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### **Ownership of Capital in Monetary Economies and the Inflation Tax on Equity**

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#### **Abstract**

Financial instruments are subject to inflation taxes on the wealth they represent and on the nominal income flows they provide. This paper explicitly introduces financial instruments into the standard stochastic growth model with money and production and shows that the value of the firm in this case is equal to the firm's capital stock divided by inflation. The resulting asset-pricing conditions indicate that the effect of inflation on asset returns differs from the effects found in other papers by the addition of a significant wealth tax.

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## I. INTRODUCTION

The invention of corporations that own their own capital is one of the key events in economic development and a distinguishing feature of developed economies. In the absence of this business structure, the household and the firm are identical, in the sense that households must invest their wealth directly into capital goods. But because capital goods are lumpy and expensive, household-firms can generally invest only in one production technology at a time. Consequently, they are poorly diversified against economic shocks and the scale of productive capacity is limited by the scale of individual household-firm wealth.

The invention of the corporation, however, relieves the household of the need to invest its wealth in physical capital. Corporations take on the role of specialized investors in production technologies. They offer households indirect ownership in capital goods through ownership of the corporation, which is infinitely divisible. Households gain two main benefits from this arrangement. One benefit is diversification of risk across different production technologies, since households can spread their wealth across many firms by holding partial ownership shares in them. The other benefit is increased growth, resulting from the corporation's ability to borrow the resources of many households.

There is also at least one significant cost to society associated with the existence of the corporation: inflation. The existence of the corporation is made possible by, and is virtually identical to, the introduction of two specialized financial instruments—stocks and corporate bonds. These instruments typically are valued in terms of the monetary unit of account and entitle their owners to periodic money payments, or cash flows. Inflation reduces the real value of these instruments, thereby introducing distortions into both households' and firms' decisions. To the extent that there is systematic inflation risk in the economy, the distortionary effects of inflation should be reflected in asset prices and returns. But because the financial markets also assist in the allocation of real resources, it is likely that inflation affects the real economy as well. Understanding the full impact of inflationary distortions in the financial markets, therefore, is an important task not only for asset pricing theory but also for monetary theory and policy.

Households will trade their physical capital for financial instruments if owning a stock or bond performs roughly the same functions for the household as those performed by owning capital goods. We believe that stocks and bonds must perform two related but distinct functions in order to make the holding of these instruments acceptable substitutes for holding physical capital. One function is to serve as a leasing agreement. Households have wealth that they want to invest for a positive return, and firms need to finance the capital they use. Generally, the financing is done through a leasing arrangement in which households lease resources to the firm, and the firm promises a sequence of payments to the households in return. For example, when households own capital goods, they lease them to the firm in return for rental payments of some kind. Financial

instruments also represent a leasing agreement between the firm and the household. In this case, however, the firm is leasing money from the household in return for future cash payments. The written evidence of this lease is the stock certificate or corporate note, and the value of the lease may be calculated from the cash flows that the instrument represents. Thus, one of the purposes of financial instruments is to facilitate lending and borrowing between households and firms.

The second function of stocks and bonds relates to ownership and control of physical capital. Ownership carries the right to control how the capital is used, and whether it will be maintained, increased, or decreased. In other words, ownership of capital gives the owner the right to make the investment and production decisions for the firm. In order for stocks and bonds to be acceptable substitutes to direct ownership of capital, therefore, they must represent ownership claims on the firm's assets. The ownership claims, furthermore, must grant their holders a degree of control over the firm's investment (production) decisions as well as control over the disposition of the firm's capital goods. It is well known that stocks and bonds do exactly this, though the ownership claims are often contingent ones. These ownership and control claims have a value that can be calculated from the value of the firm's capital, which is the output to be had by putting the firm's physical capital to its best use. Distributing ownership and control of the firm's assets across households, then, is the second function of financial instruments.

The distinction between these functions becomes clear through two examples. The first example is the case of bankruptcy, when the firm no longer honors its commitments to deliver cash flows<sup>2</sup>. When this happens, stocks and bonds do not usually lose all of their value. Instead, the value of these instruments then derives solely from the value (and legal priority) of their ownership claims on the assets of the firm. The value of the real assets, in turn, depends on their productivity in uses outside the particular bankrupt firm.

The second example illustrates the opposite extreme: stocks and bonds serve only the financing function. This is the case in the standard asset pricing models based on Brock and Mirman's (1972) stochastic growth model. In these models, households own the capital stock and firms do not own assets. Instead, households lease their physical capital goods to firms in exchange for rental payments. Stocks and bonds (and markets for them) do not actually exist in these models, because the financing of the firm takes place through the rental of physical capital. But the rental income stream earned by households is analogous to the dividends from a stock, so the discounted value of the rental payments is taken as the implied price of the stock. Similarly, bonds are priced as if they are claims to a fixed rental payment, such as one dollar every period in the future. Since the value of a perpetual lease on the capital good is taken as the value of equity in these models, we refer to the value of equity determined solely through its financing function as the pure lease value of the stock.

When a single financial instrument fulfills two or more roles, it is necessary to explicitly add a financial market to the model. The financial market values each function performed by the financial instrument. Then the entire economic system must reconcile these values in order to prevent arbitrage. The same instrument can have only one market price. If a share's pure lease value (the value of the claim on the stream of cash flows) does not equal its ownership value (the value of the ownership claim on the firm's physical capital), there will be arbitrage between the

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<sup>2</sup>This example is for illustrative purposes only. Bankruptcy is not modeled in this paper.

two values of the instrument until one common value is established. In terms of modeling, the possibility for arbitrage between the two distinct functions of financial instruments implies that a no-arbitrage condition must always be satisfied in equilibrium. The no-arbitrage condition will state that the pure lease value of the instrument equals the ownership value. When one side of the condition is disturbed, arbitrage will take place until the no-arbitrage condition is reestablished. Or, if a shock affects both sides of the no-arbitrage equation but in different proportions, arbitrage will also take place.

This paper shows that inflation is just such a shock to the no-arbitrage condition. Inflation creates arbitrage opportunities, we argue, because it has different effects on the two functions of financial instruments. Inflation reduces both the ownership and the pure lease values, but its effect on the ownership value of stocks and bonds is greater than its effect on the instruments' pure lease value. The differential effect of inflation on these two values creates an arbitrage opportunity, which we discuss below.

It is well known that inflation reduces the pure lease value of stocks and bonds by reducing the real value of the cash flows (dividends) that accrue to the holders of the instruments. This is often referred to as the dividend tax effect of inflation. What has not been recognized in the literature, however, is that inflation also reduces the ownership value of stocks and bonds. This occurs because stocks and bonds are nominally-denominated ownership claims. Although stocks and bonds are claims to real assets, the value of the claims themselves is expressed in terms of money<sup>3</sup>. Therefore, when the price level increases, it lowers the real value of these nominally-denominated claims while leaving the real value of the underlying capital goods intact. In this way, inflation serves as a tax on the value of the capital goods that are held indirectly by the households. The difference between the real value of the nominal claims and the real value of the capital goods is the government's revenue from this tax.

Given that the rate of tax on the ownership value of financial instruments is the same as the rate on the pure lease value, the total size of each tax then depends on the size of the tax base. Basically, inflation functions as a property tax on the ownership value of stocks and bonds, and as an income tax on the pure lease value of these instruments. The property tax is levied on the entire property—the ownership value of the stock or bond—during each period, while the income tax is collected only from the current income—the current cash flow from the instrument. If we assume the usual cash flow profiles for stocks and bonds, such as quarterly dividend payments and semiannual bond payments, then during any given period the cash flow from the stock or bond will have a money value that is less than the money value of the entire instrument. Therefore, the total amount of the property tax will exceed the amount of income tax from inflation. Assuming that the no-arbitrage condition was satisfied before the increase in the price level took place, this implies that the after-tax value of the ownership function of the stock or bond will be less than the after-tax value of the leasing function of the instrument. In this way, inflation disturbs the no-arbitrage condition and prevents the stock market from clearing.

For example, suppose that inflation reduces the ownership value of stock below its pure lease value, but the market price of the stock is set at its ownership value. Then a clever household

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<sup>3</sup>The nominal claims must be equal to the nominal value of the capital stock, evaluated during the appropriate period.

could sell claims identical to the cash flows of the firm, for a price equal to the pure lease value of the stock. The household would then use the proceeds of this sale to purchase all of the firm's stock, for which it pays the ownership value of the stock. Since the household now owns and controls the firm, the household can use the cash flows of the firm to pay off the claims against it. This transaction would return a profit to the household equal to the difference between the pure lease value and the ownership value of the stock. Of course, all households would want to perform this arbitrage, and the demand for the stock would exceed supply.

How is this arbitrage prevented? In financial markets, arbitrage usually leads to changes in the market prices of financial instruments. But in this case, changes in the market price of the stock will not restore balance between the lease value of the stock and its ownership value. Recall that the lease value of the stock depends on the cash flows generated by the stock, while the ownership value depends on the capital of the firm. Neither of these quantities is affected by changes in the price of the stock. Therefore, changes in the stock price will not help to equate the two values. Rather, adjustments need to be made inside the firm if the no-arbitrage condition is to be maintained. The firm must either alter the cash flows it pays to stockholders, change the productivity of its assets, or change both until the lease value of the stock equals the ownership value.

Again, an example helps to illustrate. Whenever the pure lease value of the stock is greater than its ownership value, a clever household can borrow by issuing claims that are identical to the financing value of the stock, and use the proceeds to purchase the ownership rights to the firm's assets from the current shareholders<sup>4</sup>. The household can then reorganize the firm's assets in a way that increases their productivity but maintains the current cash flow profile. Once the household does this, it can sell the ownership claims for a higher price, pay off its creditors with the proceeds, and pocket a profit.

What we have just described is a version of the classic takeover strategy executed by corporate raiders<sup>5</sup>. But this strategy could just as well have been executed internally, by the current managers of the corporation. The managers could reallocate the firm's capital in order to restore the balance between the ownership value of the stock and the financing value. Instead of doing this to earn arbitrage profits, however, they may have a simpler motivation: to keep shareholders happy and thus retain their jobs.

The previous two examples are intended only to illustrate how a real adjustment could take place. In this paper, we are presenting an equilibrium model and consequently do not model the actual adjustment. Nonetheless, we will show that equilibrium requires that an adjustment in the firm's assets take place as a result of inflation. And regardless of whether it is accomplished from outside the firm or within the firm, this adjustment of the firm's physical assets implies changes in the market price of the financial instruments issued by the firm as well as changes in the quantities of investment and consumption.

The mechanism that we described above contributes to our understanding of the inflation's effect on financial markets in two ways. First, we identify an additional inflation tax on financial

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<sup>4</sup>The latter transaction described here is similar to the "stripping" of interest and principal payments on a portfolio of mortgages.

<sup>5</sup>This example is meant only to be suggestive. Corporate raiders are not modeled in this paper.

instruments that has not previously been addressed in the literature. This is the inflation tax on the ownership value of financial instruments that is analogous to a property tax and is significantly larger than the income tax on dividends and cash flows. The second contribution is the arbitrage framework that comes from the functional view of financial instruments. According to this framework, shocks will affect asset prices to the extent that they disturb the no-arbitrage condition. Restoring the no-arbitrage condition must be accomplished through adjustments of real quantities. Therefore this framework shows the path through which inflation and other shocks to financial markets affect real economic activity. In other words, this framework identifies a transmission path from financial markets to the goods and labor markets.

In this paper, we develop an equilibrium model of a monetary economy that formalizes the basic ideas described above. We start with the standard stochastic growth model with cash-in-advance money. We restrict the model so that households own financial instruments and corporations own physical capital. The financial market is explicitly present in our model in the form of a no-arbitrage condition.

These changes to the standard model result in one significant change in the theoretical predictions of the model. We find that the real value of the firm is equal to the capital stock of the firm divided by inflation. Other asset pricing models find that the real value of the firm is equal to the capital stock only. We also demonstrate that this result has the Modigliani-Miller property that stocks and bonds have identical effects on the value of the firm. This implies that managers will be indifferent between issuing one or the other.

The increased role for inflation in valuing the firm increases the sensitivity of asset prices and real activity to inflation. We simulate the effect of an inflation shock on real stock returns, consumption growth, and capital accumulation using both our model and the standard asset pricing model. Our model generates much greater responses to inflation than those obtained using the standard model.

Our paper proceeds in the following way. In the first part of Section 2, we develop the model. To keep the presentation simple, we include one financial instrument initially—stocks—and analyze the addition of other financial instruments in a later section. Since our model is very similar to the standard stochastic growth model, we emphasize three key differences between our model and the standard model in our presentation. First, the household's budget constraint is different because the households own stocks rather than physical capital. Second, the firm's problem is more complex because it must allocate its profits between dividends and retained earnings, which are used to finance new investment. Finally, an explicit stock market is present in the model.

