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WORKING PAPER

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Abstract

Would the introduction of a Central Bank Digital Currency (CBDC) lead to lower deposits (disintermediation) and lending in the banking sector? This paper develops a model where households heterogeneous in wealth allocate between an illiquid asset and assets that can be used for payments: bank deposits, cash, and CBDC. CBDC is more efficient as a means of payment and has lower access cost than deposits. Deposits are offered by an imperfectly competitive banking sector which raises deposit interest rates after CBDC introduction to prevent substitution away from deposits to CBDC. We find that there are two opposing margins of impact on the level of aggregate deposits: (1) the *intensive margin* gain in deposits by richer households increasing their holdings of deposits because of higher interest rates, and (2) the *extensive margin* loss of deposits among poorer households who switch from deposits to the CBDC. The extensive margin loss in deposits is more likely to dominate (yielding a fall in aggregate deposits) when the mass of poorer households is large and when it is relatively costly to access bank accounts. This tends to be the case in developing and emerging market economies. However, even when the extensive margin loss of deposits dominates and there is disintermediation, the impact on lending is quantitatively small if banks have access to other forms of funding, such as wholesale or central bank financing.

JEL Classification: E50, E58, G21

Keywords: CBDC; banking disintermediation; financial inclusion; monetary policy

[*] The views expressed in this paper are those of the authors and therefore do not necessarily reflect those of the IMF or the ECB. This work was performed when Huifeng Chang was an IMF intern. This work has benefited from insightful comments by David Andolfatto, Chris Erceg, Tommaso Mancini-Griffoli, Rob Townsend, Pierre-Olivier Weill, and seminar participants at Banco Central do Brasil, Banco de Mexico, Banca d'Italia, the European Central Bank, the Federal Reserve Board, the IMF, and Riksbank.

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WORKING PAPERS

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Contents

1 Introduction

Recent years have seen a surge in the use of new kinds of privately issued digital money.[1] In response, more and more central banks have initiated work on exploring the issuance of their own digital money, denominated central bank digital currency (CBDC).[2]

While different central banks pursue different objectives with CBDC[3], a common key concern policy makers have is the whether the introduction of CBDC could lead to bank disintermediation (Bindseil, 2020; Mancini-Griffoli et al., 2018). Disintermediation would be a result of CBDC crowding out commercial bank deposits. Deposits are a cheap and stable source of funding for banks, so if CBDC becomes successful in substituting bank deposits as a payment instrument, this could have a negative impact on banks' overall funding, and thus, their ability to lend.

In this paper we present a standard portfolio choice model with banks, in the spirit of Monti (1972); Klein (1971) and Drechsler et al. (2017), to analyze whether CBDC can actually generate bank disintermediation and, if so, how big this effect may be. In the model, households choose how to allocate their wealth between an illiquid asset and three imperfectly substitutable liquid assets: cash, bank deposits and CBDC. Households' utility depends on their final wealth and on services provided

[1] For example, payment systems provided by mobile network operators, new payment system providers, and stablecoins. See Adrian and Mancini-Griffoli (2019) for a detailed discussion.

[2] For example, China, Canada, Sweden, The Bahamas, and European Union. See Soderberg et al. (2022) for a discussion on these central banks' policy objectives for considering a CBDC.

[3] Central banks have indicated that CBDC may provide a more efficient, secure, and modern central bank money available to everyone, and that it can also increase resilience, availability, efficiency and contestability of retail payments, as well as broaden financial inclusion.

by the liquid assets. Deposits are offered by banks who then can invest funds in bonds that pay a fixed return or lend to firms. Banks have market power in deposits, which allows them to charge a positive spread between the return on bonds and the deposit rate.

We measure the impact of CBDC on bank intermediation by comparing the size of the bank deposit base and the size of bank lending before and after the introduction of CBDC. In the model, CBDC is simply a new and imperfect substitute for the other two liquid assets: deposits and cash. When it is costless to access the liquid assets, households would like to use all three of them, as households derive utility from variety. We show analytically that in this case the introduction of CBDC does not lead to bank disintermediation. The reason is that CBDC reduces banks' market power, to which banks optimally respond by increasing the rate of return on deposits. Thus, households choose to hold even more of bank deposits and the aggregate deposit base increases. We call this effect the *intensive margin* of CBDC introduction, and it is always positive. At the same time, due to the lost market power the net effect of CBDC introduction on bank profits is negative.[4]

However, there can be barriers and costs to access financial assets. While getting cash usually has no costs for retail users[5], gaining access to deposits can be

[4]In a related model of Andolfatto (2021) the introduction of CBDC also has a non-negative impact on aggregate bank deposits. Similarly to our setup, this happens because competition from CBDC makes banks offer higher rates of return on deposits. However, in Andolfatto (2021) agents cannot hold positive amounts of cash, CBDC and deposits at the same time, but strategically choose only one means of payment. As a result, also the extensive margin (see below) has always a positive impact on the size of the bank deposit base, which is not the case in our model. Allowing deposits, cash and CBDC to be imperfect substitutes allows us to study the consequences of CBDC introduction for a more general set of household preferences.

[5]Although this is most certainly the case for retail users and small amounts of cash, storage

cumbersome and costly.[6].

We introduce household heterogeneity in wealth in the model together with fixed costs of holding bank deposits and CBDC. When access costs to CBDC are lower than those of bank deposits, the introduction of CBDC can generate bank disintermediation. This happens when (i) CBDC is more liquid than deposits and cash and (ii) the high cost of access to bank deposits leads poorer households to abandon deposits and to use CBDC and cash only, even with higher deposit rates. We call this the *extensive margin* of CBDC. If this effect is large enough, the extensive margin can more than offset the *intensive margin*.

We calibrate the model with heterogeneous households to US data and solve it numerically. We find that aggregate bank deposits fall following CBDC introduction when it is easy to access CBDC and when the mass of poorer households is large. In this case, more households find deposits fixed costs too high and switch completely to CBDC. Banks do not aggressively increase deposit rates to prevent the outflow of customers due to the relatively small wealth held by the poor households. This leads to an aggregate decrease in bank deposits of 4 percent.

To analyze the impact of CBDC on bank lending, we extend the model to firms that demand credit from banks. We find that, even when CBDC generates deposit disintermediation, the negative effect on lending is quantitative small and below 0.2 percent. Banks' access to other forms of funding, such as wholesale or central bank

costs can be non-negligible when they involve larger amounts. Cash also pays a lower real return compared to bank deposits (as long as interest rates on deposit are positive).

[6]For example, banks may require customers to provide a proof of residence and employment in order to open an account. Some banks may also charge fixed fees for setting up an account

financing, allows banks to compensate the decline in deposits without having to reduce lending too much. On the one hand, when these alternative funding sources are relatively cheap, it is easy for banks to substitute away from deposits.[7] On the other hand, when alternative forms of funding are expensive, banks fight for deposits more aggressively, further increasing deposit rates, and thus reducing their loss of deposits.[8]

Overall, our results show the importance of taking into account households characteristics, market structure, and banks' strategic responses when assessing the impact of CBDC on the banking system. Policymakers aiming to examine resiliency of bank lending to the introduction of potentially very attractive means of payment should take into account the mechanisms that we unveil. Our findings point also to the need for quality data on households' preferences over means of payment to estimate the demand for liquid assets.

Related Literature Our paper contributes to the emerging literature on CBDCs. Our main focus is on the effects that CBDC introduction can have on the deposit base of commercial banks and on bank lending. Other papers that have also considered this question include Andolfatto (2021), Chiu et al. (2023) and Agur et al. (2022).

In Andolfatto (2021), banking sector is also monopolistic, but CBDC is a perfect substitute for currency and bank deposits. Thus, agents choose to hold only one

[7] The model ignores the effect that the changes in the funding structure may have on regulatory ratios or, more generally, on financial stability.

[8] An important caveat is that our model is static, so the compression of bank profits and capital erosion does not affect lending. This is clearly a channel that can have an impact on lending in a dynamic setting. See, for instance, (Van den Heuvel et al., 2002).

means of payment and banks always match the rate paid on deposits with the return on CBDC. Additionally, costs of accessing bank deposits and CBDC are the same. As a result, while there is an extensive margin of CBDC introduction similar to our setting, Andolfatto (2021) always has a *positive* impact on bank deposits leading to *intermediation* of bank deposits.

Chiu et al. (2023) consider a model calibrated to the US economy where cash and deposits are used for different transactions and where a remunerated CBDC is a perfect substitute for bank deposits only. They show that if banks have market power in the deposit market, CBDC can enhance competition, raising the deposit rate and expanding intermediation.

In our model CBDC can increase bank deposit base by reducing the market power of banks — as in Andolfatto (2021) and Chiu et al. (2023). Our model also nests key mechanisms of these two papers while our richer set up unveils that CBDC's effect on deposits depends on parametrization of households preferences, wealth distribution, and the costs of accessing CBDC relative to bank deposits. In contrast to these two papers, we model CBDC as an imperfect substitute for both cash and bank deposits in a simple portfolio choice model. Thus, CBDC is held regardless of whether it pays an interest or not.[9,10] Different from Chiu et al. (2023) and as in Andolfatto (2021), in our model households are heterogeneous in wealth which leads to an intensive and extensive margin of deposit holding. Different from Andolfatto (2021), we show that

[9]This is relevant as most central banks are not considering paying an interest rate on a retail CBDC.

