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Optimal Monetary and Macroprudential Policies under Fire-Sale Externalities

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ABSTRACT: I provide an integrated analysis of monetary and macroprudential policies in a model economy featuring a financial friction and a nominal wage rigidity. In this set-up, the monetary authority faces a trade-off between macroeconomic and financial stability: While expansionary counter-cyclical monetary policy prevents involuntary unemployment, it also amplifies an inefficient reallocation of capital across sectors. The main contribution of the analysis is threefold: First it highlights a novel channel through which monetary policy can impact financial stability. Second, it shows that, by itself, monetary policy can significantly mitigate the wedge between the constrained efficient and the competitive allocation. Third, regardless of the availability of macroprudential tools, stabilizing demand is usually not optimal for monetary policy.

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I. INTRODUCTION

The 2008 financial crisis has led to the wide agreement that new tools are needed to mitigate excessive risk taking in the financial sector. This perception has fostered the design and implementation of macroprudential tools around the world.[1] While macroprudential policies focus on the mitigation of systemic risks in the financial sector, ongoing debates in academia and politics have highlighted critical spillovers between financial stability and monetary policy objectives which have traditionally focused on macroeconomic stability. Empirical evidence suggests that monetary policy can affect risk taking in the financial sector while tight macroprudential regulations may curb economic activity.[2] This raises important questions about the interaction of macroprudential and monetary policy, including: (1) How does optimal macroprudential policy depend on the monetary policy regime? (2) Which objectives should monetary policy pursue? (3) And how does optimal monetary policy depend on the availability of macroprudential tools?

In this paper, I address these questions in a model economy with a rationale for macroprudential policy due to a pecuniary externality (based on Kara and Ozsoy (2019)) and with a non-neutral role for monetary policy due to a downward nominal wage rigidity (as in Schmitt-Grohé and Uribe (2016)). The combination of a financial friction and a nominal rigidity induces a trade-off for monetary authorities between macroeconomic and financial stability: While expansionary monetary policy can prevent involuntary unemployment in crises states, it also reduces asset prices and thereby amplifies an inefficient capital reallocation across sectors. The theoretical analysis yields three main findings. First, the article highlights a novel channel through which monetary policy impacts financial stability. Second, I show that, by itself, monetary policy can significantly mitigate the wedge between the constrained efficient and the decentralized allocation. Optimal macroprudential interventions are thus critically shaped by the monetary policy regime. Third, I find that regardless of the availability of macroprudential tools, fully stabilizing demand is usually not optimal for monetary policy.

To provide an integrated analysis of macroprudential and monetary policy, I develop a three-period model economy populated by four different types of agents: Households, banks operating in the banking sector (BS), entrepreneurs operating in the traditional sector (TS) and a central bank (CB). Households consume one tradable consumption good, save in bank de-

[1] See, for example, Cerutti, Claessens, and Laeven (2016) for a documentation of the use of macroprudential tools for a large set of emerging and advanced economies.

[2] Empirical evidence on the effects of monetary policy on risk taking is presented by Dell'Ariccia, Laeven, and Suarez (2017), Gabriel Jimenez (2014), among others. Richter, Schularick, and Shim (2019) estimate the costs of macroprudential policy.

posits and supply labor inelastically to entrepreneurs who combine labor and capital to produce output in the final period. Banks use deposits and their own capital to invest in liquid assets and productive bank projects which yield a save return in the final period but are subject to stochastic restructuring costs in the interim period.

Uncertainty takes the form of a liquidity shock and is fully revealed in period one: In normal times, bank projects do not require any additional funds. However, in case of a liquidity shock, bank projects require additional funds and banks must use their liquid assets or sell a share of their projects to entrepreneurs to finance the restructuring costs. The sell-off of bank projects takes the form of fire-sales as entrepreneurs are less productive in managing bank projects and have limited funds available. If entrepreneurs have to absorb more projects from the BS, this crowds out resources from the TS which further reduces the fire-sale price due to the concave production function of the TS. Additionally, fire-sales cause involuntary unemployment in the TS because a lower capital stock also reduces labor demand. Thereby, the model provides a mechanism how a shock originating in the financial sector can spread to the real economy, causing large declines in domestic production, employment, and asset prices.

The interaction of the financial friction and the nominal wage rigidity confronts the CB with the following monetary policy trade-off: By committing to expansionary counter-cyclical monetary policy in case of a liquidity shock, i.e., a large price level, the CB can devalue real wages and increase employment in the TS. At the same time, however, a larger level of employment increases the productivity of capital in the TS which lowers entrepreneurs' demand for bank projects and thereby reduces the fire-sale price. The nominal rigidity thus induces a relative price change which alleviates the financial friction as the decline in employment mitigates the decline in asset prices in crises states.[3]

Two features of monetary policy in this model contrast with the existing literature: First, monetary policy is time consistent regardless of the availability of macroprudential policies.[4] Second, there is no trade-off between ex-post financial and macroeconomic stability. The reason for this result is that all investment decisions are made in the initial period. These investment decisions predetermine the resource allocation by entrepreneurs which also defines financial stability in the current framework. Therefore, ex-post monetary policy actions have no effect on financial stability and can solely focus on macroeconomic stability, i.e., full employment, which generally requires an increase in the price level. A higher price level makes risky in-

[3]Without the nominal and financial friction acting in concert, monetary policy optimally focuses on employment and output stability.

[4]Seminal contributions on time inconsistency and monetary policy include Kydland and Prescott (1977), Calvo (1978), and Barro and Gordon (1983).

vestment more costly for banks which discourages them from taking excessive risks ex-ante. This feature makes monetary policy time consistent and removes the trade-off between financial and ex-post macroeconomic stability.

Crucially, monetary policy by itself can significantly mitigate the wedge between the constrained efficient and the competitive allocation. This wedge arises due to the presence of a *pecuniary externality*:[5] Banks do not consider the incremental effect of their investment decisions on the asset price. As a result, private banks overinvest in bank projects and underinvest in liquid assets. To address these inefficiencies, the CB needs to use additional macroprudential policies targeting private banks‘ holdings of liquid assets and investment projects.[6] Importantly, however, the wedge between the constrained efficient and the competitive allocation decreases as the CB commits to more expansionary counter-cyclical monetary policy. A higher price level increases employment and thereby the productivity of capital in the TS. This lowers the equilibrium fire-sale price and increases the costs of fire-sales for private banks. Banks anticipate their exposure towards more severe fire-sale losses and invest less in bank projects and accumulate more liquid assets to insure against these adverse events. Therefore, the need for macroprudential interventions surprisingly increases as monetary policy focuses more on financial stability regulation and is largest under a constant price level regime.

While macroprudential policies are part of the optimal policy mix, they are generally not sufficient to fully address financial frictions in the current set-up. If the costs of the financial friction are sufficiently large, i.e., the return difference between the BS and the TS is sufficiently large, optimal monetary policy deviates from fully stabilizing employment, even if the central bank has access to the full set of macroprudential instruments. In the absence of macroprudential tools, the CB uses monetary policy to "lean against the wind". This implies that the CB commits to expansionary counter-cyclical monetary policy to mitigate excessive risk taking in the BS ex-ante. Thereby, the monetary authority can minimize the wedge between the decentralized and the constrained efficient allocation.

(a) Relation to the Literature. This paper is related to several strands of the literature. First, it relates to a literature which evaluates monetary policy in economies featuring finan-

[5]Inefficiencies due to the presence of pecuniary externalities have been extensively studied in the literature. Important contributions include, but are not limited to, Lorenzoni (2008), Mendoza (2010), Korinek (2018) and Dávila and Korinek (2018).

[6]Monetary policy, by itself, is generally insufficient to restore constrained efficiency.

cial frictions.[7] In contrast to this literature, I highlight the effect of monetary policy on the wedge between the constrained efficient and the competitive allocation and derive analytical results on optimal policy. Similar to the current study, Ottonello (2021) characterized a monetary policy trade-off between macroeconomic and financial stability and shows that deviations from the full employment regime can be optimal in certain states. In this study, I highlight another mechanism through which monetary policy can affect financial stability in a closed economy framework and underline the effect of monetary policy on the inefficiencies which may arise in the unregulated allocation. The finding that the private sector's rewards of insuring against crises periods depend on expected monetary policy further relates the current study to Caballero and Krishnamurthy (2003). Importantly, however, in their framework, monetary policy loses its potency in times of crises because of a sharp external resource constraint. Here, monetary policy is useful exactly in crises times when fire-sales crowd out productive capital in the TS and thereby lead to a reduction of employment.[8]

A few studies have provided an integrated analysis of macroprudential and monetary policies. [9] In contrast to the current analysis, these studies to not characterize optimal policy but focus on the evaluation of simple rules. A characterization of the jointly optimal policy mix was offered by Collard and others (2017) in a New Keynesian model. In their framework, however, monetary policy by itself can not deter excessive risk taking by banks. As a result, monetary policy optimally focuses on macroeconomic stability and optimal regulatory policies do not depend on the monetary policy regime. In this study, I highlight a mechanism through which monetary policy can affect risk-taking in the private sector and highlight the implications for monetary and macroprudential policy.

[7]Important contributions include Woodford (2012), Stein (2012), Borio (2014), Svensson (1985), Caballero and Simsek (2019), among others. Adrian and Liang (2018) provide a literature review. Lawrence Christiano (2004), Lane and Michael B. Devereux (2005), Gertler, Gilchrist, and Natalucci (2007), Fornaro (2015) study small open economies; Curdia (2008), Braggion, Christiano, and Roldos (2009) and Coulibaly (2019) focus on sudden stop prone economies and Chang and Velasco (2017) on unconventional monetary policy. Another literature has studied foreign exchange interventions with imperfect capital markets (see e.g., Gabaix and Maggiori (2015), Cavallino (2019), Fanelli and Straub (2018), Itskhoki and Mukhin (2022), Amador and others (2020)).

[8]In Caballero and Krishnamurthy (2003), monetary authorities also face a serious time-inconsistency issue while this is not the case in the current framework.

[9]Otrok and others (2012) studied monetary and macroprudential policies in a stylized model. Loisel (2014) provide a summary of the main findings of several quantitative contributions. More recent normative perspectives are provided by, e.g., Gersbach, Hahn, and Liu (2017), Adrian and Liang (2018), Van der Ghote (2021). Kiley and Sim (2017) characterized optimal monetary or macroprudential policy but not the jointly optimal policy mix.

Third, the paper relates to a literature which studies macroprudential policies in the presence of liquidity risk.[10] These studies highlighted the strong precautionary motive to invest in liquid assets and derived conclusions regarding optimal liquidity policies to smooth credit access in periods of financial distress. The current framework builds on the model developed by Kara and Ozsoy (2019) but expands on the existing studies by considering a nominal model with a non-trivial role for monetary policy due to a nominal rigidity. The link between financial frictions, relative prices, leverage and aggregate outcomes further relates the present study to the seminal work by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) as well as a recent extension .

Finally, the paper is also related to literature which highlights the importance of nominal rigidities in exacerbating downturns during financial crises.[11] Schmitt-Grohé and Uribe (2016) documented a strong nominal wage rigidity in emerging markets and show that the combination of rigid nominal wages, free capital mobility and a fixed exchange rate regime creates an aggregate demand externality.[12] In contrast to the current study, however, monetary authorities can implement the first-best allocation via large exchange rate depreciations and do not face a policy trade-off between macroeconomic and financial stability. Finally, we share with Boz and others (2020) the approach of studying multiple policy instruments in a stylized three period model.

The rest of the article proceeds as follows. Section 2 lays out the model framework. In section 3 and 4, I characterize the unregulated competitive equilibrium and the constrained-efficient allocation, respectively. The main insights of the analysis are illustrated in section 5 and 6 which characterize the inefficiencies, their dependence on monetary policy and their implications for optimal policy. Section 7 provides a numerical example and section 8 concludes.

[10] Important empirical contributions include Obstfeld, Shambaugh, and Taylor (2010) and Calvo, Izquierdo, and Loo-Kung (2012). Aizenman (2011), Jeanne (2016), Acharya and Krishnamurthy (2019), Kara and Ozsoy (2019), Bocola and Lorenzoni (2020), Lutz and Pichler (2020) and Jeanne and Sandri (2020a) have studied static models while dynamic models have been analyzed by Bianchi, Hatchondo, and Martinez (2018), Jeanne and Sandri (2020b) and Benigno, Fornaro, and Wolf (2021) .

[11] See, for example, Eichengreen and Sachs (1985), Bernanke and Carey (1996) and Schmitt-Grohé and Uribe (2016), among others.

[12] Similarly, Farhi and Werning (2016), Korinek and Simsek (2016) and Bianchi and Sosa-Padilla (2020) have analyzed frameworks featuring aggregate demand externalities.

II. The Model

Consider a three-period model economy with $t = 0, 1$ and 2 and four types of agents: Households, banks operating in the banking sector, entrepreneurs operating in the traditional sector and a central bank that sets the price level. Households consume a single tradable consumption good and there are two states of the world, $\theta \in \{g, b\}$, which I refer to as the good and the bad state, respectively.