After we present the model, we define the competitive equilibrium and write down the equations that are used to solve the model. In addition to the standard Euler equations, resource constraints, and market clearing conditions, a no-arbitrage condition is needed to solve for equilibrium. Although we do not solve for equilibrium analytically, we prove a Theorem stating that equilibrium requires that the value of the firm is related to the capital stock divided by inflation. This theorem implies as a corollary that the real stock return is equal to the marginal return on investment. We use the Theorem to reduce the system

of equations that define equilibrium down to a two-equation system that we then simulate.

In order to compare our theoretical results with those obtained in previous work, we present a standard asset pricing model and point out the key differences in the equilibrium conditions, solution method, and in the equations that are simulated. We show that separating household ownership of financial instruments from firm ownership of physical capital alters the timing and lag structure of the two-equation system that is simulated. This presentation foreshadows, and helps explain, the differences in the simulation results that are presented in Section 3.

In Section 4, we perform two robustness checks on the model. First, we replace the cash-in-advance money of Section 2 with a “shopping time” motivation for holding money, as in Marshall (1992). This change to the model does not alter Theorem 1. As a second check on robustness, we add a bond to the model that acts like commercial paper, so that firms may finance themselves either through equities or bonds. We show again that Theorem 1 still holds in this setup, and moreover that firms will be indifferent between issuing equity and debt. This is the famous Modigliani-Miller result. Section 5 concludes.

## II. MODEL

The model is a combination of Lucas’ (1982) and Lucas and Stokey’s (1987) cash-in-advance (CIA) model with Brock and Mirman’s (1972) stochastic growth model. Preferences take the form of a CRRA utility function, while production is represented by a neoclassical production function with partial depreciation of the capital stock. The timing convention is similar to Sargent (1987) and Cooley and Hansen (1989), rather than to Svensson (1985) or Hodrick, Kocherlakota and Lucas (1991), in that asset decisions are made before goods are purchased. A single representative firm and representative household comprise the private economy.

Since the components of the model are all standard, we will specify the model without much discussion except to point out the effects of separating the ownership of physical capital from ownership of nominal shares of stock. We first describe the assumptions regarding timing, technology, and stochastic processes, then use these assumptions to derive the constraints of the model. Then we write down and solve the consumer’s problem and the firm’s problem.

### A. Timing, Technology, and Stochastic Processes

With regard to timing, we follow the convention established by Sargent (1987), in which there are three sessions to each time period. At the beginning of the period, the gross rate of money growth and the shock to productivity for time period  $t$  are revealed to all. In the first session, the following events take place: the government prints additional money and imposes a lump-sum tax on households; households reallocate their assets and supply labor inelastically to the firms; and firms combine capital and labor to produce output.

In the second session, firms sell output to the households, subject to a cash-in-advance constraint. In the third session, the firms pay wages and dividends to the households. Output is produced according to the production function



$$Y_t = F(\theta_t H_t, K_t), \quad (1)$$

where  $\theta_t$  is the temporary shock to productivity,  $H_t$  is inelastically supplied labor, and  $K_t$  is the quantity of capital.<sup>6</sup>

The production function is assumed to be continuously differentiable, homogenous of degree one, and increasing as well as concave in labor and capital. In addition, the cross derivative between labor and capital is positive,  $F'(\theta_t H_t, 0) = 0$  and the Inada conditions hold.

Next period's real capital stock is a function of the firm's capital stock last period and last period's real investment in new capital, ( $I_t$ ):

$$K_{t+1} = A(K_t, I_t). \quad (2)$$

We assume that the capital accumulation function is continuously differentiable, homogenous of degree one, and increasing as well as concave in both of its arguments<sup>7</sup>,  $A_{12}(K_t, I_t) \geq 0$  and  $A(0, 0) = 0$ .<sup>8</sup>

The stochastic shocks to the model are the growth rate of money and the temporary productivity shock. Following Cooley and Hansen (1995), money growth  $\mu_{t+1} = M_{t+1}/M_t$  and the productivity shock are assumed to follow the stationary Markov processes

$$\begin{aligned} \mu_{t+1} &= \Psi_{10} \mu_t^{\Psi_{11}} \exp[u_{t+1}] \\ \theta_{t+1} &= \theta_t^{\Psi_{22}} \exp[\epsilon_{t+1}]. \end{aligned} \quad (3)$$

We assume that the innovations in money growth,  $u_{t+1}$ , and productivity,  $\epsilon_{t+1}$ , are mean-zero, i.i.d. random variables drawn from bounded sets.<sup>9</sup>

## B. Budget and Balance-Sheet Constraints

In addition to the technology constraints it faces, the firm must allocate its profits between dividends and retained earnings. The retained earnings are used to finance the firm's investment. The firm earns profits at time  $t$  given by

$$PR_t = P_t [F(\theta_t H_t, K_t) - \mathcal{W}_t H_t], \quad (4)$$

where  $P_t$  is the price level,  $\mathcal{W}_t$  is the real wage, and  $H_t$  is the quantity of labor chosen by

<sup>6</sup>In an earlier version, we also included a deterministic trend in production, as is done in King, Plosser, and Rebelo (1988). This does not affect our result, so it is omitted for the sake of clarity.

<sup>7</sup>Throughout the paper, the partial derivative of a general function with respect to its  $i$ th argument is denoted by the subscript  $i$ .

<sup>8</sup>These assumptions for the capital accumulation equation are identical to Restoy and Rockinger's (1994).

<sup>9</sup>See Altuğ and Labadie (1994, pp 144 and 224) for a discussion of this assumption.

the firm. These profits are either paid out to the shareholders as dividends or retained by the firm, so that

$$PR_t = RE_t + D_t S_t, \quad (5)$$

where  $RE$  represents retained earnings,  $D$  is nominal dividends, and  $S = 1$  is the number of shares.

The firm finances investment with retained earnings:

$$P_t I_t = RE_t. \quad (6)$$

Therefore, nominal dividends satisfy

$$D_t = P_t [Y_t - I_t - W_t H_t]. \quad (7)$$

The derivation of equation (7) shows the first important difference between this paper and previous work. The firm's constraint reflects the separation of the firm from the household. Instead of having the quantity of investment dictated to it by the household, the firm chooses the share of profits it will reinvest in itself and the share it will pay out as dividends. As equation (7) shows, the shareholders in this model are residual claimants, since dividends are the revenues left over after wages and investment funds have been paid out.

Money enters the economy when the government collects a real lump sum tax ( $\mathcal{T}_t$ ) on the household and simultaneously distributes additional money ( $M_t - M_{t-1}$ ) to it. Since the change in the money stock is assumed to be an exogenous stochastic process, the tax collections of the government must satisfy the government budget constraint

$$\mathcal{T}_t = -\frac{M_t - M_{t-1}}{P_t}. \quad (8)$$

The consumer faces a separate constraint in each session during a time interval. The consumer begins the period with given nominal wealth ( $W_t$ ), which consists of the following: the stocks of the firm purchased in the previous time period ( $S_{t-1}$ ), which pay nominal dividends ( $D_{t-1}$ ); currency carried over from the previous time period ( $M_{t-1}$ ); and new money balances printed by the government ( $M_t - M_{t-1}$ ). Given this initial wealth, the consumer decides how much money ( $M_t^p$ ) and stock ( $S_t$ ) to purchase, and pays the real lump sum tax ( $\mathcal{T}_t$ ) owed to the government. The consumer's purchases of assets are carried out subject to the constraint

$$\frac{P_t^s S_t}{P_t} + \frac{M_t^p}{P_t} \leq \frac{W_t}{P_t} - \mathcal{T}_t, \quad (9)$$

where  $P_t^s$  is the price of a share of stock at time  $t$ .

In the second session, the consumer purchases output in the amount  $P_t C_t$  subject to the CIA constraint

$$P_t C_t \leq M_t^p. \quad (10)$$

Thus, the total sales of the firm to the consumer are

$$P_t[Y_t - I_t] = P_t C_t \leq M_t^p. \quad (11)$$

We will henceforth concern ourselves with equilibria where the CIA constraint is binding, such that (10) and (11) hold with equality.<sup>10</sup>

In the final session, the firm pays wages and dividends to the consumer, so that the consumer enters the next time period with real wealth

$$\frac{W_{t+1}}{P_{t+1}} = \frac{R_{t+1}^s P_t^s}{P_{t+1}} S_t + \frac{M_t^p - P_t C_t}{P_{t+1}} + \frac{P_t \mathcal{W}_t H_t^s}{P_{t+1}}, \quad (12)$$

where

$$R_{t+1}^s \equiv \frac{P_{t+1}^s + D_t}{P_t^s} \quad (13)$$

is the *ex post* nominal gross return on stock and  $H_t^s$  is the quantity of labor chosen by the household.

Equations (9) and (12) show the second important difference between this paper and previous work. In standard asset pricing models, physical capital is the main store of household wealth. In this model, financial instruments replace physical capital in household wealth, so that household wealth is held exclusively in nominal instruments. Thus there is no longer an investment available to the household that serves as an “inflation hedge”.

### C. The Consumer Optimization Problem

The consumer chooses the amount of money, stocks and consumption in each time period to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\gamma}}{1-\gamma} \quad (14)$$

subject to the constraints (9), (10), and (12).

Labor is supplied inelastically such that  $0 \leq H_t^s \leq 1$ . Here  $\gamma$  is the constant relative risk aversion parameter and  $\beta$  is the consumer’s subjective rate of discount.  $E_t(z)$  is the household’s expectation of  $z$  conditional on its information at time  $t$ .

Although the consumer chooses money, stocks, and goods during the period, we focus only on the stock purchase decision. The optimal demand for stocks is<sup>11</sup>

$$1 = E_t \left[ mrs_{t+1} \frac{R_{t+1}^s}{\Pi_{t+1}} \right], \quad (15)$$

<sup>10</sup>See Sargent (1987, pp. 161), and Cooley and Hansen (1989 p. 736).

<sup>11</sup>Please refer to the mathematical appendix for the proper derivation of equation (15).

where  $mr s_{t+1} \equiv \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$  is the intertemporal marginal rate of substitution, and  $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$  is the inflation rate. The consumer must pay the ownership value of the firm this period in exchange for the ownership value of the firm next period plus the dividend (lease) payment next period. Thus, the consumer decides to purchase stocks until the utility of consumption given up today is just equal to the expected discounted utility next period of the proceeds from the stocks.

Using the definition of nominal stock returns (13) in the optimal condition (15) allows us to solve (15) forward for the consumer's valuation of stock price

$$\frac{P_t^s}{P_t} = E_t \sum_{j=0}^{\infty} \left\{ \left[ \prod_{i=0}^j mr s_{t+i+1} \right] \frac{D_{t+j}}{P_{t+j+1}} \right\}, \quad (16)$$

which is the discounted value of all future dividends. The consumer's intertemporal marginal rate of substitution is the stochastic discount factor used by the investor.

#### D. The Firm Optimization Problem

The Firm is the owner of the capital stock, which it finances with retained earnings. The firm cannot pay nominal dividends until current goods are sold so that dividends produced this period are not available until next period. The firm chooses labor and capital to maximize the discounted value of current and future real dividends. The firm's stochastic discount factor,  $sd_{t+i}, i = 1 \cdot \infty$  is exogenously given to the firm. This stochastic discount factor is determined in equilibrium.

The choice of discount factor for the firm is an important feature of this model. Having separate discount factors for the firm and the household creates the separation between the household and the corporation we described in the introduction. When the firm and the household are identical, then the firm knows the household's discount factor and it is appropriate for the firm and the household to use the same discount factor. But if the household and the firm are distinct entities, then the firm does not necessarily know the household's marginal rate of substitution. Thus the firm should have a discount factor that differs from the household's discount factor. We specify the firm's discount factor as an endogenous variable whose value is determined in equilibrium.

As we indicated in the introduction, we believe that the firm exists for the benefit of the household shareholders. Therefore we can assume that the firm's objective is to maximize the discounted value of all future dividends (lease payments) which is distributed to the shareholders.<sup>12</sup> We refer to this objective as the lease value of the firm. Thus, the firm's present value maximization problem is

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<sup>12</sup>This implies that the firm may try to learn about the household's preferences in order to choose the correct discount rate. For the sake of simplicity, and because this is an equilibrium model, we do not introduce firm learning about household time preferences.

$$V_t^F \equiv \max_{K_{t+1}, H_t} E_t \sum_{j=0}^{\infty} \left\{ \left[ \prod_{i=0}^j sd_{t+i+1} \right] \frac{D_{t+j}}{P_{t+j+1}} \right\}, \quad (17)$$

subject to (1), (2) and (7). Here  $E_t(z)$  is the firm's expectation of  $z$  conditional on current and past  $\theta_t$ , as well as on current and past government policy.

The firm's optimal labor decision satisfies the condition

$$\mathcal{W}_t = \theta_t F_1(\theta_t H_t, K_t), \quad (18)$$

so that the real wage is equal to the marginal product of labor.

