

**IMF Working Paper**

© 1999 International Monetary Fund

This is a *Working Paper* and the author(s) would welcome any comments on the present text. Citations should refer to a *Working Paper of the International Monetary Fund*. The views expressed are those of the author(s) and do not necessarily represent those of the Fund.

WP/99/16

INTERNATIONAL MONETARY FUND

Asia and Pacific Department

**Interest Rate Arbitrage in Currency Baskets: Forecasting  
Weights and Measuring Risk**

Prepared by Peter Christoffersen and Lorenzo Giorgianni<sup>1</sup>

Authorized for distribution by Ranjit Teja

January 1999

**Abstract**

When constructing hedged interest rate arbitrage portfolios for basket currencies, two issues arise: first, how are the unknown future basket weights optimally forecasted from past exchange rate data? And, second, how is risk—in terms of the conditional variance of expected profits from the interest rate arbitrage portfolio—appropriately measured when the basket weights are time-varying? Answers to these questions are provided within a time-varying parameter modeling framework estimated through the Kalman filter. An empirical application is devoted to the experience of the Thai baht currency basket (January 1992–February 1997).

JEL Classification Numbers: C53, F37

Keywords: Time-varying Parameters, Cointegration, Exchange Rates

Author's E-Mail Address: christop@management.mcgill.ca, lgiorgianni@imf.org

---

<sup>1</sup>We are particularly indebted to Mark Watson and two anonymous referees for detailed comments. We are also grateful to Fabio Canova, Frank Diebold, Rob Engle, Eric Ghysels, Hashem Pesaran, Jim Stock, Rene Stulz, Norm Swanson, and participants at the NBER Summer Institute and numerous university seminars. We are responsible for all remaining errors.

	Contents	Page
I.	Introduction .....	3
II.	The Basket Hedge Portfolio .....	4
	A. Defining the Basket Currency .....	4
	B. Constructing the Basket Hedge .....	5
	C. Calculating Profits and Risk .....	7
III.	Allowing for Time-Varying Parameters .....	8
	A. Defining the Time-Varying Currency Basket .....	8
	B. Constructing the Basket Hedge with Time-Varying Parameters .....	9
	C. Models with Time-Varying Parameters .....	10
IV.	An Application to the Thai Baht Basket .....	12
	A. Basket Exchange Rates and Nonstationarity: Specification And Estimation Issues .....	12
	B. Empirical Results: Estimation of the Baht Basket Weights .....	13
	C. Ex-Post Evaluation of Ex-Ante Risk and Return Measures .....	16
V.	Summary and Extensions .....	19
	Appendix I .....	27
	References .....	29

## I. INTRODUCTION

In a number of emerging and developed economies, the monetary authorities follow a policy of linking their currency to a basket of so-called hard currencies. In several cases the authorities do not disclose the basket weights and apparently change them over time.

The increasing integration of international capital markets has enabled private investors to take advantage of often very profitable interest rate spreads arising from the relatively high local interest rates vis-à-vis lower interest rates on instruments denominated in the hard currencies belonging to the basket.

For investors, knowledge of conditional risk and return of a speculative position in the basket is of paramount importance. For monetary authorities, it is crucial as well because short-term capital flows are a function of the risk-adjusted return on basket-hedges. If the risk-adjusted returns are large, managing the size and volatility of capital inflows can present a severe challenge to monetary policy makers.

When constructing hedged interest rate arbitrage portfolios for basket currencies, two issues arise: first, how are the unknown future basket weights optimally forecasted from past exchange rate data accounting for possible non-stationarities? Second, how is the riskiness of the arbitrage portfolio (a long position in the local basket currency hedged against a particular short position in the underlying hard currencies) appropriately measured?

We address these two issues in a time-varying parameter modeling framework using Kalman filtering techniques. The time-series models applied allow us to optimally forecast the time-varying weights, and to properly and conveniently measure risk in terms of the conditional variance of the associated portfolio profits. We explicitly derive the optimal portfolio weights that eliminate hedgeable risk, and define a conditional Sharpe-ratio risk-measure which allows investors to compare the riskiness of basket trades with other investment opportunities.

An empirical application at the end of the paper is devoted to the experience of the Thai baht exchange rate. In the past few years, billions of dollars have poured in and out of Thailand from foreign investors, particularly from the United States, attempting to take advantage of interest rate spreads sometimes exceeding 500 basis points. We apply the methodology sketched above to Thai baht basket trades, compute ex-ante measures of profits and risk, and suggest formulas for computing risk-adjusted returns which appropriately account for the time-variability of the weights. We evaluate the performance of competing models using profit-based criteria in the tradition of Henriksson and Merton (1981), Leitch and Tanner (1991), and Pesaran and Timmermann (1994).

Many countries other than Thailand have made use of currency baskets at one time or another, and the literature on optimal basket regimes dates back to at least Flanders and Helpman (1979), and Lipschitz and Sundararajan (1980). It has since been extended by Turnovsky (1982), Bhandari (1985), and Edison and Vaardal (1990), and it typically implies

that the optimal weights are changing over time with macroeconomic fundamentals. Conversely, the financial literature on basket regimes usually assumes fixed weights, see Graf von der Schulenburg (1984), Horngren and Vredin (1989), Klein (1989), and Pikkariainen (1991). An exception is Klein and Muller (1992) who study an anticipated one-time change in the ECU basket. We model the basket weights as evolving smoothly over time, a phenomenon consistent with the macroeconomic theory, and often observed in actual basket regimes.

The remainder of the paper is structured as follows. In section 2 we define the basket currency framework with static weights and solve for the optimal hedge position. Section 3 generalizes the problem to allow for time-varying weights, suggests particular models with time-varying parameters, and computes the explicit risk and return formulas. Section 4 is devoted to a thorough empirical study of Thailand's recent experience with a basket currency regime. We estimate basket weights as well as profit and risk measures under different assumptions about the basket weight dynamics. In section 5 we conclude and suggest further developments of the methodology.

## II. THE BASKET HEDGE PORTFOLIO

Hedging an investment in a (local) basket currency provides a way to remove the implicit exposure to the (hard) currencies against which the investment currency is managed so that only pure local-currency risk remains. For example, to hedge the implicit exposure of an investment in a currency pegged to a two-currency basket, all that is needed is to quantify the exposure arising from movements in the cross exchange rate (i.e., the rate at which the hard currencies belonging to the basket are exchanged) and generate a trade of opposite exposure: the so-called basket hedge. As local-currency risk is deliberately not removed, this hedge is not perfect, and for this reason it is sometimes referred to as a proxy-hedge.

We will now briefly describe the typical basket currency regime with fixed weights, and then map out the details of basket hedge positions, including explicit measures of profitability and risk.

### A. Defining the Basket Currency

Consider a currency 0 constructed as a weighted sum of K other currencies with  $\alpha_j$  being the (possibly unobserved) units of currency j included in the basket. Taking currency K as the numeraire, we can write the currency K value of the basket as

$$e_0 = \alpha_K + \sum_{j=1}^{K-1} \alpha_j e_j, \quad (1)$$

where  $e_j$  is the amount of currency K per unit of currency j, i.e., the cross-currency exchange rate. Observing relationship (1) over time, and adding a stationary error term ( $\epsilon$ ), which

accounts, e.g., for movements within a small prefixed band around the basket rate, yields the estimable equation

$$e_{0,t} = \alpha_K + \sum_{j=1}^{K-1} \alpha_j e_{j,t} + \epsilon_t \quad \text{with } \text{var}(\epsilon_t) \equiv \sigma^2. \quad (2)$$

The individual (floating) exchange rates are generally believed to be well characterized by random walk processes. Therefore, under stationarity of  $\epsilon_t$ , the basket exchange rate ( $e_{0,t}$ ) will be cointegrated with the hard exchange rates belonging to the basket (the  $e_{j,t}$ 's for  $j > 0$ ).

The assumption of stationarity of the error term attached to equation (2) is justifiable, e.g., whenever the actual exchange rate is allowed to fluctuate within a narrow band centered at the basket parity. In this case, the error term represents the deviation-from-the-central-parity stochastic process which is mean reverting whenever the central bank is committed to defending the fluctuation band.

### B. Constructing the Basket Hedge

In this section, we show how to construct basket hedge portfolios under the assumption that the currency weights are fixed over time.

Formally, a basket hedge portfolio constructed at time  $t$  is the combination of a long position of a given size,  $M_{0,t} > 0$ , in currency 0 for  $h$  days, with a corresponding short position in the hard currencies of sizes  $M_{j,t} > 0$ ,  $j = 1, \dots, K$ . The  $M_{j,t}$ 's are denominated in currency  $j$ , and are to be determined below.

The cash flow denominated in currency  $K$  from the hedge position is

<b>Cash</b>	<b>time <math>t</math></b>	<b>time <math>t+h</math></b>
<b>Inflow</b>	$\sum_{j=1}^{K-1} M_{j,t} e_{j,t} + M_{K,t}$	$M_{0,t} (1 + r_{0,t}) e_{0,t+h}$
<b>Outflow</b>	$M_{0,t} e_{0,t}$	$\sum_{j=1}^{K-1} M_{j,t} (1 + r_{j,t}) e_{j,t+h} + M_{K,t} (1 + r_{K,t})$

At time  $t$ ,  $M_{0,t}$  units of currency 0 are deposited. The deposit is financed by a composite loan in currencies 1 through  $K$ . At time  $t+h$ , the currency 0 deposit is withdrawn with interest,  $r_{0,t}$ , and the composite loan is repaid with interest rates,  $r_{1,t}, \dots, r_{K,t}$ . All amounts are converted into currency  $K$  using the appropriate time  $t$  and time  $t+h$  exchange rates.