A. Households

Households are risk neutral and like consumption in period $t = 0, 2$, such that

$$U = c_0 + \delta \mathbb{E} c_2(\theta), \tag{1}$$

where δ denotes the discount factor. Period 1 can be interpreted as an interim stage where agents cannot consume but the state of the world is revealed and agents can trade in asset markets.[13] In period 0, households receive an initial endowment x denominated in domestic consumption goods. Note that in the following, I use capital letters to denote nominal variables and lowercase letters for real variables. The period-0 nominal value of household's initial endowment is thus denoted by X. Households can use their endowment to save in bank deposits, $\tilde{D}$, with a nominal risky return $R^d(\theta)$ in state θ or to finance consumption, c_0. The period-0 budget constraint in nominal terms reads

$$P_0 c_0 \quad \leq \quad X - \tilde{D}, \tag{2}$$

where P_t denotes the domestic price level in period t. In the final period, households are endowed with $h = 1$ units of labor and spend all their savings $\tilde{D}R^d(\theta)$, wage income $W(\theta)\tilde{h}(\theta)$ and profits Π_j from sector $j \in \{BS, TS\}$, to finance period-2 consumption. The period-2 budget constraints hence reads

$$P_2(\theta)c_2(\theta) \quad \leq \quad R^d(\theta)\tilde{D} + W(\theta)\tilde{h}(\theta) + \Pi_2^{BS}(\theta) + \Pi_2^{TS}(\theta). \tag{3}$$

In the following, I set the domestic price level in period $t = 0$ to one, i.e., $P_0 = 2$, such that all nominal and real variables coincide in period 0.[14]

[13] See, for example, Stein (2012) for a similar utility specification.

[14] This assumption simplifies notation but is without loss of generality.

In period 0, households choose consumption $\{c_0, c_2(\theta)\}$, savings $\tilde{D}$ and labor supply $\tilde{h}$ to maximize utility (1) subject to the budget constraints (2) and (3) taking prices, profits and endowments $\{X, R^d(\theta), W(\theta), \Pi_j(\theta), P_t(\theta)\}$ as given which yields the following first-order condition

$$\mathbb{E}\left[\frac{R^d(\theta)}{P_2(\theta)}\right] \leq \frac{1}{\delta}. \tag{4}$$

Condition (4) holds as strict equality if $\tilde{D} > 0$. Accordingly, households save in bank deposits if their net expected return compensates them for postponing consumption. Households always want to supply their total labor endowment, $\tilde{h} = 1$, as they have no disutility from working.

B. The Traditional Sector

Entrepreneurs in the TS are endowed with $\bar{l} = 1$ units of land and e units of the consumption good in period 1 and have access to the following two investment opportunities:[15] First, they can transform the endowment into capital goods k at no costs and use the capital good together with land and labor from the household sector to produce output subject to the technology,

$$y(k(\theta), h(\theta), l) = Ak(\theta)^{\alpha} h(\theta)^{\beta} l^{1-\alpha-\beta},$$

where $y(.)$ denotes period-2 output of domestic consumption goods and A denotes the productivity level.[16] [17] Second, entrepreneurs can use their endowment to buy existing projects from the BS. If projects are managed by banks, they yield a linear return r^H. Entrepreneurs, however, are less efficient in managing bank projects and only receive a linear return r^L, where $r^H > r^L > 1$.[18] The relative period-1 price of a bank project delivering r^L consumption goods in period 2 in terms of the consumption good is denoted by $q(\theta)$.[19] Entrepreneurs choose la-

[15]For simplicity I assume that the endowment of the entrepreneurs is exogenous. In Appendix B I show that the findings are robust to environments with endogenous e. Importantly, however, period-1 resources must be fixed and limited in the interim period (see e.g., Stein (2012) for a similar assumption).

[16]In Appendix B, I provide a discussion of the implications of a standard TFP shock.

[17]Entrepreneurs can be interpreted as firms without access to the financial sector but with a limited amount of internal funds.

[18]The assumption that the TS absorbs projects from the BS in case of financial distress but is less efficient in managing them is also present in Lorenzoni (2008) and Kara and Ozsoy (2019), among others. This assumption is essential for the presence of a pecuniary externality in the model.

[19]The price of bank projects at t = 2 will be 0 because all goods fully depreciate at this point.

bor and capital given the real prices $\{w(\theta), q(\theta)\}$ to maximize real profits given by

$$\pi^{TS}(\theta) = Ak(\theta)^{\alpha} h(\theta)^{\beta} l^{1-\alpha-\beta} - w(\theta)h(\theta) + (e - k(\theta)) \frac{r^L}{q(\theta)},$$

where $w(\theta) = \frac{W}{P_2(\theta)}$. Total profits are given by period-2 output minus the labor costs plus the return from buying projects from the BS. The first order conditions read

$$w(\theta) = A\beta h(\theta)^{\beta-1} k(\theta)^{\alpha} l^{1-\alpha-\beta}, \quad (5)$$

$$\frac{r^L}{q(\theta)} \leq A\alpha h(\theta)^{\beta} k(\theta)^{\alpha-1} l^{1-\alpha-\beta}, \quad (6)$$

which equalize the marginal productivity of labor and capital with their real marginal costs $w(\theta)$ and $\frac{r^L}{q(\theta)}$, respectively. Condition (6) holds with equality if entrepreneurs buy a positive amount of bank projects, i.e., $k < e$. Entrepreneurs always use their total endowment of land such that $l = \bar{l} = 1$. Throughout the following analysis, I assume the following:

Assumption 1 (Scarce Resources): $A\alpha e^{\alpha-1} h^{\beta} \geq 1$.

Assumption 1 implies that the marginal productivity of capital is greater than or equal 1 even if the TS transforms its total endowment into capital. As will become clear below, this assumption ensures that the period-1 price of bank-projects is always below the fundamental value.[20]

(a) Nominal Wage Rigidity. The central friction in the model, which introduces a rationale for monetary policy, is a downward nominal wage rigidity.[21] Specifically, I impose

$$W(\theta) \geq \bar{W} = A\beta e^{\alpha}, \quad (7)$$

where $\bar{W}$ denotes the optimal full employment wage ($h = 1$) when capital is equal to the total endowment e and the CB does not adjust the price level, i.e., $P_0 = P_2 = 1$. Wages are thus constant and independent of the period-2 state: In the aggregate good state, the TS does not have to absorb any projects from the BS, wages are at their market clearing level and labor demand is equal to 1. In the aggregate bad state, entrepreneurs use parts of their endowment e to buy fire-sold projects from the BS. This reduces the marginal productivity of labor and,

[20]This is a sufficient but not a necessary condition.

[21]The importance of nominal wage rigidities in exacerbating downturns during financial crises was highlighted by Eichengreen and Sachs (1985), Bernanke and Carey (1996) and Schmitt-Grohé and Uribe (2016), among others. A growing literature has further emphasized the importance of wage rigidities as key transmission channel for monetary policy (see e.g. Lawrence Christiano (2004); Olivei and Tenreyro (2007)).

because nominal wages are fixed, labor demand declines. Using condition (5) together with (7) yields the following expression for labor demand

$$h(\theta) = \left[\frac{k(\theta)^{\alpha} P_2(\theta)}{e^{\alpha}}\right]^{\frac{1}{1-\beta}}. \tag{8}$$

Note that in the case where $k(\theta) < e$, labor demand depends positively on the period-2 price level and the available capital stock in the TS. The CB can increase employment by increasing the price level which reduces the real costs of labor. Finally, note that the price level which implements full employment is given by $P_2(\theta) = (\frac{e}{k(\theta)})^{\alpha}$. In the following, I refer to this policy as the full employment regime.

(b) The Period-1 Asset Price. Condition (6) shows that entrepreneurs are only willing to purchase shares in bank projects if their return compensates them for the lower capital stock. This pins down real period-1 price of bank projects,

$$q(\theta) = \frac{r^L}{A\alpha h(\theta)^{\beta} k(\theta)^{\alpha-1}}.$$

Note that as the capital stock in the TS falls this increases the productivity of capital and consequently the asset price must adjust downward such that the returns from both investment opportunities are still equalized. Further, because capital and labor provide complementary inputs, the asset price is also a decreasing function of employment.

When the CB commits to expansionary counter-cyclical monetary policy to stabilize employment, this, hence, also raises the return on capital and thereby reduces the equilibrium asset price $q(\theta)$. While expansionary monetary policy is thus a powerful tool to stabilize employment and output in the TS, it also lowers the asset price and thereby increases the share of projects which are redistributed from the BS to the less efficient TS.

C. The Banking Sector

Banks are endowed with b units of the consumption goods (equity) worth B units of domestic currency in the initial period. Further, banks can raise D units of funds by issuing deposits to the household sector such that the total funds of the BS in the initial period are $D+B$. Banks can use their funds to invest in bank projects I and liquid assets L. Bank projects that are managed by the bank until the last period yield a linear return r^H. Bank projects are risky as they are subject to stochastic restructuring costs. In the bad state, bank projects require additional funds c per unit of investment in the interim period. If the additional funds c are not invested, projects are scrapped and yield zero return. More details are provided below.[22] Bank's investment technology satisfies the following condition:

Assumption 2 (Technology): $\frac{1}{\delta} + pc < r^H < \frac{c(1-p)}{c-(1+c)p}$.

The first inequality of the technology assumption implies that the net expected return on bank projects is positive. The second inequality ensures that scrapping is never optimal. Intuitively, scrapping is never optimal if $q(b) > c$, i.e., if the gain of selling bank projects to the TS exceeds the period-1 liquidity needs. I provide a derivation of this claim in the proof of lemma 2 in Appendix C.

Liquid assets L yield a zero net return but can be liquidated at any point in time.[23] Further, banks face a real cost of operating the bank $\Phi(I+L)$. The operational costs are increasing in the balance sheet size and convex to ensure that the balance sheet size is well-defined, i.e., $\Phi'(.) > 0, \Phi''(.) > 0$. The period-0 budget constraint of the bank is given by

$$I + L = D + B. \tag{9}$$

In period 1, the aggregate state of the world is revealed. With probability $1-p$, the aggregate good state is realized, and bank projects do not require any additional funds. With probability p, however, a liquidity shock realizes, and banks projects require additional funds. In this case, banks use their liquid assets and sell a fraction $1-\gamma(b)$ of their projects to the TS:

$$cI \leq L + (1-\gamma(\theta))Iq(\theta). \tag{10}$$

In the final period, returns in the TS and in the BS are realized, all debts are repaid, and all profits are distributed to the household sector who spends all remaining funds on consumption.

[22] Bank projects can also be interpreted as banks' lending to entrepreneurs which operate in the financial sector and have access to bank financing. For similar interpretations see e.g., Lorenzoni (2008) and Gersbach, Hahn, and Liu (2017).

[23] Banks cannot rent out their liquidity holdings to the TS in the good state. This is not an essential assumption but simplifies the analysis as the net real return of liquid assets is pinned down at zero.

D. The Central Bank

The CB implements monetary policy by setting the final period price level $P_2(\theta)$.[24] In the following, I focus on a CB that credibly commits to a policy in $t = 0$ before decisions are made by households and the BS. I later show that this policy is time-consistent, i.e., the CB has indeed never an incentive to deviate from its initial choice once uncertainty is revealed in the interim period. Finally, note that the non-crisis policy is a trivial no-action policy because there are no frictions in the model.

III. The Unregulated Competitive Equilibrium

This section analyses the unregulated competitive equilibrium where agents maximize their individual objectives but are unaware of their incremental effect on aggregate prices and outcomes. In the following, I use * to denote the equilibrium choices of private agents.

(a) Asset Markets in Period 1. I solve the model backwards and first characterize the unregulated equilibrium of the asset market in period 1 given bank's initial choices $\{I^*, L^*, D^*\}$ and the CB's choice of $P_2^*(b)$. Recall that in the good state banks require no additional funds and no projects are sold to the TS. In this case, entrepreneurs use their total endowment to invest in capital. Recall further, that if capital is equal to the endowment e, labor demand is equal to labor supply such that the economy operates at full capacity, $y^*(g) = Ae^\alpha$, independent of banks' initial choices. In case of a liquidity shock, however, banks need to finance their liquidity needs cI^* using their liquid assets L^* or by fire-selling shares of bank projects to the less efficient TS. Given assumption 1 together with $r^H > r^L$, the return of bank projects r^H strictly exceeds the return from selling projects to the TS, i.e., $q(b) < r^H$, which implies that constraint (10) holds with strict equality in the bad state. Accordingly, the share of projects that remains in the BS is given by

$$\gamma^*(b) = 1 - \frac{cI^* - L^*}{q^*(b)I^*}. \tag{11}$$

[24] In the current framework, this is equivalent to setting the level of employment $h(\theta)$ directly. In an open economy setting, where the foreign price level is normalized to one, this is also equivalent to a CB which sets the nominal exchange rate.

Entrepreneurs operating in the TS absorb these projects such that the remaining capital stock is given by $k^*(b) = e - cI^* + L^*$. Note that this also implies a decline in employment as capital and labor provide complementary inputs in the TS and nominal wages cannot adjust due to the nominal wage rigidity. Using condition (8), equilibrium labor demand in the aggregate bad state is given by

$$h^*(b) = \left[\frac{(e - cI^* + L^*)^\alpha P_2^*(b)}{e^\alpha}\right]^{\frac{1}{1-\beta}} .$$

Combining this condition with the entrepreneur's optimality condition for capital (6), yields the following expression for the period-1 asset price

$$q^*(b) \quad = \quad \frac{r^L}{\alpha A}\left[\frac{e^\alpha}{P_2^*(b)}\right]^{\frac{\beta}{1-\beta}} \left[e - (cI^* - L^*)\right]^{\frac{1-\alpha-\beta}{1-\beta}} . \tag{12}$$

Clearly, the equilibrium period-1 asset price depends on the liquidity shortage in the BS $(cI^* - L^*)$. In particular, there are two counteracting effects present in the model. First, fire-sales crowd out productive capital investment in the TS and thereby increase the return on capital and lower the equilibrium asset price $q^*(b)$. Additionally, a lower capital stock reduces labor demand $h^*(b)$ which lowers the real return on capital and thereby increases the equilibrium asset price in period-1. Hence, by lowering the real return on capital, the nominal rigidity induces a relative price change which mitigates the distortions arising from the financial friction.[25] The overall effect of a higher liquidity shortage in the BS on the equilibrium asset price $q^*(b)$ is strictly negative given the assumption $\alpha + \beta < 1$ (see section 2.2).[26] Private banks, however, are too small to internalize this effect giving rise to a pecuniary externality which is discussed in more detail in sections 4 and 5.

