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The Macroeconomic Returns of Investment in Resilience to Natural Disasters under Climate Change: A DSGE Approach

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ABSTRACT: This paper presents a Markov switching dynamic stochastic general equilibrium model designed to evaluate the macroeconomic return of adaptation investment to natural disasters (NDs) and the impact of climate change. While the model follows the existing literature in assuming that NDs destroy a share of the public and private capital stocks and a government that can invest in adaptation at an additional cost, it adds several features that are key to the analysis, both in the near (transition) and long (steady state) terms. Those include incomplete markets, financial frictions with collateral constraints, foreign remittances, full menu of tax and government spending instruments, and endogenous climate risk premium. The model is calibrated to the case of Dominica. It finds that NDs have large and persistent negative effects on output and public finances. It also shows that adaptation investment has large returns in terms of private investment, employment, output and tax revenue in the long term, especially under climate change.

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WORKING PAPERS

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Introduction

Climate-related natural disasters (NDs), which are on trend to grow with increase in their intensity and frequency under global warming, recurrently cause human and economic loss. This impact has been particularly catastrophic in small states located in highly vulnerable areas, especially in the Caribbean and in the Pacific where, given their small size, NDs affect the entire economy, leading to large losses of macroeconomic proportions (e.g. Raddatz, 2009; Mendelsohn and others, 2011; Loayza and others, 2012, Fomby and others, 2013; Acevedo, 2014, Lian and others, 2021, Akyapi and others, 2022 among others).[2] Given this, many affected economies are planning extensive adaptation investment to achieve physical and financial resilience to NDs. However, these investments are typically significantly costlier than the non-resilient type and, particularly in Small Developing States (SDS), the total required amount can be very large relative to the size of the economy.[3] It is therefore important to assess the extent to which the return to investment in resilient infrastructure outweigh its cost, which is important to evaluate the financial sustainability of the large investment needs. This is critical considering that resilient investments require large up-front costs while the returns materialize in the long-term. This assessment is critical for the design of macroeconomic policy. It can imply the need to mobilize additional revenue and to re-prioritize spending to create space for resilient investment. Also, it can prove critical to access financing and its terms within fiscal and external sustainability bounds.

This paper presents a dynamic stochastic general equilibrium model (DSGE) suitable for the quantification of the return of investment in infrastructure resilient to NDs- such as cyclones, tornadoes, and floods in SDS. The stochastic properties of the model also make it suitable for the analysis of the macroeconomic impact of climate change given the explicit parametrization of frequency and intensity of natural disasters, which affect not only the near-term dynamic responses of the economy after NDs but also the long run equilibrium (steady state). Under the stochastic properties of the model reflected in its multiple regimes, agents internalize the possibility of future NDs with associated output loss and destruction of assets, affecting investment, employment, and output even if no ND has taken place.[4] The model is calibrated to the Dominica economy, an SDS in the Caribbean. The calibration is used to quantify the short-run and long-run impact of NDs on macroeconomic outcomes, including the return of costly investments in resilient structures and the impact of climate change.[5]

The paper builds on earlier work that develop DSGE models to assess both the impact of ND and climate adaptation, seeking to explain the propagation channels of NDs. Bevan and Adam (2016) and Marto and

[2] While NDs do have a negative impact on economic activity on impact, the evidence on whether growth is subsequently impacted is mixed, with some empirical studies point to subsequently positive impacts on growth as the economy rebounds from the ND. Kousky (2014) and Lazzaroni and van Bergeijk (2014) provide surveys on economic growth, Skidmore and Toya (2002) find a positive correlation of climate disasters with productivity and growth, Loayza and others (2012) and Cunado and Ferreira (2014) also find positive evidence for the case of floods, and Cavallo and others (2013) do not find a significant effect on long-run growth from catastrophic disasters. Likewise, the literature has found that geophysical and meteorological disasters increase the sovereign risk premium, but hydrological and climatic disasters do not (Klomp, 2015).

[3] For example, detailed analysis in small states in the Caribbean indicates that the total cost of the resiliency needs can be in the range of 1 to 5 times the size of their GDP (see e.g., Disaster Resilience Strategies for Dominica and Grenada).

[4] This contrasts with more standard DSGE models that approximate natural disaster shocks through events that destroy capital or productivity through unanticipated shocks. In that set-up, the model agents only respond to climate events in the aftermath of a shock. In the setup of the model presented in this paper, agents know ex-ante the probability of climate events and their impact on the economy and therefore their decision rules incorporate this information.

[5] A similar approach to that used in this paper was applied to Bahamas, Costa Rica, Dominican Republic, Grenada, Honduras, Jamaica, Nicaragua, Panama and St. Vincent and the Grenadines and reported in IMF (2021b). The largest impact and gains from adaptation are for Dominica.

others (2018) are early contributors to the literature. They take the work of Buffie and others (2012) as a starting point, enriching the fiscal sector by considering additional sources of taxation and public expenditures, and by incorporating NDs shocks that destroy non-resilient capital stock, reducing output. Both papers consider the costs of reconstruction following a ND and evaluate the gains of adaptation investment in resilient public capital under various financing options (i.e., additional taxation, donor contributions, budget reallocation of alternative public spending). Mendes-Tavares, Guo and Guerson (2022) also include NDs and the destruction of public infrastructure incorporating key features of SDS, including labor migration and remittances, an informal service sector, and alternative labor skills. Their model includes a broad set of fiscal policy instruments, sector-specific labor allocation according to skills which enables the analysis of income distribution impact of alternative policies. However, these models include NDs shocks as one-off surprises, which only makes them suitable to examine the transitional dynamics as opposed to long-term (steady state) implications of adaptation investment and climate change, which are based on expected losses in the future.

Our work is closest to Cantelmo and others (2019) who also study the transmission channels of NDs in a stochastic environment and focus on tax policy and financing for resilient investment. They calibrate their model to an “average” small-open economy (both emerging and developed) subject to ND shocks and find sizeable (over six percent) and persistent (over three years) output effects and significant increases in the public debt to GDP ratio, thereby emphasizing the role for donor grants both for post-disaster reconstruction and the financing of resilient investment. Guerson (2019) finds net-positive returns to investment in costly resilient capital, with increases in private investment, the capital stock, and employment leading to output gains in the range of 3-11 percent for countries in the Eastern Caribbean Currency Union (ECCU). However, they present a simplified menu of fiscal policy instruments and omit the destruction of private capital by NDs.

Table 1 presents a summary of the key ingredients of the models that have evaluated the impact of investment in adaptation to natural disasters. The key common feature of the existing models is the explicit incorporation of NDs that destroy a share of public capital, within a small open economy setup. The table highlights the innovative nature of the model in this paper, which include the following critical assumptions that make it better suited for the quantitative evaluation of the returns of investment in resilient infrastructure:

- *A Markov-Switching DSGE* to introduce NDs in the economy, resulting in agents that fully internalize climate-related risk as recurrent and form expectations accordingly. The model is solved using a perturbation method for regime-switching rational expectations models described in Maih (2015). This methodology is fast and efficient, relatively easy to implement and scales easily to larger models.

- *Incomplete markets and financial frictions with collateral constraints on credit access by Ricardian agents*. This is important in the analysis of the recovery from NDs, which destroy collateral and therefore limits borrowing, slowing the recovery. Lending rates to investors are affected by the risk of NDs and the destruction of collateral. It is also important in affecting the steady state given the expectations of collateral destruction, constraining lending, and investment.

- *A share of non-Ricardian (hand-to-mouth) agents*, which rely on government support after natural disasters. This allows the simulation of fiscal pressures to support the population after NDs, which ultimately impacts tax expectations and economic dynamics.

- *A full menu of tax and non-tax revenue instruments and government spending allocations, including transfers to the Ricardian and non-Ricardian agents*, in addition to the investment in resilient and non-resilient infrastructure. The addition of non-tax revenue is important because it can be calibrated to capture resource-based revenue and also Citizenship-by-Investment revenue, which is sizable in several SDS affected by NDs. This enables the internalization of the efficiency and distortionary implications of the tax mix and the dynamic impact of the alternative expenditure uses, including a comparison between ex-ante investment in resilience vs. ex-post reconstruction and support to the population (which are larger when infrastructure is not resilient).

- *Foreign remittances*, which are set to vary according to the state of the economy. This is a crucial feature in several SDS affected by natural disasters, including in the Caribbean, Central America, and the Pacific, with material impact on the recovery from NDs.

Table 1: Models Incorporating NDs and Adaptation Investment

Model Features		Bevan and Adam (2016)	Marto, Papageorgiou and Klyuev (2018)	Guerson, Guo, Mendes-Tavares (2022)	Guerson (2019)	Cantelmo, Melina and Papageorgiou (2019)	Fernandez-Corugedo, Gonzalez-Gomez and Guerson (2022)
Production	ND shock destroys public capital	Yes	Yes	Yes	Yes	Yes	Yes
	ND shock destroys private capital	Yes	Yes	No	Yes	Yes	Yes
	ND shock affects growth	No	Yes	No	No	Yes	Yes
	Labor migration and remittances 1/	No	No	Yes	Yes	No	Yes
	Disaster affects long-term output level	No	No	No	Yes	Yes	Yes
Fiscal Policy	Full set of fiscal instruments	Yes	Yes	Yes	No	No	Yes
	Government Optimizes	No	No	No	No	No	No
	Social Impact / income distribution / informality	No	No	Yes	No	No	No
	Suitable for Debt Sustainability Analysis	No	Yes	No	Yes	No	No
	Endogenous risk premium	Yes	Yes	No	No	Yes	Yes
Suitable analysis of climate change (stochastic)		No	No	No	Yes	Yes	Yes
Incomplete Markets 1/		No	Yes	Yes	Yes	No	Yes
Financial frictions 1/		No	No	No	No	No	Yes

1/ Affect returns to adaptation investment.

As in the previous literature, NDs are found to have large and persistent effects on the economy. This paper, however, underscores the important amplification effects of these shocks with migration, credit frictions and incomplete markets. In addition, the impact of climate change is assessed based on estimates of the intensity of natural disasters to global warming as well as the gains from adaptation, highlighting the important nonlinearities noted by Cantelmo and others (2019).

The rest of the paper is structured as follows. Section II provides a non-technical description of the model. Section III presents the calibration to Dominica's economy and the solution technique. Section IV describes the impact of a ND in the short and the long-run and examines the key transmission channels. Section V illustrates the returns of resilient investment. Section VI displays a counterfactual climate change experiment, quantifying the impact of an increase intensity of NDs and reassessing the returns of adaptation investment. Section VII shows a sensitivity analysis of the results. Section VIII concludes. Two appendices describe the model and the model solution in more detail.

Model Description

The model comprises four sectors: households, firms, government, and the external sector. There are two types of households: *investor/saver households* that invest in non-resilient capital and hire labor, and *worker households* that supply labor, receive remittances but cannot save. All households maximize utility over their consumption and labor input bundle. Households delegate labor and wage decisions to a union that negotiates with firms. The labor market is assumed to be non-competitive such that wages for Ricardian and non-Ricardian are equal and set in a centralized manner by an economy-wide union. Hours are then determined by firms (rather than being chosen optimally by households) given the wage set by the union with households willing to meet the demand from firms. Such non-competitive labor market structure implies that there is a wedge between the marginal rate of substitution and the real wage. [6] All households are affected by natural disaster events since these tend to lower output, employment and wages and hence affect the consumption set. Households purchase consumption goods and pay lump-sum taxes to the government. They receive labor income, foreign remittances, and government transfers. In the case of remittances, these are modelled as exogenously determined though dependent on the model regime, such that these are higher when a ND event hits the economy.[7]

There are two types of firms. *Final good firms* produce a homogenous good that can be transformed into consumption, investment, and export goods. Production firms choose labor and capital inputs, taking as given the stock of public capital, real wages, and the price of output. Firms borrow to finance investment and labor input expenses and use the value of its capital as collateral. Final good firms are directly affected by NDs since these destroy a share of the public and private capital stocks and are also assumed to temporarily reduce total factor productivity (TFP). They pay taxes on their profits. *Investment and consumption firms* transform the final good to both investment and consumption goods. They take the price of final goods as given and combine these goods with imported investment (consumption) goods to produce homogeneous investment (consumption) goods that are sold to final good firms (consumers). Investment and consumption firms are not directly affected by natural disaster events.

The *government* collects revenues from consumption, firms' profits, wages, and lump-sum taxes and receives external grants. It spends on purchases of goods and services, transfers to households, public capital, and interest on public debt. Crucially, public investment can be of two types: *resilient and non-resilient to NDs*. It is assumed that investment in resilient public capital is costlier relative to the non-resilient type (assuming a premium of 25 percent over non-resilient investment based on estimates in the

[6] No skill differences are assumed between Ricardian and non-Ricardian households. The union mechanism therefore mitigates the wealth effect of consumption on labor supply, with hours determined by labor demand for a given wage.

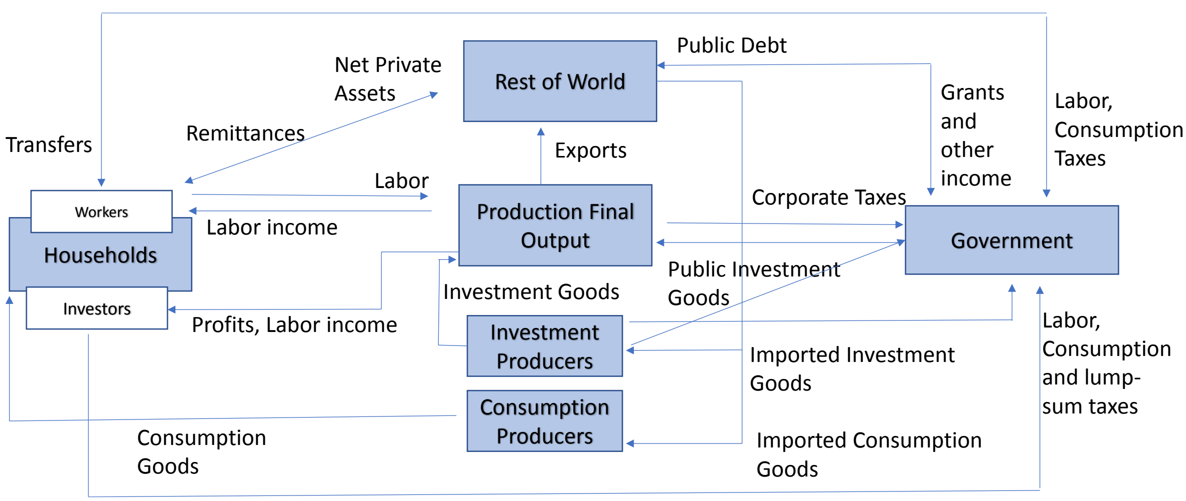
[7] Migration is not modelled and is thus assumed constant. An extension of this model with endogenous migration is being developed.

2017 Dominica Post Disaster Needs Assessment produced by the World Bank).[8] Both types of investment are assumed to be perfect substitutes in production. This implies that both types have the same contribution to output, but the resilient type is costlier—for example, a resilient bridge produces the same service as a non-resilient one as long as there are no NDs. This specification is particularly important because it enables the assessment of the cost-return trade-off of investment in resilient infrastructure. It is also the empirically relevant specification, as typically the construction of resilient structures implies additional cost in design, engineering, and materials.[9] The government is directly affected by NDs as non-resilient public capital is destroyed and is indirectly affected as tax revenues decrease with the decline in output and labor income.

