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Monetary Policy and Inflation Scares

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Christopher Erceg[1] , Jesper Lindé[2] , and Mathias Trabandt[3]

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ABSTRACT: A salient feature of the post-COVID inflation surge is that economic activity has remained resilient despite unfavorable supply-side developments. We develop a macroeconomic model with nonlinear price and wage Phillips curves, endogenous intrinsic indexation and an unobserved components representation of a cost-push shock that is consistent with these observations. In our model, a persistent large adverse supply shock can lead to a persistent inflation surge while output expands if the central bank follows an inflation forecast-based policy rule and thus abstains from hiking policy rates for some time as it (erroneously) expects inflationary pressures to dissipate quickly. A standard linearized formulation of our model cannot account for these observations under identical assumptions. Our nonlinear framework implies that the standard prescription of "looking through" supply shocks is a good policy for small shocks when inflation is near the central bank's target, but that such a policy may be quite risky when economic activity is strong and large shocks drive inflation well above target. Moreover, our model implies that the economic costs of "going the last mile" – i.e. a tight stance aimed at returning inflation quickly to target – can be substantial.

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1. Introduction

Following several decades of low and stable inflation in advanced economies, the post-COVID inflation surge caught central banks, economic forecasters, and academic economists by surprise. A vibrant debate has ensued about the causes of the surge, on why models failed to predict it, and on how to refine policy strategies.[1]

Supply shocks were widely viewed as generating much of the initial spike in inflation after the pandemic, including from COVID-related supply disruptions and escalating energy prices. However, while the shocks were clearly very large and drove headline inflation to levels not seen in decades, both the shocks and their effects on underlying inflation were forecast to be transient. The view that "second round" effects were likely small - implying weak endogenous propagation of supply shocks - was supported by a wide body of empirical evidence, and attributed to well-anchored inflation expectations as well as to structural features such as relatively flexible labor markets (Blanchard and Gali, 2007). Accordingly, advanced economy central banks choose to largely "look through" these temporary shocks and maintain highly accommodative policies even as realized inflation ran far above their targets.

The growing perception headline inflation pressures were reverberating to wages and core inflation eventually led to a rapid tightening in monetary policy that has helped markedly slow inflation. But the experience has left many open questions. Why did such a broad range of forecasters and forecasting models predict that high inflation would rapidly dissipate? What modeling features could help account for why second-round effects have generally been small since the start of the Great Moderation, but much higher in the recent surge? Are large shocks different in their effects? And, if transmission is more state-dependent than recognized, what are the implications for policy strategy, including for "looking through" supply shocks and how policy responds to inflation forecasts?

Our paper develops a modeling framework that aims to help address these questions. Following Harding, Lindé and Trabandt (2022, HLT), we start by embedding a nonlinear Phillips Curve into a fairly standard DSGE model that arises from a quasi-kinked demand schedule. The Phillips curve is flat when inflationary pressures are subdued and steepens as inflationary pressures rise. As shown by HLT (2023), these features imply that large shocks can have an outsized effect on inflation when inflation is already running above target.

We extend the analysis in HLT along several important dimensions. First, we introduce gradual

[1] See e.g. Federal Reserve Chair J. Powell speech at the 2021 Jackson Hole conference as well as the debate between L. Summers and P. Krugman that took place since early 2021. See also Gopinath (2022).

learning about the nature of the cost push shocks that are assumed to (mainly) drive inflation. Specifically, agents can't tell if the shocks are transitory or persistent (only observing the sum of the components), and must solve a signal extraction problem to estimate the underlying shock. The initial misperception of the shock as transitory - when it in fact turns about to be much more persistent - is one factor helping our model account for the large inflation forecast errors observed during the early phases of the inflation surge.

Second, we allow for endogenous indexation of prices and wages by firms and households. In particular, the fraction of prices and wages set in a backward-looking manner is higher when inflation runs persistently and significantly above target. Thus, the stock of past inflation forecast errors has important consequences for inflation persistence in the Phillips Curve. Our calibration implies that this "intrinsic" persistence is very low if inflation has been reasonably close to target in recent years, consistent with estimates during the Great Moderation period; but implies much higher intrinsic persistence if inflation rises significantly above target for some time, as during the recent inflation surge.

Third, we consider the implications of a forecast-based targeting rule in which the central bank responds to an inflation forecast one or two years ahead, and compare this to a standard instrument rule with contemporaneous inflation. The forecast-based rule aims to capture how inflation-targeting central banks typically respond to medium-term forecasts of inflation and thus "look through" transient supply shocks (consistent with a wide literature on forecast-based Taylor rules).[2] An average inflation targeting (AIT) targeting rule, based on a sufficiently long window of average past and future inflation outcomes, shares many features with forecast inflation targeting in our framework, provided the AIT rule is not paired with escape clauses.

Interactions of these three key features – imperfect information about the nature of the adverse cost-push shock, endogenous indexation, and the policy reaction function - allow our model account for key facets of the persistent post-COVID inflation surge and the monetary policy response. Given that the shock is initially perceived as transitory and intrinsic persistence is viewed as low, inflation is expected to quickly revert to baseline, and the central bank under the forecast-based rule keeps policy rates nearly unchanged. When inflationary pressures turn out to be more pervasive than initially projected the central bank starts to hike rates materially, but at this stage the "inflation ghost" is already out of the bottle. In particular, the state-dependent sensitivity of inflation to economic activity (from the nonlinear Phillips Curve) interacts with endogenous

[2] Ashley et al. (2020) provides empirical evidence in favor of such behavior by the U.S. Federal Reserve, arguing that the Fed has focused on persistent movements in inflation but ignored transient inflation impulses.

indexation mechanism to produce a full "inflation cycle" in our nonlinear model.

Importantly, the inflation cycle is associated with an initial expansion in economic activity followed by a subsequent contraction later in the inflation cycle when real interest rates rise. Because nominal rates depend on the medium-term forecast of inflation and only rise gradually, short-term real interest rates actually fall for some time (since inflation in the very near-term rises more). This stimulus contrasts with a standard full-information model in which an adverse supply shock triggers higher inflation along with a sizeable contraction in output. Thus, our model can account for some of the observed resilience in economic activity that accompanied the initial inflation surge even without demand shocks playing a material role.

We highlight how the implications of our nonlinear model contrast sharply with the those of a linearized formulation embedding otherwise identical assumptions. While the linear model provides a good approximation if supply shocks are small, the disparity in both the quantitative and qualitative responses becomes large in the event of large shocks or when shocks occur when inflation has already been running persistently high. We also show that our nonlinear model – when fed with a combination of large adverse supply shocks and moderate demand shocks – does a good job of accounting for the overall contours of U.S. data in 2021-2023. Our nonlinear model also accounts well for the forecast revisions of professional forecasters. By contrast, the linearized model fails to account for the data and forecasts.

From a policy perspective, our analysis suggests that the standard monetary policy prescription of "looking through" adverse supply shocks should be applied cautiously. While there is a strong case for the merits of such a policy for small adverse cost-push shocks which do not drive inflation far away from its target, our nonlinear model illustrates how such a policy may prove costly when the economy is hit with larger shocks, especially if inflation is initially running above the central bank's target.[3]

Relatedly, our model has important implications for the use of forecast-based rules as a guidepost for monetary policy. In particular, our analysis suggests reasons for caution in putting a high weight on inflation point forecasts when shocks are large and intrinsic inflation persistence is high (Schnabel, 2024). Forecast errors – misperceiving a persistent shock as transitory – can be very costly and drive a very costly runup in inflation. Accordingly, we show how a more aggressive approach that puts some weight on realized inflation can lead to much better inflation outcomes and may also be preferable on welfare grounds despite implying bigger output declines. On the flip

[3] Our calibration of the unobserverd components representation of the cost-push shock places a predominant role on transitory realizations of cost-push shocks and hence rationalizes that the central bank uses a forecast-based policy rule in normal times.

side, the risk of inflation taking off due to misperceiving the shock is much smaller when inflation has been running close to target and intrinsic persistence is low, so that forecast-based rules consistent with "looking through" perform well and help avoid excessive volatility in output.

Another key policy implication of our nonlinear model is that the central bank faces a more severe trade-off between inflation and output stabilization if it aims to push inflation quickly back to target aggressively when inflation has peaked and is gradually receding. While the Phillips curve appears steeper – which in some models would make it less costly to disinflate – it isn't possible to simply "ride down" the steeper Phillips Curve if the higher inflation also works to amplify inflation persistence. This feature of our model makes it costly for the central bank to aggressively attempt to push inflation all the way back to target quickly unless a sizeable part of the underlying adverse supply shock reverses and helps to push inflation back to target. Put differently, we find that the economic costs of "going the last mile" and bringing inflation quickly back to target can be sizeable.

These findings rest importantly on the interaction between Phillips Curve nonlinearity and indexation mechanisms. Thus, cost-push shocks generate conditional heteroscedasticity in inflation and inflation risk in our nonlinear model, consistent with the seminal paper by Engle (1982) and the more recent work by López-Salido and Loria (2020). As these shocks appear to have played an important role during the post-COVID period, we argue that our model can account better for inflation dynamics during this period than a standard linearized macroeconomic model. This is corroborated by recent empirical analysis, which supports the view that cost-push-type shocks have a larger impact on inflation if inflation is initially high (see e.g., Gelos and Ustyugova 2017; Forbes, Gagnon and Collins 2021a; Forbes, Gagnon and Collins 2021b; and Ball, Leigh and Mishra 2022).

We establish our main results in a nonlinear variant of the benchmark Erceg, Henderson and Levin (2000, henceforth EHL) model with sticky wages and prices. The EHL model shares most of the model features in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007, henceforth SW), though it excludes endogenous capital accumulation. To parameterize the model, we follow HLT (2023) and allow for a more prominent role for Kimball (1995) quasi-kinked demand in goods markets. HLT shows that this feature increases the marginal data density in the basic SW model provided that the average markup aligns with micro- and macroeconomic empirical evidence. Recent work by Dupraz (2017) and Ilut et al. (2022) provides a microfounded theory of kinked demand.

The remainder of the paper is organized as follows. Section 2 presents and discusses cross-country data for inflation and policy rates as well as the projections for the survey of professional forecaster forecasts for the U.S. economy. Section 3 presents the quantitative macroeconomic model with real rigidities in a dynamic stochastic general equilibrium framework with nominal price and wage stickiness. Section 4 discusses our results. Section 5 discusses the related literature. Finally, Section 6 provides some concluding remarks.

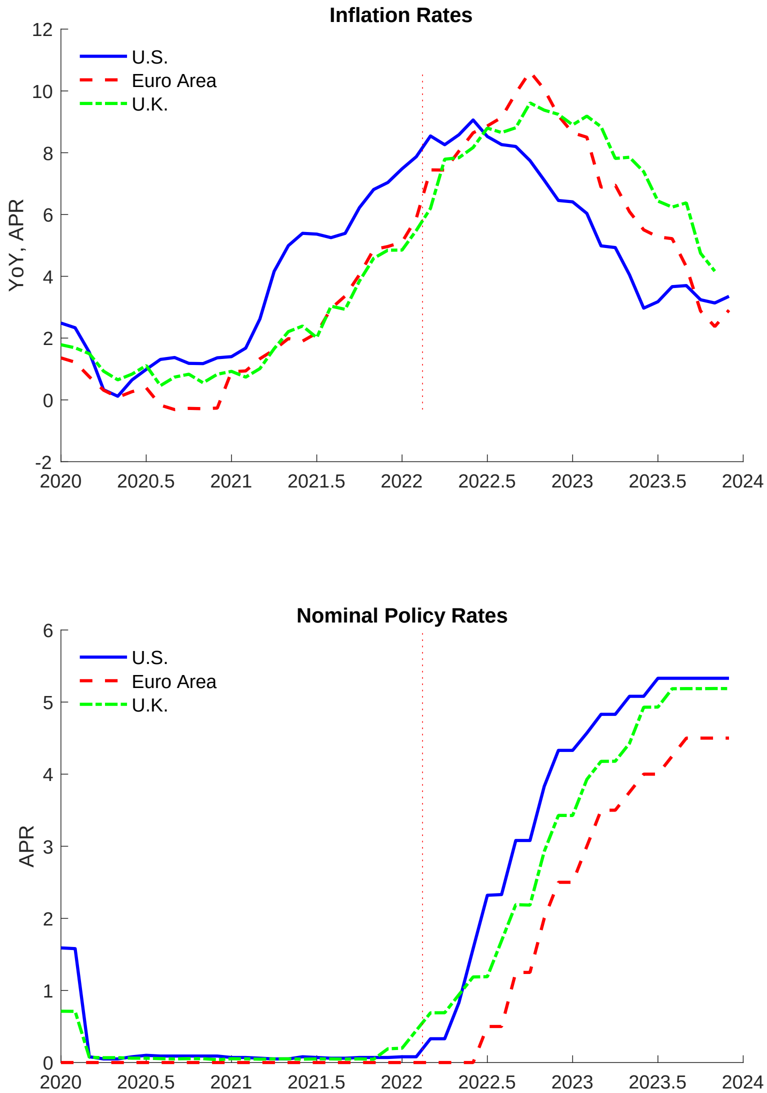

Figure 1: Inflation rates and nominal policy rates in the U.S., Euro Area, and United Kingdom.

2. Data

Figure 1 depicts monthly data for inflation (12-month change) and nominal policy rates for the U.S., the Euro Area and the U.K. Several facts are evident from the figures. First, inflation has surged in all these major economies, with US inflation leading that in the Euro Area and UK by roughly half a year. Second, the central banks in these economies – the US Federal Reserve, the European Central Bank (ECB) and the Bank of England (BoE) – kept policy rates unchanged for about 1 to 1.5 years after inflation rose above their 2 percent targets. In late 2021 and into 2022, the BoE began raising rates followed by the Fed and ECB (the vertical dotted line in the figure shows when the Fed began tightening in March 2022).

Figure 2 shows quarterly U.S. data for PCE inflation, PCE growth and the three-month T-bill rate. In addition, the figure shows projections from the Philadephia's Federal Reserve's Survey of Professional Forecasters. Several facts emerge from Figure 2. First, professional forecasters underestimated both the size and persistence of the increase in U.S. inflation beginning in early 2021. Second, professional forecasters underestimated the resilience of economic activity initially. Third, professional forecasters underestimated how much the Fed would need to raise policy rates (proxied here by the forecast for 3-month T-bills) to cool the economy and bring inflation back to target. We next develop a model that can account for these facts and use it to consider implications for policy.

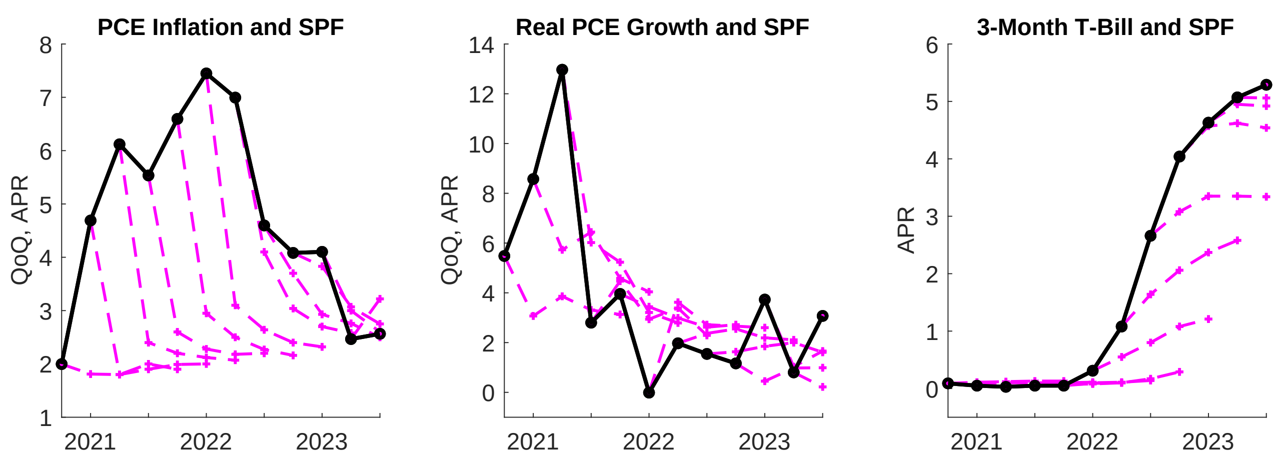

Figure 2: U.S. PCE inflation, PCE growth and 3-months T-Bill rate (solid: data; dashed: survey of professional forecasters (SPF) data.)

3. Quantitative Model

The model developed below modifies and extends the model in Lindé and Trabandt (2018) and Harding, Lindé and Trabandt (2022, 2023).

3.1. Households

There is a continuum of households $j \in [0,1]$ in the economy. Each household supplies a specialized type of labor n_{jt} to the labor market. The j^{th} household maximizes

$$\max_{c_t, n_t, b_t} E_0 \sum_{t=0}^{\infty} \beta^t \varsigma_t \left\{ \ln\left(c_t - hC_{t-1}\right) - \frac{1}{1+\chi} n_{j,t}^{1+\chi} \right\} \tag{1}$$

Here E_0 denotes the conditional expectation operator, while c_t is consumption and n_{jt} is hours worked by household j with the discount factor β and the habit persistence parameter h satisfying $0 \leq \beta < 1$ and $0 \leq h < 1$.

The household faces the budget constraint

$$P_t c_t + B_t = W_{j,t} n_{j,t} + R_{t-1} B_{t-1} - T_t + \Gamma_t + a_{j,t}$$

where B_t are the holdings of risk-free bonds (which are in zero net supply), P_t denotes the aggregate price level, $W_{j,t}$ the wage rate of household j, and R_t the gross nominal interest rate on bonds purchased in period $t-1$ which pay off in period t. T_t are lump-sum taxes net of transfers and Γ_t denotes the share of profits that the household receives. While households set wages in Calvo-style contracts (as described below), there are insurance markets – with $a_{j,t}$ denoting the payments and receipts – which ensure that all households have the same consumption.[4] Finally, the variable ς_t is an exogenous shock to the discount factor. We assume that $\delta_t = \frac{\varsigma_{t+1}}{\varsigma_t}$ is exogenous and follows an AR(1) process:

$$\delta_t - \delta = \rho_\delta \left(\delta_{t-1} - \delta\right) + \varepsilon_{\delta,t},$$

3.2. Labor Contractors and Wage Setting

Competitive labor contractors aggregate specialized labor inputs $n_{t,j}$ supplied by households into homogenous labor n_t which is hired by intermediate good producers. Labor contractors maximize profits

[4] In principle, the presence of wage setting frictions implies that households have idiosyncratic levels of wealth and, hence, consumption. However, we follow EHL in supposing that each household has access to perfect consumption insurance. Because of the additive separability of the family utility function, perfect consumption insurance at the level of households implies equal consumption across households. Given this, we have simplified our notation and not include a subscript, j, on the j^{th} family's consumption (and bond holdings).

$$\max_{n_{t,j},n_t} W_t n_t - \int W_{t,j} n_{t,j} dj, \text{ or } \max_{n_{t,j}/n_t} 1 - \int \frac{W_{t,j}}{W_t}\frac{n_{t,j}}{n_t} dj$$

where $W_{t,j}$ is the wage paid by the labor contractor to households for supplying type j labor. W_t denotes the wage paid to the labor contractor for homogenous labor. Maximization of profits is subject to

$$\int G_w\left(\frac{n_{t,j}}{n_t}\right) dj = 1$$

where

$$G_w\left(\frac{n_{t,j}}{n_t}\right) = \frac{\omega_w}{1+\psi_w}\left[(1+\psi_w)\frac{n_{t,j}}{n_t} - \psi_w\right]^{\frac{1}{\omega_w}} - \frac{\omega_w}{1+\psi_w} + 1$$

is the Kimball aggregator specification as used in Dotsey and King (1995) or Levin, Lopez-Salido and Yun (2007) adapted for the labor market. Note that $\omega_w = \frac{(1+\psi_w)\phi_w}{1+\phi_w\psi_w}$ and $\phi_w = 1+\theta_w$ where $\theta_w \geq 0$ denotes the net wage markup, $\phi_w \geq 1$ denotes the gross wage markup and $\psi_w \leq 0$ is the Kimball parameter that controls the degree of complementarities in wage setting. Let ϑ_t^w denote the multiplier on the labor contractor's constraint. The appendix contains detailed derivations that result in the following equations:

$$\frac{n_{t,j}}{n_t} = \frac{1}{1+\psi_w}\left(\left[\frac{W_{t,j}}{W_t}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} [\vartheta_t^w]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \psi_w\right) \quad (2)$$

$$W_t\vartheta_t^w = \left[\int W_{t,j}^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} dj\right]^{-\frac{\theta_w}{1+\psi_w+\theta_w\psi_w}} \quad (3)$$

$$\vartheta_t^w = 1+\psi_w - \psi_w \int \frac{W_{t,j}}{W_t} dj \quad (4)$$

Here equation (2) denotes the demand for labor, equation (3) is the aggregate wage index and equation (4) is the zero profit condition for labor contractors. Note that for $\psi_w = 0$ we get the standard Dixit-Stiglitz expressions $\frac{n_{t,j}}{n_t} = \left[\frac{W_{t,j}}{W_t}\right]^{-\frac{(1+\theta_w)}{\theta_w}}$, $W_t = \left[\int W_{t,j}^{-\frac{1}{\theta_w}} dj\right]^{-\theta_w}$, and $\vartheta_t^w = 1$.

