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# Modeling the Reserve Demand to Facilitate Central Bank Operations

Zhuohui Chen, Nikolaos Kourentzes, and Romain Veyrune

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Prepared by Zhuohui Chen, Nikolaos Kourentzes, and Romain Veyrune**Authorized for distribution by Jihad Alwazir  
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# Modeling the Reserve Demand to Facilitate Central Bank Operations

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## Abstract

Implementing monetary policy largely consists in controlling short-term interest rates which supposes having a good understanding of banks' demand for liquidity on their account at the central bank also called "reserves". This work aims to offer a modeling methodology for estimating the demand for reserves that itself is influenced by various macro and market structure variables. The model can help central banks to identify "stable points" on the demand for reserves, which correspond to the levels of reserves for which the short-term interest rate volatility is minimal. Both parametric and non-parametric approaches are provided, with a particular focus on capturing the modeling uncertainty and, therefore, facilitating scenario analysis. A method is proposed to test the forecasting performances of different approaches and exogenous regressors combination, finding that simpler parametric expressions provide on balance better performances. Adding variables to both parametric and non-parametric provides better explanations and predictions. The proposed methodology is evaluated using data from the Euro system and the US Federal Reserve System.

*Keywords:* Reserve demand, scenario analysis, short-term interest rate, sigmoid

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## 1. Introduction

The objective of this paper is to help central banks model the demand for banks' reserves, which the central bank targets to implement monetary policy. Reserves are the balance on banks' demand deposits at the central bank. They are the most liquid asset in a financial system and are, thus, sometimes referred to as "liquidity". Monetary policy implementation is inexorably connected with controlling the short-term interest rates, which represent the marginal cost of funding financial intermediaries.

Although the importance of correctly calibrating the short-term interest rates is well recognised (Moschitz, 2004; Beirne, 2012; Valimaki, 2008; Bindseil and Jablecki, 2011; Veyrune et al., 2018; El Gemayel et al., 2022), there is limited work to aid with the modeling of the reserve demand, and by extension setting the short-term interest rates. This constitutes a gap in the literature, and limiting factor in having data-driven decision support for the operations of central banks. Previous work has recognised the logistic curve as a stylised representation (Valimaki, 2001, 2008; Bindseil, 2014) but does not offer guidance on the calibration and estimation of the model. Veyrune et al. (2018) contributes by discussing the estimation of the reserve demand curve as a logistic curve, albeit for the bivariate case, excluding potential covariates from the model. Åberg et al. (2021) further demonstrate the effectiveness of the method by Veyrune et al. (2018). Lopez-Salido and Vissing-Jorgensen (2022) consider another modification to the logistic curve to include additional variables, however they fix one of its asymptotes to facilitate the model estimation. They found that the demand for reserve has likely move due to structural change in the financial sector driven mainly by prudential regulation (such as liquidity coverage ratio and leverage ratio) regulation and changes to the Federal Reserve operational framework. El Gemayel et al. (2022) use a double exponential instead of a logistic curve. Afonso et al. (2022) focus on estimating interest rate elasticity to changes in reserves, based

on a locally bi-variate linear approximation of the demand for reserves rather than estimating the curve itself. They later infer the demand curve by imposing an arc-tangent functional form, but do not assess its goodness of fit to the data. Nonetheless, these represent relatively rare examples in the literature that focuses on the modeling aspects of the problem, with the majority of the work focusing on the policy and economic aspects.

This work provides multiple contributions:

- Proposes several alternative parametric sigmoid (and benchmark) models for the reserve demand that are able to account for external factors. Non-parametric alternatives are provided to demonstrate the advantages and weaknesses of the proposed parametric curves.
- Prescribes a methodology to select between the alternative reserve demand curve representations and regressors, automatically specifying the appropriate model.
- Provides asymmetric quantile estimates for the curves, both communicating the modeling confidence to the analyst and indicating the risk of deviating from the estimated interest rate for a given reserve level. This is the first work that models the uncertainty of the reserve demand.
- Facilitates scenario analysis for changes in the external factors, while providing conditional predictive distributions to the analyst, demonstrating both shifts in the estimated reserve demand curve and its uncertainty. This helps to forecast the change in the demand and calibrate monetary operations accordingly.

The proposed methodology provides approaches to overcome estimation complications for the various models, obtain quantiles with minimal assumptions, and overcome typical issues in scenario analysis. It is believed that, this contribution is the

first that provides a complete set of tools to estimate the demand for reserves and, therefore, facilitate central banks to exercise monetary policy. Our work subsumes previous work by Veyrune et al. (2018), El Gemayel et al. (2022), and Afonso et al. (2022) by embedding their logistic, double exponential, and arctangent approximations to a complete modeling framework.

The rest of the paper is structured as follows. Section 2 provides background context, highlighting the importance of the modeling task. Section 3 describes the proposed models and modeling methodology. Section 4 presents the two empirical cases studies that are used for evaluation, followed by Section 5 that discusses the use of the models for scenario analysis, followed by concluding remarks.

## 2. Background on Central Bank Policy Rates

Controlling short-term interest rates is paramount for monetary policy implementation. Short-term interest rates represent the cost of the most liquid and risk-free asset in financial system; that is banks' on-demand account at the central bank also referred to as "reserve" or "liquidity". They indicate what is the cost for intermediaries, e.g., banks, to borrow an additional unit of reserves in the market (or the return of placing one additional unit in the market). As such, they represent the marginal cost of funding that would influence the funding cost at longer maturities<sup>1</sup> and, via funding cost, how much intermediaries would charge when they lend to corporation and individuals. Central banks implement monetary policy by keeping short-term rates at the appropriate level to pursue their inflation objective, increasing them if inflation exceeds the target and lowering them if inflation is below target. The monetary policy implementation challenge is, thus, to keep short-term rates stable, close

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<sup>1</sup>That is, the marginal cost plus how much counterparties would request to forgo the benefit of liquidity (the liquidity premium).

to level deemed appropriate generally called the "policy rate".

Reserves are instruments of the central banks. They issue them and decide how much reserves are on the accounts of the banks in their books. Some factors influence the stock of reserves and are not under the direct control of the central bank. Those are the 'autonomous factors' such as the demand for bank notes, which is another instrument issued by the central bank that depends on the public need for transaction purpose, or the government account at the central bank. However, central banks could add or withdraw reserves from the banks' accounts via their monetary policy operations. The autonomous factors should, thus, be forecasted and possibly compensated by monetary operations.

The demand for reserves describes the rate at which banks are ready to borrow and lend reserve as a function of aggregated reserves in the system. Banks demand reserves for different reasons that could vary with time (Afonso et al., 2022; Hamilton, 1997; Poole, 1968). They seek to keep a certain buffer of reserves to absorb unexpected payments out of their account at the central bank. Their demand for reserves chiefly depends on (i) the risk (distribution of payment shocks); (ii) predictability of the payment shocks (the central bank liquidity forecast if publicly available); (iii) interbank market functioning (how easy is it to fund a short-term funding need in the market); (iv) the cost of liquidity management errors (the cost of being short or long of reserves); and (iv) regulatory demand for reserve such as the reserve requirement.

Reasonably assuming that central banks operate standing lending and deposit facilities, short-term rates are expected to decline as reserves increase, converging to the deposit facility rate. In the opposite case, when there is not enough liquidity, the banks would need to borrow at the lending facility. This connection is exemplified in Figure 1. In the figure, a series of 'stable points' are provided, which correspond to the level of liquidity where the available reserve buffer can absorb liquidity shocks without impact on short-term rates. Outside of those stable points, liquidity shocks or changes

in the demand would have an impact on short-term rates, triggering interest rate volatility, with adverse effects. These are the product of three alternative monetary policy implementation frameworks (King, 2018; Bindseil, 2014; Ennis and Keister, 2008):

- In the "mid-corridor" system, the stable point is at an equal distance from the deposit facility rate and the lending facility rate of the central bank. Therefore, there is no more incentive of being either short or long reserves. If the reserve requirement imposes the minimum buffer necessary for inter-temporal smoothing<sup>2</sup> of the liquidity shocks, only a small amount of excess reserves is usually necessary for the stability of short-term rates close to the middle of the corridor.
- In a "floor" system, the inter-temporal smoothing role previously played by the reserve requirement is fulfilled by enough recourse to the deposit facility (the minimum buffer). In that case, rates would be stable and close to the deposit facility rate, which is the effective policy rate.
- In a "ceiling system", banks finance the shortfall compared with their minimum buffer at the standing lending facility which rate, then, become a stable point.

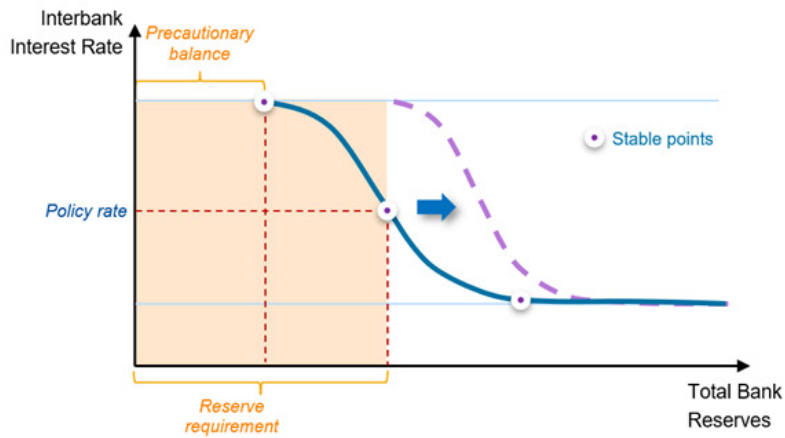
Central banks need to know the demand for reserve ex-ante to calibrate their operations and the reserve requirement as parsimoniously as possible. As with any insurance, liquidity has a cost, which means that the more is not necessary desirable. Too much buffer would impose low-yielding assets in the banks' balance sheet; a cost that will typically be passed to their clients, thereby introducing a form of liquidity premium. It is also detrimental for the money market, especially for transactions between counterparts with access to the central bank's facilities, such as banks. The

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<sup>2</sup>Ability for banks to offset shortfall of one day with surplus on the other days over the course of the maintenance period



Figure 1. Stable Points on the Demand Curve for Reserve



Source: Authors' Calculation

Notes: The dots on the demand curve represent the stable points.

marginal cost of funding, which central banks want to control, moves from transaction between counterparts with access to these central banks' facilities to transactions between counterparts with access and counterparts without access, such as non-bank financial institutions. Prudential regulation, such as the leverage ratio, could affect the willingness of banks to accomplish this intermediation. As a result, short-term rates fluctuate below the deposit facility rate with less central bank control, i.e., for a certain level of excess reserves short-term rates leave the stable points defined by the deposit facility rate. This weakens the effectiveness of central banks' policy implementation.

