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Optimal Taxation of Inflation

Damien Capelle and Yang Liu

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Optimal Taxation of Inflation
Prepared by Damien Capelle and Yang Liu*

Authorized for distribution by Maria Soledad Martinez Peria
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ABSTRACT: When inflation originates from distributional conflicts, shifts in inflation expectations, or energy price shocks, monetary policy (MP) is a costly stabilization instrument. We show that a tax on inflation policy (TIP), which would require firms to pay a tax proportional to the increase in their prices, would effectively correct externalities in firms' pricing decisions, tackle excessive inflation and reduce output volatility, without exacerbating price distortions. While proposals from the 1970s saw TIP as a substitute to MP, we find that it is a complement, with TIP addressing markups and inflation expectation shocks, and MP addressing demand shocks.

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Author's E-Mail Address:	dcapelle@imf.org, yang.liu.7@bc.edu

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WORKING PAPERS

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1 Introduction

Monetary Policy (MP) is a powerful tool to lean against fluctuations in aggregate demand. But the episode of high and persistent inflation in the wake of the pandemic has highlighted that it faces significant challenges when confronted with other sources of shocks. Shocks to energy prices and disruptions in supply chains and shocks to inflation expectations have introduced a trade-off for central banks between letting inflation rise temporarily to preserve employment or accepting a recession to stabilize inflation. Distributional conflicts, either opportunistic increases in profit margins, known as "greedflation," or persistent wage-price spirals as emphasized by Werning and Lorenzoni (2023), have further contributed to the challenges of managing inflation. Financial instability triggered by a fast monetary tightening has also been a source of concerns among central bankers and complicates the conduct of monetary policy.

In the perspective of broadening the set of tools to regulate inflation, this paper analyzes the effectiveness of a tax on inflation policy (TIP), which would require firms to pay a tax proportional to the increase in their prices or wages. By giving direct incentives to firms to moderate their price increases without exacerbating relative price distortions, we show that TIP is an effective instrument to control aggregate inflation, especially in the face of markups and shocks to inflation expectations. We do so by embedding a TIP in a workhorse New Keynesian model that includes several exogenous drivers of inflation and by deriving the optimal combination of MP and TIP in response to these shocks.

Starting with the proposal by Wallich and Weintraub (1971), TIP was widely discussed in the 1970s in the U.S. and in Western Europe, at a time when persistent inflation was the main concern of policymakers. In a few countries, versions of TIP were even briefly implemented. Because TIP has been absent from recent policy discussions and the recent literature, the paper reviews in details these earlier proposals and attempts in section 2. Building on the ideas developed at the time, we then leverage the tremendous advancements of models with sticky prices made in the last decades to formalize TIP and characterize the optimal conduct of TIP in a fully microfounded framework. Our analysis yields several contributions.

We first show that combining TIP with conventional MP can implement the first best allocation in which inflation is zero and the output gap remains closed at all times under any path of shocks. This is in sharp contrast with a setting where only MP is available, because markup and inflation expectation shocks cannot be entirely addressed with MP. Then, we show that instruments should completely specialize: MP should track the neutral rate of interest, which varies with aggregate demand and productivity shocks, to keep output at its efficient level, and TIP should rise with markup and inflation expectation shocks. By introducing a wedge between the private and the social returns to price increases, these shocks create an externality in the firms' pricing decision. By giving direct incentives to moderate price increases, TIP can re-align the private with the social valuations and correct excessive inflation. In contrast with the view of the 1970s which saw TIP as a substitute for MP, we stress that TIP and MP are complement, each specializing in specific drivers of inflation.

We then formalize the equivalence between TIP and production or payroll subsidies—the more traditional tools to address the distortion implied by markups considered in the literature. While both instruments can help implement the first best, we show that subsidies entail large and persistent fiscal costs. In addition, we show that the first-best allocation could be equivalently implemented with other instruments that have a similar flavor as TIP, such as a feebate combining a tax on inflation with a rebate to all firms, and a market for inflation permits on which firms trade rights to increase their prices—a proposal first made by Lerner (1978). The appeal of a feebate is to provide incentives without increasing the average tax burden while the appeal of the market is that it minimizes the involvement of the fiscal authority.

In stark contrast with price controls, we find that TIP does not exacerbate distortions in relative prices. In an extension with sector-specific TFP shocks, which requires adjustments in relative prices, we show analytically, that under some conditions, TIP has no effect on relative prices across sectors and we provide numerical evidence that this independence applies more generally. Intuitively, this is because TIP is linear in price increases and symmetric across firms, contrary to price controls which are convex and asymmetric: in the presence of TIP, while firms that are hit by a negative productivity shock moderate their price increases, firms that would otherwise not change their prices are incentivized to decrease them to get a subsidy from TIP. The linearity of TIP keeps relative prices across sectors broadly unchanged.

The stabilization properties of TIP continue to hold in a setting in which MP follows a Taylor rule and TIP targets inflation. Based on simulations of a calibrated model, we show that the stabilization gains from using TIP are substantial. Consistent with the first-best setting, these gains are especially large for markup and inflation expectation shocks: a reasonably calibrated TIP could lower the variance of inflation by 45% and of output by 44%. Welfare gains are smaller for TFP and demand shocks, because the lower inflation volatility is partially mitigated by higher output gap volatility. Also consistent with the first-best policies, we find that some specialization is desirable: TIP should target inflation and MP should increase its focus on the output gap.

Finally, we address issues related to the implementation of TIP. First, a feebate or a market for inflation permits could arguably be easier to implement, the first one because it avoids increasing the tax burden on firms, the second one because it avoids relying on the tax administration. As discussed above, they can both achieve the same allocation as TIP. Second, a TIP restricted to the largest 1% of firms, which would be easier to administer, would still be very effective. Third, in an extension with sticky wages, we find that a TIP on wages is very effective for markup and inflation expectation shocks arising from the setting of wages (but not from the setting of prices independently of wages). The conclusion highlights other implementation issues deserving further research.

The remainder of this paper is organized as follows. Section 2 reviews the brief history of TIP and the related literature. Section 3 introduces a conventional sticky price model augmented with a TIP. Section 4 analyzes the optimal design of TIP and MP and section 5 discusses the equivalence with other fiscal tools. Section 6 analyzes the macro-implications of an inflation-targeting TIP. Section 7 analyzes the implications of TIP for relative price distortion. Section 8 addresses several concerns related to its implementation. Section 9 concludes.

2 Brief History of TIP and Literature

Early Proposals of TIP during the Great Inflation. In the context of high and accelerating inflation due to the combined food and oil price shocks of 1973 and 1979, and persistent wage-price spirals, Wallich and Weintraub (1971) formulated the first proposal for a permanent tax on wage increases. At this time, TIP stood for "Tax-based Incomes Policies" and according to their proposal it would be levied on wage increases in excess of a pre-announced target and it would be paid by employers.

This proposal started a literature analyzing the theoretical rationale for a TIP. Kotowitz and Portes (1974) build a microeconomic model in which a union sets wages and firms set prices and find that imposing such a tax does reduce the rate of change of wages. They show that this result is robust to assuming that the union is myopic or forward-looking. Latham and Peel (1977) show that the tax on wage increases is less effective when the firm is a monopsony. From a normative perspective, Seidman (1978a) argues that when increasing their price, firms don't take into account their economy-wide inflationary effects. Like a tax on pollution, TIP would signal to firms the social costs of their actions and would make them internalize the externalities of their price-setting behaviors.

A stream of papers studied other macroeconomic implications of TIP. In a macroeconomic model, Peel (1979) shows how the tax could reduce the likelihood of business cycles. Scarth (1983) finds that an employer TIP based on price increases, and an employee TIP based on wage increases are destabilizing, while an employer TIP based on wage increases is stabilizing. Oswald (1984) discusses the conditions for an inflation tax to be equivalent to an employment subsidy and to result in higher employment. Jackman and Layard (1989) show how TIP could reduce the NAIRU and increase welfare even when the tax has an effect on workers' effort.

Issues of Implementation. The initial proposal by Wallich and Weintraub (1971) also started a large literature on the most effective ways to implement TIP. At the 25th Panel on Economic Activity organized by the Brookings Institution in 1978 on how to cure inflation, TIP was the most debated issue and the discussion was centered around its optimal design (Okun and Perry, 1978): should TIP be based on price increases or on wage increases, paid by employers or by employees, paid by all firms or only by large firms, be continuous or discontinuous, be a penalty for large increases or a reward for moderation?

Dildine and Sunley (1978) argue in favor of taxing wage changes at large private corporates to minimize administrative costs and in favor of a penalty instead of a reward since rewards would have to apply to all firms to avoid preferential treatment. They recommend against exemptions of overtime and bonuses and support a hurdle which is easier to audit rather than a continuous tax. On the contrary, Seidman (1978a), Seidman (1978b) and Seidman (1979) argue that a continuous system with penalties and rewards which would reward firms that pay wage increases below target would be more efficient than a hurdle. Similarly, Layard (1982) and Jackman and Layard (1982) argue in favor of a TIP on wage increases based on the average hourly earnings of a firm, which incorporates most of the relevant information and which is easy to observe. Other papers in this literature include Slitor (1979) and Nichols (1979). Overall, the idea of a tax on wage increases, levied on employers, and on large firms seemed to receive broad support at the time as wages were easily observable and large firms already filled a detailed tax return.

Lerner (1978) proposes a market-based plan that gives similar incentives to firms to slow wage inflation through the issuance and exchange of permits. Firms willing to grant wage increases could purchase permits. The government would determine the total supply of permits and the price would be set freely by the market reflecting the marginal value of wage increases at any given point in time. This market plan is further developed in Lerner and Colander (1980).

Contributions to the Literature on TIP. It was acknowledged at the time that a better understanding of price-setting behaviors and their macro implications was required to design TIP effectively (Koford and Miller, 1992). Our first contribution is to leverage the tremendous progress made in sticky price models since then to re-assess the effectiveness of TIP.

We share with the earlier literature the conclusion that TIP is an effective tool to control inflation, but while the earlier literature saw TIP as a substitute for MP, we find that MP and TIP are complementary instruments, each specializing in their area of comparative advantage. TIP should focus on markup and inflation expectation shocks, and MP should focus on demand shocks. We also confirm the idea that a TIP in steady-state increases welfare by decreasing markup and increasing employment, but we find that TIP should vary over time. It should increase when inflation rises, and decrease as inflation reaches its long-term target.

On implementation, we contribute by analyzing the robustness of TIP to several alternative instruments (feebate and a market for inflation permits), to the use of targeting rules for MP and TIP and to alternative tax bases (tax on wage increases, or tax on large firms), which could be easier to administer. We discuss in conclusion a few other important issues of implementation that deserve more attention in future research.

Optimal Tax Policies. Our paper contributes to the rich literature on tax policies in New Keynesian models, in settings where monetary policy can't implement the first-best allocation, in particular when it is constrained by the ZLB. Papers have found a welfare-enhancing role for tax increases aimed at restricting supply (Eggertsson and Woodford, 2006), tax cuts aimed by stimulating demand (Eggertsson, 2011), temporary government spending (Woodford, 2011), cuts in marginal labour tax rates that boost confidence (Mertens and Ravn, 2014), and well-designed paths of consumption and labor taxes, import and export tariffs (Correia et al., 2013; Farhi et al., 2014). We generalize the set of fiscal instruments, showing that, like payroll subsidies in these two papers, TIP helps implement the first best. We further argue that TIP is much less costly for the government budget than traditional subsidies.

TIP in Practice. In the 1970s, versions of TIP were implemented. From 1974 to 1977, the French governments implemented the "prélèvement conjoncturel", which covered the largest 1500 firms, representing 60% of the economy, and was based on the excess increase in value-added in nominal terms relative to an announced threshold, with an adjustments for fast-growing firms. Other versions of TIP were implemented in Mexico, Belgium, Italy, as mentioned in

Paci (1988), and in the Netherlands as explained in OECD (1975). More research is needed to analyze their institutional details and impacts.

TIP came close to be implemented in the U.S. as well. In 1978, the Carter administration proposed to Congress the "real wage insurance" to supplement the wage-price guidelines which included voluntary limits on nominal wage and price increases of 7% and 5.75% respectively. This program meant to give incentives to workers to enforce the guidelines: a worker belonging to an employee group whose earnings increased by less than 7% in a year would receive a tax-credit proportional to the difference between the realized inflation rate and 7% (Colander, 1981).

Why have discussions around TIP stopped in the U.S.? We see three causes. First, afraid of its uncertain costs for the Federal budget (the proposal was more a feebate than a tax), Congress didn't support the plan. Second, the plan combined TIP with price controls which were unpopular, especially since their use by Nixon in the early 1970s. Finally, and most importantly, with Volcker's successful anti-inflation policy in the 1980s, MP has emerged as the sole legitimate instrument for achieving price stability up until the present day. While these may explain why TIP was not further pursued at the time, this paper reassesses its potential role as an additional stabilization tool.

In the context of the transition of formerly Soviet countries to market economies, TIP came back to the forefront of policy discussions and versions of TIP were implemented in Bulgaria, Poland and Romania. Koford et al. (1993) put forward an anti-inflation plan and incentive policies to stabilize prices and output in transition economies. Bogetic and Fox (1993) analyze the design, implementation and enforcement of these policies in Bulgaria and Romania and concludes that they helped stabilize output and prices. Enev and Koford (2000) find a fairly substantial inflation-reducing effect from the Bulgarian policy but no significant results from the Polish policy. Another analysis of the effects of the tax on inflation and on the employment behavior of Polish firms can be found in Crombrugghe and de Walque (2011). These examples suggest that TIP is implementable, effective, and not too costly to administer (Paci, 1988).

3 A Model with Sticky Prices and a Tax on Inflation

We start by introducing a tax on inflation policy in an otherwise conventional New Keynesian model. After describing the households' problem, we present the one of firms maximizing their discounted sum of profits subject to Rotemberg (1982)-adjustment costs and a tax on price increases.

3.1 Households

There is a continuum of mass one of identical infinitely-lived households, indexed by h. Households preferences are given by

$$\mathcal{U}(B_{t-1h}) = \max_{C_{th},N_{th},B_{th}} \left\{ \frac{C_{th}^{1-\sigma}}{1-\sigma} - \frac{N_{th}^{1+\psi}}{1+\psi} + \beta_t E_t \mathcal{U}(B_t) \right\} \tag{1}$$

where C_{th}, N_{th} and B_{th} denote consumption, labor supply and nominal wealth in period t, $\beta \in (0,1)$ is the time discount factor, σ and ψ denote the inverse of the elasticity of intertemporal substitution of consumption and labor respectively.

Households choose consumption of good $C_{th} > 0$, labor $N_{th} > 0$ and one-period bonds in the next period B_{th} subject to the following budget constraint:

$$P_t C_{th} + Q_t B_{th} = B_{t-1h} + W_{th} N_{th} + T_t \tag{2}$$

where Q_t is the price of one-period bonds, W_{th} denotes nominal wages and T_t includes transfers from the government as well as the firms' profits. We also assume that households are subject to a solvency condition which rules out Ponzi schemes $\lim_{T\to+\infty} \Pi_{j=0}^{T} Q_j B_{Th} \geq 0$.

Households are differentiated by their idiosyncratic labor type. When choosing their labor supply, households take into account the firms' demand for their type:

$$N_{th} = \left(\frac{W_{th}}{W_t} \right)^{-\epsilon_{Nt}} N_t. \tag{3}$$

The problem of the households is to maximize (1) subject to their budget constraint (2) and the no-ponzi condition, the labor demand of firms (3) and taking prices as given. The optimality conditions are given by

$$\frac{W_{th}}{P_t} = \mathcal{M}_t^w C_{th}^\sigma N_{th}^\psi \tag{4}$$

$$Q_t = E_t \left[\beta_t \left(\frac{C_{t+1h}}{C_{th}} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{5}$$

for all $t = 0, 1, 2....$ The first equation determines the optimal labor supply given consumption C_{th} and the real wage W_{th}. The optimal markup on wages by households, $\mathcal{M}_t^w = \frac{\epsilon_{Nt}}{\epsilon_{Nt}-1}$, is allowed to vary over time and it is one of the two distributional conflict shocks in the economy: it captures attempts by workers to increase their real wage for a given supply of labor. The second equation is the traditional Euler equation which determines the optimal path of consumption given real returns on bonds. The discount factor β_t is also allowed to vary to capture demand shocks.

3.2 Final Good Competitive Firms

The final good is produced competitively by a continuum of firms. The production technology uses a continuum of varieties of intermediate goods supplied by monopolistic firms described in the next section, which we index by $i \in [0, 1]$

$$Y_t = \left(\int_0^1 Y_{ti}^{1-1/\epsilon_t} di \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{6}$$

where ϵ_t is the elasticity of substitution across varieties. Taking the price of the final good P_t and the prices of inputs $\{P_{ti}\}_i$ as given, final good firms maximize profits $\max_{Y_{ti}} P_t Y_t - \int_0^1 P_{ti} Y_{ti} di$ subject to the technology (6). The optimality condition for variety i is given by

$$Y_{ti} = \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon_t} Y_t \tag{7}$$

In equilibrium, the free entry of firms implies a no-profit condition $\int_0^1 P_{ti} Y_{ti} di = P_t Y_t$, which in turn, and after substituting out for Y_{it} using the demand from final goods firm, gives the following expression for the consumption price index $P_t = \left(\int_0^1 P_{ti}^{1-\epsilon_t} di \right)^{\frac{1}{1-\epsilon_t}}$.

3.3 Intermediate Good Monopolistic Firms

3.3.1 Technology and Market Structure

There is a continuum of mass one of firms indexed by $i \in [0, 1]$. Each firm specializes in the production of a single variety of good which they sell to the final good firms. The technology to produce goods displays decreasing marginal returns:

$$Y_{ti} = A_t N_{ti}^{1-\alpha} \tag{8}$$

where A_t denotes total factor productivity, which is common across all firms, and $1 - \alpha$ is the elasticity of output to labor. Productivity shocks capture supply-chain disruptions and technological progress, but also changes in the prices of intermediate goods and energy. For example, increases in energy prices would translate into negative TFP shocks.

Firms are in monopolistic competition and choose the price P_{ti} at which they sell their good taking into account the final good firms' demand given by (7). They also face adjustment costs to price changes, described by a function $\mathcal{C}(P_{t-1i}, P_{ti})$ which is differentiable, strictly convex and equal to 0 when prices are unchanged, $\mathcal{C}(x, x) = 0$ for all $x > 0$. A firm's i gross profit is given by

$$P_{ti} Y_{ti} - W_t N_{ti} - \mathcal{C}_t \left(P_{t-1i}, P_{ti} \right).$$

Following Rotemberg (1982), we assume that adjustment costs are quadratic:

$$\mathcal{C}_t \left(P_{t-1i}, P_{ti} \right) = \frac{\theta}{2} \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right)^2 P_t Y_t \tag{9}$$

where the assumption that the adjustment costs scale with the nominal level of output P_tY_t, is made for tractability and captures in a reduced-form way the notion that firms need to buy the final good to change their prices.

An alternative microfoundation of price rigidities is the time-dependent Calvo (1983) frictions whereby firms are allowed only occassionally to reset their price. While both microfoundations are used in the literature, Rotemberg adjustment costs turn out to be more tractable in our setting. In addition, it is appealing to introduce TIP in a framework with adjustment costs to price changes because it will allow us to shed light on how TIP resembles but also differs from the technological adjustment costs. We show in Appendix C that our results hold with time-dependent Calvo-type frictions.

3.3.2 The Firm's Problem with a TIP

The key novelty of our framework is that firms pay a tax proportional to the increase in their price $\tau_t(P_{ti} - P_{t-1i})$ where $\tau_t \in \mathbb{R}$ is the tax rate which is allowed to vary over time. In addition, we specify that a firm pays the tax on the price increase of each unit of goods sold so that a firm's total tax payment scales with its output, Y_{ti}. Hence profits net of taxes are given by

$$\Pi(P_{t-1i}, P_{ti}) = P_{ti}Y_{ti} - W_tN_{ti} - \tau_t(P_{ti} - P_{t-1i})Y_{ti} - \mathcal{C}_t\left(P_{t-1i}, P_{ti}\right). \tag{10}$$

where labor demand N_{it} is given by the technology (8) and output Y_{ti} by (7). Because the adjustment costs and the tax payment depend on a firm's current and past prices, the firm's problem is dynamic. In recursive form, it is given by

$$V(P_{t-1i}) = \max_{P_{ti}} \Pi(P_{ti}, P_{t-1i}) + E_t^f\left[Q_tV(P_{ti})\right] \tag{11}$$

where profits are given by (10). After substituting for labor using the production function, and for output using the demand schedule, one can take the first order condition for the optimal price. After imposing that all firms are identical in equilibrium, $P_{ti} = P_t$, and defining the rate of inflation $\pi_t = P_t/P_{t-1} - 1$, the optimality condition is given by

$$(\epsilon_t - 1)\left(\mathcal{M}_t MC_t - 1\right) + E_t^f\left[Q_t \frac{Y_{t+1}}{Y_t}\left(\tau_{t+1} + \theta(\pi_{t+1}+1)\pi_{t+1}\right)\right] = \tau_t\left(1 - \epsilon_t \frac{\pi_t}{1+\pi_t}\right) + \theta\pi_t(\pi_t+1) \quad (12)$$

where the marginal cost is given by $MC_t = \frac{W_t}{P_t(1-\alpha)} \frac{Y_t^{\frac{\alpha}{1-\alpha}}}{A_t^{\frac{1}{1-\alpha}}}$ and the ideal markup in the flexible price equilibrium is given by $\mathcal{M}_t = \frac{\epsilon_t}{\epsilon_t - 1}$. The markup is allowed to vary over time and it is the second distributional conflict shock in the economy: it captures attempts by firms to increase their prices for a given marginal cost, as in Clarida et al. (1999). Together with the time-varying wage markup, it is a reduced-form way to model conflicting aspirations of workers and firms over relative wages and prices, as in Werning and Lorenzoni (2023).

Finally, we allow the firms' expectations, $E^f()$, to deviate from rational expectations, $E()$, to capture the idea that exogenous shifts in inflation expectations may drive inflation. We model these deviations in a very simple way: for any function g, we denote the inflation expectation shock u_t^E such that $E_t^f\left[g(\pi_{t+1})\right] = E_t\left[g(\pi_{t+1} + u_t^E)\right]$.[1] Condition (12) can thus be rewritten as

$$(\epsilon_t - 1)\left(\mathcal{M}_t MC_t - 1\right) + E_t\left[Q_t \frac{Y_{t+1}}{Y_t}\left(\tau_{t+1} + \theta(\pi_{t+1}+1)\pi_{t+1}\right)\right] + E_t\left[Q_t \frac{Y_{t+1}}{Y_t}\theta u_t^E(2\pi_{t+1}+1)\right] = \tau_t\left(1 - \epsilon_t \frac{\pi_t}{1+\pi_t}\right) + \theta\pi_t(\pi_t+1) \quad (13)$$

3.3.3 The Mechanics of TIP at the Micro-level

To see how TIP works at the micro-economic level, we look at the version of the first-order condition (13) linearized around a zero inflation steady-state:

$$(\epsilon - 1)\hat{mc}_t + u_t + \theta u_t^E + \beta E_t\left[\tau_{t+1} + \theta\pi_{t+1}\right] = \tau_t + \theta\pi_t \quad (14)$$

[1]We are not the first ones to introduce inflation expectation shocks. For example, Hazell et al. (2022) rationalize the drop in inflation in the 1980s with similar shifts to inflation expectations (see p.1339).

where u_t is the firm's markup shock. To get more intuition on how TIP operates, we consider a simple case in which TIP is positive at the initial period and zero forever after—$\tau_0 > 0$, and $\tau_t = 0$ for all $t > 0$. In addition, we assume that there is no inflation in the future $\pi_{t+1} = 0$ and no change in the marginal cost so that $\hat{mc}_t = 0$.[2] Using equation (14) we obtain

$$p_0 = p_{-1} + \frac{u_0 + \theta u_0^E - \tau_0}{\theta} \tag{15}$$

which shows that the optimal reaction of firms to an increase in TIP τ_0 is to decrease their (log) price p_0. This reaction is very intuitive: with a tax on price increases, firms have an incentive to moderate their price increases. If there is a positive markup, or an inflation expectation shock $u_0, u_0^E > 0$, TIP can thus be used to give firms incentives to attenuate the pass-through of these shocks to prices. Consistent with this intuition, in the absence of shocks, $u_0 = u_0^E = 0$, a positive TIP would lead firms to lower their price to receive a subsidy, and hence to deflation. Interestingly, the smaller the adjustment cost θ, the stronger the disinflationary effect of a given level of TIP, τ_0.

3.4 Equilibrium

Definition and linearized equilibrium. The market clearing conditions and the definition of equilibrium are given in Appendix A.1. In Appendix A.2, we derive the log-linear approximation of the model which we will use to illustrate the optimal policies in the next section and to simulate the economy when policies follow targeting rules in section 6. We denote $\hat{y}_t^e = \log(y_t/y_t^e)$ the log-deviation of the output gap from its value y^e in the efficient flexible price equilibrium. The efficient flexible price equilibrium is the allocation that would arise if there were no adjustment costs to price changes, $\theta = 0$, and no markup and inflation expectation shocks. In this equilibrium, the only sources of disturbances are the productivity A_t and discount factor shock β_t. In our economy, prices are sticky $\theta > 0$, and there are firm's and worker's markup

[2] These assumptions are consistent with an equilibrium of the model where there is no shock other than u_t and where monetary policy sets the real rate equal to the neutral rate.

and inflation expectation shocks, u_t, u_t^w, u_t^E.

Lemma 1. *The linearized Euler equation and the Phillips curve are given by:*

$$\hat{y}_t^e = E_t \hat{y}_{t+1}^e - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^e\right) \tag{16}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{y}_t^e + \frac{1}{\theta}\left[\beta E_t \tau_{t+1} - \tau_t + \tilde{u}_t\right] \tag{17}$$

where $\kappa = \frac{\epsilon - 1}{\theta}\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$, $r_t^e = -\log \beta_t + \sigma E_t\left[\frac{1+\psi}{(1-\alpha)\sigma+\psi+\alpha}(a_{t+1} - a_t)\right]$ *and* $\tilde{u}_t = u_t + u_t^w + \theta u_t^E$.

In Appendix C, we show that Calvo-type frictions deliver the same first-order approximation and macroeconomic dynamics. This implies that the results on optimal TIP derived in the linearized model in the next sections are robust to using these frictions instead.[3]

4 Optimal Tax on Inflation and First Best Allocation

In this section, we characterize the combination of MP and TIP that implements the first-best allocation, with no inflation and no output gap. We show that TIP should increase with markup and inflation expectation shocks, because they introduce an externality in the firms' pricing decision. TIP re-aligns private with social valuations and corrects excessive aggregate inflation.

4.1 First Best: Complete Stabilization and Specialization

Social Planner's First Best Allocation. The social planner seeks to maximize the household utility given by (1) subject to the resource constraints (36), the technology (8), and (37). The optimal condition is that at any time t the marginal rate of substitution between labor and consumption is equal to the marginal

[3]The equivalence holds for a first-order linearization around a zero inflation steady-state and no indexing. The same Phillips curve also arises as a linear approximation of the model of staggered price contracts by Taylor (1979, 1980) and the state-dependent pricing model with Ss foundations by Gertler and Leahy (2008). The exact equivalence breaks when inflation is strictly positive in steady-state or there is incomplete indexing (Ascari and Sbordone, 2014).

product of labor: $(C_t^e)^\sigma (N_t^e)^\psi = (1-\alpha)A_t(N_t^e)^{-\alpha}$. After substituting N_t^e using the technology one obtains the optimal consumption and output:

$$C_t^e = (1-\alpha)^{\frac{1-\alpha}{(1-\alpha)\sigma+\psi+\alpha}} A_t^{\frac{1+\psi}{(1-\alpha)\sigma+\psi+\alpha}} \quad and \quad Y_t^e = C_t^e \tag{18}$$

Externalities of Price Changes and Limitations of Monetary Policy. If there are no markup or inflation expectation shock, it is well-known that monetary policy alone can implement the first best by tracking the neutral rate of interest

$$i_t = (Q_t^e)^{-1} - 1 \quad with \quad Q_t^e = E_t\left[\beta_t \left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\sigma(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}}\right] \tag{19}$$

for all t. This policy closes both the output gap $Y_t = Y_t^e$ and maintain price stability $P_t = P_{t-1}$ at all times. In this case, there is naturally no role for TIP.

However, when firms' inflation expectations rise beyond their rational levels, u_t^E, or when firms (workers) would like to increase their markup, $\mathcal{M}_t$ ($\mathcal{M}_t^w$), at the expense of consumers, the firms' returns to increasing prices are larger than their social returns. Distributional conflict and inflation expectation shocks open a wedge between the private and the social returns to increasing prices and thus create an externality in the pricing decision of firms. This wedge can be seen by taking the difference between the private—the left-hand side of the first-order condition of firms (13)–and the social planner's returns, equal to 0, of price increases:

$$\text{Private} - \text{Social} = (\epsilon_t - 1)\left(\mathcal{M}_t\mathcal{M}_t^w - 1\right) + E_t Q_t^e \left(\frac{A_{t+1}}{A_t}\right)^{\frac{(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}} \theta u_t^E \tag{20}$$

where we set N_t and C_t equal to their first-best value given by (18) and $\pi_{t+1} = 0$ for conciseness. The wedge (20) is clearly positive for $\mathcal{M}_t > 1, \mathcal{M}_t^w > 1$ or $u_t^E > 0$, or any combination of the three. If in addition, economic agents expect excessive inflation in the future, $\pi_{t+1} > 0$, it would generate excessive inflation in the current period as well.

Monetary policy alone cannot address these externalities and as a result cannot implement the first best and fully stabilize inflation and the output gap. This limitation is more commonly known in the literature through its macro-policy implication: the trade-off between inflation and output in the face of cost-push shocks. To correct these externalities giving rise to excessive inflation, policymakers need an instrument that directly affects the price-setting behavior of firms.

First-Best Implementation with TIP. By giving direct incentives to moderate price increases when these shocks hit, TIP can re-align the private with the social valuations of price increases and therefore correct excessive aggregate inflation. Using the Euler equation (5) and the Phillips curve (13), the proposition below derives necessary conditions on MP and TIP such that the decentralized equilibrium coincides with the first best allocation with no inflation and no output gap.