The household faces a standard monopoly problem of selecting $W_{j,t}$ to maximize the welfare, (1) subject to the demand for labor (2). Following EHL, we assume that the household experiences Calvo-style frictions when setting $W_{j,t}$. In particular, with probability $1-\xi_w$ the j^{th} family has the opportunity to re-optimize its wage rate. With the complementary probability, the family must set its wage rate according to the following rule:

$$W_{j,t} = \tilde{\Pi}_t^w W_{j,t-1} \quad (5)$$

where $\tilde{\Pi}_t^w$ is an indexation factor whose determinants we discuss in Section 3.5 below.

Let Λ_t denote the Lagrange multiplier on the household budget constraint. To compute the optimal choice for $\tilde{W}_{j,t}$, the household seeks to maximize:

$$\max_{\tilde{W}_{j,t}} E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \varsigma_{t+i} \left\{ -\frac{1}{1+\chi} n_{j,t+i}^{1+\chi} + \Lambda_{t+i} \tilde{W}_{j,t} \left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w \right) n_{j,t+i} \right\}$$

subject to labor demand:

$$n_{t+i,j} = \frac{1}{1+\psi_w} \left(\left[\frac{\tilde{W}_{j,t} \left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w \right)}{W_{t+i}} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\vartheta_{t+i}^w \right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \psi_w \right) n_{t+i}$$

In what follows, we assume that $\chi = 0$ for simplicity. The appendix provides detailed derivations for optimal wage setting.

3.3. Final Goods Producers

Competitive final goods producers maximize profits:

$$\max_{y_{t,i}/y_t} 1 - \int \frac{P_{t,i}}{P_t} \frac{y_{t,i}}{y_t} di$$

subject to

$$\int G \left(\frac{y_{t,i}}{y_t} \right) di = 1$$

where

$$G \left(\frac{y_{t,i}}{y_t} \right) = \frac{\omega_p}{1+\psi_p} \left[\left(1+\psi_p \right) \frac{y_{t,i}}{y_t} - \psi_p \right]^{\frac{1}{\omega_p}} - \frac{\omega_p}{1+\psi_p} + 1$$

is the Kimball (1995) aggregator specification used by Dotsey and King (1995) and Levin, Lopez-Salido and Yun (2007). Note that $\omega_p = \frac{(1+\psi_p)\phi_p}{1+\phi_p\psi_p}$ and $\phi_p = 1+\theta_p$ where $\theta_p \geq 0$ denotes the net price markup, $\phi_p \geq 1$ denotes the gross price markup and $\psi_p \leq 0$ is the Kimball parameter that controls the degree of complementarities in firm's pricing decisions. Let ϑ_t denote the multiplier on the constraint. The appendix contains detailed derivations that result in the following equations:

$$\begin{aligned}
\frac{y_{t,i}}{y_t} &= \frac{1}{1+\psi_p} \left(\left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \psi_p \right) \\
P_t \vartheta_t &= \left[\int P_{t,i}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} di \right]^{-\frac{\theta_p}{1+\psi_p+\psi_p\theta_p}} \\
\vartheta_t &= 1+\psi_p - \psi_p \int \frac{P_{t,i}}{P_t} di
\end{aligned}$$

where $\phi_p = 1+\theta_p$ and $\varepsilon_p = \frac{\phi_p\left(1+\psi_p\right)}{1-\phi_p}$. Note that for $\psi_p = 0$ we get the standard Dixit-Stiglitz expressions $\frac{y_{t,i}}{y_t} = \left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}}$, $\vartheta_t = 1$ and $P_t = \left[\int P_{t,i}^{-\frac{1}{\theta_p}} di \right]^{-\theta_p}$.

3.4. Intermediate Goods Producers

Intermediate goods firms have the following production function:

$$y_{t,i} = n_{t,i}$$

Total costs for the firm are:

$$TC_{t,i} = \tau_t^{1/\kappa} W_t n_{t,i} \tag{6}$$

where $\tau_t^{1/\kappa}$ is an exogenous shifter of firms' total costs, and might capture e.g., a tax shock (with lump-sum redistribution to households). In what follows, we refer to a shock to τ_t as a cost-push shock (which we use interchangeably as a markup shock, too). Note that the shock to marginal cost is scaled by the inverse slope of the price Phillips curves once the model is log-linearized, i.e. $\kappa_p = \frac{1}{1+\beta\varkappa}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\frac{1}{1-(1+\theta_p)\psi_p}$. This way, after log-linearization, the shock enters the Phillips curve additively separable with a unit coefficient as in Smets and Wouters (2007). An additional attractive feature with this scaling is that small shocks to τ_t propagate identically in the nonlinear and linearized formulation of the model. Moreover, τ_t has a mean of unity so that the scaling does not affect the steady state. Consistent with the literature, e.g. Smets and Wouters (2007) and Galí, Smets and Wouters (2012), we assume that the cost-push shock only affects the actual economy but not potential output, i.e. the cost-push shock is inefficient.[5]

Firms minimize costs (or maximize negative cost) subject to production:

$$\max_{n_{t,i}} -\tau_t^{1/\kappa} W_t n_{t,i} + MC_{t,i}\left[n_{t,i} - y_{t,i}\right]$$

$MC_{t,i}$ denotes the lagrange multiplier and coincides with firm marginal cost. Since all firms face the same marginal costs, the first order condition reads as:

$$MC_{t,i} \equiv MC_t = \tau_t^{1/\kappa} W_t$$

Accordingly, the profit maximization facing intermediate goods-producing firms may be expressed as:

$$\max_{\tilde{P}_{t,i}} E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \varsigma_{t+j}\Lambda_{t+j}\left[\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right)\tilde{P}_{t,i} y_{t+j,i} - MC_{t+j} y_{t+j,i}\right]$$

[5] It would be interesting to consider other types of adverse supply shocks (such as e.g. adverse total factor productivity shocks reflecting e.g. supply chain disruptions or energy price hikes) that affect the potential economy with flexible prices and wages, as well as alternative processing to demand and supply shocks (as suggested by our discussant Jean-Paul L'Hullier). Our choice to rely on one specific shock in eq. (6) implies that we need a large adverse effective cost-push shock $\tau_t^{1/\kappa}$ to account for the inflation surge, although the nonlinear formulation of our model moderates the required size.

subject to

$$y_{t+j,i} = \frac{1}{1+\psi_p}\left(\left[\frac{\left(\tilde{\Pi}_{t+j}\times ... \times \tilde{\Pi}_{t+1}\right)\tilde{P}_{t,i}}{P_{t+j}}\right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \psi_p\right) y_{t+j}$$

where $\tilde{\Pi}_t$ is an indexation factor whose determinants we discuss in Section 3.5 below. The appendix provides a detailed set of derivations of optimal price setting.

3.5. Endogenous Price and Wage Indexation

Define the gross inflation rate $\Pi_t = P_t/P_{t-1}$. As noted earlier, $\tilde{\Pi}_t$ and $\tilde{\Pi}_t^w$ are the price- and wage-setting indexation inflation rates for non-optimizing firms and labor unions. We consider the following state-dependent price and wage indexation specification:

$$\tilde{\Pi}_t = \tilde{\Pi}_t^w = \bar{\Pi}^{1-\varkappa_t}\Pi_{t-1}^{\varkappa_t} \tag{7}$$

where

$$\varkappa_t = e^{-\frac{\varrho}{\max\left(\Pi_t^* - \bar{\Pi},\ 0.0001\right)}} - e^{-\frac{\varrho}{0.0001}} \tag{8}$$

and

$$\Pi_t^* = \left(\Pi_{t-1}^*\right)^{\omega}\left(\Pi_{t-1}\right)^{1-\omega} \tag{9}$$

with $0 \leq \omega < 1$ and $\varrho \geq 0$. $\bar{\Pi}$ denotes steady state inflation. Note that prices and wages are indexed by the same factor which is a function of a geometric lag of past inflation rates, provided $\omega > 0$.[6] We will henceforth use the term 'endogenous' indexation since it depends on the aggregate rate of inflation which is an endogenous variable in our model.[7]

To illustrate the indexation specification, consider the following parameters. We set $\varrho = 0.002$ and $\bar{\Pi} = 1.005$. The number 0.0001 inside the max operator is set for numerical stability. Note that $\varkappa_t$ is zero in steady state. Figure 3 provides a graphical illustration of the endogenous indexation feature of our model. Specifically, varying Π_t^* on a grid and calculating the resulting indexation factor $\varkappa_t$ results in the relationship between Π_t^* and $\varkappa_t$ displayed in Figure 3.

[6] It would be interesting to study the implications of making indexation dependent on annual lagged inflation $\Pi_{t-1}^a = \left(\Pi_{t-1}\Pi_{t-2}\Pi_{t-3}\Pi_{t-4}\right)^{1/4}$ and compare the results to our our baseline specifiation with quarterly lagged inflation, Π_{t-1}.

[7] Note that our indexation scheme offers one way of rationalizing state dependence. An alternative mechanism could be that firms re-optimize prices and wages more frequently in high-inflation environments, i.e. state-dependent Calvo probabilities. This would be consistent with many S-s type models. We favour the endogenous indexation feature of our model since it implies that inflation may remain persistently high in response to inflation surges. In other words, the endogenous inflation indexation feature generates endogenous inflation persistence. In addition, our inflation indexation feature implies that disinflations are more costly if inflation has been running persistently above target. By contrast, in models where prices and wages become more flexible as a consequence of high inflation, disinflations become less costly in terms of economic activity the higher the rate of inflation.

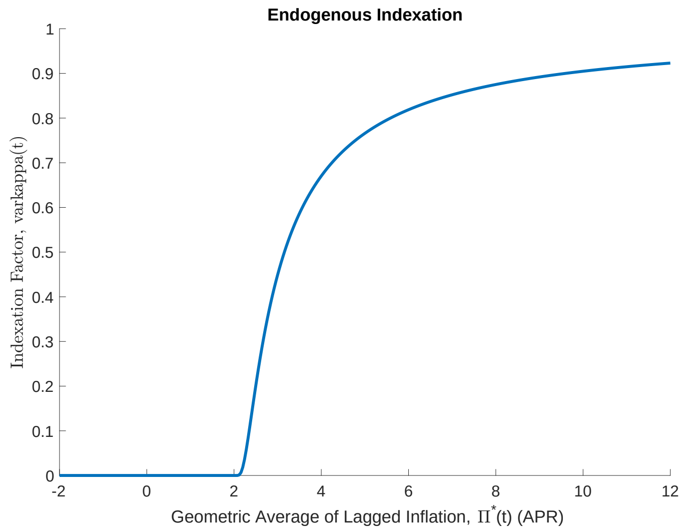

Figure 3: Endogenous Indexation

Note that when taking logs of the indexation equation $\tilde{\Pi}_t = \bar{\Pi}^{1-\varkappa_t}\Pi_{t-1}^{\varkappa_t}$ we get

$$\ln\frac{\tilde{\Pi}_t}{\bar{\Pi}} = \widehat{\tilde{\Pi}}_t = \varkappa_t \ln\frac{\Pi_{t-1}}{\bar{\Pi}} = \varkappa_t\hat{\Pi}_{t-1}$$

Taking a first order Taylor series approximation gives:

$$\widehat{\tilde{\Pi}}_t = \varkappa\hat{\Pi}_{t-1} \tag{10}$$

that is, the state-dependency feature of endogenous indexation disappears in the log-linearized model variant. The reason for eq. (10) is that $\bar{\Pi} = \Pi_{t-1}$ in eq. (7) in the steady state, which is the point of approximation. This implies no dynamic indexation in the linearized model, since $\varkappa = 0$. By contrast, in the nonlinear model, indexation to past inflation is state-dependent. Consistent with the empirical evidence in Smets and Wouters (2007) and Fernandez-Villaverde and Rubio-Ramirez (2008), there is little or no dynamic indexation when inflation is close or below the steady state, but dynamic indexation arises endogenously when inflation runs up well above the central banks' inflation target.[8]

[8] As as alternative to the indexation scheme described above, we have also examined the implications when adopting the following indexation rule: $\tilde{\Pi}_t = \left(\frac{\Pi_{t-1}}{\bar{\Pi}}\right)^{\varkappa_t}$. This indexation rule has similar implications in terms of dynamic indexation as the rule in the main text, i.e. it implies no dynamic indexation after log-linearization and endogenous dynamic indexation in the nonlinear model. In addition to these properties, the indexation rule also implies no indexation in the steady state, i.e. the model then features price dispersion in the steady state. Our qualitative and quantitative results are very little affected with this alternative indexation scheme, although it implies that the average time that is takes for firms to change their prices becomes endogenous in our nonlinear model. When inflation is close or below the steady state, firms change their prices once every three quarters. But in response to large persistent shocks that drives inflation well above the central banks' inflation target, firms change their prices more often. This model feature is consistent with recent micro evidence documented in Cavallo, Lippi, and Miyahara (2023, Figure 2).

3.6. Aggregate Resources

The aggregate resource constraint can be written as:

$$c_t = y_t = (p_t^*)^{-1} (w_t^*)^{-1} l_t$$

where p_t^* and w_t^* are measures of price and wage dispersion. See the appendix for detailed derivations and expressions for these variables.

Note that the payments and receipts associated with the insurance associated with wage stickiness are in zero net supply, i.e.

$$\int a_{j,t} dj = 0$$

and bonds are in zero net supply

$$B_t = 0.$$

3.7. Monetary Policy

Consider the following monetary policy rule which defines the so-called notional interest rate:

$$\frac{R_t^{not}}{R} = \left(\frac{R_{t-1}^{not}}{R}\right)^{\rho} \left(\frac{\mathrm{E}_t \Pi_{t+4}}{\bar{\Pi}}\right)^{(1-\rho)\gamma_\pi} \left(\frac{Y_t}{Y} / \frac{Y_t^{pot}}{Y^{pot}}\right)^{(1-\rho)\gamma_x} e^{\varepsilon_{R,t}} \tag{11}$$

where the monetary policy shock $\varepsilon_{R,t}$ is assumed to be i.i.d. zero mean with positive variance.[9]

Monetary policy is subject to the zero lower bound on interest rates, i.e. the actual nominal interest rate is

$$R_t = \max(0, R_t^{not})$$

Regarding the fiscal authority we assume that net lump-sum taxes adjust to balance the government budget. Given Ricardian equivalence, we don't spell out the government budget constraint or specify a particular fiscal rule for lump-sum transfers.

3.8. Learning about the Cost-Push Shock

We adopt the following unobserved components representation for the cost-push shock. First, define: $a_t \equiv \tau_t - 1$, i.e. a_t is the deviation of τ_t from its steady state. Now suppose that the stochastic process a_t consists of a transitory part $a_{T,t}$ and a persistent part $a_{P,t}$. Agents can observe a_t but not $a_{T,t}$ or $a_{P,t}$. Below we set up the signal-extraction problem that households and firms solve

[9] Our results are robust to replacing $\frac{E_t \Pi_{t+4}}{\Pi}$ in the monetary policy rule with $\frac{E_t \Pi_{t+4}^{1yoy}}{\Pi}$ where $\Pi_t^{1yoy} = (\Pi_{t+4}\Pi_{t+3}\Pi_{t+2}\Pi_{t+1})^{1/4}$ denotes the annual rate of change (at quarterly rate) of the price level.

to infer a_t. Our formulation is similar to Erceg and Levin (2003), who solved a signal-extraction problem involving transitory and persistent shocks to the central bank's inflation target, and to Edge, Laubach and Williams (2007), who applied it to productivity.

Following Erceg and Levin (2003), Hamilton (1994), Ljungqvist and Sargent (2018), and Canova (2007), we set up the following state space system:

$$a_t = \underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix}}_{H'} \begin{bmatrix} a_t^P \\ a_t^T \end{bmatrix}$$

$$\begin{bmatrix} a_{t+1}^P \\ a_{t+1}^T \end{bmatrix} = \underbrace{\begin{bmatrix} \rho_P & 0 \\ 0 & \rho_T \end{bmatrix}}_{F} \begin{bmatrix} a_t^P \\ a_t^T \end{bmatrix} + \underbrace{\begin{bmatrix} \sigma_P & 0 \\ 0 & \sigma_T \end{bmatrix}}_{Q} \begin{bmatrix} \varepsilon_{t+1}^P \\ \varepsilon_{t+1}^T \end{bmatrix}$$

Then, using the Kalman filter, the estimate of state variables is (state update):

$$\begin{bmatrix} \hat{a}_{t|t}^P \\ \hat{a}_{t|t}^T \end{bmatrix} = \begin{bmatrix} \hat{a}_{t|t-1}^P \\ \hat{a}_{t|t-1}^T \end{bmatrix} + L_t \left(a_t - H' \begin{bmatrix} \hat{a}_{t|t-1}^P \\ \hat{a}_{t|t-1}^T \end{bmatrix} \right)$$

where

$$\begin{aligned} L_t &= P_{t|t-1} H (H' P_{t|t-1} H)^{-1} \\ P_{t|t} &= P_{t|t-1} - L_t H' P_{t|t-1} \end{aligned}$$

The optimal forecast of state variables is (state forecast):

$$\begin{bmatrix} \hat{a}_{t+1|t}^P \\ \hat{a}_{t+1|t}^T \end{bmatrix} = F \begin{bmatrix} \hat{a}_{t|t-1}^P \\ \hat{a}_{t|t-1}^T \end{bmatrix} + K_t \left(a_t - H' \begin{bmatrix} \hat{a}_{t|t-1}^P \\ \hat{a}_{t|t-1}^T \end{bmatrix} \right)$$

where

$$\begin{aligned} K_t &= F P_{t|t-1} H (H' P_{t|t-1} H)^{-1} \\ P_{t+1|t} &= \left(F - K_t H' \right) P_{t|t-1} \left(F' - H K_t' \right) + Q Q'. \end{aligned}$$

Note that:

$$\begin{bmatrix} \hat{a}_{t+j|t}^P \\ \hat{a}_{t+j|t}^T \end{bmatrix} = F^j \begin{bmatrix} \hat{a}_{t|t}^P \\ \hat{a}_{t|t}^T \end{bmatrix} \text{ for } j = 1, ...\infty$$

The forecast of a_{t+1} is then given by:

$$\hat{a}_{t+j|t} = H' \begin{bmatrix} \hat{a}_{t+j|t}^P \\ \hat{a}_{t+j|t}^T \end{bmatrix} \text{ for } j = 1, ...\infty$$

We assume that the agents have an infinite amount of past data available so that they run the recursions of K_t and $P_{t+1|t}$ until convergence. In other words, we solve for the fixed point $P_{t+1|t} = P_{t|t-1} = P$.

Table 1: Model parameter values

Parameter	Value	Description
$\bar{\Pi}$	1.005	Steady state gross inflation rate
θ_p	0.1	Net price markup in steady state
ξ_p	2/3	Calvo price stickiness parameter
ψ_p	-12	Parameter Kimball aggregator prices
ϱ	0.002	Curvature parameter endogenous indexation
ω	0.8	Parameter in endogenous. indexation
$\varkappa$	0	Inflation indexation parameter in linear model
θ_w	0.1	Net wage markup in steady state
ξ_w	0.75	Calvo wage stickiness parameter
ψ_w	-6	Parameter Kimball aggregator wages
ρ	0.85	Taylor rule: interest rate smoothing
γ_π	1.5	Taylor rule: coef. on expected inflation
γ_x	0.125	Taylor rule: coef. on output gap
β	0.995	Household discount factor
h	0.7	Household consumption habit
χ	0	Inverse Frisch elasticity of labor supply
ρ_P	0.9	AR(1) persistent markup shock
ρ_T	0	AR(1) transitory markup shock
σ_P	1	Standard deviation persistent markup shock
σ_T	10	Standard deviation transitory markup shock
ρ_δ	0.9	AR(1) discount factor shock

3.9. Equilibrium, Solution and Parameters

The appendix provides full set of equilibrium equations for the nonlinear and linear model, as well as the steady state and details about how we solve the model.

Table 1 contains the parameter values that we use in the analysis. Most of the values are taken from HLT (2022, 2023) and considered standard in the literature. But parameters related to the learning of the cost-push shock and dynamic indexation features of the model necessitate further discussion. Beginning with the parameters related to the unobserved components representation (ρ_P , ρ_T, σ_P and σ_T) for the cost-push shock, we calibrate them such that the observed cost-push shock process $a_t = a_t^T + a_t^P$ follows closely the estimated ARMA(1,1) process in Smets and Wouters (2007).[10] This implies setting $\rho_P = 0.9$, $\rho_T = 0$, and $\sigma_P/\sigma_T = 1/10$. This parameterization also enables the model to account well for the inflation projections and forecast errors made by the Survey of Professional Forecasters (as we discuss in the next section). The indexation parameters ω and ϱ are chosen such that the nonlinear model captures well the hump-shaped dynamics of inflation in the data.

