IMF Working Paper

Real Exchange Rates and Productivity: Closed-Form Solutions and Some Empirical Evidence

Jahanara Begum

INTERNATIONAL MONETARY FUND

IMF Working Paper

Monetary and Exchange Affairs Department

Real Exchange Rates and Productivity: Closed-Form Solutions and Some Empirical Evidence

Prepared by Jahanara Begum [1]

Authorized for distribution by R. Barry Johnston

June 2000

Abstract

This paper examines the impact of productivity shocks on real exchange rate fluctuations in a dynamic international general equilibrium model with nontraded goods. The model predicts a close association between relative technology shocks and bilateral real exchange rate movements. Empirical results based on the data for Group of Seven countries are consistent with the predicted theoretical correlations. Using Johansen and Juselius (1990) multivariate cointegration tests the study finds that a statistically significant relationship exists between bilateral real exchange rates and international productivity differentials in the traded and nontraded sectors.

JEL Classification Numbers: E3, F31, G15

Keywords: Technology Shocks; Real Exchange Rates; Cointegration.

Author's E-Mail Address: Jbegum@imf.org

Contents   Page

I. Introduction

Deviations from purchasing power parity (PPP) [2] and real exchange rate movements have been a major focus of empirical research since the return to floating exchange rates by major currencies in the early 1970s. A substantial body of evidence (for example, Isard (1977), Roll and Solnik (1979), Frankel (1981), and Mussa (1986)) indicates that nominal price indexes and foreign exchange rates display large – and permanent – departures from PPP. Several explanations are put forward for the observed volatility of real exchange rates. One explanation for this phenomenon is the evidence of exogenous shocks to the world economy. To explain exchange rate movements under the float, many economists resort to models in which real shocks play the dominant role. Some of the important channels that allow real shocks to affect exchange rate movements are sectoral relative prices (i.e., relative price of nontraded to traded goods), technology shocks, changes in taste, and fiscal policy shocks.

The motivation for this study comes from the empirical evidence that large movements in real exchange rates and the failure of PPP to hold in the 1970s can be explained by the predominance of real shocks. A number of recent empirical studies (e.g., Dutton and Strauss (1997), Mark and Choi (1997), Bahmani and Rhee (1996) and Strauss (1996)) find evidence of a significant relationship between the real exchange rate and real variables (e.g., productivity differentials between domestic and foreign economy in the traded and nontraded sectors, relative price of nontraded goods, and taste shocks). Other empirical studies (Amano and Van Norden (1992), Lastrapes (1992), and Manzur (1991)) also find support for real shocks causing exchange rate movements. A number of theoretical models (Balvers and Bergstrand (1997), Kollmann (1995), Backus and Smith (1993), Cardia (1991), and Stulz (1987)) also establish the link between real factors and exchange rate movements. For example, the recent work by Balvers and Bergstrand (1997) presented closed-form solutions of real exchange rate in terms of relative productivity of non-tradeables and the rate of time preference. Backus and Smith (1993) used a dynamic exchange economy with nontraded goods to account for deviation from PPP and to explain real exchange rate movements.

A goal of this paper is to examine the role of relative productivity shocks in explaining real exchange rate movements in a two-country dynamic, general equilibrium framework with nontraded goods. Compared to previous studies, in particular those which focus on an exchange economy (e.g., Balvers and Bergstrand (1997) and Backus and Smith (1993)), there are two major contributions of the present study. First, it incorporates production technologies, which depend on both labor and physical capital. This is an important extension because the introduction of technological processes allows the adjustments in both consumption and output, whereas in exchange economies without production, the level of output is exogenous.

---

[2] This entails the theory that a long-run equilibrium relationship exists between nominal exchange rates and domestic and foreign prices.

Secondly, the model can predict explicitly the impact of technology shocks on real exchange rate movements.

While the existing literature suggests a number of real variables may have significant effects on the real exchange rate, there are a number of reasons for focusing on relative productivity differentials. *First*, total factor productivity (measured in terms of the Solow residual) is an important determinant of the long-term economic growth and prosperity of a country. *Second*, productivity shocks are exogenous variables that may play a key role in explaining movements in the real exchange rates. In recent years, the extension of real business cycle models (RBC) to the open economy has stimulated interest in technology shock effects.[3] *Third*, productivity differentials between domestic and foreign economy have long been emphasized as a significant determinant of real exchange rate movements (Balassa (1964)).

To study the relationship between sectoral productivity differentials (or ratios) (between home and foreign country) and the real exchange rate we rely on a two-country, two-sector model with nontraded goods. The presence of nontraded goods allows for permanent deviations from aggregate PPP.[4] Assuming logarithmic preferences, full depreciation of capital, and some restrictions on the technology shock process, closed-form theoretical solutions are derived for optimal policy functions for consumption, investment, and employment. The main reason for focusing on analytical solutions is that the analytical method[5] is capable of generating explicit expressions for equilibrium real exchange rates which are easy to interpret and allow empirical testing of the model. Numerical solution techniques (which provide quantitative predictions of the model using a set of parameter values) are unable to produce such simple and interpretable expressions. However, analytical method may require some restrictive assumptions (e.g., full depreciation of capital and a stationary shock process) and thereby may cause loss of many of the interesting cross country dynamics.

The model predicts a close link between bilateral real exchange rates and productivity differentials (ratios) (defined as domestic productivity in traded (nontraded) goods relative

---

[3] Implications of technology shocks in equilbrium environments for evolutions for real variables across countries have been investigated by, among others, Backus,Kehoe, Kydland (1993), Cantor and Nelson (1988), Mendoza (1991) and Stockman and Tesar (1995). Performance of these models, however, is not very satisfactory in accounting for volatility of real exchange rates observed in the data. The standard deviation of the terms of trade is found to be considerably smaller in the model than it is in the data.

[4] Since Balassa's (1964) and Samuelson's (1964) seminal work, non-traded goods have been cited as a major source of deviations in the real exchange rate from its PPP value (see, e.g, Stockman (1987), Backus and Smith (1993)).

[5] In the existing literature, two different approaches are frequently used for modeling (and testing) exchange rate movements. Under the first approach, known as an "artificial economy approach," an artificial model economy is calibrated and simulated and then the simulated moments are compared with the corresponding moments in the data. The second approach, known as "analytical solution method," on the other, hand relies on the first-order-conditions and Euler equations to generate expressions for equilibrium prices and exchange rates in terms of the parameters of preferences, technology and the forcing processes.

to foreign productivity in traded (nontraded) goods) in the traded and nontraded sectors. In particular, the theory suggests a positive relationship between the real exchange rate and productivity ratio in the nontraded sector and conversely, a negative relationship with the corresponding ratio in the traded sector. The empirical results, using time series data for the G-7 countries, are consistent with the predicted theoretical signs. The cointegration tests suggest that a statistically significant cointegrating relationship exists between domestic and foreign productivity differentials in each sector and the real exchange rate. We conclude that relative productivity shocks are an important determinant of real exchange behavior.

The remaining sections of the paper are organized as follows. Section II presents a brief review of related literature. Section III provides a two-country dynamic general equilibrium model with nontraded goods. Quantitative implications of the exchange rate dynamics and effects of relative productivity shocks are examined in Section IV. Finally, some concluding remarks are made in section V.

II. Literature Review

There have been numerous studies, both theoretical and empirical, attempting to examine what accounts for real exchange rate fluctuations. This section briefly reviews some recent literature on fundamental determinants of real exchange rate. The analysis is divided into two parts. *First*, it focuses on the theoretical models of exchange rate determination which predict a close relationship between real exchange rate and real factors, i.e., terms of trade, relative price of nontraded goods, productivity differentials, and the like. *Second*, the study critically analyzes some recent empirical research devoted to quantifying the real exchange rate dynamics.

A. Theoretical Predictions

PPP is the foundation of many long-run exchange rate models and has a long history of theoretical support. Some commonly cited causes for PPP violations and permanent movements in real exchange rates are differences in productivity between traded and nontraded sectors. Balassa (1964) and Samuelson (1964) first identified productivity differentials between two countries as a major factor contributing to the deviation of the PPP-based exchange rate from the equilibrium exchange rate.[6] They argued that when productivity differentials are greater in the production of traded goods between two countries, the differences in wages and prices will also be larger, which will lead to a larger gap between PPP

---

[6]   In the "productivity differential" model (a two-country, one-factor model where the supply of labor is fixed and mobile across sectors; thus, nominal wages, W, are equalized in both the traded and non-traded sectors) of Balassa (1964), productivity is measured in terms of the marginal product of labor (MPL) in the traded and non-traded sectors.

and the equilibrium exchange rate. This has come under the heading of the "productivity-bias hypothesis" in the PPP theory.

Recent work of Balvers and Bergstrand (1997) generated closed-form theoretical solutions for real exchange rates (in terms of relative non-tradeable productivity, taste shocks and rates of time preference) in a two-country stochastic dynamic general equilibrium model. Their solutions are consistent with equilibrium exchange rate theories and the productivity-differentials models of Balassa (1964) and Samuelson (1964). The empirical studies of this group of models using classical estimation techniques are rare, as noted in Taylor (1995). Backus and Smith (1993) use the moments of growth rates of relative consumption levels and of real exchange rates to examine the implications from a dynamic, stochastic, two-country equilibrium model. Their theoretical model suggests a positive relationship between relative consumption (between the home and foreign) of nontraded goods and bilateral real exchange rates. Empirical evidence for eight OECD countries, however, finds little support for this pattern. Kollmann(1995) examines the relationship among per capita consumptions and real exchange rates implied by a similar model, but in the absence of non-tradable goods.

Over the past decade and a half, several equilibrium models of exchange rate determination have explained the movements in real exchange rates and PPP departures in terms of relative productivity shocks, e.g., Stockman (1980 and 1987), Lucas (1982), Helpman and Razin (1982), Stulz (1987), Stockman and Svensson (1987), and Stockman and Dellas (1989). In optimizing frameworks, most of these models employ the "perfect pooling equilibrium" attributable to Lucas (1982). Stockman (1987) asserts that exchange rate volatility reflects the greater variance of real shocks in the flexible exchange rate era. Some open economy RBC models (e.g., Backus,Kehoe, and Kydland (1994) , Stockman and Tesar (1995), and Costello and Pranschik (1993) and Mendoza (1995)) attempted to capture the impact of real shocks on exchange rate movements. Empirical support for these models, however, has been very limited.

B.   Empirical Studies

A number of recent empirical studies find evidence of a significant relationship between the real exchange rate and its fundamental determinants, including terms of trade, relative price of nontraded goods, productivity differentials etc. For example, Balvers and Bergstrand (1997), Bahmani and Rhee (1996), and Strauss (1996) have provided unanimous support to the productivity-differentials model of Balassa (1964) and Samuelson (1964). Using Johansen conintegration tests, they all find that a statistically significant cointegrating relationship exists between domestic and foreign productivity differentials and the real exchange rate. Dutton and Strauss (1997) and Strauss (1995) presented additional evidence that changes in the relative price of non-tradeable and productivity differentials between

economies are significant in explaining real exchange rate movements in the short-run and in the long-run, and thus explaining the source of violations of PPP. All these studies use OECD data and, generally speaking, they all focus on the existence of relationships between productivity differentials and real exchange rate, rather than on the explicit pattern of correlations over time. Chinn (1997) documented the evidence in support of a stable, long-run relationship between the real dollar/yen exchange rate, productivity and government spending differentials, and the real price of oil. The study finds limited support for sectoral productivity differentials due to the high multicollinearity between the two productivity variables. Using monthly data for some OECD countries, Mark and Choi (1997) reported that differentials in productivity, real interest rates, and per capital income display some predictive power in explaining the deviation of the real exchange rate from its long-run equilibrium value. Stein (1995) provided evidence that the fundamental determinants of the evolution of the medium-to-longer-run of the real effective exchange rate of the United States relative to the rest of the G-7 countries are productivity and thrift in the sample countries.

