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Smuggling, Currency Substitution and Unofficial Dollarization: A Crime-Theoretic Approach

Alex Mourmouras and Steven H. Russell

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Smuggling, Currency Substitution and Unofficial Dollarization: A Crime-Theoretic Approach

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Abstract

Large stocks of U.S. dollars and other hard currencies circulate in the transition economies, in Latin America, and in other countries that have experienced macroeconomic mismanagement. Using a monetary model that combines the legal restrictions and crime-theoretic traditions, this paper demonstrates how leaky exchange controls lead to currency substitution and progressive dollarization. The paper also analyzes the impact of dollarization on the ability of governments to earn seigniorage, the dynamics of dollarization in a growing economy, and the central role of expectations—specifically, confidence in the domestic currency—in determining the extent of dollarization and, potentially, in reversing it.

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I. INTRODUCTION

Large quantities of U.S. dollars and other hard currencies circulate alongside domestic currencies in many countries of the former Soviet Union, in Central, Eastern, and Southeastern Europe, the Middle East, Latin America and elsewhere. Over half of all U.S. currency outside banks is located abroad, and the foreign stock of U.S. currency has been growing about 3 times as fast as the domestic stock during the 1990s. Similarly, 30-40 percent of the deutsche marks outstanding are held outside Germany.[2] The high degree of unofficial dollarization is one reason some emerging market policymakers sought to go the extra step of *officially* dollarizing their economies.[3]

Macroeconomic instability, especially high and volatile inflation, is a very important reason why ordinary citizens in these countries have resorted to holding U.S. dollars and other hard currencies. Moreover, dollarization is a hysteretic phenomenon: because of the long-term damage macroeconomic instability has inflicted on confidence in domestic currencies, demand for hard currencies in countries experiencing dollarization tends to be buoyant even after these countries have achieved macroeconomic stabilization. In order to focus squarely on the macroeconomic factors behind progressive dollarization, this paper abstracts from tariffs, quotas and other trade restrictions, and also from tax evasion, narcotics trafficking and other illegal smuggling activities, which all lead to smuggling and are independent sources of black market demand for hard currencies. The paper attributes the foreign use of hard currencies to restrictions in the capital account—such as surrender or advance import requirements—that prevent domestic residents in developing and transition countries from officially building foreign currency balances and maintaining them over time. Indeed, while the last twenty years have witnessed much progress in eliminating restrictions on current international transactions, restrictions involving the capital account remain in the majority of IMF member countries.[4]

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[2] Some US$200-250 billion was circulating abroad at the end of 1995, more than half of the $375 billion in U.S. currency outstanding outside banks (Porter and Judson (1996)). Over US$60 billion was shipped to countries of the former Soviet Union, and another $60 billion to Argentina. See also Rogoff (1998). The foreign circulation of Deutsche Mark is discussed in Seitz (1995). Sahay and Végh (1995) present evidence of dollarization in transition economies.

[3] On official dollarization, see Baliño et. al. (2000), Berg and Borensztein (2000), Summers (1999), Cooley and Quadrini (2000), Calvo (2000), Bencivenga et. al. (2000), Schmitt-Grohe and Uribe (2000), and Sims (2000).

[4] At the end of 1997, 131 IMF member countries (70 percent of the membership) had agreed to refrain from imposing restrictions on current account transactions, up from 50 members (36 percent of the membership) in 1982. Capital account transactions were unrestricted in only 55 countries (30 percent of the Fund's membership) in 1998, up from 33 members (24 percent of the membership) in 1982. See Macedo (1982); Johnston and others (1999), p. 6. Johnston and others, *ibid.*, and *Annual Report on Exchange Restrictions*, 1998, Appendix I.

Curiously, macroeconomists have lagged behind real trade and public finance theorists in incorporating crime-theoretic elements in their analyses.[5] In real trade models, smuggling and black markets in foreign exchange have long been analyzed as natural applications of Allingham and Sandmo's (1972) model of tax evasion. In the presence of trade restrictions, legal and illegal trade and a black market in foreign currencies all arise as natural consequences of risk aversion whether or not there are smuggling costs—see Huizinga (1991) and Martin and Panagariya (1984). By contrast, models of currency substitution feature multiple valued fiat currencies by appealing to the special role of domestic currencies in exchange, preferences, or technology.[6] Alternatively, in the legal restrictions approach associated with Wallace (1983), demand schedules for national currencies that would be perfect substitutes in laissez-faire are rendered determinate by appealing to legal restrictions, such as reserve requirements and current or anticipated capital controls. One important limitation of the legal restrictions approach is the counterfactual assumption that the relevant legal restrictions are evasion-proof.[7] Moreover, as shown below, ignoring growing stocks of foreign currencies imparts an important bias in analyses of fiscal and monetary policies by making it relatively easy for governments to resort to financing strategies featuring perpetual money-financed deficits while maintaining stable demand schedules for domestic currencies and a stable steady state rate of inflation.

In this paper we present an overlapping generations model of currency substitution-cum-dollarization that allows for growing stocks of hard currencies. The model combines the legal restrictions tradition of monetary economics with the crime-theoretic approach of real trade and public finance theory. Smuggling takes place for the sole purpose of financing what Abalkin and Whalley (1999) call "domestic capital flight"—accumulation of hard currency hoards that improve domestic agents' intertemporal terms of trade.[8] Hard currency acquired by selling goods abroad end up circulating domestically into perpetuity, the result of intergenerational trades which place the evolving stock of hard currencies in the center of the analysis. Rising hard currency stocks crowd out domestic currency balances and introduce

[5] However, see Nowak (1984) for a macroeconomic analysis of exchange controls and black currency markets, and Williamson and Lessard (1987) for a related discussion of capital flight in promoting smuggling and black currency markets.

[6] Examples include models featuring cash-in-advance requirements, shopping-time technologies, or models where money appears in the utility function.

[7] For various examples of the first approach see Hercowitz and Sadka (1987), Canzoneri and Rogers (1990), Kimbrough (1986), Végh (1995), and Siebert and Liu (1998). Models following the perfect substitution assumption include Nickelsburg (1980) and Lapan and Enders (1983), while the legal restrictions approach is used in Preston and Wallace (1985), Freeman (1987), and Mourmouras and Russell (1992).

[8] The model of this paper abstracts from smuggling to finance domestic consumption. Mourmouras and Barnett (1999) focus on the role of hard currency smuggling and hoarding in order to finance smuggling of imported goods but do not allow for gradual dollarization, as they rule out intergenerational trades in hard currency.

instability in the domestic currency demand relationship. As in the real world, the extent of unofficial dollarization in the model economy is crucial in determining the menu of available macroeconomic policy choices. As already alluded to, dollarization makes steady state money-financed deficits more difficult or impossible. A strong version of the proposition that dollarization precludes the government from earning seigniorage in the steady state is demonstrated under the realistic assumption that domestic intergenerational hard currency trades are riskless.[9] The finding that seigniorage cannot finance an *infinite* stream of fiscal deficits must be contrasted with the position taken in existing open-economy monetary models, where it is customary to begin by assuming that the government faces a fixed, perpetual real deficit sequence and then proceed by deriving the paths of inflation and money creation that are consistent with its budget constraint. However, while ruling out steady states with money-financed deficits, it is nevertheless feasible for governments in dollarizing economies to run money-financed deficits temporarily, with seigniorage revenue gradually declining in relation to GDP.

The model features asset demand indeterminacy which, as known from the work of Kareken and Wallace (1981), is a prominent feature of multiple fiat currency regimes when the role of money as a store of value is important. In such an indeterminacy-ridden world, expectations are crucial determinants of both the dynamics and the steady state extent of dollarization. In a reversal of Gresham's law, good money ends up driving out bad money if confidence in the domestic currency is initially low. Low initial confidence in the domestic currency raises the demand for hard currencies, which results in a high domestic premium over domestic currency and stimulates out-smuggling of goods and in-smuggling and hoarding of hard currencies. In the absence of population growth, no capital accumulation, and no technical progress, the flow of smuggling ebbs over time, but the damage is done: demand for real domestic currency balances is permanently lowered because of the large stock of hard currencies that has already been accumulated in the domestic economy. It is possible, however to engineer re-export of the entire domestic stock of foreign currency by restoring confidence in the domestic currency to a sufficiently high level.

The rest of this paper is organized as follows. Section II develops the basic model. Section III analyzes equilibria with in-smuggling of hard currencies and demonstrates the existence of a continuum of perfect foresight, partially or fully dollarized equilibria. Section IV examines the conditions under which unofficial dollarization may be reversed. Section V analyzes how dollarization limits the ability of governments to extract seigniorage, while Section VI examines the dynamics of dollarization in the context of a growing economy. Section VII presents a few concluding remarks.

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[9] In Mourmouras and Russell (2000), we show that a permanent money-financed deficit may be feasible if the authorities are willing and able to intensify their currency control monitoring in the interior of the country to render domestic hard currency transactions risky.

II. A Model of Currency Smuggling, Dollarization, and De-Dollarization

The Economic Environment

Consider a small open economy populated by two-period-lived overlapping generations and featuring a single nonstorable, internationally tradable good. Population is held constant until Section VI which examines the dynamics of dollarization in a small but growing economy. Each generation is composed of a continuum of individuals in the interval $[0,1]$. Each young agent is endowed with W units of the consumption good; consumption takes place in their second period of life. The utility function, $u(c_{t+1})$, has positive and diminishing marginal utility. The only assets potentially available to households are a domestic fiat currency and a foreign hard currency. Domestic and foreign currency are available in the home-country currency markets at unit prices $1/p_t$ and q_t, in terms of goods, that are determined by domestic market conditions. In addition, households may use part or all of their endowments to purchase foreign currency in the world market at a goods price that is normalized to unity.

Each young household has one opportunity to import foreign currency each period. If the household succeeds (see below), then it must hold the foreign currency until next period.[10] Whether it succeeds or fails, it may not conduct any additional transactions during the period. Old households that enter a period t with foreign currency may sell all or part of this currency to young households at its domestic price q_t, or they may try to export it for goods at the world price of unity. Each old household must decide, at the beginning of the period, how much of its holdings of foreign currency to sell in each market, and each old household has only one opportunity to engage in each type of transaction.

Possession of foreign currency violates the laws of the home country. Law-breaking of this type has no effect on the utility of households, except to the extent that they fear the consequences of the country's law-enforcement activities. The country's system of enforcing the laws is (partly) effectual only when households that possess foreign currency cross the country's border in an attempt to import or export the currency. A household that tries to export foreign currency (smuggle it out) or to import foreign currency (smuggle it in) during a period faces a probability $\alpha \in (0,1)$ of being apprehended by the enforcement authorities. The border apprehensions are independent events across households and periods.

The legal restrictions just described may be interpreted as export *surrender requirements* intended to boost demand for domestic currency by prohibiting households from storing

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[10] The analysis would be slightly more complicated, but without altering the essence of the results, if proceeds from smuggling were contemporaneously available in the domestic foreign currency market. See Mourmouras and Barnett (2000).

(accumulating) hard currency receipts from exports. In this interpretation, all hard currency export proceeds must be exchanged at the central bank for domestic currency as soon as payment from foreign sources is received. In addition, domestic agents are not allowed to hold cash foreign currency or own foreign currency accounts in domestic financial intermediaries.[11]

To summarize, despite the prohibition against accumulation of hard currency, agents may acquire and hoard such currency from two distinct sources—by smuggling exports abroad and by buying hard currency from old agents in the home-country hard currency market. Intergenerational hard currency trades taking place in the interior of the country are impossible to monitor whereas smuggling activities that involve crossing potentially guarded, but variously leaky, national borders are only partially detected. The penalty for unsuccessful smugglers is confiscation of contraband; fine revenue from the customs office is assumed to just cover this office's operations. Stated differently, the government exports all the foreign currency seized by the authorities at the border and does not return any of the export proceeds to the households. Old agents may in principle dispose of hard currency balances in two ways. First, they could use these balances to obtain consumption goods in the domestic market from young agents. Secondly, old agents could sell hard currency in the international market in exchange for consumption goods, but this activity is assumed to be as risky as it is for young agents to smuggle goods out and smuggle hard currency in.