Proposition 1 (First Best Policies)**.** *For any paths of distibutional conflict $\mathcal{M}_t, \mathcal{M}_t^w$ and inflation expectation u_t^E, aggregate demand β_t and TFP shocks A_t, there exists a combination of TIP and MP that can implement the first best. The nominal policy interest rate is given by* (19) *and TIP is given by*

$$\tau_t = (\epsilon_t - 1)\left(\mathcal{M}_t \mathcal{M}_t^w - 1\right) + E_t Q_t^e \left(\frac{A_{t+1}}{A_t}\right)^{\frac{(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}} \left[\tau_{t+1} + \theta u_t^E\right] \tag{21}$$

This proposition highlights two important results. First it says that with TIP and MP, policy makers can completely stabilize the output gap and inflation. This is in sharp contrast with an economy where only MP is available. Second, it says that MP should specialize in shocks that affect the neutral rate of interest—aggregate demand β_t, and TFP shocks A_{t+1}/A_t, and that TIP should specialize in markup $\mathcal{M}_t, \mathcal{M}_t^w$ and inflation expectation shocks u_t^E.[4]

It is also clear from equation (21) that TIP is most effective at mitigating current markup and inflation expectation shocks when it is expected to decrease in the future. When these shocks hit, the government should temporarily raise

[4]The combination of TIP and MP that implements the first-best allocation in a setting with time-dependent Calvo frictions is given in Appendix C.

TIP and announce it will decrease it later (if no new shocks occur). This commitment makes it very appealing for firms to postpone their price adjustments to avoid the tax in the current period. If firms expected the policy to be permanent, these incentives would be significantly weaker and the short-term pass-through of the shock to inflation would be stronger.

First-best with TIP in the Linearized Model. To build intuition, we now give a simpler expression for the optimal TIP in the linearized model.

Corollary 1 (First Best Policies in Linearized Model.)**.** *In the linearized model, the first best allocation and policies are characterized by*

- *Complete stabilization of the output gap and inflation* $\hat{y}_t^e = \pi_t = 0$.[5]
- *Complete specialization of MP and TIP*

$$i_t = r_t^e \quad \textit{and} \quad \tau_t = \beta E_t \tau_{t+1} + u_t + u_t^w + +\theta u_t^E$$

The corollary makes it clear that the optimal TIP rises with current and future markup and inflation expectation shocks. To be concrete, suppose all three shocks u_t, u_t^w, u_t^E are AR(1) processes with identical auto-regressive coefficient $\rho_u < 1$ and with disturbances distributed jointly normal with mean zero.

Example 1. *The optimal TIP is given by*

$$\tau_t = \frac{u_t + u_t^w + \theta u_t^E}{1 - \beta \rho_u}. \tag{22}$$

It increases with the current level ($u_t + u_t^w + \theta u_t^E$) *and the persistence* (ρ_u) *of the shock.*

The more persistent the shocks the stronger the reaction to give sufficient incentives to postpone raising prices into the future. On the contrary, if the shocks are i.i.d. over time, TIP should simply match the current ones $\tau_t = u_t + u_t^w + \theta u_t^E$.

[5] Recall that a second-order approximation of the loss function of the households' utility function is given by $\mathcal{L} = \sum_{t=0}^{\infty} E_0 \beta^t \left[\pi_t^2 + \eta_y \left(\hat{y}_t^e \right)^2 \right]$ for a positive constant $\eta_y > 0$. The allocation with zero inflation and no output gap thus achieves the first best.

The set of policies described in corollary 1, although necessary, may not be sufficient to implement the first-best allocation. These policies may be consistent with several equilibria that are not all first best. To ensure uniqueness, we add the requirement that the central bank reacts strongly to inflation, which is the usual condition for determinacy in this class of model: $i_t = r_t^e + \phi_\pi \pi_t$, with $\phi_\pi > 1$. With this rule, inflation is always zero in equilibrium, $\pi_t = 0$, and the interest rate tracks the neutral rate of interest, $i_t = r_t^e$.[6]

4.2 Remarks

An Efficient Steady-state. In steady state, the market power of firms and workers introduce two wedges between the rate of substitution of consumption and labor and the marginal product of labor which distorts the allocation and lowers output and consumption. While it is usually argued that a production or payroll subsidy a_w such that $\mathcal{M}\mathcal{M}^w(1 - a_w) = 1$ can address the distortion, we find that TIP can also correct it with

$$\tau = \frac{\epsilon - 1}{1 - \beta} \left(\mathcal{M}\mathcal{M}^w - 1 \right). \tag{23}$$

The mechanism behind this result is intuitive. Start from a steady state in which TIP is zero and firms markup $\mathcal{M} > 0$ is optimal, and suppose that the government increases TIP forever. Firms now face an incentive to decrease their price, expand production and hire more labor. In general equilibrium, this leads to an increase in production and in the real wage. But the firms' original incentive to decrease prices is now exactly counterbalanced by their lower-than-optimal markup, which implies that the economy settles in a new steady state with higher output.

Time-consistency of TIP. An appealing property of using TIP is that the combination of MP and TIP is time-consistent. The best MP and TIP under discretion are identical to the best policies under commitment, and therefore

[6] In a setting with TIP, the Taylor principle may not be the only way to ensure determinacy. We investigate whether TIP can provide new ways to ensure uniqueness in section 6.

also deliver full stabilization of the output gap and inflation. This is in sharp contrast with a setting without TIP, where discretionary policies deliver higher inflation, larger output gap and lower welfare (Clarida et al., 1999). To see why, recall that committed policies are more effective than discretionary ones only when commitment about future actions can help alleviate current trade-offs. But given that TIP eliminates the trade-off between output and inflation in the short-run, discretionary policies become as good as committed ones.

Policy Coordination. Although we solved jointly for the optimal MP and TIP, coordination is not required to implement the first best. In other words, the policies described in proposition (1) are also a Nash equilibrium of a game between two hypothetical authorities controlling MP and TIP separately.[7]

Second-best and TFP Shocks. According to the previous analysis, there is no role for TIP when inflation stems from TFP shocks. But if one were to incorporate additional realistic frictions, such labor market imperfections, or financial stability concerns, the clear specialization result we obtained previously may need to be amended and TIP could supplement MP even in the face of demand and TFP shocks. If for example the central bank couldn't increase its interest rate enough to implement the first best after a negative energy price shock due to financial stability concerns, firms would pass through too much of the cost increase. In that situation, it would arguably be valuable to use TIP besides MP to make firms internalize the social gains of a temporary price moderation and thereby limit an excessive inflation.

5 Equivalence with Alternative Fiscal Instruments

Production subsidies are the traditional tools employed in the literature to address the distortion caused by the firms' markup. In this section we formalize the equivalence between these instruments and TIP, and discuss their relative

[7] An important underlying assumption for this result is that both authorities share the same objective of maximizing the households' welfare.

strengths and weaknesses. We then show that a feebate on inflation, and a market for inflation permits could also implement the first best allocation.

5.1 Production and Payroll Subsidies

Traditionally, the literature has emphasized the role of production or payroll subsidies in implementing the first-best allocation.[8] Denoting a_w the rate of payroll subsidies,

$$\Pi(P_{t-1i}, P_{ti}, Y_{ti}) = P_{ti}Y_{ti} - (1 - a_t^w)W_t N_{ti} - \mathcal{C}\left(P_{t-1i}, P_{ti}\right). \tag{24}$$

the following proposition formalizes the equivalence with TIP:

Proposition 2. *Given a path of exogenous shocks, the equilibrium paths of outputs Y_t, employment N_t, wages W_t, profits Π_t and prices P_t are identical in the economy with a payroll subsidy and in the economy with TIP if and only if the path of subsidies follows*

$$a_t^w = \frac{\tau_t - E_t Q_t^e \left(\frac{A_{t+1}}{A_t}\right)^{\frac{(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}} \tau_{t+1}}{(\epsilon_t - 1)\mathcal{M}_t \mathcal{M}_t^w}$$

This proposition directly stems from the comparison of the first-order condition (13) with the one with a payroll subsidy:

$$(\epsilon_t - 1)\left(\mathcal{M}_t \mathcal{M}_t^w (1 - a_w) - 1\right) + E_t Q_t^e \left(\frac{A_{t+1}}{A_t}\right)^{\frac{(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}} \theta u_t^E = 0. \tag{25}$$

The advantages of payroll and production subsidies is that they are more conventional and easier to communicate. However, we highlight three weaknesses. The most important one is that subsidies imply large and persistent fiscal costs for the government's budget. Using the calibrated version of the model of section 6, we show in Appendix A.3 that the fiscal costs associated with a production subsidy that achieves the same macroeconomic path as

[8] Beyond the traditional optimal payroll subsidy to correct the distortion implied by the firms' markup, Correia et al. (2013) and Farhi et al. (2014) show that a payroll subsidy can help achieve the first best in combination with other conventional fiscal instruments when monetary policy is limited by the ZLB or a fixed exchange rate in an economy subject to demand shocks.

TIP following a markup or inflation expectation shock are very large, and an order of magnitude higher than the fiscal revenues—the tax burden for firms—implied by TIP. In addition, subsidies need to be much more persistent than TIP to achieve the same inflation path.

A second limitation is that in practice these subsidies usually cover a limited set of inputs, such as energy consumption or labor, which leads firms to over-consume these inputs relative to others, and distorts the efficient allocation of factors of production. A third issue is the political acceptability of subsidies: policy-makers may find it hard to justify transferring resources to firms that are already extracting monopolistic rents.

5.2 Feebate on Inflation Policy (FIP)

TIP increases the tax burden on firms in periods of high inflation. Although we find in the calibrated version of the model in section 6 that the tax burden on firms implied by TIP is small, we now show that it is possible to design a system that is budget-neutral on average, in which firms whose prices increase less than the average firm would receive a subsidy and firms whose prices increase more would pay a tax. Such combination of a tax on inflation with a well-designed rebate, which we call a feebate on inflation policy (FIP), would preserve firms' profits on average.

We now formally show that FIP can implement the same macroeconomic outcomes as TIP. Denoting F_t the lump-sum rebate, firms' profits are given by:

$$\Pi(P_{t-1i}, P_{ti}, Y_{ti}) = P_{ti}Y_{ti} - W_t N_{ti} - \tau_t(P_{ti} - P_{t-1i})Y_{ti} - \mathcal{C}\left(P_{t-1i}, P_{ti}\right) + F_t \quad (26)$$

$$\text{with} \quad F_t = \tau_t(P_t - P_{t-1})Y_t. \quad (27)$$

It is easy to see that firms whose prices increase less than the average firm would receive a subsidy and firms whose prices increase more would pay a tax. In addition, the FIP is by construction budget neutral, i.e. all the receipts from the TIP are given back to firms, and households' transfers are net of the rebate:

$$T_t = \int_0^1 \Pi_{it} + \tau_t(P_{ti} - P_{t-1i})Y_{ti} - F_t di = \int_0^1 \Pi_{it} di \quad (28)$$

The definition of an equilibrium is the same as before except that firms now maximize (11) subject to the new definition of profits (26) and the definition of transfers to households is given by (28). The following proposition establishes that the allocation in the economy with FIP is exactly the same as in an economy with TIP, except for profits. In the proposition, we denote x_t^{TIP} the value of variable x at time t in the economy with a TIP.

Proposition 3. *Given a path of shocks and of TIP τ_t, the equilibrium paths of output Y_t, employment N_t, wages W_t and prices P_t are identical in the economy with a FIP and a TIP, i.e. $Y_t^{TIP} = Y_t^{FIP}$, $N_t^{TIP} = N_t^{FIP}$, $W_t^{TIP} = W_t^{FIP}$ and $P_t^{TIP} = P_t^{FIP}$. In addition, the path of profits in FIP is higher than in TIP by $F_t : \Pi_t^{FIP} = \Pi_t^{TIP} + F_t$.*

To understand this proposition, it is important to see that the rebate F doesn't affect the firms' behaviors since it is lump-sum and that it doesn't affect the households' income since the lower receipts from the tax on inflation are exactly offset by the higher profits they receive from firms. As a result, no behavior is changed and the equilibrium allocations are the same in both settings. However, profits are higher, which is exactly why FIP is appealing.

5.3 Market for Inflation Permits (MIP)

An alternative instrument, initially proposed by Lerner (1978), is the market for inflation permits (MIP), where firms would issue and trade rights to increase their prices. With a MIP, the quantity of permits is controlled by the government and the price for a firm to change its price is an endogenous clearing price instead of an exogenous tax rate. Relative to a tax on inflation, a MIP could provide more certainty on the level of inflation and would not require approval from the fiscal authority, allowing for quicker reaction when inflation rises. It turns out that, like the FIP, a well-designed MIP may achieve exactly the same macroeconomic outcomes as a TIP.

Let's denote q_t the price of one permit and H_t the quantity of such permits. We assume that H_t is issued every period by the government and that firms can't accumulate permits over time.[9] Under a MIP, profits net of taxes are

$$\Pi(P_{t-1i}, P_{ti}, Y_{ti}) = P_{ti}Y_{ti} - W_t N_{ti} - q_t(P_{ti} - P_{t-1i})Y_{ti} - \mathcal{C}\left(P_{t-1i}, P_{ti}\right). \tag{29}$$

In addition, the market for permits should clear

$$\int_0^1 (P_{ti} - P_{t-1i})Y_{ti}di = H_t \tag{30}$$

and the receipts from the sale of permits are given back to households $T_t = \int_0^1 \Pi_{it}di + q_t H_t$. The definition of an equilibrium is the same as before except that firms now maximize their discounted sum of profits (11) subject to the definition of profits (29). All markets clear including the MIP (30). The following proposition establishes that the allocation in the economy with a MIP is exactly the same as in an economy with TIP, provided that the path of permits issued by the government is appropriate.

Proposition 4. *Given a path of shocks, the equilibrium paths of outputs Y_t, employment N_t, wages W_t, profits Π_t and prices P_t are identical in the economy with a MIP and in the economy with TIP if and only if the path of permits follows*

$$H_t = \frac{\pi_t^{TIP}}{1 + \pi_t^{TIP}} P_t^{TIP} Y_t^{TIP}$$

To understand this proposition, first observe that the definition of profits in MIP (29) is the same as its definition in TIP (10) if and only if $\tau_t = q_t$. In turn, this equality is true if and only if the supply of permits in the economy with MIP H_t is equal to the total units of price changes in the economy with a TIP, which is equal to

$$H_t = \int_0^1 (P_{ti}^{TIP} - P_{t-1i}^{TIP})Y_{ti}^{TIP}di = \int_0^1 \frac{\pi_{it}^{TIP}}{1 + \pi_{it}^{TIP}} P_{it}^{TIP} Y_{it}^{TIP} di = \frac{\pi_t^{TIP}}{1 + \pi_t^{TIP}} P_t^{TIP} Y_t^{TIP}$$

where the last equality used the fact that in equilibrium all firms are identical.

[9] The allocation of permits at the beginning of each period across firms and between the government and firms doesn't affect the equilibrium level of output and inflation, it only affects the distribution of profits across firms.

This shows that the allocation with TIP can be replicated by issuing the value of permits equal to the increase in nominal output in the economy with TIP.

6 Macro Implications of an Inflation-Targeting TIP

The model analyzed in section 4 assumed that policymakers perfectly observe the underlying shocks driving inflation. We now relax this assumption and suppose instead that both TIP and MP follow rules targeting inflation and the output gap:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t^e \tag{31}$$

$$\tau_t = \varphi_\pi \pi_t + \varphi_y \hat{y}_t^e. \tag{32}$$

We analyze their macroeconomic implications using simulations of the linearized model. When evaluating social welfare and characterizing optimal policies, we consider the following loss function:

$$\mathcal{L} = \sum_{t=0}^{\infty} E_0 \beta^t \left[\pi_t^2 + \eta_y \left(\hat{y}_t^e\right)^2 + \eta_i i_t^2 \right] \tag{33}$$

for some $\eta_y, \eta_i \geq 0$. A second-order approximation of the household welfare loss around the efficient steady-state is consistent with this objective function for $\eta_i = 0$ as shown in Rotemberg and Woodford (1999).

6.1 Calibration.

In our calibration, a period is a quarter. The list of parameters is in Table 1. We follow Galí (2015) to calibrate the elasticity of output to labor $(1-\alpha)$, the discount factor β, the elasticity of intertemporal substitution σ, the Frisch elasticity ψ and the elasticity of substitution ϵ.

We choose the adjustment cost parameter θ such that the slope of the linearized Phillips curve, κ, in our model is equal to the slope in Galí (2015). If $\bar{\phi}$ denotes the Calvo parameter, the slope with Calvo pricing is given by $\kappa = \frac{(1-\bar{\phi})(1-\bar{\phi}\beta)}{\bar{\phi}} \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \left(\sigma + \frac{\psi+\alpha}{1-\alpha} \right)$. In our model, the slope of the Phillips curve

Parameters	Description	Value
α	One minus the elasticity of output to labor	0.25
β	Time discount factor	0.99
σ	Elasticity of intertemporal substitution	1
ψ	Inverse Frish elasticity of labor	5
ϵ	Elasticity of substitution across varieties	9
ϵ_N	Elasticity of substitution across labor types	∞
θ	Adjustment cost	372.8
ρ_a	Autocorrelation of productivity shock	0.5
ρ_u	Autocorrelation of markup shock	0.5
ρ_{mp}	Autocorrelation of monetary shock	0.5
η_y	Preference for output stability	0.113
η_i	Preference for interest rate stability	0.687

Table 1: Model parameters

is instead given by $\kappa = \frac{\epsilon-1}{\theta}\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$. Equating the two gives:

$$\theta = (\epsilon - 1)\frac{\bar{\phi}}{(1-\bar{\phi})(1-\bar{\phi}\beta)}\frac{1-\alpha+\alpha\epsilon}{1-\alpha} \tag{34}$$

A Calvo parameter of 0.75 which corresponds to an average price duration of one year implies $\kappa = 0.17$, and a Rotemberg parameter of 372.8.

To calibrate the parameters in the welfare loss function η_y, η_i, we choose η_y and η_i such that the Taylor rule with coefficients $\phi_y = .125$ and $\phi_\pi = 1.5$ minimizes welfare losses (33) in the absence of TIP. Both η_y, η_i depend on the persistence of the shocks, and for an interior solution to exist, the persistence of the shocks cannot be too high (Giannoni and Woodford, 2003; Giannoni, 2014). Therefore, we set all persistence parameters to 0.5.[10]

6.2 Determinacy with TIP

To ensure uniqueness of the equilibrium path, it is well-known that in the baseline New Keynesian model, MP should implement the Taylor principle

[10]We have checked that the impulse response functions and welfare implications of TIP are similar when ρ is larger, even when there is no interior solution for the optimal MP.

according to which the policy rate reacts strongly to inflation. Without determinacy, the economy is subject to coordination failures: for example, if all firms expect high inflation and high output gap, the economy could shift to a self-fulfilling equilibrium with excessive inflation. These coordination failures are a potential source of excessive inflation that is distinct from inflation expectations shocks, u_t^E, in that they are consistent with rational expectations.

A natural question is whether TIP could guarantee the uniqueness of the equilibrium path. We show in Appendix A.4 that theoretically it is possible to obtain determinacy if TIP reacts strongly to the output gap, and MP reacts passively to inflation. However, in the calibrated version of the model, we also find that the necessary strength of the reaction is beyond what could be realistically implemented. This leads us to the conclusion that the Taylor principle remains the only way to ensure determinacy in practice.

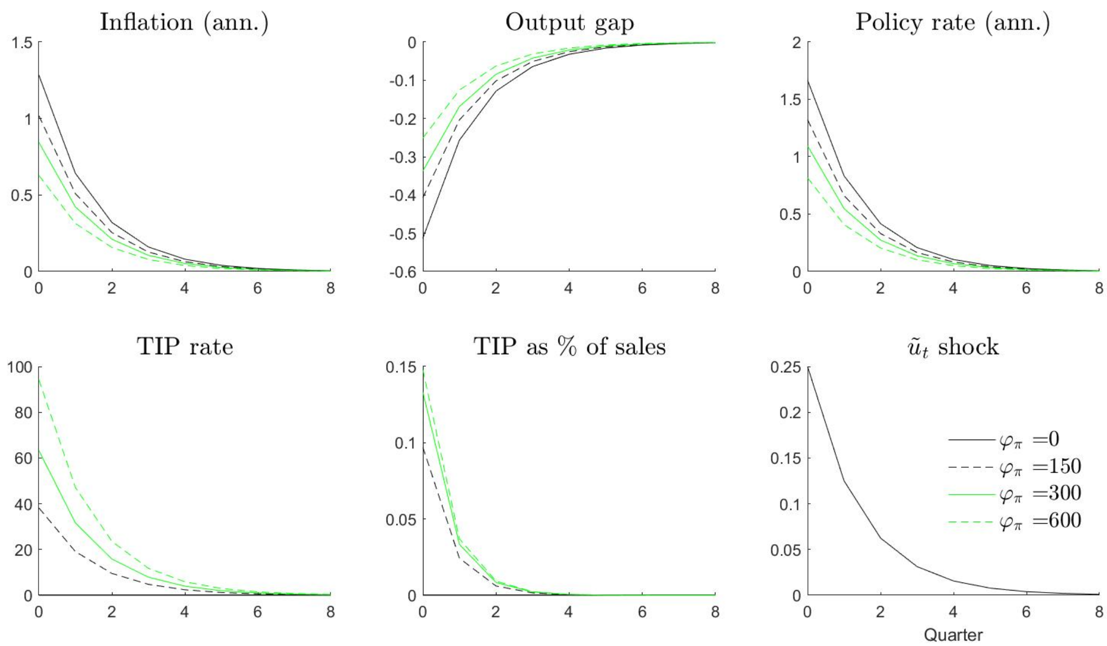
6.3 Stabilization Properties of TIP

To assess the effectiveness of TIP in reducing the volatility of inflation and the output gap, we start by drawing the IRFs of (annualized) inflation, the output gap, the (annualized) policy rate, the TIP rate τ_t and TIP as a percent of total sales following three types of shocks: markup, monetary and productivity shocks. We report the TIP tax normalized by sales because it is a better measure of the effective tax burden on firms than the tax rate. In a separate exercise, we also evaluate the implications for the volatility (standard deviations) of inflation, the output gap, the policy rate, TIP and the total welfare losses in an economy that is stochastically hit by these shocks, one type of shocks at a time. Unless otherwise mentioned, monetary policy follows a standard Taylor rule (31) with $\phi_\pi = 1.5$ and $\phi_y = 0.125$ and TIP only targets inflation, i.e. $\varphi_y = 0$. We explain at the end of the section that setting this parameter to zero is a reasonable assumption.

6.3.1 Markups and Inflation Expectation Shocks

Impulse Response Functions. Figure 1 shows the IRFs for different values of φ_π, following a 0.25pp distributional or expectation shock ($\frac{\tilde{u}_t}{\theta} = 0.25$pp).

Unlike monetary policy, TIP can very effectively reduce both inflation and the output gap after such shocks. This result is analogous to the "divine coincidence" for monetary policy after a shock to the neutral rate of interest. In the absence of TIP, inflation initially rises to 1.3% and output drops by slightly over 0.51%. A moderate TIP with $\varphi_\pi = 150$ reduces the initial inflation response to 1.0% and the decline in output to -0.40% by imposing a 38% tax on price changes in the first quarter. A stronger TIP with $\varphi_\pi = 300$ brings down the initial inflation response to 0.84% and the output gap to -0.34% by imposing a 63% tax on price changes.



Notes: The initial markup or inflation expectation shock is 0.25pp. Inflation and the policy rate are annualized.

Figure 1: Effects of TIP following a markup and inflation expectation ($\tilde{u}$) shock

The mechanism behind this divine coincidence is intuitive. The impact of TIP on inflation reduces the need to increase the policy rate, which mitigates the negative impact on output. Relative to a baseline situation with no TIP where the policy rate increases by 170 basis points on impact, a strong TIP with $\varphi_\pi = 300$ reduces the increase in the policy rate to 110 basis points.

While tax rates of 38%, 63% and 94% may all seem very high, even if temporary, the actual tax burden on firms is extremely low as a percent of sales because the tax only applies to price changes. For example, the tax burden of the strong TIP represents 0.13% of sales in the first quarter (or 0.04% in the first year). Interestingly, the tax burden relative to sales does not increase linearly as φ_π increases. This can be seen by looking at expression for the tax burden

$$\frac{\tau_t(P_{ti} - P_{t-1i})Y_{ti}}{P_{ti}Y_{ti}} = \frac{\tau_t \pi_t}{1+\pi_t} \approx \varphi_\pi \pi_t^2,$$

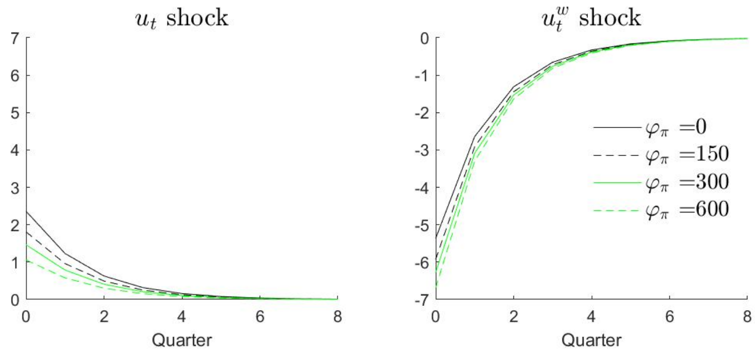
and its expectation, $E\pi_t\tau_t = \varphi_\pi V(\pi_t)$. As φ_π quadruples from 150 to 600, both π_t^2 and $V(\pi_t)$ decrease, yielding a small increase in taxes relative to sales.[11]

TIP affects corporate profits not only through its direct effect on the tax burden, but also through the general equilibrium changes in inflation and aggregate demand. Figure 2 displays the change in the corporate profits from its steady-state as a percentage of steady-state output, $\left(\frac{\Pi_t}{P_t} - \frac{\Pi}{P}\right) Y^{-1}$, after the two types of distributional shocks.[12] When inflation is driven by the firms' desire to increase markups, firms obtain higher profits as a share of output in equilibrium as shown in the left panel, and TIP effectively moderates the increase in profits. When the rise of inflation is driven by an increase in wage markups, profits decrease due to the increased labor cost as shown in the right panel, and TIP further worsens firms' profitability. Finally, when the rise of inflation is driven by inflation expectation shocks, u_t^E, firms expect *ex ante* their profit share to fall like after a wage markup shock but *ex post* profits actually rise like after a firm markup shock. This wedge between expected and realized profits reflects the fact that firms' expectations are not fully rational.

Sensitivity to the Degree of Price Flexibility. Contrary to monetary policy, we find that the effectiveness of TIP is increasing in the degree of price flexibility. To see this, we decrease the Rotemberg parameter θ such that it implies the

[11]While these issues are outside of the scope of the paper, the result that the tax burden of TIP is very small has an important implication: it should lead firms to comply with the tax, because the costs and legal risks of tax evasion are likely to outweigh the reduction in the tax burden.

[12]An approximation of the share of profit in output is derived in Appendix A.2.



Notes: The figure plots the deviation of corporate profits from steady state as a percentage of the steady-state output, following a price (left-hand side) and a wage (right-hand side) markup shock.

Figure 2: Profits following markup shocks

same slope of the Phillips curve as a decrease in the Calvo parameter $\bar{\phi}$ from 0.75 to 0.5. We also re-scale the $\tilde{u}_t$-shock so that the inflation response in the baseline without TIP is the same as in Figure 1. Figure F3 reports the results.

The intuition behind this result is that when the adjustment cost parameter θ is lower, firms find it less costly to adjust prices, which implies that TIP becomes a relatively stronger obstacle to price changes. Conversely, if prices changes are already very costly for technological reasons, TIP is relatively less important in the decisions of firms to adjust prices.

Stochastic Simulations. In Table 2 we report the results of a stochastic simulation with markup and inflation expectation shocks. Consistent with the IRFs just discussed, the standard deviations of inflation and the output gap uniformly decrease as φ_π increases. With a standard Taylor rule and a strong TIP ($\varphi_\pi = 300$), the variance of inflation and the output gap shrinks from 2.52 and 1.26 to 1.87 and .94, respectively. As a result, the welfare losses ($\mathcal{L}$) shrink by 47% from 7.91 when there is no TIP to 4.38. Based on unreported simulations, we confirm that this holds across many Taylor rules.

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ_π	1.5	1.5	1.5	1.5
ϕ_y	0.125	0.125	0.125	0.125
$\sigma(\pi_t^{ann})$	0.74	0.59	0.49	0.36
$\sigma(\hat{y}_t^e)$	0.59	0.47	0.39	0.29
$\sigma(i_t^{ann})$	0.96	0.76	0.63	0.47
$\sigma(\tau_t)$	0	44.1	73.0	108.8
E$(\pi_t\tau_t)$	0	0.13	0.18	0.20
$\mathcal{L}^* \times 10^4$	1.22	0.77	0.53	0.29

Notes: Markup and inflation expectation shocks only. The standard deviation of $\frac{u_t}{\theta}$ is 0.25pp, as in Figure 1. $\mathcal{L}^*$ is defined in equation (33) using quarterly variables (i.e., not annualized). All standard deviations and $E(\pi_t\tau_t)$ have been multiplied by 100.

Table 2: Evaluation of rules under markup and inflation expectation shocks

6.3.2 Aggregate Demand, Productivity and Monetary Shocks.

Impulse Response Functions. In Figure F1 in Appendix, we report the IRFs corresponding to an aggregate demand or a productivity shock. The size of the shock is chosen such that the inflation response in the baseline without TIP is the same as in Figure 1.

We find that TIP mitigates the inflationary effect of the shock to the same extent it does for markup and inflation expectation shocks. Similarly, it also reduces the need to increase the policy rate. With a strong TIP ($\varphi_\pi = 300$), inflation is 0.34pp lower and the hike in the policy rate is about 57 bps smaller.

However, the presence of TIP amplifies the initial widening of the output gap which becomes 0.18pp larger relative to a setting without TIP. This is a key difference with the conflict and inflation expectation shocks: TIP faces a trade-off between inflation and output under aggregate demand and productivity shocks (just as monetary policy faces a trade-off under conflict and inflation expectation shocks). As discussed in the section on first-best policies, ideally, aggregate demand and productivity shocks should be addressed with monetary policy. When both TIP and MP follow targeting rules, TIP can still complement MP in stabilizing inflation, but it faces a trade-off.[13]

[13] Figure F2 reports the IRFs corresponding to a monetary policy shock. TIP has a qualitatively

While we have treated aggregate demand and TFP shocks symmetrically, there are reasons to think that in reality they should not. Policy-makers typically don't want to trigger an aggregate consumption fall by raising the interest rate when a large negative TFP shock, for example an energy price surge, occurs. If instead of targeting the output gap relative to the efficient level of output, policymakers were targeting the output gap relative to the steady-state level, TFP shocks would be more akin to markup or inflation expectation shocks than to aggregate demand shocks. In this case, the divine coincidence shown for markup or inflation expectation shocks would also apply to TFP shocks.

