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Public Debt Management and Bailouts

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Abstract

This paper addresses how public debt should be managed to reduce the cost of private sector bailouts. It uses a tax smoothing model to show that bailouts affect the timing of government deficits and surpluses as well as the composition of public debt. In general, public debt managers will have to monitor the private sector's leverage and portfolio composition in order to design the tax smoothing policy. This contrasts with Ricardian models where households monitor the government's debt. The moral hazard aspect of defaults is also shown to be important in determining an optimal government debt strategy.

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I. INTRODUCTION

The motivation for this paper is the observation that bail out costs constitutes one of the most significant shocks to governments' budgets and thus warrants special attention from a fiscal policy and public debt management (PDM) perspective. The paper uses a standard model of tax smoothing to analyze how both the timing of taxes and the optimal government portfolio will depend on the existence of defaults and bail outs (for public debt management in tax smoothing models see, for example, Barro 1979, Bohn, 1990, Goldfajn 1995, Missale 1996). Although countries have taken measures to reduce the probability of bail outs and their impact on the budget, bail outs will happen again, perhaps at a lower frequency and cost, but it seems highly unlikely that the bail out problem has been fixed once and for all.

Dealing with bail outs and its cost to the budget can be seen as multilayer strategy, with prudential regulation and supervision there to minimize the probability of "unnecessary" bail outs, the legal institutions and bankruptcy procedures to minimize the financial cost once it occurs, and finally the tax smoothing strategy designed to minimize the excess burden associated with the fact that the government most likely will have to fill the private sector's financing gaps with taxes.

In this paper, the phrase "bail out" is meant to cover all cases where the government steps in and takes over liabilities that has been created in the non-government sector. The most common cases of bail outs are associated with the financial sector, but recently, the close links between banks and non-financial companies has lead to bail outs that are more related to the private sector in general. The paper will therefore use the more general words "firms" and "companies" to describe the entities that may have to be bailed out, instead of "banks" or "financial institution."

Table 1 illustrate the quantitative significance of bail out costs in connection to banking crises in emerging and mature markets. The fiscal and quasi-fiscal cost of these banking crises, measured as a percent of annual GDP during the reconstruction period, ranges from the single digits to over 40 percent. Reducing the impact of such financing operations on the government's budget certainly seem to be worthwhile. To further stress the importance of considering bail out costs from a fiscal policy and debt management point of view, banking crises are usually concurrent with serious output losses (estimated to average around 10 to 12 percent for industrial and emerging markets respectively, according to the International Monetary Fund, 1998), which put further pressure on the government's budget and the tax rate.

The remainder of the paper is organized as follows. The next section extends the standard tax smoothing model to include bail out costs. Section III then introduces uncertainty and the debt management decision. The following section analyzes the firms portfolio choice. Section V presents a numerical example to illustrate the main points in the paper, before Section VI concludes.

Table 1. Fiscal and Quasi- scal Costs of Banking Crises					
Emerging Markets			**Mature Markets**		
Country	**Year**	**Costs**^{a}	**Country**	**Year**	**Costs**^{a}
Argentina	1980-82	13-55	Finland	1991-93	8-10
	1985	...	Japan	1990s	3
Brazil	1994-96	4-10	Norway	1988-92	4
Chile	1981-85	19-41	Spain	1977-85	15-17
Columbia	1982-87	5-6	Sweden	1991-93	4-5
Indonesia	1994	2	United States	1984-91	5-7
Malaysia	1985-88	5			
Mexico	1994-95	12-15			
Philippines	1981-87	3-4			
Sri Lanka	1989-93	9			
Thailand	1983-87	1			
Turkey	1983-85	3			
Uruguay	1981-84	31			
Venezuela	1980-83	...			
	1994-95	17			
^{a}The cost is estimated as a percent of annual GDP during the reconstruction period Source: International Monetary Fund (1998)					

II. TAX SMOOTHING AND BAILOUTS

By now it is well known that if the conditions for Ricardian equivalence are present, the level and composition of government debt does not matter, and public debt management becomes irrelevant. However, deviations from the assumptions underlying Ricardian equivalence create a possible role for the debt manager, and here the Ricardian assumption regarding lump-sum taxes will be removed.[2] The introduction of distortionary taxes has given rise to a relatively rich literature exploring how the government should smooth taxes in order to minimize the excess burden from taxation, see, for example, Barro (1979) and (1995), and Bohn (1993). The general idea of tax smoothing is that excess burdens from taxation is a convex function of the tax rate, and thus it is optimal to keep the tax rate constant, both over time and across states of nature.

There are two conclusions from the tax smoothing literature worth noting at this stage, rst, tax smoothing will potentially lead to de cits and surpluses over time, not because this stabilizes the economy in a Keynesian sense, but because it minimizes the excess burden associated with taxation.

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[2]Whether or not Ricardian equivalence is a good approximation has received considerable attention in the empirical literature, with papers by, e.g., Feldstein (1982) and Bernheim (1987) arguing against Ricardian equivalence in favor of more Keynesian effects of budget de cits, while, e.g., Plosser (1982), Becker (1997), Becker and Paalzow (1996) and Giavazzi and Pagano (1990) nds support of Ricardian equivalence or even of expansionary scal contractions.

Secondly, the incentive to smooth taxes will be greater for higher average tax rates since the function is convex. In other words, the tax smoothing objective of the PDM operations should be more important in economies with a high average tax rate.

Some back of the envelope calculations suggests that a relatively high average tax rate in combination with a fairly convex excess burden function is needed to generate a quantitatively convincing argument for tax smoothing (see Sweden, Ministry of Finance, 1997). However, the PDM policies implied by the tax smoothing argument may have other beneficial effects that are not explicitly modeled in standard tax smoothing models. For example, the risk of government default can potentially be reduced as a side effect of using tax smoothing policies. Futhermore, there may be other cost (than the once that arise due to the convexity of the excess burdens function) associated with changing the tax rate, for example, legislative costs and creating uncertainty about tax rates for the household.

