda^{k-\delta}(y-1)df(y) + \int_{y_s}^{b} \lambda^{k}(y-p_s)df(y) \tag{18}$$

where $\delta = 1$ when there is no counterfeiting. Total welfare in the economy, W, will be given by the sum consumer welfare and profits to the state-of-the-art firm (Π_s).

$$W = W_C + \Pi_S \tag{19}$$

Proposition 2: The introduction of a counterfeit good always increases consumer welfare, and the better the quality of the counterfeit good the greater the welfare increase.

Proof. Taking the derivative of W with respect to r_s, and remembering that δ decreases as r_s increases and therefore,

$$\frac{d\lambda^{k-\delta}}{dr_s} > 0 \quad \text{and} \quad \frac{d^2\lambda^{k-\delta}}{dr_s^2} > 0$$

and noting furthermore that

$$\frac{d^2 p_s}{dr_s^2} > 0$$

$$\frac{dW_C}{dr_s} = \int_a^{y_s} \frac{d\lambda^{k-\delta}}{dr_s} df(y_s) + \int_{y_s}^{b} \lambda^{k+1}\left(-\frac{dp_s}{dr_s}\right) df(y_s) > 0 \tag{20}$$

Both terms are always positive, their actual values will depend on the distribution of income within the economy. **q.e.d.**

B. A Special Case: Uniform Distribution

Consider the special case in which consumers' income distribution is described by the uniform distribution, with

$$\begin{aligned} f(y) &= S \;\; for \;\; 0 < a \leq y \leq b \\ &= 0 \quad otherwise \end{aligned} \tag{21}$$

where S is the measure of the size of the economy. The smaller the difference $b - a$, the greater the income equality of the economy. This is not the most realistic depiction of income distribution, but it is the one most commonly used in the literature on vertical product differentiation.

The state-of-the-art firm chooses price so as to maximize profits, given by

$$\Pi_s = S(p_s - 1)(b - y_s) \tag{22}$$

This yields an equilibrium price of

$$p_s = \frac{b-1}{2r_s} + 1 \tag{23}$$

Substituting p_s into the definition of y_s

$$\begin{aligned} y_s &= r_s\left(\frac{b-1}{2r_s} + 1\right) - (r_s - 1) \\ &= \frac{b+1}{2} \end{aligned} \tag{24}$$

Therefore the state-of-the-art firm's market share is $S(b - y_s)$, i.e. $S\left(\frac{b-1}{2}\right)$. The market share of the older generation good is instead $S(y_s - a)$, i.e. $S\left(\frac{b-2a+1}{2}\right)$. Note that the market share of the state-of-the-art firm depends only on how many "rich" people there are.

Enter at this point an counterfeit good. From Proposition 1, the price charged by the state-of-the-art firm will *decrease* due to competition from the counterfeit firm

$$\frac{dp_s}{dr_s} = -\frac{b-1}{2r_s^2} \tag{25}$$

Note however that $dy_s/dr_s = 0$ i.e. the state-of-the-art firm's market share *does not change*, since y_s is independent of r_s. Profits for the state-of-the-art firm however will decrease

$$\frac{d\Pi}{dr_s} = \frac{-(b-1)^2}{8r_s^2} < 0 \tag{26}$$

So competition with imitating firms of inferior quality brings the state-of-the-art firm to lower price but maintains market share. The extent of these effects will depend on the actual quality of the counterfeit good: the smaller δ, i.e., the closer the quality of the counterfeit is to the state-of-the-art, the stronger will be competition and therefore the stronger the price effect.

This is contrary to what one would expect: that the holder of a patent *lose* market share to an imitator, even if the product offered is inferior. Instead in this case the state-of-the-art firm's market share does not change: this is due in part to the fact that the counterfeit good competes not only with the state-of-the-art but also with the older generation good (in fact the latter is completely driven out of the market). Consumers gain for two separate reasons: poorer consumers are able to substitute the higher quality counterfeit good for the older generation good, and richer consumers are now charged a lower price.