Importantly, note that in the current framework, the fire-sale price also depends on the price level $P_2^*(b)$, i.e., on the monetary policy regime in place. In particular, if the CB commits to expansionary counter-cyclical monetary policy, real wages decline and labor demand increases. As a result, employment increases which raises the return on capital and thereby lowers the asset price. The negative effect of a larger price level on the asset price is also evident from condition (12). Bank's incentive to insure against fire-sales, and hence bank's initial investment decisions, may thus be critically shaped by the monetary policy regime. This potential effect of monetary policy on initial investment decisions in the BS is, I believe, a very

[25] This finding is in line with Otrok and others (2012).

[26] By contrast, if $\alpha + \beta > 1$, i.e., if there are increasing returns to scale in the TS, the second effect dominates and the fire-sale price increases with the liquidity shortage in the BS. Finally, the fire-sale price is independent of $cI^* - L^*$ if $\alpha + \beta = 1$.

interesting new insight of the current framework and is discussed in detail in section 5. Finally note that, the equilibrium fire-sale price increases with the TS's endowment e and the productivity level r^L.

(b) The Bank's Period-0 Maximization Problem. I now characterize bank's decision problem in the initial period. Bank's period-0 expected real profits are given by

$$\begin{aligned}\mathbb{E}_0\pi^B &= r^H I + L - \frac{1}{\delta}(I+L-B) - \Phi(I+L) \\ &\quad - p\left\{cI + \mathbb{I}_{cI>L} I \left(r^H - q(b)\right)(1-\gamma(b))\right\},\end{aligned}$$

where I used the period-0 budget constraint (9) to replace D. In the good state, profits are given by the return on bank projects plus the liquidity holdings minus the borrowing costs and the balance sheet costs. In the bad state, banks additionally incur the restructuring costs and the fire-sale losses if liquid assets are not enough to finance the period-1 liquidity needs. In the following, I assume that $cI > L$ and later characterize the condition under which this condition holds (see lemma 1). Banks choose $\{I, L, D\}$ to maximize profits subject to the period-1 optimality condition (11) given the price $q(\theta)$. The first order conditions with respect to I and L are given by

$$\frac{1}{\delta} + pc + \Phi'(I+L) = r^H - pc\left(\frac{r^H}{q(b)} - 1\right), \tag{13}$$

$$\frac{1}{\delta} + \Phi'(I+L) \geq 1 + p\left(\frac{r^H}{q(b)} - 1\right). \tag{14}$$

Condition (13) equalizes the marginal expected real costs of investing in bank projects with their marginal return given by r^H minus the fire-sale losses in the aggregate bad state. Hence, bank projects become less attractive as the fire-sale price declines. By contrast, the net return of liquid assets increases as the fire-sale price falls because they provide additional resources in states of financial distress. Banks hold a positive amount of liquid assets if their net return compensates them for the costs of holding liquidity $1/\delta + \Phi(I+L)$. In this case, condition (14) has to hold with strict equality.

An unregulated competitive equilibrium can be defined as follows:

Definition: An unregulated competitive equilibrium is a collection of allocations $\{c_0^*, c_2(\theta)^*, I^*, L^*, \tilde{D}^*, D^*, \gamma^*(\theta), k(b)^*, h(\tilde{b})^*, h(b)^*\}$ and prices $\{W(\theta)^*, q^*(b), R^*(\theta)\}$ which satisfy

1. The optimality conditions of households (4), entrepreneurs (5)-(6) and banks (13) - (14),

2. The budget constraints of households (2) and (3) and banks (9),

3. Nominal wage rigidity (7),

4. Bank's period one budget constraint (10),

5. Market clearing: $k^* = e - cI^* + L^*$, $\tilde{h}^* = h^*$ and $\tilde{D}^* = D^*$,

given the exogenous parameters and functions $\{A, \alpha, \beta, \bar{l}, \bar{h}, e, b, x, r^H, r^L, \delta, p, \Phi(.)\}$, and the CB's choice of $P_2^*(\theta)$.

(c) Characteristics of the Competitive Equilibrium. From the perspective of an individual bank, fire-sales are harmful because the fire-sale price is strictly below the return of bank projects, i.e., $q(b) < r^H$. This fire-sale loss arises because of the lower productivity r^L and the limited amount of resources in the interim period. Banks can fully insure against fire-sale risk by accumulating a sufficient amount of liquid assets such that $L^* = cI^*$. In this case, the BS does not have to sell any projects to the TS and the economy always operates at full capacity. Lemma 1, however, shows that this is never optimal and private banks strictly prefer to take on fire-sale risk as long as the endowment in the TS is not too small. In the following, I use $y_k'(.)$ and $y_h'(.)$ to denote the marginal derivative of the production function with respect to capital and labor, respectively. The following result holds:

Lemma 1: Define $\hat{e}$ such that

$$\frac{\left[r^H - (1-p)\right] r^L}{pr^H(1+c)} = y_k'(\hat{e}, 1, 1),$$

Private banks take on fire-sale risk, i.e., $cI^* > L^*$, if and only if $e \geq \hat{e}$. The cut-off value $\hat{e}$ is an increasing function of the level of employment and, hence, of the price level $P_2(b)$.

Intuitively, banks fully insure against fire-sale risk if the marginal loss of fire-sales at $cI^* = L^*$ is sufficiently high. This marginal loss is large if the endowment of the TS is small such that the marginal productivity of capital in the TS is high. The cut-off value increases with the level of employment as capital and labor provide complementary inputs in the TS. Finally, the cut-off value depends positively on the productivity loss of the TS $(r^H - r^L)$, the probability of a liquidity crises p and the liquidity needs c.[27] In the following I assume that the endowment in the TS is sufficiently large such that $e > \hat{e}$ and banks never choose to fully insure against fire-sale risk.

I next state the conditions under which banks invest a positive amount of resources in liquid assets. First note that private banks always invest in bank projects given their positive net return (assumption 2). By contrast, banks may decide to not invest in liquid assets if their net return does not compensate them for the costs of holding liquidity given by $\frac{1}{\delta} + \Phi'(I+L)$. The net return of liquid assets is larger the higher the probability of a liquidity shock and the lower the asset price in the aggregate bad state. As highlighted before, the fire sale price increases with the entrepreneurs' endowment e and productivity r^L. Accordingly, private banks may decide to hold no liquid assets if the entrepreneurs' endowment or productivity level r^L is sufficiently large.

The fire-sale price, and thus net return of liquid assets, further depends on the monetary policy regime. In particular, the CB can maximize the real return on capital by committing to the full employment regime. Everything else constant, this implies a lower fire-sale price and thereby increases bank's incentive to invest in liquid assets. The formal conditions under which private banks hold a strictly positive amount of liquid assets as well as their dependence on the monetary policy regime are summarized by lemma 2.

Lemma 2: There exists a real number $\bar{e}$ such that banks hold a positive amount of liquid assets if and only if $e < \bar{e}$. The threshold level $\bar{e}$ increases with the price level $P_2(b)$ and is maximized under the full employment regime.

The formal definition of $\bar{e}$ is provided in the proof of lemma 2 in Appendix C. Lemma 2 already hints to the effect of monetary policy on bank's initial investment decisions which I now discuss in more detail. In particular, assume that the CB commits to expansionary monetary policy in case of a liquidity shock. This policy increases employment and thereby the return of capital in the TS which lowers the expected asset price and makes fire-sales more expensive for the BS. Consequently, banks increase their holdings of liquid assets and invest less in bank projects. Proposition 1summarizes the dependence of private investment decisions on the price level $P_2(b)$.

Proposition 1: If the CB commits to a more expansionary counter-cyclical monetary policy, i.e., a higher price level, private banks hold more liquid assets L, invest less in bank projects I and less capital k is crowded out from the TS.

[27] Note that the fire-sale price is not continuous at the point $cI^* = L^*$ since $r^L < r^H$ and $y'(e,1) \geq 1$.

This result shows that private decisions by banks are critically shaped by monetary policy. By committing to expansionary counter-cyclical monetary policy, the CB can use monetary policy to "lean against the wind" and reduce risk taking in the BS ex-ante. Whether or not this captures a welfare improving policy intervention is, however, unclear at this point and is discussed in more detail in section 6.

IV. THE CONSTRAINED-EFFICIENT ALLOCATION

I next consider a benevolent planner with constrained planning abilities. Specifically, the planner sets the price level $P_2(b)$ in the initial period and instructs banks on their initial choices $\{D, L, I\}$ but let's all remaining markets clear competitively. Compared to private agents, however, the planner anticipates the equilibrium responses of the other agents, i.e., households and entrepreneurs.[28] The aim of the social planner is to maximize social welfare (1) which is given by the discounted sum of household consumption. Total consumption in the initial period is given by the real endowment minus real savings, i.e., $c_0 = x - d$. Period-2 consumption depends on the realization of the aggregate state and is given by

$$c_2(g) \quad = \quad r^H I + y(e,1,1) + L - \Phi(I+L), \tag{15}$$

if the good state prevails and by

$$c_2(b) \quad = \quad \gamma(b) r^H I + (1-\gamma(b))\, r^L I + L - cI - \Phi(I+L) + y\,(k(b), h(b), 1) + cI - L, \tag{16}$$

if the bad state prevails. Social welfare can thus be expressed

$$\begin{aligned} U \quad = \quad & \frac{1}{\delta}(X-D) + r^H I + L - \Phi(I+L) + y(e,1,1) \\ & + p\left\{ y\,(k(b), h(b), 1) - y(e,1,1) - cI - (1-\gamma(b))\left(r^H - r^L\right) I + cI - L \right\}. \end{aligned} \tag{17}$$

In line with the literature, I assume that the planner is subject to the same market constraints as private agents.[29] In particular, the planner respects the wage rigidity and the cash-in-advance constraint of banks in the bad state. The planner further respects the private decisions made by households and the TS. However, unlike banks, the constrained planner understands how the asset price and labor demand are formed in equilibrium and takes the effect of initial in-

[28] This concept of a constrained planner is similar to the concept used in other studies including, e.g., Ottonello (2021).

[29] If markets were complete, there would be no fire-sales and the first best allocation would be established. I derive the first-best allocation in Appendix A.

vestment decisions in the BS on the asset price and labor demand into account. The pricing and labor demand equation respectively are given by

$$
\begin{aligned}
q(\theta) &= \mathscr{Q}\left((e-cI+L), h\left((e-cI+L), P_2(b)\right)\right) \\
&= \begin{cases} \frac{r^L}{y'_k((e-cI+L),h((e-cI+L),P_2(b)),1)} & \text{if } cI > L, \\ r^H & \text{if } cI = L, \end{cases}
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
h(\theta) &= \mathscr{H}\left((e-cI+L), P_2(\theta)\right) \\
&= \begin{cases} \left[\frac{(e-cI+L)^{\alpha} P_2(b)}{e^{\alpha}}\right]^{\frac{1}{1-\beta}} & \text{if } cI > L, \\ 1 & \text{if } cI = L. \end{cases}
\end{aligned}
\tag{19}
$$

The planner chooses the price level $P_2(b)$ and instructs bank's on their choices $\{I, L, D\}$ in the initial period such as to maximize aggregate welfare (17) subject to the bank's period one budget constraint (10), the competitive pricing and labor demand equations (18) and (19), banks' balance sheet constraint (9) and the technological constraint that banks cannot lend their liquid assets to the TS, i.e. $k \leq e$. Using constraint (10) to replace $\gamma(b)$ and banks' balance sheet condition (9) to replace D the social planner's maximization problem can be summarized by the following Lagrangian function

$$
\begin{aligned}
\mathscr{L}^P \max_{\{I,L,P_2(b)\}} &= \frac{1}{\delta}(X - I - L + B) + r^H I + L - \Phi(I+L) + y(e,1,1) \\
&\quad + p\left\{ y(k, \mathscr{H}(.), 1) - y(e,1,1) - cI - \frac{(cI-L)}{\mathscr{Q}(.)}(r^H - r^L) + cI - L \right\} - \zeta(L - cI),
\end{aligned}
$$

where $\zeta > 0$ denotes the Lagrange multiplier of the technological constraint $k \leq e$. Taking the derivatives with respect to $\{I, L, P_2(b)\}$ yields

$$
\begin{aligned}
\frac{1}{\delta} + pc + \Phi'(I+L) &= r^H - pc\left(\frac{r^H}{\mathscr{Q}(.)} - 1\right) + p\frac{(cI-L)}{\mathscr{Q}(.)^2}\frac{\partial \mathscr{Q}(.)}{\partial I}\left(r^H - r^L\right) \\
&\quad - pcy'_h(.)\frac{\partial \mathscr{H}(.)}{\partial k} + cp\zeta,
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\frac{1}{\delta} + \Phi'(I+L) &\geq 1 + p\left(\frac{r^H}{\mathscr{Q}(.)} - 1\right) + p\frac{(cI-L)}{\mathscr{Q}(.)^2}\frac{\partial \mathscr{Q}(.)}{\partial L}\left(r^H - r^L\right) \\
&\quad + py'_h(.)\frac{\partial \mathscr{H}(.)}{\partial k} - p\zeta,
\end{aligned}
\tag{21}
$$

$$
y'_h(.)\frac{\partial \mathscr{H}(.)}{\partial P_2} \geq -\frac{(cI-L)}{\mathscr{Q}(.)^2}\frac{\partial Q(.)}{\partial P_2}\left(r^H - r^L\right).
\tag{22}
$$

Equations (20) and (21) are the planner's optimality conditions for bank projects and liquid assets. Compared to private banks, the planner internalizes two additional effects: First, the planner understands that more investment (less liquidity) crowds out productive capital investment in the TS which has a negative effect on the asset price and thereby increases the share of bank projects held by the less productive TS. Second, the planner understands that a lower level of capital investment in the TS also reduces employment and thereby output in the TS which further increases the costs of bank projects and the gain of liquid assets.