Before introducing uncertainty and PDM, the case of intertemporal tax smoothing a la Barro (1979) will be analyzed with bail out costs added to an otherwise standard model. Throughout the paper, a two period version of the infinite horizon model will be used to keep the model as simple as possible. Nothing fundamental is lost by this assumption, as long as one keeps in mind that second period variables in this model corresponds not to a single period, but to the present value of the variable over all future periods in the multiperiod model.

The economy consists of a large number of identical **households** where the representative household maximizes utility over two periods subject to a budget constraint where an excess burden component has been included in the budget constraint. Formally the problem is

$$\max_{c_1} U = u(c_1) + \rho u(c_2)$$
$$\text{s.t. } c_1 = Y_1(1 - \tau_1 - h(\tau_1)) - A \tag{1}$$
$$c_2 = Y_2(1 - \tau_2 - h(\tau_2)) + (1 + r)A,$$

where c_1 and c_2 are consumption in period one and two, $u(.)$ is a standard utility function with $u'(.) > 0$, ρ is the discount factor, A is the asset carried between period 1 and 2 with real return r (since $u'(.) > 0$ there is no need to consider wealth accumulation in period 2), and Y represent income that is taxed at the rate τ. The function $h(\tau)$ represents the excess burden created by taxation.

The benevolent **government** is characterized by its budget constraint and seeks to maximize the household's welfare. The budget constraints in period 1 and 2 are

$$T_1 = \tau_1 Y_1 = G_1 + BO_1 + D_0 - D_1 \tag{2}$$
$$T_2 = \tau_2 Y_2 = G_2 + BO_2 + (1 + r)D_1,$$

where T is tax revenue that comes from taxing income Y at the rate τ, G is government consumption, D_0 is the amount of initial debt and D_1 is the amount carried between period 1 and

2. BO_1 and BO_2 represents the bail out costs in period one and two and are the only element that have been added to the standard tax smoothing model (again, no government debt is left after the second period since no one would be willing to provide a loan that cannot be repaid).[3]

At this stage, the bail out cost is an exogenous component in the model, but later on, firm behavior will be included in the model.[4] It may be worth noting that the type of bail out considered here is one where the government/country bears the cost of the bail out and not the type of bail out where the cost is born by an outside agent/agency/country, however, it does not seem plausible that bail outs by a third party would be designed in a way that the government would be left with no bail out costs at all, so to the extent that this is true, the present set up is still valid.

One initial observation is that if the government were free to chose all components in the budget, the task of minimizing excess burden would be extremely simple; just set all expenditures to zero and the need for distortionary taxes disappear. This conclusion is obviously reached because there is no feature of the model that motivates government expenditures. In general, the tax smoothing literature deals with this by simply assuming that these expenditures are exogenously given, which will be the maintained assumption here. However, there are papers that take the expenditure side more seriously by for example including government consumption in the households utility function. In these cases the more simple minded tax smoothing policy arrived at here and in the vast majority of this literature is extended to generate utility smoothing by adjusting both the expenditure side and the tax rates to arrive at an optimal policy (see, for example, Barnett, 1997).

To make the analysis relatively straight forward, we follow, for example, Bohn (1990) in assuming a quadratic excess burden function, so that $h(\tau) = \tau^2$, however, any convex function would deliver the general message of tax smoothing. By inserting the individual budget constraint in the maximization problem and dropping irrelevant terms, we can see that the government should maximize $U = -Y_1\tau_1^2 - \rho Y_2\tau_2^2$ subject to its budget constraint in order to maximize the representative household's welfare. By multiplying the maximization problem by minus one it is turned into the problem of minimizing the present value of excess burdens, that is, minimize $X = Y_1\tau_1^2 + \rho Y_2\tau_2^2$ subject to the governments budget constraint. This type of expression is familiar to the reader of Missale (1996). The first order condition for minimizing excess burdens leads to the condition

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[3]This model does not address the issue of solvency or default of the government, and thus abstracts from, e.g., default risk premia. Default risk is obviously a concern also for government's in many cases, but the issues involved warrant separate studies and introduces too much complexity to be considered as part of this model.

[4]The paper will not address the fundamental question of whether or not bail outs are optimal from a social planners point of view. This is obviously an important question that has received much attention lately in many different contexts, but well beyond the scope of this paper. For the present purpose, it is sufficient to note that bail outs are real world phenomena in governments' budgets all over the world.

$$(1+r)\rho\tau_2 \;=\; \tau_1 , \tag{3}$$

which together with the standard assumption that $\rho(1 + r) = 1$ leads to the perfect tax smoothing conclusion, namely that the tax rate should be constant. The budget constraint then determines the path of deficits and surpluses, or equivalently, the level of public debt.

We can now state the first result regarding bail outs and tax smoothing. If bail out cost are included in the tax smoothing decision, these would tend to increase (reduce) the deficits (surpluses) in the periods they occur to be consistent with tax smoothing. For example, if there are no bail out cost now but the government expects that there will be in the future, this would tend to create a budget surplus today.

If we think that a highly leveraged corporate sector would tend to increase the likelihood of future bail outs, an empirical question would thus be to see whether governments in countries with highly leverage corporate sectors tend to run smaller deficits (or larger surpluses) than comparable countries with a less leveraged corporate sector.

