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Regime-Switching Factor Models and Nowcasting with Big Data

Omer Faruk Akbal

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Regime-Switching Factor Models and Nowcasting with Big Data*

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ABSTRACT: This paper shows that the Expectation-Maximization (EM) algorithm for regime-switching dynamic factor models provides satisfactory performance relative to other estimation methods and delivers a good trade-off between accuracy and speed, which makes it especially useful for large dimensional data. Unlike traditional numerical maximization approaches, this methodology benefits from closed-form solutions for parameter estimation, enhancing its practicality for real-time applications and historical data exercises with focus on frequent updates. In a nowcasting application to vintage US data, I study the information content and relative performance of regime-switching model after each data releases in a fifteen year period, which was only feasible due to the time efficiency of the proposed estimation methodology. While existing literature has already acknowledged the performance improvement of nowcasting models under regime-switching, this paper shows that the superior nowcasting performance observed particularly when key economic indicators are released. In a backcasting exercise, I show that the model can closely match the recession starting and ending dates of the NBER despite having less information than actual committee meetings, where the fit between actual dates and model estimates becomes more apparent with the additional available information and recession end dates are fully covered with a lag of three to six months. Given that the EM algorithm proposed in this paper is suitable for various regime-switching configurations, this paper provides economists and policymakers with a valuable tool for conducting comprehensive analyses, ranging from point estimates to information decomposition and persistence of recessions in larger datasets.

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WORKING PAPERS

Regime-Switching Factor Models and Nowcasting with Big Data

Prepared by Omer Faruk Akbal[1]

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Contents

1. Introduction

GDP is arguably the most important macroeconomic indicator that researchers and policymakers closely follow to measure economic activity. A large literature examines various ways of nowcasting aggregate output, since GDP estimates are typically released with a delay. Among other approaches, exploiting information from higher frequency data via common components, i.e., factor model approaches, has become highly popular.

In the context of nowcasting, Giannone et al. (2008) proposed a systematic approach for using large-dimensional data to estimate economic activity, which became the platform for nowcasting at the New York FED until its suspension during the COVID-19 pandemic. These factor models showed their fruitfulness in processing the large number of data to estimate aggregate economic activity by circumventing the degrees of freedom issues in traditional empirical models. However, the COVID-19 pandemic revealed the shortcomings of single regime dynamic factor models (DFMs) that do not accommodate the turning points in business cycle and changes in the underlying economic structures1. On the other hand, the regime-switching setup offers an intuitive solution since it inherently accommodates the new information under different underlying economic structures and captures the turning points endogenously.

Turning points in business cycles have been a topic of interest since Burns & Mitchell (1946). Harding & Pagan (2002), Stock & Watson (2010), and Stock & Watson (2014) provided novel approaches to understand these turning points, contributing to our understanding of economic cycles. Hamilton (1989), Diebold & Rudebusch (1996), and Kim & Nelson (1999) proposed regime-switching approaches, measuring the economic state using factor models with asymmetries across recessions and expansions, and introduced a new dimension to the analysis of economic states. The latter estimated their model with a one-step approach that combines the Kalman filter and the Hamiltonian filter to estimate the factors and regime probabilities in a unified framework. Even though increasing the data dimension has the benefit of working with additional information, since the one-step approach proposed by Kim & Nelson (1999) relies on numerical maximization of the approximate likelihood, its computational cost becomes exorbitant as the model dimension increases. To deal with this problem, Diebold & Rudebusch (1996), Camacho et al. (2015) and Doz & Petronevich (2016) later proposed a two-step approach, or a “short-cut” one-step approach where the first step is to extract the factors under the linear model assumption, and the second step is to estimate the model via the same methodology as in Hamilton (1989) since, given the factor, the dynamic factor model can be written as a VAR. I show that this “short-cut” assumption might be too strong and may miss turning points. The studies point out the necessity of carefully considering the trade-off between accuracy and speed in nowcasting. To overcome this without relying on numerical routines, the paper suggests an iterative process with closed-form solutions to provide an efficient and accurate estimation methodology.

This paper contributes to the literature by proposing the expectation maximization (EM) approach to estimate the regime-switching state-space models that can accommodate mixed frequency data and missing data problems.

[1] *To deal with this problem, some suggested ex-post solutions as to eliminate or downplay the extreme event data as in (Schorfheide & Song 2021, Cascaldi-Garcia 2022). See Cascaldi-Garcia et al. (2023) for a detailed survey of literature.*

To employ, I first derive the required missing terms to complete filter and smoother design of Kim & Nelson (1999) for the EM application. Then, I show that the EM algorithm provides satisfactory performance relative to other estimation methods and delivers a good trade-off between accuracy and speed, which makes it especially useful for real-time applications involving large dimensional data. In an application to vintage US data, I show that the regime-switching modification promises improved forecasting performance even in the periods after the COVID-19 break, where the NY FED nowcasting model suspended its releases. The regime-switching model demonstrates superior nowcasting performance, particularly when key economic indicators are released. In addition, the regime-switching nowcasting model is able to closely match the recession dating of the NBER despite having less information than actual committee meetings. This allows policymakers to act preemptively. In a backcasting exercise, I show that the fit between actual NBER recession dates and model estimates of recession becomes more apparent with the additional available information.

The paper is organized as follows. Section 2 reviews the literature on dynamic factor models and nowcasting. Section 3 outlines the estimation methodology for the regimeswitching DFM, discusses modifications to handle mixed frequency and missing data problems. Section 4 applies the methodology to US data. Finally, section 5 concludes.

2. Related Literature

In single-regime or regime-switching, there are mainly two approaches for estimating factor models: the frequentist approach based on the maximization of the likelihood and the Bayesian approach, which works with the posterior of parameters rather than the likelihood.

Considering the computational burdens, some authors, such as Bai & Ng (2002), Stock & Watson (2002a), and Stock & Watson (2002b), prefer to work with principal component analysis, citing that both approaches are computationally burdensome. Forni et al. (2009) and Onatski (2012) are some of the well-known applications of the principal component methodology in the context of factor models.

However, the proponents of the likelihood-based method argue that there are advantages to working with factor models over non-parametric methods. For instance, the likelihoodbased estimation of factor models has the advantage of deriving restrictions from economic theory. Bernanke et al. (2005), Boivin & Giannoni (2002), and Reis & Watson (2010) are some of the examples of this. Moreover, it is possible to estimate structural models using the maximum likelihood, which makes it a system-based estimator.

Expectation-maximization is a popular iterative algorithm for maximizing the likelihood. An early contribution of this approach is Dempster et al. (1977). In the context of developments of nowcasting DFMs, Doz et al. (2011) propose a two-step approach to estimate the factor models with big datasets. In the first step, this approach assumes that the principal component is the actual unobservable factors and estimates model parameters with OLS. In the second step, given the model parameters, the Kalman filter and smoother are used to estimate the factors. Building on this idea, Banbura & Modugno (2014) and Bok et al. (2018) propose to use the EM where the two steps above are repeated until convergence.