The last equation summarizes the trade-off faced by the constrained planner when setting the price level $P_2(b)$: First, expansionary monetary policy devalues real wages and thereby increases employment and production in the TS. This effect is captured by the left-hand side of condition (22). However, a larger price level also increases the real return on capital in the TS and thereby lowers the fire-sale price. This, in turn, increases the share of projects hold by the less productive TS and crowds out additional real capital investment. These costs of expansionary monetary policy are captured by the right-hand side of condition (22). If condition (22) holds with equality it pins down the equilibrium capital stock given by

$$k^{**}(b) = \frac{e\alpha(r^H - r^L)}{r^L + \alpha(r^H - r^L)},$$

and there is a closed form solution for the contained efficient allocation with an optimal employment level below one. If the left-hand side is strictly greater than the right-hand side, the planner prefers to implement the full employment regime. The full-employment regime is clearly optimal if $r^H = r^L$ because a higher price level is not associated with any costs.[30] If the financial friction is sufficiently costly, optimal monetary policy deviates from fully stabilizing employment in favor of financial stabilization, even with the full set of macroprudential tools available. Finally note that the right-hand side of condition (22) can never exceed the left-hand side as the planner could, in principle, implement an arbitrarily low level of $P_2(b)$. Thereby, she could raise the fire-sale price such that only an ε share of projects has to be redistributed from the BS to the TS. Minimizing the share of redistributed projects becomes more valuable as the efficiency loss $(r^H - r^L)$ increases. However, a low price level also implies a low level of employment and since $y(.)$ satisfies the Inada condition, in particular $y'(x) = \infty$, for $x \to 0$, there must always exist a sufficiently low level of $P_2(b)$ such that the marginal cost of an additional decrease in $P_2(b)$ dominates the associated marginal gain.

Importantly, note that condition (22) can be simplified to

$$k \geq \frac{(e-k)\alpha(r^H - r^L)}{r^L},$$

[30] Note that in this case the competitive allocation also coincides with the constrained efficient allocation.

which is independent of the price level $P_2(b)$, the asset price $q(b)$ and employment $h(b)$. This implies that in period 1, once bank's balance sheet and hence the entrepreneur's capital stock ($k = e - cI + L$) is fixed, monetary policy does not have any effect on the trade-off summarized by condition (22). Accordingly, monetary authorities have no incentives to deviate from their initial choice of $P_2(b)$ once uncertainty is revealed making monetary policy time consistent in the current framework.

Definition: The constrained efficient allocation is a collection of prices $\{W^{**}, q^{**}(b), R^{**}, P_2(b)^{**}\}$ and allocations $\{c_0^{**}, c_2(g)^{**}, c_2(b)^{**}, I^{**}, L^{**}, D^{**}, \tilde{D}^{**}, \gamma(b)^{**}, k(b)^{**}, h(b)^{**}, \tilde{h}(b)^{**}\}$ which satisfy

1. The optimality condition of households (4)
2. The equilibrium asset price (18) and labor demand function (19),
3. The planner's optimality conditions (20)-(22),
4. Bank's period one budget constraint (10),
5. Nominal wage rigidity (7),
6. The resource constraints of households ($c_0 = x - d$), (15), (16) and the BS (9),
7. Market clearing: $\tilde{h}^{**} = h^{**}$, $\tilde{D}^{**} = D^{**}$, and $k^{**} = e - cI^{**} + L^{**}$

given the exogenous parameters and functions $\{A, \alpha, \beta, \bar{l}, \bar{h}, e, b, x, r^H, r^L, \delta, p, \Phi(.)\}$.

(a) Characteristics of the Constrained Efficient Allocation. Just as private agents, a benevolent social planner can fully insure against fire-sale risk by setting $cI^{**} = L^{**}$. While the planner neglects the costs of fire-sales from a private bank's perspective due to their purely redistributive effect she understands that fire-sales are harmful because they reduce period-2 consumption via three channels: First, purchases of bank assets crowd out productive capital investment in the TS as resources are limited in period 1. Second, absent monetary policy interventions, a lower capital stock in the TS also decreases labor demand which further reduces output in the TS. Finally, fire-sales imply that a positive share of bank projects γ is hold by the less efficient TS where the efficiency loss is given by $r^H - r^L$. As more projects need to be absorbed by the TS the fire-sale price declines which further increases the share of projects which need to be sold to the TS. These three effects are also clearly visible from the right-hand side of the planner's first-order conditions (20) and (21).

Lemma 3 shows that, like private agents, the constrained planner only fully insures against fire-sale risk if the marginal costs of fire-sales at full insurance $y(e,1,1)$ are sufficiently high. Importantly, however, the planner prefers to fully insure against fire-sale risk already for larger endowments in the TS because she takes all three channels through which fire-sales can reduce period-2 consumption, and hence welfare, into account. This finding is summarized by the following lemma.

Lemma 3: Define $\hat{e}^{CEA}$ such that

$$\frac{\left[r^H - pc - 1\right] r^L}{(1+c)r^H p} = y_k'(\hat{e}^{CEA}, 1, 1).$$

The planner takes on fire-sale risk if and only if $e \geq \hat{e}^{CEA}$. The cut-off value $\hat{e}^{CEA}$ is an increasing function of the level of employment and of the price level $P_2(b)$. Further, note that $\hat{e}^{CEA} > \hat{e}$.

I next describe the condition under which the benevolent planner invests in liquid assets. Recall that the net return on liquid assets is zero in the aggregate good state. In case of a liquidity shock, liquid assets reduce the liquidity shortage in the BS and thereby reduce fire-sales and increase period-2 consumption via the three channels described above. The planner invests in liquid assets if their gain compensates her for the costs of holding liquidity which are equivalent to the private costs $\frac{1}{\delta} + \Phi'(I+L)$. Lemma 4 shows that the planner prefers to invest in liquid assets if the resources in the TS are not too large and certainly invests in liquidity if $L^* > 0$.

Lemma 4: There exists a real number $\bar{e}^{CEA}$ such that the social planner instructs banks to hold a positive amount of liquid assets if and only if $e < \bar{e}^{CEA}$, where $\bar{e}^{CEA} > \bar{e}$. The planner thus certainly instructs banks to hold a positive amount of liquid assets in states where unregulated banks hold liquidity.

Finally, I state the conditions under which it is optimal to depart from the full employment policy. In this case, equation (22) holds as strict equality, and the optimal allocation is characterized by involuntary unemployment.

Proposition 2: There exists a real number $\tilde{e}$ such that optimal monetary policy does not fully stabilize employment if and only if $e < \tilde{e}$. In this case, condition (22) holds as strict equality, $P_2^{**} < P_2^{full}$ and $h^{**} < 1$.

The formal conditions for $\bar{e}^{CEA}$ and $\tilde{e}$ are provided in the poofs of lemma 4 and proposition 2, respectively. When the CB implements monetary policy, it equates the costs of unemployment with the costs of a lower asset price. The latter decrease with the endowment of the TS such that full employment is clearly optimal for sufficiently larger levels of e. Vice versa, the constrained efficient allocation is characterized by involuntary unemployment in crises periods if the endowment in the TS is below the cut-off value $\bar{e}$. In this case, optimal monetary policy is set to equalize the costs of fire-sales with the costs of involuntary unemployment and less output in the TS. If the costs of the financial friction are sufficiently large, fully stabilizing employment and output during periods of financial distress is hence not the optimal target for monetary policy, even if the central bank has access to the full set of macroprudential tools. Instead, monetary policy also takes financial stability considerations into account. The cut-off value $\tilde{e}$ also depends on the efficiency loss $(r^H - r^L)$ since a higher efficiency loss directly increases the costs of fire-sales. In environments where $r^H = r^L$, optimal monetary policy is characterized by the full employment regime and the decentralized equilibrium coincides with the constrained-efficient allocation.

V. Social Inefficiencies and Optimal Policy

This section first compares the social planner's choices with the competitive allocation and derives results regarding the inefficiencies and optimal macroprudential regulation. It then discusses one of the key findings of the current study: the dependence of social inefficiencies on the prevailing monetary policy regime. The first observation is the following:

Lemma 5: The unregulated competitive equilibrium is constrained inefficient. Compared to private agents, a constrained social planner holds (weakly) more liquid assets ($L^{**} \geq L^*$) and invests less in bank projects ($I^{**} < I^*$). The fire-sale price and the capital stock of the TS in the bad state are thus larger in the constrained efficient allocation, i.e., $k(b)^{**} > k(b)^*$ and $q(b)^{**} > q(b)^*$.

The intuition is as follows: Because of the atomistic nature of individual agents, banks do not understand the incremental effect of individual investment decisions on the period-1 asset price. If the efficiency loss in the TS is positive, i.e., $r^H > r^L$, a lower asset price reduces period-2 consumption by increasing the share of bank projects held by the less efficient TS. A benevolent planner understands this and insures more against liquidity shocks by (1) holding more liquid assets and (2) investing less in bank projects. This reduces the liquidity gap $cI - L$, increases the fire-sale price and thereby reduces the share of projects held by the TS γ.

The result that fire-sale externalities distort both investment and liquidity choices of private banks is not new but well in line with the findings of Kara and Ozsoy (2019), among others. Here, I study a nominal framework including an additional friction arising from a nominal wage rigidity. As a consequence, the competitive allocation may be constrained inefficient even if the efficiency loss in the TS is zero, i.e. $r^H = r^L$. This is because by crowding out capital investment in the TS, fire-sales also reduce employment in the TS. If the CB does not commit to the full employment regime, this channel introduces an additional wedge between the constrained efficient and the competitive allocation.

Importantly, note that, under the assumption $r^H > r^L$, the unregulated equilibrium is constrained inefficient independent of the monetary policy regime in place. This is because bank's investment decisions affect the asset price $q(b)$ and thereby the share of projects which need to be redistributed to the less efficient TS for any given price level $P_2(b)$. Contrary to the findings of Otrok and others (2012) monetary policy is thus not sufficient to restore constrained efficiency. Monetary policy can however mute the effect of fire-sales on employment by committing to a full employment regime. In the following, I characterize the macroprudential policy instruments necessary to implement constrained efficiency and describe their dependence on monetary policy decisions.

A. Implementing the Constrained Efficient Allocation

The constrained efficient allocation can be decentralized using a tax on bank investment τ and an interest rate r on liquid assets where the equilibrium interest rate on liquid assets strictly exceeds the equilibrium tax rate, i.e., $\tau^{**} > r^{**}$. This result is summarized in the following lemma:

Lemma 6: The constrained efficient allocation can be decentralized using a positive tax on investment, $\tau^{**} > 0$, and a positive subsidy on liquid assets, $r^{**} > 0$, set to satisfy

$$\begin{aligned}
\tau^{**} &= \delta pc\left(\frac{-(cI^{**}-L^{**})}{r^L}\left(y_k''(.)+y_{k,h}''(.)\frac{\partial \mathscr{H}(.)}{\partial L}\right)(r^H-r^L)+y_h'(.)\frac{\partial \mathscr{H}(.)}{\partial L}\right),\\
r^{**} &= \frac{\tau^{**}}{c\delta},
\end{aligned}$$

where $\tau^{**} < r^{**}$.

In line with the findings of Kara and Ozsoy (2019), lemma 6 shows that a benevolent planner needs two independent instruments and detailed information about the economic fundamentals to restore constrained efficiency. In the following, I refer to this as the full regulation regime. By contrast, regulating only one of the two assets individually is not sufficient to achieve constrained efficiency. This is also true when the planner can set the liquidity ratio $l = \frac{L}{I}$ instead of the asset holdings I and L which is still not sufficient to implement constrained efficiency. In this case, all equilibrium conditions remain unchanged and the insights from lemma 6 sill apply. One instrument is sufficient to restore constrained efficiency only if the CB can target the liquidity shortage in case of a liquidity shock $cI - L$ directly. In particular, the banks and the planner's maximization problem can be rewritten as a function of the liquidity shortage LS and the investment level I. In this case, it is sufficient to set a tax on the liquidity shortage to satisfy

$$\tau^{LS} = p\left\{y'_h(.) - \left(y''_k(.) + y'_{k,h}(.)\frac{\partial \mathscr{H}(.)}{\partial k}\right)\frac{r^H - r^L}{r^L}\right\},$$

to restore constrained efficiency. Once the choices for $cI - L$ are aligned, private choices coincide with the planner's choice.

B. Social Inefficiencies and Monetary Policy

I next study the effect of monetary policy on optimal macroprudential interventions. In particular, the following proposition shows that the optimal tax τ^{**} and subsidy rate r^{**} decrease when the CB commits to a more expansionary counter-cyclical monetary policy.

Proposition 3: The macroprudential tax on debt and subsidy on liquid assets necessary to implement constrained efficiency fall as the price level $P_2(b)$ increases. The tax and the subsidy are lowest when the CB commits to the full employment regime.