Consider now the role played by income inequality. In the case of uniformly distributed income, a rise in income inequality can be measured by an increase in the difference between a and b, defined $c = a - b$. This allows the price and profit functions to be rewritten as:

$$p_s = \frac{c-a-1}{2r_s} + 1, \quad \Pi_s = S\frac{(c-a-1)^2}{4r_s} \tag{27}$$

Both p_s and Π_s are increasing with c, i.e., the state-of-the-art firm's optimal price and profit level increase with the degree of income inequality.

Furthermore, the effect on prices and profits of the introduction of a counterfeit good *decreases* with an increase in income inequality:

$$\frac{d^2 p_s}{dr_s\, dc} < 0 \cdot \frac{d^2 \Pi_s}{dr_s\, dc} < 0 \tag{28}$$

The problem with using the uniform distribution to explain income distribution is that it assumes that there are always as many "rich" people as "poor" people. The lognormal distribution and the Pareto distribution are considered the most appropriate for studying income distribution. The lognormal distribution gives a better description of lower income levels, while the Pareto distribution is more suitable for describing higher levels of income. Both distributions give similar qualitative results, but since the Pareto distribution is easier to work with it will be presented first.

C. The Pareto Distribution

"Pareto's Law" is often used to describe income distribution, in particular right tail behavior. Pareto's Law applied to income translates to the assertion that 80 percent of income is held by 20 percent of the population. The density of the Pareto distribution is given by

$$f(y) = \beta a^{\beta} y^{-(\beta+1)} \qquad when\ y > a \tag{29}$$

and $f(y) = 0$ otherwise. β can be interpreted as an index of the level of *equality* of the distribution of income (the smaller β, the greater the degree of income *inequality*).

The cumulative is given by $F(y)$ where

$$F(y) = 1 - \left(\frac{y}{a}\right)^{-\beta} \tag{30}$$

From (13), for the state-of-the-art firm the profit maximizing price will be

$$p_s = \frac{y_s}{\beta r_s} + 1 \tag{31}$$

The first thing to note is that even in the case of perfect patent protection, in other words no possibility of counterfeit goods being introduced, the optimal price charged by the firm will be decreasing in β: the more unequal the distribution of income in the economy, the greater the profits to the firm that offers the state-of-the-art product. With an increase in income inequality (i.e., a decrease in β) the price charged by the state-of-the-art firm increases, and while it's market share decreases. The cut-off income will increase. In fact[8]

$$\frac{dp_s}{d\beta} < 0, \qquad \frac{dy_s}{d\beta} < 0 \qquad \text{and} \qquad \frac{d[1-F(y_s)]}{d\beta} > 0.$$ [9]

At this point a counterfeit good is again introduced. As in the case of the uniform distribution,

$$\frac{dp_s}{dr_s} < 0 \quad \frac{dy_s}{dr_s} = 0 \tag{32}$$

i.e., the price of the state-of-the-art good decreases while its market share remains unchanged. Note that if the firm opted to keep price constant and instead allow market share to adjust, market share would decrease and the effect on profits would be the same in both cases.

Note however that the magnitude of these effects increases with β. In other words, the greater the degree of income inequality the smaller the effect of the introduction of a counterfeit good on the price:

$$\frac{d^2 p_s}{dr_s d\beta} > 0$$

Therefore, the effect of the introduction of a counterfeit good on the level of profits of the state-of-the-art firm will be negative, but will be smaller in the case of greater income inequality.

$$\frac{d\Pi}{dr_s} < 0 \qquad \text{and} \qquad \frac{d^2\Pi}{dr_s d\beta} > 0$$

The effect on consumer welfare will be positive:

$$\frac{dW_C}{dr_s} > 0$$

and furthermore will increase with the degree of income inequality in the market in question

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[8]See appendix for mathematical derivations.