Private Sector Optimization

The decision problem facing a young household entering period t is to maximize $E_t\{u(c_{t+1})\}$ subject to the following constraints: At date t,

$$\frac{h_t}{p_t} + q_t f_{1t} + n_{1t} = W; \tag{1}$$

At date t+1, with probability 1-α,

$$f_{21t} + n_{21t} = f_{1t} + n_{1t} \tag{2}$$

and

$$c_{t+1}^{11} = \frac{h_t}{p_{t+1}} + q_{t+1} f_{21t} + n_{21t} \qquad \text{with probability } 1\text{-}\alpha \tag{3}$$

$$c_{t+1}^{12} = \frac{h_t}{p_{t+1}} + q_{t+1} f_{21t} \qquad \text{with probability } \alpha; \tag{4}$$

at t+1, with probability α,

$$f_{22t} + n_{22t} = f_{1t} \tag{5}$$

and

[11] This assumption is made for convenience only. A positive probability of losing access to domestic dollar accounts would force risk averse agents to act similarly.

$$c_{t+1}^{21} = \frac{h_t}{p_{t+1}} + q_{t+1} f_{22t} + n_{22,t+1} \qquad \text{with probability } 1-\alpha \tag{6}$$

$$c_{t+1}^{22} = \frac{h_t}{p_{t+1}} + q_{t+1} f_{22t} \qquad \text{with probability } \alpha. \tag{7}$$

Here h_t>0 and f_{1t}>0 represent the quantities of domestic and foreign currency, respectively, that the household purchases in the domestic market at *t*. In addition, n_{1t}>0 represents the quantity of foreign currency the household tries to smuggle into the country at date t; At date *t*+1, f_{21t} and n_{21t} represent the quantities of foreign currency that the household sells in the domestic market and tries to smuggle out of the country, respectively, in the event that it was not caught smuggling at date *t* (state 1). Similarly, f_{22t}>0 and n_{22t}>0 represent the quantities of foreign currency that the household sells in the domestic market and tries to smuggle out of the country, respectively, in the event that it was caught smuggling at date *t* (state 2). Also, let $m_t \equiv \frac{h_t}{p_t}$ denote a household's real domestic currency balances, and, if $p_t < \infty$, $\pi_t \equiv \frac{p_{t+1} - p_t}{p_t}$ the home-country's rate of inflation, and $r_t \equiv \frac{1}{1+\pi_t}$ the gross return of domestic currency. The (net) premium paid on hard currency is denoted $x_t \equiv q_t - 1$.

With this background, equation (2) can be used to write $f_{21t} = f_{1t} + n_{1t} - n_{21t}$, producing

$$c_{t+1}^{11} = r_t m_t + q_{t+1}(f_{1t} + n_{1t} - n_{21t}) + n_{21t} \tag{8}$$

$$c_{t+1}^{12} = r_t m_t + q_{t+1}(f_{1t} + n_{1t} - n_{21t}). \tag{9}$$

Similarly, equation (4) may be used to write $f_{22t} = f_{1t} - n_{22t}$, producing

$$c_{t+1}^{21} = r_t m_t + q_{t+1}(f_{1t} - n_{22t}) + n_{22t} \tag{10}$$

$$c_{t+1}^{22} = r_t m_t + q_{t+1}(f_{1t} - n_{22t}). \tag{11}$$

And since equation (1) can be written as

$$f_{1t} = \frac{W - m_t - n_{1t}}{q_t}, \tag{12}$$

it follows that

$$\begin{aligned} c_{t+1}^{11} &= r_t m_t + q_{t+1}\left(\left[\frac{W - m_t - n_{1t}}{q_t}\right] + n_{1t} - n_{21t}\right) + n_{21t} \\ &= \frac{q_{t+1}}{q_t} W + \left(r_t - \frac{q_{t+1}}{q_t}\right) m_t + q_{t+1}\left(1 - \frac{1}{q_t}\right) n_{1t} + (1 - q_{t+1}) n_{21t} \end{aligned} \tag{13}$$

$$
\begin{aligned}
c_{t+1}^{12} &= r_t m_t + q_{t+1}\left(\left[\frac{W - m_t - n_{1t}}{q_t}\right] + n_{1t} - n_{21t}\right) \\
&= \frac{q_{t+1}}{q_t}W + \left(r_t - \frac{q_{t+1}}{q_t}\right)m_t + q_{t+1}\left(1 - \frac{1}{q_t}\right)n_{1t} - q_{t+1}n_{21t}
\end{aligned}
\tag{14}
$$

and

$$
\begin{aligned}
c_{t+1}^{21} &= r_t m_t + q_{t+1}\left(\left[\frac{W - m_t - n_{1t}}{q_t}\right] - n_{22t}\right) + n_{22t} \\
&= \frac{q_{t+1}}{q_t}W + \left(r_t - \frac{q_{t+1}}{q_t}\right)m_t + q_{t+1}\left(1 - \frac{1}{q_t}\right)n_{1t} + (1 - q_{t+1})n_{22t}
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
c_{t+1}^{22} &= r_t m_t + q_{t+1}\left(\left[\frac{W - m_t - n_{1t}}{q_t}\right] - n_{22t}\right) \\
&= \frac{q_{t+1}}{q_t}W + \left(r_t - \frac{q_{t+1}}{q_t}\right)m_t - \frac{q_{t+1}}{q_t}n_{1t} - q_{t+1}n_{22t}.
\end{aligned}
\tag{16}
$$

The household's decision problem can be expressed as to maximize expected utility

$$
(1-\alpha)\left[(1-\alpha)u(c_{t+1}^{11}) + \alpha u(c_{t+1}^{12})\right] + \alpha\left[(1-\alpha)u(c_{t+1}^{21}) + \alpha u(c_{t+1}^{22})\right]
$$

by choosing m_t, n_{1t}, and the n_{2it}, i=1,2, subject to the last four constraints, plus non-negativity constraints, plus the constraints $n_{1t} \le W - m_t$, $n_{21t} \le f_{1t} + n_{1t}$ and $n_{22t} \le f_{1t}$. The first order conditions are:

$$
\begin{aligned}
m_t &: \quad \left(r_t - \frac{q_{t+1}}{q_t}\right)\left[(1-\alpha)^2 u'(c_{t+1}^{11}) + (1-\alpha)\alpha\left[u'(c_{t+1}^{12}) + u'(c_{t+1}^{21})\right] + \alpha^2 u'(c_{t+1}^{22})\right] \le 0 \\
n_{1t} &: \quad \left(1 - \frac{1}{q_t}\right)q_{t+1}\left[(1-\alpha)^2 u'(c_{t+1}^{11}) + (1-\alpha)\alpha u'(c_{t+1}^{12})\right] - \frac{q_{t+1}}{q_t}\left[(1-\alpha)\alpha u'(c_{t+1}^{21}) - \alpha^2 u'(c_{t+1}^{22}\right] \le 0 \\
n_{21t} &: \quad (1-\alpha)^2 u'(c_{t+1}^{11})(1 - q_{t+1}) - (1-\alpha)\alpha u'(c_{t+1}^{12})q_{t+1} \le 0 \\
n_{22t} &: \quad (1-\alpha)\alpha u'(c_{t+1}^{21})(1 - q_{t+1}) - \alpha^2 u'(c_{t+1}^{22})q_{t+1} \le 0,
\end{aligned}
$$

with equality being required, in all cases, if the relevant choice variable is positive.

The first condition produces the familiar relationship

$$r_t = \frac{q_{t+1}}{q_t}, \tag{17}$$

which must hold as long as m_t is positive. The second condition will be negative whenever $q_t \le 1$. It follows that $q_t > 1$ is a necessary condition for $n_{1t} > 0$: foreign currency will be smuggled into the country only if it is trading at a premium in the domestic currency market. The third condition implies

$$\frac{\alpha}{1-\alpha}\frac{u'(c_{t+1}^{12})}{u'(c_{t+1}^{11})} = \frac{1-q_{t+1}}{q_{t+1}}, \tag{18}$$

which must hold when n_{21t} is positive. It follows that $q_{t+1} < 1$ is a necessary condition for n_{21t} >0: foreign currency will be smuggled out of the country if and only if it is trading at a discount in the domestic market. Note that when $q_{t+1} \ge 1$ the value of the first condition must be negative, so we must have $n_{1t} = 0$. Similar results hold for n_{22t}.[12]

Equilibrium

The following implications can be drawn from this analysis.

- Equilibria in which $q_t > 1$, $t \ge 1$,will never involve out-smuggling of foreign currency.
- Equilibria in which $q_t < 1$, $t \ge 1$, will never involve in-smuggling of foreign currency.
- There may be equilibria in which $q_t < 1$ for "early" values of *t*. In these equilibria, foreign currency is gradually smuggled out—that is to say, in these equilibria the economy is gradually "de-dollarized".
- There are no steady states in which *r*<1. In steady states of this type, q_t would eventually get arbitrarily small. This would mean n_{21t} would have to get arbitrarily large to cause $u'(c_{t+1}^{11})$ to be arbitrarily small relative $u'(c_{t+1}^{12})$.

Two distinct types of equilibria are possible. In *fully dollarized* equilibria, $1/p_t = 0$ for all $t \ge 1$ and there is no demand for domestic currency. In *partially dollarized* equilibria, $1/p_t > 0$ for all $t \ge 1$ and domestic currency demand is positive at all dates. Such equilibria are said to have become *fully de-dollarized* at date T if $f_{1t} = 0$ for all $t \ge T$. The law of motion of the domestic stock of foreign currency at the beginning of date *t*, F_{1t}, is

[12] The decision problem of the initial old (they may have one!) is considered in Section IV. Their decision problem is nontrivial only in equilibria that involve out-smuggling of foreign currency.

$$F_{1,t+1} = F_{1,t} + (1-\alpha)n_{1t} - n_{2t}. \tag{19}$$

Here $(1-\alpha)n_{1t}$ represents currency successfully smuggled in by the young agents at date t and added to the foreign currency supply at date t+1, while n_{2t} represents the foreign currency the old agents try to smuggle out at date t+1.

III. Equilibria with Hard Currency In-Smuggling

We will first analyze in-smuggling equilibria in which $q_t > 1$ for all t.

Individual Optimization

The individual decision problem reduces to reduce to selecting n_t, $s_t \equiv m_t + q_t f_t$ and a pair of contingent consumption claims (c^1_{t+1}, c^2_{t+1}) to maximize

$$(1-\alpha)\,u(c^1_{t+1}) + \alpha u\,(c^2_{t+1}) \tag{20}$$

subject to

$$c^1_{t+1} \le r_t m_t + q_{t+1}(f_t + n_t) \qquad \text{with probability } 1\text{-}\alpha \tag{21}$$

$$c^2_{t+1} \le r_t m_t + q_{t+1} f_t \qquad \text{with probability } \alpha, \tag{22}$$

where $f_t \equiv f_{1t}$, $n_t \equiv n_{1t}$, $s_t \equiv m_t + q_t f_t$, and $n_t \equiv W - s_t$. The budget line in (c^1_{t+1}, c^2_{t+1}) space is defined by

$$c^1_{t+1} + (q_t - 1)c^2_{t+1} = q_{t+1} W. \tag{23}$$

The point where (23) intersects the 45^0 certainty line corresponds to a non-diversified portfolio in which n=0 and s=W (Figure 1). The point where the same line intersects the vertical axis corresponds to a portfolio in which the entire endowment is smuggled, s=0 and n=W. Since risk-averse agents do not participate in actuarially fair bets, smuggling is attempted if and only if its expected gross return, $(1-\alpha)q_{t+1}$, exceeds q_{t+1}/q_t, which is the gross return of domestically acquired foreign currency. Equivalently, smuggling will be attempted if and only if the premium x_t exceeds the odds of failure in smuggling, $\alpha_t/(1-\alpha_t)$. Note that demand for riskless assets (that is, domestic currency or domestically obtained hard currency) is positive so long as domestic inflation is not too high. Specifically, s_t>0 if the marginal rate of substitution at $(c^1_{t+1}, c^2_{t+1}) = (0, q_{t+1}W)$ is sufficiently high. An interior solution obtains if the net premium satisfies

$$\frac{\alpha}{1-\alpha} < q_t - 1 < \frac{\alpha}{1-\alpha}\frac{u'(0)}{u'(q_{t+1}W)}. \tag{24}$$