Fiscal policy is anchored by a debt rule and does not follow an optimization process. All government expenditures, including public investment, are set as a constant share of nominal GDP and marginal tax rates are assumed unchanged in response to a ND. Other than the aforementioned increase in grants in response to the ND, (non-distortionary) lump-sum taxes levied on households are used to raise revenue to allow to match the public debt target over the medium term. The use of non-distortionary lump sum taxes to return to the debt level set in the rule allows the isolation of pure climate shock impact without the additional cost of distortionary taxation. The sensitivity analysis section below scrutinizes the additional costs from distortionary taxation.

Exporters purchase final goods and transform them input export goods which are sold to the rest of the world, but they do not price their goods to reflect conditions in external markets (i.e., they do not price to market). The relationships among all participants in the model is presented in Figure 1.

Figure 1: Model Structure



[8] The 2017 Dominica Post Disaster Needs Assessment by The World Bank that was conducted after hurricane Maria includes estimates of replacement cost of destroyed non-resilient structures and estimated by sector cost of rebuilding with resilience, often referred as "build back better."

[9] Other papers in the literature consider a range of substitutability vs. complementarity of resilient capital. See for example Marto et. al. (2018).

Model Calibration and Solution

The model is calibrated to match the main macroeconomic ratios of Dominica's economy, the probability of occurrence of a climate related disaster and the average economic impact of a natural disaster (Box 1). The macroeconomic ratios come from the October 2021 World Economic Outlook using both historical data from 2013 to 2019 plus macroeconomic projections to 2026. The average impact of a climate disaster and the probability of a natural disaster are computed using the estimates produced by the Caribbean Catastrophe Risk Insurance Facility (CCRIF).[10] The expected annual losses using this methodology are of the order of 2.2 percent of GDP. Note that these are average annual losses and not the expected losses *per event*. According to the EMDAT database, there have been 13 cyclones that have hit Dominica since 1960, implying an average loss of around 10½ percent of GDP per event (i.e., with an annual probability of around 21 percent per year or once every 5 years, the expected cost per event as a share of GDP is 2.2/0.21).[11]

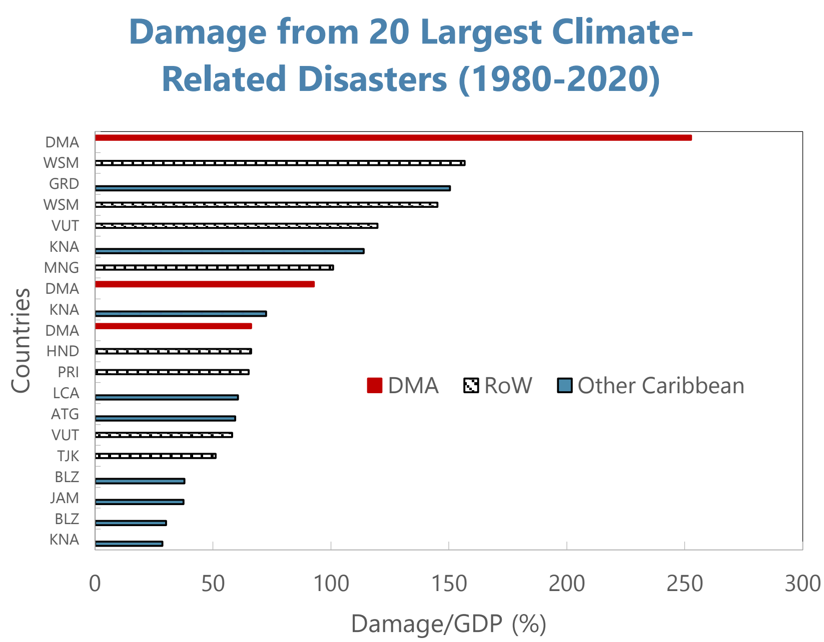
[10] CCRIF estimates seek to fit the historical cyclone activity (using statistics of previous events such as wind speed and storm surge) with a vulnerability module that estimates the replacement cost and vulnerability of structures and an exposure module that estimates the possible impact of a tropical cyclone on different assets (using the relationship between wind intensity and surge and the repair cost of damaged structures). We use the estimates of the Tropical Cyclone (TC) risk profile for Dominica. These reported losses are comparable with those of the EMDAT.

[11] In the EMDAT database, the damage per event since 1960 is twice as large as the one reported by CCRIF at around 25 percent of GDP. This is because the database includes extreme tropical storms such as hurricane Maria with estimated losses of over 90 percent of GDP. Excluding this hurricane lowers the average damage per event to around 15 percent of GDP.

Box 1. Macro Critical Climate Related Natural Disasters: the Case of Dominica

Dominica is one of the most vulnerable countries to natural disasters and climate change in the world. As noted by IMF (2021a), between 1997-2017, Dominica "was the country with highest GDP losses to climate-related natural disasters and ranked in the top 10 percent among 182 countries for climate-related fatalities." In 2017, Dominica was struck by Hurricane Maria, a Category 5 storm, that resulted in damages estimated at around 226 percent of GDP, causing damage to public infrastructure such as roads and bridges, housing and affecting both the tourism and agriculture sectors.

Not only are climate-related natural disasters sizeable but they are also relatively frequent. Since 1963, the Emergency Events Database (EMDAT) database[1] reports 13 climate-related natural disasters, implying that Dominica is hit by NDs recurrently, with a frequency of about one ND every five years.



Source: EMDAT and IMF staff calculations

Turning to the model's parameters, Table 2 presents the list of parameters used in the paper. Starting with households' preferences, parameter η is calibrated so the long run labor supply is 1/3 in the long run, in line with Cantelmo and others (2019). ξ is calibrated such that the Frisch elasticity is 2 consistent with other studies including those on emerging markets (e.g., Christoffel and others, 2008, Neumeyer and Perri, 2005, Boz and others, 2012). The elasticity of labor varieties is set up at 6 in line with Adolfson and others (2005). The average consumption shares (α_c) and the share of imported good for investment (α_I) are used to match the imports to GDP ratio.[12] We set the elasticities of substitution between domestic and imported consumption and investment goods, η_c, η_I, at 0.75 in line with Kumhof and others (2010) or Coenen and others (2018). The share of workers is set at 0.8 in line with Guerson (2019).

Firms produce with Cobb-Douglas technology, with labor, private and public capital as inputs. The share of private capital (α^k) and the average of depreciation rates are calibrated to match the investment and

[12] Disaggregated data on imported consumption and investment goods is not available for this economy.

consumption ratios in the economy. The estimated value of $\alpha^k = 0.17$ and the average depreciation rate of capital is close to 10 percent. This large depreciation rate is associated with the NDs' impact and probability. The share of public capital in the production function, α^{kg}, is set at 0.05 in line with Guerson (2019). The share of capital on the production function is in line with values typically reported in the literature at 1/3.[13] The average growth rate is set at 2.6 percent, the average observed in the data, which includes periods when NDs have hit the economy. The parameter governing the financial constraint, σ, is calibrated to match a 4 percent credit spread observed in corporate rates. Adjustment costs to capital are set to 6 as Gonzalez and others (2011).

Government revenue and expenditure parameters are set to match central government data (Table 3). Taxes on consumption (τ^C), personal income (τ^w) and corporates (τ^π) are calibrated to yield revenue close to effective revenue collections. Grants (Gr) are aligned with observed capital grants and are calibrated such that nontax revenue is close to zero in steady state. Transfers to households (T^W) are calibrated to match actual government spending on transfers. Government consumption and investment rates are calibrated to match the public sector wage bill with observed compensation of government employees and investment spending. The interest rate on government debt (r^*) is calculated as the ratio of interest expenditure to the stock of debt in the previous year, capturing the implicit interest rate on outstanding debt. There are two fiscal parameters that partly govern the response of the public sector to a ND. The first one is, ϕ_b, the fiscal reaction function, which we set at 0.1, and ϕ_{gi} which corresponds to the reconstruction response following a natural disaster and which we set at zero in the baseline.

The external interest rate is set at 5 percent. It comprises a 1 percent external risk-free rate and 4 a percent external risk premium. β , the parameter scaling risk, is adjusted such that the domestic interest rate equals the external interest rate. Remittances received by workers households match the average remittances observed in the data. The calibration assumes that remittances increase by 20 percent following a ND.[14] Foreign grants are assumed to increase by 20 percent following a ND.[15]

[13] See for example King and Rebelo (1999), Aguiar and Gopinath (2007) and Cantelmo and others (2019).

[14] There are insufficient observations to allow a full evaluation of the increase in remittances following a natural disaster for Dominica. Estimates from Bettin and Zazzaro (2017) suggest that remittances tend to be 33 percent higher in the aftermath of a ND relative to countries without natural disaster. Available data for Dominica suggest that remittances rose by around 1.2 percentage points of GDP in the aftermath of hurricane Maria in 2017, a 17 percent increase relative to the 2016 level.

[15] In the aftermath of Hurricane Maria in 2017, grants rose by 8 percentage points of GDP. Becerra and others (2014) find that foreign aid increases by around 18 percent following a ND.

Table 2. Model's Parameters

Parameter		
ω	Share of workers	0.8
η	Scale factor in the utility function	3.45
ξ	Inverse Frisch elasticity	0.5
β	Discount factor	0.99
ϵ_w	Elasticity of substitution between labor varieties	6
α_g	Share of public capital on the production function	0.1
α_K	Share of private capital on the production function	0.2
Σ	Collateral constraint	2.15
δ_{gr}	Depreciation rate of public resilient capital	9.6
α_I	Share of imported goods for investment	0.68
α_C	Share of imported goods in the consumption basket	0.67
η_I	Elasticity of import substitution for investment goods	0.75
η_C	Elasticity of import substitution for consumption goods	0.75
η_X	Elasticity of exports to the exchange rate	0.75
a^{Gr}	Price mark-up for resilient investment goods	1.25
τ^C	Consumption tax rate	20.4
τ^L	Labor income tax rate	6.0
τ^π	Corporate tax rate	15.5
ϕ_b	Fiscal reaction function parameter	0.1
ϕ_{kg}	Public Investment reconstruction parameter	0
$\bar{R}_t^*$	External interest rate (annual)	5
	Regime Switching parameters (1 normal times, 2 climate events)	
g^{A} (1)	Growth rate (annual)	2.6
g^{A} (2)		-1
δ_{Y} (1)	Depreciation rate private capital (annual)	6
δ_{Y} (2)		57.9
δ_{gnr} (1)	Depreciation rate of public non-resilient capital	6
δ_{gnr} (2)		74.8
$k_{gran/Y}$ (1)	Government Grants over GDP	21
$k_{gran/Y}$ (2)		23
$k_{rem/Y}$ (1)	Remittances over GDP	6
$k_{rem/Y}$ (2)		7

The economy is assumed to be in either of two states, indexed by *s*. State 1 is the state with no ND. In this state, the depreciation rates of non-resilient capital, productivity, remittances, and grants are close to, but below, their long-run averages. State 2 is the state with a ND. In this case, the depreciation rates of non-resilient capital increase and are set to match the average capital destruction observed in the data in EMDAT and the CCRIF. The drop in TFP is set to match the observed output decline after NDs, taking as given the higher depreciation rate in those states. Likewise, remittances, grants are set above their long-run values as observed in the data and discussed above. Table 3 summarizes the calibration moments that we seek to match for the ergodic steady state.

Table 3. Moment Calibration Summary

Moment	Model	Data
(Percent of GDP unless specified)		
Share of remittances	6	6
Net Exports	-21	-21
Imports	65.4	65.5
Exports	44.4	44.3
Total hours	1/3	1/3
Risk spread (bp)	6.2	6.1
Consumption	71	71.1
Investment	27	28
Fiscal Policy		
Revenue from consumption tax	4.3	4.3
Revenue from income tax	2.5	2.5
Revenue from corporate tax	14.4	14.3
Other revenues (grants, etc.)	26	24
Public consumption	23	23
Public Investment	15	15
Public Debt	81	81

The model is solved using the perturbation method for regime-switching rational expectations models developed by Maih (2015). The method finds a solution around a steady state that allows the economy to be in different regimes at different points in time, with each regime being governed by certain rules specific to the regime. Since the model has two regimes, it may contain multiple steady states (a particular one for each regime). Consequently, the approximation can be done around the steady-state of each regime or around an arbitrary steady-state. This paper works with a steady state associated with the ergodic mean of the switching parameters. The ergodic mean allows the calibration of the steady state that matches the long-run ratios. The transition matrix through both states s_t is

$$P_{s_t,s_{t+1}} = \begin{bmatrix} p_{1,1} & p_{1,2} \\ p_{2,1} & p_{2,2} \end{bmatrix}$$

where $p_{1,2}$ is the probability of transitioning from the state where there are no NDs in period t to a ND in t+1, $p_{1,1} = 1 - p_{1,2}$ is the probability of remaining in the state without a ND, $p_{2,1}$ is the probability of moving from the state with a ND in period t to the state without NDs in period t+1, and $p_{2,2} = 1 - p_{1,2}$ is the probability of remaining in the ND state in t+1. These probabilities are calibrated to replicate the observed frequency of NDs discussed above.

Given the specification of ND shocks with two states to simulate recurrent climate shocks, as opposed to specific climate shock perturbations, traditional impulse response functions cannot be computed. To estimate the dynamic impact of a ND, a local linear projection (LLP) method proposed by Jorda (2005) is used. The LLP approach consists of running a sequence of predictive regressions of a variable of interest on a structural shock for different prediction horizons. The dynamic response is then obtained from the sequence of regression coefficients of the structural shock. In our case, the structural shock is measured by the occurrence of a climate disaster in the model.

To compute the LLP, an MS model is simulated multiple times and then the dynamic responses are recovered using the regression coefficients $\beta(h)$ associated with the following set of h-step-ahead predictive regressions,

$$y_{t+h} = \alpha(h) + \beta(h)x_t + u_{t+h}$$

where u_{t+h} is a prediction error term and x_t is a dummy variable that equals one when there is a ND event in the simulation. Jordá (2005) proposes to recover the dynamic impact $\beta(h)$ by running $h+1$ least squares regressions. We simulate the MS model for a sample of size T, i,e t=1,…,T and computed for each simulated sample the dynamic impact. Since the impact of a climate event on the economy may be influenced by the state of the economy (stock of capital, productivity shocks, etc.), we report the average estimated impact across all possible states. To do this, we carried out the mentioned simulation exercise several times and report the average value $\beta(h)$ across all simulations.

