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A Proposal to Improve Country-Level Data on Total Factor Productivity Growth

Andrew Warner

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A Proposal to Improve Country-level Data on Total Factor Productivity Growth
Prepared by Andrew Warner

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ABSTRACT: The assumption behind popular data on national capital stocks, and therefore total factor productivity, is that countries were in a steady state in the first year that investment data became available. This paper argues that this assumption is highly implausible and is necessarily responsible for implausible data on the ratio of capital to output and productivity growth. It is not credible that countries with similar incomes had huge differences in their capital stocks. This paper claims, with evidence, that implausible features of the data can be greatly reduced by using data on electricity usage or national stocks of road vehicles.

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WORKING PAPERS

A Proposal to Improve Country-level Data on Total Factor Productivity Growth

Prepared by Andrew Warner

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Executive Summary

This paper argues that frequently used data on the capital stock by country has too much measurement error to be plausible, and the data can be significantly improved by adding information on electricity usage or automobile stocks to the mix. The issue is important because data on national capital stocks are used to construct data on total factor productivity growth which in turn is used to draw conclusions on the efficacy of policy and the determinants of long-term prosperity. The implausibility of the capital stock data can be illustrated in several ways: one of the simplest is to note that there is huge dispersion in estimates of capital per capita even among countries with similar incomes. In one data set, the reported capital stock per person in the United Kingdom in 1975 was 692 percent higher than that of Japan even though GDP per capita in the two countries differed by only 10 percent. In other data the capital stock per capita reported for Zambia in 1960 was 1,469 percent higher than that of Mozambique; and Algeria's was 673 percent higher than Egypt's. Such high dispersion despite similar incomes is caused by the use of an assumption that lacks strong motivation, namely that countries were in a steady state in the year in which investment data is first available. The paper shows that errors in these initial estimates will necessarily influence the data decades later. Further, the mathematics hardwired into the perpetual inventory equation mean that this high dispersion in estimates of the initial level of capital across countries will inevitably translate into high dispersion in growth of capital, and thus high dispersion in growth of total factor productivity. The paper concludes by showing that these implausible features of the data can be mitigated by grounding estimates of the capital stock in data on electricity usage or by using proxies for the capital stock such as stocks of automobiles.

I. Introduction

This paper examines why some estimates of total factor productivity growth by country appear implausible and are inconsistent across studies, even for the same country. Implausible data exist in several commonly used data sets. In one recent example, estimated TFP growth between 1960 and 1985 was 5.16 percent per annum for the Republic of Congo, 1.92 percent for Bangladesh, 2.12 for West Germany and 0.03 for the United States.¹ The issue is not only that estimates for the Republic of Congo seem implausibly high compared with the US, but also that some of the data seem to clash with known historical events. The relatively good performance of Bangladesh occurred during a period that included a famine, a civil war, and a devastating cyclone (in 1970).

In other data, the capital stock of one country sometimes exceeds that of another by a factor of 8, despite the two countries having similar income levels. In widely used data sets the dispersion in the capital-output ratio across countries is extremely high at low levels of GDP per-person.

This paper argues that these two puzzling features of the data have common roots, namely the imposition of a steady-state condition that capital growth equals GDP growth, deployed for computational convenience rather than plausibility. This steady-state condition does have some a-priori justification as the outcome of the Solow model, but that justification may be insufficient considering other consequences of the use of this method. It will be called SSMA, for Steady State Method A, to distinguish it from other steady state conditions referenced later in the paper.

Section 2 shows several computations using alternative versions of the steady state method, alternative starting years, and alternative data sets, to argue that practical applications of SSMA inevitably produce a common triangular pattern: high and implausible dispersion of the capital-output ratio at low levels of GDP per-capita, and lower dispersion at higher GDP. Furthermore, since the steady state approach is silent on a number of unavoidable decisions (over which time period to compute growth, whether to use GDP growth or investment growth) and these are unavoidably different if one is starting in, say, 1960 versus 1970, it produces a second implausibility: inconsistent estimates of the capital-output ratio for the same country in different data sets.

Section 3 discusses the automatic transmission from errors in estimating initial capital to errors or bias in the estimates of capital growth. The two are necessarily inversely related, given the mathematical properties of the perpetual inventory equation. Section 4 shows that the bias in capital growth is large for specific countries and is also asymmetric: negative errors in initial capital stocks impart higher errors to the growth rate of capital than positive errors of equal magnitude.

Section 4 also shows that the bias on the growth rate of capital does not necessarily decay rapidly, an argument that is often made to defend the steady state method of estimating initial capital. The key is to focus on the growth rate of capital rather than the level of capital since it's the growth in capital that affects TFP growth estimates. The section shows that errors in initial capital significantly affect TFP growth data 40 years after initial capital is estimated.

¹ Data are taken from p. 99, Appendix Data, Final column, $g(A)$, in Klenow and Rodriguez-Clare "The Neoclassical Revival in Growth Economics: has it gone too far?" NBER Macroeconomics Annual. Ben Bernanke and Julio Rotemberg eds. January 1997."

Section 5 presents alternative methods for estimating initial capital. When examining historical capital stock data constructed for the United States and the United Kingdom, David Landes (1969, p. 293), citing A. G. Frank (1959), noted the extremely high statistical correlation between time series on power consumption and time series on capital stocks.

“The coefficient of correlation between energy consumption and such calculations as have been made of industrial capital stock is astonishingly high – for the United States from 1880 to 1948, 0.9995; for the United Kingdom from 1865 to 1914, 0.96 or 0.99, depending on the series employed. Indeed, one is almost tempted to ask whether direct, composite measurement of capital formation is worth the effort.”

Section 5 introduces two examples of estimating capital stocks using first, data on electricity consumption, and second, data on automobile usage. Section 6 compares these methods, primarily on the criterion of which one reduces the implausible variation in capital-output ratios. The results suggest that either the method based on electricity and automobile data or the imposition of constant capital-output ratios by asset class at the beginning greatly reduces the dispersion in capital-output ratios for countries of similar incomes, and thus also reduce bias in capital growth and TFP data. The two methods proposed in this paper eliminate some of the puzzling examples mentioned at the outset. TFP growth in the United States between 1970 and 2014 is estimated at 2.6 percent per-year, much higher than the 0.03 estimate mentioned above, and in line with other industrialized countries. And TFP growth rates for many developing countries are revised in plausible directions: extraordinarily high or low values are brought back towards the mean and data for countries of similar incomes are much closer to each other. The key driving force behind these changes is that the revisions greatly reduce the problematic error-correction effect that plagues the estimates based on the steady state assumptions.

There has been extensive previous research developing capital stock data both for single countries and internationally, including Jorgenson (1963), Summers and Heston (1991) and Harberger (1978). Improvements, extensions, and refinements include Summers and Heston (1991), Nehru and Dhareshwar (1993), M. Berlemann and J.-E. Wesselhoft (2014), and extensive revisions summarized in Feenstra, Inklaar and Timmer (2015). Bosworth and Collins (2003) update and refine Nehru and Dhareshwar (1993). A related paper, taking issue with implausible features of the GDP data, and suggesting improvements is Johnson, Larson, Papageorgiou, and Subramanian (2013). Well-known previous papers that either use capital stock data or draw important conclusions from it include Young (1995), Klenow and Rodriguez Clare (1997) and Caselli (2005).

II. Empirical Puzzles Originating from Imposition of the Steady State Method

The Steady State Method

This paper starts by reviewing the most common method of estimating either the initial capital stock or the initial capital-output ratio in cross-country data. The raw data consists in time-series on GDP and investment, denoted I . Such data are rarely available before 1950, and usually start somewhere between 1950 (for developed economies) and 1970.

The common procedure is to start with the perpetual inventory equation:

$$\dot{K} = I - \delta K. \quad (0.1)$$

And transform it so that capital growth appears on the left-hand side:

$$g_k = \frac{\dot{K}}{K} = \frac{I}{Y} \frac{Y}{K} - \delta. \quad (0.2)$$

Where, following common usage, g_k is capital stock growth, Y is GDP, I/Y the investment ratio, K/Y the capital-output ratio and δ depreciation.

Next, the frequently used assumption is that the country is in a steady state in which capital stock growth equals GDP growth (this result holds in well-known models)

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = g_y \quad (0.3)$$

Substituting “ g_y ” for \dot{K}/K in equation (1.2) and solving for the initial K or the initial capital-output ratio yields:

$$K(0) = I \left(\frac{1}{g_y + \delta} \right) \quad (0.4)$$

Or, for the capital-output ratio:

$$\frac{K}{Y}(0) = \frac{I}{Y} \left(\frac{1}{g_y + \delta} \right)$$

Research papers and data sets that use these equations differ in the choices for “ I ” and “ g_y ” to implement these equations with real data. Harberger (1978) recommended three-year averages for both I and country-specific growth rates. The data in Caselli (2005) are based on $K(0)=I(0)/(g+d)$, where $d=0.06$, $I(0)$ stands for the first year of availability of real investment data, and “ g ” stands for real annual investment growth between the first year of data availability and 1970. Mankiw, Romer and Weil (1992) used the slightly different $K/Y=I/Y/(g+d+n)$, in which I/Y is the average investment ratio over 1960-1985 from version 5.6 of the Penn

World Tables; “g” was fixed at 0.02 (an estimate of average global growth of output per worker), “d” was fixed at 0.03 and “n” was county-specific population growth. Klenow and Rodriguez-Clare (1997) followed Mankiw, Romer and Weil (1992).

Nehru and Dhareshwar (1993) base their estimates on a regression of real investment on time to back-cast an estimate of $I(0)$ for all countries in the same year, and then used equation (1.4) to estimate the initial capital stock. The fact that researchers adopt different auxiliary assumptions inevitably results in different estimates for initial capital in otherwise similar countries. By itself, the steady state method offers no way to resolve this issue.

High Dispersion and Inconsistency

Tables of Selected Countries

Two implausible features emerge from application of the steady state method (SSM). One is that capital-output ratios vary extensively among countries with similar levels of GDP per-capita. The second is that capital-output ratios for the same country vary considerably across data sets of differing vintages and authors, because of the use of differing auxiliary assumptions.

To show the variation in capital estimates at the country level, Caselli’s (2005) version of SSM was applied to data from version 6.1 of the Penn World Tables to calculate the capital stock and the capital-output ratio in 1960. For each country, the initial capital stock was calculated for the first year in which the investment series was available, and then updated using the perpetual inventory equation to arrive at the 1960 value. Table 1 shows what this method produces for 13 countries at the lower-end of the income scale. Each of these countries had similar income levels in 1960 (using estimates of GDP at purchasing power parity prices from the Penn World Tables, shown in the second column). Compared to Botswana, the capital output ratio in Chad was 15 times higher; Zambia, 29 times higher; Zimbabwe 66 times higher (final column).

Benchmark economic theory predicts that richer countries should have *higher* capital-output ratios, rather than the triangular-shaped scatter diagrams shown later in this paper. With Cobb-Dougllass production, constant returns to scale and competitive conditions, countries with higher w/r ratios will have higher capital-labor ratios, higher GDP per worker, and higher capital-output ratios, holding technology constant. In other words, all the associations between these variables should be positive, and there should be a positive association in scatter diagrams between the capital-output ratio and GDP per-worker. And unless labor force participation rates are hugely different across countries, there should also be a positive association with GDP per-capita. Instead, the scatter diagrams in Figure 1 and Figure 2 and depict the triangular relationship that does not have a strong positive slope.