The firm's optimal investment decision satisfies the condition<sup>13</sup>

$$E_t \left[ \frac{sd_{t+1}}{\Pi_{t+1}} \right] = E_t \left[ sd_{t+1} \frac{sd_{t+2}}{\Pi_{t+2}} \left( F_2(\theta_{t+1} H_{t+1}, K_{t+1}) + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} \right) A_2(K_t, I_t) \right]. \quad (19)$$

Denote by  $mri_{t+1} \equiv \left( F_2(\theta_{t+1} H_{t+1}, K_{t+1}) + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} \right) A_2(K_t, I_t)$  the firm's marginal return on investment at time  $t + 1$ .

We can interpret the Euler condition for capital in the following way. The firm's investment decision affects the dividends (lease) paid between time  $t + 1$  and  $t + 2$ . At the margin, an increase in investment in period  $t$  reduces the discounted dividend (lease payments) stream at time  $t + 1$  and hence the value of the firm falls. This is shown by the left hand side of equation (19). However, the extra investment increases the capital stock and subsequently output at time  $t + 1$ , which is shown by the  $mri_{t+1}$  term on the right-hand side of equation (19). The increase in output increases the discounted value of dividends (lease payments) at time  $t + 2$ . The increase in the value of the firm from the future increase in dividends is shown by the RHS of (19). Because the dividends are paid after the goods market closes, the extra dividend earned in period  $t + 1$  cannot be spent until period  $t + 2$ .

## E. Competitive Equilibrium

An equilibrium in this economy consists of a set of initial conditions for capital stock,  $K_1 > 0$ , money stock  $M_0$  and consumption,  $C_0 > 0$ ,<sup>14</sup> exogenous stochastic processes for  $(\mu_t, \theta_t)_{t=1}^{\infty}$  which satisfy (3); endogenous stochastic processes for the quantity variables  $(M_t^p, \mathcal{T}_t, C_t, K_{t+1}, S_t, Y_t, D_t, I_t, H_t)_{t=1}^{\infty}$ ; and endogenous stochastic processes for prices  $(P_t, P_t^s, R_{t+1}^s, \mathcal{W}_t)_{t=1}^{\infty}$  such that

1. *Government Solvency*: The government budget constraint (8) is satisfied for all  $t > 0$ .
2. *Household Optimizing Behavior*: Given the stochastic processes for prices and the initial conditions, the stochastic processes for money, stocks and consumption solve the consumer's problem (14) subject to the constraints (9), (10) and (12) for all  $t > 0$ .

<sup>13</sup>See Cochrane (1991) for the derivation.

<sup>14</sup>The last two conditions insure that there is an initial price level,  $P_0$ .

3. *Firm Optimizing Behavior*: The investment and labor decisions of the firm solve the firm's problem (17) subject to the constraints (1),(2) and (7) for all  $t > 0$ .
4. *Market Clearing*: The endogenous stochastic processes for money, labor, consumption, investment, and output satisfy the market clearing conditions for the money market ( $M_t = M_t^p$ ), the labor market ( $H_t = H^s$ ) and the goods market ( $Y_t = C_t + I_t$ ), such that these conditions are binding for all  $t > 0$ .
5. *No Arbitrage Condition*: The real value of firm's equity is equal to the value of the firm ( $\frac{P_t^s}{P_t} S_t = V_t^F$ ) and the number of shares is constant ( $S_t = 1$ ) for all  $t > 0$ .

The extra condition (5) for equilibrium in the economy is the final important difference between this model and previous work. Condition (5) reflects the fact that there is a nontrivial equity market in this model. The household rebalances its wealth portfolio each period, so its demand for the stock must be equal to the supply of equity each period. At the same time, the firm is investing each period and changing the value of the physical assets owned by the shareholders. The equity market reconciles the investment decision of the firm with the financial investment decision of the household, i.e., the lease value and ownership value of the firm. Market clearing in the stock market is equivalent to the absence of arbitrage opportunities between these two values.

## F. Solving for Competitive Equilibrium Under Certainty

When solving stochastic growth models, it is often helpful to solve the model under certainty (perfect foresight) and use this as a benchmark from which to compare the results obtained in the stochastic case. In this particular case we use the LQ procedure of Christiano to simulate the model. Consequently, the simulation of the stochastic version is related to the perfect foresight model because of the certainty equivalence property of LQ problems. Therefore, we will write down the system of equations for the model under certainty and then give the certainty version of the main Theorem of the paper. Then we will show how to use the Theorem to reduce the seven equations of the model to a two-equation system that can be simulated. Finally, we show the relation between the perfect foresight and stochastic models.

The perfect foresight version of our model amounts to a modification of Abel (1985), in which capital is specified as a credit good and consumption as a cash (in advance) good. We modify Abel by separating firm ownership of physical capital from household ownership of stocks, and by introducing labor into the production function.

The five conditions for competitive equilibrium in the model imply the following set of equations that must be solved simultaneously. Equilibrium in the money market and the government budget constraint together imply that

$$P_t C_t = M_t. \tag{20}$$

In the goods market, the linear homogenous capital accumulation equation (2) implies by Euler's Theorem

$$I_t = \frac{K_{t+1}}{A_2(K_t, I_t)} - \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t. \quad (21)$$

Equilibrium in the goods and labor markets and the above equation yield

$$C_t = F(\theta_t H_t, K_t) - \frac{K_{t+1}}{A_2(K_t, I_t)} + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t. \quad (22)$$

The firm's optimal labor decision together with Euler's Theorem applied to (1) implies

$$Y_t - W_t H_t = F_2(\theta_t H_t, K_t) K_t. \quad (23)$$

Consequently, real dividends from (7) are given by

$$\frac{D_t}{P_t} = F_2(\theta_t H_t, K_t) K_t - \frac{K_{t+1}}{A_2(K_t, I_t)} + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t. \quad (24)$$

The consumer's problem is satisfied when

$$P_t^s = \frac{mrs_{t+1}}{\Pi_{t+1}} (P_{t+1}^s + D_t). \quad (25)$$

The firm's optimal capital decision follows the condition

$$\frac{sd_{t+1}}{\Pi_{t+1}} = sd_{t+1} \frac{sd_{t+2}}{\Pi_{t+2}} \left( F_2(\theta_{t+1} H_{t+1}, K_{t+1}) + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} \right) A_2(K_t, I_t). \quad (26)$$

Finally, the equity market clears so that

$$\frac{P_t^s}{P_t} = V_t^F. \quad (27)$$

The above equations (20), (22), (23), (24), (25), (26), and (27) form a system of seven equations. The endogenous variables to be solved by this system are  $K_{t+1}$ ,  $C_t$ ,  $P_t$ ,  $P_t^s$ ,  $W_t$ ,  $D_t$ , and  $sd_t$ . We cannot solve this system analytically, so we will have to simulate the solution. In order to do this, we try to simplify the system as much as possible.

The key result that allows us to simplify the system is to reconcile the ownership decision of the investor (25) with the condition that maximizes the lease value of the firm (26). This reconciliation is complicated since the ownership decision at time  $t$  compares payments between  $t$  and  $t + 1$  while the lease decision at time  $t$  compares payments between  $t + 1$  and  $t + 2$ . Our approach is to identify the two conditions in the equity market that reconcile the ownership decision with the lease decision. These conditions are then substituted into the ownership decision in a way that allows us to collapse the system down to two equations, which we then simulate. This characterization is given by Theorem 1.

**Theorem 1** *The conditions for equilibrium in the stock market are (1) the firm's real stock price at any time  $t$  is  $\frac{P_t^s}{P_t} = \frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})}$ , and (2) the firm's stochastic discount factor at any time  $t$  is equal to the consumer's intertemporal marginal rate of substitution, i.e.,  $sd_t = mrs_t$ .*

**Proof:** Equilibrium in the stock market occurs when the demand for stocks determined by (25) is in agreement with the firm's investment decision (26) so that the equity market clears, satisfying (27). We start with (25) and determine the conditions under which it agrees with (26). Because of the difference in timing of the decisions, increase the time period by 1 in equation (25) so that

$$P_{t+1}^s = \frac{mrs_{t+2}}{\Pi_{t+2}} (P_{t+2}^s + D_{t+1}).$$

Multiplying both sides of the above equation by  $\frac{mrs_{t+1}}{P_{t+1}}$ ,

$$\frac{mrs_{t+1}}{P_{t+1}} P_{t+1}^s = \frac{mrs_{t+1}}{P_{t+1}} \frac{mrs_{t+2}}{\Pi_{t+2}} (P_{t+2}^s + D_{t+1}). \quad (28)$$

Substituting (24) updated one time period into (28) yields

$$mrs_{t+1} \frac{P_{t+1}^s}{P_{t+1}} = mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} \left[ F_2(\theta_{t+1} H_{t+1}, K_{t+1}) K_{t+1} - \frac{K_{t+2}}{A_2(K_{t+1}, I_{t+1})} + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} K_{t+1} \right] + mrs_{t+1} mrs_{t+2} \frac{P_{t+2}^s}{P_{t+2}}. \quad (29)$$

Finally, regroup terms in (29) such that

$$mrs_{t+1} \frac{P_{t+1}^s}{P_{t+1}} = mrs_{t+1} mrs_{t+2} \left[ \frac{P_{t+2}^s}{P_{t+2}} - \frac{K_{t+2}}{\Pi_{t+2} A_2(K_{t+1}, I_{t+1})} \right] + mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} \left[ F_2(\theta_{t+1} H_{t+1}, K_{t+1}) K_{t+1} + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} K_{t+1} \right]. \quad (30)$$

A comparison of the investor's optimal behavior (30) with the optimal investment decision of the firm (26) reveals that the two decisions concur when conditions (1) and (2) of the theorem are satisfied. Substitution of (16) and (17) into (27) demonstrates that the no arbitrage condition is also satisfied.

Condition (1) of the Theorem has two implications that are important for asset pricing. First, the Condition equates the real stock price with the value of the firm's capital stock, which from our earlier discussion is the ownership value of the firm. Condition (1) combined with the consumer's Euler equation implies that in equilibrium, the lease value of the stock must be equal to the ownership value of the stock, which is the no-arbitrage condition discussed in the Introduction. The second implication of Condition (1) is that even under certainty, inflation affects the real value of the firm and the real stock price. Recently, Sharpe (1999) has documented that expected inflation has significant effects on stock returns. Both the model and the empirical evidence suggest that the separation of ownership of stock from ownership of capital exposes holders of stock to inflation risk that they must be compensated for. We will have more to say about this after we discuss the stochastic case of the Theorem.

Next we show how we use Conditions (1) and (2) of Theorem 1 to collapse the system of seven equations down to two conditions that can be simulated. Condition (2) of Theorem 1 implies that if the household Euler equation is satisfied, then the firm's Euler equation (26) is also satisfied. Therefore we start with the household's Euler equation for stock ownership, (25):

$$\frac{P_t^s}{P_t} = mrs_{t+1} \left( \frac{P_{t+1}^s}{P_{t+1}} + \frac{D_t}{P_{t+1}} \right). \quad (31)$$



We use condition (1) from Theorem 1 to substitute  $\frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})}$  for  $\frac{P_t^s}{P_t}$  on both sides of (25), obtaining

$$\frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})} = mrs_{t+1} \left( \frac{K_{t+1}}{\Pi_{t+1} A_2(K_t, I_t)} + \frac{D_t}{P_{t+1}} \right). \quad (32)$$

Using the Theorem to eliminate  $\frac{P_t^s}{P_t}$  implies that we are using equation (27). Next we use the equilibrium equation in the money market, (20), to replace  $\Pi_t$  and  $P_t$  with functions of  $C_t$  and  $M_t$ , which yields

$$\frac{K_t C_t}{\mu_t C_{t-1} A_2(K_{t-1}, I_{t-1})} = mrs_{t+1} \left( \frac{K_{t+1} C_{t+1}}{\mu_{t+1} C_t A_2(K_t, I_t)} + \frac{D_t C_{t+1}}{M_{t+1}} \right). \quad (33)$$

Now we multiply and divide  $D_t$  by  $P_t$  and use equations (24) and (20) to replace  $\frac{D_t}{P_{t+1}}$  with a function of  $K_t$ ,  $C_t$  and  $M_t$ :

$$\frac{K_t C_t}{\mu_t C_{t-1} A_2(K_{t-1}, I_{t-1})} = mrs_{t+1} \frac{C_{t+1}}{\mu_{t+1} C_t} \left( \frac{K_{t+1}}{A_2(K_t, I_t)} + F_2(\theta_t h_t, K_t) K_t - \frac{K_{t+1}}{A_2(K_t, I_t)} + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t \right). \quad (34)$$

Note that by using (24) we are also using equation (23).