The regime-switching extension of the otherwise standard dynamic factor model is an essential aspect of tracking economic activity and determining the current state of the economy, i.e., recession or expansion. This strand of the literature is connected to the examination of turning points. For instance, the NBER business cycle dating committee announces ex-post turning points for the US economy. Since Burns & Mitchell (1946), many studies have focused on determining the turning points of business cycles. Harding & Pagan (2002), Stock & Watson (2010), and Stock & Watson (2014) provide novel approaches to understand turning points. Using the factor models with asymmetries across recessions and expansions, Hamilton (1989), Diebold & Rudebusch (1996), and Kim & Nelson (1999) propose to measure the economic state. In these models, regime changes occur in the intercept term of the underlying factors. His approach was further extended by Kholodilin et al. (2002b), and Bessec & Bouabdallah (2015) to a setting where the slope of the factors also are subject to regime-switching. Chauvet (1998) and Kholodilin et al. (2002a) study regime-switching on the volatility term of the factors.

Hamilton (1989)'s filter design is the early influential work of regime-switching literature. The work is based on a vector autoregression (VAR) with an unobserved regime indicator. Following this line of work, Kim & Nelson (1999) provides a one-step approach that combines the Kalman and Hamiltonian filters to estimate the factors and regime probabilities in a unified framework. The one-step approach proposed by Kim & Nelson (1999) numerically maximizes the approximate likelihood obtained from the Kalman filter. Since the one-step approach utilizes numerical maximization, its computational cost becomes exorbitant as the model dimension increases. To deal with this problem, Diebold & Rudebusch (1996), Camacho et al. (2015) and Doz & Petronevich (2016) propose a two-step approach, or a "short-cut" one-step approach. Here, the first step is to extract the factor by assuming that the model is linear, i.e., single-regime. Next, the model is estimated via the same methodology as in Hamilton (1989) since, given the factor, the dynamic factor model can be written as a VAR. Doz & Petronevich (2017) study the consistency of the two-step estimator and Camacho et al. (2018) apply the idea to a mixed-frequency nowcasting setup of Banbura & Modugno (2014). Section 3.3 shows that this "short-cut" assumption might be too strong and may lead to misses in capturing turning points.

To overcome this without relying on numerical routines, the paper suggests an iterative process with closed-form solutions to provide an efficient and accurate estimation methodology[2]. This paper contributes to the literature by deriving and applying the EM approach to estimate the regime-switching version of the nowcasting model, which can accommodate mixed frequency data and missing data problems.

3. Methodology

In this section, I first give the notation and definitions. Then, I present the modified EM approach for regime-switching and discuss how to incorporate mixed frequency data and missing observation problems into this setting. Finally, Section 3.3 studies the performance of methodology with two replication exercises.

[2] *Murphy (1998) and Zhou & Shumway (2008) are early discussions of regime-switching state-space models with EM approach with uniform data frequency with a balanced data. Recently Urga & Wang (2024) shows the inference in regime-switching factor model loadings without explicitly modeling the state equation evolution, i.e., by not allowing the named factor identification as discussed in Stock & Watson (2016) sense.*

3.1. Definition and Notation

Let Y_t be an $N \times 1$ vector of observables where each element, $y_{i,t}$, represents a different time series. Define f_t as $q \times 1$ vector of unobservable factors. Let e_t be a vector of "possibly serially correlated" innovations, and ε_t and v_t be normally distributed i.i.d shocks to the time series $t = 1,2, \dots, T$. A dynamic form of the DFM is[3]

$$Y_t = c_0[s_t]f_t + c_1[s_t]f_{t-1} + \cdots + c_s[s_t]f_{t-s} + e_t \quad (1)$$

$$f_t = \mu_0[s_t] + a_1[s_t]f_{t-1} + a_2[s_t]f_{t-2} + \cdots + a_h[s_t]f_{t-h} + v_t \quad (2)$$

where equation (1) is called the measurement equation which represents the observables as a linear combination of unobservable common factors, i.e., f_t. Here, the $N \times q$ coefficients c_i are called factor loadings, and equation (2) is called the state equation, which indicates that factors follow a VAR. This is what makes this factor model dynamic as today's factors depend on the lags of the factors. Here, s_t is the regime indicator, which is discrete, and $\mu_0[s_t]$ is the regime-dependent shift in the unobservable factors. $q \times q$ coefficients a_i are vector autoregressive coefficients for f_t.

The regime transition probabilities are assumed to follow a first-order Markov process where, for M regimes, the exogenous regime-switching probability from the i^{th} regime, i.e., $s_{t-1} = i$, to the j^{th} regime, i.e., $s_t = j$ is denoted by $\pi_{i,j}$. These form the transition matrix Z. Throughout the paper, I will use $Pr(.)$ to denote probabilities where

$$Pr(s_t = j | s_{t-1} = i) = \pi_{i,j} \quad (3)$$

The definitions of exact and approximate DFM frameworks depend on the assumptions regarding to the innovation to the observation equation, i.e., et. Stock & Watson (2016) define the exact DFM as the model where et is uncorrelated cross-sectionally and serially. The assumption of the exact DFM may be untenable for some applications, especially in the nowcasting context where the shocks to the high-frequency indicators might be related. Define a define a vector with a lag length p:

$$F_t = \begin{bmatrix} 1 & f_t & f_{t-1} & \cdots & f_{t-p} \end{bmatrix}'.$$

Then, the equations (4) represents an approximate static form DFM where $N \times N$ coefficients δ_i are persistence coefficients for innovations e_t when it follows an autoregressive process itself.

[3] *One can also define a regime-dependent shift only in measurement equation, and the derivations in the following sections are straightforward to modify as motivated in Hamilton (1989), and Kim and Nelson (1999).*

$$Y_t = C[s_t]F_t + e_t$$

$$F_t = \mu[s_t] + A[s_t]F_{t-1} + v[s_t]$$

$$e_t = \delta_1[s_t]e_{t-1} + \delta_2[s_t]e_{t-2} + \cdots + \delta_p[s_t]e_{t-p} + \varepsilon_t$$

$$\varepsilon[s_t] \sim \mathcal{N}(0, R[s_t])$$

$$v[s_t] \sim \mathcal{N}(0, Q[s_t]) \quad (4)$$

This paper is mainly concerned with an approximate DFM as represented in (4) given its focus on nowcasting and forecasting for which this is more relevant. However, it is straightforward to apply following filtering, smoothing, and maximization steps for the exact form.

3.2. Estimation of Regime-Switching DFMs

The DFM, either defined in exact or approximate form, has two independent sets to estimate, i.e. model parameters and latent variables. The regime-switching DFM, in addition to what is already defined in its single-regime counterpart, promotes an additional latent variable s_t to estimate[4].

The Expectation-maximization algorithm suggests iterating a modified Kalman filter and smoother conditional on model parameters and maximizing likelihood conditional on the latent variables until the latent variables $[F_t, s_t]$, and model parameters are jointly estimated.

