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Reconciling Random Walks and Predictability: A Dual-Component Model of Exchange Rate Dynamics

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Prepared by Bas B. Bakker*

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ABSTRACT: This paper addresses a key puzzle in international finance: whether exchange rates follow a random walk or exhibit predictable patterns. We demonstrate that exchange rates can possess a unit root while maintaining substantial predictability over certain horizons. Our model combines a stochastic trend—representing the slowly moving equilibrium exchange rate—and a stationary cyclical component capturing temporary deviations, reconciling long-term random walk behavior with medium-term predictability.
This dual-component framework is essential for capturing three key features of exchange rate dynamics: expected exchange rate changes are not zero, they are highly persistent, and there is a strong relationship between exchange rate levels and expected future changes. Without the stationary component, expected exchange rate changes would be zero, and if the stochastic trend evolved too quickly, this relationship would break down. To illustrate, we extend the Bacchetta and van Wincoop (2021) framework (which generates a stationary component of the exchange rate) with a stochastic trend. Our model generates an inverted U-shaped pattern where forecast accuracy peaks at intermediate horizons and predicts that multi-year exchange rate changes are increasing multiples of one-year changes. Using data from 2000–2024 for nine inflation-targeting countries with freely floating exchange rates, we find strong empirical support for these predictions, with our model consistently outperforming the random walk benchmark in out-of-sample tests.

JEL Classification Numbers:	F31, F37, and F41
Keywords:	Exchange rate dynamics; Random walk hypothesis; Medium-term predictability; Unit root; Stationary component; Stochastic trend; Mean reversion; Forecasting accuracy; Exchange rate modeling; Exchange rate puzzles

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* Previous versions of this paper have benefitted from comments by Emine Boz, Nicolas Fernandez-Arias, Emilio Fernandez Corugedo, Pierre-Olivier Gourinchas, Russell Green, Leslie Lipschitz, Jim Morsink, and Maylin Sun.

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Reconciling Random Walks and Predictability: A Dual-Component Model of Exchange Rate Dynamics

Prepared by Bas B. Bakker[1]

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Contents

1 Introduction and Executive Summary

A long-running debate over exchange rate dynamics centers on whether they are stationary or follow a random walk. Traditional models assume that exchange rates exhibiting a unit root follow a random walk, making their future movements inherently unpredictable. This assumption aligns with the Efficient Market Hypothesis (EMH), which posits that asset prices fully reflect all available information.

However, this paper challenges this view by demonstrating that exchange rates can simultaneously possess a unit root while maintaining predictable components. We show that exchange rate dynamics can be decomposed into two parts: a slowly moving stochastic trend and a stationary cyclical component. The stochastic trend represents the long-run equilibrium exchange rate, evolving as a random walk driven by fundamental economic factors like differential inflation rates and productivity growth. In contrast, the stationary cyclical component captures temporary deviations from this equilibrium path, which diminish over time as the exchange rate converges back toward equilibrium. By combining these elements, exchange rates can exhibit both long-term unpredictability (driven by the stochastic trend) and medium-term predictability (due to the cyclical component). In this framework, expected exchange rate changes depend solely on the cyclical component—the larger the cyclical component, the higher the expected exchange rate depreciation.

This dual-component framework captures three key features of exchange rate dynamics: expected exchange rate changes are not zero, they are highly persistent, and there is a strong relationship between exchange rate levels and expected future changes. Without a stationary component, expected exchange rate changes would by definition be zero. Furthermore, if the stochastic trend did not evolve slowly, the relationship between exchange rate levels and expected changes would break down, and the cyclical component—along with the persistence of expected exchange rate changes—would diminish.

The interplay between the stochastic trend and the stationary cyclical component produces an inverted U-shaped predictability pattern, where exchange rates are least predictable in the short and long term but exhibit significant forecastability in the medium term. This reflects the dominance of noise in short-term fluctuations, the stochastic trend in long-term movements, and the stationary component in medium-term dynamics.

To illustrate, this paper extends the Bacchetta and van Wincoop (2021) framework (which is an example of how mean-reverting interest rate differentials and gradual portfolio adjustments can generate a stationary component) with a stochastic trend.

To test the model, we align its theoretical predictions with data from 2000 to 2024 for nine inflation-targeting countries with freely floating exchange rates. These countries are chosen because their interest rate differentials are exogenous to expected exchange rate changes. Our empirical analysis shows that twelve-month exchange rate forecasts significantly predict multi-year changes, with predictive power increasing over extended horizons. This confirms the model's prediction of an inverted U-shaped pattern, with medium-term changes being the most predictable. Moreover, our model significantly outperforms the random walk benchmark in out-of-sample forecasting. This improvement in predictive accuracy is particularly notable over longer horizons, where traditional models struggle to balance short-term predictability with long-term uncertainty. Finally, we conduct a sensitivity analysis to demonstrate that the model's outperformance remains robust across a range of parameter values, underscoring the stability of the forecasting framework.

While our model incorporates portfolio adjustment frictions within the Bacchetta and van Wincoop (2021) framework to generate the stationary component, it is important to note that this is only one example of a mechanism that can produce mean-reverting dynamics. Other approaches, such as models emphasizing transaction costs, behavioral biases, or deviations from purchasing power parity (PPP), could similarly lead to mean-reversion in exchange rates. The ability to incorporate alternative mechanisms highlights the broader applicability of the dual-component framework, accommodating various theoretical foundations to capture stationary behavior.

In summary, this paper introduces a hybrid model that reconciles the random walk hypothesis with mean reversion. By combining a stochastic trend with a stationary component, the model captures key features of exchange rate dynamics and demonstrates strong out-of-sample predictive power across different horizons. This dual-component framework, adaptable to various theoretical perspectives, lays the foundation for further research on alternative mechanisms for generating mean-reverting dynamics.

2 Literature review

Mean Reversion of Real Exchange Rates

One of the longstanding debates in international finance is whether exchange rates are stationary or possess a unit root, thereby following a random walk. Stationarity implies that exchange rates revert to a long-term mean, suggesting predictability over the long run. In contrast, a unit root indicates that exchange rates follow a random walk, where shocks have permanent effects, leading to unpredictability.

Early empirical studies predominantly supported the random walk hypothesis, suggesting that exchange rates follow a unit root process and are therefore unpredictable. Meese and Rogoff (1983) famously demonstrated that random walk models often outperform economic models in exchange rate forecasting. Their seminal work showed that models based on economic fundamentals failed to outperform a simple random walk model in out-of-sample predictions. This finding has been corroborated by numerous studies, such as Mark (1995), which found that the predictive power of economic models over short horizons was limited, reinforcing the view that exchange rates follow a random walk. The random walk hypothesis aligns with the Efficient Market Hypothesis (EMH), which posits that exchange rates reflect all available information and are thus inherently unpredictable.

In the 1980s, researchers employed unit root tests to assess whether real exchange rates are mean-reverting—a necessary condition for PPP to hold over time. These early tests frequently failed to reject the null hypothesis of a unit root for major economies, suggesting that real exchange rates might follow a random walk rather than exhibit mean reversion Taylor and Taylor (2004), Rogoff (1996), Nelson and Plosser (1982) and Stock and Watson (1988) provided early empirical evidence that many macroeconomic time series, including exchange rates, exhibit unit root behavior, supporting the random walk hypothesis.

Unit root tests often have low power in small samples, making it difficult to distinguish between a unit root and a near-stationary process. DeJong et al. (1992) highlighted the sensitivity of these tests to sample size and structural breaks, which can significantly impact conclusions.

Recognizing the low statistical power of these unit root tests in small samples (Lothian and Taylor, 1997, Sarno and Taylor, 2002), researchers have explored several enhancements. These include extending the time series data (Lothian and Taylor, 1996) and employing panel unit root tests that aggregate data across different countries (Pedroni, 2001). Such methodologies have generally provided more support for the mean reversion of real exchange rates, offering empirical backing for PPP.

Structural breaks and regime changes, such as shifts in economic policy or external shocks, also play a crucial role. Perron (1989) showed that ignoring structural breaks can lead to incorrect conclusions about the presence of unit roots. Lumsdaine and Papell (1997) found that allowing for multiple structural breaks often changes the inference about stationarity and unit roots. They demonstrated that accounting for breaks can reveal periods of mean reversion within an overall unit root process.

Non-linear Adjustment

The introduction of nonlinear adjustment models marked a significant advancement. These models propose that the speed of mean reversion increases as deviations from the equilibrium level grow larger, suggesting faster mean reversion than previously recognized and aligning better with PPP predictions (Kilian and Taylor, 2003, Taylor and Taylor, 2004).

Moreover, the literature discusses the impact of transaction costs and market frictions, which can create "bands of inaction." In these bands, arbitrage does not occur until deviations exceed transaction costs, leading to periods of apparent non-reversion. This complexity has been explored in models incorporating elements such as nonlinear adjustments and transaction costs, showing that real exchange rates do revert to a mean under more realistic market conditions (Michael et al., 1997, Sarno et al., 2004).

Implications for Nominal Exchange Rates

Several authors have concluded that mean reversion of real exchange rates implies mean reversion of nominal exchange rates. Cheung et al. (2004) challenge the conventional view that price adjustment determines the PPP reversion rate. They argue that nominal exchange rate adjustment, not price adjustment, is the key engine governing the speed of PPP convergence.

Kilian and Taylor (2003) show that deviations of the nominal exchange rate from PPP equilibrium can forecast changes in nominal exchange rates. They show that while nominal rates approximate a random walk near equilibrium, significant deviations from fundamentals trigger mean-reverting behavior. The predictability of nominal exchange rates, therefore, improves over longer horizons. Zorzi and Rubaszek (2020) show that real exchange rates in advanced countries with flexible regimes are mean-reverting, as implied by the Purchasing Power Parity model. They also show that the adjustment takes place via nominal exchange rates. They propose forecasting the nominal exchange rate change by using the level of the real exchange rate and show that this has significant predictive power.

Conclusion

The ongoing debate over the stationarity of exchange rates remains unresolved due to the complex nature of exchange rate dynamics and the limitations of statistical tests. Both sides present compelling arguments: early studies predominantly supported the random walk hypothesis and the unpredictability of exchange rates, while later studies, leveraging longer time series and advanced techniques, find more support for mean reversion. A nuanced understanding that incorporates both perspectives may offer the most comprehensive insight into exchange rate behavior.

3 Exchange rates have a unit root

To analyze the time series properties of exchange rates, we conducted tests for unit roots using two common approaches: the Augmented Dickey-Fuller (ADF) test and the KPSS (Kwiatkowski-Phillips-Schmidt-Shin) test.