[10]In both Chiu et al. (2023) and in Andolfatto (2021) CBDC increases competition in the deposit market by serving as an outside option to depositors and setting the interest rate on deposits and CBDC is not held in equilibrium.

the extensive margin can be stronger and lead to deposit disintermediation and lower bank lending if both the cost of setting a CBDC account is smaller than setting up a bank deposit account and if there is larger mass of poorer households.

Agur et al. (2022) consider a setup where households choose the means of payments depending on their preferences over the level of anonymity and security of transactions. While cash offers most anonymity, bank deposits provide most security. Similarly to our model, variety in payment instruments increases welfare, but this is because of the heterogeneity in household preferences. Contrary to our setup, Agur et al. (2022) do not consider the role of banks market power in the deposits. There, banks are modeled as price-takers in both deposit and loan markets. The implications of CBDC introduction crucially depend on how close it resembles cash or deposits: a cash-like CBDC can reduce the demand for cash beyond the point where network effects cause the disappearance of cash, while a deposit-like CBDC can cause an increase in deposit and loan rates, and a contraction in bank lending to firms. The optimal design of CBDC involves a trade-off between loss of utility from variety when CBDC crowds out cash and loss of bank intermediation in the presence of severe lending frictions.

Other related papers on macroeconomic implications of CBDC, include Barrdear and Kumhof (2022),Burlon et al. (2022), Keister and Sanches (2023), Brunnermeier and Niepelt (2019), Williamson (2019), Piazzesi and Schneider (2022), Garratt et al. (2021), Wang and Hu (2022), Gross and Letizia (2023), Whited et al. (2023), and Li et al. (2023). In particular, Keister and Sanches (2023) show that by choosing a proper interest on CBDC, policymakers can ensure that CBDC introduction never decreases welfare. Barrdear and Kumhof (2022) introduce CBDC in a DSGE model with competitive but regulated banking sector. They find that CBDC always spurs economic activity, lowers the policy and deposit rates and increases bank lending. Burlon et al. (2022) also utilize a DSGE model where CBDC, cash, and deposits are, as in our model, imperfect substitutes for the representative household. They find that CBDC exerts a smoothing effect as lending and real GDP by stabilizing deposit holds when there are shocks to liquidity services. Garratt et al. (2021) consider a model with banks that have heterogenous market shares, and analyze how an interest-bearing CBDC can affect concentration in the banking system. They find that the impact crucially depends on the design of CBDC. Wang and Hu (2022) study the link between CBDC and financial development. They argue that in less financially developed economies, retail CBDCs can be useful for promoting financial inclusion, while in countries with high levels of financial development, CBDC can enhance financial stability by substituting out more risky non-bank e-money. Gross and Letizia (2023) develop an agent-based model where households have random utility over CBDC, cash, and deposits and calibrated to the US economy that generates more or less disintermediation depending on whether CBDC is designed to be more to deposits or to cash. Whited et al. (2023) and Li et al. (2023) estimate potential demand of CBDC using household data and structural choice models. While the approach of these papers is very different than ours, their baseline results for deposits and lending are in line with ours.[11]

Our paper also belongs to the vast literature studying implications of imperfect

[11] For example, Whited et al. (2023) find that the impact of CBDC on lending is much smaller than on deposits as banks have the option to use wholesale funding.

competition in banking system (Drechsler et al., 2017, Repullo et al., 2020). In particular, we build on at model developed Drechsler et al. (2017) to study the deposit channel of monetary policy. The model contains two features that make it suitable for our purposes: i) imperfect substitution between liquid assets as a means of payment, and ii) imperfectly competitive banking system. Finally, our work builds on models that distinguish between the extensive and intensive margins of adjustment as in Hopenhayn (1992) and Melitz (2003).

The rest of the paper is organized as follows. Section 2 introduces the baseline model with homogeneous households and no fixed costs for holding bank deposits and CBDC. Section 3 discusses the enriched model with heterogeneous households and fixed costs of holding deposits and CBDC. Section 4 adds lending and wholesale funding for banks to the model. Section 5 concludes.

2 Baseline model with homogeneous households

This section introduces the baseline model with homogeneous households. The purpose is to introduce some of the mechanisms that are at play in the larger model with heterogeneous agents. We show that when households are homogeneous in wealth, the introduction of CBDC will always lead to an increase in total bank deposits.

2.1 Setup

We consider a portfolio choice model with an imperfectly competitive banking sector. There are three types of agents in the model: households, banks, and a central bank.

Households Households are homogenous and have initial wealth of W_0, which they allocate among four types of assets: (i) notes (cash), denoted by N, earns no return; (ii) CBDC, denoted by C, earns return $r_C \geq 0$; (iii) deposits, denoted by D, earn r_D; and (iv) bonds, earn a non-negative rate f. The bonds are risk-free, and f is the risk-free rate set by the central bank. Bonds are also "illiquid" as they are not useful as means of payment. Cash, CBDC, and deposits can instead be used for payments, creating liquidity services value in households' utility function.

Households' utility is a function of final wealth W and liquidity services L:

$$U(W_0) = \max \left(W^{\frac{\rho-1}{\rho}} + \lambda L^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \tag{1}$$

where wealth and liquidity are complements, with the elasticity of substitution $\rho < 1$.[12] Liquidity services arising from holding cash, CBDC and deposits are defined by:

$$L(N, C, D) = (N^{\frac{\epsilon-1}{\epsilon}} + \delta_C C^{\frac{\epsilon-1}{\epsilon}} + \delta_D D^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}. \tag{2}$$

The three liquid assets are imperfect substitutes for households, hence the elasticity of substitution is greater than one, $\epsilon > 1$. δ_C and δ_D represent the relative usefulness

[12] We abstract from adjustment cost in non liquid wealth. If these costs were taken into account, impacts would be more muted. See Kaplan et al. (2018) for an example of an illiquid assets portfolio choice model in general equilibrium.

of CBDC and deposits as means of payments compared to cash.

Households face the following budget constraint:

$$W = W_0(1+f) - Nf - C(f - r_C) - D(f - r_D), \tag{3}$$

rearranged to highlight the opportunity costs of holding the liquid assets with respect to bonds. As cash earns no return, households face an opportunity cost of f, the return on bonds, when holding cash. The opportunity costs of holding CBDC and deposits are lower than for cash, as they guarantee non-negative returns r_C and r_D, respectively.

Banks Aggregate deposits (D) are a composite good produced by a set of J banks (D_j), indexed by $j \in \{1, 2..., J\}$:

$$D = \left(\frac{1}{J} \sum_{j=1}^{J} D_j^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{4}$$

where $\eta > 1$ is the elasticity of substitution between deposits of different banks. It is greater than one, reflecting imperfect substitutability between bank deposits.

To focus on the effect that CBDC has on the deposit market, for now we assume that banks are fully funded by deposits and can only invest in bonds. These assumptions are relaxed in Section 4. As deposits are imperfect substitutes, banks have market power and set the return on deposits $r_{D,j}$ with the objective of maximizing their profits, $(f - r_{D,j})D_j$, subject to deposits demand. The return on aggregate deposits is defined by the weighted average of each bank's rate of return, i.e. $r_D = \frac{1}{J}\sum_{j=1}^{J}\frac{D_j}{D}r_{D,j}$.

Central bank The central bank chooses the risk-free rate f, i.e. remuneration on bonds, and the interest rate on CBDC, r_C. It also supplies bonds and CBDC with an infinite elasticity.

2.2 Equilibrium

The behaviour of households is characterized by four first-order conditions. First, households choose between liquid assets and bonds according to:

$$\frac{L}{W} = \lambda^{\rho} s_L^{-\rho}, \tag{5}$$

where $s_L \equiv (f^{1-\epsilon} + \delta_D^{\epsilon}(s^*)^{1-\epsilon} + \delta_C^{\epsilon}(f - r_C)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$ is the foregone interest of holding liquid assets. A higher forgone interest decreases the share of wealth kept in liquid assets.[13]

Second, households choose between liquid assets according to the two first-order conditions

$$\frac{C}{N} = {\delta_C}^{\epsilon}\left(\frac{f - r_C}{f}\right)^{-\epsilon}, \tag{6}$$

$$\frac{C}{D} = \left(\frac{\delta_C}{\delta_D}\right)^{\epsilon}\left(\frac{f - r_C}{f - r_D}\right)^{-\epsilon}. \tag{7}$$

[13] Note that the households' budget constraint can be also rewritten as $W = W_0(1 + f) - Ls_L$.

It follows that households will want to hold more CBDC if it is more useful as means of payments relative to other liquid assets, and if it earns a higher return.

