IMF Working Paper

WP/99/158

INTERNATIONAL MONETARY FUND

African Department

Linkages Among Asset Markets in the United States: Tests in a Bivariate GARCH Framework

Prepared by Salim M. Darbar and Partha Deb[1]

Authorized for distribution by Jürgen T. Reitmaier

November 1999

Abstract

This paper develops a bivariate GARCH model that allows for time-varying conditional correlations and simultaneous testing of two Granger-causal linkages: the impact of return volatility in a market on intermarket correlation and the impact of return volatility in one market on the volatility of another. Using daily data from stock, bond, currency, and commodity markets in the United States, the paper finds evidence of each form of linkage. Furthermore, the conditional correlations change over time and exhibit considerable persistence. The estimated time-varying conditional correlations provide insight into the nature of the stock market crash of 1987.

JEL Classification Numbers: C32, C51, G14

Keywords: Logistic Exponential GARCH, conditional correlation, Granger causality

Author's E-Mail Address: sdarbar@imf.org and pdeb@iupui.edu

---

[1] Partha Deb is an Associate Professor in the Department of Economics at Indiana University Purdue University Indianapolis. We would like to thank Charles Kramer for helpful comments.

Contents Page

I. Introduction

Since the U.S. equity market crash in October 1987, there has been a renewed interest in empirical and theoretical investigations of links between asset markets. However, while there are a number of studies of relationships among international stock markets and among foreign exchange markets, we know of no such investigations for the different asset markets within the U.S. In this paper we consider four major financial markets in the U.S.: the commodity, foreign exchange, bond and stock markets, and examine two linkages: (i) the impact of return volatility in a market on the correlation between that and another market, and (ii) the impact of volatility in one market on the volatility of another market.

There are a number of reasons for studying second-moment links between U.S. financial markets. Asset volatilities and cross-asset correlations are important, in general, because they affect the volatilities of portfolios and have implications for the CAPM. In particular, each of the four asset markets we have chosen is quite large and just as investors diversify internationally, they also diversify across assets within the U.S. Therefore the results of this study could be used to improve portfolio management strategies. More importantly, a study of links between U.S. asset markets has implications for regulatory policy. This feature is not shared by studies of international stock or foreign exchange markets because of the problem of policy coordination across national boundaries. In a single country, on the other hand, it is quite conceivable that a coordinated regulatory policy could be implemented across asset markets; such a policy might dominate a separate policy for each market. Finally, the analysis of links across markets in the U.S. is less likely to be contaminated by issues of market timing (the absence of synchronous trading) and heterogeneity caused by different holidays, cultural differences, etc. - features which are inevitably present in cross-country studies.

There is growing empirical evidence on the contemporaneous relationship between international stock return volatilities and associated correlation among pairs of returns. Bennett and Kelleher (1988) show that higher daily volatility within a month is associated with a larger cross-country correlation. King and Wadhwani (1990) find support for this hypothesis using hourly stock market returns in London, New York, and Tokyo from July 1987 to February 1988. King, Sentana and Wadhwani (1994) study sixteen national stock markets using daily data for the period January 1970 through October 1988. They also find a positive contemporaneous relationship between volatility and correlation. These studies examine contemporaneous relationships, whereas we examine a Granger-causal relationship from volatility to correlation.

The empirical evidence on volatility spillovers - a Granger-causal relationship from the volatility in one market to the volatility in another - is mixed. Hamao, Masulis, and Ng (1990) study stock markets in Tokyo, London, and New York using a univariate GARCH model. They find evidence of volatility spillovers in daily data for the period April 1, 1985 through March 31, 1988. Susmel and Engle (1994) find much less evidence of volatility spillovers. They attribute the strength of the Hamao, Masulis and Ng results to the standard errors of relevant parameters produced by a non-robust estimator, which are typically too small. Theodossiou and Lee (1993) use a multivariate GARCH model to study weekly stock market data from the U.S., Japan, the U.K., Canada, and Germany for the period January 11, 1980 through December 27, 1991. They also find evidence of volatility spillovers. For foreign exchange rates, Engle, Ito, and Lin (1990) find that there is little evidence for their heat-wave hypothesis (country-specific volatility spillovers) but more evidence for a meteor shower hypothesis (cross-market volatility spillovers).

At first glance the existence of Granger-causal spillover relationship from volatility to correlation and from the volatility of one market to the volatility of another suggests that markets are not informationally efficient. However, the theoretical models presented in King and Wadhwani (1989) and King et al. (1994) suggest that such links could arise from rational processing of information by agents if the information structure is such that prices are not fully revealing signals.

There is overwhelming evidence of the time-variation in conditional volatility of asset returns (see Bollerslev, Chou and Kroner, 1992, for an extensive survey) and growing evidence on the time-variation in the conditional correlation between assets (Longin and Solnik, 1995). These features of the data, combined with the desirability of a model in which the hypotheses of interest are conveniently tested led us to the development of a new bivariate GARCH model. Previous parameterizations of bivariate GARCH models are not adequate for a number of reasons. Some specify conditional covariance matrices which are not globally positive definite. Others specify constant correlations or directly parameterize the conditional covariance which make testing hypotheses on the conditional correlation impossible or very inconvenient. Finally, if the popular linear GARCH specification is used to parameterize the variance, the volatility spillover must be constrained to be non-negative in order for the conditional variance to be globally positive. But there is no theoretical reason to rule out negative spillovers thus stressing the importance of a specification that validly allows negative spillovers.[1]

In section 2 we describe the shortcomings of previous specifications in greater detail and introduce a bivariate Logistic Exponential GARCH (LEGARCH) model which overcomes the above objections. In section 3 we describe the data and present some characteristics of asset returns. Empirical results based on the LEGARCH model are presented in section 4. We conclude in section 5.

---

[1] Hamao, et al. (1990) and Susmel and Engle (1994) report negative spillover coefficients.

II. The Model

Univariate GARCH models have been widely used in describing the statistical behavior of economic and financial time-series, especially those sampled at high frequencies. Early work on volatility spillovers was done using univariate models in which the volatility of one market is introduced as an exogenous variable in the variance of the second market. At a minimum this approach is not efficient; if the information matrix is not block-diagonal then the estimates will be inconsistent. Moreover, when testing for volatility spillovers in one direction the univariate GARCH approach assumes that there is no reverse spillover. If there is indeed bidirectional spillover the test statistics may be misleading.

