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The Economics of Sovereign Debt, Bailouts, and the Eurozone Crisis

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ABSTRACT: Despite a formal 'no-bailout clause,' we estimate significant net present value transfers from the European Union to Cyprus, Greece, Ireland, Portugal, and Spain, ranging from roughly 0.5% (Ireland) to a whopping 43% (Greece) of 2010 output during the Eurozone crisis. We propose a model to analyze and understand bailouts in a monetary union, and the large observed differences across countries. We characterize bailout size and likelihood as a function of the economic fundamentals (economic activity, debt-to-gdp ratio, default costs). Our model embeds a 'Southern view' of the crisis (transfers did not help) and a 'Northern view' (transfers weaken fiscal discipline). While a stronger no-bailout commitment reduces risk-shifting, it may not be optimal from the perspective of the creditor country, even ex-ante, if it increases the risk of immediate insolvency for high debt countries. Hence, the model provides a potential justification for the often decried policy of 'kicking the can down the road.' Mapping the model to the estimated transfers, we find that the main purpose of the outsized Greek bailout was to prevent an exit from the eurozone and possible contagion. Bailouts to avoid sovereign default were comparatively modest.

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Prepared by Prepared by Pierre-Olivier Gourinchas, Philippe Martin, and Todd Messer [1]

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1 Introduction.

'The markets are deluding themselves when they think at a certain point the other member states will put their hands on their wallets to save Greece.'
ECB Executive Board member, Júrgen Stark (Reuters, 2010, January 10)

'The euro-region treaties don't foresee any help for insolvent countries, but in reality the other states would have to rescue those running into difficulty.'
German finance minister Peer Steinbrueck (The Financial Times, 2009, February 18)

'No, Greece will not default. Please. In the euro area, the default does not exist.'
Economics Commissioner Joaquin Almunia (Reuters, 2009, January 29)

As the Eurozone crisis of 2010-2015 highlighted, a potential default on government debt within a monetary union comprised of sovereign members involves unique features that affect the potential costs and benefits to debtor and creditor countries. A monetary union facilitates financial integration. With large within-union cross-border holdings of financial assets, including government debt held by banks, the exposure of creditor countries inside the union to sovereign risk is high. In addition, monetary and economic unions usually go hand in hand, fostering real integration. Should a country experience a severe fiscal crisis, this has the potential to disproportionately disrupt its trading partners inside the union. Further, a sovereign default could be the first step towards a potential exit from the monetary union, potentially impairing the credibility of other members or of the monetary union as a whole. For creditor countries inside the union, there is significant direct and indirect exposure if a fiscally weak member defaults.

Should these costs for the group of creditor countries exceed the resources the debtor country needs to avoid a default, a unilateral transfer -in the form of a bailout- might be a preferred outcome for the union, helping to achieve (ex-post) efficiency. It follows that a monetary union, through the direct and collateral linkages it generates among members, creates conditions where creditor countries may desire (ex-post) to bail-out weaker members. For the defaulting country, not surprisingly, the possibility of future bailouts distorts (ex-ante) incentives to issue debt by softening the debtor's budget constraint. This results in excessive debt issuance that can make the union more fragile. Such problems did not end with the Eurozone debt crisis. More recently, discussions about unilateral transfers in the Eurozone have been revived in the context of the COVID-19 crisis (Bénassy-Quéré, Corsetti, Fatás, Febelmayr, Fratzscher, Fuest, Giavazzi, Marimon, Martin, Pisani-Ferry et al., 2020).

This paper empirically and theoretically investigates the sizes and determinants of unilateral transfers within a monetary union. Empirically, we document the support that fragile Eurozone countries received during the Eurozone crisis. We show substantial heterogeneity in the sizes of these bailouts across five Eurozone countries: Cyprus, Greece, Ireland, Portugal, and Spain. We then present a model that captures the trade-off between ex-post bailout and ex-ante borrowing incentives. The model allows us to characterize the likelihood and size of bailouts in equilibrium as a function of country characteristics. The model also explores the conditions under which it is beneficial (ex-ante) for the creditor country to allow for the possibility of (ex-post) bailouts.

We estimate the implicit transfers arising from official European Union financing to five Eurozone crisis countries and three non-Eurozone countries. These countries received funding from euro area organizations and the International Monetary Fund (IMF). We comb official sources to construct country-lender specific disbursements, interest payments, and repayment profiles (both realized and expected). If the Eurozone programmes simply provided risk-free funding at the market risk-free rate, there would be no implicit subsidy and bailouts would be zero. More generally, these programmes include an implicit subsidy (bailout) if they charge a rate that is below the risk-adjusted market rate reflecting the risk of the programme.

Our methodology estimates the subsidy component relative to that risk-adjusted counterfactual. A key assumption is the choice of a relevant risk-adjusted market rate. It is immediate that this should not be the current market rate since official lending has generically a much lower risk profile. Instead we use the internal rate of return on the International Monetary Fund (IMF) lending to these countries.[1] This assumption is justified since IMF programs are super senior, even relative to the European Union institutions. Importantly, this assumption yields a *lower bound* on the true size of transfers from the European Union to recipient countries for at least four reasons. First, since the IMF is super senior, the true risk-adjusted rate for European institutions is likely to be higher than the IMF. Second, IMF programs are relatively short term (between three and nine years) compared to European packages, with a duration ranging from 10 years to 30 years. Adjusting the IMF internal rate for a term-premia component would increase the estimates of the transfers. Third, IMF program themselves may include a transfer component, although the evidence in Joshi and Zettelmeyer (2005) suggests that they do not incorporate a substantial transfer component, except for concessional lending to poor and highly indebted countries. Fourth, we ignore any potential transfer component arising from the European Central Bank policies such as the Securities Market Program or the Asset Purchase Program.

Our estimates indicate substantial variation in the implicit transfers received by the fiscally weak Eurozone countries, from roughly 0.4 percent of output for Ireland or Spain, to roughly 3 percent of output for Cyprus and Portugal, to a very substantial 43.7 percent of output for Greece. By contrast, non-Eurozone countries that received a transfer, such as Hungary, Latvia[2] or Romania, did not receive any measurable direct transfer from the European Union. It is clear, based on these estimates, that unilateral

[1] Spain did not have an IMF program, so we use an average of the IMF's internal rate of return for the other four countries.
[2] Latvia adopted the Euro on January 1, 2014.

transfers did happen, or said differently, that the no-bail out clause was not enforced.

To understand the nature of bailouts in the Eurozone, we then present a simple model that captures the trade-off between ex-post bailouts and ex-ante borrowing incentives inside a monetary union. The model features three countries. Two countries are members of a monetary union, one of which is fiscally strong and the other fiscally fragile. A third country represents the rest of the world, outside the monetary union. Each region issues sovereign debt and private portfolio holdings are determined endogenously. A sovereign default inflicts direct costs on bondholders, but also indirect costs on both the defaulting country and its monetary union partner. The structure of these collateral costs, together with the realization of output and the composition of portfolios, determine the conditions under which the fiscally strong country may prefer to bailout its fiscally weak partner.

Our model generates a number of important results. First, because of the indirect costs a default inflicts on the creditor country, it is always preferable for that country to bailout the debtor country (i.e. an external bailout), rather than absorb the losses of its own bondholders (an internal bailout). Second, the fiscally strong country realizes that any bailout also serves to repay debt held by the rest of the world. This limits that country's willingness to offer financial assistance. Third, while ex-post bailouts are ex-post efficient from the joint perspective of the two members of the monetary union, they transfer all the surplus to the fiscally strong country, leaving the debtor country no better off than with a default. We call this the *'Southern view'* of the crisis. That financial assistance to a country that is close to default does not improve its fate may seem surprising. However, in the absence of political integration, there is no reason creditor countries would offer more than the minimal transfer required that leaves the debtor country indifferent between default and no default. Seen through the lens of our model, even if Greece receives a transfer equal to 44 percent of its output, this does not necessarily make Greece better off ex-post.

However, the anticipation of a bailout creates ex-ante a typical moral hazard problem: a higher likelihood of a bailout lowers the effective cost of borrowing for the fiscally weak country, generating excessive borrowing, at the expense of the fiscally strong country. We call this weakening of ex-ante fiscal discipline the *'Northern view'* of the crisis. In the context of the Eurozone crisis, this position has been articulated many times by the German Treasury and other countries who have pointed to fiscal laxity as a root cause of the crisis in some countries. Our analysis very naturally reconciles the *'Northern'* and *'Southern'* views of the crisis as the two sides of the same coin: risk shifting by the debtor country occurs in the first period because of the anticipation of the bailout, even if, ex-post, the creditor country captures all the efficiency gains from avoiding a default. This suggests a simple fix: if the creditor country could credibly commit to a *no bail-out clause*, this would eliminate ex-ante risk shifting and over-borrowing.

A central result of our analysis is that such commitment may not be optimal, even from the ex-ante perspective of the creditor country. Instead, we find that if the debtor country's initial debt level is sufficiently elevated, the fiscally strong country may prefer not to commit to a no-bailout clause. The reason is that the announcement of a strict no-bailout clause would push the debtor country into immediate insolvency, with indirect costs for the creditors. Instead, if a future bailout remains possible, the debtor country may be able to roll-over its initial debt today, avoiding immediate default. While this can lead to excessive borrowing, the scope for risk shifting is more limited, the higher the initial debt is, as the debtor country finds itself already close to the maximum of its Laffer curve. Hence the fiscally strong country faces a meaningful trade-off between the immediate insolvency of the borrower and a rollover with the possibility of a future default. Thus, our model provides conditions under which it is optimal for creditor countries to 'kick the can down the road,' in official EU parlance, by remaining evasive about the strictness of the no-bailout clause. This part of our model provides a way to interpret what happened between 2000 and 2008 when spreads on sovereign debts in the Eurozone were severely compressed.

We then allow for two separate decisions by fiscally weak countries: whether to default on sovereign debt and whether to exit the monetary union. This extension is motivated by the fact that Greece received the largest transfer from EU institutions in 2012, the year it defaulted on its debt. In the extended model, a country can receive a bailout in order to avoid exit, even if it defaults on its sovereign debt, as was the case in Greece. Viewed through the lens of the extended model, the supersized 2012 Greek bailout was not intended to prevent a default on public debt, but was necessary to ensure that Greece would remain in the Eurozone. As in our baseline model, however, the ex-post welfare gains from that transfer accrue entirely to the creditor countries inside the Eurozone. The model delivers sharp characterizations of the size of ex-post bailouts, and whether they are feasible, as a function of a few key observable such as the initial debt-to-gdp ratio, the size of the recession, and the share of the debt held domestically or by the rest of the world. We map the transfers implied by the model to the empirically estimated transfers to show the model performs reasonably well in matching the observed response to the Eurozone debt crisis.

Following a review of the literature, the remainder of the paper is organized as follows. In Section 2, we review how bailouts unfolded during the Eurozone debt crisis in the different countries and estimate transfers implicit in lending from European countries to Greece, Ireland, Portugal, Cyprus and Spain. The possibility of such transfers is a key element of our theoretical model which we present in Section 3. Section 4 analyzes the incentives for defaults and bailouts and Section 5 studies how these incentives shape optimal debt issuance. Section Section 6 incorporates the possibility exit the Eurozone to relate the model back to the empirically estimated transfers. Section Section 7 concludes.

Literature Review

The theoretical literature on sovereign debt crises has focused on the following question: why do countries repay their debt? Two different approaches have emerged (see the survey by Bulow and Rogoff (2015)). On the one hand, Eaton and Gersovitz (1981) focus on the reputation cost of default for countries that value access to international capital markets to smooth consumption. On the other hand, Cohen and Sachs (1986), Bulow and Rogoff (1989a), Bulow and Rogoff (1989b) and Fernandez and Rosenthal (1990) focus on the direct costs of default in terms of disruption of trade for example. Our model clearly belongs to this second family of models. Output loss for the country that defaults comes from trade and financial disruptions but may also come from the risk of exit from the monetary union. Empirically, Rose (2005) shows that debt renegotiation results in a decline in bilateral trade of around 8 percent a year, which persists for around 15 years. Collateral damage of a sovereign default plays an important role in our analysis of the euro crisis. We are not the first to make this point. Bulow and Rogoff (1989a) show that because protracted debt renegotiation can harm third parties, the debtor country and its lenders can extract side-payments. Mengus (2021) shows that if the creditor's government has limited information on individual domestic portfolios, direct transfers to residents cannot be perfectly targeted so that it may be better off honoring the debtor's liabilities.

Cross-border linkages are increasingly being incorporated into models of sovereign default. Our paper analyzes how linkages in a monetary union can lead to ex-post solidarity, and connects most closely to Tirole (2015) and Farhi and Tirole (2018). Tirole (2015) investigates ex ante and ex post forms of solidarity. Farhi and Tirole (2018) adds a second layer of bailout in the form of domestic bailouts of the banking system by the sovereign to analyze the "deadly embrace" or two-way link between sovereign and financial balance sheets. As in our paper, the impacted countries may stand by the troubled country because they want to avoid the collateral damage inflicted by the latter. However, our paper builds on these two papers along a number of important dimensions. First, we show how ex-post solidarity may not generate any welfare gain for the debtor country. Second, our analysis includes a third country that is not a member of the monetary union. This allows us to explore important issues related to debt re-nationalization and debt monetization. Second, we explore how exit, and not just default, affects optimal debt issuance and the formation of bailouts. Finally, we are able to bridge theory with empirics by connecting the model to estimated bailouts in order to test the theoretical predictions of our model. As in Horn, Reinhart and Trebesch (2020), financial integration is important in determining the scope of bailouts.

Beyond these two papers, our work relates to other studies focusing on the Eurozone debt crisis.[3]

[3] A subset of this literature focuses on bank bailouts and country solidarity. Dovis and Kirpalani (2020) also analyze how expected bailouts change the incentives of governments to borrow but concentrate on the conditions under which fiscal rules can correct these incentives in a reputation model. Niepmann and Schmidt-Eisenlohr (2013) analyze how bank

Broner, Erce, Martin and Ventura (2014) analyze the Eurozone sovereign crisis through a model which features home bias in sovereign debt holdings and creditor discrimination. Our model shares with Broner et al. (2014) the first feature but not the second. In their model, creditor discrimination provides incentives for domestic purchases of debt which itself generate inefficient crowding-out of productive private investment. Uhlig (2014) analyzes the interplay between banks holdings of domestic sovereign debt, bank regulation, sovereign default risk and central bank guarantees in a monetary union. Contrary to this paper, we do not model banks explicitly but the home bias in sovereign bonds plays an important role in the incentive to default. Our model also relates to Corsetti, Erce and Uy (2018) who introduce official lending institutions in a model of sovereign debt and default motivated by the euro crisis but focus on different questions as they analyze how official lending and its conditions (maturity and rates) may restore debt sustainability and affect a government's decision to default. A related paper is also Dellas and Niepelt (2016) who show that higher exposure to official lenders improves incentives to repay due to more severe sanctions but that it is also costly because it lowers the value of the sovereign's default option. Our model does not distinguish private and official lenders.

Our paper is also related to Azzimonti and Quadrini (2021). Like us, that paper explores conditions under which bailouts can be Pareto improving not just ex-post but also ex-ante. Their paper emphasizes a different mechanism than ours: in their model, public debt provides public liquidity and is undersupplied in equilibrium. The promise of a bailout increases the ex-ante supply of debt, which can be Pareto improving. Our paper offers a complementary analysis, focusing on the trade-off the creditor country faces between triggering immediate insolvency or a higher likelihood of future default. Our paper also emphasizes the importance of the 'rest of the world,' in shaping equilibrium bailouts. Finally, we offer empirical estimates of the bailouts in the eurozone crisis and relate them to the fundamentals of our model.

Since the seminal paper of Calvo (1988), a large part of the literature on sovereign default has focused on an analysis of crisis as driven by self-fulfilling expectations (see for example Cole and Kehoe (2000)).[4] In this framework, the crisis abates once the ECB agrees to backstop the sovereign debt of Eurozone members. We depart from this literature and do not focus on situations with potential multiple equilibria or liquidity issues. This is not because we believe that such mechanisms have been absent. In a framework where the crisis is solely driven by self-fulfilling expectations, the bad equilibrium can be eliminated by a credible financial backstop and transfers should remain "off the equilibrium path". However, we

bailouts are affected by cross-border contagion costs.

[4]For example, de Grauwe (2012), Aguiar, Amador, Farhi and Gopinath (2015) and Corsetti and Dedola (2016)) interpret the episode as a rollover crisis where some governments (e.g. Spain) experienced a liquidity crisis. An important difference between Aguiar et al. (2015) and our work is that they exclude the possibility of transfers and concentrate on the lack of commitment on monetary policy that makes the central bank vulnerable to the temptation to inflate away the real value of its members' nominal debt. We view the lack of commitment on transfers as an distinctive feature of a monetary union and analyze the interaction between the monetary policy and transfers in a situation of possible sovereign default.

will show in the next section that transfers (from the EFSF/ESM) to the periphery countries have been substantial and not only to Greece.

2 Implicit Transfers During the Euro area crisis

This section briefly describes the financial assistance programmes for the major crisis countries (Cyprus, Greece, Ireland, Portugal, and Spain). We also review the terms of EU financial assistance programmes to three non-Eurozone members of the European Union (Hungary, Latvia and Romania). We refer the reader to Table A.4 in Appendix A and the Online Appendix for details of each program, to Corsetti, Erce and Uy (2020) for a description of the development of a euro area crisis resolution framework, and to Abbas, Arnold, Dacheva, De Broeck, Forni, Guerguil and Versailles (2014) for further discussion on the causes and development of the euro area crisis..

2.1 Historical Overview

Bailouts in the Euro area. The founders of the European Monetary Union sought explicitly to limit the potential for excessive debt issuance. Article 125 of the Treaty on the Functioning of the European Union (TFEU), often referred to as the *no-bailout clause*, prevents any form of liability of the Union or Member States for another Member's debt obligations.[5] Of course, assistance could still be provided via debt monetization. While the benefits and costs of inflation are borne all members, their distribution is not uniform. Surprise inflation reduces the ex-post real value of debt for all members. This benefits disproportionately highly indebted countries, while the costs of inflation are more uniformly distributed. Article 123 of the same treaty sought to address this by expressly prohibiting the European Central Bank from directly purchasing member countries' public debt.[6]

Yet the same European Treaty did not close the door entirely on the possibility of (ex-post) financial assistance. For instance, Article 122 of the TFEU allows the Union or other Member States to provide assistance to other members under exceptional circumstances.[7] Indeed, at various points during the

[5]Article 125 stipulates that 'The Union shall not be liable for or assume the commitments of central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of any Member State, without prejudice to mutual financial guarantees for the joint execution of a specific project. A Member State shall not be liable for or assume the commitments of central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of another Member State, without prejudice to mutual financial guarantees for the joint execution of a specific project' EU (2012b).

[6]Article 123 stipulates 'Overdraft facilities or any other type of credit facility with the European Central Bank or with the central banks of the Member States ('national central banks') in favor of Union institutions, bodies, offices or agencies, central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of Member States shall be prohibited, as shall the purchase directly from them by the European Central Bank or national central banks of debt instruments.' EU (2012b)

[7]Article 122 stipulates "..Where a Member State is in difficulties or is seriously threatened with severe difficulties caused by natural disasters or exceptional occurrences beyond its control, the Council, on a proposal from the Commission, may

Eurozone sovereign debt crisis, Greece, Ireland, Portugal, Spain, and Cyprus lost market access and had to ask for the support of other Eurozone members in order to avoid a default or a collapse of their domestic banking sector. This financial support was mainly provided through the creation of the European Financial Stability Facility (EFSF) and its successor, the European Stability Mechanism (ESM), who lent large amounts to these countries.

Lenders. Financial assistance to distressed countries in the Eurozone was provided by the so-called "Troika", a tripartite group consisting of the International Monetary Fund (IMF), the European Commission (EC), and the European Central Bank (ECB). This paper, for reasons that will become clear shortly, focuses on the first two legs of the Troika: the IMF and the EC.[8] The EC provided financial support using different lending institutions, including the European Financial Stability Mechanism (EFSM), the European Financial Stability Facility (EFSF) the European Stability Mechanism (ESM), the Greek Loan Facility (GLF), and the Balance of Payments (BoP) facility.

Since the countries in crisis are all high-income, IMF loans came through non-concessional Stand-by Arrangements or Extended Fund Facilities, and such loans were subject to structural reforms and macroeconomic conditionality. Loans were medium-term with maximum maturities of up to 10 years.[9] Under normal access, the maximum amount that countries can borrow depends on their IMF quota.[10] Many countries were given "exceptional access," allowing them to override the borrowing limit.

At the beginning of the crisis, the EC had few tools with which to manage debt crises within the Eurozone. The March 2010 programme for Greece, through the EC's newly created GLF, marked the beginning of large-scale EC lending. In June 2010 the EC created the EFSF, which would provide support to Greece, Ireland, and Portugal. In 2011 the EC further created the European Financial Stability Mechanism (EFSM), which provided assistance to Ireland and Portugal. Finally, in September 2012 the EC created the ESM, a permanent facility to replace the EFSF and to continue providing support to Greece, Spain, and Cyprus. With the exception of the GLF, which used bilateral loans from Eurozone members, all EC programmes typically use an on-lending scheme in which the lender borrows from financial markets and extends the fund to the distressed country.

Greek Financial Assistance Programmes. Greece was by far the largest recipient of financial assistance during the European sovereign debt crisis. Greece entered the first of three successive pro-

grant, under certain conditions, Union financial assistance to the Member State concerned.' EU (2012b)

[8]The role of the ECB has been discussed at length in, for example, Shambaugh (2012) or Giannone, Lenza, Pill and Reichlin (2012).

[9]Early programmes were mostly medium-term maturity Stand-By Arrangements, with repayment typically due within 3-5 years. Later programmes were designed under the Extended Fund Facility (EFF), with slightly longer maturity of 4-10 years.

[10]Both Stand-By Arrangement (SBA) and EFF loans stipulate that, under normal access, the maximum amount a country can borrow is 145% of a their quota annually or 435% over the lifetime of a programme.

grammes following sharp rises in government yields and revelations that the Greek government had materially understated its public debt and deficit figures in late 2009.[11] Programme 1 resulted in assistance from the EC via the Greek Lending Facility (GLF) and the IMF in the period 2010-2011. Programme 2 resulted in assistance from the EFSF and the IMF in 2012-2015. Programme 3 resulted in assistance from the ESM in 2015-2018. The first three rows of Table A.4 summarize the financial assistance measures to Greece.

The size of Programme 1 was approximately €110 bn with €80 bn coming from the GLF and €30 bn from an IMF's SBA programme. Actual disbursements by the IMF totaled €20.1 bn in six tranches between May 2010 and December 2011. Actual disbursements from the EC totaled €52.9 bn also over six tranches between May 2010 and December 2011. The disbursements were apportioned to each European country, with Germany (€15.17 bn), France (€11.39 bn), and Italy (€10.00 bn) contributing the bulk of the funding (European Commission, 2014).[12]

Towards the end of Programme 1, it became apparent that the financial assistance and structural reforms were not sufficient to restore growth and financial market access. The IMF and the EC agreed to a second programme, with the EC this time operating through the EFSF, coinciding with a restructuring of Greek debt.[13] This programme included the un-disbursed amount from Programme 1, plus an additional €130 bn, for a total of almost €180 bn (European Commission, 2012b). Actual disbursements by the IMF for Programme 2 totaled around €11.6 bn out of a total planned contribution of €28 bn (IMF, 2014). IMF loans were now given through the EFF beginning with the first tranche in March 2012 through the last in June 2014. The EFSF committed a total of €144.7 bn of which approximately €141.8 bn was disbursed (European Commission, 2012b).[14]

Programme 2 was interrupted at the end of 2014 by Greek elections. Following the election of Syriza and six months of failed negotiations, Programme 2 was allowed to expire in June 2015 during the fifth review. That August, Greece and the EC, through the ESM, agreed to a third programme, this time without the IMF, which was concluded in August 2018.[15] Programme 3 consisted of new loans by the ESM only, who committed €86 bn to Greece and disbursed €61.9 bn in total.

[11]There is by now a substantial literature discussing the unfolding of the Greek crisis. See Zettelmeyer, Trebesch and Gulati (2013), Blustein (2015), Schumacher and Mauro (2015), Gourinchas, Philippon and Vayanos (2017), and Chodorow-Reich, Karabarbounis and Kekre (2021) among others.

[12]Originally, Ireland and Portugal were also slated to contribute to Programme 1. However, their own fiscal struggles caused them to eventually drop out.

[13]See Zettelmeyer et al. (2013) for a summary of the restructuring.

[14]There is a net outstanding amount of €130.9 bn as of October 2018, due to the return of €10.9 bn. The €10.9 bn returned consisted of bonds that were to be used to recapitalize Greek banks through the Hellenic Financial Stability Fund.

[15]Greece received one bridge loan from the European Financial Stability Mechanism (EFSM) when it missed a payment on its loans to the IMF in July 2015. This was a three-month loan for €7.16 bn given to allow Greece time to transition to Programme 3 and receive assistance from the ESM.

Other Eurozone Financial Assistance Programmes. Three of the other four Eurozone crisis countries, Cyprus, Ireland, and Portugal, had similar, albeit less extreme, patterns of financial assistance by the IMF and the European Union. Ireland's request for financial assistance in November 2010 was approved in December 2010. External funding was comprised of €17.7 bn from the EFSF, €22.5 bn from the EFSM, €22.5 bn from the IMF, and €4.8 bn from Bilateral Loans (United Kingdom, Sweden, and Denmark).[16,17] Portugal received financial assistance from the EC (via the EFSF and the EFSM) and the IMF between May 2011 and June 2014. The three groups each committed approximately €26 bn for a total of €78 bn (European Commission, 2016), however Portugal allowed the programme to lapse without taking the final tranche of funding from the IMF. Cyprus officially asked for assistance in 2012 and was approved for a joint IMF/ESM programme in April/May 2013. Cyprus officially exited its programme in March 2016. The programme's total approved financing was €10 bn, with the ESM committing €9 bn and the IMF approximately €1 bn. The ESM disbursed only €6.3 bn between May 2013 and October 2015, with expected repayment between 2025 and 2031. The IMF disbursed a little less than its commitment through the EFF.

Spain received assistance from the ESM only, with no assistance from the IMF.[18] A total of €100 bn was committed in July 2012. Only €41.3 bn were disbursed in December 2012 and February 2013. The assistance was used to recapitalize the banking sector. Expected repayment should take place between 2022 and 2027, although Spain has made voluntary early repayments on these loans.

Other European Bailout Programmes. Financial assistance was also offered to three European Union countries outside of the monetary union: Hungary, Latvia, and Romania. Assistance was primarily made under the EC's medium-term Balance of Payment (BoP) assistance programme and by the IMF through SBAs. Assistance was granted to Hungary in November 2008 and lasted through the end of 2009. The total amount available was €6.5 bn in assistance from the EC and approximately €12.5 bn from the IMF.[19] However, only €5.5 bn was disbursed by the EC.[20] The realized EC loans comprised three installments given over the course of December 2008 to July 2009, each with a maturity between three and six years. For the IMF, approximately €8.7 bn (SDR7.6 bn) was disbursed between November 2008 and September 2009 and was repaid between February 2012 and August 2013. Latvia received assistance in January 2009. A total of €7.5 bn was made available, with €3.1bn from the EC and ap-

[16] Ireland also had to commit €17.5 bn

[17] Sweden provided €600 million, the United Kingdom provided €3,830 million (£3.23 million), and Denmark provided €400 million.