Another strand of literature has focused on the terms-of-trade shocks to explain fluctuations in real exchange rates and deviations from PPP since the collapse of the Bretton Woods system. Mendoza (1995) provided evidence for G-7 countries and a group of developing countries that terms-of-trade shocks account for nearly half of actual GDP variability, and cause real appreciations and positive interest differentials. De-Gregorio and Wolf (1994) present empirical evidence for a sample of 14 OECD countries that faster productivity growth in the tradable relative to the nontradeable sector and an improvement in the terms of trade induce a real appreciation. Amano and Van Norden (1992) also reported that terms of trade shocks can potentially explain much of the variations in the Canadian-U.S. exchange rate over the past 19 years. Decomposing the overall terms-of-trade index into two components: the price of exported energy and the price of exported non-energy commodities (each divided by the price of imported manufactured goods) Amano and Van Norden find that changes in energy prices played a critical role in predicting changes in real exchange rates. For example, they find that the cumulative effect of the rise in energy prices from early 1970 to mid-1981 was to move the real exchange rate by 35 percent. Other empirical studies (Lastrapes (1992); Manzur (1991)) also find support for real shocks causing exchange rate movements. Comparing the relative importance of monetary and real sources of fluctuations in exchange rates over the current flexible rate period, they find that much of the long-run movement in real exchange rates is the result of real disturbances.

In light of the theoretical predictions and the empirical evidence of a close association between the real exchange rate and real variables, this seems an important exercise to explore further the impact of relative productivity shocks on real exchange rate fluctuations. Compared to earlier studies, the closed-form theoretical solutions in this paper for equilibrium real exchange rates (in terms of the parameters of preference and technology, and the sectoral productivity ratios) are easy to interpret and are potentially estimable.

III. A Two-Country Equilibrium Model with Nontraded Goods

In the following section we first describe the model and then derive the intertemporal equilibria for logarithmic preference. Estimable closed form solutions are obtained under some restrictive assumption regarding the technology shock process.

This is a simple two-country, two-sector equilibrium real business cycle model that is capable of being solved for analytical decision rules. The model is formulated in discrete time with an infinite horizon. In the world economy, each country produces the same internationally traded good and one nontraded good for domestic consumption and investment in each period. The countries have identical preferences and follow identical production techniques, but each country's techniques are subjected to country-specific productivity shocks. Let the countries be labeled as "home" and "foreign" and denote all foreign variables with an asterisk.

The preferences of a representative household in home country are defined by a logarithmic time separable utility function,

$$U = \sum_{t=0}^{\infty} \beta^t E_t \left[\gamma \log(C_{tT}) + (1 - \gamma) \log(C_{tN}) + \mu \log(L_t) \right] \tag{1}$$

where C_{tT} is consumption of the traded good, C_{tN} is consumption of the nontraded good, L_t is leisure; $\beta \in (0,1)$ is the subjective discount factor, $\gamma(1 - \gamma)$ is the share of traded good(nontraded good) in the aggregate consumption bundle, μ is the weight given to leisure in the utility function; E_t is the expectation operator.

Similarly, the preferences of a representative agent in the foreign country is defined as

$$U^* = \sum_{t=0}^{\infty} \beta^t E_t \left[\gamma \log(C_{tT}^*) + (1 - \gamma) \log(C_{tN}^*) + \mu \log(L_t^*) \right] \tag{2}$$

The technologies used to produce the traded and nontraded goods are assumed to be constant-returns-to-scale Cobb-Douglas, that is,

Traded Sector:

$$Y_{tT} = \theta_{tT} K_{tT}^{\alpha^T} N_{tT}^{(1-\alpha^T)} \tag{3}$$

$$Y_{tT}^* = \theta_{tT}^* K_{tT}^{*\alpha^T} N_{tT}^{*(1-\alpha^T)} \tag{4}$$

nontraded Sector:

$$Y_{tN} = \theta_{tN} K_{tN}^{\alpha^N} N_{tN}^{(1-\alpha^N)} \tag{5}$$

$$Y_{tN}^* = \theta_{tN}^* K_{tN}^{*\alpha^N} N_{tN}^{*(1-\alpha^N)} \tag{6}$$

where Y_{tT}, Y_{tT}^*, Y_{tN}, Y_{tN}^* are output levels for home and foreign country in the traded and nontraded sector respectively; K_{tT}, K_{tT}^*, K_{tN}, K_{tN}^* are capital stocks at the beginning of period t; N_{tT}, N_{tT}^*, N_{tN}, N_{tN}^* are the amount of work supplied at period t; Z_{tT}, Z_{tT}^*, Z_{tN}, Z_{tN}^* are technology shocks; α^T and α^N are the shares of capital in each sector. It is assumed that capital is perfectly mobile across countries in the traded sector, but there is no inter-sectoral mobility within the country.

In each country, the rate of physical depreciation of capital stock in each sector is assumed to be unity. Therefore, the laws of motion are: $I_{tT} = K_{(t+1)T}$, $I_{tT}^* = K_{(t+1)T}^*$, $I_{tN} = K_{(t+1)N}$, and $I_{tN}^* = K_{(t+1)N}^*$. This assumption is required to derive the closed-form analytical solution for optimal decision rules.

In equilibrium the goods and labor markets clear. In the market for traded goods, the aggregate resource constraint requires that the world supply must be exhausted by world consumption and investment demand,

$$C_{tT} + C_{tT}^* + I_{tT} + I_{tT}^* = Y_{tN} + Y_{tN}^* \tag{7}$$

The equilibrium conditions for the nontraded goods sectors imply that the domestic supply of the goods be equal to domestic consumption and investment demand,

$$C_{tN} + I_{tN} = Y_{tN} \tag{8}$$

$$C_{tN}^* + I_{tN}^* = Y_{tN}^* \tag{9}$$

It is assumed that labor is perfectly mobile between the traded and nontraded sectors within a country, but immobile internationally. We normalize each country's population and the endowment of time of the representative household in each country, to one. Therefore, the labor market clearing conditions are given in,

$$N_{tT} + N_{tN} + L_t = 1 \tag{10}$$

$$N_{tT}^* + N_{tN}^* + L_t^* = 1 \tag{11}$$

The technology shocks are modeled as a quadrivariate autoregressive process, AR(1),

$$\theta_{t+1} = \Omega\theta_t + \varepsilon_t \tag{12}$$

where $\theta_t=[\theta_{tT},\theta_{tN},\theta_{tT}^*,\theta_{tN}^*]$ is the vector of exogenous disturbances, Ω is a 4x4 matrix describing the autoregressive components of the disturbances. Finally, ε_t is a vector $[\varepsilon_{tT},\varepsilon_{tN},\varepsilon_{tT}^*, \varepsilon_{tN}^*]$ describing the contemporaneous components of the shocks which are independently and identically distributed with mean zero and variance σ_ε^2.

A. Equilibrium Allocation

We can define and derive a competitive equilibrium for this two-country world economy. Since there are no distortions in the economy, and production exhibits constant returns to scale, the equivalence between competitive equilibrium and a social planning optimum allows us to solve instead a social planner's problem in which a weighted sum of national utilities is maximized, subject to technologies and the aggregate resource constraint. Treating each country symmetrically (so that the weights in the social planner's objective function are equal) leads us to formulate the social planning problem as follows:

Choose $\left(C_{tT}, C_{tN}, C_{tN}^*, N_{tT},N_{tN},N_{tT}^*,N_{tN}^*, K_{(t+1)T},K_{(t+1)N}, K_{(t+1)T}^*, K_{(t+1)N}^*\right)$ to maximize

$$\sum_{t=0}^{\infty} \beta^t E_t \left[U(C_{tT},C_{tN},L_t) + U^*(C_{tT}^*,C_{tN}^*,L_t^*) \right]$$

subject to the production technologies in equations (3)-(6) and the market clearing conditions given in equations (7)-(11) and the technology shock process in equation (12).

Let the state vector for the planning problem be,
$$s_t = \left[K_{tT},K_{tN}, K_{tT}^*, K_{tN}^*, \theta_{tT},\theta_{tN},\theta_{tT}^*,\theta_{tN}^* \right]$$

To solve the planning problem we can define the value function $V(s_t)$ as

$$V(s_t) = \max \left[\begin{array}{c} \gamma\log(C_{tT}) + (1-\gamma)\log(C_{tN}) + \mu\log(L_t)+ \\ \gamma\log(C_{tT}^*) + (1-\gamma)\log(C_{tN}^*) + \mu\log(L_t^*) + \beta E_t V(s_{(t+1)}) \end{array} \right] \tag{13}$$

Substituting for C_{tT} using equation (7), for C_{tN} using (8), for C_{tN}^* using (9), for L_t using (10) and for L_t^* using (11), the value function is expressed as

$$V(s_t) = \max \left[\begin{array}{c} \gamma\log(Y_{Tt} + Y_{Tt}^* - C_{Tt}^* - K_{(t+1)T} - K_{(t+1)T}^*)+ \\ (1-\gamma)\log(Y_{Nt} - C_{Nt} - K_{(t+1)N}) + \mu\log(1 - N_{Tt} + N_{Nt})+ \\ \gamma\log(C_{tT}^*) + (1-\gamma)\log(Y_{Nt}^* - C_{Nt}^* - K_{(t+1)N}^*)+ \\ \mu\log(1 - N_{Tt}^* + N_{Nt}^*) + \beta E_t V(s_{(t+1)}) \end{array} \right]$$

subject to resource constraints and technology.

The first-order-conditions are:

$$C_{tT}^* : \frac{\gamma}{C_{tT}} = \frac{\gamma}{C_{tT}^*} \tag{14}$$

$$K_{(t+1)T} : \frac{\gamma}{C_{tT}} = \beta E_t V_{k_{(t+1)T}} \tag{15}$$

$$K_{(t+1)N} : \frac{1-\gamma}{C_{tN}} = \beta E_t V_{k_{(t+1)N}} \tag{16}$$

$$K_{(t+1)T}^* : \frac{\gamma}{C_{tT}^*} = \beta E_t V_{k*_{(t+1)T}} \tag{17}$$

$$K_{(t+1)N}^* : \frac{1-\gamma}{C_{tN}^*} = \beta E_t V_{k*_{(t+1)N}} \tag{18}$$

$$N_{tT} : \frac{\mu}{(1 - N_{tT} - N_{tN})} = \frac{\gamma(1 - \alpha^T)Y_{tT}}{C_{tT}N_{tT}} \tag{19}$$

$$N_{tN} : \frac{\mu}{(1 - N_{tT} - N_{tN})} = \frac{(1 - \gamma)(1 - \alpha^N)Y_{tN}}{C_{tN}N_{tN}} \tag{20}$$

$$N_{tT}^* : \frac{\mu}{(1 - N_{tT}^* - N_{tN}^*)} = \frac{\gamma(1 - \alpha^T)Y_{tT}^*}{C_{tT}^*N_{tT}^*} \tag{21}$$

$$N_{tN}^* : \frac{\mu}{(1 - N_{tT}^* - N_{tN}^*)} = \frac{(1 - \gamma)(1 - \alpha^N)Y_{tN}^*}{C_{tN}^*N_{tN}^*} \tag{22}$$

These FOCs implicitly define optimal decision rules for consumption, investment and hours.