The ADF test examines the null hypothesis that a time series has a unit root. If the null is rejected, it indicates that the series is stationary. In our analysis, the ADF test results showed (Table 3.1) that we could not reject the null hypothesis for the examined exchange rates, suggesting that the exchange rate has a unit root.

To complement the findings from the ADF test, we applied the KPSS test, which assumes the opposite null hypothesis: that the series is stationary. Our KPSS test results reject the null hypothesis, reinforcing the conclusion that the exchange rate series are not stationary.

The results from both the ADF and KPSS tests consistently indicate that the examined exchange rate series are non-stationary.

Table 3.1: ADF and KPSS Test Results for Exchange Rates vis-a-vis US dollar, 2000-2024

	ADF Statistic	ADF p-value	KPSS Statistic	KPSS p-value
EUR	-2.314	0.444	0.867	0.01
JPN	-1.108	0.920	0.894	0.01
GBR	-2.633	0.310	2.973	0.01
CAN	-1.677	0.712	0.976	0.01
AUS	-1.855	0.637	1.092	0.01
NZL	-2.137	0.519	1.522	0.01
CHE	-2.098	0.535	3.820	0.01
SWE	-2.281	0.458	1.389	0.01
NOR	-2.106	0.531	2.038	0.01

[a] Monthly data. The p-values indicate the significance level for the null hypotheses.

4 Why Exchange Rates Combine a Stationary Component with a Slowly Moving Trend

4.1 Intuition

If exchange rates followed a random walk, expected exchange rates should be zero. To test whether this is true, we use monthly data of 12-month expected exchange rate changes vis-à-vis the US dollar for the period 2000–2024, from a dataset of monthly survey data kindly provided by Das et al. (2022). Several key observations emerge:

- Expected exchange rate changes are not zero. They can be quite substantial (Figure 4.1).
- Expected exchange rate changes exhibit high persistence over time (Figure 4.2).
- There is a notable negative relationship between exchange rate levels and expected future changes. This link is strong for most countries, though weaker for a few, including Great Britain (Figure 4.3).

Thus, exchange rates exhibit characteristics that seem paradoxical. On the one hand, they have a unit root, suggesting a random walk with no long-term predictability. On the other hand, survey data indicate that expected exchange rate changes are not zero and exhibit strong persistence, suggesting medium-term predictability.

This apparent contradiction can be resolved by recognizing that exchange rates are composed of two distinct components: a slowly evolving stochastic trend and a stationary cyclical component. The stochastic trend governs long-term, unpredictable movements, while the stationary component introduces mean-reverting dynamics. Together, these components allow exchange rates to display both a unit root and medium-term predictability.

If the stochastic trend dominates, exchange rate changes are largely unpredictable. However, when the stationary component is significant, the cyclical behavior of the exchange rate creates medium-term predictability. Expected changes are then determined by the current value of the cyclical component, with greater deviations leading to stronger mean reversion.

Figure 4.1. Expected 12-months change log exchange rate, 2000-2024
(Monthly data)

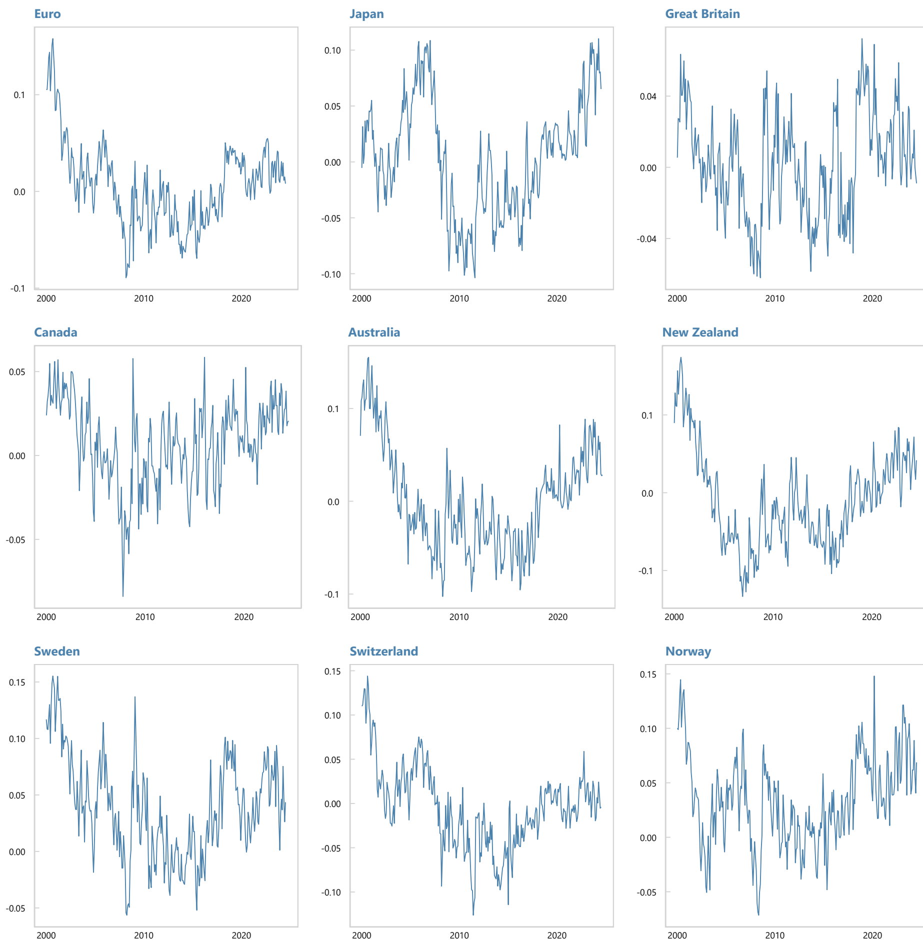


Figure 4.2. ACF of Expected 12-months change log Exchange Rate, 2000-2024

(Monthly data, up to 48 lags)

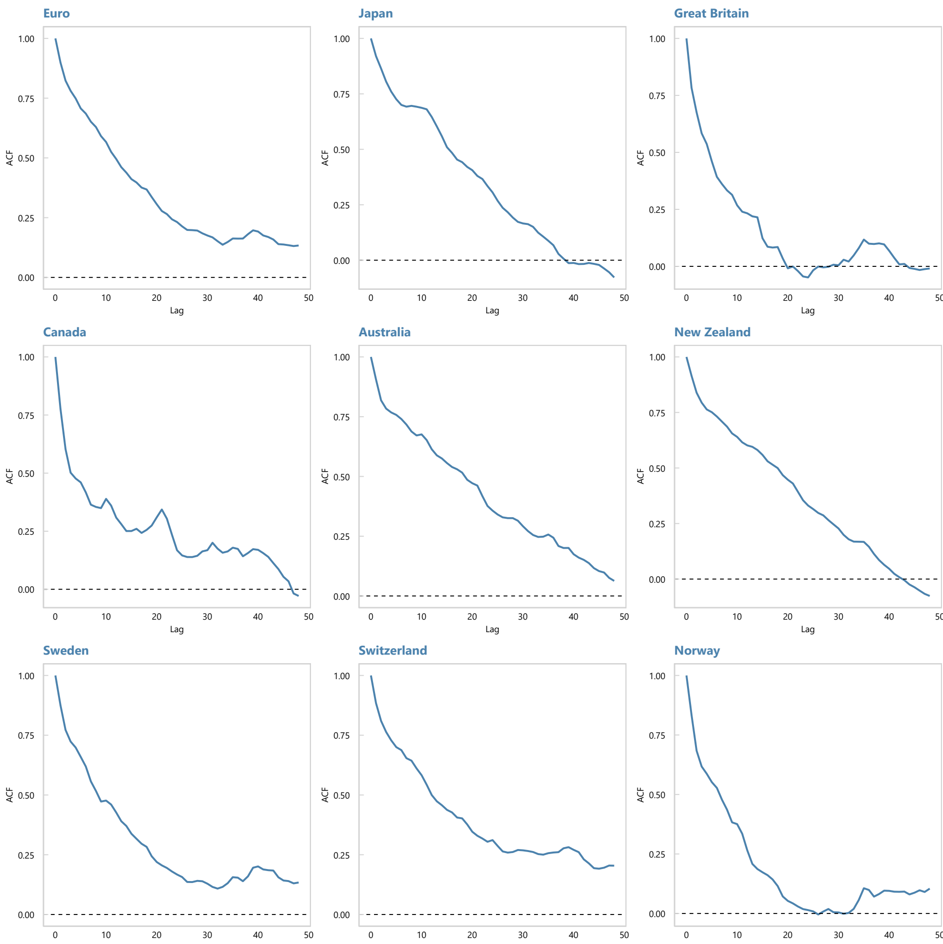
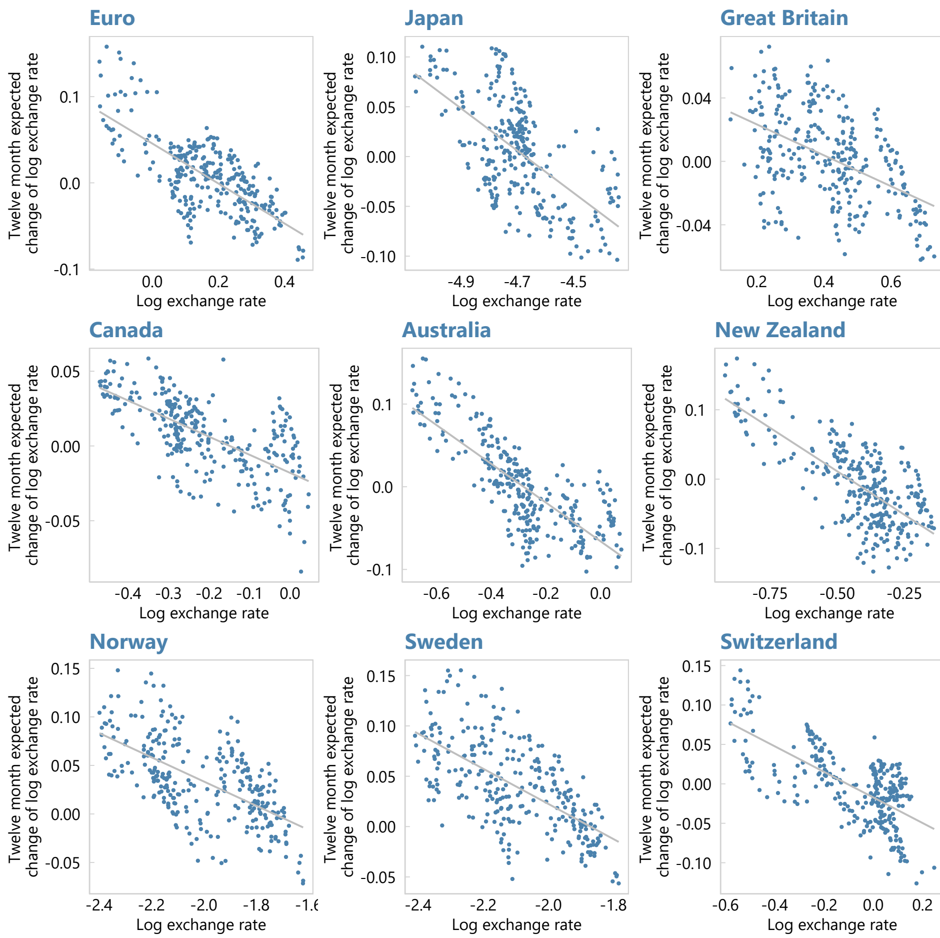


Figure 4.3. Log Exchange rate level and
Expected 12-month Change Log Exchange Rate, 2000-24
(Monthly data)



To illustrate, consider two scenarios in which an initially equilibrium exchange rate appreciates by 20%:

1. **Scenario 1: Predominance of the Cyclical Component (90%)** In this scenario, 90% of the 20% appreciation—equivalent to 18 percentage points—originates from the cyclical component (s_t^c), while the remaining 2 percentage points reflect a shift in the equilibrium level (μ_t). Since s_t^c is a stationary component that reverts to its mean over time, the expected future depreciation would be approximately 18%. This scenario demonstrates strong mean reversion, as the appreciation is predominantly attributable to the transitory cyclical component.