More recently, a number of multivariate GARCH processes that parameterize the covariances between multiple time-series have been utilized in studies of volatility spillovers. In the multivariate GARCH model, the M-element residual vector ε_t is specified as follows:

$$\boldsymbol{\varepsilon}_t \mid \boldsymbol{\Omega}_{t-1} \sim D(\mathbf{0}, \mathbf{H}_t) \tag{1}$$

where $\mathbf{H}_t : (M \times M)$ is the time-varying conditional covariance matrix and D some distribution. Several specifications of $\mathbf{H}_t$ have been discussed in the econometric literature. In the diagonal vech form of Bollerslev, Engle and Wooldridge (1988), the components of $\mathbf{H}_t$ in a GARCH(p,q) process are given by

$$h_{ij,t} = c_{ij} + \sum_{k=1}^{q} a_{ij,k}\varepsilon_{i,t-k}\varepsilon_{j,t-k} + \sum_{k=1}^{p} g_{ij,k}h_{ij,t-k}, \quad i,j = 1,2,..M, \quad i \neq j. \tag{2}$$

This form is parsimonious and has a potentially appealing property that each element of the covariance is a function of only its past history. But this variant does not guarantee positive definiteness of $\mathbf{H}_t$ for all realizations of ε_t. In the Engle and Kroner (1995) generalized positive definite form, the conditional covariance matrix is specified as:

$$\mathbf{H}_t = \mathbf{C}^* + \sum_{k=1}^{K}\sum_{i=1}^{q} \mathbf{A}_{ik}^{'}\boldsymbol{\varepsilon}_{t-i}\boldsymbol{\varepsilon}_{t-i}^{'}\mathbf{A}_{ik} + \sum_{k=1}^{K}\sum_{i=1}^{p} \mathbf{G}_{ik}^{'}\mathbf{H}_{t-i}\mathbf{G}_{ik}. \tag{3}$$

The choice of K determines the generality of the process. If $K = 1$, one obtains the positive definite GARCH formulation that has been utilized in a number of applications but is undesirable here because it does not permit convenient tests of the hypotheses of interest. However, one can use this form to estimate impulse response functions of interest which might substitute for formal tests (Karolyi, 1995).

In each of the above parameterizations, the conditional correlation processes are complicated functions of the underlying parameters and variables in the conditional covariance matrix. They cannot be conveniently used to test hypotheses on the conditional correlation because the hypotheses of interest, in general, cannot be generated by restrictions on parameters alone. However, these forms allow convenient testing of hypotheses on the conditional covariance (Deb, Trivedi and Varangis, 1996).

In order to test hypotheses on the conditional correlation between a pair of assets, it is preferable to specify a bivariate GARCH process with a parametric form for the conditional correlation instead of the conditional covariance. The bivariate constant correlation GARCH model (Bollerslev, 1990), which is such a specification, parameterizes the conditional correlation as a constant. The obvious drawback of this model is that it allows for no dynamics in the conditional correlation function. The evidence in Joy, et al. (1976) and Longin and Solnik (1995), among others, suggests that the conditional correlation between asset returns is time-varying. If the conditional correlation is time-varying then the constant correlation GARCH model is misspecified and so are, in general, any tests based on that specification.

Finally, all the previous specifications use a linear GARCH parameterization of the conditional variance. A serious drawback of this form is that the spillover terms must be nonnegative for the conditional variance to be globally well defined, i.e., if the spillover coefficient is negative, the conditional variance might become negative for some values of the random variables. Therefore, we adopt the EGARCH specification for the conditional variance (Nelson, 1991). Because the conditional variance function is exponential in parameters, no sign restrictions are needed to maintain global positivity.

Braun, Nelson and Sunier (1995) use a bivariate version of the EGARCH model to estimate a CAPM relationship between asset (portfolio) and market returns with a time-varying conditional beta. Because of their specifications of the time-varying conditional beta and conditional variances, their model implies a time-varying conditional correlation. However, their model is inadequate for our purposes for two reasons. First, the CAPM relationship is special and does not apply to the relationship between two arbitrary assets. Second, it is impossible to test specific hypotheses on the correlation without imposing severe restrictions on the variances and/or beta.

A. The Logistic Exponential GARCH model

Let $\eta_{i,t} = h_{ii,t}^{-1/2}\varepsilon_{i,t}$ denote the standardized disturbances. The conditional variance function of the bivariate LEGARCH(p, q) process is given by

$$\log(h_{ii,t}) = c_{ii} + \sum_{k=1}^{q} a_{ii,k}|\eta_{i,t-k}| + \sum_{k=1}^{p} g_{ii,k}\log(h_{ii,t-k}), \quad i = 1,2. \tag{4}$$

We do not model asymmetric responses in this paper (a feature of the model given in Nelson, 1991) to avoid additional complexity and because the asymmetry parameter in preliminary univariate EGARCH analysis was insignificant for all four of our data series.

The conditional covariance function $h_{12,t} = \rho_{12,t}\sqrt{h_{11,t}h_{22,t}}$, is the product of the conditional correlation $\rho_{12,t}$ and the relevant conditional standard deviations. In the Logistic EGARCH(p,q) model the conditional correlation $\rho_{12,t}$ is specified as a logistic transformation of an index function $\xi_{12,t}$, i.e.,

$$\rho_{12,t} = 2(\frac{1}{1+\exp(-\xi_{12,t})}) - 1, \tag{5}$$

where

$$\xi_{12,t} = c_{12} + \sum_{k=1}^{q} a_{12,k}\eta_{1,t-k}\eta_{2,t-k} + \sum_{k=1}^{p} g_{12,k}\xi_{12,t-k}. \tag{6}$$

The function $\xi_{12,t}$ is linear in past values of the cross-products of the standardized errors, $\eta_{1,t-k}\eta_{2,t-k}$. These cross-products are random variables that are proxies for past conditional correlations. The transformation in equation (5) ensures that the conditional correlation is in (-1,1), a sufficient condition for *global* positive definiteness of the conditional covariance. The general multivariate analog of this model does not guarantee global positive definiteness of $\mathbf{H}_t$. However, in-sample positive definiteness can be ensured during estimation by checking the eigenvalues of the conditional covariance matrix at each point in time.

The vector of errors is assumed to follow a bivariate normal distribution so the conditional log likelihood for each observation is given by

$$l_t(\boldsymbol{\theta}) = -\log(2\pi) - \frac{1}{2}\log|\mathbf{H}_t(\boldsymbol{\theta})| - \frac{1}{2}\boldsymbol{\varepsilon}_t'(\boldsymbol{\theta})\mathbf{H}_t^{-1}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t(\boldsymbol{\theta}). \tag{7}$$

where $\boldsymbol{\theta}$ is the vector of all parameters. The log likelihood to be maximized is

$$l(\boldsymbol{\theta}) = \sum_{t=1}^{T} l_t(\boldsymbol{\theta}). \tag{8}$$

In the case of non-normal errors, this method will yield quasi-maximum likelihood estimates (QMLE). Under suitable regularity conditions, QMLE parameter estimates are consistent and asymptotically normal (Gouriéroux, Monfort and Trognon, 1984). The LEGARCH model is estimated using the Broyden, Fletcher, Goldfarb and Shanno (BFGS) algorithm with numerical derivatives. As with other GARCH models, it is relatively straightforward to evaluate the conditional covariance matrix recursively. Upon convergence, a robust estimator of the covariance matrix of $\widehat{\boldsymbol{\theta}}$, the MLE of $\boldsymbol{\theta}$, is estimated as