B.   Closed-Form Solutions

To derive closed-form analytical solutions for optimal policy functions, we add some restrictive assumptions to the basic model. However, the simple restrictive models that are capable of being solved for analytical decision rules may lose many of the interesting cross-country dynamics, particularly those arising from the persistence of shocks. In particular, we can derive closed-form solutions under three alternative restrictions: *first*, technology shocks are independently and identically distributed (i.i.d.) processes; *second*, shocks processes are persistent but (a) there is no capital and technology is linear and (b) agents have perfect foresight.[7]

---

[7]   In a deterministic model (with perfect foresight), Cantor and Nelson (1988) derived closed-form solutions when shocks to technology are persistent.

1. Productivity shocks are i.i.d. process

Under this assumption the technology shock processes are serially independent across time, and identical and independent across countries. Thus, there is no persistence in the shock processes. In this situation, home country's relative share of world output of the traded good is constant.[8] Employment is constant too, because income and substitution effects cancel out. The optimal decision rules (see Appendix II for details of solution method) for consumption and investment are now:

$$C_{tT} = C^*_{tT} = \frac{(1 - \beta\alpha^T)}{2} H_T^{(1-\alpha^T)} \left[\theta_{tT} K_{tT}^{\alpha^T} + \theta^*_{tT} K_{tT}^{*\alpha^T} \right] \tag{23}$$

$$K_{(t+1)T} = K^*_{(t+1)T} = \frac{\beta\alpha^T}{2} H_T^{(1-\alpha^T)} \left[\theta_{tT} K_{tT}^{\alpha^T} + \theta^*_{tT} K_{tT}^{*\alpha^T} \right] \tag{24}$$

$$C_{tN} = (1 - \beta\alpha^N) H_N^{(1-\alpha^N)} \theta_{tN} K_{tN}^{\alpha^N} \tag{25}$$

$$C^*_{tN} = (1 - \beta\alpha^N) H_N^{*(1-\alpha^N)} \theta^*_{tN} K_{tN}^{*\alpha^N} \tag{26}$$

$$K_{(t+1)N} = \frac{\beta\alpha^N}{2} H_N^{(1-\alpha^N)} \theta_{tN} K_{tN}^{\alpha^N} \tag{27}$$

$$K^*_{(t+1)N} = \frac{\beta\alpha^N}{2} H_N^{*(1-\alpha^N)} \theta^*_{tN} K_{tN}^{*\alpha^N} \tag{28}$$

where $\overline{N}_T = \overline{N}^*_T = H_T$ and $\overline{N}_N = \overline{N}^*_N = H_N$ and

$$H_T = \frac{\gamma(1 - \alpha^T)}{\mu(1 - \beta\alpha^T) + \gamma(1 - \alpha^T) + (1 - \gamma)(1 - \beta\alpha^T)(1 - \beta\alpha^N)^{-1}(1 - \alpha^N)}$$

$$H_N = \frac{(1 - \gamma)(1 - \alpha^N)}{\mu(1 - \beta\alpha^N) + (1 - \gamma)(1 - \alpha^N) + \gamma(1 - \beta\alpha^N)(1 - \beta\alpha^T)^{-1}(1 - \alpha^T)}$$

2. Productivity shocks are persistent

The assumption made above that technology shocks are i.i.d. is very strong and empirically not plausible due to the evidence of large persistence in the technology shocks. The absence of persistence in the shock processes implies that current shocks provide no signal about the rate of return on current investment, nor do they provide information about where to invest. With country-specific persistent shocks, current shocks do provide information about the profitability of investment. In this case, with perfect mobility of capital, the occurrence of shocks in one country will lead to a redirection of investment expenditure towards that country and away from the other country. Therefore, with persistence in the shock processes, the home country's relative share of world output of traded goods is no longer constant. It also allows for fluctuations in employment in equilibrium. Thus, the persistence of shocks leads to interesting cross-country dynamics. However, we are unable to derive closed-form analytical

---

[8]   In particular, under i.i.d. shocks each country on the average produces half of the world output of the traded good.

solutions under persistence, unless we put two alternative restrictions on the standard RBC model described above.

(i) Labor is the only factor of production, and the technology is linear:.

Now, the technologies take the simple form

$$Y_{tT} = \theta_{tT} N_{tT}$$
$$Y_{tN} = \theta_{tN} N_{tN}$$
$$Y_{tT}^* = \theta_{tT}^* N_{tT}^*$$
$$Y_{tN}^* = \theta_{tN}^* N_{tN}^*$$

In the absence of physical capital or investment, traded consumption in each country is half of the world output, and nontraded consumption is equal to the domestic production of the nontraded good. In this case the optimal policy functions for consumption and hours in the traded and nontraded sector are

$$C_{tT} = C_{tT}^* = \frac{1}{2} \left[\frac{\theta_{tT}}{1 + A(\delta_t)^{-1}} + \frac{\theta_{tT}^*}{1 + A(1 - \delta_t)^{-1}} \right] \tag{29}$$

$$C_{tN} = \frac{\theta_{tN}}{1 + B + D\delta_t} \tag{30}$$

$$C_{tN}^* = \frac{\theta_{tN}^*}{1 + B + D(1 - \delta_t)} \tag{31}$$

$$N_{tT} = \frac{1}{1 + A(\delta_t)^{-1}} \tag{32}$$

$$N_{tT}^* = \frac{1}{1 + A(1 - \delta_t)^{-1}} \tag{33}$$

$$N_{tN} = \frac{1}{1 + B + D(\delta_t)^{-1}} \tag{34}$$

$$N_{tN}^* = \frac{1}{1 + B + D(1 - \delta_t)^{-1}} \tag{35}$$

where,

$$\delta_t = \frac{1 + A(1 - \theta_{tT}^* \theta_{tT}^{-1})}{1 + \theta_{tT}^* \theta_{tT}^{-1}}$$

$$A = \frac{\mu + (1 - \gamma)}{2\gamma}; \qquad B = \frac{\mu}{1 - \gamma}; \qquad D = \frac{2\gamma}{1 - \gamma}$$

(ii) The model is deterministic, i.e., the agents have perfect foresight..

Perfect foresight implies that the evolution of the productivity disturbances is known with certainty. Under this assumption the utility function takes the following deterministic form

$$U = \sum_{t=0}^{\infty} \beta^t \left[\gamma \log(C_{tT}) + (1 - \gamma) \log(C_{tN}) + \mu \log(L_t) \right]$$

Now the optimal decision rules for consumption, investment, and hours are:

$$C_{tT} = C_{tT}^* = \frac{1-\beta\alpha^T}{2}\left[\begin{array}{c} \theta_{tT}K_{tT}^{\alpha^T}\left(\frac{1}{1+G\omega_t^{-1}}\right)^{(1-\alpha^T)} + \\ \theta_{tT}^*K_{tT}^{*\alpha^T}\left(\frac{1}{1+G(1-\omega_t)^{-1}}\right)^{(1-\alpha^T)} \end{array} \right] \tag{36}$$

$$C_{tN} = (1-\beta\alpha^N)\theta_{tN}K_{tN}^{\alpha^N}\left[\frac{1}{1+Q+R\omega_t}\right]^{(1-\alpha^N)} \tag{37}$$

$$C_{tN}^* = (1-\beta\alpha^N)\theta_{tN}^*K_{tN}^{*\alpha^N}\left[\frac{1}{1+Q+R(1-\omega_t)}\right]^{(1-\alpha^N)} \tag{38}$$

$$K_{(1+t)T} = \beta\alpha^T\omega_{t+1}\left[\begin{array}{c} \theta_{tT}K_{tT}^{\alpha^T}\left(\frac{1}{1+G\omega_t^{-1}}\right)^{(1-\alpha^T)} + \\ \theta_{tT}^*K_{tT}^{*\alpha^T}\left(\frac{1}{1+G(1-\omega_t)^{-1}}\right)^{(1-\alpha^T)} \end{array} \right] \tag{39}$$

$$K_{(1+t)T}^* = \beta\alpha^T(1-\omega_{t+1})\left[\begin{array}{c} \theta_{tT}K_{tT}^{\alpha^T}\left(\frac{1}{1+G\omega_t^{-1}}\right)^{(1-\alpha^T)} + \\ \theta_{tT}^*K_{tT}^{*\alpha^T}\left(\frac{1}{1+G(1-\omega_t)^{-1}}\right)^{(1-\alpha^T)} \end{array} \right] \tag{40}$$

$$K_{(t+1)N} = \beta\alpha^N\theta_{tN}K_{tN}^{\alpha^N}\left[\frac{1}{1+Q+R\omega_t}\right]^{(1-\alpha^N)} \tag{41}$$

$$K_{(t+1)N} = \beta\alpha^N\theta_{tN}K_{tN}^{\alpha^N}\left[\frac{1}{1+Q+R(1-\omega_t)}\right]^{(1-\alpha^N)} \tag{42}$$

$$N_{tT} = \frac{1}{1+G\omega_t^{-1}} \tag{43}$$

$$N_{tT}^* = \frac{1}{1+G(1-\omega_t)^{-1}} \tag{44}$$

$$N_{tN} = \frac{1}{1+Q+R\omega_t} \tag{45}$$

$$N_{tN}^* = \frac{1}{1+Q+R(1-\omega_t)} \tag{46}$$

where

$$\omega_t = \frac{1+G\left[1-(\theta_{tT}^*\theta_{tT}^{-1})^{\frac{1}{1-\alpha^T}}\right]}{1+(\theta_{tT}^*\theta_{tT}^{-1})^{\frac{1}{1-\alpha^T}}}$$

$$G = \frac{\mu(1-\beta\alpha^T)+(1-\gamma)(1-\alpha^N)(1-\beta\alpha^T)(1-\beta\alpha^N)^{-1}}{2\gamma(1-\alpha^T)}$$

$$Q = \frac{\mu(1-\beta\alpha^T)}{(1-\gamma)(1-\alpha^N)}; \qquad R = \frac{2\gamma(1-\beta\alpha^N)(1-\alpha^T)}{(1-\gamma)(1-\alpha^N)(1-\beta\alpha^T)}$$

The closed-form solutions derived above can be used to relate real exchange rate to the primitives of the underlying economic structure. For example, the parameters of utility, technology and the productivity shock processes.

C. Determination of Real Exchange Rate

The real exchange rate is defined as the nominal exchange rate adjusted for changes in the domestic and foreign price levels, and represents the real price of a foreign basket of goods relative to domestic one:

$$e_t^R = \frac{e_t P_t^*}{P_t} \tag{47}$$

where e_t^R is the real exchange rate, e_t is the nominal exchange rate (defined as the price of foreign currency in terms of a unit of domestic currency), and P_t^* is the foreign general price level and P_t is the general price level in the domestic economy. Under this definition, an increase in nominal exchange rate (real exchange rate) is equivalent to a depreciation (real depreciation) of domestic currency. Conversely, a fall in the nominal exchange rate (real exchange rate) is equivalent to an appreciation (real appreciation) of domestic currency.