III. PUBLIC DEBT MANAGEMENT WITH BAILOUTS

So far, the model does not incorporate uncertainty and public debt management, but focused only on ex ante tax smoothing that relies on transferring tax revenues over time with a single debt instrument with known real return. In the case there is no uncertainty about the future tax base or future expenditure components there would not be a role for the public debt manager from a tax smoothing perspective (more than that the entire debt portfolio would consist of price indexed bonds). However, to achieve ex post tax smoothing, or tax smoothing over states of nature, the government's debt portfolio would have to include instruments that hedge the risk in the other components of the budget, that is, in the tax base or non-interest expenditures. Issuing bonds contingent on the tax base (GDP) or expenditures has been discussed by for example Shiller (1993) and Barro (1995). These bonds would be constructed such that the return is low when the tax base (expenditures) is smaller (greater) than expected. Alternatively, the bonds would be linked to the primary deficit in a way that the deficit inclusive of interest payments would be stabilized.

Barro (1995) shows that in an infinite horizon model, the government has to issue state contingent consoles that eliminate all future deficits inclusive of interest payments in order to avoid not only unpredictable changes in the present value of the tax base and expenditures but also variations in the real interest rate. This is clearly a very strong conclusion with respect to debt management, but the practical implications are restricted by the lack of such state contingent instruments. The creation of instruments contingent on the primary deficit would face serious moral hazard problems in most countries, since it would require investors to buy an instrument that delivers a low return when the primary deficit is large, creating serious temptations for the politicians controlling the budget.

However, debt indexed to GDP, as mentioned in the introduction, has been argued to be relatively realistic, although there are no examples (as far as the author knows) of governments issuing such bonds.[5] The literature on tax smoothing has therefore come to focus on how existing debt instruments can be used to achieve something similar to the (slightly unrealistic) case of perfect ex post tax smoothing, see for example Bohn (1988, 1990), Missale (1996), Paalzow (1998) and Goldfajn (1995). This strand of the literature focuses in general on three types of instruments, nominal bonds, price indexed (or real) bonds and bonds denominated in foreign currency (which in the following will be referred to, somewhat incorrectly, as foreign bonds just to make the language less tedious.) From a tax smoothing perspective, the real bond is the risk free asset, since the government's resource use as well as the excess burden is supposed to be related to real quantities (which in most instances seems to be a reasonable assumption, although some government expenditures can certainly be defined in nominal terms.)[6]

In the following part of the paper, the households problem will be suppressed, and we will work directly with the government minimizing the expected present value of excess burden according to

$$
\begin{aligned}
&\min_{\tau_1} E[Y_1 \tau_1^2 + \rho Y_2 \tau_2^2] \\
&\text{s.t. } T_1 = \tau_1 Y_1 = G_1 + BO_1 + D_0 - D_1 \\
&\quad T_2 = \tau_2 Y_2 = G_2 + BO_2 + \\
&\quad [(1 + r_0)(1 - m - m^*) + (1 + r_1)m + (1 + r_2)m^*]D_1.
\end{aligned} \tag{4}
$$

First note that the model now incorporates the issue of uncertainty, thus the introduction of the expectations operator E. All the second period variables, Y_2, G_2, and BO_2 are stochastic (in the numerical example later on they will be assumed to be multivariate normally distributed). The variables m and m^* are the portfolio shares of nominal and foreign bonds respectively, r_0 is the certain real return on price indexed bonds, while r_1 and r_2 are the realized real returns on nominal and foreign bonds respectively, which are stochastic.

Following the literature, the paper will assume that the expected real return is the same on all types of instruments. If price setting of the instruments were incorporated in a closed economy model, the agents would obviously have to be risk neutral to obtain this result. An alternative interpretation that will be used here is that it is a small open economy with returns determined by

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[5] Islamic banking laws prevent the use of ex ante fixed interest rates and instead call for different profit sharing arrangements that in some cases, at least in theory, come close to the contingent returns here. For a discussion of Islamic banking and government financing, see Sundararajan et al. (1998).

[6] In addition to the choice of instruments, there is of course also the important question of maturity of the instruments. However, in this theoretical exercise little is lost from abstracting from maturity issues, since it would be equivalent to introducing another debt instrument. I.e. maturity considerations does not add any new questions or solutions in the present framework, although it is worth noting that the risk free instrument would be a consol, as discussed in Barro (1995), since only then will variations in the real interest rate be avioded.

risk neutral international investors, or by international investors that can fully hedge the risk of these instruments in other markets. In other words, we assume that

$$r_0 = E[r_1] = E[r_2].$$ (5)

Later on we will discuss how the returns on the stochastic instruments are determined. The timing is the standard one used in the tax smoothing literature, where the exogenous variables are first realized and then the government chooses the tax rate to fulfill its budget constraint, which implies that the budget constraint holds with certainty in both the first and the second period. That is, once the government chooses its first period tax rate, the level of public debt and the second period tax rate can be derived by the first and second period budget constraints, and since the tax base and expenditures are stochastic in the second period, so is the second period tax rate. The first order conditions can now be stated without further assumptions, and are

$$E[\tau_2] = \tau_1$$ (6)

$$Cov(\tau_2, r_1) = 0$$ (7)

$$Cov(\tau_2, r_2) = 0,$$ (8)

which we recognize from, for example, Bohn (1990). The first condition is the standard tax smoothing over time, while the latter conditions comes from tax smoothing over states of nature. The intuition for the additional conditions are straight forward. If the tax rate covaries with the returns on a certain instrument, ex post tax smoothing can be improved by issuing more or less of that instrument depending on if the covariance is negative or positive.