Intuitively, this implies that the wedge between the unregulated and the constrained efficient allocation shrinks as the CB commits to a more expansionary monetary policy during periods of financial distress. As outlined before, this wedge arises because of two effects: First, the planner anticipates how investment decisions affect the share of redistributed projects via the fire-sale price. Second, the planner anticipates that fire-sales reduce employment by crowding out productive capital investment in the TS. When the CB commits to large currency depreciation's both, private banks, and the planner, expose themselves to less fire-sale risk which also increases the available capital stock in the TS in case of a liquidity shock. Because of the concavity assumption on the technology of the TS, this lowers the marginal return on capital $y'_K(.)$ and thereby also the marginal effect of additional bank projects and liquid assets on the fire-sale price. Clearly, the first effect hence becomes smaller as the price level increases. The second effect $y_h'(.)\frac{\partial \mathscr{H}(.)}{\partial k}$ can be rewritten as $\frac{\beta}{1-\beta}y_k'(.)$ and thus also clearly decreases with the price level $P_2(b)$. Overall, a more expansionary monetary policy in case of a liquidity shock must hence lower the wedge between the constrained efficient and the decentralized allocation.

VI. Optimal Monetary Policy with Limited Macroprudential Instruments

This section aims to analyze optimal monetary policy in environments where the CB has access to no or a limited set of macroprudential policy instruments. This is relevant because optimal macroprudential regulation, as outlined above, requires (1) detailed information about economic fundamentals, (2) an active management of the tools and (3) a legal environment which enables the CB to implement the required tools. By contrast, uninformed or ad-hoc macroprudential regulation may cause large welfare costs.[31] In the following I assume $e < \bar{e}$ such that liquidity holdings are strictly positive in the decentralized, $L^* > 0$, and the constrained efficient allocation, $L^{**} > 0$.

A. No Access to Macroprudential Policy Tools

I first consider the case where the CB has no access to macroprudential policy tools at all but only sets the period-2 price level in the aggregate bad state $P_2(b)$. Importantly, note that bank's period-0 balance sheet choices $\{I, L, D\}$ critically depend on the expected period-2 price level $\mathbb{E}\{P_2(b)\}$ due to its effect on the asset price, $q(b)$. This asset price, in turn, determines the costs of investing in risky versus save assets for private banks. By committing to a monetary policy regime $P_2(b)$ in the initial period the CB can thereby guide bank's initial investment decisions. Since the CB has no macroprudential instruments available it can instead use monetary policy to "lean against the wind".

In particular, I consider a CB that can set $P_2(b)$ only but understands how the price level affects bank's investment decisions $\{I, L, D\}$. As before, the CB also understands how the asset prices and labor demand are formed in equilibrium. Formally, the CB's maximization problem is identical to the one described in section 4 with the two additional implementability

[31] Otrok and others (2012) and Bianchi and Mendoza (2018), among others, have demonstrated the potentially larger welfare costs of simple macroprudential rules.

constraints (13) and (14) and can be summarized by

$$
\begin{aligned}
\mathcal{L}^P &= \frac{1}{\delta}(X-I-L+B)+(1-p)\left\{r^H I+L-\Phi(I+L)+y(e,1,1)\right\} \\
&\quad +p\left\{r^H I-\frac{(cI-L)}{\mathcal{Q}(.)}(r^H-r^L)+L-cI-\Phi(I+L)+y(k,\mathcal{H},1)+cI-L\right\} \\
&\quad -\zeta\,(L-cI)-\zeta_I\left(1/\delta+pc+\Phi'(I+L)-r^H+pc(\frac{r^H}{\mathcal{Q}(.)}-1),)\right) \\
&\quad -\zeta_L\left(1/\delta+\Phi'(I+L)-1-p(\frac{r^H}{\mathcal{Q}(.)}-1)\right)-\zeta(L-cI),
\end{aligned}
$$

where ζ_I and ζ_L denote the multipliers of the participation constraints for I and L from the competitive equilibrium. Importantly note that, when evaluated at the same allocation, the implementability constraints (13) and (14) are binding constraints for the planner as the social cost (gain) of investment (liquid assets) strictly exceeds the private costs (gain) given the assumption $r^H > r^L$.

In the previous analysis, I showed that private agents overinvest in risky assets and underinvest in liquid assets compared to the constrained efficient allocation due to the pecuniary externality present in the model. In the full regulation regime, the planner thus uses a tax on investment and a subsidy on liquid assets to align individual investment decisions. In the absence of such macroprudential instruments, however, the CB may use monetary policy to "lean against the wind" and mitigate inefficiencies in bank's initial investment decisions. Intuitively, the CB may have an incentive to commit to a more expansionary monetary policy in case of financial distress to lower the expected asset price $q(b)$ and thereby increase the precautionary behavior of private banks. The following proposition shows that, without macroprudential tools, the CB indeed commits to a more expansionary counter-cyclical monetary policy upfront and thereby minimizes the necessary amount of fire-sold assets $cI-L$ in case of a liquidity shock.

Proposition 4: In the absence of macroprudential tools, the CB strictly prefers to implement the full employment regime.

Note that "leaning against the wind" is a third-best policy and is only optimal under the constraint that no macroprudential tools are available. The monetary policy trade-off characterized by the planner's optimality condition (22) is still present. While a higher price level increase period two production in the TS, it also increases the share of projects which need to be redistributed to the less efficient TS (given bank's initial investment decisions). Without macroprudential tools, however, the full employment regime is optimal as it allows the CB to minimize the wedge between the constrained efficient and the competitive allocation.

B. Limited macroprudential policy.

Next consider the case where the CB has access to a limited set of macroprudential instruments: Either (1) the CB can only implement the efficient investment level but has no instrument to regulate liquidity holdings in the BS or (2) the CB sets the liquidity holdings but cannot regulate bank's investment decisions. First consider a situation (1). In this case, the planner's first-order conditions read

$$\begin{aligned}
\frac{1}{\delta}+pc+\Phi'(I+L) &= r^H - pc\left(\frac{r^H}{\mathscr{Q}(.)}-1\right)+p\frac{(cI-L)}{\mathscr{Q}(.)^2}\frac{\partial \mathscr{Q}(.)}{\partial I}(r^H-r^L) && (23)\\
&\quad -py_h'(.)\frac{\partial \mathscr{H}(.)}{\partial I}+cp\zeta-\zeta_L\left\{\Phi''(I+L)+\frac{pcr^H}{r^L}\left(y_k''(.)+y_{k,h}''(.)\frac{\partial \mathscr{H}(.)}{\partial k}\right)\right\},\\
\frac{1}{\delta}+\Phi'(I+L) &\geq 1+p\left(\frac{r^H}{\mathscr{Q}(.)}-1\right)+p\frac{cI-L}{\mathscr{Q}(.)^2}\frac{\partial \mathscr{Q}(.)}{\partial L}(r^H-r^L)+py_h'(.)\frac{\partial \mathscr{H}(.)}{\partial L} && (24)\\
&\quad -p\zeta-\zeta_L\left\{\Phi''(I+L)-\frac{pr^H}{r^L}\left(y_k''(.)+y_{k,h}''(.)\frac{\partial \mathscr{H}(.)}{\partial k}\right)\right\},\\
y_h'(.)\frac{\partial \mathscr{H}(.)}{\partial P_2} &\geq -\frac{(cI-L)}{\mathscr{Q}(.)^2}\frac{\partial Q(.)}{\partial P_2}((r^H-r^L-\zeta_L r^H), && (25)
\end{aligned}$$

where ζ_L denotes the multiplier of condition (14) which enters the planners' problem as implementability constraint because liquidity is still chosen by private banks. Combining equation (24) and (25) then yields

$$\Phi'(I^{**}+L^{**})=\frac{r^H-(1+c)\left(\frac{1}{\delta}+\zeta_L\Phi''(I+L)\right)+c(1-p)}{(1+c)}.$$

Since $\zeta_L>0$ this implies that bank's balance sheet is strictly smaller compared to the full regulation regime. This is because, the planner invests less in bank projects to compensate for the low liquidity holdings of private agents and to reduce the costs of fire-sales in a crisis. Further, (25) shows that with limited macroprudential tools, a benevolent planner values expansionary counter-cyclical monetary policy more compared to the full regulation regime. Evaluated at the same allocation, the benevolent planner hence implements more expansionary monetary policy compared to the full regulation regime.

A similar argument holds for the case where the planner sets the liquidity holdings, but bank's chose their individually optimal investment levels. In this case, the first-order conditions read

$$\begin{aligned}
\frac{1}{\delta}+pc+\Phi'(I+L) &= r^H - pc\left(\frac{r^H}{\mathscr{Q}(.)}-1\right)+p\frac{(cI-L)}{\mathscr{Q}(.)^2}\frac{\partial\mathscr{Q}(.)}{\partial I}(r^H-r^L), \qquad (26)\\
&\quad -py_h'(.)\frac{\partial\mathscr{H}(.)}{\partial I}+cp\zeta-\zeta_I\left\{\Phi''(I+L)-\frac{pcr^Hc}{r^L}\left(y_k''(.)+y_{k,h}''(.)\frac{\partial\mathscr{H}(.)}{\partial k}\right)\right\}\\
\frac{1}{\delta}+\Phi'(I+L) &\geq 1+p\left(\frac{r^H}{\mathscr{Q}(.)}-1\right)+p\frac{cI-L}{\mathscr{Q}(.)^2}\frac{\partial\mathscr{Q}(.)}{\partial L}(r^H-r^L)+py_h'(.)\frac{\partial\mathscr{H}(.)}{\partial L}, \qquad (27)\\
&\quad -p\zeta-\zeta_I\left\{\Phi''(I+L)+\frac{pcr^H}{r^L}\left(y_k''(.)+y_{k,h}''(.)\frac{\partial\mathscr{H}(.)}{\partial k}\right)\right\}\\
y_h'(.)\frac{\partial\partial H(.)}{\partial P_2}r^L &\geq -\frac{(cI-L)}{\mathscr{Q}(.)^2}\frac{\partial Q(.)}{\partial P_2}((r^H-r^L+\zeta_I c r^H), \qquad (28)
\end{aligned}$$

where $\zeta_I < 0$ denotes the multiplier of condition (13). Combining equation (26) and (27) now yields

$$(I^{**}+L^{**})=\frac{r^H-(1+c)\left(\frac{1}{\delta}+\zeta_I\Phi''(I+L)\right)+c(1-p)}{\Phi''(I+L)(1+c)}.$$

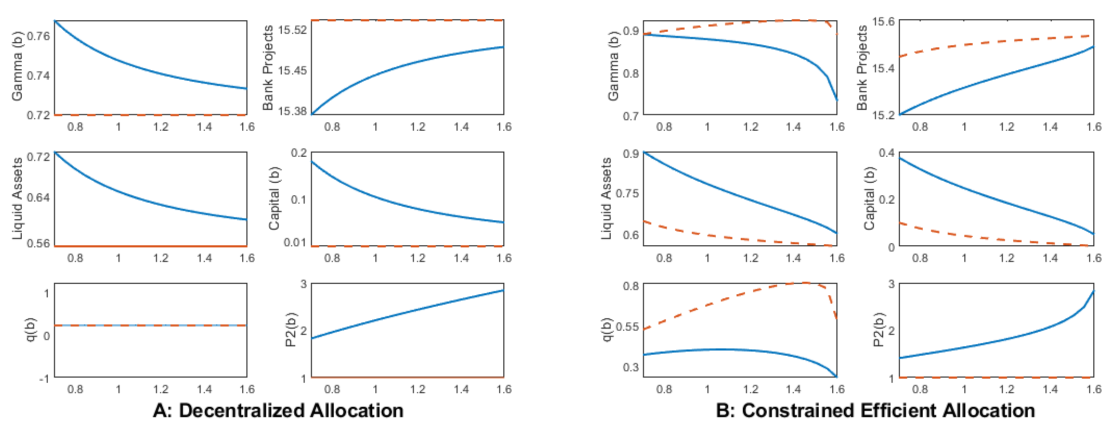
Accordingly, the balance sheet size is larger compared to the full regulation regime. Since the planner cannot regulate investment, she mitigates the negative effects of fire-sales by accumulating even more liquid assets. Finally, condition (28) again shows that when evaluated at the same allocation, the benevolent planner implements a more expansionary monetary policy compared to the full regulation regime.

VII. Numerical Illustration

This section illustrates the main results of the analysis by means of a numerical example. Following Kara and Ozsoy (2019), I set one model period to 2 years so that the total length is four years, the cost function to $\Phi(I,L)=\xi(I+L)^2$ with $\xi=.01$, the magnitude of the liquidity needs to $c=.1$, and the initial equity in the BS to $b=1$. Further, I set the final period return of bank projects to 1.6, which corresponds to an annual return of around 12%, the (biennial) probability of a crisis to $p=9\%$ and the domestic real interest rate to $R^b=1.2155$, which implies an annual interest rate of 5%. The parameters in the TS are set to $\alpha=.35$, $\beta=.6$ and $e=1$ (without loss of generality). The parameter capturing productivity, A, is set such that the marginal return on capital is one in case of full employment. Finally, I do not choose a specific value for r^L but instead vary the parameter in the interval $[.7,r^H]$ such that the efficiency loss of the TS in managing bank projects varies between 0.9 and 0 percentage points.

