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Explaining Economic Growth with Imperfect Credit Markets

Luis Carranza

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Explaining Economic Growth with Imperfect Credit Markets[1]

Prepared by Luis Carranza[2]

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Abstract

The purpose of this paper is to explain the humped-shaped behavior of the growth rate. Within a dynamic general equilibrium framework, it is found that, in the early stages of development, the source of growth is the reallocation of resources from sectors low-productivity sectors to high-productivity sectors ("extensive growth"), resulting in increasing growth rates. In the middle and mature stages of development, the source of growth is the higher average productivity achieved by the competition among entrepreneurs ("intensive growth"). As a result, the growth rate could be increasing in the middle stage and then displays a decreasing pattern during the mature stage.

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Author's E-Mail Address: lcarranza@grupobbv.com.pe

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I. Introduction

This paper studies how financial markets and accumulation of financial assets interact with economic growth, change of industry structure, productivity and distribution of wealth across households. To carry out this study, a dynamic general equilibrium model with imperfections in the credit market is developed.

The main purpose is to explain an empirical regularity of the development process that has been overlooked or not well addressed in existing economic theories: the nonlinear, humped-shaped behavior of the growth rate. Nevertheless, such regularity has been acknowledged in the empirical literature.

Kuznets (1971), for example, found that developed countries have experienced a significant acceleration in growth rates when they entered the stage known as modern economic growth. More recently, Japan, Korea, among other countries are examples of the nonlinear pattern displayed by the growth rate[3]. The same generalization about the growth rate can be drawn from cross-countries comparisons. According to the World Development Report 1983, middle-income countries tend to grow faster that both lower and high-income countries (see table 1). More empirical evidence of this pattern is found in Cho (1992) and Chenery, et al. (1986).

TABLE 1

Growth of Total and Per Capita Income of All but Centrally Planned Countries by Income Group

Per Capita Income of Country Group	Annual Growth Rate of Per Capita Income (%) 1960-81	Annual Growth Rate of GDP (%) 1960-81
India	1.4	3.5
China	3.4	5.4
Other Low-Income	0.8	4.1
Lower Middle-Income	3.4	5.3
Upper Middle-Income	4.2	6.0
Industrial Market Economies	3.4	4.0

Source: Chenery, et. al. (1986)

Our model is based on the literature on distributional dynamics, which emphasizes the role of occupational choice and intergenerational transition of wealth in the development process. In particular, Banerjee and Newman (1993) showed that an economy can either become

[3] The behavior of the growth rate in Japan has been studied by Parente and Prescott (1994), Minami (1973), among others.

prosperous or poor depending on the initial wealth distribution; Galor and Zeira (1993) pointed out that with credit constraints and indivisibility in human capital investment, the initial wealth distribution affects the growth rate in the short and long run; while Bernhardt and Lloyd-Ellis (1994) focused on the evolution of inequality in an economy.

This paper describes the income distribution dynamics and explains the process of economic take-off. If we eliminate credit-constraints, the result would be exactly that of the Solow model, where the economic equilibrium depends on the aggregate level of wealth and not on the distribution of wealth across households. On the other hand, for a complete credit-constraint economy, we would have a development trap (as in Banerjee and Newman, 1993). Thus, economic growth is affected by the level of imperfection in credit markets and the mechanism that produces economic take-off is endogenous in the model.

The rest of the paper is organized as follows. In Section 2, we will discuss our model economy. Section 3 is devoted to explain the occupational choice in this economy. Section 4 studies the intergenerational evolution of wealth. In Section 5, the equilibrium is presented. Section 6 discusses the development process in the economy. Section 7 focuses on the results from simulation. Finally, the conclusions are presented in Section 8.

II. Model Economy

There is a continuum of one-period-lived agents with measure one. At the end of the period, agents die and have children that live next period. Each agent is endowed with one unit of labor and some units of the consumption good that were inherited from his parent. As agents care about their children and their own consumption, the utility function is represented by:

$$c^{\delta}b^{(1-\delta)} \tag{1}$$

Where c stands for consumption, b for bequest and $0 < \delta < 1$. Note that the utility depends on the level of bequest and not on the child's utility[4].

There are three technologies in the economy:

i. Subsistence technology. Using this technology, agents can transform one unit of labor into D units of the consumption good.

ii. Storage technology. Using this technology at the beginning of the period, agents can transform one unit of the consumption good into z units of the consumption good by the end of the period. It is assumed that $(1-\delta)z < 1$.

[4] This is the model structure used in Andreoni (1989).

iii. Advanced technology. To use this technology, agents must pay a fixed entry cost equal to ϕ. After this, they observe an idiosyncratic productivity shock a with support on [0, a^*], probability distribution function $g(a)$ and cumulative distribution function $G(a)$. Given this shock, entrepreneurs hire labor (l) and rent capital (k) to produce the consumption good at the end of the period, according to the production function $af(k,l)$. This production function has the following properties: it is twice differentiable, it exhibits decreasing returns to scale, and $f_k > 0, f_l > 0, f_{kl} > 0, f_{kk} < 0, f_{ll} < 0$. For simplicity, it is assumed that capital goods have a depreciation rate equal to 100%, and that consumption goods can be transformed into capital goods without cost.

Credit markets in this economy are imperfect due to an enforcement problem. First, borrowing is only available for working capital, meaning that the entry cost must be self-financed. Then, the lenders cannot force the borrowers to repay the debt, but they can seize a fraction θ of the borrower's final income. Therefore, the amount of goods that the agent has to repay cannot be greater than his enforceable income, i.e.:

$$rC \leq \theta I \qquad (2)$$

Where r is the gross real interest rate, C is the amount of credit and I is the final income.

In this model, the decisions are sequential. In the first stage, at the beginning of the period, agents have to decide whether or not to pay the entry cost, taking as given their endowments and the equilibrium prices in the economy.

At the second stage, those who paid the entry cost will draw a productivity shock and depending on its realization and the borrowing constraint, agents will decide whether or not to use this technology; while those who did not pay the entry cost will be workers. At the final stage, given the net output (after financial transactions), agents must decide between consumption and bequest (see figure 1).

Note that at the beginning of the period individuals can be indexed by their level of wealth (w), while at the second and final stages, individuals can be indexed by (w,a), because the realization of the shock is also a source of differentiation among agents.

III. Occupational Choice

Given the three stages within the period, optimal policies can be found by solving the model backwards. The solution in the final stage is trivial: given the net income, agents will choose consumption and bequest policies to maximize their utility function.