Impact of Climate Events

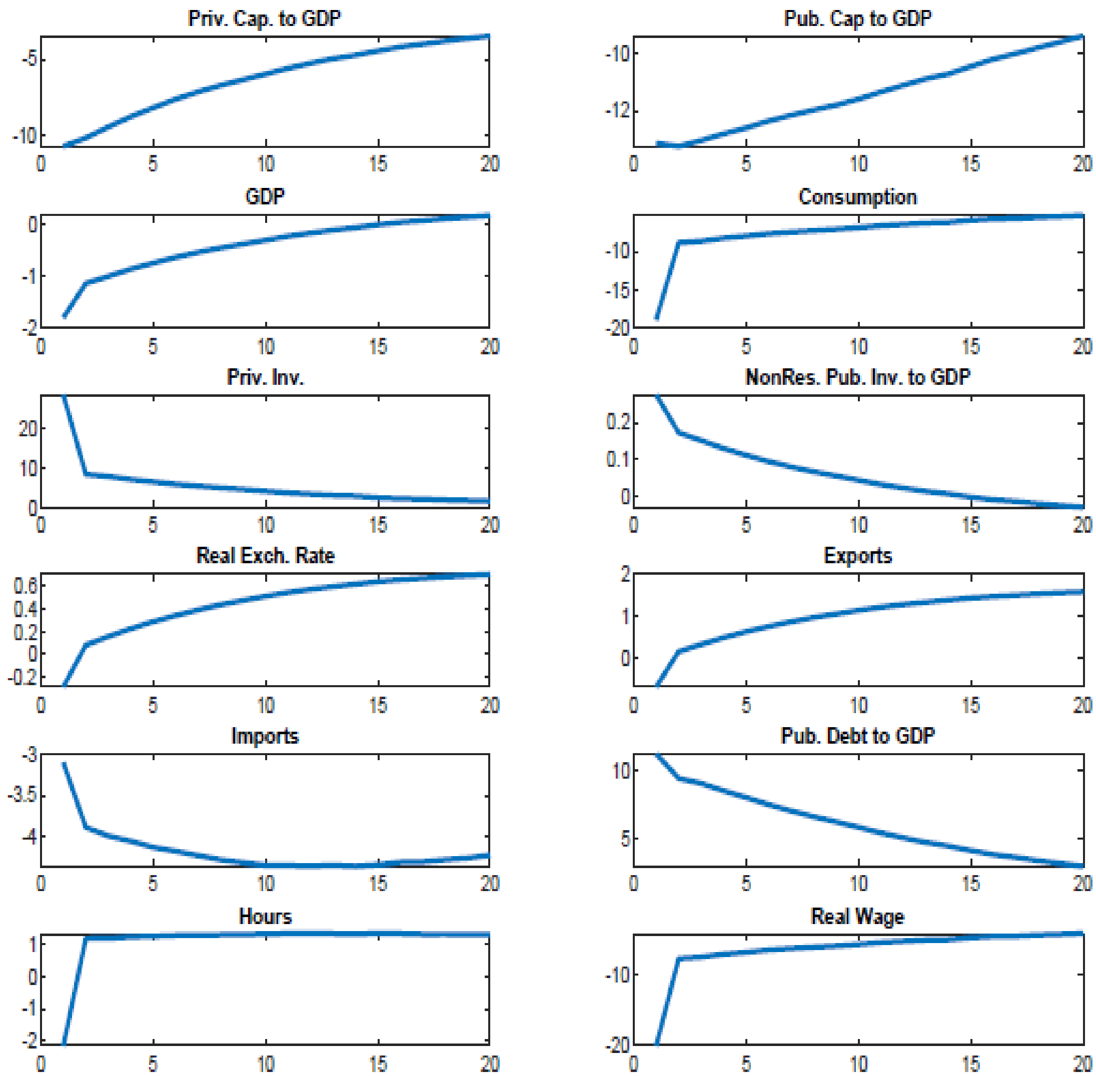
This section presents the results of NDs shocks for the calibration to Dominica's economy. It begins by examining the short-run properties following a ND and then it documents the long-run (steady-state) properties compared to an economy without NDs, otherwise identical. The ND considered is similar to that in Cantelmo and others (2019), which is assumed to destroy a share of the capital stock[16] that is not resilient, plus a transitory decline in TFP. The results are shown in Figure 2. As explained above, the increase in the depreciation rate in a climate event results in approximately a 10 percent decline of the capital stock in the year of the shock. It is assumed there is no additional reconstruction spending in the baseline.

The destruction of a share of the capital stock and the reduction in TFP following a ND results in a decline of output. This lowers the demand for labor and the wage rate. In this context, agents reduce consumption and increase investment to rebuild the stock of private capital. The destruction of public capital has a persistent impact, as the stock of the public capital is gradually rebuilt at the set government investment rate, resulting in a gradual recovery of the private capital stock. Private investment and capital are also slowed by the collateral constraint affecting borrowing, after a share of private capital has been destroyed by the ND. This gradual rebuilding of public and private capital stocks explains a protracted recovery of output, which takes around five years to return to its pre-ND level. This result is consistent with Lian and others (2021). The real exchange rate appreciates on impact,[17] but then it depreciates as output recovers while domestic demand remains depressed due to the decline of private consumption. Imports fall in response to the decline in consumption and the real exchange rate depreciation as imports of capital increase for investment. Exports fall on impact with the decline of output, but subsequently they increase in line with the depreciation of the exchange rate. The decline in GDP, consumption, and labor income results in lower tax revenue. With public spending assumed to be exogenous, the decline of output leads to an increase in expenditures as a percentage of GDP. Overall, the level of public debt significantly increases in response to the natural disaster event, taking around 5 years to return to the pre-ND level.

[16] These are also similar to depreciation shocks in DSGE models, as detailed by Furnaletto and Seneca (2014).

[17] The exchange rate appreciates in response to an increase in the domestic price level. The domestic price increases in response to a significant increase in the rental price of capital due to the decline in capital (which exceeds the fall in the real wage). As a result, costs increase and hence prices.

Figure 2: Short-run Response to ND Event



Note: y-axes are percent deviations from the ergodic steady state except ratios to GDP which are percentage point deviations. The time scale on the x-axes are quarters.

Table 4 presents a counter factual experiment to measure the cost to the economy of NDs in the long run (steady state). Taking the calibrated values of the parameters selected to match the key moments in the presence of NDs as given, it computes the impact of removing the NDs shocks. The results indicate that the level of GDP is about 3.5 percent higher without NDs. This is despite a lower level of private investment in the economy without NDs—firms do not have to periodically rebuild the capital, implying a de facto lower depreciation rate. Since consumers forego consumption to rebuild capital following a ND, the level of consumption is higher in the absence of NDs. While investment declines, the capital to GDP ratio is around 7 percent higher when there are no NDs. This explains why the real wage is higher as labor productivity rises. Finally, there is a significant improvement in tax revenue in line with the higher levels of activity, consumption, and wages.

Table 4. Long-run Impact of no Natural Disasters

Dominica	
(Difference in percent unless otherwise specified)	
GDP	3.4
Private Consumption	7.1
Private Investment	-6.3
Real Wage	6.8
Consumption Tax Revenue	7.1
Corporate Tax Revenue	4.6
Labor Income Tax Revenue	6.7
Corporate interest spread (bp)	330

Climate Adaptation: Return of Investing in Infrastructure Resilient to Natural Disasters

This section presents estimates of the return of investment in resilient structures, or investment in adaptation. It shows that investing in resilient infrastructure leads to long-run net gains (steady state) because the return outweighs the additional cost of resilience. It also leads to near-term gains after a ND (transitional dynamics). Notice that the return has three components which are additive: (i) the reduced need to rebuild the capital destroyed (stock loss); (ii) the smaller decline of output, employment, and revenue as a result (near term flow loss); and (iii) the higher steady-state level of output, employment, and tax revenue (long term flow loss).

In line with IMF (2019, 2021b), it is assumed that the share of resilient public investment and hence the public capital stock is increased to 80 percent from zero.[18] The calibrated parameters reported in Table 3 are maintained; the only change is the share of resilient public investment. The fiscal behavioral assumptions also remain in operation, including the long-term public debt target.[19]

Table 5 presents the long-run gains from investing in resilient public capital. The level of GDP is about 6½ percent higher than without resilient capital. The lower share of capital destruction reduces the output loss after NDs. Notice that resilient investment has a multiplicative effect on output. The decline in GDP losses and destruction implies higher expected returns to private investment when public infrastructure is resilient, boosting private investment in the long-run by around 18 percent. Moreover, higher investment and capital increase labor productivity and the real wage. Ultimately, higher investment and employment reinforce each other with positive feedback, resulting in a potentially large multiplicative effect on output.

[18] The simulations evaluate the potential gains once the 80 percent share is reached. They do not consider the transition path while resilient capital is gradually built. This is akin to assuming that, when resilient investment is in place, the capital stock destruction by a ND is reduced to 20 percent of that without resilient structures.

[19] Given endogenous changes to output, investment, consumption and employment, tax revenue would differ when resilient investment is in place compared with without resilient investment. These returns to resilient investment, however, need not match the assumed additional cost of resilient infrastructure, and therefore can take public debt to a different level. To achieve the same level of debt in the exercise, any residual (positive or negative) fiscal need is assumed to be covered with non-distortionary lump sum taxes or transfers, to ensure the neutrality of the results.

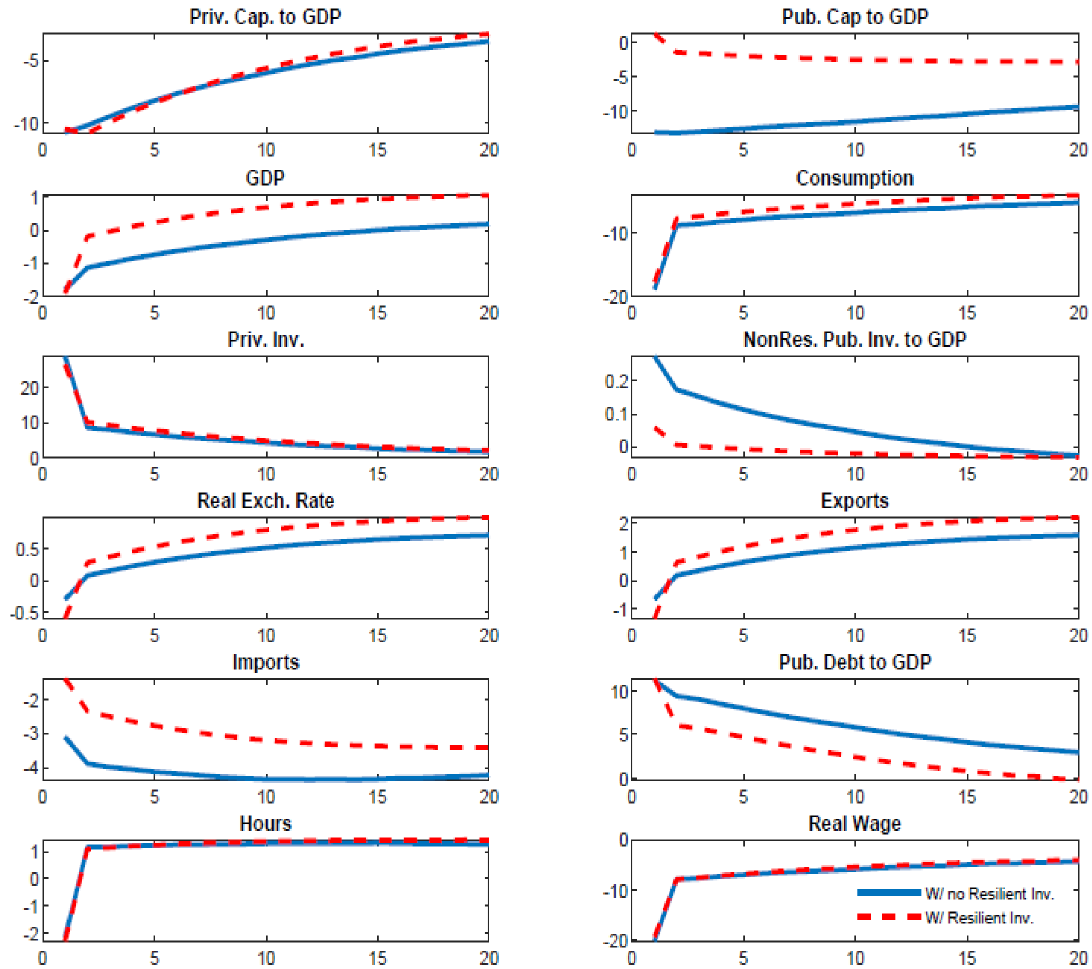
Table 5. Long-run Estimates of Adaptation

Dominica	
(Percent difference vs baseline with no adaptation)	
GDP	6.3
Private Consumption	1.9
Private Investment	17.9
Real Wage	3.2
Consumption Tax Revenue	1.9
Corporate Tax Revenue	6.7
Labor Income Tax Revenue	6.3

Figure 3 presents the short-run gains. The destruction of public capital following a ND is significantly lower with resilient investment. While the GDP contraction is comparable on impact, output returns to the steady state faster, around a year later. While the increase in private investment is comparable in the resilient and non-resilient cases on impact as firms seek to rebuild destroyed capital,[20] the share of private capital in GDP is lower in the near term in the economy with resilient capital given the lower contraction in GDP. In terms of public capital to GDP, despite the lower contraction to the level of GDP, public capital to GDP is significantly higher with resilient capital because less capital is destroyed. Real wages and employment are slightly higher with resilient capital due to the higher capital stock. Private consumption is also marginally higher when there is resilient capital due to higher labor income and lesser deterioration of public finances.

[20] The level of private investment is higher when there is a larger share of public capital since the returns from investing in private capital are larger. This explains the faster convergence to the steady state despite the higher level of GDP.

Figure 3: Short-run Gains to Resilient Capital after ND Event



Note: y-axes are percent deviations from the ergodic steady state except ratios to GDP which are percentage point deviations. The time scale on the x-axes are quarters.