Figure 1 compares the full employment with the fixed price regime in the decentralized (left panel) and the constrained efficient allocation (right panel). First note that given the choice of functional forms and parameters both, private agents and the benevolent planner, never fully insure against fire-sale risk, i.e. $e > \hat{e}^{DE}$ and $e > \hat{e}^{CEA}$ but always invest a positive amount of resources in liquid assets, i.e. $e < \bar{e}^{DE}$ and $e < \bar{e}^{CEA}$. Further, figure 1 confirms the effect of monetary policy on agent's incentive to accumulate liquid assets and insure against liquidity risk as highlighted by the analytical analysis. Both private banks and the planner accumulate more liquid assets and invest less in bank projects if the CB commits to the full employment regime. Consequently, fire-sales crowd out less productive capital investment in the aggregate bad state. Despite the larger capital stock in the TS, the fire-sale price in the constrained efficient allocation is still smaller under the full employment regime due to the larger employment level. For very low levels of r^L, the full employment regime therefore also displays a lower share of bank projects which remains in the BS. In the competitive allocation the two counteracting effects cancel out such that the fire-sale price is identical in the two allocations. Finally, note that as r^L increases both agents insure less against fire-sale risk which increases the severity of a potential liquidity shock and increases the price-level necessary to restore full employment.

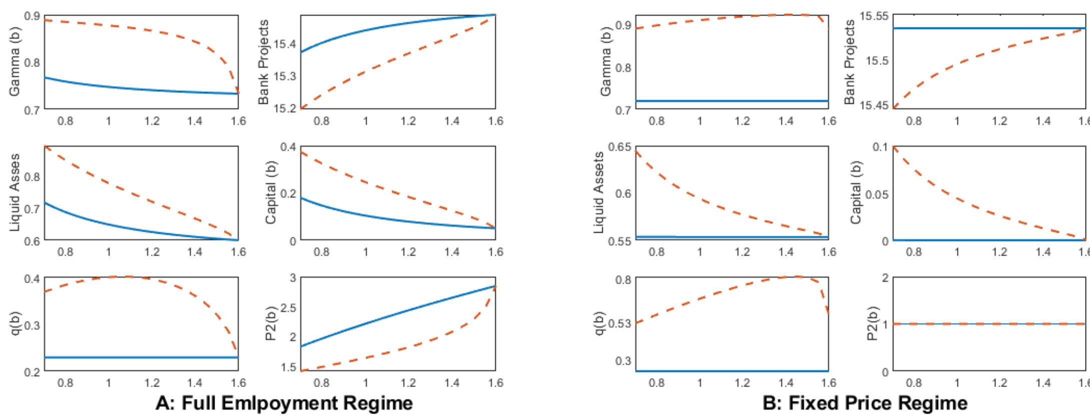
Figure 1. Full Employment vs. Fixed Price Regime



Note: The blue solid line corresponds to the full employment regime, the red dashed line to the fixed price regime. The left panel plots the unregulated equilibrium, the right panel the constrained efficient allocation.

Next, figure (2) compares the competitive (blue line) with the constrained efficient allocation (red dashed line) given the two different monetary policy regimes. Again, the results confirm the general findings discussed in the analytical analysis. Most notably, the benevolent planner invests less in bank projects and holds more liquid assets compared to private banks independent of the prevalent monetary policy regime. Hence, liquidity crises in the constrained efficient allocation are characterized by a higher asset price, a larger capital stock in the TS and a lower redistribution of bank projects to the TS. It is further worth highlighting that the wedge between the constrained efficient and the decentralized allocation decreases as r^L increases. When the efficiency loss in the TS is zero $r^H = r^L$, both allocations coincide in the full employment regime but there remains a gap between the two allocations otherwise. This is because the initial investment decisions in the BS still affect the level of employment and hence output in the TS in case of a liquidity shock if the CB does not commit to the full employment regime.

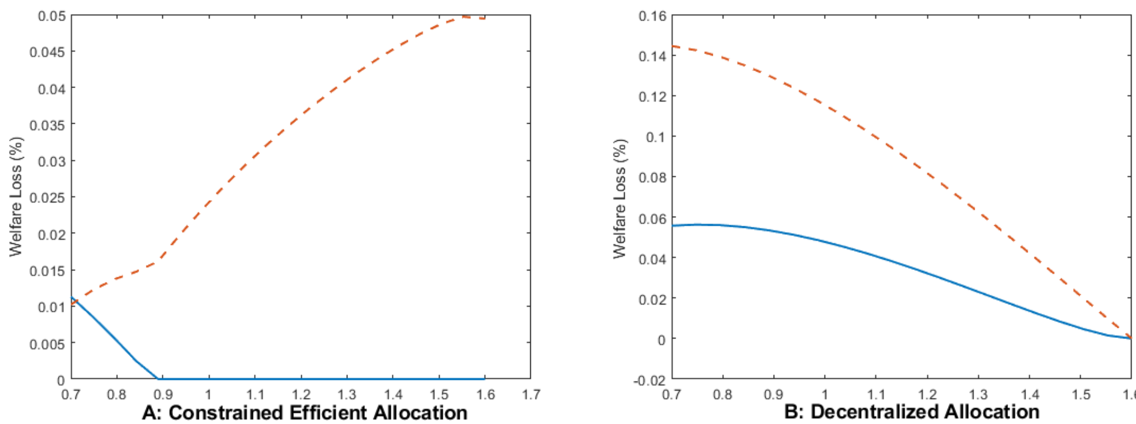
Figure 2. Competitive vs. Constrained Efficient Allocation



Note: The blue solid line corresponds to the unregulated competitive equilibrium, the red dashed line to the constrained-efficient equilibrium. In the left panel the CB implements a full employment regime, in the right panel a fixed employment regime.

So far, I compared two very distinct and ad-hoc monetary policy regimes, i.e., the full employment with the fixed price regime. The left panel of figure (3) now compares welfare in the constrained efficient allocation when the CB commits to the two ad-hoc rules with the optimal monetary policy regime. The following observations are particularly worth noting: First, the full employment regime represents the optimal monetary policy regime for $r^L > .83$. In this case, condition (22) holds as strict inequality, and the CB strictly prefers to stabilize employment rather than the asset price. In the current framework, the welfare gains of stabilizing the asset price are thus comparably small even for large efficiency losses in the TS. While the current exercise does not provide a calibrated quantitative analysis, this result is in line with the findings of Beningio et at. (2011) showing that the welfare costs of financial frictions are generally small compared to the costs of nominal rigidities.[32] Second, fixed price regimes become more expensive as the efficiency loss in the TS becomes smaller. This is intuitive, as a fixed price level stabilizes asset prices and thereby reduces the share of bank projects which need to be redistributed to the TS in case of a liquidity shock.

Figure 3. Welfare

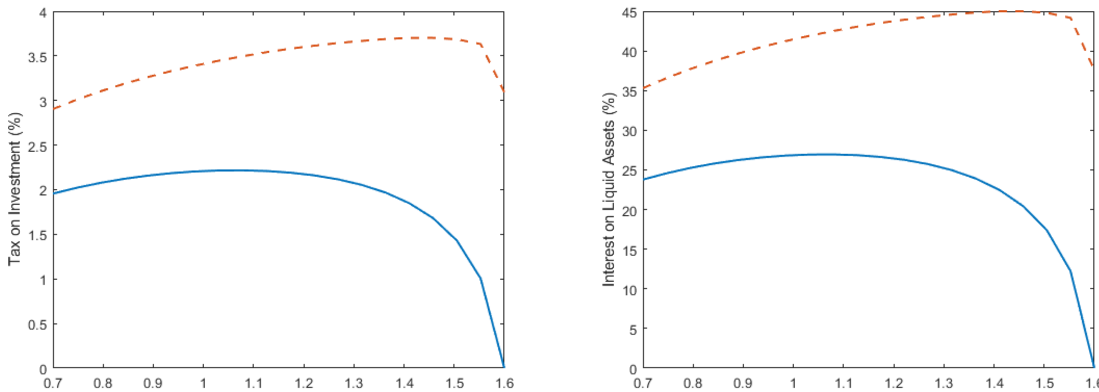


Note: The left panel plots the welfare loss in the constrained efficient allocation given the full employment (blue solid line) and the fixed price (red dashed line) regime relative to the optimal monetary policy regime. The right panel plots the welfare loss in the decentralized allocation relative to the constrained efficient allocation given the full employment and the fixed price regime.

Panel B of figure (3) plots the welfare loss of the decentralized equilibrium relative to the constrained efficient allocation given the full employment and the fixed price regime. Recall that the full employment regime is always optimal if the CB has no macroprudential policy instruments available, i.e., the decentralized allocation is realized. Further, the figure confirms the finding of section 5 that the wedge between the competitive and the constrained efficient allocation is minimized in the full employment regime. This finding is also evident from figure (4) which shows that the optimal tax rate τ^{**} and interest rate r^{**} necessary to restore constrained efficiency are larger in the fixed price regime.

[32]This is also consistent with studies showing that the welfare costs of crises generated by financial frictions are generally small (see e.g., Bianchi (2011) and Bianchi and Mendoza (2018)).

Figure 4. Optimal Macroprudential Regulation



Note: The blue solid line corresponds to the full employment regime, the red dashed line to the fixed employment regime.

VIII. CONCLUSION

In this paper, I develop a model with a nominal rigidity and a financial friction to provide an integrated analysis of macroprudential and monetary policies. In this framework, monetary authorities face a trade-off: While a more expansionary counter-cyclical monetary policy reduces real wages and thereby stabilizes employment in crises states, they also increase fire-sale discounts and thereby exacerbate the distortions which arise from the financial friction. While monetary policy is generally insufficient to restore constrained efficiency, optimal macroprudential regulation critically depends on the monetary policy regime. In particular, I show that as monetary policy is more targeted towards stable employment rather than financial stability this reduces the need for additional macroprudential interventions. This is because the anticipation of expansionary monetary policy during periods of financial distress increases fire-sale discounts and thereby private agent's incentives to insure against crises periods. In the absence of macroprudential tools, monetary authorities strictly prefer to focus on macroeconomic stability as this leaves the economy better insured against periods of financial distress.

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APPENDIX A. FIRST-BEST ALLOCATION

This section characterizes the first-best allocation. Markets are complete and there are no fire-sales. In this situation, the planner is not subject to any frictions but can always access short-term funding to finance cI in the interim period. Accordingly, no projects are redistributed to the less efficient TS, i.e., no fire-sales occur, no capital is crowded out in the TS, employment and production is at full capacity and there is no need to invest in liquid assets as short-term credit markets always provide the funds necessary to finance the liquidity needs at no costs. More specifically, the first-best allocation is given by the following closed form solution:

$$
\begin{aligned}
q(b)^{FB} &= r^H, \\
\gamma^{FB} &= 1, \\
k^{FB} &= e, \\
L^{FB} &= 0, \\
h^{FB} &= 1, \\
I^{FB} &= (\Phi'(I+L))^{-1}\left(r^h - \frac{1}{\delta} - c\right).
\end{aligned}
$$

Note that as employment is always at potential monetary policy is neutral in this setup.

APPENDIX B. MODEL EXTENSIONS

B.1. Endogenous Traditional Sector Endowment e

The previous results have been derived under the assumption that the endowment of the TS e is exogenous. I now relax this assumption and consider a TS that chooses its period-1 level of resources endogenously by borrowing from the household sector in the initial period. In the following I show that all main results are robust to this alternative specification. In particular, in an allocation where $L^* > 0$ private agents choose the same level of e^* as the planner such that $e^* = e^{**}$. This is summarized by the following proposition

Proposition 5: Let e be chosen endogenously by investors. In an unregulated competitive equilibrium where banks hold a positive amount of liquidity, $L^* > 0$ (and thus $L^{**} > 0$), the privately optimal choice of e is given by

$$e^{*} = \left(\frac{1/\delta(1+c) - r^{H} + (1-p)(1+c)}{(1-p)(1+c)\alpha A}\right)^{\frac{1}{\alpha-1}},$$

which is socially efficient, i.e., $e^{*} = e^{**}$ and satisfies assumption 1 ($y'(e^{*},1,1) > 1$).

Given the identical choices of the TS's resources in the decentralized and the constrained efficient allocation it is immediate that the main results of our analysis with exogenous e carry over to environments with an endogenous choice of e. In particular, consider an environment where p is such that such that full insurance is never optimal and both agents hold a positive amount of liquid assets ($\hat{e}^{CEA} < e < \bar{e}$). In this case, it immediately follows that all results from above still apply, in particular, the dependence of private choices on monetary policy, the results regarding social inefficiencies and the dependence of optimal macroprudential policy on the monetary policy regime.

B.2. Stochastic Total Factor Productivity

In the previous analysis productivity in the TS was constant. This section expands on the baseline analysis by discussing the effects of a standard TFP shock and showing that all main results are robust to this extension. For this purpose, consider a two state TFP process with A $\in [A_H, A_L]$, where $A_H > A_L$ and $A = A^H$ in the baseline model. The TFP process is uncorrelated with the liquidity shock. This exposes the economy to two different uncertainties, TFP and the restructuring costs which can lead to the realization of four different states in period $t = 1$: $\{[\theta = g, A = A_H], [\theta = g, A = A_L], [\theta = b, A = A_H], [\theta = b, A = A_L]\}$. For simplicity, I focus on cases with $L^{*} > 0$.

How does the introduction of a TFP shock affect optimal monetary policy? First consider the case where $\theta = g$, such that no capital is crowed out of the TS and $e = k$. If TFP is high, the economy is characterized by full employment and optimal monetary policy is a trivial non-action strategy as discussed in the baseline framework. By contrast, if TFP is low employment falls below potential and optimal monetary policy is characterized by an increase in the price level to restore full employment. Note that in this situation there is no policy trade-off involved and stabilizing demand is the optimal strategy for monetary policy.

As in the baseline analysis, the policy trade-off arises in states where bank projects are subject to additional restructuring costs. Importantly, note that this trade-off is entirely independent of TFP which can be directly seen by rearranging condition (22) to:

$$k^{**}(b)[r^L + \alpha(r^H - r^L)] = e\alpha(r^H - r^L).$$

This expression determines the socially optimal level of capital independent of TFP, employment and the price level as discussed in more detail in section 4. Optimal monetary policy still reacts to a low realization of TFP by implementing a larger price level to mitigate the decline in employment. This can be seen from the closed form solution of the constrained efficient allocation presented in the proof of Proposition 2.