The  $\frac{K_{t+1}}{A_2(K_t, I_t)}$  terms inside the parentheses cancel, and we divide through both sides of (34) by  $\frac{K_t}{A_2(K_{t-1}, I_{t-1})}$ , leaving

$$\frac{C_t}{\mu_t C_{t-1}} = mrs_{t+1} \frac{C_{t+1}}{\mu_{t+1} C_t} \left( F_2(\theta_t h_t, K_t) + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} \right) A_2(K_{t-1}, I_{t-1}). \quad (35)$$

Finally, we substitute the definition of  $mrs_{t+1}$  in and collect terms, obtaining

$$\frac{C_t (C_t)^{-\gamma}}{\mu_t C_{t-1}} = \beta \frac{C_{t+1} (C_{t+1})^{-\gamma}}{\mu_{t+1} C_t} \left( F_2(\theta_t H_t, K_t) + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} \right) A_2(K_{t-1}, I_{t-1}). \quad (36)$$

Equation (36) is a function of  $K_t$ ,  $C_t$  and exogenous variables. To obtain this equation, we have used equations (20), (23), (24), (25), (26) and (27). We take the remaining equation in the system, the goods market equilibrium condition (22), as the second equation in the two-equation system that is used to solve for equilibrium:

$$C_t = F(\theta_t H_t, K_t) - \frac{K_{t+1}}{A_2(K_t, I_t)} + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t. \quad (37)$$

Equations (36) and (37) yield a third order difference equation in the capital stock from time  $t-1$  to  $t+2$ . This difference equation is solved for the equilibrium capital stock,  $K_{t+1}$  as a function of the capital stock in the previous two periods.

At this point, it is useful to compare the above result from those obtained from a

standard perfect foresight model<sup>15</sup>. Abel (1985) examines the case of consumption being a cash good and capital being a credit good when the household owns the capital stock rather than the firm. In this case the ownership of the firm is not an issue so that (25) is no longer relevant. Only the lease value of the firm is important so that equilibrium capital stock is based on (26) when the stochastic discount factor is the consumer's *mrs*. Abel shows that this system reduces to (36), updated one time period, and (37).<sup>16</sup> Abel goes on to demonstrate that these conditions form a third order difference equation in the capital stock from time  $t$  to  $t+3$ . This difference in timing arises because the lease value of the firm is determined by a comparison of cash flows between time  $t + 1$  and  $t + 2$ . Abel solves this difference equation for the equilibrium capital stock,  $K_{t+1}$  as a function of the capital stock in the previous period. The difference between the two systems is the result of the separation of ownership of capital and the implied necessity of ensuring that the stock market clears.

### G. Solving for Competitive Equilibrium in the Stochastic Case

Now we return to the solution of the stochastic version of the model. We proceed as above, first laying out the system of equations to be solved, then stating and proving the stochastic version of Theorem 1. Then we discuss how we use the Theorem to simplify the seven equations of the model to the two-equation system that we simulate in Section 3.

Five of the seven equations needed to solve for equilibrium in the stochastic case are identical to the equations used in the certainty case. These are equations (20), (22), (23), (24) and (27), which are derived from constraints, market-clearing conditions and within-period optimization (i.e., the firm's labor demand). The remaining two equations are the Euler equations describing the household's and firm's intertemporal optimization.

The consumer's problem is satisfied when

$$P_t^s = E_t \left\{ \frac{mrs_{t+1}}{\Pi_{t+1}} (P_{t+1}^s + D_t) \right\}. \quad (38)$$

The firm's optimal capital decision follows the condition

$$E_t \left[ \frac{sd_{t+1}}{\Pi_{t+1}} \right] = E_t \left[ sd_{t+1} \frac{sd_{t+2}}{\Pi_{t+2}} \left( F_2(\theta_{t+1}H_{t+1}, K_{t+1}) + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} \right) A_2(K_t, I_t) \right]. \quad (39)$$

The equations (20), (22), (23), (24), (38), (39), and (27) form the system of seven equations for the stochastic case. The endogenous variables to be solved by this system again are  $K_{t+1}$ ,  $C_t$ ,  $P_t$ ,  $P_t^s$ ,  $\mathcal{W}_t$ ,  $D_t$ , and  $sd_t$ . As before, we cannot solve this system analytically, so we collapse the system down to two equations for simulation, through the use of a Theorem. Next we restate Theorem 1 for the stochastic case, and compare it with the perfect foresight version of the Theorem. Then we state a corollary that highlights this result's implications for the behavior of the financial markets.

<sup>15</sup>We develop a standard stochastic growth model and compare its results with ours in Section 2.8.

<sup>16</sup>See Abel's equation (14) and (15).

**Theorem 2** *The conditions for equilibrium in the stock market are (1) the firm's real stock price at any time  $t$  is  $\frac{P_t^s}{P_t} = \frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})} + B_t$ , where  $B_t$  is a rational bubble as defined in Diba and Grossman (1988); and (2) the firm's stochastic discount factor at any time  $t$  is equal to the consumer's intertemporal marginal rate of substitution plus an i.i.d. shock with mean zero and uncorrelated across time,  $e_t$ , i.e.,  $sd_t = mrs_t + e_t$ .*

**Proof:**

Equilibrium in the stock market occurs when the demand for stocks determined by (38) is in agreement with the firm's investment decision (39) so that the equity market clears, satisfying (27). We start with (38) and determine the conditions under which it agrees with (39).

First, increase the time period by 1 in equation (38) so that

$$P_{t+1}^s = E_{t+1} \left\{ \frac{mrs_{t+2}}{\Pi_{t+2}} (P_{t+2}^s + D_{t+1}) \right\}.$$

Multiplying both sides of the above equation by  $\frac{mrs_{t+1}}{P_{t+1}}$ , and using the law of iterated expectations yields

$$E_t \left\{ \frac{mrs_{t+1}}{P_{t+1}} P_{t+1}^s \right\} = E_t \left\{ \frac{mrs_{t+1}}{P_{t+1}} \frac{mrs_{t+2}}{\Pi_{t+2}} (P_{t+2}^s + D_{t+1}) \right\}. \quad (40)$$

Substituting (24) updated one period into (40) yields

$$E_t \left\{ mrs_{t+1} \frac{P_{t+1}^s}{P_{t+1}} \right\} = mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} \left[ E_t \left\{ mrs_{t+1} mrs_{t+2} \frac{P_{t+2}^s}{P_{t+2}} + F_2(\theta_{t+1} H_{t+1}, K_{t+1}) K_{t+1} - \frac{K_{t+2}}{A_2(K_{t+1}, I_{t+1})} + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} K_{t+1} \right\} \right]. \quad (41)$$

Finally, regroup terms in (41) such that

$$E_t \left\{ mrs_{t+1} \frac{P_{t+1}^s}{P_{t+1}} \right\} = E_t \left\{ mrs_{t+1} mrs_{t+2} \left[ \frac{P_{t+2}^s}{P_{t+2}} - \frac{K_{t+2}}{\Pi_{t+2} A_2(K_{t+1}, I_{t+1})} \right] \right\} + E_t \left\{ mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} \left[ F_2(\theta_{t+1} H_{t+1}, K_{t+1}) K_{t+1} + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} K_{t+1} \right] \right\}. \quad (42)$$

A comparison of the investor's optimal behavior (42) with the optimal investment decision of the firm (39) reveals that the two decisions concur when Conditions (1) and (2) of the Theorem are satisfied. Substitution of (16) and (17) into (27) demonstrates that the no arbitrage condition is also satisfied.

Theorem 2 also has an important Corollary, which we present for the stochastic case of the model. Following Restoy and Rockinger (1994), we have

**Corollary 1**  $R_{t+1}^s = \Pi_t mri_t$

**Proof:** The definition of the marginal return on investment implies

$$\Pi_t mri_t = \Pi_t \left[ \left( F_2(\theta_t H_t, K_t) K_t + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t \right) \frac{A_2(K_{t-1}, I_{t-1})}{K_t} \right]. \quad (43)$$

Using the expression for real dividends (24) in (43) yields

$$\Pi_t mri_t = \frac{\Pi_t A_2(K_{t-1}, I_{t-1})}{K_t} \left[ \frac{D_t}{P_t} + \frac{K_{t+1}}{A_2(K_t, I_t)} \right]. \quad (44)$$

Finally, using Condition (1) from Theorem 2, that  $\frac{P_t^s}{P_t} = \frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})}$  yields

$$\Pi_t mri_t = \Pi_{t+1} \frac{\frac{D_t}{P_{t+1}} + \frac{P_{t+1}^s}{P_{t+1}}}{\frac{P_t^s}{P_t}} = R_{t+1}^s. \quad (45)$$

When the production technology displays constant returns to scale, and when the firm has access to complete contingent claims markets, Altuğ and Labadie (1994) and Restoy and Rockinger (1994) prove in a production based asset pricing model that the firm's value function is equal to the capital stock. Moreover, Restoy and Rockinger (1994) prove that the real stock return is equal to the same period's marginal return on investment. Our result is in nominal terms because of the possibility of inflation, while the one period delay is due to the delay in the payment of nominal dividends.

One implication of Theorem 2 that needs to be addressed is that the stock return is known at time  $t$ . This is simply an artifact of the information structure we choose for this model. In particular, it is a direct consequence of including time  $t$ 's dividend payment in the household's time  $t$  information set, which enables the household to infer the productivity and monetary shocks. We choose this specification for clarity and convenience. When we make the more realistic assumption that the period  $t$  dividend is not observed by the household until time  $t + 1$ , this does make the stock return random, but it also makes the stock return depend on a shock to the marginal productivity of labor. This additional term complicates the model without altering Theorem 2 or any of the relationships between inflation and stock returns discussed below. Since our purpose is to highlight the effect of inflation on stock returns, we chose the information structure that yielded the simpler (and hence more transparent) expressions linking stock returns and inflation.

Now we show how we use Conditions (1) and (2) of Theorem 2 to collapse the system of seven equations down to two conditions that can be simulated. This derivation is similar to the certainty case, but is complicated by the presence of a rational bubble. Condition (2) of Theorem 2 implies that if the household Euler equation is satisfied, then the firm's Euler equation (39) is also satisfied. Therefore we start with the household's Euler equation, (38):

$$\frac{P_t^s}{P_t} = E_t \left[ mrs_{t+1} \left( \frac{P_{t+1}^s}{P_{t+1}} + \frac{D_t}{P_{t+1}} \right) \right]. \quad (46)$$

We use condition (1) from Theorem 2 to substitute  $\frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})} + B_t$  for  $\frac{P_t^s}{P_t}$  on both sides of (38), obtaining

$$\frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})} + B_t = E_t \left[ mrs_{t+1} \left( \frac{K_{t+1}}{\Pi_{t+1} A_2(K_t, I_t)} + B_{t+1} + \frac{D_t}{P_{t+1}} \right) \right]. \quad (47)$$

Again, using the Theorem to eliminate  $\frac{P_t^s}{P_t}$  implies that equation (27) is also satisfied.

If we focus only on the bubble terms in (47) we have

$$B_t = E_t [mrs_{t+1} B_{t+1}]. \quad (48)$$

This expectational difference equation is identical to the rational bubble condition in Diba and Grossman (1988). As a result, the rational bubble will have the same properties given in Diba and Grossman: first, the bubble can never be negative; and second, once a bubble ends it cannot be started again. For the rest of the paper we ignore the possibility of a rational bubble by setting the initial bubble to zero.

We now proceed to solve for the fundamental solution to (47). We use the equilibrium equation in the money market, (20), to replace  $\Pi_t$  and  $P_t$  with functions of  $C_t$  and  $M_t$ , which yields

$$\frac{K_t C_t}{\mu_t C_{t-1} A_2(K_{t-1}, I_{t-1})} = E_t \left[ mrs_{t+1} \left( \frac{K_{t+1} C_{t+1}}{\mu_{t+1} C_t A_2(K_t, I_t)} + \frac{D_t C_{t+1}}{M_{t+1}} \right) \right]. \quad (49)$$

Now we multiply and divide  $D_t$  by  $P_t$  and use equations (24) and (20) to replace  $\frac{D_t}{P_{t+1}}$  with a function of  $K_t$ ,  $C_t$  and  $M_t$ :

$$\frac{K_t C_t}{\mu_t C_{t-1} A_2(K_{t-1}, I_{t-1})} = E_t \left[ \left( \frac{K_{t+1}}{A_2(K_t, I_t)} + F_2(\theta_t h_t, K_t) K_t - \frac{K_{t+1}}{A_2(K_t, I_t)} + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t \right) \frac{mrs_{t+1} \frac{C_{t+1}}{\mu_{t+1} C_t}}{A_2(K_{t-1}, I_{t-1})} \right]. \quad (50)$$

Note that by using (24) we are also using equation (23).