The next issue to address is how the portfolio weights should be chosen in order to fulfill the first order conditions, that is, to optimally use the hedging properties of the different instruments. Since the problem and returns are stated in real terms we have to transform nominal and foreign bond returns to real returns. The realized real return on nominal bonds is determined by

$$1 + r_1 = \frac{1 + i}{1 + \pi}$$ (9)

and on foreign bonds by

$$1 + r_2 = \frac{(1 + i^*)(1 + e)}{1 + \pi},$$ (10)

where i and i^* are domestic and foreign nominal interest rates, π and π^* domestic and foreign inflation and e is the depreciation of the nominal exchange rate, which is normalized to one in the first period as is the price level. Following, for example, Goldfajn (1995), we let the nominal

interest rate be determined by the Fisher equation

$$1 + i = (1 + r)(1 + \overline{\pi}), \tag{11}$$

where $\overline{\pi}$ is the expected inflation rate. The foreign instrument is also a nominal bond but in the foreign currency, with the foreign nominal interest rate i^* determined in the same way as

$$1 + i^* = (1 + r)(1 + \overline{\pi}^*). \tag{12}$$

The real interest rate r is assumed to be the same across countries. These expressions allow us to derive the expected real returns on these instruments, and the realized returns are then obviously given by the same expressions but with the realized values rather than expected, so that the ex post returns will differ if the realized values differ from the expected. For the domestic bond we obtain

$$1 + r_1 = \frac{(1 + r)(1 + \overline{\pi})}{1 + \pi}. \tag{13}$$

To determine the real return on the foreign bond, the assumption of PPP is added to the Fisher equation, that is, $1 + e = (1 + \pi) / (1 + \pi^*)$ (note that the exchange rate and price levels are normalized to 1 in the first period, so that PPP also holds then). Together these assumption yield

$$1 + r_2 = \frac{(1 + r)(1 + \overline{\pi}^*)}{1 + \pi^*}. \tag{14}$$

If PPP is not assumed to hold in terms of realized values but only in expectations, that is, $1 + \overline{e} = (1 + \overline{\pi}) / (1 + \overline{\pi}^*)$, this can be used to derive the expression $1 + r_2 = (1 + r)(1 + q) / (1 + \overline{q})$, where $1 + q = (1 + e) / (1 + \pi)$ is the nominal depreciation in terms of the domestic price level. This formulation allows for changes in the real exchange rate and could be used to perform the same analysis as the one here that will maintain the assumption of actual PPP. For the reader who prefers to not impose the PPP restriction, q and $\overline{q}$ can readily be substituted for $-\pi^*$ and $-\overline{\pi}^*$ in the following analysis. This implies that the interpretation would be that the risk associated with foreign bonds is an exchange rate risk rather than a foreign inflation risk. The presentation will however stick to the latter interpretation to avoid too much confusion.

By using the above derived real returns on the instruments and by dividing the second period budget constraint with $Y_2 = (1 + n) Y_1$, we get

$$\tau_2 = g_2 + bo_2 + \frac{1 + r}{1 + n} \left(1 - m - m^* + \frac{1 + \overline{\pi}}{1 + \pi} m + \frac{1 + \overline{\pi}^*}{1 + \pi^*} m^* \right) d, \tag{15}$$

where lower case letters indicate that the variable is in terms of Y_2. This expression can be linearized around the expected values (see Goldfajn, 1995 and Missale, 1996) to yield the expression

$$\tau_2 = g_2 + bo_2 + (1 + r - n - \widehat{\pi}m - \widehat{\pi}^*m^*)\, d, \tag{16}$$

where $\widehat{\pi} = \pi - \overline{\pi}$ is the surprise in domestic inflation and the foreign inflation surprise is defined equivalently. Note that the tax rate (or burden of debt) is reduced by a high real growth rate, since then the tax base is increased, and by positive inflation surprises both at home and abroad, since then the real value of nominal and foreign debt is reduced. The optimal portfolio shares can be obtained by using the first order conditions above. However, a slightly more intuitive way of deriving the portfolio shares is by noting that minimizing excess burden here is equivalent to minimizing the variance of the tax rate, which we will use here. The variance of the tax rate is

$$
\begin{aligned}
\sigma_\tau^2 &= \sigma_g^2 + \sigma_{bo}^2 + d^2\sigma_n^2 + d^2 m^2 \sigma_\pi^2 + d^2 m^{*2}\sigma_{\pi^*}^2 + 2\sigma_{g,bo} \\
&\quad - 2d\left(\sigma_{g,n} + m\sigma_{g,\pi} + m^*\sigma_{g,\pi^*}\right) - 2d\left(\sigma_{bo,n} + m\sigma_{bo,\pi} + m^*\sigma_{bo,\pi^*}\right) \\
&\quad + 2d^2\left(m\sigma_{n,\pi} + m^*\sigma_{n,\pi^*}\right) + 2d^2 m m^*\sigma_{\pi,\pi^*}\,,
\end{aligned}
\tag{17}
$$

and minimizing the variance with respect to the portfolio shares, the following conditions are obtained

$$\frac{\partial \sigma_\tau^2}{\partial m} = 2d^2 m \sigma_\pi^2 - 2d\sigma_{g,\pi} - 2d\sigma_{bo,\pi} + 2d^2\sigma_{n,\pi} + 2d^2 m^*\sigma_{\pi,\pi^*} = 0 \tag{18}$$

$$\frac{\partial \sigma_\tau^2}{\partial m^*} = 2d^2 m^*\sigma_{\pi^*}^2 - 2d\sigma_{g,\pi^*} - 2d\sigma_{bo,\pi^*} + 2d^2\sigma_{n,\pi^*} + 2d^2 m\sigma_{\pi,\pi^*} = 0. \tag{19}$$

Solving for the optimal portfolio shares we get

$$m = \frac{\sigma_{\pi^*}^2\left(\sigma_{g,\pi} + \sigma_{bo,\pi} - d\sigma_{n,\pi}\right) - \sigma_{\pi,\pi^*}\left(\sigma_{g,\pi^*} + \sigma_{bo,\pi^*} - d\sigma_{n,\pi^*}\right)}{d\left(\sigma_\pi^2\sigma_{\pi^*}^2 - \sigma_{\pi,\pi^*}{}^2\right)} \tag{20}$$

$$m^* = \frac{\sigma_\pi^2\left(\sigma_{g,\pi^*} + \sigma_{bo,\pi^*} - d\sigma_{n,\pi^*}\right) - \sigma_{\pi,\pi^*}\left(\sigma_{g,\pi} + \sigma_{bo,\pi} - d\sigma_{n,\pi}\right)}{d\left(\sigma_\pi^2\sigma_{\pi^*}^2 - \sigma_{\pi,\pi^*}{}^2\right)}. \tag{21}$$

The interpretations of these portfolio shares are relatively intuitive. The first terms in the numerator describe the hedging value of nominal and foreign bonds respectively. For example, if (foreign) inflation is positively correlated with g_2 and bo_2, and negatively correlated with n, this would lead to a large share of nominal (foreign) bonds in the portfolio. The second term in the numerator adds the consideration of correlation between the risky instruments, while the denominator indicates that high variance in the risky instruments that is not accompanied by desirable hedging properties would lead to a reduced share of the risky instruments and thus to a large share of indexed bonds.