Stochastic Simulations. Table F1 reports the results of a stochastic simulation with aggregate demand or productivity shocks. It confirms that TIP faces a trade-off between inflation and the output gap when these shocks hit the economy. Increasing φ_π to 300 lowers the variance of inflation from 2.52 to 1.87 but raises the variance of the output gap from 0.94 to 1.27. Because inflation is more volatile than output in the baseline without TIP, the reduction in the variance of inflation outweighs the increase in the variance of the output gap, resulting in higher welfare (lower $\mathcal{L}$).

6.4 Optimal Monetary Policy with TIP

How should the introduction of TIP change the design of the Taylor rule? Should it lead to specialization and a stronger emphasis on the output gap? To answer this question, we now simulate many economies that differ in the four weights in the rules $(\phi_\pi, \phi_y, \varphi_\pi, \varphi_y)$. For clarity, we report results for a small subset of these parameters, but the full matrices are available upon request. In particular, we focus on rules for TIP that put no weight on the output gap, $\varphi_y = 0$. We explain at the end of the section why this is reasonable. In each simulation, economies are stochastically hit by demand and markup shocks with the same standard deviation equal to .25.

similar effect on inflation and on the gap between output and its efficient level in the case of a monetary and productivity shock. This suggests that when monetary policy needs to deviate from the standard targeting rule (31) to address financial markets disruptions, or to manage the exchange rate, TIP can greatly mitigate the cost of these interventions.

Consistent with the first-best policies, we find that some specialization of monetary policy is desirable. Table 3 gives the optimal weights on inflation and the output gap in the Taylor rule for different specifications of TIP when demand shocks and markup shocks are independently distributed and have equal variance. Monetary policy should focus relatively more on output and less on inflation. The Taylor coefficient ϕ_π decreases from 1.5 to .99 and .98 for a moderate and strong TIP, respectively. By contrast, the coefficient on output increases from .125 to .126, and .162 for a moderate and strong TIP, respectively. Monetary policy should focus relatively more on the output gap when TIP is used because it is its comparative advantage: the interest rate directly affects aggregate demand, hence output, while it affects inflation only through changes in output. TIP on the other hand gives direct incentives to firms to moderate their price changes, and thus directly affects aggregate inflation. It is natural that each instrument should specialize in their area of relative effectiveness.[14,15]

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ_π^*	1.500	0.990	0.983	0.980
ϕ_y^*	0.125	0.126	0.162	0.134
$\sigma(\pi_t^{ann})$	0.77	0.71	0.58	0.41
$\sigma(\hat{y}_t^e)$	0.66	0.47	0.43	0.44
$\sigma(i_t^{ann})$	1.03	0.67	0.55	0.41
$\sigma(\tau_t)$	0	53.3	86.3	124.0
$\text{E}(\pi_t \tau_t)$	0	0.19	0.25	0.26
$\mathcal{L}^* \times 10^4$	1.38	0.84	0.56	0.31

Notes: Demand and markup shocks are independently distributed with the same standard deviation equal to .25. Monetary policy (ϕ_π^*, ϕ_y^*) is determined by minimizing $\mathcal{L}^*$ in equation (33) conditional on TIP. All standard deviations and $E(\pi_t \tau_t)$ have been multiplied by 100.

Table 3: Optimal MP

[14]Specialization becomes more desirable as the variance of markup shocks increases. In Table F3 where we double their standard deviation, ϕ_π decreases to .96 and ϕ_y rises further to .38 for a strong TIP.

[15]ϕ_y does not necessarily increase monotonically with φ_π. When TIP goes from strong to extreme in Table 3, we find that the optimal ϕ_y decreases slightly. Similarly, when we reduce the markups shocks variance in Table F4, stronger TIP lowers both ϕ_π and ϕ_y substantially. This is because it leads to a lower cost of i_t^2 in the loss function.

On the Choice of the Weight on the Output Gap in TIP (φ_y)**.** We now explain why it is reasonable to focus on settings with $\varphi_y = 0$. In the presence of markup and inflation expectation shocks, we find in unreported simulations that TIP should react strongly and positively to inflation and strongly but negatively to the output gap. This result stems from the divine coincidence we discussed earlier. However, it is unlikely that in the real world, policymakers would ever adopt a rule with $\varphi_y < 0$ and it is thus reasonable to restrict our attention to policy rules with $\varphi_y \geq 0$. Given this restriction, it is best to set $\varphi_y = 0$.

In the presence of aggregate demand or productivity shocks, we find that for the standard Taylor rule, there is a range of optimal TIP that delivers the same outcome: a TIP that reacts strongly to inflation and weakly to the output gap would generate the same outcome and equivalent to a TIP that reacts strongly to the output gap and weakly to inflation. It is therefore always possible to design a TIP with $\varphi_y = 0$ as long as the reaction to inflation is strong enough.

7 TIP and Relative Price Distortions

One concern with TIP is that it could impede the adjustment of relative prices leading to a misallocation of resources. To assess these effects, we extend the model to include a large number of sectors facing specific TFP shocks that require relative price adjustments. In contrast with price controls, we find that TIP doesn't exacerbate relative price distortions across sectors.

7.1 An Extended Model with Multiple Sectors

The economy is made of a continuum of sectors, indexed by $s \in [0,1]$. The production technology for final goods combines sector goods, with an unitary elasticity of substitution, $Y_t = \exp\left(\int_0^1 \gamma_s \ln Y_{ts} ds\right)$ with $\int_0^1 \gamma_s ds = 1$. Each sector is populated by a continuum of firms in monopolistic competition and sector goods combine varieties produced in their sector with a CES production function $Y_{ts} = \left(\int y_{tis}^{1-1/\epsilon_t} di\right)^{\frac{\epsilon_t}{\epsilon_t - 1}}$.

Firms face sector-specific TFP shocks. The production technology of a monopolist producing variety i is sector s is given by, $Y_{tis} = A_{ts} N_{tis}^{1-\alpha}$, where

A_{ts} is sector-specific and stochastic. Sector prices change over time, because of aggregate and sector shocks and for future reference, we denote the (log) relative productivity of sector s, $\tilde{a}_{st} = \log(A_{st}/A_t)$ where A_t is the average productivity. Relative to the setting in section 3, the equilibrium now features a non-degenerate distribution of relative prices across sectors, which we denote, in log, $\tilde{p}_{st} = p_{st} - p_t$.[16] We give more details in Appendix B.

Firms would ideally like to pass through variations in productivity to prices but they face quadratic adjustment costs. This implies that, even without TIP, relative prices depart from their values in the flexible price equilibrium because nominal frictions slow down their adjustment. Furthermore, and consistent with empirical evidence, we allow pricing frictions to differ across sectors and we denote θ_s the sector-specific degree of price stickiness.

The distortions in relative prices implied by nominal frictions misallocate sector outputs which decreases welfare. The lemma below shows that besides the output gap $(\hat{y}^e_s)^2$ and the average price changes, $E(\theta_s \pi_s^2)$, which captures the costly adjustments of prices *within* sectors as in the previous sections, the second-order approximation of the household welfare loss now also depends on the deviation of relative prices across sectors from their efficient levels, $\hat{\tilde{p}}^e_{st} = \tilde{p}_{st} - \tilde{p}^e_{st}$.

Lemma 2. *The second-order approximation of the households' welfare loss is given by*

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\alpha + \psi}{1-\alpha} \right) (\hat{y}^e_t)^2 + E[\theta_s \pi^2_{st}] + \frac{E\left[\left(\hat{\tilde{p}}^e_{st} \right)^2 \right]}{1-\alpha} \right] \tag{35}$$

where E denotes the cross-sectional mean under the density γ_s.

To investigate whether TIP amplifies price distortions, we proceed in two steps. First we uncover conditions such that TIP has exactly no effect on relative prices and the economy behaves like in the previous sections. Second, we numerically simulate the model to assess the robustness of this independence result.

[16] While in our setting "relative prices" unambiguously refer to relative prices across sectors, in a setting with Calvo frictions, there are also distributions of relative prices within sectors across firms. Later in the section, we discuss the robustness of our findings to Calvo frictions.

7.2 Independence of Sector Relative Prices from TIP: an Analytical Result

Following Rotemberg and Woodford (1999), we start with evaluating TIP using the definition of welfare (35) and the first order approximation of the economy. It turns out that under some conditions, TIP affects welfare only through the output gap and aggregate inflation, but not through changes in relative prices.

Proposition 5. *Assume θ_s is common across sectors and steady-state TIP is zero. Then*

- *Relative prices across sectors are independent of TIP. They depend only on the adjustment cost θ, other parameters (ϵ, α, β), and on the sector-specific productivity shocks process ($\tilde{a}_{st}$).*
- *The responses of the output gap and inflation, π_t, to aggregate shocks and TIP are the same as in the linearized economy of Section 4. The output gap and inflation, π_t, are sufficient statistics to evaluate the welfare impacts of TIP.*

The proof is in Appendix B.

The independence of relative prices stems from the linearity of TIP. On the one hand, firms that face a negative TFP shock, and would like to increase their price, will moderate their price increases because of TIP. On the other, firms that would not change their price without TIP, will lower their price to benefit from the subsidy. The resulting effect on relative prices is ambiguous. But it turns out that relative prices remain exactly unchanged when all sectors face the same adjustment cost parameter $\theta_s = \theta$ and the steady-state TIP is zero.[17] Price controls, in contrast, distort relative prices, because they are akin to a convex cost. This further highlights the importance of the linearity and symmetry of TIP: if TIP were convex, or if it applied only to positive price changes and didn't subsidize price decreases, it would distort sector prices.

The fact that the aggregate economy behaves like in the linearized model of the previous sections means that the TIP given in Corollary 1 perfectly stabilizes inflation and the output gap in response to aggregate markup and

[17] If we had initially allowed for heterogeneous elasticities of substitution, ϵ_{st}, and hence markups, they would have had to be equal across sectors for the independence result to hold.

inflation expectation shocks. Similarly, all results derived in Sections 6 and 8 remain quantitatively exactly the same. Finally, the result that welfare varies with TIP only through the output gap and average inflation stems from the independence of relative prices from TIP. Indeed this independence implies that the tax doesn't affect the deviations of relative prices from their efficient levels $E\left((\tilde{p}_{st}-\tilde{p}^e_{st})^2\right)$, and that it changes the cost due to price adjustments, $\theta E[\pi^2_{st}]$, only through its average effect on prices, $\theta\pi^2_t$.[18]

7.3 Independence Result: Numerical Simulations

Although Proposition 5 is quite general since it holds at and outside of the steady-state, in deterministic and in stochastic settings, it relies on a first-order approximation, and on the homogeneity of price stickiness across sectors. To assess its robustness, we now turn to numerical simulations in the non-linear version of the model.

Calibration. For parameters that are common across sectors, we closely follow the calibration in section 6 (Table 1). In addition, we assume that (log) TFP in each sector follows an AR(1) process $\log(A_{ts}) = (1-\rho)\log A_s + \rho\log(A_{t-1s}) + \nu_{st}$ where ν_{st} are i.i.d. across time and sectors. The standard deviation of the shock ν_{st} is set to .05 and the persistence of the sector-specific TFP, ρ, to .5. To calibrate the values of θ_s, we follow Nakamura and Steinsson (2008) who document the heterogeneity of price stickiness across sectors. We start from Table II in their paper and we group industries into three categories: durable, non-durable and services. We compute the average duration of regular prices within these categories. In the model, we split sectors into three segments, in proportion to the expenditure weights of the three categories given in Table II.

[18]We show in Appendix C that proposition 5 also holds for time-dependent adjustment frictions. With Calvo frictions, the cost $E[\theta_s\pi^2_{st}]$ corresponds to the dispersion within sectors between firms that get to reset their price and those that don't. Under the conditions in proposition 5, TIP affects welfare through within-sector distortions in prices $E\left[\theta_s\pi^2_s\right]$—but not through distortions across sectors—and this effect is the same across sectors and captured by the average inflation rate π^2_t. By using TIP, policy-makers can minimize the average within-sector distortions implied by aggregate shocks, as in corollary 1.

All sectors in a given category share the same Rotemberg parameter θ_s, which we compute using the empirical average duration and equation (34).

In our main exercise, we simulate the effect of an unexpected one-period markup shock in the fully non-linear model. Starting from a steady-state with no TIP, we assume the elasticity of substitution decreases to $\epsilon_0 = 7.5$ at $t = 0$ and goes back to its steady-state value $\epsilon_t = 9$ for $t > 0$. We then compare this baseline scenario in which TIP doesn't respond ($\tau_0 = 0$) to two alternative scenarios in which TIP responds moderately ($\tau_0 = 10\%$) and more strongly ($\tau_0 = 15\%$). Table F5 reports the values of the three terms in the loss function (35) at time $t = 0$.

Results. We find that the average distortion of relative prices across sectors, $E\left((\tilde{p}_{st} - \tilde{p}^e_{st})^2\right)$, remains broadly unaffected by the size of TIP, and if anything it decreases very slightly with it. This confirms proposition 5: because it is a linear tax, TIP doesn't exacerbate price distortions across sectors, in contrast with price controls.[19] Consistent with the findings in sections 4 and 6, we find that a higher TIP leads to lower aggregate inflation and lower average squared price changes ($E[\theta_s \pi^2_{st}]$), as well as a lower output gap which unambiguously improves welfare.

To assess the role of the price stickiness heterogeneity, we do a first robustness exercise in which we assume that all sectors face the same adjustment cost parameter $\theta = 372.8$ as in Table 1. In addition, to make sure that our results are robust to the steady-state level of TIP $\bar{\tau}$, we do a second robustness exercise with $\bar{\tau} = 15\%$. As shown in Table F5, our results are similar to what we obtained in the main exercise: TIP has no effect on relative price distortions, and improves welfare by reducing inflation and the output gap.

[19] To understand why the average distortion can even decrease, recall that TIP influences the pricing decision through two channels which can be seen in the term $\tau_t\left(1 - \epsilon_t \frac{\pi_t}{1+\pi_t}\right)$ in the optimal pricing condition (12): a linear first-order channel—raising one's price increases taxes owed—and a second-order channel—increasing one's price reduces a firm's demand, which lowers the tax bill. Since the latter effect is proportional to the firm's price increase, firms that want to increase their price more see a stronger reduction in their tax bill as the demand for their good decreases. This gives them more, not less, incentives to adjust prices, which facilitates the overall relative price adjustments process and lowers price distortions.

8 Robustness Analysis and Implementation

We now assess the quantitative importance of two concerns related to the implementation of TIP. Would TIP be as effective if it covered only the biggest firms in the economy? Would a TIP on wages be as effective as a TIP on prices?

8.1 Taxing Only Large Corporates

An argument against TIP is that it would be costly to implement because the tax administration would have to collect additional information from all firms. A solution that would limit these costs would be to apply TIP only to very large corporations (Dildine and Sunley, 1978).

In this section, we extend the model in section 6 to allow for two types of firms: small firms (S) don't pay TIP and large firms (L) pay TIP. We solve the problem of each type separately and give the full list of equations describing the equilibrium in Appendix D. We calibrate the mass of large firms to match the fact that the top 1% of firms in terms of size accounts for 75% of all sales in the U.S. (Crouzet and Mehrotra, 2020). Figure F7 shows the IRFs for different values of φ_π, following a 0.25pp $\tilde{u}$-shock.

It is striking that TIP is almost as effective as in the baseline case, even if the tax covers only 1% of firms. A stronger TIP with $\varphi_\pi = 300$ brings down the initial inflation response from 1.7% to 1.3%, compared to 1.2% when TIP covers all firms, and the output gap to -.65, compared to -.6%. This result is not only driven by the fact that the TIP covers 75% of sales, but also by the fact that large firms' initial inflation response is more muted than in the baseline economy. This muted response stems from a slightly higher tax rate, at 98% of price changes, compared to 93% in the baseline economy, which is due in turn to a slightly higher inflation rate than in the baseline economy.

8.2 Sticky Wages and Tax on Wage Increases

It was argued in the early literature on TIP that taxing wage changes would be easier than taxing price changes because it is easier to observe the quantity of days worked separately from wages and more difficult to mis-report wages.

We now look at the effects of a tax on wage inflation. To introduce a meaningful wage decision, we extend the model to a setting where households face an adjustment cost when changing their nominal wages, in the spirit of Erceg et al. (2000). We also use this extension of the model to assess the robustness of the tax on price increases investigated in section 4.

Following the literature, we assume that households can change their wage subject to a Rotemberg (1982)-type adjustment costs. We solve the model and report the full set of equations describing the economy in Appendix E. Following Galí (2015), we assume the elasticity of substitution across labor types to be lower than the substitution across goods at $\epsilon_N = 4.5$, and we set the wage adjustment cost parameter θ_w to match the slope of the wage Phillips curve implied by a Calvo parameter for wages of 0.75.

TIP on prices. Figure F8 reports the IRF following a shock to the price Phillips curve and to the wage Phillips curve. It shows that a TIP on prices is as effective when wages are sticky as in the baseline case. A stronger TIP with $\varphi_\pi = 300$ brings down the initial inflation response from 2.5% to 1.6% and the output gap from -.1.1% to -.65% following a shock in the price Phillips curve. TIP is effective at controlling price inflation independently of the sources of the shocks because it affects directly the incentives of firms to changes their prices. However, TIP on prices is effective at controlling wage inflation only when the shock is to the price Phillips curve.[20] The reason why it is ineffective when the shock is to the wage Phillips curve is that the feedback effect of prices on wages is small relative to the direct effect of the shock on wages.

TIP on wages We now turn to the effectiveness of a TIP on wages. Figure F9, which reports the IRF following a shock in the price Phillips curve and following a shock to the wage Phillips curve, shows that the effectiveness of TIP on wages at controlling price inflation crucially depends on the source of the shock. When the shock is in the wage Phillips curve, a stronger TIP on

[20]In a setting with sticky wages, the welfare loss function includes wage inflation besides price inflation and the output gap (Erceg et al., 2000). A lower wage inflation thus improves welfare.

wages with $\varphi_\pi = 300$ is as effective as a TIP on prices. It brings down the initial inflation response from .94% to .75% and the output gap from -.65% to -.55%. The reason why TIP is effective is that it offsets the transmission of the shock to wage inflation. In turn, a lower wage inflation implies a lower price inflation given that wages are the main component of firms' costs.

However, when the shock is in the price Phillips curve, TIP on wages is ineffective at controlling price inflation or at reducing the output gap. Although a stronger TIP on wages with $\varphi_\pi = 300$ is effective at controlling wage inflation, it is not sufficient to reign in price inflation. As in the case of a TIP on prices with a shock on wages, the reason is that the feedback effect of wages on prices—the wage-price spiral—is small relative to the direct effect of the shock. In unreported simulations, we find that this weak wage-price spiral is sensitive to the calibration of the elasticity of substitution across labor types ϵ_L: the relatively low elasticity implies a high wage markup, and in turn a slow response of wages to changes in the economy.

9 Conclusion

In the face of markup and inflation expectation shocks, policymakers have only imperfect instruments to stabilize inflation. In this paper, we put forward a tax on inflation policy (TIP), which would require firms to pay a tax proportional to the increase in their prices or wages. By giving direct incentives to firms to moderate their price increases without exacerbating relative prices distortions, we find that TIP is an effective instrument to control inflation. We show that combining TIP with MP can significantly lower the volatility of inflation and output relative to an economy where only MP is available and that TIP should specialize in addressing markups and inflation expectation shocks, and MP in addressing demand shocks. These results are robust to several alternative designs of TIP.

Our paper opens avenues for future research. First, our analysis has focused on how TIP can complement MP in the face of markups and inflation expectation shocks, but there are other challenges faced by MP that TIP could help address. For example, a negative TIP could help avoid a deflationary spiral at the ZLB.

Second, a few implementation issues deserve a more in-depth quantitative inquiry. Tax avoidance, for example, warrants more attention. The main risk is that firms relabel old products as seemingly new ones, or that they shrink their quality. The paper has argued that these risks are small given the remarkably low tax burden implied by TIP. To further mitigate these concerns, one approach could involve taxing wages as analyzed in the paper, or taxing changes in a price index at the firm-level. Quantifying the effects of alternative designs of TIP in a framework with endogenous product creation, information asymmetries about product quality and costly monitoring by the tax administration is an important next step. This framework could also help assess the related concern that firms may postpone quality innovation if tax authorities fail to distinguish price increases from quality changes. Although this issue is confined to periods of elevated TIP (i.e. of high inflation) and may not significantly impede long-run growth, future research should investigate it further.

Delving into the political economy of TIP is another important avenue for future research. What are the risks that TIP be used for objectives other than macroeconomic stabilization? Could it lead to less independent monetary policy in countries with weak institutional frameworks? To ensure that TIP is dedicated to inflation stabilization, it seems natural that its conduct should be given to an independent committee. This governing committee could be appointed by elected representatives but conduct its policy independently, like central banks or energy and telecommunication commissions which in many countries have discretionary power to set interest rates, prices and regulations. This independence from elected officials would be justified on the ground that revenues generated by TIP are very small (a feebate would even be fiscally neutral) and that it has a clear and narrow mandate, like central banks.

References

Ascari, G. and Sbordone, A. M. (2014). The Macroeconomics of Trend Inflation. *Journal of Economic Literature*, 52(3):679–739.

Bogetic, Z. and Fox, L. (1993). Incomes Policy During Stabilization: A Review and Lessons from Bulgaria and Romania. *Comparative Economic Studies*, 35(1):39–57.

Calvo, G. A. (1983). Staggered Prices in a Utility-maximizing Framework. *Journal of Monetary Economics*, 12(3):383–398.

Clarida, R., Galí, J., and Gertler, M. (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature*, 37(4):1661–1707.

Colander, D. (1981). *Incentive Anti-Inflation Plans: A Study for the Use of the Joint Economic Committee Congress of the United States.* U.S. government printing office.

Correia, I., Farhi, E., Nicolini, J. P., and Teles, P. (2013). Unconventional Fiscal Policy at the Zero Bound. *American Economic Review*, 103(4):1172–1211.

Crombrugghe, A. D. and de Walque, G. (2011). Wage and employment effects of a wage norm : The Polish transition experience. Working Paper Research 209, National Bank of Belgium.

Crouzet, N. and Mehrotra, N. R. (2020). Small and Large Firms over the Business Cycle. *American Economic Review*, 110(11):3549–3601.

Dildine, L. L. and Sunley, E. M. (1978). Administrative Problems of Tax-Based Incomes Policies. *Brookings Papers on Economic Activity*, 9(2):363–400.

Eggertsson, G. B. (2011). What Fiscal Policy Is Effective at Zero Interest Rates? In *NBER Macroeconomics Annual 2010, volume 25*, NBER Chapters, pages 59–112. National Bureau of Economic Research, Inc.

Eggertsson, G. B. and Woodford, M. (2006). Optimal Monetary and Fiscal Policy in a Liquidity Trap. In *NBER International Seminar on Macroeconomics 2004*, NBER Chapters, pages 75–144. National Bureau of Economic Research, Inc.

Enev, T. and Koford, K. (2000). The Effect of Incomes Policies on Inflation in Bulgaria and Poland. *Economic Change and Restructuring*, 33(3):141–169.

Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics*, 46(2):281–313.

Farhi, E., Gopinath, G., and Itskhoki, O. (2014). Fiscal Devaluations. *Review of Economic Studies*, 81(2):725–760.

Galí, J. (2015). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications Second edition*. Number 10495 in Economics Books. Princeton University Press.

Gertler, M. and Leahy, J. (2008). A Phillips Curve with an Ss Foundation. *Journal of Political Economy*, 116(3):533–572.

Giannoni, M. P. (2014). Optimal Interest-Rate Rules and Inflation Stabilization versus Price-Level Stabilization. *Journal of Economic Dynamics Control*, 41:110–129.

Giannoni, M. P. and Woodford, M. (2003). Optimal Interest-Rate Rules: II. Applications. *NBER Working Paper*, (9420).

Hazell, J., Herreño, J., Nakamura, E., and Steinsson, J. (2022). The Slope of the Phillips Curve: Evidence from U.S. States. *The Quarterly Journal of Economics*, 137(3):1299–1344.

Jackman, R. and Layard, R. (1982). An inflation tax. *Fiscal Studies*, 3(2):47–59.

Jackman, R. and Layard, R. (1989). The Real Effects Of Tax-Based Incomes Policies. Technical report.

Koford, K. J. and Miller, J. B. (1992). Macroeconomic Market Incentive Plans: History and Theoretical Rationale. *American Economic Review*, 82(2):330–334.

Koford, K. J., Miller, J. B., and Colander, D. C. (1993). Application of Market Anti-inflation Plans in the Transition to a Market Economy. *Eastern Economic Journal*, 19(3):379–393.

Kotowitz, Y. and Portes, R. (1974). The 'Tax on Wage Increases' : A Theoretical Analysis. *Journal of Public Economics*, 3(2):113–132.

Latham, R. and Peel, D. (1977). The Tax on Wage Increases when the Firm is a Monopsonist. *Journal of Public Economics*, 8(2):247–253.

Layard, R. (1982). Is Incomes Policy the Answer to Unemployment? *Economica*, 49(195):219–239.

Lerner, A. P. (1978). A Wage-Increase Permit Plan to Stop Inflation. *Brookings Papers on Economic Activity*, 9(2):491–505.

Lerner, A. P. and Colander, D. C. (1980). Map a Market Anti-Inflation Plan. *New York: Harcourt Brace Jovanovich*.

Mertens, K. R. S. M. and Ravn, M. O. (2014). Fiscal Policy in an Expectations-Driven Liquidity Trap. *The Review of Economic Studies*, 81(4 (289)):1637–1667.

Nakamura, E. and Steinsson, J. (2008). Five Facts about Prices: A Reevaluation of Menu Cost Models. *The Quarterly Journal of Economics*, 123(4):1415–1464.

Nichols, D. A. (1979). Comparing TIP to Wage Subsidies. *American Economic Review*, 69(2):207–211.

OECD (1975). *Socially responsible wage policies and inflation : a review of four countries' experience* . OECD.

Okun, A. M. and Perry, G. L. (1978). *Curing Chronic Inflation*.

Oswald, A. J. (1984). Three Theorems on Inflation Taxes and Marginal Employment Subsidies. *Economic Journal*, 94(375):599–611.

Paci, P. (1988). Tax-based incomes policies: will they work? have they worked? *Fiscal Studies*, 9(2):81–94.

Peel, D. A. (1979). The Dynamic Behaviour of a Simple Macroeconomic Model with a Tax-based Incomes Policy. *Economics Letters*, 3(2):139–143.

Rotemberg, J. J. (1982). Sticky Prices in the United States. *Journal of Political Economy*, 90(6):1187–1211.

Rotemberg, J. J. and Woodford, M. (1999). Interest Rate Rules in an Estimated Sticky Price Model. In *Monetary Policy Rules*, NBER Chapters, pages 57–126. National Bureau of Economic Research, Inc.

Scarth, W. M. (1983). Tax-related Incomes Policies and Macroeconomic Stability. *Journal of Macroeconomics*, 5(1):91–103.

Seidman, L. S. (1978a). Tax-Based Incomes Policies. *Brookings Papers on Economic Activity*, 9(2):301–361.

Seidman, L. S. (1978b). Would Tax Shifting Undermine the Tax-Based Incomes Policy? *Journal of Economic Issues*, 12(3):647–676.

Seidman, L. S. (1979). The Role of a Tax-Based Incomes Policy. *American Economic Review*, 69(2):202–206.

Slitor, R. E. (1979). Implementation and Design of Tax-Based Incomes Policies. *The American Economic Review*, 69(2):212–215.

Taylor, J. B. (1979). Staggered Wage Setting in a Macro Model. *The American Economic Review*, 69(2):108–113.

Taylor, J. B. (1980). Aggregate Dynamics and Staggered Contracts. *Journal of Political Economy*, 88(1):1–23.

Wallich, H. C. and Weintraub, S. (1971). A Tax-Based Incomes Policy. *Journal of Economic Issues*, 5(2):1–19.

Werning, I. and Lorenzoni, G. (2023). Inflation is conflict. Technical report.

Woodford, M. (2011). Simple Analytics of the Government Expenditure Multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35.

For Online Publication - Appendix

A Baseline Model with TIP

A.1 Market Clearing and Equilibrium Definition

Market Clearing We now show the market clearing conditions. In equilibrium, the markets for each intermediate good Y_{ti} and for the final good should clear

$$Y_t = C_t + \int_0^1 \frac{\theta}{2}\left(\frac{P_{ti}}{P_{t-1i}} - 1\right)^2 Y_t di \tag{36}$$

The sum of labor hired in all firms should be equal to the supply of labor by households:

$$\left(\int N_{th}^{1-1/\sigma_N} dh\right)^{\frac{\sigma_N}{\sigma_N - 1}} = N_t = \int_0^1 N_{ti} di \tag{37}$$

With no government debt, holdings of bonds by households are zero: $B_t = 0$. Finally, transfers received by households include profits and tax receipts:

$$T_t = \int_0^1 \Pi_{it} + \tau_t (P_{ti} - P_{t-1i}) Y_{ti} di. \tag{38}$$

Equilibrium. An equilibrium is a path of output, labor, bonds, wages, price level, bond prices and TIP $\{\{C_{th}, N_{th}, B_{th}, W_{th}\}_h, \{Y_{ti}, N_{ti}, P_{ti}\}_i, W_t, P_t, Q_t, \tau_t\}_{t=0,1,2..}$, such that

- Taking TIP as given, intermediate firms maximize their discounted sum of profits (11) subject to the definition of profits (10), the technology (8) and the demand schedule (7).
- Taking prices as given, final good firms maximize their profits subject to the technology (6).

- Taking prices and transfers (38) as given, households maximize their utility (1) subject to their budget constraint (2), no-ponzi condition (**??**) and demand for labor (3).
- The markets for final good (36), intermediate goods, labor (37) and bonds clear.