The general price level (typically given by the consumer price index,[9] is embodied in the expenditure function associated with the solution to the consumers problem) in both economies is comprised of traded goods prices, P_{tT}, and nontraded goods prices, P_{tN}, as follows:

$$P_t = P_{tT}^{\gamma} P_{tN}^{(1-\gamma)} \tag{48}$$
$$P_t^* = P_{tT}^{*\gamma} P_{tN}^{*(1-\gamma)} \tag{49}$$

Combining equations (48) and (49), the ratio of traded goods prices can be expressed as

$$\frac{P_{tT}}{P_{tT}^*} = \left[\frac{\frac{P_{tN}^*}{P_{tT}^*}}{\frac{P_{tN}}{P_{tT}}} \right]^{(1-\gamma)} \frac{P_t}{P_t^*} \tag{50}$$

The purchasing power parity (PPP) hypothesis states that the nominal exchange rate adjusts one-for-one to movements in domestic and foreign price levels. PPP relies on the existence of arbitrage of goods and services between the domestic and foreign economy when the prices (denominated in a common currency) in the two economies diverge. However, if large price divergences can be arbitrated only for traded goods not for the nontraded goods,

---

[9] In empirical work, the consumer price index provides a convenient basis for measuring the real exchange rate.

PPP will exist only for traded goods in the long run. We model the assumption that PPP holds only for traded goods,

$$e_t P_{tT}^* = P_{tT} \tag{51}$$

Combining (47), (50) and (51) yields the following

$$e_t^R = \left[\frac{\frac{P_{tN}^*}{P_{tT}^*}}{\frac{P_{tN}}{P_{tT}}} \right]^{(1-\gamma)} \tag{52}$$

or, in logarithms,

$$q = (1-\gamma) \ln\left(\frac{P_{tN}^*}{P_{tT}^*}\right) - (1-\gamma) \ln\left(\frac{P_{tN}}{P_{tT}}\right) \tag{53}$$

where q is the real exchange rate in logarithms. Equation (53) states that the real exchange rate is a function of the relative price of non-tradeable in the domestic and foreign economy. An increase in the domestic relative price of nontradeables decreases q (the price of foreign currency in real terms), appreciating the domestic currency in real terms. Economies that experience an increase in the relative prices of nontraded goods are predicted to have an appreciating real exchange rate.

One method to model and explain the price levels of traded and nontraded goods is in terms of the consumer's optimization conditions.[10] The setup of the optimization problem in section III suggests that the competitive equilibrium is equivalent to the social planner's optimum, and hence prices are simply the Lagrange multipliers on the resource constraints. Therefore the first-order conditions of consumer's optimization can be used to solve for prices,

$$P_{tT} = \frac{\partial U(C_{tT}, C_{tN})}{\partial C_{tT}}$$

$$P_{tN} = \frac{\partial U(C_{tT}, C_{tN})}{\partial C_{tN}}$$

$$P_{tT}^* = \frac{\partial U(C_{tT}, C_{tN})}{\partial C_{tT}^*}$$

$$P_{tN}^* = \frac{\partial U(C_{tT}, C_{tN})}{\partial C_{tN}^*}$$

The first-order conditions illustrate some of the features of the equilibrium. In general PPP does not hold because output and, hence, prices of nontraded goods differ across countries. However, because utility functions are additively separable between traded and nontraded goods, and homothetic and identical across countries, marginal utilities of consumption, and hence the prices of traded goods will be equalized across countries. So bilateral real

---

[10] A number of recent empirical studies (Strauss (1997)) have modeled the price levels of traded and non-traded goods in terms of the "productivity differentials" hypothesis of Balassa (1964) and Samuelson (1964).

exchange rates between home and foreign countries depend only on the relative consumption of nontraded goods

$$e_t^R = \left(\frac{C_{tN}}{C_{tN}^*}\right)^{(1-\gamma)} \tag{54}$$

Equation (54) is similar to the exchange rate equation reported by Backus and Smith (1993) in that it shows a direct positive relation between bilateral real exchange rates and the ratio of nontraded consumption between domestic and foreign economies. Now, substituting the analytical solutions to optimal policy functions for consumption of nontraded goods derived above we can express real exchange rates in terms of the parameters of preference, technology, and the productivity differentials (or ratios) between home and foreign country in the traded and nontraded sectors. The exchange rate equations under the three alternative restrictive assumptions discussed earlier are derived as follows.

1.   Technology shocks are i.i.d:

Substituting the closed-form solution to optimal consumption of nontraded goods from equations (24) and (25) into the exchange rate equation (54) we get,

$$e_t^R = \left(\frac{\theta_{tN} K_{tN}^{\alpha^N}}{\theta_{tN}^* K_{tN}^{*\alpha^N}}\right)^{(1-\gamma)} \tag{55}$$

In this case real exchange movements depend only on the productivity differentials(ratios) between home and foreign country in the nontraded sector.

2.   Persistence of shock processes: Case 1: No capital and a linear technology:

Substituting the analytical solution from equations (33) and (34) into equation (54), we get the following exchange rate equation,

$$e_t^R = \left[\frac{\theta_{tN}\left[(1+B-AD)+(1+B+D+AD)(\theta_{tT}^*\theta_{tT}^{-1})\right]}{\theta_{tN}^*\left[(1+B+D+AD)+(1+B-AD)(\theta_{tT}^*\theta_{tT}^{-1})\right]}\right]^{(1-\gamma)} \tag{56}$$

3.   Case 2: Perfect foresight i.e. no uncertainty:

Substituting equations (45) and (46) into equation (54) yields the following expression for real exchange rate,

$$e_t^R = \left[\frac{\theta_{tN}K_{tN}^{\alpha^T}\left[\{(1+Q-GR)+(1+Q+R+GR)(\theta_{tT}^*\theta_{tT}^{-1}\}^{\frac{1}{1-\alpha^T}}\right]^{1-\alpha^N}}{\theta_{tN}^*K_{tN}^{\alpha^T}\left[\{(1+Q+R+GR)+(1+Q-GR)(\theta_{tT}^*\theta_{tT}^{-1}\}^{\frac{1}{1-\alpha^T}}\right]^{1-\alpha^N}}\right]^{(1-\gamma)} \tag{57}$$

In equations (55)-(57), real exchange rate depends on the parameters of preference and technology and the productivity differentials (ratios) in the traded and nontraded sectors. Therefore the implications of our simple model are striking because they provide expressions for equilibrium prices and real exchange rates which are easy to interpret and do not require restrictions on parameter values.

D. Relationship Between Productivity Shocks and Real Exchange Rate

Some comparative static exercises will now be performed to analyze the impacts of the relative productivity shocks in the traded and nontraded sectors on the bilateral real exchange rate movements. By partially differentiating the exchange rate equations (55)-(57), the effects of these exogenous disturbances on the level of real exchange rate can be derived.

1. Effects of productivity differentials in the nontraded sectors

In this case, we assume that shocks to the traded sector are constant. For all the exchange rate equations, we find that

$$\frac{\partial e_t^R}{\partial \left(\frac{\theta_{tN}}{\theta_{tN}^*} \right)} > 0$$

Thus, there exists a direct positive relationship between bilateral real exchange rates and the relative productivity shocks (between home and foreign country) in the nontraded industries. The economic intuition is clear: increases in domestic productivity growth in the nontraded sector, $[(\ln \theta_{tN} - \ln \theta_{tN}^*) > 0]$ (relative to foreign nontraded productivity and to domestic traded productivity) cause workers to move from the traded to the more productive nontraded sector, so that domestic output of the nontraded goods, Y_{tN}, increases and home firms are able to reduce P_{tN}. The fall in domestic relative price of nontraded goods, $[(P_{tN} - P_{tT}) < 0]$ leads to a rise in e_t^R, implying a real depreciation of home currency. Therefore, a rise in the relative productivity differentials (or ratios) in the nontraded sector will cause a real appreciation of foreign currency and a domestic depreciation of the real exchange rate.

2. Effects of productivity differentials in the traded sectors:

In this model, the productivity shocks in the traded sector appear as a determinant of real exchange rates only under the persistence of shock processes. From the exchange rate equations (56) and (57) we find that

$$\frac{\partial e_t^R}{\partial \left(\frac{\theta_{tT}}{\theta_{tT}^*}\right)} \quad < \quad 0$$

which indicates an inverse relation between the relative productivity shocks in the traded sector and bilateral real exchange rates.(See Appendix II for proof.) The economic intuition is straightforward: an increase in domestic productivity of traded goods, $[(\ln \theta_{tT} - \ln \theta_{tT}^*) > 0]$ (relative to foreign traded productivity) causes both labor and capital to become more productive in that sector, so that domestic output of the traded good Y_{tT} increases,[11] which allows domestic firms to reduce P_{tT}. The consequent rise in the relative price of non-tradeables, $[(P_{tN} - P_{tT}) > 0]$ will cause an appreciation of the real exchange rate, implying a fall in e_t^R. Therefore, an increase in the productivity differentials (or ratios) in the traded sector implies a depreciation of the foreign currency in real terms and a real domestic appreciation.

IV.   Empirical Results

This section presents empirical results using quarterly data for G-7 countries for the period 1960:QI-1997:QIII. The following discussion contains a brief description of the definitions, construction, and sources of the data; the summary statistics (standard deviation, autocorrelation and correlation) of nominal and real exchange rate and the productivity differentials; the results of Augmented Dickey-Fuller tests for unit roots of relevant variables; and the examination of the impact of relative productive shocks on the real exchange rate using the multivariate cointegration methodology proposed by Johansen (1988) and Johansen and Juselius (1990 and 1992).

A.   Data Descriptions

This paper constructs total factor productivity[12] for the traded and nontraded sectors for the G-7using OECD data from the International sectoral Database. The sector breakdowns follow the International Standard Industrial Classifications (ISIC) currently used in the OECD National Accounts (ANA) publication. Under this sectoral decomposition, nontraded goods account for half of the aggregate output. Following the work by the OECD (1988), the open or traded good sectors comprise only manufacturing, and the nontraded goods are the service sectors comprising: (1) electricity, gas and water, (2) construction, (3) wholesale and retail trade, restaurants and hotels, (4) transport, storage and communications, (5) financial service

---

[11]   Domestic output of traded goods, Y_{tT} rises due to movements of labor from domestic non-traded sector and also movements of capital from foriegn traded-sector.

[12]   Total factor productivity, also known as Solow residual, is defined as output per unit of weighted capital and labor. For Cobb-Douglas technology it is given by : $\theta = \frac{Y_t}{K_t^\alpha N_t^{1-\alpha}}$ and in logarithmic terms, $\log \theta_t = \log Y_t - \alpha \log K_t - (1 - \alpha) \log N_t$.

and insurance, (6)community, social and personal services and (7) government services.[13] Productivity data for the traded(nontraded) sector are obtained by dividing the OECD's figures for GDP in constant prices for the traded (nontraded) sector by total labor employment for that sector, yielding labor productivity or real output per worker.[14]

Home and foreign productivity differentials (or ratios) are estimated relative to the United States i.e. using the United States as the benchmark. Productivity differentials (or ratios) in the traded sector is defined as : $\ln\left(\frac{\theta_{iT}^i}{\theta_{tT}^{US}}\right)$ or $(\ln\theta_{tT}^i - \ln\theta_{tT}^{US})$ where θ_t is the total factor productivity and $i = Canada, France, Germany, Italy$, and $United\ Kingdom$. Similarly, for the nontraded sector: $\ln\left(\frac{\theta_{tN}^i}{\theta_{tN}^{US}}\right)$ or $(\ln\theta_{tN}^i - \ln\theta_{tN}^{US})$.