From a technical point of view, the addition made in this paper so far compared with the previous literature, for example, Bohn (1990) and Goldfajn (1995), is the covariance terms between the bail out cost and domestic and foreign inflation respectively. These covariances are basically what the public debt manager can exploit to reduce the impact of bail outs. It is therefore of interest to understand what the correlations are under different assumptions regarding the private sector's behavior.

In most of the tax smoothing literature, the correlations are determined by empirical rather than theoretical studies, predominantly by using VARs to gauge the correlations of interest. However, making statements on how to structure debt from empirically estimated correlations (in reduced form models) encounter severe difficulties if all relevant events are not included in the observed data (or if the reduced form disguises some important structural relationship generating the data). This is of particular concern when dealing with infrequent events like bail outs. An alternative way is to continue with the theoretical modeling to see if that can provide some insights to how the optimal debt portfolio may depend on the firms behavior. This is done in the following part of the paper.

IV. THE CORPORATE PORTFOLIO

This section will address the questions of what determines the expected bail out costs and the correlation between bail out costs and the relevant elements in the government's budget. The first question is important to understand in deciding the level of public debt, while the other question is related to the composition of public debt. To make the analysis interesting from a PDM perspective, the private sector is modeled in terms of a representative company that is subject to aggregate shocks. If we only allowed for idiosyncratic risk and a large number of small firms, there would be no risk from the government's perspective and thus no role for the public debt manager (only for intertemporal tax smoothing). However, since real world bail outs most often seem to be associated with aggregate risks, like recessions or floating of exchange rates etc., introducing aggregate risk in the model seems reasonable.

Assume that the firms are primarily interested in maximizing its real profits, but for a given profit level they also seek to minimize the variation in profits. (Preferences are lexicographic, risk neutral in first stage then risk averse). The choice the firm has is limited here to how they finance their operations. Each firm is assumed to have a certain amount of equity and a revenue generating operation (or project) that is given by history. Furthermore, the financing need of the firm is not covered by (issuing more) equity, but in each period a given amount of debt financing is used. Like the government, the firms can chose between real, nominal or foreign bonds, where c and c^* now are the shares of nominal and foreign bonds in the corporate portfolio respectively. Formally, let the profit function in the second period be described by[7]

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[7]Since we are only interested in default and the uncertainty regarding defaults, we will ignore the profit in the first period. This can be viewed as being part of the overall financing need, but since it will not affect the portfolio choice for the second period in the present set up, it will be omitted here to reduce notation, thus there is no need to use a time index here. If the financing is made contingent on first period performance for some reason not modelled here, that would obviously imply that first period profits had to be included. To keep the model relatively simple and focus

$$P = R - [(1 + r_0)(1 - c - c^*) + (1 + r_1)c + (1 + r_2)c^*] B \qquad (22)$$

where P is the profit derived from real revenues R minus the financing cost of its corporate bond stock B. The amounts of nominal and foreign bonds in the corporate portfolio are denoted c and c^* (it would be straight forward to add other cost components, but it does not add anything in terms of the present analysis, alternatively we could simply interpret R as the net result before interest and amortization payments). Under the maintained assumption of equal expected returns on all instruments, the choice of the corporate debt structure will not affect the expected returns if we for the moment ignore the default element. The firm will then chose the portfolio share that minimize the variance in profits. If we first linearize the profit function in the same way the government's budget constraint was linearized we get that profits can be approximated by

$$P = R - (1 + r - c\widehat{\pi} - c^*\widehat{\pi}^*) B . \qquad (23)$$

Real profits will thus increase (decrease) with positive inflation shocks if nominal and foreign bonds have positive (negative) weights in the corporate portfolio. The variance of profits (σ_P^2) is given by

$$\sigma_P^2 = \sigma_R^2 + B^2 c^2 \sigma_\pi^2 + B^2 c^{*2} \sigma_{\pi^*}^2 + 2B \left(c\sigma_{R,\pi} + c^* \sigma_{R,\pi^*} \right) + 2B^2 cc^* \sigma_{\pi,\pi^*} , \qquad (24)$$

where σ's with one subscript and the exponent 2 denotes the variance of a variable and σ's with two subscripts denotes a covariance. Minimizing the variance of profits with respect to the portfolio shares, we get

$$\frac{\partial \sigma_P^2}{\partial c} = 2B^2 c\sigma_\pi^2 + 2B\sigma_{R,\pi} + 2B^2 c^* \sigma_{\pi,\pi^*} = 0 \qquad (25)$$

$$\frac{\partial \sigma_P^2}{\partial c^*} = 2B^2 c^{*2} \sigma_{\pi^*}^2 + 2B\sigma_{R,\pi^*} + 2B^2 c\sigma_{\pi,\pi^*} = 0 . \qquad (26)$$

Solving for the portfolio shares we get

$$c = \frac{-\sigma_{\pi^*}^2 \sigma_{R,\pi} + \sigma_{\pi,\pi^*} \sigma_{R,\pi^*}}{B \left(\sigma_\pi^2 \sigma_{\pi^*}^2 - \sigma_{\pi,\pi^*}^2 \right)} \qquad (27)$$

$$c^* = \frac{-\sigma_\pi^2 \sigma_{R,\pi^*} + \sigma_{\pi,\pi^*} \sigma_{R,\pi}}{B \left(\sigma_\pi^2 \sigma_{\pi^*}^2 - \sigma_{\pi,\pi^*}^2 \right)} \qquad (28)$$

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on one issue, these considerations are left out of the model.

