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# THE MIRAGE OF FALLING *R*-STARS

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**The Mirage of Falling *R*-stars**

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**ABSTRACT:** Was the recent decline in real interest rates driven by a diminishing natural real interest rate, or have we observed a long sequence of shocks that have pushed market rates below the equilibrium level? In this paper we show on a sample of 12 open economies that once we account for equilibrium real exchange rate appreciation/depreciation, the natural real interest rate in the 2000s and 2010s is no longer found to be declining to near or below zero. The explicit inclusion of equilibrium real exchange rate appreciation in the identification of the natural rate is the main deviation from the Laubach-Williams approach. On top of that, we use a full-blown semi-structural model with a monetary policy rule and expectations. Bayesian estimation is used to obtain parameter values for individual countries.

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WORKING PAPERS

# The Mirage of Falling *R*-stars

Prepared by Aleš Bulíř and Jan Vlček[1]

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# Contents

# Introduction

Was the recent decline in real interest rates over the past few decades driven by a decline in $r^*$, the natural real interest rate? Or have we observed a long sequence of shocks—from monetary and fiscal policy, demographics, and globalization—that have pushed the market and policy interest rates below the equilibrium level? The answer to this question matters for the conduct of monetary and fiscal policies, past and future. In this paper we argue that once we account for equilibrium real exchange rate appreciation/depreciation, the natural real interest rate in the 2000s and 2010s is no longer found to be declining to near or below zero in most countries. If anything, during the first two decades of the 21st century the natural rate of interest for many advanced and emerging market economies appears to have been stable at around 2-3 percent, with a modest decline in the early 2020s.

The calculation of $r^*$matters a lot for policymaking, even if such an estimation involves considerable uncertainty and central bankers consider a variety of indicators in their deliberations. Fed Chairman Powell (2018) likened monetary policymakers to sailors, navigating by the "stars" when plotting a course for the economy, referring to concepts such as the natural real interest rate, $r$*. Blanchard (2022) wrote: "The mandate of central banks is to set the actual safe real interest rate, $r$, as close as they can to the neutral interest rate, $r^*$, and in so doing, keep output close to potential output."[2] Swiss National Bank Chairman Thomas Jordan (2024) argued that "$r^*$ estimates help in evaluating different monetary policy options". Underestimate the natural rate for your country and your country's monetary conditions will be too loose and likely lead to overheating and higher-than-desired inflation. Overestimate $r^*$ and monetary conditions will be too tight and likely lead to higher unemployment and undershooting of the central bank's inflation target.

What is $r^*$ and how can we measure it? In new Keynesian models the natural real interest, $r^*$, is equivalent to the growth rate of output in the closed-economy Euler equation. Over time, $r^*$ should converge to the long-run average of observed real interest rates, either as the theoretical return absent policies and shocks, or as the actual return without cyclical factors, or simply as the real interest rate path at very low frequencies, perhaps driven by the integration of financial markets across regions (Miranda-Agrippino and Rey 2022). Laubach and Williams (2003) and Holston, Laubach, and Williams (HLW 2017) brought together these two strands of literature by extracting trends in real interest rates and in output growth and tying them together using the closed-economy IS schedule. Their methodology has been replicated for many countries, although most of them are classified as small open economies. The HLW closed-economy approach is problematic for open economies that have received massive capital flows which have (1) influenced the structure of their economies and increased real growth and (2) led to real appreciation of their currencies.

While real interest rates have declined relative to the 1990s and stayed low up until, say 2022, there is little agreement on why $r^*$ should have declined to close to zero. The secular decline has been attributed to four developments (Goodhart and Pradhan 2020): (1) changes in the natural rate of interest; (2) effects of the "savings glut"; (3) easy monetary policy and unconventional monetary policy in the aftermath of the Global Financial Crisis (GFC); and (4) lower and more stable inflation prior to the COVID-19 shock. Goodhart and Pradhan were skeptical that these conditions could prevail in the long run. In contrast, IMF (2023) and

[2] In the literature the terms *natural rate* and *neutral rate* have often been used interchangeably. Our preference is for the former, where the natural real interest rate refers to the real rate of interest that would prevail in a long-run equilibrium where there are no price rigidities or other frictions (Obstfeld 2023).

Blanchard (2022) have argued that once the current inflationary episode has passed interest rates are likely to revert toward pre-pandemic levels in advanced countries. Reis (2022) has reasoned that the long-standing focus on the decline in short-term bond rates, or safe rates, is misplaced and has demonstrated that measures of the return on private aggregate capital have remained roughly constant, or have even increased slightly, since the Global Financial Crisis. Similarly, there is no consensus on whether $r^*$ is rising back to the 1990s level during the post-pandemic period or why. The case for permanently higher natural real rates rests on the exceptional investment needs arising from structural challenges related to the climate transition, the digital transformation, and geopolitical shifts (Schnabel 2024).

Our contribution has three dimensions. First, we extend the equation for the natural rate of interest for appreciation of the equilibrium real exchange rate (the “Penn effect”). To this end, the $r^*$ equation contains both the rate of growth of potential output and equilibrium appreciation. In addition, we use an open-economy IS schedule, which is more relevant for small open economies with $\beta$-convergence. Most economies are not in a steady state and experience some convergence or divergence either in per capita income or in equilibrium real exchange rates vis-à-vis their advanced-economy peers. These convergence gains have been approximated by trend (equilibrium) real appreciation (Samuelson 1994). In countries with equilibrium appreciation, $r^*$ should be lower as nonresidents collect a part of the “convergence gain” through the return on the exchange rate (rather than through higher GDP growth). In contrast, equilibrium depreciation should lower yields in the foreign currency and nonresidents would demand a higher return on country assets, thus pushing $r^*$ up.

Our extensions differ significantly from previous attempts to incorporate the exchange rate component into the HLW framework. While Berger and Kempa (2014) included the exchange rate gap in the Phillips curve, Kupkovič (2020) added it to both the Phillips curve and IS curve. Arena et al. (2020) used the real effective exchange rate gap to explain the residual in the closed-economy equation for the natural rate of interest. To the best of our knowledge, our paper is the first to extended the natural rate of interest equation for the open-economy environment.

Second, we use a full-blown semi-structural model closed by a monetary policy rule, in contrast to the HLW framework, which has no expectations in the model and no policy rule. Third, although we impose a canonical semi-structural model on all sample countries, we use Bayesian estimation to obtain parameter values for individual countries. The equations are modified to capture alternative monetary regimes and features of advanced, emerging market, and low-income countries.

Third, we extend the $r^*$ estimation sample period to 2022Q4, that is, till the end of the pandemic. The Bayesian regressions are run from 2002Q1 to 2019Q4 to obtain the coefficients of interest, however, the Kalman filter estimates are derived for the full-sample period of 2002Q1–2022Q4. The reason for a shorter sample for the Bayesian estimates is that the Metropolis-Hasting procedure become unstable during the pandemic period.

The framework allows us to answer two policy-relevant questions in the small open-economy framework. First, has the natural rate of interest been broadly stable before, during, and after the Great Recession or has if drifted downward? Second, conditional on the natural rate estimates, has monetary policy been too loose in response to the Great Recession and the accompanying low-inflation environment?

Our results suggest that the natural rate of interest has been stable and higher than when estimated using the HLW framework, with the difference being mostly attributable to real equilibrium exchange rate developments. The average $r^*$ difference of about 150 basis points between the HLW framework and the open-economy framework suggests that monetary policy informed by HLW estimates of natural rate of interest may have been too loose during the pre- and post-pandemic period. Only in a couple of countries do we find $r^*$ dipping below the zero lower bound and only at the very end of the sample, at the height of the pandemic.

We begin by outlining our extensions to the HLW framework, Bayesian estimation, and our sample. The next section compares alternative estimates of the natural real rate of interest, comparing our estimates with those obtained using the HLW framework. The penultimate section suggests some policy implications and the final section concludes.

# The Analytical Framework

## The HLW Framework and Its Applications

The unobserved component model proposed by HLW has been the empirical workhorse of the $r^*$ literature. The framework starts with the neoclassical growth model and its Euler equation: the natural rate of interest varies over time in response to shifts in preferences and the growth rate of output. To model $r^*$ as a time-varying process, HLW combine a new Keynesian Phillips curve with an IS schedule, arriving at

$$r_t^* = g_t^* + z_t,$$

where $g_t^*$ is the trend growth rate of the natural rate of output and $z_t$ captures all other determinants of $r^*$, including, for example, the real exchange rate. Empirically, the closed-economy real output gap is modeled as an autoregressive process that is also affected by movements in the short-term real interest rate, $r_t$, and the natural real interest rate, $r^*$. External developments are captured by the rest-of-the-world output gap.

The HLW framework did not explicitly capture the salient features of small open economies, specifically the role of the real exchange rate in output and inflation developments, and it was soon extended.[3] Berger and Kempa (2014) introduced the real effective exchange rate, $q_t$, expressing it as a deviation from its equilibrium level.[4] The exchange rate enters the Phillips and IS schedules and thus affects inflation directly through its first difference, and indirectly through the output gap (Ball 1999). They close the model by tying the real interest rate gap, $\hat{r}_t$, to the real exchange rate gap, $\hat{q}_t$: $\hat{r}_t = \gamma \hat{q}_{t-1} + \kappa_{t-1}$, where the error term, $\kappa_t$, captures time-varying risk premia and other distortions in international capital markets. However, by leaving the equilibrium HLW relationship as $r_t^* = g_t^* + z_t$, the authors create an inconsistency between the exchange rate channel in the IS schedule in the underlying structural model and the HLW relationship.

The modifications of the HLW framework did not question the assumption that $r^*$ depends solely on economic growth. The implication is that fast-growing, emerging market countries exhibit higher $r^*$ by construction and the HLW framework then struggles to explain the decline in observed real rates in emerging market countries. Arena and others (2020) reported that around one-half of the overall estimated

[3] HLW-inspired extensions for small open economies include, for example, Canada (Berger and Kempa 2014), the Czech Republic (Hlédik and Vlček 2018), Denmark (Pedersen (2015), and Slovakia (Kupkovič 2020).

[4] The real exchange rate, $q_t$, is the nominal rate, $s_t$, adjusted for the ratio of the foreign price level, $p_t^F$, to the domestic price level, in logs: $q_t = s_t + p_t^F - p_t$. A negative value of $\Delta q_t$ implies appreciation of the real exchange rate.

decline in $r^*$ since the GFC must be attributed to the behavior of the unexplained component, $z_t$, rather than to a change in $g^*$. In their view the unexplained component is mostly equilibrium real appreciation/depreciation.

We account explicitly for the impact of equilibrium movements in the exchange rate. The “Penn effect” established a link between equilibrium real growth and equilibrium appreciation: a country “converging” in per capita GDP terms experiences real appreciation (Samuelson 1994). The gain from the “convergence process” in West Germany in the 1960s and 1970s, in the Asian Tigers in the 1970s and 1980s, and in the European transition countries in the 1990s and 2000s was split between a faster rate of growth of real GDP and real exchange rate appreciation. De Broeck and Sløk (2006) argued that about ⅖ of the real convergence was realized through equilibrium real appreciation. Ignoring equilibrium appreciation in the closed-economy HLW framework will overestimate the natural rate of interest in fast-growing countries that face equilibrium appreciation, and underestimate *r** in countries with equilibrium depreciation.

## Open-Economy Extension of the HLW Framework

How does the Penn effect relate to $r_t^*$? On the one hand, trend GDP growth accounts for capital yields from production. On the other hand, real appreciation accounts for investment yields realized in foreign currency. Empirically, real exchange rate appreciation has been closely linked to FDI inflows and associated productivity gains (Bulíř and Šmídková 2005; Bini Smaghi 2007; Babecký, Bulíř, and Šmídková 2009) and convergence-related capital inflows (Lipschitz, Lane, and Mourmouras 2006). First, following Hlédik and Vlček (2018) we extend HLW for equilibrium real appreciation, $q_t^*$. Second, we assume no stochastic term in the natural rate equation with persistence:

$$r_t^* = \rho r_{t-1}^* + (1-\rho)\big(2c_1(c_2 g_t^* + (1-c_2)q_t^*)\big), \tag{1}$$

where the contributions of the potential growth and equilibrium exchange rate components add up to unity. We estimate parameters $c_1$ and $c_2$ below and cannot reject the null hypothesis of them being equal to roughly 1 and ½, respectively, in our sample countries (Arena and others 2020, De Broeck and Sløk 2006).

The addition of the real exchange rate variable—in both trends and gaps—necessitates extending the underlying modeling framework also for uncovered interest rate parity and the policy reaction function. The full model therefore resembles the semi-structural quarterly projection model (QPM) used in many central banks—see Berg and others (2006) for a canonical version. The model used here consists of four key equations as follows:

*The Phillips curve*

$$\pi_t = a_1\pi_{t-1} + (1-a_1)\pi_{t+1} + a_2 RMC_t + \varepsilon_t^\pi, \tag{2}$$

where $\pi$ is quarter-on-quarter inflation, $RMC$ are real marginal costs, and $\varepsilon^\pi$ is a cost push shock. The $RMC$ variables are expressed as deviations from the long-term trends, denoted by “^”. Specifically, they are defined as a weighted sum of the output gap, $\hat{y}$, and the real exchange rate gap, $\hat{q}$:

$$RMC_t = a_3\hat{y}_t + (1-a_3)\hat{q}_t. \tag{3}$$

*Aggregate demand*

$$\hat{y}_t = b_1 \hat{y}_{t-1} - b_2 MCI + b_3 \hat{y}_t{}^F + \varepsilon_t^{\hat{y}}, \tag{4}$$

where $MCI$ is the monetary conditions index, $\hat{y}^F$ is the foreign output gap, and $\varepsilon^{\hat{y}}$ is a demand shock. The monetary conditions index combines the real interest rate gap, $\hat{r}$, and the real exchange rate gap, $\hat{q}$, with the weight of $b_4$ of the real interest rate gap and $(1 - b_4)$ of the real exchange rate gap:

$$MCI_t = b_4 \hat{r}_t + (1 - b_4)(-\hat{q}_t). \tag{5}$$

*Uncovered interest rate parity*

$$\begin{aligned} s_t = {} & h_2(s_{t-1} + \Delta s^{TAR}) \\ & + (1 - h_2)\left((1 - e_1)s_{t+1}^e + e_1\big(s_{t-1} + 2(\pi^{TAR} - \pi^{TAR,US} + \Delta q^*)\big) \right. \\ & \left. + \frac{-i_t + i_t^{EA} + prem_t}{4}\right) + \varepsilon_t^s, \end{aligned} \tag{6}$$

where $s$ is the nominal exchange rate, $\Delta s^{TAR}$ is the exchange rate depreciation target, $\pi^{TAR}$ is the domestic inflation target, $i$ is the domestic policy interest rate, $i$ is the domestic policy rate, $i^{EA}$ is the ECB policy rate, $prem$ is the risk premium, and $\varepsilon^S$ is a UIP shock. The specification allows us to model a range of alternative exchange rate regimes, with and without foreign exchange interventions that either smooth the exchange rate trajectory ($e_1$) or target a specific exchange rate trajectory ($h_2$).

*Policy reaction function*

$$i_t = h_1(4(s_{t+1} - s_t) + i_t^{EA} + prem_t) + (1 - h_1)\Big(g_1 i_{t-1} + (1 - g_1)\big((r_t^* + \Delta_4 \pi_{t+3}) + g_2(\Delta_4 \pi_{t+3} - \pi^{TAR}) + g_3 \hat{y}_t\big)\Big) + \varepsilon_t^i, \tag{7}$$

where $\varepsilon^i$ is a monetary policy shock. Again, this extended specification allows us to model rate-setting environments that are either focused on inflation stability ($h_1$=0) or combine inflation and exchange rate stability objectives. In Annex I we provide a complete overview of our parametrization of the model under different monetary policy regimes.