[18] Technically, Spain agreed to a programme with the EFSF. The ESM was set up before any disbursements were given, and so the assistance was transferred.

[19] The World Bank made approximately €1bn available as well.

[20] We focus on actual disbursements, so the second round of assistance in July 2012, which did not result in any assistance, is excluded from our analysis.

proximately €1.7 bn from the IMF.[21] For the EC, realized financial assistance came in four installments of €2.9 bn between February 2009 and October 2010. The IMF disbursed approximately SDR1 bn between 2008 and 2010 and was repaid in 2013. Finally, assistance to Romania was granted in May 2009. A total of €20 bn was made available, comprised of €5 bn from the EC and approximately €12.95 bn from the IMF.[22]

2.2 Transfers estimates

2.2.1 Motivation and Methodology

To estimate the implicit transfers in financial assistance for the programmes described above, we follow Joshi and Zettelmeyer (2005) who perform a similar exercise for transfers implicit in IMF programmes. A critical input is the risk-adjusted discount rate on public assistance programmes to crisis countries, which can be very different from market rates. To put this point in sharp relief, imagine that EU loans are risk free and that the EU charges the risk-free rate. In that case, there is no implicit transfer from lenders to borrowers, regardless of the market rate on risky loans. Using the private sector (risky) interest rate to discount public sector (risk-less) loans would erroneously conclude that there is a large concessional component in these programmes.[23]

Our approach consists in assuming that the default risk on EU funding is similar or higher than the default risk of IMF loans to these same countries, during the same period. Importantly, we do not need to assume that the IMF loans themselves are risk-free. As long as the risk profile of IMF loans and EU loans are similar, then we can use the internal rate of return on IMF programmes to each country to discount the term sheet of European loans to that same country.

To estimate the Net-Present-Value (NPV) of total transfers, $Tr_t^{i,j}$, for borrower i from creditor j at time t, we calculate the difference between the present value of the sequence of net transfers discounted at our benchmark IMF internal rate of return and the present value of the sequence of net transfers discounted at its actual internal rate of return. By definition, this latter term is zero, and so we can write the NPV of transfers as

$$Tr_{t_0}^{i,j} = \sum_{t=t_0}^{T} \frac{1}{(1+\bar{r}^i)^t} NT_t^{i,j}, \tag{1}$$

[21] Additionally, €1.9 bn were made available by Nordic countries (Sweden, Denmark, Finland, Estonia, and Norway), the World Bank, the European Bank of Reconstruction and Development, the Czech Republic, and Poland.

[22] Additionally, there was €1 bn from the World Bank, and €0.5 bn each from the European Investment Bank (EIB) and the European Bank for Reconstruction and Development (EBRD).

[23] A first estimate of transfers was attempted by the ESM itself (see ESM (2014, 2015) reports). However, the discount rate used was the market interest rate that crisis countries would have paid had they been able to cover their financing needs from private investors. As argued above, this overlooks the fact that the risk profile of public loans is very different from private loans.

in which t_0 refers to year 2010 and T is the date of the last net transfer flow. As explained above we use the internal rate of return on the IMF's lending for borrower i during the Eurozone crisis, $\bar{r}^i$, as the discount rate for country i. $NT_t^{i,j}$ are net transfers from lender j to borrower i at time t, constructed as:

$$NT_t^{i,j} = D_t^{i,j} - R_t^{i,j} - \sum_{\tau=1} i_{t-\tau}^{i,j} D_{r,t-\tau}^{i,j},$$

where $R_t^{i,j}$ are principal repayments from j to i at time t, and $D_t^{i,j}$ denotes disbursements from i to j at time t. τ denotes the maturity of each disbursement and $D_{r,t-\tau}^{i,j}$ is the *outstanding* principal balance remaining at time t on a disbursement effected at time $t-\tau$, and $i_{t-\tau}^{i,j} D_{r,t-\tau}^{i,j}$ the corresponding interest payments at time t. The internal rate of return $irr^{i,j}$ is the value that sets the sequence of net transfers to zero.

The IMF interest rate is an attractive choice for three reasons. First, while both IMF and EU lending are expected to enjoy preferred creditor status, IMF loans are legally senior to loans from the EFSF and the ESM.[24] Therefore, the correct discount rate on European loans should be higher than the IMF's internal rate. This biases our bailout estimates *downwards*. Second, European loans are time-varying, based on the cost of funds. The relevant benchmark rate must also be time-varying in order to account for changes in global lending conditions. As discussed above, the interest rate on IMF loans is time-varying and related to the IMF's cost of funding and to the size of the loan relative to the country's quota. Third, IMF programmes tend to have a medium term maturity (3 to 9 years) while European loans have a longer duration (10 to 30 years). To the extent that there is a positive term premium, this also biases our bailout estimates *downward*.

One concern with our methodology is that the IMF programmes themselves may include a transfer component. We do not consider that this is a serious issue for two reasons. First, Joshi and Zettelmeyer (2005), estimating the realized transfers in IMF lending from 1973 to 2003 find no evidence of a significant transfer component for high and middle income countries, unlike low income countries or Highly Indebted Poor Countries. Second, one might be concerned that IMF lending to European countries during the Eurozone crisis may have adopted lending criteria that deviated from past practice, resulting in a potential concessional component from IMF lending. In that case, however, using the IMF interest rate would again bias our transfer estimates *downwards*.

To summarize, our estimated transfers provide a lower bound on the true transfer for three reasons: because EU programmes are riskier than the IMFs; because they have longer maturity; and because IMF programmes themselves may be subsidized. Conversely, our estimates would be biased upwards if either EU lending was senior to the IMF, was shorter maturity or if the IMF was lending at a penalty

[24] Article 13 of the ESM Treaty states "ESM loans will enjoy preferred creditor status in a similar fashion to those of the IMF, while accepting preferred creditor status of the IMF over the ESM" EU (2012a).

rate. For reasons argued above, we do not view these scenarios as serious possibilities.

We compile data on disbursements, repayments, and interest payments for the eight borrowing countries and five official international lenders described above. We make two key assumptions. First we assume that the most current specification of future repayments and interest rates coincides with future realized outcome. This assumes that there will be no more debt renegotiations in the future. Any changes to the current agreement, such as delaying interest payments or extending the overall maturity, could only make the terms more favorable for the borrower and would result in a larger transfer than we estimate. Second, for loans with variable interest rates, we construct a proxy for future interest rates by adding an estimate of the term premium to the current interest rate.[25]

2.2.2 Results

Table 1 reports our results for Eurozone borrowers. Column (1) reports the calculated internal rate of return for each given borrower-lender pair i, j, $irr^{i,j}$. Column (2) reports the IMF internal rate of return for borrower i, $\bar{r}^i$, which is used as the discount rate.[26] Column (3) calculates the difference between the IMF internal rate of return and the loan's internal rate of return (Column (2) minus Column (1)). With the notable exception of the EFSM loan to Ireland, the IMF internal rate of return is always higher, which implies a transfer element from European institutions.

The IMF internal rate varies from 1.73% for Cyprus to 3.26% for Greece. There are at least two reasons for this variation across countries. First, countries borrowed at different points in time. Greece's IMF programme started in May 2010, while Cyprus' programme started in May 2013 (see Table A.1). Over that period, the IMF's basic rate of charge decreased from 1.25% to 1.06%.[27] Second, countries borrowed various fraction of their quotas, with a corresponding penalty adjustment in their borrowing rate (see Table A.2). Greece, which borrowed 3,374% of its 2010 quota, faced a higher rate than Cyprus which borrowed 'only' 547% of its 2010 quota.[28] As discussed above, these differences can be interpreted as risk premia on IMF lending, and do not affect our calculations as long as EC lending remained less senior than IMF lending *to that country*.[29]

Column (4) displays the duration of the lending cycle, d, following the methodology in Joshi and Zettelmeyer (2005). The duration of the lending cycle between borrower i and lender j, $d^{i,j}$, is cal-

[25]We construct this term premium at various maturity from the difference between forward 3-month Euribor rate and the current 3-month Euribor. See the Online Appendix for details.

[26]Note that this is simply repeated for reference from the IMF row by country. For Spain, who did not receive any IMF loans, we take the simple average of the other IMF internal rates for the other countries.

[27]The IMF's historical basic rate of charge is available at https://www.imf.org/external/np/fin/data/query.aspx.

[28]Given the size of its programme, Cyprus also spent a lot less time above its quota, at a penalty rate, relative to Greece.

[29]In addition, since we use the realized IMF schedule of disbursements, repayments, and interest rates, countries like Ireland, which repaid their IMF debt early mechanically faced a lower IMF internal rate of return.

Borrower i	Lender j	$irr^{i,j}$ (1)	$\bar{r}^i$ (2)	$\Delta irr^{i,j}$ (3)	$d^{i,j}$ (4)	$\sum D^{i,j}$ (5)	$Tr^{i,j}$ (6)	$Tr^{i,j}/GDP^i$ (7)
Cyprus	ESM	0.82	1.73	0.90	15.48	6.30	0.74	3.62%
Cyprus	IMF	1.73	1.73		7.26	0.95		
Greece	EC	0.68	3.26	2.58	20.66	52.90	18.49	8.18%
Greece	EFSF	1.16	3.26	2.11	37.20	171.20	66.82	28.19%
Greece	ESM	1.83	3.26	1.43	33.81	61.90	16.64	7.30%
Greece	IMF	3.26	3.26		6.70	31.99		
Ireland	EFSF	1.83	2.66	0.83	21.98	17.70	2.22	1.29%
Ireland	EFSM	3.23	2.66	-0.57	16.04	22.50	-1.51	-0.88%
Ireland	IMF	2.66	2.66		4.59	22.61		
Portugal	EFSF	1.78	3.25	1.46	21.92	26.00	5.47	2.93%
Portugal	EFSM	3.10	3.25	0.14	15.64	24.30	0.38	0.21%
Portugal	IMF	3.25	3.25		5.65	26.39		
Spain	ESM	0.93	2.65	1.73	9.14	41.33	5.55	0.49%

Table 1: Implicit Transfers from Eurozone Funding programmes

The table reports the internal rate of return ($irr^{i,j}$) for each recipient country i and funding agency j, the duration of the programme ($d^{i,j}$), the total (nominal) amount disbursed ($\sum D^{i,j}$), the implicit transfer $Tr^{i,j}$ in billions of euros and scaled by 2010 nominal GDP.

Borrower i	Lender j	$irr^{i,j}$	$irr^{i,IMF}$	$\Delta irr^{i,j}$	$d^{i,j}$	$\sum D^{i,j}$	$Tr^{i,j}$	$Tr^{i,j}/GDP^i$
Hungary	BoP	3.56	2.42	-1.13	5.24	5.50	-0.28	-0.31%
	IMF	2.42	2.42		4.22	8.75		
Latvia	BoP	3.09	2.55	-0.53	6.92	2.90	-0.09	-0.49%
	IMF	2.55	2.55		3.85	1.11		
Romania	BoP	3.00	2.70	-0.30	7.96	5.00	-0.10	-0.08%
	IMF	2.70	2.70		4.65	11.87		

Table 2: Implicit Transfers from non-Eurozone Funding programmes

The table reports the internal rate of return ($irr^{i,j}$) for each recipient country i and funding agency j, the duration of the programme ($d^{i,j}$), the total (nominal) amount disbursed ($\sum D^{i,j}$), the implicit transfer $Tr^{i,j}$ in billions of euros and scaled by 2010 nominal GDP.

culated as:

$$d^{i,j} = \sum_{t=1}^{T} \frac{R_t^{i,j}}{\sum_s R_s^{i,j}} \cdot t,$$

where $R_t^{i,j}$ are the principal repayments from country i to lender j at time t. The duration of IMF programmes, between 4.59 years (Ireland) and 7.26 years (Cyprus), is much shorter than the duration of EC programmes, between 9.14 years (Spain) and 33.81 years (Greece, ESM). As discussed above, these duration differences suggest a transfer element.

Column (5) shows the sum of all *nominal* disbursements $\sum D^{i,j}$, in €billion. They vary between 7.25 €billion for Cyprus representing 36% of its 2010 GDP, to 317.82 €billion for Greece, representing 140% of its 2010 GDP. Columns (6) and (7) of Table 1 show our estimates of the net present value of transfers, calculated according to Eq. (1), first in 2010 billions of Euros and then as a percentage of the country's 2010 GDP. There are striking differences in the estimated transfers across countries. Two countries stand out. First, we find that the size of Ireland's transfer was positive but very small, at 0.69€billion, representing 0.41% of its 2010 GDP. At the other extreme, Greece received a very substantial transfer of 98.6 €billion, representing 42.3% of its 2010 GDP. For Portugal and Cyprus the transfer is positive and quite sizable, between 3.14 and 3.62% of GDP. In the case of Spain, where lending was directed towards bank recapitalization and therefore of a different nature from the other countries, the transfer is less than 0.5% of GDP.[30]

We also calculate the transfer component included in the financial assistance programmes that the European Commission extended to three EU members outside the Eurozone: Hungary, Latvia and Romania. All three countries benefitted also from a concurrent IMF programme. Table 2 reports the results. Since these countries were not in the Eurozone, they constitute an interesting control group with which to compare the Eurozone bailouts. We highlight three important results. First, the internal rates of return for the EC programmes - between 3.00% and 3.56%– are never lower than the IMF internal rate of return – between 2.42% and 2.70%. Second, the IMF loans are again always shorter, even though in some cases the IMF lent more than the EC. Finally, in the last two columns, we see that the transfer is always *negative*, between -0.08% (Romania) and -0.49% (Latvia). Taken together, we interpret this result as saying there was no transfer element embedded in the EC lending to these countries.

We draw the following conclusions from our empirical analysis. First, we estimate implicit transfers of very different sizes to different eurozone countries during the euro crisis. They vary from small (Ireland) to extremely large (Greece). We view these transfers as a central part of the crisis resolution. Second, non-eurozone EU countries did not receive any significant transfer from the EC: sizable transfers are tied to membership in the monetary union, not to membership of the European Union.

[30] As noted above, our calculation for Spain is a bit less precise since the country did not have an IMF programme and we are using an average of the IMF internal rate of return for the other countries.

3 Model

We now present a simple model to study the conditions under which a monetary union might agree to bail out one of its members and characterize the determinants of that transfer. The model emphasizes the collateral cost of default and/or exit from the monetary union in case of default on the other members of the union.

3.1 Assumptions

The baseline model is a simple extension of Calvo (1988). There are 2 periods, $t \in \{0, 1\}$, and three countries, $j \in \{g, i, u\}$. Countries g and i belong to a monetary union while country u does not. Countries g and u are fiscally strong in the sense that their government debt is risk-free. In contrast, country i is fiscally fragile: its government may be unable or unwilling to repay its debts either in period 0 or period 1.[31] We denote ω^j the relative size of country j, with $\sum_j \omega^j = 1$.

Each country j receives an exogenous per capita endowment in period t denoted y_t^j. This endowment represents the maximum fiscal resources available in each period and country. The only source of uncertainty is the realization of the endowment in country i in period 1, y_1^i. We define expected output $\bar{y}_1^i \equiv E[y_1^i]$, where $E[.]$ denotes the expectation operator, and $\epsilon_1^i = y_1^i/\bar{y}_1^i$ is an endowment shock that satisfies $E[\epsilon_1^i] = 1$. ϵ_1^i is distributed according to a cumulative distribution function $G(\epsilon)$ with pdf $g(\epsilon)$, over a bounded support $[\epsilon_{\min}, \epsilon_{\max}]$, with $0 < \epsilon_{\min} \leq \epsilon_{\max} < \infty$.

The only traded financial assets are sovereign bonds issued by each country. In country j, a representative agent —representing a composite of the country's households and banks— has preferences defined over aggregate consumption sequences $\{c_0^j, c_1^j\}$ and over holdings of sovereign bonds, as follows:

$$U^j = c_0^j + \beta E[c_1^j] + \omega^j \left(\lambda^s \ln b_1^{s,j} + \lambda^{i,j} \ln b_1^{i,j}\right). \tag{2}$$

In Eq. (2), $b_1^{k,j}$ denotes the face value of debt issued at $t = 0$ by country k, held by agent j. We assume that households are risk-neutral over consumption sequences. We further assume that government bonds provide 'money-like' liquidity services. We model these liquidity services by including the face value of bonds maturing next period directly in the utility function, as in Krishnamurthy and Vissing-Jorgensen (2012).[32]

[31] We do not take a stand on *why* country i is fiscally fragile, although it is important. If it is related to structural issues such as low productivity growth, then the transfers that we estimate in the model may not be sufficient to prevent future debt crises, which may be frequent. The conditionality embedded in IMF lending works to put countries on sustainable paths with structural reforms. We use this to motivate our focus on debt dynamics.

[32] A number of recent models have introduced bonds in the utility function. This captures, in a simple reduced form, the demand for safe stores of value that would arise in presence of idiosyncratic risk. For a discussion, see Kaplan and Violante (2018). An alternative is offered by Azzimonti and Quadrini (2021) who model public debt as a liquid asset that facilitates

Crucially, we assume that bonds from different countries provide different levels of liquidity services, depending on how 'money-like' these government bonds are perceived to be by different investors. As countries u and g are both fiscally sound and their debt is risk-free, we treat their bonds as perfect substitutes and so they provide identical liquidity services. We can consider the total demand for *safe* assets by households in country j as $b_1^{s,j} = b_1^{g,j} + b_1^{u,j}$. The liquidity services provided by these 'safe assets' is the same in all countries, hence $\lambda^{s,j} = \lambda^s$. Since country i is fiscally fragile, its debt is not necessarily safe. Moreover, the liquidity services from holding i's government debt may vary across investors. We assume that i-bonds provide higher liquidity services to i-investors, then g-investors, then u-investors: $\lambda^{i,i} > \lambda^{i,g} > \lambda^{i,u}$.

We view this assumption as reasonable. First, i-investors are likely to perceive i-debt as more liquid/safe than other investors if risk-shifting leads them to ignore the states of the world where both they and their government default. i-investors may also be coerced into holding debt of their own sovereign, through moral suasion or outright financial repression. In our set-up, this would be captured by a higher $\lambda^{i,i}$. Second, collateral policy (i.e. which government debt is accepted for collateral and at which haircut) inside the monetary union may make i-debt more desirable for foreign investors also located inside the monetary union (i.e. g-investors) than for investors located outside the monetary union (i.e. u-investors). In the Eurozone, for instance, the Eurosystem collateral framework was criticized before the crisis by Buiter and Sibert (2005) for reducing risk premia and thereby contributing to the insufficient differentiation of sovereign risk.[33] During the crisis, the ECB implemented numerous changes to its collateral framework, including in the application of its minimum rating threshold.[34] For the time being, these liquidity services are taken as given but we will later discuss how changes in collateral policy can affect the equilibrium bailouts.

In order to simplify a number of expressions, we will often consider the *bondless limit* that obtains when $\lambda^s \to 0$ and $\lambda^{i,j} \to 0$, while keeping the ratios $\omega^j \lambda^{i,j} / \sum_k \omega^k \lambda^{i,k}$ constant.[35] In this limit, as we will see, the bond portfolios remain well defined, but the liquidity services become vanishingly small, so the level of debt does not affect utility.

As mentioned above, countries i and g differ in their fiscal strength. While g is fiscally sound, i is fiscally fragile: it needs to refinance maturing debt b_0^i in period $t = 0$, and can decide to default in

production.

[33]See Bindseil, Corsi, Sahel and Visser (2017) for a description of the Eurosystem collateral framework.

[34]Before the crisis only high-rated government debt (A- at minimum) were accepted as collateral with a haircut of 3% (for a residual maturity of 5-7 years). Lower-rated government bonds (BBB- and above) became eligible on October 2008 with a haircut of 8%. In September 2013, these haircuts were changed to 2% and 10% respectively for high-rated and lower-rated government bonds. Furthermore, the ECB often waived the minimum rating threshold for low grade sovereign bonds. See Bindseil et al. (2017).

[35]The terminology here is by analogy with Woodford (1998)'s 'cashless limit' where the direct utility gains from money holdings become vanishingly small.

either period. Should a default occur, we follow the literature and assume that i suffers an output loss in the period of default equal to Φy_t^i, with $0 \leq \Phi \leq 1$. This output loss captures the disruption to the domestic economy caused by the default event. With a default cost increasing in output, a default is more costly, and therefore less likely, when the economy is doing well.

In the event of a default, we assume that creditors can recover an amount ρy_t^i where $0 \leq \rho < 1$ on their original claims. This assumption captures the fact that country i's decision not to repay its debt does not generally result in a full expropriation of outstanding creditor claims. Importantly, the recovery amount is proportional to output, and not to the outstanding debt. Hence, ρy_t^i is economically equivalent to the implicit collateral value of the outstanding debt. The recovery payment is distributed *pari passu* among all creditors, domestic or foreign, in proportion to their claims. We restrict our analysis to the case where $\Phi + \rho < 1$ so that the country always has enough resources for the recovery amount in the event of default.

A default in i creates direct economic losses on foreign bondholders (both g and u), through their portfolio holdings of i's debt, $b_1^{i,j}$. In addition to this direct portfolio exposure, we assume that country g also suffers a *collateral output loss* when country i defaults, equal to κy_t^g, with $0 \leq \kappa < 1$. Country u, by contrast, does not suffer any collateral damage. This assumption captures the idea that, since i and g belong to a monetary union, their economies are deeply intertwined. It is reasonable to expect that a default in country i would severely disrupt economic activity in country g as well, to a greater extent than in country u.

Finally, we allow for voluntary transfers from g to i in either period, denoted τ_t. Crucially, we consider an environment where g can make ex-post transfers conditional on the realization of output, and therefore on i's default decision. These transfers are voluntary and therefore must satisfy: $\tau_t \geq 0$. There is no reason for g to make a transfer to i in case of default, so the optimal transfer in that case is zero.[36]

3.2 Resource Constraints and Market Clearing

The government of country j issues one-period bonds with face value b_1^j in period $t = 0$. It also raises aggregate lump-sum taxes T_t^j on domestic residents in period t. To preserve space, we directly present the consolidated budget constraint in country i. The budget constraints of both countries are presented in Appendix B.1.

[36]While we allow for unilateral transfers from g to i, we do not need to consider transfers from u to i. There are two reasons for this. First, g has more exposure to i's default than u, both through larger direct portfolio holdings and indirectly through the collateral damage. Therefore, g has a stronger incentive than u to bailout i. Conditional on a bailout from g, u does not need to intervene since a default is already prevented. Second, in the event that g declines to intervene, we will see that u will have no incentive to intervene either, given its lower exposure. We implicitly rule out the case where u and g could coordinate their bailout efforts. This last assumption is plausible and does not preclude IMF-style programmes, as long as these do not include a concessional component.

Assuming that no default occurs at $t=0$, country i's resource constraint takes the form:

$$c_0^i + b_1^{s,i}/R^* = y_0^i + \tau_0 - (b_0^i - b_0^{i,i}) + \left(b_1^i - b_1^{i,i}\right)/R^i + b_0^{s,i}. \tag{3}$$

Country i's resources equal output y_0^i, plus possible transfers $\tau_0 \geq 0$, minus the net repayment of initial domestic debt to the rest of the world, $(b_0^i - b_0^{i,i})$, plus the net issuance of new debt, $\left(b_1^i - b_1^{i,i}\right)/R^i$ and the safe asset holdings, $b_0^{s,i}$. These resources are split between current consumption c_0^i and safe asset investments $b_1^{s,i}/R^*$.

In period $t=1$ the resource constraint becomes:

$$\begin{cases} c_1^i & = y_1^i + \tau_1 - (b_1^i - b_1^{i,i}) + b_1^{s,i} \qquad\qquad \text{if } i \text{ repays} \\ c_1^i & = y_1^i(1-\Phi) - \rho y_1^i \left(1 - \frac{b_1^{i,i}}{b_1^i}\right) + b_1^{s,i} \quad \text{if } i \text{ defaults.} \end{cases} \tag{4}$$

In case of repayment, country i's representative agents consumes output y_1^i, plus any transfer τ_1, minus the repayment of domestic debt to the rest of the world $(b_1^i - b_1^{i,i})$, plus holdings of safe assets, $b_1^{s,i}$. In case of default, output is reduced by the default cost Φy_1^i, while foreign investors recover $\rho y_1^i(b_1^i - b_1^{i,i})/b_1^i$.

Finally, the market clearing condition for safe bonds and i-bonds takes the form:

$$\sum_j b_1^{s,j} = b_1^s \quad ; \quad \sum_j b_1^{i,j} = b_1^i. \tag{5}$$

3.3 Optimal Portfolios

Denote $\mathcal{P}^j \leq 1$ the expected payment per unit of i's sovereign debt for j's household, given the optimal default decision at $t=1$. If i cannot discriminate between different types of bondholders when defaulting, this expected payoff is the same for all investors: $\mathcal{P}^j \equiv \mathcal{P}$. The optimal bond portfolio maximizing U^j satisfies:

$$\frac{1}{R^i} - \beta\mathcal{P} = \frac{\omega^j \lambda^{i,j}}{b_1^{i,j}}, \qquad\qquad \frac{1}{R^*} - \beta = \frac{\omega^j \lambda^s}{b_1^{s,j}}. \tag{6}$$

The first equation characterizes the demand for i bonds. The left-hand side of that equation represents the expected monetary cost from purchasing one more unit of i's debt: a cost $1/R^i$ and an expected discounted return $\beta\mathcal{P}$. The right hand side of that equation represents the additional liquidity benefit. The second equation characterizes the demand for safe bonds. Denote $\bar{\lambda}^i \equiv \sum_k \omega^k \lambda^{i,k}$ the weighted average of liquidity services provided by i-debt.

Combining Eqs. (5) and (6) the next proposition characterizes optimal portfolio shares, $\alpha^{i,j}$, $\alpha^{s,j}$, and equilibrium yields. Portfolio shares of i-debt are proportional to the relative liquidity benefits of i-debt across investor classes and size, while portfolio holdings of s debt reflect only size differences. Further, the yield on safe debt incorporates a liquidity premium while that on i-debt incorporates also a risk-premium. In the *bondless limit*, the liquidity premium on safe debt disappears and the spread on i's debt reflects entirely default risk ($\mathcal{P} \leq 1$).

Proposition 1 (Optimal Portfolios and Equilibrium Yields)**.** *In the absence of selective default, and given an expected repayment $\mathcal{P}$ per unit of i-debt,*

- *Equilibrium bond portfolios are independent from yields and reflect relative liquidity services:*

$$\alpha^{i,j} \equiv \frac{b_1^{i,j}}{b_1^i} = \frac{\omega^j \lambda^{i,j}}{\bar{\lambda}^i} \quad ; \quad \alpha^{s,j} \equiv \frac{b_1^{s,j}}{b_1^s} = \omega^j$$

  where $\bar{\lambda}^i \equiv \sum_k \omega^k \lambda^{i,k}$ denotes the weighted average of liquidity services provided by i-debt.