2. **Scenario 2: Predominance of the Stochastic Trend (90%)** Here, only 10% of the 20% appreciation—2 percentage points—can be ascribed to s_t^c, while the remaining 18 percentage points are driven by the stochastic trend (μ_t). Unlike the cyclical component, the stochastic trend evolves slowly and lacks mean-reverting properties, leading to a predicted future depreciation of merely 2%. This scenario reflects weaker mean reversion and reduced predictability.

These scenarios illustrate how the relative contributions of the stochastic trend and the cyclical component influence the degree of predictability in exchange rates. When the stochastic trend evolves slowly and the cyclical component plays a dominant role, exchange rates exhibit considerable medium-term predictability. This dual-component framework reconciles the unit root hypothesis with the observed persistence in expected exchange rate changes.

4.2 The Math

We will now show that a combination of a slowly evolving trend and a cyclical (stationary) component can account for these patterns.

Why Exchange Rates Should Have a Stationary Component

We assume that the exchange rate s_t at time t is equal to a stochastic trend plus a stationary component:

$$s_t = \mu_t + s_t^c \tag{4.2.1}$$

Here, μ_t represents the stochastic trend, and s_t^c is the stationary component. The stochastic trend μ_t evolves as a random walk:

$$\mu_t = \mu_{t-1} + \eta_t \tag{4.2.2}$$

where η_t is a white noise error term with zero mean and constant variance σ^2_η. This random walk component allows for persistent, unpredictable changes in the long-term level of the exchange rate.

Expectations of future exchange rates are formed based on both the stochastic trend and the stationary component:

$$E_t[s_{t+h}] = E_t[\mu_{t+h}] + E_t[s^c_{t+h}] \tag{4.2.3}$$

We assume that expectations of future values of the cyclical component are based on the current value of the cyclical component and the autocorrelation function (ACF) of the stationary process:

$$E_t[s^c_{t+h}] = \rho(h)s^c_t \tag{4.2.4}$$

Expected future values of the stochastic trend are equal to the current value:

$$E_t[\mu_{t+h}] = \mu_t \tag{4.2.5}$$

It follows that:

$$E_t[s_{t+h} - s_t] = -(1 - \rho(h))s^c_t \tag{4.2.6}$$

When exchange rates incorporate both a random walk (trend) and a cyclical component, the best forecast for future changes will be the expected change in the cyclical component, which itself depends on the current value of this cyclical component.

Thus, the expected exchange rate change will be zero only if the cyclical component is zero, meaning that the exchange rate behaves purely as a random walk. If the cyclical component is not zero, neither will the expected exchange rate change.

Why the Stochastic Trend Must Be Moving Slowly

Next, we explain why the stochastic trend must be moving slowly. This is so for two reasons: if it were not, expected exchange rate changes would not be highly persistent, and there would not be a strong link between exchange rate levels and expected exchange rate changes.

The persistence of expected exchange rate changes is influenced by the persistence of the cyclical component, which varies depending on whether exchange rate changes are driven primarily by the stochastic trend or by the cyclical component. If exchange rate changes are largely influenced by the cyclical component, the stochastic trend will evolve slowly, resulting in potentially large and persistent expected changes. If the stochastic trend drives most of the exchange rate changes, the cyclical component will remain small, leading to relatively small and less persistent expected changes.

We demonstrate this in Figure 4.4, where we decompose the New Zealand-US dollar exchange rate into the trend and the cyclical component using an HP-filter. For smaller values of λ, the exchange rate changes are mostly driven by the trend, and the movements of the cyclical component are less persistent. By contrast, if λ is large, the exchange rate changes are mostly driven by the cyclical component, and the movements of the cyclical component are more persistent.

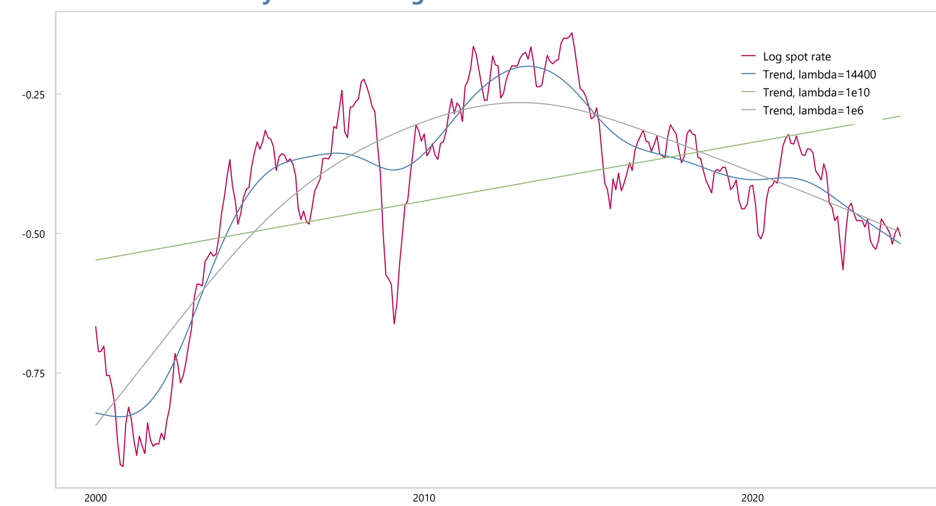
A slowly moving stochastic trend can also explain the link between the exchange rate level and the expected exchange rate change. The top panel of Figure 4.5 shows the simulation of a combination of a stochastic trend with a stationary AR(1) process with a coefficient of 0.97. We assume that the variance of the stochastic trend is much smaller than the variance of the cyclical component. In a small sample of 300 (equivalent to 25 years of monthly data), the link between the exchange rate level and the cyclical component is very strong. The link weakens in very large samples (bottom of Figure 4.5), but remains significant. By contrast, with a significantly higher variance of the stochastic trend, the link between exchange rate levels and expected exchange rate changes becomes weak even in small samples (Figure 4.6).

In sum, we can explain the three observations (a) exchange rates have a unit root; (b) expected exchange rate changes are highly persistent; (c) there is a strong link between exchange rate levels and expected exchange rate changes by assuming that exchange rates combine a slowly moving stochastic trend with a stationary component.

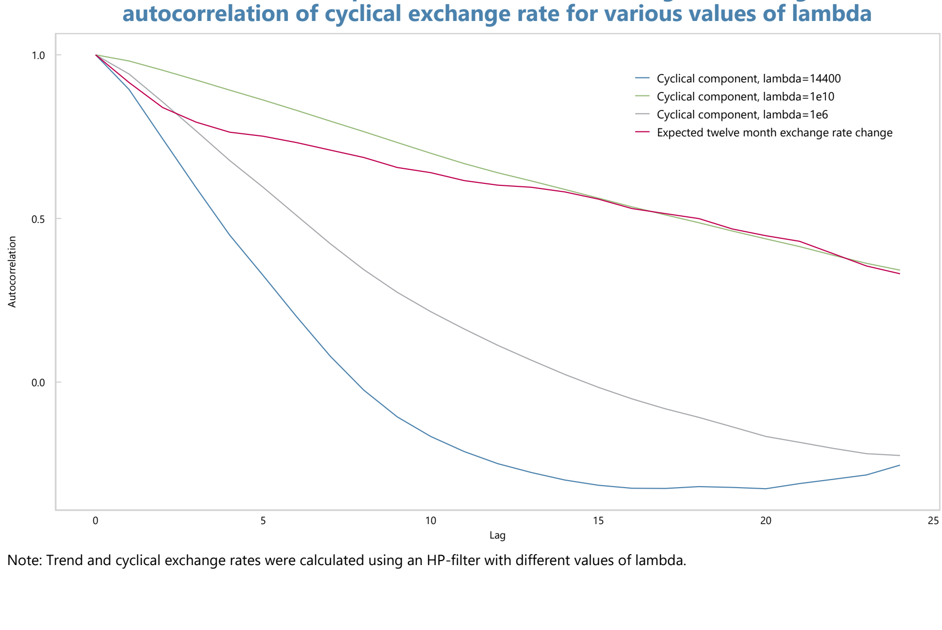
Figure 4.4. Persistence of Cyclical Part of New Zealand-US Dollar Exchange Rate and Persistence of One-Year Expected Exchange Rate Change, 2000-24

(Monthly data)

Non-cyclical exchange rate for various values of lambda



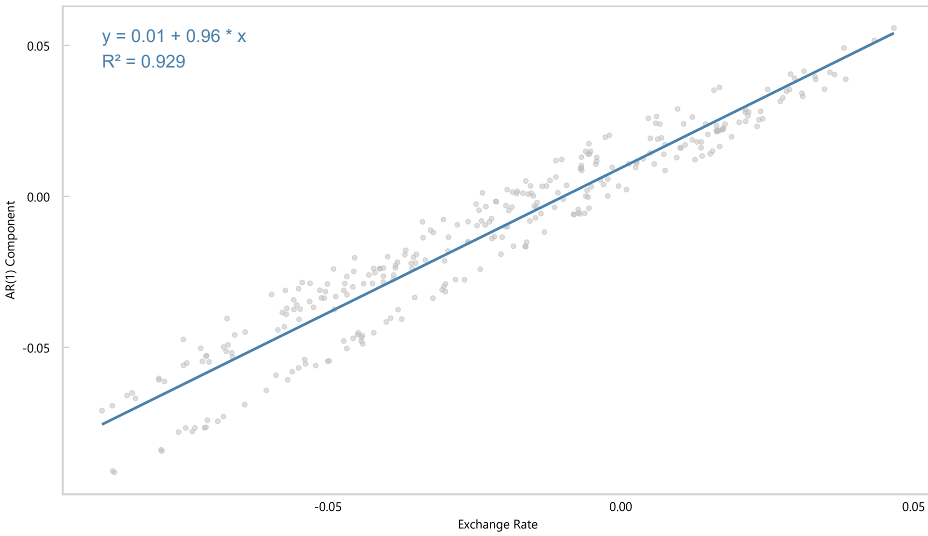
Autocorrelation of expected twelve month exchange rate change and autocorrelation of cyclical exchange rate for various values of lambda