## Estimation

We strive to minimize country-specific adjustments to the model and its coefficients, limiting our ad hoc changes to modifications to the policy rule and to the UIP to reflect the exchange rate and monetary policies. To this end, Bayesian estimation gives us a disciplined rule-based calibration and estimation framework. We start by pre-filtering the foreign data, namely real GDP and the real rate, using the Hodrick-Prescott filter.[5] The rest of the trends and gaps, including the inflation objective/target, are identified in the full model by the Kalman filter jointly with the model parameters, including their standard deviations, which

[5] Prefiltering of the foreign variables ensures that foreign gaps are the same across the sample countries when the Kalman filter is eventually applied.

are identified using Bayesian estimation.[6] The prior distributions of the individual parameters are set at either ¼ or ½ of the initial parameter calibration or the empirical standard deviation of the series (Annex I), with robustenss checks in Annex II. Country-specific results are summarized in Annex III.

## The Sample and Data

Our sample contains 12 countries in or near Europe that trade principally with the euro area countries. Their currencies are either floating or pegged to the euro, covering a range of monetary policy regimes, from floats and inflation targeting through "stabilized regimes" to a currency board, Table 1. The sample is also diverse in terms of economic development—the richest country, Switzerland, has a GDP per capita more than eight times higher than the poorest, Morocco. The rest-of-the-world trading partner is therefore the euro area, and the relevant exchange rate is the domestic currency against the euro.

We use seasonally adjusted quarterly data for our sample countries from 2002Q1 to 2022Q4 from Eurostat and national statistical offices. The series for Türkiye, Serbia, North Macedonia, Bosnia and Herzegovina, and Morocco are slightly shorter, based on availability. Inflation is the annualized first difference of the log of seasonally adjusted core CPI,[7] output is quarterly real GDP in natural logs multiplied by 100, and the exchange rate is the country's real exchange rate in domestic currency terms (a decline denotes appreciation of the domestic currency against the euro). Our measure of the (ex post) average real interest rate is based either on the policy rate or on an interbank rate that has closely mirrored the policy rate (see Annex III for the interest rate definitions for our sample countries). All data are either from Eurostat or from central bank websites.

[6] The identification of the inflation objective begins with the official inflation target, if available. However, the final estimate of the inflation objective is obtained using the Kalman filter as in Ireland (2007). Intuitively, should inflation stay above or below the target for an extended period, the official target may cease to be credible.

[7] The definition of the core CPI is country specific. The estimates using headline inflation were not materially different from those using core inflation. Except Bosnia and Hercegovina where no core inflation was available, and we used headline CPI.

**Table 1. The Sample Countries and Their Characteristics**

Notes: 2021 GDP per capita is measured in purchasing power parity, 2017 international dollars; the real interest rate is measured as the mean of the policy interest rates *minus* the one-period ahead rate of inflation during 2008–2022; IT denotes inflation targeting. The classification of the monetary regime is based on the IMF's 2021 AREAER Database (IMF 2022).

| Country | ISO code | GDP per capita, PPP US$ | Average real interest rate | Exchange rate and monetary framework | Sample period |
|---|---|---|---|---|---|
| Switzerland | CHE | 70,764 | -0.3 | Crawl-like arr. | 2002Q1-2022Q4 |
| Norway | NOR | 64,443 | 0.3 | Float/IT | 2002Q1-2022Q4 |
| Sweden | SWE | 54,240 | -0.2 | Float/IT | 2002Q1-2022Q4 |
| United Kingdom | GBR | 45,989 | -0.7 | Float/IT | 2002Q1-2022Q4 |
| Czech Republic | CZE | 40,917 | -0.9 | Float/IT | 2002Q1-2022Q4 |
| Poland | POL | 34,587 | 1.4 | Float/IT | 2002Q1-2022Q4 |
| Hungary | HUN | 33,863 | 1.8 | Float/IT | 2002Q1-2022Q4 |
| Türkiye | TUR | 31,753 | -2.6 | Float/IT | 2010Q1-2022Q4 |
| Serbia | SRB | 19,695 | 3.3 | Stabilized ER/IT | 2007Q1-2022Q4 |
| North Macedonia | MKD | 16,372 | 6.4 | Soft peg | 2006Q1-2022Q4 |
| Bosnia and Herzegovina | BIH | 14,813 | 1.7 | Currency board | 2015Q1-2022Q4 |
| Morocco | MAR | 8,353 | 1.1 | Peg with a band | 2008Q1-2022Q4 |

Source: IMF, World Economic Outlook Database, October 2022; IMF, Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER), 2021; Eurostat; authors' calculations.

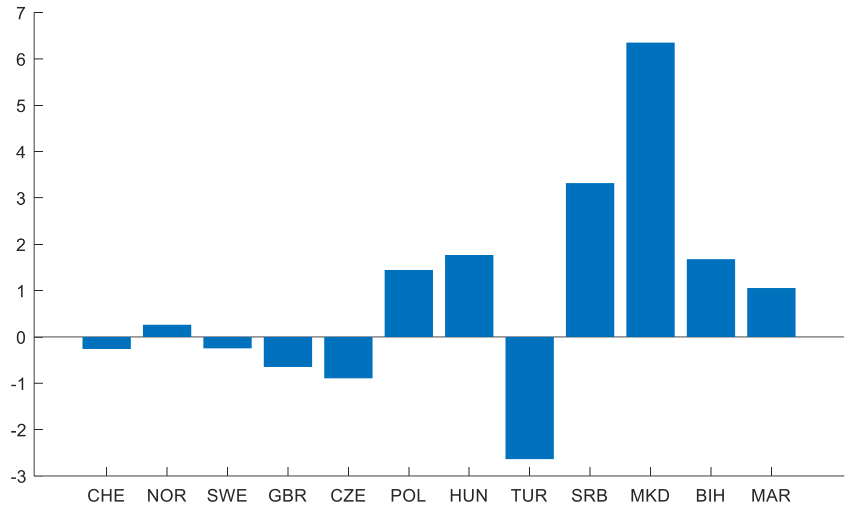
The decline in global real interest rates—policy rates as well as short- and long-term market rates—is a well-established fact (Gamber 2020; Goodhart and Pradhan 2020; and Rogoff, Rossi, and Schmelzing 2022). After peaking at about 4 percent in the early 1990s, real rates have continued their downward trend over the following three decades. After the GFC, real policy and short-term interbank rates dropped close to or below zero in almost all industrial countries.

The sample central banks chose to keep low their post-GFC real interest rates during the pandemic and early post-pandemic period (Figure 1). Excluding Bosnia and Herzegovina with its domestic political instability and correspondingly high sovereign risk premium, only in Serbia and North Macedonia in 2008–2022 did real policy rates stay close to 3 percent, a level that used to be considered appropriate for emerging market countries. In the subsample of EU countries—all with central banks practicing inflation targeting—the average real policy rate was minus 50 basis points. Such low real rates mostly reflect nominal policy rates being brought close to the zero lower bound rather than above-target inflation. Only at the beginning and end of the sample period was the rate of inflation higher than targeted.

**Figure 1. Real Policy Interest Rate, 2008–2022**

(In percent)

Notes: The average ex-post real policy interest rate is calculated as the mean of the nominal policy rate minus one-period ahead inflation. The sample for Türkiye starts in 2010 and that for Bosnia and Herzegovina in 2015.




Source: Eurostat, national central banks; authors' calculations.

Moreover, market participants—such as the respondents to the Consensus Forecast surveys—have expected real interbank and bond rates to stay low well past the pandemic. In each of the four sample countries for which Consensus Forecasts nominal rate projections are available—Norway, Sweden, Switzerland, and the UK—the 2022 average of five-year ahead expectations of real interest rates was well below 1 percent. Five-year ahead projections of interest rates and inflation should be free of cyclical effects, so the real interest rate series are a good proxy for market expectations of the natural rate. It is reasonable to assume that market expectations of $r^*$ would be comparable for the rest of our sample.

# Estimates of the Natural Rate of Interest $(r^*)$

In this section we present our estimates of the natural real interest rate. These fall into two groups: (1) time-invariant estimates (assuming that both trend real output growth and trend real appreciation/depreciation were constant during the sample period); (2) time-varying estimates (assuming that both trends were stochastic). We then compare our results with those of HLW.[8] To avoid polluting our estimates with the COVID-19 shock, our Bayesian estimation of the model parameters employs data up to the first quarter of 2020.

[8] All Matlab codes are presented in an online appendix, available at: https://ales-bulir.wbs.cz/bulir_vlcek_code.zip.

## Time-Invariant Estimates of $r^*$

The time-invariant $r^*$ was positive in all countries and well above 1 percent (Table 2). These estimates are based on a restrictive assumption that both the trend (potential) rate of real output growth, $g^*$, and the trend (equilibrium) real exchange rate appreciation, $q^*$, were constant at the sample average during 2002–2022. The advantage of this assumption is that we can easily compute the sample contribution of potential growth and equilibrium real exchange rate appreciation to the $r^*$ estimate. We will relax the restrictive assumption of constant trend rates of growth in the next section.

Our starting hypothesis that $r^*$ ought to be lower, other things being equal, in countries with equilibrium appreciation—that is, “converging countries” experiencing the Penn effect—is broadly confirmed, see Table 2. But let us mention the outliers first. Poland and Türkiye, the two fast-growing countries, bucked the Penn effect—their currencies depreciated in real terms during the sample period. As a result, their period-average $r^*$ estimates are high at 5.4 percent and 9.5 percent, respectively. Switzerland, the richest country in the sample, experienced persistent real appreciation, as the Swiss franc acted as a “safe haven” currency. The (presumably disequilibrium) appreciation pushed $r^*$ down by 80 basis points.[9]

**Table 2. Country Estimates of Time-Invariant $r^*$ and Its Components, 2002Q1-2022Q4**

(In percent)

Modified $r^*$equation: $\overline{r^*} = 2c_1(c_2\overline{g^*} + (1-c_2)\overline{q^*})$

Estimation period: 2002Q1-2022Q4

Notes: The estimate of the average $\overline{r^*}$ is evaluated at the sample average for potential growth ($g^*$) and appreciation ($q^*$). A negative value of $q^*$ implies real appreciation as the exchange rate is expressed in units of the domestic currency for one euro, with parameters $c_1$ and $c_2$ obtained from the Bayesian estimates. We multiply the second term by 2 to check that we obtain a $c_1$ of 1 as in HLW and to test whether the contribution of equilibrium real appreciation is indeed about ½ (we use the $\beta$-distribution to test the hypothesis that $c_2 = 0.5$). See Appendix I and II for the Bayesian priors and estimation results.

| Country | $r^*$ | $g^*$ | $q^*$ | $c_1$ | $c_2$ |
|---|---|---|---|---|---|
| Switzerland | **0.6** | 1.7 | −1.0 | 0.8 | 0.5 |
| Norway | **2.8** | 1.6 | 0.8 | 1.1 | 0.6 |
| Sweden | **2.8** | 2.1 | 0.8 | 1.0 | 0.4 |
| United Kingdom | **1.9** | 1.4 | 0.6 | 1.0 | 0.4 |
| Czech Republic | **1.5** | 2.8 | −1.1 | 0.9 | 0.5 |
| Poland | **5.4** | 3.8 | 0.7 | 1.1 | 0.6 |
| Hungary | **2.5** | 2.4 | 0.1 | 1.0 | 0.5 |
| Türkiye | **9.5** | 5.7 | 5.0 | 0.9 | 0.5 |
| Serbia | **2.0** | 2.3 | −0.6 | 1.0 | 0.5 |
| North Macedonia | **3.4** | 2.3 | -0.1 | 1.3 | 0.6 |
| Bosnia and Herzegovina | **2.2** | 3.0 | -0.7 | 1.1 | 0.5 |
| Morocco | **2.3** | 3.0 | −0.4 | 1.0 | 0.5 |

Source: Authors’ calculations.

[9] The calculation of the contribution of trend appreciation to $r^*$ is as follows: the equilibrium appreciation of the Swiss franc, $q^*$, is estimated at –1.0 percent per annum, $c_1$ is estimated at 0.8, and $c_2$ is estimated at 0.5. The contribution of $q^*$ to $r^*$ is therefore $2c_1(1-c_2)\overline{q^*}$ or 2*0.8*(1-0.5)*-1 = -0.8 percent, or 80 basis points.

For the rest of the sample, we cannot reject our hypothesis of lower $r^*$ in fast-growing, converging countries as compared to slower-growing, advanced countries. The average $r^*$ for the Czech Republic, Poland, Hungary, Türkiye, Serbia, North Macedonia, Bosnia and Herzegovina, and Morocco is 3.6 percent. However, if we exclude both Türkiye and Poland, it declines further to 2.3 percent. In contrast, the average advanced-economy $r^*$ in Norway, Sweden, and the United Kingdom is about 2.5 percent, largely on account of real depreciation of their currencies.

Two findings stand out. First, we do not observer the sharp decline in $r^*$ reported in past literature. Second, after excluding the three outliers, Switzerland, Poland, and Türkiye, the advanced country estimates of $r^*$ seem to be marginally higher than those of the faster-growing, converging countries, with the difference mostly attributable to the real appreciation-to-depreciation differential.

What is the reason for such markedly different estimates of $r^*$ than those obtained in previous studies? On the one hand, equilibrium real appreciation lowered $r^*$ in some of the converging countries as nonresidents realized higher yields in euro terms. In the Czech Republic, where FDI inflows gradually brought the trade balance into surplus (Bulíř and Šmídková 2005), equilibrium appreciation of more than 1 percent per annum pushed $r^*$ down by a full 100 basis points. On the other hand, equilibrium depreciation would lower yields in euro terms, and nonresidents would correspondingly request higher returns on country assets. We observed this effect, for example, in the UK and Türkiye: long-term depreciation of the pound and the lira pushed the estimate of $r^*$ 54 basis points and 540 basis points higher, respectively.

## Time-Varying Estimates of $r^*$

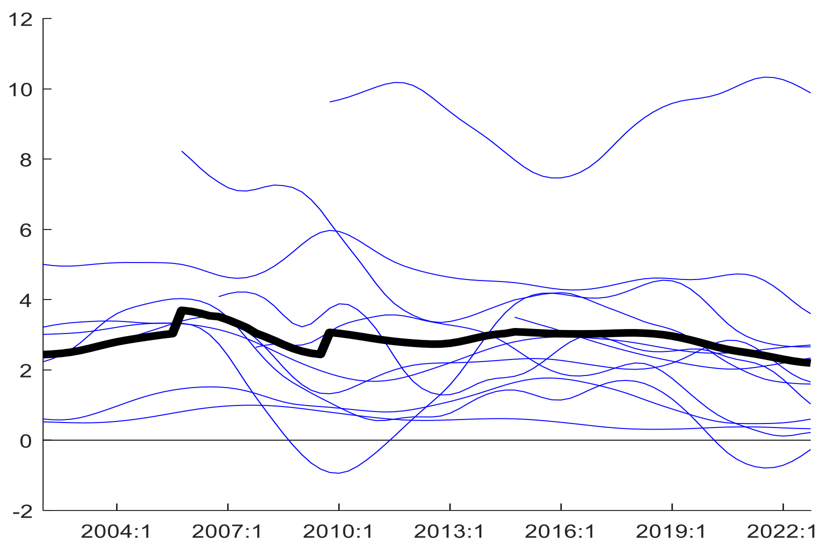
Both the time-varying, individual-country estimates of $r^*$ (Figure 2, thin blue lines) and the sample average (the thick black line) are rather stable. We bring together the market real interest rate, $r$, and the time-invariant and time-varying estimates of $r^*$ in Figure 3. See Annex II for individual-country charts showing the various interest rate variables.