The interpretation of the shares is straight forward and analogous to the interpretation of the shares in the government's portfolio. The first term in the numerator describes the direct hedging properties of the instrument, while the second term describes the hedging properties of the other risky instrument weighted by the covariance of the risky instruments. The denominator scales the shares with the riskiness of these instruments compared with the riskless instrument. In the case the cross correlation between π and π^* is zero, the share of an instrument is simply minus the covariance over the variance times the debt level, which makes the trade off between the risky instruments hedging properties and general riskiness very transparent.[8]

The company is assumed to default when its profits are sufficiently negative to wipe out its equity (E) completely, and the excess is what the government bails out in this model, that is, the bail out cost is defined as

$$BO \ = \ -\min\left[E + P, 0\right] \geq 0. \tag{29}$$

We will focus on two cases, first the case with **no moral hazard**, which is defined as the case when the firms use the actual distributions of the various instruments for making its portfolio decision and the portfolio shares are those derived above. The **moral hazard** case is defined as the case where firms include the default option in their decision. Then their portfolio is no longer described by minimizing the variance, and the portfolio shares will change compared to the no moral hazard case. The question is then what the portfolio will look like with the moral hazard problem. In general, the above two step strategy of determining portfolio shares does not give an interior solution. The basis for understanding this is to realize that the first step of maximizing profits leads to the conclusion that the firms want to maximize (rather than minimize) the variance in profits to obtain the highest expected profits once the default option is included in the firm's decision. This is due to the fact that all instruments have the same expected return, while the lower tail of the profit distribution is truncated; the most spread out distribution will thus benefit most from the truncation in terms of the after truncation expected value.

Ignoring correlation between the returns on the risky instruments, the above argument would lead the firm to chose a positive weight on the most risky instrument and a negative weight on the risk free instrument, with the weights being infinitely positive or negative. As a result, the variance would also go to infinity (see equation 24). Instead of using these infinite weights that are unreasonable for several reasons, not least for the fact that we are using returns given by a world market in the model, the moral hazard case will simply be defined as the case when the firms chose a portfolio that leads to a larger variance in profits than in the no moral hazard case.[9]

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[8]To simplify the interpretation further, the correlation between, e.g., the revenue and domestic inflation is $\rho_{R,\pi} = \sigma_{R,\pi} / \left(\sigma_R \sigma_\pi\right)$ and by normalizing the variance in revenues and the debt level to one, the portfolio share becomes a simple function of the correlation and the variance according to $c = -\rho_{R,\pi} / \sigma_\pi$.

[9]An alternative way of limiting the variances and portfolio shares would have been to assume that the firms are risk averse rather than risk neutral, then the usual trade off between risk and return would have been present in the moral hazard case to limit the variance. However, that would make the algebra a little more tedious without adding too much to the current analysis, and

The question is what the two types of firm behavior implies in terms of expected bail out costs its correlations with the instruments in the public debt portfolio.

Starting with the question of the expected bail out cost, we know that the expected values of the profit functions including the default range is the same for both cases. However, the variance is larger for the moral hazard case, which implies that this distribution has fatter tails than the no moral hazard case. For a given level of equity, which defines the default range, the fatter tails of the moral hazard case implies that the expected value of bail outs with moral hazard is higher than in the case without moral hazard. The implication for the tax smoothing government is thus that the level of the public debt should be lower in the case there is a moral hazard problem in the firms' portfolio choice to compensate for the higher expected bail out cost. This is the intertemporal tax smoothing part of the government's problem that only involves the level of public debt, not the composition of the debt portfolio. In other words, for the expectations part of the tax smoothing problem the result is unambiguous, moral hazard implies that the excess burden is higher than it would be without the moral hazard aspect.

The next question to address is how firm behavior affects tax smoothing over states of nature, which involves the composition rather than level of public debt, and in order to understand this, we need to analyze how the relevant correlations change between the two cases. In particular, what will matter is the correlations between bail out costs and the instruments in the government's portfolio. These correlations will determine how well the government can hedge the uncertainty regarding bail out costs in the two cases.

In the **no moral hazard case**, the firms portfolio choice makes profits and the inflation rates uncorrelated, since this is the way firms minimize the variance in profits. The distribution of bail out costs is then derived from the distribution of profits such that profits are shifted by the amount of equity and then truncated at zero, so the bail out cost is the remaining negative part of the equity adjusted profit distribution. As long as the correlation does not depend on the part of the distribution we are looking at, this implies that the correlation between bail out costs and inflation are zero. However, in the government's budget constraint it is the bail out cost in relation to the tax base (GDP) that matters, and this measure of bail out costs will be correlated with inflation if the tax base is. If, for example, supply shocks dominate, the correlation between inflation and output is negative, so when output is high inflation is low, but when output is high, the relevant measure of the bail out cost is low, so inflation will in this case be positively correlated with the bail out cost. This correlation structure would therefore reinforce the argument for using nominal debt in the portfolio compared to the case when bail out costs are ignored.