$$V(\widehat{\boldsymbol{\theta}}) = \mathcal{H}^{-1}(\widehat{\boldsymbol{\theta}})[\mathcal{G}'(\widehat{\boldsymbol{\theta}})\mathcal{G}(\widehat{\boldsymbol{\theta}})]\mathcal{H}^{-1}(\widehat{\boldsymbol{\theta}}) \tag{9}$$

where $\mathcal{G}(\widehat{\boldsymbol{\theta}})$ is the matrix of contributions to the gradient and $\mathcal{H}(\widehat{\boldsymbol{\theta}})$ is the Hessian. Both $\mathcal{G}(\widehat{\boldsymbol{\theta}})$ and $\mathcal{H}(\widehat{\boldsymbol{\theta}})$ are computed numerically.

B. Tests of Linkages

The following extension of the specification of the conditional correlation given in equation (6) allows for a Granger-causal relationship from the conditional variance to the conditional correlation, i.e.,

$$\xi_{12,t} = c_{12} + \sum_{k=1}^{q} a_{12,k}\eta_{1,t-k}\eta_{2,t-k} + \sum_{k=1}^{p} g_{12,k}\xi_{12,t-k} + d_1|\varepsilon_{1,t-1}| + d_2|\varepsilon_{2,t-1}|. \tag{10}$$

Tests of the hypothesis that conditional volatility Granger-causes conditional correlation are tests of $d_1 = 0$ and $d_2 = 0$. When the variance specification in equation (4) is extended such that

$$\log(h_{ii,t}) = c_{ii} + \sum_{k=1}^{q} a_{ii,k}|\eta_{i,t-k}| + \sum_{k=1}^{p} g_{ii,k}\log(h_{ii,t-k}) + s_j|\varepsilon_{j,t-1}|, \tag{11}$$

a test for a Granger-causal relationship from the volatility of market j to the volatility of market i is given by a test of $s_j = 0$. We use the unscaled residual $\varepsilon_{j,t-1}$ to capture spillover effects in equations (10) and (11) to maintain consistency with prior work on volatility spillovers (see, for example, Susmel and Engle, 1994).[2]

III. The Data

The data consists of ten years (01/03/84 through 12/31/93) of daily prices for four financial assets in the U.S.: a commodity index (COM) that is an average of 4 Commodity Research Bureau (CRB) commodity sub-indexes (energy, grains, raw industrials and metals), the CRB currency index (CUR), the CRB interest rate index (INT), and Standard and Poor's 500 stock index (STO). All data was obtained from the CRB InfoTech database published by Knight-Ridder. We define daily returns as logarithmic differences of close prices (multiplied by 100). Following Karolyi (1995), we eliminate returns on October 16, 19, 20 and 21, 1987 because they are likely to be unduly influential due to the crash of '87. The full period, consequently, consists of 2528 observations. In the analysis below, we also consider two sub-periods, first from 01/03/84 to 10/13/87 ($T = 961$) and second from 10/22/87 to 12/31/93 ($T = 1567$).

Table 1 presents summary statistics of the asset returns in all three samples. The average daily return is positive in each period for CUR, INT and STO but negative for COM. Only stocks are consistently negatively skewed in each period but other markets show some significant skewness as well. Each series displays evidence of significant excess kurtosis and ARCH. However, there is no evidence of serial correlation. Overall, the summary statistics are consistent with the stylized facts of financial time series data.

---

[2] The results reported in this paper are reasonably robust to using squared instead of absolute residuals.

IV. Results

In an attempt to identify an adequate mean specification for the bivariate GARCH analysis, we estimate bivariate VARMA models for each sub-period and report the Akaike Information Criterion (AIC) and the Schwarz Bayesian Criterion (SBC) in Table 2. The SBC always selects the VARMA(0,0) model - for each pair and in each sub-period. The AIC is consistent with the SBC for sub-period 2 but there is some evidence in favor of VARMA(0,1) and VARMA(1,0) in sub-period 1.[3] However, given the preponderance of evidence in favor of VARMA(0,0), we estimate LEGARCH models with mean equations specified simply as constants.

Univariate EGARCH models estimation suggested that the EGARCH(1,1) specification adequately explained the conditional variance process for each market. Consequently, we chose to estimate only LEGARCH(1,1) models for each pair of markets and check for model adequacy post-estimation. We also estimated 4-variate LEGARCH models for the baseline case and find similar results for structural change in the aggregate and similar parameter estimates. However, given the usual problem of parameter proliferation in these high-dimensional models, we report and interpret results from bivariate LEGARCH models.

Table 3 contains robust Wald tests of structural change in the data between the first sub-period (before the crash) and the second (after the crash). There is evidence of overall structural change for all but two pairs–COM-STO and CUR-STO. Further tests of structural change in specific components of the model, i.e., in the variance and correlation specifications taken separately, suggest considerable evidence of a structural change in the conditional variance of INT and evidence of changing correlation structures in the COM-CUR and CUR-INT pairs. Based on these results we examine the linkages of interest separately for the two sub-periods.

Parameter estimates and residual diagnostics of the baseline model given by equations (4) - (6) for each period separately are reported in Table 4. The parameters of the variance equations are consistent with the stylized facts of GARCH models, i.e., the ARCH parameters (a_{11}, a_{22}) are small (ranging from 0.049 to 0.139) while the GARCH parameters (g_{11}, g_{22}) are large (ranging from 0.971 to 0.998) suggesting a high degree of persistence in the conditional variance. The series appear to be adequately modeled as the heteroskedasticity-consistent Q-statistics (for residuals and squared-residuals) indicate no significant autocorrelation. The excess kurtosis in the standardized residuals are considerably lower than in the raw data, but they are still significant, suggesting that the conditional density of the asset returns are non-normal.

---

[3] In the 4-variate VARMA system (results not reported), both AIC and SBC select VARMA(0,0) as the best model.

A. Conditional Correlations

The sample correlations of pairs of asset returns are reported in Table 5. There is significant positive correlation between COM and CUR and significant negative correlation between COM and INT. The INT-STO pair also displays significant positive correlation. These sample correlations are, however, unconditional estimates. Given the growing evidence of time-variation in conditional correlations of international stock returns (Longin and Solnik, 1995), we test for the constancy of conditional correlations using the LEGARCH(1,1) model.