Table 1. Estimated Capital-Output Ratio in 1960 using Steady State Method (PWT 6.1 Data)

| | GDP per-capita (PPP estimate from PWT6.1) | Estimated Capital- Output Ratio (Steady State Method 1) | Index (Botswana=100) |
|-----------------|---|---|-------------------------|
| Madagascar | 1239.57 | 0.33 | 330 |
| Zimbabwe | 1231.78 | 6.65 | 6650 |
| Chad | 1212.39 | 1.50 | 1500 |
| Zambia | 1206.58 | 2.93 | 2930 |
| Thailand | 1091.12 | 1.15 | 1150 |
| Bangladesh | 1057.28 | 0.33 | 330 |
| Nigeria | 1032.72 | 0.19 | 190 |
| Cape Verde | 994.47 | 2.32 | 2320 |
| Mali | 982.62 | 0.27 | 270 |
| Zaire | 979.89 | 0.56 | 560 |
| Botswana | 958.01 | 0.10 | 100 |
| Rwanda | 937.83 | 0.13 | 130 |
| Indonesia | 936.08 | 0.30 | 300 |

Evidence from firms or industries, thought scant, does not corroborate the idea that capital stocks vary across countries by the ratios seen in these tables; in fact, it's not even close. At face value, the estimates in Table 1 suggest that the value of structures and infrastructure and machinery in Zambia is 29 times that of neighboring Botswana; or Zimbabwe is 35 (6.65/0.19) times that of Nigeria. The McKinsey manufacturing productivity study comparing industries in different countries did not cite differences in the capital stocks or the capital-labor ratio as important factors explaining labor productivity differences (McKinsey Global Institute, 1993), and thus usually choose to present data on other sources of productivity differences. In one of the instances in which capital data is reported (auto assembly, US vs Japan), the capital-labor ratio in the US was a mere 4 percent higher, a far cry from the 100-3000 percent differences implied by the country-level data in this paper. Further, the data on automobile usage or electricity consumption presented later in this paper, which are of course partial and imperfect indicators, also do not corroborate cross-country differences on the level of 2000-3000 percent.

Applying the Steady-State Method to a later vintage of the data (PWT7.1) one decade later (1970), Table 2 shows similar results: highly variable numbers for the capita output ratio for countries at similar levels of GDP per-capita. Now, the second implausible feature is apparent. Botswana, which had an estimated capital-output ratio of 10 percent in 1960 when the method was applied using PWT6.1 data, now has an estimated capital output ratio of 190 percent in 1970 when the same method was applied using PWT7.1 data.

Table 2. Estimated Capital-Output Ratio in 1970 using Steady State method (PWT7.1 Data)

| | GDP per- capita (PPP estimate from PWT7.1) | Estimated Capital- Output Ratio (Steady State Method 1) | Index (The Gambia=100) |
|--------------------|---|--|---------------------------|
| Cote d'Ivoire | 1396.29 | 0.98 | 364 |
| Botswana | 1383.36 | 1.90 | 706 |
| Congo, Republic of | 1348.29 | 5.72 | 2128 |
| Haiti | 1310.59 | 0.51 | 188 |
| Gambia, The | 1283.65 | 0.27 | 100 |
| Senegal | 1255.10 | 0.81 | 302 |
| Comoros | 1212.04 | 2.37 | 881 |
| Guinea-Bissau | 1202.00 | 3.33 | 1239 |
| Egypt | 1170.71 | 0.86 | 318 |
| Madagascar | 1160.89 | 1.22 | 452 |
| Togo | 1157.28 | 1.12 | 418 |
| Cape Verde | 1117.18 | 4.51 | 1678 |
| Kenya | 1026.14 | 1.67 | 620 |

Graphs of All Countries

The finding of high dispersion in capital for given values of GDP per-person also holds for broad samples of countries. Figure 1 graphs the same data as in Table 1 excluding outliers Congo and Zimbabwe. Note that the variability is larger for lower-GDP countries.

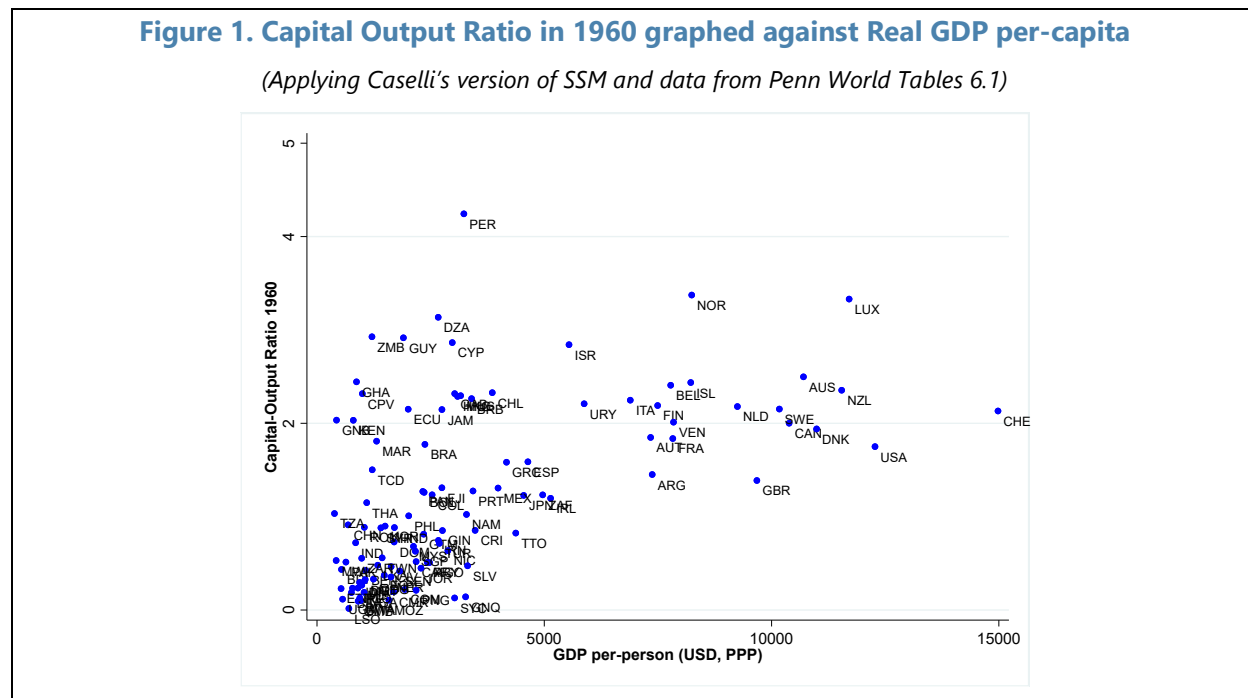
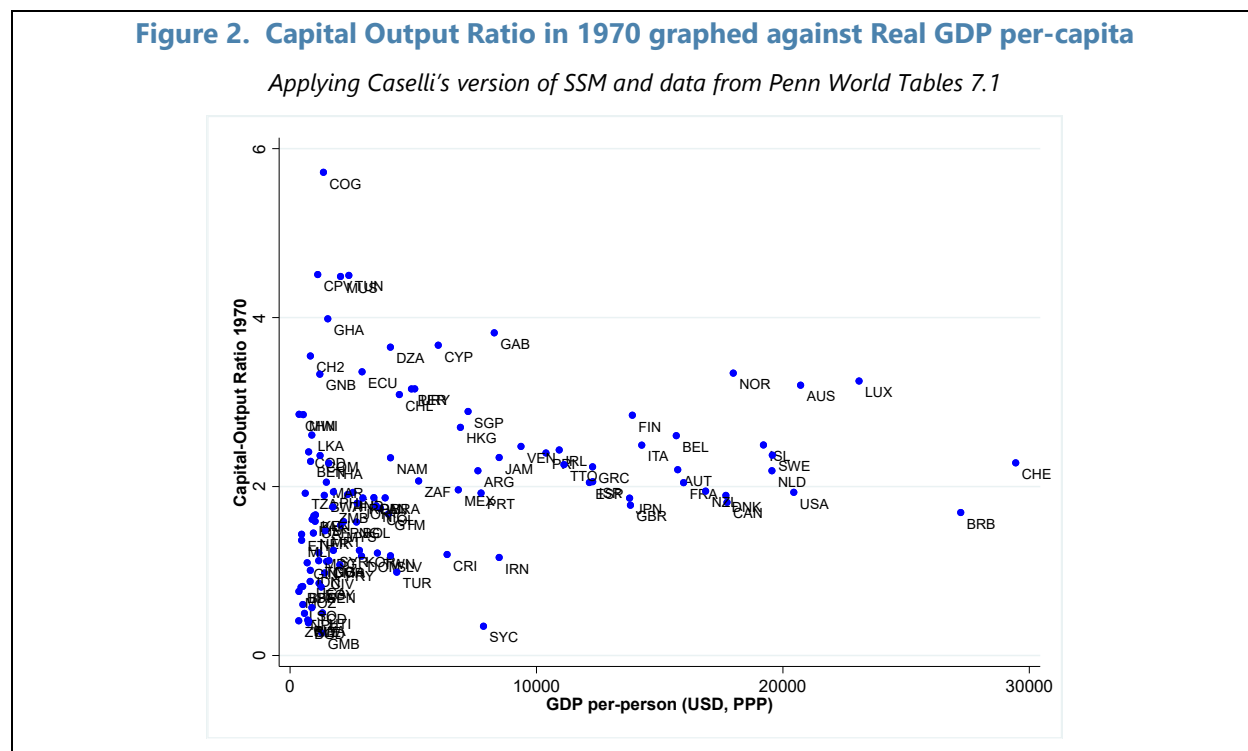
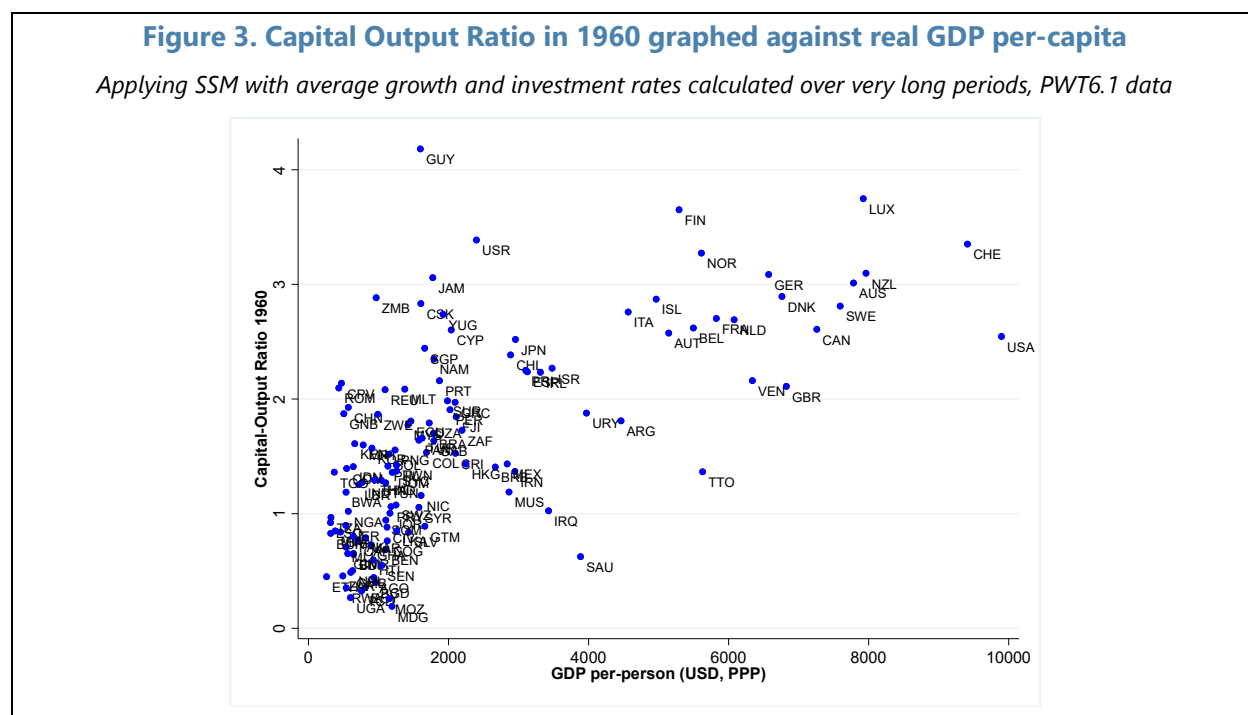


Figure 2 graphs the data for all countries using the method behind Table 2. The high dispersion in capital-output estimates for given levels of GDP is once again apparent (Equatorial Guinea is excluded as an outlier).



The results do not change dramatically when longer spans of time are used to calculate growth. Instead of using the period T0 to 1970 to calculate growth, as in Caselli, Figure 3 shows the estimated capital-output ratio when real GDP growth is calculated over 1960-1985, the mean investment to GDP ratio is calculated over 1965-1985, and depreciation is 0.06. The formula used is $K/Y=(I/Y)/(g+d)$ and results are shown for 1960. As can be seen, there remains a high degree of dispersion as, for example, Uganda's capital output ratio is only 0.27 but Zambia's is more than ten times higher at 2.88. The outlier is Guyana with a capital output ratio above 4.0.