In the range given by (24), demand schedules for the risky and safe assets are positive: $s(q_t, q_{t+1}; W) > 0$ and $n(q_t, q_{t+1}; W) > 0$ are characterized by $W = s_t + n_t$ and the tangency condition

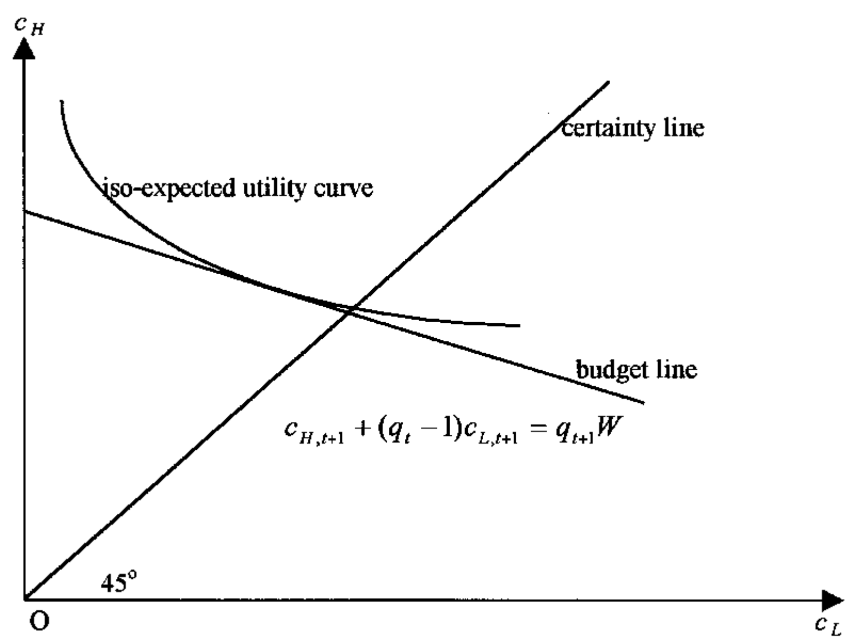
$$\frac{\alpha}{1-\alpha} \frac{u'\left(\frac{q_{t+1}}{q_t} s_t + q_{t+1} n_t\right)}{u'\left(\frac{q_{t+1}}{q_t} s_t\right)} = q_t - 1. \tag{25}$$

Example. If u(c)=*ln*(c), and provided $(1-\alpha)q_t > 1$, a diversified equilibrium involves

$$s_t = W \frac{\alpha q_t}{q_t - 1}; \qquad n_t = W \frac{(1-\alpha)q_t - 1}{q_t - 1}; \tag{26}$$

$$c_{t+1}^1 = (1-\alpha)q_{t+1} W; \quad c_{t+1}^2 = \alpha \frac{q_{t+1}}{q_t - 1} W. \tag{27}$$

Figure 1. Consumer Optimization



Fully Dollarized Equilibria

In-smuggling equilibria in which $1/p_t = 0$ for all $t \geq 1$ correspond to a fully dollarized economy in which the equilibrium law of motion for foreign currency is $F_{t+1} = F_t + (1-\alpha)n_t$. The equilibrium price of foreign currency is obtained by substituting the demand functions

(26) in the law of motion for F_t and recalling that in equilibrium[13] $f_t = F_t$, and $m_t = 0 \Leftrightarrow s_t = q_t f_t$:

$$\frac{\frac{q_{t+1}}{q_{t+1}-1}\alpha W}{q_{t+1}} = \frac{\frac{q_t}{q_t-1}\alpha W}{q_t} + (1-\alpha)W\frac{(1-\alpha)q_t-1}{q_t-1} \tag{28}$$

The above equation eventually reduces to the following first-order difference equation:

$$q_{t+1}-1 = \frac{q_t-1}{1+\frac{1-\alpha}{\alpha}\left[(1-\alpha)(q_t-1)-\alpha\right]}. \tag{29}$$

The steady states of (29) are $q_{t+1}-1=0$ and $q_{t+1}-1=\frac{\alpha}{1-\alpha}$. The premium declines over time whenever its starting value exceeds the positive steady state $\alpha/(1-\alpha)$. For $q_{t+1} < q_t$ we need $q_t - 1 > \frac{\alpha}{1-\alpha}$. It must also be established that $q_{t+1}-1 > \frac{\alpha}{1-\alpha}$ whenever $q_t - 1 > \frac{\alpha}{1-\alpha}$. $q_{t+1}-1=\frac{\alpha}{1-\alpha}$ when $q_t - 1 > \frac{\alpha}{1-\alpha}$. Differentiating (29) with respect to $q_t - 1$ yields $\frac{dv_{t+1}}{dv_t} = \frac{\alpha^3}{\left[(1-\alpha)^2 v_t + \alpha^2\right]} > 0.$

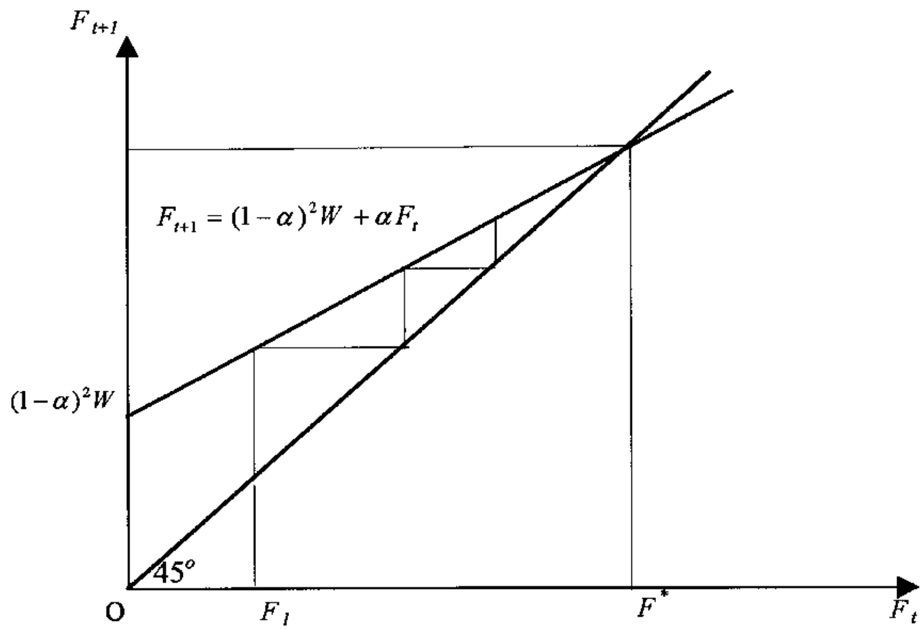
The equilibrium law of motion for F is a simple linear difference equation (Figure 2). This is established from $F_{t+1} = F_t + (1-\alpha_t)n_t$ and $F_t = f_t = \frac{s_t}{q_t} = \frac{\alpha W}{q_t - 1}$, which imply $q_t = 1 + \frac{\alpha W}{F_t}$ and $n_t = W\frac{(1-\alpha)q_t - 1}{q_t - 1}$. Substituting out q_t gives

$$\begin{aligned} F_{t+1} &= F_t + (1-\alpha)[(1-\alpha)W - F_t] \\ &= \alpha F_t + (1-\alpha)^2 W. \end{aligned} \tag{30}$$

The steady state stock of domestically circulating hard currency is $F^* = (1-\alpha)W$. No smuggling takes place in the stationary state—recall that a necessary condition for $n_t>0$ is $(1-\alpha)q_t > 1$. If hard currency is scarce in the economy in the initial period, ($F_1 < F^*$), both q_t and n_t will be high initially. As the stock of hard currency is augmented over time through smuggling, the market price and the level of smuggling will both decline and the stock of hard currency rises. In other words, we have $F_{t+1} > F_t$ as long as $F_t < (1-\alpha)W$.

[13] As the proceeds from smuggling enter the domestic market with one period lag, the supply of dollars at t, F_t, consists of dollar hoards carried over by the members of generations t-1.

Figure 2. Law of Motion of Hard Currency Stock



Requirements for Equilibrium

This equilibrium exists if $0 \le F_0 \le (1-\alpha)W$, that is if the stock of foreign currency in the hands of the initial old is not "too high" relative to the steady state. This is established by noting that q_1 is determined from the date 1 equilibrium condition $s_1 = q_1 F_1$, or

$$W\alpha \frac{q_1}{q_1 - 1} = q_1 F_0 \Leftrightarrow q_1 - 1 = \frac{\alpha W}{F_0}. \tag{31}$$

It is also required that at date 1, $0 \le n_1 \le W$, where $n_t = (1-\alpha)W - F_t$ along the equilibrium path. It follows that the boundaries for $F_1 = F_0$ are $0 \le F_0 \le (1-\alpha)W$. We cannot have $F_0 = 0$, since q_1 would be undefined. But we can have q_1 very large if F_0 is very small. And if $F_0 = (1-\alpha)W$ we have $q_1 = (1-\alpha)^{-1}$ and we have reached the steady state immediately. We have no problems along the transition path in this case, since as long as $F_t \le (1-\alpha)W$ we have $n_t > 0$ and hence $s_t < W$.

Welfare Comparisons

The welfare of all agents $t \ge 1$ is increasing in the intertemporal terms of trade

$$\frac{q_{t+1}}{q_t}=\frac{1+\dfrac{\alpha W}{\alpha F_t+(1-\alpha)^2 W}}{1+\dfrac{\alpha W}{F_t}}.$$

$q_{t+1}/q_t=1$, when $F_t=(1-\alpha)W$; $q_{t+1}/q_t=0$ when $F_t=0$; and that $q_{t+1}/q_t\in(0,1)$ is globally increasing in F_t. Thus the higher F_0 is the higher the welfare of the members of generations $t\geq 1$ will be. Yet, the fully dollarized equilibrium is *inefficient*: the equilibrium rate of return is lower than unity for all $t\geq 1$. The inefficiency is evidenced by the economy's overaccumulation of foreign currency: the value of the steady state stock of hard currency circulating in the economy, $q^*F^*=W$, exceeds F^*, the value of foreign currency in the world market. However, the asymptotic gross risk-free rate of interest approaches unity, meaning that the level of welfare enjoyed by agents living near the steady state is arbitrarily close to the level they would enjoy in the stationary, laissez-faire equilibrium with valued domestic money, a constant money supply and zero inflation.

The initial old are also better off the higher F_0 is: their consumption is $c_0=q_1F_0=F_0+\alpha W$. The prohibition against accumulation of foreign currency creates a monopoly for domestic residents who happen to own foreign currency at the initial date. The real price of domestically available hard currency rises to ration the available supply, creating rents that are captured by the original owners of foreign currency. These rents diminish only gradually over time as smuggling and the importation of foreign reduces hard currency premiums.

Note that even if $F_0=(1-\alpha)W\Leftrightarrow c_0=W$, the initial old are not as well off as they would be in a fully de-dollarized laissez faire steady state. In that steady state, they would export their foreign currency endowment plus sell domestic currency with a real value of $m_1=W$, enjoying $c_0=F_0+W$. So no one is as well off, in the fully dollarized equilibria, as they are in the best laissez faire steady state. Finally, also note that if $F_0>(1-\alpha)W$ we have corner-solution steady states in which $q_1<(1-\alpha)^{-1}$, there is no in-smuggling of foreign currency, and the initial old sell all their foreign currency at a total real price of *W*. If $F_0=W$, for example (a limiting case, under our assumptions), then $q_1=1$.