Returns to Adaptation Investment with Climate Change

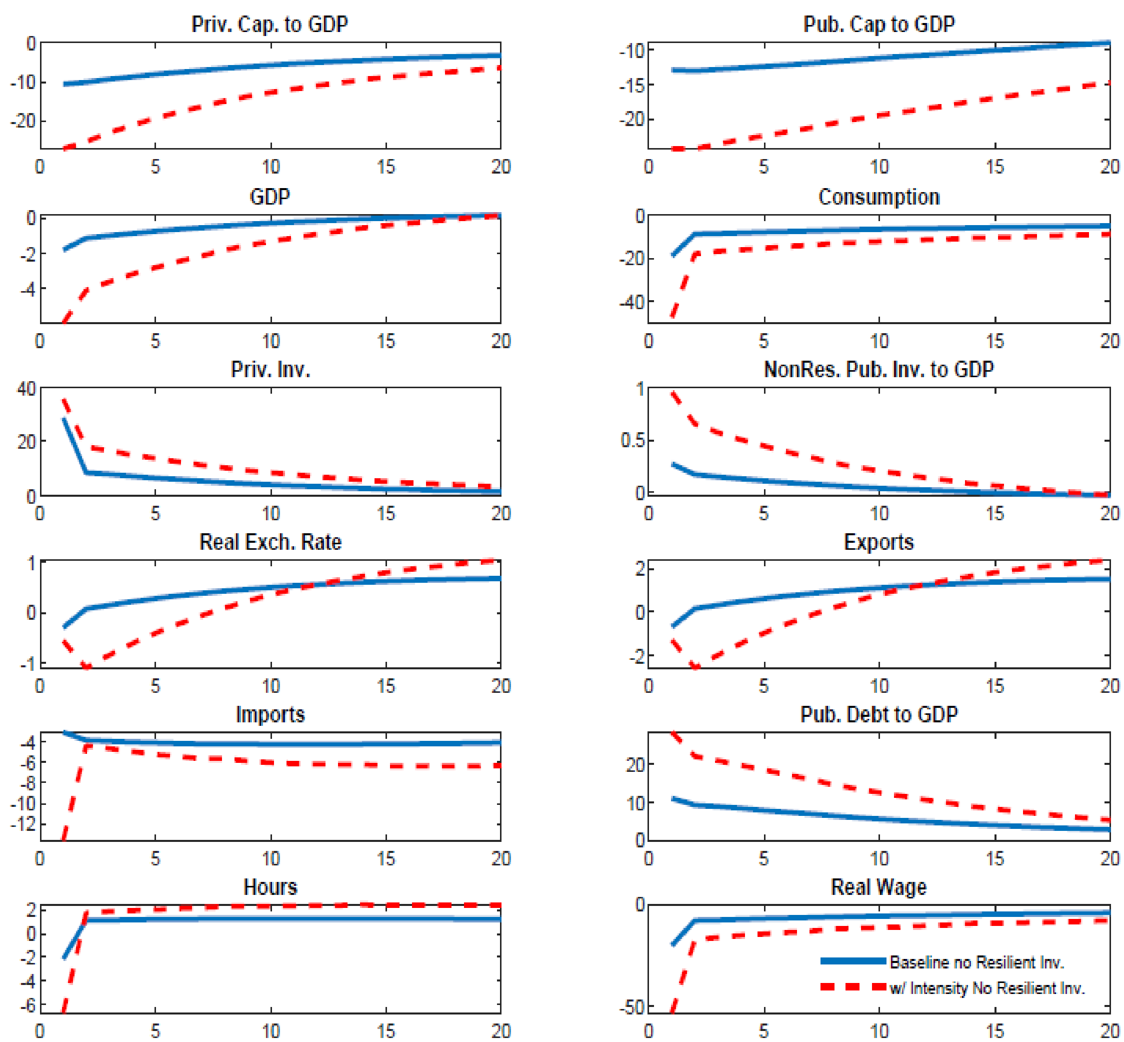
As noted by Acevedo (2016), the consensus among climate scientists is that sustained carbon (CO2) emissions at current levels would lead to an increase in athmosferic and sea level temperatures, and as a result to more intense extreme weahther events like hurricanes and excess rainfall. Building on the work of Nordhaus (2010), Acevedo (2016) estimates the increase in storm damages for a sample of Caribbean countries due the predicted increase in global temperatures under alternative CO2 emissions scenarios. Using Acevedo's estimates for the RCP 8.5 scenario[21] of the United Nations, which assumes no change in current CO2 emission trends, NDs damages in Dominica would increase by 14 percent by 2050 and by 36

[21] The RCP8.5 scenario assumes that emissions grow faster than what current developments and policies seem to imply. The RCP4.5 scenario is closer to present trends (including observed trends in the decline of carbon intensity of GDP).

percent by 2100. To simulate climate change, the calibrated model is run assuming a 14 percent increase in the NDs damages. Figure 4 presents the near term results and Table 6 the long-run gains. The results are presented against the baseline results in Figure 2.

Predicatbly, the larger capital destruction under climate change results in a greater decline in output, consumption, real wages and employment, partially offset by a larger response of investment for reconstruction. Due to the larger decline in output and tax revenue, public debt increases more than without climate change. The impact, however, is non-linear: while capital destruction increases by 14 percent, the decline in output is more than twice as large. This is partly due to the complementarity between public and private capital in production which amplifies the output loss of private investment. In addition, the additional expected capital destruction with climate change tightens collateral constraints on firms' borrowing further, delaying the recovery. However, while the persistence of the response to the ND shock is also present in the face of more intense climate events, the differences diminish over time. Table 7 presents the long-run estimates of the impact on key variables from more intense climate events.

Figure 4: Short-run Impact of more Intense ND Event with Climate Change



Note: y-axes are percent deviations from the ergodic steady state except ratios to GDP which are percentage point deviations. The time scale on the x-axes are quarters.

In the long-run, the level of GDP is around 3½ percent lower relative to the baseline. This decline is underpinned by two effects of opposite sign. On the one hand, the level of private capital relative to GDP is lower with climate change because the increase in NDs' intensity results in higher expected losses and, as a result, lower expected returns to investment (determined by the global interest rate and the corporate interest rate through the impact of financial frictions). With climate change, the capital/output ratio is smaller (by 16 percentage points of GDP[22] but the level of investment is higher as firms undertake more investment given they internalize additional capital destruction with climate change. On the other hand, the increase in investment is more than offset by the decline in private consumption driven by the households' recurrent need to save and rebuild capital and by the lower labor income with a lower capital stock. Overall, the decline of consumption, labor income and output results in lower tax revenue as climate events get more intense with climate change.

Table 6. Long-run Impact of more Intense Natural Disasters under Climate Change vs. Baseline

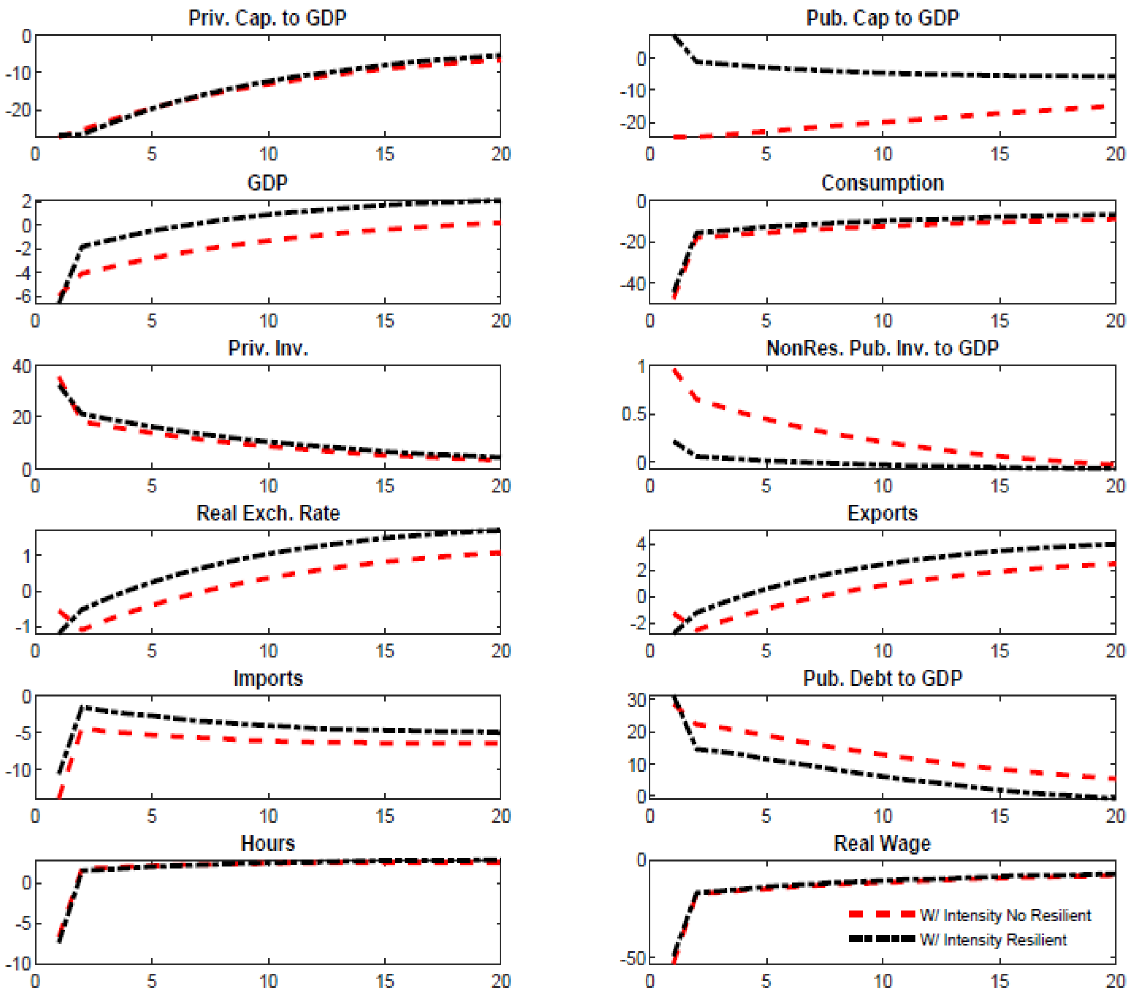
Dominica	
(Percent change relative to the baseline without climate change)	
GDP	-3.3
Private Consumption	-7.6
Private Investment	7.9
Real Wage	-7.3
Consumption Tax Revenue	-7.6
Corporate Tax Revenue	-6.2
Labor Income Tax Revenue	-6.9

Figure 5 and table 7 show that the gains from adaptation with more intense NDs increase with climate change. In the near term, the decline in GDP is slightly larger with resilient capital on impact due to the lower response of investment and also the larger capital stock held as a precaution in anticipation of destruction. However, output recovery is significantly faster and protracted when capital is resilient, with returns that largely exceed the small initial loss and returning to the pre-climate event level after around one year. As a result, the debt to GDP ratio returns to its steady-state level more rapidly. This implies a significant improvement in debt sustainability prospects by reducing the dispersion of public debt outcomes. In the long run, the returns from public investment in resilience increase with higher intensity of NDs under climate change, with estimated gains of about 10 percent of the level of GDP. Consumption, investment and the real wage also increase, leading to higher tax revenue.

Overall, notice that this result with quantitatively large steady-state impact of more intense NDs under climate change highlight the importance of the stochastic framework used in the model in this paper: the ability to account for climate change is not only a feature of the model in this paper, but it is also important in quantitiative terms for both the steady state and the near-term dynamics.

[22] The level of the capital stock is around 7½ lower when climate events are more intense.

Figure 5: Short-run Gains from more Resilient Capital with more Intense ND Event



Note: y-axes are percent deviations from the ergodic steady state except ratios to GDP which are percentage point deviations. The time scale on the x-axes are quarters.

Table 7. Estimated Long-run Benefits of Adaptation

Dominica (percent)		
	Baseline	More intense events
GDP	6.3	10.3
Private Consumption	1.9	5.4
Private Investment	17.9	21.3
Real Wage	3.2	6.3
Consumption Tax Revenue	1.9	5.4
Corporate Tax Revenue	6.7	10.3
Labor Income Tax Revenue	6.3	10.3

Sensitivity Analysis: Inspecting the Mechanism

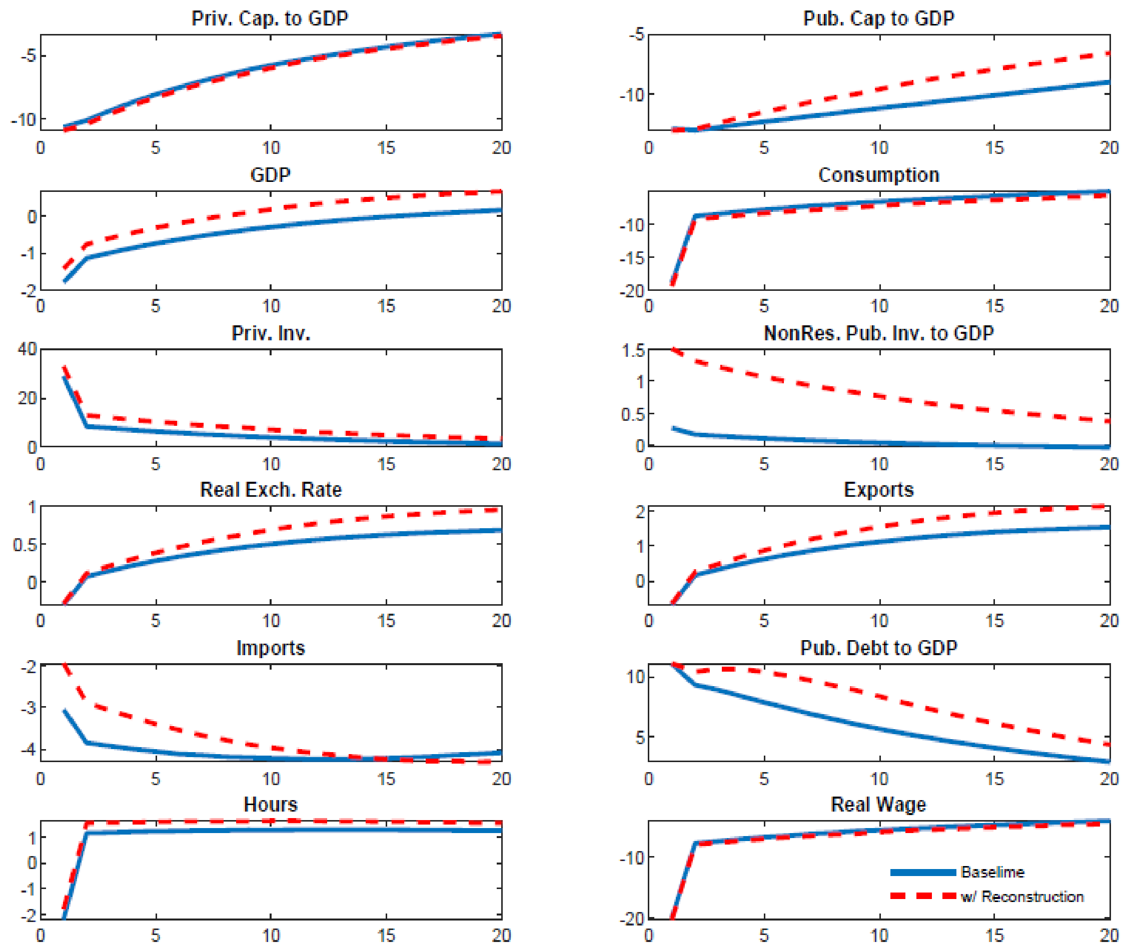
This section considers alternative model specifications to better understand the transmission mechanism of a ND shock and to evaluate the gains from adaptation investment. To this end, the section focuses on specific model parameters that do not affect the steady state but affect the short-term dynamics: an increase of public reconstruction with non-resilient investment, changes to the fiscal accommodation rule with consumption or labor income taxes as instrument to meet the budget constraint and reach the debt target, and alternative values for the adjustment cost of capital.

A. Increasing Public Reconstruction

Faster government reconstruction increases the response of non-resilient public investment, thus dampening the impact on the public capital stock. Figure 6 shows the impact of further increasing reconstruction spending following a ND.[23] The experiment shows the results after an increase of the coefficient ϕ_{kg} to 0.8 in line with Cantelmo and others (2019). By boosting public investment and thus the public capital stock, additional reconstruction after a ND has a positive impact on output, which returns to its steady-state almost ten quarters sooner than in the baseline. Given the complementarity of public and private capital, private investment is also higher, though the private capital to GDP ratio is slightly lower given that the level of GDP is now higher. Despite the improvement in output, reconstruction investment worsens the public finances and public debt increases more following a ND. As a result, lump-sum taxes increase and private consumption falls by more in response.

[23] In the baseline, public investment to GDP increases in the aftermath of the climate event. This is because it is assumed that public investment is constant and equal to its steady-state level. Since GDP falls on impact, public investment as a percent of GDP increases.