In the LEGARCH framework, the null hypothesis of constant conditional correlation is given by $a_{12} = 0, g_{12} = 0$ (equation 6). The estimates of a_{12} and g_{12} are generally individually significant (Table 4). However, a joint test is preferable in this context. But testing $g_{12} = 0$ is only meaningful if $a_{12} \neq 0$ because ARCH effects are a prerequisite for testing for GARCH effects. This is an example of what is sometimes known as the "Davies problem" (Davies 1977, 1987). In this case it implies that the appropriate distribution for the LR test statistic is a $\chi^2(1)$ as opposed to a $\chi^2(2)$ which is what the "counting rule" would suggest. In the univariate GARCH(1,1) context, Lee (1991) has shown that a test for GARCH(1,1), which also possesses the Davies problem, has a $\chi^2(1)$ distribution. The Wald tests presented in Table 5 show that the hypothesis of constant correlation is soundly rejected for each pair of markets in each period.

Much like the parameters obtained from estimation of conditional variances, the ARCH parameter (a_{12}) in the conditional correlation equation is positive and small but generally significant. The GARCH parameter (g_{12}) is also usually large and close to one, suggesting a high degree of persistence in correlations as well. But there are some notable exceptions to this feature. For COM-CUR, there is no persistence in period 1 while g_{12} is significantly negative in period 2. For INT-STO, there is positive persistence in both periods but it is much smaller than the typical persistence for daily conditional variances.

In order to further characterize the conditional correlations, we graph the estimated conditional correlations from the LEGARCH(1,1) model for each pair of markets in Figures 1-6. While each figure plots the conditional correlations for the entire sample period, they are based on separate models for the two sub-samples demarcated in the figures by the vertical line. Considerable daily variation in the conditional correlation is displayed in both periods for each pair of markets. For COM-CUR (Figure 1) and INT-STO (Figure 6) for which the sample correlations are significantly positive, there are some days of negative conditional correlations in the second period. For COM-INT (Figure 2) whose sample correlation is negative, there are a number of positive correlations. Since the sign of the correlation plays an important role in portfolio asset allocation, this information is likely to be quite useful to practitioners. Another feature that emerges from the figures is the high persistence of the correlations of COM-INT, COM-STO, CUR-INT and CUR-STO relative to the correlations of COM-CUR and INT-STO.[4] How often a portfolio manager would wish to adjust asset allocations on the basis of changing conditional correlations might depend on the persistence of the correlations.

B. Tests of Linkages

Robust t-statistics for the various tests of linkages between the different markets are presented in Table 6. In the first two columns we report the statistics for the test of a Granger-causal relationship from the volatility of a market to the correlation between it and another market. There appears to be considerable evidence of this kind of linkage between the commodity market (COM) and the other markets (CUR, INT and STO) but these latter three markets show less evidence of such interrelationships. Generally, the linkage from volatility to correlation is a negative one, i.e., an increase in the volatility in a market causes a decrease in the correlation between it and the other. Furthermore, in most cases, while the signs of the statistics are consistent across periods, the occurence of significant spillovers (volatility to correlation) is higher in the second period, supporting our earlier finding that these markets experienced significant structural changes between the early and late 1980's.

The last two columns of Table 6 report the t-statistics for volatility spillovers, i.e., a Granger-causal relationship from the volatility of one market to the volatility of another. In four cases, we find that an increase in the volatility in one market causes a decrease in the volatility in another market. Note that if a linear GARCH equation were used, such a result would not be feasible because of constraints on the parameters of the variance equation. Only the COM-CUR pair exhibits bi-directional volatility spillovers. Except in one case, the source of every spillover is the volatility of CUR. This suggests that trading rules designed to curb volatility based purely on within-market indicators may not be optimal.

Overall we find more evidence of linkages across markets in the second period. In particular, in the first period we find significant linkages in only two pairs of markets, whereas in the second period, five of the six pairs exhibit evidence of significant linkages. We find evidence of linkages between the commodity and currency markets and between stock and bond markets. Neither result is surprising because of the close connection between commodity exports and imports and foreign exchange rates, and between stocks and bonds as investors commonly diversify across these two markets. On the other hand, there is little evidence of linkages between the bond and commodity markets. Since storage costs are negotiated for months at a time, daily fluctuations in interest rates will not, in general, affect the costs of storage and hence will have little impact on daily commodity prices.

---

[4] Note that the observed differences in persistence are consistent with the estimates of the "persistence" parameter g_{12} in the correlation function (Table 4); it is close to 1 for the pairs which display persistence and smaller for the other two pairs.

C. Evidence of a Structural Cause of the Crash of 1987

In the days and months following the stock market crash of 1987, numerous articles by journalists and studies by commissions and academics ascribed the crash to a variety of sources ranging from technological ones such as inadequate computer systems, disequilibrium or bubbles in associated futures markets, or the existence of portfolio insurance. These, and many other explanations, are discussed in detail by Roll (1989) who finds little empirical support for the arguments. The Report of the Presidential Task Force on Market Mechanisms (1988) suggests an additional cause - that the magnitude of the U.S. trade deficit triggered worldwide events and caused a large stock market correction. This potential cause does not appear to have been given attention in previous work.

The graphs of the conditional correlations for CUR-INT and CUR-STO (Figures 4 and 5 respectively) reveal a startling fact: beginning early in 1987, the conditional correlations declined steadily from historical levels until the crash after which they appear to have reverted quickly back to the pre-1987 levels. Since CUR is the common variable in the two figures it is safe to assume that movements in the foreign exchange market caused these deviations from the norm. The "crash" appears to be a rapid correction to these deviations, bringing the relationships between the currency market and the bond and equity markets back to historical levels.

In fact, one can point to a potential impediment to market forces in the currency market in the months before the crash which may explain the deviations from the norms. Between January, 1986 and January, 1987, the U.S. Dollar had declined by almost 24% . In an attempt to arrest this decline, officials from the G-7 nations forged a deal at a meeting in Paris on February 22, 1987 to support the tumbling U.S. Dollar via central bank intervention. Through that year, there was "$100 billion of net official dollar purchases by the monetary authorities of 14 major countries, including the United States" (p. 21, Board of Governors of the Federal Reserve System, 1988). Since exchange rate movements naturally reflect divergences in economic conditions in different countries, attempts to suppress such movements could alter the relationships between it and other financial markets. The sharp deviations in the conditional correlations prior to the crash are consistent with intervention in the foreign exchange market. Therefore, these movements may be interpreted as structural deviations from the norm, followed by a rapid correction in October 1987.

V. Conclusion

In this paper we examine two forms of second-moment linkages between asset markets using daily data from four major financial markets in the U.S.: the commodity, foreign exchange, bond and stock markets. There is overwhelming evidence of time-variation in the conditional volatility of asset returns and growing evidence of time-variation in the conditional correlation between markets. For these reasons and because of the desirability of a model in which the hypotheses of interest are conveniently tested, we develop a new bivariate GARCH model called the Logistic Exponential GARCH model.