[9]Note that $1-y_s$ does not represent the market share of the state-of-the-art firm, $1-F(y_s)$ does. A decrease in y_s will only correspond to an increase in $[1-F(y_s)]$ if β is kept constant.

$$\frac{d^2 W_C}{dr_s \, d\beta} < 0 .$$

D. The Lognormal Distribution

The lognormal distribution is considered the best for studying income, in particular lower levels of income. It is however more cumbersome to work with than the Pareto distribution. Furthermore, since this paper focuses on the market share of the state-of-the-art firm, which caters only to the right end of the income distribution, the Pareto distribution is most appropriate. However, the results seen in the case of the Pareto distribution will hold largely for the lognormal as well.

The density function for the lognormal distribution is:

$$f(y) = \frac{1}{y\sigma\sqrt{2\pi}} \exp\left\{ -\frac{1}{2\sigma^2} (\log y - \mu)^2 \right\} \tag{33}$$

Similarly to the Pareto distribution, in this case σ^2 is a measure of income inequality: the greater σ^2, the more unequal the distribution of income. (Note therefore that σ^2 is negatively correlated with β.)

Given the relative complexity of the density and cumulative function of the logarithmic distribution of the logarithmic, it is not useful to derive an expression for the price function of the state-of-the-art firm. However, using the general formulas for the price function (13) and for the effect on price of the introduction of a counterfeit good (17), one can derive the following results. (See appendix.)

$$\frac{dp_s}{dr_s} < 0 \qquad \frac{dy_s}{dr_s} = 0 \qquad \frac{d\Pi_s}{dr_s} < 0 \tag{34}$$

As in the previous cases, the introduction of a counterfeit good induces the state-of-the-art firm to lower prices. The net effect on the state-of-the-art firm's profits will be negative. Once again, if the firm prefers to fix prices and allow market share to adjust, market share will shrink such that the effect on profits is unchanged.

Also, as in the case of the Pareto distribution, *in general* the magnitude of these effects decreases with income inequality. In particular, the greater σ, the smaller the effect of the introduction of a counterfeit good on the state-of-the-art.

$$\frac{d^2 p_s}{dr_s \, d\sigma_s} < 0 \tag{35}$$

The only case in which the magnitude of the effect will increase with income inequality is for values of income very far from the mean. For example, if in equilibrium the state-of-the-art firm sells even to very poor segments of the economy, and therefore controls a very large share of the market, which is a very particular situation. Or, if the state-of-the-art firm sells only to the richest segment of the population, in which case the Pareto distribution would be more appropriate. (See Appendix.)

The effect on welfare will always be positive, i.e., the introduction of a counterfeit good increases consumer welfare. In the general case where the magnitude effect on price is as in the Pareto distribution, the welfare effect of the introduction of a counterfeit good will be increasing with income inequality.

IV. Concluding Remarks

Optimal IPR protection and the effective cost of imitation to innovating firms depends fundamentally on the extent of the price and profit effects. Qualitatively, these results do not change depending on the different distribution of income of the economy. Independently from the functional form used to represent income distribution, although it is true in general that profits decrease for the state-of-the-art firm in the face of imitation, the magnitude of this effect decreases with the degree of income inequality. The effect of the introduction of counterfeit goods on welfare instead increases with inequality.

These results have important policy implications for developing countries, where counterfeit goods are particularly widespread (although counterfeit goods are not restricted to these areas!). Developing countries are often characterized by high levels of income inequality. In such countries, counterfeit goods have much to contribute to national welfare, with state-of-the-art firms losing less than counterfeiting would (and does) cost them in more developed markets with a higher degree of income equality. These considerations should be taken into account both by developing countries looking to reform their IPR regimes, and by firms in developed countries concerned by loss of revenue due to IPR infringement. In practice, a uniform international level of IPR protection might not be the best solution, and allowing laxer IPR protection in some developing countries should be accepted by the international community, at least in the short term.

The story could be made richer by considering that counterfeit firms also have some power to choose the quality level of their products. Another possible extension would be to consider an international market for counterfeit goods, with counterfeits of varying quality being offered in different markets based on different income levels and income distribution in those markets. These topics are left for future research.