For the second stage, after the entry cost has been paid and the productivity shock has been observed, agents must choose capital, labor and financial assets (borrowing or lending) in order to maximize profits. Formally, an agent type (w,a) chooses k, l and C such that:

$$\pi(w,a,r_t,v_t) = \max_{k,\,l,\,C} \; [af(k_t,l_t) - r_t C_t] \qquad (3)$$

$$\text{s.t.} \quad v_t l_t + k_t \leq C_t + (w-\phi) \tag{4}$$

$$r_t C_t \leq \theta a f(k_t, l_t) \tag{5}$$

Where w is the initial wealth of the agent, the productivity level is given by a, and v_t stands for wages. Equation (4) is simply the budget constraint, which it will be assumed to hold with equality (C_t can be positive or negative), while equation (5) is the borrowing constraint. Note that the cost of inputs includes the financial gains foregone for using internal funds in the production process (see Christiano and Eichenbaum, 1992).

If the borrowing constraint is binding, capital and labor demand functions, $k(w,a,r_t,v_t)$ and $l(w,a,r_t,v_t)$, respectively, depend positively on the level of wealth w. On the other hand, when the borrowing constraint is not binding, the unrestricted (optimal) capital and labor demand functions, $k^*(a,r_t,v_t)$ and $l^*(a,r_t,v_t)$, respectively, do not depend on the level of wealth w.

Definition 1.- Let $\varpi(a, r_t, v_t)$ be the minimum level of wealth needed to get sufficient credit to hire the unrestricted levels of labor and capital, i.e.:

$$\varpi(a, r_t, v_t) = k^*(a,r_t,v_t) + v_t l^*(a,r_t,v_t) - (\theta/r_t)\, a f(k^*,l^*) + \phi \tag{6}$$

An agent with wealth lower than $\varpi(a, r_t, v_t)$ is said to be "credit constrained." If, after the productivity shock a is realized, an agent decides not to be an entrepreneur, his final income will be:

$$r_t (w-\phi) + v_t \tag{7}$$

Definition 2. At stage two, the final income of an individual type (w,a) is represented by:

$$V_2(w,a,r_t,v_t) = \max\{\pi(w,a,r_t,v_t),\ r_t(w-\phi) + v_t\} \tag{8}$$

Proposition 1.- For any $w > \phi$, there is a threshold level for the technological shock $\underline{a}(w,r_t,v_t)$, such that if the realized productivity shock is greater (lower) than this level, then the individual will (will not) choose to be an entrepreneur.

Proof: (See appendix).

Definition 3.- Let us define the function $e(w,a,r_t,v_t)$ as the occupational choice function such that:

$$e(w,a,r_t,v_t) = \begin{cases} 1 & \text{if } a \geq \underline{a}(w,r_t,v_t) \\ 0 & \text{if } a < \underline{a}(w,r_t,v_t) \end{cases} \tag{9}$$

Finally, at the first stage, the decision is whether to pay the entry cost.

Definition 4.- The expected final income at the first stage of an agent who pays ϕ is:

$$V_e(w,a,r_t,v_t) = E_a\, V_2(w,a,r_t,v_t) \tag{10}$$

Definition 5.- The final income of an agent who decides to be a worker will be:

$$V_s(w,r_t,v_t) = r_t\, w \;+\; v_t \tag{11}$$

Definition 6.- Let us define the payment function $\phi(w,r_t,v_t)$ as:

$$\phi(w,r_t,v_t) = \begin{cases} 1 & \text{if } V_e > V_s \\ 0 & \text{if } V_e < V_s \end{cases} \tag{12}$$

Since, we want to assure that at the earliest stages of development, any agent with wealth greater than the entry cost will be willing to pay it, if the credit constraint is not too severe, it is assumed that $V_e(\phi,z,D) \;>\; V_s(\phi,z,D)$.

Now, it can be proved that if an equilibrium exists, the prices that support that equilibrium belong to a bounded set, denoted by **B**. Formally:

Proposition 2.- The equilibrium prices belong to a bounded set.

Proof: (See appendix).

Regarding the choice of paying the entry cost, it is possible to find a threshold level for initial wealth, $\omega(r_t,v_t)$, such that, any agent with wealth greater (lower) than this level will choose to (not to) pay the entry cost. The following proposition formalizes this statement.

Proposition 3.- For any (r,v) in B', there is a critical level of wealth $\omega(r_t,v_t)$, such that:

$$V_e(w,a,r_t,v_t) \geq V_s(w,r_t,v_t) \qquad \text{if } w \geq \omega(r_t,v_t), \text{ and}$$
$$V_e(w,a,r_t,v_t) < V_s(w,r_t,v_t) \qquad \text{if } w < \omega(r_t,v_t)$$

Proof: (See appendix).

The intuition behind this result is that, for any pair of prices (r',v') outside the set, all the individuals would be better off as workers (v relatively too high) or as lenders (r relatively too high). Consequently, the excess demand function in at least one market would be negative, pushing down the relevant price. The same logic applies for $r_t < z$ and $v_t < D$.

IV. Intergenerational Wealth Dynamics

The intergenerational evolution of wealth depends on the prices, the occupational decisions, productivity shocks, and the level of initial wealth. There are three cases:

i. If an agent chooses to be a worker the off-spring will have wealth equal to:

$$w_{t+1} = (1-\delta)(r_t w_t + v_t) \tag{13}$$

ii. If an agent chooses to pay the entry cost, but the productivity shock realized is too low, the off-spring will have wealth equal to:

$$w_{t+1} = (1-\delta)(r_t (w_t-\phi) + v_t) \tag{14}$$

iii. If an agent becomes an entrepreneur, the off-spring will have wealth equal to:

$$w_{t+1} = (1-\delta)(\pi(w,a,r_t,v_t) - r_t C_t) \tag{15}$$

The transition diagram in figure 2 represents the dynamics of the dynasty's wealth, given prices and productivity shock. If the level of wealth is lower than $\omega(r_t,v_t)$, the agent will be a worker and wealth will move along the segment AB, with intercept given by $(1-\delta)v_t$ and slope equal to $(1-\delta)r_t$.

Now, if the agent decides to pay the entry cost and $a < \underline{a}(w,r_t,v_t)$; the evolution of wealth will be described by the line CD. This line starts for wealth equal or greater to $\omega(r_t,v_t)$ and the slope is the same as in the previous case. Point C would be at the level of point A if $\omega(r_t,v_t) = \phi$. However, if the agent's productivity is greater than $\underline{a}(w,r_t,v_t)$, the wealth dynamics will be described by EFG. Notice that in the interval $[\omega(r_t,v_t), \varpi(a, r_t, v_t)]$, the agent is credit-constrained, this means that, given the prices, the capital and labor demand functions are lower than they would be if there were no enforcement problem. If $w \geq \varpi(a, r_t, v_t)$, credit restriction is not binding and additional increments in wealth will be allocated to financial assets and not to hire productive factors. The slope of the curve for this case will be the same as before $(1-\delta)r_t$.