We find, using the LEGARCH model, that there is evidence of the parameters of the model changing between two sub-periods of the data. We also find that the conditional correlations change significantly over time and that there is, in a majority of cases, evidence of considerable persistence in the conditional correlation. There is evidence for some form of linkage for every pair of markets, though the results are not generally consistent across sub-periods. Contrary to earlier findings on the positive contemporaneous relationship between volatility and correlations, we find evidence of volatility Granger-causing correlation to increase in some cases and to decrease in others. In addition, we find some evidence of volatility spillovers.

Furthermore, the graphs of conditional correlations between currency and interest rates and between currency and stocks show a persistent downward trend in 1987, followed by a correction immediately after the stock market crash. This intriguing movement in correlations prior to the crash may be seen as a potential structural deviation from the norm followed by a rapid correction - the stock market crash of 1987.

Table 1: Summary Statistics of Daily Asset Returns

Market	Mean	Var.	S	K	Q(20)	$Q^2(20)$
	Full Period: 01/03/84 - 12/31/93 (T = 2528)					
COM	-0.008	0.639	-0.809*	7.432*	26.26	154.13*
CUR	0.012	0.327	0.223*	2.427*	16.39	133.53*
INT	0.011	0.165	0.023	1.629*	25.29	488.45*
STO	0.049	0.810	-0.733*	7.600*	18.18	311.78*
	Period 1: 01/03/84 - 10/13/87 (T = 961)					
COM	-0.008	0.756	-0.063	1.475*	16.75	113.11*
CUR	0.028	0.349	0.633*	3.338*	24.27	81.01*
INT	0.010	0.242	-0.075	0.599*	16.66	127.13*
STO	0.068	0.721	-0.178*	2.016*	16.55	28.09
	Period 2: 10/22/87 - 12/31/93 (T = 1567)					
COM	-0.007	0.567	-1.501*	13.482*	24.20	84.88*
CUR	0.002	0.313	-0.079	1.651*	15.46	65.16*
INT	0.012	0.117	0.207*	2.313*	21.85	60.11*
STO	0.038	0.865	-0.983*	9.830*	24.23	202.95*

Notes:

COM, CUR, INT, and STO refer to commodities, currencies, interest rates, and stocks, respectively.

S is the skewness statistic; K is the excess kurtosis statistic.

Q(20) is a heteroskedasticity consistent Ljung-Box statistic for serial correlation up to 20 lags. $Q^2(20)$ is a test for ARCH up to 20 lags. Both statistics are asymptotically distributed as $\chi^2(20)$.

* indicates that the statistic is significant at the 5% level.

Table 2: Model Selection Criteria for Bivariate ARMA Models

		VARMA(0,0)	VARMA(0,1)	VARMA(1,0)
		Period 1: 01/03/84 - 10/13/87		
COM-CUR	AIC	4027.29*	4034.88	4034.89
	SBC	4051.63+	4078.69	4078.70
COM-INT	AIC	3802.44*	3806.20	3806.37
	SBC	3826.78+	3850.01	3850.18
COM-STO	AIC	4879.12	4877.71	4877.69*
	SBC	4903.46+	4921.52	4921.50
CUR-INT	AIC	4019.69*	4029.86	4027.51
	SBC	4044.03+	4073.67	4071.32
CUR-STO	AIC	4136.12	4135.73*	4136.02
	SBC	4160.46+	4179.54	4179.84
INT-STO	AIC	3609.86	3606.68*	3606.77
	SBC	3634.20+	3650.49	3650.58
		Period 2: 10/22/87 - 12/31/93		
COM-CUR	AIC	6174.52*	6176.31	6176.51
	SBC	6201.31+	6224.52	6224.72
COM-INT	AIC	4568.21*	4571.14	4571.46
	SBC	4594.99+	4619.35	4619.67
COM-STO	AIC	7746.15*	7750.30	7750.54
	SBC	7772.94+	7798.51	7798.75
CUR-INT	AIC	4896.09*	4906.14	4903.37
	SBC	4922.88+	4954.35	4951.59
CUR-STO	AIC	6847.41*	6851.01	6850.77
	SBC	6874.20+	6899.23	6898.98
INT-STO	AIC	5128.32*	5132.44	5132.42
	SBC	5155.10+	5180.66	5180.64

Notes:

COM, CUR, INT, and STO represent commodities, currencies, interest rates, and stock prices, respectively.

* indicates the selected model under the AIC criteria.

+ indicates the selected model under the SBC criteria.

Results for the full period were very similar.

Table 3: Tests of Structural Change based on the LEGARCH(1,1) Model

Hypothesis	d.f.	COM-CUR	COM-INT	COM-STO	CUR-INT	CUR-STO	INT-STO
Overall	11	37.79*	23.37*	7.48	25.94*	5.19	18.23+
Var(1)	3	2.79	4.92	4.38	1.72	1.65	15.21*
Corr(1,2)	3	30.76*	1.18	1.38	7.98*	1.36	0.36
Var(2)	3	1.77	16.85*	0.73	17.05*	0.87	0.90

Notes:

COM, CUR, INT, and STO stand for commodities, currencies, interest rates, and stocks, respectively.

Overall is the Wald statistic for equality of parameters in sub-period 1 (01/03/84 - 10/13/87) and sub-period 2 (10/22/87 - 12/31/93).

Var(1) is the Wald statistic for equality of parameters in the variance equation for the first market in the pair in sub-period 1 (01/03/84 - 10/13/87) and sub-period 2 (10/22/87 - 12/31/93).

Corr(1,2) is the Wald statistic for equality of parameters in the correlation equation in sub-period 1 (01/03/84 - 10/13/87) and sub-period 2 (10/22/87 - 12/31/93).

Var(2) is the Wald statistic for equality of parameters in the variance equation for the second market in the pair in sub-period 1 (01/03/84 - 10/13/87) and sub-period 2 (10/22/87 - 12/31/93).

* indicates that the statistic is significant at the 5% level.

+ indicates that the statistic is significant at the 10% level.