Partially Dollarized Equilibria

The fully dollarized equilibrium is a limiting case of a partially dollarized equilibria (PDE), to which we now turn. With valued home-currency, $s_t=m_t+q_tf_t$. Substituting this into the equilibrium law of motion for F_t, equation (19), and employing (26) yields the following system of equations that must be satisfied in a PDE, with m_t>0, t=1,2,...

$$s_t=m_t+q_tF_t=W\alpha q_t/(q_t-1) \tag{32}$$

$$n_t = W - s_t = W\left[(1-\alpha)q_t - 1\right]/\left[q_t - 1\right] \tag{33}$$

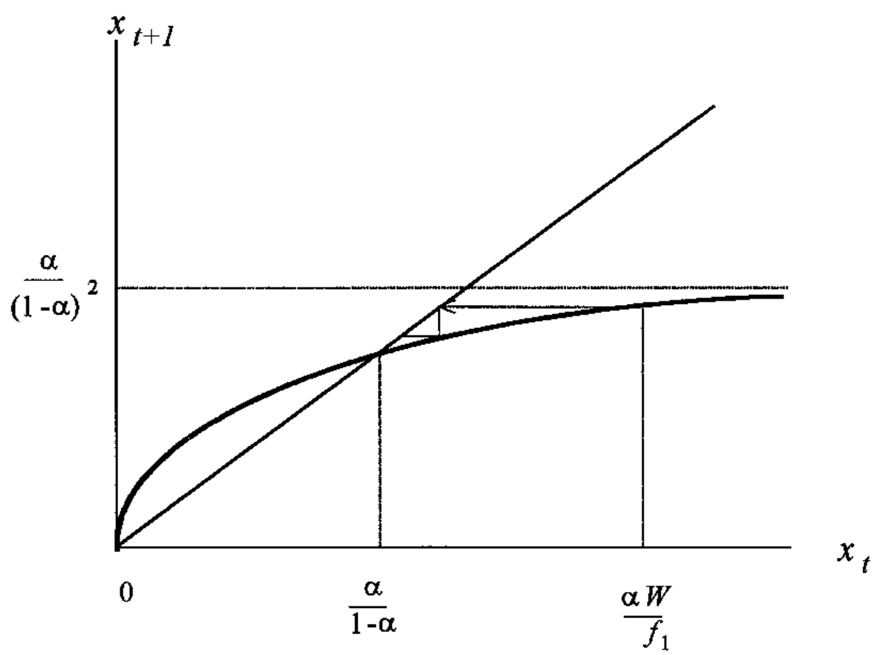
$$m_{t+1} / m_t = q_{t+1} / q_t = r_t \tag{34}$$

$$F_{t+1} = F_t + (1-\alpha_t)n_t . \tag{35}$$

Equation (32) is the equilibrium condition in the domestic market for hard currency ; equation (33) is the flow demand curve for imports of foreign currency; equation (34) is the equilibrium condition for the domestic currency market; equation (35) is the law of motion of the stock of foreign currency. Using (32) to eliminate m_t in (34) yields the following difference equation in the premium $x_t \equiv q_t - 1$ (Figure 3):

$$q_{t+1} - 1 = \frac{q_t - 1}{1 + \frac{1-\alpha}{\alpha}\left[(1-\alpha)(q_t - 1) - \alpha\right]} . \tag{36}$$

Figure 3. Equilibrium With In-Smuggling of Hard Currency



Requirements for Equilibrium

The stationary points of (36) are 0 and $\alpha/(1-\alpha)$. Equation (36) defines a continuum of perfect foresight stationary monetary equilibria, each indexed by q_1. The permissible values of $q_1 \in [(1-\alpha)^{-1}, 1+\alpha W/F_0]$. Assuming $F_0 < W(1-\alpha)$, the lower bound of m_1 is 0 (the fully dollarized steady state analyzed above), which obtains when q_1 reaches its upper bound

$q_1 = 1 + \frac{\alpha W}{F_0}$. The upper bound of m_1, $m_1 = W - F_0/(1-\alpha) > 0$, is obtained when q_1 attains its lower bound, namely the steady state, $q_1 = (1-\alpha)^{-1}$, and corresponds to the no additional dollarization steady state $F_t = F_0$, for all t. Values of q_1 in between the two extremes give rise to a continuum of PDEs where q converges to $(1-\alpha)^{-1}$.

In order to verify that we have a bona fide equilibrium, we must show that m_t is not driven above s_t at some point along the transition path. That is, we need to show that $m_{t+1} < s_{t+1}$ whenever $m_t < s_t$, or that $m_t < \frac{q_t}{q_t - 1}\alpha W$ implies $\frac{q_{t+1}}{q_t} m_t < \frac{q_{t+1}}{q_{t+1} - 1}\alpha W$. That this is indeed the case can been shown somewhat laboriously using the difference equation for q.

Partially Dollarized Steady States

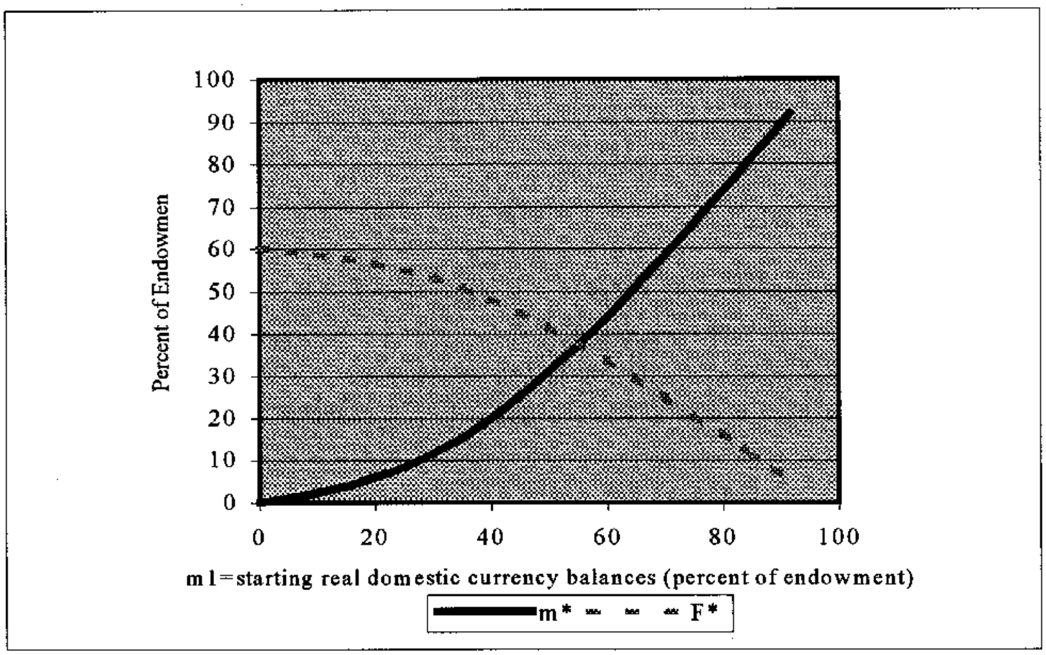
In PDEs, an additional dimension of the equilibrium concerns the steady state values of m and F satisfying $m^* + q^* F^* = W$. Recursion on the equilibrium condition for domestic real currency balances, $m_{t+1} = \frac{q_{t+1}}{q_t} m_t$, yields

$$m_{t+1} = \frac{q_{t+1}}{q_1} m_1 \Rightarrow m^* = \lim_{t\to\infty}\left(\frac{q_{t+1}}{q_1} m_1\right) = \frac{q^*}{q_1} m_1 = \frac{m_1}{(1-\alpha)q_1}. \tag{37}$$

We have $m_1 = s_1 - q_1 f_1 = \frac{q_1}{q_1 - 1}\alpha W - q_1 F_0$, which demonstrates that as q_1 increases, m_1 declines.

PDEs have the property that, holding the starting value F_1 constant, higher levels of q_1 are associated with lower m_1 and higher n_1, a higher stock of hard currency in the steady state, F^*, and lower steady state real domestic currency balances, m^* (Figure 4). In PDEs, steady state real domestic currency balances are positive but less than the maximum level attained in the no-dollarization steady state. Stated somewhat differently, the nonstationary monetary equilibria indexed by the initial value of q_1 (or m_1) converge to steady states in which the long-run value of real domestic currency balances is higher the higher the initial value of m_1 and the lower q_1 is.

Figure 4. Steady State Distributions of Real Currency Holdings



We interpret high values of q_1 and low values of m_1 as situations in which home-country residents do not have confidence in the domestic currency. This lack of confidence becomes a self-fulfilling prophecy in the sense that it induces high demand for foreign currency and raises the premium at the initial date, leading to more out-smuggling of exportables. The resulting smuggling of foreign currency into the country gives rise to a stock of foreign currency that continues to circulate in the economy forever, resulting in a *permanently* lower level of real domestic currency balances.

Table 1 demonstrates the sensitivity of the steady state level of real domestic currency balances in a PDE to changes in expectations about the initial value of the currency. Whereas agents living at or near the steady state are indifferent as regards the composition of their holdings of safe assets, all agents during the transition would prefer to live in the no-additional dollarization equilibrium in which no smuggling ever occurs. At the level of real domestic balances achieved in that equilibrium, the intertemporal terms of trade are at the biological optimum level (unity) starting at t=1.

Table 1. Limiting Distributions of Domestic Real Currency and Dollar Balances

$m(1)$	$q(1)$	$\pi(1)$	$n(1)$	m^*	F^*	q^*F^*
			In percent			
0.0	900.0	338.5	55.0	0.0	60.0	100.0
5.0	812.3	297.1	54.4	1.0	59.4	99.0
10.0	727.5	257.0	53.6	2.3	58.6	97.7
15.0	646.4	218.8	52.7	3.9	57.7	96.1
20.0	570.2	182.9	51.5	5.8	56.5	94.2
25.0	500.0	150.0	50.0	8.3	55.0	91.7
30.0	437.2	120.6	48.1	11.4	53.1	88.6
35.0	382.8	95.3	45.9	15.2	50.9	84.8
40.0	337.2	74.2	43.1	19.8	48.1	80.2
45.0	300.0	57.1	40.0	25.0	45.0	75.0
50.0	270.2	43.6	36.5	30.8	41.5	69.2
55.0	246.4	33.0	32.7	37.2	37.7	62.8
60.0	227.5	24.7	28.6	44.0	33.6	56.0
65.0	212.3	18.2	24.4	51.0	29.4	49.0
70.0	200.0	13.0	20.0	58.3	25.0	41.7
75.0	189.9	8.9	15.5	65.8	20.5	34.2
80.0	181.5	5.6	10.9	73.5	15.9	26.5
85.0	174.5	2.9	6.3	81.2	11.3	18.8
91.7	166.7	0.0	0.0	91.7	5.0	8.3

Source: Staff Calculations

IV. Equilibria Involving Out-Smuggling of Foreign Currencies

We now investigate the existence and other properties of equilibria where, following the restoration of confidence in the domestic currency, foreign currency is re-exported abroad.