In order to ensure that the position is self-financed, the time- $t$  net cash flow is required to be zero,

$$\sum_{j=1}^{K-1} M_{j,t} e_{j,t} + M_{K,t} - M_{0,t} e_{0,t} = 0. \quad (6)$$

The expression for currency-K profits at time t+h from an h-day hedge made at time t is then simply

$$\Pi_{t+h} = M_{0,t}(1+r_{0,t})e_{0,t+h} - \sum_{j=1}^{K-1} M_{j,t}(1+r_{j,t})e_{j,t+h} - M_{K,t}(1+r_{K,t}), \quad (7)$$

where the interest rates are expressed as h-day returns (for notational convenience), so that the standard simple annual yields are obtained as  $r^*(360/h)$ . Profits arise from the interest rate on currency 0 being higher than those of the hard currencies, and potential losses arise from the local currency devaluating.

Normalizing  $M_{j,t}$  by  $M_{K,t}$  to get the normalized weights,  $m_{j,t}$ , we can obtain the normalized profit,  $\pi_{t+h}$ , which is denominated in currency K, and thus interpretable as the currency-K excess return,

$$\pi_{t+h} = m_{0,t}(1+r_{0,t})e_{0,t+h} - \sum_{j=1}^{K-1} m_{j,t}(1+r_{j,t})e_{j,t+h} - (1+r_{K,t}). \quad (8)$$

Substituting for the time t+h basket exchange rate, we get

$$\pi_{t+h} = m_{0,t}(1+r_{0,t}) \left( \alpha_K + \sum_{j=1}^{K-1} \alpha_j e_{j,t+h} + \epsilon_{t+h} \right) - \sum_{j=1}^{K-1} m_{j,t}(1+r_{j,t})e_{j,t+h} - (1+r_{K,t}). \quad (9)$$

**Definition:** In order to construct a *hedge portfolio* that only leaves local currency 0 risk we need to remove the currency  $j = 1, \dots, K-1$  risk, i.e., we need to find the  $m_{j,t}$ 's which satisfy the following K-1 first-order conditions of the hedging problem,

$$\frac{\partial \pi_{t+h}}{\partial e_{j,t+h}} = 0, \quad j = 1, 2, \dots, K-1. \quad (10)$$

Using estimated weights, the K-1 first-order conditions yield

$$m_{j,t} = \frac{1+r_{0,t}}{1+r_{j,t}} \hat{\alpha}_j m_{0,t}, \quad j = 1, 2, \dots, K-1. \quad (11)$$

The currency 0 position is given from the normalized accounting equation,

$$\sum_{j=1}^{K-1} m_{j,t} e_{j,t} + 1 - m_{0,t} e_{0,t} = 0, \quad (12)$$

and using the optimal  $m_{j,t}$ 's:

$$m_{0,t} = \left( e_{0,t} - \sum_{k=1}^{K-1} \frac{1+r_{0,t}}{1+r_{k,t}} e_{k,t} \hat{\alpha}_k \right)^{-1} \approx \left( \hat{\alpha}_K + \sum_{k=1}^{K-1} (r_{k,t} - r_{0,t}) e_{k,t} \hat{\alpha}_k \right)^{-1}. \quad (13)$$

Plugging the relative currency 0 position back in the first-order conditions (11) yields the final expression for the hedge portfolio weights

$$m_{j,t} = \frac{1+r_{0,t}}{1+r_{j,t}} \hat{\alpha}_j \left( e_{0,t} - \sum_{k=1}^{K-1} \frac{1+r_{0,t}}{1+r_{k,t}} e_{k,t} \hat{\alpha}_k \right)^{-1}, \quad j = 1, 2, \dots, K-1. \quad (14)$$

Note that the hedge-portfolio weights do not equal the basket weights, but instead allocate adjusting for interest rate differentials, thus mimicking forward positions.

### C. Calculating Profits and Risk

Assuming independence between the estimation errors of the basket weights and the innovations to the hard-currency exchange rates, the optimal expected excess returns can be calculated from (9), using the optimal portfolio weights in (14), as

$$\pi_{t+h|t} \equiv E_t[\pi_{t+h}] = m_{0,t} (1+r_{0,t}) \hat{\alpha}_K - (1+r_{K,t}). \quad (15)$$

Since the cross-currency risk is hedged away, and the weights are assumed to be constant through time, the only source of uncertainty in profits arise from the error term in the basket hedge, and estimation uncertainty in the basket weights. We therefore have the following result:

**Proposition 1:** The conditional variance of profits from holding the basket portfolio for  $h$  days is

$$\sigma_{t+h|t}^2 \equiv \text{Var}_t(\pi_{t+h}) = m_{0,t}^2 (1+r_{0,t})^2 \left( \text{tr} \left\{ \text{Var}_t(\hat{\alpha}) \Gamma_{t+h|t} \right\} + \sigma^2 \right), \quad (16)$$

where  $\text{tr}(\cdot)$  is the trace operator, and  $\Gamma_{t+h|t}^*$  is the *uncentered* time- $t$  second moment matrix of  $\tilde{\epsilon}_{t+h}$ , with  $\tilde{\epsilon}_{t+h}' = [e_{1,t+h} \ e_{2,t+h} \ \dots \ e_{K-1,t+h} \ 1]$ .

**Proof:** See the appendix

The conditional risk-return trade-off in the hedge position is naturally measured by a Sharpe-ratio giving the risk-adjusted return

$$\frac{\pi_{t+h|t}}{\sigma_{t+h|t}} = \left( \alpha_K - \frac{1+r_{K,t}}{m_{0,t}(1+r_{0,t})} \right) \text{tr}(\text{var}(\alpha)\Gamma_{t+h|t}^* + \sigma^2)^{-1/2}. \quad (17)$$

Notice that, contrary to standard practice in the construction of Sharpe ratios, we do not subtract the risk-free rate as the position is self-financed.

A stark conclusion emerges: if the weights are constant and are known, or can be estimated precisely with a long sample of past observations, profits from the basket hedge seem virtually riskless, as  $\text{var}(\alpha)$  will be relatively small.

Of course the above analysis obscures the fact that the basket weights might be changing over time. In this case, to correctly assess the risk in carrying out basket hedges, one must properly account for the time variability in the currency weights. Thus we now develop the basket hedge formulae with time-varying weights, and demonstrate that time-variability plays a crucial role in measuring conditional volatility.

### III. ALLOWING FOR TIME-VARYING PARAMETERS

#### A. Defining the Time-Varying Currency Basket

In some instances, the authorities managing basket pegs let the weights change over time in response to changes in economic fundamentals, speculative pressures, or in pursuit of policy objectives. Instead of (2), we then have

$$e_{0,t} = \alpha_{K,t} + \sum_{j=1}^{K-1} \alpha_{j,t} e_{j,t} + \epsilon_t, \quad (18)$$

and the econometric challenge is to model the time-varying parameter structure keeping the cointegration feature of the data intact.

Practitioners routinely respond to the time-varying weights feature by simply estimating rolling OLS regressions on a certain window of the past data (see J.P. Morgan, 1997, and Goldman Sachs, 1997). The result is potentially poor estimates of the weights, and worse, a grossly understated measure of uncertainty, as the time-variability in the weights is merely averaged out.



## B. Constructing the Basket Hedge with Time-Varying Parameters

In the following we make up for the deficiencies of the constant-weight analysis, and consider a proxy hedge which incorporates uncertainty arising from the time-variability of the basket weights.

Allowing for time-varying weights, the profit formula becomes

$$\pi_{t+h} = m_{0,t}(1+r_{0,t}) \left( \alpha_{K,t+h} + \sum_{j=1}^{K-1} \alpha_{j,t+h} e_{j,t+h} + \epsilon_{t+h} \right) - \sum_{j=1}^{K-1} m_{j,t}(1+r_{j,t}) e_{j,t+h} - (1+r_{K,t}), \quad (19)$$

which yields the following stochastic first-order conditions

$$E_t \left( (1+r_{0,t}) \alpha_{j,t+h} m_{0,t} - m_{j,t} (1+r_{j,t}) \right) = 0, \quad j = 1, 2, \dots, K-1. \quad (20)$$

As the  $\alpha_{j,t+h}$ 's are unknown at time  $t$ , we take conditional expectations of the first-order conditions and solve for  $m_{j,t}$  to get the (imperfect) hedge portfolio weights,

$$m_{j,t} = \frac{1+r_{0,t}}{1+r_{j,t}} \hat{\alpha}_{j,t+h|t} m_{0,t}, \quad j = 1, 2, \dots, K-1, \quad (21)$$

where  $\hat{\alpha}_{j,t+h|t}$  is the estimated time- $t$  conditional expectation of  $\alpha_{j,t+h}$ . Again we use the accounting equation (12) to get

$$m_{0,t} = \left( e_{0,t} - \sum_{k=1}^{K-1} \frac{1+r_{0,t}}{1+r_{k,t}} e_{k,t} \hat{\alpha}_{k,t+h|t} \right)^{-1}, \quad (22)$$

and consequently to obtain the optimal weights

$$m_{j,t} = \frac{1+r_{0,t}}{1+r_{j,t}} \hat{\alpha}_{j,t+h|t} \left( e_{0,t} - \sum_{k=1}^{K-1} \frac{1+r_{0,t}}{1+r_{k,t}} e_{k,t} \hat{\alpha}_{k,t+h|t} \right)^{-1}, \quad j = 1, 2, \dots, K-1. \quad (23)$$

Assuming independence between the future cross-currency exchange rates and the future innovations to the basket weights, the expected profit formula is similar to that in the constant weight case

$$\pi_{t+h|t} = m_{0,t}(1+r_{0,t}) \hat{\alpha}_{K,t+h|t} - (1+r_{K,t}). \quad (24)$$

But the conditional profit variance now reflects the basket weight uncertainty, and we must find

$$\sigma_{t+h|t}^2 = m_{0,t}^2(1+r_{0,t})^2 \text{Var}_t \left( \sum_{j=1}^{K-1} (\alpha_{j,t+h} - \hat{\alpha}_{j,t+h|t}) e_{j,t+h} + (\alpha_{K,t+h} - \hat{\alpha}_{K,t+h|t}) + \epsilon_{t+h} \right), \quad (25)$$

which will depend on the particular parametric model assumed for the  $\alpha$ 's. In the next section, we consider some candidates, and work out the corresponding variance expressions.