The single-regime Kalman filter and smoother equations are modified in Kim & Nelson (2006) to be able to estimate a regime-switching state space model. Since their approach based on numerical likelihood maximization, their smoother design does not require to calculate the lagged auto-covariance term for the unobservable factors, $Cov_{\hat{\theta}}\left[F_t^j\left(F_{t-1}^j\right)'\middle|\Omega_T\right]$In the next section, I derive the required missing terms to complete the expectation step. Then, I will promote the closed-form solutions to the expected maximum-likelihood maximizing parameters.

3.2.1. The Expectation Step

The standard Kalman filter is modified for each possible transaction from regime $s_{t-1} = i$ to $s_t = j$. Let P_t be the variance-covariance matrix of unobservable factor F_t, and the expected values of the unobservable factors and

[4] *Online Appendix 3.2 discusses the estimation procedure of single-regime DFM and its mapping to the regime-switching model.*

their variance-covariance matrices for period t' conditional on the information set at time t, i.e., Ω_t, be $E[F_{t'}|\Omega_t] = F_{t'|t}$ and $E[P_{t'}|\Omega_t] = P_{t'|t}$[5]

$$F_{t|t-1}^{(i,j)} = A_t^j F_{t-1|t-1}^i \quad (5)$$

$$P_{t|t-1}^{(i,j)} = A_t^j P_{t-1|t-1}^i (A_t^j)' + Q_j \quad (6)$$

$$\eta_{t|t-1}^{(i,j)} = Y_t - C_j F_{t|t-1}^{(i,j)} \quad (7)$$

$$f_{t|t-1}^{(i,j)} = C_j P_{t|t-1}^{(i,j)} C_j' + R_j \quad (8)$$

$$F_{t|t}^{(i,j)} = F_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} C_j' \left[f_{t|t-1}^{(i,j)}\right]^{-1} \eta_{t|t-1}^{(i,j)} \quad (9)$$

$$P_{t|t}^{(i,j)} = \left(I - P_{t|t-1}^{(i,j)} C_j' \left[f_{t|t-1}^{(i,j)}\right]^{-1} C_j\right) P_{t|t-1}^{(i,j)} \quad (10)$$

where Ω_t contains observables up to time t[6]. However, the curse of dimensionality does not allow for all possible paths of regimes s_t to be considered. To overcome this issue, the latent factors and their variances need to be approximated and collapsed. Noting higher order dependency derivations straightforward, I use the generalized Pseudo Bayesian algorithm of order $r = 2$ as in Kim and Nelson (1999):

$$F_{t|t}^j = \sum_{i=1}^{M} \Pr(s_{t-1} = i | s_t = j, \Omega_t) F_{t|t}^{(i,j)} \quad (11)$$

$$P_{t,t}^j = \sum_{i=1}^{M} Pr(s_{t-1} = i | s_t = j, \Omega_t) \left[P_{t,t}^{(i,j)} + \left(F_{t,t}^j - F_{t,t}^{(i,j)}\right)\left(F_{t,t}^j - F_{t,t}^{(i,j)}\right)'\right] \quad (12)$$

where M is the number of regimes and equations take conditional on $s_{t=1}$, Ω_{t-1} does not contain any additional information. In the smoothing step, I calculate the expected values of the latent factors and their variance-covariance using the whole sample information. This exercise by definition is retrospective, inferring the state at a particular point in time using information beyond the period. And it makes use of the output from the filtering step as the input. Kalman smoother for each transition from regime $s_{t-1} = i$ to $s_t = j$ conditional on the full information set at time t is

$$J_{t+1,t}^{(j,k)} = P_{t,t}^j A^{k\prime}{}_t \left[P_{t+1,t}^{(j,k)}\right]^{-1} \quad (13)$$

$$F_{t,T}^{(j,k)} = F_{t,t}^j + J_{t+1,t}^{(j,k)}\left(F_{t+1,T}^k - A^k{}_t F_{t,t}^j\right) \quad (14)$$

[5] *While conceptually well-defined, this may not be computationally feasible. More on this below. The term $\eta_{t|t-1}$ is one-step ahead prediction error for the observable and $f_{t|t-1}$ is the variance-covariance matrix of this. As usual, $'$ stands for matrix transpose. See Online Appendix for detailed derivations.*

[6] *Note that this makes use of information at time t. The notation features conditioning on $t-1$ to emphasize that the conditional expectation of the observable uses information up to $t-1$.*

$$P_{t,T}^{(j,k)} = P_{t,t}^{j} + J_{t+1,t}^{(j,k)}\left(P_{t+1,T}^{k} - P_{t+1,t}^{(j,k)}\right)J_{t+1,t}^{(j,k)\,\prime} \quad (15)$$

where similar to the regime-switching Kalman Filter, needs to be collapsed in each iteration due to the curse of dimensionality. I approximate $F_{t|T^{(j,k)}}$ and $P_{t|T^{(j,k)}}$ as

$$F_{t,T}^{j} = \sum_{k=1}^{M} \Pr(s_{t+1} = k, s_t = j, \Omega_T) F_{t,T}^{(j,k)} \quad (16)$$

$$P_{t,T}^{j} = \sum_{k=1}^{M} \Pr(s_{t+1} = k, s_t = j, \Omega_T)\left[P_{t,T}^{(j,k)} + \left(F_{t,T}^{j} - F_{t,T}^{(j,k)}\right)\left(F_{t,T}^{j} - F_{t,T}^{(j,k)}\right)'\right] \quad (17)$$

The equations (5) – (17) constitute the standard modified regime-switching filter and smoother. Since Kim and Nelson (2006) uses a one-step numerical approach to estimate the state-space model, the modified smoother does not require the auto-covariance term. In order to complete the EM algorithm, I calculate the auto-covariance term $Cov_{\hat{\theta}}\left[F_t^{j}\left(F_{t-1}^{j}\right)'\middle|\Omega_T\right]$. First, define

$$P_{t,t+1,T}^{(j,k)} = E\left[\left(F_{t+1} - F_{t+1,t}^{(j,k)}\right)\left(F_{t+1} - F_{t+1,t}^{(j,k)}\right)' |\Omega_T, s_t = j, s_{t+1} = k\right]$$

I extend the single-regime smoothing step of the covariance term in Byron et al. (2004) to the regime-switching framework using

$$P_{t,t+1,T}^{(j,k)} = P_{t+1,T}^{k} J_{t+1,t}^{(j,k)\,\prime} \quad (18)$$

Then the covariance term can be calculated by integrating out $P_{t,t+1,T^{(j,k)}}$ to $P_{t,t+1,T^{k}}$[7].