After fluctuating between 2 percent and 3 percent prior to the COVID-19 pandemic, our estimates of average $r^*$ declined to close to 2 percent in the early 2020s. The end-of-sample decline in average $r^*$ can be attributed to the pandemic-induced growth shock; that shock was partly offset by initially lower inflation during 2020-2021, which depreciated the real exchange rate in our sample countries. Recall that consumer price inflation peaked in 2022/2023, hence the negative contribution to $r^*$ from the real appreciation of the national currencies is not yet pronounced in the sample data.

**Figure 2. Time-Varying Estimates of $r^*$, 2002Q1–2022Q4**

(In percent)

Notes: The thick black line is the unweighted mean of $r^*$ in our 12 sample countries; the thin blue lines are individual-country estimates.




Source: Authors' calculations.

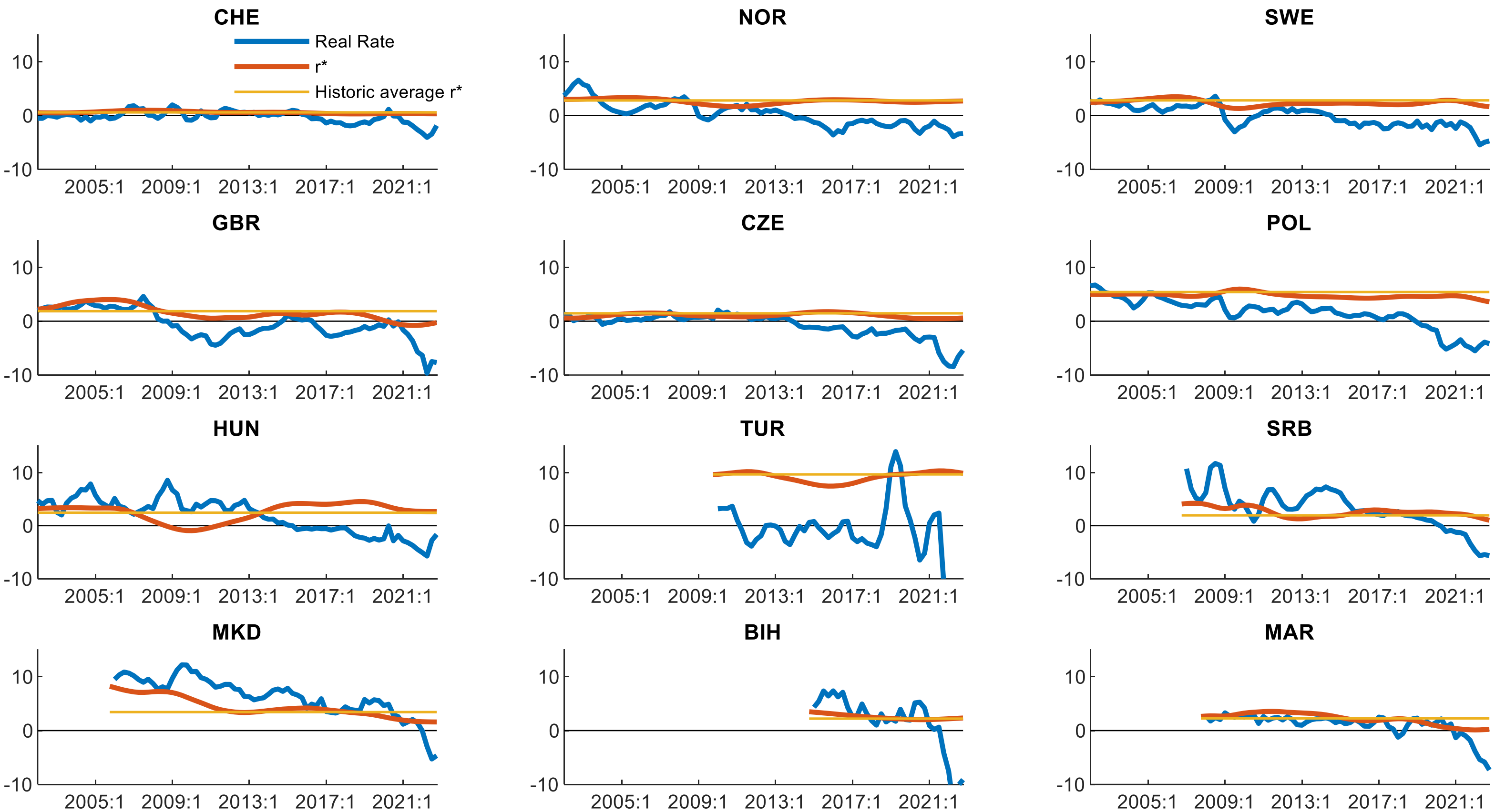
We highlight three key findings. First, the time-varying estimates of $r^*$ are close to or fluctuating around the time-invariant estimates of $r^*$ (Figure 3). This result gives us some assurance that the impact of trend appreciation has been largely stable across time and sample countries. Second, we see only limited evidence of a secular decline in $r^*$. Only in the UK, the Czech Republic, Serbia, and Morocco did $r^*$ trend visibly to or below zero toward the end of the sample. The Swiss $r^*$ estimates were close to zero for most of the sample period. This result has intriguing policy implications—the frequently voiced concerns about economies getting close to the effective lower bound do not seem validated by our estimates. Natural real rates may have been a lot higher than policymakers thought.

Third, in every advanced country the observed real rate has been well below our estimate of the natural rate during the post-GFC period, sometimes to the tune of 70–100 basis points. This was not true for some of the emerging countries: in Serbia, North Macedonia, Bosnia and Herzegovina, and Morocco the observed real rate was very close to or even above $r^*$ until 2015 or even 2020. The observed rates declined well below $r^*$ afterward, however.

**Figure 3. Country-Specific Real Time-Varying Estimate of $r^*$, 2001Q1–2022Q4**

(In percent)

Notes: The blue line is the ex-post real policy rate, the red line is the country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ when both $g^*$ and $q^*$ are set equal to the sample average, that is, the time-invariant estimate of $r^*$.




Source: Authors' calculations.

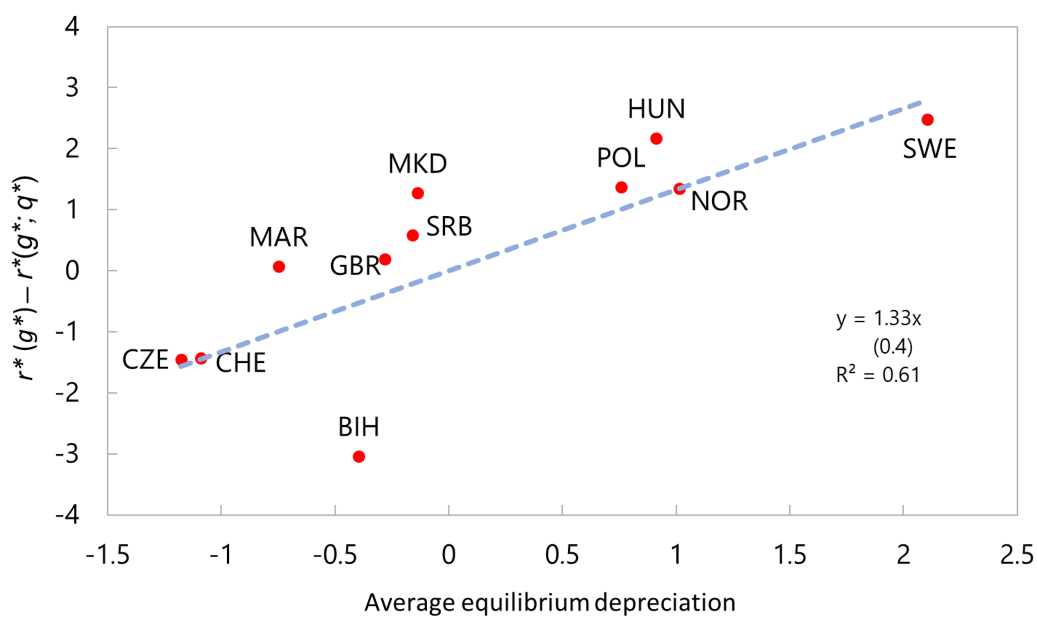
## Robustness Check: Comparison with HLW

The HLW model is a natural benchmark against which to compare our extended framework. We applied the HLW methodology to our sample countries and estimated the natural rate, asking two questions. First, does equilibrium appreciation lower the open-economy estimate of the natural interest rate? Second, does the extended, open-economy framework imply a different "average" real natural interest rate as compared to the closed-economy (HLW) framework?

Regarding the former, we hypothesized that our estimate of $r^*$ should be below the HLW estimate in countries that experience equilibrium real appreciation and above the HLW estimate in countries with equilibrium depreciation.[10] We find clear evidence of this effect for our sample countries during 2013–2019: every percentage point of real equilibrium appreciation ($q^*$) is associated with the natural rate being about 1.3 percentage points lower, Figure 4.[11]

**Figure 4. Exchange Rate Appreciation and *r** Adjustment, 2013–2019**
(Percentage points)

Notes: The horizontal axis measures the mean equilibrium exchange rate, with negative numbers indicating appreciation and positive numbers depreciation. The vertical axis measures the difference between the HLW (closed-economy) estimate of $r^*$, denoted as $r^*(g^*)$ and this paper's (open-economy) estimate of $r^*$, denoted as $r^*(g^*; q^*)$. The point estimate for the intercept was not significantly different from zero, hence the linear regression shown on the chart is estimated without the intercept; the standard error of the point estimate is in parentheses. The sample excludes Türkiye, although its inclusion does not change the slope and increases the coefficient of determination only marginally.




Source: Authors' calculations.

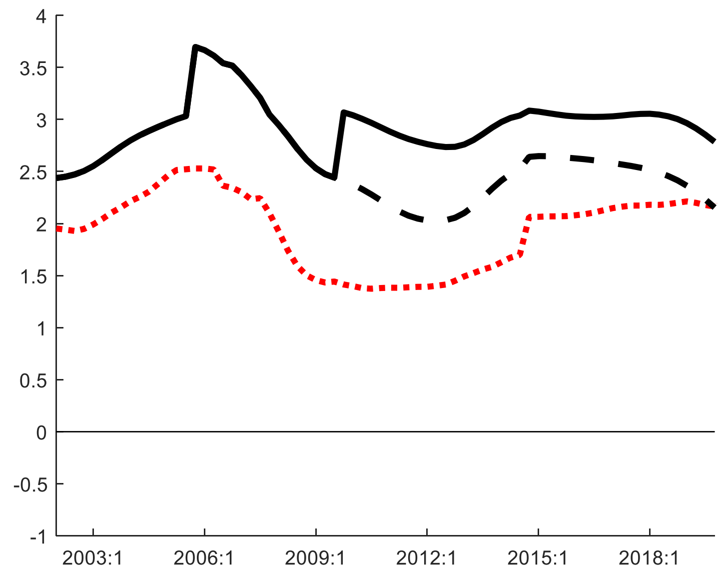
[10] The HLW estimation procedure in R was replicated using their code transformed to Matlab. Our version of the HLW Matlab codes is a part of the package available online.

[11] Extending the sample to 2002–2019 does not change the results materially—the slope coefficient is somewhat steeper at 1.6. Of course, the sample becomes unbalanced as the series for Türkiye, Serbia, North Macedonia, Bosnia and Herzegovina, and Morocco are available from different dates.

Regarding the latter, the magnitude of the difference between the HLW and our estimates of $r^*$ surprised us. We find that the closed-economy HLW estimates of $r^*$ are some 150 basis points lower on average than the open-economy estimates of $r^*$, Figure 5.[12] In our view, the key difference between the HLW and our estimates of $r^*$ stems from the different weights each framework imposes on the actual data. The lower HLW estimates of $r^*$ suggest that their methodology puts a greater emphasis on actual real interest rates. Indeed, to a casual observer, the HLW estimates may look a lot like a moving average of the observed real interest rates, $r$. In contrast, our extended model puts more weight on the inflationary pressures determining $r^*$, the estimates of which can thus deviate significantly from the actual real rates.

**Figure 5. Estimates of $r^*$ Using HLW and Our Methodology, 2002Q1–2019Q4**
(In percent)

Notes: The solid black line replicates the unweighted mean of $r^*$ for our extended, open-economy model from Figure 3 and the dashed black line excludes Türkiye. The dotted red line is the unweighted mean of $r^*$ using the closed-economy HLW methodology, including Türkiye.




Source: Authors' calculations.

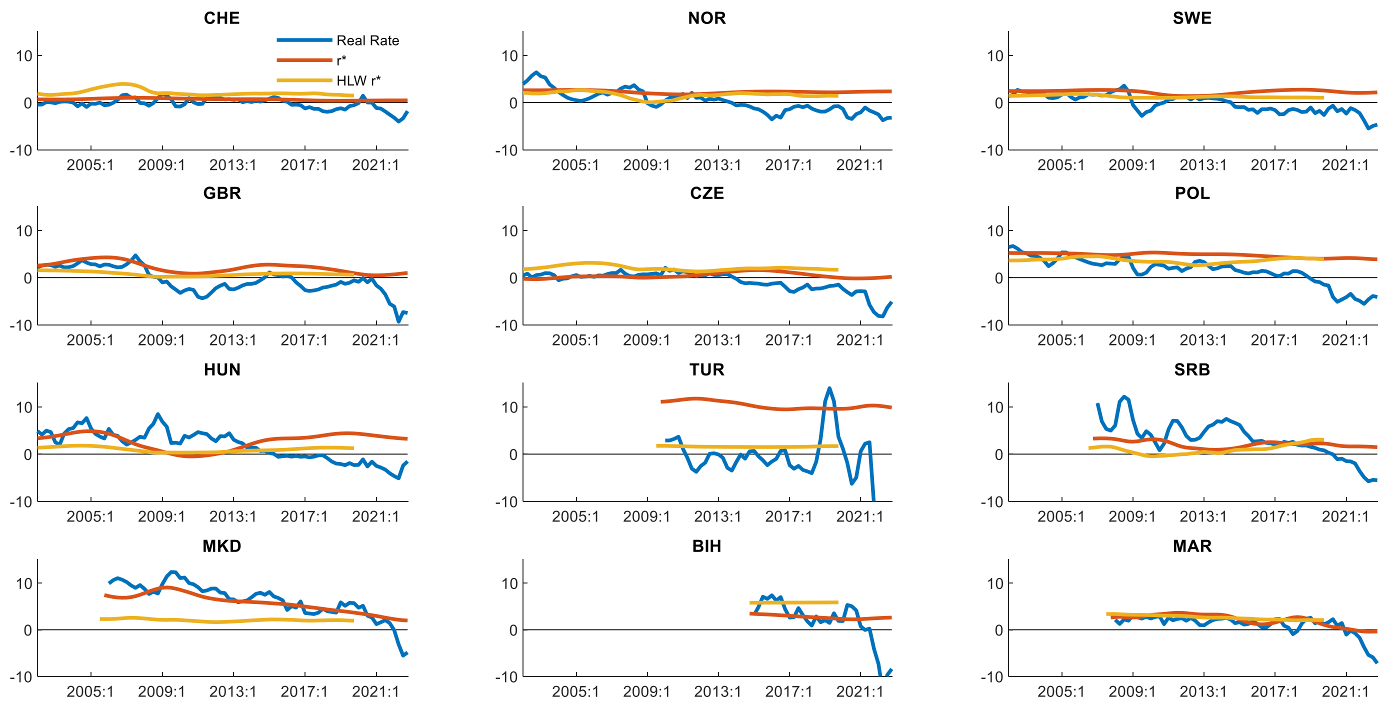
Comparing the individual country estimates of $r^*$, only in Switzerland, the Czech Republic, and Bosnia and Herzegovina do we find the HLW estimate to be consistently and visibly above the open-economy estimate, Figure 6. These three countries recorded, of course, fast trend real appreciation of their domestic currencies, pushing the natural rate downward. For the remainder of the sample, either the two estimates overlap (Morocco and the post-GFC U.K.) or the HLW estimates are lower than the open-economy ones. Interestingly, neither approach suggests a secular decline in $r^*$ after 2000.