The GDP price deflators are constructed by dividing the nominal GDP for the traded (nontraded) by the real GDP for that sector. The nominal exchange rate for each country is the bilateral exchange rate vis-a-vis the U.S dollar. [15] Real exchange rates are measured using the nominal exchange rates and individual country consumer price indexes(CPI). Prices and the exchange rate data are from the International Financial Statistics, IFS. Definitions of the variables and the sources are given in Appendix I.

B. Summary Statistics

Table 1 presents of Hodrick-Prescott filtered quarterly observations of nominal and real exchange rates and productivity differentials for G-7countries. Three regularities are evident. *First,* for each country standard deviation of both nominal and real exchange rates are much higher during the recent flexible exchange rate period (1973-1997) compared to the estimates for the entire sample period (1960-97) and for the fixed exchange rate period (1960-73).This result extends support to one of the stylized facts of exchange rate behavior well documented by empirical studies[16]. *Second,* in all cases (except for Canada) the volatility

---

[13] Agriculture and mining are considered neither traded nor nontraded since intercountry trade is partially hindered in some economies by large tariffs and informal barriers; thus these sectors are excluded from the sample.

[14] We omit capital due to the unavailability of the capital stock data on a quarterly basis. This may not be a serious omission, since evidence indicates that capital stock does not contribute significantly to the cyclical fluctuations of output. (See, e.g., Kydland and Presscott (1982) and Backus,Kehoe, and Kydland (1993).)

[15] Bilateral rates were used to avoid issues associated with: (1) the statistical properties of multilateral aggregates, (2) the sensivitiy of results of shifting weights, (3) the omission of relevant countries in the multilateral aggregate, and (4) collinearity between exchange rates because a multilateral index is related to the index of other economies. For example, a weighted sum of a nonstationary and stationary series is a nonstationary series, and hence the exchange rate would be very sensitive to the construction and statistical properties of the relevant rates (Strauss(1997)).

[16] It has been well documented in the empirical studies since at least Mussa (1986) that volatility of nominal and real exchange rate markedly differ under different exchange rate regimes.

Table 1: Summary Statistics of Exchange Rates and Productivity Differentials, Quarterly data, 1960-1997

Country	Sample Period 1/	Standard Deviation				Persistence 2/				Correlation		
		e	q	$\ln\left(\dfrac{\theta^i_{tT}}{\theta^{US}_{tT}}\right)$	$\ln\left(\dfrac{\theta^i_{tN}}{\theta^{US}_{tN}}\right)$	e	q	$\ln\left(\dfrac{\theta^i_{tT}}{\theta^{US}_{tT}}\right)$	$\ln\left(\dfrac{\theta^i_{tN}}{\theta^{US}_{tN}}\right)$	$\begin{array}{c}corr\\(e,q)\end{array}$	$corr\left(q,\ln\left(\dfrac{\theta^i_{tT}}{\theta^{US}_{tT}}\right)\right)$	$corr\left(q,\ln\left(\dfrac{\theta^i_{tN}}{\theta^{US}_{tN}}\right)\right)$
Canada	I	2.2	2.5	2.2	2.1	0.8	0.9	0.8	0.8	1.0	-0.5	0.5
	II	1.3	1.2	2.0	1.3	0.8	0.8	0.8	0.8	0.9	-0.3	0.4
	III	3.0	3.5	2.9	2.9	0.8	0.8	0.8	0.8	1.0	-0.6	0.6
France	I	5.3	5.3	3.4	3.1	0.9	0.9	0.8	0.8	0.9	-0.6	0.5
	II	3.1	4.1	2.8	2.1	0.8	0.8	0.7	0.7	0.9	-0.5	0.4
	III	7.3	8.1	3.9	3.2	0.8	0.8	0.8	0.8	0.9	-0.7	0.6
Germany	I	6.9	6.8	2.3	2.0	0.8	0.8	0.8	0.8	1.0	-0.6	0.6
	II	4.2	4.0	3.5	3.1	0.8	0.8	0.8	0.8	0.9	-0.5	0.5
	III	8.9	8.3	3.7	3.0	0.9	0.8	0.8	0.7	0.9	-0.5	0.5
Italy	I	5.9	5.3	3.6	3.2	0.8	0.8	0.8	0.8	0.8	-0.5	0.6
	II	3.9	3.5	2.9	2.1	0.8	0.8	0.8	0.8	0.9	-0.5	0.3
	III	7.0	6.8	3.9	2.9	0.8	0.8	0.8	0.8	0.8	-0.6	0.5
Japan	I	7.2	7.0	2.3	2.1	0.8	0.8	0.8	0.7	0.9	-0.6	0.5
	II	4.7	4.5	1.8	1.7	0.8	0.8	0.7	0.7	0.9	-0.4	0.3
	III	9.2	8.3	2.7	2.1	0.8	0.8	0.7	0.7	0.9	-0.7	0.5
United Kingdom	I	6.9	7.0	1.9	2.0	0.9	0.8	0.8	0.7	0.9	-0.6	0.6
	II	4.1	4.0	1.1	1.1	0.8	0.8	0.8	0.8	0.9	-0.5	0.5
	III	10.0	9.0	2.2	2.3	0.9	0.8	0.7	0.8	1.0	-0.6	0.6

Notes: 1/ I: 1960:01-1997:03; II:1960:01-1973:03; III: 1973:03-1997:03.
  2/ Persistence measure is the first-order autocorrelation coefficient.

Source: OECD Quarterly National Accounts and International Financial Statistics.

of nominal and real exchange rates arebetween two to five times higher than the volatility of productivity differentials in traded and nontraded sectors[17].Third, exchange rate movements are highly persistence as indicated by first-order autocorrelation coefficients for real and nominal exchange rate of approximately 0.84. On the average, productivity differentials are less persistent than those of real exchange rate .The estimates of sample correlations suggest the following features of association among exchange rates and productivity differentials: first, nominal and real exchange rates are significantly positively correlated with the correlation coefficient close to one; second, there is positive relationship between productivity differentials(between home and foreign country) in the nontraded sector and the real exchange rate; third, the correlations between real exchange rate and productivity differentials in the traded sector is negative. The direction of correlations are consistent with the predictions of the theoretical model regarding the relationship between real exchange rate and home and foreign productivity differentials.

C.   Test for Stationarity

The first step in the cointegration analysis is to test the unit roots in each variable.[18] To this end we apply Augmented Dickey-Fuller (ADF) stationarity tests on the real exchange rate (q), productivity differential in the traded sector $(\ln \theta^i_{tT} - \ln \theta^{US}_{tT})$ and differential in the nontraded sector $(\ln \theta^i_{tN} - \ln \theta^{US}_{tN})$. The ADF tests are performed assuming a constant, one difference and a time trend. Table 2 reports the results of the ADF test for the level as well as for the first-difference of the relevant variables. For all countries, the tests fail to reject at the 5 percent level the null hypothesis of a unit root in the data-generating process. The null hypothesis is rejected when the series are first-differenced i.e., all variables are first-differenced statationary. This implies that all the series are integrated of order one, $I(1)$ variables.

D.   Cointegration tests

We examine the relationship between the real exchange rate and the sectoral productivity differentials between home and foreign country using the multivariate cointegration methodology proposed by Johansen (1988) and Johansen and Juselius (1990 and 1992).

---

[17]   Using data for OECD countries, Backus, Kehoe and Kydland (1993) reported that volatitity of terms of trade (which they assume as equivalent to real exchange rate) is much higher compared to the output volatility.

[18]   Since the conintegration methodology involves finding a stationary, linear combination of a set of variables which are themselves non-stationary, a precondition is that all variables are non-stationary.

$$DZ_t = \alpha_1 + \alpha_2 Z_{t-1} + \alpha_4 trend$$

$$H_0 : \alpha_2 = 0(UnitRootExists)$$

Country	Level			First Difference		
	q	$ln\left(\dfrac{\theta^i_{tT}}{\theta^{US}_{tT}}\right)$	$ln\left(\dfrac{\theta^i_{tN}}{\theta^{US}_{tN}}\right)$	q	$ln\left(\dfrac{\theta^i_{tT}}{\theta^{US}_{tT}}\right)$	$ln\left(\dfrac{\theta^i_{tN}}{\theta^{US}_{tN}}\right)$
Canada	-2.34	-2.65	-3.01	-4.44	-6.44	-6.78
France	-2.59	-3.13	-2.76	-5.66	-5.93	-4.98
Germany	-2.35	-2.69	-2.89	-4.98	-4.77	-4.24
Italy	-3.07	-2.51	-2.43	-5.23	-4.34	-5.64
Japan	-2.01	-2.97	-2.95	-4.94	-5.22	-5.07
United Kingdom	-2.56	-2.16	-2.77	-5.67	-4.67	-4.78

Notes: 1/ The Mackinnon (1991) critical value of the ADF test at the 5% level of significance is -3.49.

Variables:

q = log of real exchange rate.

$ln\left(\dfrac{\theta^i_{tT}}{\theta^{US}_{tT}}\right)$ = relative productivity in the traded sector.

$ln\left(\dfrac{\theta^i_{tN}}{\theta^{US}_{tN}}\right)$ = relative productivity in the non-traded sector.

Source: OECD Quarterly National Accounts and International Financial Statistics.

The Johansen maximum likelihood approach has some advantages over the traditional Engle-Granger procedure: (i) it allows testing in a multivariate framework, (ii) considers the error structure of the data processes, (iii) allows for interactions in the determination of the relevant economic variables independent of the choice of the endogenous variable and (iv) allows explicit hypotheses tests of parameter estimates and rank restrictions using likelihood ratio tests that employ Chi-Square statistics.[19]

Johansen tests can determine the rank of exchange rate equation (56) by testing the following vector

$$J = \left[q \left(\ln \theta_{tT}^i - \ln \theta_{tT}^{US} \right) \left(\ln \theta_{tN}^i - \ln \theta_{tN}^{US} \right) \right] \tag{58}$$

where all variables are in logarithms. Johansen tests suggest that if a linear combination of vector (58) is a rank one (i.e., a linear combination of relevant vector is stationary), then a single cointegrating relationship exists between productivity differentials (between home and foreign country) in the traded and nontraded sector and the real exchange rate. Chi-Square restrictions tests are then imposed to test hypotheses regarding the cointegration space of the vector.

Table 3 reports Johansen cointegration and Chi-Square restriction tests for real exchange rates and domestic and foreign productivity differentials in the traded and nontraded sectors. Column III indicated that the null of no cointegrating linear combinations between real exchange rate and sectoral productivity differentials between domestic and foreign country can be rejected at the 1percent confidence level for all countries except Germany, where the null is rejected at 5 percent significance level. Cointegrating ranks of less than one or two cannot be rejected. Therefore, at least one cointegrating relationship is supported by the data generating process.

The next step is to show that the cointegrating linear combination arises not solely between the productivity differentials, but between real exchange rates and domestic and foreign productivity differentials. Column IV reports the reduced rank restriction tests $[0 \ \beta_2 \ \beta_3]$ for the cointegrating vector $[\beta_1 q \ \beta_2(\ln \theta_{tT}^i - \ln \theta_{tT}^{US}) \ \beta_3(\ln \theta_{tN}^i - \ln \theta_{tN}^{US})]$. The test results reject the no significance of real exchange on the cointegrating vector. This implies that the cointegrating results are not due to linear combination between productivity differentials between domestic and foreign country in the traded sector and in the nontraded sector. Therefore, the cointegrating results reported in column III arise due to a linear combination between real exchange rates and sectoral productivity differentials between home and foreign country.