In summary, given w_t at the beginning of the period, the individual wealth evolves according to the following stochastic process:

$$w_{t+1} = \Gamma(w_t; a, r_t, v_t) \tag{16}$$

Where Γ is a stochastically monotone operator, in the sense that given $w_1 < w_2$, $\Gamma(w_2; .)$ dominates $\Gamma(w_1; .)$ in the first order stochastic sense. The transition function for the entire wealth distribution H is given by:

$$H_{t+1}(w') = \int\int_{[(w,a):\, \Gamma(w,a,r,v) < w']} dG(a)\, dH_t(w) \tag{17}$$

V. EQUILIBRIUM

Definition 7.- Let $\xi_c(r_t,v_t)$ and $\xi_l(r_t,v_t)$ be the excess demand functions for credit and labor, respectively.

$$\xi_c(r_t,v_t) = \int_{\omega(r_t,v_t)}^{\infty}\int_{\underline{a}(w,r_t,v_t)}^{a^*}[v_t l(a,w,r_t,v_t) + k(a,w,r_t,v_t) - (w-\phi)]dG(a)dH_t(w)$$

$$\int_{\omega(r_t,v_t)}^{\infty}\int_{0}^{\underline{a}(w,r_t,v_t)}[w-\phi]dG(a)dH_t(w) - \int_{0}^{\omega(r_t,v_t)} wdG(a)dH_t(w) - x \qquad (18)$$

The first term is the demand for credit by agents who paid the entry cost and got high productivity shocks, while the second term is the supply represented by those that paid the entry cost but got low productivity shocks. The last term is the supply of credits of those who did not pay the entry cost. Note that for an agent with $w_t > k^*(w_t,r_t,v_t) + v_t\, l^*(w_t,r_t,v_t) + \phi$, the demand for credit will be negative. That is, they start to have positive investment in financial assets. Defining in the same way the excess demand function for labor, we have:

$$\xi_l(r_t,v_t) = \int_{\omega(r_t,v_t)}^{\infty}\int_{\underline{a}(w,r_t,v_t)}^{a^*} l(a,w,r_t,v_t)dG(a)dH_t(w) - \int_{0}^{\infty} dH_t(w) - \int_{\omega(r_t,v_t)}^{\infty}\int_{\underline{a}(w,r_t,v_t)}^{a^*} dG(a)dH_t(w) \qquad (19)$$

The first term is the demand for labor given by the entrepreneurs, while the second and third term represent the fraction of people that are willing to work.

Definition 8.- An equilibrium is the sequences of consumption and bequest decisions $\{c_t(w,a,r_t,v_t),\ b_t(w,a,r_t,v_t)\}$, the occupational choice and payment function $\{e_t(w,a,r_t,v_t),\ \phi_t(w,r_t,v_t)\}$, the demand for labor, capital and the credit decisions $\{l_t(w,a,r_t,v_t),\ k_t(w,a,r_t,v_t),\ C_t(w,a,r_t,v_t)\}$; the sequences for distribution of wealth H_t and the prices r_t and v_t, such that:

- Given $e_t(w,a,r_t,v_t)$, $\phi_t(w,r_t,v_t)$, $k_t(w,a,r_t,v_t)$, $l_t(w,a,r_t,v_t)$, $C_t(w,a,r_t,v_t)$, r_t, v_t and H_t; the individual type (w,a) chooses $c_t(w,a,r_t,v_t)$ and $b_t(w,a,r_t,v_t)$ to maximize the utility.

- Given $\phi_t(w,r_t,v_t)$, $e_t(w,a,r_t,v_t) = 1$, r_t, v_t and H_t; the individual type (w,a) chooses $l_t(w,a,r_t,v_t)$, $k_t(w,a,r_t,v_t)$ and $C_t(w,a,r_t,v_t)$ to maximize the final income.

- Given $\phi_t(w,r_t,v_t)$, r_t, v_t, H_t; the individual type (w,a) chooses $e_t(w,a,r_t,v_t)$ to maximize the expected final income.

- Given r_t, v_t, H_t; the individual type (w) chooses $\phi_t(w,r_t,v_t)$ to maximize expected final income.

- Given H_t; r_t and v_t clear the markets, i.e.:

$\xi_c(r_t,v_t) \leq 0$; with equality if $r_t > z$
$\xi_l(r_t,v_t) \leq 0$; with equality if $v_t > D$

- The transition rule for the wealth distribution:

$$H_{t+1}(w') = T_t(H_t)$$

where T is the transitional operator defined in equation (17).

VI. AGGREGATEE DYNAMICS: DEVELOPEMNT PROCESS

1.- Underdevelopment Stage

This is the initial stage in the economic process. A characteristic of this stage is that some positive fraction of the population is using the subsistence and the storage technology. Some societies can stay in this phase of development (underdevelopment trap) and never take-off. The next proposition formalizes this statement.

Proposition 4.- If the credit constraint is too severe (θ closed to zero), the economy will converge in the long run to a subsistence economy.

Proof: (See appendix).

The key to understanding this phenomenon is the persistence in wealth patterns ("if you are poor, so will be your child"). Given that θ is closed to zero, you are constrained by your own funds to finance the entry cost as well as the cost of capital and labor inputs. The level of wealth that makes you indifferent between being an entrepreneur or not is greater than ϕ, even if prices are at their lowest level. If the dynasty's initial wealth is less than the threshold level of wealth and the aggregate conditions in the economy do not generate increments in prices, the dynasty's wealth is going to be a sequence converging to a fixed point.

On the other hand, considering a dynasty with initial wealth greater than $\omega(.)$, at some point in time, when a member of this dynasty draws a low productivity shock, such that his wealth falls below $\omega(.)$, he will choose to be at the subsistence sector. From then on, every member of the dynasty will remain in the subsistence sector.

If the condition for growth is satisfied, meaning that the credit restrictions are not too severe, a poor economy can developed into a rich one. In the early stages of development, as there are only few entrepreneurs, labor and credit are abundant, yielding wages and interest rates at their lowest levels, D and z, respectively. As input prices are low, a large fraction of potential entrepreneurs can actually become entrepreneurs. As the fractions of entrepreneurs increases, the demand for labor and capital also increase. This structural change is the main source of growth in this period. The reallocation of resources from these traditional sectors to the modern sector produces a positive impact in aggregate output, resulting in increasing growth rates of the economy. This is formalized in the next proposition.

Proposition 5.- During the early stage of development, as $r_t=z$ and $v_t=D$:

i. The fraction of entrepreneurs and workers are strictly increasing.
ii. The demand function for credit is also strictly increasing.
iii. The growth rate could be increasing

Proof: (See appendix).

Notice that the imperfection in the credit market is the reason of the inefficient allocation of resources in this economy. A corollary from this proposition is that the number of people in the subsistence sector declines monotonically during this stage. Given that the population is constant at each period, we have that the size of the subsistence sector (S_t) would be given by:

$$S_t = 1 - E_t - L_t$$

Since E_t and L_t are increasing, S_t must be decreasing.