### C. Models with Time-varying Parameters

We will use the flexible state space representation as a convenient framework for modeling the time-variability in the basket weights. The measurement equation is provided directly by the definition of the basket currency,

$$e_{0,t} = \alpha_{K,t} + \sum_{j=1}^{K-1} \alpha_{j,t} e_{j,t} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2), \quad (26)$$

The evolution of the vector of parameters  $\alpha \equiv [\alpha_1, \dots, \alpha_K]$ , i.e., the transition equation, will be written as

$$\alpha_t = \mu + T\alpha_{t-1} + v_t, \quad v_t \sim N(0, \sigma^2 Q), \quad (27)$$

taking the initial conditions of the parameter vector to be  $\alpha_0 \sim N(0, \sigma^2 P_0)$ , and defining the variance matrix of the parameters at time  $t$  as  $\sigma^2 P_t$ .

Within this general class of Time-Varying Parameter (TVP) models, probably the most standard and simple specification is the ‘‘adaptive regression’’ approach introduced by Cooley and Prescott (1973). They assume  $T=I$ , the identity matrix, a zero drift term,  $\mu=0$ , and let  $Q$  be a diagonal matrix with each element representing the relative variability of the corresponding regression coefficient. According to their specification, the regression coefficients evolve as independent random walk processes without drifts. Notice also that the standard constant-parameter case emerges if all the elements in  $Q$  equal zero.

The general specification of the time variation in the basket weights, together with the assumption that the hard-currency cross-exchange rates behave like driftless random walks, will be applied to construct measures of expected profits and risk of basket hedge portfolios. Other specifications for the time-varying parameters could be considered. For example, if the weights jump repeatedly, the regime-shift model (Hamilton, 1994) is an obvious competitor. Due to the institutional features of the data analyzed in the empirical investigation presented in Section 4, we will focus on relatively smoothly evolving TVP models. In this regard, it is important to emphasize that the basket weights in our model are measured directly in the hard

currencies to which they pertain. Thus a large simultaneous negative shock to the weights in the basket corresponds to a sharp depreciation of the basket vis-à-vis the hard currencies. In this way, the multi-shock feature of the model is potentially capable of generating large movements in the basket rate.

The optimal expected excess return formula is similar to the constant parameter case, where  $\hat{\alpha}_{K,t+h|t}$  is the estimated weight obtained from the Kalman filter. The conditional profit variance now reflects the basket weight uncertainty, and we have:

$$\pi_{t+h|t} = m_{0,t}(1+r_{0,t})\hat{\alpha}_{K,t+h|t} - (1+r_{K,t}). \quad (28)$$

**Proposition 2:** Assuming independence between the innovations to the weights and the innovations to the hard currency exchange rates, the conditional variance of the h-period basket hedge is

$$\sigma_{t+h|t}^2 = m_{0,t}^2(1+r_{0,t})^2 \sigma^2 \left( \text{tr} \left\{ \mathbf{P}_{t+h|t} \mathbf{\Gamma}_{t+h|t} \right\} + 1 \right). \quad (29)$$

where  $\text{tr}\{\cdot\}$  is the trace operator,  $\sigma^2 \mathbf{P}_{t+h|t}$  is the h-step-ahead signal extraction error variance matrix from the Kalman filter, and  $\mathbf{\Gamma}_{t+h|t}$  is the *uncentered* time-t second moment matrix of  $\tilde{\mathbf{e}}_{t+h}$ , with  $\tilde{\mathbf{e}}_{t+h}' = [e_{1,t+h} \ e_{2,t+h} \ \dots \ e_{K-1,t+h} \ 1]$ .

**Proof:** See the appendix.

The Kalman recursions readily yield the necessary expression for  $\mathbf{P}_{t+h|t}$ , as

$$\mathbf{P}_{t+h|t} = \mathbf{T} \mathbf{P}_{t+h-1|t} \mathbf{T}' + \mathbf{Q}, \quad \text{with } \mathbf{P}_{t+1|t} = \mathbf{T} \mathbf{P}_t \mathbf{T}' + \mathbf{Q}.$$

Even though  $\mathbf{\Gamma}_{t+h|t}$  might contain conditional heteroskedasticity, it will probably be safe to ignore it in most empirical applications. The value for h is typically at least 30 days, thus the h-step-ahead forecast for the conditional centered second moment should be close to its unconditional mean, even for relatively persistent volatility series.  $\mathbf{\Gamma}_{t+h|t}$  will still be time-varying by the fact that it contains the (square of the) exchange rate conditional means.

The interaction between the cross exchange rate moments and the innovations to the weights in the term  $\mathbf{P}_{t+h|t} \mathbf{\Gamma}_{t+h|t}$  is potentially very powerful for measuring the risk of basket hedges. Notice that this interaction emerges even under the assumption of independence between the innovations to the weights and the currencies.

The risk-return trade-off can again be measured by the Sharpe ratio

$$\frac{\pi_{t+h|t}}{\sigma_{t+h|t}} = \left( \hat{\alpha}_{K,t+h|t} - \left( \frac{1+r_{K,t}}{m_{0,t}(1+r_{0,t})} \right) \right) \sigma^{-1} \left( \text{tr} \left\{ \mathbf{P}_{t+h|t} \mathbf{\Gamma}_{t+h|t} \right\} + 1 \right)^{-1/2}, \quad (30)$$

which will enable investors to compare basket trades to other investment opportunities with different risk and return profiles.

#### IV. AN APPLICATION TO THE THAI BAHT BASKET

The focus of this empirical investigation is the Thai baht, but several other applications could be considered. At the end of December 1996, at least thirty countries were officially reported to be pegging their exchange rate to a basket of hard currencies (see International Monetary Fund, 1997). Other currencies, such as the Singapore dollar and the South African rand, even if not officially pegged, were informally known to be managed tightly. From 1985 until its suspension on July 2, 1997, following a speculative attack, the baht was pegged to a basket of currencies consisting of Thailand's main trading partners. In order to gain greater discretion in setting monetary policy, the Bank of Thailand followed a policy of neither disclosing the currencies in the basket nor their weights.

According to the institutional setting for the baht, the Bank of Thailand announced the baht/dollar intervention rate every morning, and defended it until noon. In the afternoon, the baht traded freely. The exchange rate policy resulted in a long period of significant stability of the baht/dollar rate (Figure 1).

In the remainder of this section, we will first highlight general econometric issues arising when estimating the unknown time-varying basket weights; second, the results from different approaches to estimating the basket weights of the Thai baht will be presented; and, third, measures of profitability and risk of baht basket hedges, constructed from the various models of the basket weights, will be compared and discussed.

##### A. Basket Exchange Rates and Nonstationarity: Specification and Estimation Issues

If the cross-exchange rates of the hard currencies are integrated of order one,  $I(1)$ , and not cointegrated, and the basket weights are constant, then the basket exchange rate will also follow an  $I(1)$  process. If the basket peg determines only an intervention rate, i.e., not the actual exchange rate, as in the Thai baht case, then the deviation of the observed basket exchange rate from the intervention rate will follow a stationary process. In this case, consistent and efficient estimates of the parameters of interest (i.e., the basket weights) can be obtained by employing cointegrating regression methods (Phillips and Hansen, 1990, and Park, 1992).

However, if the weights of the basket change over time according to a random walk, then the basket exchange rate will follow an  $I(2)$  process. To see this, let  $T=I$ ,  $\mu=0$ , and  $u_{k,t} \equiv \Delta e_{k,t}$ , where  $\Delta$  denotes the first-difference operator. Then equation (26) can be rewritten (see Harvey, 1989, p. 45) as

$$e_{0,t} = \frac{v_{K,t}}{\Delta} + \sum_{k=1}^{K-1} \frac{v_{k,t}}{\Delta} \frac{u_{k,t}}{\Delta} + \epsilon_t$$

which implies that stationarity in the left hand side is achieved only after differentiating the relationship twice. In this case, application of neither standard cointegration techniques nor

standard TVP techniques (Granger and Swanson, 1997; Stock and Watson, 1998) warrants success.

In order to cope with the time-variability in the baht basket weights, we apply the general *TVP cointegration* model outlined in Section 3.3, which can be estimated using Kalman filter recursions. The application of the Kalman filter to estimate cointegrating regressions in the context of TVP models was originally suggested by Granger (1986) and has been applied by Canarella, Pollard and Lai (1990) to test purchasing power parity.

## **B. Empirical Results: Estimation of the Baht Basket Weights**

The focus of this section is on the estimation of the time-varying Thai baht basket weights for the period January 2, 1992 to February 12, 1997, containing 1329 daily observations. Thus far we have assumed that the currencies belonging to the baht basket arrangement are known. In the remainder, we will rely on the assumption (confirmed by numerous press accounts and informal conversations with Bank of Thailand officials) that these currencies are the U.S. dollar, the Japanese yen and the German mark. Other studies focusing on the Thai baht, see for example J.P. Morgan (1997) and Goldman Sachs (1997), have similarly assumed that these are the currencies belonging to the baht basket.