In the **moral hazard case**, profits will be correlated with the most risky instrument if only this instrument is used. If foreign bonds is the risky instrument for example, profits will be positively correlated with the foreign inflation rate, since a high foreign inflation rate will lead to high real profits. Thus, bail out costs will be higher the more negative the foreign inflation shock is, making the total bail out costs negatively correlated with foreign inflation. The total bail out cost is then divided by output to get the tax base adjusted bail out cost that is relevant in the

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the present exogenous limitations on the portfolio shares can be viewed as a crude but not too harmful short cut.

government's budget. Without output shocks, the bail out component in the government's budget will be negatively correlated with foreign inflation, which implies that the government should have a negative weight on foreign bonds in the public debt portfolio to hedge the private sector's financing decision (disregarding other factors, see equation 21).

V. A NUMERICAL EXAMPLE

To illustrate how the level and variance of the tax rate, and the government's portfolio depend on assumptions regarding correlations and firm behavior, a numerical example is presented below. In general, we need to find the variance and covariances of the relevant bail out cost to compute the government's optimal portfolio and level and variance of the tax rate. More precisely, $E(bo)$ and $Cov(bo, x)$ is needed, where x represents all the elements in the government's budget. This requires several steps: first of all, the variance and covariances of the profit function must be computed, then this has to be mapped into the corresponding characteristics for the total bail out cost, and finally this has to be converted into the characteristics of the bail out cost per tax base unit, which is the relevant object in the government's budget. These three steps are outlined in more detail in Appendix I.[10]

A few things are worth to note about the equations in the Appendix. First, since moral hazard is defined as a behavior that increases the variance in the firm's profit distribution, we can immediately confirm that the mean of the bail out cost is higher in the moral hazard case, since the mean is a linear increasing function in the standard deviation. For the same reason, the variance in the bail out cost also increases in the moral hazard case. Finally, the total bail out cost and the two inflation rates are uncorrelated in the case of no moral hazard, so that the relevant covariances in the government's budget is only determined by the covariance of the tax base with the inflation rates. On the other hand, in the moral hazard case, the covariances depend on the firms portfolio choice; since the shares are not chosen to minimize the variance, the returns on the instruments will in general be correlated with profits and thus with the bail out cost.

Table 2. Different Cases of Government and Firm Behavior		
	Firms	
Government	1. No moral hazard	2. Moral hazard
1. Include bail out	Case 1.1	Case 1.2
2. Ignore bail out	Case 2.1	Case 2.2

The presentation of the results will focus on four cases by using our two ways of modeling firm behavior from the previous section, and by comparing what happens in these cases if the government includes or ignores the bail out element in the budget in designing their debt portfolio. The different cases can be sorted according to Table 2. The assumptions are summarized in Table 3 and the results are presented in Table 4.

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[10]Since closed form solutions can be obtained by the methods described in Appendix I, the example can be produced by a relatively simple spreadsheet where the assumptions can be easily modified. For the interested reader who would like to experiment with other assumptions, the spreadsheet is available from the author.

Table 3. Assumptions for Numerical Example								
		Correlation/Variance						
Variables	**Means**	R	π	π^*	g	n	**Constants**	**Value**
R	0.3	0.3					Y_1	1
π	0	-0.1	1				B	0.3
π^*	0	0	0	0.1			d_0	0.4
g	0.3	0	0	0	0.1		g_1	0.3
n	0	1	0	0	0	0.1	r	0

Table 4 shows that a firm seeking to minimize the variance in profits would chose slightly less than a fifth of the portfolio to be nominal bonds, and use no foreign bonds. This is due to the assumptions that domestic (foreign) inflation is negatively (not) correlated with revenues. The moral hazard case is simply defined as a case where the firms uses only foreign bonds, which leads to a higher variance in profits, a higher expected profit when the default option is considered, but also higher expected bail out costs for the government. In this example, the moral hazard problem is relatively minor, only increasing the expected bail out cost by half a percent of output. However, the public debt portfolio changes substantially when the moral hazard aspect is added to firm behavior. The correlations are chosen such that the risky instruments cannot be used to hedge expenditure or tax base risk when bail outs are ignored. Furthermore, in the case the firm is minimizing the variance in profits, the returns on the risky instruments are uncorrelated with the bail out costs in the governments budget, and therefore, the government uses only real bonds in Case 1.1.

Table 4. Results from Numerical Example				
Variables	**Case 1.1**	**Case 1.2**	**Case 2.1**	**Case 2.2**
c	0.1826	0^a	0.1826	0^a
c^*	0	1^a	0	1^a
$E(bo_2)$	0.1185	0.1233	0.1185	0.1233
$Cor(bo, \pi)$	0	0.0765	0	0.0765
$Cor(bo, \pi^*)$	0	-0.1325	0	-0.1325
m	0	0.1979	0	0
m^*	0	-1.0841	0	0
τ_1	0.5592	0.5616	0.5000	0.5000
d_1	0.1408	0.1384	0.2000	0.2000
$E(\tau_2)$	0.5592	0.5616	0.6185	0.6233
σ_τ^2	0.2529	0.2545	0.2665	0.2718
[a] Definition of moral hazard case.				

In the moral hazard case (1.2), the fact that the firm does not minimize the variance in profits leads the risky instruments to be correlated with the bail out cost. This correlation implies that it is optimal for the government to use both nominal and foreign debt in its portfolio to hedge the uncertain bail out costs, and the portfolio shares are such that the government issues 19 percent

nominal bonds and aquires 108 percent foreign bonds. Thus the government internalizes the firms hedging decision in its portfolio choice by reducing the combined exposure to foreing bonds, while taking advantage of the fact that issuing nominal bonds reduces the variance in the firm's profits. In other words, the government partly "recreates" the firms minimum variance portfolio by its own portfolio choice. However, the fact that the moral hazard case increases the expected bail out cost is not something the government can undo by clever debt management, although the costs can be minimized by adjusting the level of public debt, which drops by half the increase in the expected bail out cost, since half the increased tax burden is carried in the first period and half in the second.