$$P_{t,t+1,T}^{k} = \sum_{j=1}^{M} \frac{\Pr(s_t = j, s_{t+1} = k, \Omega_T)}{\Pr(s_{t+1} = k, \Omega_T)}\left(P_{t,t+1,T}^{(j,k)} + \left(F_{t+1,T}^{k} - F_{t,t+1,T}^{(j,k)}\right)\left(F_{t+1,T}^{k} - F_{t,t+1,T}^{(j,k)}\right)'\right) \quad (19)$$

where

$$F_{t+1,T}^{k} = E[F_{t+1}|\Omega_T, s_{t+1} = k]$$

$$F_{t,t+1,T}^{(j,k)} = E[F_{t+1}|\Omega_T, s_t = j, s_{t+1} = k]$$

Finally, approximating $F_{t+1,T^{k} \approx F_{t,t+1,T^{(j,k)}}}$. The expression reduces to

$$P_{t,t+1,T}^{k} = \sum_{j=1}^{M} \frac{\Pr(s_t = j, s_{t+1} = k|\Omega_T)}{\Pr(s_{t+1} = k|\Omega_T)} P_{t,t+1,T}^{(j,k)} \quad (20)$$

[7] *See Online Appendix for detailed derivations.*

$$= \sum_{i=1}^{M} Pr(s_t = j | s_{t+1} = k, \Omega_T) P_{t,t+1,T^{(j,k)}}$$

3.2.2. The Maximization Step

Consider the general case of the model in (4) where the intercept term explicitly written on the factor as follows:[8]

$$Y_t = C[s_t]F_t + e_t$$

$$F_t = \mu[s_t] + A[s_t]F_{t-1} + v[s_t]$$

$$\varepsilon[s_t] \sim \mathcal{N}(0, R[s_t])$$

$$v[s_t] \sim \mathcal{N}(0, Q[s_t]) \quad (21)$$

For any parameter X, define $X[s_t = j] = X_j$. Given the model parameter set of $\theta = \{(A_j, Q_j, C_j, R_j, \mu_j): 1 < j < M; \pi_{i,j} \in Z\}$, let $E_\theta[.|\Omega_T]$ be the expectations operator for the parameters $\hat{\theta}$ conditional on complete observable data, and $\log L_c(\theta)$ be the log-likelihood function conditional on parameters θ.

In the maximization step I solve the expected value of $\log L_c(\theta)$ at the parameter value $\hat{\theta}$

$$\hat{\theta} = \arg\max_{\theta} E_\theta [\log L_c(\theta) | \Omega_T] \quad (22)$$

where the expected value of the likelihood, the Q-function, on the right-hand side is based on the outcome from the previous iteration, and in the form of

[8] *Note that the equation for Ft now features the potentially regime-switching intercept term μ[st]. This is for the expositional convenience. This equation can always be rewritten in the form of equation (4) where the vector of factors is augmented with the constant of one. Henceforth, both notations will be used without loss of generality.*

$$Q = E_{\hat{\theta}}[logL_c(\theta), \Omega_T] = -\frac{T(N+r)}{2}log(2\pi)$$
$$-\frac{1}{2}\sum_{t=1}^{T}\sum_{j=1}^{M}[\,log|R_j|\mathbf{Pr}_{\hat{\theta}}(\mathbf{s_t = j}|\mathbf{\Omega_T})$$
$$+tr(R_j^{-1}(Y_tY_t' - Y_t\mathbf{E}_{\hat{\theta}}[(\mathbf{F_t^j})'|\mathbf{\Omega_T}]C_j' - C_j\mathbf{E}_{\hat{\theta}}[\mathbf{F_t^j}|\mathbf{\Omega_T}]Y_t'$$
$$+C_j(\mathbf{E}_{\hat{\theta}}[\mathbf{F_t^j}|\mathbf{\Omega_T}]\mathbf{E}_{\hat{\theta}}[(\mathbf{F_t^j})'|\mathbf{\Omega_T}] + \mathbf{Cov}_{\hat{\theta}}[\mathbf{F_t^j}(\mathbf{F_t^j})'|\mathbf{\Omega_T}])C_j'))$$
$$\times\,\mathbf{Pr}_{\hat{\theta}}(\mathbf{s_t = j}|\mathbf{\Omega_T}) + log|Q_j|\mathbf{Pr}_{\hat{\theta}}(\mathbf{s_t = j}|\mathbf{\Omega_T})$$
$$+tr(Q_j^{-1}((\mathbf{E}_{\hat{\theta}}[\mathbf{F_t^j}|\mathbf{\Omega_T}]\mathbf{E}_{\hat{\theta}}[(\mathbf{F_t^j})'|\mathbf{\Omega_T}] + \mathbf{Cov}_{\hat{\theta}}[\mathbf{F_t^j}(\mathbf{F_t^j})'|\mathbf{\Omega_T}]) \tag{23}$$
$$-\mathbf{E}_{\hat{\theta}}[\mathbf{F_t^j}|\mathbf{\Omega_T}]\mu_j' - (\mathbf{E}_{\hat{\theta}}[\mathbf{F_t^j}|\mathbf{\Omega_T}]\mathbf{E}_{\hat{\theta}}[(\mathbf{F_{t-1}^j})'|\mathbf{\Omega_T}] + \mathbf{Cov}_{\hat{\theta}}[\mathbf{F_t^j}(\mathbf{F_{t-1}^j})'|\mathbf{\Omega_T}]])A_j'$$
$$-\mu_j\mathbf{E}_{\hat{\theta}}[(\mathbf{F_t^j})'|\mathbf{\Omega_T}] + \mu_j\mu_j' + \mu_j\mathbf{E}_{\hat{\theta}}[(\mathbf{F_{t-1}^j})'|\mathbf{\Omega_T}]A_j'$$
$$-A_j(\mathbf{E}_{\hat{\theta}}[\mathbf{F_{t-1}^j}|\mathbf{\Omega_T}]\mathbf{E}_{\hat{\theta}}[(\mathbf{F_t^j})'|\mathbf{\Omega_T}] + \mathbf{Cov}_{\hat{\theta}}[\mathbf{F_{t-1}^j}(\mathbf{F_t^j})'|\mathbf{\Omega_T}])$$
$$+A_j\mathbf{E}_{\hat{\theta}}[\mathbf{F_{t-1}^j}|\mathbf{\Omega_T}]\mu_j' + A_j(\mathbf{E}_{\hat{\theta}}[\mathbf{F_{t-1}^j}|\mathbf{\Omega_T}]\mathbf{E}_{\hat{\theta}}[(\mathbf{F_{t-1}^j})'|\mathbf{\Omega_T}]$$
$$+\mathbf{Cov}_{\hat{\theta}}[\mathbf{F_{t-1}^j}(\mathbf{F_{t-1}^j})'|\mathbf{\Omega_T}])A_j')) \times \mathbf{Pr}_{\hat{\theta}}(\mathbf{s_t = j}|\mathbf{\Omega_T})$$
$$+\sum_{i=1}^{M}\mathbf{Pr}_{\hat{\theta}}(\mathbf{s_t = j}, \mathbf{s_{t-1} = i}|\mathbf{\Omega_T})\,logPr_{\hat{\theta}}(s_t = j|s_{t-1} = i)]$$

where the expressions in bold are the smoothed variables obtained in the expectation step.