### **1. Univariate Properties of the Data**

To confirm the conjectured presence of nonstationarity in the exchange rates, unit root tests were performed on the levels of the individual exchange rates belonging to the baht basket, (the mark/dollar and the yen/dollar rates—see Figures 2 and 3) and on the level of the baht/dollar rate itself. Table 1 presents the results of Augmented Dickey-Fuller (ADF) and Phillips and Phillips-Perron  $Z_a$  and  $Z_t$  unit root tests and their critical values. All the test statistics fail to reject the null hypothesis of a unit root at the 5 percent significance level for all exchange rates. Nonstationarity is also detected when using test statistics which take stationarity as the null hypothesis. Table 2, which presents the results of the J- and G-tests of Park and Choi for different assumptions about deterministic components under the null and the alternative hypotheses, suggests that the hypothesis of stationarity is strongly rejected for all exchange rates, including the baht/dollar rate. Furthermore, for the three exchange rates under consideration, unity is always contained in the 95 percent confidence intervals around the median unbiased estimates of the largest autoregressive root, computed as in Stock (1991).

The last step in the characterization of the univariate properties of the hard cross-exchange rates belonging to the baht basket is the fitting of different autoregressive-moving average models and the ranking of these against a driftless random walk benchmark. According to the data-based procedure adopted, which is based on a posterior odds criterion (PIC) (see Phillips and Ploberger, 1994), both the yen/dollar and mark/dollar exchange rates follow a driftless random walk. When applying the same data identification procedure to the baht/dollar rate, the PIC chooses an AR(3) model with no deterministic components and with

the largest AR root equal to one. The presence of high order dynamics is expected for this time series if it is sampled from a time-varying distribution.

## 2. Stability of Full-sample, Constant-weights Cointegrating Regressions

The second step in the empirical analysis is to test for the presence of instability in the cointegrating relationship linking the baht/dollar rate to (a constant plus) the yen/dollar and the mark/dollar exchange rates. If indeed the policy of the Bank of Thailand was to vary the basket weights over time, we should be able to detect such time-variation from the exchange rate data. For this purpose, the following full-sample cointegrating regression

$$e_{us/th,t} = \alpha_{us} + \alpha_{gm} e_{us/gm,t} + \alpha_{jp} e_{us/jp,t} + \epsilon_t \quad (31)$$

was estimated using three different methods: OLS, Phillips and Hansen's (1990) Fully Modified-OLS (FM-OLS), and Park's (1992) Canonical Cointegrating Regression (CCR). Both FM-OLS and CCR are superior methodologies to straight OLS because they do not suffer from second-order biases and, unlike OLS, allow for standard asymptotic Gaussian inference even in the presence of endogeneity and autocorrelation in the estimated residuals. Table 3 presents the estimation results for the period January 2, 1992 to February 12, 1997. The OLS estimates are practically identical to both FM-OLS and CCR estimates, suggesting an extremely low degree of endogeneity. All coefficients are positive and significant at the usual 5 percent significance level. The constant term represents the amount of dollars in the baht basket, and the slope coefficients give the units of marks and yen entering in the basket. Recall that the weights are measured in hard-currency units. Thus a U.S. dollar weight of approximately 3.4 cents, and a dollar per basket exchange rate of about 4 cents, indicates that 85 percent of the basket consisted of dollars, confirming the common wisdom that the baht was predominately tied to the U.S. dollar.

The residual unit-root test presented in Table 3 provides support for the presence of cointegration among the three exchange rates. Furthermore, it suggests that all the relevant exchange rates have been included in the basket regression, as an omitted random walk variable would likely appear as a unit root in the error term. In order to assess the adequacy of the constant-weight specification, we now test for the presence of instability in the cointegration regressions.

Hansen (1992) proposed three tests of stability of a cointegrating regression: the  $L_c$ , the Mean-F, and the Sup-F tests. All three tests have the same null hypothesis, i.e., that the sample sequence of the cointegrating parameters  $\alpha_t = [\alpha_{1,t} \dots \alpha_{K,t}]$  (for some  $T_0$  and  $T_1$  such that  $T_0 < t < T_1$ ) is constant, but the  $L_c$  and the Mean-F are designed against a different alternative than the Sup-F test. Under the alternative of the  $L_c$  and Mean-F tests, the cointegrating vector follows a smooth martingale process. In the case of the Sup-F test, the alternative hypothesis is an abrupt shift in regime. In practice, however, all the tests will tend to have power in similar directions. The Mean-F and Sup-F tests are obtained by time-averaging and taking the sup of a so-called F-sequence, which is simply a recursive F-type Chow test of structural stability computed over the  $[T_0, T_1]$  region using the recursive FM-OLS estimator. Figure 4 plots the calculated F-sequence along with the 5% percent

asymptotic critical values of the Sup-F and Mean-F statistic respectively. When comparing the F-sequence to either critical value, the null hypothesis of parameter constancy in the estimated cointegrating regression is soundly rejected. The same conclusion arises from the  $L_c$  test value reported in Table 3.

### 3. Rolling Cointegrating Regressions

A simple first approach to incorporating time variation in the basket weights is to estimate the cointegrating equation (33) over a rolling sample of fixed length. This approach is often recommended by investment banks (see, for example, J.P. Morgan, 1997). The rolling regression methodology, however, captures only the *average* time variation in the basket weights, and the estimated parameters depend crucially on the length of the estimation window. Objective statistical criteria to guide the choice of the size of such window are not available, thus any choice is arbitrary. Figures 5, 6 and 7 present rolling FM-OLS estimates ('Rolling') of the constant term and slope coefficients of equation (33), for a window of 25 percent of the available observations (roughly corresponding to 300 observations). We also show the weights estimated from an OLS recursion on the entire past sample ('Recursive'). In each iteration, we reestimate all parameters, and select a new bandwidth following the data-based procedure of Andrews and Monahan (1992), and imposing a Bartlett kernel smoother. The plots clearly confirm the presence of significant time variation in the baht basket weights. In the case of the German mark, the rolling regression procedure yields negative estimates of the weights in some instances.

### 4. Cointegrating Regressions in a TVP Modeling Framework

In an attempt to overcome the arbitrariness inherent to the rolling regression approach, we estimate the cointegrating regression (33) within the simple TVP model framework presented in Section 3.3 (equations 26 and 27). This approach allows the data to reveal the degree of time variation in the basket weights, and provides more accurate measures of the conditional variance.

Before proceeding with the illustration of the estimation results of this model, a couple of computational issues warrant mention. In order to initialize the recursion, the variance of the prior distribution,  $P_0$ , must be provided. We follow the common strategy of estimating the variance from the first  $K$  observations of the sample which are then discarded. Also, before proceeding to the estimation, conjectures must be made on the structure of the transition matrix,  $T$ , of the basket weights, and on the variance-covariance matrix of the innovation to the basket weights,  $Q$  (see equation 27). In the first specification estimated we fixed both matrices to  $I$ , the identity matrix. This specification is the simplest possible, but is highly restrictive, because it imposes zero correlation and variance equalization across the innovations to the weights. The second specification, while maintaining the assumption  $Q=I$ , imposes a diagonal structure for the  $T$  matrix, and estimates the diagonal elements of  $T$  by ML. Because the estimated autoregressive parameters of the basket weights turned out to be equal to 1 to the third decimal, we proceeded under the assumption that the basket weights evolved according to random walks (i.e.,  $T=I$ ), but calibrated the  $Q$  matrix to the data. To

ensure a fair out-of-sample model comparison, the Q matrix was computed recursively each day by taking the sample variance-covariance matrix of the first difference of the past weights estimated by recursive OLS, and dividing it by the square of the OLS standard error of regression. (The first three hundred estimated weights were discarded to remove the effect of the initialization of the OLS recursions on the computation of Q). Notice that this procedure allows for contemporaneous correlations between basket weight innovations, i.e., the off-diagonal elements of the Q matrix are not restricted to be equal to zero. The average of the recursive estimates of  $\sigma^2Q$  is shown in Table 4.

Alternative procedures to calibrate the Q matrix could be considered. Notice, however, that the standard practice of approximating Q with the inverse of the sample mean of the time-t cross-product (moment) matrix of the vector of the right-hand-side variables (see, e.g., Stock and Watson, 1996) could not be followed in the present context because of the nonstationarity of the right-hand-side variables.

Figure 5, 6, and 7 include plots of the estimated basket weights for the time-varying model with Q calibrated recursively from past data ('TVP-Q'). Notice that the variability of the estimated weights differs sensibly across estimation methods. The most variable sequences of weights are estimated by the TVP-Q model, followed by the rolling, with the recursive weights being the smoothest.

### C. Ex-Post Evaluation of Ex-Ante Risk and Return Measures

The lack of broadly accepted statistical tools to evaluate time-varying regression models prompts us to carry out an evaluation of the performance of the estimated models in terms of the end-goal of profitability of the basket hedges executed from the estimated weights. Once estimates of the baht basket weights are available, profitability and risk measures of basket hedges can be computed by applying the formulae introduced in Section 3. Ex-ante profits (or excess returns) from 30- and 90-day basket hedges, as well as risk measures and risk-adjusted profitability indices were computed using daily observations on nominal interbank interest rates on dollar, mark, yen and baht deposits. A common source for interest rate and exchange rate data was used (Bloomberg, L.P.—the mnemonics and the raw data can be obtained from the authors), to guarantee their proper alignment. The period under analysis is April 1993 to November 1996.

In order to compare the different ex-ante measures of profits (or excess returns) to their ex-post realizations we computed the realized profits from method 'x' as

$$\pi_{t+h}(x) = m_{0,t}(x)(1+r_{0,t})e_{0,t+h} - \sum_{j=1}^{K-1} m_{j,t}(x)(1+r_{j,t})e_{j,t+h} - (1+r_{K,t}), \quad (32)$$



where  $x = \text{'Recursive'}$ ,  $\text{'Rolling'}$ , and  $\text{'TVP-Q'}$ . This measure provides an estimate of the realized excess return on an  $h$ -day basket hedge for an exposure in currency 0 of relative size  $m_{0,t}$ .