Notice that dynamic price and wage indexation is absent (i.e. $\varkappa = 0$ in eq. 7) in steady state in both the nonlinear and linear models and also absent in the linearized model away from the steady state. This is consistent with many studies showing that there is little evidence in favor of dynamic indexation in micro and macro data when inflation is close to target. However, once inflation surges, endogenous indexation kicks in our nonlinear model, whereas this model feature is absent in the linearized model.

4. Results

In this section, we report our results. We begin in the next subsection by showing that our model can capture the key features of the post-COVID inflation surge, including the behavior of realized and forecasted inflation, real activity, and the federal funds rate. Given the critical role of cost push shocks, the following subsections focus heavily on the features influencing their transmission in our nonlinear framework – including the size of the shock and initial conditions – and contrast these results with a standard linear setting. We also consider the implications for monetary policy – including to gauge the merits of "looking through" supply shocks, and evaluate different disinflation strategies.

4.1. U.S. Data-Model Comparison

Figure 4 provides a U.S. data-model comparison for the nonlinear model. The data in the left column show actual realizations (red solid with dots) and projections (green dashed with dots) made by the Survey of Professional Forecasters (SPF). They are the same as presented in Figure 2, except that the trajectories in this figure are expressed in changes relative to 2020Q4.[11] In the model simulation underlying the right column in Figure 4, we use both an adverse cost-push shock and an expansionary demand shock (i.e., to the discount factor). Both shocks are phased-in over 2-3 quarters using forward guidance on the policy rate so that the liftoff of the policy rate that agents

[10] Smets and Wouters (2007) estimate the following cost-push shock process: $a_t = .9a_{t-1} + \varepsilon_{a,t} - 0.75\varepsilon_{a,t-1}$.
[11] We choose 2020Q4 as a reference date since inflation – the key object of interest in our paper – was close to its target of two percent.

in the model expect at each date matches the expectation from the SPF (again, as proxied by the 3 month T-bill rate). In what follows, model variables are expressed relative to their respective steady states except for inflation and the policy rate.

Although we have not optimized the fit of the model relative to the data with a formal criterion, our nonlinear model does a good job of accounting for the overall contours of the U.S. data and the inflation forecasts from the Survey of Professional Forecasters. This includes the progressive ratcheting up of inflation through mid-2022 as well as for the "optimistic" forecasts that projected that inflation would recede fairly quickly.

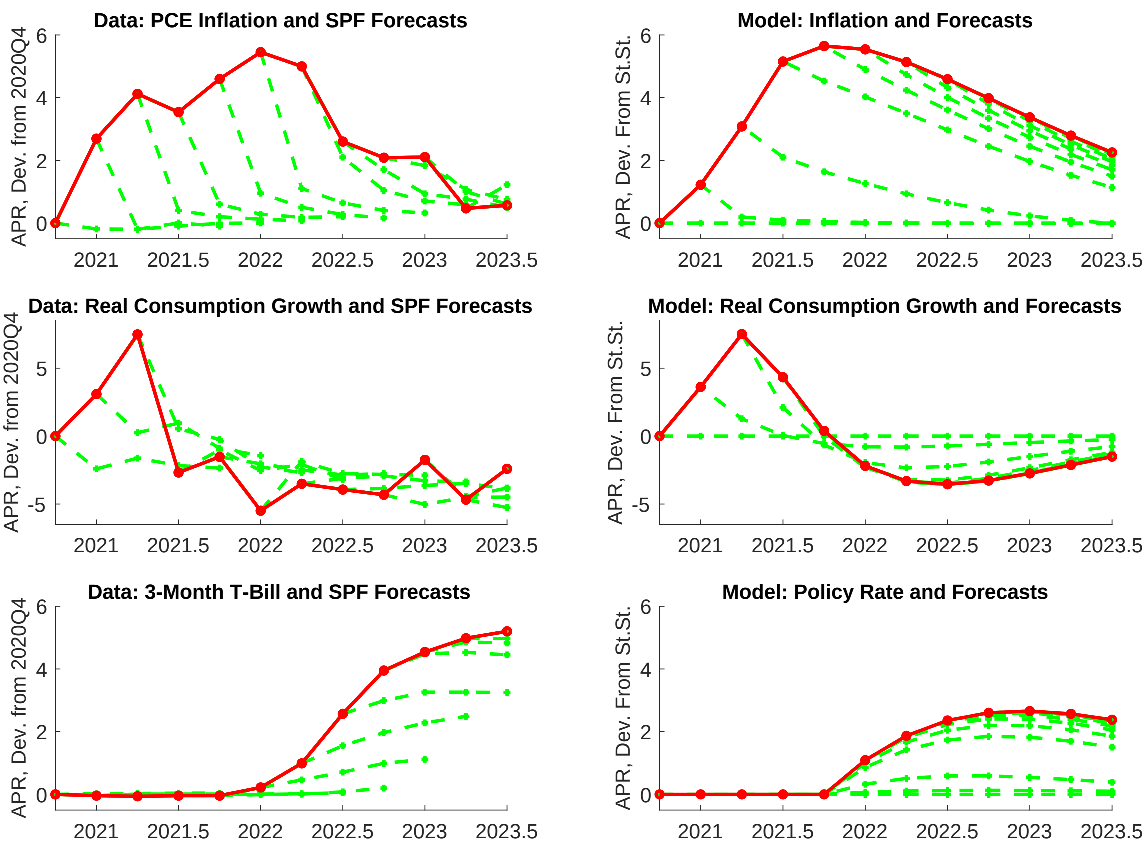

Figure 4: Comparison of U.S. data vs. nonlinear model.

Two implications of this analysis are important to highlight that we will explore more fully in subsequent analysis. First, our model implies a very strong endogenous propagation of the underlying demand shocks through the two sources of nonlinearity in our model. In particular, the shocks required to match the observed runup in inflation over 2021-23 are big enough that the nonlinearities in our model really "kick in." As we illustrate in Appendix B.1, the same structural shocks in a linearized version of the model would generate only a small and relatively transient rise in inflation.

The second implication is that while many of the salient features of the data can be accounted for by large but temporary cost-push shocks even without the need for outsized positive demand shocks (in some contrast to Giannone and Primiceri 2024). As we show, adverse cost-push shocks can have expansionary short-run output effects while triggering an inflation cycle. In the following, we explore further how large persistent cost-push shocks transmit to the economy and the conditions that can markedly affect their transmission relative to a standard linear framework.

4.2. How Adverse Cost-push Shocks can be Expansionary

Figure 5 provides the impulse responses in the nonlinear model to an adverse cost-push shock.

Specifically, we assume $\varepsilon_0^P = 0.0025$ and $\varepsilon_0^T = 0$ so that the realized cost-push shock is driven by the persistent component of the unobserved components representation. The central bank follows the inflation-forecast based Taylor rule in equation (11) but can only observe the sum of the persistent and transitory components and hence has to sequentially filter (in periods $t = 0, 1, 2, ...$) which of the two components drives the cost-push shock. Figure 5 shows the realized impulse responses as well as real-time forecasts of households, firms, and the central bank at each point in time. In line with the SPF forecast patterns following the COVID pandemic discussed in Figure 4, the figure shows that it takes considerable time for households, firms, and the central bank to adjust their beliefs and come to see more of the observed cost-push shocks as driven by the persistent component.

The figure shows that the cost-push shock drives a hump-shaped inflation boom and induces a gradual rise in the policy rate. A striking aspect of the responses is that output increases in the short-run – in contrast to the familiar result that a markup shock causes output to contract.

Three features in the model account for this interesting result. First, agents have misperceptions about the underlying shock. Given that the model is calibrated so that cost-push shocks are usually transitory, private agents and the central bank forecast the cost-push shock to recede soon and hence that inflation and output will return quickly to the steady state (as can be seen by the dashed lines). Second, the central bank's forecast rule (11) responds to 4-quarter ahead inflation with some degree of gradualism or inertia; consistent with estimated forecast-based rules, the central bank raises the nominal interest rate very little initially. Because expected inflation rises in the short-term, this can

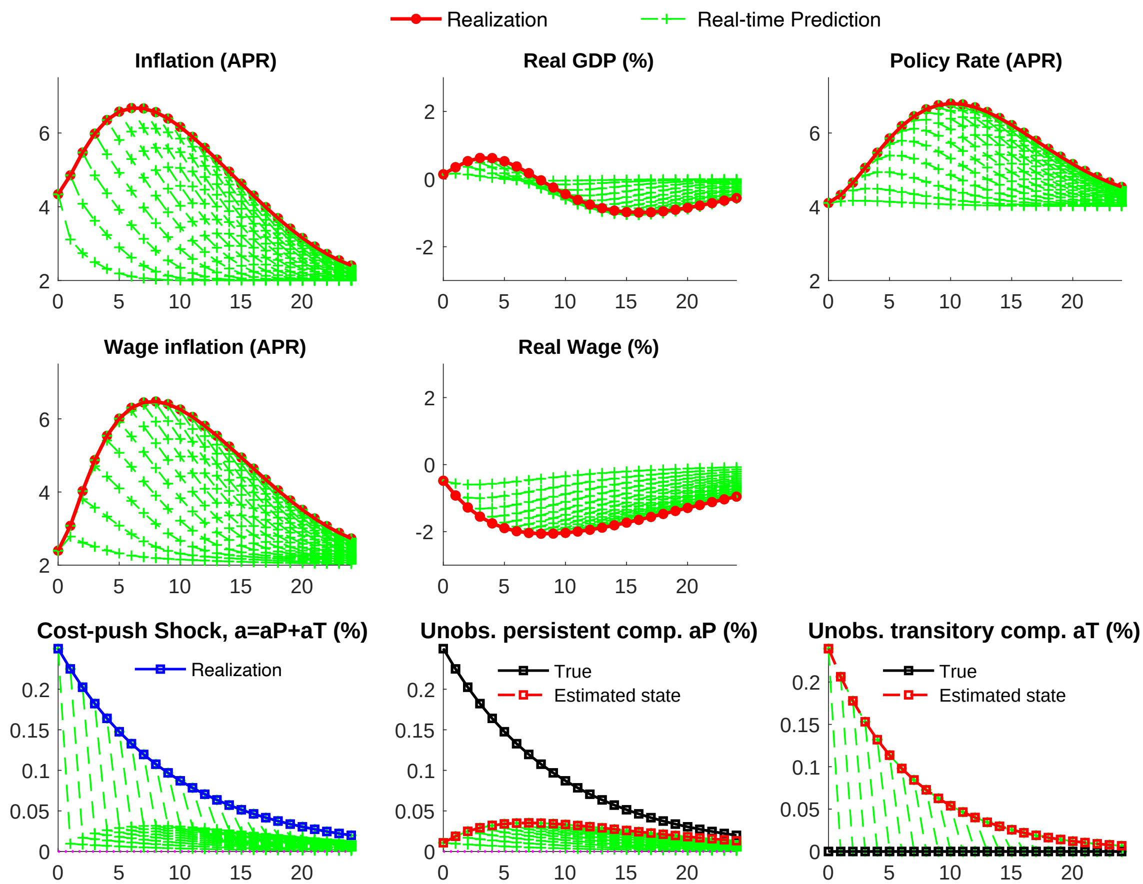

Figure 5: Impulse responses to an adverse cost-push shock in the baseline nonlinear model.

cause output to expand if it dominates the dampening effect of higher real rates at longer horizons. The third factor reflects that intrinsic persistence in the Phillips Curve rises in response to large and ongoing misses of the inflation target – a key form of nonlinearity in model, and what accounts mainly for the output expansion. This feature induces markup-shock surprises to have progressively bigger effects on inflation and inflation expectations. With the gradualist forecast-based rule, real rates fall markedly, generating the sizeable short-term output expansion seen in Figure 5.

While we will later explore the role of monetary policy and misperceptions, we turn first to explore more deeply how the nonlinear features of our model give rise to a sizeable and persistent output expansion as well as hump-shaped inflation response. In this vein, Figure 6 provides a comparison of the impulse responses to a cost-push shock in both the nonlinear and linearized versions of our model. Notably, the linear model implies a front-loaded response of inflation that

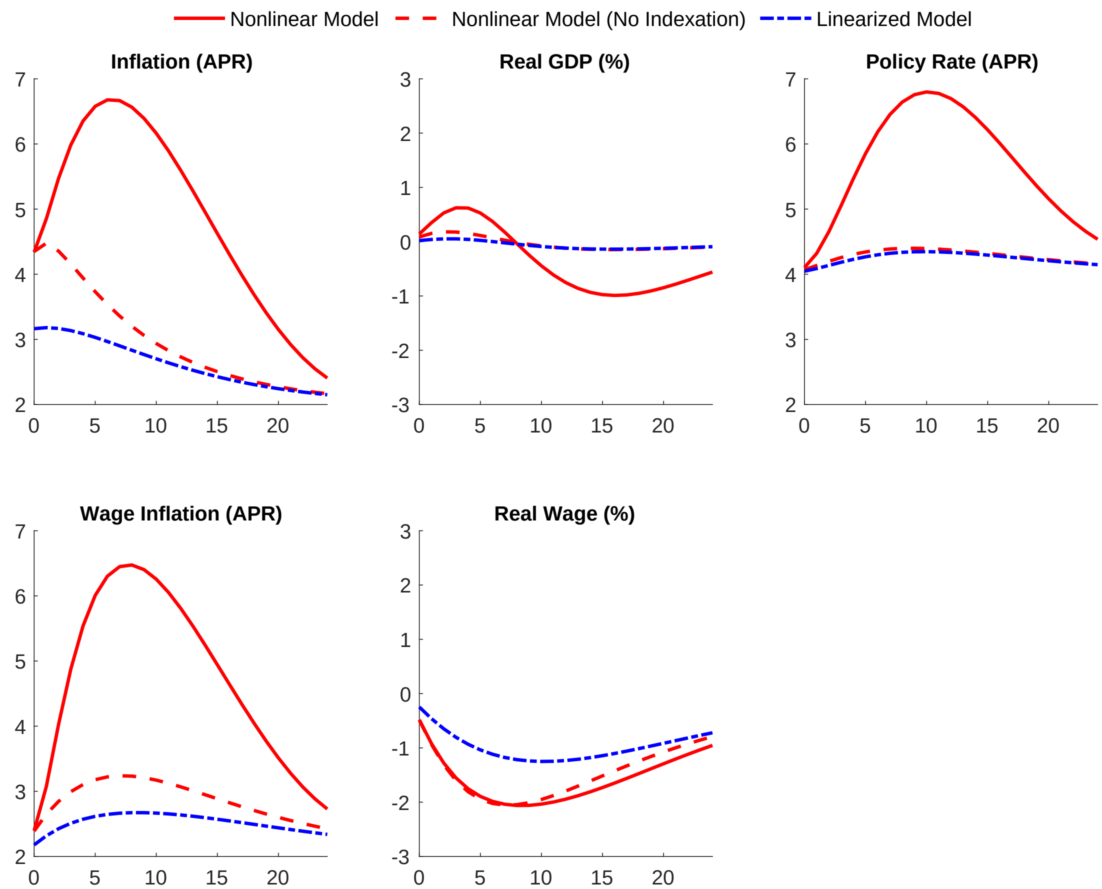

Figure 6: Impulse responses to a cost-push shock in the nonlinear model with and without indexation, and in the linearized formulation of the model.

is also far smaller than in the nonlinear model. Even a modest and gradual rise in the policy rate is enough to basically keep output at potential. Thus, even with misperceptions, the gradualist reaction function does pretty well in bringing inflation monotonically back to target: monetary policy doesn't get "behind the curve."

The dash-dotted line shows how the inflation response is considerably amplified by allowing for a nonlinear Phillips Curve through a Kimball aggregator as in HLT (2023). Even so, it is the combination of the Kimball aggregator and the endogenous indexation feature (allowing $\varkappa_t > 0$ in eq. 7) that drives the large and hump-shaped inflation response (as can be seen from the difference between the red solid and red dashed lines).[12] With the nonlinearity from the Kimball aggregator

[12] Note that due to endogenous indexation, medium-term inflation expectations become more elevated in the nonlinear than the linearized model. Our nonlinear framework implies that inflation expectations become more sensitive to realized inflation rates above target due to endogenous indexation ($\varkappa_t > 0$). Thus, our nonlinear model

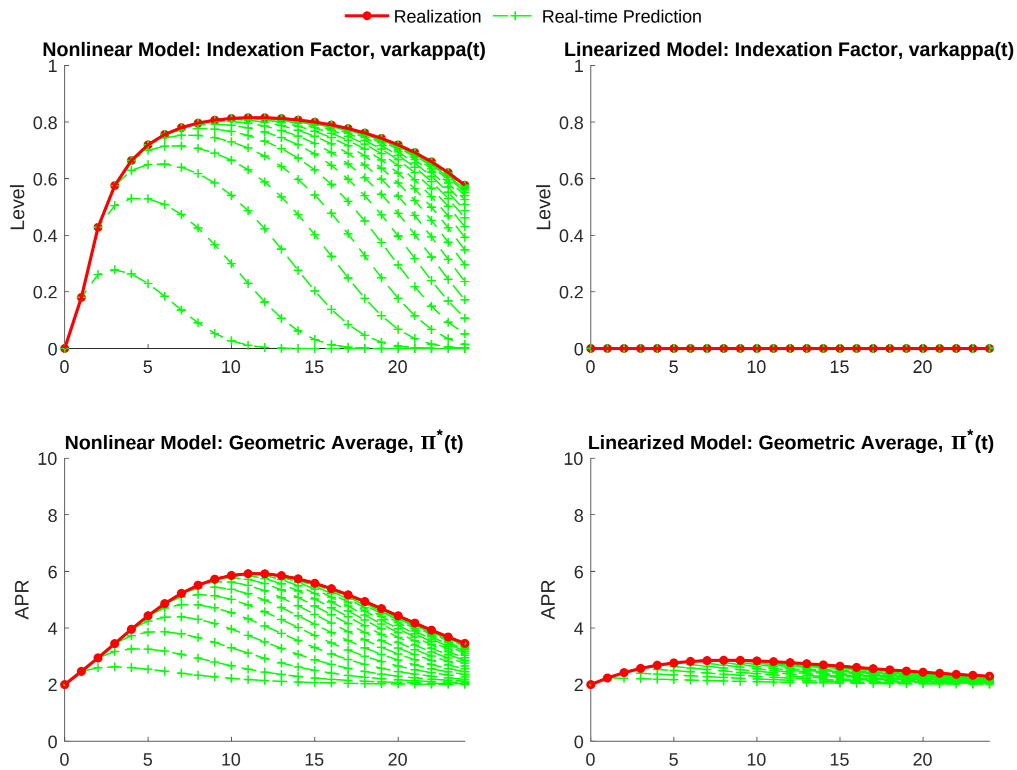

Figure 7: Impulse responses of indexation variables to the adverse cost-push shock in the baseline nonlinear and linearized models.

alone, the inflation response is bigger, but relatively little policy tightening is needed to keep output close to potential and inflation on a monotonically declining path. When endogenous indexation is added to the mix, the large deviations of realized inflation from target cause the inflation indexation parameter to rise so that subsequent markup shocks have bigger effects on inflation and inflation expectations. In this vein, Figure 7 shows the impulse responses of the endogenous indexation variables $\tilde{\Pi}_t$ and Π_t^* to the cost-push shock in the nonlinear and linearized model. When inflation is low, there is very little (if any) indexation. But once inflation rises well above the central bank's inflation target and stays high, indexation starts to kick in.

Given that the indexation parameter depends on the size of the geometric average of inflation deviations from target, the size of the cost-push shocks plays a critical role in accounting for the hump-shaped inflation dynamics in Figure 5. To underscore this, Figure 8 compares impulse responses of inflation in the nonlinear and linearized model for different sizes of the cost-push

implies more 'de-anchoring' of medium-term inflation expectations than the linearized model.