2.-Economic Take-Off

The next phase of development is when the economic break-through takes place. At some point in time, as the fraction of entrepreneurs increases, demanding more labor and capital, labor and capital resources become scarce, resulting in higher prices. This has two opposite effects on the economy. On one hand, this increment in prices would put some potential entrepreneurs out of the market. On the other hand, as higher wages now implies more potential entrepreneurs next period, the fraction of entrepreneurs could actually rise (especially at the beginning of this stage) since this new generation of managers is more productive than the last generation. As the "productivity effect" dominates the "competition effect", we will observe increasing growth rates in this economy. We formalize this in the following proposition:

Proposition 6.- At this stage:

i. The interest rate and wages are increasing.
ii. The growth rate could be increasing.

Proof: (See appendix).

3.- Convergence

Eventually, more and more entrepreneurs become wealthy enough to self finance their projects and to invest in financial assets. As aggregate wealth continues to increase, capital becomes an abundant factor again, resulting in the declining of interest rates. Meanwhile, wages continue to increase as there is no population growth in this economy. At this stage, the "competition effect" and the "productivity effect" work in the same direction, resulting in a decreasing sequence of the growth rate.

The following proposition formalizes this statement.

Proposition 7.- At this stage,

i. The interest rate r_t is decreasing and converges to r*.
ii. The wage v_t is increasing and converges to v*.

Proof: (See appendix).

In this stage, a large fraction of population is unconstrained entrepreneurs, who are operating their firms at the optimal scale: $vl^* + k^*$. When most of the agents overcome their credit constraints, the economy behaves as the neoclassical growth model. Now, to find the long run equilibrium in this economy, the wealth distribution must converge to a unique distribution. The following theorem establishes this fact.

Theorem .- The wealth distribution will converge to a unique invariant distribution.

Proof: (See appendix).

The intuition behind this theorem is that, given that the prices converge and that the transition of wealth is (stochastically) monotone, if the wealth process is globally ergodic, meaning that one agent can move from one wealth interval to another in a finite time period, then H_t converges to a unique invariant distribution. In the case where credit constraints were too severe, there was no mobility from one state to another, collapsing the wealth distribution into a one-point distribution.

VII. A Numerical Example

This section presents a numerical exercise. Since we are interested in the qualitative features of the growth rate, there is not explicit intention to choose parameters to match real data of any particular country experience. The same claim applies to the time-period of the model economy, which in this case can be equivalent to 30 to 40 years. Keeping this in mind, some numerical values are presented in table 2, while the functional form of the technology in the advanced sector is a Cobb-Douglas production function: $f(k,l) = k^{\alpha} l^{\beta}$.

TABLE 2

Parameters

$\delta = 0.5$	$\theta = 0.4$	$\alpha = 0.25$
$\beta = 0.5$	$\phi = 1$	$D = 0.5$
H_0 (w) unif. on [0,0.9]	f(a) unif. on [0,3]	$z = 1.2$

In figure 3, we can see that the growth rate displays a non-monotone behavior. In the first three periods, the growth rate is increasing and then tends to decline. However, there is an increment in the growth rate from period five to six. The reason for this is that in the sixth period the productivity effect dominates the population effect. After this, the growth rate converges monotonically to zero.

In figure 4, we can observe that from period one to five the population moves from the subsistence sector to the modern sector. Fewer people are using the subsistence technology and become workers or managers. Both fractions, managers and workers are increasing in this stage. This is the stage of "extensive growth": The economy grows because there are more entrepreneurs. From period six, the fraction of entrepreneurs is strictly decreasing because they are competing for the scarce resources. This is a period of "intensive growth": The economy grows because the entrepreneurs are richer and more productive.

VIII. Conclusions

The paper presents a complete description of the development process, focusing on the behavior of the growth rate. It is found that in the early stages of development the economy experiences a period of "extensive growth," in which the growth rates are increasing and the fraction of entrepreneurs is positively correlated with the level of aggregate output. The engine of growth in this stage comes from the reallocation of resources from low to high productivity sectors. In the middle and mature stages of development, the economy experiences "intensive growth." At the middle stage, the growth rate could be increasing if there is a larger fraction of credit-constrained entrepreneurs using the modern technology when it exhibits increasing returns to scale. At the mature stage, the growth rate will be decreasing because more and more people overcome the credit constraint, using the modern technology when there are decreasing returns to scale. During the middle and mature stages, the fraction of entrepreneurs is negatively correlated with the level of output and the source of growth is the higher average productivity achieved by the competition among entrepreneurs.

There are implications for other variables in the economy. The real wage is strictly increasing during the process of development and the real interest rate follows the same nonlinear pattern as the growth rate. The explanation for this is that the labor force becomes scarce as the demand for labor increases and the population remains constant. In addition, credit becomes scarce at the early stages when the entrepreneurs are competing against each other for credit. Then, as they accumulate wealth, most of the entrepreneurs become net lenders. The capital-labor ratio will be weakly increasing during the entire process. The reason for this is that this ratio depends only on the real wage. The wealth distribution dynamic resembles the Kuznets's hypothesis: At the early stages of development, income inequality tends to expand with economic growth but then inequality tends to decline.

A final comment about growth and development in an open economy: In a world with two countries, the country with less inequality (given the same aggregate wealth) or the country with higher aggregate wealth (given the same wealth distribution) will lead the world economy. Both countries will converge to the long-run steady state faster than in the case of closed economies. However, the disparity, in terms of total output, between these two countries grows bigger at the beginning, but also declines faster with respect to the closed eonomy case.