Table 4: Parameter Estimates and Specification Tests of LEGARCH(1,1) Models (equations 4 - 6)

	COM-CUR	COM-INT	COM-STO	CUR-INT	CUR-STO	INT-STO
	Period 1: 01/03/84 - 10/13/87					
c_{11}	-0.102*	-0.089*	-0.091*	-0.132*	-0.134*	-0.134*
	(0.022)	(0.018)	(0.019)	(0.030)	(0.032)	(0.023)
c_{12}	0.815*	-0.002	0.001	-0.002*	-0.001	0.264
	(0.231)	(0.003)	(0.002)	(0.001)	(0.001)	(0.502)
c_{22}	-0.134*	-0.130*	-0.038*	-0.135*	-0.040*	-0.038*
	(0.032)	(0.023)	(0.010)	(0.022)	(0.011)	(0.012)
a_{11}	0.126*	0.107*	0.111*	0.139*	0.139*	0.119*
	(0.028)	(0.023)	(0.025)	(0.036)	(0.037)	(0.022)
a_{12}	0.131*	0.043*	0.036*	0.029*	0.022*	0.088
	(0.039)	(0.022)	(0.015)	(0.007)	(0.008)	(0.080)
a_{22}	0.136*	0.118*	0.051*	0.123*	0.053*	0.049*
	(0.034)	(0.022)	(0.014)	(0.022)	(0.015)	(0.016)
g_{11}	0.980*	0.976*	0.976*	0.975*	0.973*	0.971*
	(0.010)	(0.010)	(0.011)	(0.012)	(0.013)	(0.010)
g_{12}	0.035	0.970*	0.979*	0.988*	0.991*	0.653
	(0.252)	(0.021)	(0.012)	(0.004)	(0.005)	(0.602)
g_{22}	0.972*	0.973*	0.998*	0.972*	0.998*	0.996*
	(0.013)	(0.011)	(0.005)	(0.011)	(0.005)	(0.005)
log likelihood	-1919.07	-1823.54	-2374.19	-1450.92	-2004.37	-1756.39
K_1	1.20	1.16	1.15	1.67	1.68	0.39
K_2	1.70	0.39	1.55	0.40	1.54	1.58
$Q_1(20)$	17.94	17.59	17.62	22.14	22.16	22.96
$Q_2(20)$	22.20	23.20	15.90	23.12	15.93	16.01
$Q_1^2(20)$	14.67	15.64	15.42	14.70	14.69	15.43
$Q_2^2(20)$	14.81	15.25	17.36	15.27	17.45	17.30

Table 4 (continued)

	COM-CUR	COM-INT	COM-STO	CUR-INT	CUR-STO	INT-STO
	Period 2: 10/22/87 - 12/31/93					
c_{11}	-0.120*	-0.117*	-0.111*	-0.096*	-0.095*	-0.065+
	(0.019)	(0.018)	(0.017)	(0.024)	(0.026)	(0.036)
c_{12}	0.399*	-0.019	-0.003	0.001	-0.007	0.345*
	(0.010)	(0.018)	(0.007)	(0.001)	(0.010)	(0.121)
c_{22}	-0.100*	-0.041	-0.056*	-0.041	-0.068*	-0.064*
	(0.026)	(0.073)	(0.021)	(0.049)	(0.031)	(0.025)
a_{11}	0.149*	0.145*	0.137*	0.084*	0.084*	0.043*
	(0.027)	(0.026)	(0.024)	(0.022)	(0.023)	(0.019)
a_{12}	0.163*	0.049+	0.024	0.021+	0.064	0.106*
	(0.065)	(0.027)	(0.026)	(0.013)	(0.041)	(0.034)
a_{22}	0.085*	0.031	0.072*	0.032	0.086*	0.081*
	(0.023)	(0.037)	(0.028)	(0.027)	(0.040)	(0.032)
g_{11}	0.992*	0.991*	0.992*	0.973*	0.973*	0.985*
	(0.005)	(0.005)	(0.006)	(0.011)	(0.012)	(0.012)
g_{12}	-0.478*	0.926*	0.974*	0.988*	0.890*	0.517*
	(0.189)	(0.053)	(0.046)	(0.008)	(0.090)	(0.182)
g_{22}	0.969*	0.992*	0.994*	0.992*	0.992*	0.991*
	(0.012)	(0.021)	(0.004)	(0.014)	(0.007)	(0.006)
log likelihood	-2850.14	-2063.68	-3527.92	-1786.23	-3245.57	-2365.62
K_1	2.72	2.79	2.80	1.52	1.52	1.99
K_2	1.52	1.96	8.05	1.96	8.04	7.98
$Q_1(20)$	24.02	24.12	24.20	16.13	16.15	25.03
$Q_2(20)$	16.09	25.18	22.44	25.29	22.01	22.01
$Q_1^2(20)$	15.98	15.81	15.62	23.42	23.29	22.34
$Q_2^2(20)$	23.49	23.09	13.08	22.76	10.92	11.64

Notes:

COM, CUR, INT, and STO refer to commodities, currencies, interest rates, and stocks, respectively.

K is the excess kurtosis statistic for standardized residuals.

$Q_1(20)$ and $Q_2(20)$ are heteroskedasticity consistent Ljung-Box statistics for serial correlation up to 20 lags in the standardized residuals of the first and second market in the pair, respectively. $Q_1^2(20)$ and $Q_2^2(20)$ are tests for ARCH up to 20 lags in the standardized residuals of the first and second market in the pair respectively. Both statistics are asymptotically distributed as $\chi^2(20)$.

* indicates that the statistic is significant at the 5% level.

Table 5: Sample Correlations and Tests of Constant Conditional Correlations

	COM-CUR	COM-INT	COM-STO	CUR-INT	CUR-STO	INT-STO
	Period 1: 01/03/84 - 10/13/87					
Sample Correlation	0.387*	-0.174*	-0.025	0.030	0.044	0.410*
Constant Correlation	11.57*	19709*	26341*	151009*	80775*	17.03*
	Period 2: 10/22/87 - 12/31/93					
Sample Correlation	0.112*	-0.225*	-0.159*	0.029	-0.074	0.332*
Constant Correlation	22.39*	1094*	5183*	162854*	323.7*	144.7*

Notes:

COM, CUR, INT, and STO stand for commodities, currencies, interest rates, and stocks, respectively.

Constant Correlation is a Wald statistic for constant conditional correlation which follows a $\chi^2(1)$ under the null.

* indicates that the statistic is significant at the 5% level.

Table 6: Tests of Linkages based on the LEGARCH(1,1) Model

		Causality from		Causality from	
Market 1	Market 2	Var(1) to Corr(1,2)	Var(2) to Corr(1,2)	Var(1) to Var(2)	Var(2) to Var(1)
		Period 1: 01/03/84 - 10/13/87			
COM	CUR	-2.968*	0.456	2.009*	-2.388*
COM	INT	-0.589	0.578	0.341	1.441
COM	STO	-1.263	-0.383	1.204	0.723
CUR	INT	0.698	-0.852	-0.788	0.227
CUR	STO	0.358	-1.823+	-2.230*	-1.121
INT	STO	-0.868	-0.701	0.551	-0.169
		Period 2: 10/22/87 - 12/31/93			
COM	CUR	-2.192*	-2.034*	1.113	-1.946+
COM	INT	-1.027	-2.016*	0.410	0.176
COM	STO	-2.364*	-0.292	-1.424	0.866
CUR	INT	0.316	-1.520	-2.133*	1.182
CUR	STO	1.099	-0.236	0.474	0.927
INT	STO	-1.210	0.127	0.641	2.428*

Notes:

Robust t-statistics are reported.