Figure 6: Short-run Response to ND Event with Public Reconstruction



Note: y-axes are percent deviations from the ergodic steady state except ratios to GDP which are percentage point deviations. The time scale on the x-axes are quarters.

B. Alternative Fiscal Instruments: Consumption and Labor Income Taxes

This section examines the near-term impact of a ND under alternative fiscal instruments to reach the debt target instead of the non-distortionary lump-sum taxes considered in the baseline. Two taxes are considered: consumption and labor income taxes, levied on both worker and saving households. This results in the following tax rules:

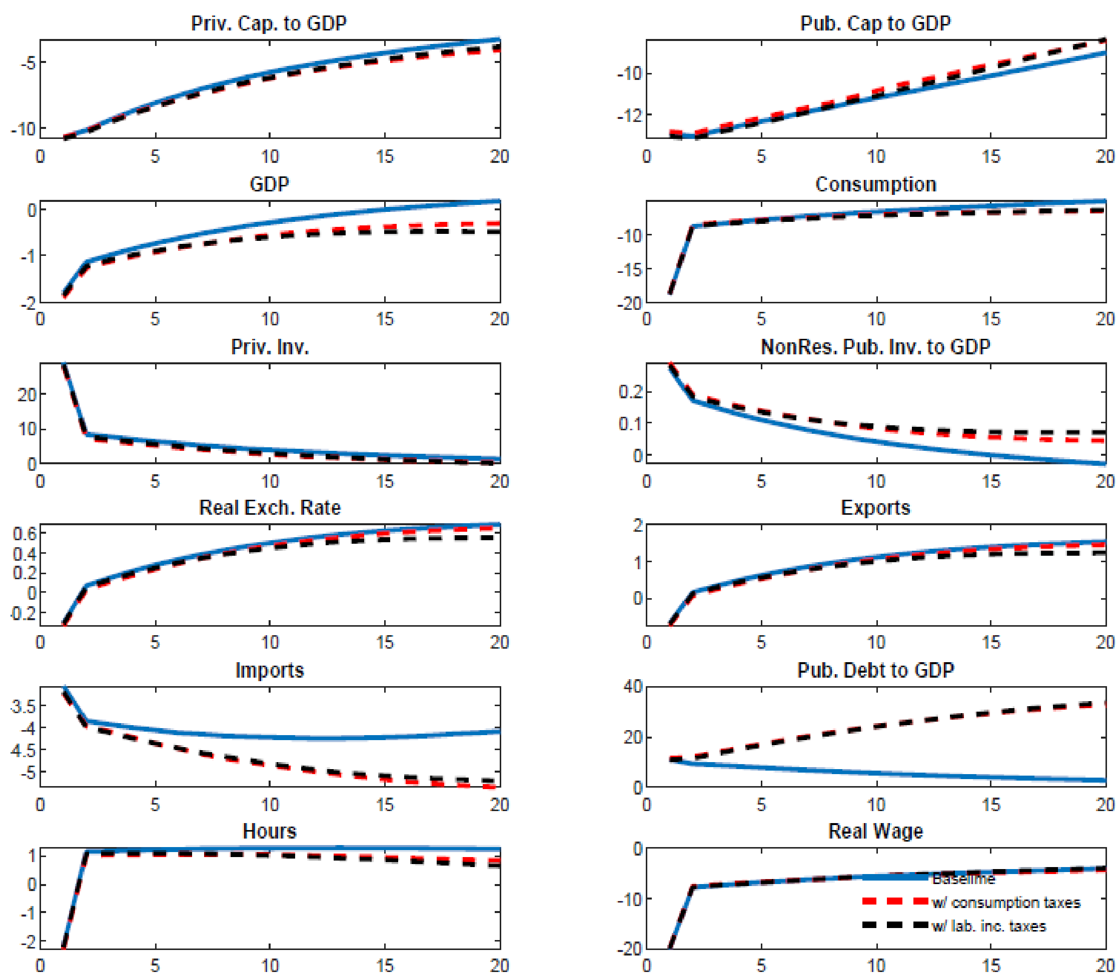
$$\tau_t^C = \rho_c \tau_{t-1}^C + \frac{\tau^C}{(1-\rho_c)} - \frac{\phi_b}{(1-\rho_c)}\left(\frac{z_t B_t^*}{Y_t} - \frac{B}{Y}\right),$$

$$\tau_t^l = \rho_l \tau_{t-1}^l + \frac{\tau^l}{(1-\rho_l)} - \frac{\phi_b}{(1-\rho_c)}\left(\frac{z_t B_t^*}{Y_t} - \frac{B}{Y}\right),$$

with the coefficients on the response of taxes to deviations of the debt level relative to its long-run steady-state, ϕ_b, being equal across specifications. It is assumed that the coefficients on lagged taxes are equal to 0.95, in line with Cantelmo and others (2019).

Using consumption and labor income taxes results in a larger contraction of GDP relative to the baseline (Figure 7). Moreover, the GDP recovery is more sluggish. This is because consumption and labor income taxes are more distortionary than lump-sum taxes, which are only levied on Ricardian (saver) households that can smooth their consumption. In the case of consumption taxes, the incidence of the tax falls both on savers and workers (financially constrained hand to mouth) households, with the latter having a larger marginal propensity to consume. This further depresses consumption and hence overall domestic demand. The larger contraction of consumption also dampens the marginal utility of consumption, further depressing labor supply. Labor income taxes are even more distortionary than consumption taxes and reduce labor supply further. As a result, the recovery of output is delayed compared with the baseline and consumption taxes. The slower output recovery results in larger public debt.

Figure 7: Short-run Response to ND Event with Alternative Fiscal Rules

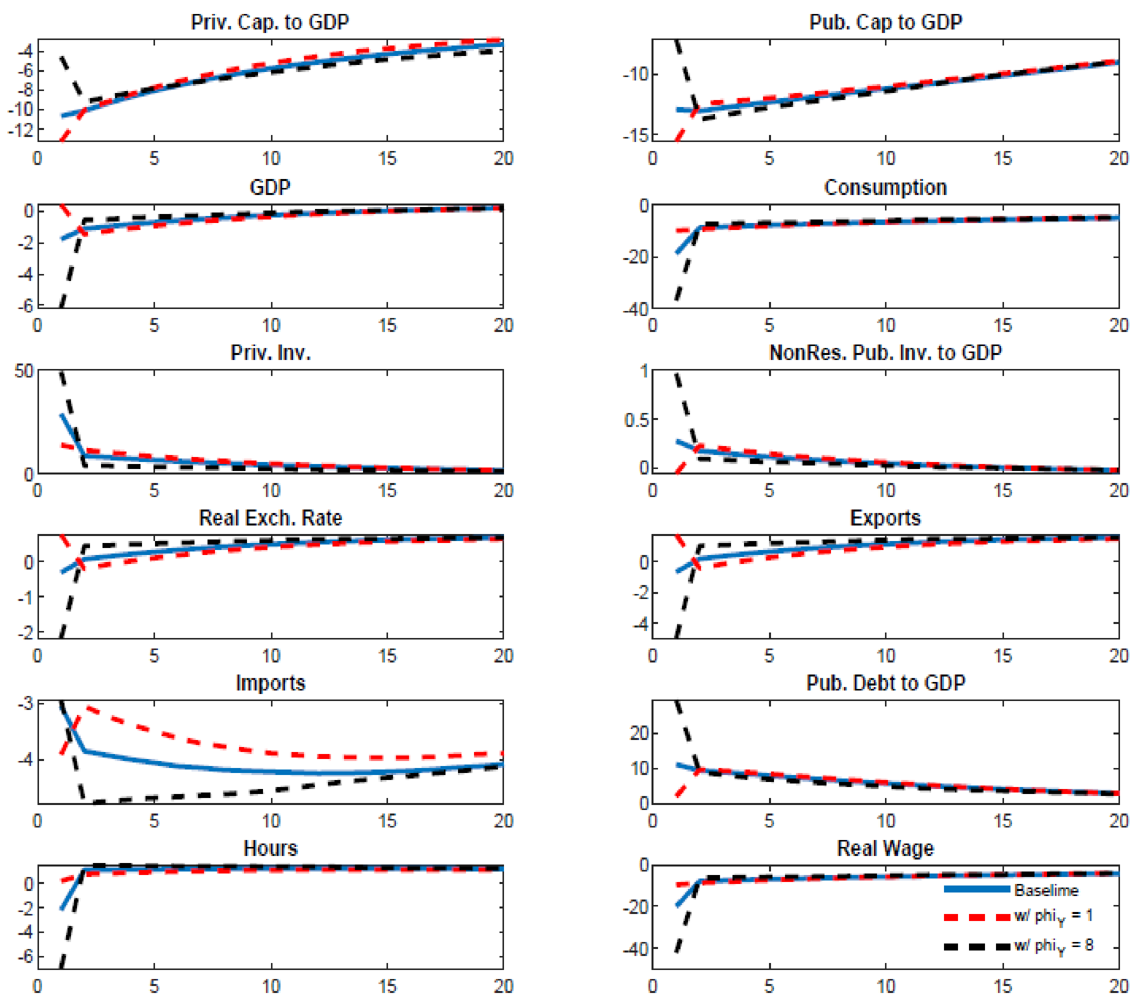


Note: y-axes are percent deviations from the ergodic steady state except ratios to GDP which are percentage point deviations. The time scale on the x-axes are quarters.

C. Alternative Adjustment Costs of Investment

This section considers the implications of alternative adjustment costs to investment, shown in Figure 8. Adjustment costs are expressed in terms of changes of investment relative to the capital stock and are incurred each period private investment changes relative to the private capital stock. The ND destroys capital relative to investment and hence the investment to capital ratio increases. Since, as explained above, this ratio rules the size of adjustment costs, firms increase their investment by more when these costs are larger (ψ_Y=8 vs ψ_Y=0) at the expense of the consumption of savers, leading to a larger contraction to consumption and GDP on impact. With the larger adjustment costs (ψ_Y=8 vs ψ_Y=0), however, the investment response is subsequently more muted, which acts to delay the recovery in GDP and of other key macroeconomic variables.

Figure 8: Short-run Response to ND Event with Alternative Capital Adjustment Costs



Note: y-axes are percent deviations from the ergodic steady state except ratios to GDP which are percentage point deviations. The time scale on the x-axes are quarters.

Conclusion

This paper presents a DSGE model that allows the evaluation of the cost-return trade-off of investment in public infrastructure resilient to NDs. The model set-up is particularly fitting for the analysis of the impact of NDs in SDS where NDs often have large costs relative of the size of the economy, of macroeconomic proportions. The model in this paper follows a growing literature by including ND shocks that destroy the stock of productive (public and private) assets. The model shows that when public infrastructure is resilient to NDs, the output loss by private investors in the aftermath of NDs declines (i.e., roads and bridges remain open or become usable more promptly, ports and airports essential for export and imports remain operational, etc.), which increases expected returns to private investment, ultimately increasing the amount of private investment, employment, and output. In addition, it can improve government finances if the return (in terms of additional tax revenue) outweighs the cost (in terms of the additional cost of resilient structures), providing space to reduce distortionary taxes and/or increase public spending in infrastructure or other development needs. The latter has critical importance for the financing of the resiliency cost and can inform decisions of financing terms.

A key distinctive feature of the model in this paper is that it accounts for the ex-ante internalization by investors and consumers of the recurrent nature of NDs. In the model in this paper agents expect NDs to occur at some random moment and intensity in the future and adjust their consumption and investment decisions accordingly. Most existing papers in the literature do not share this feature, treating NDs as one-off surprises. This feature is important because the returns of costly investments in resilient infrastructure are not only important to quantify losses in the aftermath of a shock by reducing the size of the damage and output drop and the recovery period, but it also accounts for the NDs' long-term impact (steady state), thereby providing a full account of the NDs' cost. Moreover, the calibration to the case of Dominica illustrate that this steady state impact is not only qualitatively important but also quantitatively very significant. The latter is critical to evaluate the net returns of costly investment in resilient public infrastructure.

This paper makes several contributions to the literature on the macroeconomic analysis of resiliency to NDs and climate change. Like other contributions in this literature, the analysis is centered around a ND shock that destroys a share of public and private productive assets, in relation to the probability (frequency) and intensity of NDs. The model in this paper contributes to the literature by combining specific features that are critical to the analysis of resilience investment and the impact of climate change in SDSs, which have not been incorporated in a holistic way in the existing literature:

i. The model accounts for the destruction of both public and private capital.

ii. The NDs shocks have permanent negative effects on output, as each ND shock reduces total TFP temporarily. This results in scarring effects with long-lasting macroeconomic implications by way of reducing the output level after each ND event, which is important for the assessment of returns to adaptation investments that reduce the NDs' impact.

iii. A full set of fiscal policy instruments, including taxes, non-tax revenue, and spending allocations (including public wages, transfers, and public investment resilient and non-resilient) that account for the distortionary impact of resilience investment financing and allow fiscal policy experiments.

iv. A Markov-switching specification of the NDs shock, which implies that rational consumers and investors fully internalize the possibility of future NDs in their decisions. NDs are not a one-time surprise, they are expected by all participants in the economy. This has important implications on the long-term (steady state) equilibrium explained above.

v. The stochastic specification enables the analysis of the macroeconomic implications of climate change. The intensity and frequency of NDs, and the corresponding damage to productive assets, are explicit parameters in the model which. As a result, the return to costly investment in structural resilience can be assessed in a context of climate change.

vi. Financial frictions in the form of collateral constraints to investors which restrict their ability to borrow to rebuild their capital stock after the destruction inflicted by a ND. The stochastic nature of the model implies that creditors internalize the risk of collateral destruction and make lending decisions accordingly. Moreover, once hit by a NDs, collateral constraints affect the transitional dynamics of the economic recovery by slowing the ability of producers to borrow to rebuild the capital stock. This feature is critical in the assessment of the returns to investment in resilience to NDs because it implies not only a reduction in the destruction of productive assets after a ND (stock loss), but also reduces the output loss (flow), plus the positive long-term return in the (steady state).

vii. There is an endogenous ND risk premium that affects consumption and investment decisions.

viii. The model includes a share of non-Ricardian hand-to-mouth consumers, important to capture the impact of NDs and resilience investments on the poor, which in most SDSs affected by NDs are a large share of the population (also typically the most exposed and vulnerable to NDs shocks).

ix. Foreign remittances that vary according with the state of the economy, and specifically in the event of a ND. Many SDSs affected by NDs show a large share of their population working abroad and sending remittances to back to their country of origin.

A calibration to the case of Dominica is used to illustrate the working of the model, enabling a quantification of the economic impact of climate-related NDs and climate change. The results indicate that NDs' destruction of the capital stock of around 10½ percent of GDP (in line with observed events in the past) results in a persistent and slow recovery, with output returning to the pre-ND level in around five years. There is an important deterioration of the public finances, with public debt increasing by around 10 percentage points of GDP. In addition to this near-term impact, NDs also affect the long run equilibrium with a reduction of steady-state output of around 3½ percent on the level of GDP. As explained above, this is because investors and consumers internalize the possibility of future NDs.