A.2 Linearization

Firms' First-Order Condition The first-order conditions associated with the firm's problem are:

$$(1-\epsilon_t)\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t} Y_t - (1-\epsilon_t)\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t} Y_t\tau_t - \tau_t\epsilon_t\frac{P_{t-1i}}{P_{ti}}Y_t\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t}$$
$$+\frac{\epsilon_t}{1-\alpha}\frac{W_t}{P_{ti}}\left[\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t}\frac{Y_t}{A_t}\right]^{\frac{1}{1-\alpha}} - \frac{\theta}{P_{t-1i}}\left(\frac{P_{ti}}{P_{t-1i}}-1\right)P_tY_t + EQ_tV'(P_{ti}) = 0$$

and

$$V'(P_{t-1i}) = \tau_t Y\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t} + \frac{P_{ti}}{P_{t-1i}^2}\theta\left(\frac{P_{ti}}{P_{t-1i}}-1\right)P_tY_t.$$

Assuming symmetry gives

$$\left((1-\epsilon_t)Y_t-(1-\epsilon_t)Y_t\tau_t-\tau_t\epsilon_t\frac{P_{t-1}}{P_t}Y_t+\frac{\epsilon_t}{1-\alpha}\frac{W_t}{P_t}\left[\frac{Y_t}{A_t}\right]^{\frac{1}{1-\alpha}}-\frac{\theta}{P_{t-1}}\pi_tP_tY_t\right)$$
$$+E_tQ_t\left[\tau_{t+1}Y_{t+1}+(\pi_{t+1}+1)^2\theta\pi_{t+1}Y_{t+1}\right]=0$$
$$\iff\left(1-\epsilon_t-\tau_t\left(1-\epsilon_t\frac{\pi_t}{1+\pi_t}\right)+\epsilon_tMC_t-\theta\pi_t(\pi_t+1)\right)$$
$$+E_tQ_t\left[\tau_{t+1}\frac{Y_{t+1}}{Y_t}+(\pi_{t+1}+1)^2\theta\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]=0$$
$$\iff\left((1-\epsilon_t)\left(1-\frac{\epsilon_t}{\epsilon_t-1}MC_t\right)-\tau_t\left(1-\epsilon_t\frac{\pi_t}{1+\pi_t}\right)-\theta\pi_t(\pi_t+1)^2\right)$$
$$+E_tQ_t\left[\tau_{t+1}\frac{Y_{t+1}}{Y_t}+(\pi_{t+1}+1)^2\theta\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]=0$$
$$\iff\frac{1}{\theta}\left((1-\epsilon_t)\left(1-\mathcal{M}_tMC_t\right)-\tau_t\left(1-\epsilon_t\frac{\pi_t}{1+\pi_t}\right)\right)-\pi_t(\pi_t+1)$$
$$+E_tQ_t\left[\frac{\tau_{t+1}}{\theta}\frac{Y_{t+1}}{Y_t}+(\pi_{t+1}+1)^2\pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]=0$$

$$\text{with}\quad MC_t=\frac{W_t}{P_t(1-\alpha)}\frac{Y_t^{\frac{\alpha}{1-\alpha}}}{A_t^{\frac{1}{1-\alpha}}}\quad\text{and}\quad\mathcal{M}_t=\frac{\epsilon_t}{\epsilon_t-1}.$$

We denote the steady-state markup $\bar{\mathcal{M}}$. The next step is to linearize this optimality condition around a steady-state with no inflation, constant output, a zero tax on price changes, a flexible price markup and no markup shocks, $\pi=0,\tau=0,Y=Y',MC_t=1/\bar{\mathcal{M}}$. Denoting mc the log of the real marginal cost MC and μ the log of $\mathcal{M}$, we obtain:

$$\frac{1}{\theta}\left[(1-\epsilon)\left(1-1*(1+mc_t-mc+\mu_t-\mu)\right)-\tau_t\right]+E_tQ_t\left[\tau_{t+1}\frac{1}{\theta}+\pi_{t+1}\right]=\pi_t$$
$$\iff\frac{1}{\theta}\left((\epsilon-1)(\hat{mc}_t+\hat{\mu}_t)-\tau_t\right)+\beta E_t\left[\frac{\tau_{t+1}}{\theta}+\pi_{t+1}\right]=\pi_t$$
$$\iff\frac{1}{\theta}\left((\epsilon-1)\hat{mc}_t+u_t-\tau_t\right)+\beta E_t\left[\frac{\tau_{t+1}}{\theta}+\pi_{t+1}\right]=\pi_t$$

where $\hat{mc}_t^e = mc_t - mc$ is the gap between the effective marginal cost and the desired marginal cost under flexible prices in steady-state, and $u_t = (\epsilon - 1)(\mu_t - \mu) = (\epsilon - 1)(mc - mc_t^n)$ is the markup shock, where mc_t^n is the marginal cost under flexible price. We thus obtain

$$\begin{aligned}
\pi_t &= \frac{1}{\theta}\left((\epsilon-1)\hat{mc}_t - \tau_t + u_t\right) + \beta E_t\left[\frac{\tau_{t+1}}{\theta} + \pi_{t+1}\right] \\
&= \beta E_t \pi_{t+1} + \frac{\epsilon-1}{\theta}\hat{mc}_t + \frac{1}{\theta}\left[\beta E_t \tau_{t+1} - \tau_t + u_t\right]
\end{aligned}$$

Equilibrium From the labor market clearing condition $N_t = \int_0^1 N_{ti} di$, we get

$$\begin{aligned}
N_t &= \int_0^1 \left(\frac{Y_{ti}}{A_t}\right)^{\frac{1}{1-\alpha}} di \\
&= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}
\end{aligned}$$

where we used the fact that all firms are ex post identical $Y_{ti} = Y_t$.

Phillips Curve and Euler Equation Since all firms are idential in equilibrium, the market clearing condition for the final goods market is given by

$$Y_t\left(1 - \frac{\theta}{2}\pi_t^2\right) = C_t.$$

Taking logs

$$y_t + \log\left(1 - \frac{\theta}{2}\pi_t^2\right) = c_t.$$

and approximating around the efficient equilibrium with zero inflation:

$$y_t^e + \hat{y}_t^e + 0 = c_t^e + \hat{c}_t^e \Rightarrow c_t = y_t.$$

We now turn to the first order conditions of the households given by

$$w_t - p_t = \sigma c_t + \psi n_t$$
$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho)$$

with $i_t = -\log Q_t$. Combining them with the market clearing condition for the final goods gives

$$w_t - p_t = \left(\sigma + \frac{\psi}{1-\alpha}\right) y_t - \frac{\psi}{1-\alpha} \log a_t$$
$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho)$$

From the definition of markup, we obtain

$$\begin{aligned} mc_t &= w_t - p_t + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t - \log(1-\alpha) \\ &= \left(\sigma + \frac{\psi}{1-\alpha}\right) y_t - \frac{\psi}{1-\alpha} \log a_t + \frac{\alpha}{1-\alpha} y_t - \frac{1}{1-\alpha} a_t - \log(1-\alpha) \\ &= \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t - \frac{1+\psi}{1-\alpha} a_t - \log(1-\alpha) \end{aligned}$$

where the second line uses the first order condition of the households and the market clearing condition for labor.

Hence

$$mc_t - mc_t^e = \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)(y_t - y_t^e)$$

where the flexible price and efficent (no markup shocks) level of output are defined as follows

$$\begin{aligned} y_t^e &= \frac{mc_t^e + \log(1-\alpha)}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} a_t \\ mc_t^e &= mc \end{aligned}$$

From the Euler equation, we obtain the flex-price rate of interest:

$$
\begin{aligned}
r_t^e &= \rho + \sigma E_t(y_{t+1}^n - y_t^n) \\
r_t^e &= \rho + \sigma E_t \left[\frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} (a_{t+1} - a_t) \right]
\end{aligned}
$$

Combining everything gives

$$
\begin{aligned}
\hat{y}_t^e &= E_t \hat{y}_{t+1}^e - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - r_t^e \right) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa \hat{y}_t^e + \frac{1}{\theta} \left[\beta E_t \tau_{t+1} - \tau_t + u_t \right]
\end{aligned}
$$

with $\kappa = \frac{\epsilon-1}{\theta} \left(\sigma + \frac{\psi+\alpha}{1-\alpha} \right)$.

For future reference we also define the natural rate of output and the neutral interest rate, which are simply equal to their value in flexible price equilibrium:

$$
\begin{aligned}
y_t^n &= \frac{mc^n + \log(1-\alpha)}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} a_t \\
r_t^n &= \rho + \sigma E_t(y_{t+1}^n - y_t^n) \\
r_t^n &= \rho + \sigma E_t \left[\frac{mc_{t+1} - mc_t}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} (a_{t+1} - a_t) \right] \\
&= \rho + \sigma E_t \left[\frac{u_{t+1} - u_t}{(\epsilon - 1)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} (a_{t+1} - a_t) \right] \\
mc_t^n &= -\mu_t
\end{aligned}
$$

Firms' profits. Real profits are given by

$$
\frac{\Pi(P_{t-1i}, P_{ti})}{P_t} = \frac{1}{P_t} \left[P_{ti} Y_{ti} - W_t N_{ti} - \tau_t (P_{ti} - P_{t-1i}) Y_{ti} - \mathcal{C}_t \left(P_{t-1i}, P_{ti} \right) \right].
$$

In a symmetric equilibrium, $P_t = P_{ti}$, and using the household's first-order condition $\frac{W_t}{P_t} = \mathcal{M}_t^w Y_t^{\left(\sigma + \frac{\psi}{1-\alpha}\right)} A_t^{-\frac{\psi}{1-\alpha}}$ gives:

$$
\begin{aligned}
\frac{\Pi(P_{t-1}, P_t)}{P_t} &= Y_t - \frac{W_t}{P_t} N_t - \tau_t \frac{\pi_t}{1+\pi_t} Y_t - \frac{\theta}{2}\pi_t^2 Y_t \\
&= Y_t - \mathcal{M}_t^w Y_t^{\sigma} \left(\frac{Y_t}{A_t}\right)^{\frac{\psi}{1-\alpha}} N_t - \tau_t \frac{\pi_t}{1+\pi_t} Y_t - \frac{\theta}{2}\pi_t^2 Y_t \\
&= Y_t - \mathcal{M}_t^w Y_t^{\sigma} \left(\frac{Y_t}{A_t}\right)^{\frac{\psi+1}{1-\alpha}} - \tau_t \frac{\pi_t}{1+\pi_t} Y_t - \frac{\theta}{2}\pi_t^2 Y_t \\
&= Y_t \left[1 - \mathcal{M}_t^w Y_t^{\sigma-1} \left(\frac{Y_t}{A_t}\right)^{\frac{\psi+1}{1-\alpha}} - \tau_t \frac{\pi_t}{1+\pi_t} - \frac{\theta}{2}\pi_t^2\right]
\end{aligned}
$$

In the steady state, the efficient level of output is given by

$$
\begin{aligned}
y_t^e &= \frac{mc + \log(1-\alpha)}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} a_t \\
&= \frac{1}{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} \left[mc + \log(1-\alpha) + \frac{1+\psi}{(1-\alpha)} a_t\right],
\end{aligned}
$$

which solves the steady-state labor share

$$
\begin{aligned}
\mathcal{M}^w (Y_t^e)^{\sigma-1} \left(\frac{Y_t^e}{A_t}\right)^{\frac{\psi+1}{1-\alpha}} &= \frac{\epsilon_N}{\epsilon_N - 1} e^{\left(\sigma - 1 + \frac{\psi+1}{1-\alpha}\right) y_t^e - \frac{1+\psi}{(1-\alpha)} a_t} \\
&= \frac{\epsilon_N}{\epsilon_N - 1} e^{\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t^e - \frac{1+\psi}{(1-\alpha)} a_t} \\
&= \frac{\epsilon_N}{\epsilon_N - 1} e^{mc + \log(1-\alpha)} \\
&= \frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1-\alpha).
\end{aligned}
$$

We can now derive the steady-state profit share:

$$
\frac{\Pi}{PY} = 1 - \frac{\epsilon_N}{\epsilon_N - 1} \frac{\epsilon - 1}{\epsilon} (1-\alpha)
$$

It is natural to simplify the profit share $\frac{\Pi(P_{t-1}, P_t)}{P_t Y_t}$ by log-linearizing the labor share. Although TIP and the adjustment costs are negligible in the first-order approximation around the zero-inflation steady state, we keep them in the expression in order to clarify the different mechanisms at play

$$
\begin{aligned}
\frac{\Pi_t}{P_t Y_t} &= 1 - \mathcal{M}^w_t Y_t^{\sigma-1}\left(\frac{Y_t}{A_t}\right)^{\frac{\psi+1}{1-\alpha}} - \tau_t \frac{\pi_t}{1+\pi_t} - \frac{\theta}{2}\pi_t^2 \\
&= 1 - \frac{\mathcal{M}^w_t}{\mathcal{M}^w}\left(\frac{Y_t}{Y^e_t}\right)^{\sigma-1+\frac{\psi+1}{1-\alpha}}\left[\mathcal{M}^w (Y^e_t)^{\sigma-1}\left(\frac{Y^e_t}{A_t}\right)^{\frac{\psi+1}{1-\alpha}}\right] - \tau_t \frac{\pi_t}{1+\pi_t} - \frac{\theta}{2}\pi_t^2 \\
&= 1 - \frac{\mathcal{M}^w_t}{\mathcal{M}^w}\left(\frac{Y_t}{Y^e_t}\right)^{\sigma-1+\frac{\psi+1}{1-\alpha}}\left[\frac{\epsilon_N}{\epsilon_N-1}\frac{\epsilon-1}{\epsilon}(1-\alpha)\right] - \tau_t\pi_t - \frac{\theta}{2}\pi_t^2 \\
&= 1 - \left[\frac{\epsilon_N}{\epsilon_N-1}\frac{\epsilon-1}{\epsilon}(1-\alpha)\right] e^{\left(\sigma-1+\frac{\psi+1}{1-\alpha}\right)\hat{y}^e_t + \frac{u^w_t}{\epsilon-1}} - \tau_t\pi_t - \frac{\theta}{2}\pi_t^2 \\
&\approx 1 - \left[\frac{\epsilon_N}{\epsilon_N-1}\frac{\epsilon-1}{\epsilon}(1-\alpha)\right]\left[1 + \left(\sigma-1+\frac{\psi+1}{1-\alpha}\right)\hat{y}^e_t + \frac{u^w_t}{\epsilon-1}\right] - \tau_t\pi_t - \frac{\theta}{2}\pi_t^2
\end{aligned}
$$

The fourth equation uses the relationship between m^w_t ($\ln \frac{\mathcal{M}^w_t}{\mathcal{M}^w}$) and u^w_t: $u^w_t = (\epsilon-1)m^w_t$. The approximated profit share allows us to analyze both the deviation of the profit share,

$$
\frac{\Pi_t}{P_t Y_t} - \frac{\Pi}{PY} \approx -\left[\frac{\epsilon_N}{\epsilon_N-1}\frac{\epsilon-1}{\epsilon}(1-\alpha)\right]\left[\left(\sigma-1+\frac{\psi+1}{1-\alpha}\right)\hat{y}^e_t + \frac{u^w_t}{\epsilon-1}\right] - \tau_t\pi_t - \frac{\theta}{2}\pi_t^2
$$

and the deviation of the real profit level normalized by steady-state output

$$
\left(\frac{\Pi_t}{P_t} - \frac{\Pi}{P}\right)Y^{-1} = \frac{\Pi_t}{P_t Y_t}\frac{Y_t}{Y} - \frac{\Pi}{PY} \approx \frac{\Pi_t}{P_t Y_t}(1+\hat{y}^e_t) - \frac{\Pi}{PY}.
$$

A.3 Fiscal cost of subsidies

We first log-linearize equation 25,

$$
\begin{aligned}
\frac{\epsilon-1}{\theta}\left(mc_t - mc + \mu_t - \mu - a^w_t\right) + E_t Q_t \pi_{t+1} &= \pi_t \\
\frac{\epsilon-1}{\theta}(\hat{mc}_t - a^w_t) + \frac{u_t}{\theta} + \beta E_t \pi_{t+1} &= \pi_t
\end{aligned}
$$

To derive the fiscal costs of payroll subsidies, we assume a targeting rule for a_t^w similar to equation 31.

$$a_t^w = \varphi_\pi^w \pi_t$$

We choose φ_π^w such that the impulse response functions of inflation and the output gap match the ones in Figure 1 after markups or inflation expectation shocks.

Figures F4, F5, and F6 report the effects of wage subsidies following a markup shock, a productivity shock, and a monetary policy shock. To achieve the same macroeconomic outcome after a markup shock as a strong TIP ($\varphi_\pi = 300$) does in Figure 1, wage subsidies would amount to 4% of total payrolls (or 3.5% of output) in the first period. By sharp contrast, a strong TIP costs below 0.15% of output in the same period. The persistence of the fiscal costs also differs across the two instruments. The fiscal cost of wage subsidies, which are proportional to π_t, decreases much slower than the cost of TIP, because they are proportional to π_t^2 instead.

Finally, as we increase the strength of the instrument, wage subsidies grow linearly while the fiscal costs implied by TIP increase at a decreasing speed. This is because a stronger TIP can significantly reduce its own tax base by moderating price changes, while a higher wage subsidy rate marginally raises its base by raising the output gap.

A.4 Determinacy

Before turning to the properties of an economy with TIP, we analyze what are the conditions ensuring the uniqueness of the equilibrium path. When there are multiple equilibria, the economy is subject to coordination failures. For example, if all firms expect high inflation and high output gap, the economy could shift to a self-fulfilling equilibrium with excessive inflation. These coordination failures are a potential source of excessive inflation which is distinct from the shocks to inflation expectations, u_t^E, in that they are consistent with rational expectations.

To ensure determinacy, it is well-known that in the baseline New Keynesian model, MP should implement the Taylor principle according to which the policy rate reacts strongly to inflation. A natural question is whether TIP could guarantee the uniqueness of the equilibrium path. Substituting out the Taylor rule (31) into the Euler equation (16) gives

$$\hat{y}_t^e = E_t \hat{y}_{t+1}^e - \frac{1}{\sigma} \left(\rho + \phi_\pi \pi_t + \phi_y \hat{y}_t^e - E_t \pi_{t+1} - r_t^e \right),$$

and substituting out the rule for TIP (32) into the Phillips curve (17) gives

$$\left(1 + \frac{\varphi_\pi}{\theta}\right) \pi_t = \beta \left(1 + \frac{\varphi_\pi}{\theta}\right) E_t \pi_{t+1} + \left(\kappa - \frac{\varphi_y}{\theta}\right) \hat{y}_t^e + \frac{\varphi_y}{\theta} \beta E_t \hat{y}_{t+1}^e + \frac{1}{\theta} \tilde{u}_t.$$

To analyze the uniqueness of the solution to this system of two difference equations, we compute its eigenvalues. This system with two non predetermined variables is determinate if and only if both eigenvalues are inside the unit circle (Blanchard and Kahn, 1980). We derive the necessary and sufficient conditions for this to hold in appendix, and show sufficient conditions in the following proposition which turn out to be more intuitive.

Proposition 6. *The equilibrium path is unique if one of the following conditions holds*

- $\phi_\pi > 1$ *and* $\varphi_y < \min\left(\frac{\theta\kappa}{1-\beta}, \frac{\theta\phi_\pi\kappa + \theta(1-\beta)(\sigma(1-\beta)+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)}{\phi_\pi - \beta} \right)$
- $\beta < \phi_\pi < 1$ *and* $\frac{\theta\kappa}{1-\beta} < \varphi_y < \frac{\theta\phi_\pi\kappa + \theta(1-\beta)(\sigma(1-\beta)+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)}{\phi_\pi - \beta}$
- $\phi_\pi < \beta$ *and* $\frac{\theta\kappa}{1-\beta} < \varphi_y$

Proof.

$$\hat{y}_t^e = E_t \hat{y}_{t+1}^e - \frac{1}{\sigma} \left(\rho + \phi_\pi \pi_t + \phi_y \hat{y}_t^e - E_t \pi_{t+1} - r_t^e \right)$$

$$\left(1 + \frac{\varphi_\pi}{\theta}\right) \pi_t = \beta \left(1 + \frac{\varphi_\pi}{\theta}\right) E_t \pi_{t+1} + \left(\kappa - \frac{\varphi_y}{\theta}\right) \hat{y}_t^e + \frac{\varphi_y}{\theta} \beta E_t \hat{y}_{t+1}^e + \frac{1}{\theta} u_t.$$

We first rewrite this system in matrix form

$$A\begin{pmatrix}\pi_t\\ \hat{y}^e_t\end{pmatrix} = B\begin{pmatrix}E_t\pi_{t+1}\\ E_t\hat{y}^e_{t+1}\end{pmatrix} + C\begin{pmatrix}r^e_t-\rho\\ \tilde{u}_t\end{pmatrix}$$

with $A = \begin{pmatrix}\phi_\pi & \sigma+\phi_y\\ 1+\frac{\varphi_\pi}{\theta} & \frac{\varphi_y}{\theta}-\kappa\end{pmatrix}$, $B = \begin{pmatrix}1 & \sigma\\ \beta\left(1+\frac{\varphi_\pi}{\theta}\right) & \frac{\varphi_y}{\theta}\beta\end{pmatrix}$ and $C = \begin{pmatrix}1 & 0\\ 0 & \frac{1}{\theta}\end{pmatrix}$

We now solve for the eigenvalues of this system. We first invert A

$$A^{-1} = \frac{1}{\phi_\pi\left(\frac{\varphi_y}{\theta}-\kappa\right)-(\sigma+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)}\begin{pmatrix}\frac{\varphi_y}{\theta}-\kappa & -\sigma-\phi_y\\ -1-\frac{\varphi_\pi}{\theta} & \phi_\pi\end{pmatrix}$$

and then multiply it by B:

$$\begin{aligned}
A^{-1}B &= \frac{1}{\phi_\pi\left(\frac{\varphi_y}{\theta}-\kappa\right)-(\sigma+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)}\begin{pmatrix}\frac{\varphi_y}{\theta}-\kappa & -\sigma-\phi_y\\ -1-\frac{\varphi_\pi}{\theta} & \phi_\pi\end{pmatrix}\begin{pmatrix}1 & \sigma\\ \beta\left(1+\frac{\varphi_\pi}{\theta}\right) & \frac{\varphi_y}{\theta}\beta\end{pmatrix}\\
&= \Omega\begin{pmatrix}\frac{\varphi_y}{\theta}-\kappa-(\sigma+\phi_y)\beta\left(1+\frac{\varphi_\pi}{\theta}\right) & \sigma\left(\frac{\varphi_y}{\theta}-\kappa\right)-(\sigma+\phi_y)\beta\frac{\varphi_y}{\theta}\\ \left(1+\frac{\varphi_\pi}{\theta}\right)\left[\beta\phi_\pi-1\right] & -\sigma\left(1+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\frac{\varphi_y}{\theta}\beta\end{pmatrix}\\
&= -\Omega\begin{pmatrix}-\frac{\varphi_y}{\theta}+\kappa+(\sigma+\phi_y)\beta\left(1+\frac{\varphi_\pi}{\theta}\right) & \sigma\left(\kappa-\frac{\varphi_y}{\theta}\right)+(\sigma+\phi_y)\beta\frac{\varphi_y}{\theta}\\ \left(1+\frac{\varphi_\pi}{\theta}\right)\left[1-\beta\phi_\pi\right] & \sigma\left(1+\frac{\varphi_\pi}{\theta}\right)-\phi_\pi\frac{\varphi_y}{\theta}\beta\end{pmatrix}
\end{aligned}$$

with $\Omega = \frac{1}{\phi_\pi\left(\frac{\varphi_y}{\theta}-\kappa\right)-(\sigma+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)}$. We denote this matrix A', and we now compute its trace and determinant.

$$\begin{aligned}
TrA' &= -\Omega\left[-\frac{\varphi_y}{\theta}+\kappa+(\sigma+\phi_y)\beta\left(1+\frac{\varphi_\pi}{\theta}\right)+\sigma\left(1+\frac{\varphi_\pi}{\theta}\right)-\phi_\pi\frac{\varphi_y}{\theta}\beta\right]\\
&= \frac{1}{(\sigma+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}\left[-\frac{\varphi_y}{\theta}(1+\beta\phi_\pi)+\kappa+(\sigma(\beta+1)+\phi_y\beta)\left(1+\frac{\varphi_\pi}{\theta}\right)\right]
\end{aligned}$$

$$
\begin{aligned}
detA' = & \left(\frac{1}{(\sigma + \phi_y)\left(1 + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right)^2 \\
& \times \left[\left[-\frac{\varphi_y}{\theta} + \kappa + (\sigma + \phi_y)\beta \left(1 + \frac{\varphi_\pi}{\theta}\right) \right] \left[\sigma \left(1 + \frac{\varphi_\pi}{\theta}\right) - \phi_\pi \frac{\varphi_y}{\theta} \beta \right] \right. \\
& \qquad \left. - \left[\sigma \left(\kappa - \frac{\varphi_y}{\theta} \right) + (\sigma + \phi_y)\beta \frac{\varphi_y}{\theta} \right] \left[\left(1 + \frac{\varphi_\pi}{\theta}\right) [1 - \beta\phi_\pi] \right] \right. \\
= & \left(\frac{1}{(\sigma + \phi_y)\left(1 + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right)^2 \\
& \times \left[\left[\kappa - \frac{\varphi_y}{\theta} \right] \sigma \left(1 + \frac{\varphi_\pi}{\theta}\right) - \left[\kappa - \frac{\varphi_y}{\theta} \right] \phi_\pi \frac{\varphi_y}{\theta} \beta + (\sigma + \phi_y)\sigma\beta \left(1 + \frac{\varphi_\pi}{\theta}\right)^2 \right. \\
& - (\sigma + \phi_y)\beta \left(1 + \frac{\varphi_\pi}{\theta}\right) \phi_\pi \frac{\varphi_y}{\theta} \beta - \sigma \left(\kappa - \frac{\varphi_y}{\theta}\right) \left(1 + \frac{\varphi_\pi}{\theta}\right) \\
& \left. + \sigma \left(\kappa - \frac{\varphi_y}{\theta}\right) \left(1 + \frac{\varphi_\pi}{\theta}\right) \beta\phi_\pi - (\sigma + \phi_y)\beta \frac{\varphi_y}{\theta} \left(1 + \frac{\varphi_\pi}{\theta}\right) [1 - \beta\phi_\pi] \right]
\end{aligned}
$$

$$
\begin{aligned}
detA' = & \left(\frac{1}{(\sigma + \phi_y)\left(1 + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right)^2 \\
& \times \left[- \left[\kappa - \frac{\varphi_y}{\theta} \right] \phi_\pi \frac{\varphi_y}{\theta} \beta + (\sigma + \phi_y)\sigma\beta \left(1 + \frac{\varphi_\pi}{\theta}\right)^2 \right. \\
& \qquad \left. + \sigma \left(\kappa - \frac{\varphi_y}{\theta}\right) \left(1 + \frac{\varphi_\pi}{\theta}\right) \beta\phi_\pi - (\sigma + \phi_y)\beta \frac{\varphi_y}{\theta} \left(1 + \frac{\varphi_\pi}{\theta}\right) \right. \\
= & \left(\frac{1}{(\sigma + \phi_y)\left(1 + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right)^2 \\
& \times \beta \left[(\sigma + \phi_y) \left(1 + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right) \right] \left[\sigma \left(1 + \frac{\varphi_\pi}{\theta}\right) - \frac{\varphi_y}{\theta} \right] \\
= & \left(\frac{1}{(\sigma + \phi_y)\left(1 + \frac{\varphi_\pi}{\theta}\right) + \phi_\pi \left(\kappa - \frac{\varphi_y}{\theta}\right)} \right) \beta \left[\sigma \left(1 + \frac{\varphi_\pi}{\theta}\right) - \frac{\varphi_y}{\theta} \right]
\end{aligned}
$$

Let's assume $\varphi_y, \varphi_\pi, \phi_\pi, \phi_y \geq 0$ and restrict our analysis to the case where the determinant is positive (both eigenvalues have the same sign and are non-imaginary), i.e. assume that $\left(\sigma\left(1+\frac{\varphi_\pi}{\theta}\right)-\frac{\varphi_y}{\theta}\right)\left((\sigma+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\kappa-\phi_\pi\frac{\varphi_y}{\theta}\right) > 0$. These restrictions are consistent with any empirically reasonable parametrization.

This system with two non predetermined variables is determinate if and only if both eigenvalues are within the unit circle (Blanchard and Kahn, 1980). There are then two sufficient and necessary conditions for both eigenvalues to be within the unit circle: $detA' < 1$ and $TrA < 1 + detA'$. The latter condition can be derived from the condition that both eigenvalues are strictly below 1 $(1-\lambda_1)(1-\lambda_2) > 0$ when both are positive, or both strictly above -1 $(-1-\lambda_1)(-1-\lambda_2) > 0$ when both are negative. The condition $detA' < 1$ gives:

$$\frac{\varphi_y}{\theta}(\phi_\pi - \beta) < \phi_\pi\kappa + (\sigma(1-\beta)+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)$$

This condition is always true if $\phi_\pi < \beta$ (recall that we assumed $\varphi_y \geq 0$). If $\phi_\pi > \beta$, then it requires φ_y to be small enough:

$$\varphi_y < \frac{\theta\phi_\pi\kappa + \theta(\sigma(1-\beta)+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)}{\phi_\pi - \beta}$$

The second necessary and sufficient condition is that $TrA' < 1 + detA'$ which gives

$$\frac{\left[-\frac{\varphi_y}{\theta}(1+\beta\phi_\pi)+\kappa+(\sigma(\beta+1)+\phi_y\beta)\left(1+\frac{\varphi_\pi}{\theta}\right)\right]}{(\sigma+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)} < 1 + \frac{\beta\left[\sigma\left(1+\frac{\varphi_\pi}{\theta}\right)-\frac{\varphi_y}{\theta}\right]}{(\sigma+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)}$$

$$\left[-\frac{\varphi_y}{\theta}(1+\beta\phi_\pi)+\kappa+(\sigma(\beta+1)+\phi_y\beta)\left(1+\frac{\varphi_\pi}{\theta}\right)\right] < (\sigma+\phi_y)\left(1+\frac{\varphi_\pi}{\theta}\right)+\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)+\beta\left[\sigma\left(1+\frac{\varphi_\pi}{\theta}\right)-\frac{\varphi_y}{\theta}\right]$$

$$-\frac{\varphi_y}{\theta}(1+\beta\phi_\pi)+\kappa-\left(1+\frac{\varphi_\pi}{\theta}\right)(1-\beta)\,\phi_y<\phi_\pi\left(\kappa-\frac{\varphi_y}{\theta}\right)-\beta\frac{\varphi_y}{\theta}>0$$
$$(\phi_\pi-1)\kappa+\frac{\varphi_y}{\theta}\left[(1+\beta\phi_\pi)-\beta-\phi_\pi\right]+\left(1+\frac{\varphi_\pi}{\theta}\right)(1-\beta)\,\phi_y>0$$
$$(\phi_\pi-1)\left[\kappa-\frac{\varphi_y}{\theta}(1-\beta)\right]+\left(1+\frac{\varphi_\pi}{\theta}\right)(1-\beta)\,\phi_y>0$$

This ends the proof. □

The first bullet point corresponds to the traditional Taylor principle according to which the reaction of monetary policy to inflation is strong enough so that the path of inflation is always unique. In this case, the reaction of TIP to the output gap can't be too strong. The second and third cases correspond to a new principle according to which the reaction of the monetary policy to inflation is weak but the reaction of TIP to the output gap is strong.