Along with the average ex-post profits,

$$\text{Ex-post}(x) = \frac{1}{T-h} \sum_{t=1}^{T-h} \pi_{t+h}(x), \quad (33)$$

we computed the ex-post profit bias for each method as

$$\text{Bias}(x) = \frac{1}{T-h} \sum_{t=1}^{T-h} (\pi_{t+h}(x) - \pi_{t+h|t}(x)). \quad (34)$$

For each method, we also computed the mean squared error over time by

$$\text{MSE}(x) = \frac{1}{T-h} \sum_{t=1}^{T-h} (\pi_{t+h}(x) - \pi_{t+h|t}(x))^2, \quad (35)$$

in order to infer about the variability of the realized profits around their expected values.

A good estimation method for the basket weights should produce a high  $\text{Ex-post}(x)$ , a  $\text{Bias}(x)$  close to zero, and a low  $\text{MSE}(x)$ . The estimated values of these statistics are presented in Table 5 for 30- and 90-day holding periods. The following results are apparent:

First, the ex-post profits are roughly equal across methods.

Second, for all estimated weights the bias is negative, meaning that profits are expected to be higher than their realizations. By far, the smallest bias is achieved by the TVP model. The rolling and recursive methods both report biases notably larger in magnitude. This is true both in the 30- and 90-day case.

Third, in terms of MSE, the worst performer is the full sample recursion for both holding periods. The rolling model has the lowest mean squared error in both cases.

Fourth, the ex-ante standard deviations differ considerably across methods. For both investment horizons, TVP-Q has by far the highest standard deviation, followed by the recursive, with the rolling method reporting the lowest standard deviation, and thus appearing the least risky ex-ante. Comparing the mean ex-ante standard deviations with the standard deviations of the ex-post profits, the rolling tend to be too low ex-ante, and the TVP-Q too high ex-ante. Thus the rolling tend to understate risk, while the TVP-Q tends to exaggerate it.

Fifth, the combination of relatively high ex-ante conditional profits and low conditional standard deviations causes the recursive and rolling models to report very large conditional Sharpe Ratios. The TVP-Q Sharpe-ratio seems more reasonable (close to one). A crude measure of the ex-post ratio can be obtained by dividing the mean of the ex-post profits

by its standard deviation for each method. The ex-post Sharpe Ratio is around one for the one-month holding period, and around 2.5 for the three-month holding period. Thus the rolling and recursive methods give excessively confident risk-adjusted measures, whereas the TVP method tends to be on the cautious side.

Finally, in terms of a purely statistical fit, the TVP model seems to dominate. When computing the 1-step-ahead out-of-sample  $R^2$  of the dollar per baht exchange rate for the three models, the TVP model reports .99, the recursive, .93, and the rolling, .59.

From the results in Table 5 and supporting statistical evidence we conclude that it seems worth the effort to estimate TVP-models: their ex-ante measures are close to unbiased estimates of the ex-post realizations, their adjusted risk measures are on the cautious rather than confident side, and a better out-of-sample statistical fit is obtained.

To conclude the empirical section, let us try to put the in-sample results in the perspective of the subsequent currency crisis. Speculation against the baht in the period following the end of our sample period forced the Bank of Thailand to end the basket regime on July 2, 1997. As a consequence, the dollar per baht rate dropped 9.8 percent on that day. A natural question arises: What would have happened to an investor who continued carrying out basket trades until July 2? The capital controls imposed by the Bank of Thailand on May 15, 1997, induced a two-tier exchange rate system, and render precise calculations impossible, but the following approximations give some guidance.

Recall that the average ex-post profits in Table 5 were around 5.5 percent per year, corresponding to .021 percent per trading day (using 250-day compounding) in the sample period. Therefore, it would take 445 trading days—21 months—of average profits to make up for the loss on July 2.

The average annualized ex-ante standard deviation is reported for each model in Table 5. Rough estimates of the average daily ex-ante standard deviation can be computed by dividing the average ex-ante standard deviations in Table 5 by  $\sqrt{250}$ . We can then divide the 9.8 percent loss on July 2 with the daily ex-ante standard deviations to get an assessment from each model of how extreme an event July 2 was. Performing such a calculation reveals that the crash represented a 29 standard deviation event in the rolling model, 50 in the recursive, and an 18 standard deviation event in the TVP-Q model. While all models render a 9.8 percent move very unlikely, the time-varying parameter model somewhat better captures the possibility of the event that unfolded on July 2.

## V. SUMMARY AND EXTENSIONS

We have explicitly laid out the mechanics of currency basket hedging from a time-series modeling perspective allowing us to compute conditional risk and return. The exact hedging position is worked out both in the case of a constant and a time-varying basket. The optimal weights in the hedge are found to depend on the conditional expectation of the

currency basket weights corrected for interest rate differentials. We compute exact analytical formulae for ex-ante profit, as well as ex-ante conditional variances of the profit in the constant and in the time-varying basket cases. Risk-adjusted returns are computed using standard Sharpe ratio measures.

To investors, proper ex-ante risk and return measures are of paramount importance. Monetary authorities should be equally interested. High ex-ante risk-adjusted basket hedge returns can serve as a warning signal of increases in short-term capital inflows which, in turn, increase the money base and cause inflationary pressures.

Over the last five years huge volumes of profitable arbitrage deals, seemingly containing very low risk, have been undertaken in the Thai baht basket. In the empirical application, we tried to shed some light on the measurement of profits and risks associated with such deals. We found a substantial advantage in applying time-varying parameter models over standard rolling regression methods when computing accurate ex-ante measures of risk and return. Furthermore, the risks associated with basket hedges were estimated to be significant in the context of TVP regression models, while they seemed severely understated with simple rolling regression methods.

Several issues are left for future research. In the above analysis, the evolution of the basket weights takes place in a purely statistical, reduced-form model. Combining the hedging analysis with an economic model of the basket weights based on macroeconomic fundamentals could be interesting and useful. On the technical side, relaxing the independence assumption between future innovations to the basket weights and the cross currency rates might be more realistic. Finally, developing a valid inference strategy for variable inclusion in TVP models would be beneficial. We leave these issues for future work.

Table 1. Unit Root Tests on the Level of the Exchange Rates:  
Null Hypothesis Is Unit Root  
Daily Observations: January 1992 - February 1997 (1329 observations)

Statistic	5% Crit. Value	baht/dollar	mark/dollar	yen/dollar
ADF (w/ intercept)	-2.87	-1.4	-1.7	-2.1
Z <sub>a</sub> (w/ intercept)	-13.7	-3.9	-6.6	-2.3
Z <sub>t</sub> (w/ intercept)	-2.87	-1.0	-1.7	-1.1
Stock's (1991) 95% Confidence Interval		0.996 - 1.003	0.992 - 1.003	0.995 - 1.003

The table presents the values of the three unit root tests: the ADF (Augmented Dikey-Fuller) test, the Phillips and Phillips-Perron Z<sub>a</sub> and Z<sub>t</sub> statistics. A data-based methodology was employed to determine the amount of correction for autocorrelation in the residuals of the auxiliary regressions. The last row reports the 95 percent confidence interval around Stock's (1991) median unbiased estimate of the largest autoregressive unit root, based on the demeaned auxiliary regression,  $\tau^u$ , in Stock's terminology.

Table 2. Unit Root Tests on the Level of the Exchange Rates:  
Null Hypothesis Is Stationarity  
Daily Observations: January 1992 - February 1997 (1329 observations)

Number of deterministic components under:		J-Test			G-Test		
null	alternative	baht/ dollar	mark/ dollar	yen/ dollar	baht/ dollar	mark/ dollar	yen/ dollar
-1	2	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
-1	3	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01
-1	4	<0.01	<0.01	<0.01	<0.01	<0.01	<0.01

The table presents the p-values (marginal significance levels) of two tests for stationarity: Park-Choi's J- and G-statistics. Hence, low p-values imply rejection of the hypothesis of stationarity. These statistics are computed for different assumptions about the number of deterministic components present under the null and the alternative hypothesis. The case labeled "-1" refers to "no deterministic components"; "2" refers to "intercept, plus linear, plus quadratic trends"; and so forth. A data-based methodology was employed to determine the amount of correction for autocorrelation in the residuals of the auxiliary regressions.

Table 3  
Basket Currency Cointegrating Regression: Equation (33)  
Daily Observations: January 1992 - February 1997 (1329 observations)

	OLS Coefficient	FM-OLS Coefficient	T-stat	CCR Coefficient	T-stat	Stability Tests
dollar weight (intercept)	0.0353	0.0350	158.8	0.0350	158.7	
mark weight	0.0025	0.0025	6.5	0.0025	6.5	
yen weight	0.2890	0.2891	18.8	0.2891	18.8	
R <sup>2</sup>	0.80	0.80		0.80		
Residual ADF Test		-6.67		-6.67		
S.E.R.		0.0005		0.0005		
MeanF						91.5
SupF						108.1
L <sub>c</sub>						12.9

The 5 percent critical value of the residual ADF test is -2.71. MeanF, SupF and L<sub>c</sub> are Hansen's (1991) stability tests discussed in the text. Their 5 percent critical values are, respectively: 6.2, 14.8 and .69. A data-based methodology was employed to determine the amount of correction for autocorrelation in the residuals of the cointegrating regressions.

Table 4. Average Recursive Estimate of  $\sigma^2Q$

	Mark	Yen	Dollar
<b>Mark</b>	0.0000000168		
<b>Yen</b>	-0.0000009981	0.0001668932	
<b>Dollar</b>	-0.0000000017	-0.0000007656	0.0000000089

Please see Section IV for a description of the numbers in this table.