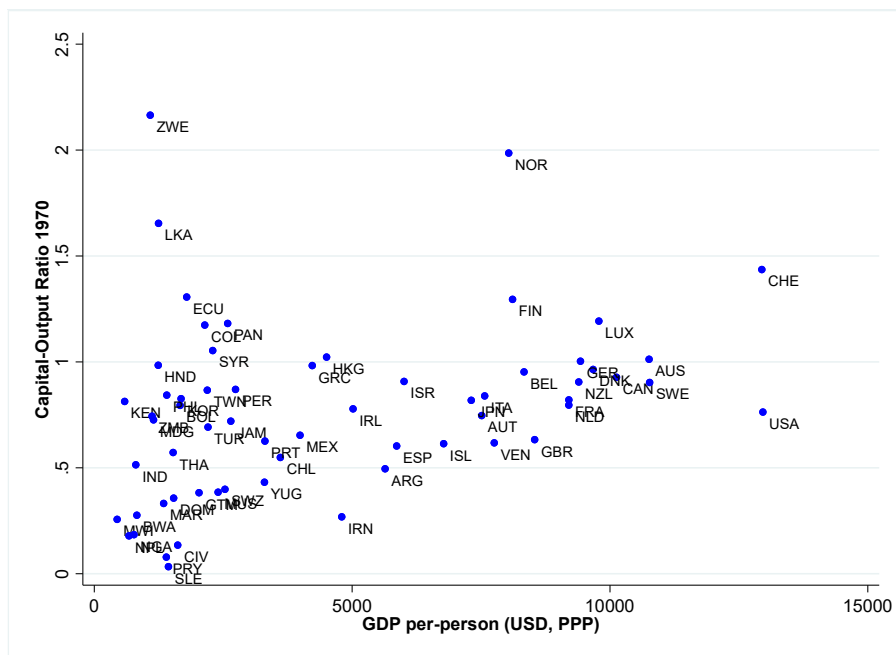


Finally, data reported directly (in research papers or in public data sets) also shows high dispersion of capital-output ratios at given values of GDP per-capita. Penn World Tables, PWT5.6 (QJE, May 1991), offered two variables, capital stock per worker (KAPW) and GDP per worker (RGDPW). The ratio of the two gives an estimate of the capital-output ratio. Data for 1970 are shown in Figure 4.² Sierra Leone, with per-capita GDP (at PPP prices) of \$1435 in 1970, had a capital output ratio of 0.034, while Madagascar, with per-capita GDP of \$1146, had a capital-output ratio of 0.727, or 21 times higher than that of Sierra Leone

² The year 1970 is chosen because earlier years lack data on many countries.

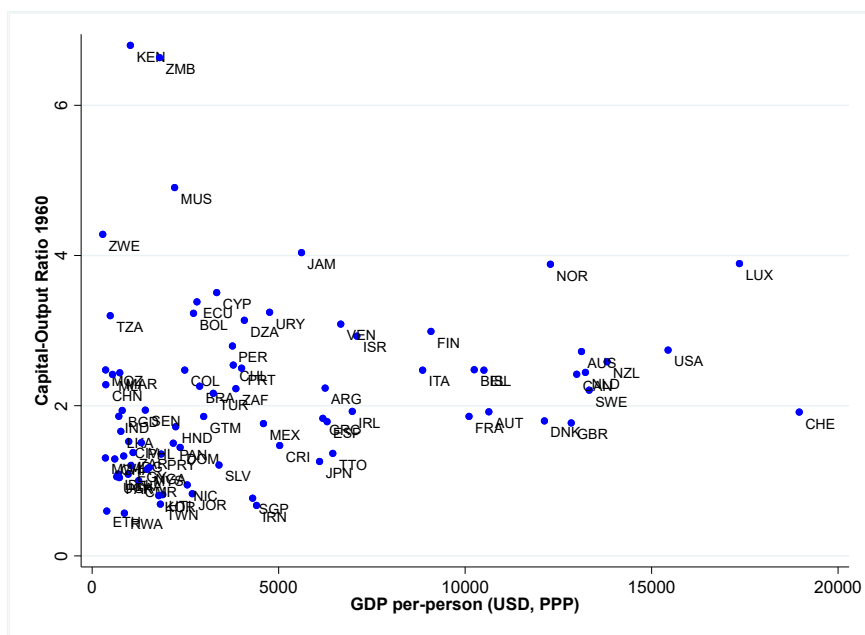
Figure 4. Capital Output Ratio graphed against real GDP per capita

Showing the data implicitly reported in Penn World Tables Mark 5.6



The data from Nehru and Dhareshwar (1993) also shows high dispersion in K/Y (Figure 5):

Figure 5. Capital Output Ratio graphed Against GDP per person, Nehru Dhareshwar (1993)



As does the data reported in Klenow and Rodriguez-Clare (1997) in Figure 6, and the data reported in version 9.0 pf the Penn World Tables, Figure 7.

Figure 6. Capital-Output ratio graphed against GDP per capita in 1985

Reported in the Appendix in Klenow and Rodriguez-Clare (1997), based on PWT version 5.6

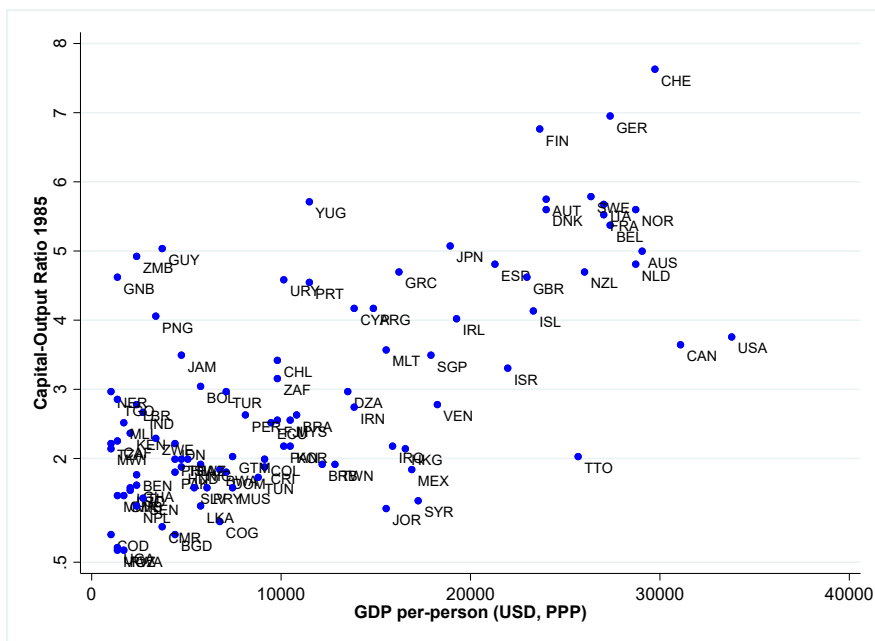


Figure 7. Capital-output ratios in 1975 taken from PWT9.0

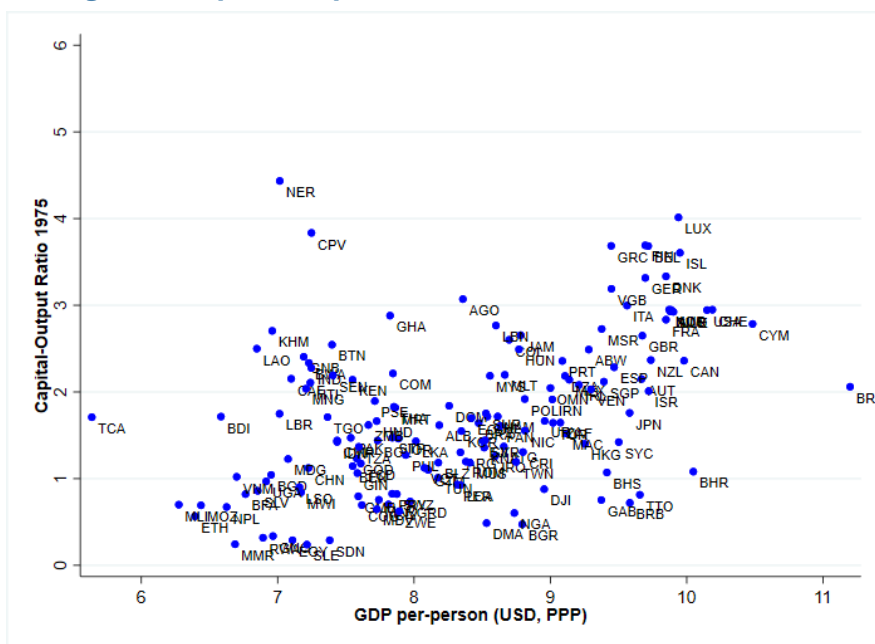


Figure 7 shows the series CK/CGDPO plotted against $\ln(\text{RGDPO}/\text{POP})$ for 1975.

Rising Capital-Output Ratios despite no Corroboration in the Investment Data

The Steady State Method of choosing initial capital produces data for some countries in which the capital-output ratio grows dramatically over time. But this can happen not because the investment rates are high or rising, but solely as a byproduct of the assumption or method for choosing initial capital. It also results in an odd and increasing discrepancy between two ways of measuring capital input: the investment ratio and growth in the capital stock. To illustrate these points, data from Jamaica are shown in Figure 8, which shows the rising capital output ratio, and Figure 9, which shows that the investment ratios are not unusually high. Figure 10 shows data on two ways of measuring capital input: the investment ratio and growth in the capital stock. In Figure 10, note that the short run changes in the data correlate with each other but there is an ever-increasing discrepancy.

This discrepancy becomes so large that by the end of the period, in 2020, while the investment ratio remains a rather normal 18 percent, the data on capital stock growth show negative growth.