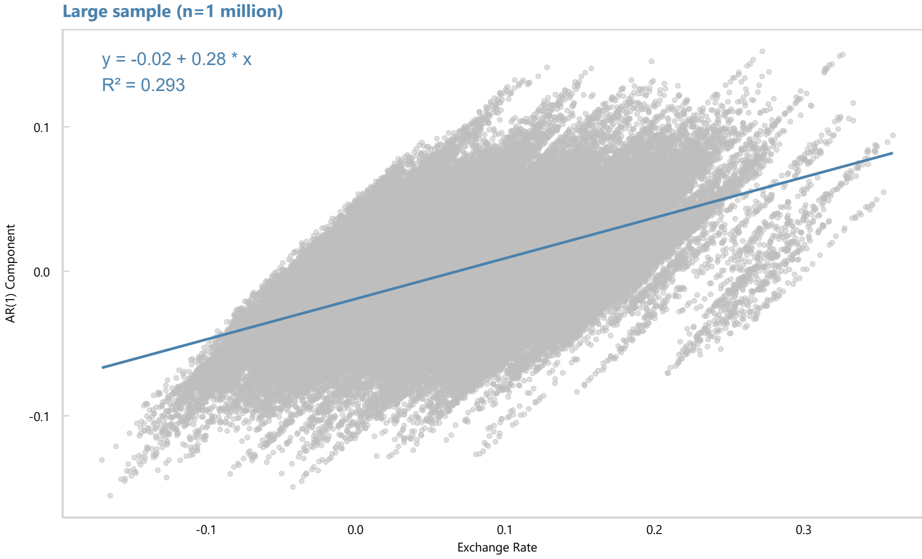
Note: Trend and cyclical exchange rates were calculated using an HP-filter with different values of lambda.

Figure 4.5. Link between Exchange Rate and Stationary Component for various sample sizes

Small sample (n=300)



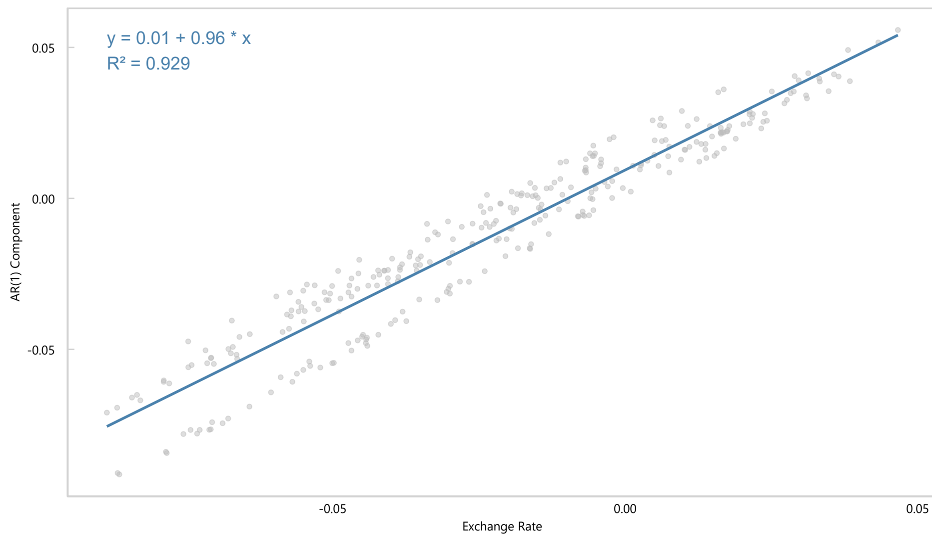
Large sample (n=1 million)



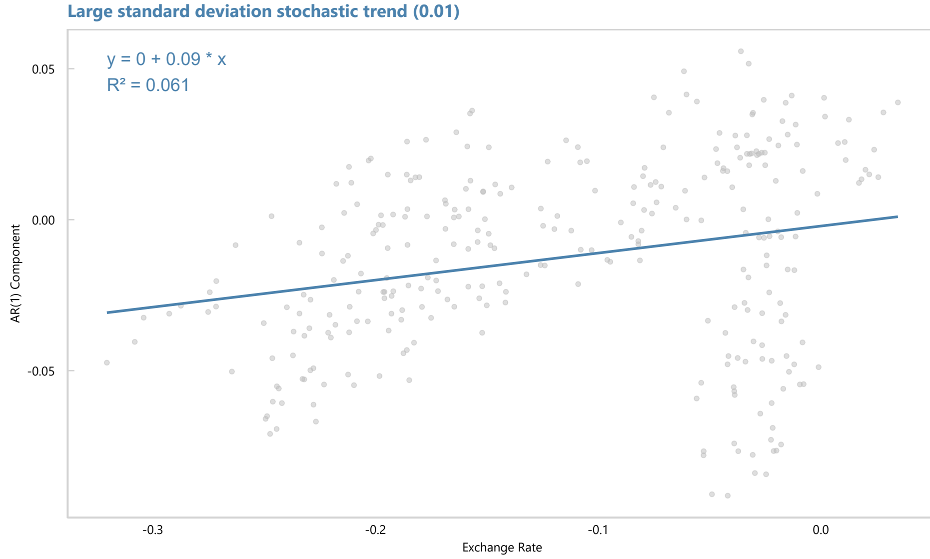
AR(1) coefficient = 0.97, standard deviation AR(1) process=0.01, standard deviation stochastic trend = 0.001.

Figure 4.6. Link between Exchange Rate and Stationary Component for various values of the standard deviation of the stochastic trend

Small standard deviation stochastic trend (0.001)



Large standard deviation stochastic trend (0.01)



AR(1) coefficient = 0.97, standard deviation AR(1) process=0.01.

4.3 Partial Reversion of Exchange Rate Shocks

Having established the mathematical foundations of the dual-component model, we now examine its implications for exchange rate shocks. The model suggests that such shocks are not entirely permanent but instead exhibit partial reversion, driven by the mean-reverting nature of the stationary component. Below, we derive and interpret these dynamics to illustrate their role in shaping medium-term predictability in exchange rates.

At time $t-1$, the expected exchange rate change between period t and $t+h$ is equal to:

$$E_{t-1}[s_{t+h} - s_t] = -(1-\rho(h))E_{t-1}[s_t^c]. \tag{4.3.1}$$

At time t, the expected exchange rate change between period t and $t+h$ is equal to:

$$E_t[s_{t+h} - s_t] = -(1-\rho(h))[s_t^c]. \tag{4.3.2}$$

It follows that

$$E_t[s_{t+h} - s_t] - E_{t-1}[s_{t+h} - s_t] = -(1-\rho(h))\left(s_t^c - E_{t-1}[s_t^c]\right) \tag{4.3.3}$$

For large h, $\rho(h)$ approaches 0, leading to:

$$\lim_{h\to\infty} E_t[s_{t+h} - s_t] - E_{t-1}[s_{t+h} - s_t] = -\left(s_t^c - E_{t-1}[s_t^c]\right). \tag{4.3.4}$$

This means that in the long run, the shock to the stationary component $(s_t^c - E_{t-1}[s_t^c])$ is expected to be fully reversed.

However, the total shock to the exchange rate also includes the shock to the stochastic trend:

$$s_t - E_{t-1}[s_t] = \left(s_c^t - E_{t-1}s_t^c\right) + \left(\mu_t - E_{t-1}\mu_t\right) \tag{4.3.5}$$

Therefore, if the exchange rate in period t is higher than expected, only the part of the shock attributable to the stationary component is expected to be reversed in the future.

The dual-component model bridges the gap between purely stationary and random walk models by incorporating both cyclical predictability and long-term stochastic behavior.

Part I

The Model

To illustrate, this paper extends the Bacchetta and van Wincoop (2021) framework (which is an example of how mean-reverting interest rate differentials and gradual portfolio adjustments can generate a stationary component) with a stochastic trend.

We first will discuss, in section 5, the stationary component. In section 6, we will combine this with a stochastic trend.

5 The Stationary Component

5.1 The model of the stationary component

Bacchetta and van Wincoop (2021) derive a model in which the exchange rate depends on the lagged exchange rate and the sum of current and expected future interest rate differentials:

$$s_t = \alpha s_{t-1} + E_t \sum_{i=0}^{\infty} \rho^i \text{dif}_{t+i} \tag{5.1.1}$$

where s_t is the log exchange rate (US dollars per foreign currency unit), and dif_t is the interest rate differential (foreign interest rate minus US interest rate). If portfolio adjustment is gradual, $0 < \alpha < 1$, otherwise $\alpha = 0$. If investors are risk-neutral, $\rho = 1$, otherwise $0 < \rho < 1$.[1]

In line with Bacchetta and van Wincoop (2021), we assume that the interest rate differential follows a stochastic AR(1) process:

$$\text{dif}_t = \beta \text{dif}_{t-1} + \epsilon_t \tag{5.1.2}$$

This implies:

$$E_t \text{dif}_{t+i} = \beta^i \text{dif}_t \tag{5.1.3}$$

Substituting (5.1.3) in (5.1.1) we get

$$s_t = \alpha s_{t-1} + \left(\frac{\rho}{1 - \rho\beta} \right) \text{dif}_t \tag{5.1.4}$$

The exchange rate depends on the lagged exchange rate and the interest rate differential only.

[1] If $\alpha = 0$ and $\rho = 1$, we get the UIP-model.

Substituting (5.1.2) into (5.1.4) and solving,[2] we obtain:

$$s_t = (\alpha + \beta)s_{t-1} - \alpha\beta s_{t-2} + \epsilon'_t \tag{5.1.9}$$

where $\epsilon'_t = \frac{\epsilon_t}{1-\beta}$. Equation (5.1.9) is an AR(2) process with roots α and β. As long as both $\alpha < 1$ and $\beta < 1$, this process is stationary, implying mean reversion in exchange rates.