Figure 1

TIMING

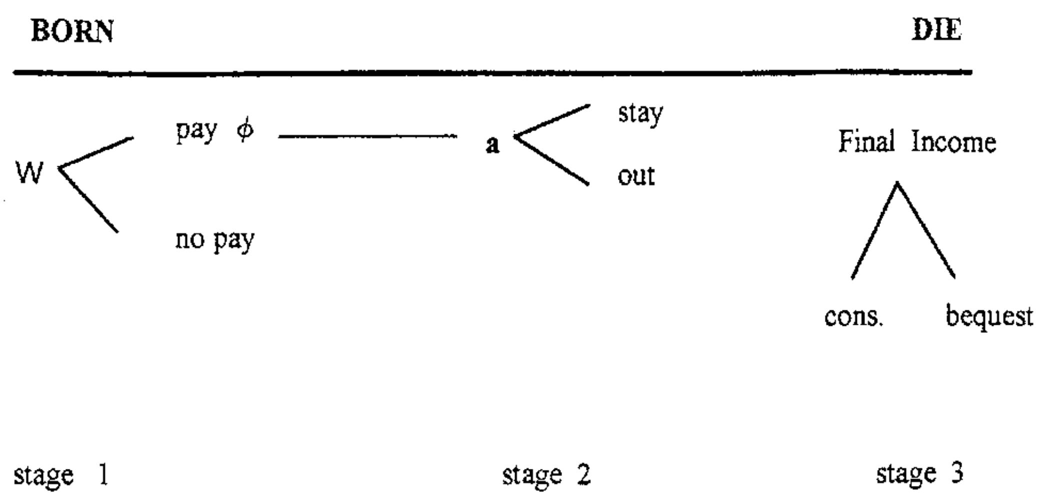


Figure 2

Intergenerational Wealth Dynamics

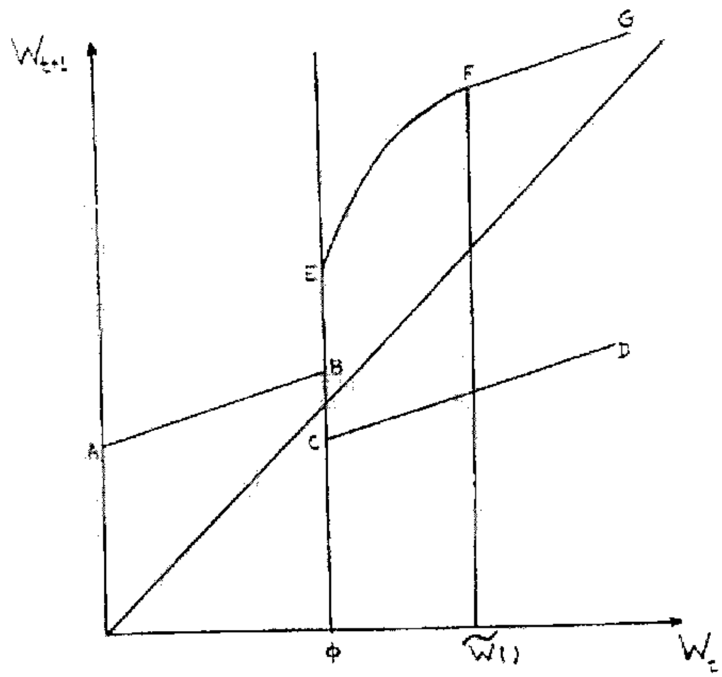
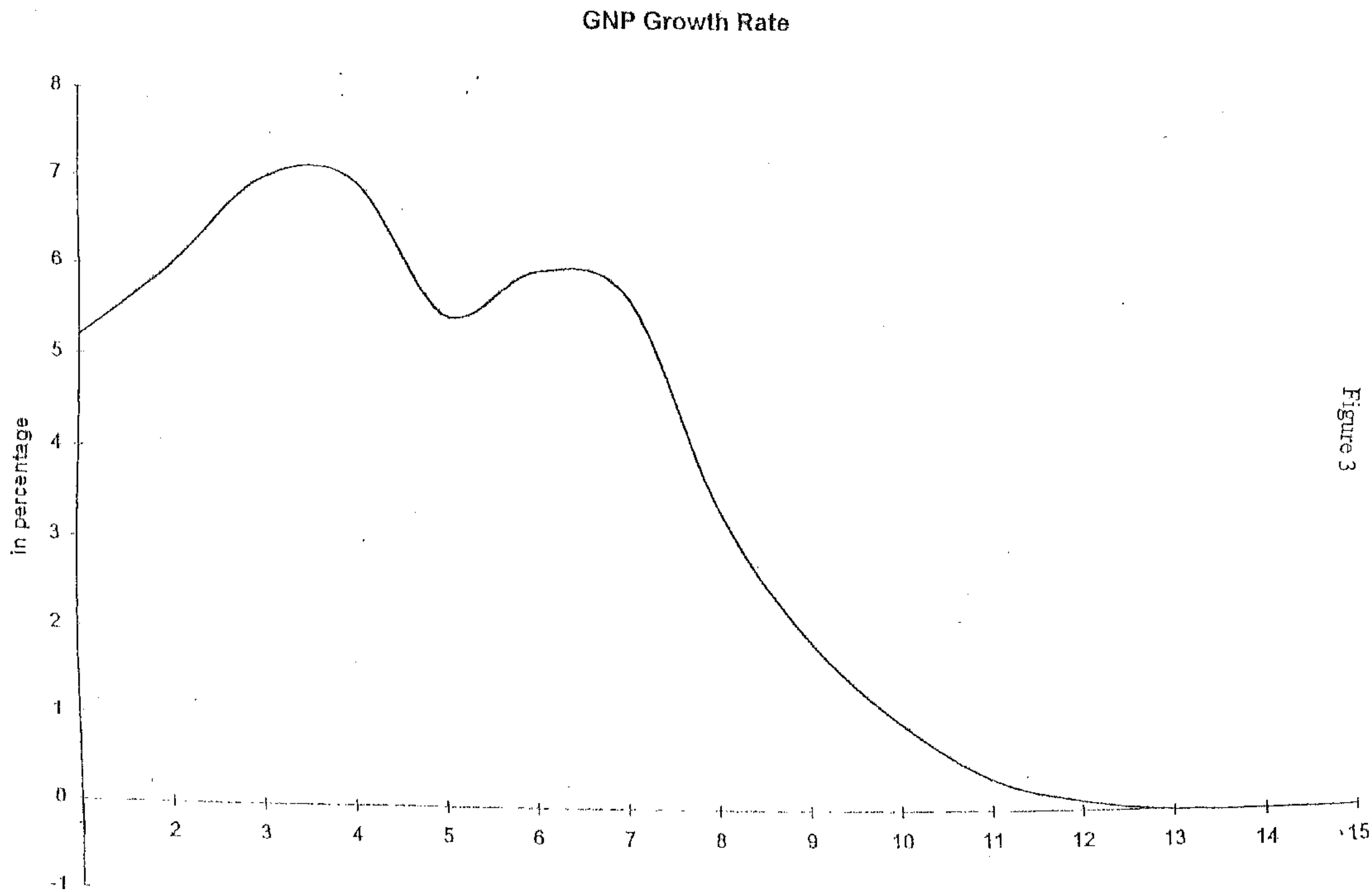


Figure 3


GNP Growth Rate
in percentage
8
7
6
5
4
3
2
1
0
-1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