- *In the bond-less limit, equilibrium yields satisfy:*

$$R^* = \beta^{-1} \quad ; \quad R^i = (\beta \mathcal{P})^{-1} \tag{7}$$

Proof. See Appendix B □

4 Ex-Post Defaults and Bailouts

We solve the model by backward induction, starting from the final period $t = 1$. In period 1, i's government can unilaterally decide to repay its debt or default, after observing the realization of the income shock ϵ_1^i, and taking as given the transfer τ_1 it receives from g's government in the event it does not default. From country i's consolidated budget constraint Eq. (4), a government maximizing the welfare of domestic agents will decide to repay its debts when:

$$y_1^i \left[\Phi + \rho(1 - \alpha^{i,i})\right] + \tau_1 \geq b_1^i (1 - \alpha^{i,i}). \tag{8}$$

This equation has a natural interpretation. The left hand side captures the cost of default for i's government. This cost has three components. First, there is the direct disruption to the domestic economy captured by Φy_1^i. Second, when a default occurs the country has to repay a fraction ρ of output to foreign investors. These foreign investors hold a fraction $(1 - \alpha^{i,i})$ of marketable debt, hence will receive $\rho y_1^i (1 - \alpha^{i,i})$. Lastly i will forego the unilateral transfer from g, τ_1. Against these costs, the benefit of default consists in not repaying the outstanding debt to foreign investors, both inside the monetary union and in the rest of the world: $b_1^i(1-\alpha^{i,i})$. Intuitively, default is more likely if the direct cost of default Φ is low, the recovery rate ρ is low, transfers τ are low, and a larger fraction of the public debt is held abroad (low $\alpha^{i,i}$).

Condition (8) defines the minimum transfer $\underline{\tau}_1$ necessary to avoid a default:

$$\underline{\tau}_1 \equiv b_1^i(1-\alpha^{i,i}) - y_1^i\left[\Phi + \rho(1-\alpha^{i,i})\right]. \tag{9}$$

Since transfers are voluntary (i.e. $\tau_1 \geq 0$), equation (9) defines a minimum realization of the output shock ϵ_1^i, which we denote $\bar{\epsilon}$, such that repayment is optimal even in the absence of transfer when $\epsilon \geq \bar{\epsilon}$:

$$\bar{\epsilon} \equiv \frac{(1-\alpha^{i,i})b_1^i/\bar{y}_1^i}{\Phi + \rho(1-\alpha^{i,i})}. \tag{10}$$

Intuitively, $\bar{\epsilon}$ increases, i.e. default without bailout becomes more likely, with the amount of debt owed to foreigners relative to expected resources, $(1-\alpha^{i,i})\, b_1^i/\bar{y}_1^i$, and decreases with the cost of default Φ or the recovery rate ρ. A larger fraction of i-debt held by i-investors makes default less appealing to i's government since a default becomes a neutral transfer from domestic bondholders to domestic taxpayers. In the limit where i-debt is entirely held domestically, $\alpha^{i,i} = 1$, and there is never any incentive to default regardless of the realization of output: $\bar{\epsilon} = 0 < \epsilon_{\text{min}}$.

This result indicates one important benefit of the 're-nationalization' of bond markets: all else equal, it decreases the *ex-post* incentive to default as emphasized by Tenreyro (2019). In our model there is no *deadly embrace* or *doom-loop* between sovereigns and bondholders, unlike Farhi and Tirole (2018). In that paper, the deadly embrace arises from the distorted incentives of domestic banks to hold debt issued by their own sovereign, creating an enhanced contagion channel from banks to sovereigns and vice-versa. This channel is absent in our model.

Let's now consider the choice of optimal ex-post transfers τ_1 by g's government. When $\epsilon_1^i < \bar{\epsilon}$, a transfer becomes necessary to avoid default. Given our assumptions, g will make the minimum transfer required to avoid a default: $\tau_1 = \underline{\tau}_1$.[37] Substituting $\underline{\tau}_1$ from Eq. (9) into g's consolidated budget constraint, g's government prefers to make a transfer as long as[38]:

$$\Phi y_1^i + \kappa y_1^g \geq \alpha^{i,u}\left(b_1^i - \rho y_1^i\right). \tag{11}$$

[37]We assume that if i is indifferent between default and no-default, it chooses not to default.
[38]g's budget constraints are presented in Appendix B.1

The left hand side of Eq. (11) measures the output loss from default for the monetary union as a whole. It consists of the sum of the direct cost Φy_1^i for i and the contagion cost κy_1^g for g. The right hand side measures the overall benefit of default: from the point of view of the monetary union, the benefits of default consists in not repaying the rest of the world, a gain of $\alpha^{i,u}(b_1^i - \rho y_1^i)$.

Eq. (11) makes clear that g's ex-post bailout restores *joint ex-post efficiency* from the perspective of the monetary union. The difference between the left and right hand side of Eq. (11) represents the surplus from avoiding a default. Moreover, under our assumption that g makes a unilateral take-it-or-leave-it offer to i, g is able to appropriate the entirety of this ex-post surplus.[39]

We can solve Eq. (11) for the minimum realization of ϵ_1^i such that a transfer (and no-default) is optimal. This defines a threshold, $\underline{\epsilon}$, such that a default is jointly optimal when $\epsilon_1^i \leq \underline{\epsilon}$:

$$\underline{\epsilon} \equiv \frac{\alpha^{i,u} b_1^i/\bar{y}_1^i - \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}}. \tag{12}$$

We make the following observations about equation Eq. (12). First, it can be immediately checked that $\underline{\epsilon} \leq \bar{\epsilon}$ as long as $\alpha^{i,g} \geq 0$ and/or $\kappa \geq 0$. In other words, as long as g is exposed directly (through its portfolio) or indirectly (through contagion) to i's default, it has an incentive to offer ex-post bailouts to i. It follows immediately that an *ex-ante* no-transfer policy —such as a no-bailout clause— is not *renegotiation proof* and therefore may prove difficult to enforce. Second, g will always be willing to bailout i regardless of its debt level if $\alpha^{i,u} = 0$ since in that case $\underline{\epsilon} \leq 0 < \epsilon_{\min}$. In other words, if all of i's debt is held within the monetary union and i's default is costly for either country, it is ex-post efficient for g to offer a full bailout.[40] The threat of collateral and direct damage to g from i's default relaxes ex-post i's budget constraint, a point emphasized also by Tirole (2015). Third, because g offers the minimum transfer $\underline{\tau}_1$ to avoid a default, i receives a positive transfer but achieves the same utility as under default. When $\underline{\epsilon} \leq \epsilon_1^i < \bar{\epsilon}$, i's consumption in period $t = 1$ is given by:

$$c_1^i = y_1^i(1 - (\Phi + \rho(1 - \alpha^{i,i}))) + b_1^{s,i}. \tag{13}$$

This result captures what we call the *Southern view* of the crisis: the ex-post support that i receives from g does not make i better off, relative to default. It avoids the deadweight losses imposed by a default, but g captures *all* the corresponding efficiency gains. We summarize these results in the following proposition.

[39]An alternative arrangement is to assume that i and g bargain over the surplus from avoiding default. We analyze this extension in Section C.2.

[40]Of course, anticipating on the results from the next section, in that case i would want to issue so much debt in period $t = 0$ that this would eventually threaten g's own fiscal capacity. In what follows we always assume that $\alpha^{i,u} > 0$ and that g has sufficient fiscal capacity to make the necessary transfers.

Proposition 2 (Optimal ex-post bailout). *Given debt level b_1^i, the following output thresholds fully characterize the optimal ex-post bailout and default policy:*

$$\underline{\epsilon} = \frac{\alpha^{i,u} b_1^i / \bar{y}_1^i - \kappa y_1^g / \bar{y}_1^i}{\Phi + \rho \alpha^{i,u}} \leq \bar{\epsilon} = \frac{(1 - \alpha^{i,i}) b_1^i / \bar{y}_1^i}{\Phi + \rho (1 - \alpha^{i,i})}$$

1. *When output is high, i.e. $\epsilon_1^i \geq \bar{\epsilon}$, country i does not default and receives no bailout, $\tau_1 = 0$*
2. *When output is intermediate, i.e. $\underline{\epsilon} \leq \epsilon_1^i < \bar{\epsilon}$, country i receives the minimal bailout $\underline{\tau}_1$ with probability $1 - \pi$ to avoid a default:*

$$\tau_1 = \underline{\tau}_1 \equiv b_1^i (1 - \alpha^{i,i}) - y_1^i \left[\Phi + \rho (1 - \alpha^{i,i}) \right]$$

3. *When output is low, i.e. $\epsilon_1^i < \underline{\epsilon}$, country i defaults and receives no transfer, $\tau_1 = 0$.*

Proof. See text. □

Observe that the optimal bailout τ_1 is non-monotonous in i's output: there is no bailout when either output is very low or very high. The optimal bailout is also discontinuous at $\epsilon_1^i = \underline{\epsilon}$. The reason is that i's gain from default is stronger at lower levels of output, and therefore avoiding a default requires a larger transfer. At the lower threshold $\underline{\epsilon}$, the joint surplus from avoiding default falls to zero, while the incentive to default is strongest for i, requiring the largest possible transfer.

5 Debt Rollover Problem at $t = 0$.

It may be difficult for g to conduct transfers. The institutional framework inside the monetary union may forbid direct transfers from one country to another under so called 'no-bailout' clauses. While direct transfers may be ruled out, indirect transfers, via common institutions -such as the common central bank- may still be possible, albeit difficult to implement in practice.[41] These 'no-bailout' clauses have repeatedly been invoked and played an important role in shaping the response to the Eurozone crisis. In particular, the legality of proposed bailout programmes has often been questioned and referred to the German federal constitutional court (commonly called the Karlsruhe court), or the European Court of Justice.[42] From our point of view, the important observation is that the political process contains a

[41] We explore this possibility in more details in section C.3.

[42] For instance, on September 12, 2012, in a landmark decision, the Karlsruhe court ruled that the European Stability Mechanism (ESM) did not violate German law, allowing for the ESM statutes to be signed into law by the German president. However, the German high court imposed strict conditions requiring approval by the German parliament before any extension of the country's total exposure to the ESM (see https://www.bundesverfassungsgericht.de/SharedDocs/Pressemitteilungen/EN/2012/bvg12-067.html). This judgement was re-affirmed on March 18, 2014 when the high court ruled that constitutional challenges to the ESM were without mer-

certain amount of uncertainty, since it is not known ex-ante how the legal authorities will rule on these matters and whether future developments will re-open legal challenges.

We capture both the political uncertainty and the attempt to achieve some form of ex-ante commitment with an exogenous parameter $0 \leq \pi \leq 1$, denoting the probability that ex-post transfers will *not* be implemented, even when they are ex-post in the best interest of both parties. By varying π, we nest the polar cases of full commitment ($\pi = 1$) and full discretion ($\pi = 0$). With these assumptions, the ex-ante probability of default is given by

$$\pi_d = G(\underline{\epsilon}) + \pi(G(\bar{\epsilon}) - G(\underline{\epsilon})) \tag{14}$$

5.1 The Debt Laffer Curve.

We now turn to the optimal debt issuance problem b_1^i at time $t = 0$, taking the initial transfer τ_0 and initial debt level b_0^i as given. If debt with notional value b_1^i has been issued at $t = 0$, the expected repayment $\mathcal{P}b_1^i$ is given by:

$$\mathcal{P}b_1^i = (1 - \pi_d)b_1^i + \rho\bar{y}_1^i \left(\int_{\epsilon_{\min}}^{\underline{\epsilon}} \epsilon dG(\epsilon) + \pi \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon dG(\epsilon) \right). \tag{15}$$

This expression has three terms. First, if country i does not default (with probability $1 - \pi_d$), it repays the face value. If default occurs, investors recover instead ρy_1^i. This can happen either because default is ex-post optimal (when $\epsilon_1^i < \underline{\epsilon}$) or when a transfer is needed (when $\underline{\epsilon} \leq \epsilon_1^i < \bar{\epsilon}$) but fails to materialize (with probability π).

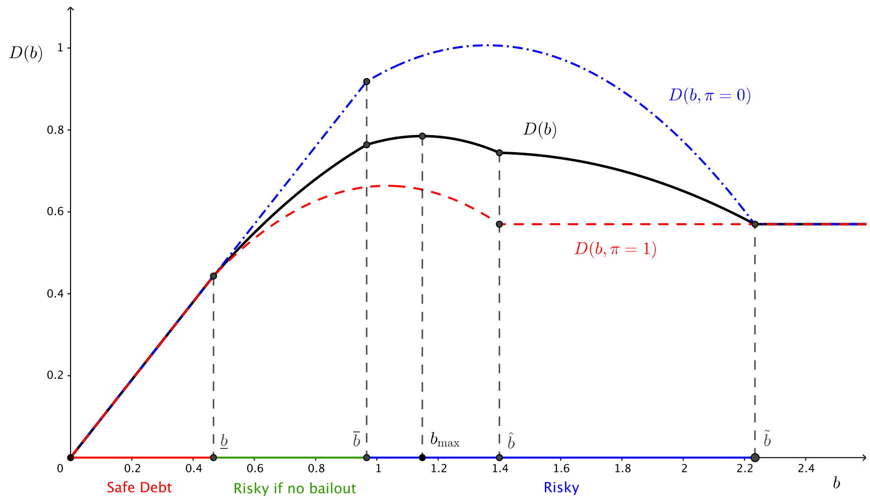
Substituting this expression into Eq. (7), we obtain an expression for the fiscal revenues $\mathcal{D}(b_1^i) \equiv b_1^i / R^i$ raised by the government of country i in period $t = 0$:

$$\begin{aligned} \mathcal{D}(b_1^i) &= \beta\mathcal{P}b_1^i + \overline{\lambda}^i \\ &= \beta b_1^i (1 - \pi_d) + \beta\rho\bar{y}_1^i \left(\int_{\epsilon_{\min}}^{\underline{\epsilon}} \epsilon dG(\epsilon) + \pi \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon dG(\epsilon) \right) + \overline{\lambda}^i \end{aligned} \tag{16}$$

The function $\mathcal{D}(b)$ defines a *Laffer curve* that plays an important role in the analysis of the optimal choice of debt issuance. Fig. 1 illustrates the shape of this Laffer curve in the bondless limit and shows how it varies with the no-bailout probability π.[43] Heuristically, we can consider the following cases:

its (see https://www.bundesverfassungsgericht.de/SharedDocs/Pressemitteilungen/EN/2014/bvg14-023.html). The legality of the ESM was also affirmed by the European Court of Justice in the Pringle Case, on November 27, 2012 (see http://curia.europa.eu/juris/liste.jsf?num=C-370/12#).

[43] This figure is drawn under the assumption that the shocks are uniformly distributed.



$\mathcal{D}(b)$ for $\pi = 0$ (discretion), $\pi = 0.5$ and $\pi = 1$ (no bailout).

[The figure is drawn in the bondless limit under the assumption that ϵ_1^i is distributed uniformly with $\rho = 0.6$, $\Phi = 0.2$, $\kappa = 0.05$, $\epsilon_{\text{min}} = 0.5$, $\epsilon_{\text{max}} = 1.5$, $\beta = 0.95$, $\bar{y}_1^i = 1$, $y_1^g = 2$, $\alpha^{i,i} = 0.4$, $\alpha^{i,g} = \alpha^{i,u} = 0.3$. $\underline{b} = 0.47$, $\bar{b} = 0.97$ and $\hat{b} = 1.4$]

Figure 1: The Debt-Laffer Curve for various values of the no-bailout probability π.

- When $0 \leq b_1^i \leq \underline{b} \equiv y_{\text{min}}^i \left(\Phi/(1 - \alpha^{i,i}) + \rho\right)$, the debt level is so low that i will always prefer to repay, without bailout and regardless of the realization of the output shock. The debt is safe, there is no default risk and $\mathcal{D}(b) = b/R^* = \beta b + \bar{\lambda}^i$. This corresponds to the left part of the graph, labelled 'Safe Debt'.
- When $\underline{b} < b_1^i \leq \bar{b} \equiv ((\Phi + \rho\alpha^{i,u})y_{\text{min}}^i + \kappa y_1^g)/\alpha^{i,u}$, the level of debt is sufficiently low that it is optimal for g to bailout i when output is too low. Default might occur if bailouts are not allowed, i.e. with probability $\pi > 0$. In that region, the Laffer curve with discretionary bailout ($\pi = 0$, in blue on the figure) is linear and lies strictly above the Laffer curve under no bailout ($\pi = 1$, in red on the figure): this is a consequence of the soft budget constraint that is induced by the transfers. Under the assumptions specified in Appendix B.3, the Laffer curve is increasing (at a decreasing rate) over that range. This corresponds to the middle part of the graph, labelled 'Risky if no bailout.'
- Since default can occur when $b > \bar{b}$, the region $b > \bar{b}$ is labelled 'Risky' on Fig. 1. It consists of three sub-regions:
  - When $\bar{b} < b_1^i \leq \hat{b} \equiv y_{\text{max}}^i(\Phi/(1 - \alpha^{i,i}) + \rho)$, it becomes optimal for g to let i default when y_1^i is sufficiently low, even if bailouts are allowed. This increases default risk and the yield on i's debt. Under the assumptions specified in Appendix B.3, the Laffer curve is convex in this region and reaches its peak at $b = b_{\text{max}}$ strictly below $\hat{b}$.
  - When $\hat{b} < b_1^i \leq \tilde{b} \equiv ((\Phi + \rho\alpha^{i,u})y_{\text{max}}^i + \kappa y_1^g)/\alpha^{i,u}$, we enter a region where default would

occur with certainty in the absence of transfers. With transfers, it is possible for default to be avoided, if output is sufficiently high. Under the assumptions in the appendix, the Laffer curve is decreasing over that region.

– Finally, for $b_1^i > \tilde{b}$, i always defaults regardless of the realization of output and bailouts are never optimal. There are no transfers and investors expect a repayment $\rho \bar{y}_1^i$.

Section B.3 provides a full characterization of these debt cut-offs and a set of necessary conditions to ensure that the Laffer curve $\mathcal{D}(b)$ is convex over the relevant range, $[0, \hat{b})$. The fact that country i can choose its repayment level b_1^i implies that it will never choose to locate itself on the 'wrong side' of the Laffer curve. It follows that we only need to consider levels of debt level such that $b \leq b_{\max}$. This eliminates Calvo (1988)-like rollover crises and multiple equilibria. Over the relevant range, the Laffer curve is convex, continuous and exhibits two non-differentiable points, $\underline{b}$ and $\bar{b}$, where it admits left and right differentials, which we denote $\mathcal{D}'_-(b)$ and $\mathcal{D}'_+(b)$ respectively.[44]

We summarize these results in the following proposition.

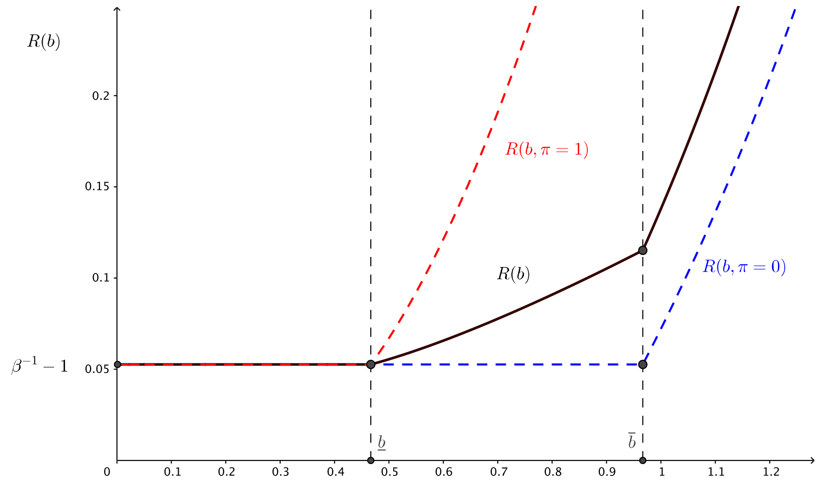
Proposition 3 (Debt Laffer Curve). *Under the regularity conditions stated in Section B.3, the Laffer curve $\mathcal{D}(b)$ has the following characteristics:*

1. *For low levels of debt, $b \leq \underline{b}$, the debt is safe there is no transfer, hence $\mathcal{D}(b) = b/R^* = \beta b + \bar{\lambda}^i$.*
2. *For intermediate levels of debt $\underline{b} < b \leq \bar{b}$, the debt is risky and a default occurs only if there is no ex-post transfer ($\pi > 0$).*
3. *For high levels of debt, $b > \bar{b}$, the debt is risky even when ex-post transfers are possible.*
4. *The Laffer curve is convex over the range $[0, \hat{b})$ where $\hat{b} > \bar{b}$ denotes the level of debt such that an ex-post default becomes unavoidable in the absence of transfers.*
5. *The Laffer curve reaches a maximum $\bar{b} \leq b_{\max} < \hat{b}$*
6. *Country i always chooses a debt level on the 'correct' side of the Laffer curve, i.e. $0 \leq b \leq b_{\max}$.*

Proof. See appendix B.3 and text. □

Fig. 2 illustrates how the contractual yield R^i varies with i's debt levels and with the probability of enforcement of no-bailout clause π. The interesting range is for $\underline{b} < b \leq \bar{b}$ where the yield remains equal to R^* if bailouts are allowed, since the debt remains safe with bailouts, but increases very rapidly –together with the ex-post probability of default– when bailouts are prohibited. This figure illustrates one possible channel for the rapid surge in yields when the crisis erupted: the perception that implicit

[44]Since $\mathcal{D}(b)$ is convex, $\mathcal{D}'_-(b) \geq \mathcal{D}'_+(b)$ with equality, denoted $\mathcal{D}'(b)$, whenever $\mathcal{D}(b)$ is differentiable.



$R(b) = (\beta\mathcal{P}(b))^{-1}$ for $\pi = 0$, $\pi = 1$ and $\pi = 0.2$.

[The figure is drawn in the bondless limit under the assumption that ϵ_1^i is distributed uniformly with $\rho = 0.6$, $\Phi = 0.2$, $\kappa = 0.05$, $\epsilon_{\min} = 0.5$, $\epsilon_{\max} = 1.5$, $\beta = 0.95$, $\bar{y}_1^i = 1$, $y_1^g = 2$, $\alpha^{i,i} = 0.4$, $\alpha^{i,g} = \alpha^{i,u} = 0.3$. $\underline{b} = 0.47$ and $\bar{b} = 0.97$]

Figure 2: Contractual Yields for various values of the no-bailout probability π

bailout guarantees were removed (i.e. a switch from $\pi = 0$ to $\pi > 0$). Similarly, one can interpret the decline in yields following President Draghi's famous pronouncement that the ECB would do 'whatever it takes' to preserve the Euro, as a sign that bailout guarantees would be reinstated, i.e. a switch from $\pi > 0$ to $\pi = 0$.[45]

5.2 Optimal Debt Issuance

We now consider the optimal choice of debt b_1^i in the *bondless limit*. This allows us to ignore the direct impact of the debt level on the utility of the agents via liquidity services. Recall that bond portfolios remain pinned down and invariant to the level of debt so we can take the portfolio shares $\alpha^{j,k}$ as given.

Substituting the optimal transfer τ_1 from Proposition 2 into the resource constraint of country i, Eqs. (3) and (4), we can express country i's aggregate consumption in both periods as a function of the face value of the debt issued in period 0 b_1^i:

$$c_0^i(b_1^i) \quad = \quad (y_0^i + \tau_0 - (1-\alpha^{i,i})b_0^i + \alpha^{s,i}b_0^s) + (1-\alpha^{i,i})\mathcal{D}(b_1^i) - \alpha^{s,i}b_1^s/R^*,$$

[45] 'Within our mandate, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough.' July 26, 2012. See https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html.

while

$$\begin{cases} c_1^i(b_1^i) = y_1^i - b_1^i(1-\alpha^{i,i}) + \alpha^{s,i} b_1^s & \text{if } \epsilon_1^i \geq \bar{\epsilon} \text{ (i repays, no transfer)} \\ c_1^i(b_1^i) = y_1^i(1-\Phi) - \rho y_1^i(1-\alpha^{i,i}) + \alpha^{s,i} b_1^s & \text{if } \epsilon_1^i < \bar{\epsilon} \text{ (i defaults or receives a transfer).} \end{cases}$$

The optimal debt issuance solves the following program:[46]

$$\begin{aligned} \max_{b_1^i} \quad & U(b_1^i) = c_0^i(b_1^i) + \beta \left(\int_{\epsilon_{\min}}^{\bar{\epsilon}} c_1^i(b_1^i) dG(\epsilon) + \int_{\bar{\epsilon}}^{\epsilon_{\max}} c_1^i(b_1^i) dG(\epsilon) \right) \\ \text{s.t.} \quad & c_0^i(b_1^i) \geq 0, \\ & 0 \leq b_1^i \leq b_{\max}, \\ & c_0^1(b_1^i) \text{ and } c_1^i(b_1^i) \text{ defined above.} \end{aligned}$$

Denoting ν_0 the multiplier on period 0 consumption and μ_1 the multiplier on $b_1^i \geq 0$, the first-order condition and complementary slackness conditions are:

$$\begin{aligned} 0 \;\in\; & \mu_1 + (1-\alpha^{i,i}) \partial \mathcal{D}(b_1^i)(1+\nu_0) - \beta(1-G(\bar{\epsilon}))(1-\alpha^{i,i}), \\ & \nu_0 c_0^i(b_1^i) = 0, \\ & \mu_1 b_1^i = 0. \end{aligned}$$

where $\partial \mathcal{D}(b)$ denotes the sub-differential of $\mathcal{D}(b)$.[47] Consider first an interior solution ($c_0^i > 0$ and $b_1^i > 0$) where the revenue curve is differentiable. The first-order condition becomes:

$$\mathcal{D}'(b_1^i) = \beta \left(1 - G(\bar{\epsilon})\right). \tag{17}$$

This condition equates the marginal gain from one additional unit of debt (at face value), $\mathcal{D}'(b_1^i)$, with its marginal cost. Eq. (17) establishes that this marginal cost is equal to the probability of repayment without transfer $1 - G(\bar{\epsilon})$, discounted back at the risk free rate $1/R^* = \beta$. In other words, country i only considers as relevant the states of the world where it is repaying the debt without default or bailout. In case of default, the repayment is proportional to output, and therefore independent of the debt level. In case of a bailout, the debt is, at the margin, repaid by country g.[48]

[46] We do not need to impose the constraint that $c_1^i \geq 0$: it is always satisfied under the assumption that $\Phi + \phi \leq 1$.

[47] The sub-differential is the derivative $\mathcal{D}'(b)$ where that derivative exists. It is the convex set $[\mathcal{D}'_+(b), \mathcal{D}'_-(b)]$ where that derivative does not exist, at $\underline{b}$ and $\bar{b}$. See Rockafellar (1972). The constraint $b \leq b_{\max}$ does not need to be imposed.

[48] From Proposition 2, the thresholds $\bar{\epsilon}$ and $\underline{\epsilon}$ are also affected by the optimal debt level. However, since these thresholds are themselves optimally chosen, the envelope theorem ensures that i does not need to consider their variation.