COM, CUR, INT, and STO stand for commodities, currencies, interest rates, and stocks, respectively.

* indicates that the statistic is significant at the 5% level.

+ indicates that the statistic is significant at the 10% level.

Figure 1: Estimated Conditional Correlations for COM-CUR

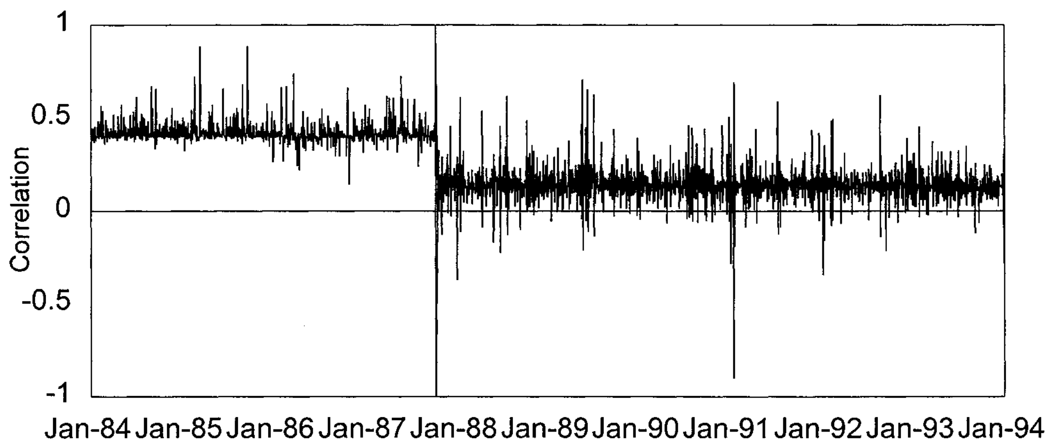
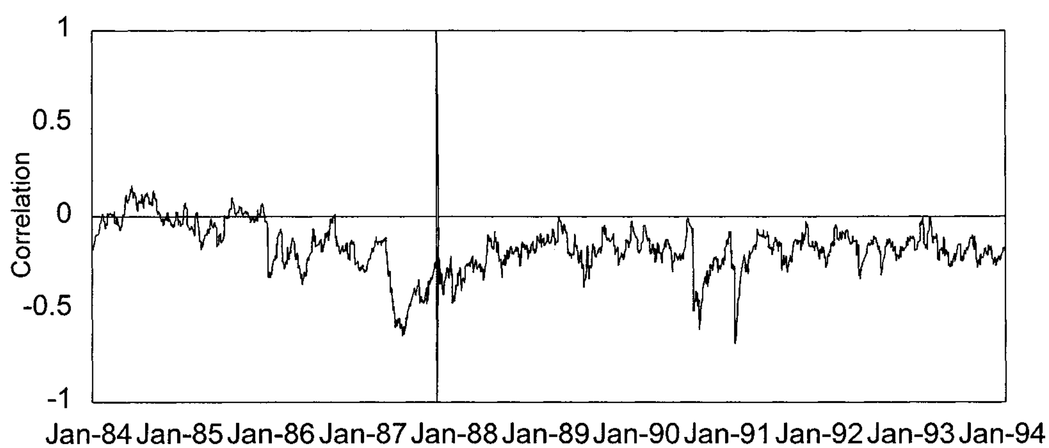


Figure 2: Estimated Conditional Correlations for COM-INT



Note: The vertical line denotes the date at which the sample is split into 2 periods.

Figure 3: Estimated Conditional Correlations for COM-STO

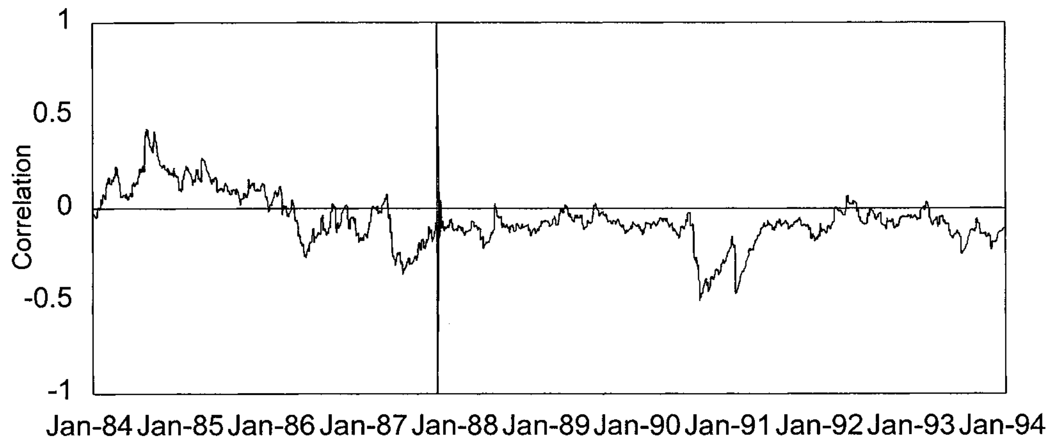
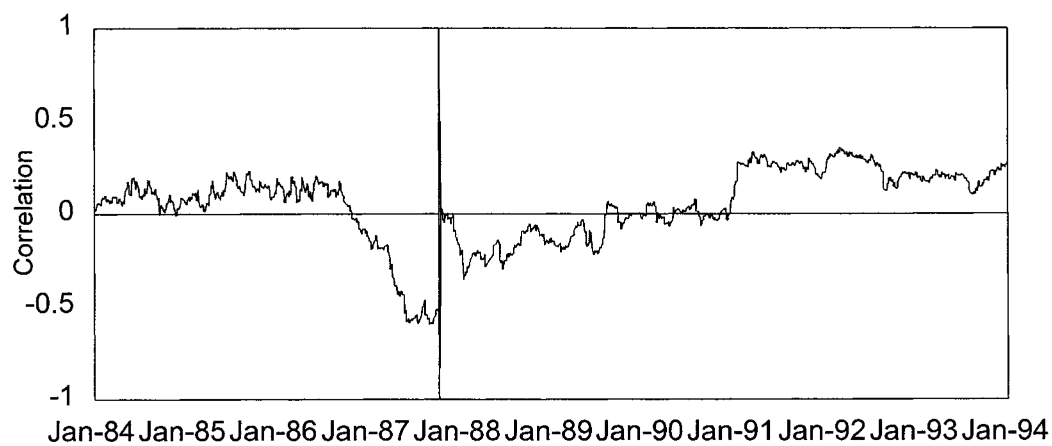


Figure 4: Estimated Conditional Correlations for CUR-INT



Note: The vertical line denotes the date at which the sample is split into 2 periods.