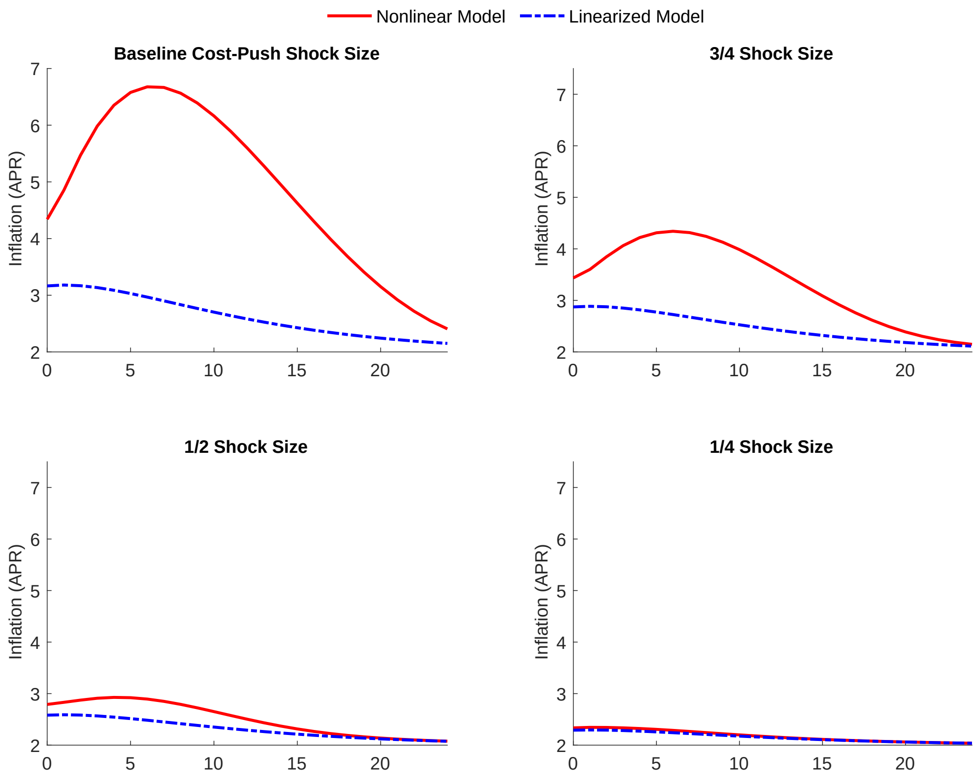

Figure 8: Impulse responses of inflation in the nonlinear and linearized model for different sizes of the cost-push shock.

shock. While the differences between the nonlinear and linear model are very large – as seen in the upper left panel, as in Figure 6 – the differences essentially disappear when the underlying shocks are considerably smaller. Thus, if cost-push shocks are small, a linearized variant captures the transmission of the shock well and there is no need to work with the nonlinear model. But, if the cost-push shock is large, it is crucial to work with the nonlinear model to capture the dynamic effects of cost-push shocks quantitatively.

4.2.1. Monetary Policy Rule

In addition to endogenous indexation, the forecast-based monetary policy reaction function markedly amplifies the effects of the markup shock on inflation and output when agents initially misperceive a persistent shock as transitory. Figure 9 provides a comparison of the impulse responses of the nonlinear model to the cost-push shock under our baseline rule – in which the central bank reacts to one-year ahead expected inflation in the simple Taylor-style rule (11) – to alternatives in which it reacts to two-year ahead expected inflation ($\mathrm{E}_t\Pi_{t+8}/\bar{\Pi}$), or to the current inflation gap ($\Pi_t/\bar{\Pi}$), in the Taylor rule (11). An average inflation targeting (AIT) rule based on a sufficiently long window of average past and future inflation rates should work similarly to the forecast-based targeting rules we consider provided that the AIT rule is not coupled with an escape clause that kicks in when current inflation exceed a certain threshold.

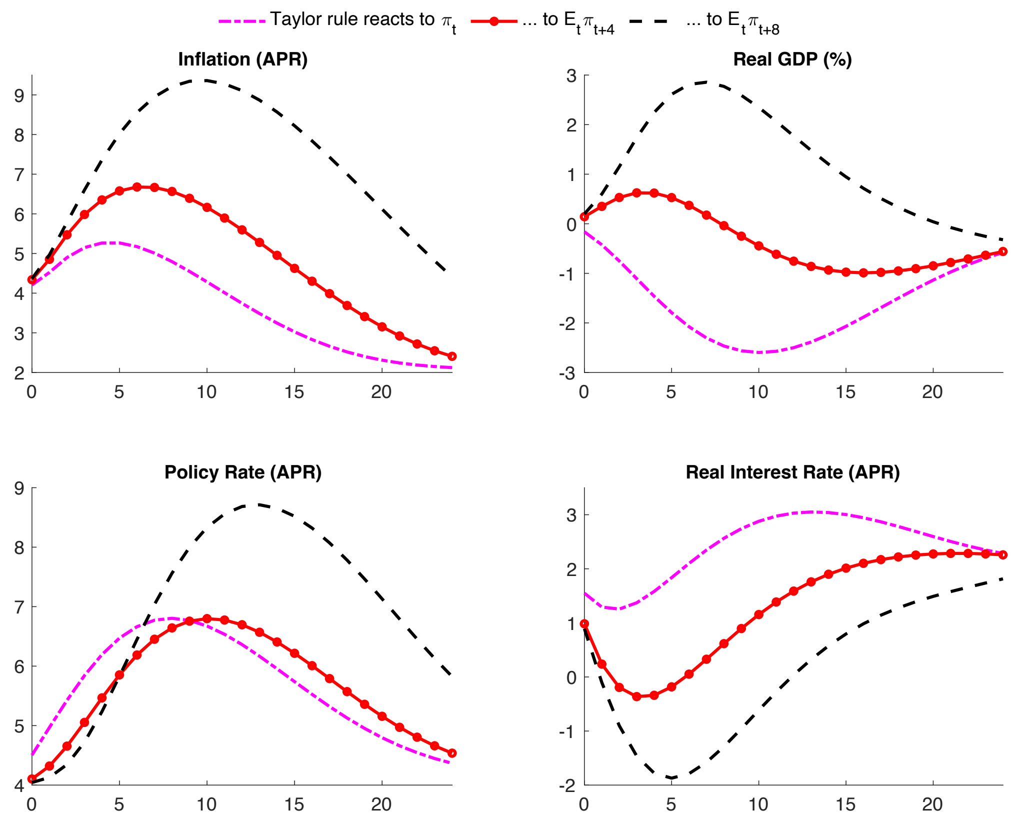

Figure 9: Impulse responses to a cost-push shock in the nonlinear model when the central bank reacts to current inflation, one-year ahead expected inflation, or two-year ahead expected inflation in the Taylor rule.

As seen from the figure, when the central bank reacts to realized inflation, the real interest rises substantially, causing output to contract while reducing both the size and persistence of the runup in inflation. This response to inflation clearly has sizeable output costs, which help explains why central banks are typically reluctant to react in this way. Even so, there are clear benefits of an aggressive response when the shock is big enough that the endogenous amplification channels kick in – such a response contains inflation and reduces vulnerability to additional shocks (as we explore below). An ad hoc loss function weighting inflation and the output gap equally would in fact imply a much smaller loss under the rule responding to contemporaneous inflation than under the forecast based rule.[13]

Conversely, the forecast rule responding to two-year ahead expected inflation allows real interest rates to decline even more sharply than in the baseline, inducing a bigger runup in inflation and output. Importantly, while agents see intrinsic persistence rising and come to forecast higher medium-term inflation, the inertia in the policy rule slows the response of the policy rate, and hence the central bank falls further "behind the curve."

These implications suggest potential shortcomings of a forecast-based rule when shocks are large and there is uncertainty about shock persistence. Even so, our model can also show the appeal of a forecast-based rule in circumstances in which shocks are smaller and fairly transient – conditions that typically prevailed in the Great Moderation period. Under these conditions, the forecast-based rule is consistent with "looking through" the shock: because the inflation is short-lived, the central bank doesn't need to respond by tightening policy.

This situation is well-captured by our model, and shown in Figure 10. While inflation initially rises to 4 percent, the central bank only raises the policy rate a tad. But even with real rates and hence output remaining roughly unchanged, inflation comes back down quickly, reflecting there is little intrinsic persistence in the Phillips Curve (i.e., indexation is close to zero). By contrast, reacting aggressively to contemporaneous inflation would seem counterproductive – causing output to contract sharply, but with little effect on the path of inflation – so that it is easy to see why the forecast-based rule is appealing. Moreover, the forecast-based rule also performs reasonably well in containing inflationary pressure in response to a persistent demand shock that is correctly recognized as such.[14] As illustrated in Appendix B.2, inflation is expected to run well above target a year or two out, so that the policy is raised quickly enough to contain the inflationary pressure, in part through contracting output via higher real rates.

All told, our results suggest caution against relying on inflation projections at longer forecast horizons in the formulation of monetary policy when shocks are large and inflation persistence is high (due to past shocks). In these circumstances, the central bank should be particularly wary about the risk of treating shocks as temporary when they may in fact be much more persistent. At the same time, the risks of misperceiving the shock and allowing inflation to get out of control is much smaller when inflation has been running closer to target and intrinsic persistence lower, so

[13] The expected loss – measured as the sum of the squared deviations of annualized inflation from target plus the sum of squared output gaps for the first 5 years – falls by about 30 percent from a loss value of about 255 under the forecast-based rule to a loss of about 174 under a rule which responds to actual inflation.

[14] Without the sizeable degree of interest rate smoothing ($\rho_R = 0.85$) that we assume in the interest rate rule (eq. 11), the difference between the rules would be further amplified.

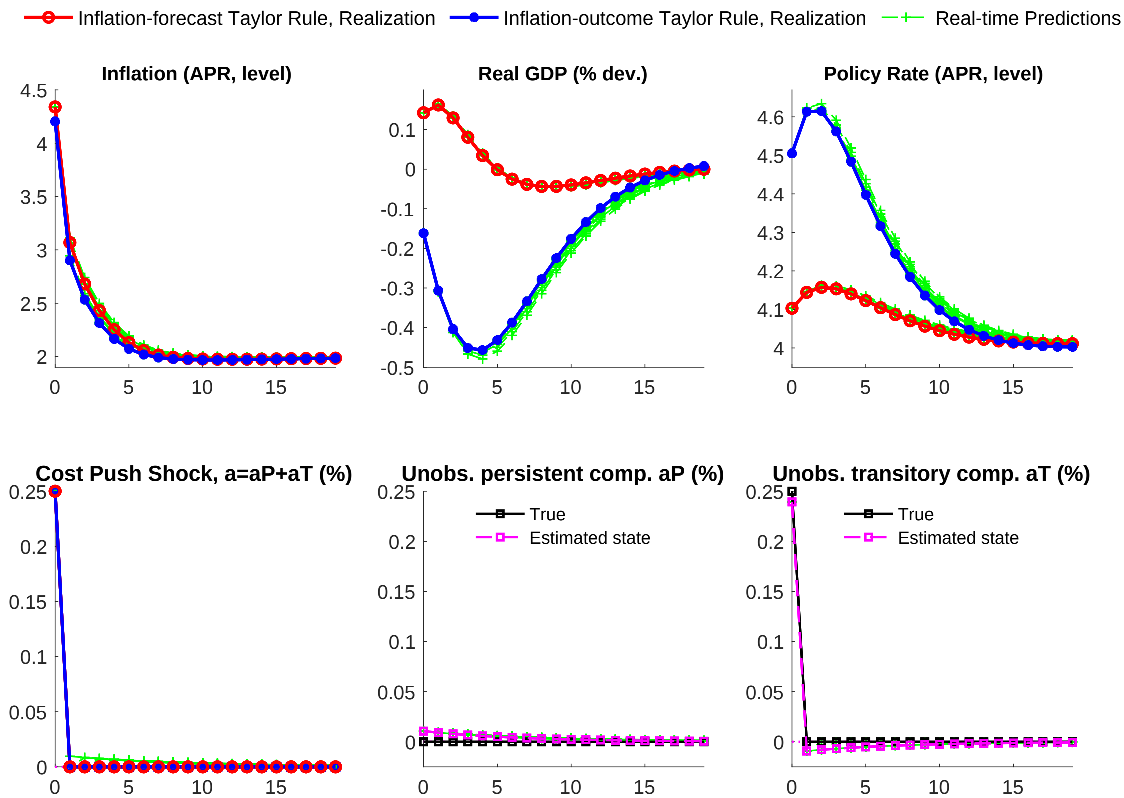

Figure 10: Impulse responses to a transient cost-push shock under alternative monetary policy rules in the nonlinear model.

that forecast-based rules consistent with "looking through" can help avoid excessive volatility in output.

4.2.2. Role of Misperceptions

Misperceptions also play a critical role in driving the inflation and output dynamics in our model. If the central bank and economic agents were able to correctly identify a persistent markup shock as such when it occurred, interest rates would rise much more quickly even under a forecast-based reaction function, and output would contract. In Appendix B.3 and Appendix B.4 we consider robustness to the "signal to noise" ratio that determines how quickly agents learn about the underlying shocks – which depends on the ratio of standard deviations (σ_P/σ_T) of the persistent and transitory components of the cost push shock. Notably, Figure B.4 shows the effects of assuming alternative values for the ratio of standard deviations (σ_P/σ_T) for the case in which the underlying shock is highly persistent. If agents are more rapidly able to filter out that the cost-push shocks are driven by the persistent component than in our baseline, the central bank would recognize more persistent upward price pressures and tighten its policy stance faster and more forcefully, causing output to fall and limiting the rise in inflation. As might be expected, the problem of misperceptions for inflation control is much less acute when the underlying shock is in fact transitory (as explored in Figure B.3 in Appendix B.3).

4.2.3. Implications for Identification of Shocks

An intriguing feature of our model simulations with misperceptions is that forecast-based rules can imply persistent positive comovement between inflation and output in response to a supply shock. In other words, output and inflation exhibit a pattern akin to demand shocks even though the driving force is an adverse cost-push shock. This implication calls into question the robustness of a standard empirical identification assumption with sign restrictions in structural vector auto regressions (see e.g. Giannone and Primiceri, 2024 for a recent prominent study) that cost-push shocks drive output and inflation in opposite directions, at least under some conditions. Relatedly, it calls into question the empirical identification assumption that demand shocks are the only driver of positive short-term co-movement of output and inflation. From the standpoint of the recent inflation surge, our model suggests that some of the strength in activity – at least initially – may have come from the relatively muted response of monetary policy to adverse supply shocks.

4.3. State-dependent Amplification of Cost-push Shocks

In this subsection, we illustrate that cost-push shocks have amplified effects in an environment in which demand is initially strong – even before the shock occurs – and inflation already elevated. To conduct this exercise, we first generate alternative baselines – shown in the first column of Table 2 – that are constructed with progressively larger demand shocks. These demand shocks are constructed with discount factor shocks, and hence affect the potential real interest rate but not potential output. The first row show the smallest (zero) positive demand shock for which inflation simply remains at its steady state value and output remains at potential, whereas the last row shows the biggest demand shock that drives up peak inflation to 3.2 percent and the output gap to 5.4 percent.[15] Although the increment of the underlying discount factor shock is the same across each row in the table, the peak impact on inflation increases with the size of the shock given the nonlinear Phillips Curve (as seen in the second column in Table 2).

[15] In the table, the underlying discount factor shock varies from 0 (no shock), -0.5, -1, -1.5 and -2.0; i.e. the increment of the underlying demand shock is constant. A fall in the discount factor implies a rise in demand.

Table 2: Amplification of Cost-Push Shocks in Nonlinear Model

Baseline: Discount Factor Shock		**Scenario:** Baseline+Same-Sized Cost-Push Shock	State-dependent Effects of Cost-Push Shock
Output Peak	Inflation Peak	Inflation Peak	Δ Inflation Peak **(Scenario-Baseline)**
0.0%	2% (Steady State)	6.7%	4.7%
1.4%	2.2%	8.0%	5.8%
2.7%	2.4%	9.4%	7.0%
4.1%	2.7%	10.8%	8.1%
5.4%	3.2%	12.2%	9.0%

Against each of these alternative baselines, we add the same-sized (i.e., identical) adverse cost-push shock. As a result, inflation surges even more in the scenario with the cost-push shock – reported in the third column – compared to the baseline simulation. The last column of Table 2 calculates the marginal impact of the cost-push shock against the alternative baselines, and clearly indicates how stronger initial demand conditions (columns 1 and 2) cause a given-sized cost shock to have bigger inflationary effects. For instance, a cost-push shock that pushes up inflation by 4.7 percent when inflation is initially at target would cause inflation to rise by 9 percent if occurring against the backdrop of an "overheated" economy in which inflation was 3.2 percent initially and the output gap slightly over 5 percent. This state-dependence has important implications for the conduct of monetary policy, which we will discuss in the next section.

4.4. Effects of Monetary Tightening

When inflation persistently exceeds the central banks' inflation target as in Figure 5, the central bank must consider the pros and cons of bringing inflation back to target more quickly. In this subsection, we study the effects of more forceful monetary tightening than implied by our baseline Taylor-style reaction function. Specifically, we let the monetary policy shock $\varepsilon_{R,t}$ in the Taylor rule (11) follow an AR(1) process with a persistence coefficient of 0.75. We size the monetary policy shock in both the nonlinear and linearized models such that inflation is reduced by one percentage point (APR) below its baseline path, where the latter is constructed using the cost-push shock in Figure 5 plus a one percent discount factor shock (to generate a "high inflation" baseline). The monetary policy intervention is assumed to start when inflation attains its peak in the nonlinear model.[16] In addition, we consider the case that in the linear model, the slope of the price and wage Phillips curves is twice as large as in steady state so that the slopes in the linear model are roughly equal to those in the nonlinear model under conditions of strong demand.[17]

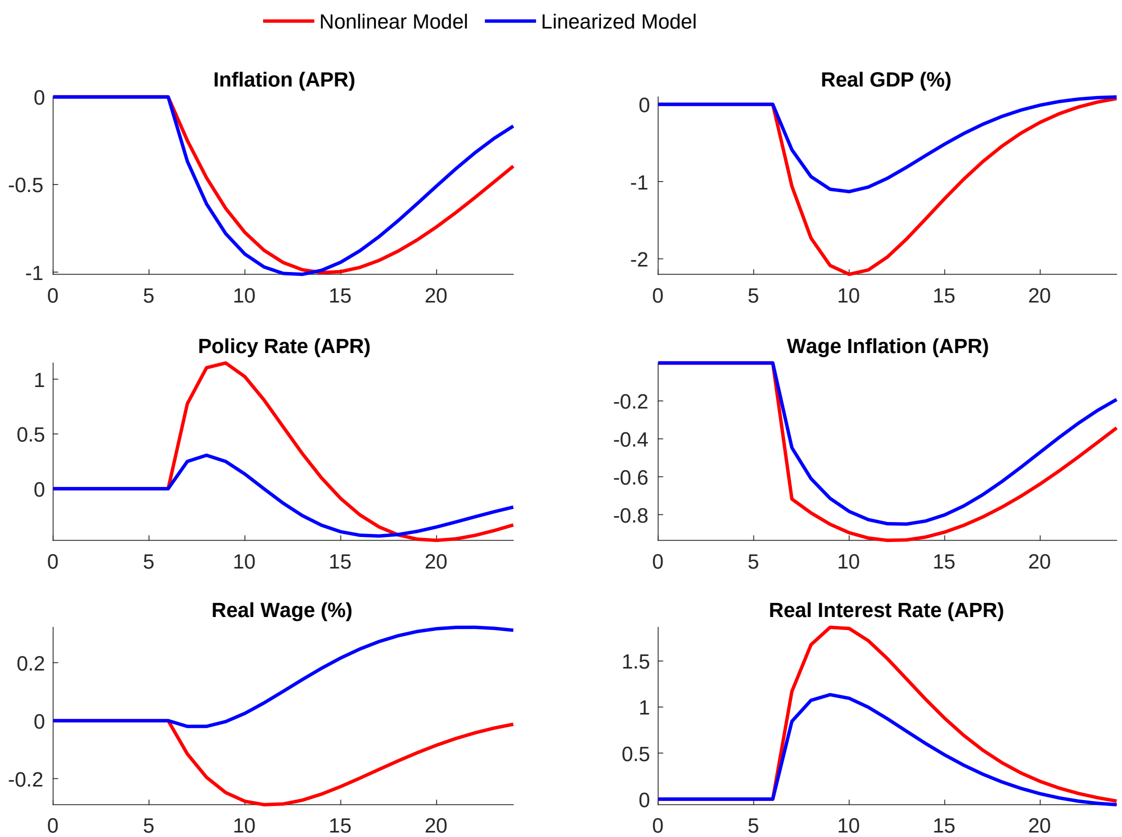

Figure 11: Effects of more aggressive monetary policy (deviation from baseline) when inflation peaks in nonlinear and linearized models.

Figure 11 shows the effects of the monetary tightening in terms of deviations of model variables from the "high inflation" baseline. The figure shows that to attain a one percentage point (APR) lower trajectory for inflation, the nonlinear model implies that the nominal policy rate has to be

[16] In Appendix B.5, we consider the implications when the central bank becomes more aggressive at different points in time. Specifically, Figure B.5 shows the simulation results for more aggressive monetary policy in the nonlinear model for different start dates of the monetary intervention. The key takeaway is that the earlier the central bank intervenes, the larger the reduction in inflation for a given hike in the policy rate. Put differently, monetary policy becomes more effective the higher inflation is to begin with. In this sense, the efficacy of monetary tightening and the associated sacrifice ratio are state-dependent in our model.