5.2 Exchange Rate Levels and Expected Exchange Rate Changes

We assume that expectations of future exchange rate are formed using the Autocorrelation Function (ACF):

$$E_t s_{t+k} = \rho(k)s_t \tag{5.2.1}$$

where $\rho(k)$ is the ACF.

We show in Annex A that for our AR(2) process:

$$\rho(k) = \frac{\alpha^{1+k}\left(1-\beta^2\right) - \beta^{1+k}(1-\alpha^2)}{(\alpha-\beta)(1+\alpha\beta)} \tag{5.2.2}$$

We also show that $0 < \rho(k) < 1$ and $\rho(k+1) < \rho(k)$. Moreover $\lim_{k\to\infty} \rho(k) = 0$.

We can rewrite equation (5.2.1) as

$$E_t s_{t+k} - s_t = (\rho(k) - 1)s_t \tag{5.2.3}$$

Equation (5.2.3) has two important implications:

- Expected exchange rate changes depend on the exchange rate level. The more appreciated the exchange rate level, the larger the expected future depreciation.

[2] It follows from equation (5.1.2) that

$$(1-\beta L)\text{dif}_t = \epsilon_t \tag{5.1.5}$$

It follows that

$$\text{dif}_t = \frac{\epsilon_t}{1-\beta L} \tag{5.1.6}$$

Substituting (5.1.6) in (5.1.4) we get:

$$(1-\alpha L)(1-\beta L)s_t = \epsilon'_t \tag{5.1.7}$$

where $\epsilon'_t = \frac{\epsilon_t}{1-\beta}$. We can rewrite this as:

$$s_t = (\alpha+\beta)s_{t-1} - \alpha\beta s_{t-2} + \epsilon' t \tag{5.1.8}$$

- The longer the lag (i.e., the higher k), the higher the coefficient. As $k \to \infty$ the coefficient of a regression of expected exchange rate changes on exchange rate levels should go to -1 .

5.3 The Exchange Rate Level and Interest Rate Shocks

Note that we can rewrite equation (5.1.4) as:

$$s_t = \left(\frac{1}{(1-\alpha L)(1-\beta L)}\right)\frac{\epsilon_t}{1-\beta} \tag{5.3.1}$$

Using

$$\frac{1}{(1-\alpha L)(1-\beta L)} = \frac{\frac{\alpha}{\alpha-\beta}}{1-\alpha L} - \frac{\frac{\beta}{\alpha-\beta}}{1-\beta L} \tag{5.3.2}$$

we can rewrite (5.3.1) as:

$$s_t = \left(\frac{1}{\alpha-\beta}\right)\left(\frac{1}{1-\beta}\right)\left(\sum_{i=0}^{i=\infty}\kappa_i\epsilon_{t-i}\right) \tag{5.3.3}$$

where $\kappa_i = \alpha^{i+1} - \beta^{i+1}$. The exchange rate level is a weighted average of current and past interest rate shocks. An interest rate shock will lead to an appreciation of the exchange rate, but over time, the impact of the shock wanes, and the exchange rate returns to its equilibrium.

Our model's representation of exchange rates as a weighted average of interest rate shocks aligns with Kekre and Lenel (2024), who emphasize the importance of persistent interest rate differentials in driving exchange rate dynamics. Their work provides empirical support for this relationship, reinforcing the importance of the dynamic interest rate-exchange rate relationship in understanding currency movements.

5.4 Economic Drivers of Interest Rate Differential Mean Reversion

The mean reversion of interest rate differentials is itself grounded in fundamental economic processes. One key driver is the behavior of output gaps, which plays a significant role in shaping interest rate dynamics across countries.

Interest rates are often closely tied to a country's output gap—the difference between actual and potential GDP. Central banks typically employ a counter-cyclical monetary policy, raising rates when the economy is overheating (positive output gap) and lowering them during economic slowdowns (negative output gap). This responsive policy approach creates a direct link between output gaps and interest rates.

Economic theory and empirical evidence suggest that output gaps tend to close over time as economies naturally adjust. This convergence towards potential output exerts a mean-reverting force on interest rates. As an economy operating above potential gradually cools down, interest rates are likely to decrease. Conversely, an economy operating below potential is expected to recover, leading to rising interest rates over time.

The convergence of output gaps becomes particularly relevant for exchange rates when considering interest rate differentials between two countries. When two economies have opposing output gaps—for example, country A with a positive gap and country B with a negative gap—their interest rate differential is likely to narrow as these gaps close. Country A's rates would be expected to decrease as its economy cools, while country B's rates would rise as its economy recovers. This differential convergence in interest rates drives mean reversion in exchange rates.

6 Combining the Stationary Component and a Stochastic Trend

Building on the discussion in Section 4, the exchange rate $s(t)$ is modeled as the sum of two components: a stochastic trend μ_t and a stationary component $s_c(t)$:

$$s_t = \mu_t + s_t^c. \tag{6.0.1}$$

The stochastic trend evolves as a random walk:

$$\mu_t = \mu_{t-1} + \eta_t, \tag{6.0.2}$$

where η_t is a white noise error term. This trend accounts for the long-term, persistent movements in exchange rates, reflecting their random walk nature.

The stationary component s_t^c, previously detailed in section 5, follows an AR(2) process:

$$s_t^c = (\alpha + \beta)s_{t-1}^c - \alpha\beta s_{t-2}^c + \varepsilon_t, \tag{6.0.3}$$

where α and β determine the degree of mean reversion. This component introduces cyclical dynamics, with exchange rates reverting toward equilibrium over time.

This hybrid structure allows exchange rates to simultaneously exhibit random walk behavior in the long run while showing mean-reverting tendencies in the medium term, reconciling the two contrasting dynamics.

Part II

Exchange Rate Changes

7 Predictability of Exchange Rate Changes and the Relationship between Short-term and Long-term Expected Changes

To test the validity of the dual-component framework, we conduct an empirical investigation using exchange rate data from nine inflation-targeting countries spanning 2000–2024. This analysis evaluates key theoretical predictions, including medium-term predictability, the relationship between short-term and long-term expected changes, and the model's outperformance over the random walk benchmark. By comparing actual exchange rate changes to their expected values across different horizons, we provide robust evidence supporting the framework's applicability.

This section investigates the predictability of exchange rate changes across different time horizons and explores the link between short-term and long-term expected changes. Contrary to the traditional view that exchange rates are largely unpredictable, especially over extended periods, our analysis presents both theoretical and empirical evidence highlighting two main findings:

- Long-term exchange rate changes are more predictable than short-term changes.
- Expected long-term changes are proportional to short-term expected changes, with the proportionality factor increasing with the time horizon.

7.1 Predictability of Exchange Rate Changes

Predictability of the Stationary Component

The expected change in the stationary component of the exchange rate is characterized by:

$$E_t\left[s^c_{t+h} - s^c_t\right] = (\rho(h) - 1)s^c_t, \tag{7.1.1}$$

where $\rho(h)$ is the autocorrelation function at horizon h.

As detailed in Annex C, if we regress actual changes in the stationary component on their expected values

$$s^c_{t+h} - s^c_t = a_h + b_h\left[(\rho(h) - 1)s^c_t\right] + \varepsilon_t. \tag{7.1.2}$$

the expected value of the coefficient of determination (R^2) is equal to:

$$R^2(h) = \frac{(1 - \rho(h))}{2} \tag{7.1.3}$$

For small h, R^2 is close to zero (as $\rho(h)$ is close to 1). R^2 increases asymptotically to 0.5 as h becomes larger.

These results suggest that, while short-term changes in the stationary component are challenging to predict, predictability improves significantly for longer-term changes.

Predictability of the Exchange Rate

The overall predictability of the exchange rate reflects the combined predictability of its stationary and stochastic trend components. While the stationary component becomes more predictable over longer horizons, the stochastic trend becomes less predictable. Consequently, the predictability of the exchange rate as a whole initially increases, reaches a peak, and then declines.

7.2 The Relationship Between Short-term and Long-term Expected Changes

The expected h-period exchange rate change is determined by:

$$E_t\left[s_{t+h} - s_t\right] = E_t\left[s^c_{t+h} - s^c_t\right]. \tag{7.2.1}$$

For a 12-month horizon ($h = 12$), this relationship becomes:

$$E_t\left[s_{t+12} - s_t\right] = E_t\left[s^c_{t+12} - s^c_t\right]. \tag{7.2.2}$$

Combining these expressions, we derive:

$$E_t\left[s_{t+h} - s_t\right] = \left(\frac{1 - \rho(h)}{1 - \rho(12)}\right) E_t\left[s_{t+12} - s_t\right]. \tag{7.2.3}$$

This equation implies that expected multi-year exchange rate changes are proportional to the expected one-year change, with the proportionality factor increasing with h.

7.3 Empirical Evidence

To test these theoretical predictions, we estimate the following regression:

$$s_{t+h} - s_t = a_h + b_h \left[E_t \left(s_{t+12} - s_t \right) \right] + \varepsilon_t. \tag{7.3.1}$$

This regression links h-period changes in the exchange rate to the expected one-year change lagged by h periods. The theoretical model yields two key expectations:

- b_h should increase with h and exceed 1 for $h > 12$.
- The behavior of R^2 depends on the relative contributions of the stationary and stochastic trend components:
  - For the stationary component, predictability increases with h.
  - For the stochastic trend, predictability decreases with h.
  - For the combined process, predictability initially rises, peaks, and subsequently declines.

We perform this analysis using monthly exchange rate data for 10 advanced economies relative to the U.S. dollar from 2000 to 2024. Regression results are presented in Tables 7.1 and 7.2:

Table 7.1: Coefficients and Standard errors of regression of x-year exchange rate change on x-year lagged one-year expected change, 2000-2024

	Coefficients					Standard errors					
	1 year	2 years	3 years	4 years	5 years	1 year	2 years	3 years	4 years	5 years	Observations
CAN	0.95	1.26	1.82	2.50	3.03	0.18	0.27	0.33	0.39	0.45	237
JPN	-0.23	0.10	0.84	1.92	2.77	0.12	0.20	0.25	0.24	0.23	237
GBR	0.84	1.94	2.28	2.38	2.76	0.18	0.22	0.25	0.30	0.32	237
SWE	0.69	1.74	2.34	2.65	2.70	0.15	0.18	0.17	0.19	0.23	237
EUR	0.59	1.44	2.08	2.42	2.58	0.12	0.15	0.15	0.17	0.19	237
AUS	0.42	1.15	1.78	2.30	2.53	0.13	0.17	0.19	0.20	0.23	237
CHE	0.46	0.94	1.42	1.87	2.10	0.09	0.11	0.11	0.11	0.12	237
NOR	0.62	1.21	1.64	1.93	1.99	0.16	0.22	0.26	0.31	0.37	237
NZL	0.41	1.10	1.62	2.02	1.97	0.12	0.15	0.16	0.17	0.19	237

[a] Monthly data.