Corollary 1. *For the number of regimes M=1, the intercept term* $\mu = 0$ *and* $P(s_t = j, s_{t-1} = i|\Omega_T) = P(s_t = j|\Omega_T) = 1\ \forall t \in [1, \dots, T]$*, the Q-function in (23) becomes*

$$Q = \mathrm{E}_{\hat{\theta}}[logL_c(\theta)|\Omega_T] = -\frac{T(N+r)}{2}log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}[\,log|R|$$
$$+tr(R^{-1}(Y_tY_t' - Y_t\mathbf{E}_{\hat{\theta}}[(\mathbf{F_t})'|\mathbf{\Omega_T}]C' - C\mathbf{E}_{\hat{\theta}}[\mathbf{F_t}|\mathbf{\Omega_T}]Y_t' + C\mathbf{E}_{\hat{\theta}}[\mathbf{F_t}(\mathbf{F_t})'|\mathbf{\Omega_T}]C')) \tag{24}$$
$$+log|Q| + tr(Q^{-1}(\mathbf{E}_{\hat{\theta}}[\mathbf{F_t}(\mathbf{F_t})'|\mathbf{\Omega_T}] - \mathbf{E}_{\hat{\theta}}[\mathbf{F_t}(\mathbf{F_{t-1}})'|\mathbf{\Omega_T}]A' - A\mathbf{E}_{\hat{\theta}}[\mathbf{F_{t-1}}(\mathbf{F_t})'|\mathbf{\Omega_T}]$$
$$+A\mathbf{E}_{\hat{\theta}}[\mathbf{F_{t-1}}(\mathbf{F_{t-1}})'|\mathbf{\Omega_T}]A'))]$$

which is equivalent to the expected log-likelihood of Banbura and Modugno (2014) for the single-regime DFM for an otherwise identical setting.

For any parameter $\hat{\theta} = \{(A_j, Q_j, C_j, R_j, \mu_j): 1 < j < M; \pi \in Z\}$, the estimators are obtained by solving the first order conditions[9].

$$\frac{\partial E_{\hat{\theta}}[logL_c(\theta)|\Omega_T]}{\partial\theta[s_t = j]} = 0 \tag{25}$$

The first order condition for $A[s_t = j]$ is

[9] *The solutions for estimators* $C[s_t = j]$, $R[s_t = j]$, $A[s_t = j]$, $Q[s_t = j]$, *and* $\mu[s_t = j]$ *are given in Online Appendix.*

$$A[s_t = j] = \frac{\sum_{t=1}^{T}\left[Pr_{\hat{\theta}}(s_t = j|\Omega_T)\left(E_{\hat{\theta}}\left[F_t^j(F_{t-1}^j)'\middle|\Omega_T\right]\right)\right]}{\sum_{t=1}^{T}\left[Pr_{\hat{\theta}}(s_t = j|\Omega_T)E_{\hat{\theta}}\left[F_{t-1}^j(F_{t-1}^j)'\middle|\Omega_T\right]\right]} - \frac{\mu[s_t = j]\sum_{t=1}^{T}\left[Pr_{\hat{\theta}}(s_t = j|\Omega_T)\left(E_{\hat{\theta}}\left[(F_{t-1}^j)'\middle|\Omega_T\right]\right)\right]}{\sum_{t=1}^{T}\left[Pr_{\hat{\theta}}(s_t = j|\Omega_T)E_{\hat{\theta}}\left[F_{t-1}^j(F_{t-1}^j)'\middle|\Omega_T\right]\right]} \quad (26)$$

where

$$\mu[s_t = j] = \frac{\sum_{t=1}^{T}\left[Pr_{\hat{\theta}}(s_t = j|\Omega_T)E_{\hat{\theta}}\left[F_t^j\middle|\Omega_T\right]\right]}{(T-1)} - \frac{A[s_t = j]\sum_{t=1}^{T}\left[Pr_{\hat{\theta}}(s_t = j|\Omega_T)E_{\hat{\theta}}\left[F_{t-1}^j\middle|\Omega_T\right]\right]}{(T-1)} \quad (27)$$

Online Appendix shows how to solve these two terms simultaneously. Similarly, the estimator for $C[s_t = j]$ is in the form of

$$C[s_t = j] = \frac{\sum_{t=1}^{T}\left[Pr_{\hat{\theta}}(s_t = j|\Omega_T)E_{\hat{\theta}}\left[Y_t(F_t^j)'\middle|\Omega_T\right]\right]}{\sum_{t=1}^{T}\left[Pr_{\hat{\theta}}(s_t = j|\Omega_T)E_{\hat{\theta}}\left[F_t^j(F_t^j)'\middle|\Omega_T\right]\right]} \quad (28)$$

The variance matrices $R[s_t = j]$ and $Q[s_t = j]$ are

$$R[s_t = j]' = (\sum_{t=1}^{T} Pr_{\hat{\theta}}(s_t = j|\Omega_T))^{-1} \times [\sum_{t=1}^{T} Pr_{\hat{\theta}}(s_t = j|\Omega_T)Y_tY_t' - C[s_t = j](\sum_{t=1}^{T} Pr_{\hat{\theta}}(s_t = j|\Omega_T)E_{\hat{\theta}}[(F_t^j)Y_t'|\Omega_T])] \quad (29)$$

$$Q[s_t = j]' = (\sum_{t=1}^{T} Pr_{\hat{\theta}}(s_t = j|\Omega_T))^{-1} [\sum_{t=1}^{T} Pr_{\hat{\theta}}(s_t = j|\Omega_T)E_{\hat{\theta}}[(F_t^j)(F_t^j)'|\Omega_T] \sum_{t=1}^{T} Pr_{\hat{\theta}}(s_t = j|\Omega_T)\mu[s_t = j]E_{\hat{\theta}}\left[(F_t^j)'\middle|\Omega_T\right] - \sum_{t=1}^{T} Pr_{\hat{\theta}}(s_t = j|\Omega_T)A[s_t = j]E_{\hat{\theta}}\left[(F_{t-1}^j)(F_t^j)'\middle|\Omega_T\right] \quad (30)$$

Finally, the transition probabilities, as shown by Hamilton (1989,) are

$$\pi_{i,j} = \frac{\sum_{t=1}^{T} Pr_{\hat{\theta}}(s_{t-1} = i, s_t = j|\Omega_T)}{\sum_{t=1}^{T} Pr_{\hat{\theta}}(s_t = i|\Omega_T)} \tag{31}$$

3.2.3. The Modifications for Mixed and Missing Data

I follow Banbura and Modugno (2014) and Bok et al. (2018) who use the approximation of Mariano and Murasawa (2003) for mixed frequency applications. While this section illustrates how to implement a quarterly to monthly approximation, the modifications for other cross-frequency equations are straightforward to implement.