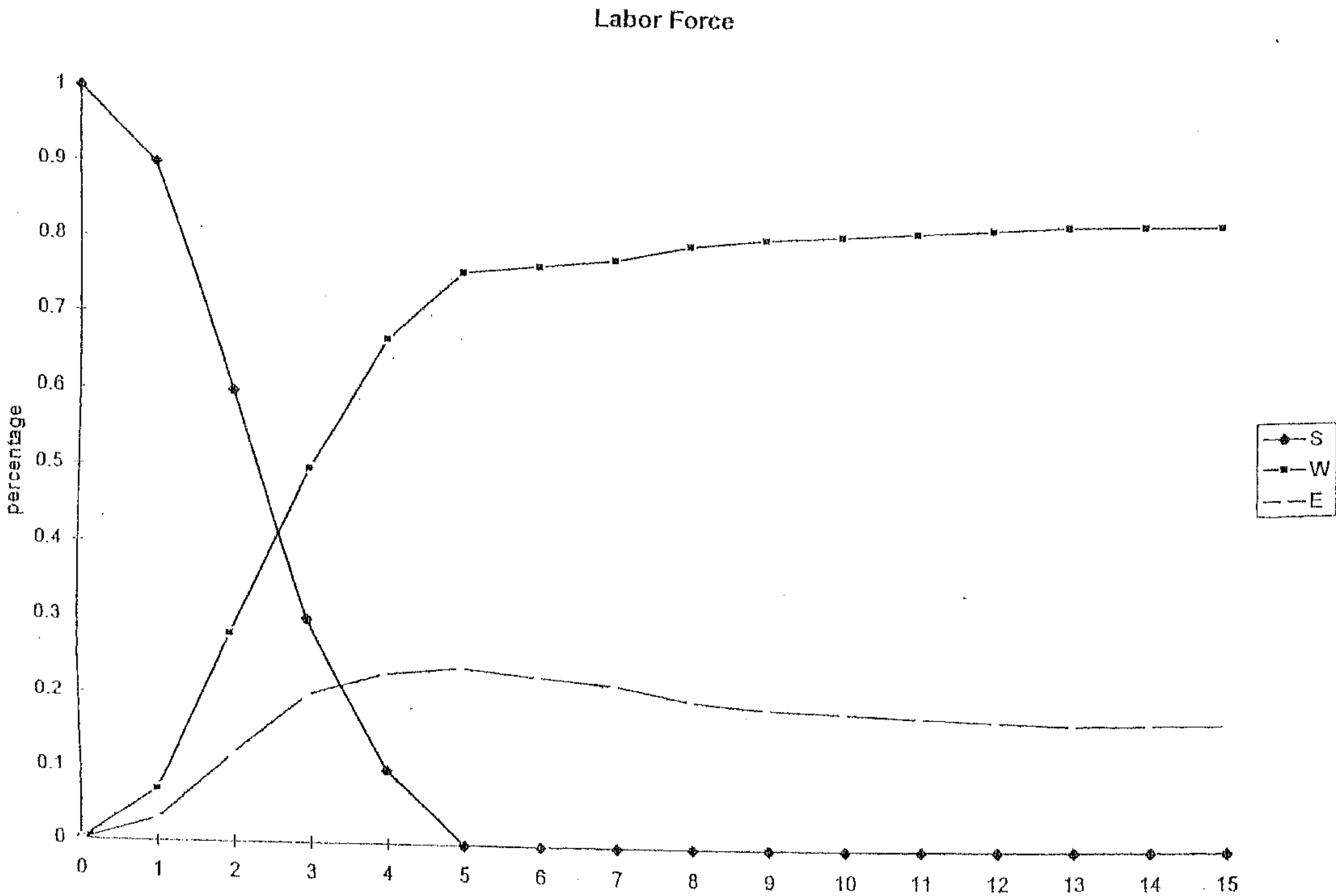

Labor Force
percentage
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
S
W
E

Figure 4

Proof of proposition 1.-

First note that $\pi(0,w,r_t,v_t) = r_t\,(w - \phi) < r_t\,(w - \phi) + v_t$. Since π is strictly increasing and continuous in a (it is trivial to show this), $\exists\ \hat{a} \in R_{++}$, s.t.

$$\pi(\hat{a},w,r_t,v_t) = r_t\,(w - \phi) + v_t$$

From this equation, the implicit function $\hat{a}(w,r_t,v_t)$ can be defined.
Now, define $\underline{a}(w,r_t,v_t) = min\ [\hat{a}(w,r_t,v_t),\ a^*]$
The proposition follows by the continuity of π in a. It is easy to see that the function $\underline{a}(w,r_t,v_t)$ is weakly increasing in r_t and v_t and weakly decreasing in w.

Proof of Proposition 2.-

First, we already know that the set (r,v) is bounded from below by (z, D).
Define for each level of w, the set $A(w)$:

$$A(w):\ \{(r,v)\ \in [z,\infty)x[D,\infty) :\ g(w,r,v) \geq 0\}$$

where $g(w,r,v) = V_e(w,r,v) - V_s(w,r,v)$

$$\frac{\partial Ve}{\partial w} = \int_{\underline{a}(w,r_t,v_t)}^{a^*} \frac{\partial \pi}{\partial w} dG(a) + \int_0^{\underline{a}(w,r_t,v_t)} r_t dG(a) \geq r_t$$

Then $g\ (w,r,v)$ is weakly increasing in w and for $w_1 < w_2$, $A(w_1) \subseteq A(w_2)$

Now, defining set C by:

$$C:\ \{(r',v')\ \in [z,\infty)x[D,\infty):\ g\ (\varpi(a^*,r',v'),\ r',v') = 0\}$$

Then we have that for any $w > \varpi(a^*,r',v')$, $g\ (w,r',v') = 0$. Since $\partial V_e/\partial w = r_t$ for any pair $(r,v) \geq (r',v')$ (where $\geq$ means that at least one component is greater); then $g(w,r,v) < 0$, for any w.
Now, applying the definition of set C to set B:

$$B = C \cup [(r,v)\ \in [z,\infty),\ [D,\infty):\ (r,v) \leq (r',v')]$$

Then for any w, $A(w) \subseteq B$.
Since A(w) is a bounded monotone sequence, $A(w)$ converges to A'.

Claim 1.- $A' = B$

Suppose not; then $\exists\ (r,v) \in B - A'$. Therefore, since $(r,v)\ \leq\ (r',v')$, then $g(\varpi(a^*,r,v),\ r,v) \geq 0$ and since $(r,v) \notin A(w)$, then $g(w,\ r,v) < 0$ for any w. A contradiction.

Claim 2.- In any equilibrium $(r,v)\ \in B$.

Suppose not. Then (r', v') are the equilibrium prices and belong to B^c.
Then the agents will not pay the entry cost. They will choose to be workers and invest in financial assets. The excess demand functions for labor and credit are negative, then prices will have to decrease. A contradiction.

Proof of Proposition 3.-

This is obvious. Just define: $w_m(r,v) = min_w\{w \geq \phi : (r,v) \in A(w)\}$ and the result follows from
proposition 2.