[17] Finally, to put the nonlinear and linearized model on a more equal footing, we allow for endogenous indexation in the linear model too, although eq. (10) implies that the linearized model does carry this feature. We will refer to this a pseudo-linearized model in which indexation is nonlinear (and endogenous) but all other model equations are linearized.

tightened notably more than in the linear model, and that this results in a considerably lower output path in the former. Put differently, if the central bank needs to disinflate faster than implied in the baseline in order to preserve credibility for the inflation target, our nonlinear model implies that the resulting output costs are notably larger than implied by a standard linearized model with Phillips curve intended to match the dynamics of inflation when it surged above the inflation target.

Figure 12 provides a graphical illustration of the intuition underlying our quantitative result that additional monetary tightening goes hand in hand with larger economic costs in an environment with nonlinear (kinked) Phillips curves. In Figure 12, the economy is initially in point A, i.e. at the intersection of a Phillips curve and a monetary policy rule. In the figure, π stands for inflation and u stands for the unemployment rate which is assumed to be proportional to the negative output gap, say $-x$. For simplicity, we assume a monetary policy rule as in Clarida, Gali, and Gertler (1999) in which the central bank conducts optimal monetary policy under discretion in response to a cost-push shock, i.e. $\pi = a * u$ or, after substituting out for the unemployment rate, $\pi = -a * x$. Now, an adverse cost-push shock shifts the Phillips curve up persistently to point B.

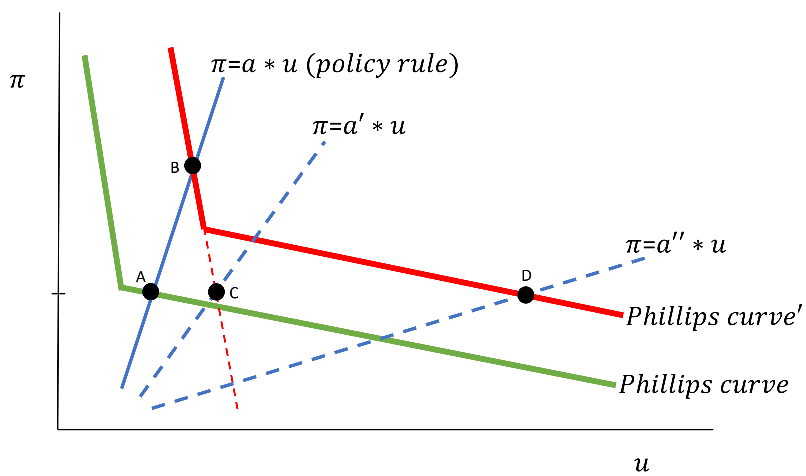

Figure 12: Intuition for transition dynamics and economic costs of more monetary tightening than embedded in the baseline.

If the central bank pursues its historical policy rule then the economy travels slowly back from point B to point A, i.e. the baseline dynamics. However, if the central bank seeks to bring the economy back to target inflation faster than implied by the baseline, the central bank can adapt a more aggressive policy stance. Point C illustrates the outcome the central bank would expect under this more aggressive rule if it erroneously based its assessment on the linear Phillips Curve as in Figure 11. Here the steep slope of the Phillips Curve – if it was linear at all levels of unemployment

– would ease the costs of disinflating. By contrast, point D illustrates the implications in the true underlying nonlinear model in which a much stronger monetary tightening is required to bring inflation back to target faster. The resulting economic costs can – depending on the slope of the nonlinear Phillips curve – become very substantial as shown in Figure 11. All told, Figure 12 illustrates that the costs of additional monetary tightening than embedded in the baseline can be substantial in a nonlinear framework while a linearized framework may suggest notably lower costs.

The left column of Figure 13 shows the implications for output and inflation when allowing for stochastic cost-push shocks and discount factor shocks around the baseline. The resulting densities shown in the figure are constructed as follows. Starting at the baseline path at $t = 8$, the economy is hit by random unexpected cost-push and discount factor shocks in each period $t \geq 8$.[18] The cost-push shocks follow the unobserved components specification embedded in the model, i.e. have realizations of transitory and persistent shocks. The variances of cost-push and discount factor shocks are chosen such that the model generates roughly the unconditional standard deviation of core PCE inflation, the unconditional standard deviation of real consumption per capita growth, and the correlation between consumption growth and inflation in post-war/pre-Covid U.S. data.

The density plots are then constructed by using 500 random sequences of these shocks, simulating the model with each sequence separately and such that in each period agents are surprised by new realizations of cost-push and discount factor shocks. The density plots in Figure 13 show the $\{2.5, 10, 20, \ldots, 90, 97.5\}$ percentiles and the median.

Strikingly, as seen in the left panel of Figure 13, the density plots are asymmetric for inflation. There are more realizations of high inflation than low inflation in the stochastic simulations. The reason for this result is due to the amplification effects of Kimball aggregation and endogenous indexation. That is, to the extent that the economy is hit by cost-push and discount factor shocks and inflation already runs above the central bank's inflation target, our results indicate that the economy will see bursts of inflation more often than inflation declines.

The right panel of Figure 13 illustrates the effects when a central bank adopts a more aggressive stance toward inflation surges. Specifically, starting in period $t = 8$, the central bank increases the weight of inflation in the Taylor rule by a factor of three and reduces the weight of the output gap by a factor of three.[19] With these changes to the systematic parts of the central bank's interest

[18] An alternative simulation setup would be to consider that the supply shock up to the time of the intervention is actually driven by observationally equivalent draws of the transitory component. This would allow for a more rapid post-intervention decay in inflation.

[19] In this experiment, we have chosen to model monetary tightening with a more aggressive response coefficient to expected inflation in the central bank's reaction function. An alternative approach would be to switch from forecasted inflation to current (observed) inflation in the policy rule whenever actual inflation exceeds a threshold inflation level. That is, whenever the observed rate of inflation becomes too high, the central bank abandons its inflation forecast-based policy rule in favor of a policy rule with actual inflaion. Given that actual inflation runs higher than the

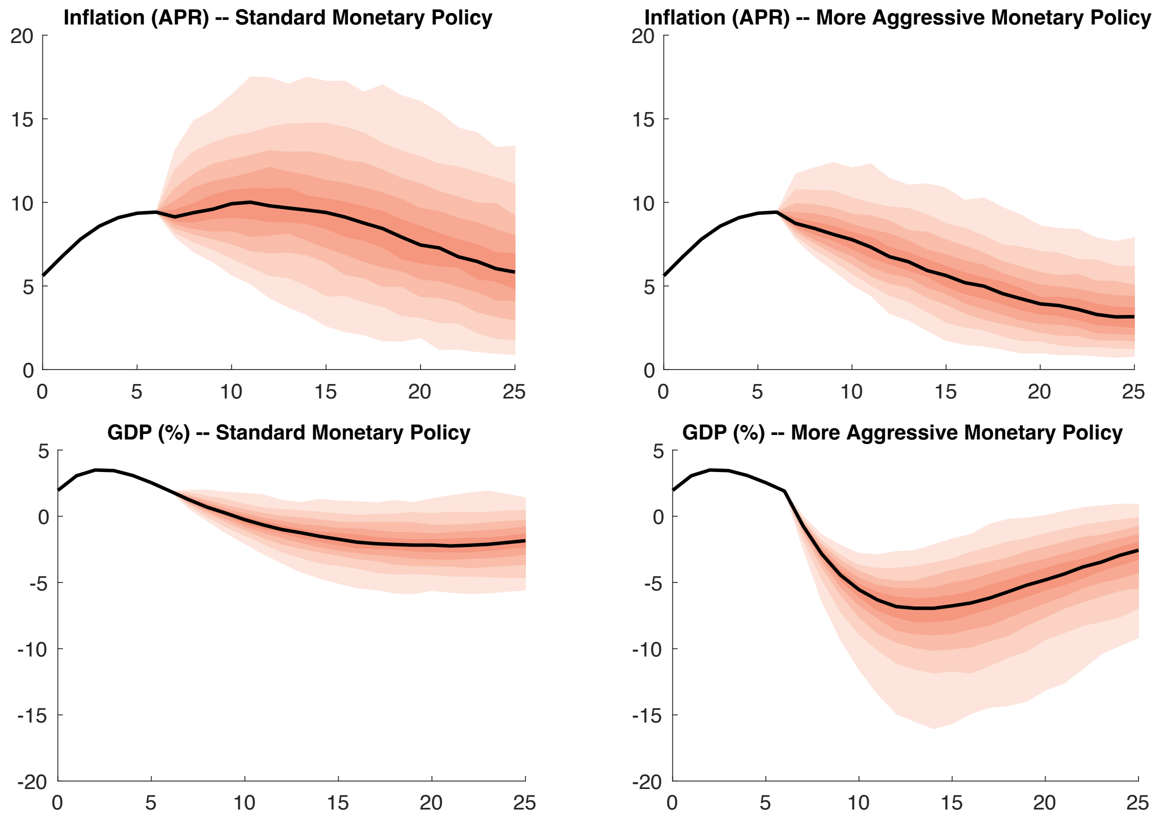

Figure 13: Distributions of inflation and output (dev from SS) with stochastic shocks for alternative monetary policy rules.

rate feedback rule, the right panel of Figure 13 suggests two important implications. First, inflation is lower on average than with the standard Taylor rule specification. Second, there is almost no asymmetry in the distribution for inflation. However, the improved stabilization of inflation (i.e. lower mean and less upward asymmetric inflation risks) comes with substantial economic costs as illustrated by the bottom right subplot for output in Figure 13. The distribution of output displays a significant lower mean and evident downside risks relative to the left panel with the historical policy rule. These results highlight the costs of disinflating late in an economic cycle once inflation and higher inflation expectations have become entrenched.

5. Related Literature

In addition to the literature discussed in the previous sections, our paper is also related to several key strands of the literature.

Many scholars are seeking to understand the causes of the recent inflation surge and the dy-

inflation forecast in Figure 13 the results of this alternative experiment should be similar to our approach to model more aggressive tightening.

namics of the labor market. Bernanke and Blanchard (2023) develop a linear model of wage-price dynamics which they use to decompose the sources of the recent inflation surge. They show that supply and energy shocks were the main drivers behind the runup in inflation, whereas labor market conditions accounted only for a small share of the inflation spike, especially early on. However, according to their analysis, the influence of the product market shocks will fade, while the tight labor market can exert more persistent effects on inflation.

Gagliardone and Gertler (2023) develop a New Keynesian model that aims to account for the inflation surge with emphasis on the role of oil price shocks and accommodative monetary policy. An important feature of this model, which includes non-linearities, is that oil is treated as a complementary good for households and as a complementary input for firms. With these model features, a upward oil price shock reduces the marginal product of labor (given the strong complementarity between oil and labor) and thereby increases marginal cost, which increases inflation . This, together with monetary policy accommodation, helps to explain the inflation surge, even after allowing for demand and labor market tightness shocks.

Lorenzoni and Werning (2023) model a wage-price spiral, highlighting conflict between workers and firms about real wages as a proximate cause of inflation. Notably, the model incorporates a scarce non-labor input with low substitutability in production and both sticky wages and prices. Their findings demonstrate that both demand and supply shocks can exhibit a similar three-phase pattern of adjustment in nominal prices, characterized by stronger price inflation early on, followed by wage inflation catching up later on.

Ball, Leigh, and Mishra (2022) analyze the recent surge in U.S. inflation, with a special focus on core and headline inflation. They argue that core inflation is influenced by a tighter labor market and past shocks from headline inflation, particularly due to higher energy prices and supply chain disruptions. The paper also explores future inflation scenarios, mainly focusing on one where unemployment rises modestly as projected by the Federal Reserve. Their analysis suggests that achieving the Fed's inflation target hinges on optimistic assumptions about inflation expectations and the relationship between unemployment and job vacancies. If these assumptions are not met, inflation may remain above the Fed's 2 percent target unless unemployment increases more than currently projected by the Federal Reserve.

Amiti et al. (2024) analyze how much the supply-side disruption and tight labor market contributed to the recent inflation surge. The authors develop a two-sector New Keynesian model with multiple input factors such as labor, domestic and foreign intermediate inputs, shocks to imported intermediate input prices, foreign competition in domestic markets, and shocks to workers' willingness to work. When all shocks hit at the same time, firms' ability to substitute between inputs is diminished, so that the total effect on inflation is amplified.

Benigno and Eggertsson (2023, 2024) develop a model that features a non-linear Phillips curve. The nonlinearity arises from asymmetries in wage setting. The authors provide evidence that the Phillips curve has a higher slope coefficient when market tightness is exceptionally high, which usually defines a labor shortage. They conclude that the key reason why policymakers failed to foresee the large persistent inflation surge was because the Phillips curve was assumed to be flat. An exceptionally tight labor market, which moved the economy on the steep segment of the Phillips curve, is, according to Benigno and Eggertsson (2023, 2024) responsible for the increase in inflation in the early 2020s.

Ferrante et al. (2023), Gudmundsson et al. (2024), Guerrieri et al. (2022), Guerrieri et al. (2024) among others study the implications for inflation of switching expenditures from services to goods during the pandemic followed by a switch back from goods to services in the aftermath of the pandemic. These papers typically find that disruptions in one sector can be helpful to understand inflation dynamics during and after the pandemic.

Hakamada and Walsh (2024) study the implications of a cost-push shock in a linear New Keynesian model when the central bank is assumed to keep the policy rate unchanged for some time. The authors report that the accommodative stance of the central bank renders the model capable of accounting for a surge in inflation and an expansion of economic activity.

Schmitt-Grohé and Uribe (2024) propose a model with heterogeneous downward nominal wage rigidity. The model delivers a nonlinear wage Phillips curve linking current wage inflation with current unemployment which the authors use to study the pattern of wage inflation and unemployment observed in the United States over the past 40 years.

Blanco et al. (2024) develop a tractable sticky price model in which the fraction of price changes evolves endogenously over time and increases with inflation. The model features an inflation accelerator – a feedback loop between inflation and the fraction of price changes – which increases the slope of the Phillips curve during periods of high inflation.

Pfäuti (2024) estimates an inflation attention threshold by the public at an inflation rate of 4% and that attention increases markedly as inflation exceeds this threshold. Notably, he estimates that adverse supply shocks become twice as inflationary in times of high attention. Using a model, he shows that shocks that are usually short lived lead to a persistent surge in inflation if they induce an increase in people's attention. The attention threshold also implies that is takes more time for inflation to return to low levels after an inflation surge.

Borio, Hofmann and Zakrajšek (2023) argues that the strength between money growth and inflation depends on the inflation regime: it is one-to-one when inflation is high and virtually non-existent when inflation is low. They argue that higher money growth preceded the recent inflation surge, and that countries with stronger money growth saw markedly higher inflation.

Relative to the body of work cited above, our model combines a unique set of features whose interplay allows us to account for the joint dynamics of inflation, output, interest rates and the real wage in the post-Covid episode. The features highlighted in this paper are: i) nonlinear price and wage Phillips curves, ii) an unobserved components representation for cost-push shocks, iii) endogenous intrinsic price and wage indexation, and iv) an inflation forecast-based Taylor rule. With these elements, we have shown that a steep surge in inflation, resilient economic activity, a slow central bank response, as well as a fall of the real wage emerge endogenously in our model. These features also imply that our model has novel implications for the amplification of cost-push shocks and the conduct and effects of monetary policy. Moreover, our model not only applies to the recent post-Covid episode but is also useful to understand deep recessions such as the Great Recession as well as 'normal' business cycles. Specifically, given the boomerang-shaped nonlinear price and wage Phillips curves embedded in our model, our framework can be used to resolve the missing deflation puzzle, see HLT (2022).

6. Conclusion

We use a macroeconomic model with nonlinear Price and Wage Phillips curves, endogenous intrinsic indexation and an unobserved components representation of a cost-push shock to explain the post-COVID inflation surge. The cost push shock can be driven by a transitory or persistent component but households, firms and the central bank can only observe the sum of both components, and hence must solve a signal extraction problem to deduce which component drives the observed markup shock. We consider the case when agents and the central bank expect cost-push shocks to be transitory most of the time, but the realized cost-push shock in fact stems from the persistent component. In this environment, when assuming a central bank which follows an inflation forecast-based policy rule to see through transient inflation movements, we show that a nonlinear formulation of our model can explain the persistent inflation surge along with an initial expansion in economic activity in response to an adverse cost-push shock. Put differently, in our model, an adverse cost-push shock is expansionary in the short run. Our finding stems from the central bank misjudging the persistence of the underlying inflationary pressures and abstains from hiking policy rates as it (erroneously) expects inflationary pressures to dissipate quickly. Under identical assumptions, a standard linearized formulation of our model does not generate an inflation cycle and the output gap remains closed.

There are two important monetary policy implications of our nonlinear framework. First, while "looking through" supply shocks may be good policy for small shocks when inflation is near the central banks target, it may be quite risky when economic activity is strong and large adverse shocks drive inflation well above target. Second, our model implies that the economic costs of "going the last mile" – i.e. with a notably tighter stance than normal behavior would prescribe attempt to returning inflation quickly to target – can be considerable.

We leave several interesting issues for future research. First, the degree of indexation to past inflation in price- and wage-setting evolves as a function of the aggregate rate of inflation which is an endogenous variable in our nonlinear model. It would be very interesting to consider a version of our model in which firms are allowed to choose a desired rate of indexation and compare the implications of this in a nonlinear vs. linear model. Second, future research might consider allowing to switch from an "intensive margin" interpretation of indexation to an "extensive margin" interpretation – i.e., rather than allowing all non-reoptimizers to partially index to past inflation, one could allow a state-dependent fraction of non-reoptimizers to (fully) index to past inflation. In this case, the indexation rule could be calibrated to match differences in the observed frequency of price adjustment across high- and low-inflation episodes, or to match more refined estimates of the empirical relationship between the frequency of price adjustment and the prevailing inflation rate. It might also be worthwhile to consider using separate indexation rules for prices and wages, in which case the parameterization of the wage rule could be disciplined to match the prevalence of cost-of-living-adjustment clauses observed during high-inflation episodes.

All told, our analysis suggests that the interaction of nonlinearities and unexpectedly persistent shocks are crucial to understand the 2021-23 post-COVID episode and are critical to formulate good policy.

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Appendix A. Derivations, Equilibrium Equations, and Additional Results

A.1. Households

Let Λ_t denote the Lagrange multiplier on the household budget constraint. The first order conditions for consumption and bonds can be written as (in scaled form):

$$
\begin{aligned}
c_t &: \quad \frac{1}{c_t - hC_{t-1}} = \lambda_t \\
B_t &: \quad \lambda_t = \beta \delta_t E_t \frac{R_t}{\Pi_{t+1}} \lambda_{t+1}
\end{aligned}
$$

where $\Pi_t = P_t/P_{t-1}$ and $\lambda_t = \Lambda_t P_t$. Note that in steady state $R = \Pi/\beta$ and that in equilibrium, $c_t = C_t$.