Therefore, what is central to effective monetary policy is the correct identification of these stable points and the reserve demand curve, or its evolution due to changes in the market environment. Unconventional monetary policies, especially those consisting in providing ample excess reserves, do not affect the proposed methodology and usually do not influence the demand for reserve that usually reflects factors that are not under the direct control of the central banks but under the control of their

monetary counterparties (i.e. banks). The next section will introduce the method to assist central bank operations, conditional on the information that would be typically available to them.

### 3. Modeling the Excess Reserves Demand Curve

#### 3.1. Functional form

The general form of the model is  $r_i = f(g(\mathbf{C}_i))$ , where  $r_i$  is the short term interest rate for observation  $i$ , and  $\mathbf{C}_i = [R_i \quad \mathbf{X}_i]$  is a matrix that contains the excess reserve  $R_i$  and  $p$  explanatory variables in  $\mathbf{X}_i$ . Let

$$g(\mathbf{C}) = c + \mathbf{C}w_g, \tag{1}$$

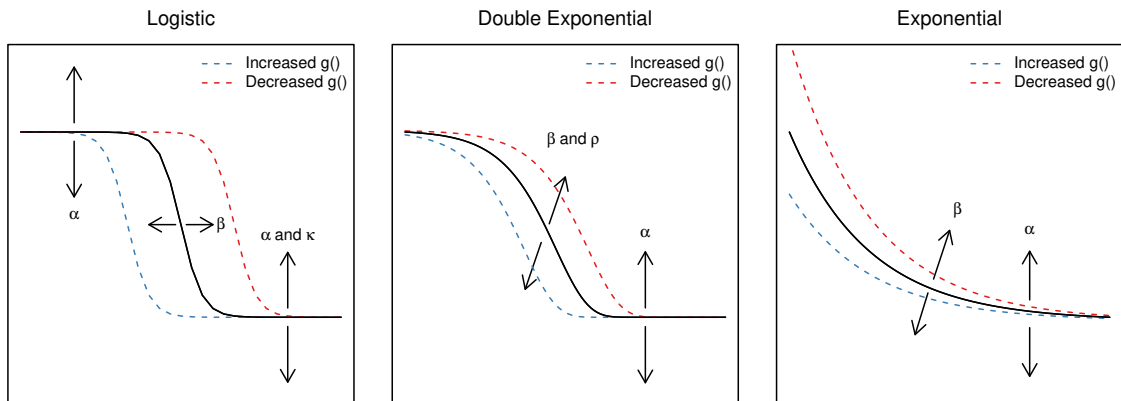
where  $w_g$  is a column vector of  $p + 1$  coefficients, and  $c$  is a constant. For  $f(\cdot)$  several alternative forms are considered, which are listed in Table 1. Therein, Logistic and the Reduced Logistic have parameters  $\alpha$ ,  $\beta$ , and  $\kappa$  controlling the shape of the curve. Veyrune et al. (2018) use a bivariate logistic curve, while Lopez-Salido and Vissing-Jorgensen (2022) include additional variables in the logistic but drop  $\alpha$  restricting somewhat the control over the asymptotes of the curve. Their modification has advantages for the estimation of the model parameters. For double exponential parameters  $\alpha$ ,  $\beta$ , and  $\rho$  control the shape (used by El Gemayel et al., 2022), while for exponential only parameters  $\alpha$  and  $\beta$  are used. The arctangent has been used by Afonso et al. (2022), with parameter  $\alpha$  changing the location of the curve, and  $\beta$  controlling its spread and ensuring that the resulting sigmoid is decreasing. The coefficients in  $g(\mathbf{C})$  are sufficient to control the phase of the arctangent, reducing the required number of parameters from the formulation by Afonso et al. (2022). The effect of the various parameters can be seen in Figure 2. The linear functional form for  $f(\cdot)$  has the role of a benchmark for the more complex non-linear models. For all

curves, an additive error term is assumed on which there is no further distributional assumptions imposed, nor homoscedasticity assumption.

Table 1. Functional Forms of  $f(\cdot)$

Curve	Function
Logistic	$r_i = \alpha + \kappa / (1 - \beta e^{g(\mathbf{C}_i)}) + \varepsilon_i$
Reduced Logistic	$r_i = \alpha + 1 / (1 - \beta e^{g(\mathbf{C}_i)}) + \varepsilon_i$
Double exponential	$r_i = \alpha + \beta e^{\rho e^{g(\mathbf{C}_i)}} + \varepsilon_i$
Exponential	$r_i = \alpha + \beta e^{g(\mathbf{C}_i)} + \varepsilon_i$
Arctangent	$r_i = \alpha + \beta \arctan(g(\mathbf{C}_i))$
Linear	$r_i = g(\mathbf{C}_i) + \varepsilon_i$

Figure 2. The Sigmoid Alternatives for  $f(\cdot)$  and the Effect of Their Parameters



Source: Authors' Calculation

None of the prescribed curves enable modeling vertical shifts in the reserve demand curve. Operational changes or otherwise market disruptions can cause such shifts, which are accounted with the addition of a parameter  $\eta$  to quantify the shift, encoded by a binary indicator  $I_i$ :  $r_i = f(g(\mathbf{C}_i)) + \eta I_i$ .

### 3.2. Selection of Explanatory Variables and Functional Form

The method allows for the inclusion of  $p \geq 0$  explanatory variables in  $g(\cdot)$ . The inclusion of predictively useful covariates is intended to explain the variance of  $r_i$ . However, the functional form of  $f(\cdot)$ , irrespectively of any variables included in  $\mathbf{X}$ , will explain the variance of  $r_i$  as well. Therefore, in selecting which variables to include in the method, it is desired to simultaneously choose the shape of the curve and the variables.

Given that there is no distributional assumptions,  $k$ -fold cross-validation (CV) will be employed to assess the functional form. Although CV is effective for this, it can make an exhaustive search very computationally expensive. To mitigate the computational requirements, the following search strategy is proposed. First, the CV data folds will be generated. Second, for each curve, follow a backwards selection for the variables. Once the best subset is identified, third, compare the CV errors of the different curves. As the folds are kept common across the search between specifications, the errors remain comparable. More detail is elaborated below.

Given a curve functional form,  $f(\cdot)$ , start by building the model that includes all possible covariates and obtain its CV error. And then the CV error will be evaluated after removing each of the variables on their own. If any such elimination improves the CV error, then that variables will be removed and repeat the process till either all variables have been eliminated or the CV cannot be improved further by eliminating any variable. At maximum, the selection routine will stop at  $p$  steps.

There is ample evidence of the good performance of the general to specific approach in model building (Hoover and Perez, 1999, 2004; Hendry and Krolzig, 2011). A detailed overview of its merits is provided by Campos et al. (2005). Nonetheless, the implementation can take several forms. In terms of criterion, the CV Mean Squared Error (MSE) will be examined as this has been shown to be an effective way to obtain well specified models for predictive purposes (Bergmeir et al., 2018). The

MSE is chosen, as it is of interest to obtain specifications that accurately model the expected  $r_i$  (Gneiting and Raftery, 2007). And the authors are testing “non-nested” models that encompass the general unrestricted model.

The described backwards iterative search can be greedy, in that it does not explore all potential specification combinations, and further a stepwise search that would permit re-inclusion of rejected variables is not allowed. This is done for computation efficiency, while there is evidence that the specification search does not suffer greatly (Hastie et al., 2009), in contrast to a forward search that has been shown to explore a very limited modeling space.

At the end of the backwards variable search the best setup for each functional form of  $f(\cdot)$  is obtained. Given that the cross-validation uses a common sampling, the errors between the best specifications are directly comparable, and therefore the selection of best curve and variables without additional computations will be achieved simultaneously.

In settling with this approach, an additional consideration is that it is trivially parallelizable, enabling the efficient use of computations resources. Attempt to model the linear  $g(\cdot)$  was made first, with variable selection using the Akaike’s Information Criterion, and using its predicted values as inputs for  $f(\cdot)$ . In that case,  $\mathbf{X}_i$ , the potentially explanatory variables, will be regressed on  $R_i$ , the excess reserve. Although this approach seems reasonable and computationally efficient, it suffers from two important drawbacks. First, the two stage modeling introduces several biases in that the first model is built on minimizing the errors in explaining  $R_i$  and not  $r_i$ , and the sequential parameter optimization of  $g(\cdot)$  and  $f(\cdot)$ . Second, even if these biases are disregarded, the regression model that takes the part of  $g(\cdot)$  can output values that are inappropriately scaled as inputs for the functional forms in Table 1 due to the exponent. This can be dealt with by introducing a scaling factor where in Table 1 the  $g(\mathbf{C}_i)$  is replaced by  $\phi g(\mathbf{C}_i)$  with  $\phi$  scaling the outputs of the regression to values that

do not saturate the exponent. However, the trials indicated that the optimization of  $\phi$  is non-trivial, failing to identify appropriate  $\phi$  and produce useful predictions of  $r_i$ .

### 3.3. Estimation of Prediction Intervals

Instead of assuming a particular distribution for  $\varepsilon_i$  it is obtained from the data. This has multiple advantages. First, the estimation is not biased by an incorrect choice of the underlying distribution, which connects to the second, assumption is not needed on symmetry from the mean, or lack of, for the upper and lower intervals. Third,  $\varepsilon_i$  is expected to be heteroscedastic, with an unknown scedastic function of  $\mathbf{C}_i$ . For instance, it is anticipated that for high volumes of  $R_i$  the variance of  $r_i$  will be small. And a similar connection is expected when  $R_i$  is very small.

To estimate the upper and the lower intervals for a given probability  $\alpha$  we re-estimate the previously specified curve using the tick loss (or pinball):

$$S = \begin{cases} \alpha|y_i - \hat{y}_i|, & \text{if } \hat{y}_i \leq y_t, \\ (1 - \alpha)|y_i - \hat{y}_i|, & \text{if } \hat{y}_i > y_t, \end{cases} \quad (2)$$

where  $y_i$  and  $\hat{y}_i$  are the observation and prediction for observation  $i$  respectively. This loss is a proper score for predicting quantiles at levels of  $\alpha \in (0, 1)$  (Gneiting and Raftery, 2007) and therefore appropriate to our task. For instance, if we are interested in the 95% interval, we optimize using  $S$  for  $\alpha = 2.75\%$  and  $\alpha = 97.25\%$ .