The simulations under climate change, with a counterfactual experiment that assumes an increase in the intensity of NDs with global warming. It is calibrated consistent with predicted increases in wind speeds and sea surface temperature in line with the United Nations' RCP 8.5 scenario which assumes no change in current trends of CO2 emission trends. The estimates in Acevedo (2016) imply a 14 percent increase in NDs-related damages with climate change. As expected, the experiment predicts a decline in steady state output of over 3 percent relative to a baseline without climate change, and declines in private consumption and real wages of about 7 ½ percent. Investment increases as the private sector is forced to rebuild more or more often, and also to cushion de loss once impacted by a NDs. The fiscal outlook worsens significantly with permanent decline in tax revenue of around 7 percent in real terms.

The returns of investing in resilient infrastructure quantify the benefits of adaptation investment.[24] The results show that the return of investing in resilient structures outweigh the cost in the case of Dominica:

- *Near-term return of resilient investment.* The Results indicate quantitatively significant returns of resilient investment which reduces output decline (flow return) and reconstruction cost (stock return). Assuming 80 percent resiliency of public capital (NDs' damage reduced to 20 percent of those without the resilient investments for the same ND intensity), the output decline after a NDs is reduced by over 2 percentage points of GDP, which proves very protracted (long lasting). Therefore, it compounds to large output loss over time, affected by time to build under investment adjustment cost and also financial frictions and collateral constraints explained above. Moreover, the increase in public debt after a ND is also reduced by near 10 percent of GDP, with this gap remaining very protracted. This implies that resilient investment reduces the dispersion of debt outcomes due to natural disasters, supporting fiscal resilience. The loss of exports and real exchange rate depreciation following a ND shock also declines significantly with investment in resilience, strengthening external sustainability.

- *Long-term return of resilient investment.* The calibration to the case of Dominica indicates that the long-term output level increases by over 6 percent, while tax revenues increase in the range of 1.9-6.7 percent depending on the tax type. The increase in tax revenue more-than-offsets the additional cost of resilient structures in the case of Dominica, a critical result considering the additional cost of resilient investments. The long-term return of costly resilient investment increase significantly if climate change increases the intensity of NDs. GDP increases by additional 4 percent relative to no climate change; consumption and investment by additional 3.5 percent; and real wages by over 3 percent relative to the returns to resilient investments without climate change.

[24] These results may be considered as a lower bound since the model does not consider some channels that may further amplify the impact of climate-related natural disasters such as those associated with endogenous productivity (e.g., Anzoategui and others, 2019, Cerra and others, 2021 or Queralto, 2020).

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Annex I. Full Model Presentation

This Annex presents a full description of the model, including a version that takes into account migration decisions. Simulations are presented including in the full model with migration.

I. Households

There are three types of households in the economy: investors, workers, and migrants. In a continuum of households indexed by $i \in [0,1]$, households in the interval $[0, \omega)$ are workers' households; these households cannot access financial markets and do not have an initial capital endowment. Workers' households consume their disposable income in each period. Investor households, in the interval $(\omega, 1]$, have access to the financial market and can smooth consumption. Savers households are also the owners of the firms.

As in Gali, Lopes, and Valles (2007), we assume that households forgo the labor supply decision. Instead, they supply labor to unions and let unions negotiate wages in a non-competitive labor market. To incorporate the non-competitive labor market, we follow, Schmitt-Grohe and Uribe (2006) and Colgiago (2011). In the remaining part of this section, we will describe the optimization problems of different households and the maximization problems of the unions.

The utility function is common across households and has the functional form $(\ln C_t) - \eta \frac{N_t^{(1+\xi)}}{1+\xi}$ where C_t denotes consumption, and N_t labor. The parameters in the utility function are the inverse of Frisch elasticity, ξ, and a scale parameter η.

Ricardian Households

Ricardian (saver) households maximize the expected utility

$$\mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \left(\ln C_t^K - \eta \frac{(N_t^K)^{1+\xi}}{1+\xi} \right)$$

subject to the budget constraint:

$$(1+\tau_t^C) C_{s,t}^K + T_{s,t}^G + B_{s,t} + z_t B_{s,t}^* \\ = (1-\tau_t^\pi) \Pi_{s,t} + \left(1-\tau_t^l\right) W_t N_{s,t}^K + R_t B_{s,t-1} + z_t R_{t-1}^* B_{s,t-1}^*.$$

τ_t^C is the tax rate on consumption expenditure, τ_t^l is the labor tax rate and τ_t^π is a tax on profits received from firms. $T_{s,t}^G$ are lump-sum taxes, $B_{s,t}$, and $B_{s,t}^*$ are net-external assets. R_t and R_t^* are the domestic and external interest rate, respectively. z_t is the real exchange rate.

The first order conditions, aggregated across the saver's households, are:

$$\frac{1}{C_t^K} - \Delta_t^K (1+\tau^C) = 0$$

$$\Delta_t^K = \beta \Delta_{t+1}^K \frac{z_{t+1}}{z_t} R_t^*$$

$$\Delta_t^K = \beta \Delta_{t+1}^K R_t$$

where Δ_t^K is the Lagrange multiplier associated with the budget constraint.

Worker Households

These households maximize expected utility subject to the budget constraint $(1+\tau_t^C)C_{s,t}^W = (1-\tau_t^l)W_t N_{s,t}^W + T_{s,t}^{GW}$ where $T_{s,t}^{GW}$ are lump sum transfers from the government. The aggregated first order conditions of these households are[1]:

$$\Delta_t^W = \frac{1}{(1+\tau^C)C_t^W}$$

$$(1+\tau_t^C)C_t^W = (1-\tau_t^l)W_t N_t^d + \mu T_t^{GW}$$

II. Union Problem

Households provide labor services to a union which negotiates on their behalf. Following Gali, Lopes, and Valles (2004,2007) and Colgiago (2011), labor types are differentiated, and each household supplies all types of labor j to a union. The union sets the wage subject to a demand function

$$N_{j,t} = \int_0^1 N_{jt}(z)dz = \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\epsilon_W} N_t^D(z)dz = \left(\frac{W_t^j}{W_t}\right)^{-\epsilon_W} N_t^D.$$

where $N_t^D(z)$ is labor demand for all labor types by firm z given by

$$N_t(z) = \left(\int_0^1 \left(N_{j,t}^{\frac{\epsilon_W-1}{\epsilon_W}}\right)dj\right)^{\frac{\epsilon_W}{\epsilon_W-1}}$$

and $N_{j,t} = \int_0^1 N_{j,t}(z)dz$ is aggregate labor demand for labor type j. In equilibrium all labor demanded by firms is supplied by households ($N_t = N_t^D$), hence $N_t^D = N_t^d = \int_0^1 N_{j,t}\,dj = \int_0^1 N_t(z)dz$.

The problem for the union is to set the wage to

$$\max_{W^j} E_t \sum_{t=0}^{\infty} \beta^t \left\{ -\eta \frac{\left[\left(\frac{W_t^j}{W_t}\right)^{-\epsilon_W} N_t^D\right]^{1+\xi}}{1+\xi} + \left((1-\omega)\Delta_t^K + \omega\Delta_t^W\right)\left[\left(1-{\tau_t^l}^l\right)W_t^j\left(\frac{W_t^j}{W_t}\right)^{-\epsilon_W} N_t^D\right]\right\}$$

The first order condition for the union problem is

$$\frac{1}{\eta}\left((1-\omega)\frac{\Delta_t^K}{(N_t^D)^\xi} + \omega\frac{\Delta_t^W}{(N_t^D)^\xi}\right)W_t\left(1-\tau_t^l\right) = \frac{\epsilon_w}{(1-\epsilon_w)}$$

that implies that, the labor supply is equal across household types and that there is a wedge between the "weighted" marginal rate of substitution and the wage.

III. Investment Producers

Private Investment Producers

Investment producers use imported and domestic investment goods as inputs for production. The production of this sector is used in the construction of private and public capital. The investment producer solves the following minimization cost

$$\min P_{H,t}I_{H,t} + P_{F,t}I_{F,t}$$

[1] Note that the aggregation up to this level is across the workers households

$$s.t. \quad I_t = \left[(1-a_I)^{\frac{1}{\eta_I}}(I_{H,t})^{\frac{\eta_I-1}{\eta_I}} + a_C^{\frac{1}{\eta_I}}(I_{F,t})^{\frac{\eta_I-1}{\eta_I}}\right]^{\frac{\eta_I}{\eta_I-1}}$$

where $I_{H,t}$ and $I_{F,t}$ are domestic and foreign investment goods, respectively. P_t^F and $P_{H,t}$ are the prices of foreign and domestic goods. a_I is the share of home inputs in investment production and η_I is the elasticity of substitution of domestic and foreign goods. The first order conditions of this minimization problem are

$$I_{H,t} = (1-a_I)\left(\frac{P_{H,t}}{P_t^I}\right)^{-\eta} I_t$$

$$I_{F,t} = a_I\left(\frac{P_{F,t}}{P_t^I}\right)^{-\eta} I_t$$

where P_t^I is the price of the investment good. P_t^I its a weighted average of the domestic and foreign prices

$$p_t^I = \frac{P_t^I}{P_t^C} = \left[(1-a_I)(p_{H,t})^{1-\eta_I} + a_I(z_t)^{1-\eta_I}\right]^{\frac{1}{1-\eta_I}}$$

where z_t is the real exchange rate[2]. I_t is the aggregate demand for investment goods given by

$$I_t = I_t^g + I_t^Y + I_t^{Gn}$$

with I_t^Y private investment, I_t^g public investment, and I_t^{Gn}. I_t^{Gn} is the demand for investment good of producers of resilient public capital (explained below).

Public Investment Producers

There are two types of public capital, resilient capital and non-resilient capital. The resilient capital stock is not affected by natural disasters. The production of the non-resilient capital stocks needs a different investment good that is produced using the following production function $I_t^{Gr} = a^{Gr} I_t^g$ with $0 < a^{Gr} < 1$.

The producers of the resilient investment good solve the following maximization problem

$$\begin{aligned} \max \quad & p_t^{Gr} I_t^{gr} - p_t^I I_t^g \\ s.t \quad & I_t^{Gr} = a^{Gr} I_t^g \end{aligned}$$

and the first order condition of the maximization problem are

$$p_t^{Gr} = \frac{p_t^I}{a^{Gr}}$$

$$I_t^{Gr} = a^{Gr} I_t^g$$

where p_t^{Gr} is the price of the resilient investment goods, p_t^{Gr} is a^{Gr} times higher than p_t^I, the price of genetic investment good in the economy.

IV. Consumption Good Producers

The final consumption basket is a composed by imported and domestically produced consumer goods. The final good producer, minimize the cost of producing the final consumption good subject to a CES production technology. The minimization problem is

[2] Note that the price of foreign goods either for investment or consumption is normalized to one. Hence the relatives prices $\frac{P_{F,t}}{P_t^I}$ and $\frac{P_{F,t}}{P_t^C}$ are $\frac{P_{F,t}}{P_t^I} = \frac{P_t^C}{P_t^I}\frac{s_t P_t^*}{P_t^C}\frac{P_{F,t}^*}{P_t^*} = z_t \frac{P_t^C}{P_t^I} = \frac{z_t}{p_t^I}$ and $\frac{P_{F,t}}{P_t^C} = z_t$, respectively.

$$\min P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \;\; s.t\; C_t = \left[(1-a_C)^{\frac{1}{\eta_C}}\left(C_{H,t}\right)^{\frac{\eta_C-1}{\eta_C}} + a_C^{\frac{1}{\eta_C}}\left(C_{F,t}\right)^{\frac{\eta_C-1}{\eta_C}}\right]^{\frac{\eta_C}{\eta_C-1}}$$

where $P_{H,t}$ is the price of the consumption domestic good[3], a_C is the share of home inputs in the consumption production function and η_C is the elasticity of substitution of domestic and foreign goods. The first order condition of the minimization problem is

$$C_{H,t} = (1-a_C)\left(p_{H,t}\right)^{-\eta_C}C_t$$

$$C_{F,t} = a_C(z_t)^{-\eta_C}C_t$$

where aggregate demand for consumption goods, C_t, equals,

$$C_t = (1-\omega)C_t^K + \omega C_t^W$$

These first order conditions together with the consumption production function imply that the price of the domestic consumption is function of the price of the domestic good and the real exchange rate

$$1 = \left[(1-a_C)\left(p_{H,t}\right)^{1-\eta_C} + a_C(z_t)^{1-\eta_C}\right]^{\frac{1}{1-\eta_C}}$$

V. Domestic Producers

Final Output

Following Schmitt-Grohe and Uribe, (2006) we assume that the labor input used by firms is a composite made of a continuum of differentiated labor services. Formally, the labor input is

$$N_{l,t}^d \;\; = \;\; \left[\int_o^1 \left(N_{j,t}\right)^{\frac{\epsilon_w-1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w-1}}$$

Firms select the optimal combination of labor varieties by min $\int_0^1 W_{j,t}\, N_{j,t} dj$ subject to the restriction above. The optimal demand for labor type j by firm z is

$$N_{z,j,t} = \left(\frac{W_{jt}}{W_t}\right)^{-\epsilon_w} N_{z,l,t}^D$$

where W_t is the wage index $W_t = \left(\int_0^1 W_{j,t}^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}} dj$. This last expression is the labor demand used in the household optimization problem. Note that,

$$N_{j,t} = \int_0^1 N_{z,j,t}\, dz$$

and that

$$N_{j,t} = \left(\frac{W_{jt}}{W_t}\right)^{-\epsilon_w} N_t^D$$

where $N_t^D = \int_0^1 N_{z,t}\, dz$ and

$$N_t^d = \int_0^1 N_{j,t}\, dj = \int_0^1 \left(\frac{W_{jt}}{W_t}\right)^{-\epsilon_w} N_t^D dj = N_t^D$$

Producer of the domestic output select the optimal combination of labor and capital services by maximizing profits subject to their constraints: A the technological constraint given by

$$Y_t^H = z_t^Y\theta(s)A_t(K_{t-1}^G)^{\alpha_g}(K_{t-1}^Y)^{\alpha_K}(N_t^d)^{1-\alpha_K}$$

[3] P_t^H is the actual price, the relative price in terms of the consumption price is $p_t^H = P_t^H/P_t^C$.

where Y_t^H denotes domestic output, K_t^g and K_t^Y are the public and private capital stocks, respectively. α_g and α_K are the output elasticities of public and private capital. z_t^Y is the exogenous transitory productivity shock and $\theta(s)$ is a parameter that captures the impact of climate disaster on productivity. $\theta(s)$ equals one in normal times and its lower than one in a natural disaster event. A_t is a permanent productivity shock, such as $A_t/A_{t-1} = g_t^A = \left(1-\rho_{g^A}\right)g^A + \rho_{g^A} + \epsilon_t^{gA}$ with $\rho_{g^a} \in (0,1)$. The second constraint is a borrowing constraint that limits borrowing to a maximum fraction σ of the value of the capital stock,

$$W_t N_t^d + p_t^I I_t^Y \leq \sigma(Q_t K_{t-1}^Y)$$

The profit maximization problem is summarized through the following maximization problem

$$\begin{array}{rr}
\max & \left(1-\tau_t^\pi\right)(p_t^H Y_t^H - W_t N_t^d - p_t^I I_t^Y) \\
\text{s.t} & Y_t^H = z_t^Y \theta(s) A_t (K_{t-1}^G)^{\alpha_g} (K_{t-1}^Y)^{\alpha_K} (N_t^d)^{1-\alpha_K} \\
\{Q_t\} & K_t^Y = (1-\delta)K_{t-1}^Y + I_t^Y - \dfrac{\psi_y}{2}\left(\dfrac{I_t^Y}{K_{t-1}^Y} - \delta_Y\right)^2 K_{t-1}^Y \\
\{\varsigma_t\} & W_t N_t^d + p_t^I I_t^Y \leq \sigma(Q_t K_{t-1}^Y)
\end{array}$$

Q_t and ς_t are the Lagrange multipliers for the capital accumulation equation and the credit constraint condition.