---

[19] For details, see Johansen and Juselius (1990,1992)

Table 3: Johansen Cointegration and Hypothesis Tests

$$vector: \left[\beta_1 q \quad \beta_2\left(\ln\theta_{iT}^i - \ln\theta_{iT}^{US}\right)\beta_3\left(\ln\theta_{iN}^{US}\right)\right]$$

Country	Null	λmax rank Statistic	Likelihood Ratio Tests $0\ \beta_2\ \beta_3$	Johansen Coeff. & S.E.	
				β_2	β_3
Canada	r=0	54.24**	8.89**	-2.67**	4.66**
	r<1	24.25		(0.56)	(0.67)
	r<2	7.92			
France	r=0	55.78**	7.67*	-3.45**	4.54**
	r<1	19.65		(1.07)	(1.09)
	r<2	9.89			
Germany	r=0	32.45*	9.67**	-2.78**	3.87**
	r<1	20.77		(0.89)	(1.22)
	r<2	5.66			
Italy	r=0	57.33**	7.05*	-4.56**	3.56**
	r<1	27.29		(0.56)	(0.79)
	r<2	9.88			
Japan	r=0	56.43**	8.95**	-5.89**	3.37**
	r<1	23.09		(0.67)	(0.85)
	r<2	6.77			
United Kingdom	r=0	58.77**	10.55**	-4.67**	5.77**
	r<1	27.09		(1.21)	(1.51)
	r<2	12.44			

Notes: ** = significance at the 1% level; * = significance at the 5% level.

Variables:
q = log of real exchange rate

$\ln\left(\dfrac{\theta_{iT}^i}{\theta_{iT}^{US}}\right)$ = domestic traded productivity relative to foreign traded productivity.

$\ln\left(\dfrac{\theta_{iN}^i}{\theta_{iN}^{US}}\right)$ = domestic non-traded productivity relative to foreign non-traded productivity.

Column II: Ho: No unit root.

Source: OECD Quarterly National Accounts and International Financial Statistics.

Columns V and VI report the test statistic for individual significance of productivity differentials in each sector on the real exchange rate. These columns contain the magnitude, sign, and adjusted Johansen standard errors for the productivity differentials in the traded and nontraded sectors. In all cases, the coefficients are significantly different from zero with the correct sign, which is consistent with the predictions of the theory. In all six economies, higher productivity differentials in the traded sector (increases in domestic traded productivity relative to foreign traded productivity, i.e., $(\ln \theta_{tT}^i - \ln \theta_{tT}^{US}) > 0$) implies a significant increase in the relative price of nontraded goods in the home country, which is equivalent to a real appreciation of the exchange rate (i.e., a decline in the price of foreign currency). Conversely, in all cases, increases in the productivity differentials in the nontraded sectors $((\ln \theta_{tN}^i - \ln \theta_{tN}^{US}) > 0$ implies a decline in the relative price of nontraded goods and consequently a real depreciation of the exchange rates.

V. Conclusion

In this paper a two-country dynamic general equilibrium model with nontraded goods has been developed to examine the impact of productivity differentials (between domestic and foreign country) in the traded and nontraded sector on the real exchange rate movements. The model yields closed-form analytical solutions for optimum consumption, investment, and hours and for equilibrium real exchange rate. The real exchange rate obtained depends on the parameters of preference, technology and the sectoral productivity differentials between home and foreign country. The theory predicts a close relationship between the real exchange rate and domestic and foreign productivity differentials. Increases in the domestic productivity of traded goods (relative to the foreign productivity of traded goods) lead to an appreciation of real exchange rates. Conversely, increases in the domestic productivity of nontraded goods cause a real depreciation of exchange rates.

Empirical results using data for G-7 countries suggest excess volatility of exchange rates and positive comovements between real and nominal exchange rates during the recent floating exchange rate period. The estimates of sample correlations (between real exchange rates and productivity differentials) coefficients are consistent with the sign predicted by the theoretical model. Johansen cointegration tests also show that a significant cointegrating relationship exists between relative (between domestic and foreign) productivity shocks in each sector and the real exchange rate. Therefore, sectoral productivity differentials (or ratios) between home and foreign country may be important determinants of real exchange rate fluctuations.

References

Amano, Robert and Simon Van Norden (1992). "A Forecasting Equation for the Canada-US Dollar Exchange Rate ," in *The Exchange Rate and the Economy*, the Bank of Canada.

Backus, David, Patrick Kehoe, and Finn Kydland (1994). "Dynamics of the trade balance and the terms of trade: The J-curve ?," *American Economic Review* Papers and Proceedings.

Backus, David, Patrick Kehoe, and Finn Kydland(1993). "International Real Business Cycles," *Journal of Political Economy*, 4:745-775.

Backus, David, and Gregor Smith (1993). "Consumption and Real Exchange Rates in Dynamic Exchange Economics with Nontraded Goods," *Journal of International Economics*, 35:297-316.

Bahmani, Oskooee M. and Hyun-Jae Rhee (1996) "Time-series Support for Balassa's Productivity-Bias Hypothesis: Evidence from Korea," *Review of International Economics*, 4:364-70.

Balassa, Bela (1964). "The Purchasing Power Parity Doctrine: A Reappraisal," *Journal of Political Economy*, December: 584-96.

Balvers, Ronald J., and Jeffrey H Bergstrand (1997), "Equilibrium Real Exchange Rates: Closed-Form Theoretical Solutions and Some Empirical Evidence," *Journal of International Money and Finance*, 16:345-366.

Cantor, Richard, and Mark Nelson (1988). "The International Transmission of Real Business Cycles," *International Economic Review*, 21: 493-507.

Cardia, Emanuela.(1991). "The Dynamics of a Small Open Economy in Response to Monetary, Fiscal and Productivity Shocks," *Journal of Monetary Economics*, 28:411-434.

Chinn, Menzie-David (1997). "Whither the Yen? Implications of an intertemporal Model of the Dollar-Yen Rate," *Journal of the Japanese and International Economics*, 11:228-46.

Costello, Donna and Jack Praschnik(1993). "The Roll of Oil Price Shocks in a Two-Sector, Two-Country Model of the Business Cycle," mimeo, University of Western Ontario.

De-Gregorio., and Wolf, Holger C (1994). "Terms of Trade, Productivity, and the Real Exchange Rate," *National Bureau of Economic Research Working Paper*, 4807:10-17.

Dutton, Marilyn and Jack Strauss (1997). "Cointegration Tests of Purchasing Power Parity: the Impact of Nontraded Goods," *Journal of International Money and Finance*, 16:433-444.

Frankel, Jeffrey A, (1981) "On the Mark: Reply [on the Mark: A Theory of Floating Exchange Rates Based on Real Interest Differential," *American Economic Review*, 71:5, 1075-82.

Johansen, Soren. (1988) "Statistical Analysis of Cointegration Vectors," *Journal of Economics Dynamics and Control* , 12:3, 231-54.

Johansen, Soren. and Katarina Juselius (1990) "Maximum Likelihood Estimation and Inference on Cointegration–With Applications to the Demand for Money," *Oxford Bulletin of Economics and Statistics*, 52:2, 169:210.

Johansen, Soren. and Katarina Juselius (1992) "Testing Structural Hypothesis in a Multivariate Cointegration Analysis of the PPP and the UIP for the UK," *Journal of Econometrics*, 53:3, 211-44.

Helpman, E. and Razin, A. (1982) "Dynamics of a Floating Exchange Rate Regime," *Journal of Political Ecomony* 90:4, 728-54.

Isard, Peter (1977). "How far Can We Push the Law of one Price," *American Economic Review* 67:5, 942-48.

Kollmann, Robert (1995). " The Correlation of Productivity Growth Across Regions and Industries in the United States," *Economic Letters*, 47:3, 437-43.

Kydland, Finn E. and Edward C Prescott (1982). "Time to Build and Aggregate Fluctuations," *Econometrica*, 50:6, 1345-70.

Lastrapes, William (1992). "Sources of Fluctuations in Real and Nominal Exchange Rates," *Review of Economics and Statistics*, 74:530-39.

Lucas, Robert (1982). "Interest rates and Currency Prices in a Two-Country World ," *Journal of Monetary Economics*, 10:335-60.

Manzur, Meher (1991). " Purchasing Power Parity and Relative Price Variability: The Missing Link?," *Australian Economic Papers*, 30:128-47.

Mark, Nelson C. and Doo-Yull Choi (1997) "Real Exchange-Rate Predictions over Logn Horizons," *Journal of International Economics*, 43:29-60.

Mendoza, Enrique G. (1995) "The Terms of Trade, The Real Exchange Rate, and Economic Fluctuations," *International Economic Review*, 36:101-137.

Mussa, Michael. (1986). "Nominal Exchange Rate Regimes and the Behavior of Real Exchange Rates," in K.Brunner and A. Meltzer, eds., *Real Business Cycles, Real Exchange Rates, and Actual Policies*, Carnegie-Rochester Conference Series.

Roll, Richard, and Solnik, Bruno (1979) "On Some Parity Conditions Encountered Frequently in International Economics," *Journal of Macroeconomics,* 1:3,267:83.

Samuelson, P.A. (1964) "Theoretical Notes on Trade Problems," *Review of Economics and Statistics* 46:145-154.

Solow, Robert. (1957). " Technical Change and the Aggregate Production Function," *Review of Economics and Statistics*, 39:312-320.

Stein, Jerome, L. (1995). " The Fundamental Determinants of the US Dollar relative the G7," Brown University, Department of Economics Working Paper, February:38-50.

Stockman, Alan. (1980). " A Theory of Exchange Rate Determination," *Journal of Political Economy,* 88:673-98.

Stockman, Alan. (1987). " The Equilibrium Approach to Exchange Rates ," *EconomicReview*, Federal Reserve Bank of Richmond, 73:12-31.

Stockman, Alan, and Linda Tesar (1995). "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements ," *American Economic Review*, 85:168-185.

Stockman, A.C. and Svensson, L. (1987) "Capital flows, Investment and Exchange Rates," *Journal of Monetary Economics* 19:171-201.

Stockman, Alan C. and Dellas, Harris (1989) "International Portfolio Nondiversification and Exchange Rate Variability," Journal of International Economics, 26:3, 271-89.

Strauss, Jack. (1996) " The Cointegrating Relationship between Productivity, Real Exchange Rates and Purchasing Power Parity," *Journal of Macroeconomics*, 18:299-313.

Strauss, Jack. (1995), "Real Exchange Rates, PPP and the Relative Price of Nontraded Goods," *Southern Economic Journal*, 61:991-1005.

Stulz, R.M. (1987). " An Equilibrium Model of Exchange Rate Determination and Asset Pricing with nontraded goods and Imperfect Information," *Journal of Political Economy*, 95:1024-1040.

Taylor, M.P. (1995) "The Economics of Exchange Rates," Journal of Economic Literature, 33:13-47.