Proof of Proposition 4.-

Take $\theta = 0$ and then define $w_0(z,D)$ implicitly from $V_e(w_0(z,D), z,D) = V_s(w_0(z,D), z,D)$. Now, if $w_0(z,D) > w^* \equiv D(1-\delta)/(1-z(1-\delta))$, then for any $w_t < w_0(z,D)$, is easy to see that w_t converges to w^*. Now, for $w_t > w_0(z,D)$, note that $\exists\, M \geq 1$:

$$Prob\,[w_{T+M} > w_0\,(z,D) \,/\, w] < 1$$

Now, for $N \geq M$:

$$\Pr ob[w_{T+N} > w_0(z,D)/w_T > w_0(z,D)] = \prod_{j=M}^{N} [1 - G(\underline{a}(w_j,z,D)]$$

fixing M and letting $N \to \infty$:

$$\lim_{N \to \infty} \Pr ob[w_{T+N} > w_0(z,D)/w_T > w_0(z,D)] = 0$$

Then $w < w_0(z,D)$ is an absorbing state.

Lemma 1.- If $r_t = z$, $v_t = D$ and H_t dominates H_{t-1} in the first order sense, then H_{t+1} dominates H_t in the first order sense.

Proof.-
Let $\varphi(w,w')$ be defined by

$$\varphi(w,w') = \int_{a:\Gamma(w,a)\leq w'} dG(a)$$

Note that $\varphi_w(w,w') \leq 0$ since $\partial\Gamma/\partial w$ and $\partial\Gamma/\partial a$ are positive.

Now,

$$H_{t+1}(w') = \int_0^\infty \varphi(w,w')dH_t(w)$$

Therefore, using integration by parts:

$$H_{t+1}(w') - H_t(w') = \int_0^{\infty} \varphi(w, w')[H_{t-1}(w) - H_t(w)]d(w)$$

Since H_t dominates (first order) H_{t-1} and $\varphi_w(w,w') \leq 0$, then H_{t+1} dominates (first order) H_t. In order to assure that H_1 dominates H_0, we need that h'_0 be uniformly bounded.

Proof of Proposition 5.-

i. The fraction of entrepreneurs and workers are strictly increasing.

Let E_t be the fraction of entrepreneurs at period t.

$$E_t = \int_{\omega(z,D)}^{\infty} \Phi(w) dH_t(w)$$

where

$$\Phi(w) = \int_{\underline{a}(w,z,D)}^{a^*} dG(a)$$

Integrating by parts:

$$E_t - E_{t-1} = \int_{\omega(z,D)}^{\infty} [H_{t-1}(w) - H_t(w)]\Phi_w(w)d(w) + [H_{t-1}(\omega) - H_t(\omega)](1 - G(\underline{a}(w,z,D))$$

Since $\Phi_w(w) > 0$ and H_t dominates (first order) H_{t-1}, then $E_t > E_{t-1}$.

Let L_t be the total number of workers, then:

$$L_t = \int_{\omega(z,D)}^{\infty} \Psi(w) dH_t(w)$$

Where

$$\Psi(w) = \int_{\underline{a}(w,z,D)}^{a^*} l(w,a,z,D) dG(a)$$

Integrating by parts:

$$L_t - L_{t-1} = \int_{\omega(z,D)}^{\infty} [H_{t-1}(w) - H_t(w)]\Psi_w(w)d(w) + [H_{t-1}(\omega) - H_t(\omega)](\Psi(\omega(z,D)))$$

Given that $\Psi_w(w) \geq 0$, $L_t > L_{t-1}$.

ii. Excess demand for credit (EC_t) is increasing.

Define $EC_t = F_t - W_t$, where F_t is the total funds used to finance investment in the advanced technology (including fixed costs) and W_t is the aggregate wealth in the economy, i.e.:

$$F_t = \int_{\varpi(z,D)}^{\infty} [\Delta(w) + \phi] dH_t(w)$$

$$W_t = \int_0^{\infty} w dH_t(w)$$

Where

$$\Delta(w) = \int_{\underline{a}(w,z,D)}^{a^*} [v_t l(w,a,z,D) + k(w,a,z,D)] dG(a)$$

After some algebra, we have the following expression for $EC_t - EC_{t-1}$

$$\int_{\varpi(z,D)}^{\infty} [H_{t-1}(w) - H_t(w)][\Delta_w(w) - 1] d(w) + [H_{t-1}(\varpi) - H_t(\varpi)](\Delta(\varpi(z,D) + \phi))$$

It is easy to see that $\Delta_w(w,a) > 1$ when $w \leq \varpi(a,z,D)$ and $\Delta_w(w,a) = 1$ when $w \geq \varpi(a,z,D)$. Then,
$EC_t > EC_{t-1}$.

iii. The growth rate in this period could be increasing.

Let Y_t be the total output in the economy. After some algebra we get:

$$Y_t = \int_{\varpi(z,D)}^{\infty} \int_{\underline{a}(w,z,D)}^{a^*} [a(f(k(w,a,z,D), l(w,a,z,D)) + v_t l(w,a,z,D)] dG(a) dH_t(w) - r_t EC_t - v_t EL_t$$

Where EC_t and EL_t are the excess demand functions in the credit and labor markets, respectively. Note that EC and EL are negative and increasing. Also note that because of the utility function $W_t = (1-\delta)Y_{t-1}$, therefore $Y_t/Y_{t-1} = (1-\delta)Y_t/W_t$. Using the credit market equilibrium condition and after some calculations, we have that $Y_t/Y_{t-1} - Y_{t-1}/Y_{t-2}$ is greater than:

$$\int_{\varpi(z,D)}^{\infty} \int_{\underline{a}(w,z,D)}^{a^*} [\chi_w(w,a)(H_{t-1}(w) - H_t(w)) dG(a) dH_t(w) + (H_{t-1}(\varpi) - H_t(\varpi)) \int_{\underline{a}(w,z,D)}^{a^*} \chi(\varpi,a) dG(a)$$

Where $\chi(w,a) = [af(k,l)+vtl]/[vtl+k] + \phi/[1-H_t(\varpi)]$. It can be proved that exists w'', such that $\chi_w(w,a) > 0$ when $w < w''$ and $\chi_w(w,a) < 0$ when $w > w''$. Agents with wealth in *[ω(z,D), w'']* are using the advanced technology when it displays increasing returns to scale. If this effect dominates the 'decreasing returns' effect the growth rate would be increasing in this period.

Lemma 2.-If $v_t > v_{t-1}$, $r_t > r_{t-1}$, $\Gamma(w,a,r_t,v_t) > w$ for any a, and H_t dominates (first order sense) H_{t-1}, then H_{t+1} dominates (first order sense) H_t.