Third, households choose between deposits of different banks according to:

$$\frac{D_j}{D} = \left(\frac{f - r_{D,j}}{f - r_D}\right)^{-\eta}. \tag{8}$$

Banks' have market power, hence they can remunerate deposits below the central bank's risk-free rate: the spread with respect to the rate f, $f - r_{D,j}$, is positive. The first-order condition for banks is given by:

$$\frac{\partial D_j}{\partial(f - r_{D,j})} \frac{(f - r_{D,j})}{D_j} = -1. \tag{9}$$

We can use the interbank margin (8) to calculate the elasticity in (9). Following Drechsler et al. (2017), we focus on the symmetric equilibrium with $D_j = D$ where it is given by:

$$\frac{\partial D_j}{\partial(f - r_{D,j})} \frac{(f - r_{D,j})}{D_j} = \frac{1}{J}\left(\frac{\partial D}{\partial(f - r_D)} \frac{(f - r_D)}{D}\right) - \eta\left(1 - \frac{1}{J}\right). \tag{10}$$

Substituting (10) into (9), we get the equilibrium condition.

$$\frac{1}{J}\left(\frac{\partial D}{\partial(f - r_D)} \frac{(f - r_D)}{D}\right) - \eta\left(1 - \frac{1}{J}\right) = -1. \tag{11}$$

It follows from (11) that when banks are at an interior optimum, the elasticity of aggregate deposit demand with respect to the spread $(f - r_D)$ is equal to:

$$-\frac{\partial D}{\partial(f - r_D)}\frac{(f - r_D)}{D} = 1 - (\eta - 1)(J - 1) = \mathcal{M}. \tag{12}$$

The elasticity of demand with respect to the spread decreases in the level of competition in the deposit market. In turn, the competitiveness of the deposit market increases with the number banks J, and with higher substitutability of deposits across banks, η.

A closed-form solution to the model can be obtained for the limit case in which $\lambda \to 0$. In this case, following proposition 1 in Drechsler et al. (2017), if $\epsilon > \mathcal{M} > \rho$, the deposit remuneration and aggregate deposits are given by:

$$f - {r_D}^* = \delta_D^{\frac{\epsilon}{\epsilon-1}} \left[\frac{\mathcal{M} - \rho}{\epsilon - \mathcal{M}}\right]^{\frac{1}{\epsilon-1}} \left[f^{1-\epsilon} + \delta_C^{\epsilon}(f - r_C)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} > 0, \tag{13}$$

and

$$D^* = \delta_D^{\frac{\epsilon(1-\rho)}{1-\epsilon}} (f - {r_D}^*)^{-\rho} \left[1 + \delta_D^{-\epsilon} \left(\frac{f - {r_D}^*}{f}\right)^{\epsilon-1} + \left(\frac{\delta_D}{\delta_C}\right)^{-\epsilon} \left(\frac{f - {r_D}^*}{f - r_C}\right)^{\epsilon-1}\right]^{\frac{\rho-\epsilon}{\epsilon-1}}. \tag{14}$$

If $\mathcal{M} < \rho$, then ${r_D}^* = f$.

Throughout the analysis we focus on the case when the return on deposits is strictly less than the policy rate f, hence we impose that $\epsilon > \mathcal{M} > \rho$. This holds as ϵ is greater than one (as liquid assets are substitutes) and $\mathcal{M}$ has an upper bound of one (when the representative bank acts as a monopolist). We focus on the empirically relevant case in which banks have market power, so r_D^* is lower than f, so $\mathcal{M}$ is larger than ρ.

As in Drechsler et al. (2017) the equilibrium spread $s^* \equiv f - r_D{}^*$ is *non-decreasing* and the amount of deposits D^* is non-increasing in the policy rate f, giving a rise to a "bank deposit channel":

$$\frac{\partial s^*}{\partial f} \geq 0 \tag{15}$$

$$\frac{\partial D^*}{\partial f} \leq 0. \tag{16}$$

In equilibrium, a higher rate f increases the opportunity cost of using cash or CBDC instead of deposits to service liquidity needs, allowing banks to increase the rate paid on bank deposits, but by not as much as f, hence the spread s^* increases. In response to the higher opportunity cost of holding deposits, the total supply of deposits by households declines because it is now more profitable to save through bonds than through deposits.

2.3 Impact of CBDC introduction

In this section we analyze the impact of the introduction of CBDC on the equilibrium deposit return and the amount of deposits. For simplicity, we will present the results in terms of the deposit spread, that is, the spread between the policy rate and the bank deposit rate, $s^* \equiv f - r_D{}^*$. Here, we focus in the case where $\lambda \to 0$ to provide intuition with a closed-form solution. In following sections, we solve the model quantitatively without taking lambda to this limit.

Equilibrium deposit interest rate and deposit base with and without CBDC. In the absence of CBDC, proxied by setting $\delta_C = 0$, the equilibrium deposit spread and aggregate amount of deposits simplify to:

$$\tilde{s}^* = \delta_D^{\frac{\epsilon}{\epsilon-1}} \left[\frac{\mathcal{M}-\rho}{\epsilon-\mathcal{M}}\right]^{\frac{1}{\epsilon-1}} \times f, \tag{17}$$

and

$$\tilde{D}^* = \delta_D^{\frac{\epsilon(1-\rho)}{1-\epsilon}} (\tilde{s}^*)^{-\rho} \left[1 + \delta_D^{-\epsilon} \left(\frac{\tilde{s}^*}{f}\right)^{\epsilon-1}\right]^{\frac{\rho-\epsilon}{\epsilon-1}}. \tag{18}$$

Comparing equations (13)-(14) and (17)-(18), the introduction of CBDC has two opposite effects. First, as long as CBDC is not a perfect substitute for deposits and cash ($\epsilon > 1$), its introduction will induce households to diversify their liquidity basket, reducing but not fully eliminating the demand for cash and deposits. At the same time, however, it will also force banks to reduce the deposit spread, that is increase remuneration of deposits, in order to keep households from substituting deposits with CBDC. This will increase households' demand for deposits.

Notwithstanding the two opposite effects, the deposit spread will always *decline* in equilibrium following introduction of CBDC, or equivalently that the return on deposits will increase, and that the aggregate deposits will always increase in equilibrium, as presented in Proposition 1.

Proposition 1 *Take $\lambda \to 0$. The equilibrium spread on deposits always declines, and the equilibrium level of deposits always increases, after CBDC is introduced (with*

$\delta_C > 0$*):*

$$s^* - \tilde{s}^* = \Delta s < 0,$$

$$D^* - \tilde{D}^* = \Delta D > 0.$$

Proof. Using (14) and (17), the ratio

$$\frac{s^*}{\tilde{s}^*} = \frac{\left[f^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}{f} = \left[1 + \delta_C^\epsilon \left(\frac{f - r_C}{f}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} < 1$$

since $\epsilon > 1$ and the quantity inside the square bracket is greater than one.

Define $s_l \equiv (f^{1-\epsilon} + \delta_D^\epsilon (s^*)^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$ such that aggregate deposits (14) can be rewritten as $D^* = \delta_D^\epsilon \left(\frac{s^*}{s_l}\right)^{-\epsilon} s_l^{-\rho}$. Then, using (13) and doing some algebraic manipulation, on obtains $\frac{s^*}{s_l} = \delta_D^{\frac{\epsilon}{\epsilon-1}} \left[\frac{\epsilon-\rho}{\epsilon-\mathcal{M}}\right]^{\frac{1}{\epsilon-1}}$. Then define equivalently $s_l(\tilde{s}^*) \equiv (f^{1-\epsilon} + \delta_D^\epsilon (\tilde{s}^*)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$. Using (17) and doing some algebraic manipulation one obtains $\frac{\tilde{s}^*}{s_l(\tilde{s}^*)} = \delta_D^{\frac{\epsilon}{\epsilon-1}} \left[\frac{\epsilon-\rho}{\epsilon-\mathcal{M}}\right]^{\frac{1}{\epsilon-1}}$. Hence $\frac{s^*}{s_l(s^*)} = \frac{\tilde{s}^*}{s_l(\tilde{s}^*)}$. Taking the ratio of aggregate deposits with and without CBDC

$$\frac{D^*}{\tilde{D}^*} = \left[\frac{s_l(s^*)}{s_l(\tilde{s}^*)}\right]^{-\rho} = \left[\frac{s^*}{\tilde{s}^*}\right]^{-\rho} > 1$$

where the first equality comes after algebraic manipulation, the second from the result previously obtained, and the inequality since $0 < \rho < 1$ and $\frac{s^*}{\tilde{s}^*} < 1$. $\square$

The intuition for the preceding proposition is the following. Since CBDC is a substitute for deposits, banks decrease the deposit spread in order to fight the competition from CBDC. As for deposits, one can rewrite aggregate deposits in the limit case of $\lambda \to 0$ as: $D^* = \delta_D^\epsilon \left(\frac{s^*}{s_l}\right)^{-\epsilon} s_l^{-\rho}$, where $s_l \equiv (f^{1-\epsilon} + \delta_D^\epsilon (s^*)^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$ represents the opportunity cost of holding liquid assets. Thus, the ratio $\left(\frac{s^*}{s_l}\right)^{-\epsilon}$ represents the substitution forces between the cost of deposits and overall liquidity, while $s_l^{-\rho}$ represents the demand for liquidity, which is decreasing in the opportunity cost of liquidity. When CBDC is introduced, the opportunity cost of liquidity, s_l, decreases. Directly, because there is a new liquid asset (*love for variety* effect), and indirectly because CBDC competition induces banks to decrease the deposits spread, s^*. But banks adjust the spread so that households are exactly indifferent between holding one more unit of CBDC and one more unit of deposits and do not substitute away from deposits (equivalently, $\frac{s^*}{s_l(s^*)} = \frac{\tilde{s}^*}{s_l(\tilde{s}^*)}$)[14]. As a result of these two effects, the increase in liquid asset demand increases the demand for deposits by the households and the aggregate amount of bank deposits in equilibrium.

The simple model points to one key implication: introducing CBDC generates a reaction from banks to keep their deposit base, namely increasing remuneration of deposits.[15] As in other works (for instance Andolfatto (2021)), this leads actually to an increase in aggregate deposits. Key for the result is that household have non-liquid assets, so that they can increase holdings of deposits together with holding CBDC. The model is however silent on another key aspect of CBDC introduction: some depositors might close their deposit account altogether just to hold CBDC. This aspect will be captured in the enriched model presented in Section 3.