Let y_t be any quarterly time series data which is the geometric mean of the underlying monthly series $\hat{y}_t$, that is

$$y_t = (\hat{y}_t \times \hat{y}_{t-1} \times \hat{y}_{t-2})^{\frac{1}{3}} \tag{32}$$

Taking the logarithm of each side,

$$\log y_t = \frac{1}{3} \times (\log \hat{y}_t + \log \hat{y}_{t-1} + \log \hat{y}_{t-2}) \tag{33}$$

Define $\Delta_i y_t = \log y_t - \log y_{t-i}$ and take the quarterly difference to obtain

$$\Delta_3 y_t = \frac{1}{3} \times (\Delta_1 \hat{y}_t + 2\Delta_1 \hat{y}_{t-1} + 3\Delta_1 \hat{y}_{t-2} + 2\Delta_1 \hat{y}_{t-3} + \Delta_1 \hat{y}_{t-4}) \tag{34}$$

Equation (33) defines a linear relation between the quarterly and monthly data. Since it is possible to augment the DFM to include the approximation above and still write the DFM in the form of equation (21), the EM algorithm above continues to apply.

To tackle the issue arising from the missing data problem, the equation (28) needs to be modified as

$$C[s_t = j] = \frac{\sum_{t\in t^*} \left[Pr_{\hat{\theta}}(s_t = j|\Omega_T) E_{\hat{\theta}} \left[Y_t \left(F_t^j\right)' \middle| \Omega_T \right] \right]}{\sum_{t\in t^*} \left[Pr_{\hat{\theta}}(s_t = j|\Omega_T) E_{\hat{\theta}} \left[F_t^j \left(F_t^j\right)' \middle| \Omega_T \right] \right]} \tag{35}$$

where t^* includes only the observed data. The estimation of $R[s_t = j]$ follows a similar modification.

3.3. Performance of the Methodology

In this section, I test the performance of EM methodology to estimate regime switching DFMs by replicating two seminal papers that use different estimation procedures[10]. The first paper is the well-known example of one-step procedure.

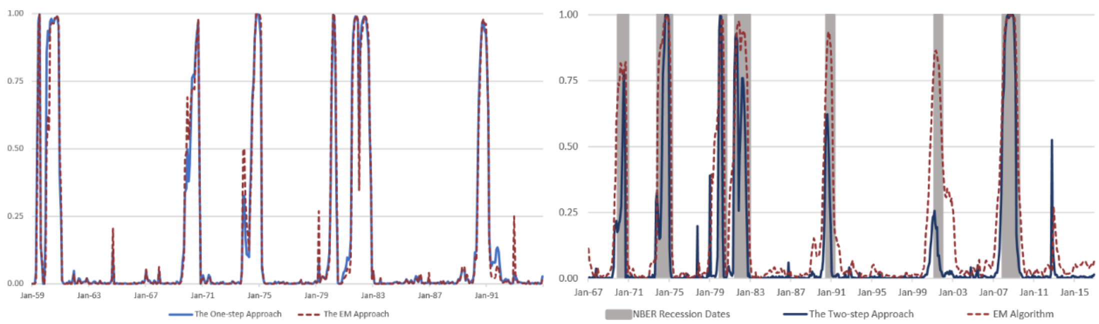
[10] *Online Appendix provides the details of replication exercises of known papers and Monte Carlo simulations on regime switching DFM using the proposed estimation methodology.*

Table 1: Estimation Time for the Two Estimation Approaches

Estimation Methodology	Total Computation Time (in Sec)
One-step Approach	56.1
EM Approach	41.0

with numerical maximization, i.e., the model proposed in Kim and Nelson (2006), and the second one is the two-step, or shortcut one-step, procedure discussed in Camacho et al. (2018). In order to test the performance of methodology discussed in this paper, I first replicated the papers with their corresponding original methodologies and then the EM methodology discussed in this paper.

Figure 1: Model replication exercises



(a) One-step Approach

(b) Two-step approach

Figure 1a compares the smoothed recession probabilities for the model estimate obtained from the one-step approach (blue solid line) and the EM approach (red dashed line), respectively. It is clear that both approaches give similar recession probability estimates.

Given that the two approaches give similar estimates for recession probabilities, the advantage of the EM approach to one-step approach is its estimation speed as summarized in Table 1. It will be shown that the computational advantage carries over to a setting with larger datasets in the section 4.

Table 2: Estimation Time for the Two Estimation Approaches

Estimation Methodology	Total Computation Time (in Sec)
Two-step Approach	110.2
EM Approach	71.5

Figure 1b plots the smoothed recession probabilities generated by the two approaches: the two-step approach of Camacho et al. (2018) (blue solid line) and the EM approach (red dashed line). I superimpose the NBER recession dates on top of these using the gray shaded areas.

The results show that two approaches mostly agree, which also closely resemble the NBER recession dates, and the EM methodology overperform in 1997 and 2003 recessions in which two-step approach mostly miss. As before, the EM approach comes with a speed advantage as shown in Table 2.

Note that all these estimates are not real-time observations with vintage time series. In section 4, I will demonstrate how the regime-switching model, estimated with the EM approach, performs against the NBER dating in real-time.

4. An Application to US Vintage Data

To show the practicality of regime-switching EM algorithm for DFMs, I use the New York FED nowcasting model which has produced popular nowcasts for the US economy until its suspension following the COVID-19 pandemic until recently. I extend the model by allowing regime-switching, which makes it potentially applicable even in the sample including events involving large nonlinearities such as the pandemic. To demonstrate the effectiveness of this approach, I compare the performance of the single-regime New York FED model with my regime-switching model in predicting the current quarter of US GDP.

Furthermore, I use this model to examine whether it produces recession dating in real-time that aligns closely with the NBER dating, which typically becomes available with up to a one-year delay.

In the New York FED model, the common components are classified into four blocks, namely global, soft, real, and labor. The model on the New York FED website consists of 22 monthly indicators and three quarterly indicators.[11] Every indicator loads on the global factor, hence the naming, and all other variables other than those related to the price level load on one additional factor. This is described in Table 3.

The dataset from the New York FED website is extended both forward and backward using vintage data from ALFRED, the archival database website of St. Louis FED, with the total span of data starting in January 2000 and ending in March 2022.

I use the first seven years of the data to train the nowcasting models. For this reason, the nowcasting exercise starts from January 2007. In what follows, all nowcasts will be in real-time, i.e., for more than 2000 data releases.