### 3.4. Parameter Estimation and Constraints

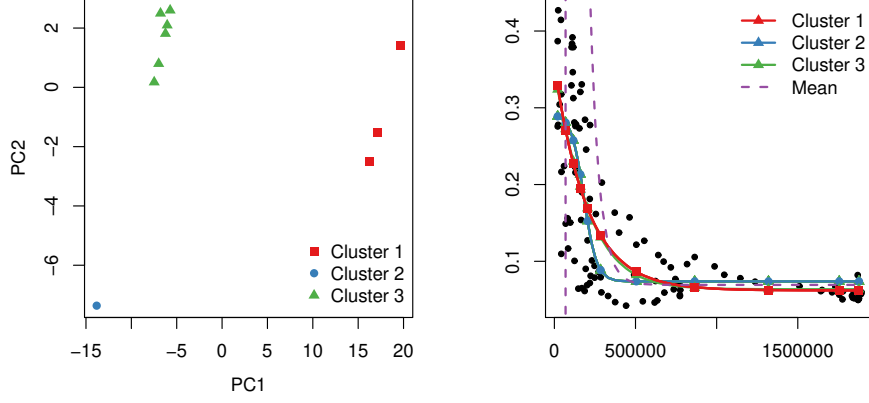
Due to the nonlinear nature of the curves the estimation of their parameters can be difficult, with the error surface exhibiting local minima. Furthermore, the various curves in Table 1 allow for more shapes than those depicted in Figure 2 that connect to our application focus. To achieve good quality estimates we start with reasonable initial values for the parameters, which already incorporate the appropriate

overall shape. To initialize the coefficients in  $g(\cdot)$  we pre-optimize the model using a standard numerical optimizer like Nelder-Mead (Powell, 1973). As the quality of the optimization is not guaranteed, we rely on the Particle Swarm optimization (PSO) metaheuristic for the final estimates. PSO provides a very effective search of the error surface. This is achieved by intialising a population of candidate solutions, the search of which is influenced by both the known local and global best positions at each iteration. This ensures that the majority of the different solutions converge towards the global best, while effectively searching the error surface (Bonyadi and Michalewicz, 2017). However, PSO has the disadvantage in that its solution is stochastic, and different random initial points for the candidate solutions may result in different outcomes. Depending on the quality of the estimation, these solutions may exhibit high variation.

The influence of the random initialization can have a substantial effect on the quality of solutions (Kourentzes et al., 2014). Note that the parameters in the various curves interact (see Figure 2), with visually similar solutions being potentially the product of different sets of parameters. Figure 3 demonstrates this. A logistic curve is optimized on the same data for ten times. The vectors of parameters are projected using principal components analysis on the left plot. Three clusters of solutions are identified, between which there are substantial differences in terms of estimated parameters. The plot on the right shows the corresponding curves. The nine curves belonging to clusters 1 and 2 are almost identical. Curve resulting from the average of the sets of parameters is also provided, which has a very poor fit. With this, both the variations between solutions, and how the parameters in these solutions can counterbalance each other to result in a very similar fit are illustrated. This effect is stronger for curves that have more parameters controlling their shape.

Two different strategies to resolve this are considered. For both, we initialize the optimization randomly multiple times. Then, we either (i) pick the intilisation

Figure 3. Two Different Visualizations of 10 Different Estimates for a Logistic Curve



Sources: Authors' Calculation

Notes: The plot of the left projects the parameters on a 2D space using principal components, demonstrating the diversity of the solutions. The plot on the right shows the resulting logistic curves from the different sets of estimated parameters. We also provided the resulting curve of the average of the estimated parameters.

that resulted in the lowest objective function value, or (ii) use a carefully designed combination of the initializations. As a simple combination of parameters will not result in a good solution, we instead rely a modification of the "islands" pooling heuristic by Kourentzes et al. (2019). For  $j = 1, \dots, u$  different initializations, we collect the resulting values of the objective function  $\mathbf{v} = v_j$ . Then:

1. Find the correlations  $\boldsymbol{\rho} = -\rho_j$  of all vectors of parameters with the solution with the minimum  $\mathbf{v}$ . Observe the negative sign that ensures that the more correlated solutions rank lower in  $\boldsymbol{\rho}$ .
2. Sort  $\boldsymbol{\rho}$  from lowest to highest.
3. Construct  $\boldsymbol{\rho}' = \{0, \Delta\boldsymbol{\rho}\}$ , where  $\Delta\boldsymbol{\rho}$  is the first difference of the sorted  $\boldsymbol{\rho}$ .
4. Include in the combination pool all initializations where  $\rho' \leq \tau$ , with  $\tau$  being a threshold calculated iteratively as  $Q3(\boldsymbol{\rho}'_j) + IQR(\boldsymbol{\rho}'_j)$ , with Q3 and IQR being the third quartile and the interquartile range respectively, and  $\boldsymbol{\rho}'_j$  being a vector



including up to the  $j$ -th value of  $\boldsymbol{\rho}'$ . This step eliminates from the combination poorly performing estimates.

5. Ensure that a second criteria of inclusion is satisfied:  $\rho_j \geq 0.8$  where solutions with very different parameters are not included. This helps avoiding combining counterbalancing estimated parameters that can perform poorly (see Figure 3).
6. Linearly combine the remaining sets of parameters using the weights:  $z_l / \sum z$ , where  $z_l$  is the  $l$ -th value of  $\boldsymbol{v}$  that was included in the combination pool.

The objective of the combination "islands" is to retain only high performing solutions in the combination, reducing the number of combination weights to be estimated that can harm the overall combination (Claeskens et al., 2016), while being able to reduce the variability of the final set of parameters due to the combination. In contrast, the first option does not mitigate the variability of the solutions, but is able to provide good quality estimates nonetheless.

An additional feature of the proposed approach is the ability to incorporate restrictions on the coefficients in  $g(\cdot)$  that model the impact of the explanatory variables. It is anticipated that for many of the explanatory variables the analyst will have some understanding whether a positive or a negative effect is to be expected, which is translated into a non-negativity or non-positivity constraint for the respective coefficient. By introducing these restrictions, faster optimization can be achieved, which also help with the selection of variables. If a variable is included in the curve spuriously, there is a good chance that its sign may also be inappropriate. Introducing these restrictions prohibits these solutions, pushing that variable out of the model. If the analyst does not have an expectation about the sign of a specific variable, then its coefficient remains unrestricted.

Beyond any statistical benefits, the ability to introduce parameter restrictions for the explanatory variables can help with the acceptability and adoption of the method

in practice. The trustworthiness of a method is positively affected by demonstrating that accepted domain knowledge is captured, as well as the ability of analysts to introduce their domain expertise (Arrieta et al., 2020). The trustworthiness is further reinforced by limiting the apparent stochasticity of the solution obtained by the PSO metaheuristic, providing stable and consistent predictions (Spavound et al., 2022).

Finally, in estimating the prediction intervals, the obtained curve parameters are used as a starting point, and restrictions to retain the same signs for the estimated parameters are imposed. An additional restriction is used to ensure that the provided intervals do not cross the fitted curve.

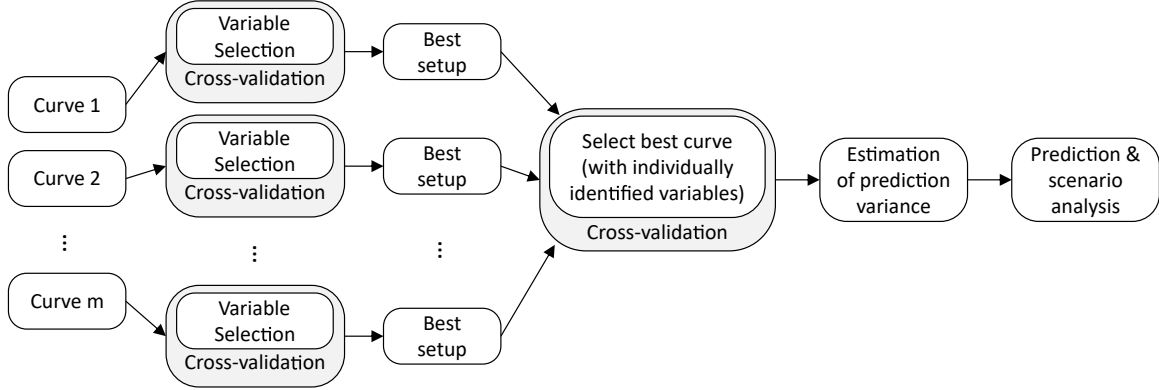
### *3.5. Use of the Method and Scenario Analysis*

The overall modeling methodology is summarised in Figure 4. Starting from a number of potential functional forms (summarised in Table 1) and explanatory variables, through cross-validation, we obtain the best curve  $f^*(\cdot)$ . This is then optimized across the complete available sample using the MSE, and twice with the tick loss for the upper and lower prediction intervals. If required, we can sample the predictive distribution in more detail by estimating the corresponding parameters for any number of quantiles. The approach resembles a quantile regression, as we obtain an estimated curve for each desired quantile.

The method can then be used to obtain predictions for  $r_i$  or to evaluate the effect of different scenarios as captured by the conditions prescribed by the included variables from  $\mathbf{X}_i$ , for instance increasing or decreasing market segmentation. This scenario analysis can be instrumental in supporting planning for a variety of uncertainties (Wright and Goodwin, 2009; Schoemaker and Tetlock, 2012).

In generating scenarios we often consider a number of alternatives, for instance *mild* and *severe* scenarios. Although the generation of the different scenarios is interesting, it is beyond this discussion and we refer the reader to Wright and Goodwin

Figure 4. The Proposed Methodology for Identifying the Appropriate Functional Form



Sources: Authors' Calculation

Notes: The proposed methodology for identifying the appropriate functional form. Each curve should be an instantiation of the general unrestricted model.