[14]Note that this is not true when $\lambda \nrightarrow 0$. However the change in the ratio is very small.

[15]Appendix A shows analytical results for changes in the policy rate, CBDC remuneration, and bank market power.

3 Heterogeneous households and the extensive margin of deposit disintermediation

So far, we have assumed that households do not differ in the amount of initial wealth they hold. As a result, the only way through which the introduction of CBDC was altering households' portfolio allocation decisions was through higher or lower holdings of different assets by the representative household.

However, many view CBDC as a means to bolster financial inclusion, particularly in countries where banking penetration is low and where cash no longer offers a viable alternative. Importantly, if CBDC introduction increased financial inclusion—understood as access to and use of formal financial services—its effect on bank intermediation through this channel would be ambiguous. On the one hand, if banks increase the return offered on deposits in response to competition from CBDC, in addition to households who already have bank account to increase their deposits, as shown in Section 2, some previously unbanked households could decide to open bank accounts. Both effect would push the total amount of deposits further up. On the other hand, if setting up a CBDC account is considerably cheaper than opening a bank account, this could encourage poorer households to switch from deposits to CBDC entirely. Thus, to enrich our analysis and to capture these potentially important effects, in this section we introduce two additional features to the model: i) heterogeneity in the initial household wealth, and ii) fixed costs of holding both CBDC and deposits.[16]

[16]Introducing fixed costs alone in the model would not be sufficient to create an extensive margin

The solution of the model will be now characterized by equilibrium wealth thresholds under which households will not hold CBDC and/or deposits. Thus, changes in aggregate deposit holdings will be driven by changes in how much deposits households hold conditional on having deposits at all (*intensive margin*) and how many households hold deposits (*extensive margin*).

3.1 Model setup

We assume that households' initial wealth W_0 has now a Pareto (Type I) distribution with the shape parameter α. The probability density function is given by $f(W_0) = \frac{\alpha \underline{W_0}^{\alpha}}{W_0^{\alpha+1}}$, where $\underline{W_0}$ is the lowest possible wealth level. For simplicity and without loss of generality we set $\underline{W_0} = 1$.

Households also need to pay a fixed cost (ϕ^D) to hold deposits and a fixed cost (ϕ^C) to hold CBDC. These costs are measured in terms of utility to simplify the model solution. The introduction of $\phi^D > 0$ and $\phi^C > 0$ allows us to capture pecuniary and non-pecuniary frictions that households face when accessing payment instruments. In addition, we assume that the cost of holding deposits is higher than the cost of holding CBDC: ($\phi^D > \phi^C$). This can be justified because introduction of CBDC—a policy intervention—would likely be aimed at increasing access to payment instruments and/or increasing financial inclusion.

Under these two new assumptions, households' utility can be written as

$$u(W_0) = \max \left[(W^{\frac{\rho-1}{\rho}} + \lambda L^{\frac{\rho-1}{\rho}})^{\frac{\rho}{\rho-1}} - \mathbb{1}(\phi) \right], \tag{19}$$

in CBDC and bank deposits holdings.

where

$$\mathbb{1}(\phi) \equiv \begin{cases} \phi^C & \text{if } C > 0 \text{ and } D = 0 \\ \phi^D & \text{if } D > 0 \text{ and } C = 0 \\ \phi^C + \phi^D & \text{if } C > 0 \text{ and } D > 0 \end{cases}$$

The rest of the model is as before, with the only difference that banks take into account the fixed costs of opening the bank account in the demand of deposits by the households.

3.2 Solution characterization

The model does not have analytical solutions and we solve it numerically. The appendix provides more details on the solution, here we only present the basic intuition for the equilibrium characterization.

With the fixed costs of setting up a CBDC or a bank deposit account, households may choose to hold neither CBDC nor deposit (cash will be always held as it has not a fixed cost), only hold CBDC, only hold bank deposit, or hold both. This choice depends on returns on assets and on the individual household's initial wealth. More specifically, the fixed costs will induce cutoffs in the distribution of households, so that household below and above some thresholds in initial wealth will demand different type of assets. For instance, in the case when CBDC does not exist, one threshold on initial wealth will arise. Households with wealth less than the threshold will hold only cash, while households with initial wealth higher than the threshold will hold both cash and deposits. Banks will maximize profits, given household's demand conditional on the wealth threshold.

Similar to Section 2, we compare the implication of the model without CBDC ($\delta_C = 0$) and when CBDC is introduced ($\delta_C > 0$). Different parameters values might give rise to different equilibria once the CBDC is introduced.[17] These equilibria differ by which segment of the population will hold which instrument. We will focus on the case when, in equilibrium, households with low initial wealth use only cash, households with medium initial wealth use both cash and CBDC, and households with high initial wealth use cash, CBDC, and deposits. This equilibrium arises in our preferred numerical calibration, and it is a plausible outcome of the introduction of a CBDC.[18]

3.3 Calibration

The parameter values used for numerical solutions are summarized in Table 1. We set the interest on bonds (policy rate) to 3 percent. We choose λ to generate a share of non-liquid assets to total household wealth of 83 percent (U.S. Census).[19] The parameter governing the liquidity services of cash (δ_N) is normalized to one.

[17] As the objective function of the household optimization problem is not strictly concave, existence and uniqueness of the equilibrium is not globally guaranteed. However, in all simulations, the equilibria was always found to be unique (and though might differ across parameters' values), and found only a very limited range of parameters' values for which the equilibrium does not exist.

[18] Other possible equilibria could have some middle-class households hold only cash and deposits, while richer ones holds both deposits and CBDC. Assuming $\phi_D > \phi_C$ is not enough to have a sorting of households, so that poorer households unambiguously only hold CBDC, while richer household hold CBDC and deposits. This is so since even though deposits have a higher fixed cost, they provide higher remuneration than CBDC. Hence there are some levels of wealth, fixed costs, and deposits returns, such that it might optimal to have deposits only and not CBDC. However this type of equilibrium arises only for a limited set of parameter values.

[19] In US, the share of wealth in checking accounts and Other Interest earning accounts at financial instituions, as a fraction of household net worth (excluding equity in own home) is $\approx 0.83\%$ in 2019. See https://www.census.gov/data/tables/2019/demo/wealth/wealth-asset-ownership.html.

Table 1: Baseline calibration

Parameter	Definition	Value
λ	Relative importance of liquid assets	$1.5 * 10^{-6}$
ρ	Complementarity b/w wealth & liquidity	0.15
ϵ	Substitutability b/w different liquid assets	3
η	Substitutability b/w deposits at different banks	1.1
J	Number of banks	8
δ_D	Share of deposits	1.5
δ_C	Share of CBDC	2
f	Interest on bonds	3%
r_C	Return on CBDC	0
ϕ^D	Fixed cost of accessing deposits	$0.06 \times \lambda^\rho$
ϕ^C	Fixed cost of accessing CBDC	$0.001 \times \lambda^\rho$
$\underline{W}$	Normalized lowest wealth	1
α	Shape of wealth distribution	1.52

We choose δ_D, so that it matches the share of wealth held in cash versus deposits in the U.S. when there is no CBDC ($\approx$ 11%, Cash/M2) and set δ_C assuming that CBDC is more helpful in providing liquidity services than deposits.[20] The fixed cost of accessing deposits is set to have a share of population that is fully banked (not unbanked nor underbanked) to around 80% (FDIC). The shape parameter of the wealth distribution (1.52) is set to match the Gini coefficient of the U.S. (0.49). Given these parameters, we choose, J, η ρ and ϵ so that the condition required for the existence of equilibrium holds ($\epsilon > \mathcal{M} > \rho$),[21] while still falling in a range consistent with the literature.

We set the fixed costs of accessing CBDC (ϕ_C) to be substantially lower that that of accessing deposits (ϕ_D). Below, we discuss how the relative cost of accessing CBDC compared to deposits ($\frac{\phi_C}{\phi_D}$) and the shape of the wealth distribution (α)

[20]See Agur et al. (2022) for a discussion on users' preferences over anonymity and security when choosing payment instruments.

[21]See discussions on this constraint following equation (14) in Section 2.2.

drive the mechanisms of the model, and how results changes with different calibrating assumptions. Finally we assume a non-interest bearing CBDC for all baseline simulations, but analyze the outcome if CBDC is interest bearing in the Robustness section.

3.4 Results

The results of the simulation with the baseline calibration are presented in Table 2. The first column reports the values of key variables when there is no CBDC in the model. Banks set remuneration on deposits at 2.25%, a spread of 75 basis points with respect to the policy rate, the measure of their market power. Given the fixed cost of accessing deposits, and its remuneration, 79.5% of the population holds a bank account (they are financially included), holding 16.6% of the their wealth in deposits; the rest of the population holds only cash. In the aggregate, 15.4% of total wealth is held in deposits, 1.8% in cash and 82.8% in the non-liquid asset.