In the case the government ignores the bail out cost (2.1 and 2.2) in its tax smoothing policy, this can lead to a situation where even without the moral hazard problem, the variation in tax rates both over time and over states of nature is larger than in the moral hazard case with a government that incorporates the bail out cost in the PDM policy. This comes from the fact that the level of public debt is not chosen optimally. Furthermore, the portfolio shares ignore the hedging that can be done in the moral hazard case, thus creating a higher variance in the tax rate. Note also that any given variance is more harmful in this case, since the average tax rate is higher in period 2.[11]

To summarize, the example has illustrated several of the results that has been discussed in the previous section: the level of public debt depends on the behavior of the firms, the composition of the corporate portfolio affects the compositions of the government's portfolio and ignoring the bail out cost leads to tax rates that fluctuate both over time and over states of nature, when it, at least to some extent, can be avoided by the debt manager.

VI. CONCLUDING REMARKS

This paper has introduced bail out costs as a consideration for the public debt manager. The main observations are that the existence of possible bail outs will affect both the timing of deficits and surpluses and the composition of the government's debt portfolio. More specifically, debt instruments should be chosen to include instruments that can hedge the uncertainty regarding future bail out cost. Determining the expected bail out cost and the appropriate hedging instruments requires that the government observes the financial position of the corporate/financial sector. This contrasts with the Ricardian paradigm, where it is the households that observes the governments portfolio in order to smooth consumption, here the government observes the private sector's portfolio in order to smooth taxes.

Several empirical implications emerges from the paper and will be the subject of future research. The first question is to investigate is whether countries with highly leveraged firms with variable profits, which implies higher expected bail out costs, have relatively small public deficits, other things equal.

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[11]Remember that excess burden is a convex function of the tax rate, and thus tax smoothing becomes more important the higher the average tax rate is.

Secondly, the results suggest that the government will hedge the uncertainty of future bail outs by taking position in the public debt portfolio that are opposite of the firms' positions. The empirical question would thus be to investigate if private sectors' portfolios help in explaining the observed differences in governments' portfolios.

BAILOUT COST CALCULATIONS

This appendix presents the method used to compute the relevant mean, variance and covariances for the bail out costs. This is a three step procedure going from the pro t function to bail out costs per tax base unit.

Step 1: Calculate the variance and covariances for the pro t distribution.

This can be done by using the following property of covariances for a set of linear functions if

$$y_1 = a_{11}x_1 + ... + a_{1j}x_j \tag{A.1}$$

$$\vdots$$

$$y_i = a_{i1}x_1 + ... + a_{ij}x_j$$

or in matrix form,

$$\mathbf{y} = \mathbf{A}\mathbf{x} \tag{A.2}$$

we have that

$$\mathrm{Cov}(\mathbf{y}) = \mathbf{A}\Sigma\mathbf{A}' \tag{A.3}$$

where Σ is the variance-covariance matrix of the $\mathbf{x}$ vector.

Step 2: calculate the mean and covariance matrix for the bail out distribution.

a) Assume that the stochastic variables generating the pro t function are normal so that the pro t function, which is a linear combination of these variables, is normally distributed. Assume also that the mean of the pro t function is equal to zero, and that equity is zero as well. The mean of the bail out cost is then a linear function of the standard deviation in the pro t function according to

$$E(BO) = \overline{BO} = -\left(\int_{-\infty}^{0} x\phi(x)\,dx + \frac{1}{2}\cdot 0\right) = \frac{1}{\sqrt{2\pi}}\sigma_P, \tag{A.4}$$

where $\phi(x)$ is the Gaussian pdf.

b) The variance is then

$$Var(BO) = \int_{-\infty}^{0} \left(x - \overline{BO}\right)^2 \phi(x)\,dx + \frac{1}{2}\left(0 - \overline{BO}\right)^2 = \frac{(\pi - 1)}{2\pi}\sigma_P^2. \tag{A.5}$$

c) Finally, if x_3 is the truncated function of x_1 at 50% of a symmetric distribution (like the normal), then if $Cov(x_1, x_2) = a$ we get that $Cov(x_3, x_2) = a/2$. This result seems intuitive if

the covariance can be computed as a weighted average of the covariances in different parts of the distribution, since then the covariance is the original covariance in half the distribution and zero for the other half, where the truncated distribution is constant (at zero).[12]

Step 3: Getting from results regarding BO to results regarding bo.

We are interested in $bo \equiv BO/Y_1(1+n)$ not in BO. To compute the covariances of sets of non linear functions, a rst order Taylor expansion around the mean can be used (see, for example, Green 1997). The $\mathbf{A}$ matrix in step 2 now contains the partial derivatives $\frac{\partial y}{\partial x}$ evaluated at the mean of x. In this case the partial derivatives of interest are

$$\frac{\partial bo}{\partial n} = -\frac{BO}{Y_1(1+n)^2} \tag{A.6}$$

and

$$\frac{\partial bo}{\partial BO} = \frac{1}{Y_1(1+n)} \tag{A.7}$$

where n and BO are replaced by their means when the covariances are computed.

Using this procedure we get that

$$\sigma_{R,P} = \sigma_R^2 + Bc\sigma_{R,\pi} + Bc^*\sigma_{R,\pi^*} \tag{A.8}$$

$$\vdots$$

and that

$$\sigma_{bo,x} = \frac{\partial bo}{\partial n}\sigma_{n,x} + \frac{\partial bo}{\partial BO}\sigma_{BO,x}, \tag{A.9}$$

$$\sigma_{bo}^2 = \frac{\partial bo}{\partial n}\sigma_n^2 + \frac{\partial bo}{\partial BO}\sigma_{BO}^2 \tag{A.10}$$

where again the x denotes the elements in the government's budget.

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[12]At this stage no formal proof has been constructed, but the result is con rmed by simulations. The corresponding results does not hold for other arbitrary cut off points other than half the distribution. This is also relatively intuitive, since the variance of the distribution is not a constant function of the probability mass, i.e., the tails have relatively high variance per mass.

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