A.2. Labor Contractors and Wage Setting

Optimization:

$$
\frac{W_{t,j}}{W_t} = \vartheta_t^w \frac{dG_w\left(\frac{n_{t,j}}{n_t}\right)}{d\frac{n_{t,j}}{n_t}}
$$

Calculate derivative and rearrange:

$$
\begin{aligned}
\frac{W_{t,j}}{W_t} &= \vartheta_t^w \left[(1+\psi_w) \frac{n_{t,j}}{n_t} - \psi_w \right]^{\frac{1-\omega_w}{\omega_w}} \\
\frac{n_{t,j}}{n_t} &= \frac{1}{1+\psi_w} \left(\left[\frac{1}{\vartheta_t^w} \frac{W_{t,j}}{W_t} \right]^{\frac{\omega_w}{1-\omega_w}} + \psi_w \right) \\
\frac{n_{t,j}}{n_t} &= \frac{1}{1+\psi_w} \left(\left[\frac{W_{t,j}}{W_t} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \psi_w \right)
\end{aligned}
$$

Substitute into aggregator which gives the aggregate price index resp. definition of lagrange multiplier:

$$
\begin{aligned}
1 &= \int G_w\left(\frac{n_{t,j}}{n_t}\right)dj \\
1 &= \int\left(\frac{\omega_w}{1+\psi_w}\left[(1+\psi_w)\frac{n_{t,j}}{n_t}-\psi_w\right]^{\frac{1}{\omega_w}}-\frac{\omega_w}{1+\psi_w}+1\right)dj \\
1 &= \int\frac{\omega_w}{1+\psi_w}\left[(1+\psi_w)\frac{n_{t,j}}{n_t}-\psi_w\right]^{\frac{1}{\omega_w}}dj-\int\frac{\omega_w}{1+\psi_w}dj+\int 1dj \\
1 &= \int\left[(1+\psi_w)\frac{n_{t,j}}{n_t}-\psi_w\right]^{\frac{1}{\omega_w}}dj \\
\vartheta_t^w &= \left[\int\left[\frac{W_{t,j}}{W_t}\right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}}dj\right]^{-\frac{\theta_w}{1+\psi_w+\theta_w\psi_w}}
\end{aligned}
$$

Note that after imposing zero profits for labor contractors (free entry), we can write

$$
\begin{aligned}
1 &= \frac{1}{1+\psi_w}\vartheta_t^w+\frac{\psi_w}{1+\psi_w}\int\frac{W_{t,j}}{W_t}dj \\
\vartheta_t^w &= 1+\psi_w-\psi_w\int\frac{W_{t,j}}{W_t}dj
\end{aligned}
$$

Substituting labor demand into the objective and re-arranging gives:

$$
\max_{\tilde{W}_{j,t}} E_t\sum_{i=0}^{\infty}(\beta\xi_w)^i\frac{\varsigma_{t+i}n_{t+i}\lambda_{t+i}}{1+\psi_w}\left\{\begin{array}{c}\frac{W_{t+i}}{P_{t+i}}\vartheta_{w,t+i}^{\varepsilon}\left[\frac{\left(\tilde{\Pi}_{t+i}^w\times\ldots\times\tilde{\Pi}_{t+1}^w\right)}{W_{t+i}}\right]^{1-\varepsilon}\tilde{W}_{j,t}^{1-\varepsilon}+\psi_w\frac{\left(\tilde{\Pi}_{t+i}^w\times\ldots\times\tilde{\Pi}_{t+1}^w\right)}{P_{t+i}}\tilde{W}_{j,t} \\ -mrs_{t+i}\vartheta_{w,t+i}^{\varepsilon}\left[\frac{\left(\tilde{\Pi}_{t+i}^w\times\ldots\times\tilde{\Pi}_{t+1}^w\right)}{W_{t+i}}\right]^{-\varepsilon}\tilde{W}_{j,t}^{-\varepsilon}-\psi_w mrs_{t+i}\end{array}\right\}
$$

where

$$
\begin{aligned}
mrs_{t+i} &= \frac{1}{\Lambda_{t+i}P_{t+i}}=\frac{1}{\lambda_{t+i}} \\
\varepsilon &= \frac{(1+\theta_w)(1+\psi_w)}{\theta_w}
\end{aligned}
$$

Differentiating:

$$
E_t\sum_{i=0}^{\infty}(\beta\xi_w)^i\frac{\varsigma_{t+i}n_{t+i}\lambda_{t+i}}{1+\psi_w}\left\{\begin{array}{c}(1-\varepsilon)\frac{W_{t+i}}{P_{t+i}}\vartheta_{w,t+i}^{\varepsilon}\left[\frac{\left(\tilde{\Pi}_{t+i}^w\times\ldots\times\tilde{\Pi}_{t+1}^w\right)}{W_{t+i}}\right]^{1-\varepsilon}\tilde{W}_{j,t}^{-\varepsilon}+\psi_w\frac{\left(\tilde{\Pi}_{t+i}^w\times\ldots\times\tilde{\Pi}_{t+1}^w\right)}{P_{t+i}} \\ +\varepsilon mrs_{t+i}\vartheta_{w,t+i}^{\varepsilon}\left[\frac{\left(\tilde{\Pi}_{t+i}^w\times\ldots\times\tilde{\Pi}_{t+1}^w\right)}{W_{t+i}}\right]^{-\varepsilon}\tilde{W}_{j,t}^{-\varepsilon-1}\end{array}\right\}=0
$$

All wage adjusters choose the same wage, i.e. $\tilde{W}_{j,t}=\tilde{W}_t$. Re-arranging:

$$
E_t\sum_{i=0}^{\infty}(\beta\xi_w)^i\frac{\varsigma_{t+i}n_{t+i}\lambda_{t+i}}{1+\psi_w}\left\{\begin{array}{c}(1-\varepsilon)w_{t+i}\vartheta_{w,t+i}^{\varepsilon}\left[\frac{\left(\tilde{\Pi}_{t+i}^w\times\ldots\times\tilde{\Pi}_{t+1}^w\right)W_t}{W_{t+i}}\right]^{1-\varepsilon}\tilde{w}_t^{1-\varepsilon}+\psi_w w_{t+i}\frac{\left(\tilde{\Pi}_{t+i}^w\times\ldots\times\tilde{\Pi}_{t+1}^w\right)W_t}{W_{t+i}}\tilde{w}_t \\ +\varepsilon mrs_{t+i}\vartheta_{w,t+i}^{\varepsilon}\left[\frac{\left(\tilde{\Pi}_{t+i}^w\times\ldots\times\tilde{\Pi}_{t+1}^w\right)W_t}{W_{t+i}}\right]^{-\varepsilon}\tilde{w}_t^{-\varepsilon}\end{array}\right\}=0
$$

where

$$\tilde{w}_t = \frac{\tilde{W}_t}{W_t}, w_t = \frac{W_t}{P_t}$$

Note that we can write the first-order condition as:

$$\begin{aligned}
0 = E_t \textstyle\sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}}\right]^{1-\varepsilon} \tilde{w}_t \\
-\frac{\varepsilon}{\varepsilon-1} E_t \textstyle\sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} mrs_{t+i} \vartheta_{w,t+i}^{\varepsilon} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}}\right]^{-\varepsilon} \\
-\frac{\psi_w}{\varepsilon-1} E_t \textstyle\sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \tilde{w}_t^{1+\varepsilon}
\end{aligned}$$

Or

$$\begin{aligned}
0 = E_t \textstyle\sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \vartheta_{w,t+i}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}}\right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} \tilde{w}_t \\
-\frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} E_t \textstyle\sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} mrs_{t+i} \vartheta_{w,t+i}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\
-\frac{\theta_w\psi_w}{\psi_w+\theta_w\psi_w+1} E_t \textstyle\sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}
\end{aligned}$$

Or

$$\begin{aligned}
S_t^w &= F_t^w \tilde{w}_t - A_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\
& \text{or in scaled terms} \\
\frac{S_t^w}{\varsigma_t} &= \frac{F_t^w}{\varsigma_t} \tilde{w}_t - \frac{A_t^w}{\varsigma_t} \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\
s_t^w &= f_t^w \tilde{w}_t - a_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}
\end{aligned}$$

where

$$\begin{aligned}
F_t^w &= E_t \sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \vartheta_{w,t+i}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}}\right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} \tilde{w}_t \\
A_t^w &= \frac{\theta_w\psi_w}{\psi_w+\theta_w\psi_w+1} E_t \sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} w_{t+i} \frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}} \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\
S_t^w &= \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} E_t \sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \varsigma_{t+i} n_{t+i} \lambda_{t+i} mrs_{t+i} \vartheta_{w,t+i}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\frac{\left(\tilde{\Pi}_{t+i}^w \times ... \times \tilde{\Pi}_{t+1}^w\right) W_t}{W_{t+i}}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}
\end{aligned}$$

Writing recursively:

$$F_t^w = \varsigma_t n_t \lambda_t w_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta\xi_w E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}}\right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} F_{t+1}^w$$

or in scaled terms

$$f_t^w = n_t \lambda_t w_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta\xi_w \delta_t E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}}\right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} f_{t+1}^w$$

$$A_t^w = \frac{\theta_w\psi_w}{\psi_w+\theta_w\psi_w+1}\varsigma_t n_t \lambda_t w_t + \beta\xi_w E_t \frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} A_{t+1}^w$$

or in scaled terms

$$\alpha_t^w = \frac{\theta_w\psi_w}{\psi_w+\theta_w\psi_w+1} n_t \lambda_t w_t + \beta\xi_w \delta_t E_t \frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \alpha_{t+1}^w$$

$$S_t^w = \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w}\varsigma_t n_t \lambda_t mrs_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta\xi_w E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} S_{t+1}^w$$

or in scaled terms

$$s_t^w = \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} n_t \lambda_t mrs_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta\xi_w \delta_t E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} s_{t+1}^w$$

and the optimality condition in scaled terms

$$s_t^w = f_t^w \tilde{w}_t - \alpha_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

A.3. Final Goods Producers

Optimization:

$$\frac{P_{t,i}}{P_t} = \vartheta_t \frac{dG\left(\frac{y_{t,i}}{y_t}\right)}{d\frac{y_{t,i}}{y_t}}$$

Calculate derivative and rearrange:

$$\frac{P_{t,i}}{P_t} = \vartheta_t \left[\left(1+\psi_p\right)\frac{y_{t,i}}{y_t} - \psi_p\right]^{\frac{1-\omega_p}{\omega_p}}$$

$$\frac{y_{t,i}}{y_t} = \frac{1}{1+\psi_p}\left(\left[\frac{P_{t,i}}{P_t}\vartheta_t^{-1}\right]^{\frac{\omega_p}{1-\omega_p}} + \psi_p\right)$$

$$\frac{y_{t,i}}{y_t} = \frac{1}{1+\psi_p}\left(\left[\frac{P_{t,i}}{P_t}\right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \psi_p\right)$$

Substitute into aggregator which gives the aggregate price index resp. definition of lagrange multiplier:

$$
\begin{aligned}
1 &= \int \left(\frac{\omega_p}{1+\psi_p} \left[\left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \right]^{\frac{1}{\omega_p}} - \frac{\omega_p}{1+\psi_p} + 1 \right) di \\
\vartheta_t &= \left[\int \left[\frac{P_{t,i}}{P_t} \right]^{1-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} di \right]^{\frac{1}{1-\frac{1+\theta_p}{\theta_p}(1+\psi_p)}}
\end{aligned}
$$

Note that after imposing zero profits, we can write

$$
\begin{aligned}
1 &= \frac{1}{1+\psi_p}\vartheta_t + \frac{\psi_p}{1+\psi_p} \int \frac{P_{t,i}}{P_t} di \\
\vartheta_t &= 1 + \psi_p - \psi_p \int \frac{P_{t,i}}{P_t} di
\end{aligned}
$$

A.4. Intermediate Goods Producers

Substituting the demand function into the profit function gives:

$$
\max_{\tilde{P}_{t,i}} E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \varsigma_{t+j} \Lambda_{t+j} \frac{y_{t+j}}{1+\psi_p} \left[\begin{array}{c} P_{t+j} \left[\frac{(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1})\tilde{P}_{t,i}}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)+1} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \\ + \left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i}\psi_p \\ -MC_{t+j} \left[\frac{(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1})\tilde{P}_{t,i}}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} - \psi_p MC_{t+j} \end{array} \right]
$$

Differentiate

$$
\begin{aligned}
0 = \; & E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \varsigma_{t+j} \Lambda_{t+j} \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p} \left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i} \\
& \times \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i}}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} y_{t+j} \\
& - E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \varsigma_{t+j} \Lambda_{t+j} \frac{\psi_p}{1+\psi_p} \theta_p \left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i} y_{t+j} \\
& - E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \varsigma_{t+j} \Lambda_{t+j} (1+\theta_p) MC_{t+j} \\
& \times \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) \tilde{P}_{t,i}}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} y_{t+j}
\end{aligned}
$$

Define

$$\tilde{p}_{t,i} = \frac{\tilde{P}_{t,i}}{P_t},\ mc_{t+j} = \frac{MC_{t+j}}{P_{t+j}}$$

Using the above definitions and after rearranging:

$$\begin{aligned} 0 \quad = \quad & \underbrace{E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right]^{-\frac{\psi_p+\theta_p\psi_p+1}{\theta_p}} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \tilde{p}_{t,i}}_{\equiv F_t} \\ & \underbrace{-E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p} mc_{t+j}}_{\equiv S_t} \\ & \underbrace{-\frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p} E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right]}_{\equiv A_t} \tilde{p}_{t,i}^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \end{aligned}$$

Or

$$S_t = F_t \tilde{p}_{t,i} - A_t \tilde{p}_{t,i}^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)}$$

Note that, in each period, all firms that reset prices face the same problem and therefore set the same price, $\tilde{p}_{t,i} = \tilde{p}_t$,

$$S_t = F_t \tilde{p}_t - A_t \tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)}$$

It is convenient to scale the above equation by ς_t

$$\begin{aligned} \frac{S_t}{\varsigma_t} &= \frac{F_t}{\varsigma_t} \tilde{p}_t - \frac{A_t}{\varsigma_t} \tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \\ s_t &= f_t \tilde{p}_t - \alpha_t \tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \end{aligned}$$

Consider the expressions for S_t, F_t and A_t :

$$\begin{aligned} S_t &= E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p} mc_{t+j} \\ F_t &= E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right]^{-\frac{\psi_p+\theta_p\psi_p+1}{\theta_p}} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \\ A_t &= \frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p} E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right] \end{aligned}$$

Note that

$$S_t = \varsigma_t \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \frac{(1+\psi_p)(1+\theta_p)}{1+\psi_p+\theta_p\psi_p} mc_t$$
$$E_t \sum_{j=1}^{\infty} (\beta\xi_p)^j \varsigma_{t+j} \lambda_{t+j} y_{t+j} \left[\frac{\left(\tilde{\Pi}_{t+j} \times ... \times \tilde{\Pi}_{t+1}\right) P_t}{P_{t+j}} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_{t+j}^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \frac{(1+\psi_p)(1+\theta_p)}{1+\psi_p+\theta_p\psi_p} mc_{t+j}$$

So that

$$\begin{aligned} S_t &= \frac{(1+\psi_p)(1+\theta_p)}{1+\psi_p+\theta_p\psi_p} \varsigma_t \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} mc_t + \beta\xi_p E_t \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} S_{t+1} \\ s_t &= \frac{(1+\psi_p)(1+\theta_p)}{1+\psi_p+\theta_p\psi_p} \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} mc_t + \beta\xi_p E_t \delta_{t+1} \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} s_{t+1} \end{aligned}$$

Similarly,

$$f_t = \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \beta\xi_p E_t \delta_{t+1} \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{(1+\psi_p+\psi_p\theta_p)}{\theta_p}} f_{t+1}$$

Finally,

$$\alpha_t = \frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p} y_t \lambda_t + \beta\xi_p E_t \delta_{t+1} \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right) \alpha_{t+1}$$

A.5. Aggregate Resources

$$\begin{aligned} y_t^{sum} &= \int y_{t,i} di \\ &= \int n_{t,i} di \\ y_t^{sum} &= n_t \end{aligned}$$

Also,

$$y_t^{sum} = y_t \int \left(\frac{1}{1+\psi_p} \left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p} \right) di$$

So that

$$\begin{aligned} y_t &= \frac{1}{\int \left(\frac{1}{1+\psi_p} \left[\frac{P_{t,i}}{P_t} \right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} \vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p} \right) di} n_t \\ y_t &= (p_t^*)^{-1} n_t \end{aligned}$$

where

$$
\begin{aligned}
p_t^* &= \frac{\vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}}{1+\psi_p}\Delta_{t,1}^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p} \\
\Delta_{t,1} &= \left[\int \left[\frac{P_{t,i}}{P_t}\right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} di\right]^{-\frac{\theta_p}{(1+\theta_p)(1+\psi_p)}} \\
\Delta_{t,1} &= \left[\int \left[\frac{P_{t,i}}{P_t}\right]^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} di\right]^{-\frac{\theta_p}{(1+\theta_p)(1+\psi_p)}} \\
\Delta_{t,1} &= \left[\left(1-\xi_p\right)\tilde{p}_t^{-\frac{(1+\theta_p)(1+\psi_p)}{\theta_p}} + \xi_p\left[\frac{\tilde{\Pi}_t}{\Pi_t}\Delta_{t-1,1}\right]^{-\frac{(1+\theta_p)(1+\psi_p)}{\theta_p}}\right]^{-\frac{\theta_p}{(1+\theta_p)(1+\psi_p)}}
\end{aligned}
$$

Denote aggregate hours worked by households by l_t. Then,

$$
\begin{aligned}
l_t &= \int n_{j,t} dj \\
&= \frac{1}{1+\psi_w} n_t \int \left(\left[\frac{W_{t,j}}{W_t}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \psi_w\right) dj \\
&= n_t \int \left(\frac{1}{1+\psi_w}\left[\frac{W_{t,j}}{W_t}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \frac{\psi_w}{1+\psi_w}\right) dj
\end{aligned}
$$

Or

$$n_t = \left(w_t^*\right)^{-1} l_t$$

where

$$
\begin{aligned}
w_t^* &= \int \left(\frac{1}{1+\psi_w}\left[\frac{W_{t,j}}{W_t}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \frac{\psi_w}{1+\psi_w}\right) dj \\
w_t^* &= \frac{\left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}}{1+\psi_w}\left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} dj + \frac{\psi_w}{1+\psi_w} \\
\Delta_{t,1}^w &= \left[\int \left[\frac{W_{t,j}}{W_t}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} dj\right]^{-\frac{\theta_w}{(1+\theta_w)(1+\psi_w)}} \\
\Delta_{t,1}^w &= \left[(1-\xi_w)\left[\tilde{w}_t\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \xi_w\left[\frac{\tilde{\Pi}_t^w}{\Pi_t^w}\Delta_{t-1,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} dj\right]^{-\frac{\theta_w}{(1+\theta_w)(1+\psi_w)}}
\end{aligned}
$$

So that the aggregate resource constraint reads as follows:

$$c_t = y_t = \left(p_t^*\right)^{-1}\left(w_t^*\right)^{-1} l_t$$

The zero profit condition for final goods producers can be written as:

$$\begin{aligned} \vartheta_t &= 1 + \psi_p - \psi_p \int \frac{P_{t,i}}{P_t} di \\ \vartheta_t &= 1 + \psi_p - \psi_p \Delta_{t,2} \end{aligned}$$

where

$$\begin{aligned} \Delta_{t,2} &= \int \frac{P_{t,i}}{P_t} di \\ \Delta_{t,2} &= (1 - \xi_p)\, \tilde{p}_t + \xi_p \left(\tilde{\Pi}_t / \Pi_t \right) \Delta_{t-1,2} \end{aligned}$$

Further, the aggregate price index equation can be rewritten as follows:

$$\begin{aligned} P_t \vartheta_t &= \left[\int P_{t,i}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} di \right]^{-\frac{\theta_p}{1+\psi_p+\psi_p\theta_p}} \\ \vartheta_t &= \Delta_{t,3} \\ \Delta_{t,3}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} &= (1-\xi_p)\tilde{p}_t^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} + \xi_p \left(\left(\tilde{\Pi}_t / \Pi_t \right) \Delta_{t-1,3} \right)^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} \end{aligned}$$

The zero profit condition for labor contractors can be written as:

$$\begin{aligned} \vartheta_t^w &= 1 + \psi_w - \psi_w \int \frac{W_{t,j}}{W_t} dj \\ \vartheta_t^w &= 1 + \psi_w - \psi_w \Delta_{t,2}^w \\ \Delta_{t,2}^w &= \int \frac{W_{t,j}}{W_t} dj \\ \Delta_{t,2}^w &= (1 - \xi_w)\, \tilde{w}_t + \xi_w \left(\tilde{\Pi}_t^w / \Pi_t^w \right) \Delta_{t-1,2}^w \end{aligned}$$

The aggregate wage index can be written as follows:

$$\begin{aligned} W_t \vartheta_t^w &= \left[\int W_{t,j}^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} dj \right]^{-\frac{\theta_w}{1+\psi_w+\theta_w\psi_w}} \\ \vartheta_t^w &= \Delta_{t,3}^w \\ \left[\Delta_{t,3}^w\right]^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} &= (1-\xi_w)\tilde{w}_t^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} + \xi_w \left(\left(\tilde{\Pi}_t^w / \Pi_t^w \right) \Delta_{t-1,3}^w \right)^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} \end{aligned}$$

Notice the following relation between the real wage, price inflation and wage inflation

$$
\begin{aligned}
\Pi_t^w &= \frac{W_t}{W_{t-1}} \\
\Pi_t^w &= \frac{P_t}{P_t}\frac{P_{t-1}}{P_{t-1}}\frac{W_t}{W_{t-1}} \\
\Pi_t^w &= \frac{P_t}{P_{t-1}}\frac{P_{t-1}}{W_{t-1}}\frac{W_t}{P_t} \\
\Pi_t^w &= \Pi_t \frac{w_t}{w_{t-1}}
\end{aligned}
$$

A.6. Solution and Implementation

We use the nonlinear ('simul') solver in Dynare to solve the model. Specifically, we will use the two-point boundary value solver that is implemented in dynare. In stylized form, the model can be written as follows:

$$
\begin{aligned}
f(y_{-1}, y_0, y_1; a_0) &= 0 \text{ in period } t = 0 \\
f(y_0, y_1, y_2; a_1) &= 0 \text{ in period } t = 1 \\
f(y_1, y_2, y_3; a_2) &= 0 \text{ in period } t = 2 \\
&\quad .... \\
f(y_{T-1}, y_T, y_{T+1}; a_T) &= 0 \text{ in period } t = T
\end{aligned}
$$

Where y_0 denotes the vector of endogenous variables of the model. Dynare's <simul> command solves this set of equations for periods $t = 0, ..., T$ using a Newton algorithm.

y_{-1} and y_{T+1} are given, and most often equal the steady state of the model.