Table 5. Summary Statistics of Profits and Sharpe Ratios  
 April 26, 1993 - November 4, 1996.  
 (In percent per year)

	1-month horizon			3-month horizon		
	Recursive	Rolling	TVP-Q	Recursive	Rolling	TVP-Q
<b>Profits</b>						
<b>Ex-ante</b>						
Mean	10.6	8.2	5.8	7.8	7.0	5.8
StDev	4.7	3.6	1.6	2.3	2.0	1.6
<b>Ex-post</b>						
Mean	5.7	5.5	5.6	5.5	5.4	5.4
StDev	4.3	4.4	5.4	2.2	2.2	2.7
<b>Bias (=Ex-post - Ex-ante)</b>						
Mean	-4.9	-2.7	-0.3	-2.3	-1.7	-0.5
StDev	4.6	4.1	4.9	1.8	1.7	2.6
<b>Squared Bias</b>						
Mean (=MSE)	45.2	24.3	24.4	8.3	5.8	6.7
StDev	74.4	78.2	63.5	12.0	11.8	11.5
<b>Risk measures</b>						
<b>Ex-ante standard deviation</b>						
Mean	5.3	3.1	8.6	1.8	1.1	5.1
StDev	4.3	4.4	5.4	4.3	4.4	5.4
<b>Sharpe ratios</b>						
Mean	2.2	3.0	0.7	4.9	7.5	1.2
StDev	1.0	1.6	0.3	1.7	3.6	0.4

Please see Section IV for an explanation of the numbers in this table.

Chart 1

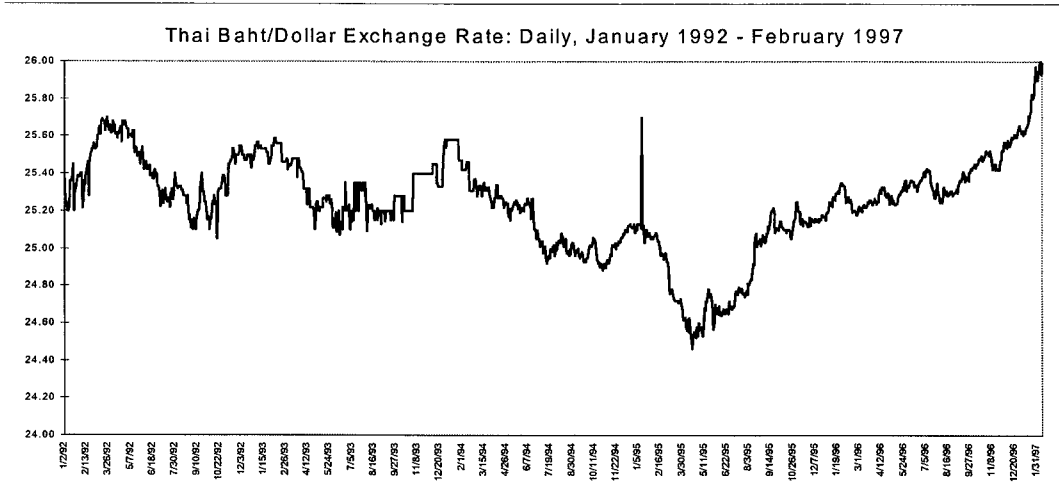


Chart 2

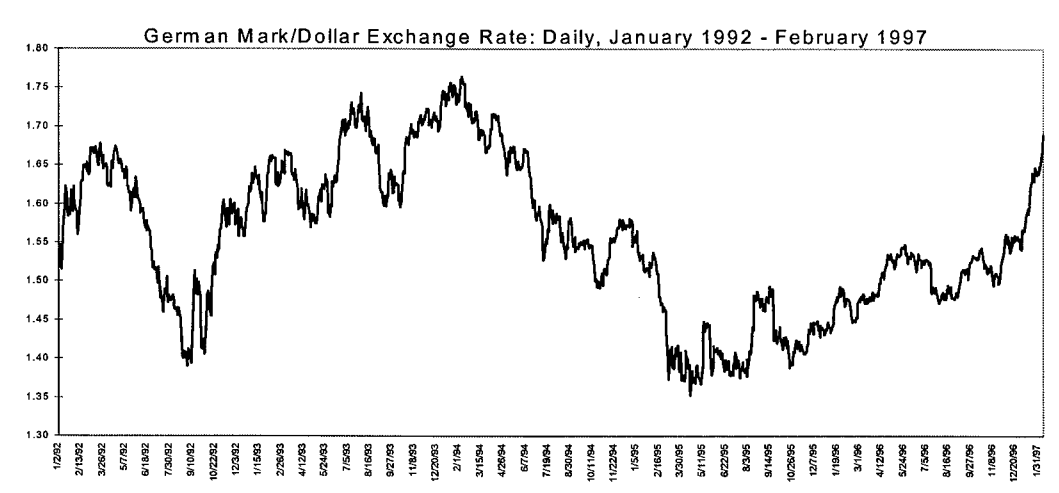


Chart 3

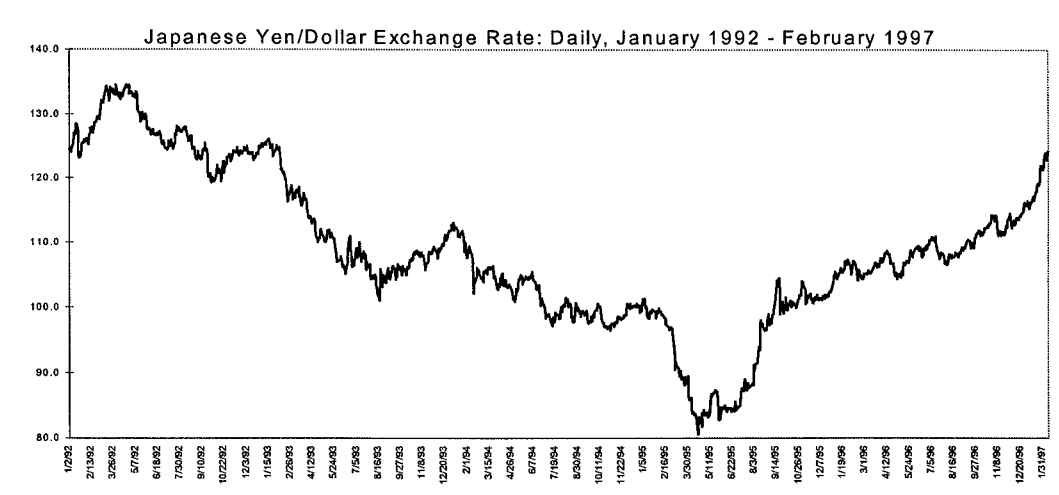
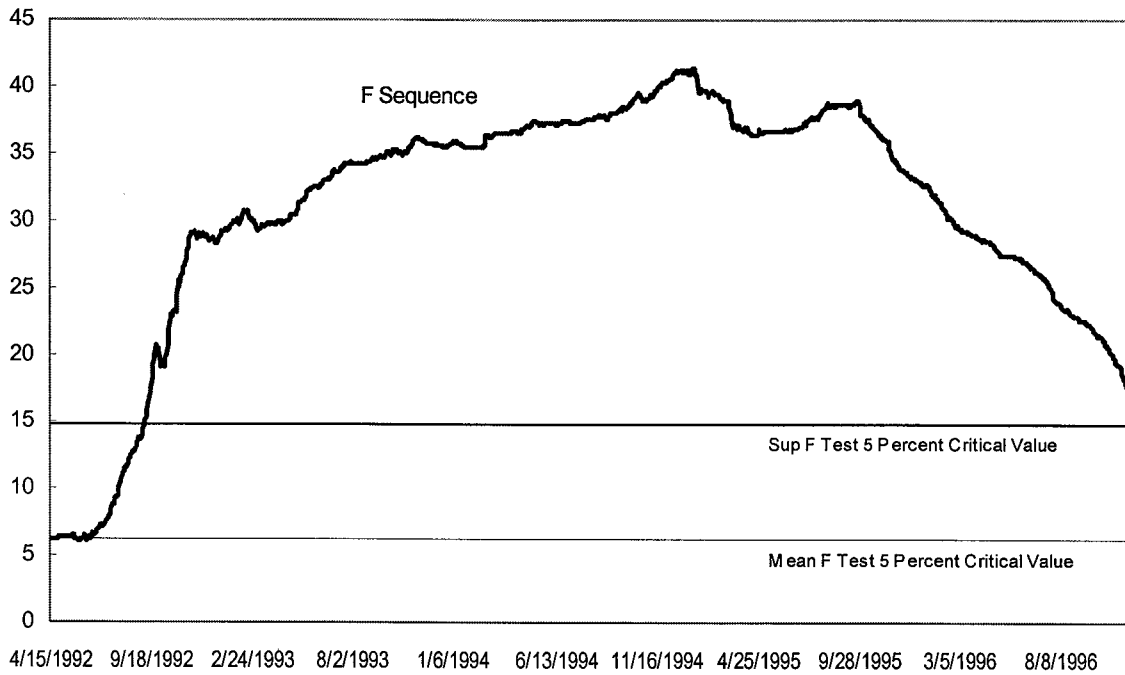


Figure 4. Hansen's F-sequence (Stability Test) and Critical Values



Notes to Figure: Plots the recursive F Sequence from Hansen's (1992) tests for parameter instability along with the 5 percent critical values for the Sup F-Test (thick horizontal line) and the Mean F-Test (thin horizontal line).