[12] Figure 5 shows the results for 2002–2019 only, as the HLW-based simulations become unstable during the COVID period. We shortened the simulation period for our methodology commensurately.

## Figure 6. Country-Specific Real Time-Varying Estimate of $r^*$ Using LHW methodology, 2001Q1–2019Q4

(In percent)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$as defined in Equation 1, and the yellow line is the estimate of $r^*$ based on the HLW methodology.




Source: Authors’ calculations.

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## $r$ and $r^*$: Too Tight or Too Loose?

Looking back, did the sample central banks keep $r$ broadly equal to $r^*$? Figure 6 suggests that the answer depends on the chosen $r^*$ estimation technique and on the chosen time period. On average, we found monetary policy to have been loose ($r^* > r$) in all countries except Serbia and North Macedonia, where the results point to tight monetary policy (Table 3). We quantify the average gap between $r$ and $r^*$ at about *minus* 120 basis points during the full sample period (2002–2022), doubling to *minus* 240 basis points during 2013–2022.[13]

**Table 3. Loose or Tight Monetary Policy: $r$ minus $r^*$**

(In percent)

Notes: Each cell contains the sample period mean of $r$ - $r^*$.

| Country | Our methodology | | HLW methodology | |
|---|---|---|---|---|
| | 2002–22 | 2013–22 | 2002–19 | 2013–19 |
| Switzerland | -0.8 | -1.2 | -2.2 | -2.3 |
| Norway | -2.4 | -4.3 | -0.9 | -2.9 |
| Sweden | -2.5 | -3.8 | -1.0 | -2.2 |
| United Kingdom | -2.2 | -2.7 | -0.8 | -1.9 |
| Czech Republic | -1.9 | -3.5 | -2.1 | -3.2 |
| Poland | -3.2 | -5.0 | -1.2 | -2.5 |
| Hungary | -0.8 | -4.7 | 1.6 | -1.3 |
| Türkiye | -12.1 | -12.8 | -1.3 | -1.2 |
| Serbia | 0.6 | -0.5 | 3.6 | 2.1 |
| North Macedonia | 1.9 | 0.9 | 5.3 | 3.4 |
| Bosnia and Herzegovina | -0.8 | -0.8 | -4.2 | -4.2 |
| Morocco | -1.1 | -1.2 | -0.8 | -0.9 |
| Unweighted average (excluding Türkiye) | -1.2 | -2.4 | -0.2 | -1.4 |

Source: Authors' calculations.

Why have a break in 2013? While it is somewhat arbitrary, we noticed two well-known policy events in 2013 that had a profound impact on interest rates in our sample economies, namely, the "taper tantrum" episode and the ECB's decision to bring its main refinancing rate close to zero in November 2023. These events made it easier and cheaper for financial market participants to obtain liquidity. Consequently, during 2013–2022 only North Macedonia's $r^*$ was higher than the observed real rate under our methodology and also Serbia's under the HLW methodology. Looking only at the EU countries and Switzerland, Norway, and the UK, the average gap between $r$ and $r^*$was a staggering *minus* 360 basis points during 2013–2022. The same-period gap in the three countries that joined European Union in mid-2000 (the Czech Republic, Poland, and Hungary) was even higher at 440 basis points.

[13] The corresponding averages for the HLW methodology are *minus* 20 and *minus* 140 basis points, smaller than the open-economy estimates but pointing in the same direction for all countries.

# Policy Implications

Monetary policy is about guiding real interest rates toward their natural level. Misjudging the natural real interest rate is likely to have long-term consequences for output, unemployment, and inflation, especially if the $r^*$ estimation error is large and long-lasting.

First, our estimates suggest that the sample central banks may have underestimated the value of $r^*$, on average by more than 100 basis points during 2002–2022 and by twice as much after the taper tantrum period. Guided by a low estimate of $r^*$, they kept the policy rate close to the zero lower bound for too long and, hence, $r < r^*$. These estimates are consistent with the narrative that the 2021–2023 period of high inflation had, at least partly, monetary roots (Bordo, Taylor, and Cochrane 2023).

Second, and related to the first point, one can ask whether central banks in small open economies can really set their policy rates independently of the dominant central banks such as the US Federal Reserve System and the European Central Bank (the “dilemma-rather-than-trilemma” argument of Miranda-Agrippino and Rey 2022). Or perhaps these banks can set rates independently but they just stopped trying after the Global Financial Crisis. The hypothesis of a near-zero $r^*$ was repeated often enough by fellow policymakers and was all too convenient for small open economy central bankers to resist its lure.

Finally, our estimates do not support the claim that all the advanced and emerging market economies have been operating close to the effective (nominal) zero lower bound. The estimates suggest that the true natural interest rate—$r^*$ plus expected inflation (or the inflation target)—may have been higher by 100 basis points on average and three times as much for some countries. Only in the UK and the Czech Republic do we observe the open-economy estimates of the natural real interest rate dipping below zero, and even then only during the COVID period.

# Conclusions

This paper challenges the conventional narrative of declining and persistently low real natural rates of interest. We modify the Holston, Laubach and Williams (2017) framework for the effects of equilibrium real exchange rate appreciation or depreciation. Our contribution is to include the real exchange rate (1) as a gap in the IS schedule, as done in previous extensions, but (2) also in the natural rate estimation as the estimated real exchange rate trend. The model parameters are estimated using Bayesian techniques and the latent variables using the Kalman filter.

Three important results emerge. First, we cannot reject the hypothesis that $r^*$ should be lower in countries with currencies appreciating in real terms. Second, the open-economy extension of the original framework suggests that in our sample of 12 countries located in and around Europe the natural real rate of interest has been stable and more than 100 basis points higher than in the original, closed-economy HLW framework. Third, the sizable negative difference between real observed interest rates and real natural interest rates, $r - r^*$, suggests that monetary policy may have been too loose during the 2013–2022 period in most of the sample countries.

# References

Arena Marco, Gabriel Di Bella, Alfredo Cuevas, Borja Gracia, Vina Nguyen, and Alex Pienkowski, 2020, "It is Only Natural: Europe's Low Interest Rates (Trajectory and Drivers)," IMF Working Paper, WP/20/116. Available at: https://www.imf.org/-/media/Files/Publications/WP/2020/English/wpiea2020116-print-pdf.ashx.

Babecký, Jan, Aleš Bulíř, and Kateřina Šmídková, 2009, "Sustainable Real Exchange Rates in the New EU Member States: Is FDI a Mixed Blessing?" *European Economy*, Economic Papers No. 368.

Ball, Lawrence, 1999, "Policy rules for open economies," In: Taylor, J. (Ed.), Monetary Policy Rules. The University of Chicago Press, 129–156.

Berg, Andrew, Philippe Karam, and Douglas Laxton, 2006, "A Practical Model-Based Approach to Monetary Policy Analysis–Overview." IMF Working Paper 06/81, Washington: International Monetary Fund. Available at: http://www.imf.org/external/pubs/ft/wp/2006/wp0681.pdf.

Berger, Tino, and Bernd Kempa, 2014, "Time-varying equilibrium rates in small open economies: Evidence for Canada," *Journal of Macroeconomics*, 39: 203–214.

Bini Smaghi, Lorenzo, 2007, "Real and nominal convergence – policy challenges," paper at the Conference on European Economic Integration, Currency and Competitiveness, Austrian National Bank, Vienna, November 20, 2007. Available at: https://www.bis.org/review/r071121e.pdf.

Blanchard, Olivier, 2022, *Fiscal Policy under Low Interest Rates.* Cambridge, Massachusetts: The MIT Press.

Bordo, Michael D., John B. Taylor, and John H. Cochrane, 2023, *How Monetary Policy Got Behind the Curve—and How to Get Back*, Stanford, California: Hoover Institution Press.

Bulíř, Aleš, and Kateřina Šmídková, 2005, "Exchange rates in the new EU accession countries: What have we learned from the forerunners?" *Economic Systems*, 29: 163–186.

Carare, Alina, and Robert Tchaidze, 2005, "The Use and Abuse of Taylor Rules: How Precisely Can We Estimate Them?" IMF Working Paper 05/148, Washington: International Monetary Fund. Available at: https://www.imf.org/-/media/Websites/IMF/imported-full-text-pdf/external/pubs/ft/wp/2005/_wp05148.ashx.

De Broeck, Mark, and Torsten Sløk, 2006, "Interpreting real exchange rate movements in transition countries," *Journal of International Economics*, 68: 368–383.

Gamber, Edward, 2020, "The Historical Decline in Real Interest Rates and Its Implications for CBO's Projections," Congressional Budget Office Working Paper 2020-09. Available at: https://www.cbo.gov/system/files/2020-12/56891-real%20-interest-rates.pdf.

Goodhart, Charles, and Manoj Pradhan, 2020, *The Great Demographic Reversal: Ageing Societies, Waning Inequality, and an Inflation Revival*, London: Palgrave Macmillan.

Hlédik, Tibor, and Jan Vlček, 2018, "Quantifying the Natural Rate of Interest in a Small Open Economy – The Czech Case," Czech National Bank Working Paper, No.7. Available at:

https://www.cnb.cz/export/sites/cnb/en/economic-research/.galleries/research_publications/cnb_wp/cnbwp_2018_07.pdf.

Holston, Kathryn, Thomas Laubach, and John C.Williams, 2017, "Measuring the natural rate of interest: International trends and determinants." *Journal of International Economics*, 108: 59–75.

International Monetary Fund (IMF), 2022, *Annual Report on Exchange Arrangements and Exchange Restrictions 2021*, Washington: International Monetary Fund. Available at: https://www.elibrary.imf.org/downloadpdf/book/9781513598956/9781513598956.xml.

International Monetary Fund (IMF), 2023, "Natural Rate of Interest: Drivers and Implications for Policy." Chapter 2 in *World Economic Outlook,* April 2023. Available at: https://www.imf.org/-/media/Files/Publications/WEO/2023/April/English/ch2.ashx.

Ireland, Peter, 2007, "Changes in Federal Reserve's inflation target: Causes and Consequences," *Journal of Money, Credit and Banking*, 39: 1851–1882.

Jordan, Thomas J., 2024, "The natural rate of interest (r*) as a reference point for monetary policy – a practitioner's view," Keynote Speech at the 2024 Bank of Korea International Conference; Seoul, May 30, 2024. Available at: https://www.snb.ch/en/publications/communication/speeches/2024/pre_20240530_tjn

Kupkovič, Patrik, 2020, "R-star in transition economies: Evidence from Slovakia," National Bank of Slovakia Working Paper, No.3. Available at: https://nbs.sk/_img/documents/publik/wp_3_2020_kupkovic_r_star_in_transition_economies_en.pdf.

Laubach, Thomas, and John C. Williams, 2003, "Measuring the Natural Rate of Interest." *The Review of Economics and Statistics*, 85: 1063–1070.

Lipschitz, Leslie, Timothy Lane, and Alex Mourmouras, 2006, "Capital Flows to Transition Economies: Master or Servant?", *Czech Journal of Economics and Finance*, 56: 202–222. Available at: https://journal.fsv.cuni.cz/storage/1054_s_202_222.pdf.

Miranda-Agrippino, Silvia, and Hélène Rey, 2022, *The Global Financial Cycle*, Handbook of International Economics 5, Amsterdam, Netherlands: North Holland Publishing Company.

Obstfeld, Maurice, 2023, "Natural and Neutral Real Interest Rates: Past and Future," NBER Working Paper 31949. Available at: https://www.nber.org/papers/w31949.

Pedersen, Jesper, 2015, "The Danish natural real rate of interest and secular stagnation," Danmarks Nationalbank Working Papers, No. 94. Available at: https://www.nationalbanken.dk/en/publications/Documents/2015/03/DNWP_94.pdf.

Powell, Jerome H., 2018, "Monetary Policy in a Changing Economy," speech at *Changing Market Structure and Implications for Monetary Policy*, a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, August 24, 2018. Available at: https://www.federalreserve.gov/newsevents/speech/powell20180824a.htm.

Reis, Ricardo, 2022, “Has monetary policy cared too much about a poor measure of r-star?” a paper presented at LSE, August 2022. Available at: https://abfer.org/media/abfer-events-2022/ampf/Ricardo-Reis-ampf-paper.pdf

Rogoff, Kenneth, Barbara Rossi, and Paul Schmelzing, 2022, “Long-Run Trends in Long-Maturity Real Rates 1311–2021,” NBER Working Paper 30475. Available at: https://www.nber.org/system/files/working_papers/w30475/w30475.pdf.

Samuelson, Paul A., 1994, “Facets of Balassa-Samuelson Thirty Years Later,” *Review of International Economics*, 2: 201–26.

Schnabel, Isabel, 2024, “R(ising) star?”, Speech at The ECB and its Watchers XXIV Conference session on: Geopolitics and Structural Change: Implications for Real Activity, Inflation and Monetary Policy, Frankfurt, March 20, 2024. Available at: https://www.ecb.europa.eu/press/key/date/2024/html/ecb.sp240320_2~65962ef771.en.html.

Annex I

# I. The Model and Its Bayesian Estimation

In this annex we describe the model structure in detail and discuss our Bayesian estimation. The model is taken to the country data by combining calibration and Bayesian estimation techniques.

First, based on the AREAER database (IMF 2022), we assign each country the relevant monetary policy and exchange rate regime, choosing parameters $h_1$ and $h_2$ in the UIP and the monetary policy reaction function, Table A.2. Second, we calibrate the remaining parameters in the equations for the UIP, the monetary policy reaction function, and the natural rate equation function identically for all countries, conditional on the monetary regime, and compute the standard deviations of shocks in these equations from the data. We choose this strategy (1) because it is tricky to estimate a policy rule from the data (Carare and Tchaidze 2005) and (2) to ensure that the shape and calibration of the policy rule is not binding for the identification of the natural rate. Third, we use Bayesian estimation to obtain all the remaining parameters and standard deviations of shocks in the Phillips curve and IS curve. Fourth, we use the Kalman filter to filter all the data and derive the unobservable variables, namely, the trends and gaps in real interest rates, the exchange rate, and output.

**Data**

The observed data encompass four variables: the short-term policy interest rate or an interbank rate mirroring the policy rate (Table A1), real GDP, the exchange rate against the euro, and core CPI.