After the introduction of CBDC two main effects arise. First, as in the basic model with homogenous households, banks will increase remuneration of deposits to fight the competition of CBDC. In the new equilibrium remuneration of deposits increases by about 50 basis points, to 2.72% (second column in table 2). Second, the wealth thresholds that define whether households hold different portfolios (and the composition of those portfolios) change. Some households will find more convenient to hold CBDC and are going to completely close their deposit account (extensive margin), while those who keep having a deposit account open, will hold more deposits, given that remuneration of deposits has increased (intensive margin). The

Table 2: Results of introducing CBDC in baseline calibration

	Baseline	
	No CBDC	CBDC
Cash (%)	1.8%	0.4%
CBDC (%)	0.0%	3.5%
Deposits (%)	15.4%	11.2%
Non liquid wealth (%)	82.8%	84.9%
Interest on deposits	2.25%	2.72%
Banks profit	0.0012	0.0003
Financial inclusion (%)	79.5%	100%
% Deposits for those with bank	16.6%	19.6%
% Wealth held by those with bank account	92.5%	56.9%
Intensive margin*		1.69 p.ps.
Extensive margin**		-5.92 p.ps.

* Intensive margin is defined as the percent change in deposits for those with a bank account times the share of total wealth held by those with a bank account after CBDC is introduced.

** Extensive margin is defined as the percent change in wealth held by those with a bank account times the percent of deposits held for those with a bank account before CBDC is introduced.

share of non-liquid wealth in the portfolio of households increases to about 85% after the introduction of CBDC, as the share of liquid assets fall.[22]

Figure 1 illustrates the change in the intensive margin and the extensive margin following the introduction of CBDC. The x-axis represents households with different initial wealth levels and the y-axis represents the share of initial wealth. The blue lines plot the share of wealth that a household will allocate to deposits (D/W) for an initial level of wealth W_0. Given that preferences are homothetic, households within the same wealth group will hold a fixed percent of their wealth in any asset, hence the flat lines.

[22]We abstract from modelling equilibrium remuneration of non-liquid wealth. In a general equilibrium framework a higher demand of non-liquid assets would decrease their return and would decrease incentives to hold fewer deposits. However, as the exercise shows, the quantitative increase in non-liquid asset demand is small, which suggests limited general equilibrium effect.

The dashed blue line represents the allocations before CBDC introduction ($\delta^C = 0$). Households with initial wealth lower than W_A (20.5% of the households, the financially excluded households) do not have a bank account and thus $D/W = 0$. Households with initial wealth higher than W_A hold both cash and deposits.

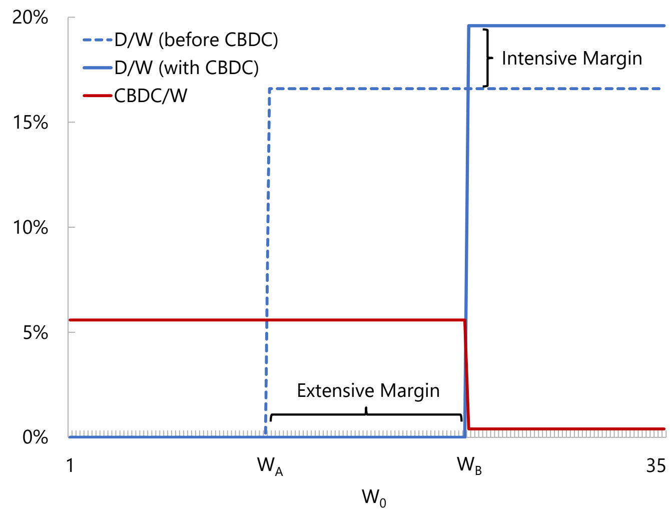

Figure 1: Portfolio adjustment when CBDC is introduced

The solid blue line represents the allocations when CBDC is introduced. Now a lower fraction of the population is willing to hold deposits, this is represented by the shift from W_A to W_B: households within these two thresholds used to hold deposits but don't after, a decrease in bank deposits via the extensive margin. On the other hand, D/W is higher for households that choose to hold deposits when CBDC is present (intensive margin). The extensive margin and the intensive margin work in different directions, and the net effect depends on the assumed parameter values. In the baseline calibration, the extensive effects dominates, leading to an about 4% loss in deposits.

The red line represents the share of wealth allocated to CBDC, C/W. All households choose to hold CBDC with our baseline choices of parameters. This is because we assume that CBDC is very easy to access (ϕ^C is small). CBDC thus improves financial inclusion as now poorest households ($W_0 < W_A$) hold CBDC and not only cash. Richer households also hold CBDC, although they allocate a much smaller fraction of their wealth to this. This is not surprising as these households have access to deposits that pay interest and are thus assets that provide liquidity and have a better return.

Figure 2 shows how aggregate deposits change when CBDC is introduced. The figure presents the level of deposits held by households at each level of wealth ($Df(W)$ on the y-axis and households' wealth W on the x-axis). The dotted blue line shows $Df(W)$ before CBDC is introduced. In this case, households with wealth between zero and W^A only hold cash and households above W^A hold cash and deposits. The full red line shows $Df(W)$ after CBDC is introduced. The blue area represents the total amount of deposits that are lost because some households now choose to hold CBDC instead of deposits (these are households with wealth between W^A and W^B). Put differently, the minimum level of wealth in which a household chooses to still hold deposits is higher. This is the extensive margin of the change in aggregate deposits. The red area represents the increase in aggregate deposits driven by richer households (W^B) deciding to increase their deposit holdings – this is driven by banks increasing deposit rates r_D. This is the intensive margin of the change in aggregate deposits. As it can be inferred from the figure, the extensive margin is a larger area than the intensive margin. More precisely, and as presented above in Figure 2, the difference in these areas represents 4.23 percentage points of initial aggregate deposits following the introduction of CBDC.

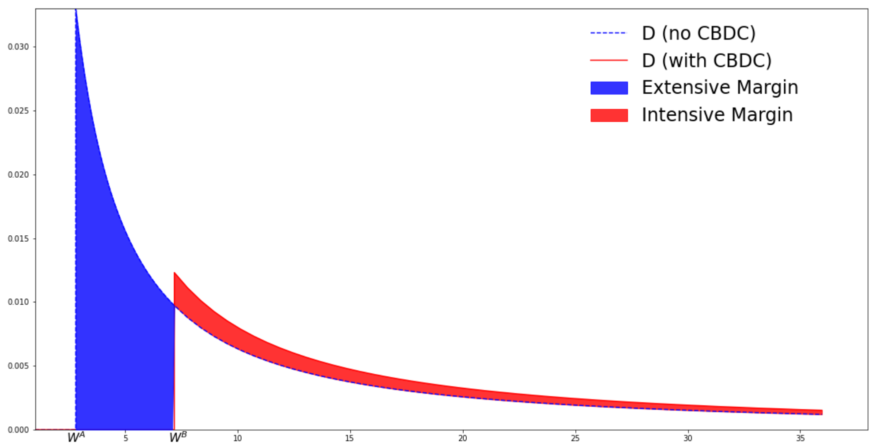

Figure 2: Aggregate deposits when CBDC is introduced

3.5 Key mechanisms driving results

There are two parameters that are crucial for the relative strength of the extensive and intensive margins. The first is given by the design of CBDC: the relative fixed costs of setting up a CBDC and a bank account (ϕ^C/ϕ^D). The second is given by the context in which CBDC is issued: the distribution of initial wealth $F(W)$ governed by α. We find that the extensive margin dominates the intensive margin and the aggregate deposits decline in equilibrium when access to CBDC is much cheaper than bank deposits $\phi^C << \phi^D$ and when the mass of poorer households is large (α

is high).

Table 3 shows how results change with a less attractive CBDC in terms of access costs relative to deposits (ϕ^C/ϕ^D increases). This leads to intermediation (higher total deposits). The table also presents results when the mass of poorer households is larger (α increases). This leads to higher disintermediation (relative to the baseline case).

Table 3: Results of introducing CBDC in alternative calibrations

	High CBDC cost $c = 0.03$		High alpha $\alpha = 1.62$	
	No CBDC	CBDC	No CBDC	CBDC
Cash (%)	1.8%	0.0%	1.9%	0.5%
CBDC (%)		1.6%	0.0%	3.9%
Deposits (%)	15.4%	15.5%	15.6%	10.3%
Non liquid wealth (%)	82.8%	82.7%	82.5%	85.3%
Interest on deposits	2.25%	2.67%	2.32%	2.74%
Banks profit	0.0012	0.0005	0.0011	0.0003
Financial inclusion (%)	79.5%	100%	80.4%	100%
% Deposits for those with bank	16.6%	19.2%	17.0%	19.9%
% Wealth held by those with bank account	92.5%	80.7%	92.0%	51.6%
Intensive margin*		2.08 p.ps.		1.52 p.ps.
Extensive margin**		-1.96 p.ps.		-6.86 p.ps.

* Intensive margin is defined as the percent change in deposits for those with a bank account times the share of total wealth held by those with a bank account after CBDC is introduced.

** Extensive margin is defined as the percent change in wealth held by those with a bank account times the percent of deposits held for those with a bank account before CBDC is introduced.

In Figure 3 we summarize how for different combination of these two dimensions (ϕ^C/ϕ^D and α) disintermediation occurs following the introduction of CBDC.[23]

The intuition for these results is the following. The introduction of CBDC increases the competition faced by banks to attract deposits, and banks will increase

[23] If $\phi^C/\phi^D \to 0$ then the model behaves qualitatevely like the model in section 2.