Figure 5. Time-varying Basket Weights: US Dollar

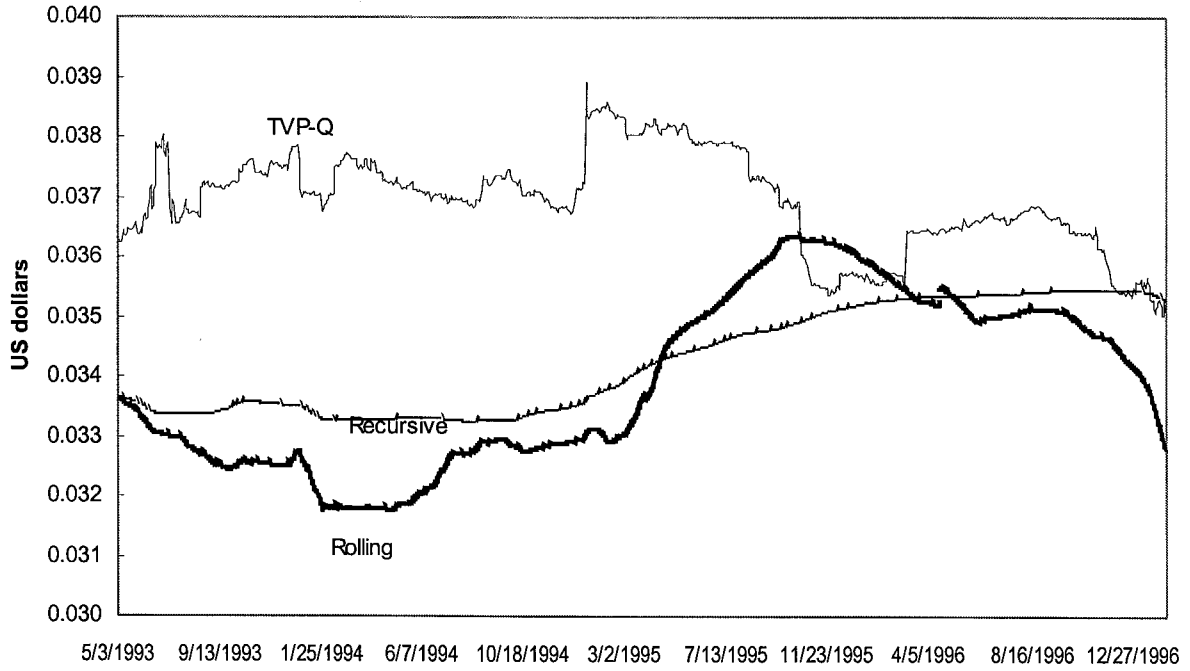


Figure 6. Time-varying Basket Weights: German Mark

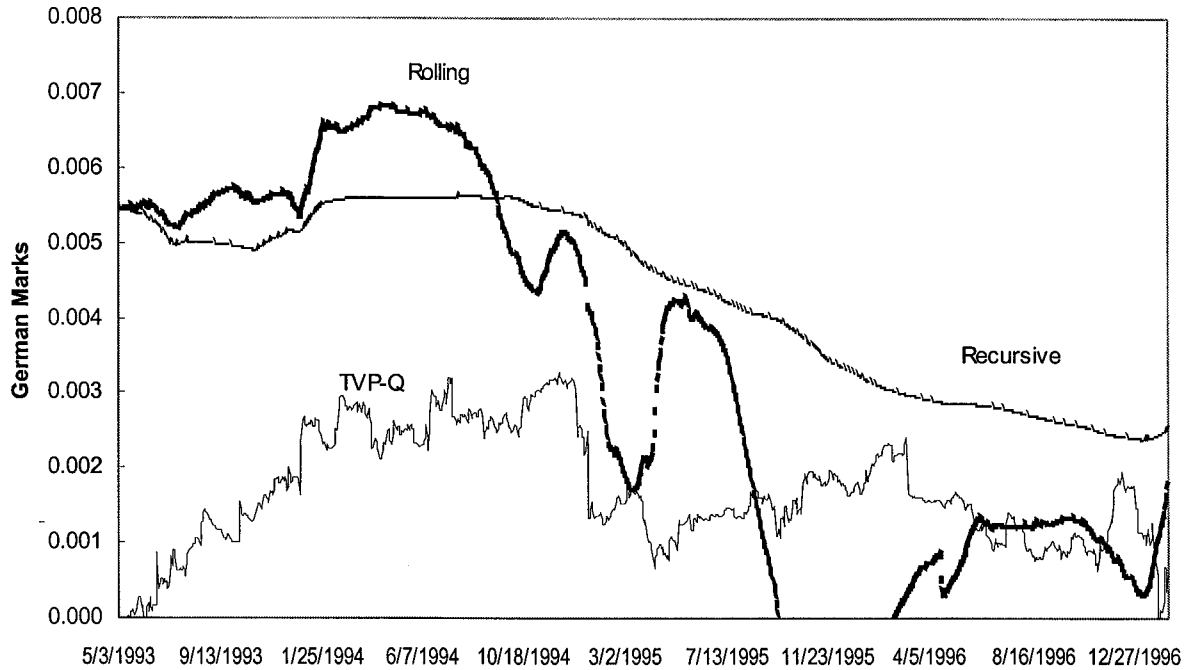
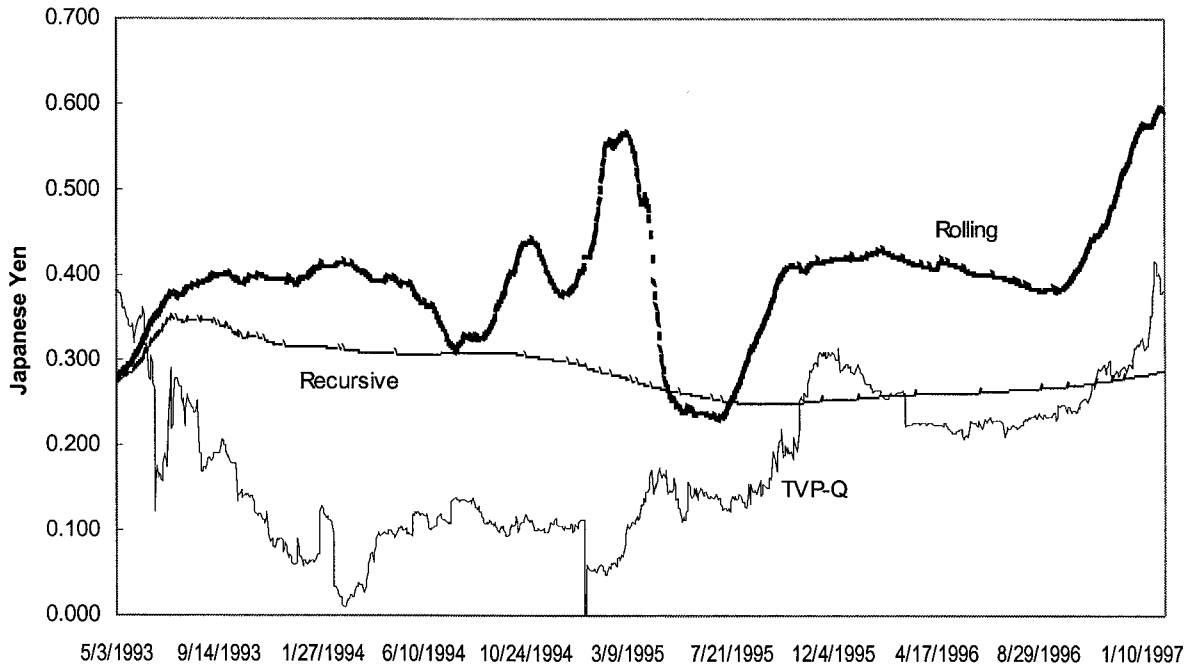


Figure 7. Time-varying Basket Weights: Japanese Yen



Notes to Figures 5-7: 'Rolling' refers to a rolling FM-OLS regression on the most recent 25 percent of the sample. 'Recursive' is an FM-OLS recursion using all past data.. TVP-Q is a time-varying parameter model with correlated innovations.

### A. PROOF OF PROPOSITION 1

We have that

$$\sigma_{t+h|t}^2 \equiv E_t \left[ \pi_{t+h} - \pi_{t+h|t} \right]^2 = E_t \left[ \mathbf{m}_{0,t} (1 + \mathbf{r}_{0,t}) \left( \sum_{j=1}^{K-1} (\alpha_j - \hat{\alpha}_j) \mathbf{e}_{j,t+h} + (\alpha_K - \hat{\alpha}_K) + \epsilon_{t+h} \right) \right]^2.$$

By the appropriate definitions, we can write

$$E_t \left( \sum_{j=1}^{K-1} (\alpha_j - \hat{\alpha}_j) \mathbf{e}_{j,t+h} + (\alpha_K - \hat{\alpha}_K) + \epsilon_{t+h} \right)^2 \equiv E_t \left( \tilde{\mathbf{e}}_{t+h}' (\alpha - \hat{\alpha}) + \epsilon_{t+h} \right)^2.$$

Since the  $\epsilon$  is assumed to be an iid innovation term we have

$$E_t \left( \tilde{\mathbf{e}}_{t+h}' (\alpha - \hat{\alpha}) + \epsilon_{t+h} \right)^2 = E_t \left( \tilde{\mathbf{e}}_{t+h}' (\alpha - \hat{\alpha}) \right)^2 + \sigma^2.$$

The square of the inner product, as it is a scalar, can be written as

$$\begin{aligned} E_t \left( \tilde{\mathbf{e}}_{t+h}' (\alpha - \hat{\alpha}) (\alpha - \hat{\alpha})' \tilde{\mathbf{e}}_{t+h} \right) &= E_t \left( \text{tr} \left\{ \tilde{\mathbf{e}}_{t+h}' (\alpha - \hat{\alpha}) (\alpha - \hat{\alpha})' \tilde{\mathbf{e}}_{t+h} \right\} \right) \\ &= E_t \left( \text{tr} \left\{ (\alpha - \hat{\alpha}) (\alpha - \hat{\alpha})' \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right\} \right) = \text{tr} \left\{ E_t \left( (\alpha - \hat{\alpha}) (\alpha - \hat{\alpha})' \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right) \right\}, \end{aligned}$$

where  $\text{tr}\{*\}$  is the trace operator. Relying on the assumed independence between the estimation errors of the  $\alpha$ 's and the innovations to the  $\mathbf{e}$ 's, and on the zero-mean property of the estimation errors, we can use the law of iterated expectations to write

$$\begin{aligned} \text{tr} \left\{ E_t \left( (\alpha - \hat{\alpha}) (\alpha - \hat{\alpha})' \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right) \right\} &= \text{tr} \left\{ E_t \left( E_t \left( (\alpha - \hat{\alpha}) (\alpha - \hat{\alpha})' \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \mid \tilde{\mathbf{e}}_{t+h} \right) \right) \right\} \\ &= \text{tr} \left\{ E_t \left( E_t \left( (\alpha - \hat{\alpha}) (\alpha - \hat{\alpha})' \mid \tilde{\mathbf{e}}_{t+h} \right) \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right) \right\} = \text{tr} \left\{ \text{Var}_t(\hat{\alpha}) E_t \left( \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right) \right\} \equiv \text{tr} \left\{ \text{Var}_t(\hat{\alpha}) \Gamma_{t+h|t}^* \right\}, \end{aligned}$$

where  $\Gamma_{t+h|t}^*$  is the *uncentered* time-t second moment matrix of  $\tilde{\mathbf{e}}_{t+h}$ . The result follows.