Individual Optimization

Recall that in the present model, old agents may obtain consumption by selling their foreign currency hoards in the international market, where the real price of hard currency is unity. However, as was shown in Section II, given a positive probability of detection, out-smuggling is not a profitable activity if the domestic price of hard currency is greater than or equal to 1. But, as shown below, out-smuggling will be profitable if the premium on domestic holdings of foreign currency turns negative, $q_t \leq 1$ for all $t \geq 1$. Under these assumptions, the household budget constraint reduces to

$$c_{t+1}^1 \leq r_t m_t + q_{t+1} f_{2t} + n_{2t} \qquad \text{with probability } 1-\alpha \tag{38}$$

$$c_{t+1}^2 \leq r_t m_t + q_{t+1} f_{2t} \qquad \text{with probability } \alpha, \tag{39}$$

where $f_{2t} + n_{2t} = f_{1t}$. As we have seen, we must have $r_t = q_{t+1} / q_t$. We also have $W = m_t + q_t f_{1t} = m_t + q_t (f_{2t} + n_{2t})$. In a manner analogous with the analysis of Sections II-III, let $s_t = m_t + q_t f_{2t}$, so that

$$W = s_t + q_t n_{2t} \Leftrightarrow n_{2t} = \frac{W - s_t}{q_t}. \tag{40}$$

These substitutions produce

$$c_{t+1}^1 \leq \frac{q_{t+1}}{q_t} s_t + \frac{W - s_t}{q_t} \tag{41}$$

$$c_{t+1}^1 \leq \frac{q_{t+1}}{q_t} s_t, \tag{42}$$

where it is required that $s_t \in [0, W]$. The first order condition is

$$(1-\alpha) u'(c_{t+1}^1) \left[\frac{q_{t+1}}{q_t} - \frac{1}{q_t} \right] + \alpha u'(c_{t+1}^2) \frac{q_{t+1}}{q_t} = (1-\alpha) u'(c_{t+1}^1) \left[q_{t+1} - 1 \right] + \alpha u'(c_{t+1}^2) q_{t+1} \leq 0, \tag{43}$$

with equality if $s_t > 0$. The first order condition makes it is clear that $q_t < 1$ for all $t \geq 1$. Assuming $u(c) = \log(c)$, an assumption that will be maintained henceforth, the interior first order condition becomes

$$(1-\alpha) \frac{q_{t+1} - 1}{\frac{q_{t+1}}{q_t} s_t + \frac{W - s_t}{q_t}} + \alpha \frac{q_{t+1}}{\frac{q_{t+1}}{q_t} s_t} = 0, \tag{44}$$

which yields the following asset demands:

$$s_t = \frac{\alpha W}{1 - q_{t+1}} \quad \text{and} \quad n_{2t} = \frac{[(1-\alpha) - q_{t+1}] W}{q_t (1 - q_{t+1})}. \tag{45}$$

Notice that if n_{2t} is to be non-negative for $t \geq 1$ then we must have $q_{t+1} \leq 1 - \alpha$ (But there is no such requirement on q_1.)

A fully dollarized equilibria with out-smuggling would require $W = q_t f_t$ for each $t \geq 1$. When $q_t \leq 1$ this requires $f_t \geq W$, which is conceivable. However, these fully dollarized equilibria with out-smuggling will be ruled out in situations (which we regard as most plausible) where the initial stock of foreign currency is not "too large": $F_0 \le W(1-\alpha)$. The equilibrium law of motion for the foreign currency stock may be utilized to derive the transition equation for q_t:

$$f_{1,t+1} = f_{2t} = f_{1t} - n_{2t} \tag{46}$$

But $f_{1t} = \dfrac{W - m_t}{q_t}$, while any equilibrium with valued domestic currency and without seigniorage must involve $m_{t+1} = \dfrac{q_{t+1}}{q_t} m_t$. Thus, the foreign currency law of motion becomes

$$\frac{W - \frac{q_{t+1}}{q_t} m_t}{q_{t+1}} = W - \frac{W - m_t}{q_t} - n_{2t}, \tag{47}$$

or

$$\frac{W_t}{q_{t+1}} = \frac{W}{q_t} - n_{2t} \Leftrightarrow n_{2t} = W\left(\frac{1}{q_t} - \frac{1}{q_{t+1}}\right) \tag{48}$$

If n_{2t} is to remain positive, the q_t-sequence must be increasing. Recall that

$$n_{2t} = \frac{[(1-\alpha) - q_{t+1}]W}{q_t(1-q_{t+1})}, \tag{49}$$

implying

$$\frac{1}{q_t} - \frac{1}{q_{t+1}} = \frac{(1-\alpha) - q_{t+1}}{q_t(1-q_{t+1})},$$

which simplifies to

$$q_{t+1} = \frac{q_t}{\alpha + q_t}. \tag{50}$$

Equilibrium

Here are some properties of this equilibrium (Figure 5):

- There are two steady states, $q^* = 0$ and $q^* = 1-\alpha$. Since $0 < q_t < 1-\alpha$ implies $q_t < q_{t+1} < 1-\alpha$, in out-smuggling nonstationary equilibria of this type, the q_t-sequence is strictly increasing and converges to 1-α from below.

- The difference equation is upward-sloping and concave:
$$\frac{dq_{t+1}}{dq_t} = \frac{\alpha}{(\alpha+q_t)^2} > 0 \text{ and } \frac{d^2 q_{t+1}}{dq_t^2} = -\frac{2\alpha}{(\alpha+q_t)^3} < 0.$$

- The m_t sequence implied by the difference equation converges to

$$m^* = \lim_{t\to\infty} \frac{q_t}{q_1} m_1 = \frac{m_1}{q_1} \lim_{t\to\infty} q_t = (1-\alpha)\frac{m_1}{q_1}. \tag{51}$$

- Since $W = q_1 f_{11} + m_1 \Leftrightarrow q_1 = \dfrac{W - m_1}{f_{11}}$, the condition $q_1 < 1-\alpha$ requires $\dfrac{W-m_1}{f_{11}} < 1-\alpha \Leftrightarrow f_{11} > \dfrac{W-m_1}{1-\alpha}$. But, as will be shown below, the value of f_{11} is actually endogenous, being a choice variable. Thus, to pin down the equilibrium value of q_1, we must consider the decision problem of the initial old.

Note that for the resulting q_t-sequence to be an equilibrium, we must have $s_t < W$ for all $t \geq 1$, which ensures that $n_{2t} > 0$. However, it is clear that $0 < s_t < W$ and $n_{2t} > 0$ as long as $q_{t+1} < 1-\alpha$. In addition, we must have $m_t < s_t$, which ensures that $f_{1,t+1} = f_{2t} > 0$.

Integrating the Initial Old

Each initial old agent is endowed with a quantity of real domestic currency balances m_1 and a quantity of foreign currency f_0. The initial old divide their foreign currency holdings into a quantity f_1 that they sell in the domestic hard currency market and a quantity n_0 that they attempt to smuggle out of the country. The budget constraints of the initial old are

$$f_1 + n_0 = f_0 \tag{52}$$

$$c_0^1 = m_1 + q_1 f_1 + n_0 \qquad \text{with probability } 1\text{-}\alpha \tag{53}$$

$$c_0^1 = m_1 + q_1 f_1 \qquad \text{with probability } \alpha. \tag{54}$$

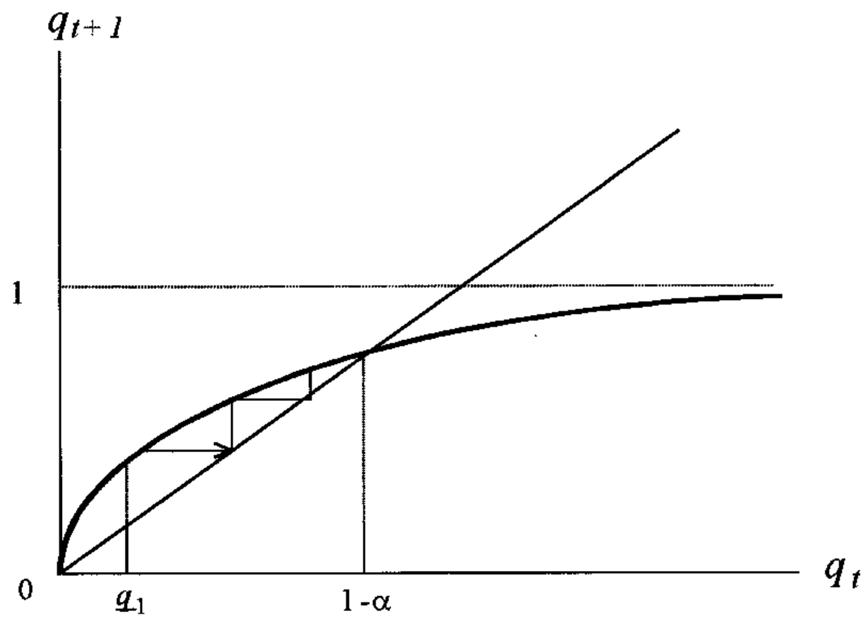
Here c_0^i denotes the consumption of the initial old in apprehension state i=1,2. The budget constraints can be rewritten

$$c_0^1 = m_1 + q_1 f_0 + (1-q_1) n_0 \tag{55}$$

$$c_0^2 = m_1 + q_1 (f_0 - n_0) \tag{56}$$

In this problem, m_1 is an endowment, not a choice variable, but n_0 is a choice variable.

Figure 5. Equilibrium Law of Motion With Hard Currency Out-Smuggling



The decision problem of an initial old household can be viewed as choosing n_0 to maximize its expected utility

$$(1-\alpha)u(c_0^1)+\alpha u(c_0^2)$$

subject to (55), (56) and $0 \le n_0 \le f_0$.

In the log-utility case, the first order conditions lead to the following interior solution

$$n_0 = \frac{[1-\alpha-q_1]}{q_1(1-q_1)}(m_1+q_1 f_0). \tag{57}$$

Thus, out-smuggling of foreign currency will be non-negative whenever $q_1 \le 1-\alpha$. Also, the supply of foreign currency by the initial old in the domestic market at date 1 is

$$f_1 = f_0 - n_0 = \frac{\alpha q_1 f_0 - [1-\alpha-q_1]m_1}{q_1(1-q_1)}.$$

We must have $f_1 \ge 0$, which is readily seen to be equivalent to $q_1 \ge \dfrac{(1-\alpha)m_1}{\alpha f_0 + m_1}$.

To summarize, an interior solution for the initial old requires

$$\frac{(1-\alpha)m_1}{\alpha f_0 + m_1} \le q_1 \le 1-\alpha\,. \tag{58}$$

Constructing Equilibria

On the demand side of the foreign currency market at date 1, we have $f_{11} = \dfrac{W-m_1}{q_1}$ from the initial young. Equilibrium requires $f_{11} = f_1$. Thus, we must have

$$W - m_1 = \frac{\alpha q_1 f_0 - [1-\alpha-q_1]m_1}{1-q_1},$$

which works out to

$$q_1 = \frac{W-\alpha m_1}{\alpha f_0 + W} \tag{59}$$

Substituting this into (58), we thus require that

$$\frac{(1-\alpha)m_1}{\alpha f_0 + m_1} \le \frac{W-\alpha m_1}{\alpha f_0 + W} \le 1-\alpha\,. \tag{60}$$

The inequality on the left-hand side holds if and only if $m_1 \le W$. The right-hand side inequality requires $m_1 \ge W-(1-\alpha)f_0$.

Note that if $f_0 = W/(1-\alpha)$, which is our upper bound on f_0, then the only legitimate initial values are $m_1 = 0$ and $q_1 = 1-\alpha$, which would give us the steady state immediately. Otherwise, there will be positive values of m_1 and lower values of q_1 that may produce out-smuggling equilibria.

The condition $m_1 \le W$, which is also a requirement for equilibrium, along with equation (59) imply that $q_1 > \dfrac{(1-\alpha)W}{\alpha f_0 + W}$, which places the ultimate lower bound on q_1.

Note that if m_1 is too low [that is, if $m_1 < W-(1-\alpha)f_0$] then the only possible equilibrium is one in which $f_1 = f_{11} = f_0$, and

$$q_1 = \frac{W-m_1}{f_0} \ge 1-\alpha\,.$$

One equilibrium of this type is a steady state with $q_t = q_1$ for all $t \geq 1$, provided m_1 is not so low that $\frac{W-m_1}{f_0} > \frac{1}{1-\alpha}$ (see below).

To summarize, a requirement for an out-smuggling equilibrium, or, indeed, for avoiding in-smuggling equilibria, is that confidence in the domestic currency should be sufficiently high.

Requirements for Equilibria

It has been already established that for a nonstationary out-smuggling equilibrium to exist

$$q_1 = \frac{W - \alpha m_1}{\alpha f_0 + W}, \quad m_1 > W - (1-\alpha) f_0 .$$

It has also been established that an equilibrium will not exist unless $q_1 \geq \frac{(1-\alpha)m_1}{W}$.

It follows that we need $\frac{W - \alpha m_1}{\alpha f_0 + W} \geq \frac{(1-\alpha)m_1}{W}$, which reduces to $m_1 \leq \frac{W^2}{W + \alpha(1-\alpha)f_0}$.

So it is required that

$$W - (1-\alpha)f_0 \leq m_1 \leq \frac{W^2}{W + \alpha(1-\alpha)f_0}. \tag{61}$$

Notice that $[W-(1-\alpha)f_0][W+\alpha(1-\alpha)f_0] \leq W^2$ for any $f_0 > 0$, so values of m_1 can be found to satisfy this inequality for any positive value of f_0.