Table 7.2: R2 of regressions of x-year exchange rate change on x-year lagged one-year expected exchange rate change, 2000-2024

	1 year	2 years	3 years	4 years	5 years	Observations
CHE	0.08	0.22	0.41	0.53	0.58	237
EUR	0.07	0.26	0.44	0.46	0.43	237
JPN	0.01	-0.00	0.04	0.20	0.38	237
SWE	0.07	0.25	0.43	0.44	0.37	237
AUS	0.03	0.14	0.26	0.34	0.33	237
NZL	0.04	0.17	0.29	0.37	0.31	237
GBR	0.07	0.22	0.24	0.20	0.23	237
CAN	0.08	0.07	0.10	0.14	0.16	237
NOR	0.05	0.10	0.13	0.13	0.11	237

[a] Monthly data.

Our findings are consistent with the theoretical predictions:

- The expected twelve-month exchange rate change exhibits substantial predictive power for subsequent multi-year changes, with predictive power increasing at longer horizons.

- The relationship between the twelve-month expected change and the actual five-year change is particularly pronounced for Switzerland, Japan, the Euro area, and Sweden.

- Multi-year changes are proportional to the twelve-month expected change, as predicted by the model.

While these results are robust, a discrepancy arises in the coefficient linking actual twelve-month changes to expected changes. This issue warrants further exploration and will be addressed in the following section.

8 Assessing Bias in Exchange Rate Change Forecasts

In our regressions of actual twelve-month exchange rate changes on the lagged twelve-month predicted changes, we observe coefficients that are significantly lower than the expected value of 1. This suggests a potential bias in the forecasts, as we would theoretically expect a one-to-one correspondence between predicted and actual changes. In this section, we explore whether this discrepancy is indicative of a systematic forecast bias or if alternative explanations account for this result.

8.1 Potential Sources of Apparent Bias: Interest Rate Differential Projections

One possible source of bias stems from the behavior of interest rate differentials. Gourinchas and Tornell (2004) showed that in advanced economies, interest rate differentials tend to converge more slowly than investors anticipate, offering an explanation for several exchange rate puzzles. If the convergence of interest rate differentials is slower than expected, this could, in theory, lead to a discrepancy between expected and actual exchange rate changes.

Slower-than-expected convergence of interest rate differentials would imply slower adjustment in exchange rate levels, potentially explaining why the observed twelve-month exchange rate change is often smaller than predicted. This is because the path of the exchange rate is closely linked to interest rate differentials, and delays in the adjustment of interest rates would naturally cause delays in exchange rate adjustments.

However, when examining the specific period from 2000 to 2024, this theoretical explanation does not hold.[3]

Contrary to the slower-than-expected convergence typically observed, we find that interest rate differentials converged *faster than expected* during this period (Table 8.1). As a result, the bias in the forecasted exchange rate changes cannot be attributed to slow-moving interest rate adjustments. Instead, other factors must be considered to explain the lower-than-expected coefficients in our regression analysis.

[3] We derive expected changes of one-year interest rate differentials from current one year differentials and current multi-year interest rate differentials, assuming that the expectations hypothesis of interest rate differentials holds. The data are from Haver, from the INTDAILY database. We leave out Australia and New Zealand for lack of availability of multi-year interest rate differentials.

Table 8.1: Coefficients and Standard errors of regression of x-year change in interest rate differential on x-year lagged expected x-year change

	Coefficients				Standard errors			
	1 year	2 years	3 years	4 years	1 year	2 years	3 years	4 years
EUR	1.66	1.65	1.54	1.43	0.12	0.10	0.08	0.07
JPN	1.04	1.19	1.08	0.98	0.12	0.13	0.11	0.09
GBR	1.63	1.77	1.21	1.20	0.11	0.09	0.10	0.08
CAN	1.36	1.37	1.49	1.22	0.12	0.11	0.11	0.10
SWE	2.10	2.06	1.68	1.51	0.13	0.13	0.11	0.09
CHE	1.13	1.63	1.43	1.11	0.12	0.12	0.11	0.10
NOR	1.73	1.49	1.24	1.17	0.12	0.11	0.11	0.08

[a] Monthly data.

Table 8.2: R2 of regression of x-year change in interest rate differential on x-year lagged expected x-year change, 2004-24

	R2			
	1 year	2 years	3 years	4 years
EUR	0.43	0.54	0.59	0.63
JPN	0.23	0.27	0.29	0.37
GBR	0.46	0.64	0.40	0.56
CAN	0.33	0.37	0.46	0.38
SWE	0.50	0.54	0.54	0.57
CHE	0.26	0.47	0.44	0.39
NOR	0.44	0.43	0.39	0.56

[a] Monthly data.

8.2 Potential Sources of Apparent Bias: Timing of exchange rate projections

The observed low coefficients do not necessarily imply biased forecasts. A plausible explanation lies in the timing of exchange rate projections. For instance, projections made in March 2023 for end-March 2024 are typically produced at the beginning of March, not at month-end (they are published in the middle of the month, and presumably made some time before that). This timing difference suggests that the expected exchange rate should be characterized by $E_{t-1}s_{t+12}$ rather than $E_t s_{t+12}$, potentially introducing a bias in the regression coefficients.

Analytical Framework

To account for this timing discrepancy, we consider regressing $x_{t+h} - x_t$ on $(E_{t-1}x_{t+h} - x_t)$, where $E_{t-1}x_{t+h} = \rho(h+1)x_{t-1}$, instead of regressing on $(E_t x_{t+h} - x_t)$.

As demonstrated in Annex F, for a stationary ARMA(p,q) process with autocorrelation function $\rho(h)$, the regression coefficient for $s_{t+h} - s_t$ on $\rho(h+1)s_{t-1} - s_t$ is given by:

$$\beta(h) = \frac{\rho(h+1)\rho(h+1) - \rho(h) - \rho(h+1)\rho(1) + 1}{\rho(h+1)^2 + 1 - 2\rho(h+1)\rho(1)} \tag{8.2.1}$$

Illustrative Example

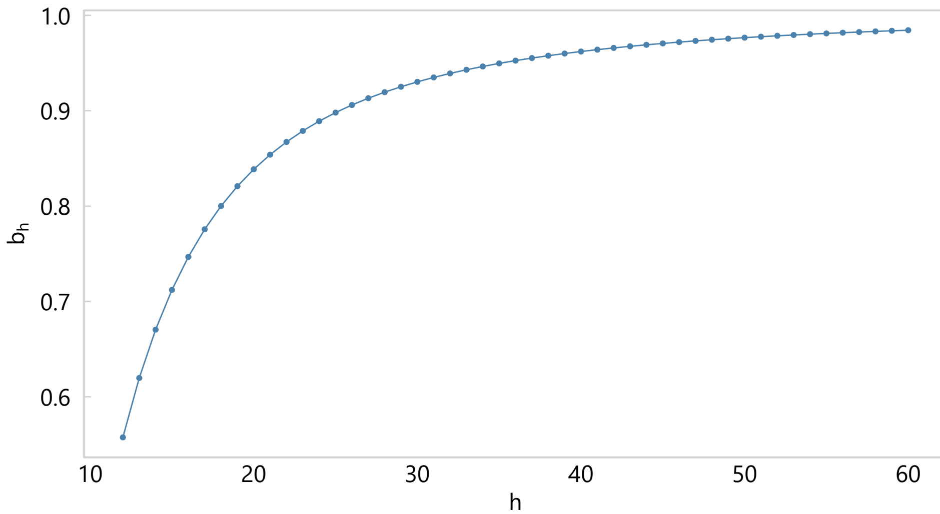
Consider an AR(2) process with roots 0.98 and 0.65:

- Regressing $s_{t+12} - s_t$ on $\rho(12)s_t - s_t$ yields an expected coefficient of 1.
- Regressing $s_{t+12} - s_t$ on $\rho(13)s_{t-1} - s_t$ yields an expected coefficient of $\beta(12) = 0.56$.

Importantly, this bias diminishes as h increases. Figure 8.1 illustrates how $\beta(h)$ converges to 1 as h increases for this AR(2) process.

Figure 8.1. b_h for regressing $s_{t+h} - s_t$ on $\rho(h+1)s_{t-1} - s_t$

($\alpha = 0.65$, $\beta = 0.98$)



9 Out-of-Sample Forecasts of Multi-Year Exchange Rate Changes

9.1 Methodology

To evaluate the predictive power of our model against a random walk benchmark, we employ out-of-sample forecasts and the Mean Squared Error (MSE) metric. We define the MSE and the MSE ratio for exchange rate changes as follows:

$$\text{MSE}_{\text{Model}} = \frac{1}{N}\sum_{i=1}^{N}(\Delta s_{t+h,i} - \Delta \hat{s}^{\text{Model}}_{t+h,i})^2 \tag{9.1.1}$$

$$\text{MSE}_{\text{RW}} = \frac{1}{N}\sum_{i=1}^{N}(\Delta s_{t+h,i})^2 \tag{9.1.2}$$

$$\text{MSE Ratio} = \frac{\text{MSE}_{\text{Model}}}{\text{MSE}_{\text{RW}}} \tag{9.1.3}$$

where:

- $\Delta s_{t+h,i} = s_{t+h,i} - s_{t,i}$ is the actual change in exchange rate from time t to $t+h$ for observation i
- $\Delta \hat{s}^{\text{Model}}_{t+h,i}$ is the predicted change from our model
- N is the number of forecasts

Note that for the random walk model, the predicted change is always zero, simplifying its MSE to the average squared actual change.

9.2 Model Specification

Our model posits that expected multi-year exchange rate changes are a multiple of the expected 12-month change:

$$E_t[s_{t+h} - s_t] = b_h[E_t s_{t+12} - s_t] \tag{9.2.1}$$

Instead of estimating the coefficients, we set them a priori as shown in Table 9.1. They are based on the insights developed earlier in the paper that expected multi-year changes should be a multiple of the expected one-year change. We will show later that our results are not very sensitive the precise value of the coefficients.

Table 9.1: A priori coefficients for different forecast horizons

Horizon (months)	Coefficient (b_h)
12	1.00
24	1.50
36	2.00
48	2.25
60	2.50

9.3 Results and Discussion

Table 9.2 shows the MSE ratios for various currencies and forecast horizons. It demonstrates that the predictive power of our model increases over time

Table 9.2: Ratio of mean squared error to that of random walk, 2005-24

	1 year	2 years	3 years	4 years	5 years
JPN	1.28	1.13	0.98	0.71	0.51
EUR	0.88	0.70	0.67	0.61	0.52
AUS	0.97	0.91	0.86	0.73	0.69
GBR	0.91	0.76	0.79	0.83	0.74
SWE	1.01	0.92	0.91	0.82	0.78
CHE	1.06	1.16	1.21	0.93	0.79
CAN	0.89	0.95	0.93	0.87	0.83
NZL	1.07	1.09	1.17	0.92	0.87
NOR	1.01	1.06	1.16	1.05	0.95

[a] Monthly data. Data start in January 2005.