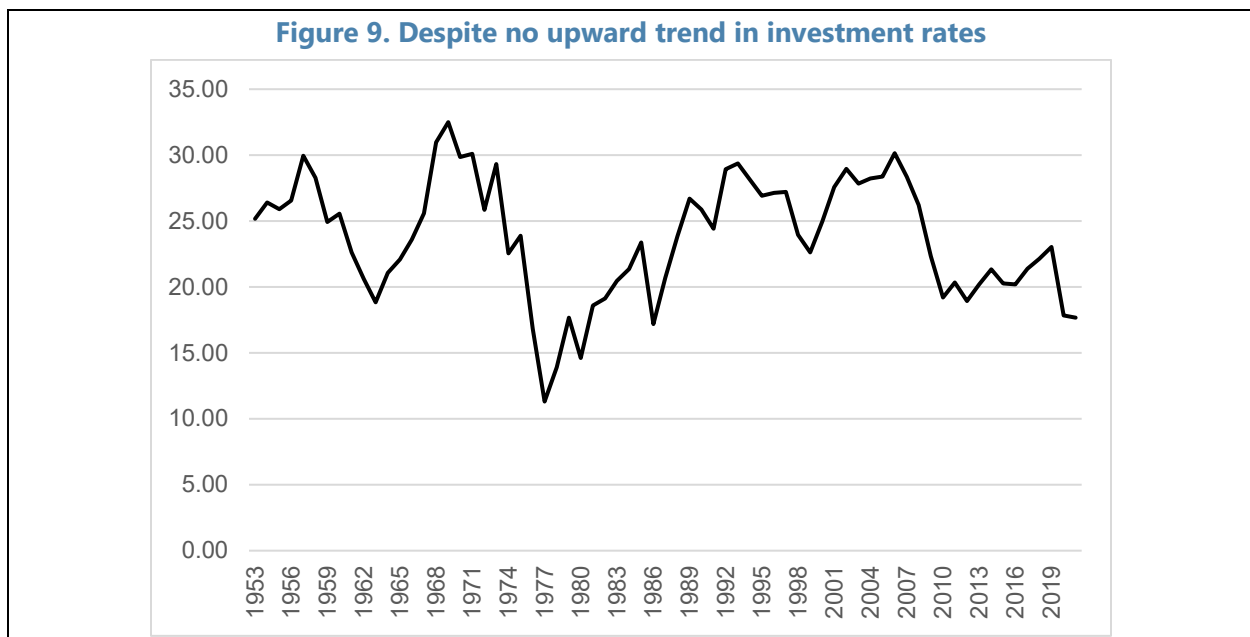
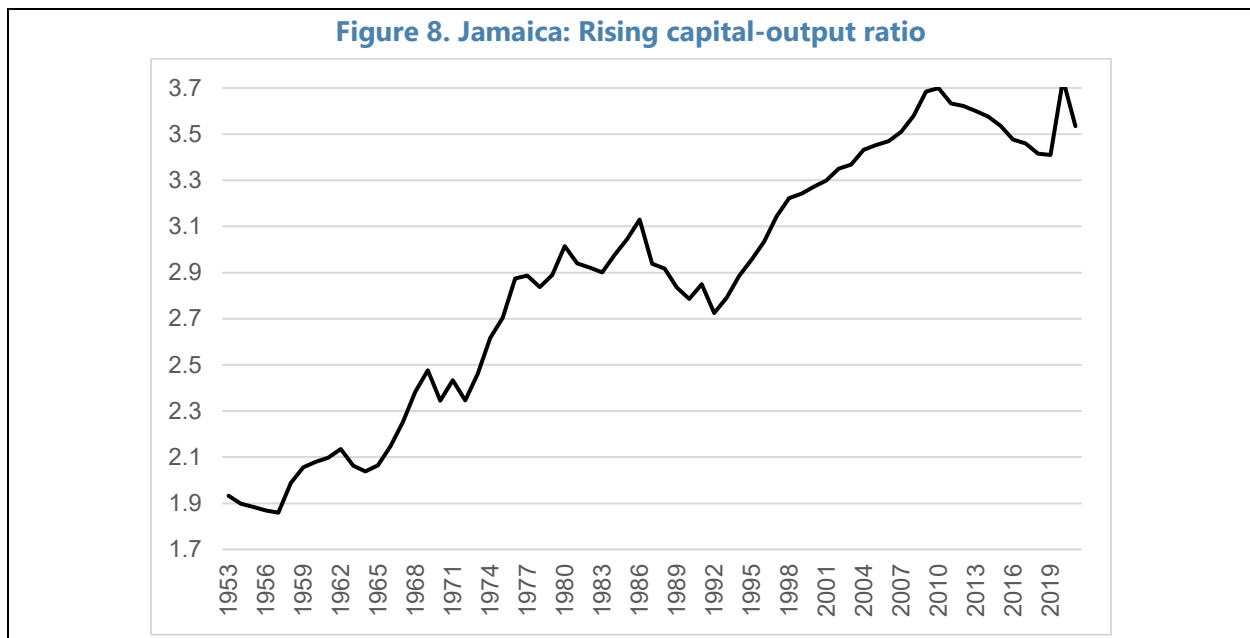
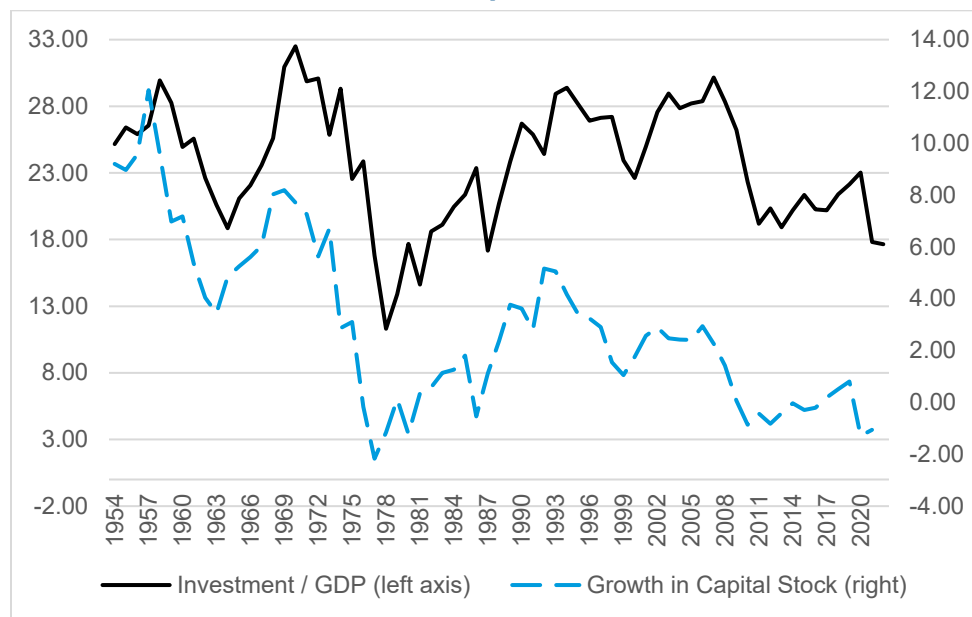


Figure 10. Increasing discrepancy between two alternative methods of measuring capital input



The Error-Correction Effect and Biased TFP Growth

The reason for the occasional disconnect between capital growth and investment data shown in the previous section is rooted in an error-correction effect that is triggered whenever the initial capital estimate is out of line with the subsequent investment data. The relevant comparison is between the estimate of the initial capital-output ratio and the steady-state capital-output ratio implied by the investment data and the perpetual inventory equation. The larger the deviation between the two, the more K/Y will trend up or down to reach its steady state value. To see this formally, note that the perpetual inventory equation is a stable differential equation:

$$\dot{K} = I - \delta K, \quad (0.5)$$

Solving this, assuming investment, I , and depreciation ($\delta > 0$) is constant, and introducing a term, μ to stand for the percentage error in estimating initial capital, $K(0)$, we have:

$$K(t) = K(0)(1 + \mu)e^{-\delta t} + \frac{I}{\delta}(1 - e^{-\delta t}) \quad (0.6)$$

Hence with constant investment, observed capital evolves between its initial value $K(0)(1 + \mu)$ when $t=0$ and its steady state value, I/δ as “ t ” approaches infinity. (Dividing through by GDP (Y) shows this relationship in terms of the capital-output ratio (K/Y) and the investment rate (I/Y)). When the initial value is lower than the steady state value capital will grow and vice versa. This dynamic explains the result for Jamaica in Figure 8. In the Jamaican data, the investment ratio averaged 24.39 percent between 1953 and 2010. Assume for the sake of illustration that this value is constant. Using 0.2439 as the constant investment rate and 0.06 as the constant depreciation rate, the steady state capital-output ratio for Jamaica was 4.06. Given that the initial estimate of the capital-output ratio (using the steady state method) was 2, far lower than 4, there was a prolonged period of adjustment with growth in the capital stock. And indeed, it is apparent from Figure 8 that the capital output ratio

in Jamaica rose from a value around 2.0 to a value close to 4.0. Such an error-correction effect can introduce a significant difference between capital growth rates and investment rates.

More generally, given the universally-used perpetual inventory equation, (1.5), capital growth will depend on a number of factors: the investment data, the assumed rate of depreciation, initial capital, and errors in estimating initial capital, which works through this error correction effect. To understand the issues explicitly, re-write the perpetual inventory equation with capital growth on the left:

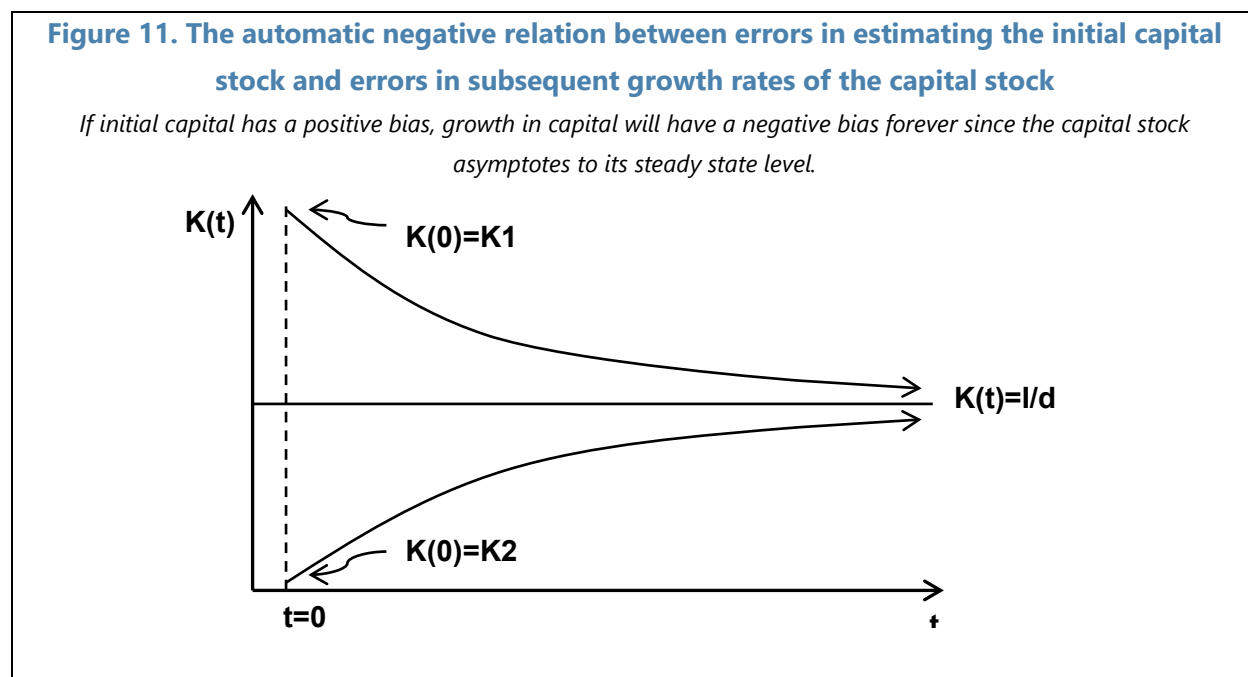
$$g_k = \frac{\dot{K}}{K} = \frac{I}{K} - \delta \quad (0.7)$$

And substitute for K:

$$g_k = \frac{I}{K(0)(1+\mu)e^{-\delta t} + \frac{I}{\delta}(1-e^{-\delta t})} - \delta \quad (0.8)$$

Again, by dividing both numerator and denominator through by Y, this expression can be equivalently analyzed in terms of the capital-output ratio K/Y and the investment rate I/Y rather than K and I alone. Also note that for ease of discussion a parameter “μ” has been introduced to distinguish between capital itself and any error in estimating capital, μ. The important point is that if the error in estimating initial capital is positive, μ>0, growth in capital will be biased downward for all “t”, and vice versa. In other words, a positive error in the initial level of capital translates into a negative error in g_k and vice versa. And, extending this to TFP growth, a positive error in “μ”, imparts a negative bias in capital growth “g” and a positive bias in TFP growth. Moreover, such biases are hardwired into the mathematics and are thus unavoidable once an error is made estimating initial capital.

Figure 11 illustrates the inverse relation between errors in the initial capital estimate and growth rates in the capital stock.



Simulations Show the Bias is Asymmetric, Large, and Potentially Persistent

Bias is Asymmetric

A further point is that the consequence of errors in estimating initial capital is not symmetric on g_k : downside errors in K translate into bigger errors in g_k than upside errors. Since it is mathematically messy to manipulate equation (1.8), consider a deliberately simplified simulation in Table 3. Let the error in estimating initial capital take one of two values: 50 percent too low and 50 percent too high, corresponding to $\mu = \pm 0.5$. Further, let $l=25$, and $d=0.05$, so that the steady state capital stock is 500. For convenience the base scenario will set $K(0) = 500$. This is done to ensure that true capital growth will be 0, so that the error can be read straight off the table without having to perform any subtraction.