Substituting the general expression for $\mathcal{D}'(b_1^i)$ from Eq. (16) into Eq. (17) we can rewrite the optimality condition as:

$$\left(G(\bar{\epsilon}) - G(\underline{\epsilon})\right)(1-\pi) = (b_1^i - \rho \bar{y}_1^i \underline{\epsilon})(1-\pi) g(\underline{\epsilon}) \frac{d\underline{\epsilon}}{db} + (b_1^i - \rho \bar{y}_1^i \bar{\epsilon}) \pi g(\bar{\epsilon}) \frac{d\bar{\epsilon}}{db}. \tag{18}$$

The left hand side of this equation has a very natural interpretation. It represents the likelihood of a bailout, i.e. the likelihood that an extra unit of debt borrowed today is not repaid by i while avoiding default. This is a direct benefit to i. The right hand side represents the cost of issuing more debt via an increase in the contractual yield R^i. It has two components. The first term captures the cost of an increase in debt due to a change in $\underline{\epsilon}$. Recall that i defaults below $\underline{\epsilon}$, and receives no bailout. An increase in b_1^i increases $\underline{\epsilon}$, making outright default more likely. If $\epsilon = \underline{\epsilon}$, lenders lose b_1^i and receive instead $\rho y_1^i \underline{\epsilon}$, with probability $g(\underline{\epsilon})(1-\pi)$. They correspondingly ask for a higher contractual yield as compensation. The second term captures the cost of an increase in debt due to a change in $\bar{\epsilon}$. Recall that, above $\bar{\epsilon}$, i repays its debts and default does not occur while below $\bar{\epsilon}$, a default can occur when bailouts are not allowed. An increase in debt increases $\bar{\epsilon}$, again making default more likely. At $\epsilon = \bar{\epsilon}$, lenders are now at risk of losing b_1^i and receiving instead $\rho y_1^i \bar{\epsilon}$, in case a bailout does not materialize, i.e. with probability $g(\bar{\epsilon})\pi$. Eq. (18) highlights that i trades off the increased riskiness of debt —and therefore higher yields— against the likelihood of a bailout, i.e. non-repayment.

Under a no-bailout clause ($\pi = 1$) the left hand side of Eq. (18) is identically equal to zero. The only interior solution is $\bar{\epsilon} \leq \epsilon_{\min}$, so that $g(\bar{\epsilon}) = g(\underline{\epsilon}) = 0$: country i has no incentives to issue risky debt. By contrast, once $\pi > 0$, country i may choose to issue risky debt (i.e. $\bar{\epsilon} > \epsilon_{\min}$) in order to maximize the chance of a bailout in period 1. Eq. (18) makes clear that the possibility of a bailout in period 1 induces country i to choose excessively elevated debt levels in period 0. This risk shifting result is a common feature of moral hazard models. Ex-post bailouts partially shield borrowers from the fiscal consequences of excessive borrowing. Not surprisingly, this provides an incentive to borrow excessively. We call this the *Northern view* of the crisis.

Note also that a lower collateral cost of default for g, i.e. a lower κ, increases $\underline{\epsilon}$. This reduces the probability i will receive a transfer from g (the left hand side of Eq. (18)) and therefore the incentive to issue debt. Hence, reducing κ has a direct positive impact on g but also serves to discipline i. This resonates with proposals to introduce orderly restructuring in case of a default in a monetary union. These proposals can be interpreted in the context of our model as lower collateral costs of default that shield the rest of the monetary union, but also reduce risk-shifting (see Bénassy-Quéré, Brunnermeier, Enderlein, Farhi, Fratzscher, Fuest, Gourinchas, Martin, Pisani-Ferry, Rey, Schnabel, Veron, Weder di Mauro and Zettelmeyer (2018)).

Section B.4 provides a full description of the optimal level of debt issued in period 0. In particular, we show that, under some mild regularity conditions, the optimal choice of debt is either $b \leq \underline{b}$, i.e. a safe level of debt, or $b_{opt} \leq b \leq b_{\max}$, where b_{opt} denotes the unique optimal level of *risky* debt that obtains when the funding needs are smaller than $\mathcal{D}(b_{opt})$. The following proposition summarizes the optimal debt level, as a function of the *funding needs* of country i in period 0, defined as $x_0^i \equiv (b_0^i(1-\alpha^{i,i}) + \alpha^{s,i} b_1^s / R^* - y_0^i - \tau_0 - b_0^{s,i})/(1-\alpha^{i,i})$.

Proposition 4 (Optimal Debt Issuance in period 0)**.** *Under the regularity conditions specified in Section B.4, the optimal choice of debt as a function of the initial funding needs x_0^i satisfies:*

- *For $x_0^i > \mathcal{D}(b_{\max})$, country i is insolvent in period 0 and must default. No level of debt can ensure solvency.*
- *For $\mathcal{D}(b_{\max}) \geq x_0^i > \mathcal{D}(b_{opt})$, country i issues $b \in [b_{\max}, b_{opt})$ such that $\mathcal{D}(b) = x_0^i$ and there is no consumption in period 0. There is no risk shifting in the sense that debt issuance is fully constrained by country i's funding needs in period 0.*
- *For $\mathcal{D}(b_{opt}) \geq x_0^i > \beta \underline{b}$, country i chooses to issue b_{opt}. In that range, the possibility of a bailout leads i to risk shiting in the sense that $\mathcal{D}(b_{opt}) > x_0^i$. The probability of default is higher than necessary to cover i's funding needs.*
- *For $x_0^i < \beta \underline{b}$, country i can choose to issue either a safe amount of debt $x_0^i/\beta < \underline{b}$ or the risky amount b_{opt}. If country i prefers to issue risky debt, then the amount of risk shifting is maximal. This will be the case if i achieves a higher level of utility at b_{opt} then by keeping the debt safe, i.e. if:*

$$\begin{aligned} U(b_{opt}) - U(x_0^i/\beta) =& (1-\alpha^{i,i})\beta(1-\pi)\left(G(\bar{\epsilon}) - G(\underline{\epsilon})\right)\left(b_{opt} - \rho \bar{y}_1^i E[\epsilon | \underline{\epsilon} \leq \epsilon \leq \bar{\epsilon}]\right) \\ & - \beta \Phi G(\bar{\epsilon}) \bar{y}_1^i E[\epsilon | \epsilon < \bar{\epsilon}] > 0 \end{aligned} \tag{19}$$

Proof. See Section B.4. □

The first term of Eq. (19) represents the expected net gain from the bailout. Since $b_{opt} > \rho \bar{y}_1^i \bar{\epsilon}$, it follows that $b_{opt} > \rho \bar{y}_1^i E[\epsilon | \underline{\epsilon} < \epsilon < \bar{\epsilon}]$. The second term represents the expected discounted cost of default for i. This cost is borne by i as soon as $\epsilon < \bar{\epsilon}$ since the bailout does not affect i's utility. It is immediate that there is no risk shifting when $\pi = 1$ or when i holds most of its own debt ($\alpha^{i,i} \approx 1$). Risk shifting is more likely the higher is the optimal debt output ratio $b_{opt}/\bar{y}_1^i$ and the lower the cost of default Φ.

5.3 Making i's Debt Safe or Gambling for Resurrection?

The previous analysis makes clear that the extent of risk shifting depends on the likelihood of a bailout, $1-\pi$. When bailouts are very likely ($\pi \approx 0$), and under the regularity conditions described in Sections B.3 and B.4, b_{opt} is larger than $\bar{b}$. In other words, i chooses a level of risky debt sufficiently high so that there might be a possibility of default, even when ex-posts bailouts are almost guaranteed. In that case, the extent of risk shifting is maximal.

As π increases, so that bailouts become less certain, the optimal level of risky debt decreases until it reaches $\bar{b}$. Section B.4 shows that there is a critical level of π, denoted π_c, such that for $\pi > \pi_c$, the optimal level of debt falls discontinuously from $\bar{b}$ to $b \leq \underline{b}$ and debt becomes safe. This indicates that it is not necessary for g to enforce a strict no-bailout policy ($\pi = 1$) to eliminate risk shifting in period 0. Any level π superior to π_c will result either in a safe debt level, or the minimum level of debt necessary to cover funding needs, i.e. $\mathcal{D}(b_1^i) = x_0^i$.

Suppose g can choose a commitment technology π in period 0. A higher π reduces the amount of risk shifting and for $\pi > \pi_c$ risk shifting is eliminated entirely. However, this also reduces resources available to country i in $t = 1$ and therefore makes a default more likely. It also makes country i less solvent in $t = 0$, by shifting down the Laffer curve. If the initial fundings needs x_0^i are sufficiently high, it could force country i to default in period 0, with direct and indirect costs for g. We now establish formally that it can be in the interest of g to choose $\pi < \pi_c$, i.e. to allow the possibility of a bailout in period 1, even if it leads to risk shifting in period 0. The reason is that g prefers to gamble for resurrection: under certain conditions, a possible default tomorrow is preferable to a certain default today.

In the bondless limit, g's utility can be expressed as a function of the optimal debt $b(\pi)$ issued by i under no-bailout policy π:

$$\begin{aligned} U_g(b(\pi), \pi) &\equiv c_0^g + \beta E[c_1^g] \\ &= y_0^g - b_0^g + b_0^{i,g} + b_0^{s,g} + \beta y_1^g + \Psi(b(\pi); \pi). \end{aligned}$$

where $\Psi(b; \pi)$ denotes the net utility gain to g from i's sovereign default decision and is defined as:

$$\begin{aligned} \Psi\left(b, \pi\right) &= -\beta\kappa y_1^g G\left(\bar{\epsilon}\right) \\ &\quad - \left(1-\pi\right)\alpha^{ig}\beta b\left(G\left(\bar{\epsilon}\right) - G\left(\underline{\epsilon}\right)\right) \\ &\quad + \left(1-\pi\right)\beta\int_{\underline{\epsilon}}^{\bar{\epsilon}}\left(y_1^i\left(\Phi + \rho\left(1-\alpha^{ii}\right)\right) - \alpha^{iu} b\right) dG\left(\epsilon\right) \\ &\quad + \left(1-\pi\right)\beta\kappa y_1^g\left(G\left(\bar{\epsilon}\right) - G\left(\underline{\epsilon}\right)\right). \end{aligned}$$

The term on the first line represents the expected utility loss to g due to collateral damage when i defaults and there is no bailout ($\pi = 1$). Notice that this loss is not baked into the borrowing rate R^i since it is not a private loss for g investors. The next three lines represent the gains/losses when π is different from 1, i.e. in the presence of bailouts. The second line represents the cost of g's transfer to i. The third line represents the fact that g captures the efficiency gains from avoiding a default and the last line captures the gain from avoiding collateral damage. It is immediate to check that if i's debt is safe, then $\Psi(b;\pi) = 0$.

The optimal choice of commitment technology maximizes g's utility and satisfies:

$$\frac{d\Psi(b(\pi);\pi)}{d\pi} = \frac{\partial\Psi(b(\pi);\pi)}{\partial\pi} + \frac{\partial\Psi(b(\pi);\pi)}{\partial b}\frac{db}{d\pi} = 0.$$

The appendix provides a full discussion of the optimal choice of commitment technology and establishes that g always prefers to choose a level of commitment that rules out risk shifting, i.e. $\pi \geq \pi_c$. The intuition is simple: when $\pi \geq \pi_c$, the optimal debt level does not depend on π: $db/d\pi = 0$. It follows that the optimal choice of π over that range is controlled by the sign of $\partial\Psi/\partial\pi$. But since i's debt is safe $\Psi = 0$ then therefore g is indifferent. For lower levels of commitment, if a default with transfer is possible, it must lower the utility of g. Hence it is strictly preferable to eliminate risk shifting, to the extent possible.

This analysis is only valid as long as i remains solvent. Denote $\mathcal{D}_{\max}(\pi)$ the maximum of the Laffer curve under commitment level π. It is immediate that $d\mathcal{D}_{\max}(\pi)/d\pi \leq 0$. If $\mathcal{D}_{\max}(\pi_c) < x_0^i$, country i cannot honor its debts and is forced to default in the initial period if g insists ruling out risk shifting. This has a direct cost for g, κy_0^g, and an indirect cost, $\alpha^{i,g}(b_0^i - \rho y_0^i)$. Assume further that, in the event of a default in the initial period, i is unable to borrow, so $b_1^i = 0$. It follows that g will choose either the maximum level of commitment π consistent with avoiding a default in the current period, i.e. such that $\mathcal{D}_{\max}(\pi) = x_0^i$, or it will choose $\pi > \pi_c$ and let i default in the initial period. It will prefer the former if the following condition is satisfied:

$$\kappa y_0^g + \alpha_0^{i,g}(b_0^i - \rho y_0^i) + \Psi(b_{\max}, \pi) \geq 0. \tag{20}$$

Eq. (20) states that it can be optimal ex-ante for g to allow ex-post bailouts if these allow i to avoid an immediate default. The logic is quite intuitive: by allowing the possibility of a future bailout, g *gambles for resurrection*: in the event that i's output is sufficiently hight in period 1, debts will be repaid and a default will be avoided in both periods. Even if a bailout is required, the cost to g as of period 0 is less than one for one. This condition is more easily satisfied the higher is g's output, and the higher its exposure to i's debt. We summarize these results with the following proposition.

Proposition 5 (Insulation and Gambling for Resurrection)**.** *Suppose g can choose a commitment policy π in period 0. Then:*

- *If $x_0^i \leq \mathcal{D}_{\max}(\pi_c)$, country i's initial funding needs are sufficiently low so country g optimally adopts a firm commitment policy: $\pi \geq \pi_c$. This rules out risk-shifting and makes $i's$ debt safe if $x_0^i < \underline{b}/\beta$.*
- *If $x_0^i > \mathcal{D}_{\max}(\pi_c)$, country g prefers to gamble for resurrection if Eq. (20) is satisfied. In that case, it chooses π such that $\mathcal{D}_{\max}(\pi) = x_0^i$, i.e. it provides just enough resources in expectation to cover i's funding needs. While this eliminates risk shifting, i's debt remains risky and i may receive a bailout in $t = 1$.*

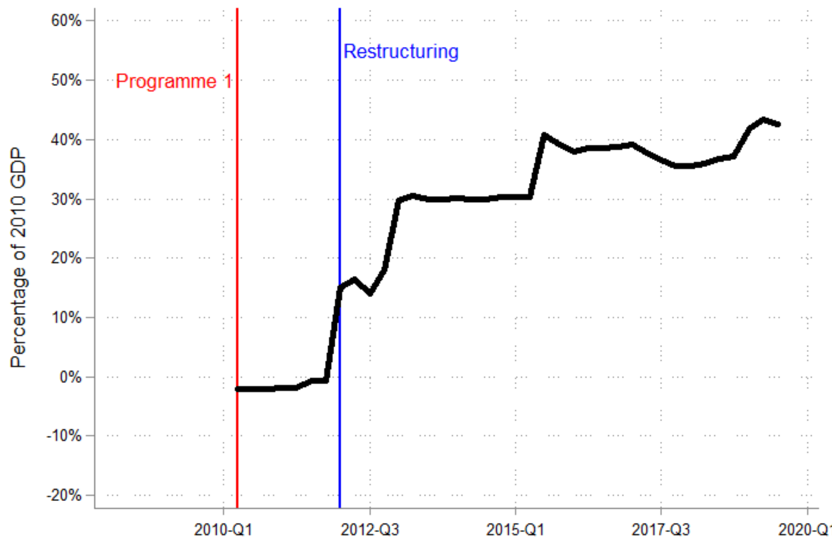
This discussion highlights that g is more likely to adopt an ex-ante lenient position on future bailouts (i.e. a low π) when i has initially a high debt level or a low output level. The proposition also highlights that g can always eliminate risk shifting, even if it does not adhere to a strict no-bailout policy. This proposition provides an interesting interpretation of the early years following the creation of the Eurozone. Countries were allowed to join the Eurozone with vastly different levels of initial public debt. The strict imposition of a no-bailout guarantee could have pushed these countries towards an immediate default and debt restructuring. Instead, it may well have been optimal to allow countries to rollover their debt on the conditional belief that a bailout might occur in the future. The fiscal cost to g of an immediate default may have exceeded the expected costs from possible future bailouts.

6 Default vs. Exit

In July 2012, Greece restructured its debt, implementing one of the largest sovereign haircuts in modern history. Yet, the country remained in the eurozone, and agreed to the terms of the bailout described in Section 2. In our baseline model, in the event that the borrower defaults, it should not receive any bailout. Fig. 3 reports the timeline of NPV transfers to Greece, estimated between 2010 and 2019 using the methodology of Section 2 and using the planned sequence of disbursements and repayments at each point in time. The figure indicates that the transfers to Greece were minimal between 2010 and 2012, before Greece defaulted, and jumped to 20% of GDP in 2012, precisely at the time at which Greece restructured its sovereign debt. Through the lens of the model, Greece should have received a transfer in 2010-11 and no transfer post-default. We now extend the model to account for the pattern we see in the data. To do so, we consider an extension of the model were a country can decide separately whether to default and/or exit the monetary union. In this extension, the other members of the monetary union may now find it in their interest to support financially one of their neighbors, so as to avoid a default, to avoid an exit from the currency union, or both.

6.1 Extended Model with Exit and Default

The extended model differentiates between the direct cost of a default for country i, denoted Φ_d, and that of an exit, denoted Φ_e, expressed as a fraction of country i's output in period 1. Similarly, we



The figure reports the timeline of combined transfers in present value from the GLF, the EFSF, the ESM and the IFM to Greece, between 2010 and 2019, were the NPV at each point in time depend on the planned sequence of disbursements and repayments in place at that time. Fraction of 2010 Greek GDP. *Source*: Authors calculations from ESM, IFM data. See online appendix for details.

Figure 3: The Time Line of Greek Transfers. Percent of 2010 Greek GDP

differentiate between the collateral cost for country g in the event of a default, denoted κ_d, and that in the event of an exit, denoted κ_e, expressed as fraction of country g's output. As in the baseline model, these costs represent the net economic disruption associated with a default, and an exit respectively on i and g. We also assume that a decision to simultaneously default and exit the currency union imposes *additive* costs $\Phi_d + \Phi_e$ on i and $\kappa_d + \kappa_e$ on g.[49]

The decision to exit the currency union brings additional benefits to i. Most importantly, it allows i to regain some monetary autonomy. We assume that this benefit is proportional to the level of expected output and express it as $\Delta \bar{y}_1^i$, with $\Delta \geq 0$.[50] The key economic assumption is that the benefits and costs of exit are not directly related to the level of debt b_1^i. As a consequence, the decision to exit will be mostly driven by the size of the recession in period 1. If anything, a larger stock of debt would make the decision to exit and devalue (but without a default) more costly, through a standard balance sheet effect. All our results would be reinforced.

In period 1, country i decides whether to repay or default and/or stay or exit the currency union.

[49]This assumption is made mostly for simplicity. An alternative assumption which we do not explore in this paper is that the cost function is super-additive in default and exit.

[50]This benefit may arise through a depreciation of the -newly created- domestic currency, and an associated expansion in domestic output. We assume that the corresponding competitiveness loss to country g is already subsumed in κ_e. In addition, one could imagine that i exiting the currency union would also confer some flexibility to g. However, we consider in what follows that the gains from this increased autonomy are negligible from g's perspective.

Country g can then decide to make a unilateral transfer conditional on i's decision package, and the realization of i's output. We further assume that g cannot commit to a no-bailout clause.[51] We already know that, under this assumption, i and g will always achieve joint ex-post efficiency. We begin by characterizing the decision choices of country i in the absence of transfers. This is summarized in the following proposition.

Proposition 6 (Optimal Default and Exit Decisions without Bailouts)**.** *In the absence of transfers, country i's default and exit decisions in period $t = 1$ are characterized by a default threshold $\bar{\epsilon}^d$ and an exit threshold $\bar{\epsilon}^e$ such that:*

1. *country i defaults if and only if:*

$$\epsilon_1^i \leq \bar{\epsilon}^d \equiv \frac{(1-\alpha^{i,i})b_1^i/\bar{y}_1^i}{\Phi_d + \rho(1-\alpha^{i,i})}. \tag{21}$$

2. *country i exits the currency union if an only if:*

$$\epsilon_1^i \leq \bar{\epsilon}^e \equiv \frac{\Delta}{\Phi_e} \tag{22}$$

Proof. See the Appendix. □

Because the gains and costs of default and exit are additive, the decisions are mutually independent. The default threshold is as in Proposition 2. Regarding the exit decision, the benefit is proportional to expected output, $\Delta\bar{y}_1^i$, while the cost is proportional to actual output $\Phi_e y_1^i$. Comparing the two yields Eq. (22).

Fig. 4 provides a graphical illustration of i's decision to default and/or exit, as a function of the ratio of debt to potential output, $b_1^i/\bar{y}_1^i$, on the horizontal axis, and the output gap ϵ_1^i on the vertical axis. The cut-offs $\bar{\epsilon}^d$ and $\bar{\epsilon}^e$ represent the cutoff for default and exit respectively. They partition the state space into the four regions described in the proposition. Higher realizations of output and lower initial debt levels make it more likely that debts will be repaid and that the country will remain in the currency union. For a given output realization, higher debt levels make it more likely that the country will want to default but don't affect exit decision. Conversely, for a given debt level, a lower realization of output makes it more likely that the country will default and exit.

Next, we consider optimal ex-post transfers from g to i. As before, we assume that g makes the minimal transfer needed to avoid default and/or exit from i. Given the additivity assumption, we can consider three possible transfers: a transfer τ_1^d to avoid a default, a transfer τ_1^e to avoid an exit. The total transfer is

[51] In terms of the baseline model, we assume that $\pi = 0$.

then $\tau_1 = \tau_1^d + \tau_1^e$. The following proposition characterizes the optimal transfers and the corresponding outcomes.

Proposition 7 (Optimal Ex-post Transfers and Default/Exit Decisions). *Under the assumptions of the model, country g implements the following optimal ex-post bailout policy:*

- *When $\epsilon^i \geq \bar{\epsilon}^d$, country i does not default and $\tau_1^d = 0$.*
- *When $\underline{\epsilon}^d \geq \epsilon_1^i < \bar{\epsilon}^d$, where*
$$\underline{\epsilon}^d = \frac{\alpha^{i,u} b_1^i / \bar{y}_1^i - \kappa_d y_1^g / \bar{y}_1^i}{\Phi_d + \rho \alpha^{i,u}} < \bar{\epsilon}^d,$$
country i does not default, and g makes a minimal transfer $\tau_1^d = \underline{\tau}_1^d \equiv (b_1^i - \rho y_1^i)(1 - \alpha_1^{i,i}) - \Phi_d y_1^i$ to avoid default,
- *When $\epsilon_1^i < \underline{\epsilon}^d$, country i defaults and country g does not make a transfer to avoid default: $\tau_1^d = 0$,*
- *When $\epsilon_1^i > \bar{\epsilon}^e$, country i does not exit and country g sets $\tau_1^e = 0$,*
- *When $\underline{\epsilon}^e \leq \epsilon_1^i < \bar{\epsilon}^e$ where*
$$\underline{\epsilon}^e = \frac{\Delta - \kappa_e y_1^g / \bar{y}_1^i}{\Phi_e} < \bar{\epsilon}^e,$$
country i does not exit and country g makes a transfer $\tau_1^e = \underline{\tau}_1^e \equiv \Delta \bar{y}_1^i - \Phi_e y_1^i$ to avoid exit,
- *When $\epsilon_1^i < \underline{\epsilon}^e$, country i exits and country g sets $\tau_1^e = 0$.*

Proof. See the Appendix. □

The intuition for the result is as follows. First, when $\epsilon_1^i \geq \max(\bar{\epsilon}^d, \bar{\epsilon}^e)$, country i prefers to repay and stay in the monetary union even in the absence of any transfer. Therefore, $\tau_1 = 0$. Second, when $\epsilon_1^i < \min(\underline{\epsilon}^d, \underline{\epsilon}^e)$, country i it is efficient to let it default and exit the monetary union, so $\tau_1 = 0$. Third, when $\epsilon_1^i < \underline{\epsilon}^d$ and $\epsilon_1^i > \bar{\epsilon}^e$, the country does not want to exit but it is optimal to let it default, so $\tau_1 = 0$. Symmetrically, when $\epsilon_1^i < \underline{\epsilon}^e$ and $\epsilon_1^i > \bar{\epsilon}^d$, the country does not want to default, but it is optimal to let it exit, so $\tau_1 = 0$. Outside of these cases, country i will receive a bailout to avoid default, exit, or both. When $\underline{\epsilon}^d \leq \epsilon_1^i < \bar{\epsilon}^d$, the joint surplus from not defaulting is positive and country g will make the minimal required transfer $\underline{\tau}_1^d$. Similarly, when $\underline{\epsilon}^e \leq \epsilon_1^i < \bar{\epsilon}^e$, the joint surplus from not exiting is positive and country g will make the minimal transfer required $\underline{\tau}_1^e$.

Fig. 4 illustrates the optimal choice of default and exit in the presence of the optimal transfers. The lines labelled $\underline{\epsilon}^d$ and $\underline{\epsilon}^e$ represent the cutoffs for default and exit respectively in the presence of transfers. Together with the original cut-offs, they partition the state in nine regions that determine both the transfer received and the outcome. The region in blue denotes the region where a bailout avoids both default and exit.