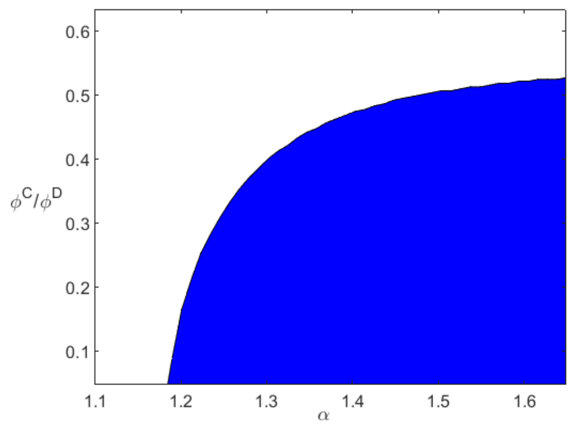

Figure 3: Parameter space for deposit disintermediation (blue area)
This figure shows that deposit disintermediation occurs for high values of α and/or high access costs to CBDC relative to the costs of deposits ((ϕ^C/ϕ^D))

the interest rate on deposits (decrease deposit spread) in response. With higher interest rate paid on deposits, households increase their deposit holding and aggregate deposit increases. This result in the intensive margin is the same as in the basic model when households are homogeneous (see Proposition 1).

With wealth heterogeneity in households and different fixed costs of accessing CBDC and deposits, the extensive margin leads to fewer aggregate deposits. Some poorer households decide to stop holding deposits and switch from deposits to CBDC. For this effect to more than offset the intensive margin, banks must have less than enough incentives to further increase deposits rates to go after households who chose not to hold deposits accounts.

In the case with relatively higher costs to access CBDC (relative to the baseline case), fewer households have incentives to stop having bank accounts and switching to only have CBDC and cash. The higher interest on deposits after the CBDC is introduced more than compensate the fixed cost of bank accounts for more households than before. Thus, the extensive margin (-1.96%) is smaller than the intensive margin (2.08%) and total deposits grow (by 0.1%). Still CBDC increases competition for banks and their profits fall due to the higher deposit remuneration but less than in the baseline case. This shows that ease of access to CBDC is likely to shape the overall outcome for banks funding and profitability.

In the case in which the mass of poorer households is large ((α is 1.62, higher than in the baseline case), disintermediation is higher. Here, more households are relatively poor and have incentives to stop having a bank account and switch to CBDC. Banks do not aggressively increase deposit rates to prevent the outflow of customers due to the relatively small wealth held by the poor households. Thus, the extensive margin and CBDC holdings are higher than in the baseline case and even higher than in the case when CBDC has a high access cost. Total deposits also fall by more. This illustrates how the country's economic structure also influences the outcome of CBDC introduction for banks.

4 Robustness and extensions

In this section we analyze the model's robustness to (i) varying remuneration rates on CBDC and (ii) different preferences for CBDC, δ_C, in the utility function. We then extend the model to include lending and wholesale funding for banks, and analyze how bank lending changes in the case when total deposits fall.

4.1 Robustness

Interest bearing CBDC Figure 4 presents the percentage drop in aggregate deposits with respect to a non-CBDC economy as a function of interest rates on CBDC. The change in deposits is linear in the remuneration of the CBDC. Intuitively, the higher the remuneration on CBDC, the more a competitive threat to banks CBDC represents. Banks, even though they increase further their deposits rate, are not able to stem the outflow of deposits. Increasing the interest rate on CBDC from zero to 0.7% make aggregate deposits drop by a further 1 percentage point compared to a non-remunerated CBDC, while the remuneration of deposits increases by a further 6 basis points (Tables 2 and 4). This leads to a further decrease in banks profits. Our model works as well if we assume negative returns on CBDC.[24] Decreasing the interest on CBDC leads to a smaller drop in aggregate deposits with respect to non-remunerated CBDC.

Comparative statics on δ_C The parameters $\{\delta_N, \delta_C, \delta_D\}$ govern how good cash, CBDC, and deposits are in terms of providing liquidity services. In the baseline calibration we normalize $\delta_N = 1$ and assume $\delta_C = 2$ and $\delta_D = 1.5$. Here we perform comparative statics with respect to these δ_C. Figure 5 shows that the better CBDC is in providing liquidity (i.e., higher δ_C), the larger the drop in aggregate deposit following the the introduction of CBDC. This is intuitive because CBDC is a better substitute for bank deposits and thus cause higher bank disintermediation when δ_C is larger.

[24] Liquidity is a CES composite of cash, CBDC and deposits, so the households still want to hold some CBDC even if the opportunity costs of holding CBDC is higher than for cash ($r_C < 0$).

Table 4: Results of introducing a remunerated CBDC

	Remunerated CBDC $r_c = 3\%$	
	No CBDC	CBDC
Cash (%)	1.8%	0.3%
CBDC (%)	0.0%	4.4%
Deposits (%)	15.4%	10.4%
Non liquid wealth (%)	82.8%	84.9%
Interest on deposits	2.25%	2.78%
Banks profit	0.0012	0.0002
Financial inclusion (%)	79.5%	100%
% Deposits for those with bank	16.6%	20.4%
% Wealth held by those with bank account	92.5%	51.2%
Intensive margin*		1.97 p.ps.
Extensive margin **		-5.88 p.ps.

* Intensive margin is defined as the percent change in deposits for those with a bank account times the share of total wealth held by those with a bank account after CBDC is introduced.

** Extensive margin is defined as the percent change in wealth held by those with a bank account times the percent of deposits held for those with a bank account before CBDC is introduced.

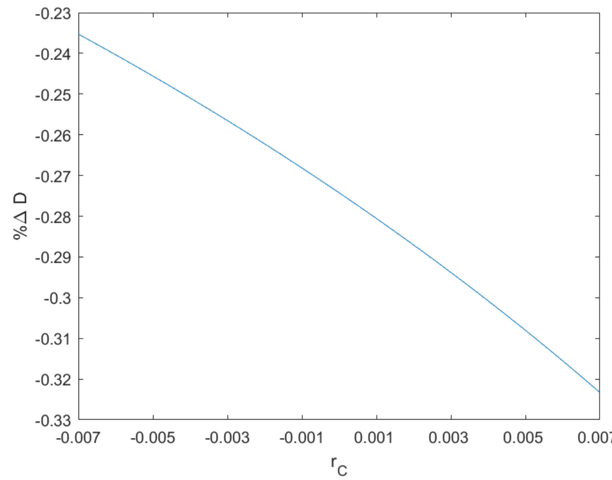

Figure 4: Percentage Drop in Aggregate Deposit for Different Interest Rate on CBDC

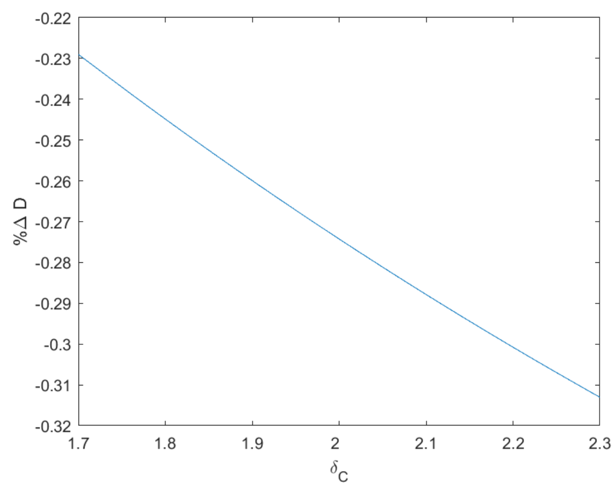

Figure 5: Percentage Drop in Aggregate Deposit for Different δ_C

4.2 Effects of disintermediation in lending

In this section we enrich the balance sheet of banks by introducing, following Drechsler et al. (2017), lending and wholesale funding. We solve the model numerically and we find that our main results still hold qualitatively, that is the introduction of

CBDC leads to a reduction in lending under the same specific conditions as in the heterogenous households model, however the drop in lending is quantitatively very small.

Everything on the consumer side is unchanged and only the banking problem is now different. Banks can now fund both with deposits (D_i) and wholesale funding (H_i). They lend L_i , which is "unproductive" and given to firms outside of the economy.

The problem of the bank is:

$$\max_{D_i,H_i} \left(f + l_0 - \frac{l_1}{2} L_i\right) L_i - \left(f + \frac{h}{2} H_i\right) H_i - (f - s_i) D_i \tag{20}$$

$$\text{s.t. } L_i = H_i + D_i$$

$l_0, l_1, h > 0$ are parameters that shape the bank's lending opportunities, and wholesale funding costs and availability. The first terms defines how banks profit from lending. $l_0 > 0$ denotes how much more lending profits banks with respect to bonds, while $l_1 > 0$ captures the fact that the bank has a limited pool of profitable lending opportunities. The second term defines the extra cost for wholesale funding, where $h > 0$ also wants to capture the limited availability of wholesale funding, which makes the cost of wholesale funding increase in the amount borrowed. Lending (L) and wholesale funding (H) are in positive net supply with infinite elasticity.