An equilibrium sequence also requires that

$$s_1 \geq m_1 \Leftrightarrow W(\alpha + q_1) \geq m_1 \Leftrightarrow W\left(\alpha + \frac{W - \alpha m_1}{\alpha f_0 + W}\right) \geq m_1 \Leftrightarrow m_1 \leq W \frac{\alpha^2 f_0 + (1+\alpha)W}{\alpha f_0 + (1+\alpha)W}$$

It can be shown that for $f_0 > 0$,

$$\frac{W^2}{W + \alpha(1-\alpha)f_0} < W \frac{\alpha^2 f_0 + (1+\alpha)W}{\alpha f_0 + (1+\alpha)W}.$$

It follows that $\bar{m}_1 \equiv \frac{W^2}{W + \alpha(1-\alpha)f_0}$ is the relevant upper bound on m_1. Note that if $m_1 = \bar{m}_1$, then the limiting steady state is fully un-dollarized, while if $m_1 \leq W - (1-\alpha)f_0$, then we have a steady state in which the quantity of foreign currency remains fixed at its initial value.

A moral of this analysis is that there is a maximum initial amount of real domestic currency balances that the economy can sustain in equilibrium when agents place confidence in the domestic currency.

Finally, notice that if $f_0 = 0$ then the only equilibrium of the type we are describing is a steady state in which $m_1 = W$ and $q_1 \geq 1-\alpha$. The reason for this is that our simplifying assumptions (no first-period consumption) fix household saving. We cannot get nonstationary equilibria via changes in the amount of saving along a nonstationary path. We can get them only through currency substitution along an equilibrium path.

Properties of Limiting Steady States

Suppose $q_1 = 1-\alpha$, so we are in the steady state immediately, but that we are not at a corner. Then we must have

$$1-\alpha = \frac{W-\alpha m_1}{\alpha f_0 + W} \Leftrightarrow m_1 = W-(1-\alpha)f_0 \text{ and } f_1 = f_0. \tag{62}$$

Of course, $m^* = m_1$ and $f^* = f_0$ as can be seen from $m^* = \dfrac{1-\alpha}{q_1} m_1 = m_1$.

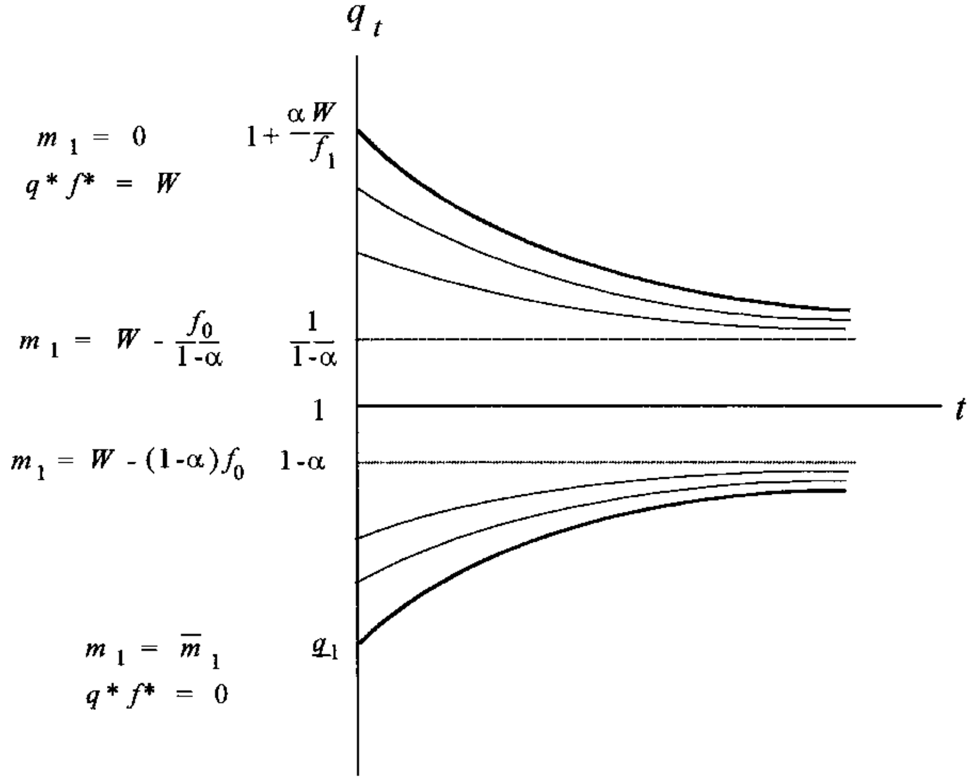
Notice that we cannot be in a steady state immediately if $f_0 > W/(1-\alpha)$, since this would produce $m_1 < 0$. But we have assumed $f_0 \leq W/(1-\alpha)$.

If, then q_1 is lower and m^* is higher; m^* is an increasing function of m_1, which is to say a decreasing function of q_1. Figure 6 juxtaposes in-smuggling and out-smuggling equilibria for ease of comparison. For a given value of f_0, the lowest possible value of q_1 is the value associated with $\bar{m}_1$, the our upper bound of m_1. The associated value of q_1 is

$$\underline{q}_1(f_0) = \frac{W-\alpha\left[\dfrac{W^2}{W+\alpha(1-\alpha)f_0}\right]}{\alpha f_0 + W} = \frac{(1-\alpha)W}{W+\alpha(1-\alpha)f_0}. \tag{63}$$

Notice that if $f_0 = 0$ then $\underline{q}_1 = 1-\alpha$ and $\bar{m}_1 = W$, and the only equilibrium is the steady state. For positive values of f_0, $\underline{q}_1$ and $\bar{m}_1$ will both be smaller. The value of m^* associated with $\bar{m}_1$ is W regardless of the value of f_0, since this is how we derived $\bar{m}_1$. Thus, the smallest possible value of q_1 produces a steady state with only domestic currency.

Figure 6. Equilibria With In-Smuggling and Out-Smuggling of Hard Currencies



Welfare Comparisons

For young agents, lower initial values of q_1 produce higher values of q_{t+1}/q_t at each date t. Note that

$$\frac{q_{t+1}}{q_t} = \frac{\frac{q_t}{\alpha + q_t}}{q_t} = \frac{1}{\alpha + q_t}, \tag{64}$$

which increases as q_t decreases. Thus, the young households do better in equilibria with high values of m_1. For the initial old, we have

$$c_0^1 = W + n^0 \text{ with probability } 1-\alpha \tag{65}$$

$$c_0^2 = W \text{ with probability } \alpha \tag{66}$$

in equilibrium, since $m_1 + q_1 f_1 = W$. So the welfare of the initial old depends on the value of n_0. We have

$$n_0 = \frac{[(1-\alpha)-q_1](m_1+q_1f_0)}{q_1(1-q_1)} \tag{67}$$

and

$$q_1 = \frac{W-\alpha m_1}{\alpha f_0+W}, \tag{68}$$

which allows us to determine n_0 as a function of f_0 and m_1. We can then determine whether the welfare of the initial old increases or decreases with the value of m_1. This turns out to be harder than it looks. Clearly, however, the welfare of the initial old is minimized when $n_0 = 0$, which occurs at the steady state with $q_1 = 1-\alpha$. So it seems likely that their welfare also increases as m_1 increases, even though the resulting decline in q_1 causes the domestic market value of their foreign currency holdings to decline.

This is an "Adam Smith" result: the best equilibrium for this economy is one in which all the foreign currency is eventually smuggled out. This equilibrium is not as good, for the initial old, as a stationary laissez-faire equilibrium in which the initial old export all their foreign currency and young agents hold only domestic currency. But it is better for everyone else. Stated differently, the restrictions on foreign currency trading operate to transfer welfare away from the initial old towards the current young and members of future generations.

V. SEIGNIORAGE

In this section we analyze equilibria in which the government earns seigniorage. Using computational methods we show that there exist geometrically declining paths for the real money-financed deficit of the form $g_t = \beta g_{t-1}$, with $0 < \beta < \bar{\beta}$ for some upper limit $\bar{\beta} < 1$, and with g_1>0 given but "not too large" relative to the aggregate endowment W.

Equilibria With In-Smuggling

We take up the case of in-smuggling equilibria first. The amount of seigniorage at date 1 is simply the difference between m_1, the demand for real balances by the date 1 young, and m_0, which is the value of the currency the initial old are endowed with. The maximum amount of seigniorage the government can earn at date 1 is by hyperinflating the currency—that is setting M_1 arbitrarily large so that $m_0 = 0$ and $g_1 = m_1$. Starting with an initial foreign currency price $q_1 > 1/(1-\alpha)$, then we have

$$s_1 = \frac{q_1}{q_1-1}\alpha W$$

and $m_1 = s_1 - q_1 f_1$, where we are also taking f_1 as given. The maximum value of q_1 consistent with equilibrium is

$$\bar{q}_1 = 1 + \frac{\alpha W}{f_1},$$

which produces $m_1 = 0$. If $1-\alpha \le q_1 \le 1/(1-\alpha)$, then we have $s_1 = W$ and $m_1 = W - q_1 f_1$.

Transition Dynamics With Seigniorage

There exist three distinct types of equilibria with seigniorage.

- Paths along which $q_t \ge 1/(1-\alpha)$ for all $t \ge 1$. These are not fundamentally different from the no-seigniorage paths we have already studied. They converge to the steady state $q^* = 1/(1-\alpha)$.

- Paths with $q_t > 1/(1-\alpha)$ for $1 \le t < T$ and $1-\alpha < q_t \le 1/(1-\alpha)$ for $t \ge T$. In this case, the difference equation describing q_t changes at date *T*, as described below. These paths converge to some $1-\alpha \le q^{**} < 1/(1-\alpha)$.

- Paths with $1-\alpha < q_t \le 1/(1-\alpha)$ for all $t \ge 1$. Along these paths there is never any smuggling of hard currency, and is described by a third difference equation. These paths also converge to some $1-\alpha \le q^{***} < 1/(1-\alpha)$.

The following considerations may be useful in motivating the existence of equilibria in which the government earns seigniorage involving $1-a < q_t \le 1/(1-\alpha)$. Suppose that along an equilibrium path q drops below $1/(1-\alpha)$ at some date $T \ge 1$. It follows that $n_T = 0$ and $f_{T+1} = f_T$. If q continues to decline after date *T*, but does not fall below $1-\alpha$, then we have $n_t = 0$ and $f_{t+1} = f_t = f_T$ for all $t \ge T$. The government earns seigniorage along such a path by printing more domestic currency and driving q down. As q is driven down, the real value of the fixed stock of foreign currency falls, creating room for additional real balances of domestic currency. In cases like this, as is shown below, two distinct transition equations describe the evolution of q, one for $t < T$ and another for $t \ge T$.