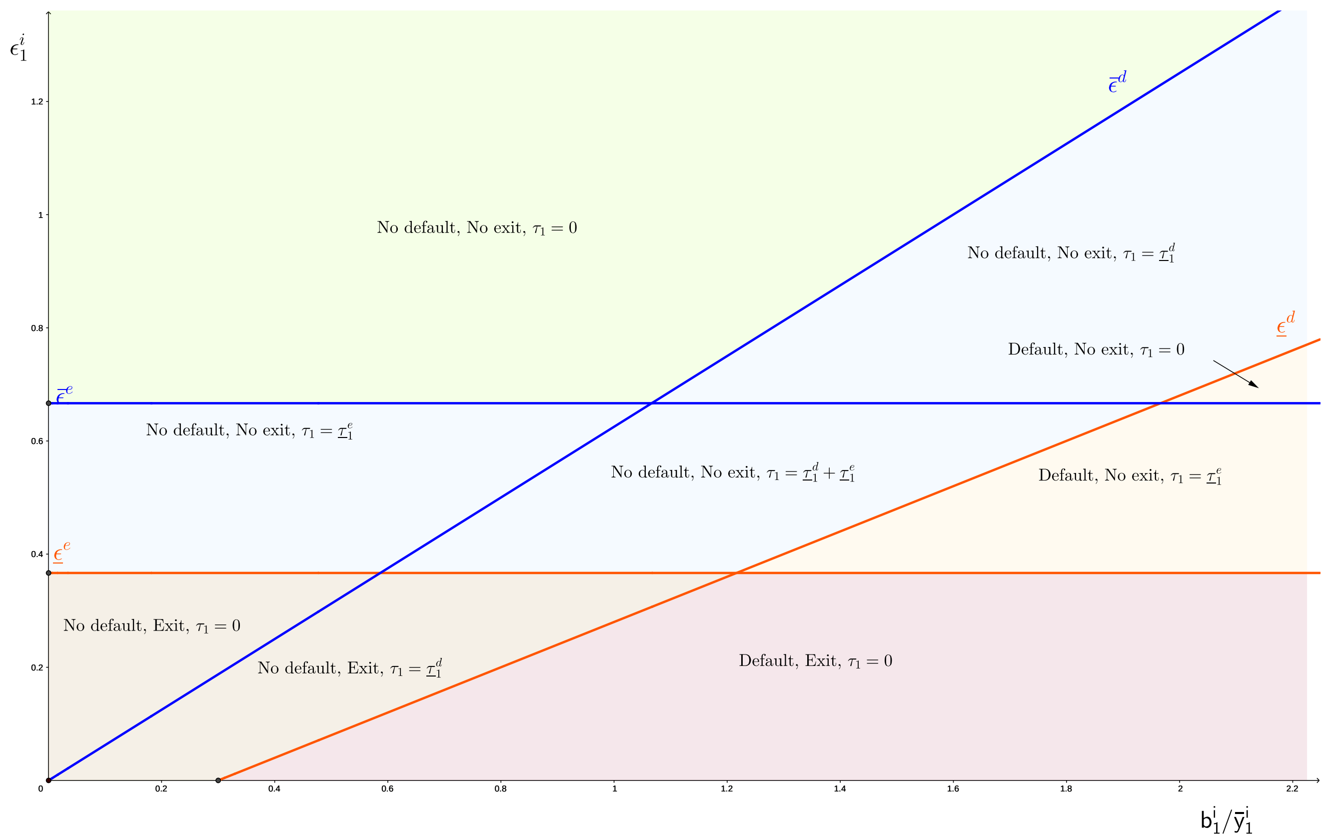


Figure 4: Optimal Ex-Post Bailout and Default vs. Exit Decisions

Notes: The figure depicts the optimal ex-post bailout and outcomes as a function of the output shock ϵ_1^i (vertical axis) and the initial debt-to-gdp ratio $b_1/\bar{y}_1^i$ (horizontal axis). The lines $\bar{\epsilon}^d$ and $\underline{\epsilon}^d$ denote the cut-off for default without and with bailout. The lines $\bar{\epsilon}^e$ and $\underline{\epsilon}^e$ denote the cut-off for exit without and with bailout.

In the case of Greece, the country defaulted but did not exit, and received a transfer τ_1 representing 44 percent of 2010 output. Through the lens of the model, this corresponds to a situation where $\underline{\epsilon}^e < \epsilon_1^i < \min(\underline{\epsilon}^d, \bar{\epsilon}^e)$, the region labeled 'Default, No exit, $\tau_1 = \underline{\tau}_1^e$' to the right in the figure. The country had elevated levels of debt, a sizable negative output shock, and received a transfer $\underline{\tau}_1^e$ to avoid exit.

The analysis contains an important message: because the exit cut-offs $\underline{\epsilon}^e$ and $\bar{\epsilon}^e$ do not depend on the country's debt level, the decision to exit and whether or not to receive a bailout $\underline{\tau}_1^e$ depends only on the realization of the output shock ϵ_1^i. This suggests that the severity of Greece's recession is a larger factor than its initial ratio of debt to output. Policies aiming to aggressively reduce public indebtedness may backfire on two fronts: they may not prevent default, and may make exit more desirable.

6.2 Empirical Counterparts

We now compare the model's implied transfer to our empirical estimates from Table 1. Four countries (Cyprus, Ireland, Portugal, and Spain) obtained a transfer that our model interprets as the minimum amount necessary to avoid default. This transfer $\underline{\tau}_1^d$ is given explicitly in Proposition 7.

While the model takes place in only two periods, our estimated transfers in Section 2 represent the present discounted value of actual and realized disbursements, repayments, and interest that have taken and will take place over many years. In the online appendix, we show how the main result of our two-period model can be extended to an infinite horizon framework. We show how we can write the present discounted value of the transfer required to avoid default as

$$\frac{\underline{\tau}_1^d/(1-\beta)}{y_0^i} = \frac{b_1^i}{y_0^i} h(1-\alpha_1^{i,i}) - \frac{y_1^i}{y_0^i}\left(\frac{\Phi_d}{1-\beta}\right) \tag{23}$$

The left hand side of this equation, $\frac{\underline{\tau}_1^d/(1-\beta)}{y_0^i}$, represents the net present value of the transfer, relative to initial output. It increases with the initial debt to GDP ratio (b_1^i/y_0^i), the share of government debt held abroad $(1-\alpha_1^{i,i})$, and the haircut (h).[52] The reason is that these variables increase the gains of default and therefore require a larger transfer. On the contrary, a higher cost of default $(\Phi_d/(1-\beta))$ applied to a larger gross increase in output(y_1^i/y_0^i) decreases the incentive to default and therefore require a smaller transfer. The key distinction between this equation and the equation in the main text is that the transfer and cost of default terms are in net present value terms, where β is the discount factor.

We obtain data on debt and GDP levels, allowing us to calculate the b_1/y_0 and y_1/y_0 for each country that received a transfer.[53] The value for the haircut h is taken from Zettelmeyer et al. (2013) and is set at $h = 0.5$. We also obtain data on the share of the debt held domestically, $\alpha_1^{i,i}$. The only remaining component is the cost of default Φ_d. We set this value to minimize the squared difference between the observed bailouts and the model-implied bailouts for the four crisis countries that avoided default.[54] We obtain a value of $\Phi_d/(1-\beta) \approx 0.09$. With a discount factor $\beta = 0.95$, this is equivalent to a flow default cost of approximately 1.5% over seven years, well within the range of estimates from existing empirical studies.[55]

Figure 5 plots the predicted value of transfers against the realized value of transfers (the black line is the 45-degree line) while Table A.3 reports the estimated transfers as well as the inputs to our calculations.

[52] In the model, we use ρ to denote the recovery rate as a share of output. The debt literature typically reports a haircut h expressed as a share of the initial debt. The relation between the two is given by $\rho y_1^i/(1-\beta) = (1-h)b_1^i$.

[53] We split the sample into two periods: 2004-2009 and 2010-2017. We choose the cutoff year 2010 as this is the first year that the first country (Greece) obtained external aid. We then take the debt to GDP ratio b_1/y_0 to be the average debt to GDP ratio in the first period, the gross change in output y_1/y_0 to be the ratio of average GDP in the second period to the first period, and we calculate the domestic share of debt as the share in the first period. See Appendix B.3 for a list of sources.

[54] We impose the constraint that transfers must be positive since they are voluntary.

[55] Estimates of the cost of sovereign default range from approximately zero (e.g. Yeyati and Panizza (2011)) to around 3-4% per year (e.g. Kuvshinov and Zimmermann (2019)) and sometimes even higher (e.g. Furceri and Zdzienicka (2012). Kuvshinov and Zimmermann (2019) find that the peak negative output effects of default lasts for at least 5 years. Given that the countries we study are advanced economies, it is reasonable that the costs of default would fall on the lower end of empirical estimates.

Our model performs well in predicting that Ireland and Spain received small transfers, and that Cyprus received a larger transfer, albeit still small. The model, however, over predicts the size of Portugal's transfer: 16% vs 3.5%. One reason may be the significantly larger debt-to-GDP ratio for Portugal (75%) relative to Cyprus, Ireland, or Spain (between 24% and 57%). The larger debt to GDP ratio may be because Portugal experienced anemic growth in the periods prior to the crisis.

Turning to Greece, our model implies that it received a transfer $\underline{\tau}_1^e$, as defined in Proposition 7, to avoid exit. Adjusted for the infinite horizon case, the transfer is given by:

$$\frac{\tau_1^i/(1-\beta)}{y_0^i} = \frac{\overline{y}_1^i}{y_0^i}\left(\frac{\Delta}{1-\beta} - \frac{\Phi_e}{1-\beta}\epsilon_1^i\right). \tag{24}$$

Importantly, this transfer does not depend on the initial debt to GDP ratio, but only on the severity of the recession ϵ_1^i. According to the model, the size of the shock and the scale of the Greek debt to GDP ratio must have been sufficiently large to obtain a transfer to avoid exit and not receive a transfer to avoid default, as shown in Figure 4. Specifically, $\underline{\epsilon}^e \leq \epsilon_1^i < \min(\bar{\epsilon}^d, \bar{\epsilon}^e)$. One way to evaluate the plausibility of the model is to consider the maximum predicted transfer that Greece could receive to avoid exit. This obtains for $\epsilon_1^i = \underline{\epsilon}^e$. Substituting the definition for $\underline{\epsilon}^e$ in the expression for $\frac{\tau_1^i/(1-\beta)}{y_0^i}$, we obtain a maximum transfer of:

$$\frac{\tau_1^i/(1-\beta)}{y_0^i} = \frac{\kappa_e}{1-\beta}\frac{y^g}{y_0^i}. \tag{25}$$

This only depends on the collateral cost that an exit would inflict to the rest of the eurozone, $\kappa_e/(1-\beta)$ and the relative size of Greece in the eurozone, y_0^i/y^g. In 2010, Greece's output was €224 bn while that of the eurozone was €9,553 bn. This suggests that a collateral cost from Greece exiting the eurozone of $\kappa_e/(1-\beta) = 0.43 \times 224/9553 \approx 1\%$ would be sufficient to justify the size of the observed bailout. This seems quite realistic. The combination of a relatively small size (high y^g/y_0^i) and a non-negligible collateral cost $\kappa_e/(1-\beta)$ deliver a very high estimate of the bailout relative to the size of the recipient economy.

7 Conclusion and policy debates

Our paper proposes a view that reconciles the "Northern" and "Southern" narratives of the crisis. The former focuses on the collateral damage of default in the EMU that reduces the credibility of the no-bailout commitment and induces excessive borrowing by fiscally fragile countries. The later stresses that the efficiency benefits of transfers and debt monetization that prevent a default are entirely captured by the creditor country. There is no "solidarity" in the transfers offered to prevent a default. We

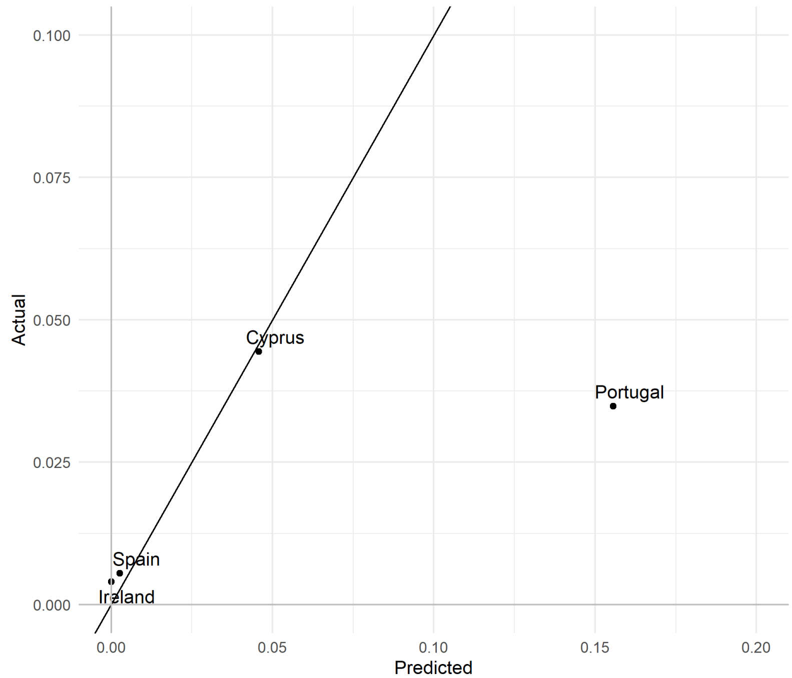


Figure 5: A Comparison of Model-Predicted and Empirically Estimated Transfers
Notes: This figure displays predicted transfers (as given in Equation 23) and realized transfers (as given in Table A.4), along with the 45-degree line. Source: See text and Appendix B.3.

show that these two views are two sides of the same coin and are necessary to understand the dynamics of the crisis at play.

Our paper also sheds light on some discussions on Eurozone reforms and why these reforms need to carefully balance these two sides by improving both market discipline and risk sharing. Proposals (see Bénassy-Quéré et al. (2018)) to introduce orderly restructuring in case of default in the Eurozone can be interpreted as lowering the collateral cost of default and also decreasing the probability of a bailout. In our model, these should reduce risk shifting, excessive borrowing and should be welcome by creditor countries. However, these proposals have been criticized (see Tabellini (2018)) as potentially destabilizing for high debt countries. This concern is indeed validated in our model because a strengthening of the no bailout commitment or any policy that increases the probability of a future default may precipitate an immediate default due to the spike in the cost of debt rollover. *"kicking the can down the road"* may have some merit after all and improving market discipline should be done very carefully and gradually especially for high-debt countries.

Our model can also speak to the debate on the creation of a fiscal capacity with macroeconomic stabilization objectives (see Kenen (1969) for the first proposal and Farhi and Werning (2017) and Bénassy-Quéré et al. (2018) among more recent ones). One criticism of such a common fiscal capacity is that it would generate transfers to fiscally fragile countries with insufficient fiscal space to use national fiscal policy during a downturn. Our model shows that these transfers are ex-post efficient in case of a shock that threatens the repayment and integrity of the eurozone. One message of our paper is that these transfers are already substantial in a monetary union with collateral exposure.

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Appendices

A Construction of the Dataset

A detailed description of the data construction is available on a not-for-publication online appendix to the paper.

- IMF Data for Cyprus, Greece, Ireland, and Portugal comes from the IMF website (https://www.imf.org/), which reports actual and projected disbursements, repayments of principal, and interest payments. Spain did not receive IMF assistance.

- EFSF and ESM Disbursements and Repayment schedules for Cyprus, Greece, Ireland, Portugal, and are available from the ESM website (https://www.esm.europa.eu/). For interest payments, we apply the blended rate for December 2019 to the series of outstanding debt over the lifetime of the lending cycle. We are grateful to Corsetti et al. (2020) for initially sharing this data, which we have extended via the ESM website.

- EFSM data for Ireland come from the Irish Treasury website. EFSM data for Portugal come from the European Commission website (https://ec.europa.eu/info/). Interest payments are calculated by applying the three-month euribor rate at the time of disbursement.

- Although we do not calculate the transfer, our information on bilateral loan data to Ireland come from the United Kingdom Treasury and the Sweden, and Denmark Ministry of Finances:

  - https://www.gov.uk/government/organisations/hm-treasury

  - https://www.government.se/government-of-sweden/ministry-of-finance/

  - https://m.fm.dk/ministryoffinance/home

- In Section 6.2, we use country level data on debt levels (domestic and total), gross domestic product, and output gaps. Nominal GDP (Mneomnic NY.GDP.MKTP.CN) and Real GDP (NY.GDP.MKTP.KN) both come from the World Bank. Debt levels come from the ECB, Total debt (GFS.A.N.XX.W0.S13.S1.C.L.LE.GD.T._Z.XDC._T.F.V.N._T, where XX is the two-letter country code) and debt held by the non-euro area (GFS.A.N.XX.W1.S13.S1.C.L.LE.GD.T._Z.XDC._T.F.V.N._T) both come from the European Central Bank. We calculate α, the share of debt held domestically, as the difference between the two series divided by the total debt series. We use data from 2005-2017, splitting the sample into two periods: 2005-2009 and 2010-2017 as desscribed in the text. All period-specific variables (GDP, etc.) are given as the average within these periods. α, which is time-invariant in the model, is taken as the average over the whole period. Table A.3 shows the resulting numbers for each borrowing country (Greece does not have data broken down by euro area and non-euro area.)

Borrower	Lender	Start Date
Cyprus	ESM	May, 2013
Cyprus	IMF	May, 2013
Greece	EC	May, 2010
Greece	EFSF	March, 2012
Greece	ESM	August, 2015
Greece	IMF	May, 2010
Hungary	BoP	December, 2008
Hungary	IMF	November, 2008
Ireland	EFSF	February, 2011
Ireland	EFSM	January, 2011
Ireland	IMF	January, 2011
Latvia	BoP	February, 2009
Latvia	IMF	December, 2008
Portugal	EFSF	June, 2011
Portugal	EFSM	May, 2011
Portugal	IMF	May, 2011
Romania	BoP	July, 2009
Romania	IMF	May, 2009
Spain	ESM	December, 2012

Table A.1: Starting Date by Programme.
Note that in the case of Greece, we treat both IMF programmes as one lending cycle. Programme 2 officially begins in May 2012. Source: compiled by authors from ESM, EFSF, IMF and European Commission. See online-appendix.

Borrower	Quota	Share of Quota
Cyprus	140	567%
Greece	823	3,374%
Hungary	1,038	735%
Ireland	838	2,322%
Latvia, Republic of	127	775%
Portugal	867	2,645%
Romania	1,030	1,026%

Table A.2: IMF Quotas (in thousands of SDR) and Share of Quotas.
Note: Share of Quotas defined as Total IMF disbursements divided by total quota as of January 2010. Source: IMF.

Borrower	Debt to GDP Ratio (b^i_1/y^i_0)	Debt Held by Euro Area (α^{ii}_1)	RGDP Growth (y_1/y_0)	Model (τ^i/y^i_0)	Empirical (τ^i/y^i_0)
Cyprus	0.57	0.51	1.04	0.046	0.045
Greece	1.09		0.80	0.46	
Ireland	0.34	0.38	1.18	0.000	0.004
Portugal	0.75	0.35	0.98	0.16	0.035
Spain	0.43	0.56	1.01	0.003	0.006

Table A.3: Inputs to model calibration, and comparison of empirical estimates and model-implied transfer values. Note that the model estimate of the transfer for Ireland is positive, yet very close to zero. Source: See text and Appendix A.

Programme	Facility	Initial Lending Rate	Initial Maturity	Disbursed	Current Lending Rate	Current Maturity
Greece I (2010)	IMF (SBA)	Standard Rate of Charge + Quota Penalty (IMF, 2016)	3-5 Years		*No Changes*	*No Changes*
	EC (GLF)	3-Month Euribor + 300bp Margin (100bp Additional After 3 Years) + Up-Front 50bp Service Fee[c]	4-Year WAM[b]		3-Month Euribor + 50bp Margin + One-Time 50bp Commission Fee	30-Year WAM
Greece II (2012)	IMF (EFF)	Standard Rate of Charge + Quota Penalty (IMF, 2017)	4-10 Years		*No Changes*	*No Changes*
	EC (EFSF)	EFSF Cost of Funding (In Kind) (ESM, 2019) + 10bp Guarantee Commission Fee[d] + 0.5bp Up-Front Service Fee[c] + 50bp Annual Service Fee + Commitment Fee (+ Step-Up Margin on Some Loans)	17.5 Year WAM		EFSF Cost of Funding (Pool Funding[a] ESM (2019) + Service Fees + Commitment Fees (+ Step-Up Margin on Some Loans, waived annually thus far)	32.5-Year WAM
Greece III (2015)	EC (ESM)	ESM Cost of Funding (Pool-Funded) + 10bp Margin + 50bp Up-Front Service Fee + 0.5bp Annual Service Fee + Commitment Fee	32.5-Year WAM		*No Changes*	32.5-Year WAM
Cyprus (2013)	IMF (EFF)	Standard Rate of Charge + Quota Penalty	4-10 Years	<€1 bn	*No Changes*	*No Changes*
	EC (ESM)	ESM Cost of Funding (Pool-Funded) + 10bp Margin + One-Time 50bp Commission Fee + Commitment Fee (Reuters, 2013)	15 Years (EFSF, 2010)	€6.3 bn	*No Changes*	*No Changes*
Ireland (2010)	IMF (EFF)	Standard Rate of Charge + Quota Penalty	4-10 Years	€22.5 bn	*No Changes*	*No Changes*
	EC (EFSF)	Fixed-Rate Back-to-Back (EFSF, 2012a)	15-Year WAM	€17.7 bn	Diversified Funding Strategy + Service Fee (ESM, 2019)	22-Year WAM (EFSF, 2013b)
	EC (EFSM)	EU Cost of Borrowing + 292.5bp Margin (Fixed-Rate Bullet) (EU, 2011e)	7.5 Years	€22.5 bn	EU Cost of Funding (Pool-Funded) (EU, 2011c)	19.5-Year WAM (EU, 2013)
Portugal (2010)	IMF (EFF)	Standard Rate of Charge + Quota Penalty	4-10 Years	<€26 bn	*No Changes*	*No Changes*
	EC (EFSF)	EFSF Cost of Funding (Fixed-Rate)	15-Year WAM (EFSF, 2012b)	€26 bn	*No Changes*	22-Year WAM (EFSF, 2013a)
	EC (EFSM)	EU Cost of Borrowing + 215bp Margin (EU, 2011b)	7.5-Year WAM	€24.3 bn	EU Cost of Borrowing (EU, 2011d)	19.5-Year WAM (EU, 2011a)
Spain (2012)	EC (ESM)	ESM Cost of Funding (In-Kind) + 10bp Guarantee Commission Fee[d] + 0.5bp Up-Front Service Fee[c] + 50bp Annual Service Fee (European Commission, 2012a)	12.5-Year WAM	€41.3 bn	*No Changes*	*No Changes*

[a] The EFSF originally used a "back-to-back" lending scheme. This scheme meant the EFSF would issue bonds of the same maturity as the loans given to borrowers. In early 2012, they formally switched to a "pool funding" scheme under the Diversified Funding Strategy. This means that the EFSF would no longer match bond issuances to borrowing countries, but would rather take advantage of different maturities in order to lower the cost of funding. The ESM always used a pooled funding scheme.

[b] WAM stands for "Weighted Average Maturity," and is calculated by taking the sum of the product of the duration and total amount of each repayment.

[c] The different types of fees on loans is as follows. One-time fees, such as the service fee by the GLF, are only assessed when loans are disbursed. From the EFSF and the ESM, there are annual fees, such as the 10bp guarantee commission fee or the 0.5bp service fee, and up-front fees, such as the 50bp service fee. The commitment fee is assessed based on changes in the borrowing strategies by the relevant institutions.

[d] The guarantee commitment fee was cancelled in November 2012.

[e] Sources: See text and online-appendix.

Table A.4: Debt Relief Summary

B Detailed Assumptions and Proofs

B.1 Budget Constraints and Market Clearing

The government of country j issues one-period bonds with face value b_1^j in period $t = 0$. It also raises aggregate lump-sum taxes T_t^j on domestic residents in period t. The budget constraints of the representative household in country i, assuming that no default occurs at $t = 0$:

$$c_0^i + b_1^{i,i}/R^i + b_1^{s,i}/R^* = y_0^i - T_0^i + b_0^{i,i} + b_0^{s,i}. \tag{B.1}$$

Country i's representative household consumes c_0^i, invests in domestic and safe debt, $b_1^{i,i}$ and $b_1^{s,i}$, with respective yields R^i and R^*. Its resources consist of after tax income $y_0^i - T_0^i$ and the face-value of maturing debt claims $b_0^{i,i}$ and $b_0^{s,i}$. In period $t = 1$, the household consumes:

$$\begin{cases} c_1^i & = y_1^i - T_1^i(r) + b_1^{i,i} + b_1^{s,i} \qquad \text{if } i \text{ repays (r)} \\ c_1^i & = y_1^i(1-\Phi) - T_1^i(d) + \rho y_1^i \frac{b_1^{i,i}}{b_1^i} + b_1^{s,i} \qquad \text{if } i \text{ defaults (d).} \end{cases} \tag{B.2}$$

The household consumes after-tax income, and liquidates its bond portfolio. In case of default, it suffers the direct cost Φy_1^i and recovers only $\rho y_1^i / b_1^i$ per unit of domestic bond purchased. Note that period 1 taxes T_1^i are state dependent and will depend on the realization of output and the decision to default or repay.

Now consider g's household. Using similar notation, the budget constraint in period $t = 0$ is

$$c_0^g + b_1^{i,g}/R^i + b_1^{s,g}/R^* \quad = \quad y_0^g - T_0^g + b_0^{i,g} + b_0^{s,g}, \tag{B.3}$$

while period $t = 1$ consumption satisfies

$$\begin{cases} c_1^g & = y_1^g - T_1^g(r) + b_1^{i,g} + b_1^{s,g} \qquad \text{if } i \text{ repays (r)} \\ c_1^g & = y_1^g(1-\kappa) - T_1^g(d) + \rho y_1^i \frac{b_1^{i,g}}{b_1^i} + b_1^{s,g} \qquad \text{if } i \text{ defaults (d).} \end{cases} \tag{B.4}$$

In the case of default, g's representative household suffers an output loss κy_1^g. As in the case of i, taxes raised in $t = 1$, T_1^g, are state contingent and may depend on whether g offers a bailout to i. A similar set of budget constraints hold for households in the rest of the world.

We now write the budget constraints of the governments in i and g. The budget constraints for i's government in periods $t = 0$ and $t = 1$ are:

$$T_0^i + b_1^i/R^i + \tau_0 = b_0^i, \tag{B.5}$$

and

$$\begin{cases} T_1^i(r) + \tau_1 & = b_1^i \qquad \text{if } i \text{ repays (r)} \\ T_1^i(d) & = \rho y_1^i \qquad \text{if } i \text{ defaults (d).} \end{cases} \tag{B.6}$$

In these expressions, τ_t denotes the direct unilateral transfer from g's government to i's government in period t. As discussed previously, these transfers can be made conditional on the decision to default by i in the same period, or *ex-ante*, so as to reduce i's debt overhang. That is, g can make a transfer τ_0 either to avoid a default in the current period or to influence default decisions in the second period.

The budget constraints for g's government are derived similarly:

$$T_0^g + b_1^g/R^* = b_0^g + \tau_0, \tag{B.7}$$

and

$$\begin{cases} T_1^g(r) & = b_1^g + \tau_1 \quad \text{if } i \text{ repays (r)} \\ T_1^g(d) & = b_1^g \qquad\quad \text{if } i \text{ defaults (d).} \end{cases} \tag{B.8}$$

Finally, the markets for safe bonds and i-bonds clear:

$$\sum_j b_1^{s,j} = b_1^s \quad ; \quad \sum_j b_1^{i,j} = b_1^i. \tag{B.9}$$

B.2 Proof of Proposition 1

Denote $\mathcal{P}^j \leq 1$ the expected payment per unit of i's sovereign debt for j's household, given the optimal default decision at $t = 1$. If i cannot discriminate between different types of bondholders when defaulting, this expected payoff is the same for all investors: $\mathcal{P}^j \equiv \mathcal{P}$. The optimal bond portfolio maximizing U^j satisfies:

$$\frac{1}{R^i} - \beta\mathcal{P} = \frac{\omega^j \lambda^{i,j}}{b_1^{i,j}}, \qquad\qquad \frac{1}{R^*} - \beta = \frac{\omega^j \lambda^s}{b_1^{s,j}}. \tag{B.10}$$

The first equation characterizes the demand for i bonds. The left-hand side of that equation represents the expected monetary cost from purchasing one more unit of i's debt: a cost $1/R^i$ and an expected discounted return $\beta\mathcal{P}$. The right hand side of that equation represents the additional liquidity benefit. The second equation characterizes the demand for safe bonds. Denote $\bar{\lambda}^i \equiv \sum_k \omega^k \lambda^{i,k}$ the weighted average of liquidity services provided by i-debt.