This finding that a high enough φ_y can ensure determinacy is intuitive and the mechanism is analogous to the Taylor principle. Assume that agents expect the economy to jump to a situation of high inflation and high output gap. Following the TIP rule, policymakers set a very high tax on inflation which leads to deflation, which in turn triggers a recession. This outcome contradicts the initial expectation. This ensures that the economy always stays on a unique equilibrium path.

However, in practice, only the Taylor principle can realistically be implemented for any reasonable calibration of the model's parameters. Following the calibration laid out in the quantitative section, the value of the coefficient φ_y necessary to ensure determinacy, $\frac{\theta\kappa}{1-\beta}$, is slightly above 6300. This coefficient implies a reaction function that is too strong to be realistically implemented. For this reason, in the rest of the paper, we restrict our attention to targeting rules where the Taylor principle applies and the reaction of TIP to the output gap is not too strong.

Full system for simulations The system of equations representating the economy used in simulations is given by

$$
\begin{aligned}
\hat{y}^e_t &= E_t\hat{y}^e_{t+1} - \frac{1}{\sigma}\left(\rho + \phi_\pi \pi_t + \phi_y \hat{y}^e_t - E_t\pi_{t+1} - r^e_t\right) \\
\left(1+\frac{\varphi_\pi}{\theta}\right)\pi_t &= \beta\left(1+\frac{\varphi_\pi}{\theta}\right)E_t\pi_{t+1} + \left(\kappa - \frac{\varphi_y}{\theta}\right)\hat{y}^e_t + \frac{\varphi_y}{\theta}\beta E_t \hat{y}^e_{t+1} + \frac{1}{\theta}u_t \\
u_t &= \rho_u u_{t-1} + \epsilon^u_t \\
a_t &= \rho_a a_{t-1} + \epsilon^a_t \\
r^e_t &= \rho + \sigma E_t\left[\frac{u_{t+1}-u_t}{(\epsilon-1)\left(\sigma+\frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma+\frac{\psi+\alpha}{1-\alpha}\right)}(a_{t+1}-a_t)\right]
\end{aligned}
$$

B Distortion of Relative Prices

B.1 Model

Setting. We extend the model to include a continuum of sectors indexed by $s \in [0,1]$. There is also a continuum of firms within each sector which are in monopolistic competition. These firms produce differentiated goods which are used in the production of a new type of goods: the sector goods, which we denote C_s. Sector goods are used for the production of final goods. They are also used by intermediate firms to pay for price increases in their relevant sector. The final good firms have the following technology:

$$
\ln Y_t = \int \gamma_s \ln C_{ts} ds
$$

where $\int \gamma_s = 1$. There are also aggregator firms in each sector with the following technology:

$$
Y_{ts} = \left(\int y_{ts}^{1-1/\epsilon} ds\right)^{\frac{\epsilon}{\epsilon-1}}
$$

Denoting $\mathcal{C}(P_{tis}, P_{t-1is})$ the nominal adjustment costs of firm i in sector s at time t, the market clearing condition in each sector is given by:

$$Y_{ts} = C_{ts}$$

The adjustment cost needs to be paid in terms of final goods.

$$Y_t = C_t + \int_s \frac{\theta_s}{2} \pi_s^2 Y_{ts}$$

Firms' First-Order Condition Consider firms in sector s. The first-order conditions associated with these firm's problem are:

$$(1-\epsilon_{st}) \left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}} Y_{st} - (1-\epsilon_{st}) \left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}} Y_{st}\tau_t - \tau_t \epsilon_{st} \frac{P_{t-1i}}{P_{ti}} Y_{st} \left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}}$$
$$+ \frac{\epsilon_{st}}{1-\alpha} \frac{W_t}{P_{ti}} \left[\left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}} \frac{Y_{st}}{A_{st}} \right]^{\frac{1}{1-\alpha}} - \frac{\theta_s}{P_{t-1i}} \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right) P_t Y_{st} + E Q_t V'(P_{ti}) = 0$$

and

$$V'(P_{t-1i}) = \tau_t Y_{st} \left(\frac{P_{ti}}{P_{st}}\right)^{-\epsilon_{st}} + \frac{P_{ti}}{P_{t-1i}^2} \theta_s \left(\frac{P_{ti}}{P_{t-1i}} - 1 \right) P_t Y_{st}.$$

Assuming symmetry gives

$$\left((1-\epsilon_{st})Y_{st} - (1-\epsilon_{st})Y_{st}\tau_t - \tau_t\epsilon_{st}\frac{P_{t-1}}{P_{st}}Y_{st} + \frac{\epsilon_{st}}{1-\alpha}\frac{W_t}{P_{st}}\left[\frac{Y_{st}}{A_{st}}\right]^{\frac{1}{1-\alpha}} - \frac{\theta_s}{P_{t-1,s}}\pi_{st}P_tY_{st}\right)$$
$$+E_tQ_t\left[\tau_{t+1}Y_{t+1s} + (\pi_{t+1s}+1)\frac{P_{t+1}}{P_{ts}}\theta_s\pi_{t+1s}Y_{t+1s}\right] = 0$$
$$\iff \left(1-\epsilon_{st} - \tau_t\left(1-\epsilon_{st}\frac{\pi_{st}}{1+\pi_{st}}\right) + \epsilon_{st}MC_{st} - \theta_s\pi_{st}\frac{(\pi_{st}+1)}{\tilde{P}_{st}}\right)$$
$$+E_tQ_t\left[\tau_{t+1}\frac{Y_{t+1s}}{Y_{st}} + \frac{(\pi_{t+1s}+1)^2}{\tilde{P}_{st+1}}\theta_s\pi_{t+1s}\frac{Y_{t+1s}}{Y_{st}}\right] = 0$$
$$\iff \left((1-\epsilon_{st})\left(1-\frac{\epsilon_{st}}{\epsilon_{st}-1}MC_{st}\right) - \tau_t\left(1-\epsilon_{st}\frac{\pi_{st}}{1+\pi_{st}}\right) - \theta_s\pi_{st}\frac{(\pi_{st}+1)}{\tilde{P}_{st}}\right)$$
$$+E_tQ_t\left[\tau_{t+1}\frac{Y_{t+1s}}{Y_{st}} + \frac{(\pi_{t+1s}+1)^2}{\tilde{P}_{st+1}}\theta_s\pi_{t+1s}\frac{Y_{t+1s}}{Y_{st}}\right] = 0$$
$$\iff \frac{1}{\theta_s}\left(\left(1-\epsilon_{st}\right)\left(1-\mathcal{M}_{st}MC_{st}\right) - \tau_t\left(1-\epsilon_{st}\frac{\pi_{st}}{1+\pi_{st}}\right)\right) - \pi_{st}\frac{(\pi_{st}+1)}{\tilde{P}_{st}}$$
$$+E_tQ_t\left[\frac{\tau_{t+1}}{\theta_s}\frac{Y_{t+1s}}{Y_{st}} + \frac{(\pi_{t+1s}+1)^2}{\tilde{P}_{st+1}}\pi_{t+1s}\frac{Y_{t+1s}}{Y_{st}}\right] = 0$$

$$\text{with} \quad MC_{st} = \frac{W_t}{P_{st}(1-\alpha)}\frac{Y_{st}^{\frac{\alpha}{1-\alpha}}}{A_{st}^{\frac{1}{1-\alpha}}} \quad \text{and} \quad \mathcal{M}_{st} = \frac{\epsilon_{st}}{\epsilon_{st}-1} \quad \text{and} \quad \tilde{P}_{ts} = \frac{P_{st}}{P_t}.$$

The optimal real marginal cost in the flexible price equilibrium is equal to the inverse of the markup. We denote the steady-state markup $\bar{\mathcal{M}}$. The next step is to linearize this Phillips curve around a steady-state with no inflation, no dispersion in productivity, constant output, a zero tax on price changes, a flexible price markup and no markup shocks, $\pi = 0, \tilde{P}_s = 1, \tau = 0, Y = Y', MC = 1/\bar{\mathcal{M}}$. Denoting mc the log of the real marginal cost MC and μ the log of $\mathcal{M}$, we obtain:

$$\frac{1}{\theta_s}\left[(1-\epsilon)\left(1-1*\left(1+mc_{st}-mc+\mu_t-\mu\right)\right)-\tau_t\right]+E_tQ_t\left[\tau_{t+1}\frac{1}{\theta_s}+\pi_{t+1s}\right]=\pi_{st}$$
$$\iff \frac{1}{\theta_s}\left((\epsilon-1)(\hat{mc}_{st}+\hat{\mu}_{st})-\tau_t\right)+\beta E_t\left[\frac{\tau_{t+1}}{\theta_s}+\pi_{t+1}\right]=\pi_{st}$$
$$\iff \frac{1}{\theta_s}\left((\epsilon-1)\hat{mc}_{st}+u_t-\tau_t\right)+\beta E_t\left[\frac{\tau_{t+1}}{\theta_s}+\pi_{t+1}\right]=\pi_{st}$$

where $\hat{mc}^e_{st} = mc_{st} - mc$ is the gap between the effective marginal cost and the desired marginal cost under flexible prices in steady-state, and $u_t = (\epsilon-1)(\mu_t-\mu) = (\epsilon-1)(mc - mc^n_{st})$ is the markup shock, where mc^n_{st} is the marginal cost under flexible price. We thus obtain

$$\begin{aligned}\pi_{st} &= \frac{1}{\theta_s}\left((\epsilon-1)\hat{mc}_{st}-\tau_t+u_t\right)+\beta E_t\left[\frac{\tau_{t+1}}{\theta_s}+\pi_{t+1}\right]\\ &= \beta E_t\pi_{t+1s}+\frac{\epsilon-1}{\theta_s}\hat{mc}_{st}+\frac{1}{\theta_s}\left[\beta E_t\tau_{t+1}-\tau_t+u_t\right]\end{aligned}$$

Final goods optimal demand for sector goods From the FOC of final goods firms,

$$Y_{ts}=\gamma_s\frac{P_tY_t}{P_{ts}}.$$

we take the logs and obtain

$$\begin{aligned}y_{ts} &= \log\gamma_s+p_t+y_t-p_{ts}\\ \Rightarrow y_{ts}-y_t &= \log\gamma_s-(p_{ts}-p_t)\\ \Rightarrow \tilde{y}_{ts} &= \log\gamma_s-\tilde{p}_{ts}\end{aligned}$$

where $\tilde{x}_s = x_s - x$ and

$$\begin{aligned}y_t &= \int\gamma_s y_{ts}ds\\ p_t &= \int\gamma_s\left(p_{ts}-\log\gamma_s\right)ds\end{aligned}$$

We can obtain the log change in the aggregate consumer price index by taking the time difference of the last equation

$$\pi_t = p_t - p_{t-1} = \int \gamma_s \pi_{ts} ds$$

Market clearing Since all firms are identical within each sector in equilibrium, the market clearing condition for the sectorial goods market is given by

$$Y_{st} = C_{st}.$$

where C_{st} denotes the consumption of sectorial good s by final goods firms. Taking logs

$$c_{st} = y_{st}$$

The market clearing for the final goods market is simply given by

$$Y_t = C_t + \int_s \frac{\theta_s}{2} \pi_{st}^2 Y_s$$

At first order the approximation is given by

$$c_t = y_t.$$

and at second order the approximation is given by

$$c_t = y_t - \frac{1}{2} E[\theta_s \pi_{st}^2].$$

From the labor market clearing condition $N_t = \int_s \int_{i\in s} N_{ti} dids$, we get

$$\begin{aligned} N_t &= \int_s \int_{i\in s} \left(\frac{Y_{ti}}{A_{st}}\right)^{\frac{1}{1-\alpha}} dids \\ &= \int_s \left(\frac{Y_{ts}}{A_{st}}\right)^{\frac{1}{1-\alpha}} ds \\ &= \int_s \left(\frac{\gamma_s P_t Y_t}{P_{st} A_{st}}\right)^{\frac{1}{1-\alpha}} ds \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_s \left(\frac{\gamma_s P_t A_t}{P_{st} A_{st}}\right)^{\frac{1}{1-\alpha}} ds \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \Delta_t \end{aligned}$$

where $\Delta = \int_s \left(\frac{\gamma_s P_t A_t}{P_{st} A_{st}}\right)^{\frac{1}{1-\alpha}} ds$ captures the costs entailed by price distortions and misallocation of sector goods, and where we used the fact that all firms are ex post identical $Y_{ti} = Y_{st}$ in each sector and the FOC of the final goods firms $Y_{st} = \gamma_s \frac{P_t Y_t}{P_{ts}}$, and where we define A_t the geometric mean of sectorial TFPs, A_{st}: $a_t = \int_s \gamma_s a_{st} ds$.

We then approximate Δ_t. We need to first obtain an expression of relative prices under flexible prices. We start from the optimal markup of a firm in sector s given by

$$\mathcal{M}_{st}^{-1} = \frac{W_t^f}{P_{st}^f} \frac{(Y_{st}^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_{st}^{\frac{1}{1-\alpha}}} = \frac{W_t^f}{P_t^f} \frac{P_t^f}{P_{st}^f} \frac{\left(\gamma_s P_t^f Y_t^f / P_{st}^f\right)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_{st}^{\frac{1}{1-\alpha}}}$$

which implies

$$\frac{P_t^f A_t}{P_{st}^f A_{st}} = \left[\left(\frac{\mathcal{M}_{st} W_t^f}{P_t^f} \right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_t^{\frac{1}{1-\alpha}}} \right]^{\alpha-1} \gamma_s^{-\alpha}$$

We next take this ratio at the power γ_s, take the product over all sectors:

$$\Pi_s^S \left(\frac{P_t^f A_t}{P_{st}^f A_{st}}\right)^{\gamma_s} = \Pi_s^S \left(\left[\left(\frac{\mathcal{M}_{st} W_t^f}{P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1} \gamma_s^{-\alpha}\right)^{\gamma_s}$$

$$\frac{P_t^f A_t}{\Pi_s^S \gamma_s^{\gamma_s} \Pi_s^S \left(P_{st}^f / \gamma_s\right)^{\gamma_s} \Pi_s^S A_{st}^{\gamma_s}} = \left[\left(\frac{W_t^f}{P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1} \Pi_s^S \mathcal{M}_{st}^{\gamma_s(\alpha-1)} \gamma_s^{-\alpha\gamma_s}$$

$$\frac{P_t^f A_t}{\Pi_s^S \gamma_s^{\gamma_s} P_t^f A_t} = \left[\left(\frac{W_t^f}{P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1} \Pi_s^S \mathcal{M}_{st}^{\gamma_s(\alpha-1)} \gamma_s^{-\alpha\gamma_s}$$

$$1 = \left[\left(\frac{\mathcal{M}_t W_t^f}{P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1}$$

with $\mathcal{M}_t = \Pi_s^S \left(\frac{\mathcal{M}_{st}}{\gamma_s}\right)^{\gamma_s}$. Hence

$$\frac{P_t^f A_t}{P_{st}^f A_{st}} = \left[\left(\frac{\mathcal{M}_{st} \mathcal{M}_t W_t^f}{\mathcal{M}_t P_t^f}\right) \frac{(Y_t^f)^{\frac{\alpha}{1-\alpha}}}{(1-\alpha) A_t^{\frac{1}{1-\alpha}}}\right]^{\alpha-1} \gamma_s^{-\alpha} = \left[\frac{\mathcal{M}_{st}}{\mathcal{M}_t}\right]^{\alpha-1} \gamma_s^{-\alpha}$$
$$\Rightarrow \tilde{p}_{st}^f = -\tilde{a}_{st} + (1-\alpha)\tilde{\mu}_{st} + \alpha \log \gamma_s$$

We can now go back to Δ and linearize it around the efficient steady-state with

$\frac{PA}{P_s A_s} = \left[\frac{\mathcal{M}_s}{\mathcal{M}}\right]^{\alpha-1} \gamma_s^{-\alpha} = \left[\Pi_s^S \gamma_s^{\gamma_s}\right]^{\alpha-1} \gamma_s^{-\alpha}$ where we used $\mathcal{M}_s = 1$ for all s.

$$\Delta_t = \int_s \left(\frac{\gamma_s P_t A_t}{P_{st} A_{st}}\right)^{\frac{1}{1-\alpha}} ds$$

$$\Delta \exp(\hat{\delta}_t) = \int_s \left(\frac{\gamma_s PA}{P_s A_s}\right)^{\frac{1}{1-\alpha}} \exp\left(-\frac{1}{1-\alpha}\left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st}\right)\right) ds$$

$$= \left[\Pi_s^S \gamma_s^{-\gamma_s}\right] \int_s \gamma_s \exp\left(-\frac{1}{1-\alpha}\left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st}\right)\right) ds$$

$$\hat{\delta}_t = -\frac{1}{1-\alpha} \int_s \gamma_s \left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st}\right) ds$$

At second order this gives

$$\hat{\delta}_t = \int_s \gamma_s \left(-\frac{1}{1-\alpha}\left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st}\right) + \frac{1}{2(1-\alpha)^2}\left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st}\right)^2\right) ds$$

$$= \frac{1}{(1-\alpha)}\left[-E_\gamma\left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st}\right) + \frac{E\left(\left(\tilde{p}_{st} - \tilde{p}_{st}^e\right)^2\right)}{2(1-\alpha)}\right]$$

$$= \frac{1}{(1-\alpha)}\left[-E_\gamma\left(\hat{\tilde{p}}_{st}^e\right) + \frac{E\left(\left(\tilde{p}_{st} - \tilde{p}_{st}^e\right)^2\right)}{2(1-\alpha)}\right]$$

where we used $\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st} = \hat{\tilde{p}}_{st}^e$ and the expectation is taken across sectors under the weights γ_s. Note that $\hat{\delta}_t$ is zero at first order

$$\hat{\delta}_t = -\frac{1}{1-\alpha} \int_s \gamma_s \left(\hat{\tilde{p}}_{st} + \hat{\tilde{a}}_{st}\right) ds = -\frac{1}{1-\alpha} \int_s \gamma_s \left(\tilde{p}_{st} - \tilde{p}_s + \tilde{a}_{st} - \tilde{a}_s\right) ds$$

$$= -\frac{1}{1-\alpha} \int_s \gamma_s \left(\tilde{p}_{st} - \tilde{p}_s - \tilde{p}_{st}^e + \tilde{p}_s^e\right) ds = 0$$

where the last line uses the definition of the price index $p = \int \gamma_s p_s$. For future reference we denote $\hat{\tilde{p}}_{st}^e = \tilde{p}_{st} - \tilde{p}_s - \tilde{p}_{st}^e + \tilde{p}_s^e$.

Phillips Curve and Euler Equation We now turn to the first order conditions of the households given by

$$w_t - p_t = \sigma c_t + \psi n_t$$
$$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \rho\right)$$

with $i_t = -\log Q_t$. Combining them with the market clearing condition for the final goods gives

$$w_t - p_t = \left(\sigma + \frac{\psi}{1-\alpha}\right) y_t - \frac{\psi}{1-\alpha}\log a_t + \psi \delta_t$$
$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - \rho\right)$$

with $\delta_t = \log \Delta_t$. From the definition of markup, we obtain

$$\begin{aligned}
mc_{st} &= w_t - p_t + p_t - p_{st} + \frac{\alpha}{1-\alpha} y_{st} - \frac{1}{1-\alpha} a_{st} - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi}{1-\alpha}\right) y_t + \psi\delta_t + p_t - p_{st} - \frac{\psi}{1-\alpha} a_t + \frac{\alpha}{1-\alpha} y_{st} - \frac{1}{1-\alpha} a_{st} - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t + \psi\delta_t - (p_{st} - p_t) - \frac{1+\psi}{1-\alpha} a_t - \frac{1}{1-\alpha}(a_{st} - a_t) \\
&\qquad + \frac{\alpha}{1-\alpha}(y_{st} - y_t) - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t + \psi\delta_t - \tilde{p}_{st} - \frac{1+\psi}{1-\alpha} a_t - \frac{1}{1-\alpha}\tilde{a}_{st} + \frac{\alpha}{1-\alpha}\tilde{y}_{st} - \log(1-\alpha) \\
&= mc_t + \psi\delta_t - \tilde{p}_{st} - \frac{1}{1-\alpha}\tilde{a}_{st} + \frac{\alpha}{1-\alpha}\tilde{y}_{st}
\end{aligned}$$

where the second line uses the first order condition of households and the market clearing condition for labor, where the third line uses $\tilde{x}_s = x_s - x$ and the last line uses $mc_t = \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) y_t - \frac{1+\psi}{1-\alpha} a_t - \log(1-\alpha)$. Finally we can use the first order condition of the final goods firms to substitute for $\tilde{y}_{st}$:

$$mc_{st} = mc_t + \psi\delta_t - \frac{1}{1-\alpha}\tilde{a}_{st} - \left(\frac{1}{1-\alpha}\right)\tilde{p}_{st}$$

We next take the difference with the steady-state:

$$\hat{mc}_{st} = \hat{mc}_t + \psi\hat{\delta}_t - \left(\frac{1}{1-\alpha}\right)(\tilde{p}_{st} - \tilde{p}_s + \tilde{a}_{st} - \tilde{a}_s)$$
$$\hat{mc}_{st} = \hat{mc}_t + \psi\hat{\delta}_t - \left(\frac{1}{1-\alpha}\right)\hat{\tilde{p}}^e_{st}.$$

Using this to substitute for the marginal cost in the sector-level Phillips curve:

$$\pi_{st} = \beta E_t \pi_{t+1s} + \frac{\epsilon-1}{\theta_s}\left[\hat{mc}_t - \left(\frac{1}{1-\alpha}\right)\hat{\tilde{p}}^e_{st}\right] + \frac{1}{\theta_s}\left[\beta E_t \tau_{t+1} - \tau_t + u_t\right] \quad (39)$$

where we used the fact that at first order $\hat{\delta}_t = 0$. Taking the (γ_s-weighted) integral of the Phillips curves over all sectors gives:

$$\pi_t = \beta E_t \pi_{t+1} + \hat{mc}_t \int_s \frac{\epsilon-1}{\theta_s}\gamma_s - \frac{1}{1-\alpha}\int_s \frac{(\epsilon-1)\gamma_s}{\theta_s}\hat{\tilde{p}}^e_{st} + [u_t + \beta E_t \tau_{t+1} - \tau_t]\int_s \frac{\gamma_s}{\theta_s} ds \quad (40)$$

Second-order approximation of welfare (proof of lemma 2) We next show that the welfare loss function of the household depends on the dispersion of relative prices around their efficient levels and on the average inflation rates across sectors.

$$\mathcal{L} = \sum_{t=0}^{\infty}\beta^t \left[\left(\sigma + \frac{\alpha+\psi}{1-\alpha}\right)(\hat{y}^e_{st})^2 + E[\theta_s \pi^2_{st}] + \frac{E\left((\tilde{p}_{st} - \tilde{p}^e_{st})^2\right)}{1-\alpha}\right]$$

The log deviation of the representative household is given by

$$\frac{U_t - U}{U_c C} = \left(\hat{c}_t + \frac{1-\sigma}{2}\hat{c}^2_t\right) + \frac{U_N N}{U_c C}\left(\hat{n}_t + \frac{1+\psi}{2}\hat{n}^2_t\right)$$

.

We next use a second order approximation of the goods market clearing condition to get an approximation for the first bracket

$$\hat{c}_t = \hat{y}_t - \frac{1}{2}E[\theta_s \pi_{st}^2].$$

We next use the labor market clearing condition to get an approximation for the second bracket:

$$\hat{n}_t = \frac{1}{1-\alpha}\left(\hat{y}_t - \hat{a}_t\right) + \hat{\delta}_t$$

Combining both, and keeping only the terms of first and second order, we obtain

$$\begin{aligned}\frac{U_t - U}{U_c C} = &\left(\hat{y}_t - \frac{1}{2}E[\theta_s \pi_{st}^2] + \frac{1-\sigma}{2}\left(\hat{y}_t\right)^2\right) \\ &+ \frac{U_N N}{U_c C}\left(\frac{1}{1-\alpha}\left(\hat{y}_t - \hat{a}_t\right) + \hat{\delta}_t + \frac{1+\psi}{2}\left(\frac{1}{1-\alpha}\right)^2\left(\hat{y}_t - \hat{a}_t\right)^2\right)\end{aligned}$$

Using also the fact that in an efficient steady-state $-\frac{U_N N}{U_C C(1-\alpha)} = 1$, we obtain:

$$\begin{aligned}\frac{U_t - U}{U_c C} &= -\frac{1}{2}E[\theta_s \pi_{st}^2] + \left(\frac{1-\sigma}{2}\left(\hat{y}_t\right)^2\right) - \left(-\hat{a}_t + \frac{1+\psi}{2(1-\alpha)}\left(\hat{y}_t - \hat{a}_t\right)^2\right) - (1-\alpha)\hat{\delta} + t.i.p \\ &= -\frac{1}{2}E[\theta_s \pi_{st}^2] - \frac{1}{2}\left(\sigma + \frac{\alpha+\psi}{1-\alpha}\right)\left(\hat{y}_t\right)^2 - \frac{1+\psi}{2(1-\alpha)}\hat{y}_t\hat{a}_t - (1-\alpha)\hat{\delta} + t.i.p \\ &= -\frac{1}{2}E[\theta_s \pi_{st}^2] - \frac{1}{2}\left(\sigma + \frac{\alpha+\psi}{1-\alpha}\right)\left[\left(\hat{y}_t\right)^2 - 2\hat{y}_t\hat{y}_t^n\right] - (1-\alpha)\hat{\delta} + t.i.p \\ &= -\frac{1}{2}E[\theta_s \pi_{st}^2] - \frac{1}{2}\left(\sigma + \frac{\alpha+\psi}{1-\alpha}\right)\left[\left(\hat{y}_t^e\right)^2 - \left(\hat{y}_t^n\right)^2\right] - (1-\alpha)\hat{\delta} + t.i.p \\ &= -\frac{1}{2}E[\theta_s \pi_{st}^2] - \frac{1}{2}\left(\sigma + \frac{\alpha+\psi}{1-\alpha}\right)\left(\hat{y}_t^e\right)^2 - (1-\alpha)\hat{\delta} + t.i.p\end{aligned}$$

note that $\hat{a}_t$, $\hat{a}_t^2$ and $(\hat{y}_t^n)^2$ are in t.i.p. because they depend only on exogenous TFP shocks. We used the definition of y_t^n, the fact that $\hat{y}_t^n = y_t^n - y^n$ and that $\hat{y}_t^e = \hat{y}_t - \hat{y}_t^n$.

Substituting the second order approximation of $\hat{\delta}_t$

$$\hat{\delta}_t = \frac{E\left((\tilde{p}_{st} - \tilde{p}^e_{st})^2\right)}{2(1-\alpha)^2}$$

into the previous expression gives the resut

$$\frac{U_t - U}{U_c C} = -\frac{1}{2}E[\theta_s \pi^2_{st}] - \frac{1}{2}\left(\sigma + \frac{\alpha+\psi}{1-\alpha}\right)(\hat{y}^e_{st})^2 - \frac{E\left((\tilde{p}_{st} - \tilde{p}^e_{st})^2\right)}{2(1-\alpha)} + t.i.p.$$

Finally the term $E[\theta_s \pi^2_{st}]$ can be decomposed into an aggregate and sector-specific components:

$$\begin{aligned}
E[\theta_s \pi^2_{st}] &= E[\theta_s(\pi_t + (\pi_{st} - \pi_t))^2] \\
&= E\left[\theta_s \pi^2_t + \theta_s(\pi_{st} - \pi_t)^2 + 2\theta_s \pi_t(\pi_{st} - \pi_t)\right] \\
&= \left(E\left[\theta_s \pi^2_t\right] + E\left[\theta_s \tilde{\pi}^2_{st}\right] + 2cov\left[\pi_t, \theta_s \tilde{\pi}_{st}\right]\right) \\
&= \pi^2_t E\left[\theta_s\right] + E\left[\theta_s \tilde{\pi}^2_{st}\right]
\end{aligned}$$

where the last line uses the fact that in the cross-section π_t is constant.

Proof of Proposition (5). Taking the difference between equations (39) and (40) to get an expression for relative price inflation $\tilde{\pi}_{st} = \pi_{st} - \pi_t$ we get a linear difference equation in $\tilde{p}_{st}$ which depends in general on aggregates, such as $\hat{mc}_t, \tau_t$ etc... A sufficient condition for the difference equation to be independent of TIP if for $\theta_s = \theta$. Indeed we show that if $\theta_s = \theta$, the distribution of relative prices depend, at first order, only on $\epsilon, \alpha, \beta, \theta$ and the process of sector-specific productivity shocks:

$$\tilde{p}_{st} - \tilde{p}_{st-1} = \beta E_t(\tilde{p}_{st+1} - \tilde{p}_{st}) - \frac{\epsilon - 1}{\theta}\frac{1}{1-\alpha}(\tilde{p}_{st} + \tilde{a}_{st}) \tag{41}$$

Relative prices are therefore independent of TIP.