## B. PROOF OF PROPOSITION 2

We have that

$$\sigma_{t+h|t}^2 \equiv \mathbb{E}_t \left[ \pi_{t+h} - \pi_{t+h|t} \right]^2 = \mathbb{E}_t \left[ \mathbf{m}_{0,t} (1 + \mathbf{r}_{0,t}) \left( \sum_{j=1}^{K-1} (\alpha_{K,t+h} - \hat{\alpha}_{K,t+h|t}) \mathbf{e}_{j,t+h} + (\alpha_{j,t+h} - \hat{\alpha}_{j,t+h|t}) + \epsilon_{t+h} \right) \right]^2.$$

By the appropriate definitions, we can write

$$\mathbb{E}_t \left( \sum_{j=1}^{K-1} (\alpha_{K,t+h} - \hat{\alpha}_{K,t+h|t}) \mathbf{e}_{j,t+h} + (\alpha_{j,t+h} - \hat{\alpha}_{j,t+h|t}) + \epsilon_{t+h} \right)^2 \equiv \mathbb{E}_t \left( \tilde{\mathbf{e}}_{t+h}' (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) + \epsilon_{t+h} \right)^2.$$

Since the  $\epsilon$  is assumed to be an iid innovation term, we have

$$\mathbb{E}_t \left( \tilde{\mathbf{e}}_{t+h}' (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) + \epsilon_{t+h} \right)^2 = \mathbb{E}_t \left( \tilde{\mathbf{e}}_{t+h}' (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) \right)^2 + \sigma^2.$$

The square of the inner product, as it is a scalar, can be written as

$$\begin{aligned} & \mathbb{E}_t \left( \tilde{\mathbf{e}}_{t+h}' (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) (\alpha_{t+h} - \hat{\alpha}_{t+h|t})' \tilde{\mathbf{e}}_{t+h} \right) = \mathbb{E}_t \left( \text{tr} \left\{ \tilde{\mathbf{e}}_{t+h}' (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) (\alpha_{t+h} - \hat{\alpha}_{t+h|t})' \tilde{\mathbf{e}}_{t+h} \right\} \right) \\ & = \mathbb{E}_t \left( \text{tr} \left\{ (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) (\alpha_{t+h} - \hat{\alpha}_{t+h|t})' \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right\} \right) = \text{tr} \left\{ \mathbb{E}_t \left( (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) (\alpha_{t+h} - \hat{\alpha}_{t+h|t})' \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right) \right\}, \end{aligned}$$

where  $\text{tr}\{\cdot\}$  is the trace operator. Relying on the assumed independence between the innovations to the  $\alpha$ 's and the innovations to the  $\mathbf{e}$ 's, and on the zero-mean property of the innovations to the  $\alpha$ 's, we can use the law of iterated expectations to write

$$\begin{aligned} & \text{tr} \left\{ \mathbb{E}_t \left( (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) (\alpha_{t+h} - \hat{\alpha}_{t+h|t})' \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right) \right\} = \text{tr} \left\{ \mathbb{E}_t \left( \mathbb{E}_t \left( (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) (\alpha_{t+h} - \hat{\alpha}_{t+h|t})' \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \mid \tilde{\mathbf{e}}_{t+h} \right) \right) \right\} \\ & = \text{tr} \left\{ \mathbb{E}_t \left( \mathbb{E}_t \left( (\alpha_{t+h} - \hat{\alpha}_{t+h|t}) (\alpha_{t+h} - \hat{\alpha}_{t+h|t})' \mid \tilde{\mathbf{e}}_{t+h} \right) \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right) \right\} = \text{tr} \left\{ \sigma^2 \mathbf{P}_{t+h|t} \mathbb{E}_t \left( \tilde{\mathbf{e}}_{t+h} \tilde{\mathbf{e}}_{t+h}' \right) \right\} \equiv \sigma^2 \text{tr} \left\{ \mathbf{P}_{t+h|t} \mathbf{\Gamma}_{t+h|t} \right\}, \end{aligned}$$

where  $\sigma^2 \mathbf{P}_{t+h|t}$  is the  $h$ -step-ahead signal extraction error variance matrix. The result follows.

## References

- Andrews, D.W.K. and J.C. Monahan (1992), "Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator," *Econometrica*, 60, 953-66.
- Bhandari, J.S. (1985), "The Flexible Exchange Rate Basket: a Macroeconomic Analysis," *Journal of International Money and Finance*, 1, 19-41.
- Canarella, G., S.K. Pollard, and K.S. Lai (1990), "Cointegration between Exchange Rates and Relative Prices: Another View," *European Economic Review*, 34, 1303-1322.
- Cooley, T.F. and E.C. Prescott (1973), "An Adaptive Regression Model," *International Economic Review*, 14, 364-371
- Edison, H.J. and Vaardal, E. (1990), "Optimal Currency Baskets for Small, Developed Economies," *Scandinavian Journal of Economics*, 92, 559-571.
- Flanders, M.J. and E. Helpman, (1979), "An Optimal Exchange Rate Peg in a World of General Floating," *Review of Economic Studies*, 46, 533-542.
- Goldman Sachs (1997), *Emerging Debt Markets Biweekly*. Various Issues.
- Graf von der Schulenburg, J.M. (1984), "Who Wants to Go Shopping with a Currency Basket?" *Jahrbuch fur Nationalokonomie und Statistik*, 19, 65-75.
- Granger, C.W.J. (1986), "Developments in the Study of Cointegrated Economic Variables," *Oxford Bulletin of Economics and Statistics*, 48, 213-28.
- Granger, C.W.J., and N.R. Swanson (1997), "An Introduction to Stochastic Unit-root Processes," *Journal of Econometrics*, 80, 35-62.
- Hansen, B.E. (1992), "Tests for Parameter Instability in Regressions with I(1) processes," *Journal of Business and Economics Statistics*, 10.
- Harvey, A.C. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge: Cambridge University Press.
- Hamilton, J. D. (1994), *Time Series Analysis*, Princeton, NJ: Princeton University Press.
- Henriksson, R. D., and R.C. Merton, (1981), "On Market Timing and Investment Performance. II. Statistical Procedures for Evaluating Forecasting Skills," *Journal of Business*, 54, 513-33.
- Horngren, L. and A. Vredin (1989), "Exchange Risk Premia in a Currency Basket System," *Weltwirtschaftliches Archiv*, 311-325.

- International Monetary Fund (1997), "Exchange Arrangements and Exchange Restrictions," Annual Report 1997, Washington, DC: International Monetary Fund.
- J.P. Morgan (1997), *Asian Local Markets*. Various Issues.
- Klein, M. (1989), "Arbitrage and Interest Rates on Currency Baskets," *Weltwirtschaftliches Archiv*, 296-310.
- Klein, M. and S.M. Muller (1992), "ECU Interest Rates and ECU Basket Adjustments: an Arbitrage Pricing Approach," *Journal of Banking and Finance*, 16, 137-153.
- Leitch, G., and J. E. Tanner (1991), "Economic Forecast Evaluation: Profits versus the Conventional Error Measures," *American Economic Review*, 81, 580-90.
- Lipschitz, L. and V. Sundararajan (1980), "The Optimal Basket in a World of Generalized Floating," *IMF Staff Papers*, 27, 80-100.
- Park, J.Y. (1992), "Canonical Cointegrating Regression," *Econometrica*, 60, 119-44.
- Pesaran, M. H., and A.G. Timmermann (1994), "A Generalization of the Non-parametric Henriksson-Merton Test of Market Timing," *Economics Letters*, 44, 1-7.
- Pikkarainen, P. (1991), "International Portfolio Diversification: the Basket-Peg Regime," *Journal of International Money and Finance*, 10, 432-442.
- Phillips, P.C.B., and B.E. Hansen (1990), "Statistical Inference in Instrumental Variables Regression with I(1) Processes," *Review of Economic Studies*, 54, 99-125.
- Phillips, P.C.B. and W. Ploberger (1994), "Posterior Odds Testing for a Unit Root with Data-Based Model Selection," *Econometric Theory*, 10, 774-808.
- Stock, J.H., (1991), "Confidence Intervals for the Largest Autoregressive Root in U.S. Macroeconomic Time Series," *Journal of Monetary Economics*, 28, 435-459.
- Stock, J.H., and M.W. Watson (1996), "Evidence on Structural Instability in Macroeconomic Time Series Relations," *Journal of Business and Economic Statistics*, 14, 11-30.
- Stock, J.H., and M.W. Watson (1998), "Median Unbiased Estimation of Coefficient Variance in a Time Varying Parameter Model," *Journal of the American Statistical Association*, 93, 349-358.
- Turnovsky, S. (1982), "A Determination of the Optimal Currency Basket. A Macroeconomic Analysis," *Journal of International Economics*, 12, 333-354.