Next, consider the effects of a TFP shock on private agent's incentives to insure against liquidity shocks and the social inefficiencies which may arise. A low realization of TFP in period $t = 1$, holding every else constant, has two reinforcing effects: First, it reduces the marginal productivity of labor in the TS which reduces labor demand and increases the period-1 asset price. Second, lower productivity reduces the marginal productivity of capital which also increases the period-1 asset price. As a result, private banks are exposed to less-severe fire-sale losses, invest more in bank project, less in liquid assets and, more capital is crowded out from the TS. This becomes immediately evident from the characterization of the competitive equilibrium in the proof of Lemma 2. For any given price level $P_2(b)$, the wedge between the constrained efficient allocation and the competitive equilibrium hence increases if declines in TFP are expected to coincide with a liquidity shock. Notably, this wedge is still decreasing as the CB commits to a more expansionary monetary policy in adverse states.

To summarize, declines in TFP generally increase the need for counter-cyclical monetary policy interventions. If the decline in TFP is expected to coincide with the liquidity shock, tighter macroprudential regulation is required to restore the second-best. The monetary-policy trade-off and the dependence of the social inefficiencies on the monetary policy regime remain unaffected.

Appendix C. Proofs Omitted in the Main Text

C.1. Proof of Lemma 1

First assume that banks fully insure against fire-sale risk, i.e., $cI = L$. In this case, banks' real profits are given by $\pi^{fi} = r^H I - 1/\delta D - \Phi(I+L) - pcI + L$. Now suppose that banks move an ε amount from L to I. Bank profits are then given by $\pi^{\varepsilon} = \pi^{fi} + \varepsilon(r^H - (1-p)) - pr^H \frac{\varepsilon(1+c)}{q(b)^{\varepsilon}}$, where $q(b)^{\varepsilon}$ denotes the price for an ε amount of fire-sales. Thus $\pi^{\varepsilon} > \pi^{fi}$ if

$$\frac{(r^H - (1-p))r^L}{pr^H(1+c)} > y_k'(e,1,1),$$

i.e., if e is sufficiently large. In the following I use $\hat{e}$ to denote the cutoff value for the TS's endowment above which full insurance is never optimal, i.e., $e \geq \hat{e}$. Further, note that the private marginal costs of fire-sales at $y_k'(e,1,1)$ are larger if the CB focuses more on stable employment as a larger level of employment also increases the marginal productivity of capital in the TS. Accordingly, the cutoff value $\hat{e}$ increases with the level of employment.

C.2. Proof of Lemma 2

First, assume that $I^* > 0$ and $L^* > 0$. In this case, the equilibrium is given by the closed form solution:

$$\begin{aligned}
q(b)^* &= r^H \frac{p(1+c)}{r^H - (1-p)}, \\
(I^* + L^*) &= (\Phi')^{-1}(r^H - pc(\frac{r^H}{q(b)^*} - 1) - \frac{1}{\delta} - pc), \\
k^*_{full}(b) &= \left(\frac{r^L}{q(b)^* \alpha A}\right)^{\frac{1}{\alpha-1}}, \\
k^*_{P_2}(b) &= (\frac{r^L e^{\frac{\alpha\beta}{1-\beta}}}{q(b)^* \alpha A P_2(b)^{\frac{\beta}{1-\beta}}})^{\frac{1-\beta}{\alpha+\beta-1}}, \\
I^*_{full} &= \frac{(I+L)^* + e - k^*(b)_{full}}{1+c}, \\
I^*_{P_2} &= \frac{(I+L)^* + e - k^*(b)_{P_2}}{1+c}, \\
L^*_{full} &= (I+L)^* - I^*_{full}, \\
L^*_{P_2} &= (I+L)^* - I^*_{P_2},
\end{aligned}$$

where subscript $full$ is used to denote the full employment regime and subscript $P_2(b)$ to denote equilibrium allocations in case the CB does not commit to the full employment regime. The first two expressions are derived from the banks' first-order conditions (13) and (14). Equilibrium capital can then be derived using condition (12) and the period-1 market clearing condition $k(b) = e - cI + L$ together with $IL = I + L$ yield the equilibrium value for investment I. This allocation is an equilibrium if $L^* > 0$, i.e. $e < (I+L)^*c + k^*(b) = \bar{e}$. Note further that the capital stock in the TS is largest in the full employment regime which implies $I^*_{full} < I^*_{P_2}$ and $L^*_{full} > L^*_{P_2}$ and $\bar{e}_{full} > \bar{e}_{P_2}$.

Finally, not that using the above expression for $q(b)^*$ directly implies the condition in assumption 2, under which scrapping can never be optimal.

C.3. Proof of Proposition 1

For the case $L^* > 0$ see lemma 2. Otherwise, the FOC of the decentralized allocation with respect to investment yields

$$\frac{1}{\delta} + pc + \Phi'(I) = r^H - pc(\frac{r^H y'_k(e-cI, h^d(e-cI, P_2(b)), 1)}{r^L} - 1),$$

which is a function of investment I only (given the CB's choice for $P_2(b)$). Note first that the lhs is strictly increasing in I due to the convexity assumptions on $\Phi(I+L)$. The rhs is strictly decreasing in I due to the concavity assumption on y(.). Further note that for $I \to 0$ the rhs is greater than the lhs (given the assumption $1/\delta + pc \leq r^H$) while the lhs clearly exceeds the rhs for sufficiently large levels of I. Thus, for each level of P_2, there exists a unique investment value I^* such that the condition holds with equality.

Finally, note that the return on capital increases with the price level and thus, for any given level of I, the rhs decreases with the price level $P_2(b)$. This implies that as the price level $P_2(b)$ increases, the equilibrium level of investment I^* decreases and since $L^* = 0$, the capital stock in the TS in the aggregate bad state increases.

C.4. Proof of Lemma 3

First assume full insurance, i.e. $cI = L$. Period-2 consumption is given by $c_2^{fi**} = r^H I - \Phi(I + L) + y(e,1) - pcI + L$. Now suppose that the planner moves an ε amount from L to I. Period-2 consumption is then given by

$$c_2^{\varepsilon**} = c_2^{fi**} + \varepsilon(r^H - pc - 1) - p\{\frac{\varepsilon(1+c)}{q^\varepsilon(b)}(r^H - r^L) + y(e,1,1) - y(e - \varepsilon(1+c),1,1)\},$$

where q^ε denotes the fire-sale price for an ε amount of fire-sales. If ε is sufficiently small,

$$c_2^{\varepsilon**} = c_2^{fi**} + \varepsilon(r^H - pc - 1) - p\{\frac{\varepsilon(1+c)}{r^L} y'_k(e,1,1)(r^H - r^L) + y'_k(e,1,1)\varepsilon(1+c)\}.$$

This movement of resources is not profitable if

$$\varepsilon(r^H - pc - 1)r^L \quad \leq \quad p\varepsilon(1+c)y_k'(e,1,1)\{(r^H - r^L) + r^L\},$$

which implies $y_k'(e,1,1) \geq \frac{[r^H - pc - 1]r^L}{(1+c)r^H p} = y_k'(\hat{e}^{CEA},1,1)$.

Again, note that a full employment regime makes it more attractive for the planner to fully insure against fire-sale risk. Compared to the decentralized allocation, the planner prefers to fully insure against fire-sale risk already for a higher endowment level in the TS.

C.5. Proof of Lemma 4

Note first that an allocation with $L = 0$ must satisfy the planner's optimality conditions

$$\begin{aligned} \frac{1}{\delta} + pc + \Phi'(I) \quad &= \quad r^H - pc(\frac{r^H}{\mathscr{Q}(.)} - 1) + p\frac{(cI - L)}{\mathscr{Q}(.)^2}\frac{\partial \mathscr{Q}(.)}{\partial I}(r^H - r^L) - pcy_h'(.)\frac{\partial \mathscr{H}(.)}{\partial k} \\ \frac{1}{\delta} + \Phi'(I) \quad &> \quad 1 + p(\frac{r^H}{\mathscr{Q}(.)} - 1) + p\frac{cI - L}{\mathscr{Q}(.)^2}\frac{\partial \mathscr{Q}(.)}{\partial L}(r^H - r^L) + py_h'(.)\frac{\partial \mathscr{H}(.)}{\partial k}, \end{aligned}$$

which together imply

$$\Phi'(I) \quad \geq \quad \frac{r^H - \frac{1}{\delta}(1+c) + c(1-p)}{(1+c)}. \tag{29}$$

Combining condition (29) and banks optimality condition for investment implies

$$\begin{aligned} \frac{1}{\delta} + pc + \frac{r^H - \frac{1}{\delta}(1+c) + c(1-p)}{(1+c)} \quad &< \quad r^H + pc \\ &+pcy_k'(.)\left(\frac{-r^H}{r^L} + \frac{(e-k(b))}{r^L}\frac{\alpha+\beta-1}{1-\beta}(r^H - r^L) - \frac{\beta}{1-\beta}\right) \end{aligned} \tag{30}$$

First note that the lhs is a constant while the rhs is, given the price level $P_2(b)$, a function of the capital stock in the TS only. Further, note that the assumptions on y guarantee that the right-hand side of (30) is monotonically increasing in $k(b)$. Moreover, for small values of k the right-hand side is clearly smaller than the left-hand side while the rhs is clearly larger than the lhs for $k(b) = e$. Hence, there exists a unique threshold value $\hat{k}$ such that (30) holds with equality. For all $k(b) > \hat{k}$, condition (30) holds as a strict inequality; for all $k(b) < \hat{k}$, condition (30) is necessarily violated.

Now note further that condition (29) together with $k^{**}(b) = e - cI^{**}$ imply

$$k^{**}(b) \leq e - c(\Phi')^{-1}\left(\frac{r^H - \frac{1}{\delta}(1+c) + c(1-p)}{(1+c)}\right). \tag{31}$$

Condition (31) and (30) hence imply that a constrained efficient allocation with $L^{**} = 0$ must satisfy $\hat{k} < k^{**}(b) < e - c\frac{r^H - \frac{1}{\delta}(1+c)+c(1-p)}{(1+c)}$. It hence follows that the planner strictly prefers to hold a positive amount of L^{**} is $\hat{k} > e - c\frac{r^H - \frac{1}{\delta}(1+c)+c(1-p)}{(1+c)}$, i.e. $e < \hat{k} + c\frac{r^H - \frac{1}{\delta}(1+c)+c(1-p)}{(1+c)} = \bar{e}^{CEA}$.

Finally note that the planner certainly holds $L^{**} > 0$ if $L^* > 0$ since $\bar{e} < \bar{e}^{CEA}$. To see this, note that $\bar{e} = k^* + c\frac{r^H - \frac{1}{\delta}(1+c)+c(1-p)}{(1+c)}$ and $\bar{e}^{CEA} = \hat{k} + c\frac{r^H - \frac{1}{\delta}(1+c)+c(1-p)}{(1+c)}$. The condition $\bar{e} < \bar{e}^{CEA}$ must hence hold if $\hat{k} > k^*$ which follows directly from the banks and the planner's optimality conditions for I and L, respectively.

C.6. Proof of Proposition 2

If condition (22) holds as strict equality, the closed form solution of the CEA is given by

$$\begin{aligned}
k^{**} &= \frac{e\alpha(r^H - r^L)}{r^L + \alpha(r^H - r^L)}, \\
(I+L)^{**} &= (\Phi')^{-1}\left(\frac{r^H - \frac{1}{\delta}(1+c) + c(1-p)}{(1+c)}\right), \\
I^{**} &= \frac{e - k^{**} + (I+L)^{**}}{(1+c)}, \\
L &= (I+L)^{**} - I^{**}, \\
P_2^{**} &= \left(\frac{(r^H - \frac{1}{\delta} - \Phi'(I+L))e^{\frac{\alpha\beta}{1-\beta}}}{pc\alpha A k^{**\frac{\alpha+\beta-1}{1-\beta}}\left(\frac{r^H}{r^L} - \frac{(e-k^{**})}{r^L}\frac{(\alpha+\beta-1)}{(1-\beta)k^{**}}(r^H - r^L) + \frac{\beta}{1-\beta}\right)}\right)^{\frac{1-\beta}{\beta}}, \\
h &= \left(\frac{P_2^{**}k^{\alpha}}{e^{\alpha}}\right)^{\frac{1}{1-\beta}},
\end{aligned}$$

This is an equilibrium if $h^{**} \leq 1$, i.e., $P_2^{**} \leq \left(\frac{r^L+\alpha(r^H-r^L)}{\alpha(r^H-r^L)}\right)^{\alpha}$ which is the case if $e \leq \tilde{e}$ where $\tilde{e}$ is the endowment for which the following condition holds as strict equality

$$\left(\frac{(r^H-\frac{1}{\delta}-\Phi'(I+L))e^{\frac{\alpha\beta}{1-\beta}}}{pc\alpha Ak^{**\frac{\alpha+\beta-1}{1-\beta}}(\frac{r^H}{r^L}-\frac{(e-k^{**})}{r^L}\frac{(\alpha+\beta-1)}{(1-\beta)k^{**}}(r^H-r^L)+\frac{\beta}{1-\beta})}\right)^{\frac{1-\beta}{\beta}} \leq \left(\frac{r^L+\alpha(r^H-r^L)}{\alpha(r^H-r^L)}\right)^{\alpha}.$$

C.7. Proof of Lemma 5

First assume that $L^* > 0$ and $L^{**} > 0$. Combining conditions (13), (14) and (20), (21) respectively yields the balance sheet size $(I+L)$ given by

$$(I+L)^* = (I+L)^{**} = (\Phi'(I+L))^{-1}\left(\frac{r^H-1/\delta(1+c)+c(1-p)}{(1+c)}\right).$$

Further, condition (13) implies that $q(b)^* = \frac{r^H pc}{r^H-1/\delta-pc-\Phi'(I+L)+pc}$ and (20) implies that $q^{**} = \frac{r^H pc}{r^H-1/\delta-pc-\Phi'(I+L)+pc+x}$, where $x = \frac{pc}{r^L}(cI-L)(r^H-r^L)(y''_k(.)+y''(.)_{k,h}\frac{\partial h}{\partial L})-pcy'_h(.)\frac{\partial h}{\partial k}$, and $x<0$. This implies that $q(b)^* < q(b)^{**}$ and thus $k^{**}(b) > k(b)^*$. Since the balance sheet size is fixed it follows that $L^{**} > L^*$ and $I^{**} < I^*$.