The optimal conditions are the constraints together with the first order conditions for capital, investment, and labor

$$Q_t\left(1-\psi_Y\left(\frac{I_t^Y}{K_{t-1}^Y}-\delta_Y\right)\right) = p_t^H\left((1-\tau^\pi)+\varsigma_t\right)$$

$$Q_t = \frac{\beta \Lambda_{t+1}^K}{\Lambda_t^K}\left((1-\tau_t^\pi)\alpha_K p_{t+1}^H \frac{Y_{t+1}}{K_t^Y} + Q_{t+1}\left((1-\delta_Y) - \frac{\psi_Y}{2}\left(\frac{I_{t+1}^Y}{K_t^Y}-\delta_Y\right)^2 + \frac{\psi_Y}{2}\left(\frac{I_{t+1}^Y}{K_t^Y}-\delta_Y\right)^2 \frac{I_{t+1}^Y}{K_t^Y} + \varsigma_{t+1}\sigma\right)\right)$$

$$(1-\tau_t^\pi)(1-\alpha_K)p_t^H \frac{Y_t}{N_t^d} = (1+\varsigma_t)W_t$$

VI. Government

The government is characterized by ten variables: public consumption C_t^g , public investment on non resilient capital, I_t^{Gn} and resilient capital I_t^{Gn}, tax rates on consumption τ_t^C, private profits τ^π and labor income τ_t^l , transfer payments T_t^{GW} to worker households, lump-sum taxes T_t^G, the stock of public bonds issued in foreign currency B_t^{G*} and grants received from abroad T_t^{G*}. Analogously to private capital, public capital is accumulated according to the following law of motions:

$$K_t^{Gr} = \left(1-\delta_g\right)K_{t-1}^{Gr} + I_t^{Gr}$$

$$K_t^{Gnr} = \left(1-\delta_g(s)\right)K_{t-1}^{Gnr} + I_t^{Gnr}$$

and $K_t^G = K_t^{Gnr} + K_t^{Gr}$. Government expenditure on I_t^{Gr} and I_t^{Gnr} is exogenous and follow AR process.

The government faces a flow budget constraint that balances its expenses on interest and debt payments, transfers, consumption, and investment with its revenues from taxes on consumption, wages and private profits, and grants and cash returns from bonds issued in the current period. For the government budget constraint, it thus:

$$\tau_t^C C_t + \tau_t^l W_t N_t^d + \tau_t^\pi \Pi_t + T_t^G + z_t B_t^{G*} + z_t T_t^{G*} = p_t^H C_t^g + T_t^{GW} + p_t^I I_t^{Gn} + p_t^{Gr} I_t^{Gr} + z_t R_{t-1}^* B_{t-1}^{G*}$$

We broadly follow Leeper and others (2010) in specifying a revenue rule for the fiscal sector of the form

$$T_t^G = T^G - \phi_b\left(\frac{z_t B_t^*}{Y_t} - \frac{B}{Y}\right)$$

where $\phi_b > 0$. This rule guarantees the stability of the public to GDP ratio. The other fiscal instruments are assumed constant and equal to their steady state value. Finally, following Schmitt-Grohe and Uribe, 2005, the interest rate that the government has to pay for its external debt equals the external risk-free interest rate plus a risk premium that depend on the debt to GDP ratio.

$$R_t^* = \bar{R}_t^*(s) + \Omega_u \left(\exp\left(\frac{z_t(B_t^* - B_t^{G*})}{GDP_t} - \frac{z(B^* - B^{G*})}{GDP} \right) - 1 \right)$$

This interest rate is also the rate at which saver households can borrow abroad.

VII. Balance of Payments and External Sector

To complete the model and define the trade balance, we need to define exports, C_t^{H*}, as

$$C_t^{H*} = (p_t^H / z_t)^{-\eta_*} Y_t^*$$

where Y_t^* is the external output and η_* is the elasticity of exports to the exchange rate.

With this equation, and the following market clearing conditions:

$$\tau_t^C C_t + \tau_t^l W_t N_t^d + \tau_t^\pi \Pi_t + z_t T_t^{G*} + z_t B_t^{G*} = T_t^G + p_t^H C_t^g + \mathrm{z}_t T_t^{GW} + p_t^I I_t^{Gn} + p_t^{Gr} I_t^{Gr} + z_t R_{t-1}^* B_{t-1}^{G*}$$

$$(1 + \tau_t^C) C_{o,i,t} + T_{o,i,t}^G + B_{i,t} + z_t B_{i,t}^* = (1 - \tau_t^\pi) \Pi_{i,t} + (1 - \tau_t^l) W_t N_{o,i,t} + R_{t-1} B_{i,t-1} + z_t R_{t-1}^* B_{i,t-1}^*$$

$$(1 + \tau_t^C) C_{s,i,t}^W = (1 - \tau_t^l) W_t N_{s,i,t}^W + T_{s,i,t}^{GW}$$

$$C_t = p_t^H C_t^H + p_t^F C_t^F$$

$$\Pi_t = p_t^H Y_t^H - W_t N_t^d - p_t^I I_t^Y$$

$$Y_t^H = C_t^H + I_t^H + C_t^g + C_t^{H*}$$

we can derive the balance of payment equation in the model as

$$z_t(B_t^* - B_t^{G*}) = \left[\left(p_t^H Y_t^H - f_e W_t^* z_t N_{m,t}^e \right) - C_t - p_t^H C_t^g - p_t^I I_t \right] + z_t R_{t-1}^* (B_{t-1}^* - B_{t-1}^{G*}) + T_t^{W*}(s) + \mathrm{z}_t T_t^{G*}.$$

where $T_t^{W*}(s)$ are remittances.

VIII. Summary of the First Order Conditions

$$\frac{1}{C_t^K} - \Delta_t^K (1 + \tau^C) = 0$$

$$\Delta_t^K = \beta \Delta_{t+1}^K \frac{z_{t+1}}{z_t} R_t^*$$

$$\Delta_t^K = \beta \Delta_{t+1}^K R_t$$

$$\Delta_t^W = \frac{1}{(1 + \tau^C) C_t^W}$$

$$(1 + \tau_t^C) \omega C_t^W = \left(1 - \tau_t^l\right) W_t \omega N_t^d + T_t^{GW} + T_t^{W*}(s)$$

$$\frac{1}{\eta} \left((1 - \omega) \frac{\Delta_t^K}{\left(N_t^d\right)^\xi} + \omega \frac{\Delta_t^W}{\left(N_t^d\right)^\xi} \right) W_t (1 - \tau^W) = \frac{\epsilon_w}{(1 - \epsilon_w)}$$

$$C_t = \left[(1-a)^{\frac{1}{\eta}}\left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + a^{\frac{1}{\eta}}\left(C_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$C_{H,t} = (1-a)\left(p_{H,t}\right)^{-\eta} C_t$$

$$C_{F,t} = a(z_t)^{-\eta} C_t$$

$$C_t = (1-\omega)C_t^K + \omega C_t^W$$

$$I_{H,t} = (1-a_I)\left(\frac{p_{H,t}}{p_t^I}\right)^{-\eta} I_t$$

$$I_{F,t} = a_I\left(\frac{z_t}{p_t^I}\right)^{-\eta} I_t$$

$$I_t = \left[(1-a_I)^{\frac{1}{\eta_I}}\left(I_{H,t}\right)^{\frac{\eta_I-1}{\eta_I}} + a_I^{\frac{1}{\eta_I}}\left(I_{F,t}\right)^{\frac{\eta_I-1}{\eta_I}}\right]^{\frac{\eta_I}{\eta_I-1}}$$

$$I_t = I_t^g + I_t^K + I_t^{Gn}$$

$$p_t^{Gr} = \frac{p_t^I}{a^{Gr}}$$

$$I_t^{Gr} = a^{Gr} I_t^g$$

$$Y_t^H = z_t^Y \theta(s) A_t (K_{t-1}^G)^{\alpha_g} (K_{t-1}^Y)^{\alpha_K} \left(N_t^d\right)^{1-\alpha_K}$$

$$Y_t^H = C_{H,t} + I_{H,t} + C_t^g + X_t + f_e N_{m,t}^e A_t^{\frac{1}{1-\alpha_k-\alpha_{kg}}}$$

$$K_t^Y = (1-\delta)K_{t-1}^Y + I_t^K - \frac{\psi_y}{2}\left(\frac{I_t^K}{K_{t-1}^Y} - \delta_Y\right)^2 K_{t-1}^Y$$

$$W_t N_t^d + p_t^I I_t^Y = \sigma(Q_t K_{t-1}^Y)$$

$$Q_t\left(1 - \psi_Y\left(\frac{I_t^K}{K_{t-1}^Y} - \delta_{KY}\right)\right) = p_t^I\left((1-\tau^\pi) + \varsigma_t\right)$$

$$Q_t = \frac{\beta \Lambda_{t+1}^K}{\Lambda_t^K}\Bigg((1-\tau_t^\pi)\alpha_K p_{t+1}^H \frac{Y_{t+1}^H}{K_t^Y}$$

$$+ Q_{t+1}\left((1-\delta) - \frac{\psi_Y}{2}\left(\frac{I_{t+1}^K}{K_t^Y} - \delta_{KY}\right)^2 + \frac{\psi_Y}{2}\left(\frac{I_{t+1}^K}{K_t^Y} - \delta_{KY}\right)^2 \frac{I_{t+1}^K}{K_t^Y} + \varsigma_{t+1}\theta\right)\Bigg)$$

$$(1-\tau_t^{\pi})(1-\alpha_K)p_t^H\frac{Y_t^H}{N_t^d}=(1+\varsigma_t)W_t$$

$$\Pi_t=p_t^HY_t^H-W_tN_t^d-p_t^II_t^K$$

$$K_t^{gr}=(1-\delta)K_{t-1}^{gr}+I_t^{gr}$$

$$K_t^{gn}=\big(1-\delta(s)\big)K_{t-1}^{gn}+I_t^{gn}$$

$$K_t^G=K_t^{Gnr}+K_t^{Gr}$$

$$\tau_t^CC_t+\tau_t^lW_tN_t^d+\tau_t^{\pi}\Pi_t+T_t^G+z_tB_t^{G*}+\mathrm{z}_tT_t^{G*}$$
$$=p_t^HC_t^g+T_t^{GW}+p_t^II_t^{Gn}+p_t^{Gr}I_t^{Gr}+z_tR_{t-1}^*B_{t-1}^{G*}$$

$$T_t^G=T^G-\phi_b\left(\frac{z_tB_t^{G*}}{GDP}-\frac{zB^{G*}}{GDP}\right)$$

$$T_t^{GW}=T^{GW}$$

$$C_t^g=C^g$$

$$I_t^{Gn}=I^{Gn}$$

$$I_t^{Gr}=I^{Gr}$$

$$Y_t^*=Y^*$$

$$X_t=(p_t^H/z_t)^{-\eta_X}Y_t^*$$

$$z_t(B_t^*-B_t^{G*})-z_tR_{t-1}^*(B_{t-1}^*-B_{t-1}^{G*})=\big(p_t^HX_t-z_tC_{F,t}+z_tI_{F,t}\big)+T_t^{W*}(s)$$

$$R_t^*=\bar{R}_t^*(s)+\Omega_u\left(\exp\left(\frac{z_t(B_t^*-B_t^{G*})}{GDP_t}-\frac{z(B^*-B^{G*})}{GDP}\right)-1\right)$$

IX. De-trended First Order Conditions

X. The model is solved along the balance growth path that requires that output divided by a deflator $A_t^{\frac{1}{1-\alpha_k-\alpha_{kg}}}$, with $\left(\frac{A_t}{A_{t-1}}\right)=g_t^A$, is constant in the steady state. $\tilde{\Delta}_t^K$ and $\tilde{\Delta}_t^W$ are stationary after the transformation $\tilde{\Delta}_t^K=\Delta_t^KA_t^{\frac{1}{1-\alpha_k-\alpha_g}}$, $\tilde{\Delta}_t^W=\Delta_t^WA_t^{\frac{1}{1-\alpha_k-\alpha_g}}$. All other variables are stationary by dividing by the "deflator".

$$\tilde{\Delta}_t^K=\frac{1}{\tilde{C}_t^K(1+\tau^C)}$$

$$\tilde{\Delta}_t^K=\beta\left(\frac{1}{g_{t+1}^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}}\tilde{\Delta}_{t+1}^K\frac{z_{t+1}}{z_t}R_t^*$$

$$\tilde{\Delta}_t^K = \beta \left(\frac{1}{g_{t+1}^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}} \tilde{\Delta}_{t+1}^K R_t$$

$$\tilde{\Delta}_t^W = \frac{1}{(1+\tau^C)\tilde{C}_t^W}$$

$$(1+\tau_t^C)\omega\tilde{C}_t^W = \left(1-\tau_t^l\right)\widetilde{W}_t\omega N_t + \tilde{T}_t^{GW}$$

$$\frac{1}{\eta}\left((1-\omega)\frac{\tilde{\Delta}_t^K}{\left(N_t^d\right)^\xi} + \omega\frac{\tilde{\Delta}_t^W}{\left(N_t^d\right)^\xi}\right)\widetilde{W}_t\left(1-\tau_t^l\quad\right) = \frac{\epsilon_w}{(1-\epsilon_w)}$$

$$\tilde{C}_t = \left[(1-a)^{\frac{1}{\eta}}\left(\tilde{C}_{H,t}\right)^{\frac{\eta-1}{\eta}} + a^{\frac{1}{\eta}}\left(\tilde{C}_{F,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$\tilde{C}_{H,t} = (1-a)\left(p_{H,t}\right)^{-\eta}\tilde{C}_t$$

$$\tilde{C}_{F,t} = a(z_t)^{-\eta}\tilde{C}_t$$

$$\tilde{C}_t = (1-\omega)\tilde{C}_t^K + \omega\tilde{C}_t^W$$

$$\tilde{I}_{H,t} = (1-a_I)\left(\frac{p_{H,t}}{p_t^I}\right)^{-\eta}\tilde{I}_t$$

$$\tilde{I}_{F,t} = a_I\left(\frac{z_t}{p_t^I}\right)^{-\eta}\tilde{I}_t$$

$$\tilde{I}_t = \left[(1-a_I)^{\frac{1}{\eta_I}}\left(\tilde{I}_{H,t}\right)^{\frac{\eta_I-1}{\eta_I}} + a_I^{\frac{1}{\eta_I}}\left(\tilde{I}_{F,t}\right)^{\frac{\eta_I-1}{\eta_I}}\right]^{\frac{\eta_I}{\eta_I-1}}$$