MATHEMATICAL DERIVATIONS

For Pareto distribution:

$$f(y) = \beta a^{\beta} y^{-(\beta+1)}$$

$$p_s = \frac{y_s}{\beta r_s} + 1$$

$$y_s = r_s p_s - (r_s - 1)$$

Regarding the sensibility of price, the cutoff income level and profits to income inequality

$$\frac{dp_s}{d\beta} = -\frac{1}{\beta^2 y_s} < 0$$

$$\frac{dy_s}{d\beta} = r_s \frac{dp_s}{d\beta} < 0$$

For the effect on market structure, *for fixed* y_s:

$$\frac{dF(y)}{d\beta} = -\left(\frac{y}{\alpha}\right)^{-\beta} \ln\frac{y}{a} < 0$$

therefore $d[1 - F(y)]/d\beta > 0$.

To find the effect of the introduction of a counterfeit good, substitute y_s into the expression for p_s:

$$\beta p_s r_s = p_s r_s - r_s + 1 + \beta r_s$$

and solving for p_s:

$$p_s = 1 + \frac{1}{r_s(\beta - 1)}$$

therefore

$$\frac{dp_s}{dr_s} < 0$$

For the effect on profits,

$$\frac{d\Pi}{dr_s} = \frac{dp_s}{dr_s}(1 - F(y_s)) < 0$$

As for the magnitude effects:

$$\frac{d^2 p_s}{dr_s d\beta} = \frac{1}{r_s^2} \frac{1}{(1-\beta)^2} > 0$$

To consider the magnitude effect on consumer welfare, remembering from equation (20):

$$\frac{dW_C}{dr_s} = \int_a^{y_s} \frac{d\lambda^{k-\delta}}{dr_s} df(y_s) + \int_{y_s}^{b} \lambda^{k+1} \left(-\frac{dp_s}{dr_s}\right) df(y_s) > 0$$

$$\frac{dW_C}{dr_s d\beta} = \int_{y_s}^{b} \lambda^{k+1} \left(-\frac{d^2 p_s}{dr_s d\beta}\right) df(y_s) < 0$$

For the lognormal distribution:

$$f(y) = \frac{1}{y\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(\log y - \mu)^2\right\}$$

$$F(y) = \int_0^{y_s} \left[\frac{1}{y\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(\log y - \mu)^2\right\}\right] dy$$

To examine the role of income inequality on the price effect, note that

$$\frac{df(y)}{d\sigma} = f(y)\left[\log(y-\mu)^2 \frac{1}{\sigma^3} - \frac{1}{\sigma}\right]$$

which is greater than zero if $\log(y-\mu)^2 > \sigma^2$, i.e. for values of income far from the mean.

Furthermore,

$$\frac{df(y)}{d\sigma} > 0 \Rightarrow \frac{dF(y)}{d\sigma} > 0 \quad \text{(case 1)}$$

$$\frac{df(y)}{d\sigma} < 0 \Rightarrow \frac{dF(y)}{d\sigma} < 0 \quad \text{(case 2)}$$

Magnitude effects

$$\frac{d^2 p_s}{dr_s d\sigma} = \frac{\frac{dF(y_s)}{d\sigma} r_s^2 f(y_s) + \frac{df(y_s)}{d\sigma}(1 - F(y_s))}{\left(r_s^2 f(y_s)\right)^2}$$

Magnitude effects will therefore be positive in case one, and negative in case two, however, since case 1 can be generally rejected for reasons discussed in the text, in general it will hold that

$$\frac{d^2 p_s}{dr_s d\sigma} < 0$$

The effect on welfare will be the same as in the case of the Pareto distribution

$$\frac{dW_C}{dr_s} > 0 \qquad \text{and}$$

$$\frac{dW_C}{dr_s d\sigma} = \int_{y_s}^{b} \lambda^{k+1} \left(-\frac{d^2 p_s}{dr_s d\sigma}\right) df(y_s) > 0$$

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