Combining the equilibrium condition Eq. (6) and the bond market clearing condition Eq. (5), the aggregate share $\alpha^{i,j}$ of i's debt held by country j satisfies:

$$\alpha^{i,j} \equiv \frac{b_1^{i,j}}{b_1^i} = \frac{\omega^j \lambda^{i,j}}{\bar{\lambda}^i}, \qquad\qquad \alpha^{s,j} \equiv \frac{b_1^{s,j}}{b_1^s} = \omega^j. \tag{B.11}$$

In the absence of selective default, the model implies that equilibrium portfolio shares are proportional to the relative liquidity benefits of i-debt across investor classes. To understand the intuition for this result, observe that all investors expect the same payment per unit of debt, $\beta\mathcal{P}$, and pay the same price, $1/R^i$. Hence, difference in equilibrium portfolios must arise entirely from differences in the relative liquidity services provided by the bonds, i.e. $\omega^j \lambda^{i,j}/\bar{\lambda}^i$. These shares don't depend on the riskiness of i's debt and remain well defined in the *bondless limit*. For safe assets, liquidity services are the same, up to size differences. It follows that equilibrium portfolios only reflect size differences with larger countries holding a larger share of safe assets.[56]

Substituting Eq. (B.11) into Eq. (B.10), we can rewrite the equilibrium conditions as:

$$R^* = \left(\beta + \frac{\lambda^s}{b_1^s}\right)^{-1} \quad ; \quad R^i = \left(\beta \mathcal{P} + \frac{\bar{\lambda}^i}{b_1^i}\right)^{-1}. \tag{B.12}$$

The first expression indicates that the yield on safe debt can be lower than the inverse of the discount rate $1/\beta$ because of a liquidity premium λ^s/b_1^s. As the supply of safe assets increases, this liquidity premium decreases, as documented empirically by Krishnamurthy and Vissing-Jorgensen (2012). Similarly, the yield on i's debt decreases with its liquidity premium equal to $\bar{\lambda}^i/b_1^i$, but increases with default risk, i.e. as the expected payoff per unit of i's debt $\mathcal{P}$ decreases.

B.3 Characterizing the Laffer Curve

This appendix provides a full characterization of the Laffer curve in the basic model.

The Laffer curve satisfies :

$$D(b) = \beta b \left(1 - \pi_d(b)\right) + \beta \rho \bar{y}_1^i \left(\pi \int_{\underline{\epsilon}(b)}^{\bar{\epsilon}(b)} \epsilon dG\left(\epsilon\right) + \int_{\epsilon_{\min}}^{\underline{\epsilon}(b)} \epsilon dG\left(\epsilon\right)\right) + \overline{\lambda}^i$$

where the cut-offs are defined as:

$$\begin{aligned}
\bar{\epsilon}(b) &= \frac{(1-\alpha^{i,i})b/\bar{y}_1^i}{\Phi + \rho(1-\alpha^{i,i})} \\
\underline{\epsilon}(b) &= \frac{\alpha^{i,u} b/\bar{y}_1^i - \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}}
\end{aligned}$$

and the probability of default is:

$$\pi_d(b) = G(\underline{\epsilon}(b)) + \pi(G(\bar{\epsilon}(b)) - G(\underline{\epsilon}(b)))$$

There are a number of cases to consider:

- When $b \leq \underline{b} \equiv y_{\min}^i \left(\Phi/(1-\alpha^{i,i}) + \rho\right)$. In that case $\bar{\epsilon} \leq \epsilon_{\min}$ and i's output is always sufficiently high that i prefers to repay even without any transfer from g. This makes i's debt riskless and

$$D\left(b\right) = \beta b + \bar{\lambda}^i$$

- If $\bar{b} \equiv ((\Phi + \rho\alpha^{i,u})y_{\min}^i + \kappa y_1^g)/\alpha^{i,u} \leq \hat{b} \equiv y_{\max}^i \left(\Phi/(1-\alpha^{i,i}) + \rho\right)$. This is a condition on the parameters. It can

[56] Since equilibrium portfolios are constant regardless of the riskiness of the bonds, our benchmark portfolio allocation cannot replicate the large shifts in cross-border bond holdings observed first after the introduction of the Euro (globalization of bond portfolios), then following the sovereign debt crisis (re-nationalization of bond portfolios). See Broner et al. (2014) for a description and a model of this portfolio rebalancing based on creditor discrimination. In the benchmark version of the model, this re-nationalization can only occur if the liquidity services provided by i's debt to i's banks ($\lambda^{i,i}$) increases, or if the liquidity services provided by i's debt to foreign investors ($\lambda^{i,g}$ or $\lambda^{i,u}$) decrease. A possible extension, left for future work, would allow for either discrimination in default or differential bailout policies, so that $\mathcal{P}^i$ becomes different from $\mathcal{P}^j$.

be rewritten as:

$$\kappa y_1^g/\bar{y}_1^i \leq \alpha^{i,u}\rho(\epsilon_{\max} - \epsilon_{\min}) + \Phi/(1-\alpha^{i,i})(\alpha^{i,u}\epsilon_{\max} - \epsilon_{\min}(\alpha^{i,u} + \alpha^{i,g}))$$

– When $\underline{b} < b \leq \bar{b} < \hat{b}$. In that case, we have $\underline{\epsilon} \leq \epsilon_{\min} < \bar{\epsilon} < \epsilon_{\max}$. When $b = \bar{b}$, $\underline{\epsilon} = \epsilon_{\min} < \bar{\epsilon} < \epsilon_{\max}$. Default can occur if $\epsilon_1^i \leq \bar{\epsilon}$ and ex-post transfers are forbidden. It follows that

$$D\left(b_1\right) = \beta[b_1\left(1 - \pi G\left(\bar{\epsilon}\right)\right) + \rho\bar{y}_1^i\pi\int_{\epsilon_{\min}}^{\bar{\epsilon}}\epsilon dG\left(\epsilon\right)] + \bar{\lambda}^i$$

and the slope of the Laffer curve is given by

$$D'\left(b_1\right) = \beta\left[1 - \pi G\left(\bar{\epsilon}\right) - \frac{\pi\bar{\epsilon}g\left(\bar{\epsilon}\right)\Phi}{\Phi + \rho(1-\alpha^{i,i})}\right]$$

For these intermediate debt levels, default is a direct consequence of the commitment *not* to bail-out country i in period $t = 1$. The derivative of the Laffer curve is discontinuous at $b = \underline{b}$ if the distribution of shocks is such that $g\left(\epsilon_{\min}\right) > 0$ and the can write the discontinuity as:

$$\begin{aligned} D'(\underline{b}^+) - D'(\underline{b}^-) &= \beta\left(-\underline{b} + \rho y_{\min}^i\right)\pi g(\epsilon_{\min})\left.\frac{d\bar{\epsilon}}{db}\right|_{b=\underline{b}} \\ &= -\beta\frac{\pi\epsilon_{\min}g(\epsilon_{\min})\Phi}{\Phi + \rho(1-\alpha^{i,i})} \leq 0 \end{aligned}$$

The intuition for the discontinuity is that at $b = \underline{b}$, a small increase in debt increases the threshold $\bar{\epsilon}$ beyond $\epsilon_{\min}$, so a default is now possible. This happens with probability $\pi g(\epsilon_{\min})d\bar{\epsilon}$. In that case, investors' discounted net loss is $\beta(-\underline{b} + \rho y_{\min}^i)$.

It is possible for the Laffer curve to *decrease* to the right of $\underline{b}$ if $\pi\epsilon_{\min}g(\epsilon_{\min})\Phi/(\Phi + \rho(1-\alpha^{i,i})) > 1$. In that case the increase in default risk is so rapid that the interest rate rises rapidly and i's revenues $D(b)$ decline as soon as $b > \underline{b}$. Given that i can always choose to be on the left side of the Laffer curve by choosing a lower b_1^i, there would never be any default or bailout. We view this case as largely uninteresting.

This case can be ruled out my making the following assumption *sufficient* to ensure $D'(\underline{b}^+) > 0$:

Assumption 1. *We assume the following restriction on the pdf of the shocks and the probability of bailout*

$$\pi\epsilon_{\min}g(\epsilon_{\min}) < 1$$

[Note: (a) this condition cannot be satisfied with a power law and $\pi = 1$ (i.e. no transfers); (b) this condition is satisfied for a uniform distribution if $\pi < \epsilon_{\max}/\epsilon_{\min} - 1$. A sufficient condition for this is $\epsilon_{\min} < 2/3$.[57]]

[57] To see this, observe that since $E[\epsilon] = 1$ we can solve for $\epsilon_{\min} < 2/(2+\pi)$.

The second derivative of the Laffer curve is:

$$D''(b) = -\beta\pi\frac{d\bar{\epsilon}}{db}\left[g(\bar{\epsilon}) + \frac{\Phi}{\Phi + (1-\alpha^{i,i})\rho}(g(\bar{\epsilon}) + \bar{\epsilon}g'(\bar{\epsilon}))\right]$$

If we want to ensure that $D''(b) < 0$ a *sufficient* condition is:

Assumption 2. *We assume that g satisfies*

$$\frac{\epsilon g'(\epsilon)}{g(\epsilon)} > -2$$

[Note: we can replace this condition by a condition on the slope of the monotone ratio: $\pi g(\epsilon)/(1-\pi G(\epsilon))$.]

[Note: (a) that sufficient condition is not satisfied for $\rho = 0$ and a power law; (b) it is always satisfied for a uniform distribution since $g'(\epsilon) = 0$.]

The value of $D'(\bar{b}^-)$ is:

$$D'(\bar{b}^-) = \beta\left[1 - \pi G\left(\bar{\epsilon}(\bar{b})\right) - \frac{\pi\Phi\bar{\epsilon}(\bar{b})g\left(\bar{\epsilon}(\bar{b})\right)}{\Phi + \rho(1-\alpha^{i,i})}\right]$$

We can ensure that this is positive (so that the peak of the Laffer curve has not been reached) by assuming that:

$$1/\pi > G\left(\bar{\epsilon}(\bar{b})\right) + \frac{\Phi\bar{\epsilon}(\bar{b})g\left(\bar{\epsilon}(\bar{b})\right)}{\Phi + \rho(1-\alpha^{i,i})}$$

This condition is always satisfied when there is no default ($\pi = 0$). Otherwise, a *sufficient* condition is:

Assumption 3. *We assume that the distribution of shocks satisfies:*

$$1 > G\left(\bar{\epsilon}(\bar{b})\right) + \bar{\epsilon}(\bar{b})g\left(\bar{\epsilon}(\bar{b})\right)$$

[Note: with a uniform distribution, the condition above becomes $\bar{\epsilon}(\bar{b}) < \epsilon_{\max}/2$. Substituting for $\bar{\epsilon}(\bar{b})$, this can be ensured by choosing $\epsilon_{\min}$ such that

$$\frac{1-\alpha^{i,i}}{\Phi + (1-\alpha^{i,i})\rho}\frac{(\Phi + \rho\alpha^{i,u})\epsilon_{\min} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}} < 1 - \frac{\epsilon_{\min}}{2}$$

This can be ensured with $\epsilon_{\min}$ sufficiently small, provided $(\Phi+(1-\alpha^{i,i})\rho)\alpha^{i,u} > (\Phi+\rho\alpha^{i,u})(1-\alpha^{i,i})\kappa y_1^g/\bar{y}_1^i$.]

Under assumptions 1 -3, the Laffer curve is upward sloping, decreasing in b, discontinuous at $\underline{b}$ and has not yet reached its maximum at $\bar{b}$.

– When $\bar{b} < b \leq \hat{b}$ then we have $\epsilon_{\min} < \underline{\epsilon} < \bar{\epsilon} \leq \epsilon_{\max}$. It's now possible to default even with optimal transfers and the Laffer curve satisfies

$$D\left(b_1\right) = \beta\left[b_1\left(1 - G\left(\underline{\epsilon}\right) - \pi\left(G\left(\bar{\epsilon}\right) - G\left(\underline{\epsilon}\right)\right)\right) + \rho\bar{y}_1^i\left(\pi\int_{\underline{\epsilon}}^{\bar{\epsilon}}\epsilon dG\left(\epsilon\right) + \int_{\epsilon_{\min}}^{\underline{\epsilon}}\epsilon dG\left(\epsilon\right)\right)\right] + \bar{\lambda}^i$$

with slope:

$$D'(b_1) \quad = \quad \beta\left[1 - \pi_d - \frac{\pi g(\bar{\epsilon})\bar{\epsilon}\Phi}{\Phi + \rho(1-\alpha^{i,i})} - (1-\pi)g(\underline{\epsilon})\frac{\Phi\underline{\epsilon} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}}\right]$$

One can check immediately that the slope of the Laffer curve is discontinuous at $b = \bar{b}$ as well, if $\pi < 1$ and $g(\epsilon_{\min}) > 0$, with:

$$\begin{aligned} D'(\bar{b}^+) - D'(\bar{b}^-) &= \beta\left(-\bar{b} + \rho y^i_{\min}\right)(1-\pi)g(\epsilon_{\min})\left.\frac{d\underline{\epsilon}}{db}\right|_{b=\bar{b}} \\ &= -\beta(1-\pi)g(\epsilon_{\min})\frac{\Phi\epsilon_{\min} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}} \leq 0 \end{aligned}$$

The interpretation is the following: when $b = \bar{b}$, a small increase in debt makes default unavoidable, i.e. default probabilities increase from π to 1, since the debt level is too high for transfers to be optimal. The probability of default jumps up by $(1-\pi)g(\epsilon_{\min})d\underline{\epsilon}$. The discounted investor's loss in case of default is $\beta(-\bar{b} + \rho y^i_{\min})$.

The second derivative of the Laffer curve is:

$$\begin{aligned} D''(b) &= -\beta\pi\frac{d\bar{\epsilon}}{db}\left[g(\bar{\epsilon}) + \frac{\Phi}{\Phi + (1-\alpha^{i,i})\rho}(g(\bar{\epsilon}) + \bar{\epsilon}g'(\bar{\epsilon}))\right] \\ &\quad -\beta(1-\pi)\frac{d\underline{\epsilon}}{db}\left[g(\underline{\epsilon}) + \frac{\Phi}{\Phi + \rho\alpha^{i,u}}g(\underline{\epsilon}) + g'(\underline{\epsilon})\frac{\Phi\underline{\epsilon} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}}\right] \end{aligned}$$

The first term is negative under assumption 2. The second term is also negative under assumption 2, unless $g'(\epsilon)$ becomes too negative.

Assumption 4. *The parameters of the problem are such that $D''(b) < 0$ for $b < \hat{b}$.*

[Note: with a uniform distribution, this condition is satisfied since $g'(\underline{\epsilon}) = 0$.]

We can check that:

$$D'(\hat{b}^-) \quad = \quad \beta\left[(1-\pi)(1-G(\underline{\epsilon})) - \frac{\pi g(\epsilon_{\max})\epsilon_{\max}\Phi}{\Phi + \rho(1-\alpha^{i,i})} - (1-\pi)g(\underline{\epsilon})\frac{\Phi\underline{\epsilon} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}}\right]$$

– As $\hat{b} < b \leq \tilde{b}$ where $\tilde{b} \equiv ((\Phi + \rho\alpha^{i,u})y^i_{\max} + \kappa y_1^g)/\alpha^{i,u}$, we have $\epsilon_{\min} < \underline{\epsilon} \leq \epsilon_{\max} < \bar{\epsilon}$ and now the only way for i to repay its debts is with a transfer from g.

$$D(b) = \beta\left(b(1-\pi)(1-G(\underline{\epsilon})) + \rho\bar{y}_1^i\left(\pi\int_{\underline{\epsilon}(b)}^{\epsilon_{\max}}\epsilon dG(\epsilon) + \int_{\epsilon_{\min}}^{\underline{\epsilon}(b)}\epsilon dG(\epsilon)\right)\right) + \overline{\lambda}^i$$

The derivative satisfies:

$$D'\left(b\right) = \beta\left[(1-\pi)(1-G(\underline{\epsilon})) - (1-\pi)g(\underline{\epsilon})\frac{\Phi\underline{\epsilon} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}}\right]$$

Evaluating this expression at $b = \hat{b}^+$, there is an *upwards discontinuity* in the Laffer curve:

$$\begin{aligned} D'(\hat{b}^+) - D'(\hat{b}-) &= \beta\left(\hat{b} - \rho y^i_{\max}\right)\pi g(\epsilon_{\max})\left.\frac{d\bar{\epsilon}}{db}\right|_{b=\hat{b}} \\ &= \beta\pi\frac{\Phi g(\epsilon_{\max})\epsilon_{\max}}{\Phi + \rho(1-\alpha^{i,i})} \geq 0 \end{aligned}$$

This upwards discontinuity arises because, at $b = \hat{b}$, an infinitesimal increase in debt pushes $\bar{\epsilon}$ above $\epsilon_{\max}$. The increase in the threshold becomes inframarginal and does not affect the value of the debt anymore (since the realizations where $\epsilon > \bar{\epsilon}$ cannot be achieved anymore).

At $b = \tilde{b}$, the derivative of the Laffer curve satisfies:

$$D'(\tilde{b}^-) = -\beta(1-\pi)g(\epsilon_{\max})\frac{\Phi\epsilon_{\max} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}} \leq 0$$

so the peak of the Laffer curve occurs necessarily at or before $\tilde{b}$.

The second derivative satisfies:

$$D''(b) = -\beta(1-\pi)\frac{d\underline{\epsilon}}{db}\left[g(\underline{\epsilon}) + \frac{\Phi}{\Phi + \rho\alpha^{i,u}}g(\underline{\epsilon}) + g'(\underline{\epsilon})\frac{\Phi\underline{\epsilon} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}}\right]$$

which is still negative under assumption 4.

The discontinuity at $\hat{b}$ could be problematic for our optimization problem. Consequently, we make assumptions to ensure that the peak of the Laffer curve occurs at or before $\hat{b}$. A sufficient assumption is that $D'(\hat{b}^+) < 0$.

Assumption 5. *We assume that the parameters of the problem are such that*

$$D'(\hat{b}^+) = \beta(1-\pi)\left[1 - G(\underline{\epsilon}) - g(\underline{\epsilon})\frac{\Phi\underline{\epsilon} + \kappa y_1^g/\bar{y}_1^i}{\Phi + \rho\alpha^{i,u}}\right] < 0$$

Under this assumption, the Laffer curve reaches its maximum at $0 < b_{\max} < \hat{b}$ such that $0 \in \partial D(b_{\max})$, where $\partial D(b)$ is the sub-differential of the Laffer curve at b. The peak of the Laffer curve cannot be reached at $\hat{b}$ or beyond since $D'(\hat{b}^-) < D'(\hat{b}^+) < 0$, so $0 \notin \partial D(\hat{b})$ and $D''(b) < 0$ for $b < \tilde{b}$. It follows immediately that $b_{\max} < \hat{b}$.

The economic interpretation of this assumption is that we restrict the problem so that the maximum revenues that i can generate by issuing debt in period 0 do not correspond to levels of debt so elevated that no realization of ϵ would allow i to repay on its own. In other words, the implicit transfer and the recovery value of debt are limited.

– As $b > \tilde{b}$ we have $\epsilon_{\max} < \underline{\epsilon}$ so that default is inevitable, even with transfers and the Laffer curve becomes:

$$D(b) = \beta\rho\bar{y}_1^i + \overline{\lambda}^i$$

which does not depend on the debt level. Note that there is an upwards discontinuity at $\tilde{b}$ since $D'(b) = 0$ for $b > \tilde{b}$.

To summarize, under assumptions 1-5, the Laffer curve reaches its peak at $b_{\max}$ with $\bar{b} \leq b_{\max} < \hat{b}$. The Laffer curve is continuous, convex and exhibits two (downward) discontinuities of $D'(b)$ on the interval $[0, b_{\max}]$. Since i will never locate itself on the 'wrong side' of the Laffer curve ($b > b_{\max}$), we can safely ignore the non-convexity associated with the upward discontinuities of the $D'(b)$ at $\hat{b}$ and $\tilde{b}$.

- For the sake of completeness, the remaining discussion describes what happens if $\bar{b} > \hat{b}$ (the reverse condition on the parameters). In that case, as b increases, the country stops being able to repay on its own first. This leads to a somewhat implausible case where the only reason debts are repaid is because of the transfer. We would argue that this is not a very interesting or realistic case.

– When $\underline{b} < b \leq \hat{b} < \bar{b}$. In that case, we have $\underline{\epsilon} < \epsilon_{\min} \leq \bar{\epsilon} < \epsilon_{\max}$. When $b = \hat{b}$, $\underline{\epsilon} < \epsilon_{\min} < \bar{\epsilon} = \epsilon_{\max}$. Default can occur if $\epsilon_1^i \leq \bar{\epsilon}$ and ex-post transfers are forbidden. It follows that

$$D\left(b_1\right) = \beta[b_1\left(1 - \pi G\left(\bar{\epsilon}\right)\right) + \rho\bar{y}_1^i\pi\int_{\epsilon_{\min}}^{\bar{\epsilon}}\epsilon dG\left(\epsilon\right)] + \bar{\lambda}^i$$

and the slope of the Laffer curve is given by

$$D'\left(b\right) = \beta\left[1 - \pi G\left(\bar{\epsilon}\right) - \frac{\pi\bar{\epsilon}g\left(\bar{\epsilon}\right)\Phi}{\Phi + \rho(1-\alpha^{i,i})}\right]$$

As before, default is a direct consequence of the commitment *not* to bail-out country i in period $t = 1$. The derivative of the Laffer curve is discontinuous at $b = \underline{b}$ if the distribution of shocks is such that $g\left(\epsilon_{\min}\right) > 0$ and $\pi > 0$.[58]

Under the same assumptions as before, the Laffer curve slopes up at $b = \underline{b}$.

The second derivative of the Laffer curve is:

$$D''(b) = -\beta\pi\frac{d\bar{\epsilon}}{db}\left[g(\bar{\epsilon}) + \frac{\Phi}{\Phi + (1-\alpha^{i,i})\rho}(g(\bar{\epsilon}) + \bar{\epsilon}g'(\bar{\epsilon}))\right]$$

and we can to ensure that $D''(b) < 0$ with:

$$\frac{\epsilon g'(\epsilon)}{g(\epsilon)} > -2$$

– When $\hat{b} < b < \bar{b}$, we have $\underline{\epsilon} \leq \epsilon_{\min} < \epsilon_{\max} < \bar{\epsilon}$. It follows that

$$D(b) \quad = \quad \beta b(1-\pi) + \beta\pi\rho\bar{y}_1^i + \overline{\lambda}^i$$

[58]To see this, observe that: $D'(\underline{b}^+) = \beta\left[1 - \frac{\pi\epsilon_{\min}g(\epsilon_{\min})\Phi}{\Phi+\rho(1-\alpha^{i,i})}\right] < \beta$ when $g(\epsilon_{\min}) > 0$ and $\pi > 0$.

which has a constant positive slope $\beta(1-\pi)$. At $b=\hat{b}$ the slope is discontinuous, with

$$D'\left(\hat{b}^{-}\right)=\beta\left[1-\pi-\frac{\pi\epsilon_{\max}g\left(\epsilon_{\max}\right)\Phi}{\Phi+\rho(1-\alpha^{i,i})}\right]$$

so there is an upwards discontinuity in the slope at $b=\hat{b}$.

– for $\bar{b}<\tilde{b}$ we have $\epsilon_{\min}<\underline{\epsilon}<\epsilon_{\max}<\bar{\epsilon}$ and it is now possible to default even with optimal transfers. The Laffer curve satisfies

$$D\left(b_1\right)=\beta\left[b_1\left((1-\pi)(1-G\left(\underline{\epsilon}\right)\right)+\rho\bar{y}_1^i\left(\pi\int_{\underline{\epsilon}}^{\epsilon_{\max}}\epsilon dG\left(\epsilon\right)+\int_{\epsilon_{\min}}^{\underline{\epsilon}}\epsilon dG\left(\epsilon\right)\right)\right]+\bar{\lambda}^i$$

with slope:

$$D'\left(b_1\right)\quad=\quad\beta(1-\pi)\left[(1-G(\underline{\epsilon})-g(\underline{\epsilon})\frac{\Phi\underline{\epsilon}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha^{i,u}}\right]$$

One can check that the slope of the Laffer curve is discontinuous also at $b=\bar{b}$ as long as $\pi<1$ and $g(\epsilon_{\min})>0$ with:

$$D'(\bar{b}^{+})-D'(\bar{b}^{-})=-\beta(1-\pi)g(\epsilon_{\min})\frac{\Phi\epsilon_{\min}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha^{i,u}}<0$$

At $b=\tilde{b}$, the derivative satisfies:

$$D'(\tilde{b}^{-})\quad=\quad-\beta(1-\pi)g(\epsilon_{\max})\frac{\Phi\epsilon_{\max}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha^{i,u}}<0$$

so the peak of the Laffer curve *needs* to occur before $\tilde{b}$.

The second derivative satisfies:

$$D''(b)\quad=\quad-\beta(1-\pi)\frac{d\underline{\epsilon}}{db}\left[g(\underline{\epsilon})+\frac{\Phi}{\Phi+\rho\alpha^{i,u}}g(\underline{\epsilon})+g'(\underline{\epsilon})\frac{\Phi\underline{\epsilon}+\kappa y_1^g/\bar{y}_1^i}{\Phi+\rho\alpha^{i,u}}\right]$$

which is still negative as long as $g'(\underline{\epsilon})$ is not too negative.

– As $b>\tilde{b}$ we have $\epsilon_{\max}<\underline{\epsilon}$ so that default is inevitable, even with transfers and the Laffer curve becomes:

$$D(b)=\beta\rho\bar{y}_1^i+\overline{\lambda}^i$$

which does not depend on the debt level.

B.4 Optimal Debt

Let's consider the rollover problem of country i. The first order condition is

$$0\quad\in\quad\mu_1+(1-\alpha^{i,i})\partial D(b_1^i)(1+\nu_0)-\beta(1-G(\bar{\epsilon}))(1-\alpha^{i,i})$$

$$\nu_0 c_0^i = 0$$
$$\mu_1 b_1^i = 0$$

We consider first an interior solution and ignore the non-continuity of $D'(b)$ at $\underline{b}$ and $\bar{b}$. The first-order condition becomes:

$$D'(b_1^i) = \beta\left(1 - G(\bar{\epsilon})\right) \tag{B.13}$$

Both sides of this equation are decreasing in b.