Proof. Substituting $\theta_s = \theta$ in the sector-s Phillips curve just derived we obtain

$$\pi_{st} = \beta E_t \pi_{t+1s} + \frac{\epsilon - 1}{\theta}\left[\hat{mc}_t^e - \left(\frac{1}{1-\alpha}\right)\hat{\hat{p}}_{st}^e\right] + \frac{1}{\theta}\left[\beta E_t \tau_{t+1} - \tau_t + u_t\right]$$

Taking the difference between with the aggregate Phillips curve, we obtain:

$$\tilde{\pi}_{st} = \beta E_t \tilde{\pi}_{t+1s} - \frac{\epsilon - 1}{\theta}\frac{1}{1-\alpha}(\tilde{p}_{st} + \tilde{a}_{st})$$

which gives

$$\tilde{p}_{st} - \tilde{p}_{st-1} = \beta E_t(\tilde{p}_{st+1} - \tilde{p}_{st}) - \frac{\epsilon - 1}{\theta}\frac{1}{1-\alpha}(\tilde{p}_{st} + \tilde{a}_{st})$$

The solution to this is independent of aggregate inflation and of TIP.

Looking now at average (squarred) price changes $E[\theta_s \pi_{st}^2] = \pi_t^2 E\left[\theta_s\right] + E\left[\theta_s \tilde{\pi}_{st}^2\right]$, under the same conditions, the second term $E\left[\theta_s \tilde{\pi}_{st}^2\right]$ is independent of aggregates and of TIP. Therefore $E[\theta_s \pi_{st}^2]$ varies with TIP only through its effect on aggregate inflation π_t^2. □

Note that if we had initially allowed for heterogeneous elasticities of substitution, ϵ, they would have had to be the same across sectors as well for the proof to go through.

B.2 Numerical simulations

Algorithm. We start with solving for the steady-state. In steady-state, we normalize the nominal wage to 1. Starting from this steady-state, we then solve for the response of the economy after an unexpected one-period increase in ϵ_0 as described in the text. The following steps describe our algorithm.

1. Start from a guess for steady-state output Y^* and price level P^*
2. Solve for the firms pricing rule $\tilde{P}_s(A_s, \tilde{P}_{s,-1})$ given aggregate output and

the price index using the following equation

$$\frac{1}{\theta_s}\left((1-\epsilon_s)(1-\mathcal{M}_s MC_s)-\tau_t\left(1-\epsilon_s\frac{\pi_s}{1+\pi_s}\right)\right)-\pi_s\frac{(\pi_s+1)}{\tilde{P}_s} + EQ\left[\frac{\tau_{+1}}{\theta_s}\frac{Y_{s,+1}}{Y_s}+\frac{(\pi_{s,+1}+1)^2}{\tilde{P}_{s,+1}}\pi_{s,+1}\frac{Y_{s,+1}}{Y_s}\right]=0$$

with $\pi_s = \log(\tilde{P}_s) - \log(\tilde{P}_{s,-1}) + \log(P^*) - \log(P^*_{-1})$ and

$$MC_s = \frac{W}{\tilde{P}_s P^*(1-\alpha)}\frac{Y_s^{\frac{\alpha}{1-\alpha}}}{A_s^{\frac{1}{1-\alpha}}} \quad \text{and} \quad \mathcal{M}_s = \frac{\epsilon_s}{\epsilon_s-1} \quad \text{and} \quad \tilde{P}_s = \frac{P_s}{P^*}.$$

and where the only source of uncertainty is the sector-specific TFP shock A_s.

For the sector-specific output, we use the demand from final goods firms:

$$Y_s = \frac{\gamma_s Y^*}{\tilde{P}_s}.$$

3. Update output and the price index using the following expressions:

$$\begin{aligned}
Y^* &= A^{(1+\psi)k}(1-\alpha)^{(1-\alpha)k}(\Gamma^*)^{-\sigma(1-\alpha)k}(\Delta^*)^{-\psi(1-\alpha)k}(\mathcal{M}^*)^{-(1-\alpha)k} \\
\text{with} \quad k &= \frac{1}{\psi+(1-\alpha)\sigma+\alpha} \\
\text{with} \quad \Gamma^* &= 1-\int \gamma_s \frac{1}{\tilde{P}_s}\theta_s \pi_s^2 ds \\
\Delta^* &= \int_s \left(\frac{\gamma_s A}{\tilde{P}_s A_s}\right)^{\frac{1}{1-\alpha}} ds \\
\mathcal{M}^* &= \exp\left(\int \gamma_s \log\left(\mathcal{M}_s\right) ds\right) \\
\mathcal{M}_s &= \frac{1}{MC_s}
\end{aligned}$$

In steady-state, the price index is obtained by dividing the nominal wage by the real wage $\frac{W}{P} = (Y^*)^{\sigma+\frac{\psi}{1-\alpha}} A^{-\frac{\psi}{1-\alpha}} (\Delta^*)^{\psi} (\Gamma^*)^{\sigma}$.

4. Iterate until output and the price index converge.

For the one-period markup shock, we assume that the shock happens at $t = 0$, and that the economy returns to the steady state at $t = 1$.

1. Set Y_1 equal to the steady state output Y^*. Inflation only lasts for one period, which implies that P_1 equals P_0. Set the relative pricing rule at $t = 1$ equal to the steady state rule $\tilde{P}_s(A_s, \tilde{P}_{s,-1})$.

2. Guess P_0 and Γ_0.

3. Compute Y_0 using the Euler equation and the Taylor rule

$$\frac{Y_0}{Y^*} = \left[\frac{\Gamma^*}{\Gamma_0}\left(\frac{P_0}{P^*}\right)^{-\phi_\pi}\right]^{\frac{1}{1+\phi_y}}$$

4. Solve for the firms optimal pricing decision at $t = 0$, given P_0, Y_0.

5. Use the optimal pricing decision to derive π_0 and then to update P_0.

6. Go back to 2 and iterate until convergence

Calibration. Calibration is explained in Section 7.3. In particular, we include three sectors: non-durable, durable, and services. The corresponding parameters are $\{20.4, 62.5, 689.7\}$ for θ_s. Their sizes are given by $\{0.4, 0.2, 0.4\}$.

Results. Panel (A) in Table F5 shows the effects of TIP in an economy with heterogeneous θ's. For comparison, we show the effects of TIP in an economy in which all sectors have the same θ in Panel (B) in the same table.

C Time-dependent Calvo pricing

In this appendix we derive the TIP that can implement the first best in a setting with time-dependent Calvo frictions and we show that the Phillips curve implied by Calvo pricing is the same at first order as in the economy with Rotemberg adjustment costs. Following the notations in Galí (2015), the maximization problem of a firm having the opportunity to reset its price is given by

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t^f \left\{ Q_{t,t+k}(P_{ti}^* Y_{t+k|ti} - \Psi_{t+k}(Y_{t+k|ti})) \right\} - \tau_t (P_{ti}^* - P_{t-1i}) Y_{t|ti}$$
$$- \sum_{k=1}^{\infty} \theta^{k-1}(1-\theta) E_t^f \left\{ Q_{t,t+k} \tau_{t+k} (P_{t+ki}^{**} - P_{ti}^*) Y_{t+k|t+ki} \right\}$$

where we denote $\Psi()$ the cost function, P_{t+ki}^{**} the optimal price of firm i when it gets the chance to reset the price in the future, and $Y_{t+k|ti}$ is the output at period $t+k$ of firm i which resets its price in period t. Note that contrary to the setting without TIP, the optimal price for a firm that gets to reset at t may differ across firms, depending on when they last reset their price. We thus need to keep the i-subscript. The F.O.C. is given by

$$\sum_{k=0}^{\infty} \theta^k E_t^f \left\{ Q_{t,t+k} \left((1-\epsilon_t) Y_{t+ki|t} + \epsilon_t \Psi'_{t+k}(Y_{t+k|ti}) \right) \right\} - \tau_t \left(1 - \epsilon_t \frac{P_{ti}^* - P_{t-1i}}{P_{ti}^*} \right) Y_{t|ti}$$
$$+ \sum_{k=1}^{\infty} \theta^{k-1}(1-\theta) E_t^f \left\{ Q_{t,t+k} \tau_{t+k} Y_{t+k|t+ki} \right\} = 0$$

Dividing by $(1-\epsilon_t)$, and multiplying by P_{ti}^*/P_{t-1} we obtain

$$\sum_{k=0}^{\infty} \theta^k E_t^f \left\{ Q_{t,t+k} Y_{t+k|ti} \left(\frac{P_{ti}^*}{P_{t-1}} - \mathcal{M}_t MC_{t+k|ti} \Pi_{t-1,t+k} \right) \right\} + \frac{\tau_t}{\epsilon_t - 1} \left(1 - \epsilon_t \frac{P_{ti}^* - P_{t-1i}}{P_{ti}^*} \right) \frac{P_{ti}^*}{P_{t-1}} Y_{t|ti}$$
$$- \sum_{k=1}^{\infty} \theta^{k-1}(1-\theta) E_t^f \left\{ Q_{t,t+k} \frac{\tau_{t+k}}{\epsilon_t - 1} \frac{P_{ti}^*}{P_{t-1}} Y_{t+k|t+ki} \right\}$$

with $MC_{t+k|ti} = \psi_{t+k|ti}/P_{t+k}$ the real marginal cost in period $t+k$ for firm i which last reset its price in period t.

First-best implementation. Together with monetary policy TIP can implement the first best. TIP should follow

$$\tau_t = \frac{\epsilon_t - 1}{Y_t}\left[\sum_{k=0}^{\infty}\theta^k E_t^f\left\{Q_{t,t+k}^e Y_{t+k}\left(\mathcal{M}_t\mathcal{M}_t^w - 1\right)\right\} + \sum_{k=1}^{\infty}\theta^{k-1}(1-\theta)E_t^f\left\{Q_{t,t+k}^e\frac{\tau_{t+k}}{\epsilon_t - 1}Y_{t+k}\right\}\right]$$

and monetary policy should target the neutral rate of interest

$$i_t = (Q_t^e)^{-1} - 1 \quad with \quad Q_t^e = E_t\left[\beta_t\left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\sigma(1+\psi)}{(1-\alpha)\sigma+\psi+\alpha}}\right].$$

First-order approximation. Going back to the first-order condition of firms that get to reset their price, we then linearize around $\frac{P_t^*}{P_{t-1}} = 1$, steady-state output Y and $\tau = 0$.

$$\sum_{k=0}^{\infty}\theta^k\beta^k Y E_t\left\{p_{ti}^* - p_{t-1} - \hat{mc}_{t+k|ti} + p_{t+k} - p_{t-1}\right\} + \frac{\tau_t}{\epsilon - 1}Y$$
$$- \sum_{k=1}^{\infty}\theta^{k-1}(1-\theta)\beta^k E_t\left\{\frac{\tau_{t+k}}{\epsilon - 1}Y\right\} = 0$$

We then show that the marginal cost is given by

$$\begin{aligned} mc_{t+k|ti} &= mc_{t+k} - \frac{\alpha}{1-\alpha}(y_{t+k|ti} - y_{t+k}) \\ &= mc_{t+k} - \frac{\epsilon\alpha}{1-\alpha}(p_{ti}^* - p_{t+k}) \end{aligned}$$

and substituting this expression into the optimal pricing decision yields

$$\sum_{k=0}^{\infty}\theta^k\beta^k E_t\left\{p_{ti}^* - p_{t-1} - \Theta\hat{mc}_{t+k} - (p_{t+k} - p_{t-1})\right\} + \Theta\frac{\tau_t}{\epsilon - 1}$$
$$- \sum_{k=1}^{\infty}\theta^{k-1}(1-\theta)\Theta\beta^k E_t\left\{\frac{\tau_{t+k}}{\epsilon - 1}\right\} = 0.$$

Rearranging the previous equation gives

$$
\begin{aligned}
p^*_{ti} - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\left\{\Theta\hat{mc}_{t+k} + (p_{t+k} - p_{t-1})\right\} - (1-\beta\theta)\Theta\frac{\tau_t}{\epsilon - 1} \\
+ (1-\beta\theta)\sum_{k=1}^{\infty}\theta^{k-1}(1-\theta)\Theta\beta^k E_t\left\{\frac{\tau_{t+k}}{\epsilon-1}\right\}
\end{aligned}
$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$. We then rewrite the sum as a difference equation

$$
\begin{aligned}
p^*_{ti} - p_{t-1} &= (1-\beta\theta)\Theta\hat{mc}_t + (1-\beta\theta)\sum_{k=0}^{\infty}(\theta\beta)^k(p_t - p_{t-1}) - (1-\beta\theta)\Theta\frac{\tau_t}{\epsilon-1} \\
&\quad + \beta\theta(1-\beta\theta)\Theta\frac{\tau_{t+1}}{\epsilon-1} \\
&\quad + (1-\beta\theta)\theta\beta\sum_{k=1}^{\infty}(\theta\beta)^{k-1}E_t\left\{\Theta\hat{mc}_{t+k} + (p_{t+k} - p_t)\right\} - \beta\theta(1-\beta\theta)\Theta\frac{\tau_{t+1}}{\epsilon-1} \\
&\quad + (1-\beta\theta)(1-\theta)\Theta\beta E_t\left\{\frac{\tau_{t+1}}{\epsilon-1}\right\} \\
&\quad + (1-\beta\theta)\beta\theta\sum_{k=2}^{\infty}\theta^{k-2}(1-\theta)\Theta\beta^{k-1}E_t\left\{\frac{\tau_{t+k}}{\epsilon-1}\right\} \\
p^*_{ti} - p_{t-1} &= (1-\beta\theta)\Theta\hat{mc}_t + \pi_t - (1-\beta\theta)\Theta\frac{\tau_t}{\epsilon-1} + \beta\theta(1-\beta\theta)\Theta\frac{\tau_{t+1}}{\epsilon-1} \\
&\quad + \beta\theta E_t(p^*_{t+1} - p_t) \\
&\quad + (1-\beta\theta)(1-\theta)\Theta\beta E_t\left\{\frac{\tau_{t+1}}{\epsilon-1}\right\} \\
p^*_{ti} - p_{t-1} &= \beta\theta E_t\{p^*_{t+1i} - p_t\} + (1-\beta\theta)\Theta\hat{mc}_t + \pi_t - (1-\beta\theta)\Theta\frac{\tau_t}{\epsilon-1} \\
&\quad + (1-\beta\theta)\beta\Theta E_t\left\{\frac{\tau_{t+1}}{\epsilon-1}\right\}
\end{aligned}
$$

We then use $\pi_t = (1-\theta)(p_t^* - p_{t-1})$ to get

$$\frac{\pi_t}{1-\theta} = \beta\theta E_t \frac{\pi_{t+1}}{1-\theta} + (1-\beta\theta)\Theta \hat{mc}_t + \pi_t - (1-\beta\theta)\Theta \frac{\tau_t}{\epsilon-1} + (1-\beta\theta)\beta\Theta E_t \left\{ \frac{\tau_{t+1}}{\epsilon-1} \right\}$$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta} \Theta \hat{mc}_t - \frac{(1-\beta\theta)(1-\theta)}{\theta(\epsilon-1)} \Theta \tau_t + \frac{(1-\beta\theta)(1-\theta)}{\theta(\epsilon-1)} \beta\Theta E_t \{\tau_{t+1}\}$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \hat{mc}_t - \zeta \left[\tau_t - \beta E_t \{\tau_{t+1}\}\right]$$

with $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}\Theta$ and $\zeta = \frac{\lambda}{(\epsilon-1)}$.

Finally, using $\hat{mc}_t = \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) \hat{y}_t$ we get

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{y}_t - \zeta \left[\tau_t - \beta E_t \{\tau_{t+1}\}\right]$$

with $\kappa = \lambda \left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)$.

This shows that using time-dependent frictions instead of state-dependent frictions leads to the same representation and macroeconomic dynamics at first order as with Rotemberg-type frictions. Therefore the results on optimal policies derived in the linearized model—corollary 1, section 6, section 8 and proposition 5—are robust to using Calvo-type frictions. The following paragraph gives more details on proposition 5.

Relative price distortions and independence from TIP. In a setting with heterogeneous sectors, we can follow the same reasoning as for Rotemberg adjustment cost and derive the following Phillips curve for each sector:

$$\pi_{st} = \beta E_t \pi_{t+1s} + \lambda_s \left[\hat{mc}_t - \left(\frac{1}{1-\alpha}\right) \hat{\tilde{p}}_{st}^e\right] + \zeta_s \left[\beta E_t \tau_{t+1} - \tau_t\right] + u_t$$

with $\lambda_s = \frac{(1-\beta\theta_s)(1-\theta_s)}{\theta_s}\Theta$ and $\zeta_s = \frac{\lambda_s}{(\epsilon-1)}$. Taking the difference between any two sectors, we see that the sufficient conditions for proposition 5 to hold, i.e. for relative prices to be independent of aggregate and on TIP, are the same, namely $\lambda_s = \lambda$ and $\zeta_s = \zeta$ so ultimately $\theta_s = \theta$. The second bullet point of the proposition is a direct corollary of the first.

D Taxing large corporates

We now consider a generalized version of the previous model with two types of firms, small (S) and large (L) which differ in two ways: L-firm face a TIP, and L-firms face a different consumers taste shock, $\bar{\xi}$. More formally, we assume that the utility is given by:

$$C = \left(\int_{i \in S} \underline{\xi}^{1/\epsilon_t} C_{ti}^{1-1/\epsilon_t} + \int_{i \in L} \bar{\xi}^{1/\epsilon_t} C_{ti}^{1-1/\epsilon_t} \right)^{\frac{\epsilon_t}{\epsilon_t - 1}}$$

with $\bar{\xi} \geq \underline{\xi}$ is the taste for large and small firms respectively. The case reported in the main text corresponds to $\bar{\xi} = \underline{\xi}$. A L-firm and S-firm faces two difference demand schedules:

$$C_{ti} = \bar{\xi} \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon_t} C_t, \quad i \in L$$
$$C_{ti} = \underline{\xi} \left(\frac{P_{ti}}{P_t} \right)^{-\epsilon_t} C_t, \quad i \in S$$

where the aggregate price index is given by

$$P = \left(\int_{i \in S} \underline{\xi} P_S^{1-\epsilon_t} + \int_{i \in L} \bar{\xi} P_L^{1-\epsilon_t} \right)^{\frac{1}{1-\epsilon_t}}$$

We start by solving the dynamic problem of large corporates. Their FOC is given by

$$(1-\epsilon_t)\bar{\xi}\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t}Y_t-(1-\epsilon_t)\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t}\bar{\xi}Y_t\tau_t-\tau_t\epsilon_t\frac{P_{t-1i}}{P_{ti}}\bar{\xi}Y_t\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t}$$
$$+\frac{\epsilon_t}{1-\alpha}\frac{W_t}{P_{ti}}\left[\left(\frac{P_{ti}}{P_t}\right)^{-\epsilon_t}\bar{\xi}\frac{Y_t}{A_t}\right]^{\frac{1}{1-\alpha}}-\frac{\theta}{P_{t-1i}}\left(\frac{P_{ti}}{P_{t-1i}}-1\right)\bar{\xi}P_tY_t$$
$$+EQ_t\left[\tau_{t+1}\bar{\xi}Y_{t+1}\left(\frac{P_{t+1i}}{P_{t+1}}\right)^{-\epsilon_t}+\frac{P_{t+1i}}{P_{ti}^2}\theta\left(\frac{P_{t+1i}}{P_{ti}}-1\right)\bar{\xi}P_{t+1}Y_{t+1}\right]=0$$

It is natural to focus on symmetric equilibria in which the prices of all goods produced by large firms are identical, $P_{ti}=P_t^L$ for $i\in L$. We define the real price of goods produced by large firms, $\tilde{P}_t^L=\frac{P_t^L}{P_t}$. We therefore get

$$(1-\epsilon_t)(\tilde{P}_t^L)^{-\epsilon_t}-(1-\epsilon_t)(\tilde{P}_t^L)^{-\epsilon_t}\tau_t-\tau_t\epsilon_t\frac{1}{1+\pi_t^L}(\tilde{P}_t^L)^{-\epsilon_t}$$
$$+\frac{\epsilon_t}{1-\alpha}\bar{\xi}^{\frac{\alpha}{1-\alpha}}\frac{1}{\tilde{P}_t^L}\frac{W_t}{P_t}\left[(\tilde{P}_t^L)^{-\epsilon_t}\frac{1}{A_t}\right]^{\frac{1}{1-\alpha}}Y_t^{\frac{\alpha}{1-\alpha}}-\frac{\theta}{\tilde{P}_t^L}(1+\pi_t^L)\pi_t^L$$
$$+EQ_t\left[\tau_{t+1}\frac{Y_{t+1}}{Y_t}(\tilde{P}_{t+1}^L)^{-\epsilon_t}+(1+\pi_{t+1}^L)^2\theta\pi_{t+1}^L\frac{Y_{t+1}}{\tilde{P}_{t+1}^LY_t}\right]=0$$

$$(1-\epsilon_t)(1-\tau_t)-\tau_t\epsilon_t\frac{1}{1+\pi_t^L}+\frac{\epsilon_t}{(1-\alpha)}\bar{\xi}^{\frac{\alpha}{1-\alpha}}\frac{1}{(\tilde{P}_t^L)^{1-\epsilon_t}}\frac{W_t}{P_t}\left[(\tilde{P}_t^L)^{-\epsilon_t}\frac{1}{A_t}\right]^{\frac{1}{1-\alpha}}Y_t^{\frac{\alpha}{1-\alpha}}$$
$$-\frac{\theta}{(\tilde{p}_t^L)^{1-\epsilon_t}}(1+\pi_t^L)\pi_t^L+EQ_t\left[\tau_{t+1}\frac{Y_{t+1}}{Y_t}\left[\frac{\tilde{P}_t^L}{\tilde{P}_{t+1}^L}\right]^{\epsilon_t}+(1+\pi_{t+1}^L)^2\theta\pi_{t+1}^L(\tilde{P}_t^L)^{\epsilon_t}\frac{Y_{t+1}}{\tilde{p}_{t+1}^LY_t}\right]=0$$

$$(1-\epsilon_t)(1-\tau_t)-\tau_t\epsilon_t\frac{1}{1+\pi_t^L}+\frac{\epsilon_t}{(\tilde{P}_t^L)^{1-\epsilon_t+\frac{\epsilon_t}{1-\alpha}}}\bar{\xi}^{\frac{\alpha}{1-\alpha}}MC_t$$
$$-\frac{\theta}{(\tilde{P}_t^L)^{1-\epsilon_t}}(1+\pi_t^L)\pi_t^L+EQ_t\left[\tau_{t+1}\frac{Y_{t+1}}{Y_t}\left[\frac{\tilde{P}_t^L}{\tilde{P}_{t+1}^L}\right]^{\epsilon_t}+(1+\pi_{t+1}^L)^2\theta\pi_{t+1}^L(\tilde{P}_t^L)^{\epsilon_t}\frac{Y_{t+1}}{\tilde{P}_{t+1}^LY_t}\right]=0$$

where we have defined $MC_t=\frac{W_t}{P_t(1-\alpha)}\left[\frac{1}{A_t}\right]^{\frac{1}{1-\alpha}}Y_t^{\frac{\alpha}{1-\alpha}}$ and denote π_t the inflation rate of the aggregate consumer price index and π_t^L the one corresponding to the L-firms. The next step is to log-linearize around $\tau=\pi=\pi^L=0$ and a constant output:

$$(1-\epsilon_t)-\tau_t+\frac{\epsilon_t}{(\tilde{P}^L_t)^{1-\epsilon_t+\frac{\epsilon_t}{1-\alpha}}}\bar{\xi}^{\frac{\alpha}{1-\alpha}}MC_t-(\tilde{P}^L)^{\epsilon_t-1}\theta\pi^L_t+EQ_t\left[\tau_{t+1}+\theta\pi^L_{t+1}(\tilde{P}^L)^{\epsilon_t-1}\right]=0$$

$$-\tau_t+u_t+(\epsilon_t-1)\left(\hat{mc}_t-\left(1-\epsilon_t+\frac{\epsilon_t}{1-\alpha}\right)\hat{\tilde{p}}^L_t\right)-(\tilde{P}^L)^{\epsilon_t-1}\theta\pi^L_t$$
$$+EQ_t\left[\tau_{t+1}+\theta\pi^L_{t+1}(\tilde{P}^L)^{\epsilon_t-1}\right]=0$$

with

$$\hat{mc}_t=mc_t-mc^e_t=mc_t-mc$$
$$u_t=(\epsilon_t-1)(mc^e_t-mc^n_t)$$
$$\hat{\tilde{p}}^L_t=\tilde{p}^L_t-\tilde{p}^L$$

We can derive a similar pricing equation for small untaxed firms:

$$(\epsilon_t-1)\left(\hat{mc}_t-\left(1-\epsilon_t+\frac{\epsilon_t}{1-\alpha}\right)\hat{\tilde{p}}^S_t\right)-(\tilde{P}^S)^{\epsilon_t-1}\theta\pi^S_t+EQ_t\theta\pi^S_{t+1}(\tilde{P}^S)^{\epsilon_t-1}=0$$

The next step is to compute aggregate inflation, π. We start from the expression of the aggregate price index P, then take the log and then take a first order approximation around the previous period set of prices:

$$P_t=\left((1-\mu_L)\underline{\xi}(P^S_t)^{1-\epsilon_t}+\mu_L\bar{\xi}(P^L_t)^{1-\epsilon_t}\right)^{\frac{1}{1-\epsilon_t}}$$
$$p_t=\frac{1}{1-\epsilon_t}\log\left((1-\mu_L)\underline{\xi}(P^S_t)^{1-\epsilon_t}+\mu_L\bar{\xi}(P^L_t)^{1-\epsilon_t}\right)$$
$$dp_t=\frac{(1-\mu_L)\underline{\xi}(P^L_{t-1})^{-\epsilon_t}}{\left((1-\mu_L)\underline{\xi}(P^S_{t-1})^{1-\epsilon_t}+\mu_L\bar{\xi}(P^L_{t-1})^{1-\epsilon_t}\right)}dP^S_t$$
$$+\frac{\mu_L\bar{\xi}(P^L_{t-1})^{-\epsilon_t}}{\left((1-\mu_L)\underline{\xi}(P^S_{t-1})^{1-\epsilon_t}+\mu_L\bar{\xi}(P^L_{t-1})^{1-\epsilon_t}\right)}dP^L_t$$

We then approximate this equation around a zero inflation steady-state, which gives:

$$\pi_t = \frac{1}{1+\frac{\mu_L}{1-\mu_L}\frac{\bar{\xi}}{\underline{\xi}}(\Delta)^{1-\epsilon}}\pi_t^S + \frac{1}{\frac{(1-\mu_L)}{\mu_L}\frac{\underline{\xi}}{\bar{\xi}}(1/\Delta)^{1-\epsilon}+1}\pi_t^L$$
$$\pi_t = \chi^S \pi_t^S + \chi^L \pi_t^L$$

where we have defined $\Delta = \frac{P_L}{P_S}$. We now compute the relative price in the flex-price equilibrium. The first-order condition of the L-firms is given by

$$(1-\epsilon_t)(\tilde{P}_t^L)^{-\epsilon_t} + \frac{\epsilon_t}{1-\alpha}\bar{\xi}^{\frac{\alpha}{1-\alpha}}\frac{1}{\tilde{P}_t^L}\frac{W_t}{P_t}\left[(\tilde{P}_t^L)^{-\epsilon_t}\frac{1}{A_t}\right]^{\frac{1}{1-\alpha}} Y_t^{\frac{\alpha}{1-\alpha}} = 0$$
$$(\tilde{P}_t^L)^{1-\epsilon_t} = \frac{\epsilon_t}{(\epsilon_t-1)}\bar{\xi}^{\frac{\alpha}{1-\alpha}}\frac{W_t}{P_t(1-\alpha)}\left[(\tilde{P}_t^L)^{-\epsilon_t}\frac{1}{A_t}\right]^{\frac{1}{1-\alpha}} Y_t^{\frac{\alpha}{1-\alpha}}$$
$$(\tilde{P}_t^L)^{1-\epsilon_t} = \frac{\epsilon_t}{(\epsilon_t-1)}(\tilde{P}_t^L)^{-\frac{\epsilon_t}{1-\alpha}}\bar{\xi}^{\frac{\alpha}{1-\alpha}} MC_t$$

Taking the ratio of this expression with the same one for the S-firms gives

$$\frac{P_t^L}{P_t^S} = \frac{\tilde{P}_t^L}{\tilde{P}_t^S} = \left(\frac{\bar{\xi}}{\underline{\xi}}\right)^{\frac{\alpha}{1-\alpha+\alpha\epsilon_t}}$$

The next step is to compute $\hat{\tilde{p}}_t^L$ ad $\hat{\tilde{p}}_t^S$. We start by deriving an expression of $\tilde{P}_t^L$ and $\tilde{P}_t^S$ as a function of Δ_t relative prices:

$$\tilde{P}_L = \frac{P_L}{P} = \left((1-\mu_L)\underline{\xi}\left(\frac{P_S}{P_L}\right)^{1-\epsilon_t} + \mu_L\bar{\xi}\right)^{-\frac{1}{1-\epsilon_t}}$$
$$\tilde{P}_L(\Delta) = \left((1-\mu_L)\underline{\xi}(1/\Delta)^{1-\epsilon_t} + \mu_L\bar{\xi}\right)^{-\frac{1}{1-\epsilon_t}}$$
$$\text{and similarly}\quad \tilde{P}_S = \frac{P_S}{P} = \left((1-\mu_L)\underline{\xi} + \mu_L\bar{\xi}(\Delta)^{1-\epsilon_t}\right)^{-\frac{1}{1-\epsilon_t}}$$

We next take a first order approximation of these two expressions around the steady-state value of Δ_t:

$$\begin{aligned}
\log \tilde{P}_t^L &= \tilde{p}_t^L = -\frac{1}{1-\epsilon_t} \log\left((1-\mu_L)\underline{\xi}(1/\Delta_t)^{1-\epsilon} + \mu_L \bar{\xi}\right) \\
\hat{\tilde{p}}_t^L &= (1-\mu_L)\underline{\xi}\frac{\Delta^{\epsilon-2}}{(1-\mu_L)\underline{\xi}(1/\Delta)^{1-\epsilon} + \mu_L\bar{\xi}}\hat{\Delta}_t \\
&= \frac{\Delta^{\epsilon-1}}{(1/\Delta)^{1-\epsilon} + \frac{\mu_L}{(1-\mu_L)\underline{\xi}}\bar{\xi}}\hat{\delta}_t
\end{aligned}$$

where we define $\delta = \log \Delta$ (which implies $\hat{\delta} = \frac{\hat{\Delta}}{\Delta}$).