Table A1. Interest Rate Definitions

| Country | Name of the series | Type | Source |
|---|---|---|---|
| Switzerland | SNB 3-Month LIBOR Target Rate | Interbank | BIS |
| Norway | Deposit Rate | Policy | BIS |
| Sweden | Repo Rate | Policy | BIS |
| United Kingdom | Official Bank Rate | Policy | BIS |
| Czech Republic | 3M PRIBOR | Interbank | CNB[1] |
| Poland | Reference Rate | Policy | BIS |
| Hungary | Hungary: Average Base Rate | Policy | BIS |
| Türkiye | 1-Week Repo Rate | Policy | BIS |
| Serbia | 1-Week Repo Rate | Policy | BIS |
| North Macedonia | Overnight Lending Rate | Policy | FSI |
| Bosnia and Herzegovina | Overnight Lending Rate | Policy | FSI |
| Morocco | Monetary Policy Rate | Policy | BAM[1] |

[1] CNB stands for the Czech National Bank and BAM for the Bank Al-Maghrib.

**The Model, Its Initial Calibration, and Bayesian Priors**

We list the model equations and prior distributions of all the parameters in Table A2. If a parameter is not estimated, only the mean of its calibration is reported, and the type and prior distribution cells are empty. Priors for estimated and non-estimated parameters are identical across all sample countries. The exception are the priors for the standard deviations of supply and demand shocks, which are set according to the variability observed in the country data. For example, std($\hat{\pi}^{obs}$) denotes the standard deviation of detrended CPI core inflation for each country and std($\widehat{\Delta y}^{obs}$) denotes that of the detrended growth rate of real GDP. We use the

Hodrick-Prescott filter to detrend of the variables. Country-specific setting of standard deviations is needed, as the differences in the actual data variability are large and the standard deviations help us determine the signal-to-noise ratios.

Table A.2 reports the mean and the standard deviation of the prior for each parameter estimated. The standard deviation of the prior is reported as multiple of its mean. In other words, this product indicates whether we have imposed a “tight” or a “loose” prior. For example, parameter $a_3$ has a prior with a mean of ½ and a standard deviation of 0.5, which we report as ½ 0.5. This is a relatively “loose” prior relative to others.

The parameter denoting inflation persistence in the Phillips curve, $a_1$, is calibrated to 0.5 for all countries. The calibration reflects (1) the standard micro foundations of the underlying model and assumed Calvo pricing with full backward indexation, and (2) the fact that the parameter cannot be identified along with $a_2$ due to their observational equivalence.

Table A2. Model Equations (cont.)

| Equation | Prior distribution | | |
|---|---|---|---|
| | Type | Mean | Standard deviation |
| **Phillips curve**<br>$\pi_t = a_1\pi_{t-1} + (1-a_1)\pi_{t+1} + a_2 RMC_t + \varepsilon_t^{\pi}$<br>$RMC_t = a_3\hat{y}_t + (1-a_3)\hat{q}_t$ | | | |
| $a_1$ | - | 0.5 | - |
| $a_2$ | Inv. $\gamma$ | 0.2 | 0.2 |
| $a_3$ | $\beta$ | 0.5 | ½ 0.5 |
| standard deviation of $\varepsilon^{\pi}$ | Inv. $\gamma$ | ¼std($\hat{\pi}^{obs}$) | ½ or ¼ std($\hat{\pi}^{obs}$) Standard deviation during COVID period (2021Q1–2022Q4) is set six times higher than before COVID |
| | | | |
| **IS curve**<br>$\hat{y}_t = b_1\hat{y}_{t-1} - b_2 MCI + b_3\hat{y}_t{}^F + \varepsilon_t^{\hat{y}}$<br>$MCI_t = b_4\hat{r}_t + (1-b_4)(-\hat{q}_t)$ | | | |
| $b_1$ | $\beta$ | 0.7 | ¼ 0.7 |
| $b_2$ | $\gamma$ | 0.1 | ¼ 0.1 |
| $b_3$ | $\gamma$ | 0.15 | ¼ 0.15 |
| $b_4$ | $\beta$ | 0.6 | ¼ 0.6 |
| standard deviation of $\varepsilon^{\hat{y}}$ | Inv. $\gamma$ | ½ std($\widehat{\Delta y}^{obs}$) | ½ std($\widehat{\Delta y}^{obs}$) |
| | | | |
| **Natural rate**<br>$r^* = \rho r_{t-1}^* + 2(1-\rho)c_1(c_2 g_t^* + (1-c_2)q_t^*)$ | | | |
| $\rho$ | - | 0.85 | - |
| $c_1$ | $\gamma$ | 1 | 1⁄15 1 |
| $c_2$ | $\beta$ | 0.5 | 1⁄15 0.5 |
| | | | |
| **UIP**<br>$s_t = h_2(s_{t-1} + \Delta s^{TAR}) + (1-h_2)\left((1-e_1)s_{t+1}^e + e_1\left(s_{t-1} + 2(\pi^{TAR} - \pi^{TAR,US} + \Delta q^*)\right) + \frac{-i_t + i_t^{EA} + prem_t}{4}\right) + \varepsilon_t^s$ | | | |
| $h_2$ | - | 1 for float or similar ER regimes (CHE, NOR, SWE, GBR, CZE, POL, HUN, TUR, SRB) and 0 for peg (BIH, MKD, MAR) | - |
| $e_1$ | - | 0.6 | - |
| standard deviation of $\varepsilon^s$ | - | ½ std($\widehat{\Delta s}^{obs}$) | - |

Table A1. Model Equations (concluded)

| Equation | Prior distribution | | |
|---|---|---|---|
| | Type | Mean | Standard deviation |
| **Monetary policy reaction function** $i_t = h_1\big((s_{t+1} - s_t)4 + i_t^{EA} + prem_t\big) + (1 - h_1)\Big(g_1 i_{t-1} + (1 - g_1)\big((r_t^* + \Delta_4\pi_{t+3}) + g_2(\Delta_4\pi_{t+3} - \pi^{TAR}) + g_3\hat{y}_t\big)\Big) + \varepsilon_t^i$ | | | |
| $h_1$ | - | 0 for float or similar ER regimes (CHE, NOR, SWE, GBR, CZE, POL, HUN, TUR, SRB) and 1 for peg (BIH, MKD, MAR) | - |
| $g_1$ | - | 0.75 | - |
| $g_2$ | - | 1.25 | - |
| $g_3$ | - | 0.25 | - |
| standard deviation of $\varepsilon^i$ | - | 4 std($\hat{\imath}^{obs}$) In aftermath of GFC, std is set two times higher than during 2010Q1–2022Q4 | - |

**The Bayesian Estimation**

To avoid polluting our estimates with the COVID-19 shock, our Bayesian estimation of the model parameters employs data up to the first quarter of 2020. The posterior distribution of the estimated parameters is constructed using Metropolis-Hasting simulation with 100,000 draws. Country-specific results, including all country-specific priors and posteriors are presented Annex II.

From the Bayesian estimation we obtain the relative weights of potential growth and equilibrium appreciation on $r^*$, $c_1$ and $c_2$ as in Equation 1. The parameter estimates come out as roughly equal to ½ for all countries, adding up to the HLW proportional relationship between $r^*$ and $g^*$. These shares are also broadly consistent with Arena and others (2020), who were looking for additional factors to explain the large $z_t$ residual during the post-GFC period.

**The Kalman Filter**

In the final step, we use the Kalman smoother to filter the historical data and obtain the latent variables. All the available data, including those for the COVID-19 covid period of 2020–2022, are used. The estimates presented are the smoothed states.

Annex II

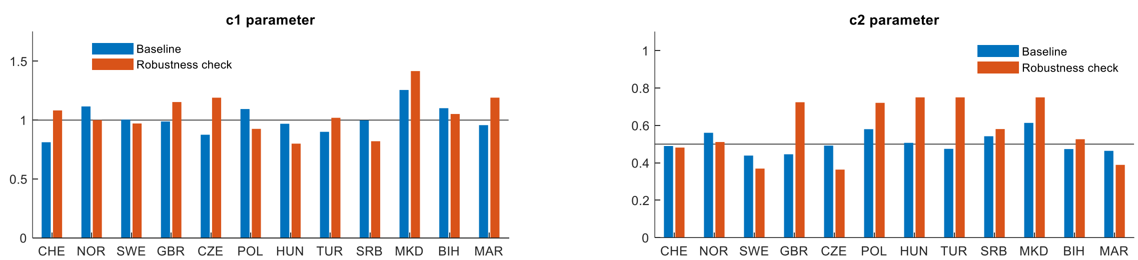
# II. Sensitivity Analysis

Estimated parameters $c_1$ and $c_2$ drive the identification of the natural rate and, for the purposes of Bayesian estimation, we started with the standard deviation of the prior being relatively small. That is, we treat these priors as “tight” ones. This both reflects the theoretical foundations of the Penn effect and provides sufficient information for the estimation procedure. This annex evaluates the implications of less tight priors of these parameters for their posterior estimates and for the natural rates identified. In summary, we find that our results are only marginally affected by an alternative choice of key priors.

Making the priors for $c_1$ and $c_2$ less tight has only a limited impact on the estimated parameters, Figure II.1. To make the “tight” priors looser, we increased the standard deviations to five times the original calibration. The table shows that the differences between the two sets of estimated parameters are not one-sided—the estimates with looser priors are larger for some countries and smaller for others. On average, the $c_1$ parameters are different by less than 20 percent and the $c_2$ parameters by about 30 percent.

Figure II.1 Sensitivity Analysis of $c_1$ and $c_2$

Note: The original parameters (blue bars) are estimated with tight priors as in Table A.2. The relaxed-prior estimates (red bars) assume that the standard deviations of the prior are five times as large.

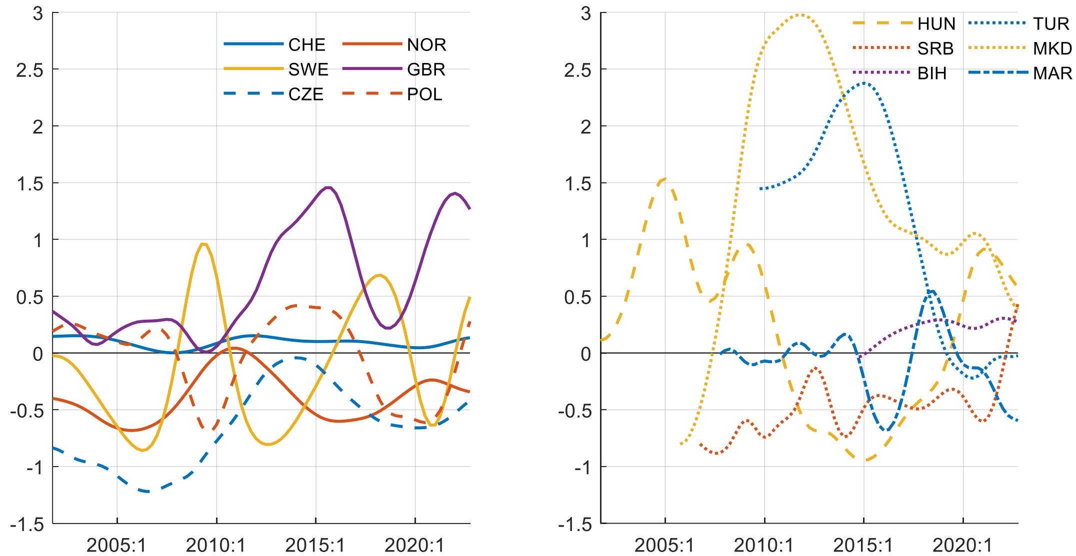



Source: Authors’ calculations.

Predictably, relaxing the priors makes the estimates of $r^*$ somewhat more volatile, but the differences are not large, Figure II.2. We highlight two observations. First, the differences are concentrated mostly within $\pm$100 basis points for all countries except North Macedonia and Türkiye. In the UK, the Czech Republic, and Hungary the differences are also occasionally outside this narrow band. Second, for one-half of the sample the relaxed priors result in a more volatile estimate of $r^*$, while for the rest we notice a directional shift. Specifically, the relaxed priors push the $r^*$ estimate down in the Czech Republic (by about 75 basis points), Norway (by less than 50 basis points), and Serbia (by about 50 basis points). The post-GFC estimates of $r^*$ are about 100 basis points higher in the U.K. and even higher in North Macedonia and Türkiye.

Figure II.2 Differences Between the $r^*$ Estimates with "Tight" and "Loose" Priors

Note: We use the new $c_1$ and $c_2$ parameters to recalculate the natural rate for our sample countries under the relaxed priors and compare them with the baseline estimates. The lines depict the difference between the original estimate of $r^*$ (under the tight priors) and the alternative estimate of $r^*$ (under the relaxed priors).



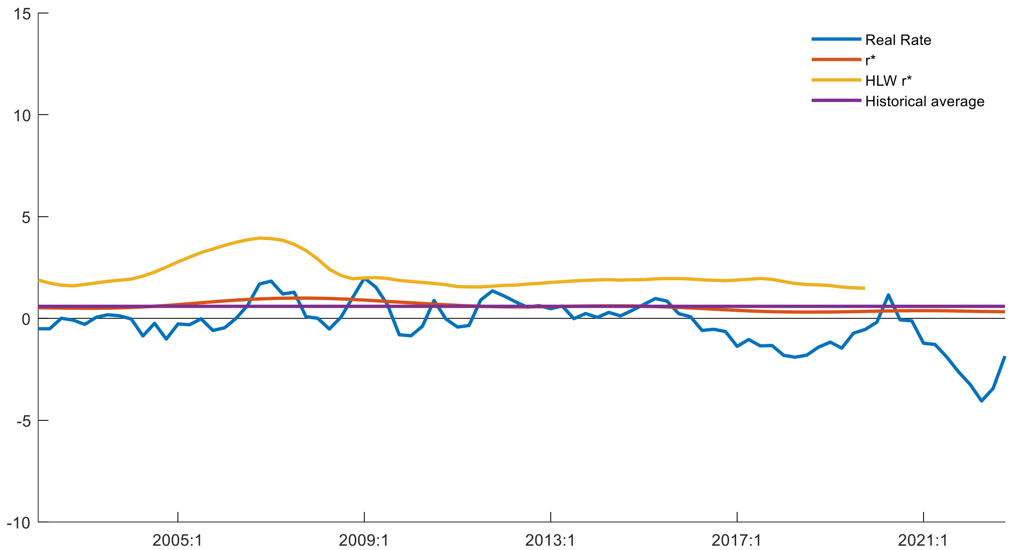

Source: Authors' calculations.

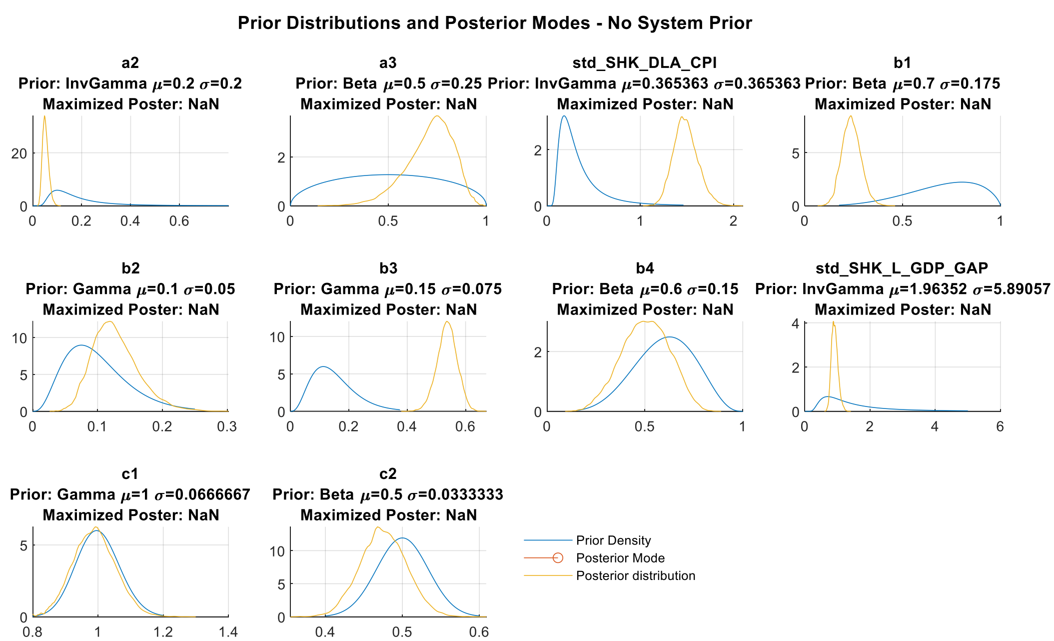
## III. Country-Specific Results

This annex summarizes both the HLW and our extended-model estimates of the natural rate of interest for all 12 sample countries and prior and posterior modes from each Bayesian estimation. The historical average for $r^*$ corresponds to the time-invariant estimate of the natural rate.