- Consider first the region $0 \leq b_1^i < \underline{b}$. Over that range, debt is safe: $D'(b) = \beta$ and $G(\bar{\epsilon}) = 0$. The first order condition is trivially satisfied: since debt is safe, risk neutral agents price the debt at β and i is indifferent as to the amount of debt it issues as long as it can ensure positive consumption.
- Consider now the interval $\underline{b} < b_1^i < \bar{b}$. We need to consider two cases.
  - when $\pi = 0$, g always bails out i and i's debt is safe. This implies $D'(b_1^i) = \beta$ and

    $$D'(b) - \beta(1 - G(\bar{\epsilon})) = \beta G(\bar{\epsilon}) > 0$$

    so there is no solution in that interval: i would always want to issue more debt.
  - when $\pi = 1$, i defaults when $b > \underline{b}$. Going back to the definition of $D'(b_1^i)$ and $\bar{\epsilon}$ we can check that

    $$D'(b) - \beta(1 - G(\bar{\epsilon})) = -\beta\frac{\Phi}{\Phi + \rho(1 - \alpha^{i,i})}g(\bar{\epsilon})\bar{\epsilon} < 0$$

    from which it follows that there is no solution in that interval: i would always want to issue less debt to remain safe.
  - In the intermediate case where $0 < \pi < 1$, it is possible to find a solution to the first-order condition. However, under reasonable conditions the second-order condition of the optimization problem will not be satisfied. This will be the case if $D'(b) - \beta(1 - G(\bar{\epsilon})$ is increasing. A sufficient condition is that g/G is monotonously decreasing. To see this, observe that for $\underline{b} < b \leq \bar{b}$, we have $\underline{\epsilon} < \epsilon_{\min}$ and therefore we can write:

    $$D'(b) - \beta(1 - G(\bar{\epsilon})) = \beta(1 - \pi)G(\bar{\epsilon})\left[1 - \frac{\pi}{1 - \pi}(b - \rho\bar{y}_1^i\bar{\epsilon})\frac{g(\bar{\epsilon})}{G(\bar{\epsilon})}\frac{d\bar{\epsilon}}{db}\right]$$

    The term in brackets is increasing in $\bar{\epsilon}$ when g/G is decreasing. If this condition is satisfied, then there is no solution in the interval $(\underline{b}, \bar{b})$. [Note: this condition is satisfied for a uniform distribution.]
- Consider next the interval $\bar{b} \leq b < \hat{b}$. We already know under the assumptions laid out in section B.3 that we only need to consider the subinterval $(\bar{b}, b_{\max})$ where $b_{\max}$ is the value of the debt that maximizes period 1 revenues. Let's consider the various values of π again:
  - for $\pi = 0$, we have $D'(\bar{b}^-) = \beta$ and $D'(b_{\max}) = 0$. Since $D'(b) - \beta(1 - G(\bar{\epsilon}))$ is continuous over that interval, then there is at least one solution to the first-order condition, possibly at $b = \bar{b}$. This solution is unique if $D'(b) - \beta(1 - G(\bar{\epsilon}))$ is strictly decreasing over that interval. Recall that over that interval we have:

$$
\begin{aligned}
D'(b) - \beta(1 - G(\bar{\epsilon})) &= \beta \left[G(\bar{\epsilon}) - G(\underline{\epsilon}) - g(\underline{\epsilon})(b - \rho \bar{y}_1^i \underline{\epsilon}) \frac{d\underline{\epsilon}}{db} \right] \\
&= \beta \left[G(\bar{\epsilon}) - G(\underline{\epsilon}) - g(\underline{\epsilon}) \frac{\Phi \underline{\epsilon} + \kappa y_1^g / \bar{y}_1^i}{\Phi + \rho \alpha^i, u} \right]
\end{aligned}
$$

The condition that $D'(b) - \beta(1 - G(\bar{\epsilon}))$ is decreasing over this range is satisfied for a uniform distribution if $\alpha^{i,g}$ is not too high.

Let's denote the unique solution b_{opt}. If $D'(\bar{b}^+) < \beta(1 - G(\bar{\epsilon}))$ then the solution is $b_{opt} = \bar{b}$.

– for $\pi = 1$ (no bailout), we can check that in that interval we can write

$$
D'(b) - \beta(1 - G(\bar{\epsilon})) = -\beta g(\bar{\epsilon})(b - \rho \bar{y}_1^i \bar{\epsilon}) \frac{d\bar{\epsilon}}{db} < 0
$$

Since $D'(\bar{\epsilon}^+) < \beta(1 - G(\bar{\epsilon}))$, it follows that there is no solution over that interval.

– For intermediate values of π, as long as π is not too high, we will have a unique solution b_{opt} as before. b_{opt} is decreasing in π for $\pi < \pi_c$. Above this critical value, this equilibrium disappears and the only remaining solutions are for $b \leq \underline{b}$. π_c is characterized by the condition that $D'(\bar{b}^-) = \beta(1 - G(\bar{\epsilon}))$. Substituting, we obtain:

$$
\pi_c = \frac{G(\bar{\epsilon})}{G(\bar{\epsilon}) + \frac{\Phi g(\bar{\epsilon}) \bar{\epsilon}}{\Phi + \rho(1 - \alpha^{i,i})}}
$$

In the case where there is no recovery, the formula for π_c simplifies to

$$
\pi_c = \frac{1}{1 + g(\bar{\epsilon}) \bar{\epsilon} / G(\bar{\epsilon})}
$$

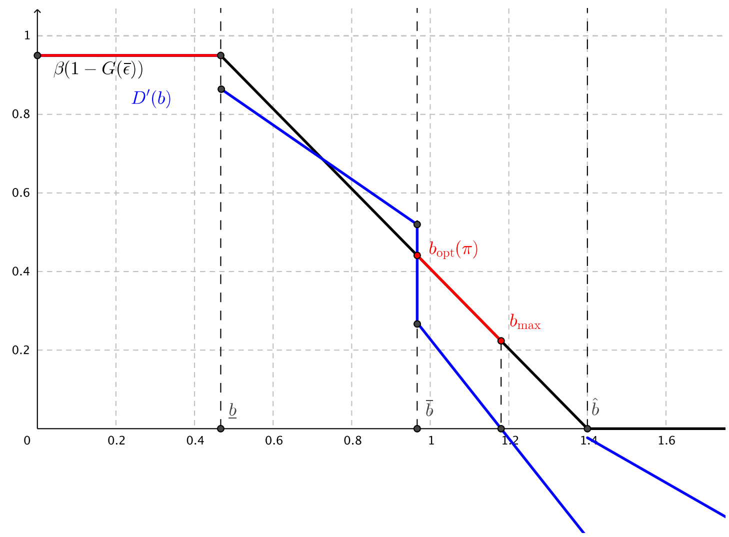
These results are summarized in Fig. B.1. The figure reports, for the case of a uniform distribution the function $\beta(1 - G(\bar{\epsilon}(b)))$ (in black) and the function $D'(b)$ (in blue). There are two discontinuities of the function $D'(b)$ at $b = \underline{b}$ and $b = \bar{b}$. In red, the figure reports the possible optimal equilibrium debt levels. For $b \leq \underline{b}$ the debt is safe and any level -if sufficient to rollover the debt– provides equivalent level of utility; $b_{opt} \geq \bar{b}$ is the optimal level of risky debt when the rollover constraint ($c_0^i \geq 0$) does not bind. Finally, $b_{opt} < b \leq b_{\max}$ obtains when the rollover constraint binds (i.e. $c_0^i = 0$ and $D(b) = x_0^i$.[59]

B.5 Exit and Default

B.5.1 Proof of Proposition 6

Proof. Denote D/ND the decision to default/repay and E/NE the decision to exit/stay in the currency union. Denote $\hat{b}$ the amount of debt held abroad, scaled by potential output: $\hat{b} = (1 - \alpha^{i,i}) b_1^i / \bar{y}_1^i$. Denote also $\hat{\rho} = \rho(1 - \alpha^{i,i})$ the foreign debt

[59] As can be seen on the figure, there is another solution to the first order condition between $\underline{b}$ and $\bar{b}$. However, this solution does not satisfy the second-order conditions.



$D'(b)$ and $\beta(1 - G(\bar{\epsilon}))$ for $\pi = 0.5$.

[Uniform distribution with $\rho = 0.6$, $\Phi = 0.2$, $\kappa = 0.05$, $\epsilon_{\min} = 0.5$, $\beta = 0.95$, $\bar{y}_1^i = 1$, $y_1^g = 2$, $\alpha^{i,i} = 0.4$, $\alpha^{i,g} = \alpha^{i,u} = 0.3$. $\underline{b} = 0.47$, $\bar{b} = 0.97$ and $\hat{b} = 1.4$]

Figure B.1: Optimal Debt Issuance

holder's recovery rate per unit of output. i prefers ND/NE to D/NE whenever:

$$-\Phi_d \epsilon_1^i + \hat{b} - \hat{\rho}\epsilon_1^i \leq 0 \iff \epsilon_1^i \geq \bar{\epsilon}^d = \frac{\hat{b}}{\Phi_d + \hat{\rho}}$$

Similarly, i prefers ND/E to D/E whenever

$$-\Phi_e \epsilon_1^i + \Delta\hat{b} \geq -(\Phi_d + \Phi_e) + (1+\Delta)\hat{b} - \hat{\rho} \iff \epsilon_1^i \geq \bar{\epsilon}^d$$

It follows that $\bar{\epsilon}^d$ represents the cut-off for default decisions, regardless of exit decisions.

Now, by a similar reasoning, we can show that i chooses to stay in the currency union whenever $\epsilon_1^i \geq \bar{\epsilon}^e$, where $\bar{\epsilon}^e$ is defined in the proposition, regardless of the decision to default. $\square$

B.5.2 Proof of Proposition 7

Proof. Let's define the minimal transfer to avoid a default $\underline{\tau}_1^d$ and the minimal transfer to avoid an exit $\underline{\tau}_1^e$. They satisfy:

$$\begin{aligned}
\underline{\tau}_1^d &= (b_1^i - \rho y_1^i)(1 - \alpha^{i,i}) - \Phi_d y_1^i \\
\underline{\tau}_1^e &= \Delta b_1^i (1 - \alpha^{i,i}) - \Phi_e y_1^i
\end{aligned}$$

Now, define $U_g(ND, NE, \tau_1)$ the utility of g if there is no default (ND), no exit (NE) and transfer τ_1. It satisfies:

$$U_g(ND, NE, \tau_1) = x_1^g + b_1^i \alpha^{i,g} - \tau_1$$

where $x_1^g = y_1^g + b_1^{s,g} - b_1^g$ is constant regardless of the transfers and i's decision. Similarly, we can define:

$$\begin{aligned}
U_g(D, NE, \tau_1) &= x_1^g - \kappa_d y_1^g + \rho y_1^i \alpha^{i,g} - \tau_1 \\
U_g(ND, E, \tau_1) &= x_1^g - \kappa_e y_1^g + b_1^i \alpha^{i,g} - \Delta b \alpha^{i,g} - \tau_1 \\
U_g(D, E) &= x_1^g - (\kappa_d + \kappa_e) y_1^g + \rho y_1^i \alpha^{i,g} - \Delta b \alpha^{i,g}
\end{aligned}$$

where we note that g will never make a transfer if i defaults and exits. Consider now the following cases:

- When $\epsilon_1^i \geq \bar{\epsilon}^d$. Since i does not want to default or exit, no transfer is necessary: $\tau_1 = 0$.
- When $\bar{\epsilon}^d > \epsilon_1^i \geq \bar{\epsilon}^e$, i prefers to default and exit. To prevent this, g must make a transfer $\underline{\tau}_1^d$. This is optimal as long as $U_g(ND, NE, \underline{\tau}_1^d) > U_g(D, NE, 0)$. This condition takes the form:

  $$\Phi_d y_1^i + \kappa_d y_1^g \geq (b_1^i - \rho y_1^i)\alpha^{i,u}$$

  or equivalently:

  $$\epsilon_1^i \geq \underline{\epsilon}^d \equiv \frac{\alpha^{i,u} b_1^i / \bar{y}_1^i - \kappa_d y_1^g / \bar{y}_1^g}{\Phi_d + \rho \alpha^{i,u}}$$

  where $\underline{\epsilon}^d < \bar{\epsilon}^d$. It follows that:

  - When $\bar{\epsilon}^d > \epsilon_1^i \geq \underline{\epsilon}^d$, g makes the transfer $\underline{\tau}_1^i$ and there is no default
  - When $\underline{\epsilon}^d > \epsilon_1^i \geq \bar{\epsilon}^e$, g does not make a transfer ($\tau_1 = 0$), i defaults, but without exiting.
- $\bar{\epsilon}^e > \epsilon_1^i$, i prefers to default and exit without transfer. g can consider two types of transfer: $\underline{\tau}_1^e$ to avoid the exit (but not the default) or $\underline{\tau}_1^d + \underline{\tau}_1^e$ to avoid both default and exit. Consider first a transfer to avoid exit. This is optimal as long as $U_g(D, NE, \underline{\tau}_1^e) > U_g(D, E)$. This condition takes the form:

  $$\Phi_e y_1^i + \kappa_e y_1^g > \Delta b \alpha^{i,u}$$

  or equivalently

  $$\epsilon_1^i \geq \underline{\epsilon}^e \equiv \frac{\Delta \alpha^{i,u} b_1^i / \bar{y}_1^i - \kappa_e y_1^g / \bar{y}_1^i}{\Phi_e}$$

  where $\underline{\epsilon}^e < \bar{\epsilon}^e$, and it yields the following utility for g:

  $$U_g(D, NE, \underline{\tau}_1^e) = x_1^g - \kappa_d y_1^g + \rho y_1^i \alpha^{i,g} - \Delta b_1^i (1 - \alpha^{i,i}) + \Phi_e y_1^i$$

  Now, within that region, g prefers to make a transfer $\underline{\tau}_1^d + \underline{\tau}_1^e$, to avoid both default and exit as long as $U_g(ND, NE, \underline{\tau}_1^d +$

$\underline{\tau}_1^e) \geq U_g(D, NE, \underline{\tau}_1^e)$ which takes the form:

$$\Phi_d y_1^i + \kappa_d y_1^g \geq (b_1^i - \rho y_1^i)\alpha^{i,u}$$

or equivalently:

$$\epsilon_1^i \geq \underline{\epsilon}^d$$

It follows that:

- When $\bar{\epsilon}^e > \epsilon_1^i > \underline{\epsilon}^e$ and $\epsilon_1^i \geq \underline{\epsilon}^d$, g prefers to make the transfer $\underline{\tau}_1^d + \underline{\tau}_1^e$ to avoid default and exit.
- When $\bar{\epsilon}^e > \epsilon_1^i > \underline{\epsilon}^e$ and $\epsilon_1^i < \underline{\epsilon}^d$, g makes the transfer $\underline{\tau}_1^e$, i defaults but stays in the currency union
- When $\underline{\epsilon}_1^e > \epsilon_1^i$, g makes no transfer ($\tau_1 = 0$), i defaults and exits.

□

C Extensions

C.1 Debt Re-Nationalization, Concentration Limits and Conditionality

The size and thresholds of bailouts are affected by the decisions of European institutions. For example, reforms of collateral rules by the ECB during the crisis changed the liquidity services provided by crisis government bonds to banks in the eurozone. In our framework, this can be modeled as a change of parameters $\lambda^{i,i}$ and $\lambda^{i,g}$ which themselves affect bond portfolio shares according to Proposition 1. As described by Bindseil et al. (2017) "Greece (2010), Ireland (2011), Portugal (2011) and Cyprus (2013) saw the suspension of the rating threshold for debt instruments issued or guaranteed by the respective governments, based on the positive assessment of the EU/IMF programmes that were ongoing at the time. However, once developments had not hinted at the successful conclusion of the programme, such waivers were lifted, as in the case of Greece and Cyprus on several occasions." The waiver allowed Eurozone banks to pledge the sovereign bonds of these countries, despite a below-investment grade credit rating. Intuitively, this increases the attractiveness of i bonds for i and g investors, relative to u investors and translates, according to Eq. (B.11), into a decrease in the equilibrium share of i bonds held outside the union, $\alpha^{i,u}$. In turn, the decrease in $\alpha^{i,u}$ lowers $\underline{\epsilon}$, reducing the possibility of default according to Proposition 2. This is intuitive: as more i bonds are held inside the union, there is less of a benefit to default on u investors. This results illustrates that with discretionary bailouts the degree of re-nationalization of debt *within* the union, i.e. the distribution of $\alpha^{i,i}$ and $\alpha^{i,g}$ for a given $\alpha^{i,u}$, is irrelevant to default outcomes. The latter are influenced by the concentration of debt holdings between the monetary union and the rest of the world as measured by $\alpha^{i,u}$.

The degree of home bias of bond holdings *within* the union does matter for the size of the bailout $\underline{\tau}_1$ and the probability of bailouts, according to Proposition 2. Some ECB measures, such as the Long Term Refinancing Operations of December 2011 and February 2012 were designed to provide long-term liquidity support to eurozone bank lending. They significantly increased the liquidity services of domestic debt to domestic banks, i.e. $\lambda^{i,i}$ (see Acharya and Steffen, 2015). Under Propositions 1 and 2, this increases $\alpha^{i,i}$, lowering $\bar{\epsilon}$ and $\underline{\tau}_1$: it reduces both the probability and the size of a bailout. Conversely, the waiver of the rating threshold described in Bindseil et al. (2017) could have contributed to a larger increase in $\lambda^{i,g}$ than $\lambda^{i,i}$, especially if domestic banks could already obtain liquidity against below-grade domestic sovereign bonds via Emergency Liquidity Assistance (ELA).[60] The waiver would decrease $\alpha^{i,i}$ relative to $\alpha^{i,g}$, increasing both the probability and

[60]From Bindseil et al. (2017), "Euro area credit institutions can receive central bank credit not only through monetary

size of a bailout. Similar effects obtain if a policy -such as concentration limits- mandates an increase in g's holdings of i's debt.

We summarize these results in the following corollary.

Corollary 1 (Debt Re-Nationalization and Concentration Limit)**.** *For a given debt level b_1^i,*

- *[Debt Re-nationalization] Higher liquidity services of domestic debt for domestic residents, $\lambda^{i,i}$: (a) increase the share of i debt held by i investors, $\alpha^{i,i}$, while reducing $\alpha^{i,g}$ and $\alpha^{i,u}$; (b) lowers the ex-post default threshold $\underline{\epsilon}$, reducing the probability of default; (c) lowers the ex-post bailout threshold $\bar{\epsilon}$, reducing the probability of a bailout and (d) reduces the size of the bailout $\underline{T}_1$.*
- *[Concentration Limit] Higher liquidity services of i debt for g residents, or concentration limits: (a) increase the share of i debt held by g investors, $\alpha^{i,g}$, while decreasing $\alpha^{i,u}$ and $\alpha^{i,i}$; (b) lowers the ex-post default threshold $\underline{\epsilon}$, reducing the probability of default; (c) increases the ex-post bailout threshold $\bar{\epsilon}$, increasing the probability of a bailout and (d) increases the size of the bailout $\underline{T}_1$.*

Proof. Immediate from Propositions 1 and 2 □

Additionally, the quote from Bindseil et al. (2017) states that the lifting of the rating thresholds was linked to the negotiation of the terms of the bailout. This suggests a trade-off between more generous support and stricter conditionality. This trade-off is potentially complex since the benefits of reform efforts can be diluted by excessive debt (debt overhang) but also stimulated, if they allow the country to escape default altogether. We analyse this question with the following variation on our main model. Suppose, to simplify things, that there is no recovery in case of default ($\rho = 0$). Further, assume that, at the beginning of period 1 once debt b_1^i has been issued but before the output shocks ϵ_1^i is realized, the government of country i can implement a 'reform' effort e_1 that increases average output according to $\bar{y}_1^i = f(e_1)$ where $f(.)$ is increasing and concave, i.e. $f'(.) > 0$ and $f''(.) < 0$. The cost of this effort is $\psi(e_1)$, which is convex to the origin with $\psi'(.) > 0$ and $\psi''(.) > 0$. Country i, in choosing its reform effort internalizes the likelihood of default and bailouts.[61] Substituting the optimal default and bailout decisions from Proposition 2, and the definition of $\bar{\epsilon}$ from Eq. (10) into Eq. (4), reform effort solves:

$$\max_{e_1} f(e_1)\left[1 - \Phi\int_{\epsilon_{\min}}^{\bar{\epsilon}} \epsilon dG(\epsilon) + \Phi\bar{\epsilon}(1 - G(\bar{\epsilon}))\right] - \psi(e_1). \tag{C.1}$$

The term in bracket represents the impact of the debt overhang on net output. When default or a bailout occurs, the country loses a fraction Φ of output. Instead, when it repays its debts, it loses $(1 - \alpha^{ii})b_1$. Importantly, because optimal bailouts leave the country indifferent between default or repayment, the debt overhang is the same in both cases (i.e. as long as

policy operations but exceptionally also through emergency liquidity assistance (ELA). ELA means the provision by a Eurosystem national central bank (NCB) of: (a) central bank money and/or (b) any other assistance that may lead to an increase in central bank money to a solvent financial institution, or group of solvent financial institutions, that is facing temporary liquidity problems, without such operation being part of the single monetary policy. Responsibility for the provision of ELA lies with the NCB(s) concerned. This means that any costs of, and the risks arising from, the provision of ELA are incurred by the relevant NCB."

[61] Note here we are considering an ex-ante reform effort, before the decision to default/bailout is made. If country i could choose its reform effort *after* output is observed, all the benefits from additional reform efforts would be captured by country g in the form of a lower transfer.

$\epsilon < \bar{\epsilon}$). Another way to see this is to observe that, according to Eq. (9), the optimal bailout $\underline{\tau}_1$ decreases in the output level y_1^i at rate Φ, i.e. the same dilution rate as under default. The optimal choice of effort e_1 satisfies:

$$f'(e_1)\left[1-\Phi\int_{\epsilon_{\min}}^{\bar{\epsilon}}\epsilon dG(\epsilon)\right]=\psi'(e_1) \tag{C.2}$$

Denote $\mathcal{J}(\bar{\epsilon})$ the term in brackets. It captures the dilution of the reform effort induced by the prospect of default or bailout.[62] Under the assumption that the second order condition of Eq. (C.1) is satisfied, the effect of bond portfolios, or debt levels, on reform effort are summarized by $\mathcal{J}'(\epsilon)$, which satisfies:[63] $\mathcal{J}'(\epsilon) = -\Phi\epsilon g(\epsilon) \leq 0$. A higher $\bar{\epsilon}$, meaning a higher likelihood of default or bailout, is associated with a lower $\mathcal{J}$, i.e. higher dilution, which reduces reform effort levels. This is a standard *debt-overhang effect*. When $\bar{\epsilon} < \epsilon_{\min}$, default or bailouts cannot occur and there is no dilution, $\mathcal{J} = 1$, and country i chooses the first-best reform effort e_1^* such that $f'(e_1^*) = \psi'(e_1^*)$. When $\epsilon_{\min} < \bar{\epsilon}$, defaults or bailouts are possible, reform benefits are diluted, $\mathcal{J} < 1$ and the reform effort level declines: $e_1 < e_1^*$.

As mentioned above, the optimal reform effort is unaffected by the bailout policy. Another way to state this same result is that bailouts do not, per se, increase the benefit of a reform for country i, despite the fact that country i receives financial assistance that helps it avoid default. This captures another aspect of the 'Southern' view: despite receiving financial assistance, the marginal benefit of reforms is unchanged. At the margin, reforms are seen as benefiting the creditor country.

A second implication is that the optimal reform effort varies with the distribution of i's debt *inside* the monetary union, as captured by $\bar{\epsilon}$. Debt-renationalization, interpreted as an increase in $\lambda^{i,i}$, increases $\alpha^{i,i}$, reducing $\bar{\epsilon}$ and the likelihood of default. This increases the optimal effort level e_1. Conversely, the waiver of rating thresholds (interpreted as an increase in $\lambda^{i,g}$), concentration limits (interpreted as a floor on $\alpha^{i,g}$), or increases in outstanding debt levels have the opposite effects: they reduce reform effort by exacerbating the debt overhang effect. In both cases, part of the reform effort benefits g since it reduces the optimal bailout τ_1. We summarize these results in the following corollary.

Corollary 2 (Optimal Reform Effort, Debt Overhang and Home Bias)**.** *Given a reform benefit function $f(e_1)$ and cost $\psi(e_1)$ as specified above,*

- *The output cost of default Φ creates a debt overhang, reducing reform efforts below their first-best level when $\epsilon_{\min} < \bar{\epsilon}$: $e_1 < e_1^*$.*
- *Optimal ex-post bailouts do not affect the optimal reform effort which remains inefficiently low: the benefits of reform are diluted via lower expected bailouts (Southern view).*
- *Optimal reform effort depends on the extent of portfolio home bias. Collateral or liquidity policies that increase debt re-nationalization increase reform efforts. Conversely, concentration limits or liquidity policies that reduce debt home bias reduce reform effort.*

Proof. See the text. □

[62] Note that the effect of the reform effort on the bailout cutoff $\bar{\epsilon}$ does not appear in this expression because of the envelope theorem: $\bar{\epsilon}$ is chosen optimally by country i.

[63] The second-order condition is satisfied if $\mathcal{S}(e_1) \equiv f''(e_1) - \Psi''(e) + f'(e_1)\mathcal{J}'(\bar{\epsilon})\partial\bar{\epsilon}/\partial e_1 < 0$. Since $\partial\bar{\epsilon}/\partial e_1 < 0$, and $\mathcal{J}'(.) < 0$, the second-order condition can always be satisfied by assuming sufficient concavity (resp. convexity) of $f(.)$ (resp. $\Psi(.)$). Full differentiation then implies that a change x that affects $\bar{\epsilon}$ will impact e_1 according to $de_1/dx = (-1/\mathcal{S}(e_1))f'(e_1)\mathcal{J}'(\bar{\epsilon})\partial\bar{\epsilon}/\partial x$.

These results help understand why the ECB may have conditioned the waiver of rating thresholds with the continued implementation of reforms. In the absence of such conditionality, Corollaries 1 and 2 indicate that waving the rating thresholds could have simultaneously reduces the probability of default, increased the likelihood and size of a bailout, but also reduced reform effort in country i.

C.2 Bargaining over the surplus

In the main model, we assumed that all the bargaining power was in the hands of the creditor country. Even though it is plausible that the creditor country has a larger bargaining weight, our assumption of a 'take it or leave it' offer may be too strong. In this section we relax this assumption and instead assume that the bargaining weight of i is $0 < \gamma < 1$, and that of country g is $1 - \gamma$. The baseline case corresponds to $\gamma = 0$. This changes the size of the transfer that i receives as it can obtain a share γ of the total surplus generated by avoiding the default. The transfer to i is now:

$$\underline{\tau}_1(\gamma) = b_1^i \left(1 - \alpha^{i,i}\right) - y_1^i \left[\Phi + \rho(1 - \alpha^{i,i})\right] + \gamma \left(\Phi y_1^i + \kappa y_1^g - \alpha^{i,u}(b_1^i - \rho y_1^i)\right). \tag{C.3}$$

The first two terms on the right hand side corresponds to the previous expression for the transfer, i.e. when i has no bargaining power. The last term represents the share of the total surplus $\Phi y_1^i + \kappa y_1^g - (b_1^i - \rho\alpha^{i,u})$ that goes to country i. It increases with the collateral damage κ inflicted on g in case of default. While the transfer increases with γ, the threshold levels $\underline{\epsilon}$ and $\bar{\epsilon}$ remain unchanged. At $\underline{\epsilon}$ the surplus is zero, hence there is no transfer and default becomes optimal. At $\bar{\epsilon}$, country i unilaterally prefers not to default even without transfers. As long as income realizations are observable, no transfer is needed. This implies a downward discontinuity in consumption for i at $\epsilon = \bar{\epsilon}$: a slight increase in income makes default non-credible and therefore eliminates the transfer.