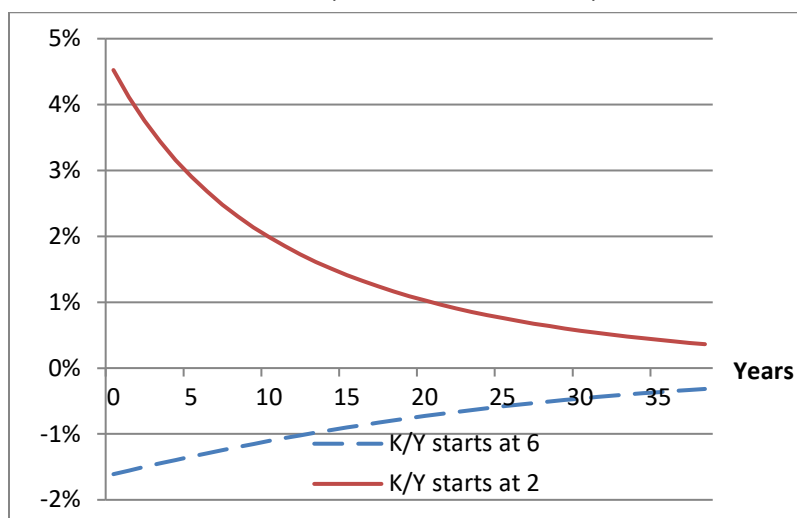
The table shows capital growth under the two cases mentioned. The first, column 2, has $K(0)$ set 50 percent too high, corresponding to $\mu = 0.5$; and the second, column 3, has $K(0)$ set 50 percent too low, corresponding to $\mu = -0.5$. Note the asymmetry. In the 50 percent too high case, the error in the growth rate is -1.45 after 5 years. In the 50 percent too low case, the error in the growth rate is 3.44 after 5 years. This asymmetry persists over time. Therefore, downside initial capital errors translate into larger errors in capital growth than upside errors.

Table 3. Simulated Errors in capital growth from symmetric Errors in estimating initial capital of +/-50 percent (true growth=0)

| Year | $\mu=0.5$ | $\mu=-0.5$ |
|------|-----------|------------|
| 5 | -1.45% | 3.44% |
| 10 | -1.20% | 2.30% |
| 15 | -0.98% | 1.61% |
| 20 | -0.79% | 1.16% |
| 25 | -0.64% | 0.85% |
| 30 | -0.51% | 0.64% |

Source: Authors calculations

This asymmetry is illustrated in Figure 12. In the simulations shown, the initial capital-output ratio is 6 and 2, and the steady-state capital-output ratio is 4. The 2 percentage-points-of-GDP negative error produces a larger impact on capital growth than the same error on the positive side.

Figure 12. Asymmetry in the Impact of Errors in Initial Capital on*Errors in Subsequent Growth Rates of Capital***Bias is large for some countries**

Illustrative simulations using actual data from specific countries shows that the bias in capital growth, and TFP growth, can be substantial, even after many years have passed. Consider the examples of Nigeria and Zambia from Table 1. Note that Nigeria's reported capital-output ratio was 0.19 and Zambia's 2.93. These are not the most extreme values in the table, as they are less extreme than Botswana (0.10) and Zimbabwe (6.65). For the sake of illustration assume that the true capital output levels should have been 1.0 in both countries. In that case the error for Zambia was $\mu=1.93$ percent and Nigeria $\mu=-.81$. Re-running the simulation in Table 3, we obtain:

Table 4. Simulated Errors in capital growth from Errors in estimating initial capital stock

| Year | $\mu=1.93$ | $\mu=-0.81$ |
|------|------------|-------------|
| 5 | -3.06% | 9.70% |
| 10 | -2.74% | 5.21% |
| 15 | -2.42% | 3.26% |
| 20 | -2.11% | 2.20% |
| 25 | -1.80% | 1.55% |
| 30 | -1.52% | 1.12% |

Source: Authors calculations

Thus, incorrectly estimating Nigeria's initial capital-output ratio to have been 0.19 rather than 1.0, results in a capital growth estimate that is, even after 30 years have passed, 1.12 percentage points too high. Furthermore, assuming a capital coefficient of $1/3$, this would translate into a TFP growth estimate that was, after 30 years, 0.37 percentage points too low. This 0.37 percentage-point figure can represent a large percent error for TFP growth estimates, which typically range between 0.5-1.5 percent per year.

Bias does not decay rapidly.

This claim is sometimes made that errors in estimating initial capital stock decay rapidly over time. However, the point is usually supported by calculations on the *level* of the capital stock, not its *growth rate*. And that's an important difference.

Consider a statistic that measures “memory”, the extent to which data on the capital stock in time “t” remembers any error in estimating capital at time “0”. With I and δ constant,³ the most common way to measure memory is to measure the importance of the initial capital estimate as a share of any subsequent year's capital stock:

$$m1(t) = \frac{K(0)e^{-\delta t}}{K(0)e^{-\delta t} + \frac{I}{\delta}(1-e^{-\delta t})} \quad (0.9)$$

Since initial capital depreciates over time the memory declines over time. Simulations show that with $I=20$, $\delta=0.05$, $K(0)=600$, the memory statistic is 29 percent after 30 years have passed, and with $I=20$, $\delta=0.05$, $K(0)=200$ the memory statistic is 12 percent after 30 years. Such calculations have supported the widely held view that errors in estimating initial capital dissipate rapidly over time.

However, what is relevant for TFP growth estimates is not the effect on the level of the capital stock but rather its growth rate. One analytical approach to examining the effect on growth rates is to refer to equation (1.8), and focus in the middle term in the denominator:

$$g_k(K(0), \mu, \delta, t) = \frac{I}{K(0)e^{-\delta t} + K(0)\mu e^{-\delta t} + \frac{I}{\delta}(1-e^{-\delta t})} - \delta. \quad (0.10)$$

There are at least four variables that influence the degree to which measurement error (μ) affects subsequent growth rates of capital: the size of the error, μ , initial capital $K(0)$, depreciation, δ , and time, t .

An alternative memory statistic might be:

$$m2(K(0), \mu, \delta, t) = g_k(K(0), \mu, \delta, t) - g_k(K(0), 0, \delta, t) \quad (0.11)$$

That is, the difference between the error-laden time series for capital growth with a non-zero measurement error μ , and the series with no measurement error, $\mu=0$.

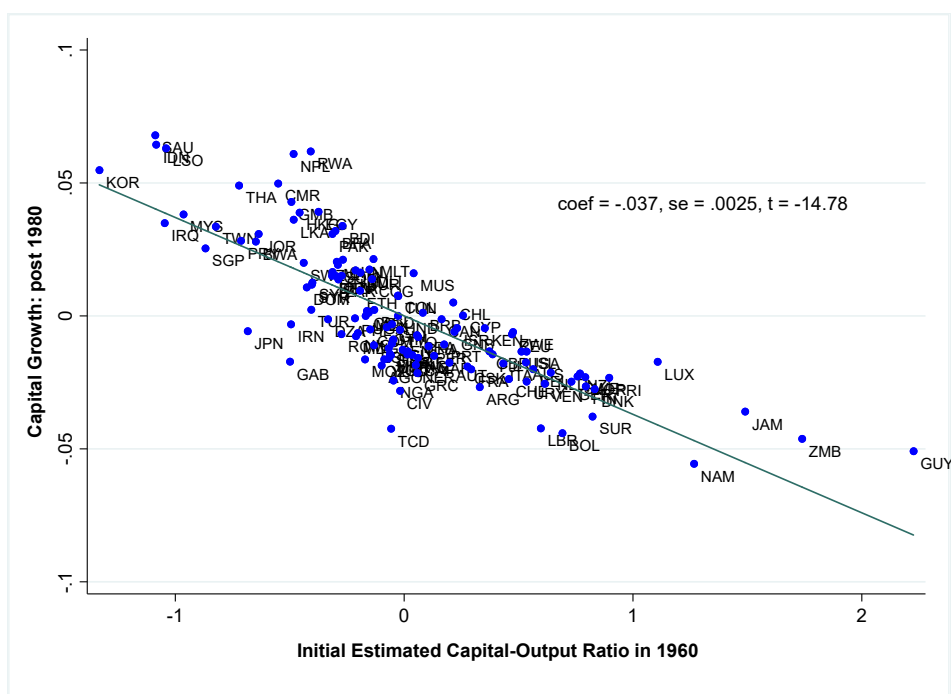
Table 4 has already shown the statistic $m2(t)$ when $I=25$, and $d=0.05$, which implied a steady-state capital stock of 500. For convenience and by design, initial capital was selected as $K(0) = 500$, so that true capital growth, namely $g_k(K(0), 0, \delta, t)$ would be 0 and measurement errors could be read directly from the table.

³ When I or δ vary over time, the statistic is more complicated and includes all previous values for investment and depreciation since the base year.

As can be seen in Table 4, with $\mu=1.93$, $m2(500,1.93,0.05,30) = -1.53$. Therefore, with an error in estimating initial capital of approximately 200 percent, even after the passage of 30 years, measured capital growth would be -1.53 percent in 1990 rather than the true value.

Consider next an empirical approach to the memory issue: to what degree do initial capital estimates still influence capital growth X years after the base year of the initial capital estimate? As previously discussed, under the commonly used steady-state method capital growth will be a function of both the data on investment and the initial estimate of the capital stock. Hence it may be of interest to examine in a regression framework how important are each of these variables in accounting for capital growth many years after the year of the initial estimate of capital. A regression was estimated of post-1980 growth rates of capital on the two variables, the 1960 capital-output ratio, and average investment since 1980. This exercise yielded the following partial regression relation:

Figure 13. Partial regression relation between post-1980 capital growth and estimates of the capital-output ratio 20 years previous, in 1960.

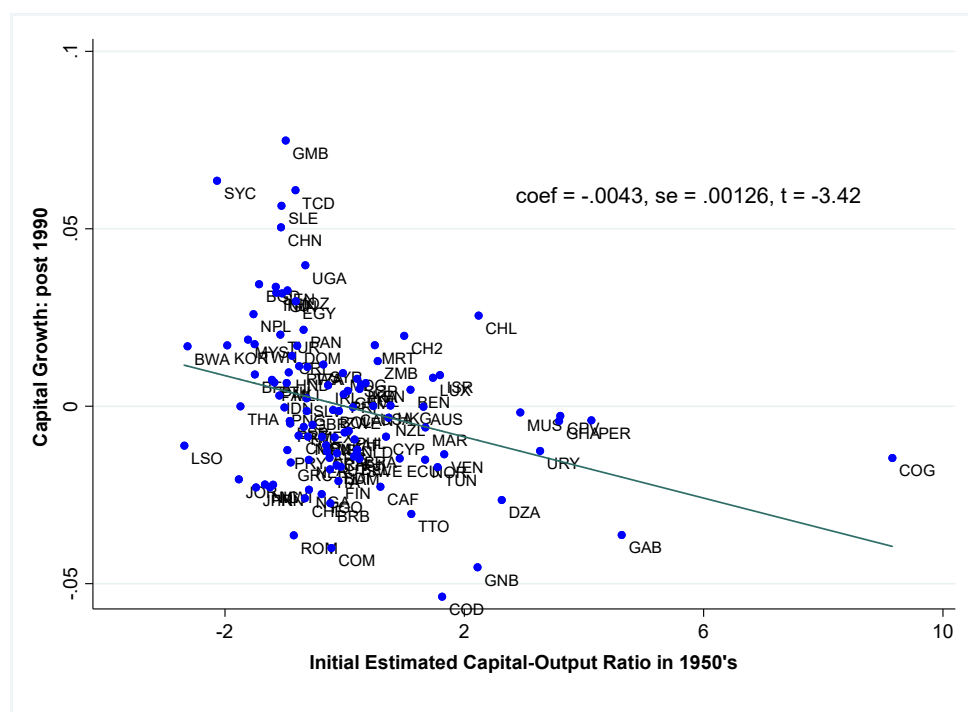


Hence, even after the passage of 20 years estimates from 1960 still strongly influence post-1980 capital growth.

Next, consider a stricter test, in which the dependent variable is capital growth after 1990, and capital output estimates from 1950, 40 years previous rather than 20 years previous.⁴

⁴ Initial capital stock was estimated in 1950 using the Caselli (2005) version of the SSM, and data from version 7.1 of the Penn World Tables.