We begin with the equilibrium law of motion for m_t: $m_{t+1} = \frac{q_{t+1}}{q_t} m_t + g_{t+1}$. Since $m_t = s_t - q_t f_t$, this becomes $s_{t+1} - q_{t+1} f_{t+1} = \frac{q_{t+1}}{q_t}(s_t - q_t f_t) + g_{t+1}$. Substituting into the law of motion for f_t, $f_{t+1} = f_t + (1-\alpha) n_t$, we have

$$s_{t+1} - q_{t+1}\left[f_t + (1-\alpha) n_t\right] = \frac{q_{t+1}}{q_t}(s_t - q_t f_t) + g_{t+1},$$

which simplifies to

$$s_{t+1} - q_{t+1}\left[(1-\alpha)n_t\right] = \frac{q_{t+1}}{q_t}s_t + g_{t+1} .$$

But $n_t = W - s_t$, implying that $s_{t+1} - q_{t+1}\left[(1-\alpha)(W - s_t)\right] = \frac{q_{t+1}}{q_t}s_t + g_{t+1}$, or

$$s_{t+1} - q_{t+1}(1-\alpha)W = q_{t+1}\left[\frac{1}{q_t} - (1-\alpha)\right]s_t + g_{t+1} .$$

If the solutions for n_t and n_{t+1} are interior—that is, if $q_t > 1/(1-\alpha)$ and $q_{t+1} > 1/(1-\alpha)$—the solution is the following quadratic difference equation:

$$\frac{q_{t+1} - 1}{\left(1 - \frac{g_{t+1}}{\alpha W}\right) + \frac{g_{t+1}}{q_{t+1}\alpha W}} = \frac{\alpha(q_t - 1)}{(1-\alpha^2)(q_t - 1) + \alpha^2} .$$

The economically relevant solution for q_{t+1} will be denoted as $\psi_1(q_t, g_{t+1})$. This solution makes sense only if $q_t > 1/(1-\alpha)$ and if $\psi_1(q_t, g_{t+1}) \geq 1/(1-\alpha)$. A substantive difference between $\psi_1(q_t, g_{t+1})$ and the analogous difference equation for the non-seigniorage case is that is quite possible to have $\psi_1(q_t, g_{t+1}) < 1/(1-\alpha)$ even though $q_t > 1/(1-\alpha)$. Suppose $\psi_1(q_t, g_{t+1}) < 1/(1-\alpha)$. In this case we need to look for a corner solution involving $q_{t+1} < 1/(1-\alpha)$ and $n_{t+1} = 0$ (no smuggling next period). If $n_{t+1} = 0$, then $s_{t+1} = W$, and thus $m_{t+1} = W - f_{t+1}q_{t+1}$. The relevant difference equation is

$$q_{t+1} \equiv \psi_2(q_t, g_{t+1}) = \frac{1 - \frac{g_{t+1}}{W}}{\frac{\alpha^2}{q_t - 1} + (1 - 2\alpha)} .$$

This solution is only valid if $q_t > 1/(1-\alpha)$ and if $\psi_1(q_t) < 1/(1-\alpha)$.

Transition Paths After Currency In-Smuggling Ceases

If $1-\alpha < q_t < 1/(1-\alpha)$ then there is a third difference equation, derived under the assumption that $1-\alpha \leq q_{t+1} < 1/(1-\alpha)$, so that $n_t = n_{t+1} = 0$. In this case we have $m_t = W - q_t f_t$, $m_{t+1} = W - q_{t+1}f_{t+1}$ and $f_{t+1} = f_t$, so that $m_{t+1} = W - q_{t+1}f_t$. The equation

$$m_{t+1} = \frac{q_{t+1}}{q_t}m_t + g_{t+1}$$

becomes

$$W - q_{t+1} f_t = \frac{q_{t+1}}{q_t}(W - q_t f_t) + g_{t+1} ,$$

or

$$W = \frac{q_{t+1}}{q_t} W + g_{t+1} \Leftrightarrow 1 = \frac{q_{t+1}}{q_t} + \frac{g_{t+1}}{W} ,$$

producing

$$q_{t+1} \equiv \psi_3(q_t, g_{t+1}) = q_t \left(1 - \frac{g_{t+1}}{W}\right) .$$

This solution is only valid if $1-\alpha < q_t \leq 1/(1-\alpha)$ and if $1-\alpha \leq \psi_3(q_t, g_{t+1}) < 1/(1-\alpha)$. The *Mathematica* notebook *gdifin.nb* (available from the authors on request) computes each of the three types of seigniorage paths described in the previous subsection. In all cases, it is assumed that $g_1 = m_1$ (hyperinflation at date 1) and $g_{t+1} = \beta g_t$, $\beta \in [0,1)$, although it is possible to assume that the initial old get something for their money, $m_0 > 0$, and require $m_1 > m_0$, with $g_1 = m_1 - m_0$. A number of experiments were conducted with the specification $W = 1$, $\alpha = 0.35$, and $f_1 = 0.1$. Initial values of $q_1 = \gamma \underline{q}_1 + (1-\gamma)\overline{q}_1$ were selected in the interval $[\underline{q}_1, \overline{q}_1]$ by varying $\gamma \in (0,1)$, where $\underline{q}_1 = (1-\alpha)^{-1}$ and $\overline{q}_1 = 1 + \alpha W / f_1$. In addition, values of q_1 between $1/(1-\alpha)$ and $(1-\alpha)$ were tried, corresponding to $\gamma > 1$. In each case, the search was for the maximum feasible value of β, which is the value that produces a path converging to $q^* = 1-\alpha$. This value maximizes total seigniorage revenue, given q_1. The results indicate that

- Higher values of q_1 entail a tradeoff: one the one hand, they produce lower values of m_1 (and thus, g_1) and they eventually generate less total seigniorage in present value terms. On the other hand, higher q_1-values permit higher values of β, spreading seigniorage over a longer period of time.

- Maximizing the present value of seigniorage revenue across all possible values of q_1 entails setting $q_1 = \underline{q}_1$, corresponding to $\gamma = 1$. Total seigniorage is maximized when real domestic currency balances are at their maximum allowable value $m_1 = W - (1-\alpha)^{-1} F_1$.

- From the point of view of maximizing total seigniorage revenue taking the value of q_1 as given, the government always wants to choose a path that converges to $q^* = 1-\alpha$.

Table 2 contains the results of the calculations which derive the values of β_{max} and $\left(\sum g\right)_{max}$ associated with each choice of γ and q_1. The time series for *q, f, n,* and *m* associated with any values of γ and β, are available from the authors upon request.

Table 2. Geometric Seigniorage Paths

γ	β	G
0.5	0.82	1.23
0.75	0.72	1.36
0.9	0.59	1.43
0.95	0.54	1.5
1	0.46	1.57
1.05	0.43	1.51
1.1	0.39	1.44
1.25	0.17	1.11

Source: staff calculations

Equilibria with Hard Currency Out-Smuggling

Next, we take up the case of out-smuggling equilibria. We have

$$m_{t+1} = \frac{q_{t+1}}{q_t} m_t + g_t, \tag{69}$$

where $m_t = s_t - q_t f_{2,t+1}$. Thus,

$$s_{t+1} - q_{t+1} f_{2,t+2} = \frac{q_{t+1}}{q_t}(s_t - q_t f_{2,t+1}) + g_t. \tag{70}$$

And since $f_{2,t+j} = f_{1,t+j} - n_{2,t+j}$, we have

$$s_{t+1} - q_{t+1}[f_{1,t+1} - n_{2,t+2}] = \frac{q_{t+1}}{q_t}(s_t - q_t[f_{1,t} - n_{2,t+1}]) + g_t. \tag{71}$$

But the law of motion for f_1 says that $f_{1,t+1} = f_{1t} - n_{2,t+1}$. So we have

$$s_{t+1} - q_{t+1}[f_{1t} - n_{2,t+1} - n_{2,t+2}] = \frac{q_{t+1}}{q_t}(s_t - q_t[f_{1,t} - n_{2,t+1}]) + g_t, \tag{72}$$

which is

$$s_{t+1} + q_{t+1} n_{2,t+2} = \frac{q_{t+1}}{q_t} s_t + g_t. \tag{73}$$

This is

$$\frac{\alpha W}{1-q_{t+2}}+q_{t+1}\left(\frac{1-\alpha-q_{t+2}}{q_{t+1}(1-q_{t+2})}W\right)=\frac{q_{t+1}}{q_t}\frac{\alpha}{1-q_{t+1}}W+g_t, \tag{74}$$

which ultimately reduces to the following difference equation in q_t.

$$q_{t+1}=\frac{q_t\left(1-\frac{g_t}{W}\right)}{\alpha+q_t\left(1-\frac{g_t}{W}\right)}. \tag{75}$$

Suppose we define $x_t=1-\frac{g_t}{W}$. Recursion on the difference equation (75) ultimately produces

$$q_n=\frac{q_1\prod_{i=1}^{n-1}x_i}{\alpha^{n-1}+q_1\sum_{i=1}^{n-1}\alpha^{n-1-i}\left(\prod_{j=1}^{i}x_j\right)}. \tag{76}$$

In addition, backward recursion on $m_{t+1}=r_t m_t+g_t$ ultimately gives us

$$m_t=m_1\prod_{i=1}^{t-1}r_i+\sum_{i=1}^{t-2}g_i\left(\prod_{j=i+1}^{t-1}r_j\right)+g_{t-1}. \tag{77}$$

Now

$$\prod_{s=k}^{t-1}r_s=\frac{q_t}{q_k},$$

and if we assume $g_t=\beta g_{t-1}$ for some $\beta\in[0,1)$ then we have

$$m_t=q_t\left[\frac{m_1}{q_1}+g_1\sum_{i=1}^{t-2}\frac{\beta^{i-1}}{q_{i+1}}\right]+\beta^{t-2}g_1. \tag{78}$$

For the purposes of the above analysis, we must interpret m_1 as real balances held by the initial young members of generation 1. When the government earns seigniorage at date 1, real balances of the initial old, which we will henceforth call m_0, is given by $m_0=m_1-g_1$. We have

$$n_0=\frac{[(1-\alpha)-q_1](m_0+q_1f_0)}{q_1(1-q_1)} \tag{79}$$

and

$$f_1=f_0-n_0=\frac{q_1(1-q_1)f_0-[(1-\alpha)-q_1](m_0+q_1f_0)}{q_1(1-q_1)}=\frac{\alpha q_1f_0-[(1-\alpha)-q_1]m_0}{q_1(1-q_1)}, \tag{80}$$

which is the foreign currency supply of the initial old. On the demand side, foreign currency demand by the initial young is $f_1=\frac{W-m_1}{q_1}$. In equilibrium, therefore,

$$W - m_1 = \frac{\alpha q_1 f_0 - [(1-\alpha) - q_1](m_1 - g_1)}{1 - q_1}. \tag{81}$$

The equilibrium price of foreign currency at date 1 is

$$q_1 = \frac{(W - g_1) - \alpha m_0}{\alpha f_0 + (W - g_1)}. \tag{82}$$

For this value to work out, $q_1 > 0$ and $f_1 \geq 0$, which are both true as long as

$$m_1 = m_0 + g_1 \leq W.$$

The condition $m_1 \leq W$ also implies

$$q_1 > \frac{(1-\alpha)W}{\alpha f_0 + (W - g_1)}, \tag{83}$$

which places a lower bound on q_1. This lower bound is approached as $m_1 \to W$. And since $q_1 < 1 - \alpha$, it is also required that

$$\frac{W - g_1 - \alpha m_0}{\alpha f_0 + (W - g_1)} < 1 - \alpha \Leftrightarrow m_0 > W - g_1 - (1-\alpha) f_0. \tag{84}$$

$f_{21} \geq 0$ is also needed, where $f_{21} = \frac{s_1 - m_1}{q_1}$. So for $f_{21} \geq 0$ we need $s_1 \geq m_1$. Now

$$s_1 = \frac{\alpha W}{1 - q_2} = W(\alpha + q_1 x_1),$$

where use was made of the difference equation for q.

VI. Dollarization in a Growing Economy

This Section examines briefly how economic growth affects the dynamics of dollarization and the possibilities for earning non-inflationary seigniorage. Consider a small, open and growing economy where Γ_t, the number of agents belonging to generation $t \geq 1$, grows according to $\Gamma_{t+1} = (1+\gamma)\Gamma_t$ ($\gamma > 0$ is the rate of population growth). The economy continues to be small in international markets, where the price of its consumption good in dollar terms is normalized to unity. For now, the nominal stock of domestic currency will be assumed constant, $M_t = M_0$, $t \geq 1$.

The main element of the earlier analysis carries over to the growing economy case: there exists a continuum of partially or fully dollarized equilibria indexed by the initial level of confidence in the domestic currency. However, a balanced growth equilibrium with a growing population features continued currency in-smuggling and a growing aggregate stock of domestically held hard currency balances. With a constant nominal stock of domestic currency, a balanced growth equilibrium path involves vanishing *per worker* real holding of domestic currency—the economy is totally dollarized asymptotically. In terms of comparative steady state results, higher population growth is reflected in a higher price for domestically circulating foreign currency, higher per worker smuggling, and lower per worker holdings of foreign currency.