Finally, we derive two expressions relating the change in relative prices $\hat{\delta}$ and the change in real prices $\hat{\tilde{p}}^L, \hat{\tilde{p}}^S$. First by definition we have

$$\hat{\delta}_t = \delta_t - \delta = \log \Delta_t - \log \Delta = \hat{\tilde{p}}_t^L - \hat{\tilde{p}}_t^S$$

Second, we also have:

$$\begin{aligned}
\delta_t - \delta_{t-1} &= \log \Delta_t - \log \Delta_{t-1} = \log P_t^L - \log P_t^S - \log P_{t-1}^L + \log P_{t-1}^S \\
&= \tilde{p}_t^L - \tilde{p}_t^S - \tilde{p}_{t-1}^L + \tilde{p}_{t-1}^S = \pi_t^L - \pi_t^S
\end{aligned}$$

Putting everything together we have three expressions for three variables as a function of π_t^L, π_t^S:

$$\begin{aligned}
\hat{\delta}_t &= \pi_t^L - \pi_t^S + \hat{\delta}_{t-1} \\
\hat{\tilde{p}}_t^L &= \chi_S \hat{\delta}_t \\
\hat{\tilde{p}}_t^S &= \hat{\tilde{p}}_t^L - \hat{\delta}_t \\
\chi_S &= \frac{1}{1 + \frac{\mu_L}{1-\mu_L}\frac{\bar{\xi}}{\underline{\xi}}\Delta^{1-\epsilon}}
\end{aligned}$$

Equilibrium We now derive an expression for the marginal cost. We start from the equilibrium conditions:

$$C_{ti} = Y_{ti} \quad \text{and} \quad N_t = \int_0^1 N_{ti} di$$

which imply

$$\begin{aligned}
N_t &= \int_0^1 \left(\frac{Y_{ti}}{A_t}\right)^{\frac{1}{1-\alpha}} di \\
&= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \left(\mu_L \bar{\xi}^{\frac{1}{1-\alpha}} (\tilde{P}_t^L)^{-\frac{\epsilon}{1-\alpha}} + (1-\mu_L)\underline{\xi}^{\frac{1}{1-\alpha}} (\tilde{P}_t^S)^{-\frac{\epsilon}{1-\alpha}}\right)
\end{aligned}$$

Taking the log gives

$$\begin{aligned}
(1-\alpha)n_t &= y_t - a_t + d_t \\
d_t &= (1-\alpha)\log\left(\mu_L \bar{\xi}^{\frac{1}{1-\alpha}} (\tilde{P}_t^L)^{-\frac{\epsilon}{1-\alpha}} + (1-\mu_L)\underline{\xi}^{\frac{1}{1-\alpha}} (\tilde{P}_t^S)^{-\frac{\epsilon}{1-\alpha}}\right)
\end{aligned}$$

We approximate d_t near the steady-state:

$$\begin{aligned}
\hat{d}_t &= (1-\alpha)\left(-\frac{\epsilon}{1-\alpha}\right) \times \left[\frac{\mu_L \bar{\xi}^{\frac{1}{1-\alpha}} (\tilde{P}^L)^{-\frac{\epsilon}{1-\alpha}} \hat{\tilde{p}}_t^L + (1-\mu_L)\underline{\xi}^{\frac{1}{1-\alpha}} (\tilde{P}^S)^{-\frac{\epsilon}{1-\alpha}} \hat{\tilde{p}}_t^S}{\mu_L \bar{\xi}^{\frac{1}{1-\alpha}} (\tilde{P}^L)^{-\frac{\epsilon}{1-\alpha}} + (1-\mu_L)\underline{\xi}^{\frac{1}{1-\alpha}} (\tilde{P}^S)^{-\frac{\epsilon}{1-\alpha}}}\right] \\
&= -\epsilon\left(\zeta^L \hat{\tilde{p}}_t^L + (1-\zeta^L)\hat{\tilde{p}}_t^S\right) = -\epsilon\left(\hat{\tilde{p}}_t^L - (1-\zeta^L)\hat{\delta}_t\right) = -\epsilon\left(\chi^S - (1-\zeta^L)\right)\hat{\delta}_t = \chi^d \hat{\delta}_t \\
\zeta^L &= \frac{1}{1 + \frac{(1-\mu_L)}{\mu_L}\left(\frac{\underline{\xi}}{\bar{\xi}}\right)^{\frac{1}{1-\alpha}} (\Delta)^{\frac{\epsilon}{1-\alpha}}} \\
\chi^d &= -\epsilon\left(\chi^S - (1-\zeta^L)\right)
\end{aligned}$$

We are now ready to derive an expression of the markup:

$$\begin{aligned}
mc_t &= w_t - p_t + \frac{\alpha}{1-\alpha}y_t - \frac{1}{1-\alpha}a_t - \log(1-\alpha) \\
&= \sigma c_t + \psi n_t + \frac{\alpha}{1-\alpha}y_t - \frac{1}{1-\alpha}a_t - \log(1-\alpha) \\
&= \left(\sigma + \frac{\psi}{1-\alpha}\right)y_t - \frac{\psi}{1-\alpha}a_t + \frac{\psi}{1-\alpha}d_t + \frac{\alpha}{1-\alpha}y_t - \frac{1}{1-\alpha}a_t - \log(1-\alpha)
\end{aligned}$$

The marginal cost relative to its efficient level is given by:

$$\hat{mc}_t^e = \left(\sigma + \frac{\psi + \alpha}{1-\alpha}\right)\hat{y}_t^e + \frac{\psi}{1-\alpha}\chi^d\hat{\delta}_t$$

where we used the fact that in the efficient equilibrium the relative price of L and S-firms is constant. Substituting back into the Phillips curve gives:

$$\begin{aligned}
\pi_t^L + \frac{\tau_t}{\theta\left(\tilde{P}^L\right)^{\epsilon-1}} &= \beta\left[E_t\pi_{t+1}^L + \frac{\tau_{t+1}}{\theta\left(\tilde{P}^L\right)^{\epsilon-1}}\right] + \frac{(\epsilon-1)}{\theta\left(\tilde{P}^L\right)^{\epsilon-1}}\left(\hat{mc}_t - \left(1-\epsilon+\frac{\epsilon}{1-\alpha}\right)\chi^S\hat{\delta}_t\right) \\
&\quad + \frac{1}{\theta\left(\tilde{P}^L\right)^{\epsilon-1}}u_t \\
&= \beta\left[E_t\pi_{t+1}^L + \frac{\tau_{t+1}}{\theta\left(\tilde{P}^L\right)^{\epsilon-1}}\right] + \kappa^L\hat{y}_t^e + \gamma^L\hat{\delta}_t + \frac{1}{\theta\left(\tilde{P}^L\right)^{\epsilon-1}}u_t \\
\kappa^L &= \frac{(\epsilon-1)}{\theta\left(\tilde{P}^L\right)^{\epsilon-1}}\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) \\
\gamma^L &= \frac{(\epsilon-1)}{\theta\left(\tilde{P}^L\right)^{\epsilon-1}}\left[\frac{\psi}{1-\alpha}\chi^d - \left(1-\epsilon+\frac{\epsilon}{1-\alpha}\right)\chi^S\right]
\end{aligned}$$

We have a similar result for small firms:

$$\begin{aligned}
\pi_t^S + \frac{\tau_t}{\theta\left(\tilde{P}^S\right)^{\epsilon-1}} &= \beta\left[E_t\pi_{t+1}^S + \frac{\tau_{t+1}}{\theta\left(\tilde{P}^S\right)^{\epsilon-1}}\right] + \kappa^S\hat{y}_t^e + \gamma^S\hat{\delta}_t + \frac{1}{\theta\left(\tilde{P}^S\right)^{\epsilon-1}}u_t \\
\kappa^S &= \frac{(\epsilon-1)}{\theta\left(\tilde{P}^S\right)^{\epsilon-1}}\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right) \\
\gamma^S &= \frac{(\epsilon-1)}{\theta\left(\tilde{P}^S\right)^{\epsilon-1}}\left[\frac{\psi}{1-\alpha}\chi^d + \left(1-\epsilon+\frac{\epsilon}{1-\alpha}\right)\chi^L\right]
\end{aligned}$$

Complete system of equations Defining $\theta^S = \theta\left(\tilde{P}^S\right)^{\epsilon-1}$ and $\theta^L = \theta\left(\tilde{P}^L\right)^{\epsilon-1}$ we have

$$
\begin{aligned}
\hat{y}_t^e &= E_t\hat{y}_{t+1}^e - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^e\right) \\
\pi_t^L + \frac{\tau_t}{\theta^L} &= \beta E_t\left[\pi_{t+1}^L + \frac{\tau_{t+1}}{\theta^L}\right] + \kappa^L\hat{y}_t^e + \gamma^L\hat{\delta}_t + \frac{1}{\theta^L}u_t \\
\pi_t^S &= \beta E_t\left[\pi_{t+1}^S\right] + \kappa^S\hat{y}_t^e + \gamma^S\hat{\delta}_t + \frac{1}{\theta^S}u_t \\
\hat{\delta}_t &= \pi_t^L - \pi_t^S + \hat{\delta}_{t-1} \\
\pi_t &= \chi_L\pi_t^L + \chi_S\pi_t^S \\
u_t &= \rho_u u_{t-1} + \epsilon_t^u \\
a_t &= \rho_a a_{t-1} + \epsilon_t^a \\
r_t^e &= \rho + \sigma E_t\left[\frac{u_{t+1} - u_t}{(\epsilon - 1)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)} + \frac{1+\psi}{(1-\alpha)\left(\sigma + \frac{\psi+\alpha}{1-\alpha}\right)}(a_{t+1} - a_t)\right] \\
i_t &= \rho + \phi_\pi\pi_t + \phi_y\hat{y}_t^e
\end{aligned}
$$

Calibration. Crouzet and Mehrotra (2020) report that in the U.S., the top 1% firms in terms of book assets account for 75% of total sales and 86% of total assets. For the calibration, we assume that large firms account for 75% of nominal output in the steady state. In the model, the relative sales of large vs small firms is given by

$$
\frac{P^L C^L}{P^S C^S} = \frac{\mu_L}{\mu_S}\left(\frac{\bar{\xi}}{\underline{\xi}}\right)^{\frac{1}{1-\alpha+\alpha\epsilon_t}}.
$$

To avoid having to calibrate four new parameters to calibrate, we consider the following case: all goods have the same appeal $\bar{\xi} = \underline{\xi} = 1$, which we see as realistic but firms have different sizes because some of them sell more varieties of goods than others. It is realistic to assume that large firms sell multiple products. This gives $\mu_L = 0.75$ and $\mu_S = 0.25$, and μ_L/μ_S should be interpreted as the ratio of the number of products produced by large firms relative to small firms.

E Sticky Wages

The problem of the households is given by

$$
\begin{aligned}
U(B_{t-1}, W_{t-t}(i)) &= \frac{C_{ti}^{1-\sigma}}{1-\sigma} - \frac{N_{ti}^{1+\psi}}{1-\psi} + \beta E_t U(B_t, W_{ti}) \\
C_{ti} &= \left(\int_0^1 C_{ti}^{1-1/\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}} \\
\int_0^1 P_{ti} C_{ti} di + E_t Q_t B_t &= B_{t-1} + W_{ti} N_{ti} + T_t - \mathcal{C}\left((W_{t-1i}, W_{ti}\right) - \tau_t (W_{ti} - W_{t-1i}) N_{ti} \\
N_{ti} &= \left(\frac{W_{ti}}{W_t} \right)^{-\epsilon_{Nt}} N_t
\end{aligned}
$$

with $\mathcal{C}_1 < 0, \mathcal{C}_2 > 0, \mathcal{C}_{11} > 0, \mathcal{C}_{22} > 0$. Following Rotemberg (1982), we assume that adjustment costs are quadratic

$$
\mathcal{C}\left((W_{t-1i}, W_{ti}\right) = \frac{\theta}{2} \left(\frac{W_{ti}}{W_{t-1i}} - 1 \right)^2 W_t N_t
$$

Finally the household is also subject to a no-Ponzi condition. The static first-order conditions are given by

$$
\begin{aligned}
C_{ti} &= \left(\frac{P_{ti}}{P_t} \right)^{\epsilon} C_t \\
\int_0^1 P_{ti} C_{ti} di &= P_t C_t.
\end{aligned}
$$

The Lagrangian associated with the dynamic problem is

$$
\begin{aligned}
\mathcal{L} = & \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\left(\left(\frac{W_{ti}}{W_t} \right)^{-\epsilon_{Nt}} N_t \right)^{1+\psi}}{1-\psi} + \beta E_t U'(B_t, W_{ti}) \\
& - \lambda \Big[P_t C_t + E_t Q_t B_t - B_{t-1} - W_{ti} \left(\frac{W_{ti}}{W_t} \right)^{-\epsilon_{Nt}} N_t - T_t \\
& + \frac{\theta}{2} \left(\frac{W_{ti}}{W_{t-1i}} - 1 \right)^2 W_t N_t + \tau_t (W_{ti} - W_{t-1i}) \left(\frac{W_{ti}}{W_t} \right)^{-\epsilon_{Nt}} N_t \Big]
\end{aligned}
$$

The first-order condition are given by

$$
\begin{aligned}
C_t^{-\sigma} &= \lambda P_t \\
\beta E_t U_1' &= \lambda E_t Q_t \\
U_1 &= \lambda \\
\frac{\epsilon_{Nt}}{W_{ti}}\left(\left(\frac{W_{ti}}{W_t}\right)^{-\epsilon_{Nt}} N_t\right)^{1+\psi} + \beta E_t U_2' &=
\end{aligned}
$$

$$
\lambda\Big[\left(\frac{W_{ti}}{W_t}\right)^{-\epsilon_{Nt}}\left((\epsilon_{Nt}-1)N_t(1-\tau_t)+\tau_t\frac{W_{t-1i}\epsilon_{Nt}}{W_{ti}}N_t\right)+\frac{\theta}{W_{t-1i}}\left(\frac{W_{ti}}{W_{t-1i}}-1\right)W_tN_t\Big]
$$

$$
U_2 = \lambda\Big[\frac{W_{ti}}{W_{t-1i}}\frac{\theta}{W_{t-1i}}\left(\frac{W_{ti}}{W_{t-1i}}-1\right)W_tN_t+\tau_t\left(\frac{W_{ti}}{W_t}\right)^{-\epsilon_{Nt}}N_t\Big]
$$

Assuming symmetry, the last two equations are given by

$$
\begin{aligned}
\frac{\epsilon_{Nt}}{W_{ti}}N_t^{1+\psi}+\beta E_t U_2' &= \lambda\left[(\epsilon_{Nt}-1)N_t(1-\tau_t)+\frac{\tau_t}{1+\pi_t^w}\epsilon_{Nt}N_t+\theta(1+\pi_t^w)\pi_t^w N_t\right] \\
U_2 &= \lambda\left[(1+\pi_t^w)^2\pi_t^w\theta N_t+\tau_t N_t\right].
\end{aligned}
$$

Putting everything together gives

$$
\begin{aligned}
\frac{\epsilon_{Nt}}{W_{ti}}N_t^{1+\psi}+\beta E_t\frac{C_{t+1}^{-\sigma}}{P_{t+1}}\left[(1+\pi_{t+1}^w)^2\pi_{t+1}^w\theta N_{t+1}+\tau_{t+1}N_{t+1}\right] &= \\
\frac{C_t^{-\sigma}}{P_t}\left[(\epsilon_{Nt}-1)N_t(1-\tau_t)+\frac{\tau_t}{1+\pi_t^w}\epsilon_{Nt}N_t+\theta(1+\pi_t^w)\pi_t^w N_t\right] & \\
\frac{\epsilon_{Nt}}{W_{ti}}N_t^{\psi}+\beta E_t\frac{C_{t+1}^{-\sigma}}{P_{t+1}}\frac{N_{t+1}}{N_t}\left[(1+\pi_{t+1}^w)^2\pi_{t+1}^w\theta+\tau_{t+1}N_{t+1}\right] &= \\
\frac{C_t^{-\sigma}}{P_t}\left[(\epsilon_{Nt}-1)(1-\tau_t)+\frac{\tau_t}{1+\pi_t^w}\epsilon_{Nt}+\theta(1+\pi_t^w)\pi_t^w\right] &
\end{aligned}
$$

The next step is to log-linearize this expression around constant consumption, constant labor supply and elasticity of substitution across labor types, zero inflation and zero tax on inflation.

$$\frac{\epsilon_{Nt}}{(\epsilon_{Nt}-1)W_{ti}}N_t^{\psi}+\beta E_t\frac{C^{-\sigma}}{P(\epsilon_N-1)}\left[\pi_{t+1}^{w}\theta+\tau_{t+1}\right]=\frac{C_t^{-\sigma}}{P_t}+\frac{C^{-\sigma}}{P(\epsilon_N-1)}\left[\tau_t+\theta\pi_t^{w}\right]$$

$$\begin{aligned}\frac{\epsilon_N}{(\epsilon_N-1)W(i)}N^{\psi}(\psi\hat{n}_t-\hat{w}_t+\hat{\mu}_t^{w})+\beta E_t\frac{C^{-\sigma}}{P(\epsilon_N-1)}\left[\pi_{t+1}^{w}\theta+\tau_{t+1}\right]=\\ \frac{C^{-\sigma}}{P}(-\sigma\hat{c}_t-\hat{p}_t)+\left[\tau_t+\theta\pi_t^{w}\right]\frac{C^{-\sigma}}{P(\epsilon_N-1)}\end{aligned}$$

$$\begin{aligned}\frac{\epsilon_N}{(\epsilon_N-1)W(i)}N^{\psi}(\psi\hat{n}_t-\hat{\omega}_t+\hat{\mu}_t^{w}+\sigma\hat{c}_t)+\beta E_t\frac{C^{-\sigma}}{P(\epsilon_N-1)}\left[\pi_{t+1}^{w}\theta+\tau_{t+1}\right]=\\ \left[\tau_t+\theta\pi_t^{w}\right]\frac{C^{-\sigma}}{P(\epsilon_N-1)}\\ \frac{\epsilon_N-1}{\theta}(\psi\hat{n}_t+\sigma\hat{c}_t-\hat{\omega}_t)+\frac{u_t^{w}}{\theta}+\beta E_t\pi_{t+1}^{w}+\frac{1}{\theta}\left[\beta\tau_{t+1}-\tau_t\right]=\pi_t^{w}\\ \frac{\epsilon_N-1}{\theta}(\psi\hat{n}_t^{e}+\sigma\hat{c}_t^{e}-\hat{\omega}_t^{e})+\frac{u_t^{w}}{\theta}+\beta E_t\pi_{t+1}^{w}+\frac{1}{\theta}\left[\beta\tau_{t+1}-\tau_t\right]=\pi_t^{w}\end{aligned}$$

where we defined the log real wage $\omega_t = w_t - p_t$, the real wage gap $\hat{\omega}_t^e = \omega_t - \omega_t^e$ and we used the fact in the flex price long-term equilibrium we have:

$$\frac{\epsilon_N}{(\epsilon_N-1)W(i)}\frac{PN^{\psi}}{C^{-\sigma}}=1$$

and we defined the wage markup shock $u_t^w = (\epsilon_N - 1)\hat{\mu}_t^w$ (fourth line) and that the fact that in the efficient equilibrium the marginal rate of substitution between consumption and leisure remains constant, $\psi n_t^e + \sigma c_t^e - \omega_t^e = \psi n + \sigma c - \omega$ (last line).

The next step is to use the aggregate production function

$$(1-\alpha)n_t = y_t - a_t \Rightarrow (1-\alpha)\hat{n}_t^e = \hat{y}_t^e$$

which gives

$$\frac{\epsilon_N - 1}{\theta}\left(\left(\frac{\psi}{1-\alpha} + \sigma\right)\hat{y}_t^e - \hat{\omega}_t^e\right) + \frac{u_t^w}{\theta} + \beta E_t \pi_{t+1}^w + \frac{1}{\theta}\left[\beta\tau_{t+1} - \tau_t\right] = \pi_t^w$$

We next derive the price Phillips curve. In this framework, it is still true that:

$$\pi_t = \frac{\epsilon - 1}{\theta}\hat{mc}_t + \beta E_t \pi_{t+1} + \frac{1}{\theta}\left[\beta E_t \tau_{t+1} - \tau_t + u_t\right]$$
$$\text{and} \quad mc_t = w_t - p_t + \frac{\alpha}{1-\alpha}y_t - \frac{1}{1-\alpha}a_t - \log(1-\alpha)$$

Combining the two gives:

$$\pi_t = \frac{\epsilon - 1}{\theta}\left(\hat{\omega}_t^e + \frac{\alpha}{1-\alpha}\hat{y}_t^e\right) + \frac{u_t - \tau_t}{\theta} + \beta E_t\left[\frac{\tau_{t+1}}{\theta} + \pi_{t+1}\right]$$

Putting the price and the wage Phillips curves together,

$$\begin{aligned}
\pi_t &= \frac{\epsilon - 1}{\theta}\left(\hat{\omega}_t^e + \frac{\alpha}{1-\alpha}\hat{y}_t^e\right) + \frac{u_t - \tau_t}{\theta} + \beta E_t\left[\frac{\tau_{t+1}}{\theta} + \pi_{t+1}\right] \\
\pi_t^w &= \frac{\epsilon_{Nt} - 1}{\theta}\left(\left(\frac{\psi}{1-\alpha} + \sigma\right)\hat{y}_t^e - \hat{\omega}_t^e\right) + \frac{u_t^w - \tau_t^w}{\theta} + \beta E_t\left[\frac{\tau_{t+1}^w}{\theta} + \pi_{t+1}^w\right] \\
\hat{\omega}_t^e &= \hat{\omega}_{t-1}^e + \pi_t^w - \pi_t - \Delta\omega_t^e \\
\Delta\omega_t^e &= \psi_{ayn}\Delta a_t \\
\psi_{ayn} &= \frac{1 - \alpha\psi_{ya}}{1-\alpha} \\
\psi_{ya} &= \frac{1+\psi}{\sigma(1-\alpha) + \psi + \alpha}
\end{aligned}$$

Finally we also get the following Euler equation:

$$\begin{aligned}
Q_t &= E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right] \\
\hat{y}_t^e &= E_t\hat{y}_{t+1}^e - \frac{1}{\sigma}\left(\rho + \phi_\pi\pi_t + \phi_y\hat{y}_t^e - E_t\pi_{t+1} - r_t^e\right)
\end{aligned}$$

where r_t^e is defined as in the previous sections.

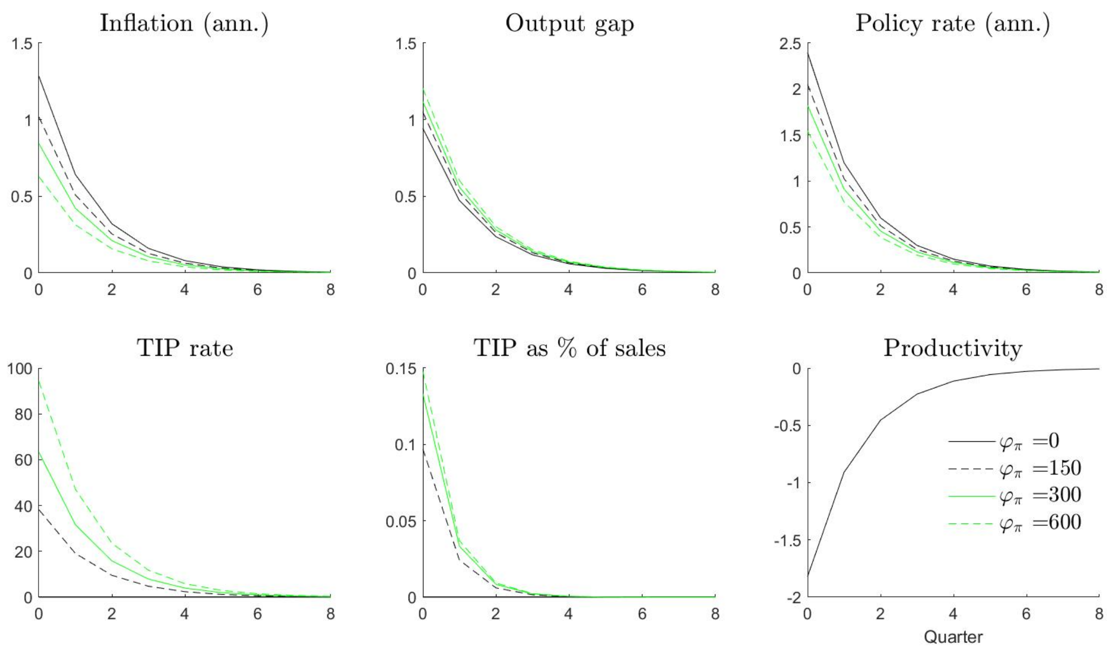
Complete set of equations

$$\pi_t = \frac{\epsilon - 1}{\theta}\left(\hat{\omega}_t^e + \frac{\alpha}{1-\alpha}\hat{y}_t^e\right) + \frac{u_t - \tau_t}{\theta} + \beta E_t\left[\frac{\tau_{t+1}}{\theta} + \pi_{t+1}\right]$$
$$\omega_t = w_t - p_t$$
$$\hat{\omega}_t^e = \omega_t - \omega_t^e$$
$$\pi_t^w = \frac{\epsilon_N - 1}{\theta}\left(\left(\frac{\psi}{1-\alpha} + \sigma\right)\hat{y}_t^e - \hat{\omega}_t^e\right) + \frac{u_t^w - \tau_t^w}{\theta} + \beta E_t\left[\frac{\tau_{t+1}^w}{\theta} + \pi_{t+1}^w\right]$$
$$\hat{\omega}_t^e = \hat{\omega}_{t-1}^e + \pi_t^w - \pi_t - \Delta\omega_t^e$$
$$\hat{y}_t^e = E_t\hat{y}_{t+1}^e - \frac{1}{\sigma}\left(\rho + \phi_\pi \pi_t + \phi_y \hat{y}_t^e - E_t\pi_{t+1} - r_t^e\right)$$

Calibration To match the slope of the Phillips and the wage Phillips curves in Galí (2015), one calculates θ and θ_w in the following way, where $\bar{\phi}$ is the corresponding Calvo parameter. Other parameters are the same as in Galí (2015).

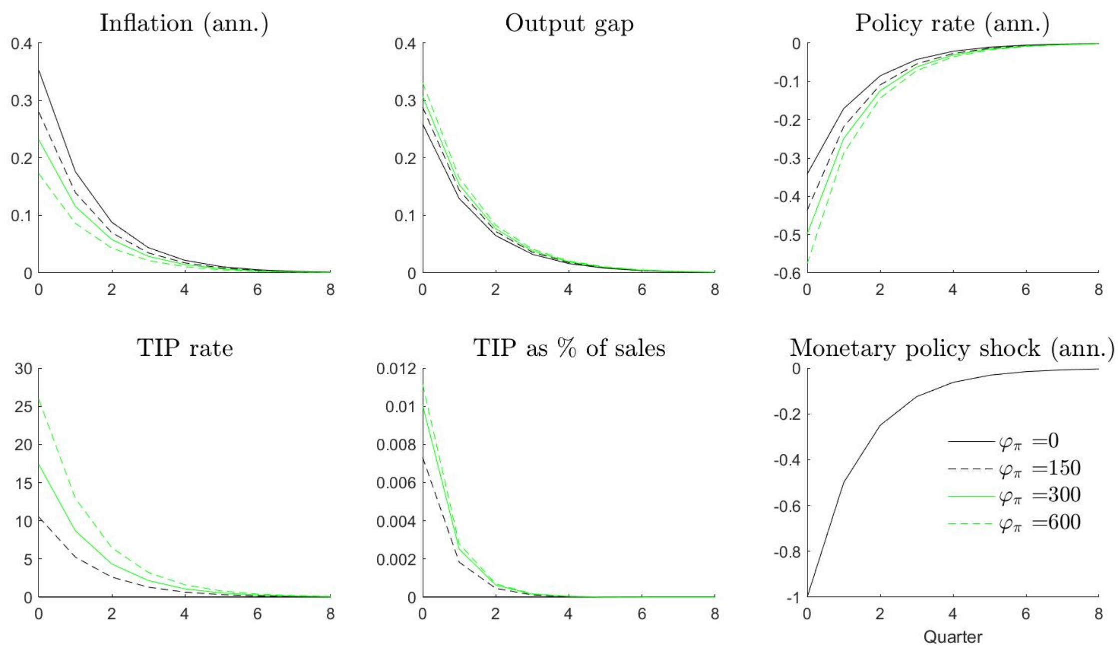
$$\theta = (\epsilon - 1)\frac{\bar{\phi}}{(1-\bar{\phi})(1-\bar{\phi}\beta)}\frac{1-\alpha+\alpha\epsilon}{1-\alpha}$$
$$\theta_w = (\epsilon_N - 1)\frac{\bar{\phi}}{(1-\bar{\phi})(1-\bar{\phi}\beta)}(1+\psi\epsilon_N)$$

F Additional Graphs - Simulations



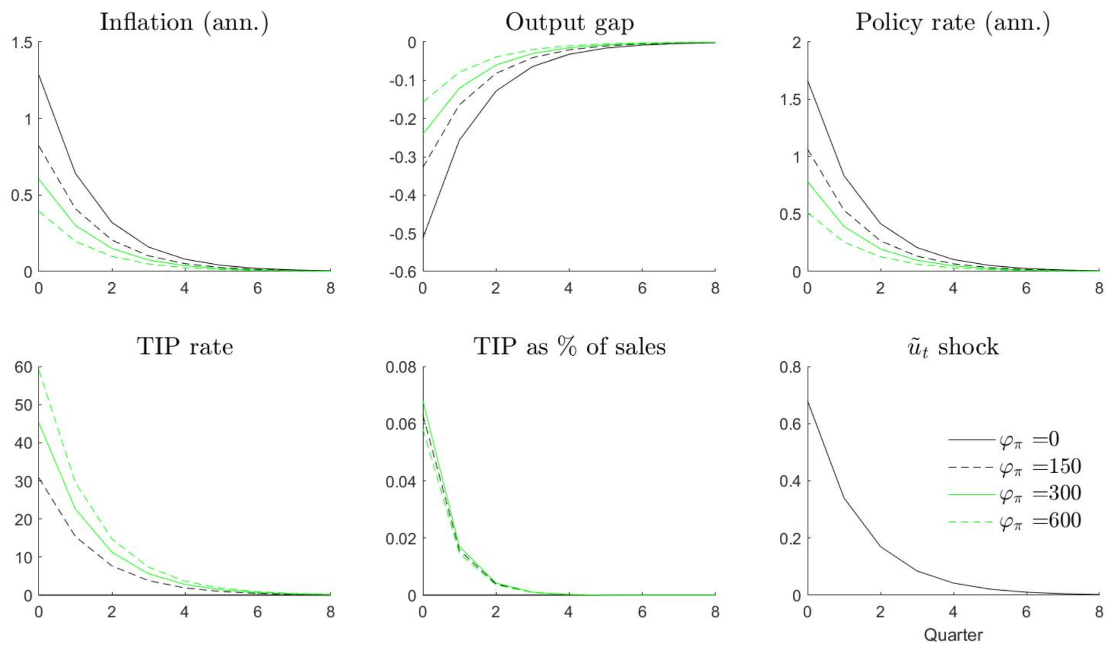
Notes: The initial productivity shock is such that the response of inflation in the baseline without TIP is the same as in the case of a markup and inflation expectation shock.