The expectation that i has some strictly positive bargaining weight in period 1 also modifies the incentive to issue debt in period 0. Because the thresholds are unchanged, the probability of default is unaffected. Hence the Laffer curve is unchanged. However, the level of debt issued, b_1^i, affects the expected transfer along two margins. In comparison to Eq. (18), the first-order condition for optimal debt has two additional terms :

$$\begin{aligned} &\left\{(G(\bar{\epsilon}) - G(\underline{\epsilon})) + \frac{\gamma}{(1-\alpha^{i,i})}\left[\Phi y_1^i(\bar{\epsilon}) + \kappa y_1^g - \alpha^{i,u}\left(b_1^i - \rho y_1^i(\bar{\epsilon})\right)\right] g(\bar{\epsilon})\frac{d\bar{\epsilon}}{db}\right\}(1-\pi) \\ = \quad & (b_1^i - \rho \bar{y}_1^i \underline{\epsilon})(1-\pi) g(\underline{\epsilon})\frac{d\underline{\epsilon}}{db} + (b_1^i - \rho \bar{y}_1^i \bar{\epsilon})\pi g(\bar{\epsilon})\frac{d\bar{\epsilon}}{db} + \frac{\gamma}{(1-\alpha^{i,i})}\alpha^{i,u}\left[G(\bar{\epsilon}) - G(\underline{\epsilon})\right](1-\pi). \end{aligned}$$

On the left hand side (marginal gain of issuing debt), in addition to the expectation that marginal debt is paid by the transfer, an additional term (the second one in the bracket) is related to the discontinuity in consumption at $\bar{\epsilon}$: increasing the level of debt raises $\bar{\epsilon}$ and therefore makes it more likely that a transfer will be needed. A share γ of the surplus is now captured by i and this additional marginal gain of debt increases risk shifting. There is however also an additional term on the right hand side (marginal cost of issuing debt). In addition to the cost of increasing debt due to higher thresholds ($\bar{\epsilon}$ and $\underline{\epsilon}$) and therefore borrowing costs, the last term on the right hand side reduces the incentive to issue debt. This is because higher debt reduces the total surplus from not defaulting on the rest of the world (as measured by the share of debt held outside the eurozone $\alpha^{i,u}$). Country i captures a share γ of that surplus, hence this reduces the incentive to issue debt. The net effect on debt issuance is ambiguous.

Relaxing the assumption that all the bargaining power is in the hand of the creditor country leaves the thresholds for default or bailout unchanged compared to our baseline case. Instead, it shares the benefits of the bailout with the debtor country, and has an ambiguous effect on the incentive to issue debt.

C.3 Debt monetization

Debt monetization is an alternative to default which we have excluded so far. Even though article 123 of the Treaty of the European Union forbids ECB direct purchase of public debt, debt monetization can still take place through inflation and euro depreciation. In this section, we analyze in a very simplified framework how the interaction of transfers and debt monetization affects the probability of default and how the ECB may be overburdened when transfers are excluded. To facilitate the analysis of this extension we simplify the model by assuming a zero recovery rate ($\rho = 0$) and by focusing on two polar cases where transfers are always possible ($\pi = 0$) and where transfers are excluded ($\pi = 1$).

There are now three players: i, g and the ECB. In addition to g's decision on the transfer, i's decision on default, the ECB decides how much and whether to monetize the debt. We assume the ECB can choose the inflation rate for the monetary union as a whole. This would be the case for example with Quantitative Easing (QE) which generates higher inflation and euro depreciation that both reduce the real value of public debt. Importantly, all public debts are inflated away at the same rate in the monetary union so that g also stands to benefit from it. We follow Aguiar et al. (2015) and assume that the ECB trades off distortionary costs of inflation against the fiscal benefits of debt reduction[64]. If z is the inflation rate, the distortion cost is $\delta z y_1^h$ for $h = i, g$. We also assume as in Aguiar et al. (2015) that the inflation rate is between 0 and a maximum rate $\overline{z}$ above which distortion costs are infinite.

The ECB can also implement targeted purchases of public debt. In this case, it would be possible to buy public debt of a specific country without any inflation cost for example if it was sterilized by sales of other eurozone countries debt. The Outright Monetary Transactions (OMT) program announced in September 2012 (but never put into place) is close to such a description. The Securities Markets Program (SMP) was put into place in May 2010 by the ECB and terminated in September 2012 to be replaced by OMT. The aim was to purchase sovereign bonds on the secondary markets. At its peak, the program's volume totaled around 210 bn euros. In the case of Greece, the Eurogroup decided in 2018 to transfer (via the ESM) part of the profits made through SMP back to the country. Such a decision can therefore be interpreted in the context of our model as similar to transfers that we analyzed above. Debt monetization at the inflation rate $\overline{z}$ is of a different nature and resembles a partial default, except that the total cost for the eurozone is $\delta\overline{z}\left(y_1^i + y_1^g\right)$ in case of inflation and $\Phi y_1^i + \kappa y_1^g$ in case of a standard default. We also reasonably assume that Φ and κ are larger than $\delta\overline{z}$, so that,in proportion to output, the costs of default are both larger than the marginal distortionary cost of inflation.

C.3.1 The case with transfers

We first analyze the case where transfers by g are possible and not subject to political risk i.e. $\pi = 0$. Remember that in presence of transfers by g to i, g captures the entire surplus of i not defaulting: g's transfers are ex-post efficient from the joint perspective of g and i. This implies that the objective of the ECB and g are perfectly aligned if, as we assume, the ECB maximizes the whole EMU welfare. The ECB will choose either zero or maximum inflation rate $\overline{z}$ depending whether the marginal benefit of inflating the eurozone debt held in the rest of the world is below or above its marginal distortion cost. Without default, the ECB will choose not to inflate the debt if the gain from inflating debt held outside the eurozone

[64] An alternative is de Ferra and Romei (2019) who analyze the interaction between sovereign default risk and monetary policy in a monetary union where debt is denominated in real terms. In their model, a looser stance of monetary policy increases debtors' incentive to repay debt.

is lower than the distortion costs of inflation. The ECB chooses a zero inflation rate if i's output realization is high enough such that:

$$\epsilon_1^i > \frac{b_1^i \alpha^{i,u} + b_1^g \alpha^{g,u}}{\delta \overline{y}_1^i} - \frac{y_1^g}{\overline{y}_1^i} \equiv \overline{\overline{\epsilon}} \tag{C.4}$$

We exclude situations such that the ECB inflates even in case of default of i (which we call fiscal dominance) which apply when g debts are very high and situations where the ECB never inflates (which we call monetary dominance) which apply when distortion costs δ are very high. This latter case is identical to the main model. The conditions on parameters are detailed in Section C.3.3. Hence, we concentrate on the interesting case where the ECB may inflate the debt for low levels of i output (below $\overline{\overline{\epsilon}}$) which we call "weak fiscal dominance". When the ECB decides not to monetize the debt, $\overline{\epsilon}' = \frac{b_1^i\left(1-\alpha^{i,i}\right)}{\Phi \overline{y}_1^i}$ defines the threshold level of shock above which i does not require any transfer and does not default. In the case of monetization, the transfer necessary to make i indifferent between default and no default becomes:

$$\tau_1 = b_1^i \left(1 - \alpha_1^{i,i}\right)(1 - \overline{z}) - y_1^i \left[\Phi - \delta \overline{z}\right] + \overline{z} b_1^g \alpha^{g,i} \tag{C.5}$$

We can compare the transfer with and without monetization. The first element on the right hand side reduces the required transfer because debt monetization weakens the incentive of i to default on debt held outside of i. The inflation distortion in the second term, proportional to output, y_1^i must be compensated by a higher transfer given that in default there is no such inflation distortion. The last term is the inflation tax on the g debt held by i which also must be compensated by a higher transfer. Hence, debt monetization allows to reduce the transfer for low levels of g debt and high levels of i debt which is the case we concentrate on. The threshold level of i output below which g prefers a default is also affected by the possibility of ECB monetization:

$$\epsilon_1^i < \frac{\alpha^{i,u} b_1^i (1 - \overline{z}) - \alpha^{g,u} b_1^g \overline{z} - y_1^g (\kappa - \delta \overline{z})}{(\Phi - \delta \overline{z}) \overline{y}_1^i} \equiv \underline{\epsilon}' \tag{C.6}$$

It can be shown that ECB monetization, if it takes place, always reduces the likelihood of default in the sense that $\frac{\partial \underline{\epsilon}'}{\partial \overline{z}} < 0$, i.e. the output realization below which i defaults falls with debt monetization. The intuition is that the net gain of inflating the debt for the eurozone is eliminated when default occurs. Hence, monetization, because it taxes agents from outside the eurozone, produces an additional incentive for g not to let i default. Another result is that the whole benefit of debt monetization (on the part of debt held outside the eurozone), if it occurs, is captured by g. The increase in consumption by g due to debt monetization can be shown to be: $\overline{z}\left[b_1^i \alpha^{i,u} + b_1^g \alpha^{g,u} - \delta\left(y_1^i + y_1^g\right)\right]$ which represents the entire surplus of monetization of eurozone debt held by the rest of the world (net of distortion costs). The intuition is that any increase in net income (through debt monetization or through an increase in ϵ_1^i) of i serves to lower the necessary transfer to avoid default.

Under reasonable parameters (see appendix) Fig. C.1 depicts how the equilibrium changes with i output realizations. As they deteriorate, the equilibrium moves from a situation with 1) no default, no transfer, no inflation, ; 2) no default, transfer, no inflation; 3) no default, inflation, transfer; 4) default, no inflation, no transfer. This case applies in particular for low levels of g debt.

C.3.2 When transfers are excluded: the overburdened ECB

The situation we described is one where a fiscal union or a strong cooperative agreement exists such that fiscal transfers are possible with full discretion ($\pi = 0$). This meant that two instruments exist for two objectives: transfers to avoid default and inflation to monetize the debt held outside the eurozone. This is an efficient use of these two instruments.

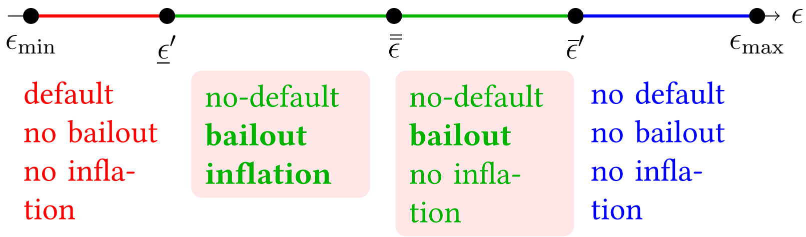


Figure C.1: Bailout and Inflation under Weak Fiscal Dominance

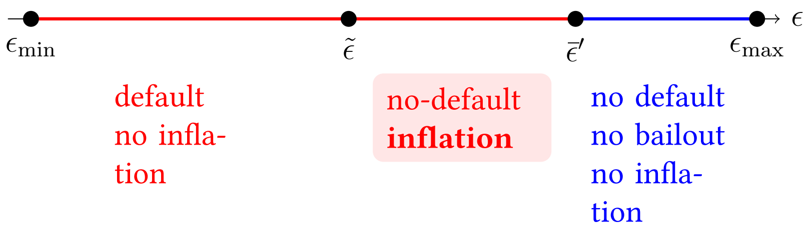


Figure C.2: Bailout with Overburdened Central Bank

These transfers may actually be hard to implement for political and legal reasons which we captured in the previous analysis with $\pi > 0$. We analyze the simplest version of this situation with $\pi = 1$. Because ex-post efficient transfers to avert a default are not possible, the ECB may now use monetary policy to avert a costly default. To make the analysis as simple and as stark as possible we assume that the ECB may choose positive inflation only because transfers are not possible and in order to avoid a default of i. In addition, we assume that $b_1^g = 0$ as we concentrate on the incentive to avert a default of i. The minimum inflation rate necessary to avoid a default is the one that leaves i indifferent between default and no default:

$$\tilde{z} = \frac{b_1^i \left(1 - \alpha^{i,i}\right) - \Phi y_1^i}{b_1^i \left(1 - \alpha^{i,i}\right) - \delta y_1^i} \tag{C.7}$$

Note that as long as $\Phi > \delta$ (which we assume), the inflation rate necessary to avert default increases as the output shock in i deteriorates. This equation also defines a threshold level of shock $\bar{\epsilon}' = \frac{b_1^i \left(1-\alpha^{i,i}\right)}{\Phi \bar{y}_1^i}$ above which i does not require any monetization and does not default. It can be shown that for $\Phi > \kappa > \delta$ the ECB is willing to accept such monetization at rate $\tilde{z}$ to avert a default but the constraint that it is below the maximum rate $\bar{z}$ defines a level of shock below which the ECB prefers to let the country default rather than monetize it:

$$\tilde{\epsilon} \equiv \frac{\left(1 - \alpha^{i,i}\right) b_1^i \left(1 - \bar{z}\right)}{\left(\Phi - \delta \bar{z}\right) \bar{y}_1^i}$$

Fig. C.2 shows that when transfers are impossible, the ECB inflates the debt for intermediate levels of output realizations to avoid default. The inflation rate is maximum just above the threshold $\tilde{\epsilon}$. Contrary to transfers, inflation generates distortion costs. Hence, using inflation rather than transfers to avoid default, a situation where the ECB is "overburdened", is inefficient.

C.3.3 Full Characterization of Debt Monetization

This appendix provides a full characterization of the different cases that arise with possible debt monetization within a monetary union. They depend on the output shock realization ϵ_1^i and on the ranking of the output thresholds. We first analyze the decision to default of i for a given transfer and inflation/monetization rate. If i repays the ECB chooses the rate z and if i defaults it chooses the rate $\hat{z}$. The budget constraint in period 1 of the i households becomes:

$$
\begin{aligned}
c_1^i &= y_1^i - T_1^i + \left(b_1^{i,i} + b_1^{g,i}\right)(1-z) - \delta z y_1^i + b_1^{u,i} \quad \text{if } i \text{ repays} \\
c_1^i &= y_1^i(1-\Phi) - T_1^i + b_1^{g,i}(1-\hat{z}) - \delta \hat{z} y_1^i + b_1^{u,i} \quad \text{if } i \text{ defaults}
\end{aligned}
$$

Government i constraint in $t = 1$ is:

$$
\begin{aligned}
T_1^i + \tau_1 &= b_1^i(1-z) \quad \text{if } i \text{ repays} \\
T_1^i &= 0 \quad \text{if } i \text{ defaults}
\end{aligned}
$$

Consolidating the private and public budget constraints, we again proceed by backward induction. At $t = 1$, i can decide to default after the shock ϵ_1^i has been revealed and the transfer τ_1 announced. Taking b_1^i and τ_1 as given, i repays if and only if:

$$
y_1^i\left[\Phi - \delta(z-\hat{z})\right] \geq b_1^i\left(1-\alpha^{i,i}\right)(1-z) + (z-\hat{z})\, b_1^g \alpha^{g,i} - \tau_1 \tag{C.8}
$$

For g, the budget constraint is:

$$
\begin{aligned}
c_1^g &= y_1^g - T_1^g + \left(b_1^{i,g} + b_1^{g,g}\right)(1-z) - \delta z y_1^g + b_1^{u,g} \quad \text{if } i \text{ repays} \\
c_1^g &= y_1^g(1-\kappa) - T_1^g + b_1^{g,g}(1-\hat{z}) - \delta \hat{z} y_1^g + b_1^{u,g} \quad \text{if } i \text{ defaults}
\end{aligned}
$$

and g government constraint in $t = 1$ is:

$$
\begin{aligned}
T_1^g - \tau_1 &= b_1^g(1-z) \quad \text{if } i \text{ repays} \\
T_1^g &= b_1^g(1-\hat{z}) \quad \text{if } i \text{ defaults}
\end{aligned}
$$

We now detail the different relevant thresholds:

- **No default, no monetization, no transfer**. Comparison made when $z = 0$ in no default and default. Necessary conditions on output shock:

$$
\begin{aligned}
\epsilon_1^i &> \frac{b_1^i \alpha^{iu} - \kappa y_1^g}{\Phi \overline{y}_1^i} \equiv \overline{\epsilon}'' \quad \text{ECB and g prefer no default to default with } z = 0 \text{ in both cases} \\
\epsilon_1^i &> \frac{b_1^i \alpha^{iu} + b_1^g \alpha^{gu}}{\delta \overline{y}_1^i} - \frac{y_1^g}{\overline{y}_1^i} \equiv \overline{\overline{\epsilon}} \quad \text{ECB prefers } z = 0 \text{ in no default} \\
\epsilon_1^i &> \frac{\alpha^{gu} b_1^g - \delta y_1^g}{\delta \overline{y}_1^i} \equiv \hat{\epsilon} \quad \text{ECB chooses } z = 0 \text{ in case of default} \\
\epsilon_1^i &> \frac{b_1^i\left(1-\alpha^{ii}\right)}{\Phi \overline{y}_1^i} \equiv \overline{\epsilon}' \quad i \text{ repays with zero transfer and } z = 0
\end{aligned}
$$

- **No default, no monetization, positive transfer** Necessary conditions on output shock:

$$\begin{aligned}
\epsilon_1^i &> \bar{\epsilon}'' \quad \text{ECB and g prefer no default to default with } z=0 \text{ in both cases}\\
\epsilon_1^i &> \bar{\bar{\epsilon}} \quad \text{ECB prefers } z=0 \text{ in case of no default}\\
\epsilon_1^i &< \bar{\epsilon}' \quad i \text{ repays only with transfer and } z=0
\end{aligned}$$

- **No default, monetization at maximum rate, no transfer** Comparison made when $z=\bar{z}$ in no default and $z=0$ in case of default.

$$\begin{aligned}
\epsilon_1^i &< \bar{\bar{\epsilon}} \quad \text{ECB prefers } z=\bar{z} \text{ in no default}\\
\epsilon_1^i &> \frac{\left(1-\alpha^{ii}\right) b_1^i\left(1-\bar{z}\right)+\alpha^{gi} b_1^g \bar{z}}{\left(\Phi-\delta \bar{z}\right) \bar{y}_1^i} \equiv \tilde{\epsilon} \quad i \text{ repays with zero transfer with } z=\bar{z}
\end{aligned}$$

- **No default, monetization at maximum rate, positive transfer** Comparison made when $z=\bar{z}$ in no default and $z=0$ in case of default.

$$\begin{aligned}
\epsilon_1^i &< \bar{\bar{\epsilon}} \quad \text{ECB prefers } z=\bar{z} \text{ in no default}\\
\epsilon_1^i &> \frac{\alpha^{iu} b_1^i\left(1-\bar{z}\right)-\alpha^{gu} b_1^g \bar{z}-y_1^g\left(\kappa-\delta \bar{z}\right)}{\left(\Phi-\delta \bar{z}\right) \bar{y}_1^i} \equiv \underline{\epsilon}' \quad g \text{ prefers no default, transfer and } z=\bar{z}\\
\epsilon_1^i &< \frac{\left(1-\alpha^{i,i}\right) b_1^i\left(1-\bar{z}\right)+\alpha^{gi} b_1^g \bar{z}}{\left(\Phi-\delta \bar{z}\right) \bar{y}_1^i} \equiv \tilde{\epsilon} \quad i \text{ repays only with transfer with } z=\bar{z}
\end{aligned}$$

  In this case, the transfer is the minimum that leaves i indifferent between default and no default (see equation C.5).

- **Default, no monetization, no transfer**

  Comparison made when $z=\bar{z}$ in no default and $z=0$ in case of default.

$$\begin{aligned}
\epsilon_1^i &< \bar{\bar{\epsilon}} \quad \text{ECB prefers } z=\bar{z} \text{ in no default}\\
\epsilon_1^i &< \frac{\alpha^{iu} b_1^i\left(1-\bar{z}\right)-\alpha^{gu} b_1^g \bar{z}-y_1^g\left(\kappa-\delta \bar{z}\right)}{\left(\Phi-\delta \bar{z}\right) \bar{y}_1^i} \equiv \underline{\epsilon}' \quad g \text{ prefers default, no transfer}\\
\epsilon_1^i &> \frac{\alpha^{gu} b_1^g-\delta y_1^g}{\delta \bar{y}_1^i} \equiv \hat{\epsilon} \quad \text{ECB chooses } z=0 \text{ in default}
\end{aligned}$$

- **Default, monetization, no transfer**

  Comparison made with $z=\bar{z}$ in both cases:

$$\epsilon_1^i < \frac{\alpha^{iu} b_1^i\left(1-\bar{z}\right)-\kappa y_1^g}{\Phi \bar{y}_1^i} \equiv \underline{\epsilon}'' \quad g \text{ prefers default, no transfer and } z=\bar{z}$$

$$\epsilon_1^i \quad < \quad \frac{\alpha^{g,u} b_1^g - \delta y_1^g}{\delta \overline{y}_1^i} \equiv \hat{\epsilon} \quad \text{ECB chooses } z = \overline{z} \text{ in default}$$

There are therefore 7 thresholds for output realizations: $\overline{\overline{\epsilon}}; \overline{\epsilon}'; \overline{\epsilon}''; \hat{\epsilon}; \tilde{\epsilon}; \underline{\epsilon}'; \underline{\epsilon}''$. In addition, we assume there is a minimum and maximum output realization ϵ^{max} and ϵ^{min}.

We can rank some of them under the assumption that $\Phi > \kappa > \delta$: $\underline{\epsilon}' < \overline{\epsilon}'; \underline{\epsilon}'' < \overline{\epsilon}''; \hat{\epsilon} < \overline{\epsilon}'; \underline{\epsilon}'' < \underline{\epsilon}'; \tilde{\epsilon} > \underline{\epsilon}'$

To simplify the analysis, we focus on parameter configurations that are most interesting and most plausible for the situation of the eurozone, we rank these thresholds based on the following general assumptions : b_1^g is small relative y_1^g and to b_1^i.

Assumptions on parameters: We can compare different cases with different degrees of fiscal dominance. **Fiscal dominance** would apply if the ECB inflates the eurozone debt even if i defaults so that only g debt remains. This is not a very interesting or plausible case so we ignore it and assume $\hat{\epsilon} < \epsilon^{min}$ which means that we concentrate as before on relatively low levels of debt to GDP levels in g and relatively high levels of the distortion costs δ. Another polar case is one of **monetary dominance**. This is a situation with low levels of g debt relative to GDP and high distortion costs δ. A sufficient condition is: $\overline{\overline{\epsilon}} < \epsilon^{min}$. The ECB never inflates the debt in a situation where transfers are possible because transfers are sufficient and the ECB would never want to avert a default if it was not in g interest which is also the interest of the Eurozone as whole. This case is identical to the one analyzed in section (4) where the role of the ECB was ignored.

- $\hat{\epsilon} < \epsilon^{min}$ which insures that the ECB will choose a zero inflation rate in the case of default. This excludes the case of strong fiscal dominance.

  $$\frac{b_1^g}{y_1^g} < \frac{\delta}{\alpha^{gu}} \left(1 + \frac{y_1^i}{y_1^g} \epsilon^{min} \right)$$

  The condition on parameters is such that the debt to GDP ratio for g is small enough.

  We then examine two cases: monetary dominance and weak fiscal dominance/.

- Monetary dominance: If $\overline{\overline{\epsilon}} < \underline{\epsilon}'$, then when transfers are possible, the ECB never chooses positive inflation. This case is valid with high y_1^g and δ, and low b_1^g.

- Weak fiscal dominance: If $\overline{\epsilon}' > \overline{\overline{\epsilon}} > \underline{\epsilon}'$, then when transfers are possible, the ECB may choose positive inflation. This is the case with intermediate levels of y_1^g and δ, and low b_1^g.

Under monetary dominance, the possible equilibria are shown in figure C.3. Only binding thresholds are indicated. Monetary policy does not affect transfers and the decision whether to default or not.

Under weak fiscal dominance, possible equilibria are shown in figure C.1. In this case, when output realization in i is sufficiently high ($\epsilon_1^i > \overline{\epsilon}'$), there is no default, no inflation and no transfer. If it is lower, i requires a transfer in order not to default ($\overline{\epsilon}' > \epsilon_1^i > \overline{\overline{\epsilon}}$) but there is no inflation. For $\overline{\overline{\epsilon}} > \epsilon_1^i > \underline{\epsilon}'$), the ECB partly inflates the debt, g makes a transfer to avoid the default. For $\epsilon_1^i < \underline{\epsilon}'$, the default is optimal and there is no more incentive to inflate the debt.

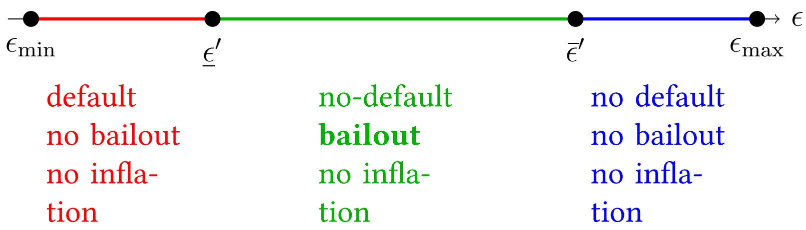


Figure C.3: Bailout under Monetary Dominance

There are several conditions on output realizations and parameters for such a situation to exist:

$$
\begin{aligned}
\epsilon_1^i &< \overline{\overline{\epsilon}} \\
\epsilon_1^i &> \frac{\alpha_1^{i,u} b_1^i \left(1-\overline{z}\right) - \alpha_1^{gu} b_1^g \overline{z} - y_1^g \left(\kappa - \delta \overline{z}\right)}{\left(\Phi - \delta \overline{z}\right) \overline{y}_1^i} \equiv \underline{\epsilon}' \\
\epsilon_1^i &< \frac{\left(1-\alpha_1^{i,i}\right) b_1^i \left(1-\overline{z}\right) + \alpha_1^{g,i} b_1^g \overline{z}}{\left(\Phi - \delta \overline{z}\right) \overline{y}_1^i} \equiv \tilde{\epsilon} \\
\hat{\epsilon} &< \epsilon^{min} < \underline{\epsilon}' < \overline{\overline{\epsilon}} < \tilde{\epsilon}
\end{aligned}
$$

The first condition says that the output realization is such that the ECB sets $z = \overline{z}$, the second that g prefers no default and transfer and the third that indeed i requires a transfer when $z = \overline{z}$. These conditions apply for intermediate levels of the output realization i. The last condition on the ranking of thresholds requires in particular intermediate levels of debt (see appendix for details).

Finally, when transfers are excluded (and $\overline{\overline{\epsilon}} < \underline{\epsilon}'$ so that monetary dominance applies with zero inflation in presence of transfers) the possible equilibria are shown in figure C.2. When output realization in i is sufficiently high ($\epsilon_1^i > \overline{\epsilon}'$), there is no default and no inflation. If it is lower, i requires a positive inflation rate in order not to default ($\overline{\epsilon}' > \epsilon_1^i > \tilde{\epsilon}$). For $\epsilon_1^i < \tilde{\epsilon}$, the default is optimal and there is no more incentive to inflate the debt.