The first order conditions for D_i, H_i are

$$[D_i] :(f + l_0) - l_1 L_i - (f - s_i) + \frac{\partial s_i}{\partial D_i} D_i = 0 \tag{21}$$

$$[H_i] :(f + l_0) - l_1 L_i - f - h H_i = 0 \tag{22}$$

From (22) one can derive

$$H_i = \frac{l_0}{l_1 + h} - \frac{l_1}{l_1 + h} D_i \quad \Longrightarrow \quad L_i = \frac{l_0}{l_1 + h} + \frac{h}{l_1 + h} D_i \tag{23}$$

and so lending co-moves with deposits, unless wholesale funding is riskless ($h = 0$), in which case lending is constant. Using the previous result in (21), after some algebraic manipulation:

$$\frac{h}{l_1 + h}(l_0 - l_1 D_i) + s_i \left(1 + \frac{\partial s_i}{\partial D_i} \frac{D_i}{s_i}\right) = 0$$

which define the individual bank demand of deposits. The second term is the same as in the baseline model, and represent the marginal profit on the bank deposits business alone. The first term is instead how much more the bank can earn by raising another dollar of deposits given the profitable lending opportunities. Basically the bank can now forego some profits on the deposits by increasing remuneration of deposits if this allows to fund a larger balance sheet and profit on lending opportunities.

To derive aggregate deposits demand, remember that in a symmetric equilibrium $s_i = s, D_i = D$ and that $\frac{\partial D_i}{\partial s_i} \frac{s_i}{D_i} = \frac{1}{N} \frac{\partial D}{\partial s} \frac{s}{D} - \eta \left(1 - \frac{1}{N}\right)$. Substituting in before and rearranging gives that equation for aggregate deposits demand:

$$-\frac{\partial D}{\partial s}\frac{s}{D} = [1-(N-1)(\eta-1)] - N\left(\frac{\frac{h}{l_1+h}(l_0-l_1D)}{\frac{h}{l_1+h}(l_0-l_1D)+s}\right) \tag{24}$$

The first term on the right hand side was already in the baseline model, while the second captures the new setup. Equalizing this to the elasticity from household side gives the equilibrium equation to solve for s, the endogenous interest rate on deposits.

4.2.1 Quantitative results

Simulations results of comparing an economy without and with CBDC are presented in Table 5. We use the same values for parameters summarized in Table 1 and pick values for new lending parameters, namely $l_0 = 0.001, l_1 = 0.001, h = 0.00002$. The economy without CBDC features slightly higher deposits as a fraction of wealth and slightly higher remuneration of deposits than the model in Section 3. This is because lending opportunities are now more profitable to banks, hence it is more profitable to collect more deposits, and to do so banks slightly increase interest rates.

We find that our main results on deposits are extended qualitatively to lending: the introduction of CBDC leads to a reduction in deposits under the baseline calibration, but the drop in lending is quantitatively small. In this economy the introduction of CBDC leads to a drop in aggregate deposits of 4 percentage points of total wealth, but lending drops only by 0.14%. By contrast, the percentage drop is 4.23 percentage points in the model with heterogenous households in Section 3.

The drop in lending is small since now the banks can use the optimal composition

Table 5: Results of introducing CBDC in an economy with bank lending

	Bank lending	
	No CBDC	CBDC
Cash (%)	1.7%	0.4%
CBDC (%)	0.0%	3.4%
Deposits (%)	15.6%	11.7%
Non liquid wealth (%)	82.7%	84.5%
Interest on deposits	2.3%	2.8%
Banks profit	0.0011	0.0003
Financial inclusion (%)	80.6%	100%
% Deposits for those with bank	16.8%	20.2%
% Wealth held by those with bank account	92.9%	57.9%
Intensive margin *		1.90%
Extensive margin **		-6.88%
Change in Lending		-0.18%

* Intensive margin is defined as the percent change in deposits for those with a bank account times the share of total wealth held by those with a bank account after CBDC is introduced.

** Extensive margin is defined as the percent change in wealth held by those with a bank account times the percent of deposits held for those with a bank account before CBDC is introduced.

between deposits and wholesale to fund the lending opportunities. More in general, drop in lending will be small as long as banks have fairly convenient source of funding alternative to deposits, or there are profitable lending opportunities. Recall from equation (23) that if wholesale funding is risklessly priced at the policy rate ($h = 0$) lending will not change after the introduction of CBDC. When $h > 0$ instead, as long as profitable opportunities for lending exist (l_0/l_1 sufficiently high), banks will try to minimize as much as they can the loss in deposits in order to continue having a large balance sheet and so providing loans.

5 Conclusions

In this paper, we set up a portfolio choice model as a laboratory to investigate the effects of the introduction of CBDC on bank deposits and lending. We find that only in special cases introducing CBDC induces bank disintermediation in deposits. In an extension to the model where banks can also lend and borrow wholesale, disintermediation in deposits due to CBDC introduction leads only to a small decrease in lending.

The model we use is fairly straightforward. There are many dimensions in which it could be extended. In our model, we specify households' preferences with a CES utility function, but other demand systems could be explored. The model could also be extended so that banks also lend to firms and fund productive projects - so lending would have a general equilibrium effect on the economy's welfare. The model could also be dynamic and have banks' capital as a state variable affected by their profitability. Banks could be heterogeneous in their efficiency to capture deposits or lend, and banks could entry and exit depending on their profitability and capitalization. We leave the explorations of these questions for future work.

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A Comparative statics for the model with homogeneous households

This appendix shows analytical results for changes in the policy rate, CBDC remuneration, and bank market power. Quantitatively, implications of CBDC introduction for bank deposits and the deposit spread will depend on the level of the policy rate f, remuneration offered by the CBDC and the level of market power in the banking sector. In what follows, we explain how these factors matters.

In line with the intuition, the decline in the deposit spread Δs is larger, and the increase in the aggregate deposits ΔD is larger, the higher the rate on CBDC is:

$$\frac{\partial |\Delta s|}{\partial r_C} > 0, \tag{25}$$

$$\frac{\partial \Delta D}{\partial r_C} > 0. \tag{26}$$

A higher rate of return on CBDC implies that banks will need to compensate households by paying a higher deposit rate in order to prevent them from switching to CBDC. Thus a higher r_C pushes the deposit spread further down and results in a larger increase in the amount of liquidity held in bank deposits.

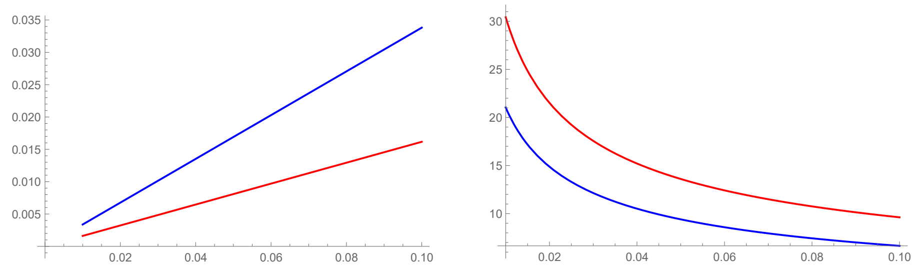
For the policy rate, which is also the rate of return on bonds in which banks invest, we can show that the decline in the deposit spread Δs is larger, and the increase in the aggregate deposits ΔD is smaller, the higher the policy rate is, i.e.:

$$\frac{\partial |\Delta s|}{\partial f} > 0, \tag{27}$$

$$\frac{\partial \Delta D}{\partial f} < 0 \tag{28}$$

The result that a higher policy rate implies a higher decline in the deposit spread follows from the comparison of equations (13) and (17). In the absence of CBDC, the deposit spread $\tilde{s}^*$ is increasing by a fixed proportion, $\delta_D^{\frac{\epsilon}{\epsilon-1}} \left[\frac{\mathcal{M}-\rho}{\epsilon-\mathcal{M}}\right]^{\frac{1}{\epsilon-1}}$, each time f is raised by one. This elasticity declines once CBDC is introduced. Intuitively, although a higher policy rate still allows banks to raise the spread, they are more constrained in the ability to raise it due to the competition from CBDC.

Figure 6: *Introduction of CBDC (red lines): impact on deposit spread and on deposits as a function of the policy rate f.*



(a) deposit spread

(b) aggregate deposits

The result that the increase in the aggregate deposits is highest for low levels of the policy rate f might seem counter-intuitive, given that the decline in the deposit spread is the largest when f is high. To understand this result, it is again useful to express aggregate deposits as a function of the deposit spread and the overall cost of liquidity: $D^* = \delta_D^\epsilon \left(\frac{s^*}{s_l}\right)^{-\epsilon} s_l^{-\rho}$ and $\tilde{D}^* = \delta_D^\epsilon \left(\frac{\tilde{s}^*}{\tilde{s}_l}\right)^{-\epsilon} \tilde{s}_l^{-\rho}$, where $\tilde{s}_l \equiv (f^{1-\epsilon} + \delta_D^\epsilon(\tilde{s}^*)^{1-\epsilon})^{\frac{1}{1-\epsilon}}$. As discussed before, following the introduction of CBDC, the deposit spread adjusts so that the ratios $\frac{s^*}{s_l}$ and $\frac{\tilde{s}^*}{\tilde{s}_l}$ are equal. Thus, a larger decline in s^* relative to $\tilde{s}^*$ when f goes up is simply necessary in order to keep the relative share of deposits in the households' liquidity basket from falling. It follows that the difference in the response of D^* and $\tilde{D}^*$ to changes in f is solely due to differences in how the overall demand for liquidity, $\tilde{s}_l^{-\rho}$ and $s_l^{-\rho}$, changes with the policy rate when there is no CBDC and when CBDC is present.