Figure F1: Effects of TIP following a negative productivity shock



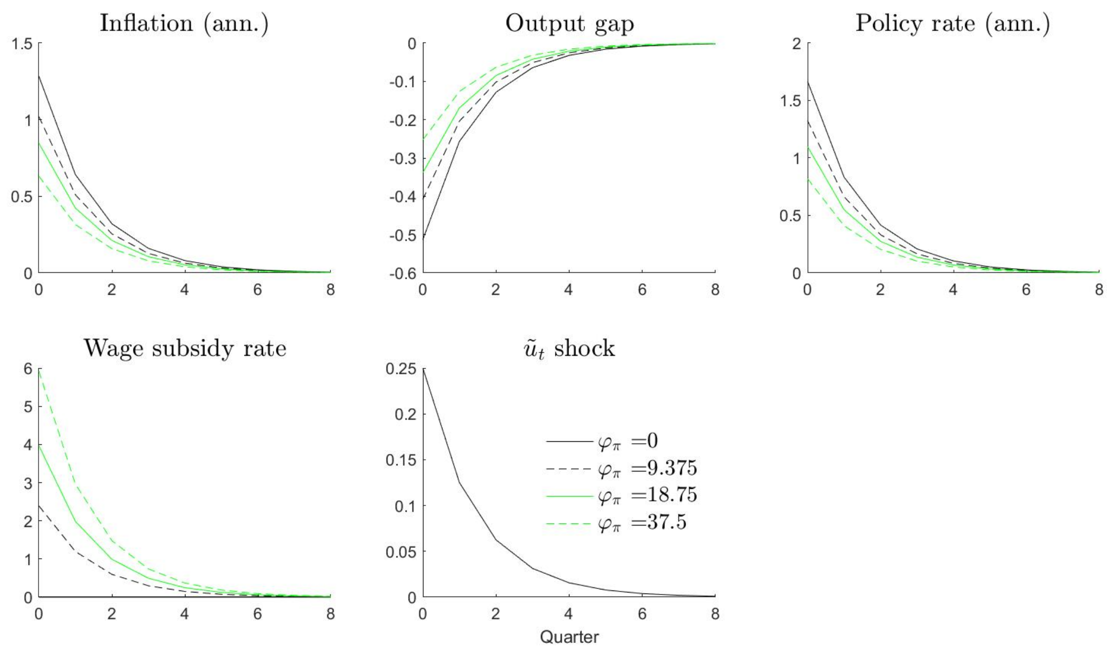
Notes: The initial monetary policy shocks is 0.25p.p. (or 1p.p. annualized).

Figure F2: Effects of TIP following a monetary policy shock



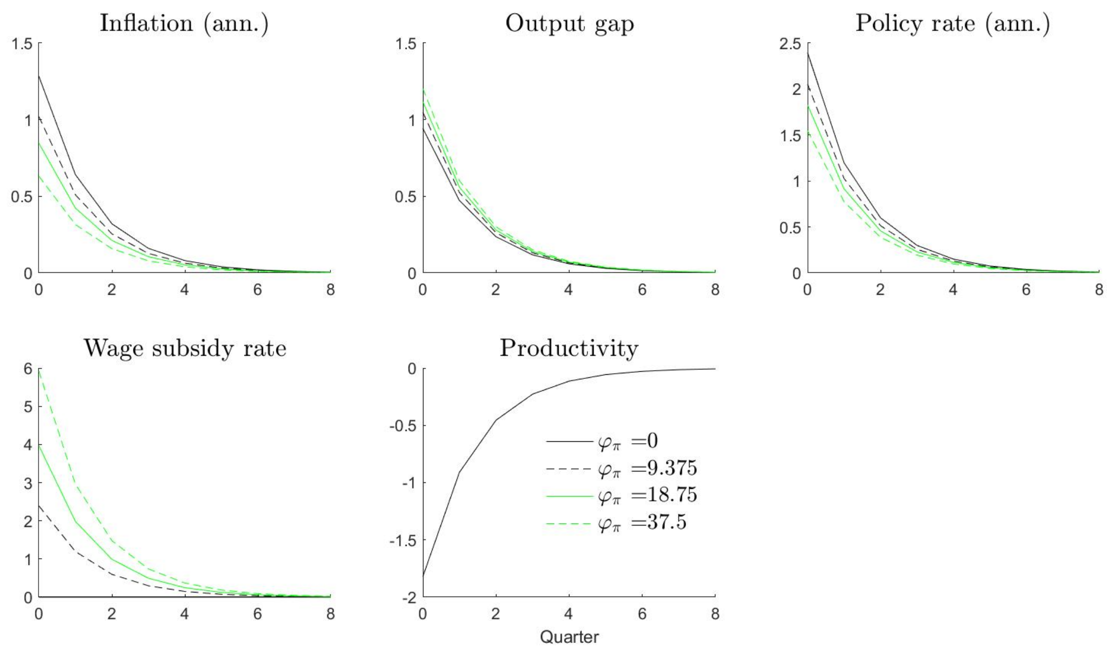
Notes: This figure corresponds to an economy with smaller adjustment costs. We set the Rotemberg parameter such that the equivalent Calvo parameter is equal to 0.5 instead of 0.75.

Figure F3: Effects of TIP following a markup and inflation expectation shock with lower price stickiness



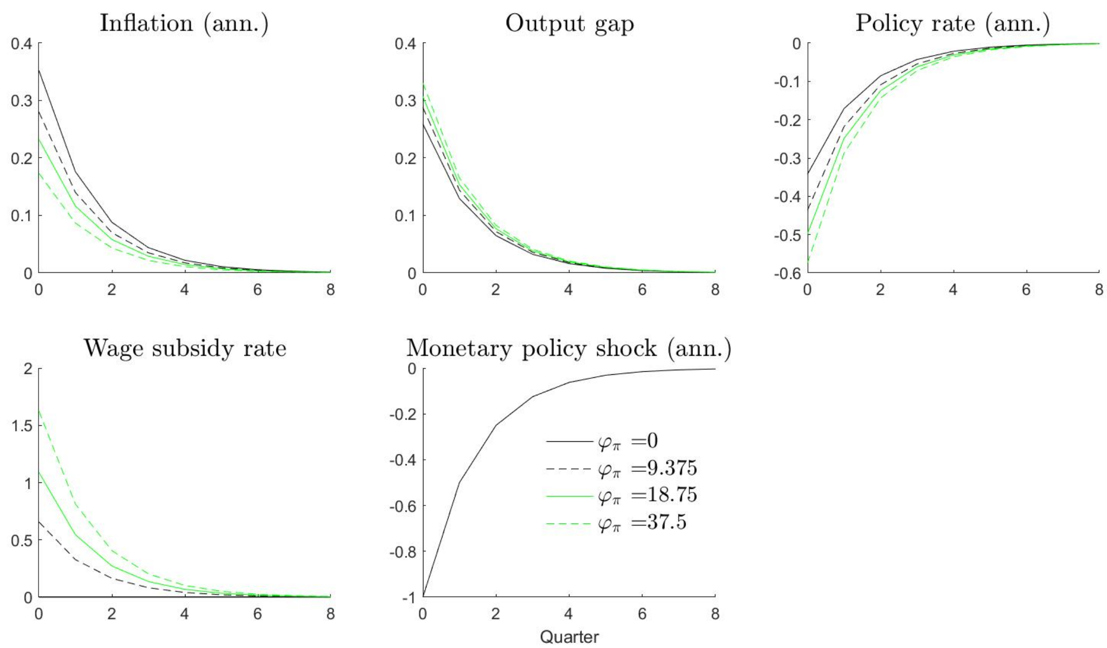
Notes: This figure corresponds to an economy with time-varying wage subsidies instead of TIP. The shock is the same as in Figure 1.

Figure F4: Effects of wage subsidies following a markup or inflation expectation shock



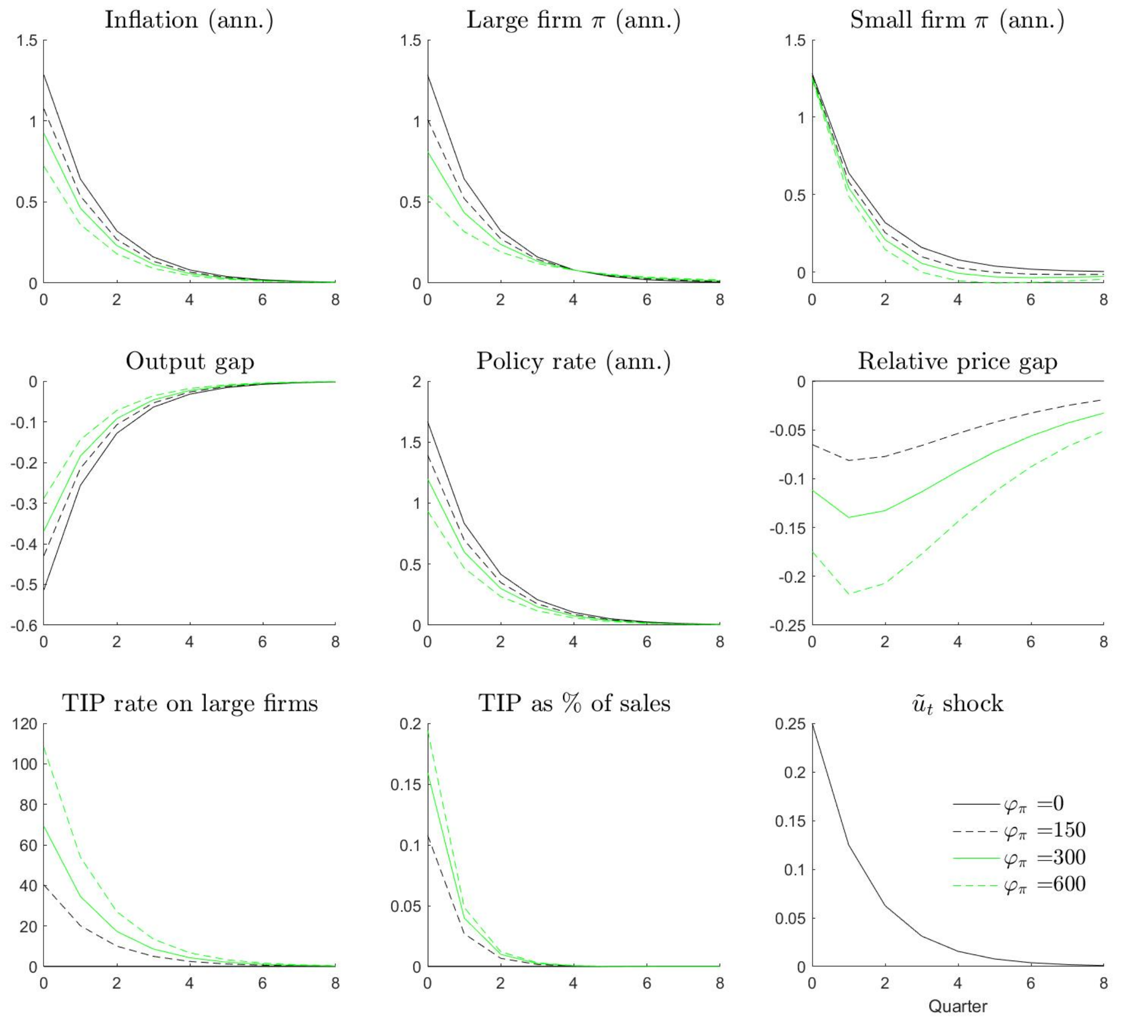
Notes: This figure corresponds to an economy with time-varying wage subsidies instead of TIP. The shock is the same as in Figure F1.

Figure F5: Effects of wage subsidies following a productivity shock



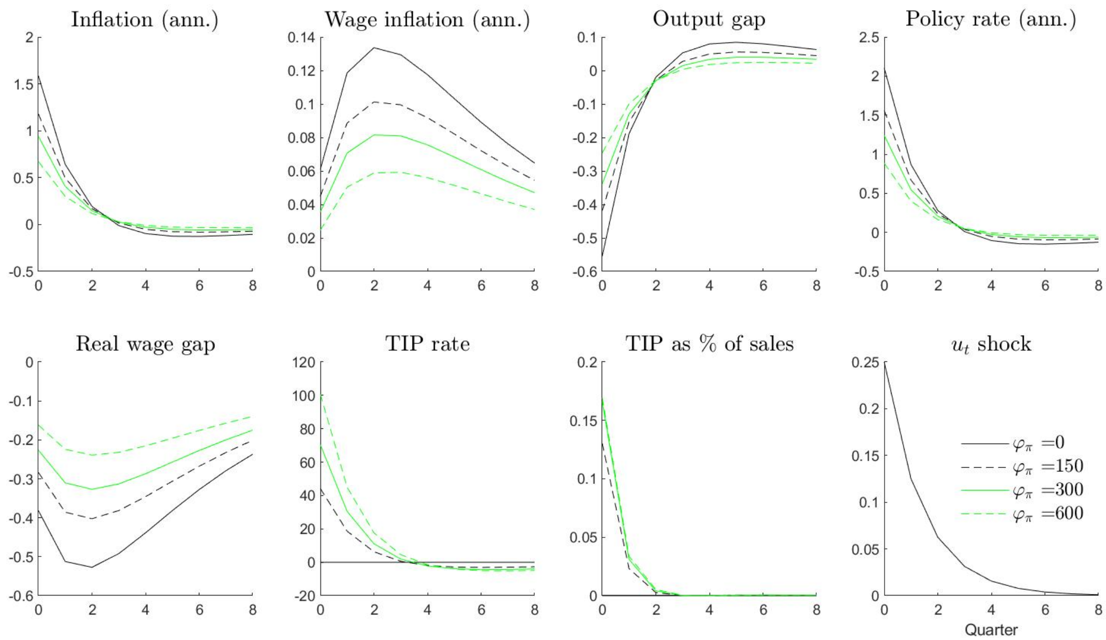
Notes: This figure corresponds to an economy with time-varying wage subsidies instead of TIP. The shock is the same as in Figure **??**.

Figure F6: Effects of wage subsidies following a monetary policy shock

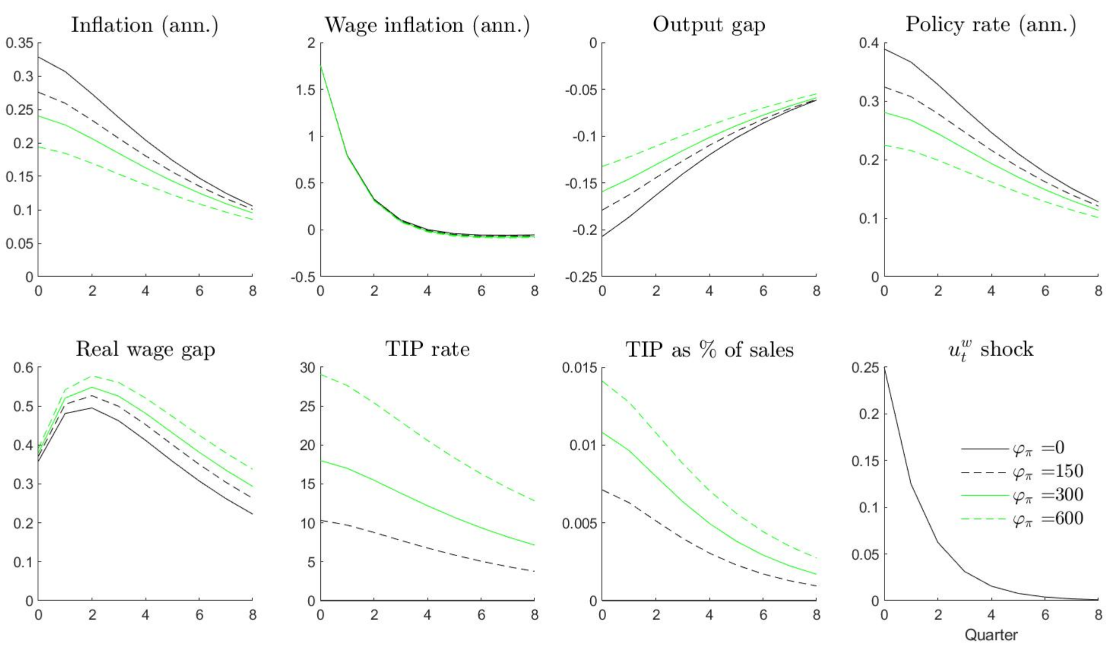


Notes: The initial markup and inflation expectation shock is 0.25pp. The shock affects all firms. TIP targets overall inflation but only applies to large firms.

Figure F7: Effects of TIP on large firms under a markup and inflation expectation shock



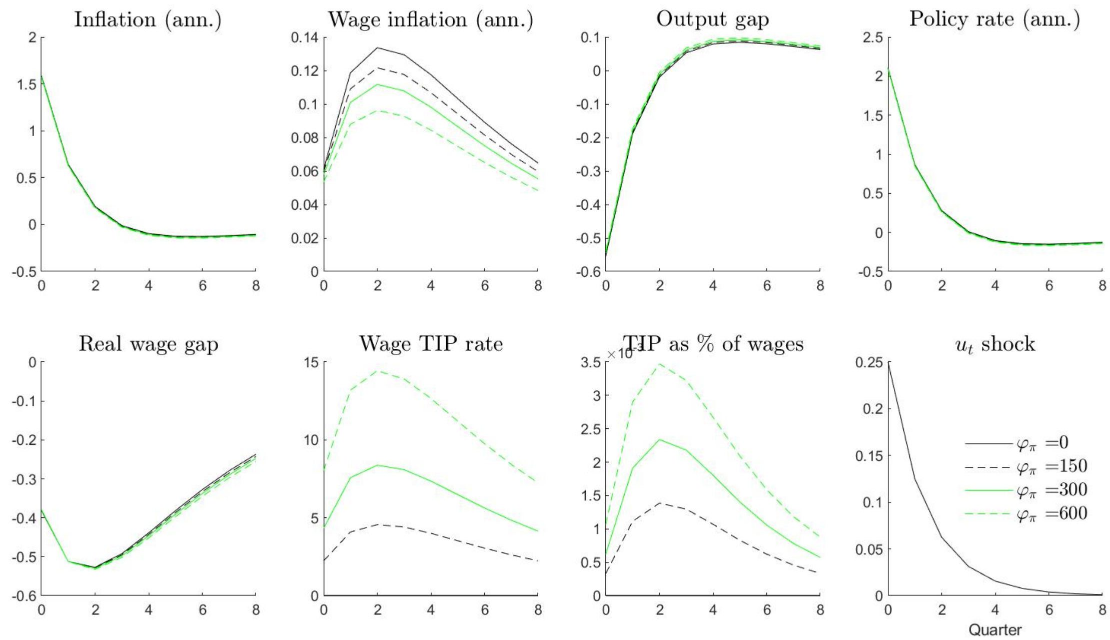
Price markup and price inflation expectation shocks



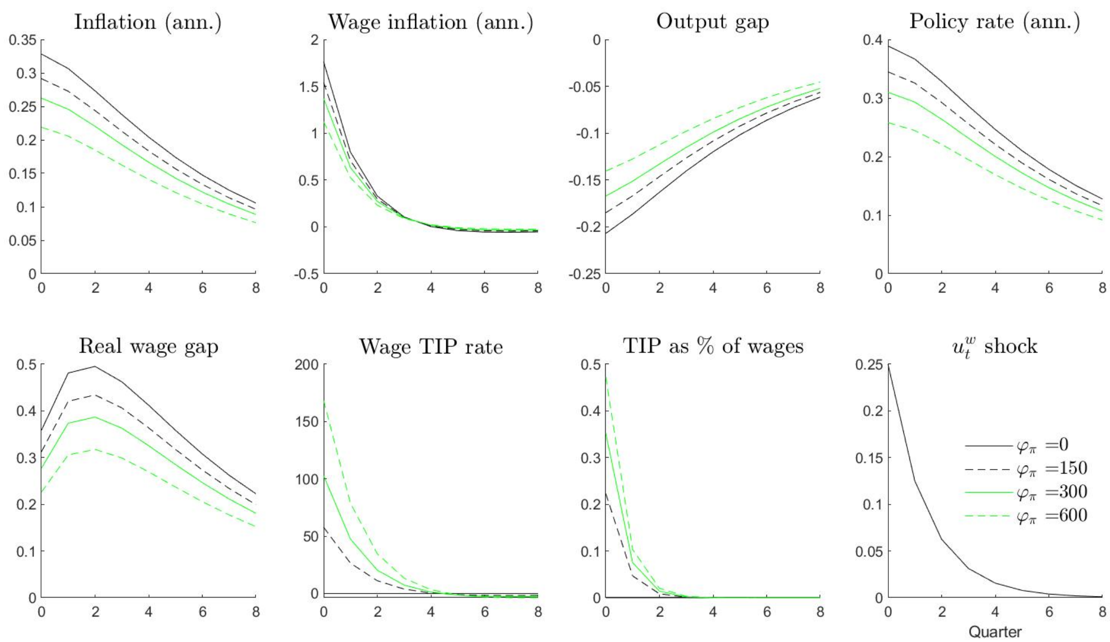
Wage markup and wage inflation expectation shock

*Notes:*Both the initial price and wage markup and inflation expectation shocks are 0.25pp. TIP targets price inflation.

Figure F8: Effects of TIP in a sticky wage model



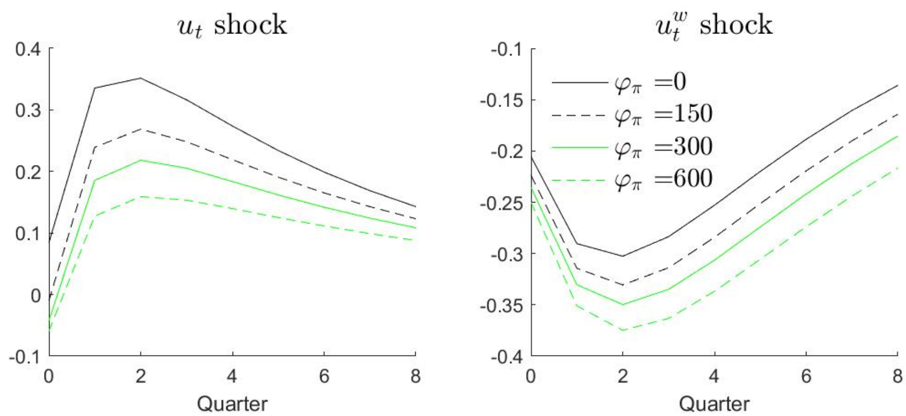
Price markup and price inflation expectation shock



Wage markup and wage inflation expectation shock

Notes: Both the price and wage markup and inflation expectation shocks are 0.25pp. Wage TIP targets wage inflation.

Figure F9: Effects of wage TIP in a sticky wage model



Notes: Changes in firms' profit share corresponding to the two shocks in Figure F8. TIP targets price inflation.

Figure F10: Effects of TIP on profits in a sticky wage model

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ_π	1.5	1.5	1.5	1.5
ϕ_y	0.125	0.125	0.125	0.125
$\sigma(\pi_t^{ann})$	0.74	0.59	0.49	0.36
$\sigma(\hat{y}_t^e)$	1.09	1.21	1.29	1.39
$\sigma(i_t^{ann})$	1.38	1.18	1.05	0.89
$\sigma(\tau_t)$	0	44.1	73.0	108.8
$\mathrm{E}(\pi_t\tau_t)$	0	0.13	0.18	0.20
$\mathcal{L}^* \times 10^4$	2.00	1.47	1.19	0.90

Notes: Productivity shocks only. The setting is the same as in Figure F1.

Table F1: Evaluation of policy rules under productivity shocks

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ_π	1.5	1.5	1.5	1.5
ϕ_y	0.125	0.125	0.125	0.125
$\sigma(\pi_t^{ann})$	0.20	0.16	0.13	0.10
$\sigma(\hat{y}_t^e)$	0.30	0.33	0.35	0.38
$\sigma(i_t^{ann})$	0.20	0.25	0.29	0.33
$\sigma(\tau_t)$	0	12.1	20.1	29.9
$\mathrm{E}(\pi_t\tau_t)$	0	0.01	0.01	0.01
$\mathcal{L}^* \times 10^4$	0.08	0.08	0.09	0.10

Notes: Monetary policy shocks only. The setting is the same as in Figure F2.

Table F2: Evaluation of policy rules under monetary policy shocks

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ^*_π	1.500	0.987	0.960	0.930
ϕ^*_y	0.125	0.155	0.380	0.460
$\sigma(\pi_t^{ann})$	1.49	1.39	1.16	0.83
$\sigma(\hat{y}_t^e)$	1.49	1.39	1.16	0.83
$\sigma(i_t^{ann})$	1.96	1.24	0.94	0.68
$\sigma(\tau_t)$	0	104.5	173.8	249.4
$\mathrm{E}(\pi_t\tau_t)$	0	0.73	1.01	1.04
$\mathcal{L}^* \times 10^4$	5.05	3.04	1.97	1.03

Notes: Demand shocks and markup shocks are independently distributed. Markup shocks have higher variance (0.5^2), while demand shocks have lower variance (0.25^2). Monetary policy (ϕ^*_π, ϕ^*_y) is determined by minimizing $\mathcal{L}^*$ in equation (33) conditional on TIP. All standard deviations and $E(\pi_t\tau_t)$ have been multiplied by 100.

Table F3: Optimal MP with Higher Markup Shocks Variance

	No TIP	Moderate TIP	Strong TIP	Extreme TIP
φ_π	0	150	300	600
ϕ^*_π	1.500	0.991	0.992	0.992
ϕ^*_y	0.125	0.105	0.076	0.050
$\sigma(\pi_t^{ann})$	0.84	0.78	0.64	0.46
$\sigma(\hat{y}_t^e)$	0.84	0.84	0.89	0.96
$\sigma(i_t^{ann})$	1.23	0.83	0.68	0.50
$\sigma(\tau_t)$	0	58.7	95.3	138.8
$\mathrm{E}(\pi_t\tau_t)$	0	0.23	0.30	0.32
$\mathcal{L}^* \times 10^4$	1.83	1.16	0.81	0.49

Notes: Demand shocks and markup shocks are independently distributed. Demand shocks have higher variance (0.5^2), while markup shocks have lower variance (0.25^2). Monetary policy (ϕ^*_π, ϕ^*_y) is determined by minimizing $\mathcal{L}^*$ in equation (33) conditional on TIP. All standard deviations and $E(\pi_t\tau_t)$ have been multiplied by 100.

Table F4: Optimal MP with Higher Demand Shocks Variance

Statistics ×100	π_0^{ann}	$\hat{y}_0^e$	$E[\theta_s \pi_{s,0}^2]$	$\left(\sigma + \frac{\alpha+\psi}{1-\alpha}\right)(\hat{y}_0^e)^2$	$\frac{E\left[\left(\hat{p}_{s,0}\right)^2\right]}{1-\alpha}$	$\mathcal{L}_0$
A. Heterogeneous sectors: $\theta_s \in \{20.4,\ 62.5,\ 689.7\}$						
1. One-period markup shock when $\bar{\tau} = 0\%$						
Steady-state	0.00	-1.58	0.44	0.20	0.28	0.92
$\tau_0 = 0\%$	2.75	-2.02	1.52	0.33	0.29	2.14
$\tau_0 = 10\%$	1.05	-1.89	0.55	0.28	0.29	1.12
$\tau_0 = 15\%$	0.18	-1.66	0.39	0.22	0.29	.90
2. One-period markup shock when $\bar{\tau} = 15\%$						
Steady-state	0.00	-1.58	0.47	0.20	0.28	0.94
$\tau_0 = \bar{\tau}$	2.87	-2.00	1.66	0.32	0.29	2.27
$\tau_0 = 25\%$	1.10	-1.89	0.59	0.29	0.28	1.16
$\tau_0 = 30\%$	0.23	-1.67	0.42	0.22	0.28	.93
B. Homogeneous sectors: $\theta = 372.8$						
1. One-period markup shock when $\bar{\tau} = 0\%$						
Steady-state	0.00	-1.65	0.21	0.22	0.39	0.81
$\tau_0 = 0\%$	0.87	-1.87	0.35	0.28	0.39	1.02
$\tau_0 = 10\%$	0.34	-1.76	0.20	0.25	0.39	0.84
$\tau_0 = 15\%$	0.03	-1.67	0.17	0.22	0.39	0.79
2. One-period markup shock when $\bar{\tau} = 15\%$						
Steady-state	0.00	-1.64	0.21	0.22	0.39	0.81
$\tau_0 = \bar{\tau}$	0.89	-1.87	0.36	0.28	0.39	1.03
$\tau_0 = 25\%$	0.33	-1.76	0.20	0.25	0.39	0.84
$\tau_0 = 30\%$	0.02	-1.66	0.17	0.22	0.39	0.78

Notes: In panels A and B, the economy is hit by an unexpected markup shock in the initial period. TIP is used in the initial period and returns to its steady-state value $\bar{\tau}$ after the shock. $\mathcal{L}$ is defined in equation 35. In panels A.1 and B.1, we set the steady-state level of TIP to 0, and in panels A.2 and B.2. we set it equal to 15%. All columns are multiplied by 100. The figures on the lines "Steady-state" correspond to steady-state values. All other figures correspond to the initial period $t = 0$.

Table F5: Effects of TIP on price distortion

References

Blanchard, O. J. and Kahn, C. M. (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48(5):1305–1311.

Crouzet, N. and Mehrotra, N. R. (2020). Small and Large Firms over the Business Cycle. *American Economic Review*, 110(11):3549–3601.

Galí, J. (2015). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications Second edition*. Number 10495 in Economics Books. Princeton University Press.

Rotemberg, J. J. (1982). Sticky Prices in the United States. *Journal of Political Economy*, 90(6):1187–1211.

PUBLICATIONS

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