The  $\frac{K_{t+1}}{A_2(K_t, I_t)}$  terms inside the parentheses cancel, and we divide through both sides of (50) by  $\frac{K_t}{A_2(K_{t-1}, I_{t-1})}$ , leaving

$$\frac{C_t}{\mu_t C_{t-1}} = E_t \left[ mrs_{t+1} \frac{C_{t+1}}{\mu_{t+1} C_t} \left( F_2(\theta_t h_t, K_t) + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} \right) A_2(K_{t-1}, I_{t-1}) \right]. \quad (51)$$

Finally, we substitute the definition of  $mrs_{t+1}$  in and collect terms, obtaining

$$\frac{C_t (C_t)^{-\gamma}}{\mu_t C_{t-1}} = \beta E_t \left[ \frac{C_{t+1} (C_{t+1})^{-\gamma}}{\mu_{t+1} C_t} \left( F_2(\theta_t H_t, K_t) + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} \right) A_2(K_{t-1}, I_{t-1}) \right]. \quad (52)$$

Equation (52) is a function of  $K_t$ ,  $C_t$  and exogenous variables. To obtain this equation, we have used equations (20), (23), (24), (38), (39) and (27). We take the remaining equation in the system, goods market equilibrium condition (22), as the second equation in the two-equation system that is used to solve for equilibrium:

$$C_t = F(\theta_t H_t, K_t) - \frac{K_{t+1}}{A_2(K_t, I_t)} + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t. \quad (53)$$

Equations (52) and (53) yield a third order difference equation in the capital

stock which is comparable with the perfect foresight model when the firm holds the capital stock, i.e., (36) and (37). In the next section we present simulations of these conditions using the LQ procedure. The solution to these conditions will represent the competitive equilibrium when the household holds equity rather than the capital stock.

The certainty equivalence property of LQ models implies that the simulation of (52) and (53) is identical to (36) and (37) when the future shocks to money,  $u_{t+i}$   $i > 0$ , and productivity,  $\epsilon_{t+i}$   $i > 0$  are replaced by their expected value of zero.

## H. Comparison With Previous Work

To see how the competitive equilibrium implied by our model differs from that implied in traditional models where the household owns the capital stock, we briefly present a representative of this standard model and make key comparisons. The model we present as the standard model is very similar to Carlstrom and Fuerst's (1995) model without portfolio rigidities, as well as to Cooley and Hansen's (1995) model with no credit good. We show that these standard models are comparable to Abel's (1985) perfect foresight model when the household owns the capital stock.

Let us assume the same household preferences, production technology, and capital accumulation function as in the model presented above. There is a cash in advance constraint on all consumption purchases, and households supply labor inelastically. Households own physical capital and lease it to the firms in return for cash payments that are comparable to dividends. Thus, the household makes the investment decision for the firm, so the household and firm are identical. Wages and dividends exhaust the firm's revenues. Stock does not exist, so there is no stock market and ownership decision for the household.

The system implied by this setup has a competitive equilibrium whose conditions can be written as four equations in the four unknowns  $K_t$ ,  $C_t$ ,  $P_t$  and  $\mathcal{W}_t$ . As before, equilibrium in the money market and the government budget constraint imply

$$P_t C_t = M_t. \quad (54)$$

Equilibrium in the goods market, labor market, and Euler's Theorem applied to the capital accumulation equation yield

$$C_t = F(\theta_t H_t, K_t) - \frac{K_{t+1}}{A_2(K_t, I_t)} + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t. \quad (55)$$

The firm's optimal labor decision together with Euler's Theorem applied to the production function imply

$$Y_t - \mathcal{W}_t H_t = F_2(\theta_t H_t, K_t) K_t. \quad (56)$$

The final equation in the system is the household Euler equation. The household chooses investment to maximize the discounted sum of its utility, subject to the budget and wealth accumulation constraints mentioned above. Thus its Euler equation is the analog of the firm's

Euler equation in our model, with the exception that the discount factor is the household's marginal rate of substitution:

$$E_t \left[ \frac{mrs_{t+1}}{\Pi_{t+1}} \right] = E_t \left[ mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} \left( F_2(\theta_{t+1}H_{t+1}, K_{t+1}) + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} \right) A_2(K_t, I_t) \right]. \quad (57)$$

There are two main differences between this system and the one implied by our model. The first difference is that the household is in control of the investment decision, so that the household and the firm are essentially the same. This means that the Euler equation in the standard model is basically the firm's Euler equation from our model, except that the firm knows exactly what the household discount factor is. This allows the firm to use  $mrs_t$  as its discount factor. The second difference is the lack of an explicit stock market. This means that there is no Euler equation describing the demand for stock and there is no market clearing condition for the stock market.

When we reduce the above system down to the two-equation system that will be simulated, we see how the differences discussed above imply further differences between the two models. We proceed with this model as we did above, using the money-market condition (54) to substitute  $P_t$  out of the Euler equation (57). Doing so yields

$$E_t \left[ \frac{C_{t+1}(C_{t+1})^{-\gamma}}{\mu_{t+1}C_t} \right] = \beta E_t \left[ \frac{C_{t+2}(C_{t+2})^{-\gamma}}{\mu_{t+2}C_{t+1}} \left( F_2(\theta_{t+1}H_{t+1}, K_{t+1}) + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} \right) A_2(K_t, I_t) \right]. \quad (58)$$

As before, the second system in the equation comes from the goods-market condition:

$$C_t = F(\theta_t H_t, K_t) - \frac{K_{t+1}}{A_2(K_t, I_t)} + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t. \quad (59)$$

The conditions for equilibrium in the economy (58) and (59) are similar to Abel's perfect foresight model when the household owns the capital stock and capital is a credit good. These conditions form a third order difference equation in the capital stock from time  $t$  to  $t + 3$ . As in the perfect foresight model this third order difference equation has two unstable roots and one stable root so that the equilibrium value of the capital stock is a function of only one lag of the capital stock.

Note that the timing of equation (58) implied by the standard model is very different from that of (52) implied by our model. As in the perfect foresight case, this is a direct consequence of the lack of a stock market. Recall that (52) is derived from the Euler equation for the household's ownership decision, condition (38). This condition and (27), which ties the stock price to the value of the firm (and hence to capital stock divided by inflation in Theorem 2), are not present when the household does not hold equity. In order to ensure stock market clearing, our solution procedure has been set up so that Condition (1) is embedded in (52) and (53). In other words, the stock price implied by the standard model has not been derived from both household optimizing behavior and stock market clearing, while the stock price generated by our model is derived from household optimization subject to stock market clearing.

This difference between the models has at least one significant consequence for asset pricing. Substituting the solutions for the capital stock and consumption from (58) and (59) into (38) yields a stock price, but this stock price does not satisfy Condition (1) of Theorem 2. As is well known, the equilibrium policy function for (58) and (59) is a function of only one lag of the capital stock, so that the stock price based on (38) is dependent on only one lag of the capital stock. On the other hand, the equilibrium stock price based on Condition (1) of Theorem 2 is a function of  $P_t$  and  $P_{t-1}$ , which means that the ownership value of the firm depends on two lags of the capital stock. This difference in the lag structures of the solutions implies differences in the dynamic behavior of the model, which we investigate below.

In the next section we discuss the differences between our model and the standard model in terms of the inflation tax. We also perform simulations to demonstrate the impact that these differences have on the empirical predictions of the model.

### III. THE INFLATION TAX ON WEALTH

To gain an intuitive understanding of the differences between the models, it is useful to think of our result in terms of the inflation taxes that are present in the model. Every monetary model features some distortion that is caused by inflation. Since money plays several roles in the economy, however, several different approaches to modeling the inflation tax on returns are possible.

Marshall (1992), for example, focuses on money's role as an additional asset that is a potential substitute for other securities. In this view, inflation taxes the total return to money, which includes the transactions services provided by money. When the after-tax return to money falls, this reduces the return to other assets, to the extent that they are good substitutes for money.

The approach of Labadie (1989) utilizes the role of money as a unit of account in that the presence of money in an economy drives a wedge between real and nominal values. This wedge creates opportunities for monetary authorities to tax the value of all nominally-denominated incomes and assets. Therefore, in Labadie's model, inflation affects stock returns by taxing dividends as well as the purchasing power of money.

Many papers that document the effects of inflation on stock returns include only this tax, including both anticipated inflation models following Stockman (1981) and stochastic inflation models such as Cooley and Hansen (1989, 1995)<sup>17</sup>.

In our model, we find three separate inflation taxes that affect stock returns. The first two are the purchasing power tax and the dividend tax from the exchange-economy literature.<sup>18</sup> The third inflation tax stems from the household's ownership of stocks.

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<sup>17</sup>Lucas (1982), Lucas and Stokey (1987), Boyle and Young (1988), Labadie (1989), Cooley and Hansen (1989), Boyle (1990), Hodrick, Kocherlakota and Lucas (1991), Giovannini and Labadie (1991), and Boyle and Peterson (1995) also study this effect.

<sup>18</sup>Mackinnon (1987) adds production to the Stockman (1981) model and separates ownership of capital from ownership of stocks. He also shows the presence of a dividend tax in addition to the purchasing power tax in a perfect foresight cash-in-advance model.



Although households own the firms and hence indirectly own the capital goods, the actual asset they hold is a nominally-denominated stock certificate. The real value of this household asset, therefore, may be taxed by a monetary authority. This tax is similar to the dividend tax in Labadie (1989) but is distinct because it is a tax on the entire value of the firm's physical capital stock rather than the flow of dividends from that stock. In other words, the distinction between the taxes is akin to that between income and property taxes.

We now show how the above result increases the effect of inflation on stock returns. One way to see this is to rewrite the Euler equation for the optimal investment decision using the formula for the real stock price in Theorem 2. Substituting the real stock price from Condition (1) of Theorem 2 into the household's Euler equation for stocks (38) yields

$$1 = E_t \left[ mrs_{t+1} \left( \frac{D_t}{P_t \Pi_{t+1}} + \frac{K_{t+1}}{\Pi_{t+1} A_2(K_t, I_t)} \right) \left( \frac{1}{\frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})}} \right) \right]. \quad (60)$$

In (60), the term  $mrs_{t+1}$  is, in Cochrane's (1991) terminology, the stochastic discount factor used to price the asset. The next three terms show two of the three inflation taxes on equity returns described above (the inflation tax on purchasing power is also present, but affects the stock return indirectly).

First, the term  $\frac{D_t}{P_t \Pi_{t+1}}$  represents the inflation tax on dividends, which is present in most asset pricing models with money. In exchange economy models such as Labadie (1989), this is the only direct inflation tax on stock returns. But in our model, the inflation tax impinges on the capital stock as well as dividends, and this tax is captured in the remaining two terms. These terms show the entire capital stock of the firm being taxed separately at two consecutive dates.

The response of asset prices to the presence of the additional inflation tax on wealth is not obvious from the above equation. As discussed in the introduction and in the previous section, the nature of the distortion captured in the model is more complex than a simple reduction in return due to an additional tax. The inflation tax on wealth taxes the capital stock of the firm, while the dividend tax falls on the household's cash flows. Thus, one way to look at the effect of the additional tax is that it upsets the balance between the ownership and leasing values of the stock, as described in the introduction. In other words, inflation disturbs the no-arbitrage condition that must hold in equilibrium. The adjustments necessary to remove arbitrage opportunities and restore equilibrium have been suggested in the introduction, but without an analytical solution to the model, they cannot be shown analytically. Instead, we use simulation evidence to demonstrate that the additional inflation tax present in our model prompts a significantly stronger adjustment to an inflation shock.

Our strategy is to simulate our model and calculate the responses of the variables of the model to a given monetary shock. We then compare these impulse responses to those generated by an identically calibrated growth model in which households own the capital stock. For ease of notation, we distinguish our specification, in which firms own the capital stock and households own stock certificates, as the "firm" model, and the standard growth model with household ownership of the capital stock as the "household" model.

The simulation proceeds as follows. For each model, we choose steady-state parameter values. We solve the capital accumulation equation for each model, using approximation techniques, and generate a time series of capital by choosing the series of shocks to the model. Then we use the equations of the model to generate time series of the other variables.

The parameters of the model were chosen both to be consistent with other work and to keep the models simple. Most of the parameters of the models were taken from Cooley and Hansen<sup>19</sup>(1995), while some were taken directly from or implied by equations in Campbell (1994). Others were chosen in order to obtain a zero-growth steady state. For example, the population growth rate was set to zero, and the subjective rate of time preference was chosen to ensure zero steady-state consumption growth<sup>20</sup>.

The capital accumulation equation for each model was found using an approach based on Christiano's (1990) LQ method. We apply this method to equations (52) and (53) from our model. As a representative of the standard model, we simulate (58) and (59) to represent Cooley and Hansen's (1995) model.<sup>21</sup>

Once the capital accumulation equations were derived, we performed the following simulation exercise. Starting from identical steady states<sup>22</sup>, each model was shocked with a two standard deviation increase in money growth, which serves as an inflation shock. The responses of the capital stock, the value of the firm, consumption, and real stock returns were calculated. The graphs of the impulse-response functions are given in Figures 1-3.

In the figures, the impact of the additional inflation tax appears as increased responses to inflation shocks relative to the responses generated by the standard model of household ownership of capital. Figure 1 graphs the response of the value of the firm to the monetary shock. The Figure shows that in the first period after the monetary shock, the value of the firm falls further when the firms own the capital stock than when the households own the capital. The relative sizes of the drop are 2.3% of value under firm ownership and 1.7% under household ownership. This difference represents the additional effects of the wealth tax discussed above.