Figure 14. Partial regression relation between post-1990 capital stock growth and estimates of the capital-output ratio in 1950



A list of alternative methods

The remaining sections of this paper will describe and compare the performance of six methods of estimating initial capital stocks. These include (a) the traditional steady-state method criticized in this paper (SSMA), (b) a second steady-state method that stipulates that the initial capital-output ratio will equal its long-run equilibrium given observed investment data (SSMB), (c) and (d) the two variants proposed by Feenstra, Inklaar, and Timmer (2015), based on assumption that all countries have the same initial capital-output ratios by type of capital good, and two new proposals in this paper that deploy either (e) cross-country electricity consumption data, or (f) automobile data.

A second steady-state method (SSMB)

The first alternative to the traditional steady-state method tackles the problematic error-correction-effect, discussed previously. The error-correction-effect is triggered whenever the initial capital-output estimate deviates from the long-run equilibrium of the perpetual inventory equation. Hence the proposed solution is to select an initial capital-output estimate to be close to that long-run equilibrium, thereby minimizing the influence of the error-correction effect on subsequent growth rates of capital. A practical issue is there is no single unique long run equilibrium as it is conditional on the investment rate. So, a feasible version of this solution is to use mean investment. Specifically, the proposal is to select the initial capital-output ratio by equation (1.13), where the numerator is an average investment rate over a suitably long time period, and depreciation takes an assumed value. This is expected to greatly reduce but not eliminate the influence of the error-correction effect.

The argument for this Steady State Method “B” (SSMB) rather than the more common Steady State Method “A” (SSMA) can be seen by comparing the SSMA equation with the proposal, equation (1.13). The key difference boils down to the use of the GDP growth rate in the former. Since the choice of “g” is arbitrary and not guided by theory, there is an unavoidable arbitrariness in SSMA that is absent from SSMB. The result is high variability in the capital-output estimates across studies and data sets.

$$K/Y = \left(\frac{\bar{I}/Y}{g_y + \delta} \right) \quad (0.12)$$

$$K/Y = \left(\frac{\bar{I}/Y}{\delta} \right) \quad (0.13)$$

Equal capital-output ratios (Feenstra Inklaar and Timmer)

The third method to be compared is the assumption of constant capital-output ratios used by Feenstra, Inklaar, and Timmer (2015), hereafter FIT. They distinguish four types of capital goods and deploy a constant capital-output ratio for each (shown in parentheses): structures (2.2); transport equipment (0.1); other machinery and assets (0.3); and ICT assets (0.0).⁵ The numbers in parentheses are the assumed capital-output ratio for each kind of capital good. Thus, for example, the quantity of structures in the economy in the initial year is assumed to be 2.2 times total GDP in that year. The authors present two series with this approach: one constructed with constant-price investment data (FITR), the other with nominal investment data (FITN). Hence, in summary, 5 series will be compared.

Use of Cross-Country Electricity and Automobile data

Although data measuring the entire capital stock are almost universally unavailable, data measuring part of the capital stock (autos, trucks) or products used by buildings and capital equipment (electricity) are frequently available. As is always the case with proxy variables, the issue is whether shortcomings in terms of coverage are more than compensated for by advantages in measurement accuracy. These data will of course be better proxies for the whole capital stock the more that components of the capital stock are used in fixed proportions.

Note that both proxy variables exhibit a strong positive log-linear relation with GDP per-capita (Figure 15 and Figure 16). Note also that the positive empirical relation between capital per-capita and GDP per-capita agrees with the standard prediction of a Cobb-Douglas framework with constant returns to scale.

⁵ See Page 14 in Feenstra, Inklaar, and Timmer (2015).

Figure 15. Cars per-person and GDP per-person, year 2000, in logs

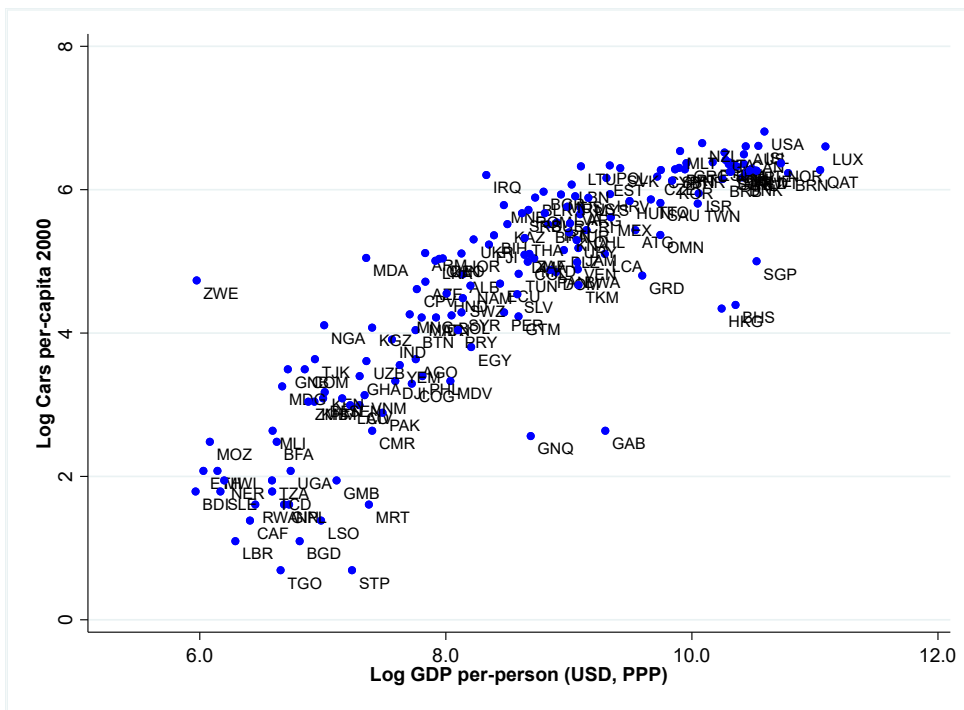
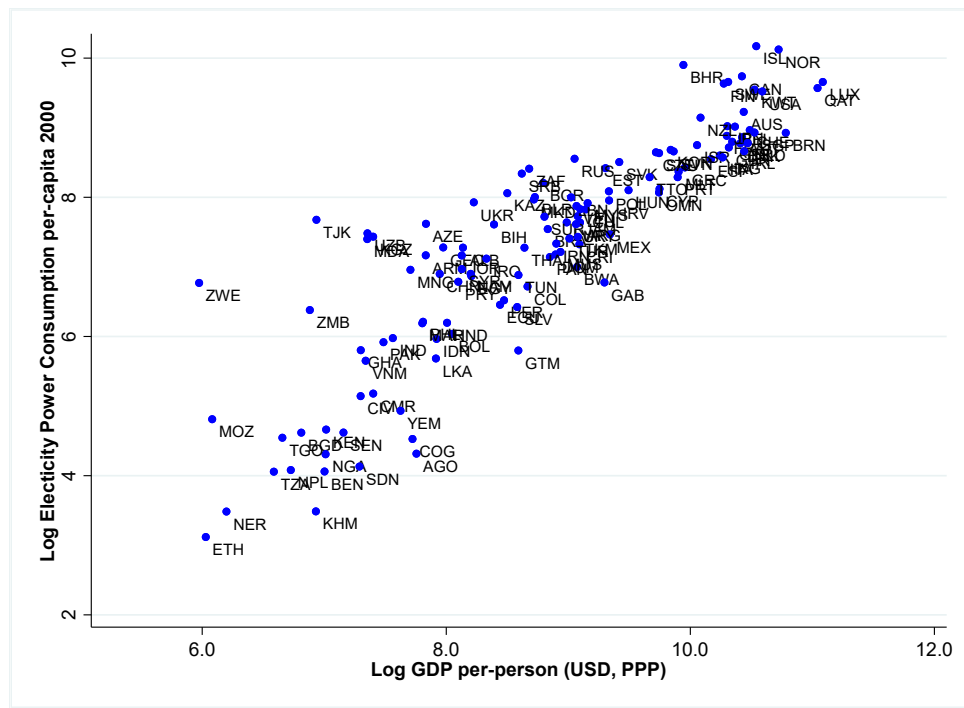


Figure 16. Electricity consumption per person and GDP per person, year 2000, in logs



Regression estimates (Table 5) show a cross-section elasticity slightly above 1.0 for both variables for the year 2000. Other years reveal similar results. A one percent increase in GDP per-person is associated with a 1.12 percent increase in electricity use per-person (1.02 percent for automobiles). Moreover, some of the outliers have ready explanations. The effects of the hyperinflation and economic crisis in Zimbabwe is apparent. Economies in which Oil production inflates GDP such as Equatorial Guinea and Gabon have high measured GDP given auto consumption. Singapore and Hong Kong have congested urban areas with extensive public transport and less need for autos and thus low auto consumption given GDP per-capita.⁶ Mozambique has Africa's largest hydropower plant enabling it to export to South Africa and Zimbabwe and boost service to urban areas domestically, hence high electricity consumption given GDP.

Table 5. Regressions of capital stock proxies on GDP. Cross-country data for the year 2000

| | | log GDP | | | |
|---|--|---------------------|------------------|-----|-----|
| | | Constant | per-person | N | R2 |
| Dependence Variable: | | | | | |
| Log Cars per-person | Estimated Coefficient (T-ratio) | -4.057 (-9.76) | 1.018 (21.18) | 162 | 74% |
| Log electricity consumption per person | Estimated Coefficient (T-ratio) | (-2.508) (-5.20) | 1.122 (20.63) | 126 | 77% |

Use of these data to estimate initial capital stocks requires the assumption that the empirical relation between total capital and GDP is the same as that for the capital proxy and GDP, as well as one data point for the capital-output ratio for a country in which the data is believed to be accurate. Since some assumption is necessary to estimate initial capital stocks, the choice between methods ultimately hinges on which assumption is deemed more plausible: the assumption that all countries are in a steady-state in which output growth equals capital growth, or the assumption that capital per-capita bears a similar relationship with GDP per-capita as automobiles per-capita?

Once the assumption is made that total capital follows a similar pattern as automobiles or electricity, little additional data is required to estimate initial capital for all countries. Using electricity data for illustration, the steps to implement this proposal are to:

1. Start with the KY ratio of a country believed to have high-quality measurement of capital. If this is the US, convert the US's capital-output ratio for a given year to capital per-person.
2. In a separate step, run a cross-sectional regression of log electricity per capita on log GDP per capita for a year in which data are available for many countries (see Figure 16 and Table 5 for an example using the year 2000).

⁶ Note also that Singapore has implemented a quota system on car ownership, with acronym COE, since 1990.

3. Assume that capital per capita of a given country falls below that of the US by the same proportion as electricity consumption per capita, using the fitted relation from the regression in step 2, not the individual data points from each country.
4. Calculate the estimated initial capital-output ratio for a given country by multiplying the fitted-value for capital per-capita by GDP per-capita for that country in the first year in which investment data are available. This becomes the estimate for the initial capital-output ratio for that country.

Repeating this with the help of equations, the proposal is to first run the following regression:

$$\ln(\text{elecpc}_j) = \alpha + \beta \ln(\text{GDPPC}_j) + \varepsilon_j$$

Referring to country “i”, go back to the first year in which that country has investment data, call this year “T0”. Observe the capital-output ratio of the US in that year, call this KY_{US,T_0} . Multiply by GDP per-person and take logs to obtain log capital per person in the US for that year, $\ln(KP_{US,T_0})$. Estimate log capital per-person for that country using the observed difference in log GDP per-person vis-a-vis the US, the estimated coefficient from the regression above, and log capital per person of the US:

$$\ln(KP_{i,T_0}) = \ln(KP_{US,T_0}) + \hat{\beta}(\ln(\text{GDPPC}_{i,T_0}) - \ln(\text{GDPPC}_{US,T_0}))$$

Then calculate the initial capital-output ratio by taking exponents and dividing by GDP per-person. The evolution of the capital-output ratio over time can then be calculated with the discreet time version of the perpetual inventory equation:

$$KY_{i,t+1} = (\text{GDP}_{i,t}/\text{GDP}_{i,t+1})(\text{INV}_{i,t} + (1 - \delta)KY_{i,t}) \text{ for } t > T_0$$

Where KY is the capital-output ratio, INV is investment as a fraction of GDP and δ is depreciation.

The case in favor of this approach compared to the Steady State Method (SSMA) is threefold: (1) it’s based on an empirically supported relationship between easily observed components or likely correlates of the capital stock and GDP; (2) the estimated relationship is consistent with a straightforward production function framework and (3) it does not require dubious assumptions.