[11] *The model of Bok et al. (2018) uses 37 variables of which three of them are at quarterly frequency and 34 of them are at monthly frequency. Because some of the data are not publicly available, I only use the portion of the data that is publicly available. This adaptation ensures the accessibility and reproducibility of the results provided in this paper. See "https://github.com/FRBNY-TimeSeriesAnalysis/Nowcasting" for the details.*

Table 3: Variables

Series Name	Frequency	Global Soft Block		Real Block	Labor Block	Transformation	Category
Payroll Employment	m	1	0	0	1	chg	Labor
Job Openings	m	1	0	0	1	chg	Labor
Real GDP	q	1	0	1	0	pca	National Accounts
Consumer Price Index	m	1	0	0	0	pch	Prices
Durable Goods Orders	m	1	0	1	0	pch	Manufacturing
Retail Sales	m	1	0	1	0	pch	Retail and Consumption
Unemployment Rate	m	1	0	0	1	chg	Labor
Housing Starts	m	1	0	1	0	pch	Housing and Construction
Industrial Production	m	1	0	1	0	pch	Manufacturing
Personal Income	m	1	0	1	0	pch	National Accounts
Exports	m	1	0	1	0	pch	International Trade
Imports	m	1	0	1	0	pch	International Trade
Construction Spending	m	1	0	1	0	pch	Housing and Construction
Import Price Index	m	1	0	0	0	pch	International Trade
Core Consumer Price Index	m	1	0	0	0	pch	Prices
Core PCE Price Index	m	1	0	0	0	pch	Prices
PCE Price Index	m	1	0	0	0	pch	Prices
Building Permits	m	1	0	1	0	chg	Housing and Construction
Capacity Utilization Rate	m	1	0	1	0	chg	Manufacturing
Business Inventories	m	1	0	1	0	pch	International Trade
Unit Labor Cost	q	1	0	0	1	pca	Labor
Export Price Index	m	1	0	0	0	pch	International trade
Empire State Mfg Index	m	1	1	0	0	lin	Surveys
Real Consumption Spending	m	1	0	1	0	pch	Retail and Consumption
Philadelphia Fed Mfg Index	m	1	1	0	0	lin	Surveys

4.1. Results and Discussion

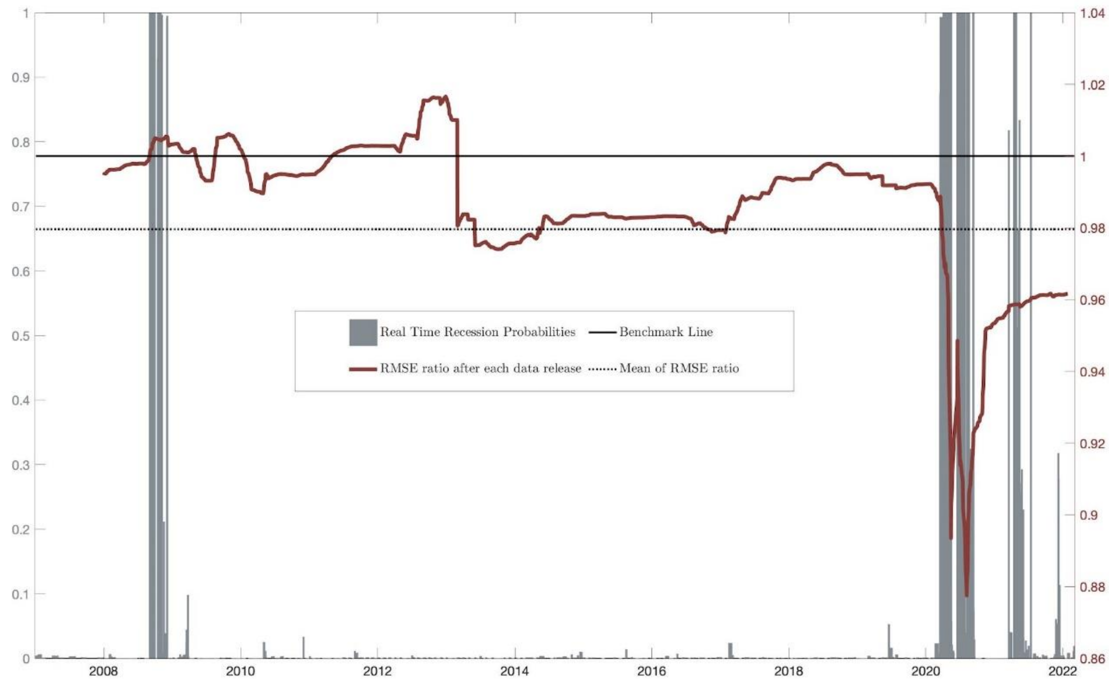
The forecast target for the nowcasting exercise is the first flash estimate of the US real GDP. I evaluate the relative nowcasting performance of the single-regime New York FED model to my regime-switching model using the relative root mean square error (RMSE):

$$RMSE_{\text{relative}, \bar{T}} = \frac{\sqrt{\sum_{t=1}^{\bar{T}} \left(\widehat{\text{GDP}}_{t,Regime\ Switching} - GDP_{\text{t, First Estimate}}\right)^2}}{\sqrt{\sum_{t=1}^{\bar{T}} \left(\widehat{\text{GDP}}_{t,Bok\ et\ al.(2018)} - GDP_{\text{t, First Estimate}}\right)^2}} \tag{36}$$

where $\bar{T}$ is the timing of data releases in the sample.

Figure 2 presents the relative RMSE (red line) which is calculated based on the expanding-window subsample. The shaded areas are the combinations of real-time (smoothed) recession probabilities from the regime-switching model. The solid horizontal line is the benchmark level of one which means two models perform equally well, and the dashed horizontal line is the mean of the relative RMSE. Note that each point in the horizontal axis is a data release which is not as regular as the calendar time.

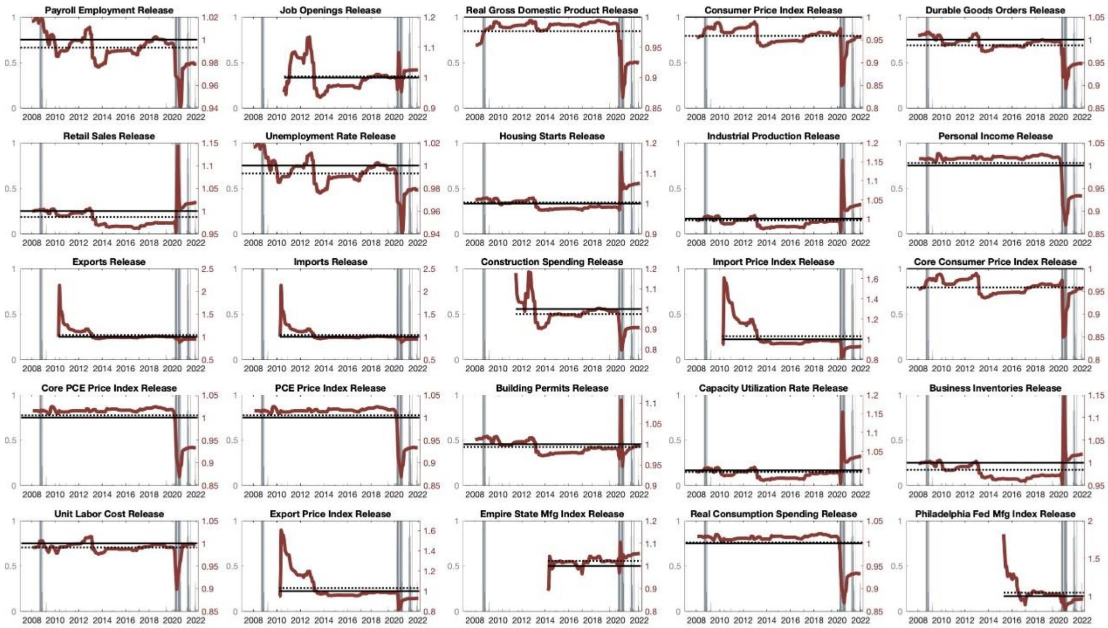
Figure 2: Expanding window RMSE and Real-Time Recession Probabilities



Recall that there are only two recessions in this sample, the first being the global financial crisis of 2008 and the second recent COVID-19 pandemic. Because the regime-switching model trains itself using the early observations, its forecasting performance is not substantially better relative to the single-regime New York FED model. However, one can see the regime-switching model's forecasting performance becomes notably superior starting in 2013, particularly evident during the onset of the COVID-19 pandemic, with the RMSE dropping substantially.