Proof:

$H_{t+1}(w') - H_t(w')$ can be expressed as:

$$\int_{\omega(r_t,v_t)}^{\infty} \varphi_w(w,w')[H_{t-1}(w) - H_t(w)]d(w) + \int_{\omega(r_t,v_t)}^{\infty}[\int_{\Delta r}\varphi_r dr + \int_{\Delta v}\varphi_v dv]dH_{t-1}(w)$$

Since $\Delta r > 0$, $\Delta v > 0$, and φ_r and φ_v are positive in some range of w, the second term in the right hand side could be positive. Since, by Lemma 1 the first term is always negative, then the sign is ambiguous. Now, taking derivatives with respect to w', we have that $h_{t+1}(w') - h_t(w')$ is given by:

$$\int_{\omega(r_t,v_t)}^{\infty} \varphi_{ww'}(w,w')[H_{t-1}(w) - H_t(w)]d(w) + \int_{\omega(r_t,v_t)}^{\infty}[\int_{\Delta r}\varphi_{rw'} dr + \int_{\Delta v}\varphi_{vw'} dv]dH_{t-1}(w)$$

Since $\varphi_{ww'}$, $\varphi_{rw'}$ and $\varphi_{vw'}$ are positives, then $h_{t+1}(w') > h_t(w')$ all w'. Then, H_{t+1} dominates H_t in the first order sense.

Proof of Proposition 6.-

i. Note that in this case there is no subsistence sector and no investments in the storage technology. If the same conditions as in proposition 5 holds and since H_{t+1} dominates H_t, then wages and interest rates must be increasing.

ii. Note that, as r_t and v_t are changing over time, $Y_t/Y_{t-1} - Y_{t-1}/Y_{t-2}$ is given by:

$$\int_{\omega t}^{\infty}\int_{\underline{a}t}^{a^*}\chi_t(H_{t-1} - H_t)dG(a)dH_t(w) - \int_{\omega t-1}^{\omega t}\int_{\underline{a}t-1}^{\underline{a}t}\chi_{t-1}dG(a)dH_{t-1}(w) + \int_{\omega t}^{\infty}\int_{\underline{a}t}^{a^*}(\chi_t - \chi_{t-1})dG(a)dH_{t-1}(w)$$

Now, the first term is positive if the same conditions as in proposition 5 holds. The second term is negative since $\underline{a}$ and ω are increasing in prices. The sign of the third is ambiguous, but when it is positive or negative but close to zero, we could observe increasing growth rates at this stage also.

Lemma 3.-

If $r_t \leq r_{t-1}$, $v_t \geq v_{t-1}$ and H_t dominates H_{t-1} in the second order sense, then H_{t+1} dominates H_t in the second order sense.

Proof.-

We can express $H_{t+1}(w') - H_t(w')$ as:

$$\int_{\omega(r_t,v_t)}^{\infty} \varphi_w(w,w')dH_t(w) - \int_{\omega(r_{t-1},v_{t-1})}^{\infty} \varphi_w(w,w')dH_{t-1}(w)$$

Integrating by parts and taking derivatives with respect to w' we get that $h_{t+1}(w') - h_t(w')$ is equal to:

$$\int_{\omega(r_t,v_t)}^{\infty} \varphi_{ww'}(w,w')[H_{t-1}(w) - H_t(w)]dw + \int_{\omega(r_{t-1},v_{t-1})}^{\infty} [\int_{\Delta r} \varphi_{rw'} dr + \int_{\Delta v} \varphi_{vw'} dv]dH_{t-1}(w)$$

If H_t dominates H_{t-1} in the second order sense and $\varphi_{www'} < 0$ and $lim_{w \to \infty} \varphi_{www'} = 0$, then it can be
shown that the first term is positive. Since most of the entrepreneurs are unconstrained the positive part in the second term offsets the negative part. Given that $h_{t+1}(w') > h_t(w')$ and for any $w > w_{max}$, where w_{max} is defined as $\Gamma(w_{max}, a^*, r, v) = w_{max}$, then H_{t+1} dominates H_t in the second order sense.

Proof of Proposition 7.-

From the labor market clearing condition we get:

$$\int_{\omega(r_t,v_t)}^{\infty} [\Psi_t(w) + 1]dH_t(w) = \int_{\omega(r_{t-1},v_{t-1})}^{\infty} [\Psi_{t-1}(w) + 1]dH_{t-1}(w)$$

Let r and v be fixed. Since H_t dominates H_{t-1}, for any given r, v_t must increase in order to clear the market.

From the credit market we have:

$$W_t = \int_{\omega(r_t,v_t)}^{\infty} [\Delta(w, r_t, v_t) + \phi v]dH_t(w)$$

As a bigger fraction of entrepreneurs becomes unconstrained, and given that H_t dominates H_{t-1} and v_t is increasing, the demand for credit is decreasing. Since $W_t > W_{t-1}$ (see below), then the interest rate must decrease in order to clear the credit market.

Now, since (r_t, v_t) are monotone sequences that belong to a bounded set (proposition 2), then r_t must converge to $r^* \geq z$ and v_t must converge to $v^* > D$.

Claim: $W_t \geq W_{t-1}$, $\forall t$

Proof.-

Since $W_t = (1-\delta)\, Y_{t-1}$, then we just need to prove that $Y_t \geq Y_{t-1}$ $\forall$ t. This result is established by induction, knowing that in the first stages $Y_t > Y_{t-1}$ and that H_t dominates stochastically H_{t-1} (in the second order sense) $\forall t$.

Proof of the Theorem.-

First of all, we need to find a compact support for H^*.
Define w_1 as: $w_1 = \omega(r^*,v^*) - \phi$, $w_1 \geq 0$ since $r^* \geq z$, $v^* > D$ and $\omega(r^*,v^*) > \omega(z,D) = \phi$
Define w_2 as: $w_2 = \Gamma(w_2,a^*,v^*,r^*)$
Then w_2 is the maximum level of wealth attainable when the prices are fixed. The minimum level of wealth is given by w_1.

Claim.- $\omega(r^*,v^*) \geq [(1 - \delta)\, v^*]/[1 - (1 - \delta)\, r^*]$

Suppose not. Then $[0,\ \omega(r^*,v^*))$ is an absorbing state. Then for any $w_t > \omega(r^*,v^*)$ $\exists$ some $N \geq 1$: $\forall \varepsilon > 0$ $Prob\,[w_{t+N} > \omega(r^*,v^*)] < \varepsilon$ (see proposition 4).

Then at some point, everybody will have wealth less than $\omega(r^*,v^*)$. This means that there will be negative excess demand for labor and credit, then prices will have to decrease. A contradiction.

Now, $[w_1,w_2]$ is the compact support for H^*.

It is easy to see that for any $w \in [w1,w2]$ there is $N \geq 1$ and $\varepsilon > 0$, such that:
$Prob\,[\Gamma^{t+N}(w_2,a^*,r^*,v^*) < w \;/\; w_t = w_2] > \varepsilon$ and $Prob\,[\Gamma^{t+N}(w_1,a^*,r^*,v^*) < w \;/\; w_t = w_1] > \varepsilon$
and since Γ is monotone, then Theorem 2 in Hopenhayn and Prescott (1992) can be applied.

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