Figure A.3 Switzerland: Alternative Estimates of $r^*$ and Bayesian Estimation

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

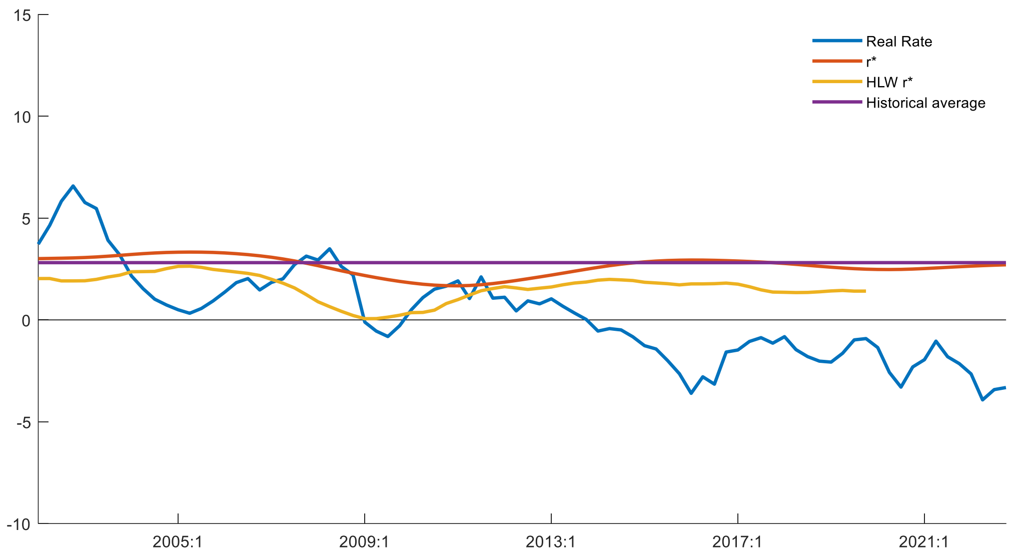


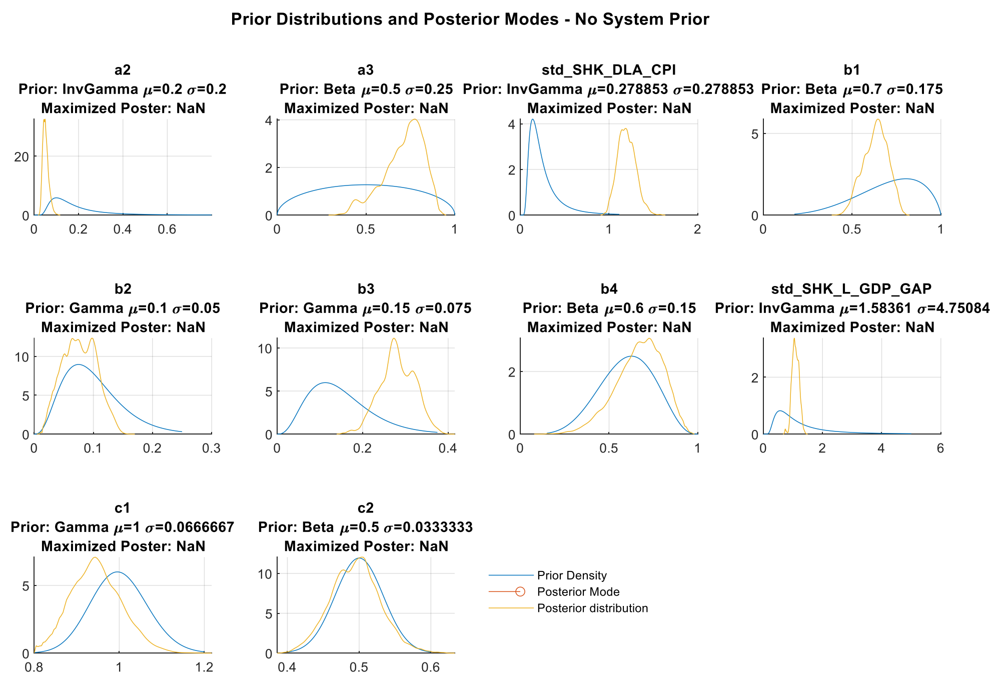




Source: Authors' calculations.

Figure A.3 Norway: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

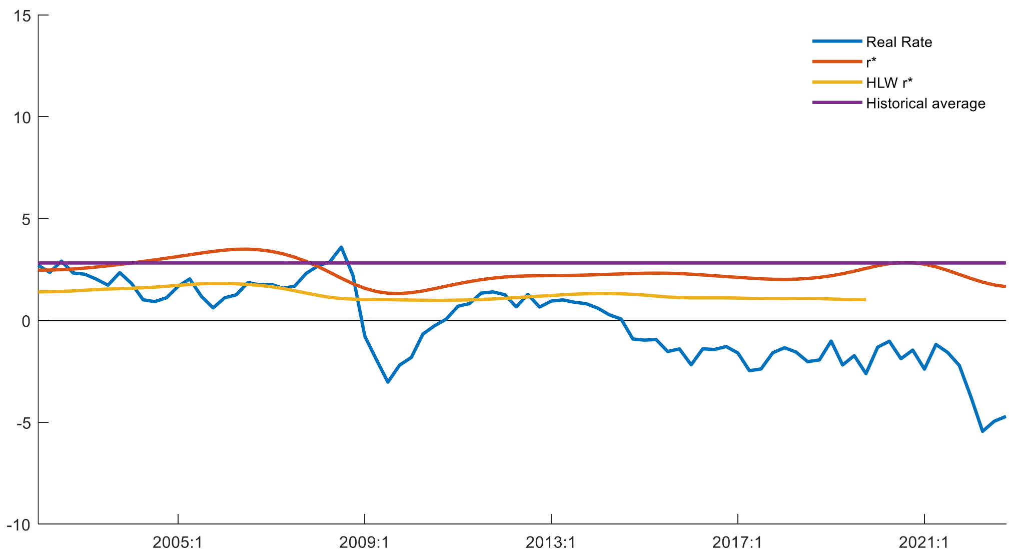


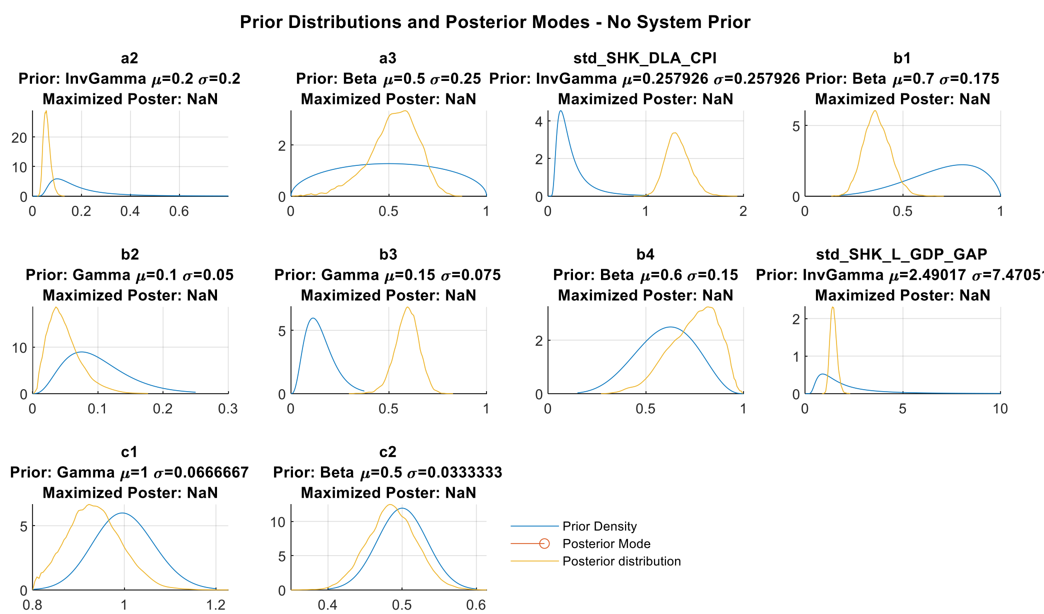




Source: Authors' calculations.

Figure A.3 Sweden: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

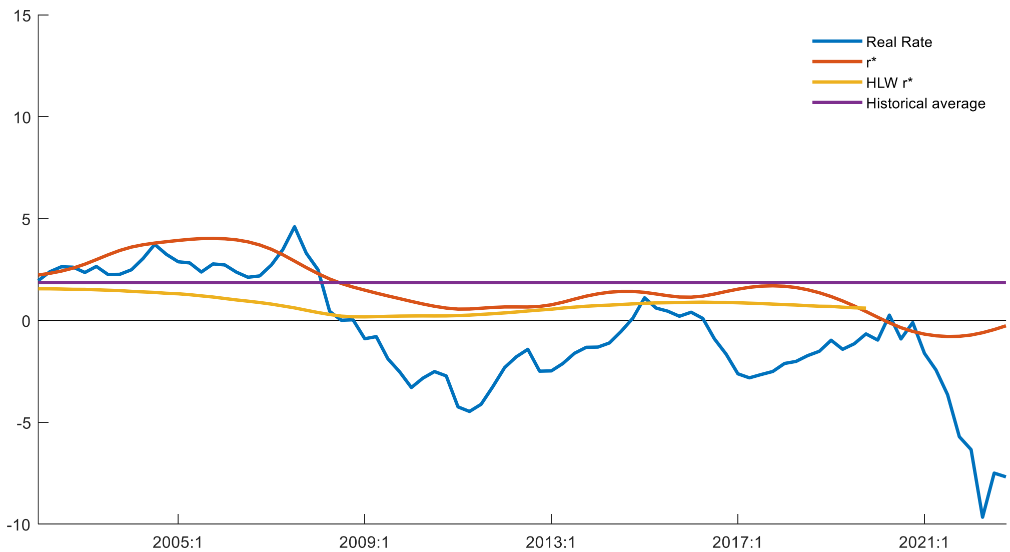


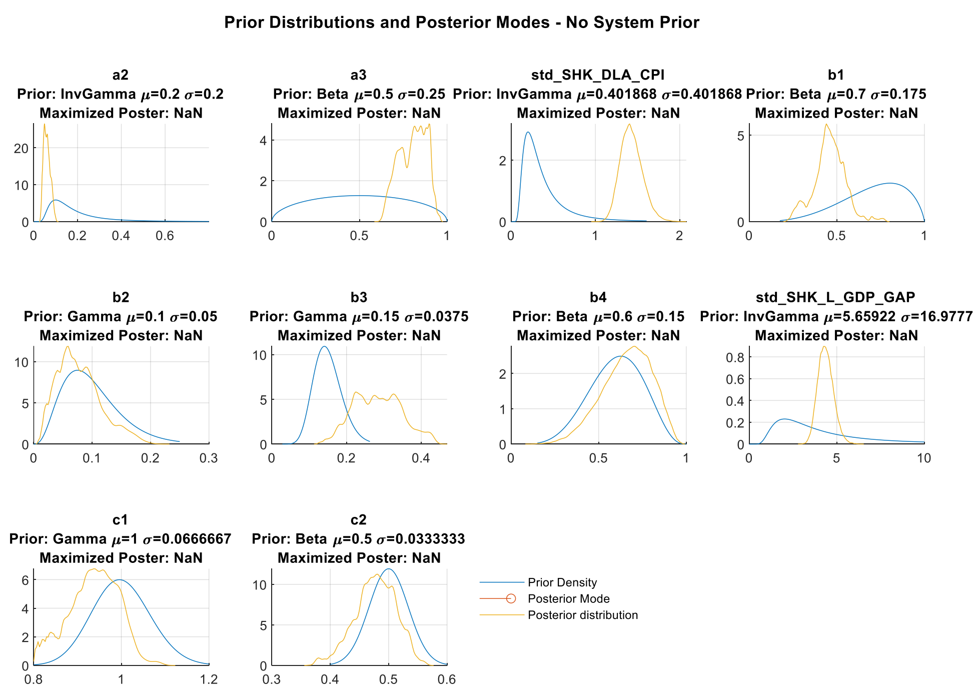




Source: Authors' calculations.

Figure A.3 United Kingdom: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

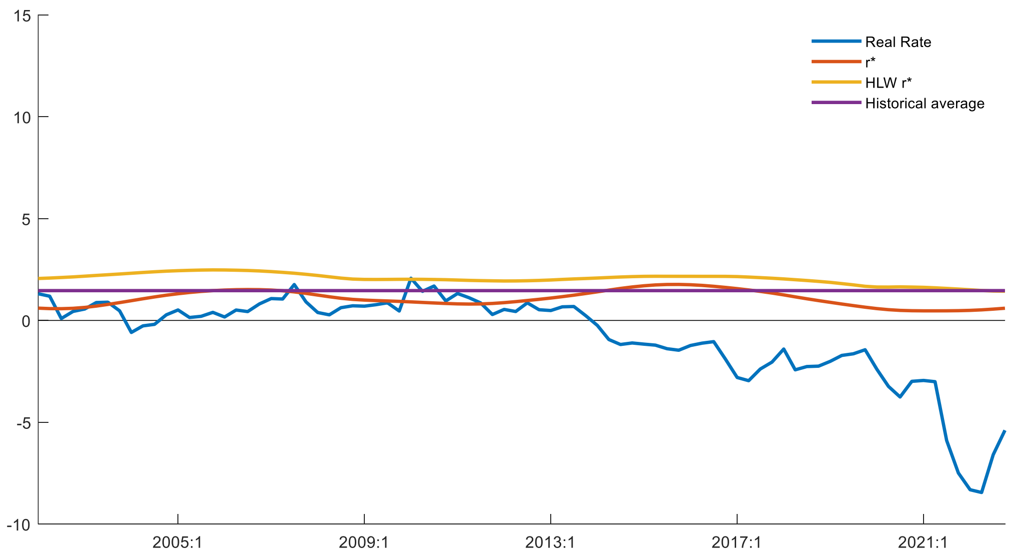


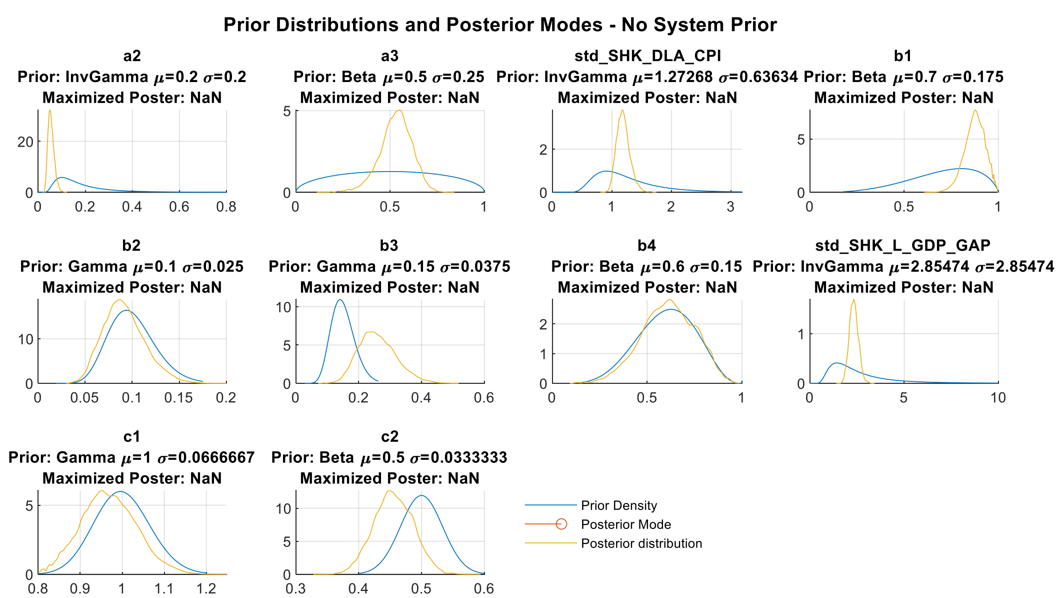




Source: Authors' calculations.