Finally, when we consider different levels of competition in the banking sector, we find that the decline in the deposit spread Δs is smaller, and the increase in the aggregate deposits ΔD is larger, the higher the elasticity of substitution among deposits (η) or the number of banks (J) is. Formally:

$$\frac{\partial|\Delta s|}{\partial \eta} < 0, \quad \frac{\partial|\Delta s|}{\partial J} < 0 \tag{29}$$

$$\frac{\partial \Delta D}{\partial \eta} > 0, \quad \frac{\partial \Delta D}{\partial J} > 0 \tag{30}$$

Higher competition has two implications. First, the deposit rate is higher, as banks have less market power. Second, the aggregate elasticity of deposits with respect to deposit rate (the negative of elasticity with respect to the deposit spread) increases as either there are more banks the household can substitute to (higher J), or there is higher elasticity of substitution across deposits. Hence, when CBDC is introduced, banks have less capacity to increase deposits rates compared to a deposit market with less competition and, even with a smaller increase in the deposit rate, the higher elasticity of aggregate deposits will imply a larger increase in deposit holdings by the households.

B Equilibrium in model with household heterogeneity

We first show that the indirect utility is linear in initial wealth W_0. The indirect utility $u(W_0)$ is defined as the maximized utility with optimal choice of assets allocation $\{N, C, D, B\}$, and is defined for each discrete choice over whether to set up an account for deposit or CBDC. Let $t(s_l) \equiv (1 + \lambda^\rho s_l^{1-\rho})^{\frac{1}{\rho-1}}$, the indirect utility can be expressed as follows:

$$U(W_0) = W_0(1+f)t\left(\mathbb{1}(s_l)\right) - \mathbb{1}(\phi), \tag{31}$$

where

$$\mathbb{1}(s_l) \equiv \begin{cases} \delta_N^{\frac{\epsilon}{1-\epsilon}} f & \text{if } C = 0 \text{ and } D = 0 \\ (\delta_N^\epsilon f^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon})^{\frac{1}{1-\epsilon}} & \text{if } C > 0 \text{ and } D = 0 \\ (\delta_N^\epsilon f^{1-\epsilon} + \delta_D^\epsilon s^{1-\epsilon})^{\frac{1}{1-\epsilon}} & \text{if } D > 0 \text{ and } C = 0 \\ (\delta_N^\epsilon f^{1-\epsilon} + \delta_D^\epsilon s^{1-\epsilon} + \delta_C^\epsilon (f - r_C)^{1-\epsilon})^{\frac{1}{1-\epsilon}} & \text{if } C > 0 \text{ and } D > 0 \end{cases}$$

The benefit of having a deposit or a CBDC account is equal to the lower cost of liquidity service, reflected in different $\mathbb{1}(s_l)$ associated with different choices. The cost is the fixed cost ϕ^C, ϕ^D.

We can compute the indirect utility for each of the four choices and compare them, in order to find the cutoff wealth values for preferred choice within each pair. Since $\phi^D > \phi^C$, there are four possible scenarios: (i) households with low initial wealth use only cash, and households with high initial wealth use cash, CBDC, and deposits; (ii) households with low initial wealth use only cash, households with medium initial wealth use both cash and CBDC, and households with high initial wealth use cash, CBDC, and deposits; (iii) households with low initial wealth use only cash, households with medium initial wealth use both cash and deposits, and households with high initial wealth use cash, CBDC, and deposits; (iv) households with low initial wealth use only cash, households with medium low initial wealth use both cash and CBDC, households with medium high initial wealth use both cash and deposits, and households with high initial wealth use cash, CBDC, and deposits. For a given parametrization, only one scenario exists.

We will focus on the second scenario, as it is the one that arises under our preferred parametrization. In scenario number 2, households with initial wealth below $\hat{W}_1 = \frac{\phi^C}{(1+f)\left(t\left(s_l^C\right)-t(s_l^N)\right)}$ would choose to hold cash only, and those with initial wealth above $\hat{W}_1$ and below $\hat{W}_2 = \frac{\phi^D}{(1+f)\left(t(s_l^B)-t(s_l^C)\right)}$ would choose to hold both cash and CBDC, while those with initial wealth above $\hat{W}_2$ would hold hold cash, CBDC, and deposits. These expressions show that the cutoff wealth levels increase with the cost of switching from/to an asset (the numerator) and fall with the benefit of switching (the denominator).

Aggregate bank deposits are now given by

$$D = \delta_D^{\epsilon}\left(\frac{s_l^B}{s}\right)^{\epsilon}\frac{\lambda^{\rho}\left(s_l^B\right)^{-\rho}}{1+\lambda^{\rho}\left(s_l^B\right)^{1-\rho}}(1+f)\times\int_{\hat{W}_2} W_0 dF(W_0), \tag{32}$$

which shows that aggregate deposits can change along both an intensive and an extensive margin (the superscript B indicates that this is the equilibrium in which some households hold both CBDC and deposits).

The deposit spread s is determined in the equilibrium by the following equation:

$$\begin{aligned}\mathcal{M} = {} & \epsilon\left(\frac{\delta_N^{\epsilon}f^{1-\epsilon}+\delta_C^{\epsilon}(f-r_C)^{1-\epsilon}}{\delta_N^{\epsilon}f^{1-\epsilon}+\delta_D^{\epsilon}s^{1-\epsilon}+\delta_C^{\epsilon}(f-r_C)^{1-\epsilon}}\right)\\ & +\rho\left(\frac{\delta_D^{\epsilon}s^{1-\epsilon}}{\delta_N^{\epsilon}f^{1-\epsilon}+\delta_D^{\epsilon}s^{1-\epsilon}+\delta_C^{\epsilon}(f-r_C)^{1-\epsilon}}\right)\\ & +\frac{(1-\rho)\left(s_l^B\right)^{1-\rho}}{\lambda^{-\rho}+\left(s_l^B\right)^{1-\rho}}\left(\frac{\delta_D^{\epsilon}s^{1-\epsilon}}{\delta_N^{\epsilon}f^{1-\epsilon}+\delta_D^{\epsilon}s^{1-\epsilon}+\delta_C^{\epsilon}(f-r_C)^{1-\epsilon}}\right)\\ & +(\alpha-1)\frac{\lambda^{\rho}\left(t(s_l^B)\right)^{2-\rho}\left(s_l^B\right)^{1-\rho}}{t(s_l^B)-t(s_l^C)}\left(\frac{\delta_D^{\epsilon}s^{1-\epsilon}}{\delta_N^{\epsilon}f^{1-\epsilon}+\delta_D^{\epsilon}s^{1-\epsilon}+\delta_C^{\epsilon}(f-r_C)^{1-\epsilon}}\right)\times\mathcal{I}_{\hat{W}_1>\underline{W}}\end{aligned} \tag{33}$$

As can be seen from Equation (31), the indirect utility of a household is a linear function of the wealth level W_0, with different slopes and intercepts given different choices over the extensive margin. Here we denote with N the choice of holding only cash, C, the choice of holding only CBDC, D the choice of holding only deposits, and finally with B the choice of holding both CBDC and deposits. Notice that the slopes are such that $B > C, D > N$, and the intercepts are such that $B < D < C < N$, so the poorest households always choose N, and the richest households always choose B, and the households in the middle might choose C or D.

Denote the cutoff wealth levels to for a household choose C, D, and B over N as $\hat{W}_{11}, \hat{W}_{12}$ and $\hat{W}_{13}$, respectively. If $\hat{W}_{13} \leq \hat{W}_{11}$ and $\hat{W}_{13} \leq \hat{W}_{12}$, the scenario would be that poor households with $W \leq \hat{W}_{13}$ choose N and other households choose B (Scenario 1). The expressions for the cutoff wealth levels are given by the following equations:

$$\hat{W}_{11} = \frac{\phi^C}{(1+f)\left(t(s_l^C) - t(s_l^N)\right)}, \quad \hat{W}_{12} = \frac{\phi^D}{(1+f)\left(t(s_l^D) - t(s_l^N)\right)}, \quad \hat{W}_{13} = \frac{\phi^D + \phi^C}{(1+f)\left(t(s_l^B) - t(s_l^N)\right)},$$

Similarly, let $\hat{W}_{21}$ and $\hat{W}_{22}$ denote cutoff wealth levels for a household to choose D and B over C, respectively:

$$\hat{W}_{21} = \frac{\phi^D - \phi^C}{(1+f)\left(t(s_l^D) - t(s_l^C)\right)}, \quad \hat{W}_{22} = \frac{\phi^D}{(1+f)\left(t(s_l^B) - t(s_l^C)\right)}.$$

Following similar reasoning, we can show that there are four possible scenarios in the distribution of households over these four choices, summarized as follows.

1. N→ B

   Conditions: $\hat{W}_{13} \leq \hat{W}_{11}$, $\hat{W}_{13} \leq \hat{W}_{12}$

2. N→ C→ B

   Conditions: $\hat{W}_{11} < \hat{W}_{13}$, $\hat{W}_{11} < \hat{W}_{12}$, $\hat{W}_{22} \leq \hat{W}_{21}$ or $t(s_l^D) \leq t(s_l^C)$

3. N→ D→ B

   Conditions: $\hat{W}_{12} < \hat{W}_{13}$, $\hat{W}_{12} \leq \hat{W}_{11}$, $t(s_l^D) > t(s_l^C)$

4. N→ C→ D→ B

   Conditions: $\hat{W}_{11} < \hat{W}_{13}$, $\hat{W}_{11} < \hat{W}_{12}$, $\hat{W}_{21} < \hat{W}_{22}$, $t(s_l^D) > t(s_l^C)$

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