For each realization of shocks from their stochastic processes, we solve the above system of equations in which the agents form expectations using the Kalman filter.[A.1]

More precisely, say in period $t = 0$ a shock is observed. Then, we solve the system of equations from $t = 0$ to $t = T$ with agents forecasting future realizations of shocks from the Kalman filter. Then, we move one period forward, i.e. $t = 1$. There, a new shock is realized. We take the state y_0 from the previous simulation as an initial value and solve the system of equations from $t = 1$ to $t = T + 1$. And so on until no new shocks are realized.

[A.1] Note that we solve the model under certainty equivalence, i.e. in each simulation period, we solve for the deterministic solution of the model. Put differently, the solution method that we are using does not take possible interactions between non-linearities and uncertainty about future shocks into account, i.e. Jensen's inequality plays no role in shaping expectations. In future work, it might be worthwile to consider the effects of shock uncertainty.

A.7. Nonlinear Equilibrium Equations

The nonlinear equilibrium equations can be written as:

$$\begin{aligned}
\text{Marginal utility (n1)} &: \quad \frac{1}{c_t - hc_{t-1}} = \lambda_t \\
\text{MRS (n2)} &: \quad mrs_t = 1/\lambda_t \\
\text{Euler equation (n3)} &: \quad \lambda_t = \beta E_t \delta_{t+1} \frac{R_t}{\Pi_{t+1}} \lambda_{t+1} \\
\text{Resource Constraint (n4)} &: \quad c_t = y_t \\
\text{Production (n5)} &: \quad y_t = \left(p_t^*\right)^{-1} \left(w_t^*\right)^{-1} l_t \\
\text{Non.lin. pricing 1 (n6)} &: \quad s_t = \frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p} \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} mc_t \\
& \quad +\beta\xi_p E_t \delta_{t+1} \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} s_{t+1} \\
\text{Non.lin. pricing 2 (n7)} &: \quad f_t = \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \beta\xi_p E_t \delta_{t+1} \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right)^{-\frac{\left(1+\psi_p+\psi_p\theta_p\right)}{\theta_p}} f_{t+1} \\
\text{Non.lin. pricing 3 (n8)} &: \quad \alpha_t = \frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p} y_t \lambda_t + \beta\xi_p E_t \delta_{t+1} \left(\tilde{\Pi}_{t+1}/\Pi_{t+1}\right) \alpha_{t+1} \\
\text{Non.lin.pricing 4 (n9)} &: \quad s_t = f_t \tilde{p}_t - \alpha_t \tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \\
\text{Zero profit condition prices (n10)} &: \quad \vartheta_t = 1 + \psi - \psi\Delta_{t,2} \\
\text{Aggregate price index (n11)} &: \quad \vartheta_t = \Delta_{t,3} \\
\text{Overall price dispersion (n12)} &: \quad p_t^* = \frac{\vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi)}}{1+\psi_p} \Delta_{t,1}^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \frac{\psi_p}{1+\psi_p} \\
\text{Price dispersion 1 (n13)} &: \quad \Delta_{t,1}^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} = \left(1-\xi_p\right) \tilde{p}_t^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} + \xi_p \left[\left(\tilde{\Pi}_t/\Pi_t\right) \Delta_{t-1,1}\right]^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} \\
\text{Price dispersion 2 (n14)} &: \quad \Delta_{t,2} = \left(1-\xi_p\right) \tilde{p}_t + \xi_p \left(\tilde{\Pi}_t/\Pi_t\right) \Delta_{t-1,2} \\
\text{Price dispersion 3 (n15)} &: \quad \Delta_{t,3}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} = (1-\xi_p)\tilde{p}_t^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} + \xi_p \left(\left(\tilde{\Pi}_t/\Pi_t\right) \Delta_{t-1,3}\right)^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}}
\end{aligned}$$

$$\text{Wage inflation (n16)} \quad : \quad \Pi_t^w = \Pi_t \frac{w_t}{w_{t-1}}$$

$$\text{Non.lin. wage setting 1 (n17)} \quad : \quad s_t^w = f_t^w \tilde{w}_t - \alpha_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$\text{Non.lin. wage setting 2 (n18)} \quad : \quad f_t^w = (w_t^*)^{-1} l_t \lambda_t w_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta \xi_w \delta_t E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} f_{t+1}^w$$

$$\text{Non.lin. wage setting 3 (n19)} \quad : \quad \alpha_t^w = \frac{\theta_w \psi_w}{\psi_w + \theta_w \psi_w + 1} (w_t^*)^{-1} l_t \lambda_t w_t + \beta \xi_w \delta_t E_t \frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \alpha_{t+1}^w$$

$$\text{Non.lin. wage setting 4 (n20)} \quad : \quad s_t^w = \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} (w_t^*)^{-1} l_t \lambda_t mrs_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$+\beta \xi_w \delta_t E_t \left[\frac{\tilde{\Pi}_{t+1}^w}{\Pi_{w,t+1}} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} s_{t+1}^w$$

$$\text{Zero profit condition wages (n21)} \quad : \quad \vartheta_t^w = 1 + \psi_w - \psi_w \Delta_{t,2}^w$$

$$\text{Agg. wage index (n22)} \quad : \quad \vartheta_t^w = \Delta_{t,3}^w$$

$$\text{Overall wage disperision (n23)} \quad : \quad w_t^* = \frac{\left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}}{1+\psi_w} \left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \frac{\psi_w}{1+\psi_w}$$

$$\text{Wage dispersion 1 (n24)} \quad : \quad \left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} = (1-\xi_w) \left[\tilde{w}_t\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$+\xi_w \left[\frac{\tilde{\Pi}_t^w}{\Pi_t^w} \Delta_{t-1,1}^w \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$\text{Wage dispersion 2 (n25)} \quad : \quad \Delta_{t,2}^w = (1-\xi_w) \tilde{w}_t + \xi_w \left(\tilde{\Pi}_t^w / \Pi_t^w \right) \Delta_{t-1,2}^w$$

$$\text{Wage dispersion 3 (n26)} \quad : \quad \left[\Delta_{t,3}^w\right]^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} = (1-\xi_w) \tilde{w}_t^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}}$$

$$+\xi_w \left(\left(\tilde{\Pi}_t^w / \Pi_t^w \right) \Delta_{t-1,3}^w \right)^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}}$$

$$\text{Indexation 1 (n27)} \quad : \quad \tilde{\Pi}_t = \bar{\Pi}^{1-\varkappa_t} \Pi_{t-1}^{\varkappa_t}$$

$$\text{Indexation 2 (n28):} \quad \tilde{\Pi}_t^w = \tilde{\Pi}_t$$

$$\text{Indexation 3 (n29)} \quad : \quad \varkappa_t = e^{-\frac{\varrho}{\max\left(\Pi_t^* - \Pi,\ 0.0001\right)}} - e^{-\frac{\varrho}{0.0001}}$$

$$\text{Indexation 4 (n30)} \quad : \quad \Pi_t^* = \left(\Pi_{t-1}^*\right)^{\omega} \left(\Pi_{t-1}\right)^{1-\omega}$$

$$\text{Marginal cost (n31)} \quad : \quad mc_t = \tau_t^{1/\kappa} w_t$$

$$\text{Taylor rule (n32)} \quad : \quad R_t^{not}/R = \left\{R_{t-1}^{not}/R\right\}^{\rho} \left\{E_t \left[\Pi_{t+4}/\Pi\right]\right\}^{(1-\rho)\gamma_\pi} \left\{ \frac{y_t}{y} / \frac{y_t^{pot}}{y^{pot}} \right\}^{(1-\rho)\gamma_x} e^{\varepsilon_{R,t}}$$

$$\text{ZLB (n33)} \quad : \quad R_t = \max(1, R_t^{not})$$

Flex-price-flex-wage (potential) economy: version of the model when prices and wages are flexible, i.e. $\xi_p = \xi_w = 0$. Also, we set the cost-push shock to zero in the potential economy. The potential economy can be summarized by the following two equations:

$$\text{Real potential rate, pot. econ (n34)} \quad : \quad \frac{1}{y_t^{pot} - h y_{t-1}^{pot}} = \beta E_t \delta_{t+1} rr_t^{pot} \frac{1}{y_{t+1}^{pot} - h y_t^{pot}}$$

$$\text{Potential output, pot. econ (n35)} \quad : \quad y_t^{pot} - h y_{t-1}^{pot} = \frac{1}{1+\theta_p} \frac{1}{1+\theta_w}$$

Note that potential output is constant and only the real potential rate moves in response to the discount factor shock.

We have 35 equations in the following 35 endogenous variables:

$$c_t \ \lambda_t \ w_t \ R_t \ R_t^{not} \ \Pi_t \ y_t \ p_t^* \ l_t \ s_t \ \vartheta_t \ \tilde{\Pi}_t \ mc_t$$

$$f_t \ \alpha_t \ \tilde{p}_t \ \Delta_{t,1} \ \Delta_{t,2} \ \Delta_{t,3} \ \Pi_t^* \ \varkappa_t \ \tilde{\Pi}_t^w \ w_t^* \ \Pi_t^w$$

$$\Delta_{t,1}^w \ \Delta_{t,2}^w \ \Delta_{t,3}^w \ \tilde{w}_t \ s_t^w \ f_t^w \ \alpha_t^w \ \vartheta_t^w \ mrs_t \ rr_t^{pot} \ y_t^{pot}$$

The variables δ_t, $\tau_t - 1 = a_t$ and $\varepsilon_{R,t}$ are exogenous.

A.8. Steady State

The following set of equations solve for the steady state of the model. Assume the central bank chooses a level of steady state inflation Π. Then:

$$(\text{n3}) : R = \frac{1}{\beta}\Pi$$

$$(\text{n27}) \quad : \quad \tilde{\Pi} = \Pi$$

$$\Pi^* \quad = \quad \Pi$$

Note that:

$$(\text{n10}) \quad : \quad \vartheta = 1 + \psi - \psi \Delta_2$$

$$(\text{n11}) \quad : \quad \Delta_3 = \vartheta$$

$$(\text{n12}) \quad : \quad p^* = \frac{\vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi)}}{1+\psi_p} \Delta_1^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p}$$

$$(\text{n13}) \quad : \quad \frac{\Delta_1}{\tilde{p}} = 1$$

$$(\text{n15}) \quad : \quad \frac{\Delta_3}{\tilde{p}} = 1$$

Then, using n10, n11, n14 and n15 we get:

$$\tilde{p} = 1$$

$$\text{(n14)}: \Delta_2 = \tilde{p}$$

$$\text{(n10)}: \vartheta = 1 + \psi - \psi\Delta_2$$

$$\text{(n11)}: \Delta_3 = \vartheta$$

$$\text{(n13)}: \Delta_1 = \tilde{p}$$

$$\text{(n12)}: p^* = \frac{\vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi)}}{1+\psi_p}\Delta_1^{-\frac{1+\theta_p}{\theta_p}(1+\psi_p)} + \frac{\psi_p}{1+\psi_p}$$

Use n6-n9:

$$mc = \left(\vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}\tilde{p} - \frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p}\tilde{p}^{1+\frac{1+\theta_p}{\theta_p}(1+\psi_p)}\right) \times \frac{1}{\frac{(1+\psi_p)(1+\theta_p)}{1+\psi_p+\theta_p\psi_p}\vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}}$$

$$\text{(n31)}: w = mc$$

Use n1, n2, n4, n5 and $mrs = 1/\lambda = w/(1+\theta_w)$ to get:

$$y = \frac{1}{1-h}\frac{w}{1+\theta_w}$$

$$\text{(n4)}: c = y$$

$$\text{(n1)}: \lambda = \frac{1}{(1-h)c}$$

$$\text{(n6)} \quad : \quad s = \frac{\frac{(1+\psi_p)(1+\theta_p)}{1+\psi_p+\theta_p\psi_p}\lambda y \vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}}{1-\beta\xi_p}mc$$

$$\text{(n7)} \quad : \quad f = \frac{\lambda y \vartheta^{\frac{1+\theta_p}{\theta_p}(1+\psi_p)}}{1-\beta\xi_p}$$

$$\text{(n8)} \quad : \quad \alpha = \frac{\frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p}y\lambda}{1-\beta\xi_p}$$

$$w^* = 1$$

$$l = n$$

$$
\begin{aligned}
\tilde{w} &= 1 \\
\Delta_1^w &= 1 \\
\Delta_2^w &= 1 \\
\Delta_3^w &= 1 \\
\vartheta^w &= 1 \\
\Pi^w &= \Pi
\end{aligned}
$$

$$
\begin{aligned}
mrs &= 1/\lambda \\
f^w &= \frac{1}{1-\beta\xi_w} n\lambda w \\
\alpha^w &= \frac{1}{1-\beta\xi_w} \frac{\theta_w\psi_w}{\psi_w+\theta_w\psi_w+1} n\lambda w \\
s^w &= \frac{1}{1-\beta\xi_w} \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} n\lambda mrs \\
s^w &= f^w-\alpha^w
\end{aligned}
$$

$$
\begin{aligned}
\tau &= 1 \\
\delta &= 1
\end{aligned}
$$

Flex-price flex-wage (potential) economy: version of the model when prices are flexible, i.e. $\xi_p = \xi_w = 0$.

$$
\text{(n34) } rr^{pot} = \frac{1}{\beta}
$$

$$
\text{(n35) } y^{pot} = \frac{1}{1-h} \frac{1}{1+\theta_p} \frac{1}{1+\theta_w}
$$

A.9. Log-Linearized Equilibrium Equations

Equations n10-15 can be expressed in log-linearized form as follows:

$$
\hat{\vartheta}_t = 0, \hat{p}_t^* = 0, \hat{\Delta}_{t,1} = 0, \hat{\Delta}_{t,2} = 0, \hat{\Delta}_{t,3} = 0, \hat{\tilde{p}}_t \ = \frac{\xi_p}{1-\xi_p}\left(\hat{\Pi}_t - \hat{\bar{\Pi}}_t\right)
$$

After some tedious math, n6-n9 can be written as:

$$\text{Non.lin. pricing 1 (n6)} \quad : \quad \hat{s}_t = \left(1-\beta\xi_p\right)\left[\hat{y}_t + \hat{\lambda}_t + \widehat{mc}_t\right] + \beta\xi_p E_t\left[\hat{\delta}_{t+1} + \frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) + \hat{s}_{t+1}\right]$$

$$\text{Non.lin. pricing 2 (n7)} \quad : \quad \hat{f}_t = \left(1-\beta\xi_p\right)\left(\hat{y}_t + \hat{\lambda}_t\right) + \beta\xi_p E_t\left[\hat{\delta}_{t+1} + \frac{\left(1+\psi_p+\psi_p\theta_p\right)}{\theta_p}\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) + \hat{f}_{t+1}\right]$$

$$\text{Non.lin. pricing 3 (n8)} \quad : \quad \hat{\alpha}_t = \left(1-\beta\xi_p\right)\left(\hat{y}_t + \hat{\lambda}_t\right) + \beta\xi_p E_t\left[\hat{\delta}_{t+1} - \left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) + \hat{\alpha}_{t+1}\right]$$

$$\text{Non.lin.pricing 4 (n9)} \quad : \quad \hat{s}_t = \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}\hat{f}_t - \frac{\psi_p\theta_p}{1+\psi_p}\hat{\alpha}_t + \frac{\xi_p\left(1-\psi_p-\psi_p\theta_p\right)}{1-\xi_p}\left(\hat{\Pi}_t - \hat{\tilde{\Pi}}_t\right)$$

Premultiply n7 and n8 by $\frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}$ and $\frac{\psi_p\theta_p}{1+\psi_p}$,respectively. Add n8 and substract n7 from n6:

$$\hat{s}_t - \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}\hat{f}_t + \frac{\psi_p\theta_p}{1+\psi_p}\hat{\alpha}_t = \left(1-\beta\xi_p\right)\widehat{mc}_t + \beta\xi_p E_t\left[\begin{array}{c} \hat{s}_{t+1} - \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}\hat{f}_{t+1} + \frac{\psi_p\theta_p}{1+\psi_p}\hat{\alpha}_{t+1} \\ +\left(1-\theta_p\psi_p-\psi_p\right)\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) \end{array}\right]$$

Use equation n9 to get:

$$\frac{\xi_p\left(1-\psi_p-\psi_p\theta_p\right)}{1-\xi_p}\left(\hat{\Pi}_t - \hat{\tilde{\Pi}}_t\right) = \left(1-\beta\xi_p\right)\widehat{mc}_t + \beta\xi_p E_t\left[\frac{\psi_p+\theta_p\psi_p-1}{\xi_p-1}\right]\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right)$$

Or

$$\hat{\Pi}_t - \hat{\tilde{\Pi}}_t = \beta E_t\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_{t+1}\right) + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\frac{1}{1-\left(1+\theta_p\right)\psi_p}\widehat{mc}_t$$

The coefficient $\frac{1}{1-(1+\theta_p)\psi_p}$ is identical to the one in Levin, Lopez-Salido and Yun (2007).

Finally, $\tilde{\Pi}_t = \Pi^{1-\varkappa_t}\Pi_{t-1}^{\varkappa_t}$ can be log-linearized to yield:

$$\hat{\tilde{\Pi}}_t = \varkappa\hat{\Pi}_{t-1}.$$

So, the log-linearized New Keynesian Phillips curve reads:

$$\hat{\Pi}_t = \frac{\varkappa}{1+\beta\varkappa}\hat{\Pi}_{t-1} + \frac{\beta}{1+\beta\varkappa}E_t\hat{\Pi}_{t+1} + \kappa\widehat{mc}_t$$

where

$$\kappa = \frac{1}{1+\beta\varkappa}\frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\frac{1}{1-\left(1+\theta_p\right)\psi_p}$$

The nonlinear wage setting equations can be log-linearized to obtain:

$$
\begin{aligned}
\hat{f}_t^w &= (1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right)+\beta\xi_w E_t\left(\hat{\delta}_t+\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{f}_{t+1}^w\right)\\
\hat{\alpha}_t^w &= (1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right)+\beta\xi_w E_t\left(\hat{\delta}_t-\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{\alpha}_{t+1}^w\right)\\
\hat{s}_t^w &= (1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\widehat{mrs}_t\right)+\beta\xi_w E_t\left(\hat{\delta}_t+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{s}_{t+1}^w\right)\\
\hat{s}_t^w &= \frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}f_t^w-\frac{\theta_w\psi_w}{1+\psi_w}\hat{\alpha}_t^w+(1-\psi_w-\theta_w\psi_w)\,\widehat{\tilde{w}}_t
\end{aligned}
$$

Also:

$$
\widehat{\tilde{w}}_t=\frac{\xi_w}{1-\xi_w}\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right)
$$

So that

$$
\begin{aligned}
\frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\hat{f}_t^w &= \frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}(1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right)\\
&\quad+\frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(\hat{\delta}_t+\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{f}_{t+1}^w\right)\\
\frac{\theta_w\psi_w}{1+\psi_w}\hat{\alpha}_t^w &= \frac{\theta_w\psi_w}{1+\psi_w}(1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right)+\frac{\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(\hat{\delta}_t-\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{\alpha}_{t+1}^w\right)\\
\hat{s}_t^w &= (1-\beta\xi_w)\left(\hat{n}_t+\hat{\lambda}_t+\widehat{mrs}_t\right)+\beta\xi_w E_t\left(\hat{\delta}_t+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{s}_{t+1}^w\right)\\
\hat{s}_t^w &= \frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}f_t^w-\frac{\theta_w\psi_w}{1+\psi_w}\hat{\alpha}_t^w+(1-\psi_w-\theta_w\psi_w)\frac{\xi_w}{1-\xi_w}\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right)
\end{aligned}
$$

Substract first and add second equation to third equation and substitute last equation to get:

$$
\begin{aligned}
(1-\psi_w-\theta_w\psi_w)\frac{\xi_w}{1-\xi_w}\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right) &= (1-\beta\xi_w)\left(\widehat{mrs}_t-\hat{w}_t\right)\\
&\quad+\beta\xi_w E_t\left(\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{s}_{t+1}^w\right)\\
&\quad+\frac{\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(-\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{\alpha}_{t+1}^w\right)\\
&\quad-\frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\hat{f}_{t+1}^w\right)
\end{aligned}
$$

Or :

$$
\begin{aligned}
(1-\psi_w-\theta_w\psi_w)\frac{\xi_w}{1-\xi_w}\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right) &= (1-\beta\xi_w)\left(\widehat{mrs}_t-\hat{w}_t\right)\\
&\quad+\beta\xi_w(1-\psi_w-\theta_w\psi_w)\left(1+\frac{\xi_w}{1-\xi_w}\right)E_t\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)
\end{aligned}
$$