Table 4. Definition of the Variables and Sources

Variable name	Definition	Source
Traded Productivity	Total factor (labor) productivity in the traded sector	OECD International Sectoral Database
Non-traded Productivity	Total factor (labor) productivity in the non-traded sector	OECD International Sectoral Database
Traded Productivity Differentials	Difference between home and foreign labor productivity in the traded sector	OECD International Sectoral Database
Nontraded Productivity Differentials	Difference between home and foreign labor productivity in the non-traded sector	OECD International Sectoral Database
Price index for output	GDP deflator	International Financial Statistics
Nominal Exchange Rate	Bilateral exchange rate in each country per U.S. dollar	International Financial Statistics
Real Exchange Rate	Nominal exchange rate multiplied by the ratio of foreign to domestic price levels	International Financial Statistics
Price Index for real exchange rate	Consumer price indexes in each country	International Financial Statistics

Social Planner's Problem (Shocks are i.i.d. process):

The social planner maximizes:

$$V(s_t) = \max \begin{bmatrix} \gamma \log(Y_{Tt} + Y_{Tt}^* - C_{Tt}^* - K_{(t+1)T} - K_{(t+1)T}^*) + \\ (1-\gamma)\log(Y_{Nt} - C_{Nt} - K_{(t+1)N}) + \mu\log(1 - N_{Tt} + N_{Nt}) + \\ \gamma\log(C_{tT}^*) + (1-\gamma)\log(Y_{Nt}^* - C_{Nt}^* - K_{(t+1)N}^*) + \\ \mu\log(1 - N_{Tt}^* + N_{Nt}^*) + \beta E_t V(s_{(t+1)}) \end{bmatrix} \qquad (1)$$

subject to resource constraints and technology.

The first-order-conditions are:

$$C_{tT}^* : \frac{\gamma}{C_{tT}} = \frac{1-\gamma}{C_{tT}^*} \qquad (2)$$

$$K_{(t+1)T} : \frac{\gamma}{C_{tT}} = \beta E_t V_{k_{(t+1)T}} \qquad (3)$$

$$K_{(t+1)N} : \frac{1-\gamma}{C_{tN}} = \beta E_t V_{k_{(t+1)N}} \qquad (4)$$

$$K_{(t+1)T}^* : \frac{\gamma}{C_{tT}^*} = \beta E_t V_{k^*_{(t+1)T}} \qquad (5)$$

$$K_{(t+1)N}^* : \frac{1-\gamma}{C_{tN}^*} = \beta E_t V_{k^*_{(t+1)N}} \qquad (6)$$

$$N_{tT} : \frac{\mu}{(1 - N_{tT} - N_{tN})} = \frac{\gamma(1-\alpha^T)Y_{tT}}{C_{tT}N_{tT}} \qquad (7)$$

$$N_{tN} : \frac{\mu}{(1 - N_{tT} - N_{tN})} = \frac{(1-\gamma)(1-\alpha^N)Y_{tN}}{C_{tN}N_{tN}} \qquad (8)$$

$$N_{tT}^* : \frac{\mu}{(1 - N_{tT}^* - N_{tN}^*)} = \frac{\gamma(1-\alpha^T)Y_{tT}^*}{C_{tT}^*N_{tT}^*} \qquad (9)$$

$$N_{tN}^* : \frac{\mu}{(1 - N_{tT}^* - N_{tN}^*)} = \frac{(1-\gamma)(1-\alpha^N)Y_{tN}^*}{C_{tN}^*N_{tN}^*} \qquad (10)$$

These FOCs implicitly define optimal decision rules for consumption, investment and hours.

Traded sector:

From equation (2) we get,

$$C_{tT} = C_{tT}^*$$

Applying Benveniste and Sheinkman formula we obtain,

$$V_{K_T} = \frac{\alpha^T \gamma Y_{(t+1)T}}{C_{(t+1)T} K_{(t+1)T}} \tag{11}$$

Substituting (10) into equation (3) yields,

$$\frac{\gamma}{C_{tT}} = \beta \alpha^T \gamma E_t \left(\frac{Y_{(t-1)T}}{C_{(t+1)T} K_{(t+1)T}} \right) \tag{12}$$

Let the guess for the optimal consumption for traded goods, C_{tT} be

$$C_{tT} = \phi(Y_{tT} + Y_{tT}^*) \qquad \qquad \phi > 0$$

Substituting the guess into (11) gives,

$$\frac{K_{(t+1)T}}{\phi(Y_{tT} + Y_{tT}^*)} = \beta \alpha^T E_t \left(\frac{Y_{(t+1)T}}{\phi(Y_{(t+1)T} + Y_{(t+1)T}^*)} \right)$$

$$\Rightarrow K_{(t+1)T} = \beta \alpha^T (Y_{tT} + Y_{tT}^*) E_t \left(\frac{Y_{(t+1)T}}{Y_{(t+1)T} + Y_{(t+1)T}^*} \right)$$

$$\Rightarrow \overline{K}_{(t+1)T} = \beta \alpha^T \delta_{t+1} (Y_{tT} + Y_{tT}^*) \tag{13}$$

where

$$\delta_{t+1} = E_t \left(\frac{Y_{(t+1)T}}{Y_{(t+1)T} + Y_{(t+1)T}^*} \right)$$

is the home countries' relative share of world of traded good. Under the assumption of i.i.d shocks and full depreciation of capital δ_{t+1} is constant. Now applying the same formula to $K_{(t+1)T}^*$ yields,

$$K_{(t+1)T}^* = \beta \alpha^T (Y_{tT} + Y_{tT}^*) E_t \left(\frac{Y_{(t+1)T}^*}{Y_{(t+1)T} + Y_{(t+1)T}^*} \right)$$

$$\Rightarrow \overline{K}_{(t+1)T}^* = \beta \alpha^T (1 - \delta_{t+1})(Y_{tT} + Y_{tT}^*) \tag{14}$$

Substituting (13) and (14) into the world resource constraint for traded goods gives the solution to optimum traded consumption

$$2C_{tT} = (Y_{tT} + Y_{tT}^*) - \beta \alpha^T \delta_{t+1}(Y_{tT} + Y_{tT}^*) - (\beta \alpha^T (1 - \delta_{t+1})(Y_{tT} + Y_{tT}^*))$$

$$\Rightarrow \overline{C}_{tT} = \frac{1}{2} \left(1 - \beta \alpha^T \right) (Y_{tT} + Y_{tT}^*) \tag{15}$$

Non-traded sector:

Applying Benveniste and Sheinkman formula to $K_{(t+1)N}$ we get,

$$\frac{1 - \gamma}{C_{tT}} = \beta \alpha^N (1 - \gamma) E_t \left(\frac{Y_{(t+1)N}}{C_{(t+1)N} K_{(t+1)N}} \right) \tag{16}$$

Let the guess for the optimal consumption for non-traded goods be
$$C_{tN} = \phi Y_{tN} \qquad\qquad \phi > 0$$

Substituting the guess into equation (4) yields,
$$K_{(t+1)N} = \beta \alpha^N Y_{tN}$$

Substituting this into the resource constraint for non-traded goods gives domestic optimum consumption of non-traded goods
$$\overline{C}_{tN} = (1 - \beta \alpha^N) Y_{tN} \tag{17}$$

Similarly for the foreign country
$$\overline{C}^*_{tN} = (1 - \beta \alpha^N) Y^*_{tN} \tag{18}$$

Solution to hours:

Equation (6) gives,
$$\frac{\mu}{1 - N_{tT} - N_{tN}} = \frac{\gamma(1 - \alpha^T)Y_{tT}}{C_{tT} Y_{tT}}$$

Substituting the solution to C_{tT} yields,
$$\frac{\mu N_{tT}}{1 - N_{tT} - N_{tN}} = \left(\frac{2\gamma(1 - \alpha^T)}{1 - \beta \alpha^T} \right) \left(\frac{Y_{tT}}{Y_{tT} + Y^*_{tT}} \right)$$
$$\Rightarrow \frac{\mu N_{tT}}{1 - N_{tT} - N_{tN}} = \left(\frac{2\gamma(1 - \alpha^T)}{1 - \beta \alpha^T} \right) \delta_t$$

where $\delta_t = \frac{Y_{tT}}{Y_{tT} + Y^*_{tT}}$ is constant under the assumption of i.i.d shocks and full depreciation of capital. In particular, $\delta_t = \frac{1}{2}$.

Therefore,
$$\frac{\mu N_{tT}}{1 - N_{tT} - N_{tN}} = \left(\frac{\gamma(1 - \alpha^T)}{1 - \beta \alpha^T} \right) \tag{19}$$

Similarly, for non-traded sector, from equation (8),
$$\frac{\mu N_{tN}}{1 - N_{tT} - N_{tN}} = \left(\frac{(1 - \gamma)(1 - \alpha^T)}{1 - \beta \alpha^T} \right) \tag{20}$$

Combining (19) and (20) yields the optimum solution to hours
$$\frac{N_{tN}}{N_{tT}} = \frac{(1 - \gamma)(1 - \alpha^N)(1 - \beta \alpha^T)}{\gamma(1 - \alpha^T)(1 - \beta \alpha^N)}$$
$$\Rightarrow \overline{N}_{tT} = \frac{\gamma(1 - \alpha^T)}{\mu(1 - \beta \alpha^T) + \gamma(1 - \alpha^T) + (1 - \gamma)(1 - \beta \alpha^T)(1 - \beta \alpha^N)^{-1}(1 - \alpha^N)}$$

$$\Rightarrow \overline{N}_{tN} = \frac{(1-\gamma)(1-\alpha^N)}{\mu(1-\beta\alpha^N) + (1-\gamma)(1-\alpha^N) + \gamma(1-\beta\alpha^N)(1-\beta\alpha^T)^{-1}(1-\alpha^T)}$$

Since $\delta = \frac{1}{2}$, this implies that $\overline{N}_{tT} = \overline{N}^*_{tT}$ and $\overline{N}_{tN} = \overline{N^*}_{tN}$.

Social Planner's Problem (Persistent Shocks: No Capital and Linear Technology)

$$V(s_t) = \max \begin{bmatrix} \gamma\log(Y_{Tt} + Y^*_{Tt} - C^*_{Tt}) + (1-\gamma)\log(Y_{Nt}) + \\ \mu\log(1 - N_{Tt} + N_{Nt}) + \gamma\log(C^*_{tT}) + (1-\gamma)\log(Y^*_{Nt}) + \\ \mu\log(1 - N^*_{Tt} + N^*_{Nt}) + \beta E_t V(s_{(t+1)}) \end{bmatrix} \qquad (21)$$

subject to the linear technologies

$$Y_{tT} = \theta_{tT} N_{tT}$$
$$Y^*_{tT} = \theta^*_{tT} N^*_{tT}$$
$$Y_{tN} = \theta_{tN} N_{tN}$$
$$Y^*_{tN} = \theta^*_{tN} N^*_{tN}$$

and the resource constraints.