Our out-of-sample forecast results reveal that at longer time horizons, the MSE ratio is consistently below one across multiple currencies, indicating superior predictive power compared to the random walk model. For instance, for the Japanese Yen at the five-year horizon, we observe an MSE Ratio of 0.51. This implies that our model's forecast errors are, on average, 51% of the random walk model's errors, representing a substantial 49% improvement in predictive accuracy. For the euro and the Danish krone, the predictive power is similar.

This substantial out-performance of the random walk benchmark challenges the widely held belief in the unpredictability of exchange rates, especially at longer horizons. It provides empirical support for our theoretical framework, which suggests increased predictability of exchange rate changes over extended periods. The improvement in predictive accuracy grows with the forecast horizon, aligning with our model's core proposition. The consistency of results across multiple major currencies strengthens the robustness of our findings.

Sensitivity analysis

Interestingly, the outperformance by our model of the random walk is not very sensitive to the precise coefficients used. Table 9.3 shows the ratio at the five-year horizon for various values of b_{60}. For all values between 1 and 3, our model outperforms the random walk (albeit marginally so for some values for some currencies).[4]

The robustness of our results to the precise value of the coefficient is not surprising when we consider the mechanics of the forecasts. For example, suppose the exchange rate is expected to appreciate by 5 percent over the next year. If the correct value of b_{60} is 2.5, this implies an expected appreciation of 12.5 percent over five years. If we instead use $b_{60} = 2$, we would forecast a five-year appreciation of 10 percent, while using $b_{60} = 3$ would yield a forecast of 15 percent. While these forecasts deviate from the "correct" one, they both predict substantial appreciation, in stark contrast to the random walk forecast of no change. Thus, even with some imprecision in the coefficient, our model captures the direction and approximate magnitude of the expected change, leading to superior performance compared to the random walk model.

[4] We did not pick any coefficients smaller than 1 as our theory suggests the coefficient should be larger than 1. We set 4 as the top of our range simply because the coefficient cannot be too large.

Table 9.3: Ratio of mean squared error to that of random walk for five years exchange rate change forecasts for various levels of b60, 2005-24

	Coefficient b60						
	1	1.5	2	2.5	3	3.5	4
EUR	0.72	0.62	0.56	0.52	0.52	0.55	0.61
JPN	0.72	0.62	0.55	0.51	0.50	0.52	0.56
GBR	0.86	0.81	0.77	0.74	0.72	0.71	0.72
CAN	0.91	0.87	0.84	0.83	0.82	0.81	0.82
AUS	0.80	0.73	0.70	0.69	0.70	0.74	0.81
NZL	0.83	0.80	0.82	0.87	0.97	1.10	1.28
CHE	0.80	0.76	0.75	0.79	0.87	0.98	1.13
SWE	0.80	0.75	0.75	0.78	0.85	0.96	1.10
NOR	0.93	0.92	0.92	0.95	0.99	1.04	1.12

[a] Monthly data. .

10 Conclusion

This paper introduces a hybrid model of exchange rate dynamics that reconciles the seemingly contradictory behaviors of random walks and mean reversion. By integrating a stochastic trend with a stationary component, the model captures key features of exchange rate behavior, offering a unified explanation for both long-term unpredictability and medium-term predictability.

Our findings reveal that exchange rates can possess a unit root while maintaining significant medium-term forecastability. This dual behavior stems from the interaction between a slowly evolving stochastic trend and a mean-reverting stationary component. The stochastic trend governs long-term movements, reflecting persistent shocks and structural changes, while the stationary component introduces cyclical dynamics, creating predictability in medium-term horizons. Notably, the model predicts an inverted U-shaped pattern of predictability, where forecast accuracy peaks at intermediate horizons, a result consistent with empirical observations.

This dual-component framework captures three key features of exchange rate dynamics: expected exchange rate changes are not zero, they are highly persistent, and there is a strong relationship between exchange rate levels and expected future changes. Without a stationary component, expected exchange rate changes would by definition be zero. Furthermore, if the stochastic trend did not evolve slowly, the relationship between exchange rate levels and expected changes would break down, and the cyclical component—along with the persistence of expected exchange rate changes—would diminish. These patterns underscore the necessity of incorporating both components to fully capture exchange rate dynamics.

We implement these insights by extending the Bacchetta and van Wincoop (2021) framework (which generates a stationary component of the exchange rate) with a stochastic trend. Our model generates an inverted U-shaped pattern where forecast accuracy peaks at intermediate horizons and predicts that multi-year exchange rate changes are increasing multiples of one-year changes.

Empirical tests using data from nine inflation-targeting countries with freely floating exchange rates confirm the model's predictions. Furthermore, the model's outperformance of the random walk benchmark in out-of-sample forecasts—particularly over multi-year horizons—challenges the conventional wisdom of inherent exchange rate unpredictability.

By combining the strengths of random walk and mean-reversion models, this framework offers a more nuanced understanding of exchange rate dynamics. It highlights the importance of medium-term horizons, where exchange rates are most predictable, and provides actionable insights for policymakers and investors. Future research could extend this framework to economies with high inflation or unconventional monetary regimes, further enhancing its applicability.

In sum, the hybrid model bridges long-standing debates in the literature, demonstrating that exchange rates—while inherently stochastic in the long run—can exhibit substantial predictability in the medium term. This dual-component approach provides a robust foundation for both theoretical and empirical advancements in the study of exchange rate behavior.

Future Research Directions

The hybrid framework developed in this paper, which combines a stochastic trend with a stationary component, is likely applicable to other economic variables characterized by persistent trends and cyclical fluctuations. A particularly compelling example is the unemployment rate, which exhibits a long-term trend shaped by structural factors such as demographic changes, technological advancements, and labor market policies, alongside short-term cyclical dynamics driven by business cycles.

Future research could extend this framework to empirically distinguish trend and cyclical components in real time, accommodate regime changes and structural breaks, and develop multivariate models to capture interactions across different variables and markets. By exploring these directions, the hybrid model could serve as a unifying framework for understanding the dynamic interplay of trends and cycles across economics.

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Part III

Annexes

A The Exchange Rate Level and Expected Exchange Rate Changes

Consider the regression of the exchange rate s_t on the k-period lagged exchange rate, s_{t-k}:

$$s_t = a + b_k s_{t-k} + \xi_t \tag{A.0.1}$$

We know that:

$$b_k = \frac{\text{cov}(s_t, s_{t-k})}{\text{var}(s_t)} \tag{A.0.2}$$

Autocovariance

The autocovariance of s_t is:

$$\gamma(k) = \text{cov}(s_t, s_{t-k}) \tag{A.0.3}$$

We can rewrite Equation (5.1.4) as:

$$\text{cov}(s_t, s_{t-k}) = (\alpha+\beta)\text{cov}(s_{t-1}, s_{t-k}) - \alpha\beta\text{cov}(s_{t-2}, s_{t-k}) + \text{cov}(\epsilon'_t, s_{t-k}) \tag{A.0.4}$$

where $\epsilon'_t = \left(\frac{\epsilon_t}{1-\beta}\right)$ This can be rewritten as:

$$\gamma(k) = (\alpha+\beta)\gamma(k-1) - \alpha\beta\gamma(k-2) \tag{A.0.5}$$

This is a second-order homogeneous difference equation. The solution is:

$$\gamma(k) = A\alpha^k + B\beta^k \tag{A.0.6}$$

Deriviation of A and B

We have the following AR(2) process:

$$s_t = (\alpha+\beta)s_{t-1} - \alpha\beta s_{t-2} + \epsilon'_t \tag{A.0.7}$$

This implies

$$\gamma(0) = (\alpha+\beta)\gamma(1) - \alpha\beta\gamma(2) + \sigma_\epsilon^2 \tag{A.0.8}$$

$$\gamma(1) = (\alpha+\beta)\gamma(0) - \alpha\beta\gamma(1) \tag{A.0.9}$$

$$\gamma(2) = (\alpha+\beta)\gamma(1) - \alpha\beta\gamma(0) \tag{A.0.10}$$

Solving the system of equations we get:

$$\gamma(0) = \left(\frac{1+\alpha\beta}{1-\alpha\beta}\right)\frac{\sigma_e^2}{(1+\alpha\beta)^2 - (\alpha+\beta)^2} = \left(\frac{1+\alpha\beta}{1-\alpha\beta}\right)\frac{\sigma_{\epsilon'}^2}{(1-\alpha^2)(1-\beta^2)} \tag{A.0.11}$$

$$\gamma(1) = \left(\frac{\alpha+\beta}{1+\alpha\beta}\right)\left(\frac{1+\alpha\beta}{1-\alpha\beta}\right)\frac{\sigma_{\epsilon'}^2}{(1-\alpha^2)(1-\beta^2)} \tag{A.0.12}$$

Recall from equation (A.0.5) that

$$\gamma(k) = (\alpha+\beta)\gamma(k-1) - (\alpha\beta)\gamma(k-2) \tag{A.0.13}$$

It follows that:

$$\gamma(0) = A + B \tag{A.0.14}$$

$$\gamma(1) = A\alpha + B\beta \tag{A.0.15}$$

It follows that:

$$A = \frac{\alpha\sigma_{\epsilon'}^2}{(1-\alpha^2)\,(\alpha-\beta)(1-\alpha\beta)} \tag{A.0.16}$$

$$B = -\frac{\beta\sigma_{\epsilon'}^2}{(\alpha-\beta)(1-\alpha\beta)\,(1-\beta^2)} \tag{A.0.17}$$

Using A and B

Note that

$$b_k = \frac{\gamma(k)}{\gamma(0)} \tag{A.0.18}$$

It follows that:

$$b_k = \frac{\alpha^{1+k}\left(1-\beta^2\right) - \beta^{1+k}(1-\alpha^2)}{(\alpha-\beta)(1+\alpha\beta)} \tag{A.0.19}$$

Note that $0 < b_k < 1$ and $b_{k+1} < b_k$. Moreover $\lim_{k\to\infty} b_k = 0$.[5]

[5]First note that

$$\frac{\alpha+\beta}{1+\alpha\beta} < 1$$

Equation (A.0.19) implies:

$$E_{t-k}s_t - s_{t-k} = a + \left(\frac{\alpha^{1+k}\left(1-\beta^2\right) - \beta^{1+k}(1-\alpha^2)}{(\alpha-\beta)(1+\alpha\beta)} - 1 \right) s_{t-k} \quad \text{(A.0.20)}$$

Equation (A.0.20) has two important implications:

- Expected exchange rate changes depend on exchange rate levels. The more appreciated the exchange rate level, the larger the expected future depreciation.
- The longer the lag (i.e., the higher k), the higher the coefficient. As $\lim_{k\to\infty} b_k = 0$, the coefficient of a regression of exchange rate changes on past exchange rate levels should go to -1 as k increases.