Turning to real stock returns in Figure 2, we see a much more dramatic difference

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<sup>19</sup>The parameter values were chosen to be the following:  $\alpha = .40$ ;  $\delta = .012$ ;  $h^s = .31$ ;  $\beta = .991$ ;  $\gamma = 5$ ;  $\ln(\psi_{10}) = .0066$ ;  $\psi_{11} = .491$ ;  $\sigma_u = .0089$ ;  $\ln(\psi_{20}) = .00$ ;  $\psi_{22} = .95$ ;  $\sigma_\epsilon = .007$ . Thomas Cooley provided the data to reproduce these parameters.

<sup>20</sup>In addition, the investment function  $K_{t+1} = A(K_t, I_t)$  was chosen to be the usual  $K_{t+1} = (1 - \delta)K_t + I_t$ .

<sup>21</sup>In Carlstrom and Fuerst (1995) the stochastic case without portfolio rigidities, equations (10) thru (12) can be reduced to (58) and (59) when the capital accumulation equation is given by  $K_{t+1} = (1 - \delta)K_t + I_t$ . Similarly, the equilibrium conditions for Cooley and Hansen's (p.198, 1995) model can be reduced to (58) and (59) when there is no credit good in their model.

<sup>22</sup>It turns out that given the same parameters, both models imply the same steady state. This occurs because it is the uncertainty over the value of household assets that distinguishes the firm model from the household model. In a steady state, there is essentially no uncertainty over the value of household assets. Abel (1985) shows that money is superneutral in this case.

in the reaction to a monetary shock across the two different models. Stock returns exhibit a stronger initial reaction to the monetary shock when firms own the capital stock. From a steady-state return of 1.48% per quarter, stock returns fall by 56 basis points when firms own the capital, while they fall only 1.2 basis points when households own the capital goods. The difference in return volatility following the shock is also pronounced. The mean squared distance between the stock return and its steady state value over the first 20 periods after the shock, which is a measure of the volatility created in the stock return as a result of the monetary shock, is .0017 when the firms own the capital but only .000036 when households own the capital. These differences show that stock returns exhibit far more sensitivity to a money (inflation) shock when firms own the capital stock and households own share certificates.

Another significant difference between the models that is clearly shown in Figure 2 is the amount of negative correlation induced in stock returns. For the first 20 periods after the shock, the first-order autocorrelation of the return is -.79 for firm ownership of capital and -.46 when households own the capital. And when the entire sample is used, the first-order autocorrelations become -.20 and .98, respectively. Not only is the negative autocorrelation in returns far larger when firms own the capital stock, but the negative autocorrelation also persists in the data over a longer horizon. This result appears to be a direct result of the tax on wealth introduced when firms own the capital stock. As mentioned above, the wealth tax reduces the capital gain in one period but increases it the next period. This wealth tax creates the oscillation in the return that is visible in Figure 2.

The monetary shock also has effects on the levels of consumption, which are graphed in Figure 3. Again, ownership of capital by firms is associated with a greater response to the monetary shock than ownership of capital by households. Consumption falls over half a percent in the first period after the monetary shock when firms own the capital stock, while consumption declines by only .012% when households own the capital stock. The larger drop in consumption is at least partially due to the larger drop in the value of the firm depicted in Figure 1. It is also related to the volatility induced in stock returns by the monetary shock, following a permanent income hypothesis argument. The increase in return volatility reduces the household's estimate of its permanent income, which in turn reduces consumption. The fall in consumption raises the level of capital in both models, but only by very small amounts. Capital rises by .01% when firms own the capital, but only by .0002% when households own the capital stock. Nevertheless, the firm ownership model generates a much larger response to the monetary shock, relative to the standard model of household ownership of capital.

Figure 1. Response of Value of the Firm to Monetary Shock

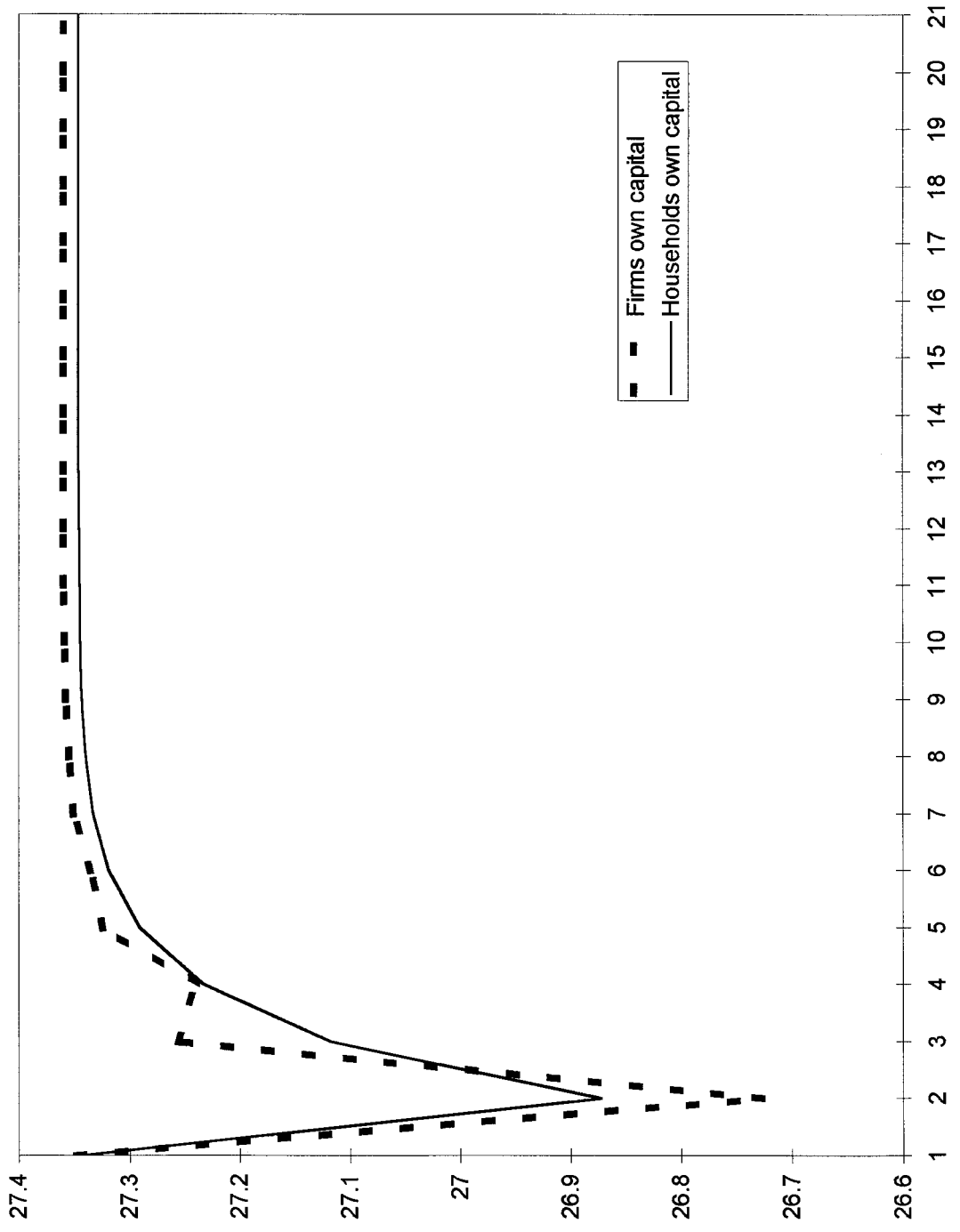


Figure 2. Response of Real Stock Returns to Monetary Shock

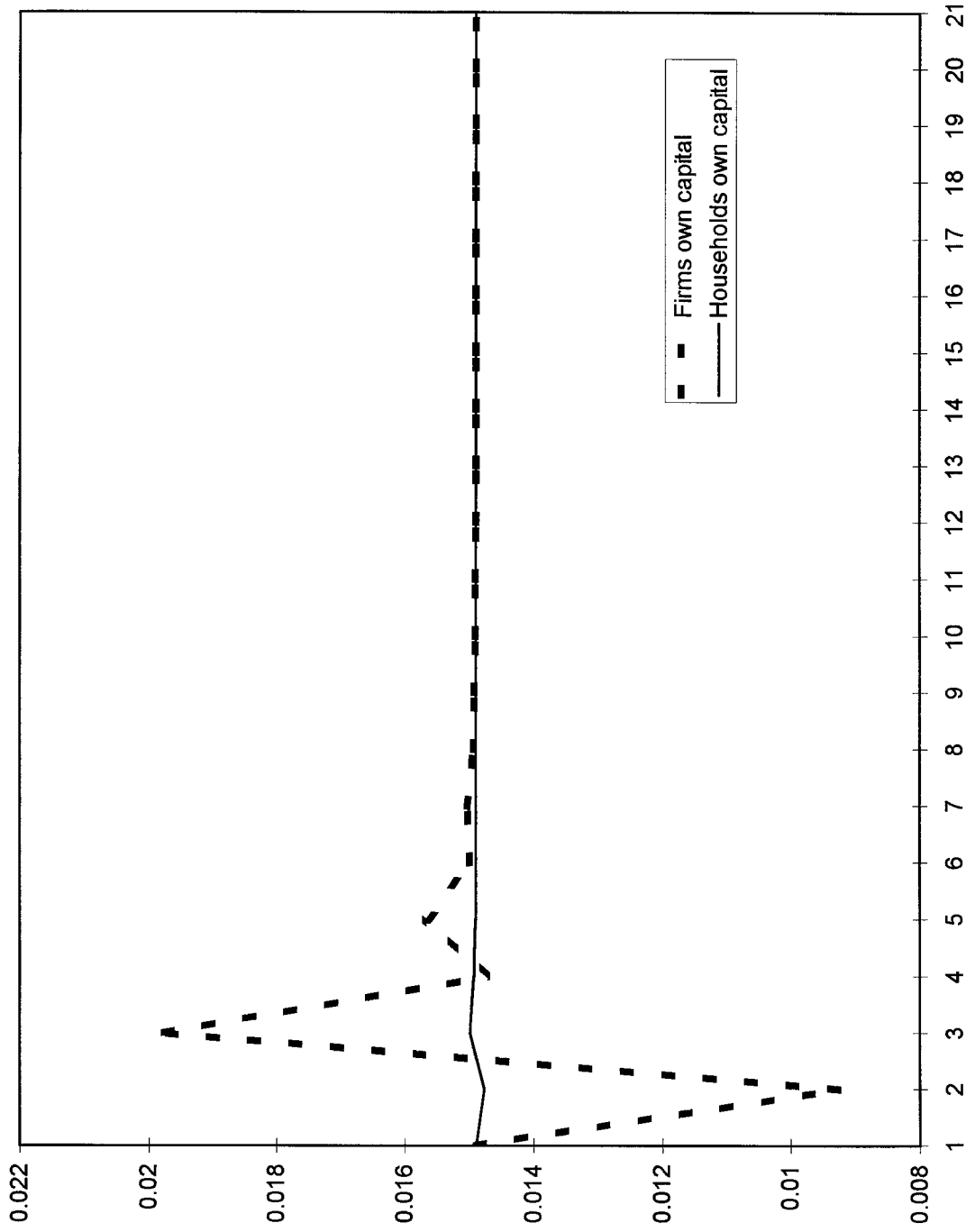
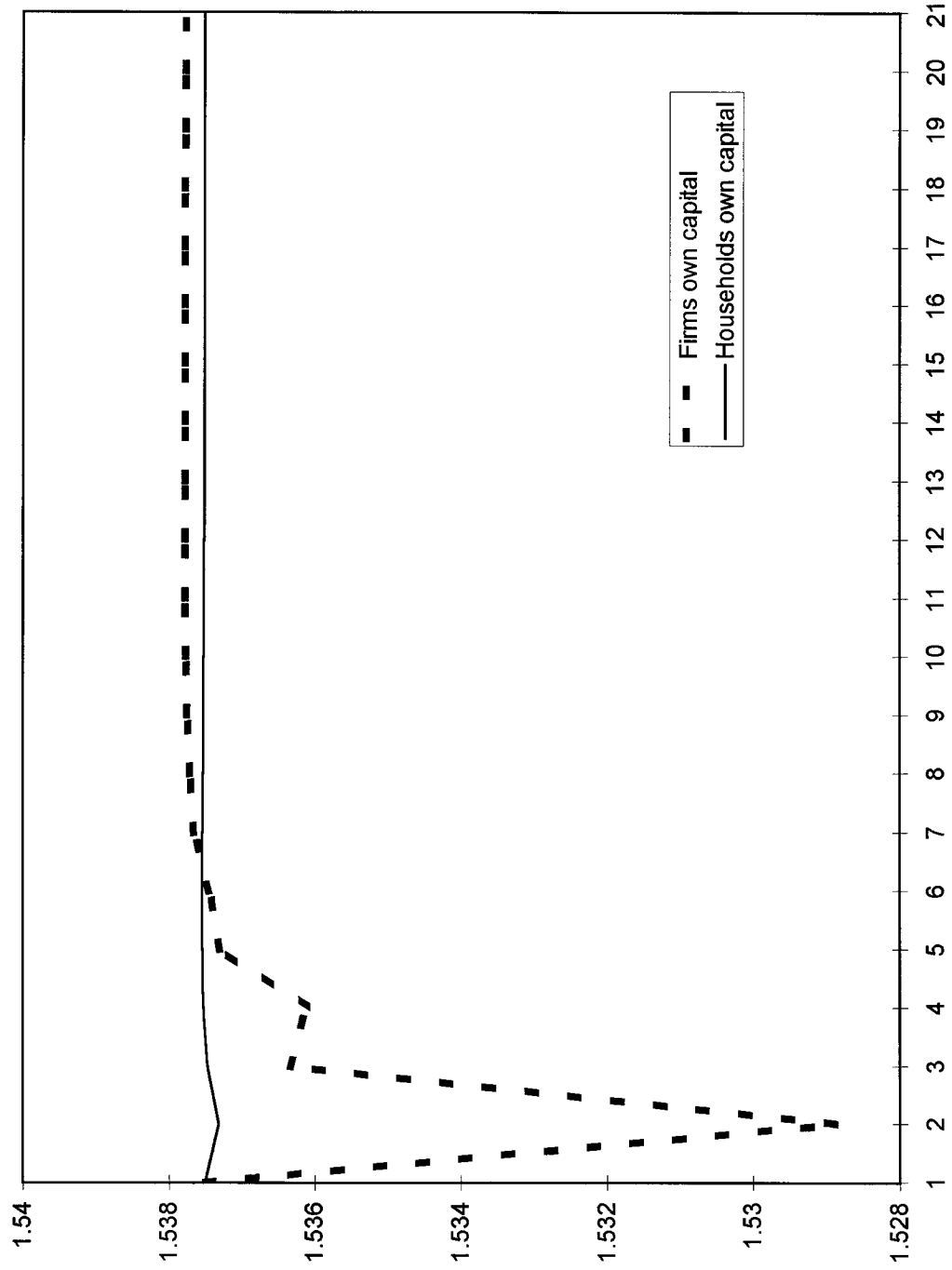