(2009) for an overview of good and bad practices. Here, we contrast scenarios with the probabilistic statements of predictive models. We exemplify the contrast using a simple bivariate regression:  $y_t = c_0 + c_1x_t + \varepsilon_t$ , where  $c_0$  is a constant,  $c_1$  is the coefficient of the predictor  $x_t$ ,  $t$  refers to a time period  $t = 1, \dots, T$  for  $T$  observations, and  $\varepsilon \sim \mathcal{N}(0, \sigma^2)$  with standard deviation  $\sigma$ . Suppose we are after a prediction for period  $t = T + 1$ , which would be the unconditional expectation equal to  $\hat{y}_{T+1} = c_0 + c_1x_{T+1}$ , with a variance of  $\sigma^2$ . In this statement we assume the value of  $x_{T+1}$  is known. In a conditional setting, where we have no knowledge of the future values, one can construct scenarios for  $x_{T+1}$  and obtain outcomes for  $y_{T+1}$  conditional on the narratives supporting each scenario. However, obtaining the conditional variance of  $y_{T+1}$  can be challenging when  $x_{T+1}$  is the outcome of scenarios. The example bivariate regression implies that  $\varepsilon_t$  is independent of  $x_t$ , and therefore would remain  $\sigma^2$ . Here lies a fallacy, as we build the scenario narratives on various conditions and assumptions that are not part of the bivariate regression. If we are willing to accept that these omitted variables will affect the expectation of  $y_t$ , then assuming that these will not affect the variance is overly restrictive. The narratives that generate the scenarios and the

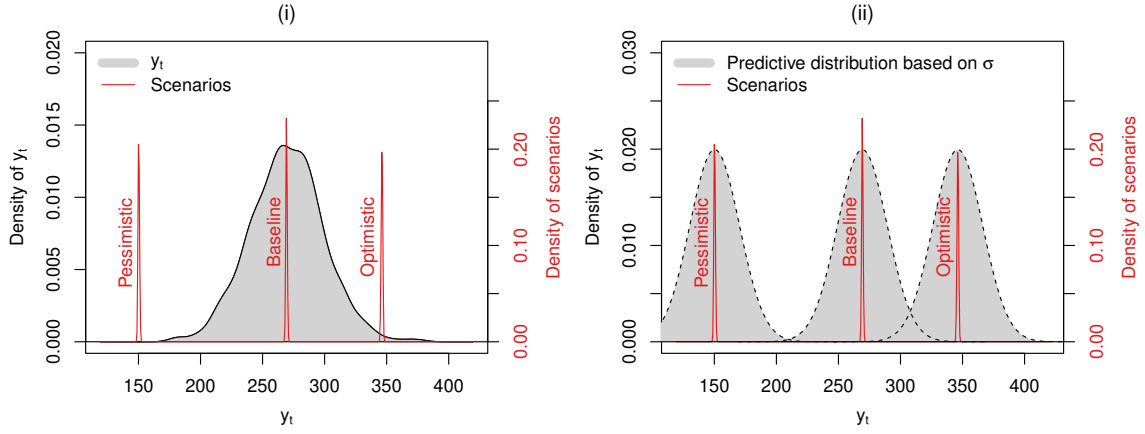
model we use to analyze the effects of the scenarios are not simultaneously valid.

This is typically overlooked and in scenario analysis we often do away with the probabilistic nature of predictions and instead obtain specific point predictions of  $y_t$ . Although these may be perceived as more credible than less involved statistical models, due to their narrative and vivid nature (Taylor and Thompson, 1982; Schnaars and Topol, 1987), they do not provide a sense of the respective uncertainty (Athanasopoulos et al., 2022) and their usefulness as predictive tools can be challenged (Fildes, 2022). Such scenarios are often associated with some ad-hoc probability of occurring, supported by the scenarios' narratives that can be skewed due to various judgmental biases (Goodwin et al., 2019a,b). Although, this can be partially mitigated by leveraging multiple experts cancelling out individual biased (Kourentzes et al., 2021), these ad-hoc probabilities are largely not based on rigorous data modeling.

Figure 5 demonstrates the issue. In panel (i) The distribution of  $y_t$ , accounting for the different  $x_t$  inputs is provided, alongside with three scenarios generated by providing three alternative values for  $x_t$ . The plotted densities are obtained using kernel density estimation. The scenarios fail to provide a probabilistic view, and when contrasted with the density of  $y_t$  their validity can be questioned, or conversely challenge the validity of the model for  $y_t$ . In panel (ii) we provide predictive distributions for the three scenarios based on the variance of  $\varepsilon_t$ , which is also problematic, as it does not account for the conditions that generated the scenarios.

Our approach enables us to construct mean point predictions for various scenarios and also sample the predictive distribution at the desired quantiles. As the intervals are generated via curves that include the estimated effect of the explanatory variables that change in the various scenarios, changes in shape and location of the quantiles are modelled. (An example is provided in Figure 13.) Athanasopoulos et al. (2022) showed that the predictive distribution can differ substantially for various scenarios. In their case, they relied on judgmental estimates from a large number of participants,

Figure 5. Density Comparison for Probabilistic Prediction



Source: Athours' Calculation

Notes: In the left panel (i) the density of the probabilistic prediction of  $y_{T+1}$  is contrasted with three scenarios generated using three values of  $x_{T+1}$ . In the right panel (ii) for the same scenario based predictions the predictive distributions based on the variance of  $\varepsilon_t$  are given.

while in our case we obtain the predictive distributions for each scenario quantitatively. This makes the approach more scalable, as it does not require collecting the views of participants. This is important, as the proposed method is meant to be used in an operational decision making context and not as a one-off tool. Nevertheless, from the proposed method an analyst can choose to extract the conventional predictive distribution, scenario based point estimates, or more completely scenario-based predictive distributions.

## 4. Application to Case Studies

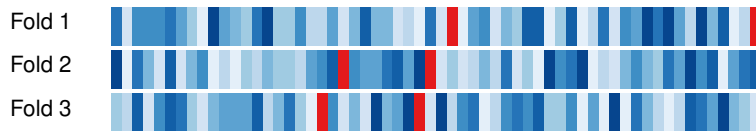
### 4.1. Experimental Design

To empirically validate the performance of the proposed parametric approaches we set up two experiments, using data from the European Central Bank, and the US Federal Reserve. The setup of the two experiments is identical.

To generate out-of-sample test data we use a 30-fold cross-validation (CV). For

each fold, the training set is used to apply the methodology described in Section 3. Figure 6 exemplifies the resulting double CV, where the outer 30-fold CV is used to iterate across test observations (in red in the Figure), and the inner 10-fold CV is used to generate validation observations to select variables and curves (the different folds shaded in blue). Eventually, all observations are used as out-of-sample test. This double CV setup is necessary to avoid specifying and evaluating the curves on the same data.

Figure 6. Three Example Folds of the Double Cross-Validation



Source: Authors' Calculation

Notes: Red observations are used as test (30-fold) and blue cross-validated sample (10-fold) are used for selecting variables and curves.

For the test set we track the following error metrics:

$$\text{RMSE} = \sqrt{\frac{1}{o} \sum_o (y_o - \hat{y}_o)^2},$$

$$\text{MAE} = \frac{1}{o} \sum_o |y_o - \hat{y}_o|,$$

where  $y_o$  is the  $o$ -th observation in the test set, and  $\hat{y}_o$  its respective prediction. The Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) measure the accuracy of the predictions. To evaluate the prediction intervals of the competing methods, the Mean Interval Score (MIS) has been used:

$$\text{MIS}_\alpha = \frac{1}{o} \sum_o \left( (U_\alpha - L_\alpha) + \frac{2}{\alpha} (L_\alpha - y_o) \mathbf{1}(y_o < L_\alpha) + \frac{2}{\alpha} (y_o - U_\alpha) \mathbf{1}(y_o > U_\alpha) \right)$$

#### 4.1.1. Methods

Parametric curves described in Table 1 are tested with the linear option acting as a benchmark. Furthermore, two non-parametric methods that attempt to approximate the underlying data structure in a data driven fashion are also experimented. These are useful to assess the restrictiveness of the parametric forms considered in our methodology.

The first non-parametric alternative is based on a Random Forest implementation. Random Forest is a popular machine learning method, applicable to both regression and classification problems, proposed by Breiman (2001). It is an ensemble learner based on decision trees, where the prediction of multiple trees is combined to achieve better accuracy and robustness than any individual tree. For each of these trees a number of variables is included randomly, providing both diversity in the combination, while avoiding the need to explicit variable selection by the modeler.

The second non-parametric approach uses Generalized Additive Models (GAMs), proposed by Hastie and Tibshirani (1990). GAMs allow for non-linear relationship between the explanatory variables and the target, using smooth spline functions to approximate the connection. The degree of smoothing can be fine-tuned by minimizing a cross-validation error, which apart from providing a data driven approximation, it also mitigates the need for variable selection, as the impact of variables that are not deemed helpful is smoothed to a degree that they have minimal contribution to the final prediction (Wood, 2006).

Finally, for each dataset two experiments are conducted. First, to model using only the excess reserve as a predictor, and second, to model using additional explanatory variables, listed separately in each case study in their respective subsections.

For the purposes of this evaluation, 10 random initializations for the PSO are used, which is implemented using the `pso` package (Bendtsen, 2022) for R (R Core Team,

2021). The variable and curve selection uses a 10-fold cross-validation in the training set. The random forests are implemented using the package `randomForest` (Liaw and Wiener, 2002) for R, with the default settings for the generation of the tree population. Quantiles for the random forecast are obtained by using the approach by Meinshausen and Ridgeway (2006), as implemented in the `quantregForest` package (Meinshausen, 2017) for R. The formulation relies on a similar direct estimation of the quantiles as described in Section 3. The GAMs are implemented using the `mgcv` package (Wood, 2006) for R, using thin-plate regression splines (Wood, 2003). Quantiles are obtained using the `qgam` package (Fasiolo et al., 2021b) for R that follows Fasiolo et al. (2021a), who tune the GAMs on the pinball loss.

## 4.2. European Central Bank

### 4.2.1. Data

The short-term interest rate used for the European Central Bank (ECB) is the volume-weighted Euro Overnight Index Average (EONIA) rate, at which commercial banks borrow from each other with a maturity of one day. Because most contributors to EONIA have access to the deposit facility, EONIA eligible transactions declined with excess reserves. The short-term interbank rates are confined within an interest rate corridor, which comprises the standing facilities as its upper and lower edges (the ECB lending rates and ECB deposit rates). The EONIA is normalised ( $\tilde{r}_i$ ) between 0 and 1 as:

$$\tilde{r}_i = \frac{r_i - \text{Deposit Rate}_i}{\text{Lending Rate}_i - \text{Deposit Rate}_i}. \quad (3)$$

The rate is also averaged over the reserve maintenance period<sup>3</sup> to smooth out intra-period volatility.