Alternative Methods Compared

Information to compare the six methods is shown in Table 6 and Figure 18 through Figure 23. In this section, the main criterion for assessing the methods is whether they produce plausible data on the capital-output ratio – meaning primarily low dispersion - both unconditionally and conditional on the level GDP.

Capital-output ratios are graphed against per-capita GDP in Figure 18 through Figure 20. The methods shown are the steady-state method A (Figure 18, top graph) steady state method B (middle graph) and the method based on electricity data (Figure 20, bottom graph). These graphs show the low dispersion of the method using electricity data compared with both steady-state methods.

Three more graphs in Figure 21 through Figure 23 show the capital-output ratios generated by the two variants of the FIT (2015) method (which assumes all countries had the same initial capital-output ratios by capital

good) along with the method based on automobile data, shown at the bottom. The comparison between Figure 21 with Figure 22 shows that the use of nominal investment data (Figure 22) yields a large reduction in dispersion compared to the use of real investment data. The method based on automobile data (Figure 23) also has low dispersion, similar to (Figure 22) and the previous result based on electricity data (Figure 20). Hence three methods, the two capital proxies, using electricity and automobile data, and the FIT method using nominal data, have noticeably lower dispersion than the other three.

Descriptive statistics in Table 6 show how the methods rank against one another in terms of dispersion. Descriptive statistics are presented for the capital-output ratio for 108 countries with GDP per-capita less than 10,000 for the year 1975. Referring first to data on the 90-10 percentile difference, the method based on electricity data has the lowest dispersion (1.53), followed by automobile data, FIT nominal, the two steady-state methods and FIT real. Based on the standard deviation data, FIT nominal comes out slightly ahead of the methods based on electricity or automobile data, followed, at a distance, by the steady-state methods.

Table 6. Comparison of the capital output ratio in 1975 for 108 low-income countries using six methods for calculating the initial capita-output ratio

| | Mean | Median | Standard Deviation | 90th percentile | 10th percentile | Difference between 90th and 10th percentile |
|---|------|--------|--------------------|-----------------|-----------------|---|
| Electricity data | 1.67 | 1.58 | 0.98 | 2.41 | 0.88 | 1.53 |
| Automobile data | 1.81 | 1.71 | 0.99 | 2.76 | 0.97 | 1.79 |
| Same initial KY by asset – FIT nominal 1/ | 1.54 | 1.44 | 0.91 | 2.50 | 0.65 | 1.85 |
| Steady State Method | 2.90 | 2.76 | 1.23 | 3.98 | 1.70 | 2.28 |
| Long Run Eq of PIE | 2.24 | 1.92 | 1.33 | 3.51 | 1.10 | 2.41 |
| Same initial KY by asset – FIT real 1/ | 2.90 | 2.42 | 2.03 | 4.99 | 1.27 | 3.72 |

Source: Authors estimates unless otherwise noted
Sample is all countries with GDP<10,000, year=1975,
N=108.

1/ FIT stands for Feenstra, Inklaar, and Timmer (2015) and PWT 9.0.

Overall, therefore, three methods deliver data that has lower variance in capital-output ratios than the steady state methods: the method based on electricity data, the method based on automobile data and the variant of the FIT method that uses nominal investment data.

The graphs serve to underline an additional point that the task of developing plausible capital stock estimates is not solely a function of the initial capital stock estimates – the plausibility of the investment data also matters. The observations that are unusually high in Figure 20, Figure 22 and Figure 23, namely Cape Verde (CPV) and Cyprus (CYP), each have unusually high investment ratios at 49 and 47.8 percent respectively. Since outlying observations can have a large influence on empirical results, and the gain in statistical power from adding a single observation to a sample of countries that typically exceeds 100 in sample size is very small, there is a case for excluding countries with very unusual investment data on prudential grounds.

Examining data for selected countries serves to emphasize two results. The first is that the difference in the methods emerges in sharp relief for low- and middle-income countries rather than OECD countries. Table 7 shows that data for the United States and the United Kingdom are similar under all three methods. An

exception to the finding that OECD countries are similar is Spain, which under the steady state method is estimated to have had a capital-output ratio 36 percent higher than the United States in 1970, and TFP growth 122 percent higher (1.62/0.73). The OECD data does not corroborate this higher TFP growth number for Spain. Albeit for a shorter time span (1985-2011), the OECD reports TFP growth of 0.30 percent and 1.0 percent for Spain and the U.S. respectively.⁷

The second result underlined by examination of specific countries is, again, the higher dispersion in estimated initial capital-output ratios for low- and middle-income countries produced by the steady state method as compared with the other two methods, and the correspondingly large discrepancies in estimated TFP growth. In Table 7 three countries are shown with highly different initial capital-output estimates, Jamaica, Cameroon and Cambodia. Note that Jamaica now has a high initial capital ratio (5.96 in 1970), under the steady-state-method, in contrast to Figure 8, which also used the steady-state method, where it was 2.4 in 1970 (and 1.9 in 1953). Although the method is the same, the starting year and thus the growth data used is different, which once again illustrates the inconsistency in the results from applying the steady state method in different time periods. In Table 7, Jamaica and Cambodia have high initial capital estimates, therefore low capital growth estimates and therefore high TFP growth estimates. Cameroon has the opposite. All three countries however have similar capital estimates under the other two methods. The differences in TFP growth across methods is substantial. Jamaica switches from a positive TFP growth country under the steady state method (0.51 percent), to a negative TFP growth country under the electricity method (-0.43 percent). Cameroon switches from a low (0.17 percent) to a high (1.02 percent) TFP growth country.

Table 7. Comparing results for six countries under three methods – estimates of capital-output ratios and total factor productivity growth.

| Country | Initial Capital-Output ratio - 1970 | | | TFP growth Estimates 1970-2014 | | |
|----------------|-------------------------------------|-------------------------------|------------------------------|--------------------------------|-------------------------------|------------------------------|
| | Steady State Method | Method using Electricity Data | Method using Automobile Data | Steady State Method | Method using Electricity Data | Method using Automobile Data |
| United States | 2.63 | 2.60 | 2.60 | 0.73% | 0.83% | 0.83% |
| United Kingdom | 2.64 | 2.71 | 2.73 | 1.14% | 1.22% | 1.23% |
| Spain | 3.57 | 1.81 | 1.84 | 1.62% | 1.06% | 1.07% |
| Jamaica | 5.96 | 1.86 | 1.92 | 0.51% | -0.43% | -0.40% |
| Cameroon | 0.87 | 1.62 | 1.88 | 0.17% | 1.02% | 1.16% |
| Cambodia | 7.79 | 1.66 | 2.30 | 1.83% | 0.67% | 0.97% |

Sources: TFP growth is estimated using annual growth in RGDP0 and EMP from PWT9.0, growth in capital from this paper, and 0.6/0.4 for the labor and capital exponents.

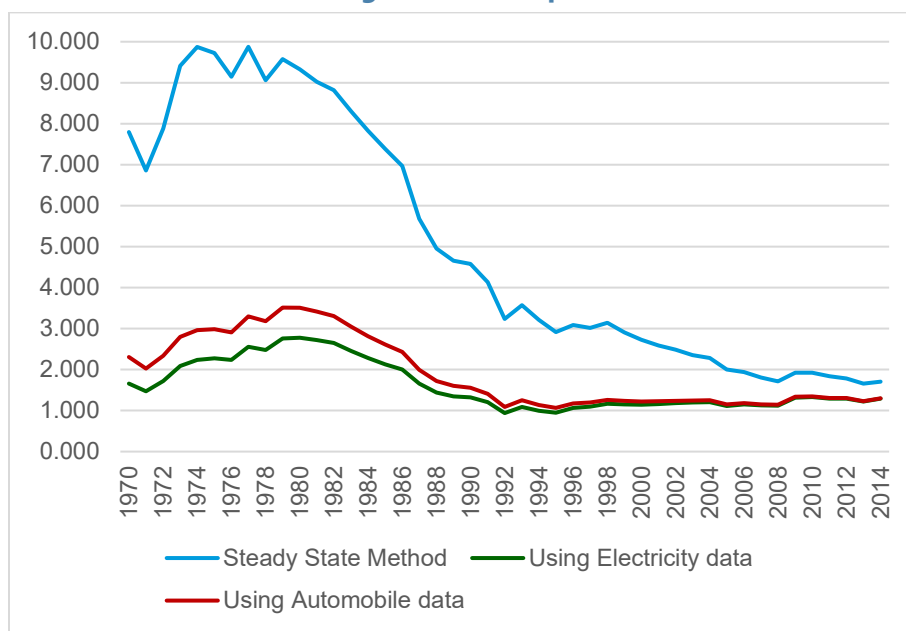
For PWT9.0, see Feenstra, Inklaar, and Timmer (2015).

The driving force behind these contrasting estimates is that the two methods proposed in this paper reduce the problematic error-correction effect that plagues data generated by the steady-state method. This can be seen by examination of the time series on the capital-output ratio for Cambodia (Figure 17). Note that under the steady-state method, the capital-output ratio starts high and rises even higher to approximately 10, before it

⁷ See multifactor productivity table in <https://stats.oecd.org/Index.aspx?DataSetCode=CS>, accessed July 14, 2019.

starts its long journey back to the equilibrium dictated by subsequent investment data. Simply put, the investment data for Cambodia justifies a long-run capital-output ratio less than 2, which is a long distance from the starting point of 10. During this period of error-correction, capital growth is negative and TFP growth is thus artificially boosted upward. As can be observed, this effect is muted when using the methods suggested in this paper.

Figure 17. Cambodia: Time series for the capital-output ratio under different methods for estimating the initial capital stock.



To understand further how the TFP estimates based on the electricity or automobile data in his paper compare with previous data, consider first the examples cited in the opening paragraph. In the data reported in the data appendix of Klenow and Rodríguez-Clare (1997), which draws in turn on PWT5.6, Republic of the Congo is listed with TFP growth of 5.16 percent between 1960 and 1985. That result is boosted by a capital growth estimate of -1.81 percent per year. Using the electricity estimates, capital growth would instead be 4.35 percent per year, with TFP growth revised downward to 3.94 percent per year. That is still a high number, but is driven by GDP growth of 7.2 percent, which probably reflects high growth in economic rents from crude oil production, rather than more traditional sources of productivity growth. In the same data set, Bangladesh's high TFP growth of 1.92 percent per year is influenced again by negative capital growth of -1.11. Using the revised estimates, capital growth would instead be virtually zero, at 0.07 percent per year, with TFP growth revised downward to 0.15 percent per year. And the U.S., rather than showing TFP growth of 0.03 percent per year, would have a more credible 1.76 percent per year.

Turning to data on the Asian Tigers, Young (1995) famously estimated that TFP growth in Singapore was -0.3 percent per year between 1966 and 1990 (see Table 8). Revised estimates in this paper put the figure much higher, at 0.9 percent per year, but that result is driven by revised higher GDP growth (9.7 percent rather than 8.5 percent) rather than major differences in estimated capital growth, which in fact go the other way (12.1 percent rather than 10.8 percent). The other Asian Tiger countries have higher or the same TFP growth between 1966 and 1990 than reported in Young (1995). However, differing capital growth numbers are not a

major reason for this. The revised data do underline an underappreciated fact from all the TFP estimates – that despite perceptions, average TFP growth estimates for the Asian Tigers are not extremely low by cross-country standards. According to the revised estimates, Hong Kong, South Korea and Taiwan achieved TFP growth rates, respectively, of 2.3, 3.2 and 2.9 percent per year. Even in Young (1995) the numbers were, respectively, 2.3, 1.6 and 2.4, not greatly different from the revised figures just quoted.