Figure 5: Estimated Conditional Correlations for CUR-STO

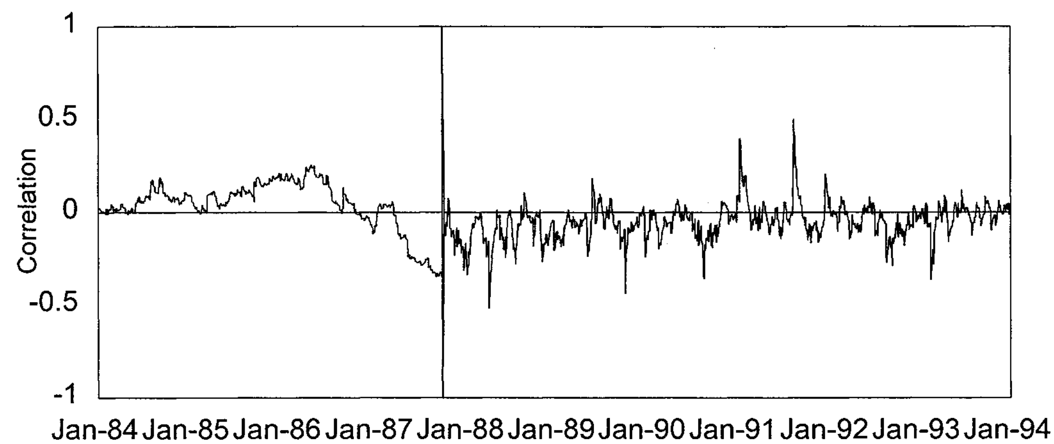
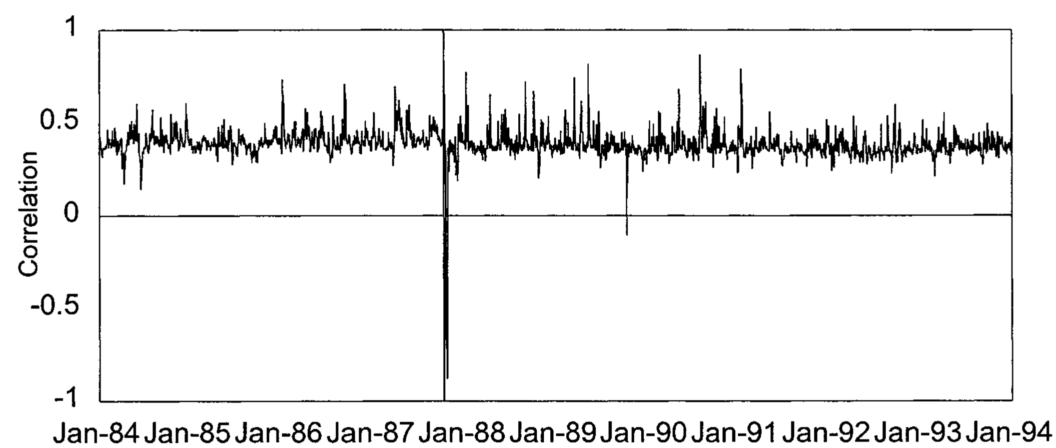


Figure 6: Estimated Conditional Correlations for INT-STO



Note: The vertical line denotes the date at which the sample is split into 2 periods.

REFERENCES

Bennett, P. and J. Kelleher (1988), "The International Transmission of Stock Price Disruptions in October 1987," *Federal Reserve Bank of New York Quarterly Review*, 13, 17-33.

Board of Governors of the Federal Reserve System (1988), "74th Annual Report 1987, Federal Reserve System.

Bollerslev, T. (1990), "Modelling the Coherence in Short-run Nominal Exchange Rates: A Multivariate Generalized ARCH Approach", *Review of Economics and Statistics*, 69, 542-547.

Bollerslev, T., R. Y. Chou, and K. F. Kroner (1992), "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence," *Journal of Econometrics*, 52, 5-59.

Bollerslev, T., R. F. Engle, and J. M. Wooldridge (1988), "A Capital Asset Pricing Model with Time-varying Covariances," *Journal of Political Economy*, 96, 116-131.

Braun, P. A., D. B. Nelson and A. M. Sunier (1995), "Good News, Bad News, Volatility, and Betas", *The Journal of Finance*, L, 1575-1603.

Davies, R. B. (1977), "Hypothesis Testing When a Nuisance Parameter is Present Only Under the Alternative," *Biometrika*, 64, 247-254.

Davies, R. B. (1987), "Hypothesis Testing When a Nuisance Parameter is Present Only Under the Alternative," *Biometrika*, 74, 33-43.

Deb, P., P. K. Trivedi and P. Varangis (1996), "The Excess Co-Movement of Commodity Prices Reconsidered", *Journal of Applied Econometrics*, 11, 275-291.

Engle, R. F. and K. F. Kroner (1995), "Multivariate Simultaneous Generalized ARCH," *Econometric Theory*, 11, 122-150.

Engle, R. F., T. Ito, and W-L. Lin (1990), "Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market," *Econometrica*, 58, 525-542.

Gouriéroux, C., A. Monfort and A. Trognon, (1984), "Pseudo-maximum Likelihood Methods: Theory," *Econometrica*, 52, 681-700.

Hamao, Y., R. W. Masulis, and V. Ng (1990), "Correlations in Price Changes and Volatility Across International Stock Markets," *The Review of Financial Studies*, 3, 281-307.

Joy, M. O., D. B. Panton, F. R. Reilly, and S. A. Martin (1976), "Co-movement of Major International Equity Markets," *The Financial Review*, 1-20.

Karolyi, G. A. (1995), " A Multivariate GARCH Model of International Transmissions of Stock Returns and Volatility: The Case of the United States and Canada", *Journal of Business and Economic Statistics*, 13, 11-26.

King, M. A., E. Sentana, and S. Wadhwani (1994), "Volatility and Links Between National Stock Markets," *Econometrica*, 62, 901-933.

King, M. A. and S. Wadhwani (1990), "Transmission of Volatility between Stock Markets," *The Review of Financial Studies*, 3, 5-33.

Lee, J. H. H. (1991), "A Lagrange Multiplier Test for GARCH Models", *Economics Letters*, 37, 265-271.

Longin, F. and B. Solnik (1995), "Is the Correlation in International Equity Returns Constant: 1960-1990?", *Journal of International Money and Finance*, 14, 3-26.

Nelson, D. B. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach", *Econometrica*, 59, 347-370.

Roll, R. (1989), "Price Volatility, International Market Links, and Their Implications for Regulatory Policies," *Journal of Financial Services Research*, 3, 211-246.

Susmel, R. and R. F. Engle (1994), "Hourly Volatility Spillovers Between International Equity Markets," *Journal of International Money and Finance*, 13, 3-25.

Theodossiou, P. and U. Lee (1993), "Mean and Volatility Spillovers Across Major National Stock Markets: Further Empirical Evidence," *The Journal of Financial Research*, 16, 337-350.