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<sup>3</sup>This is the length of time during which the central bank requires banks and other depository institutions to maintain a specified level of funds.



The genuine marginal funding rate moved to transactions between counterparts with and without access to the deposit facility. EONIA was, thus, discontinued in October 2019 and was replaced with the Euro short-term rate (ETSR) that includes transactions between banks and counterparts without access to the ECB deposit facility. ESTR, on the other hand, is not bound by the deposit facility and may drift below the deposit facility rate.

Both macroeconomic and financial variables are considered. Monetary aggregate, domestic production, and consumer prices help capture the money demand (Lopez-Salido and Vissing-Jorgensen, 2022). Financial indicators such as volatility index, foreign exchange rate, sovereign bond yields, and credit default swap are used to gauge the impact from market sentiment and movement (El Gemayel et al., 2022). A detailed list of variables, data source, and their description is available in Table A.7. Data are collected from 1999 to 2019, resulting in 239 maintenance periods where all data is available.

#### *4.2.2. Using Only the Excess Reserves*

Table 2 summarizes the results when only excess reserves is used as an input, and the best parameter vector is used. The objective of this experiment is to evaluate the fitness of the curves when there are no additional covariates to model the data. The table provides the mean RMSE, MAE, and MIS over the 30-fold CV test errors. The columns "Valid" and "Test" provide the mean RMSE rank in the validation and test CV errors respectively. The best performing result in each column is highlighted in bold. The table provides the values for the metrics for the case that we select the best optimal set of parameters, and the percentage gains over these values when we use a combination of curve parameters. The "St. Dev" column reports the percentage reduction of the average standard deviation of distribution of the curve parameters over the CV samples.

In terms of accuracy (RMSE and MAE), the Arctangent performs best, followed by the other parametric curves. The non-parametric methods are close to the worst-performing parametric curves, with the linear being substantially less accurate than all alternative specifications. In terms of MIS, the random forest ranks first, with the parametric curves following, and the spline regression ranking last. To better understand the results we provide the different curves with their 90 percent intervals in Figure 7. The poor performance of the linear is apparent and expected. The Random Forest provides a very non-smooth approximation of the data, yet quite narrow intervals. The various non-linear curves exhibit different degrees of fitness, but uniformly fairly good intervals, reflected in the MIS values. In contrast, the Spline Regression achieves a good fit, yet rather wide intervals. This is an expected trade-off, as non-parametric methods require an abundance of data. This is reflected in the Spline Regression with increased modeling uncertainty, while in the Random Forest with non-smoothness. Arguably, the good fit of the spline regression comes at the cost that it does not exhibit the expected behaviour (observe the increase of interest rates for high excess reserves). The validation and test mean ranks exhibit high correlation (0.76), suggesting that our approach to selection is effective in this case in picking a useful model.

Proceeding to the percentage gains (negative values are losses) for the combination of curve parameters, with the exception of the reduced logistic and the arctangent, for the rest of the curves we observe a reduction in the standard deviation, typically with a small loss of accuracy, suggesting that the combination of parameters stabilises the estimates as intended.

#### *4.2.3. Using Additional Predictors*

Table 3 provides the summary performance statistics when using all possible covariates. We provide only the results for selecting the best set of parameter estimates,

Table 2. Out-of-Sample Performance for the ECB using Only Excess Reserves

Method	Selection					Combination (gains %)			
	RMSE	MAE	MIS	Valid.*	Test <sup>†</sup>	St. Dev.	RMSE	MAE	MIS
Logistic	0.0631	0.0495	0.1750	3.77	4.43	59.03	-2.82	-0.86	2.65
Red. Logistic	0.0637	0.0497	0.1740	<b>2.10</b>	4.33	-11.94	0.04	1.10	0.21
Double Exp.	0.0624	0.0489	0.1750	4.73	3.93	14.68	-6.93	-4.73	-26.80
Exponential	0.0639	0.0498	0.1770	3.20	4.47	2.24	-1.31	-1.13	-1.78
Arctangent	<b>0.0616</b>	<b>0.0484</b>	0.1730	4.50	<b>3.70</b>	-36.40	-7.89	-5.01	-7.04
Linear	0.0811	0.0694	0.2430	8.00	6.10	0.00	0.00	0.00	0.00
Random Forest	0.0676	0.0510	<b>0.1320</b>	6.70	5.10	-	-	-	-
Spline Regr.	0.0635	0.0487	0.3140	3.00	3.93	-	-	-	-

\*validation CV RMSE rank; <sup>†</sup>test CV RMSE rank

Source: Authors' Calculation

as the combination approach behaves similar to the previous case. The best performing approach per column is highlighted in bold.

Table 3. Out-of-Sample Performance for ECB using All Regressors

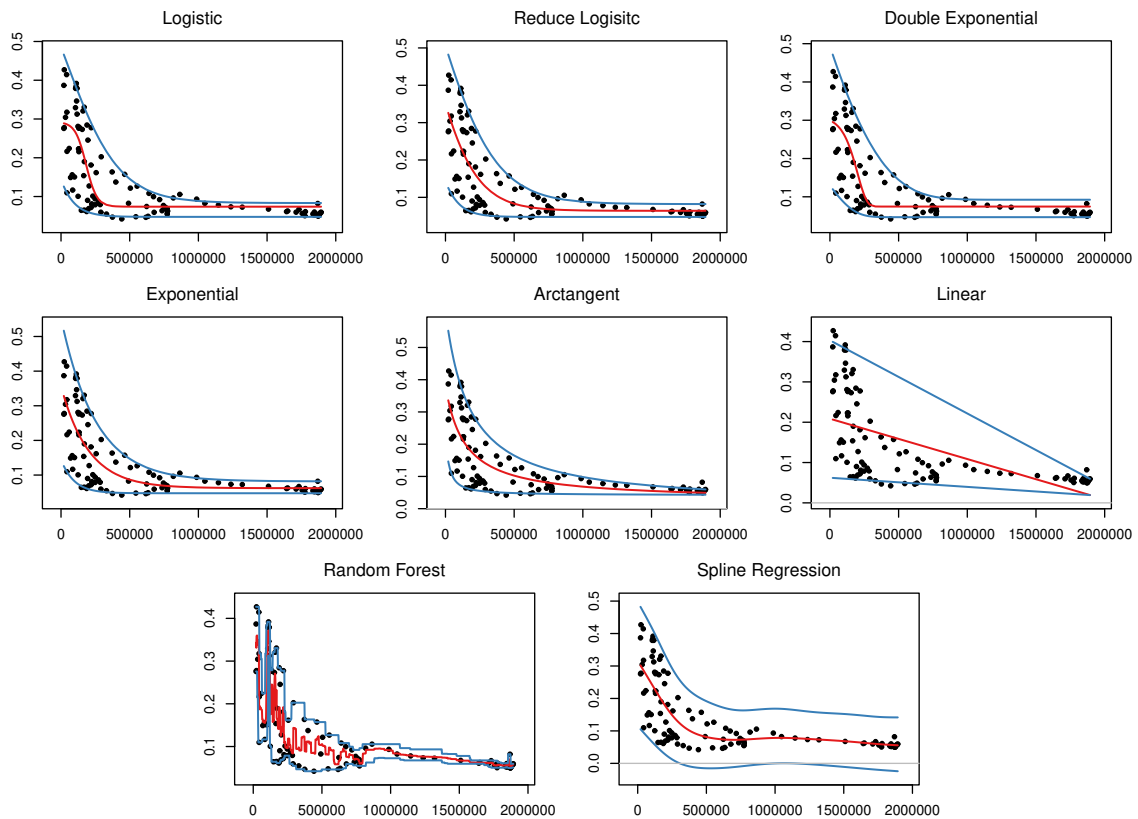
Method	RMSE	MAE	MIS	Valid*	Test <sup>†</sup>
Logistic	0.0471	0.0372	0.1840	2.97	4.00
Red. Logistic	0.0527	0.0417	0.3430	4.40	4.50
Double Exp.	0.0483	0.0389	0.1800	3.40	4.33
Exponential	0.0701	0.0591	47.5000	6.73	6.00
Arctangent	0.0653	0.0529	0.1950	6.60	6.13
Linear	0.0557	0.0459	0.2420	6.57	4.87
Random Forest	<b>0.0385</b>	<b>0.0295</b>	<b>0.1520</b>	<b>2.10</b>	<b>2.83</b>
Spline Regr.	0.0390	0.0320	0.1720	3.23	3.33

\*validation CV RMSE rank; <sup>†</sup>test CV RMSE rank

Source: Authors' Calculation

The random forest provides the most accurate results across RMSE, MAE, ME, and MIS, followed by the other non-parametric alternative, the spline regression. With the exception of the exponential and the arctangent curves, the performance of the parametric curves improves with the inclusion of additional regressors. To better understand the differences between the bivariate and the multivariate case, Figure 8 provides the beanplots of the RMSE and MIS for the two cases. Beanplots are rich

Figure 7. Fitted Parametric and Non-Parametric Alternatives, with 90 Percent Prediction Intervals



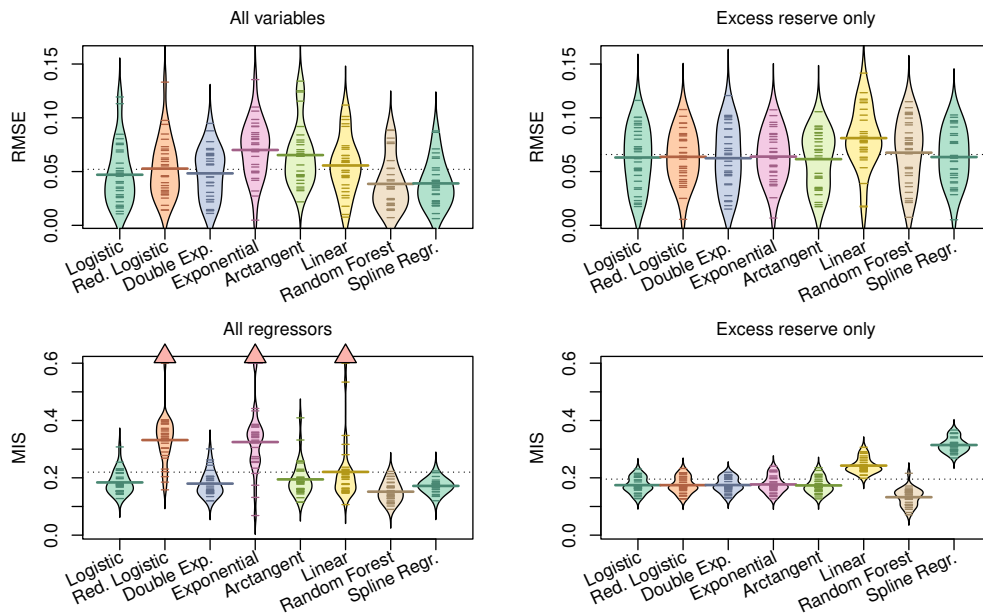
Source: Authors' Calculation

Notes: X axis is excess reserve in unit and y axis indicates normalized short-term interest rate.

variants of violin plots that provide an empirical representation of the distribution of the plotted quantities, together with indications of the actual values and means of different beans, allowing a better representation of outlying values and multimodal distributions (Kampstra, 2008). That is particularly for the case of MIS, the reduced logistic, exponential, and linear exhibit outliers. The better performing curves have more parameters and are able to make better use of the additional regressors. The error distributions are both narrower and shifted lower.