Figure A.3 Czech Republic: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

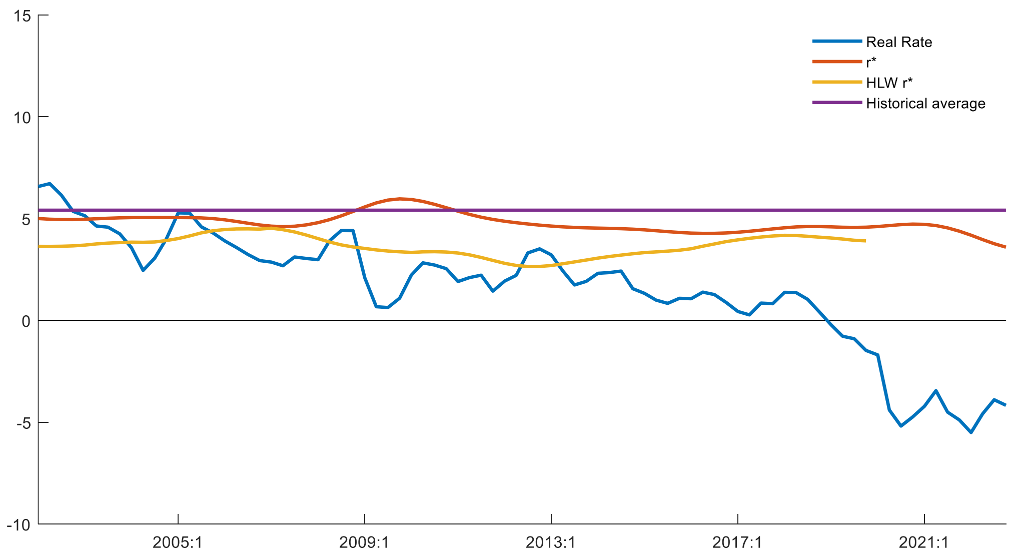


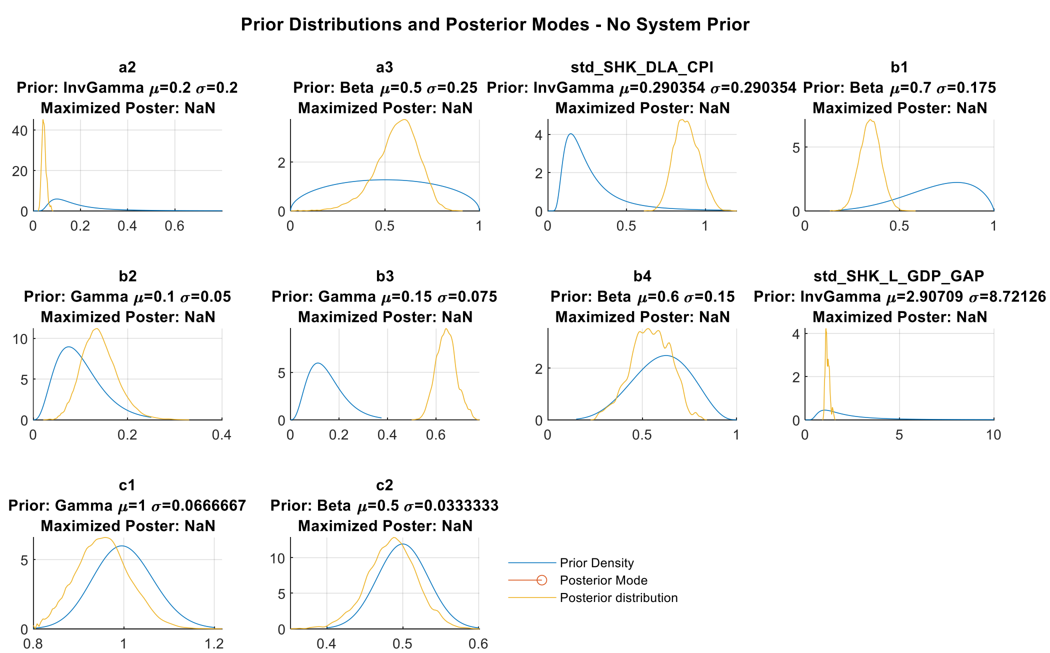




Source: Authors' calculations.

Figure A.3 Poland: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

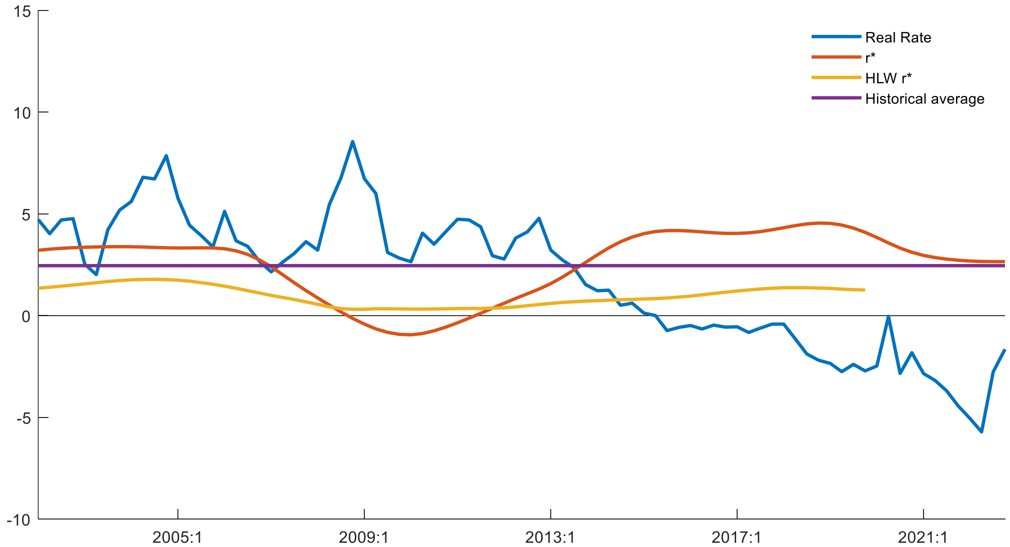


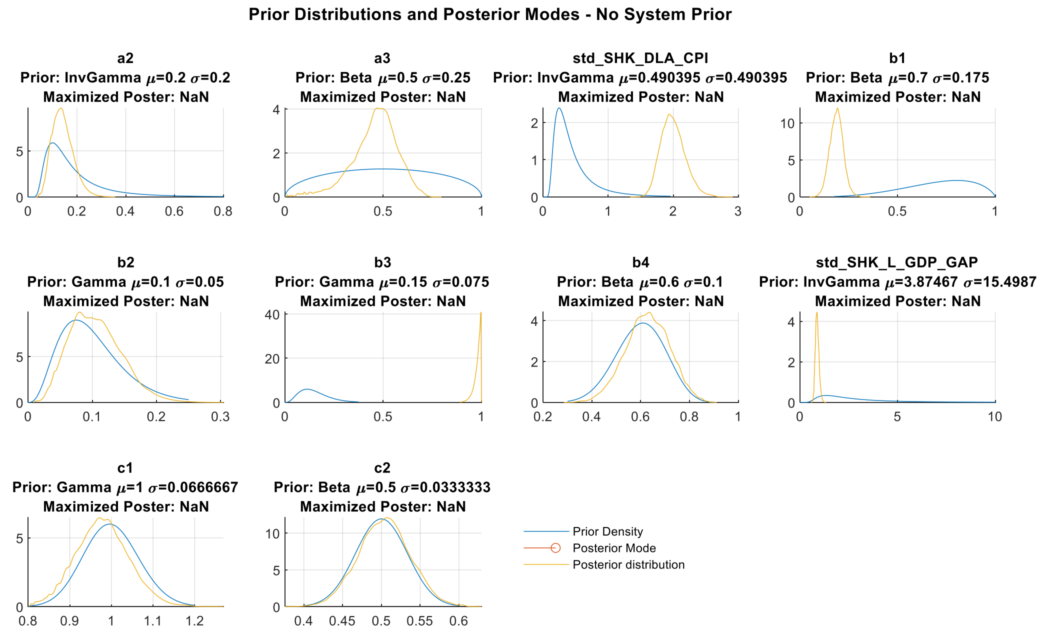




Source: Authors' calculations.

Figure A.3 Hungary: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

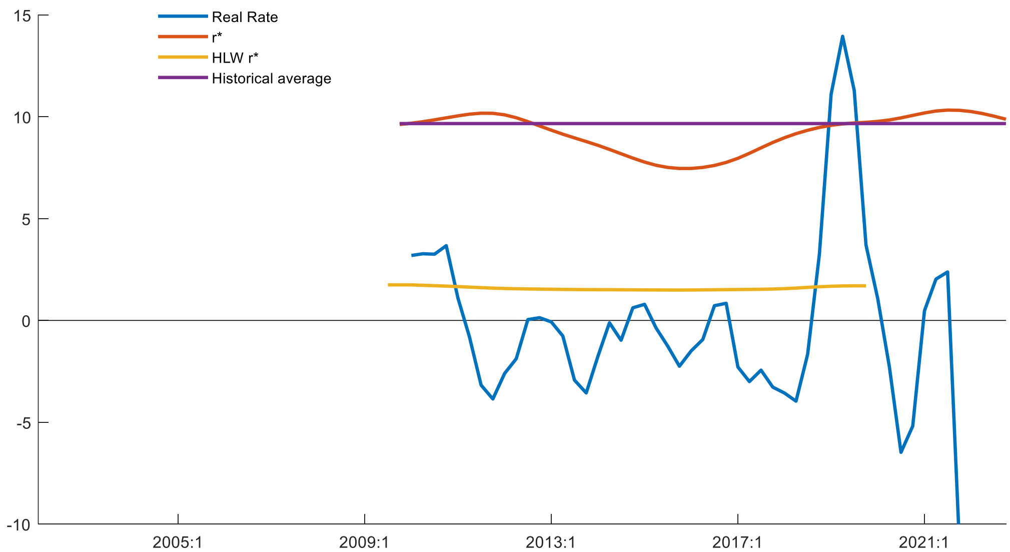


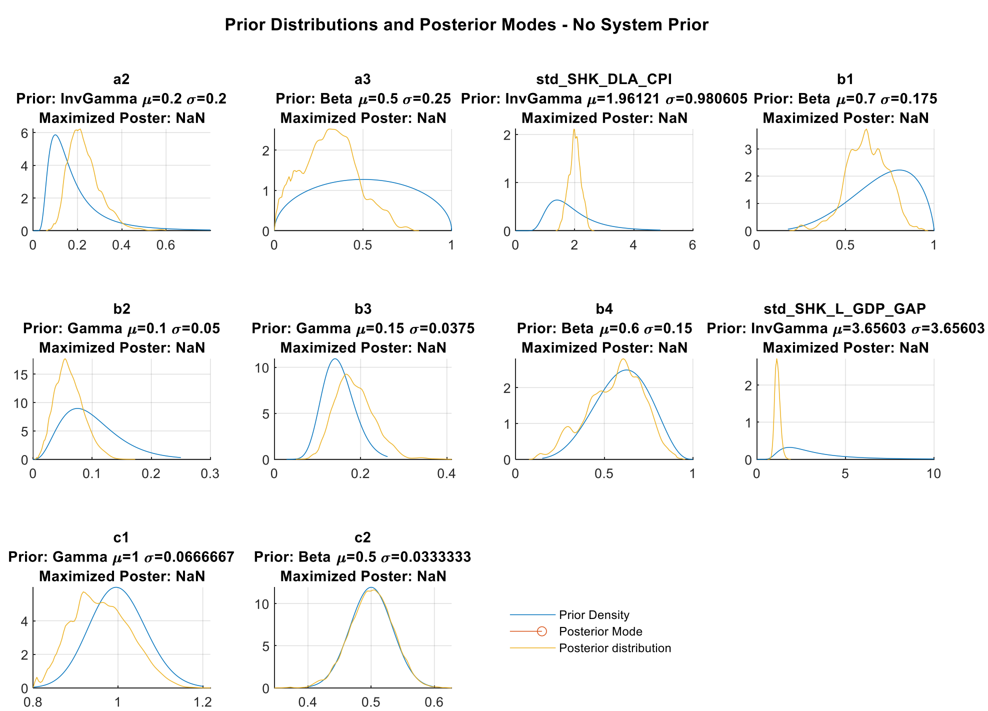




Source: Authors' calculations.