The individual choice problem is as described in Section II. The conditions for equilibrium are given by the system of equations (85)-(88), which are analogous to (32)-(35).

$$S_t \equiv \frac{M_t}{p_t} + q_t F_t = \Gamma_t W \frac{\alpha q_t}{q_t - 1} \tag{85}$$

$$\Gamma_t n_t \equiv \Gamma_t W - S_t = \Gamma_t W \frac{(1-\alpha) q_t - 1}{q_t - 1} \tag{86}$$

$$\frac{M_t}{p_t} = \frac{M_{t+1}}{p_{t+1}} \frac{p_{t+1}}{p_t} \tag{87}$$

$$F_{t+1} = F_t + \Gamma_t (1-\alpha) n_t . \tag{88}$$

Dividing through by Γ_t leads to the following system, expressed in per worker terms.

$$s_t \equiv m_t + q_t f_t = W\alpha \frac{q_t}{q_t - 1} \tag{89}$$

$$n_t \equiv W - s_t = W \frac{(1-\alpha) q_t - 1}{q_t - 1} \tag{90}$$

$$(1+\gamma) m_{t+1} = r_t m_t \left[= \frac{q_{t+1}}{q_t} m_t \right] \tag{91}$$

$$(1+\gamma) f_{t+1} = f_t + (1-\alpha) n_t . \tag{92}$$

The system (89)-(92) can be reduced to a single difference equation in $q_t - 1$ by employing the procedure used earlier to analyze equations (32)-(35). The final expression is

$$q_{t+1} - 1 = \frac{(1+\gamma)(q_t - 1)}{1 + \frac{1-\alpha}{\alpha}\left[(1-\alpha)(q_t - 1) - \alpha\right]} . \tag{93}$$

When γ =0 equation (93) collapses to equation (36), the corresponding expression for the no-growth case. Higher values of γ rotate the phase diagram around the origin in a counter-clockwise direction (Figure 3). As in the no-growth case, there exist two stationary states, given here by

$$q - 1 = 0, \qquad q^* - 1 = \frac{\alpha}{1-\alpha} \frac{1+\gamma-\alpha}{1-\alpha} > 0. \tag{94}$$

The steady state of a growing dollarized economy is characterized by a constant price of foreign currency, a constant domestic price level, constant per worker rate of inflow of hard currency, and constant per worker domestic and foreign currency balances:

$$n = W\frac{\gamma(1-\alpha)}{1+\gamma-\alpha} > 0; \qquad f = W\frac{(1-\alpha)^2}{1+\gamma-\alpha}; \qquad m = 0. \tag{95}$$

A continuum of equilibria exist, each indexed by $q_1 \in [q^*, \bar{q}]$ and the law of motion (93), where $\bar{q} \equiv 1 + \frac{\alpha W}{F_1/\Gamma_1}$, the maximum feasible value of q_1, produces full dollarization at date 1 while if $q_1 = q^*$ the economy reaches its steady state immediately and the stock of per worker foreign currency does not rise over time.[14]

Each equilibrium in the continuum converges to the positive stationary state given in (94)-(95). In steady state, q and p are constant and positive, while real per worker home-currency balances are zero, as can be seen from equation (91). In a constant-q steady state, the constant over time aggregated stock of domestic real currency becomes arbitrarily small relative to the nominal and real stocks of foreign currency, which is growing at rate $\gamma > 0$. Stated differently, the economy becomes fully dollarized in the limit as $t \to \infty$.

As alluded to already, whereas imports of foreign currency cease in the zero population growth steady state, foreign currency imports per worker are positive and constant in the steady state of the growing economy. A growing population drives up aggregate demand for hard currency in the dollarized economy, raising its equilibrium price and inducing more hard currency in-smuggling. As might be expected, the steady state value of n is increasing in γ: the intensity of foreign currency imports per worker increases in the face of faster population growth, so as to keep next period's per worker level of foreign real balances constant. On the other hand, f is decreasing in γ: a permanently higher population growth rate is partly reflected in a higher steady state value of q, which lowers demand for domestically available foreign currency.

Non-inflationary Seigniorage

In the steady state with constant domestic currency stock analyzed in the previous subsection the economy becomes fully dollarized asymptotically, and all the seigniorage derived from home-residents accrues to the issuer(s) of foreign currency(ies). In this section we show that there exist steady state equilibria in the growing-population economy in which the home-government earns positive non-inflationary seigniorage revenue. Depending on the initial

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[14] Alternatively, the equilibrium could be expressed in terms of the $\{p_t\}_{t=1}^{\infty}$ sequence: given $q_1 \in [q^*, \bar{q}]$, p_1 must satisfy equation (85) at $t = 1$, with initial conditions $M_1 = M_0$, $F_1 = F_0$, and Γ_1. The equilibrium $\{p_t\}_{t=1}^{\infty}$ sequence can then be generated from the arbitrage condition $r_t \equiv \frac{p_{t+1}}{p_t} = \frac{q_{t+1}}{q_t}$.

confidence in the home-currency, part or all the total seigniorage revenue generated by home-country residents holdings of currencies can be captured by the issuer of the domestic currency. In particular, there exists a steady state equilibrium in which the economy becomes fully de-dollarized in the steady state.

Partially Dollarized Equilibria

The government's budget constraint in per worker terms is

$$g_t \equiv \frac{G_t}{\Gamma_t} = \frac{M_t - M_{t-1}}{p_t \Gamma_t} = m_t - \frac{r_{t-1} m_{t-1}}{1+\gamma}. \tag{96}$$

Equation (96) replaces equation (91) of the previous section. The other equations in the system (89)-(92) remain unchanged. The difference equation in the premium is now

$$\frac{q_{t+1} - 1}{1 - \frac{g_t}{\alpha W} + \frac{\frac{g_t}{\alpha W}}{1 + (q_{t+1} - 1)}} = \frac{(1+\gamma)(q_t - 1)}{1 + \frac{1-\alpha}{\alpha}\left[(1-\alpha)(q_t - 1) - \alpha\right]}. \tag{97}$$

Equation (97), which collapses to equation (93) when the seigniorage sequence is zero, is a first order quadratic difference equation in $q_{t+1} - 1$. As in earlier sections, there is an infinite number of solutions $\{q_t\}_{t=1}^{\infty}$, each indexed by $q_1 \in [(1-\alpha)^{-1}, \bar{q}_1]$, where the lower limit corresponds to the no in-smuggling equilibrium at date 1, while $\bar{q}_1$ is the value of q that drives demand for domestic real balances to zero at date 1 (Table 3). The q-sequences converge to *partially dollarized* steady states in which $q_t = q_{t+1} = q^* > (1-\alpha)^{-1}$, where q^* solves the following quadratic equation in q:

$$(1-\alpha)^2 q^2 - \left(1 - \alpha + \alpha\gamma - (1+\gamma)\frac{g}{W}\right) q - (1+\gamma)\frac{g}{W} = 0. \tag{98}$$

In the steady states defined by equation (98), $n^*>0$, $f^*>0$, $m^*>0$ and $m^*+q^*f^*+n^*=W$. The values of (n^*, m^*, f^*) satisfy the steady state versions of equations (89)-(92) and (96):

$$\gamma f^* = (1-\alpha)n^*; \quad m^* + q^* f^* = W\frac{\alpha q^*}{q^*-1}; \quad n^* = W\frac{(1-\alpha)q^* - 1}{q^* - 1}; \quad g = m^*\frac{\gamma}{1+\gamma}. \tag{99}$$

A decline in confidence lowers steady state real domestic balances and seigniorage while raising in-smuggling and per capita holdings of foreign currency (Table 3).

Fully De-Dollarized Equilibria

If $q_1 = (1-\alpha)^{-1}$, there is another equilibrium sequence involving *full de-dollarization.* Starting at date 1, $q_t = q^* = (1-\alpha)^{-1}$ and $p_t = p_1$, $t \geq 1$. In this equilibrium, the price level is constant over time, no in-smuggling of foreign currency ever takes place, $n_t = 0$, and $F_t = F_0$, $t \geq 1$. The sequences for per capita real domestic and foreign currency balances and for seigniorage become

$$f_{t+1} = (1+\gamma)^{-1} f_t; \qquad m_t = W - (1-\alpha)^{-1} f_t; \qquad g_{t+1} = \frac{\gamma}{1+\gamma} W,$$

$t \geq 1$, where $f_1 = F_0 / \Gamma_1$ is a given initial condition. In steady state, n*=*f**=0, *m**=*W*, $g^* = \frac{\gamma}{1+\gamma} W$, so that the economy becomes *fully de-dollarized* in the limit and the home-government earns the maximum possible rate of non-inflationary seigniorage. The constant price level satisfies $g_1 = (M_1 - M_0)/(\Gamma_1 p_1) = W - (1-\alpha)^{-1} f_1 - M_0 /(\Gamma_1 p_1)$, which varies inversely with the size of seigniorage to be extracted at t=1. While the constant over time price level thus calculated is a matter of indifference for all members belonging to generations $t \geq 1$, it is a matter of conflict between the government and the initial old: a higher value of *p* increases seigniorage at date 1 by exactly the amount it lowers aggregate consumption by the initial old.

Table 3. Limiting Distributions of Currency Balances in a Growing Economy

m_1	q_1	q^*	n^*	m^*	q^*f^*	f^*	g/W
0.000	4.500	1.989	0.296	0.355	0.349	0.175	0.186
0.067	4.000	1.927	0.272	0.417	0.311	0.161	0.218
0.140	3.500	1.872	0.249	0.476	0.276	0.147	0.249
0.225	3.000	1.825	0.226	0.531	0.244	0.134	0.278
0.333	2.500	1.785	0.204	0.581	0.216	0.121	0.304
0.500	2.000	1.750	0.184	0.626	0.190	0.109	0.328
0.642	1.750	1.730	0.170	0.655	0.175	0.101	0.343
0.680	1.700	1.724	0.167	0.663	0.170	0.099	0.347
0.723	1.650	1.718	0.162	0.673	0.165	0.096	0.352
0.773	1.600	1.710	0.157	0.684	0.159	0.093	0.358
0.831	1.550	1.700	0.150	0.699	0.151	0.089	0.366
0.846	1.538	1.697	0.148	0.703	0.149	0.088	0.368

Source: staff calculations.

Logarithmic utility; α=0.35; γ=0.025 per year.

VII. CONCLUDING REMARKS

This paper has developed some of the analytical implications of unofficial dollarization using a framework that combines the legal restrictions approach of monetary economics with the crime-theoretic tradition of trade theory and public finance. Hard currency smuggling, currency substitution and progressive dollarization in developing and transition economies were explained by low confidence in domestic currencies—itself a result of recent or chronic macroeconomic instability that compromised domestic currencies' effectiveness as stores of value. This macroeconomic source of demand for U.S. dollars and other hard currencies is independent from their use in (legal or illicit) trade in goods and services. Macro-driven partial or full dollarization emerged as an equilibrium phenomenon under plausible assumptions about "leakages" in the enforcement of legal restrictions against foreign currency accumulation. The dynamics and steady state extent of dollarization depend on various physical and legal fundamentals, including risk aversion, the enforcement of exchange controls, economic growth, and the size of the fiscal deficit. In addition, expectations—the level of confidence in domestic currencies—play a crucial role in the process of dollarization and de-dollarization.

An important task for future research is to extend the crime-theoretic model of domestic capital flight described in this paper in the direction of incorporating *external* capital flight, which is widespread in practice. Capital flight can be analyzed by adopting the Sargent-Wallace (1982) device of dividing each generation of domestic agents into two groups, the rich and the poor, depending on the size of their endowments. The large denominations in which U.S. Treasury securities are issued and the absence of effective financial intermediaries in many developing and transition countries prohibit small savers from purchasing interest-bearing dollar assets. As in the model of this paper, the poor would continue to acquire and hold U.S. currency—a safe asset which is available in affordable low denominations. External capital flight would be conducted by rich savers who would presumably be better able to circumvent legal and denomination restrictions and take advantage of the superior investment opportunities offered by dollar-denominated interest-bearing assets. Lastly, the model could also be used to analyze the welfare implications of official dollarization, which has been sought by some emerging market policymakers in the aftermath of the crises of the 1990s, and which has been debated in the Fund and elsewhere.

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