To explain the substantial improvement in the performance of the non-parametric methods, Figure 9 provides plots of the values for the random forest and the logistic

Figure 8. Beanplots for the ECB Data for MSE and MIS, using All Regressors or Only the Excess Reserve

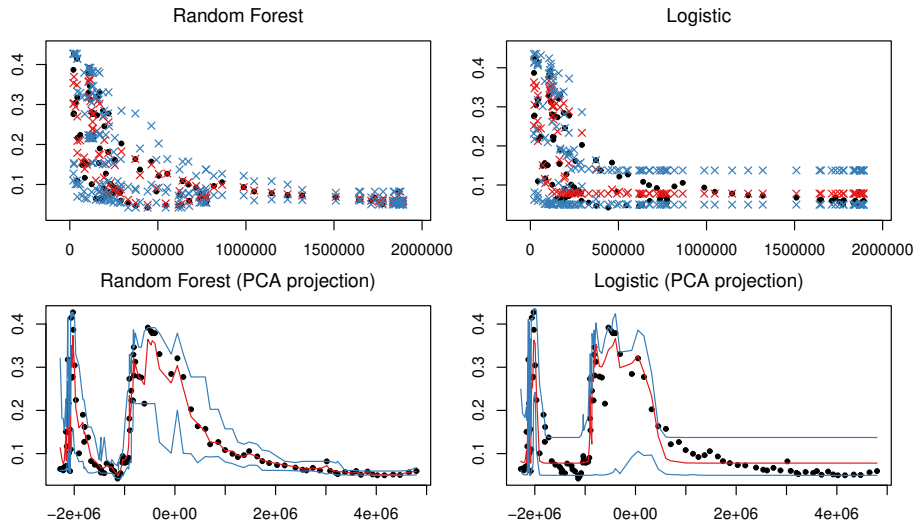


Source: Authors' Calculation

Notes: The beanplots are comprised by a violin plot, showing the empirical distribution of errors, with the narrow lines indicating observations, and the wider lines the mean of each bean. The  $\Delta$  indicates the presence of outliers beyond the plotted range. The inclusion of exogenous regressors improved forecast accuracy but not the distribution of the errors.

curves, the best performer overall and parametric curve respectively. The figure provides a plot of the model output (upper row) and there is a projection of the first two principal components, given the multivariate nature of the data. Note that the non-parametric curve takes advantage of the additional information, but similarly to Figure 7, it over-explains the variability in the data, suggesting a raise in the interest rates at high volumes of excess reserves that is contrary to the theory. Therefore, we interpret the non-parametric curves as useful methods to indicate how well the data could be explained in a theory-unrestricted case. When there is an abundance of data, we anticipate well performing curves to be between the non-parametric methods and the linear case.

Figure 9. Plots of the Random Forest and the Logistic Curves with all Selected Regressors and their PCA Projections



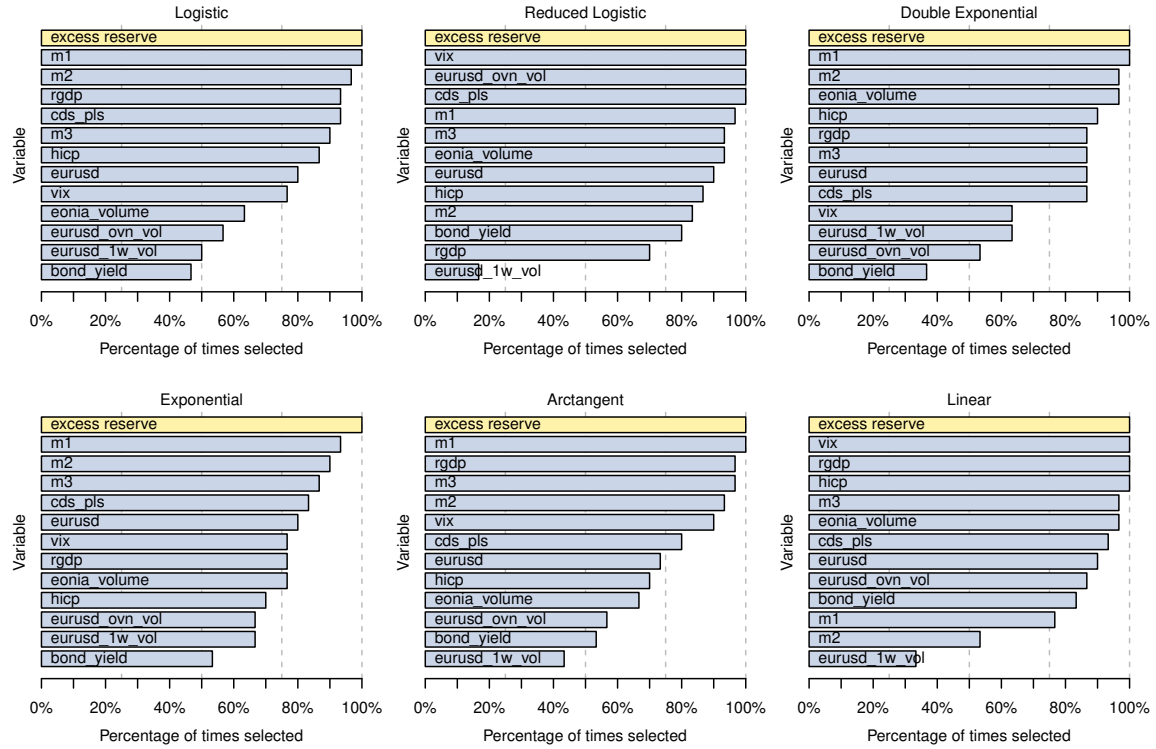
Sources: Authors' Calculation

Notes: X axis is excess reserve in unit and y axis indicates normalized short-term interest rate.

Figure 10 provides barcharts with the percentage of the times a variable was included, per curve, across the CV samples. Exogenous regressors aim at capturing factors that can influence the demand for reserves and are not under the central bank's control. They revolve around the insurance motive for holding reserve, the most liquidity asset, and the insurance cost (reserve pay less than less-liquid alternative assets). In agreement Lopez-Salido and Vissing-Jorgensen (2022), we find monetary aggregates (M1, M2, M3) to be useful, assuming that banks keep a certain amount of reserves to cover their deposits, real GDP growth to control for the increase in the volume of transactions in the economy, and the yield of alternative less-liquid assets such as government securities. Like El Gemayel et al. (2022), we find variables reflecting changes in risk perception, such as the CDS, VIX, and exchange rate volatility, useful. Concerns about loss of market access encourage banks to hoard more reserve, thereby increasing the demand for reserves. In addition, the estimate

includes variables corresponding to market functioning such as market turnover.

Figure 10. Barplots of the percentage of times (across CV samples) that variables were included for the ECB



Source: Authors Calculation

### 4.3. US Federal Reserve

#### 4.3.1. Data

Prior to the 2008 financial crisis, the US Federal Reserve intervened daily in the short-term fund market, absorbing or injecting reserves via the purchase or sale of government securities, as necessary to keep the Effective Federal Funds Rate (EFFR), which is a volume-weighted median of overnight federal funds transactions among commercial banks, near the Federal Reserve policy target (Ennis and Keister, 2008). In response to the financial crisis, which resulted in a large increase in demand for

reserves, the Federal Reserve tolerated larger excess reserve balance on accounts that were not remunerated before 2008. Therefore, it introduced a remuneration of excess reserves, the Interest Rate on Reserve Balance (IOER), to implement monetary policy by changing the IOER. Starting in 2013, the high level of excess reserves reduced transactions between counterparts with access to the IOER, and increased transaction between them and counterparts with no access. As a result, the floor system started "leaking" and the Federal Reserve introduced a new operation, the overnight reverse repurchase agreements (repo), at a slightly lower rate than the IOER (currently 10 basis point) that is available to counterparts with no access to the IOER (such as the Federal Home Loan Banks and money market funds) to cap by how much the EFRR could drift from the IOER. At end-April 2022, the amount deposited at the reserve repo reached levels close to the balance on the IOER account, making the former as important as the later regarding short-term rate convergence as excess reserves increases.

Regarding the normalization, we use the primary credit rate of the Federal Reserve as the ceiling and 0 as the floor. After 2008, we replace 0 by the IOER. Similar explanatory variables are collected, listed in Table A.7. In addition, we introduced a binary indicator to distinguish the period before and after the introduction of the overnight reverse repo program. Data are collected from 2003 to 2022 and are of weekly basis, resulting in 1042 observations where all variables are available.

#### *4.3.2. Using Only the Excess Reserves*

The presentation of the results follows from Section 4.2, with Table 4 summarising the results for the case that uses only excess reserves, with Figure 11 providing the corresponding plots. For the FED case the non-parametric spline regression appears to perform best in terms of accuracy, closely followed by the parametric curves that all exhibit similar performance. However, in terms of MIS, which evaluates the uncer-



tainty of the estimated curve, spline regression performs on average about 32% worse. In Figure 11 we can observe that neither non-parametric methods follow the expected behaviour, with the excess reserve curve exhibiting multiple increases and decreases. Therefore, the good summary statistics are misleading. It is also evident from both the figure and the summary statistics that the random forest has overfit the data. We can also observe for the parametric curves that the indicator variable influences the mean prediction and the quantiles differently. In all cases the mean exhibits a shift as expected, modeling the "leaky" floor described in Section 4.3.1. Finally, when we look at the percentage gains from using the combined curve parameters, the behaviour is similar to the one for the ECB case, with the parameters becoming less volatile on average, at the cost of accuracy.