$$\tilde{I}_t = \tilde{I}_t^g + \tilde{I}_t^K + \tilde{I}_t^{Gn}$$

$$p_t^{Gr} = \frac{p_t^I}{a^{Gr}}$$

$$\tilde{Y}_t^H = (g_t^A)^{-\frac{\alpha_k+\alpha_g}{1-\alpha_k-\alpha_g}} z_t^Y\theta(s)\left(\widetilde{K}_{t-1}^G\right)^{\alpha_g}\left(\widetilde{K}_{t-1}^Y\right)^{\alpha_K}\left(N_t^d\right)^{1-\alpha_K}$$

$$\tilde{Y}_t^H = \tilde{C}_{H,t} + \tilde{I}_{H,t} + \tilde{C}_t^g + \tilde{X}_t$$

$$Y_t = \tilde{C}_t + p_t^I\tilde{I}_t + p_t^H\tilde{C}_t^g + \left(p_t^H\tilde{X}_t - z_t\tilde{C}_{F,t} + z_t\tilde{I}_{F,t}\right)$$

$$Y_t = p_t^H Y_t^H$$

$$\widetilde{K}_t^Y = (1-\delta)\widetilde{K}_{t-1}^Y\left(\frac{1}{g_t^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}} + \tilde{I}_t^K$$

$$-\frac{\psi_y}{2}\left(\frac{\tilde{I}_t^K}{\widetilde{K}_{t-1}^Y}(g_{t+1}^A)^{\frac{1}{1-\alpha_k-\alpha_g}} - \delta_Y\right)^2 \widetilde{K}_{t-1}^Y\left(\frac{1}{g_t^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}}$$

$$\widetilde{W}_t N_t^d + p_t^I \tilde{I}_t^K = \sigma\left(Q_t \widetilde{K}_{t-1}^Y\left(\frac{1}{g_t^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}}\right)$$

$$Q_t\left(1-\psi_Y\left(\frac{\tilde{I}_t^K}{\widetilde{K}_{t-1}^Y}\left(\frac{1}{g_t^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}} - \delta_Y\right)\right) = p_t^I\big((1-\tau_t^\pi) + \varsigma_t\big)$$

$$Q_t = \frac{\beta\tilde{\Delta}_{t+1}^K}{\tilde{\Delta}_t^K}\left(\frac{1}{g_{t+1}^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}}\left((1-\tau_t^\pi)\alpha_K p_{t+1}^H \frac{\tilde{Y}_{t+1}^H}{\widetilde{K}_t^Y}(g_{t+1}^A)^{\frac{1}{1-\alpha_k-\alpha_g}} + \right.$$

$$Q_{t+1}\left((1-\delta) - \frac{\psi_Y}{2}\left(\frac{\tilde{I}_{t+1}^K}{\widetilde{K}_t^Y}(g_{t+1}^A)^{\frac{1}{1-\alpha_k-\alpha_g}} - \delta_Y\right)^2\right) +$$

$$\left. Q_{t+1}\frac{\psi_Y}{2}\left(\frac{I_{t+1}^K}{K_t^Y}(g_{t+1}^A)^{\frac{1}{1-\alpha_k-\alpha_g}} - \delta_Y\right)^2 \frac{\tilde{I}_{t+1}^K}{\widetilde{K}_t^Y}(g_{t+1}^A)^{\frac{1}{1-\alpha_k-\alpha_g}} + \varsigma_{t+1}\sigma\right)$$

$$(1-\tau_t^\pi)(1-\alpha_K)p_t^H\frac{\tilde{Y}_t^H}{N_t^d} = (1+\varsigma_t)\widetilde{W}_t$$

$$\widetilde{\Pi}_t = p_t^H\tilde{Y}_t^H - \widetilde{W}_t N_t^d - p_t^I\tilde{I}_t^K$$

$$\widetilde{K}_t^{gr} = \big(1-\delta_{gr}\big)\widetilde{K}_{t-1}^{gr}\left(\frac{1}{g_t^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}} + \tilde{I}_t^{gr}$$

$$\widetilde{K}_t^{gn} = \big(1-\delta_{gn}(s)\big)\widetilde{K}_{t-1}^{gn}\left(\frac{1}{g_t^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}} + \tilde{I}_t^{gn}$$

$$\widetilde{K}_t^G = \widetilde{K}_t^{Gnr} + \widetilde{K}_t^{Gr}$$

$$\tau_t^C\tilde{C}_t + \tau_t^l W_t\widetilde{N}_t^d + \tau_t^\pi\widetilde{\Pi}_t + \tilde{T}_t^G + z_t\widetilde{B}_t^{G*} + \mathrm{z_t}\tilde{T}_t^{G*}$$

$$= p_t^H\tilde{C}_t^g + \tilde{T}_t^{GW} + p_t^I\tilde{I}_t^{Gn} + p_t^{Gr}\tilde{I}_t^{Gr} + z_t R_{t-1}^*\widetilde{B}_{t-1}^{G*}\left(\frac{1}{g_t^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}}$$

$$\tilde{T}_t^G = T^G - \phi_b\left(\frac{z_t\widetilde{B}_t^{G*}}{\tilde{Y}_t} - \frac{zB^{G*}}{Y}\right)$$

$$\tilde{T}_t^{GW} = T^{GW}$$

$$\tilde{C}_t^{g} = \tilde{C}^{g}$$

$$\tilde{I}_t^{Gn} = \tilde{I}^{Gn}$$

$$\tilde{I}_t^{Gr} = \tilde{I}^{Gr}$$

$$\tilde{Y}_t^{*} = Y^{*}$$

$$\tilde{X}_t = (p_t^H/z_t)^{-\eta_X} Y_t^{*}$$

$$C_{m,t}^{a} = C_m^{a}(s)$$

$$W_t^{*} = W^{*}$$

$$z_t\left(\tilde{B}_t^{*} - \tilde{B}_t^{G*}\right) - z_t R_{t-1}^{*}\left(\frac{1}{g_t^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}}\left(\tilde{B}_{t-1}^{*} - \tilde{B}_{t-1}^{G*}\right) = \left(p_t^H \tilde{X}_t - z_t \tilde{C}_{F,t} + z_t \tilde{I}_{F,t}\right) + \tilde{T}_t^{W*}(s)$$

$$R_t^{*} = \bar{R}_t^{*}(s) + \Omega_u\left(\exp\left(\frac{z_t\left(\tilde{B}_t^{*} - \tilde{B}_t^{G*}\right)}{\tilde{Y}_t} - \frac{z(B^{*} - B^{G*})}{Y}\right) - 1\right)$$

$$z_t^{Y} = \rho_z z_{t-1}^{z} + \epsilon_t^{z}$$

$$g_t^{A} = \left(\frac{A_t}{A_{t-1}}\right)$$

$$g_t^{A} = \left(1-\rho_{gA}\right) g^{A} + \rho_{gA} g_{t-1}^{A} + \epsilon_t^{gA}$$

Growth rates

$$\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}} = \frac{Y_t}{Y_{t-1}}\left(\frac{A_{t-1}}{A_t}\right)^{\frac{1}{1-\alpha_k-\alpha_{kg}}}$$

$$\left(\frac{Y_t}{Y_{t-1}}\right)^{4} = \left((g_t^{A})^{\frac{1}{1-\alpha_k-\alpha_{kg}}}\frac{\tilde{Y}_t}{\tilde{Y}_{t-1}}\right)^{4} - 1$$

X. The Steady State

The steady state is given by the following equations:

$$z = 1$$

$$g_t^{A} = g^{A}$$

$$\frac{I^{Gr}}{Y} = k_{Igr/Y}$$

$$\frac{I^{Gn}}{Y} = k_{Ign/Y}$$

$$\frac{C^g}{Y} = k_{Cg/Y}$$

$$\frac{T^*(s)}{Y} = k_{rem/Y}$$

$$\frac{(B^* - B^{G*})}{Y} = k_{NFA/Y}$$

$$\frac{zB^{G*}}{Y} = k_{bg/y}$$

$$\frac{T^{GW}}{Y} = k_{trW/Y}$$

$$\bar{R}^*(s) = \bar{R}^*$$

$$R^* = \bar{R}^*(s)$$

$$\frac{\left(p^H \tilde{X} - z\tilde{C}_F - z\tilde{I}_F\right)}{Y} = \left(1 - R^*\left(\frac{1}{g^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}}\right)\left(\frac{z\tilde{B}^*}{Y} - \frac{z\tilde{B}^{G*}}{Y}\right) - \frac{z\tilde{T}^*(s)}{Y}$$

$$\frac{\tilde{I}^Y}{\tilde{K}^Y} = \delta(s)$$

$$\frac{\tilde{K}^{gr}}{Y} = \frac{\tilde{I}^{gr}}{Y}\delta_{gr}^{-1}$$

$$\frac{\tilde{K}^{gn}}{Y} = \frac{\tilde{I}^{gn}}{Y}\delta_{gn}^{-1}(s)$$

$$p^H = 1$$

$$p^I = 1$$

$$p^{Gr} = \frac{p^I}{a^{Gr}}$$

$$\frac{K^g}{Y} = \frac{K^{gr}}{Y} + \frac{K^{gn}}{Y}$$

$$Q = p^I\left((1 - \tau^\pi) + \varsigma\right)$$

$$\frac{\tilde{Y}^H}{\tilde{K}^Y} = \frac{\dfrac{Q}{\beta (g^A)^{\frac{-1}{1-\alpha_k-\alpha_g}}} - Q\left((1-\delta) + \varsigma\sigma\right)}{(1-\tau^\pi)\alpha_K p^H (g^A)^{\frac{1}{1-\alpha_k-\alpha_g}}}$$

$$\frac{\tilde{I}^Y}{\tilde{K}^Y} = \frac{(1-\tau^{\pi})\alpha_K\left(1-(1-\delta)\left(\frac{1}{g^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}}\right)p^H(g^A)^{\frac{1}{1-\alpha_k-\alpha_g}}}{\frac{Q}{\beta(g^A)^{\frac{-1}{1-\alpha_k-\alpha_g}}} - Q\big((1-\delta)+\varsigma\sigma\big)}$$

$$\tilde{Y}^H = \left((g^A)^{-\frac{\alpha_k+\alpha_g}{1-\alpha_k-\alpha_g}} z^Y \theta(s)\left(\frac{\tilde{K}^G}{Y^H}\right)^{\alpha_g}\left(\frac{\tilde{K}^Y}{Y^H}\right)^{\alpha_K}(N^d)^{1-\alpha_K}\right)^{\frac{1}{1-\alpha_g-\alpha_k}}$$

$$\tilde{W} = \frac{(1-\tau^{\pi})(1-\alpha_K)}{(1+\varsigma)}p^H\frac{\tilde{Y}^H}{N^d}$$

$$\tilde{C}^W = \frac{(1-\tau^l)\tilde{W}\omega N^d + T^{GW}}{(1+\tau^C)\omega}$$

$$\frac{I^Y}{Y} = \delta\frac{K^Y}{Y}$$

$$Y = Y^H$$

$$\frac{C}{Y} = 1 - \frac{p^I I}{Y} - \frac{p^H C^g}{Y} - \frac{Xn}{Y}$$

$$\tilde{C}_H = (1-a)\tilde{C}$$

$$\tilde{C}_F = a\tilde{C}$$

$$\tilde{C}_t^K = \frac{\tilde{C}_t - \omega\tilde{C}^W}{(1-\omega)}$$

$$\tilde{I}_H = (1-a_I)\tilde{I}$$

$$\tilde{I}_F = a_I\tilde{I}$$

$$\tilde{X} = Nx + \big(\tilde{C}_F + \tilde{I}_F\big)$$

$$Y^* = X$$

$$B^{G*} = \left(\frac{zB^{G*}}{GDP}\right) * GDP$$

$$\tilde{T}^G = p^H\tilde{C}^g + \tilde{T}^{GW} + p^I\tilde{I}^{Gn} + p^{Gr}\tilde{I}^{Gr} + zR^*\tilde{B}^{G*}\left(\frac{1}{g^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}} - z\tilde{B}^{G*} - \tau^C\tilde{C} - \tau^l\tilde{W}N - \tau^{\pi}\tilde{\Pi} - zT^{G*}$$

We solve the steady state numerically by finding the solution for (σ, N_m^e and N^d) in the following set of equations:

$$\tilde{Y}^H - \left(\tilde{C}_H + \tilde{I}_H + \tilde{C}^g + \tilde{X}\right) \approx 0$$

$$\frac{1}{\eta}\left((1-\omega)\frac{\tilde{\Delta}^K}{(N^d)^\xi} + \omega\frac{\tilde{\Delta}^W}{(N^d)^\xi}\right)\widetilde{W}(1-\tau^l) - \frac{\epsilon_w}{(1-\epsilon_w)} \approx 0$$

$$\widetilde{W}N + p^I\tilde{I}^Y - \sigma\left(Q\widetilde{K}^Y\left(\frac{1}{g^A}\right)^{\frac{1}{1-\alpha_k-\alpha_g}}\right) \approx 0$$

Annex II: Technical Description of the Toolbox

We solve our model using a perturbation method for regime-switching rational expectations models as in Maih (2015). This solution approach has several advantages. First, agents are aware of the climate risk and the probability of been impacted by an adverse climate event and form expectations accordingly. Second, agents are aware that this climate events recurring.

Following Maih (2015), the model outlined above can be cast in a general Markov Switching DSGE framework

$$\mathrm{E}_t \sum_{s_{t+1}=1}^{h} P_{s_t,s_{t+1}} f_{s_t}(x_{t+1}(s_{t+1}), x_t(s_t), x_t(s_t), x_{t-1}, \epsilon_t) = 0$$

where f_{s_t} is a $n \times 1$ vector of the nonlinear first order conditions, $s_t = 1{,}2$ is the regime at time t, x_t is a vector of variables in the system and $\epsilon_t$is a vector of shocks. $P_{s_t,s_{t+1}}$ is the transition probability from going from one regime s_t in the current period to s_{t+1}in the next period.

Maih (2015) develops a perturbation algorithm to find a solution of the form

$$x_t(s_t) = T^{s_t}(z_t)$$

where z_t is a vector of state variables find $T^{s_t}(z_t)$. This type of solution makes it clear that the framework allows the model economy to be in different regimes at different points in time, with each regime being governed by certain rules specific to the regime.

For the purpose of this paper, we use a first-order perturbation. That is, we approximate the solution with a first-order perturbation:

$$T^{s_t}(z_t) \approx T^{s_t}(\bar{z}) + T_z^{s_t}(z_t - \bar{z})$$

where z_t is vector of state variables $z_t \equiv (x'_{t-1} \; \zeta \; \epsilon_t)'$ where ζ is a perturbation parameters, $\bar{z}$ is the ergodic mean of the state vector. In particular, $\bar{z}$ is the solution of system of equations $f_{s_t}(x_t(s_t), x_t(s_t), x_t(s_t), x_t, 0; \bar{\theta}) = 0$ where $\bar{\theta}$ is the ergodic mean of the future switching parameters. Note that approximation can be done around the steady-state of each regime or around an arbitrary steady-state. We use the ergodic mean because this approach allows us to calibrate the steady state of the economy and match some long-run ratios of the modeled economies.

PUBLICATIONS

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