Figure 3. Response of Consumption to Monetary Shock



## IV. ROBUSTNESS

### A. Shopping Time Model

Explicitly modeling the ownership of stock is a robust way to introduce additional inflation effects on stock returns. To demonstrate this, we show that Theorem 2 holds when money is introduced into the economy through a “shopping time” or transaction cost technology. We will show that the definition of the stock return is the same under this model as under the CIA regime, that the Euler conditions used to price the assets are the same, and that Theorem 2 holds so that the value of the firm is equal to the capital stock divided by inflation. Inflation will therefore have the same effects on stock returns as under the CIA regime. The two approaches do imply different inflation processes, but this difference is not relevant to our purpose.

We modify the model by replacing the Cash-in-Advance constraint with a shopping time motive for holding money following Marshall (1992). In a shopping time model, the cash purchase requirement is replaced with a transactions technology that reflects the real resource costs of making purchases. The function  $\varphi\left(C_t, \frac{M_t^P}{P_t}\right)$  is the cost of purchasing commodities from the firm. Assume following Marshall that  $\varphi_1\left(C_t, \frac{M_t^P}{P_t}\right) > 0$ ,  $\varphi_2\left(C_t, \frac{M_t^P}{P_t}\right) < 0$ ,  $\varphi\left(0, \frac{M_t^P}{P_t}\right) = 0$ ,  $\varphi_{11}\left(C_t, \frac{M_t^P}{P_t}\right) \geq 0$ ,  $\varphi_{22}\left(C_t, \frac{M_t^P}{P_t}\right) \geq 0$ , and  $\varphi_{12}\left(C_t, \frac{M_t^P}{P_t}\right) \leq 0$ .

The government imposes the same tax and transfer scheme as before. The household budget constraint thus becomes

$$C_t + \varphi\left(C_t, \frac{M_t}{P_t}\right) + \frac{P_t^s S_t}{P_t} + \frac{M_t^P}{P_t} + \mathcal{T}_t \leq \frac{W_t}{P_t}. \quad (61)$$

The production technology and dividend distribution requirements remain the same, as do the stochastic shock equations of motion and the utility function of the household. Therefore, the stock return definition is the same since dividends are still paid at the end of the period.

The transactions cost technology does change the wealth accumulation equation. Real wealth next period becomes

$$\frac{W_{t+1}}{P_{t+1}} = \frac{R_{t+1}^s P_t^s}{P_{t+1}} S_t + \frac{M_t^P}{P_{t+1}} + \frac{P_t W_t H_t^s}{P_{t+1}}. \quad (62)$$

In the appendix we show that the optimal decisions for money, stocks and the financial asset are given by

$$1 = E_t \left[ mrs_{t+1} \frac{\left(1 - \varphi_2\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)\right)}{\Pi_{t+1}} \right], \quad (63)$$

$$1 = E_t \left[ mrs_{t+1} \frac{R_{t+1}^s}{\Pi_{t+1}} \right]. \quad (64)$$

$$\text{Here } mrs_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( \frac{1+\varphi_1 \left( C_t, \frac{M_t}{P_t} \right)}{1+\varphi_1 \left( C_{t+1}, \frac{M_{t+1}}{P_{t+1}} \right)} \right).$$

A comparison of the optimal stock decision under CIA (15) and Shopping Time (64) reveals that the stochastic discount factor is modified to incorporate the loss of consumer goods in the exchange process. Thus the conditions for competitive equilibrium are the same, with the new measure of the stochastic discount factor for the consumer. The exact form of the stochastic discount factor is not important for the proof of Theorem 2.

So long as the consumer's marginal rate of substitution is positive each period, the proof of Theorem 2 still holds and in fact is identical. Therefore the real stock price of the firm is given by

$$\frac{P_t^s}{P_t} = \frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})}, \quad (65)$$

which allows us to write the return on stocks as

$$R_{t+1}^s = \Pi_t mri_t. \quad (66)$$

While (65) and (66) appear to be identical under CIA and Shopping Time, they are not since the equilibrium stochastic process for inflation implied by the shopping time model is significantly different. Nonetheless, the effects of inflation on stock returns in the shopping time model will be similar to those in the CIA model: inflation and stock returns are negatively correlated.

On the other hand, the shopping time model implies that the velocity of money will change in response to changes in short-term interest rates, while velocity is fixed in the CIA model. This creates a positive correlation between stock returns and the quantity of money in the shopping time model<sup>23</sup>, in contrast to the negative correlation between money and stock returns in the CIA model. Recently, Thorbecke (1997) and Patelis (1997) have further documented the positive correlation between money and stock returns empirically.

## B. The Modigliani and Miller Property

In Section 3 we showed that the ownership of capital goods by the firm and equity by the households introduces a new inflation tax on wealth that changes the response of the economy to an inflation shock. We now want to show that the competitive equilibrium in the economy will not change, if the firm finances a fraction of its investments with bonds rather than stocks. As we discussed in the introduction, the key innovation in our model is the introduction of nominally-denominated financial instruments. The type of financial instrument introduced does not matter, basically because they all perform the same functions for the household and firm.

We go back to the model of Section 2 and allow the firm, following Altuğ and Labadie (1994), to issue commercial paper at the beginning of each time period<sup>24</sup>. The commercial paper is paid off at the end of the period. At the beginning of each period the firm sells  $a_t$  in nominally

<sup>23</sup>See Marshall (1992).

<sup>24</sup>We call this bond commercial paper because it is short-term, unsecured corporate debt that is usually paid off by issuing new paper. These characteristics correspond closely to those of



denominated commercial paper, which promises to pay back  $R_t^a a_t$  dollars. At the same time the firm pays dividends on the equity of the firm. The firm earns profits at time  $t$  which is given by

$$PR_t = P_t [F(\theta_t H_t, K_t) - \mathcal{W}_t H_t]. \quad (67)$$

These profits are distributed to claim holders of the firm

$$PR_t = RE_t + D_t s_t + R_t^a a_t, \quad (68)$$

where  $RE$  represents retained earnings.

The firm may finance investment with retained earnings or by issuing more commercial paper.

$$P_t I_t = RE_t + a_{t+1}. \quad (69)$$

Thus the net cash flow for the firm,  $N_t$ , is

$$N_t = PR_t - P_t I_t = D_t s_t + R_t^a a_t - a_{t+1}, \quad (70)$$

so that the firm's problem becomes

$$V_t^F \equiv \max_{K_{t+1}, H_t} E_t \sum_{j=0}^{\infty} \left\{ \left[ \prod_{i=0}^j s_{t+i+1} \right] \frac{N_{t+j}}{P_{t+j+1}} \right\}, \quad (71)$$

subject to (1), (2) and (7). This means that the firm's optimal decisions (23) and (39) are still true.

The consumer's problem is modified since they can now spend a fraction of their real wealth on commercial paper. Purchases of assets are carried out subject to

$$\frac{P_t^s S_t}{P_t} + \frac{a_t}{P_t} + \frac{M_t^p}{P_t} \leq \frac{W_t}{P_t} - \mathcal{T}_t, \quad (72)$$

which replaces (9), while the household's wealth at the end of the period is

$$\frac{W_{t+1}}{P_{t+1}} = \frac{R_{t+1}^s P_t^s}{P_{t+1}} S_t + \frac{R_t^a a_t}{P_{t+1}} + \frac{M_t^p - P_t C_t}{P_{t+1}} + \frac{P_t \mathcal{W}_t H_t^s}{P_{t+1}}, \quad (73)$$

which replaces (12). Thus, the optimal demand for commercial paper is

$$1 = E_t \left[ mrs_{t+1} \frac{R_t^a}{\Pi_{t+1}} \right]. \quad (74)$$

Competitive equilibrium is now satisfied by the conditions given in Section 2.7, with the addition of (74). We now have the new version of Theorem 2.

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commercial paper.

**Theorem 3** *The conditions for equilibrium in the stock and bond markets are (1) the firm's real stock price at any time  $t$  is  $\frac{P_t^s}{P_t} = \frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})} - \frac{a_t}{P_t}$ , and (2) the firm's stochastic discount factor at any time  $t$  is equal to the consumer's intertemporal marginal rate of substitution plus an i.i.d. shock with mean zero and zero correlation across time,  $e_t$ , i.e.,  $sd_t = mrs_t + e_t$ ,  $E(e_t) = 0$ ,  $E(e_t e_{t+1}) = 0$ ,  $I \neq 0$ .*

**Proof:** The Proof is given in an Appendix.

Following Restoy and Rockinger (1994), we have

**Corollary 2**  $q_t R_{t+1}^s + (1 - q_t) R_t^a = \Pi_t mri_t$ , where  $q_t = \frac{P_t^s}{P_t^s + a_t}$

**Proof:** The proof is similar to the proof of Corollary 1 and is given in an Appendix.

The main implication of Theorem 3 is that the conditions for equilibrium in the competitive economy collapse to the same two conditions (52) and (53). Thus, the equilibrium behavior of capital and consumption will be the same whether or not the firm holds commercial paper in addition to equity. Consequently, the real value of the firm will be identical as well. On the other hand the financing decision of the firm does determine how the value of the firm is allocated to equity and bond holders. In Corollary 2 the nominal value of the marginal return on investment is now allocated to a return on equity and a return on commercial paper based on the market value of equity relative to the total market value of the firm.

In summary, the property first stated by Modigliani and Miller (1958), that the value of the firm is independent of the type of financing used by the corporation, is replicated in monetary economies. On the other hand, the production based asset pricing model developed by Cochrane (1991) is modified in the presence of debt financing by the firm. Based on Corollary 2, the nominal value of the marginal return is equal to the weighted cost of capital when the firm has debt financing rather than just the firm's stock return.

## V. CONCLUSION

The punchline in many economic models is that ownership affects behavior. This model also has that implication. In particular, we show that the identity of the owner of the physical capital goods in the economy significantly affects the behavior of stock returns. Stock returns, the value of the firm, and consumption display a dramatically higher level of sensitivity to inflation shocks when ownership of capital rests with firms rather than households. The reason why something as simple as transferring ownership of capital goods from households to firms has such a significant impact on stock returns is because it creates a new nominally denominated asset—stock certificates—whose value is related to the capital stock. The existence of stock certificates creates a new channel through which the inflation tax can affect stock returns. This additional channel is the monetary authority's ability to tax the entire value of the capital stock during each period. Because the value of the capital stock is quite large, this tax is large. The inflation tax on wealth also creates arbitrage opportunities between the ownership value and the leasing value of financial instruments. Removal of these

arbitrage opportunities demands significant adjustments in both real and nominal variables.