Table 4. Out-of-Sample Performance for the FED using Only Excess Reserves

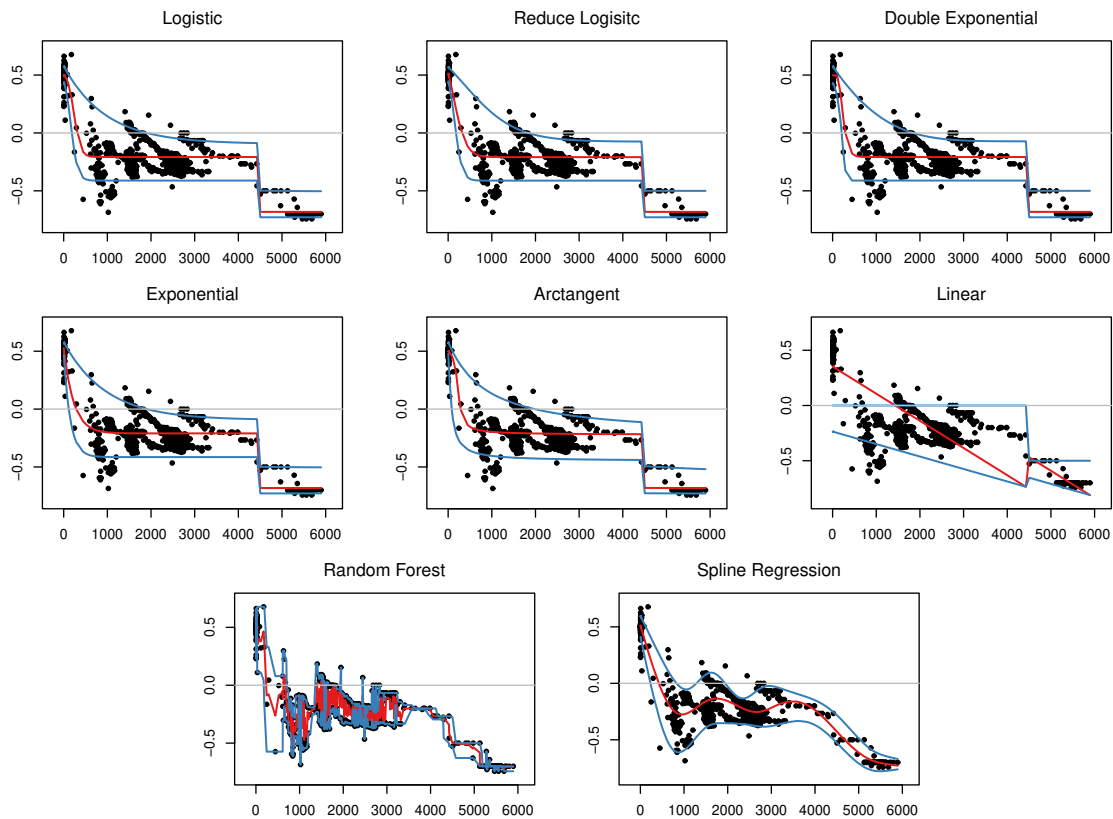
Method	Selection					Combination (gains %)			
	RMSE	MAE	MIS	Valid.*	Test†	St. Dev.	RMSE	MAE	MIS
Logistic	0.1150	0.0785	<b>0.3450</b>	2.97	3.60	43.29	-0.91	-4.25	-2.41
Red. Logistic	0.1150	0.0790	0.3460	3.30	4.00	-1.31	0.02	-0.50	0.01
Double Exp.	0.1150	0.0787	<b>0.3450</b>	3.93	3.40	17.28	-7.59	-11.27	-115.80
Exponential	0.1160	0.0807	0.3490	5.57	5.00	23.98	-1.11	-2.75	-1.09
Arctangent	0.1160	0.0786	0.3570	4.60	3.97	-38.39	-3.03	-5.42	-1.39
Linear	0.2040	0.1550	0.7180	8.00	8.00	0.00	0.00	0.00	0.00
Random Forest	0.1360	0.1130	0.8210	6.63	6.97	-	-	-	-
Spline Regr.	<b>0.1080</b>	<b>0.0753</b>	0.3940	<b>1.00</b>	<b>1.07</b>	-	-	-	-

\*validation CV RMSE rank; †test CV RMSE rank  
Source: Authors' Calculation

#### 4.3.3. Using Additional Predictors

Table 5 summarizes the results for the Federal Reserve when additional regressors are used. Across all methods we observe improvements in accuracy. The non-parametric methods remain best, with the random forest ranking first. This is matching the ECB case, and likewise the non-parametric methods provide a fit that does

Figure 11. Fitted Parametric and Non-Parametric Alternatives, with 90 Percent Prediction Intervals



Source: Authors' Calculation

Notes: X axis is excess reserve in unit and y axis indicates normalized short-term interest rate. The higher variance in negative rate range in the US compared to the Euro area reflect an implicitly narrower interest rate corridor in the former compared with the later (see section 4.3.1. for a description of the operational framework).

not follow the expectation from theory. From the parametric methods the arctangent ranks first, followed by the reduced logistic, and the logistic. Observe that the improvements in accuracy are not reflected in the MIS, which improves only for the non-parametric methods. For the remaining, the upper interval remains wide, resulting in the high MIS values.

Figure 12 summarizes the selection of regressors similarly to Figure 10. We observe that some regressors, such as *m1* and *hicp*, are consistently selected across curves,

Table 5. Out-of-sample performance for Federal Reserve using all regressors

Method	RMSE	MAE	MIS	Valid*	Test†
Logistic	0.1040	0.0708	0.5840	5.30	5.20
Red. Logistic	0.1000	0.0702	0.7160	4.60	4.57
Double Exp.	0.1070	0.0740	0.5910	6.53	5.73
Exponential	0.1030	0.0720	0.7620	5.60	4.97
Arctangent	0.0984	0.0668	0.5480	3.00	4.43
Linear	0.1360	0.0921	1.0100	7.97	7.87
Random Forest	<b>0.0468</b>	<b>0.0252</b>	<b>0.1350</b>	<b>1.00</b>	<b>1.03</b>
Spline Regr.	0.0671	0.0402	0.2540	2.00	2.20

\*validation CV RMSE rank; †test CV RMSE rank

Source: Authors' Calculation

while others, such as the *m2* and *rgdp*, are consistently selected only by some curves.

The overall conclusions for the selection of the variables are similar to the ECB.

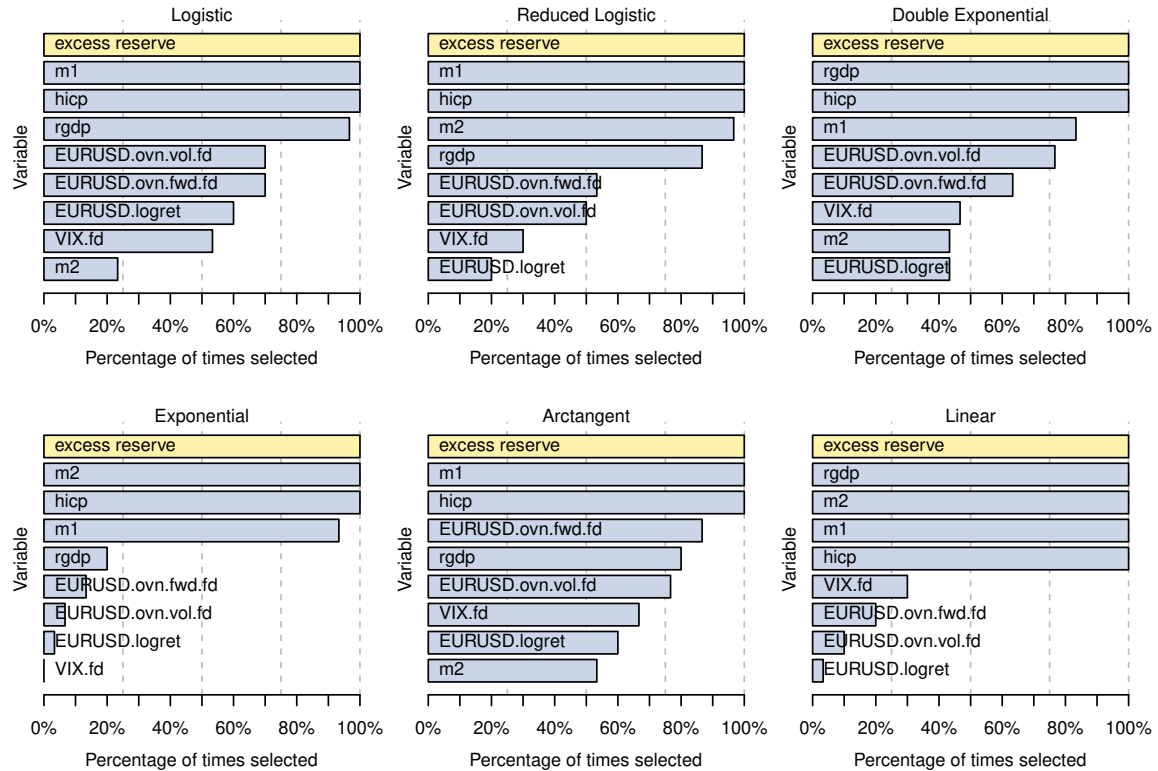
With the Federal Reserve case we demonstrate the efficacy of the inclusion of an indicator to model changes in the excess reserve curve, both in the bivariate and the multivariate case. In Section 5.2 we investigate further the stability of the estimated curve, given the changes that have occurred over the data sample.

## 5. Discussion

### 5.1. Scenario Analysis

The modeling objective is to provide tools for the central banks to calibrate their decisions and operations. To this end, the ability to analyze different scenarios becomes useful for making explicit any implicit assumptions of policy makers. In Figure 13 we exemplify this by providing two scenarios for the ECB case. The alternative scenario assumes parity between USD and euro exchange rate, differing from the baseline that is based on a stronger euro. As discussed in Section 3, each scenario is accompanied by its upper and lower quantiles that correspond to the implied uncertainty. When transitioning from the baseline to the alternative scenario, the curve is shifted, implying different stable points, but also the shape and width of the

Figure 12. Barplots of the Percentage of Times (across CV Samples) that Variables were Included for the Federal Reserve.