Figure A.3 Türkiye: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

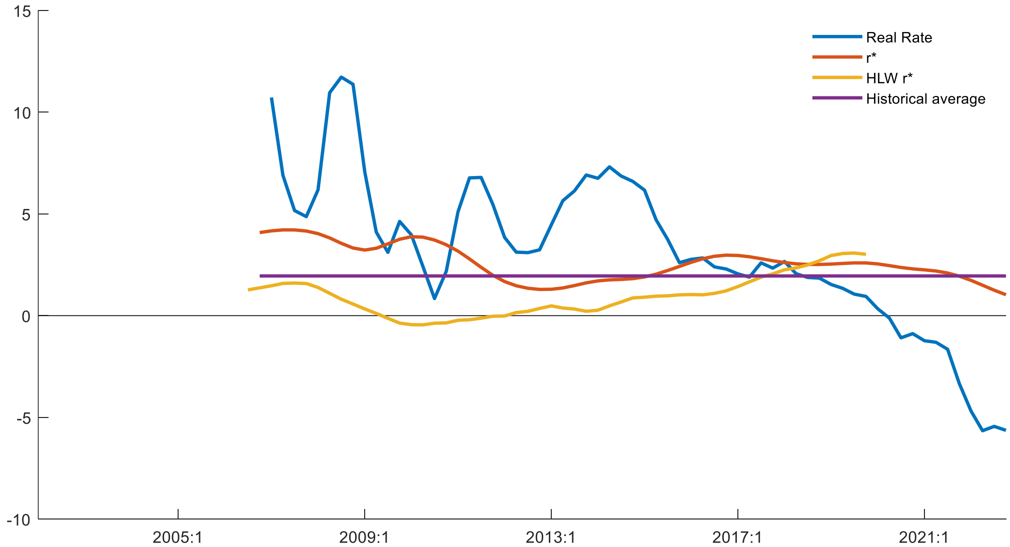


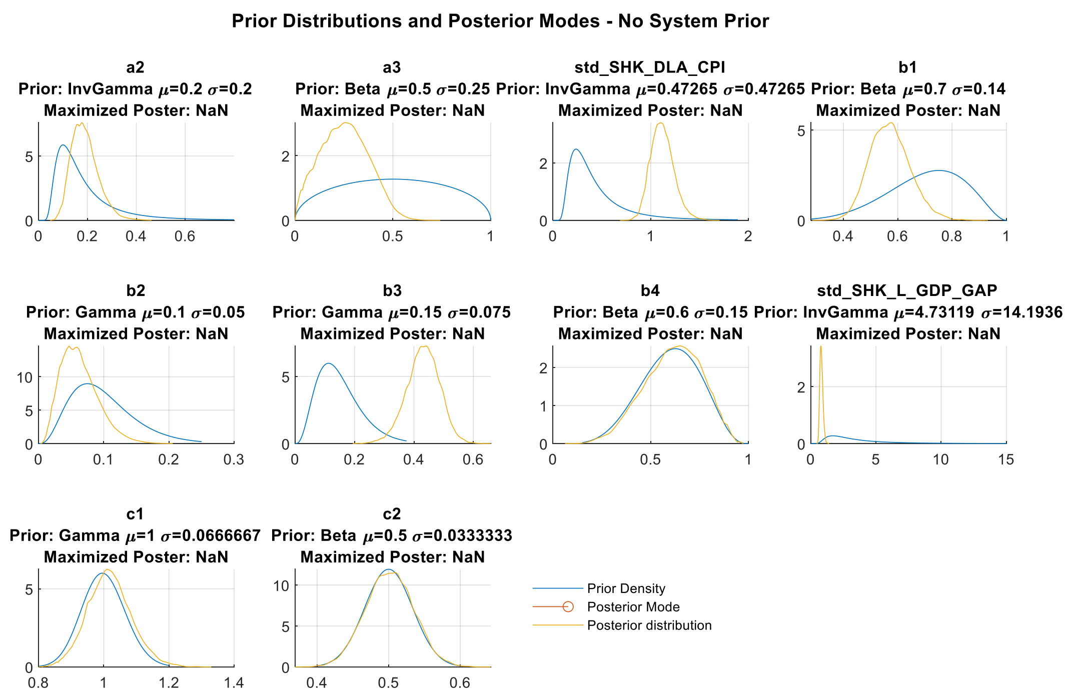




Source: Authors' calculations.

Figure A.3 Serbia: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

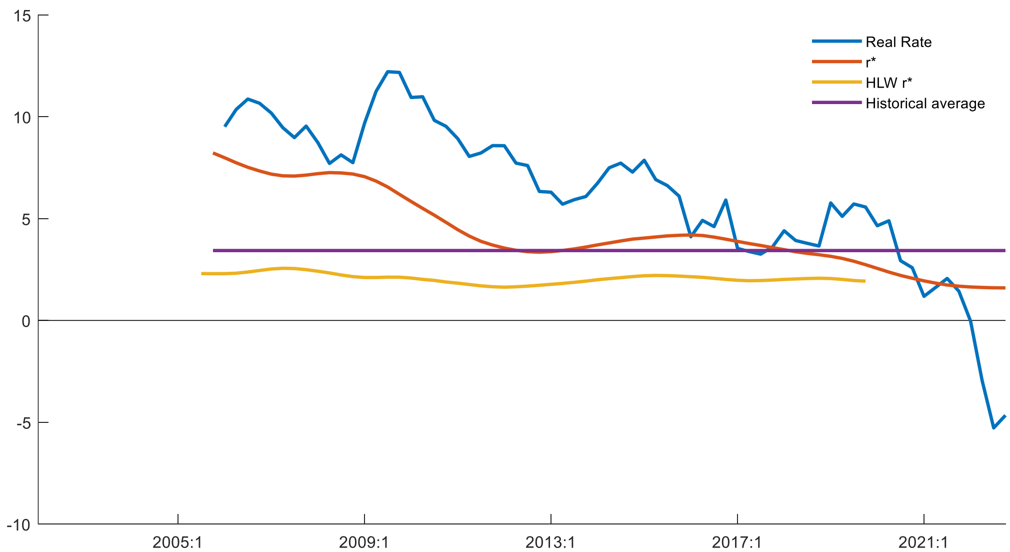


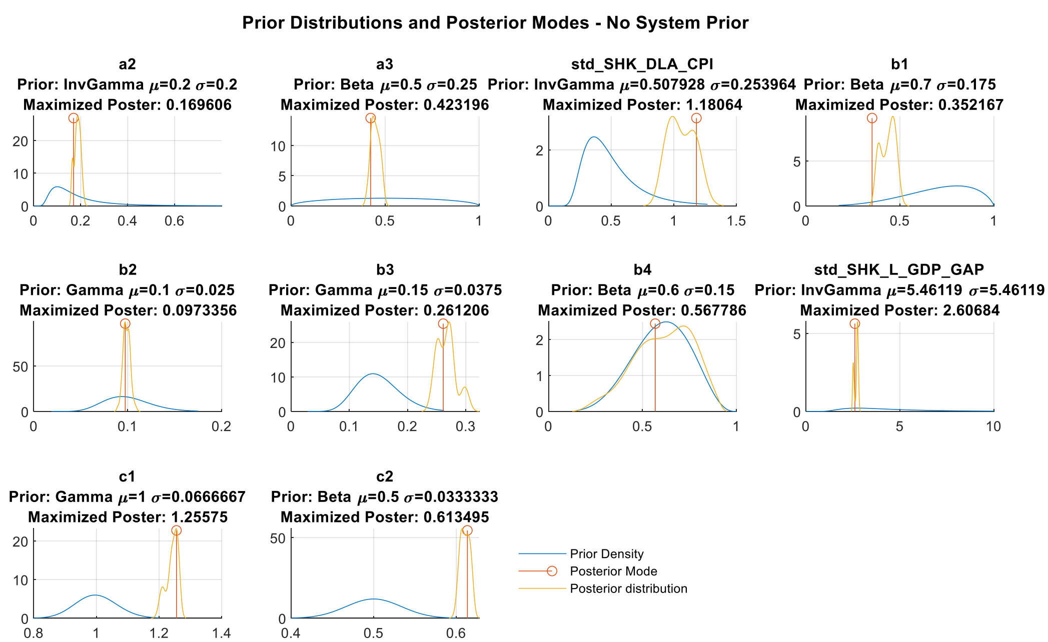




Source: Authors' calculations.

Figure A.3 North Macedonia: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

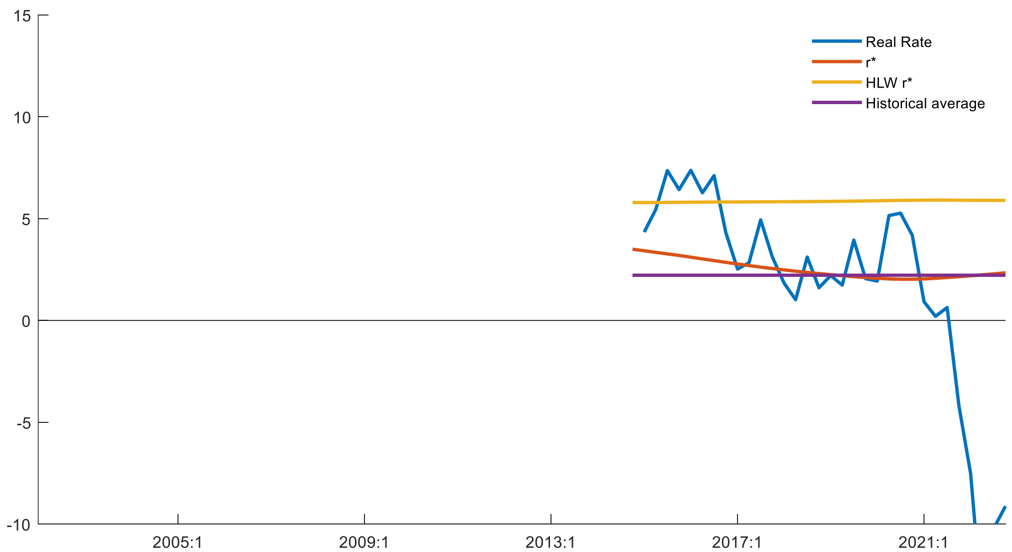


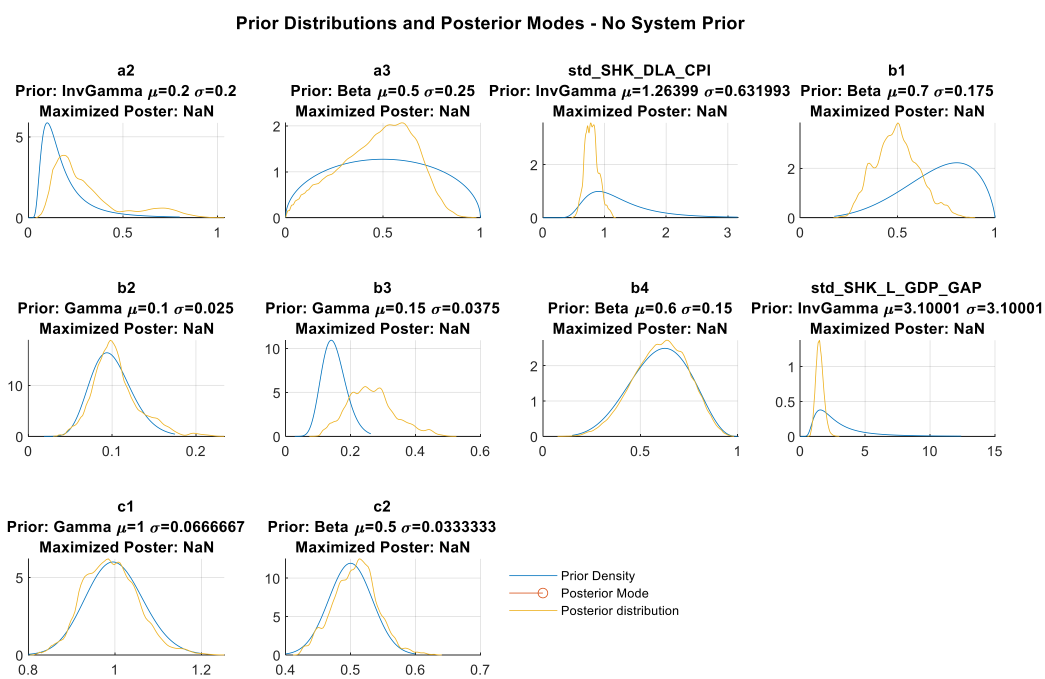




Source: Authors' calculations.

Figure A.3 Bosnia and Hercegovina: Alternative Estimates of $r^*$ and Bayesian Estimation (cont.)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.

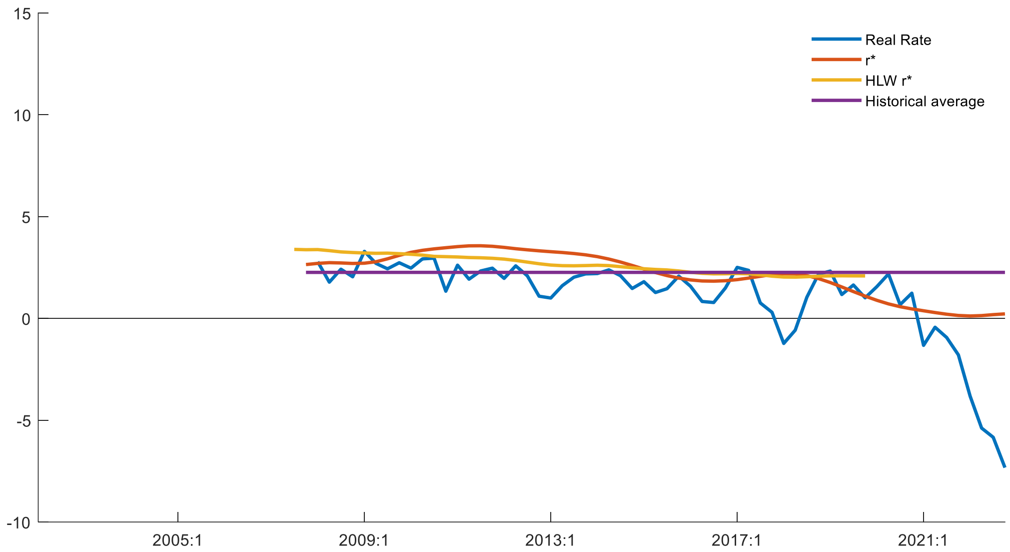


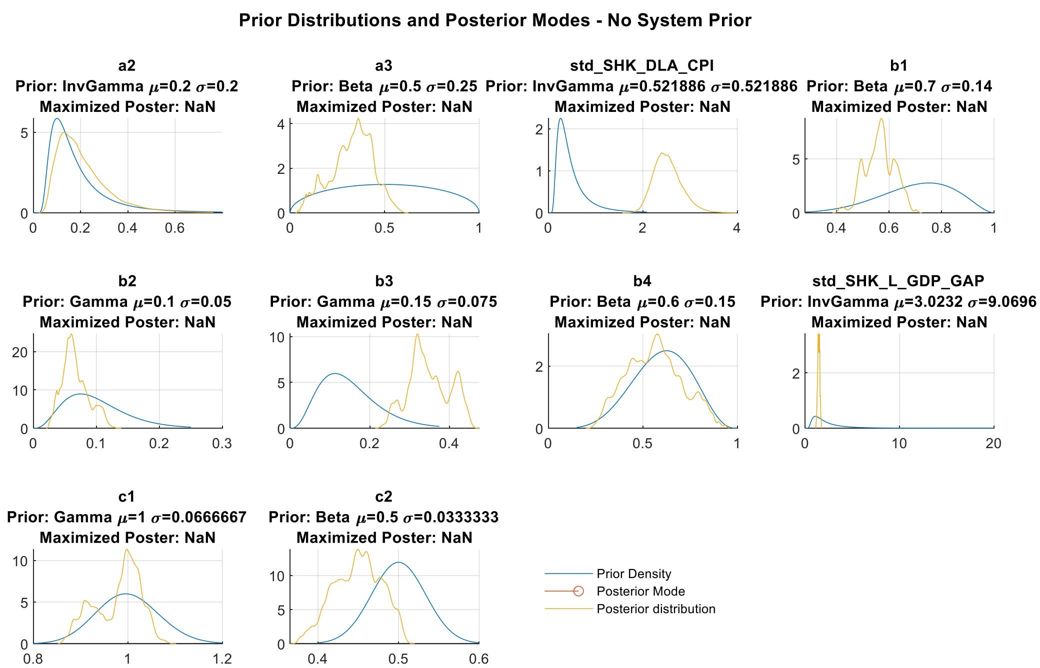




Source: Authors' calculations.

Figure A.3 Morocco: Alternative Estimates of $r^*$ and Bayesian Estimation (Concluded)

Notes: The blue line is the ex-post real policy rate, the red line is our country-specific estimate of $r^*$, and the yellow line is the estimate of $r^*$ based on the HLW methodology.




Source: Authors’ calculations.

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