In addition, our results also suggest that changing the ownership assumption may also help the standard growth model match other characteristics of equity returns that have been documented in data but not predicted well by theory. For example, our simulations do a better job of inducing the negative autocorrelation observed in stock returns than the standard growth model with household ownership of capital. In a related point, monetary shocks in this model induce a higher level of return volatility than they do in the standard model. This may help shed some light on the so-called “excess volatility” problem in stock returns. Investigating whether the model can better explain these additional characteristics of stock returns is a task for further research.

## APPENDIX A

### A. Consumer's Problem under CIA

The consumer's problem is a dynamic programming problem with the value function given by

$$V(\mu_t, \theta_t, \frac{W_t}{P_t}) = \max \left[ \frac{C_t^{1-\gamma}}{1-\gamma} + \eta_t \left( \frac{M_t^p}{P_t} - C_t \right) + \lambda_t \left( \frac{W_t}{P_t} - \frac{P_t^s S_t}{P_t} - \frac{M_t^p}{P_t} - \mathcal{T} \right) + \beta E_t \left[ V(\mu_{t+1}, \theta_{t+1}, \frac{W_{t+1}}{P_{t+1}}) \right] \right].$$

Here  $\lambda_t$  and  $\eta_t$  are the Lagrange multipliers for the constraints (9) and (11), respectively.

The consumer chooses money balances in period  $t$  according to the rule

$$\eta_t = \lambda_t - \beta E_t \left( \frac{\partial V}{\partial \left( \frac{W_{t+1}}{P_{t+1}} \right)} \frac{P_t}{P_{t+1}} \right)$$

Finally, the consumer chooses stocks to satisfy the condition

$$\lambda_t = \beta E_t \left( \frac{\partial V}{\partial \left( \frac{W_{t+1}}{P_{t+1}} \right)} R_{t+1}^s \frac{P_t}{P_{t+1}} \right).$$

In the second session the consumer chooses consumption to satisfy the consumer's optimal condition

$$C_t^{-\gamma} = \eta_t + \beta E_t \left( \frac{\partial V}{\partial \left( \frac{W_{t+1}}{P_{t+1}} \right)} \frac{P_t}{P_{t+1}} \right).$$

In making each of these decisions the consumer uses the expected marginal value of wealth. Using the Mirman-Zilcha (1975) condition,<sup>25</sup> the expected marginal value of wealth is given by

$$\frac{\partial V}{\partial \left( \frac{W_t}{P_t} \right)} = \lambda_t.$$

Furthermore, the above optimal conditions for money and consumption imply that

<sup>25</sup>See Stokey and Lucas with Prescott (1989, pp.266-267).

$$C_t^{-\gamma} = \frac{\partial V}{\partial \left( \frac{W_t}{P_t} \right)} = \lambda_t.$$

Substituting the marginal utility of consumption into the optimal condition for stocks

$$1 = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \frac{(P_{t+1}^s + D_t)}{P_t^s} \right].$$

The above condition corresponds to condition (15).

### B. Consumer's Problem under Shopping Time

The consumer's problem under shopping time is a dynamic programming problem with the value function given by

$$V(\mu_t, \theta_t, \frac{M_t}{P_t}, \frac{W_t}{P_t}) = \max \left[ \frac{C_t^{1-\gamma}}{1-\gamma} + \lambda_t \left( \frac{W_t}{P_t} - C_t - \varphi \left( C_t, \frac{M_t}{P_t} \right) - \frac{P_t^s S_t}{P_t} - \frac{M_t^p}{P_t} - \mathcal{T} \right) + \beta E_t \left[ V(\mu_{t+1}, \theta_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \frac{W_{t+1}}{P_{t+1}}) \right] \right].$$

Here  $\lambda_t$  is the Lagrange multipliers for the constraint (61). The consumer chooses money balances in period  $t$  according to the rule

$$\lambda_t = \beta E_t \left( \frac{\partial V}{\partial \left( \frac{M_{t+1}}{P_{t+1}} \right)} \frac{P_t}{P_{t+1}} \right) + \beta E_t \left( \frac{\partial V}{\partial \left( \frac{W_{t+1}}{P_{t+1}} \right)} \frac{P_t}{P_{t+1}} \right). \quad (75)$$

The optimal consumption decision yields

$$C_t^{-\gamma} = \lambda_t \left( 1 + \varphi_1 \left( C_t, \frac{M_t}{P_t} \right) \right). \quad (76)$$

The investor chooses stocks to satisfy the condition

$$\lambda_t = \beta E_t \left( \frac{\partial V}{\partial \left( \frac{W_{t+1}}{P_{t+1}} \right)} R_{t+1}^s \frac{P_t}{P_{t+1}} \right). \quad (77)$$

In making each of these decisions the consumer uses the marginal value of money and wealth. Using the Mirman-Zilcha (1975) condition,<sup>26</sup> the marginal value of money is given by

<sup>26</sup>See Stokey and Lucas with Prescott (1989, pp.266-267).

$$\frac{\partial V}{\partial \left(\frac{M_t}{P_t}\right)} = -\lambda_t \varphi_2 \left(C_t, \frac{M_t}{P_t}\right). \quad (78)$$

and the marginal value of wealth is

$$\frac{\partial V}{\partial \left(\frac{W_t}{P_t}\right)} = \lambda_t. \quad (79)$$

Now combining the optimal decisions for consumption (76) and the envelope conditions (78 and 79) with the optimal decision for money (75) yields

$$1 = \beta E_t \left[ \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{1 + \varphi_1 \left(C_t, \frac{M_t}{P_t}\right)}{1 + \varphi_1 \left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)}\right) \left(1 - \varphi_2 \left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)\right) \frac{P_t}{P_{t+1}} \right]. \quad (80)$$

This result corresponds to (1.6) of Marshall and (63) in the text.

Next we combine the optimal decisions for consumption and the envelope conditions with the optimal decision for stocks (77) to yield

$$1 = \beta E_t \left[ \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{1 + \varphi_1 \left(C_t, \frac{M_t}{P_t}\right)}{1 + \varphi_1 \left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right)} \frac{P_t}{P_{t+1}} \frac{(P_{t+1}^s + D_t)}{P_t^s} \right]. \quad (81)$$

This equation corresponds to (1.7) of Marshall and (64) in the text.

## APPENDIX B

### A. Proof of Theorem 3 and Corollary 2

Equilibrium in the stock and bond markets occurs when the demand for stocks determined by (38) and bonds (74) is in agreement with the firm's investment decision (39) so that the equity and bond markets clear (27). We start with (38) and determine the conditions under which it agrees with (74) and (39).

First, increase the time period by 1 in equation (38) so that

$$P_{t+1}^s = E_{t+1} \left\{ \frac{mrs_{t+2}}{\Pi_{t+2}} (P_{t+2}^s + D_{t+1}) \right\}.$$

Multiplying both sides of the above equation by  $\frac{mrs_{t+1}}{P_{t+1}}$ , and using the law of iterated expectations yields

$$E_t \left\{ \frac{mrs_{t+1}}{P_{t+1}} P_{t+1}^s \right\} = E_t \left\{ \frac{mrs_{t+1}}{P_{t+1}} \frac{mrs_{t+2}}{\Pi_{t+2}} (P_{t+2}^s + D_{t+1}) \right\}. \quad (82)$$

Now, by (23) we have

$$Y_t - \mathcal{W}_t H_t = F_2(\theta_t H_t, K_t) K_t \quad (83)$$

In addition, the linear homogenous capital accumulation equation (2) implies by Euler's Theorem

$$I_t = \frac{K_{t+1}}{A_2(K_t, I_t)} - \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t. \quad (84)$$

Now solve (69) for retained earnings and substitute the result into (68) to find

$$PR_t = P_t I_t + D_t S_t + R_t^a a_t - a_{t+1} \quad (85)$$

Now use (67) to see that dividends are

$$D_t S_t = P_t [Y_t - \mathcal{W}_t H_t] - P_t I_t - R_t^a a_t + a_{t+1} \quad (86)$$

Now use (83) and (84) to show that real dividends are given by

$$\frac{D_{t+1}}{P_{t+1}} = F_2(\theta_{t+1} H_{t+1}, K_{t+1}) K_{t+1} - \frac{K_{t+2}}{A_2(K_{t+1}, I_{t+1})} + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} K_{t+1} - \frac{R_{t+1}^a a_{t+1}}{P_{t+1}} + \frac{a_{t+2}}{P_{t+1}}. \quad (87)$$

Substituting (87) into (82) yields

$$E_t \left\{ mrs_{t+1} \frac{P_{t+1}^s}{P_{t+1}} \right\} = E_t \left\{ mrs_{t+1} mrs_{t+2} \frac{P_{t+2}^s}{P_{t+2}} + mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} \left[ F_2(\theta_{t+1} H_{t+1}, K_{t+1}) K_{t+1} - \frac{K_{t+2}}{A_2(K_{t+1}, I_{t+1})} + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} K_{t+1} - \frac{R_{t+1}^a a_{t+1}}{P_{t+1}} + \frac{a_{t+2}}{P_{t+1}} \right] \right\}. \quad (88)$$

Now express the investor's optimal decision for commercial paper (74) at time t+1

$$1 = E_t \left[ mrs_{t+2} \frac{R_{t+1}^a}{\Pi_{t+2}} \right]. \quad (89)$$

Multiplying both sides of the above equation by  $\frac{mrs_{t+1} a_{t+1}}{P_{t+1}}$ , and using the law of iterated expectations yields

$$E_t \left\{ \frac{mrs_{t+1} a_{t+1}}{P_{t+1}} \right\} = E_t \left\{ \frac{mrs_{t+1}}{P_{t+1}} \frac{mrs_{t+2}}{\Pi_{t+2}} R_{t+1}^a a_{t+1} \right\}. \quad (90)$$

Use this result in (88) to yield

$$E_t \left\{ mrs_{t+1} \left[ \frac{P_{t+1}^s}{P_{t+1}} + \frac{a_{t+1}}{P_{t+1}} \right] \right\} = E_t \left\{ mrs_{t+1} mrs_{t+2} \frac{P_{t+2}^s}{P_{t+2}} + mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} \left[ F_2(\theta_{t+1} H_{t+1}, K_{t+1}) K_{t+1} - \frac{K_{t+2}}{A_2(K_{t+1}, I_{t+1})} + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} K_{t+1} + \frac{a_{t+2}}{P_{t+1}} \right] \right\}. \quad (91)$$

Finally, regroup terms in (91) such that

$$E_t \left\{ mrs_{t+1} \left[ \frac{P_{t+1}^s}{P_{t+1}} + \frac{a_{t+1}}{P_{t+1}} \right] \right\} = E_t \left\{ mrs_{t+1} mrs_{t+2} \left[ \frac{P_{t+2}^s}{P_{t+2}} + \frac{a_{t+2}}{P_{t+2}} - \frac{K_{t+2}}{\Pi_{t+2} A_2(K_{t+1}, I_{t+1})} \right] \right\} + E_t \left\{ mrs_{t+1} \frac{mrs_{t+2}}{\Pi_{t+2}} \left[ F_2(\theta_{t+1} H_{t+1}, K_{t+1}) K_{t+1} + \frac{A_1(K_{t+1}, I_{t+1})}{A_2(K_{t+1}, I_{t+1})} K_{t+1} \right] \right\}. \quad (92)$$

A comparison of the investor's optimal behavior (92) with the optimal investment decision of the firm (39) reveals that the two decisions concur when condition (1) and (2) of the theorem are satisfied. Substitution of (16) and (17) into (27) demonstrates that the no arbitrage condition is also satisfied

### Proof of Corollary 2

The definition of the marginal return on investment implies

$$\Pi_t mri_t = \Pi_t \left[ \left( F_2(\theta_t, H_t, K_t) K_t + \frac{A_1(K_t, I_t)}{A_2(K_t, I_t)} K_t \right) \frac{A_2(K_{t-1}, I_{t-1})}{K_t} \right]. \quad (93)$$



Using the expression for real dividends (87) in (93) yields

$$\Pi_t m r i_t = \frac{\Pi_t A_2(K_{t-1}, I_{t-1})}{K_t} \left[ \frac{D_t}{P_t} + \frac{K_{t+1}}{A_2(K_t, I_t)} + \frac{R_t^a a_t}{P_t} - \frac{a_{t+1}}{P_t} \right]. \quad (94)$$

Finally, using condition (2) in Theorem 2 that  $\frac{P_t^s}{P_t} = \frac{K_t}{\Pi_t A_2(K_{t-1}, I_{t-1})} - \frac{a_t}{P_t}$  yields

$$\Pi_t m r i_t = \Pi_{t+1} \frac{\frac{D_t}{P_{t+1}} + \frac{P_{t+1}^s}{P_{t+1}}}{\frac{P_t^s}{P_t} + \frac{a_t}{P_t}} + \frac{\frac{R_t^a a_t}{P_t}}{\frac{P_t^s}{P_t} + \frac{a_t}{P_t}} = q_t R_{t+1}^s + (1 - q_t) R_t^a. \quad (95)$$

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