Figure 18. Capital output ratio – Steady State Method

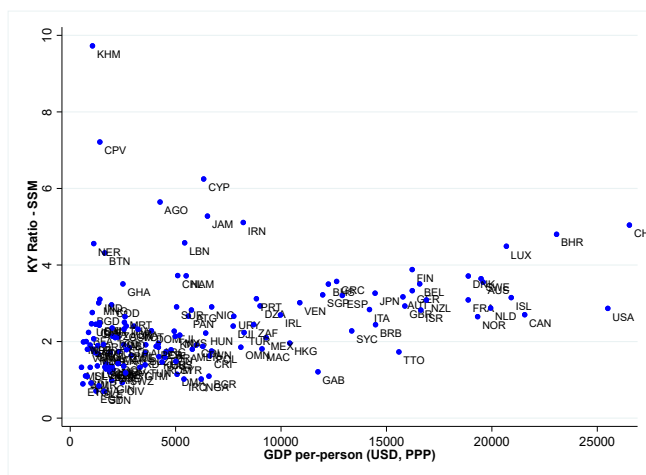


Figure 19. Capital output ratio – Long Run Eq. of PIE

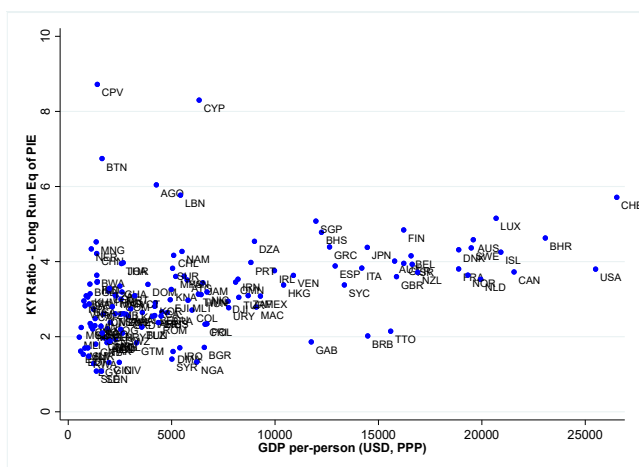


Table 8. TFP growth estimates in Young (1995) compared with estimates using capital growth from this paper and updated data

| | Time Period | Output 1/ | Aggregate Capital 2/ | Weighted Capital | Aggregate Labor 1/ | Weighted Labor | TFP | Labor Share | Table in Young (1995) | |
|---|-------------|-----------|----------------------|------------------|--------------------|----------------|-------|---------------|-----------------------|-----------|
| Estimates from Young (1995) | | | | | | | | | | |
| | Hong Kong | 66-91 | 0.073 | 0.077 | 0.080 | 0.026 | 0.032 | 0.023 | 0.628 | Table V |
| | Singapore | 66-90 | 0.085 | 0.108 | 0.115 | 0.045 | 0.057 | -0.003 | 0.470 | Table VI |
| | South Korea | 66-90 | 0.104 | 0.129 | 0.137 | 0.054 | 0.064 | 0.016 | 0.680 | Table VII |
| | Taiwan | 66-90 | 0.096 | 0.118 | 0.123 | 0.046 | 0.051 | 0.024 | 0.710 | Table VII |
| Revised estimates using capital growth from the electricity method in this paper and output and labor force growth from PWT 9.0 1/ | | | | | | | | | | |
| | Hong Kong | 66-91 | 0.073 | 0.069 | | 0.045 | | 0.023 | 0.628 | |
| | Singapore | 66-90 | 0.088 | 0.121 | | 0.051 | | 0.009 | 0.470 | |
| | South Korea | 66-90 | 0.091 | 0.127 | | 0.052 | | 0.032 | 0.680 | |
| | Taiwan | 66-90 | 0.087 | 0.116 | | 0.049 | | 0.029 | 0.710 | |

1/ The variable used for output growth is "rgdpna"; for aggregate labor growth "emp*hc". Source is PWT9.0.

2/ Capital growth uses electricity-based capital stock estimates from this paper.

Figure 20. Capital output ratio – Electricity data

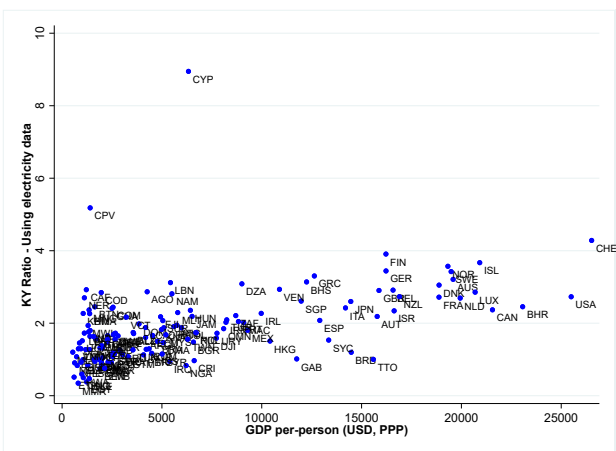


Figure 21. Capital output ratio - same initial K/Y by asset for all countries, real data

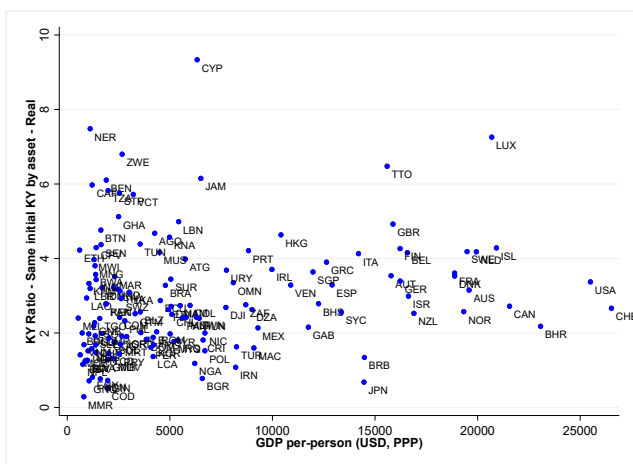


Figure 22. Capital output ratio - same initial K/Y by asset for all countries, nominal data

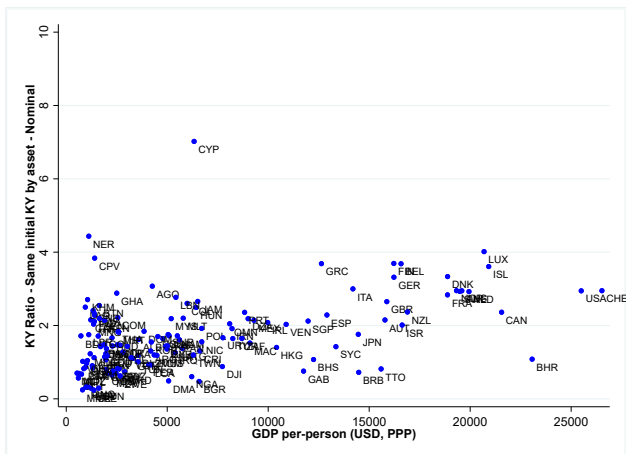
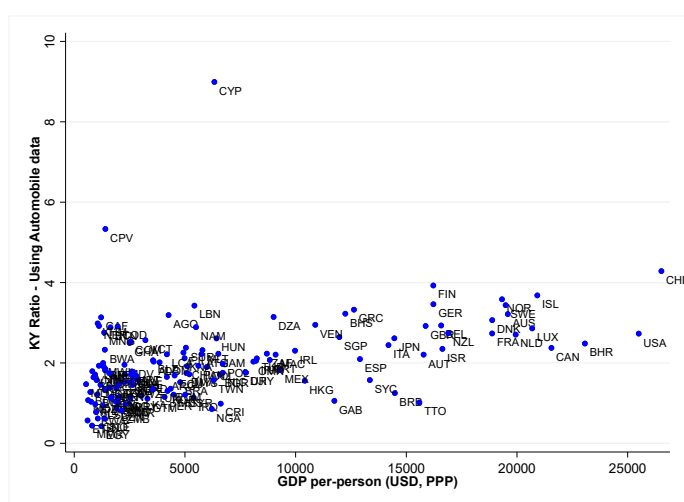


Figure 23. Capital output ratio – Automobile data



Summary of Points

This paper has argued that some of the capital-output data from commonly used data sets seem odd and strain the bounds of credulity. For two countries with similar GDP's per-capita, large differences in the K/Y ratio must be reflected in large differences in the K/L ratio.⁸ Basic reasoning, under a Cobb-Dougllass framework, would suggest that the capital-labor ratio would be proportional to the product of the wage-rental rate and technology parameters, $K/L = (w/r) \cdot (a/b)$, where w is the wage, r is the rental rate on capital, and a and b are exponents on capital and labor in the production function. If K/L varies by 8:1 in two countries of similar income levels, and the theory holds, it implies that wages and/or technological parameters vary by similarly large amounts, which is not corroborated by available evidence and seems implausible.

The paper indicts the steady-state-method for calculating initial capital stocks. It argues that the steady-state method is unavoidably arbitrary, not only because of the inherently difficult-to-justify steady state assumption but also because the method is silent on which auxiliary assumptions to deploy and what data and what period to use in calculating growth. Even using a consistent methodology, the same country sometimes has very different TFP growth data across data sets. This in turn unnecessarily introduces confusion into the meta-debate within the economics profession. Two studies using TFP growth data can reach different conclusions, or a study using TFP growth data and a study using plain GDP growth data can reach different conclusions, but not because of genuine differences in the underlying data but rather because of arbitrary differences in auxiliary assumptions used to calculate initial capital and hence TFP growth. Recent vintages of popular data sets, i.e. Feenstra, Inklaar, and Timmer (2015) appear to acknowledge these points by offering data that replaces the steady state assumption with the assumption that all countries have the same initial capital-output ratio by asset class.

The paper also argues that errors in estimating initial capital unavoidably introduce errors in estimating capital growth and TFP growth, which may argue for a risk-averse approach to estimating initial capital stocks. This is

⁸ Since $K/Y = K/L \cdot L/Y$, if L/Y is similar in two countries and K/Y is different it follows that K/L must also be different.

essentially because the universally used perpetual inventory equation is a stable differential equation that causes the capital-output ratio to strive to revert to its long-run equilibrium of (I/Y) . This produces an error-correction effect that distorts capital growth and is automatically triggered by errors in estimating initial capital. The result is asymmetric and error-laden data on capital growth, which do not necessarily dissipate quickly with the passage of time. In one empirical result presented in the paper, the initial capital-output data still affected capital growth data 40 years after the initial estimates. The resulting bias in estimates of TFP likely has the unintended consequence of penalizing rapidly growing countries in empirical studies. All else constant, with use of the SSM, rapidly growing countries will have low estimates of initial K/Y , high estimates of the growth rate in the capital stock, and thus low estimates of the growth of TFP. The net result is to falsely penalize rapidly growing economies into appearing to have low TFP growth; and to falsely raise the TFP growth of economies with low economic growth.

Because of the serious consequences of making a mistake in the estimation of initial capital, there is a case for estimating initial capital in a manner that is not likely to produce highly variable estimates across countries. It also seems desirable to take advantage of data that is likely to be correlated with the unobserved capital stock. This paper thus examines proxy-variables as a possible solution. The two used for illustration are electricity usage and automobile usage, per-capita. The drawbacks with this approach are transparent, (i.e. the data capture both final consumption as well as investment), but the approach also has several advantages. It is grounded in empirical evidence, the data are likely to be correlated with capital usage, and the method greatly reduces dispersion compared to the steady state method and thus minimizes the distorting error-correction effect. Growth in capital over time is largely driven by the country's investment data, as it should be.

The new method can be briefly described. There is an empirically robust cross-country log-linear relationship between both electricity use per-capita and auto use per-capita on the one hand and GDP per-capita on the other. If the unobserved capital stock follows a similar cross-country pattern, the regression estimates may be used to estimate the capital stock (per-capita) for any given level of GDP per-capita. From estimates of the capital stock per-capita, estimates of the capital-output ratio may be computed for any country and year, but more specifically for the present purposes, for the year which corresponds to the start of investment data. The perpetual inventory method can then be applied together with investment data to calculate a full time-series on the capital stock and the capital-output ratio.

The paper concludes by comparing six methods for estimating the initial capital-output ratio. It concludes that the method using automobile and electricity data, as well as one of the estimates in Feenstra, Inklaar, and Timmer (2015) (which assume that capital-output ratios, for four kinds of capital goods, are equal in the beginning for all countries) produce more plausible data on capital stocks and TFP growth than those based on other methods. These estimates have lower dispersion in the capital output ratio, which conforms better with theory and independent evidence. The key reason for the more credible estimates is that the methods minimize the problematic and distorting error-correction effect that plagues some of the existing data.

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