Or:

$$
\left(\hat{\Pi}_{w,t}-\widehat{\tilde{\Pi}_t^w}\right)=\beta E_t\left(\hat{\Pi}_{w,t+1}-\widehat{\tilde{\Pi}_{t+1}^w}\right)+\frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w}\frac{1}{1-(1+\theta_w)\psi_w}\left(\widehat{mrs}_t-\hat{w}_t\right)
$$

where

$$\widehat{mrs}_t = -\hat{\lambda}_t$$

So, the set of log-linearized equilibrium equations can be written as:

$$\begin{aligned}
\text{Euler equation (l1)} &: \quad \hat{y}_t = \frac{1}{1+h}E_t\hat{y}_{t+1} + \frac{h}{1+h}\hat{y}_{t-1} - \frac{1-h}{1+h}E_t\left[\hat{R}_t - \hat{\Pi}_{t+1} + \hat{\delta}_{t+1}\right] \\
\text{Marginal cost (l2)} &: \quad \widehat{mc}_t = \frac{1}{\kappa}\hat{\tau}_t + \hat{w}_t \\
\text{Marg. rate of subst.(l3)} &: \quad \widehat{mrs}_t = \frac{1}{1-h}\left(\hat{y}_t - h\hat{y}_{t-1}\right) \\
\text{Taylor rule (l4)} &: \quad \hat{R}_t^{not} = \rho\hat{R}_{t-1} + (1-\rho)\left[\gamma_\pi E_t\hat{\Pi}_{t+4} + \gamma_x\left(\hat{y}_t - \hat{y}_t^{pot}\right)\right] + \varepsilon_{R,t} \\
\text{ZLB (l5)} &: \quad \hat{R}_t = \max(-\ln R, \hat{R}_t^{not}) \\
\text{Price Phillips Curve (l6)} &: \quad \hat{\Pi}_t - \hat{\tilde{\Pi}}_t = \beta E_t\left(\hat{\Pi}_{t+1} - \hat{\tilde{\Pi}}_t\right) + (1+\beta\varkappa)\,\kappa\widehat{mc}_t \\
\text{Wage Phillips Curve (l7)} &: \quad \hat{\Pi}_{w,t} - \hat{\tilde{\Pi}}_t = \beta E_t\left(\hat{\Pi}_{w,t+1} - \hat{\tilde{\Pi}}_t\right) + \kappa^w\left(\widehat{mrs}_t - \hat{w}_t\right) \\
\text{Wage inflation (n8)} &: \quad \hat{\Pi}_{w,t} = \hat{\Pi}_t + \hat{w}_t - \hat{w}_{t-1} \\
\text{Real potential rate (l9)} &: \quad \hat{rr}_t^{pot} = -E_t\hat{\delta}_{t+1} \\
\text{Potential output (l10)} &: \quad \hat{y}_t^{pot} = 0
\end{aligned}$$

Log-linearizing the indexation equations gives:

$$\text{Indexation (l11)}: \hat{\tilde{\Pi}}_t = 0$$

Finally, the slopes of the Phillips curves are defined as:

$$\begin{aligned}
\kappa &= \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\frac{1}{1-(1+\theta_p)\,\psi_p}\frac{1}{1+\beta\varkappa} \\
\kappa^w &= \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w}\frac{1}{1-(1+\theta_w)\,\psi_w}
\end{aligned}$$

We have 13 equations in the following 13 endogenous variables:

$$\hat{R}_t \; \hat{R}_t^{not} \; \hat{\Pi}_t \; \hat{\tilde{\Pi}}_t \; \hat{y}_t \; \hat{w}_t \; \widehat{mc}_t \; \widehat{mrs}_t \; \hat{\Pi}_{w,t} \; \hat{rr}_t^{pot} \; \hat{y}_t^{pot}$$

The variables $\hat{\delta}_t$, $\hat{\tau}_t = \tau_t - 1 = a_t$ and $\varepsilon_{R,t}$ are exogenous.

Appendix B. Additional Results

This appendix contains additional results cited in the main text.

B.1. Data-Model Comparison in Linearized Model

Figure B.1 provides a comparison between the data and the linearized model.

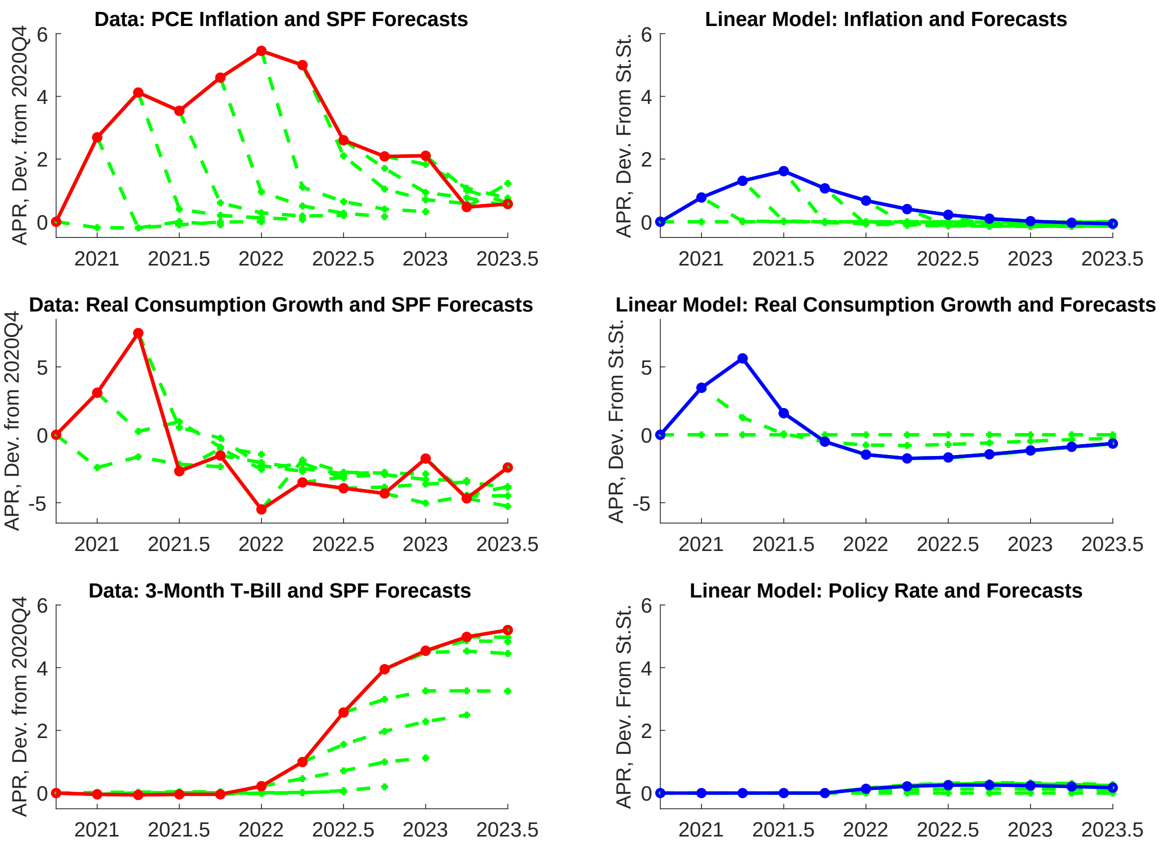

Figure B.1: Comparison of data vs. linear model.

B.2. Transmission of Discount Factor Shock

Specification: discount factor shock of $\varepsilon_{\delta,0} = -0.01$, i.e. fall in discount factor of 1 percent (quarterly) or 4 percent (annualized). Fall in discount factor implies rise in demand (but no effects on potential output).

Figure B.2 provides the impulse responses of the nonlinear model to a discount factor shock.

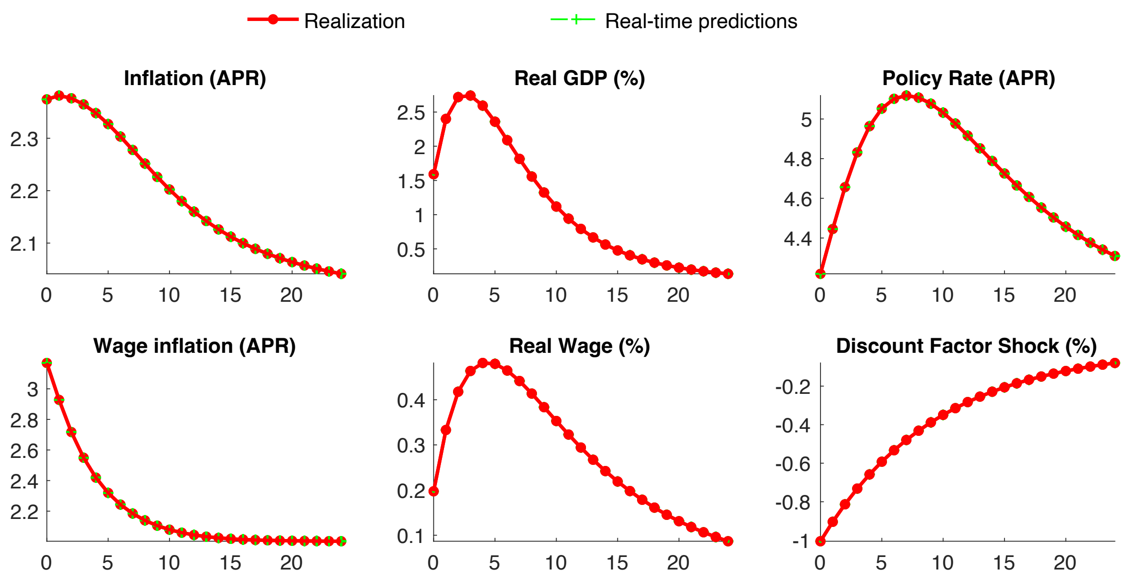

Figure B.2: Impulse responses to a discount factor shock in the nonlinear model.

B.3. Transmission of Transitory Cost-push Shock

Here, we consider the case when the cost push shock is transitory, i.e. $\varepsilon_0^T = 0.0025$ shock to the transitory component of the cost push shock. I.e. 1/4 percent (quarterly) or 1 percent (annualized) cost-push shock. This analysis could be extended to show (or argue based on the the results below) that it is more optimal to 'look through' transitory cost push shocks.

Figure B.3 in the appendix provides the impulse responses of the nonlinear model to a cost-push shock when the latter is driven by the transitory (iid) component.

B.4. Further Details on the Unobserved Components Representation

Figure B.4 shows the effects of assuming alternative values for the ratio of standard deviations (σ_P/σ_T) of the persistent and transitory components of the unobserved components representation of the cost push shock in the nonlinear model.

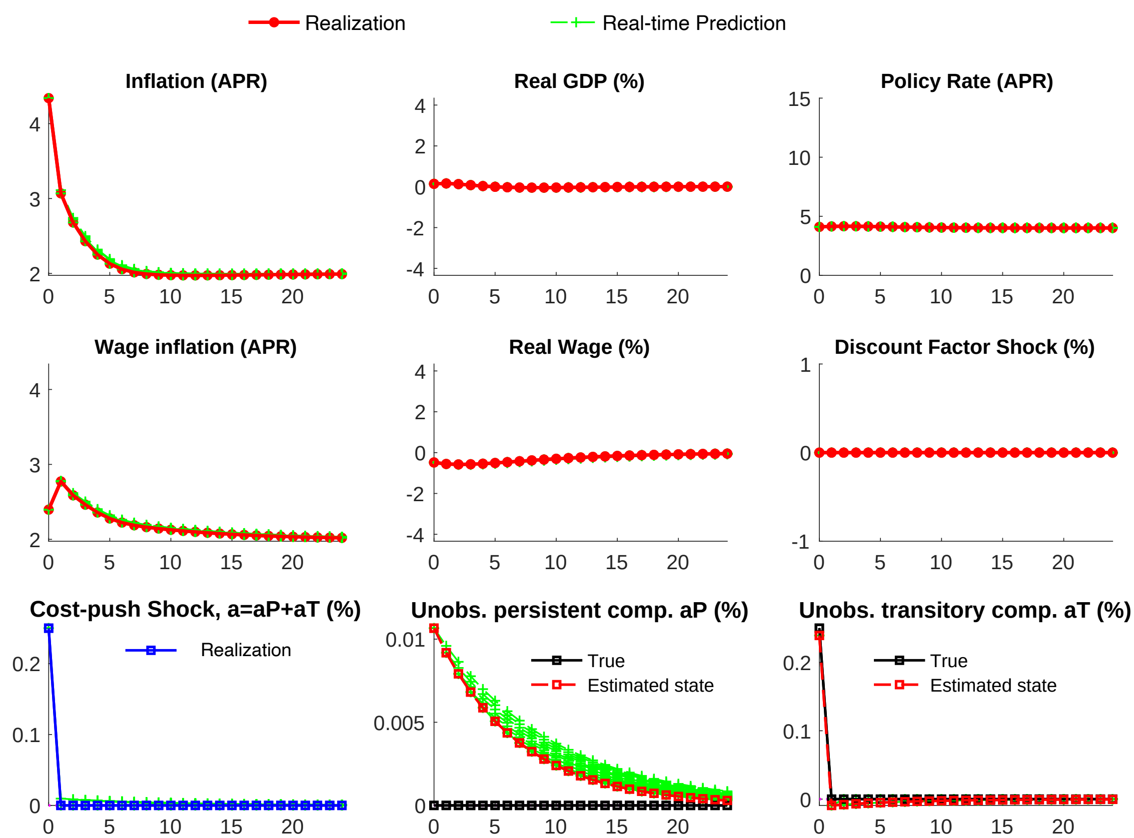

Figure B.3: Impulse responses to a transitory (iid) cost-push shock in the nonlinear model.

B.5. Timing of Monetary Policy Intervention

Here, we consider the implications when the central bank becomes more aggressive at different points in time. Specifically, Figure B.5 shows the simulation results for more aggressive monetary policy in nonlinear model for different start dates of the monetary policy intervention. All impulse responses are displayed in deviations from baseline. The key takeaway is that the earlier the central bank intervenes, the larger the reduction in inflation for a given hike in the policy rate. Put differently, monetary policy becomes less effective the higher inflation is to begin with. In this sense, the efficacy of monetary tightening and the sacrifice ratio are state-dependent in our model.

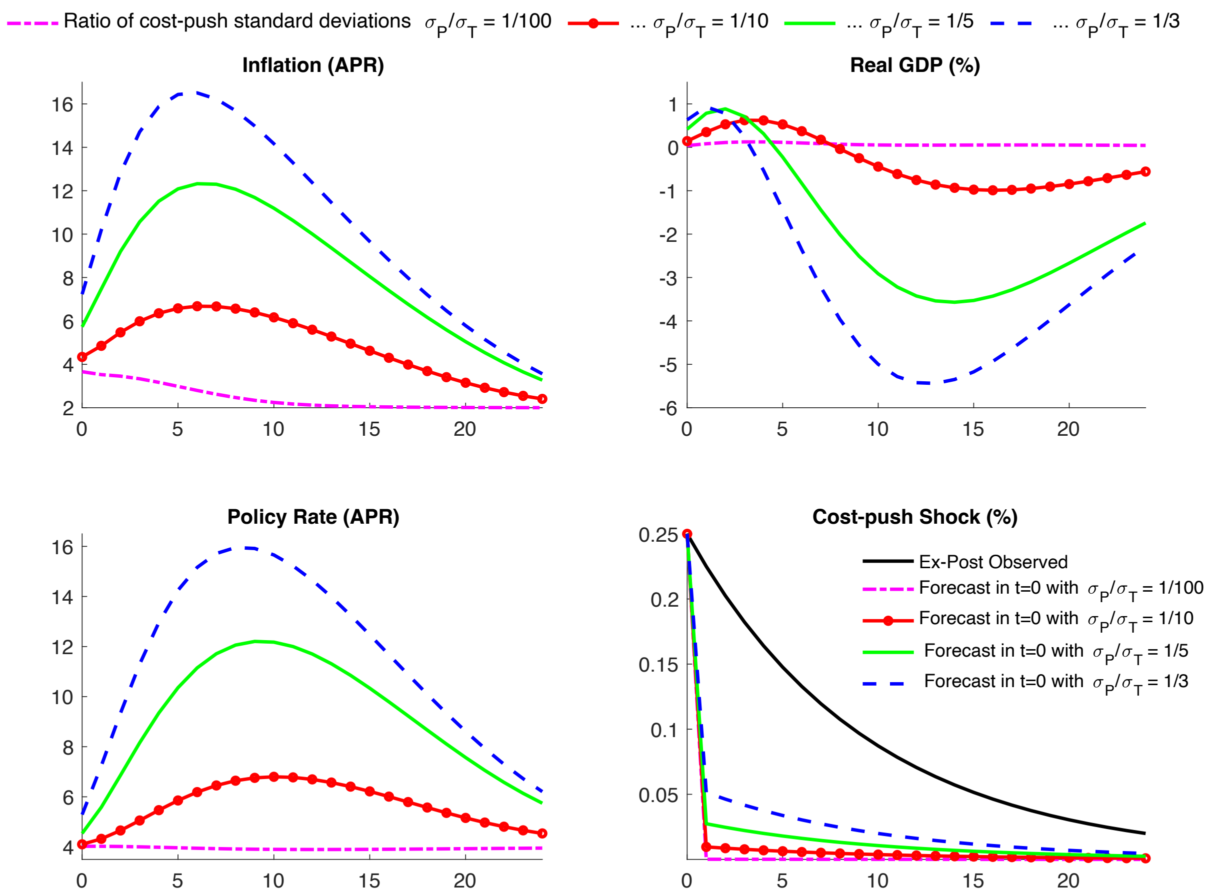

Figure B.4: Effects of alternative values for the ratio of standard deviations of the persistent and transitory components of the unobserved components representation of the cost push shock in the nonlinear model.

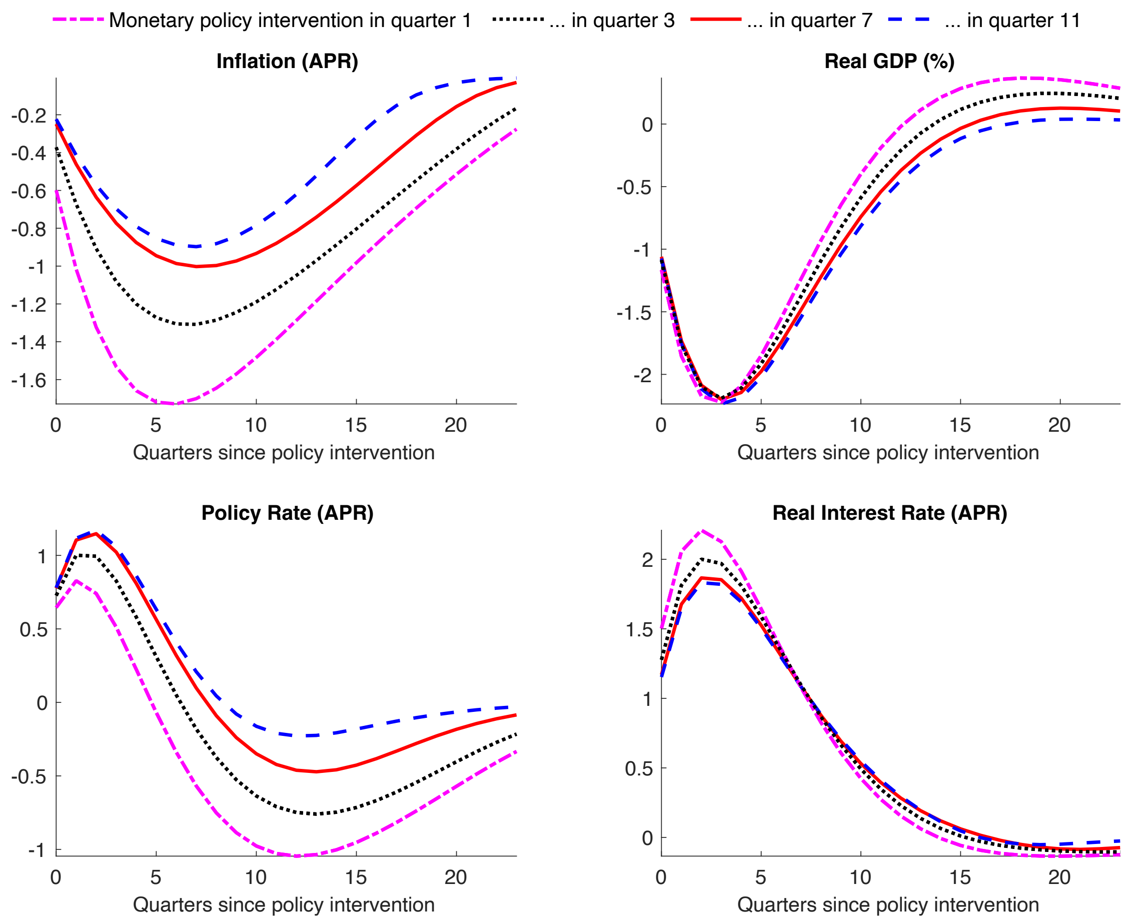

Figure B.5: Simulation results for more aggressive monetary policy in nonlinear model with different start dates for the monetary policy intervention. All responses in deviations from baseline.

PUBLICATIONS

Monetary Policy and Inflation Scares

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