This follows from

$$1+\alpha\beta > \alpha+\beta \to 1+\alpha\beta-\alpha-\beta > 0 \to (1-\alpha)(1-\beta) > 0$$

Assume that $\alpha > \beta$. To see that $b_k > 0$

$$b_k = \frac{\alpha^{1+k}\left(1-\beta^2\right) - \beta^{1+k}(1-\alpha^2)}{(\alpha-\beta)(1+\alpha\beta)} > \frac{\beta^{1+k}\left(\alpha^2-\beta^2\right)}{(\alpha-\beta)(1+\alpha\beta)} = \frac{\beta^{1+k}\left(\alpha+\beta\right)}{1+\alpha\beta} > 0$$

To see that $b_k < 1$

$$b_k = \frac{\alpha^{1+k}\left(1-\beta^2\right) - \beta^{1+k}(1-\alpha^2)}{(\alpha-\beta)(1+\alpha\beta)} < \frac{\alpha^{1+k}\left(\alpha^2-\beta^2\right)}{(\alpha-\beta)(1+\alpha\beta)} = \frac{\alpha^{1+k}\left(\alpha+\beta\right)}{1+\alpha\beta} < 1$$

Finally, note that

$$b_k = \frac{\alpha^{1+k}\left(1-\beta^2\right) - \beta^{1+k}(1-\alpha^2)}{(\alpha-\beta)(1+\alpha\beta)} < \frac{\beta\alpha^{k}\left(1-\beta^2\right) - \beta\beta^{k}(1-\alpha^2)}{(\alpha-\beta)(1+\alpha\beta)} = \beta\left(\frac{\alpha^{k}\left(1-\beta^2\right) - \beta^{k}(1-\alpha^2)}{(\alpha-\beta)(1+\alpha\beta)}\right) = \beta b_{k-1} < b_{k-1}$$

The proof is almost identical when $\alpha < \beta$.

B Regressing the Change in the Stationary Part on the Total Change

The exchange rate S_t is composed of a stationary ARMA(p, q) component X_t and a stochastic trend Z_t. Hence,

$$S_t = X_t + Z_t \tag{B.0.1}$$

where X_t is a stationary ARMA(p, q) process:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q} \tag{B.0.2}$$

with ϵ_t being white noise with mean zero and variance σ_ϵ^2 and Z_t is a stochastic trend, which can be represented as a random walk:

$$Z_t = Z_{t-1} + \eta_t \tag{B.0.3}$$

with η_t being white noise with mean zero and variance σ_η^2. The change in the exchange rate is:

$$\Delta S_t = \Delta X_t + \Delta Z_t \tag{B.0.4}$$

To regress the change in the stationary component (ΔX_t) on the change in the exchange rate (ΔS_t), we will calculate the regression coefficient b using:

$$b = \frac{\text{cov}(\Delta X_t, \Delta S_t)}{\text{var}(\Delta S_t)} \tag{B.0.5}$$

First, let's find $\text{cov}(\Delta X_t, \Delta S_t)$:

$$\Delta S_t = \Delta X_t + \Delta Z_t \tag{B.0.6}$$

$$\text{cov}(\Delta X_t, \Delta S_t) = \text{cov}(\Delta X_t, \Delta X_t + \Delta Z_t) \tag{B.0.7}$$

$$\text{cov}(\Delta X_t, \Delta S_t) = \text{cov}(\Delta X_t, \Delta X_t) + \text{cov}(\Delta X_t, \Delta Z_t) \tag{B.0.8}$$

Since ΔX_t and ΔZ_t are uncorrelated (as ΔX_t is derived from the stationary ARMA(p, q) process and ΔZ_t from the stochastic trend, which are independent),

$$\text{cov}(\Delta X_t, \Delta Z_t) = 0 \tag{B.0.9}$$

Therefore,

$$\text{cov}(\Delta X_t, \Delta S_t) = \text{var}(\Delta X_t) \tag{B.0.10}$$

Next, we need to calculate $\text{var}(\Delta X_t)$ for the ARMA(p,q) process. Given the ARMA(p,q) process:

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q} \quad \text{(B.0.11)}$$

The change in X_t is:

$$\Delta X_t = X_t - X_{t-1} \quad \text{(B.0.12)}$$

The variance of the ΔX_t is a constant times the variance of the white noise:

$$\text{var}(\Delta X_t) = \gamma \sigma_\epsilon^2 \quad \text{(B.0.13)}$$

γ depends on the specific value of ϕ_i and θ_j.[6] Next, we need $\text{var}(\Delta S_t)$:

$$\text{var}(\Delta S_t) = \text{var}(\Delta X_t + \Delta Z_t) \quad \text{(B.0.14)}$$

$$\text{var}(\Delta S_t) = \text{var}(\Delta X_t) + \text{var}(\Delta Z_t) \quad \text{(B.0.15)}$$

Since ΔX_t and ΔZ_t are uncorrelated:

$$\text{var}(\Delta Z_t) = \text{var}(\eta_t) = \sigma_\eta^2 \quad \text{(B.0.16)}$$

Thus:

$$\text{var}(\Delta S_t) = \gamma \sigma_\epsilon^2 + \sigma_\eta^2 \quad \text{(B.0.17)}$$

Now, we can find the regression coefficient (b):

$$b = \frac{\text{cov}(\Delta X_t, \Delta S_t)}{\text{var}(\Delta S_t)} = \frac{\gamma \sigma_\epsilon^2}{\gamma \sigma_\epsilon^2 + \sigma_\eta^2} \quad \text{(B.0.18)}$$

So, the regression coefficient (b) is:

$$b = \frac{\gamma \sigma_\epsilon^2}{\gamma \sigma_\epsilon^2 + \sigma_\eta^2} \quad \text{(B.0.19)}$$

This regression coefficient b represents the proportion of the change in the exchange rate that can be attributed to the change in its stationary component. It ranges from 0 to 1, where values closer to 1 indicate that changes in the exchange rate are predominantly driven by its stationary component, while values closer to 0 suggest that changes are mainly due to the stochastic trend.

[6] For example, for an AR(1) process $x_t = 0.98 x_{t-1} + \epsilon_t$, $\text{var}(\Delta x_t)$=$1.0101\sigma_\epsilon^2$, so $\gamma = 1.0101$.

C Regressing $Z(t+h) - Z(t)$ on $(\rho(h) - 1)Z(t)$

Consider a stochastic ARMA(p,q) process $Z(t)$ with autocorrelation function $\rho(h)$. We aim to show that in the regression of $Z(t+h) - Z(t)$ on $(\rho(h) - 1)Z(t)$, the regression coefficient is 1 and the R^2 approaches 0.5 as h increases.

C.1 Regression Coefficient

Let $Y = Z(t+h) - Z(t)$ and $X = (\rho(h) - 1)Z(t)$. The regression coefficient β is given by:

$$\beta = \frac{\text{Cov}(X, Y)}{\text{Var}(X)} \tag{C.1.1}$$

We calculate $\text{Cov}(X, Y)$:

$$\begin{aligned}
\text{Cov}(X, Y) &= \text{Cov}((\rho(h) - 1)Z(t), Z(t+h) - Z(t)) && \text{(C.1.2)} \\
&= (\rho(h) - 1)\text{Cov}(Z(t), Z(t+h)) - (\rho(h) - 1)\text{Var}(Z(t)) && \text{(C.1.3)} \\
&= (\rho(h) - 1)\rho(h)\sigma^2 - (\rho(h) - 1)\sigma^2 && \text{(C.1.4)} \\
&= (\rho(h) - 1)^2\sigma^2 && \text{(C.1.5)}
\end{aligned}$$

Next, we compute $\text{Var}(X)$:

$$\begin{aligned}
\text{Var}(X) &= \text{Var}((\rho(h) - 1)Z(t)) && \text{(C.1.6)} \\
&= (\rho(h) - 1)^2\text{Var}(Z(t)) && \text{(C.1.7)} \\
&= (\rho(h) - 1)^2\sigma^2 && \text{(C.1.8)}
\end{aligned}$$

Therefore, the regression coefficient is:

$$\beta = \frac{(\rho(h) - 1)^2\sigma^2}{(\rho(h) - 1)^2\sigma^2} = 1 \tag{C.1.9}$$

C.2 R^2 Analysis

To analyze R^2, we use the formula:

$$R^2 = \frac{(\text{Cov}(X, Y))^2}{\text{Var}(X) \cdot \text{Var}(Y)} \tag{C.2.1}$$

We've already calculated Cov(X, Y) and Var(X). Now we compute Var(Y):

$$\begin{aligned}
\text{Var}(Y) &= \text{Var}(Z(t+h) - Z(t)) && \text{(C.2.2)} \\
&= \text{Var}(Z(t+h)) + \text{Var}(Z(t)) - 2\text{Cov}(Z(t+h), Z(t)) && \text{(C.2.3)} \\
&= \sigma^2 + \sigma^2 - 2\rho(h)\sigma^2 && \text{(C.2.4)} \\
&= 2(1-\rho(h))\sigma^2 && \text{(C.2.5)}
\end{aligned}$$

Substituting these into the R^2 formula:

$$\begin{aligned}
R^2 &= \frac{((\rho(h)-1)^2\sigma^2)^2}{(\rho(h)-1)^2\sigma^2 \cdot 2(1-\rho(h))\sigma^2} && \text{(C.2.6)} \\
&= \frac{(\rho(h)-1)^2\sigma^2}{2(1-\rho(h))\sigma^2} && \text{(C.2.7)} \\
&= \frac{(\rho(h)-1)^2}{2(1-\rho(h))} && \text{(C.2.8)} \\
&= \frac{(1-\rho(h))^2}{2(1-\rho(h))} && \text{(C.2.9)} \\
&= \frac{(1-\rho(h))}{2} && \text{(C.2.10)}
\end{aligned}$$

As h increases, $\rho(h)$ approaches 0 for stationary ARMA processes. Taking the limit:

$$\lim_{h\to\infty} R^2 = \lim_{\rho(h)\to 0} \frac{(1-\rho(h))^2}{2(1-\rho(h))} = \frac{1^2}{2 \cdot 1} = 0.5 \qquad \text{(C.2.11)}$$

This proves that R^2 approaches 0.5 as h increases.

PUBLICATIONS