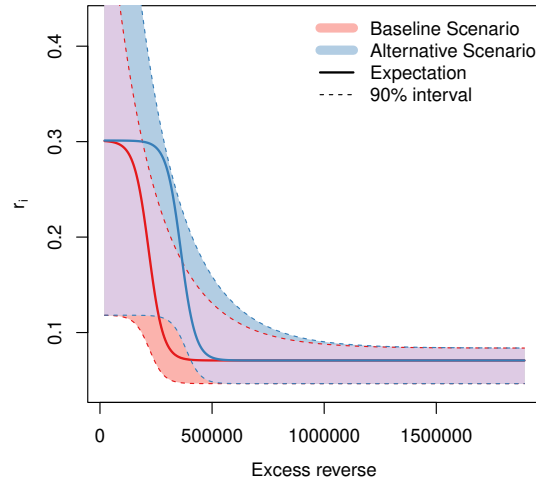


Source: Authors' Calculation

uncertainty changes. This provides a more complete view to the decision makers than merely a change in the location of the curve.

It is beyond the scope of this study to investigate the best ways to generate these scenarios. Nonetheless, it is relevant to highlight potential limitations in the scenario analysis. As discussed in the challenges associated with the parameter estimation of the curve, there may be multiple sets of parameters that correspond to similar looking curves (see Figure 3). This implies that the expectation of the analyst on the impact of a change of a variable may be invalidated. A remedy to this effect is to impose restrictions on the signs of the coefficients for the covariates. This will restrict the

Figure 13. Example of Two Scenarios with Their Uncertainties



Source: Authors' Calculation

solution to the one that corresponds to the economic understanding of the analyst and eliminate other approximations.

A further consideration is the generation of the scenarios themselves. In the provided example we vary the exchange rate between USD and euro by assuming ad-hoc parity. In practice, the analyst is expected to provide a vector of the future values of the different variables that are used by the curve. For some, it is reasonable to assume that they will remain unchanged, while for others forecasts will be required. These forecasts may be judgement or model based, and ideally will have corresponding predictive distributions. Therefore, the analyst can provide expectations, or quantiles of the desired probabilities as inputs for the scenario generation. By evaluating the scenarios for the quantiles, the complete uncertainty due to the curve modeling and the forecasts of the variables will be revealed.

Finally, a limitation of our approach is that it cannot guarantee that the quantiles of the scenarios will not cross the mean prediction or the other quantile. In the parameterisation of the baseline case there are restrictions to ensure that this does

not happen, however, due to the nonlinear nature of the curves, this does not ensure that different values used in the scenarios, which were not seen in the fitting data, will not result in this problem.

### *5.2. Stability of Reserve Demand Curve*

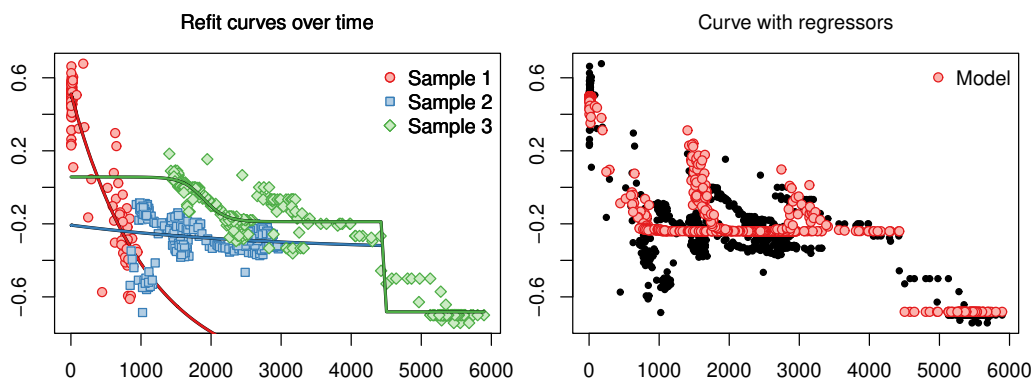
In Section 4.3.1 we outlined changes that have occurred over time in the Federal Reserve case, with a reasonable question being, whether the curve has shifted over time, or conversely whether it is reasonable to assume a consistent curve and use the complete sample for its estimation. Afonso et al. (2022) argue that the curve has shifted over time, however they do not provide an empirical evaluation of the fitted curves and instead demonstrate this through clustering of the observations. Our methodology allows for an investigation of this question, and we argue that the inclusion of additional regressors effectively explains changes in the curve.

To demonstrate this we split the available sample into three consecutive periods, and fit separately bivariate curves to each. We contrast that with a multivariate fit across all periods that relies on additional regressors. Figure 14 visualizes the results for the logistic curve. We argue that although the bivariate solutions suggest a shift over time, in agreement with Afonso et al. (2022) who also demonstrate this in the bivariate case, when additional regressors are available, a single curve is sufficient. Therefore, we argue that the regressors effectively control for the changes in the reserve demand, and that the curve has not fundamentally shifted.

### *5.3. Endogeneity*

Considering the mechanism of the reserve demand, it can be argued that the excess reserve and the short term rates are endogenous (for a discussion, see Afonso et al., 2022). These confounding factors are expected to be high-frequency; for example, central banks will react within days to increased volatility in the money market by

Figure 14. Comparison of Biivariate Logistic Curve Fit in Three Consecutive Samples and a Single Multivariate Fit



Source: Authors' Calculation

changing the supply of reserves. In principle, it is easy to include lags of the rate as regressors to cater for the endogeneity. However, there are two relevant considerations. First, endogeneity is critical for inference purposes. If it is not accounted for, it acts as an omitted variable, biasing the remaining estimates, and by extension, the selection of regressors. Due to the nonlinear nature of the sigmoid curves we do not rely on inferential statistics, but rather on a predictively focused CV procedure. The nonlinearity also limits the interpretability of any single coefficient. Therefore, endogeneity is relevant to our methodology in indicating potentially helpful additional inputs, i.e., lags of the rate, to improve the CV errors, and therefore the quality of the final curves. Second, as we are not modeling daily data, these high-frequency confounding factors may be unobservable, and instead evident in the additional regressors.

To assess the importance of including lags of the rate we repeat the empirical evaluation by providing the curves with this additional input. A better out-of-sample performance will indicate the importance of treating endogeneity in the curves. Table 6 provides the percentage gains (loses in negative) over the results from Table 3 for the ECB. For the majority of cases this leads to a loss of performance. On the

other hand, the linear case improves by almost 20%. This supports our initial assessment that for the various curves there is little benefit to be had, partially due to their nonlinearity, and partially due to the inclusion of the rest of the regressors.

Table 6. Percentage Gains by Including Endogeneity

Method	RMSE gain %
Logistic	-10.75
Red. Logistic	2.19
Double Exp.	-8.94
Exponential	-3.72
Arctangent	9.14
Linear	19.36
Random Forest	-0.98
Spline Regr.	-19.94
Average	-1.71
Average w/o Linear	-4.71

Source: Authors' Calculation

## 6. Conclusion

The control over the short-term interest rate is paramount for monetary policy implementation and depends on estimating the demand for reserves. Given its importance, its modeling has received surprisingly limited attention in the literature. This work aims to address this by introducing a complete modeling framework to identify the appropriate demand curve, and useful regressors, and provide estimates of quantiles that reflect the reserve demand uncertainty for different conditions. The proposed parametric curves were contrasted with non-parametric approaches, demonstrating the merits of each in two cases from the Euro area and the US. Finally, our approach facilitates scenario analysis, where we also contribute by providing predictive distributions that reflect the conditions imposed by the various scenarios.

The work opens multiple avenues of future research. First, given the capabilities to investigate the outcomes of scenarios, their generation becomes of interest.



Our modeling approach allows for estimating the reserve demand given a variety of regressors. However, the more complex the model, the more complex the scenario generation can become. Therefore, future work could make use of the model to explore different strategies for scenario generation. Crucially, as discussed above, our approach allows incorporating quantitatively the uncertainty of regressors that is typically overlooked in scenarios. On the other hand, our approach has the limitation that it cannot guarantee that the quantiles will not cross in all scenarios. The model contains appropriate restrictions to ensure this is not an issue in fitting to the data, but the nonlinear nature of the curves does not guarantee that generally. This remains an open question in non-linear quantile models more generally.

It is demonstrated that the nonlinear nature of the parametric curves introduces estimation challenges. Various steps to fortify against these issues have been proposed, however, these come at a substantial computational cost. Arguably, as our approach is easily parallelisable this is not a practical issue, however it opens the question of alternative estimation procedures, potentially making use of Bayesian or shrinkage estimators.

Finally, with this work, we aim to draw the attention of the wider modeling community to the problem of reserve demand modeling. The supported decisions are both high-impact and frequent enough to warrant more research in the area.

## Appendix A. Additional Regressors

Table A.7 lists the variables used in the two case studies. The first column provides the name of the variable, the second and third columns indicate whether this was used for modeling the ECB or Federal Reserve case, followed by the abbreviated data source, and finally a description. For the euro area the CDS is the first component of a partial least square regression analysis on Italy, Netherlands, Germany, Portugal, Belgium, France, and Spain CDS. In the case of the Federal Reserve the CDS, US 10 Year Yield, and Fed Fund Transaction Volume are eventually excluded from the analysis due to the limited range of periods for which we could obtain data.

Table A.7. Additional Regressors

Variable	ECB	Fed	Data*	Description
CDS	✓	✓	B	Credit Default Swaps
EONIA Volume	✓		E	Overnight interbank lending volume in the euro area
EUR/USD	✓	✓	B	EUR/USD foreign exchange rate
EUR/USD Forward Overnight		✓	B	Forward points for EUR/USD overnight prices
EUR/USD Implied Volatility	✓	✓	B	Implied overnight volatility derived from currency options prices. For ECB also 1 week
Fed Fund Transaction Volume		✓	F	Overnight interbank federal fund transaction volume in the US
HICP <sup>†</sup>	✓	✓	E/F	Harmonized Index of Consumer Prices in the Euro area/US.
M1, M2 and M3 <sup>†</sup>	✓	✓	E/F	Monetary aggregates in the Euro area
RGDP <sup>†</sup>	✓	✓	E/F	Real GDP in the Euro area/US
Sovereign Bond Yield Difference	✓		B	Difference between German 10 Year Bond Yield and Italy 10 Year Bond Yield
US 10 Year Yield		✓	B	US 10 Year treasury notes yield
VIX	✓	✓	B	Chicago Board Options Exchange's CBOE Volatility Index

\* Data source. B: Bloomberg, E: ECB, F: FRED

<sup>†</sup> Linear interpolation is employed to convert to higher frequency

Source: Authors' Calculation

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