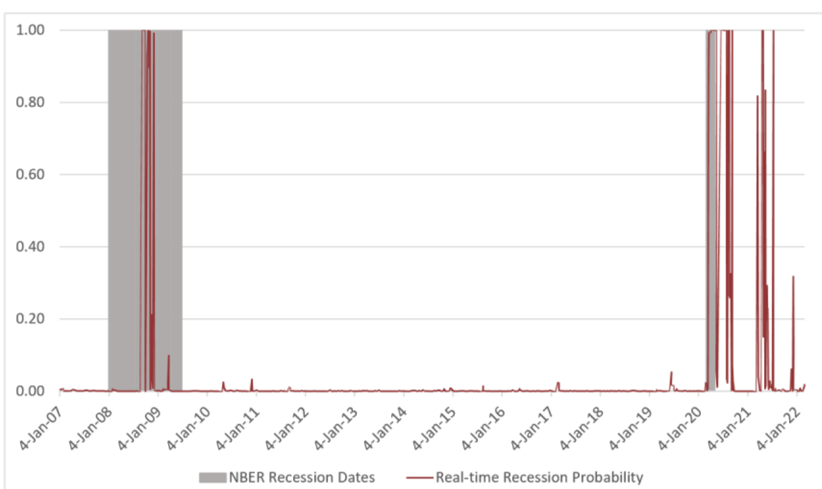
Next, I break down Figure 2 release by release. Figure 3 shows that the regime switching model has clear superior forecasting performance with the following indicators: nonfarm payroll employment, consumer price index, durable good orders, and unemployment rate. These are some of the most important indicators for monitoring the state of the economy and this is why it is not surprising to see that they are particularly informative for nowcasting real GDP. On the other hand, I observe that some indicators produce noisier information during recession regimes which would signal the sensitivty content of the indicators.

Figure 3: Expanding window RMSE for Each Data Release and Real-Time Recession Probabilities



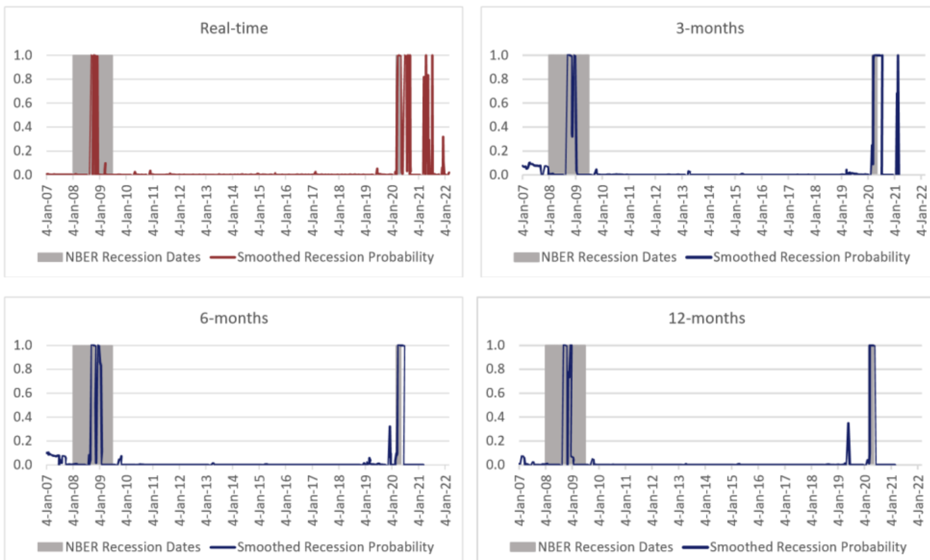
As a next step, I compare the recession probabilities out of the regime-switching model to the recession dating of the NBER in Figure 4. The former is a real-time object based on the latest release of the data which by definition is based on less information than the latter because it becomes available with delays of a quarter to a year. Despite the information disadvantage of my model, the two datings sufficiently overlap for both recessions in the US data, demonstrating the practical utility of the regime-switching nowcasting model.

Figure 4: Real-time Recession Probabilities and NBER Recession Dates



Finally, for a fair comparison, I use the nowcasting model to backcast recessions. I allow using the information up to one year out relative to the period being dated. Figure 5 shows that for the COVID-19 period, the model-based dating and the NBER dating of the recession become closer as more information becomes available and recession end dates are fully covered with a lag of three to six months, within the range of publication of the following quarter GDP. This indicates that the regime-switching nowcasting model holds a good promise as an alternative automated recession dating tool.

Figure 5: Backcasting of Recession Probabilities



5. Conclusion

This paper discusses how to implement the EM algorithm to estimate regime switching DFMs and how to use this toolkit for nowcasting with big data applications. After deriving the EM steps, the methodology is applied to nowcast the US real GDP in real-time, which as a by-product also provides real-time probabilistic recession dating. I demonstrate the usefulness of the methodology in nowcasting by showing that the regime-switching model outperforms the single-regime New York FED nowcasting model in relative terms and matches the NBER recession dating closely. The dynamic factor model form discussed in this paper is flexible enough to be of use for other applications beyond nowcasting.

The results presented in this paper are particularly useful for several reasons. First, they provide a robust and efficient solution to the challenges of nowcasting with big data applications in the context of regime-switching. Second, the regime switching model demonstrated superior nowcasting performance, particularly with key economic indicators. This suggests that the model can provide more accurate and timely forecasts, which are crucial for policy-making and economic planning. Third, the regime-switching nowcasting model was able to closely match the recession dating of the NBER, despite having less information. This indicates that the model can provide real-time probabilistic recession dating, which can be invaluable for monitoring the state of the economy and making informed decisions. While detecting when the crisis starts is relatively easier, this approach also offers policymakers a timely assessment on when the period ended which may be useful by immediately effecting the policy stance. By changing the regime-switching structure of the model, the framework can allow central bankers to evaluate whether the economy is subject to an upcoming recession, or experiencing a persistent or transitory inflationary regime, and gives opportunity to act preemptively, more hawkish or dovish if necessary.

Overall, the paper demonstrates the potential of the EM algorithm in estimating regime switching DFMs and its practical application in nowcasting U.S. GDP with big data. The dynamic factor model form discussed in this paper is flexible enough to be of use for other applications beyond nowcasting, i.e., estimation of any model that can be represented in the state-space form, opening new avenues for future research and advances in the literature.

Annex

See Online Annexes.

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