First-order-conditions are:

$$C^*_{tT} : \frac{\gamma}{C_{tT}} = \frac{1-\gamma}{C^*_{tT}} \qquad (22)$$

$$N_{tT} : \frac{\mu}{(1 - N_{tT} - N_{tN})} = \frac{\gamma Y_{tT}}{C_{tT} N_{tT}} \qquad (23)$$

$$N^*_{tT} : \frac{\mu}{(1 - N^*_{tT} - N^*_{tN})} = \frac{\gamma Y^*_{tT}}{C^*_{tT} N^*_{tT}} \qquad (24)$$

$$N_{tN} : \frac{\mu}{(1 - N_{tT} - N_{tN})} = \frac{(1-\gamma)Y_{tN}}{C_{tN} N_{tN}} \qquad (25)$$

$$N^*_{tN} : \frac{\mu}{(1 - N^*_{tT} - N^*_{tN})} = \frac{(1-\gamma)Y^*_{tN}}{C^*_{tN} N^*_{tN}} \qquad (26)$$

Since there is no capital in the model, the solution to optimum consumption of traded and non-traded goods are,

$$\overline{C}_{tT} = \overline{C}^*_{tT} = \frac{1}{2(Y_{tT} + Y^*_{tT})}$$
$$\overline{C}_{tN} = Y_{tN} ; \qquad\qquad \overline{C}^*_{tN} = Y^*_{tN}$$

Solution to hours:

Substituting $\overline{C}_{tT}$ into equation (23) gives,

$$\frac{\mu N_{tT}}{1 - N_{tT} - N_{tN}} = \left(\frac{2\gamma Y_{tT}}{Y_{tT} + Y^*_{tT}}\right) \qquad (27)$$

and substituting $\overline{C}_{tN}$ into equation (25) yields,

$$\frac{\mu N_{tN}}{1 - N_{tT} - N_{tN}} = 1 - \gamma \tag{28}$$

Combining them gives,

$$\frac{N_{tN}}{N_{tT}} = \frac{1 - \gamma}{2\gamma\delta_t} \tag{29}$$

where $\delta_t = \frac{Y_{tT}}{Y_{tT} + Y_{tT}^*}$. Substituting (29) into (27) and (28) gives the optimal solution to hours,

$$\overline{N}_{tT} = \frac{1}{1 + A(\delta_t)^{-1}} \; ; \qquad\qquad \overline{N}_{tN} = \frac{1}{1 + B + D\delta_t}$$

Similarly for the foreign country,

$$\overline{N}_{tT}^* = \frac{1}{1 + A(1 - \delta_t)^{-1}} \; ; \qquad\qquad \overline{N}_{tN}^* = \frac{1}{1 + B + D(1 - \delta_t)}$$

where

$$A = \frac{\mu + (1 - \gamma)}{2\gamma} \; ; \qquad\qquad B = \frac{\mu}{1 - \gamma} \; ; \qquad\qquad D = \frac{2\gamma}{1 - \gamma}$$

Solution to δ_t:

$$\delta_t = \frac{Y_{tT}}{Y_{tT} + Y_{tT}^*}$$

$$\Rightarrow \delta_t = \frac{\theta_{tT} N_{tT}}{\theta_{tT} N_{tT} + \theta_{tT}^* N_{tT}^*}$$

$$\Rightarrow \delta_t = \frac{\theta_{tT}\left(\frac{\delta_t}{A + \delta_t}\right)}{\theta_{tT}\left(\frac{\delta_t}{A + \delta_t}\right) + \theta_{tT}^*\left(\frac{\delta_t}{A + \delta_t}\right)}$$

$$\Rightarrow \delta_t = \frac{1}{1 + \left(\frac{\theta_{tT}^*(1 - \delta_t)}{\theta_{tT}\delta_t} \frac{A + \delta_t}{A + 1 - \delta_t}\right)}$$

$$\Rightarrow \frac{1}{\delta_t} = 1 + \left(\frac{\theta_{tT}^*(1 - \delta_t)}{\theta_{tT}\delta_t} \frac{A + \delta_t}{A + 1 - \delta_t}\right)$$

$$\Rightarrow \frac{\theta_{tT}^*}{\theta_{tT}} = \frac{A + 1 - \delta_t}{A + \delta_t}$$

$$\Rightarrow \delta_t = \frac{1 + A(1 - \theta_{tT}^*\theta_{tT}^{-1})}{1 + \theta_{tT}^*\theta_{tT}^{-1}}$$

Social Planner's Problem (Persistent Shocks: No Uncertainty/Perfect Foresight)

$$V(s_t) = \max \left[\begin{array}{c} \gamma \log(Y_{Tt} + Y_{Tt}^* - C_{Tt}^* - K_{(t+1)T} - K_{(t+1)T}^*) + \\ (1 - \gamma) \log(Y_{Nt} - C_{Nt} - K_{(t+1)N}) + \mu \log(1 - N_{Tt} + N_{Nt}) + \\ \gamma \log(C_{tT}^*) + (1 - \gamma) \log(Y_{Nt}^* - C_{Nt}^* - K_{(t+1)N}^*) + \\ \mu \log(1 - N_{Tt}^* + N_{Nt}^*) + \beta E_t V(s_{(t+1)}) \end{array} \right]$$

subject to technologies and resource constraints.

The first-order-conditions are:

$$C_{tT}^* : \quad \frac{\gamma}{C_{tT}} = \frac{1 - \gamma}{C_{tT}^*} \tag{30}$$

$$K_{(t+1)T} : \quad \frac{\gamma}{C_{tT}} = \beta_t V_{k_{(t+1)T}} \tag{31}$$

$$K_{(t+1)N} : \quad \frac{1 - \gamma}{C_{tN}} = \beta_t V_{k_{(t+1)N}} \tag{32}$$

$$K_{(t+1)T}^* : \quad \frac{\gamma}{C_{tT}^*} = \beta_t V_{k_{*(t+1)T}} \tag{33}$$

$$K_{(t+1)N}^* : \quad \frac{1 - \gamma}{C_{tN}^*} = \beta_t V_{k_{*(t+1)N}} \tag{34}$$

$$N_{tT} : \quad \frac{\mu}{(1 - N_{tT} - N_{tN})} = \frac{\gamma(1 - \alpha^T)Y_{tT}}{C_{tT}N_{tT}} \tag{35}$$

$$N_{tN} : \quad \frac{\mu}{(1 - N_{tT} - N_{tN})} = \frac{(1 - \gamma)(1 - \alpha^N)Y_{tN}}{C_{tN}N_{tN}} \tag{36}$$

$$N_{tT}^* : \quad \frac{\mu}{(1 - N_{tT}^* - N_{tN}^*)} = \frac{\gamma(1 - \alpha^T)Y_{tT}^*}{C_{tT}^* N_{tT}^*} \tag{37}$$

$$N_{tN}^* : \quad \frac{\mu}{(1 - N_{tT}^* - N_{tN}^*)} = \frac{(1 - \gamma)(1 - \alpha^N)Y_{tN}^*}{C_{tN}^* N_{tN}^*} \tag{38}$$

These FOCs implicitly define optimal decision rules for consumption, investment and hours.

Using the same techniques as in Appendix A, we can derive the following solutions.

Traded sector:

$$\overline{C_{tT}} = \overline{C}_{tT}^* = \frac{1}{2} \left(1 - \beta \alpha^T\right) (Y_{tT} + Y_{tT}^*)$$

$$\overline{K}_{(t+1)T} = \beta \alpha^T \omega_{t+1}(Y_{tT} + Y_{tT}^*)$$

$$\overline{K}_{(t+1)T}^* = \beta \alpha^T (1 - \omega_{t+1})(Y_{tT} + Y_{tT}^*)$$

$$\overline{N}_{tT} = \frac{1}{1 + G(\omega_t)^{-1}}$$

$$\overline{N^*}_{tT} = \frac{1}{1 + G(1 - \omega_t)^{-1}}$$

Non-traded sector:

$$\overline{C}_{tN} = (1 - \beta \alpha^N) Y_{tN}$$

$$\overline{C}_{tN}^* = (1 - \beta \alpha^N) Y_{tN}^*$$
$$K_{(t+1)N} = \beta \alpha^N Y_{tN}$$
$$K_{(t+1)N}^* = \beta \alpha^N Y_{tN}^*$$
$$\overline{N}_{tN} = \frac{1}{1 + Q + R\omega_t}$$
$$\overline{N}_{tN}^* = \frac{1}{1 + Q + R(1 - \omega_t)}$$

where,

$$G = \frac{\mu(1 - \beta \alpha^T) + \gamma(1 - \alpha^N)(1 - \beta \alpha^T)(1 - \beta \alpha^N)^{-1}}{2\gamma(1 - \alpha^T)}$$

$$Q = \frac{\mu(1 - \beta \alpha^N)}{(1 - \gamma)(1 - \alpha^N)} ; \qquad R = \frac{2(1 - \gamma)(1 - \beta \alpha^N)(1 - \alpha^T)}{(1 - \gamma)(1 - \alpha^N)(1 - \beta \alpha^T)}$$

Solution to ω_{t+1} :

$$\omega_{t+1} = \frac{\theta_{(t+1)T} K_{(t+1)T}^{\alpha^T} N_{(t+1)T}^{1-\alpha^T}}{\theta_{(t+1)T} K_{(t+1)T}^{\alpha^T} N_{(t+1)T}^{1-\alpha^T} + \theta_{(t+1)T}^* K_{(t+1)T}^{*\alpha^T} N_{(t+1)T}^{*1-\alpha^T}}$$

Substituting the solution for $K_{(t+1)T}$, $K_{(t+1)T}^*$, N_{tT}, and N_{tT}^* yields the following solution to ω_{t+1}

$$\omega_{t+1} = \frac{1 + G \left[1 - \theta_{(t+1)T}^* \theta_{(t+1)T}^{-1} \right]^{\frac{1}{1-\alpha^T}}}{\left[1 + \theta_{(t+1)T}^* \theta_{(t+1)T}^{-1} \right]^{\frac{1}{1-\alpha^T}}}$$

Proof: $\frac{\partial e_t^R}{\partial (\frac{\theta_{tT}}{\theta_{tT}^*})} < 0$

Consider the exchange rate equation (56):

$$e_t^R = \left[\frac{\theta_{tN} \left[(1 + B - AD) + (1 + B + D + AD)(\theta_{tT}^* \theta_{tT}^{-1}) \right]}{\theta_{tN}^* \left[(1 + B + D + AD) + (1 + B - AD)(\theta_{tT}^* \theta_{tT}^{-1}) \right]} \right]^{(1-\gamma)}$$

partially differentiating e_t^R with respect to the traded productivity ratio, $(\frac{\theta_{tT}^*}{\theta_{tT}})$ yields,

$$\frac{\partial e_t^R}{\partial (\frac{\theta_{tT}^*}{\theta_{tT}})} = (1 - \gamma) \Phi^{\gamma-2} \left(\frac{\theta_{tT}}{\theta_{tT}^*} \right) \frac{1}{Z^2} \left[\left\{ M + N \left(\frac{\theta_{tT}}{\theta_{tT}^*} \right) \right\} M - \left(N + M \left(\frac{\theta_{tT}}{\theta_{tT}^*} \right) N \right) \right]$$

since,

$$\Phi = \left[\left[\frac{\theta_{tN} \left[(1 + B - AD) + (1 + B + D + AD)(\theta_{tT}^* \theta_{tT}^{-1}) \right]}{\theta_{tN}^* \left[(1 + B + D + AD) + (1 + B - AD)(\theta_{tT}^* \theta_{tT}^{-1}) \right]} \right]^{(1-\gamma)} > 0 \right]$$

$$Z^2 = \left[(1 + B + D + AD) + (1 + B - AD)(\theta_{tT}^* \theta_{tT}^{-1}) \right]^2 > 0$$

$$M = (1 + B + D + AD) > N = (1 + B - AD)$$

Therefore,

$$\frac{\partial e_t^R}{\partial (\frac{\theta_{tT}^*}{\theta_{tT}})} > 0 \text{ and } \frac{\partial e_t^R}{\partial (\frac{\theta_{tT}}{\theta_{tT}^*})} < 0$$