Now consider the case where $L^* = 0$ and $L^{**} > 0$. By construction, this implies $L^{**} > L^*$. Combining the first order conditions of the decentralized allocation (13) and (14) as well as the planner's FOCs (20) and (21) yields

$$\begin{aligned}(I+L)^* &> (\Phi'(I+L))^{-1}\left\{\frac{r^H-\frac{1}{\delta}(1+c)+c(1-p)}{(1+c)}\right\},\\ (I+L)^{**} &= (\Phi'(I+L))^{-1}\left\{\frac{r^H-\frac{1}{\delta}(1+c)+c(1-p)}{(1+c)}\right\},\end{aligned}$$

which immediately implies $I^* > I^{**}$, $k(b)^{**} > k(b)^*$ and $q(b)^{**} > q(b)^*$.

Finally consider the case $L^* = 0$ and $L^{**} = 0$ and thus $L^{**} = L^*$. Further, the first-order conditions with respect to investment in the decentralized (13) and the constrained efficient allocation (20) can be written as a function of I only:

$$\frac{1}{\delta}+pc+\Phi(I) = r^H - pc\left(\frac{r^H y'(e-cI,h(e-cI,P_2(b)),1)}{r^L}-1\right),$$
$$\frac{1}{\delta}+pc+\Phi(I) = r^H - pc\left(\frac{r^H y'(e-cI,H(e-cI,P_2(b)),1)}{r^L}-1\right)+x,$$

where $x=\frac{pc}{r^L}cI(y''_k(.)+y''_{k,h}(.)\frac{\partial\mathscr{H}(.)}{\partial k})-pcy'_h(.)\frac{\partial\mathscr{H}(.)}{\partial k}$. Note first that the lhs is strictly increasing in I due to the convexity assumptions on $\Phi(I+L)$. The rhs is strictly decreasing in I due to the concavity assumption on $y(.)$. Further note that for $I\to 0$ the rhs is greater than the lhs (given the assumption $1/\delta+pc\leq r^H$) while the lhs clearly exceeds the rhs for sufficiently large levels of I. Thus, for each level of P_2, there exists a unique investment value I^* and I^{**}, respectively, such that the above conditions hold with equality. Since $x<0$, the rhs is strictly lower in the constrained efficient allocation which implies a lower level of I, i.e. $I^{**}<I^*$ and $k(b)^{**}>k(b)^*$ and $q(b)^{**}<q(b)^*$.

C.8. Proof of Lemma 6

With τ and r bank's FOCs yield

$$\Phi'(I+L)+pc+1/\delta(1+\tau) = r^H - pc(\frac{r^H}{q(b)}-1)$$
$$\Phi'(I+L)+1/\delta = (1+r)+p(\frac{r^H}{q(b)}-1)$$

The conditions coincide with the planner's FOCs if $\tau=\tau^{**}$ and $r=r^{**}$.

C.9. Proof of Proposition 3

The optimal tax rate can be rewritten as a function of h and k:

$$\tau^{**}=\delta qcy'_k(.)\left(\frac{(e-k^{**})}{r^L k^{**}}(\frac{1-\alpha-\beta}{1-\beta})(r^H-r^L)+\frac{\beta}{1-\beta}\right)$$

The derivative of the optimal tax rate with respect to the price level $P_2(b)$ is given by (dropping the constant δqc)

$$\begin{aligned}\frac{\partial \tau}{\partial P_2(b)} &= \frac{\partial k(b)}{\partial P_2(b)}\Bigg\{\left(y''_k(.)+y''_{k,h}(.)\frac{\partial \mathscr{H}(.)}{\partial k}\right)\left(\frac{(e-k^{**})}{r^L k^{**}}(\frac{1-\alpha-\beta}{1-\beta})(r^H-r^L)+\frac{\beta}{1-\beta}\right)\\ &\quad -y'_k(.)\frac{e}{r^L k^2}\frac{1-\alpha-\beta}{1-\beta}(r^H-r^L)\Bigg\}\\ &\quad +y''(.)_{k,h}\frac{\partial \mathscr{H}(.)}{\partial P_2}\left(\frac{(e-k^{**})}{r^L k^{**}}(\frac{1-\alpha-\beta}{1-\beta})(r^H-r^L)+\frac{\beta}{1-\beta}\right) \end{aligned} \tag{32}$$

First note that the term in curly brackets can be simplified to

$$\alpha A k^{\alpha-2}h^{\beta}\left[(\alpha-1)+\frac{\alpha\beta}{1-\beta}\right],$$

which is negative under the assumption $\alpha+\beta<1$. Further, the equilibrium capital stock k^{**} is increasing in the price level $P_2(b)$, i.e. $\frac{\partial k}{\partial P_2(b)}>0$. However, because the last term of equation (32) is strictly positive the sign of the expression is generally unclear as there is no closed form solution for the capital stock k^{**}. In the following, I proceed by first splitting $\frac{\partial k}{\partial P_2(b)}$ in two terms which are both strictly positive. I then use the term which I can write down explicitly to show that the positive term in condition (32) cancels out with one negative term to show that the overall sign of condition (32) must be negative.

Using condition (6), the equilibrium capital stock can be expressed as

$$k(b)^{**}=\Big(\frac{r^L}{q(b)A(\frac{P_2(b)}{e^{\alpha}})^{\frac{\beta}{1-\beta}}}\Big)^{\frac{1-\beta}{\alpha+\beta-1}}. \tag{33}$$

The marginal derivative of the capital stock with respect to the period-2 price level can then be split into

$$\begin{aligned}\frac{\partial k(b)}{\partial P_2(b)} &= \frac{\partial k(b)}{\partial P_2(b)}+\frac{\partial k(b)}{\partial q(b)}\frac{\partial q(b)}{\partial P_2(b)}\\ &= \frac{\beta}{1-\alpha-\beta}\frac{k(b)}{P_2(b)}+\frac{1-\beta}{1-\alpha-\beta}\frac{k(b)}{q(b)}\frac{\partial q(b)}{\partial P_2(b)}.\end{aligned}$$

Accordingly, all terms of this expression are strictly positive if $\frac{\partial q(b)}{\partial P_2(b)}>0$. To see that this is the case, note that condition (21) can be written as:

$$\frac{1}{\delta}+\Phi'(I+L) = 1+p(\frac{r^H}{q(b)}-1)-p\frac{(e-k)}{r^L}\left(\alpha A k^{\alpha-2}h^{\beta}\frac{\alpha+\beta-1}{1-\beta}(r^H-r^L)\right)+\frac{\alpha\beta}{1-\beta}Ak^{\alpha-1}h^{\beta}.$$

Replacing h using (5) yields

$$\frac{1}{\delta}+\Phi'(I+L) = 1+p(\frac{r^H}{q(b)}-1)-p\frac{(e-k)}{r^L}\left(\alpha A k^{\alpha+2\beta-2}(\frac{P_2(b)}{e^{\alpha}})^{\frac{\beta}{1-\beta}}\frac{\alpha+\beta-1}{1-\beta}(r^H-r^L)\right) + \frac{\alpha\beta}{1-\beta}Ak^{\alpha+\beta-1}(\frac{P_2(b)}{e^{\alpha}})^{\frac{\beta}{1-\beta}}, \quad (34)$$

which is, for a given level of $P_2(b)$ and using condition (33), a function of $q(b)$ only. Now note that the lhs of (34) is a constant because $\Phi'(I+L)^{**}=\frac{r^H-\frac{1}{\delta}(1+c)+c(1-p)}{(1+c)}$. Further the rhs is strictly decreasing in $q(b)$ and the rhs strictly exceeds the lhs for q sufficiently small whereas the lhs strictly exceeds the rhs for q sufficiently large ($1/\delta+\Phi(I+L)>1$). Thus, there is a unique value of $q(b)^{**}$ which solves condition (34). Finally, note that as the price level increases the rhs increases too which implies that $q(b)^{**}$ increases and thus $\frac{\partial q(b)}{\partial P_2(b)}>0$.

In the last step of the proof, I now show that the following two terms of condition (32) coincide

$$y''(.)_{k,h}\frac{\partial\mathcal{H}(.)}{\partial P_2}\left(\frac{(e-k^{**})}{r^L k^{**}}(\frac{1-\alpha-\beta}{1-\beta})(r^H-r^L)+\frac{\beta}{1-\beta}\right)=$$
$$\left(y''_k(.)+y''_{k,h}(.)\frac{\mathcal{H}(.)}{\partial k}\right)\left(\frac{(e-k^{**})}{r^L k^{**}}(\frac{1-\alpha-\beta}{1-\beta})(r^H-r^L)+\frac{\beta}{1-\beta}\right)\frac{\beta}{1-\alpha-\beta}\frac{k(b)}{P_2(b)}$$

which can be simplified to

$$\frac{\alpha\beta}{1-\beta}\frac{Ak^{\alpha-1}h^{\beta}}{P_2(b)}=\left(\alpha A k^{\alpha-2}h^{\beta}\frac{1-\alpha-\beta}{1-\beta}\right)\frac{\beta}{1-\alpha-\beta}\frac{k(b)}{P_2(b)}.$$

Because the second part of the marginal derivative of capital with respect to the price level is positive and all other parts of condition (32) are negative this hence proves that the marginal derivative of the optimal tax rate with respect to the price level is negative, i.e. $\frac{\partial\tau}{\partial P_2(b)}<0$, showing that the equilibrium tax rate decreases as the price level $P_2(b)$ increases. The same result must then apply for the optimal interest rate on liquid assets r^{**}.

C.10. Proof of Proposition 4

To proof this, first note that the decentralized first order conditions (13) and (14) hold with equality and determine the asset price $q(b)$ and the balance sheet size $(I+L)$ independent of the price level $P_2(b)$, given by

$$\begin{aligned} q(b)^* &= r^H \frac{p(1+c)}{r^H-(1-p)}, \\ (I+L)^* &= (\Phi')^{-1}(r^H - pc(\frac{r^H}{q(b)}^* - 1) - 1/\delta - c). \end{aligned}$$

Using the definition of the asset price and the labor demand function the capital stock can be written as a function of the price level, i.e., $k(b)^* = (\frac{r^L}{q^*(b)A(\frac{P_2(b)}{e^{\alpha}})^{\frac{\beta}{1-\beta}}})^{\frac{1-\beta}{\alpha+\beta-1}}$ which also determines the level of investment $I^* = \frac{e+(I+L)^*-k(b)^*}{(1+c)}$ and liquid assets $L^* = (I+L)^* - I^*$. The objective function of the CB can then be written as a function of $P_2(b)$ only

$$\begin{aligned} U &= \frac{1}{\delta}(X-(I+L)^*-B)+r^H I^*(P_2)+(1-p)L^*(P_2)+y(e,1,1)(1-p)-\Phi((I+L)^*) \\ &\quad +p\left\{\frac{-(e-k^*(P_2))}{q(b)}(r^H-r^L)+y(k^*(P_2),h^*(P_2),1)\right\}. \end{aligned}$$

Taking the derivative with respect to $P_2(b)$ yields[33]

$$y_h'(.)\frac{\partial h}{\partial P_2(b)}p+\frac{\partial k}{\partial P_2(b)}\left(\frac{(1-p)}{(1+c)}-\frac{r^H}{(1+c)}+\frac{pr^H}{q(b)}\right) \geq 0$$

Using the definition of $q(b)$, this condition reduces to $y_h'(.)\frac{\partial h}{\partial P_2(b)}p \geq 0$. Since the lhs is strictly positive, the CB strictly prefers to implement the full employment regime. From the definition of $k(b)$ it is straightforward to see that a higher price level also increases the capital stock in the bad state and since the balance sheet size is fixed and independent of the price level this implies a lower level of investment and a higher level of liquid assets.

C.11. Proof of Proposition 5

The first order condition of the TS with respect to their period 1 resources e is given by

$$\frac{1}{\delta}-1=(1-p)(y'(e,1,1)-1)+p(\frac{r^H}{q(\theta)})-1),$$

which yields

$$\frac{\frac{1}{\delta}-1-p(\frac{r^H}{q(\theta)})-1)}{1-p}+1=y'(e,1,1).$$

[33] Note that private agents never fully insure against fire-sale risk if the endowment in the TS is sufficiently high (see lemma 5 and thus $\zeta = 0$).

Using the definition of $q(b)$ for $L^* > 0$ then immediately implies the above condition.

Similarly, taking the derivative of Lagrangian (20) with respect to e yields

$$\frac{1}{\delta} - 1 = (1-p)(y'(e,1)-1) + p\left\{(\frac{r^H}{q(\theta)} - 1) - \frac{cI-L}{r^L}(r^H - r^L)(y''_k(.) + y''_{k,h}(.)\frac{\partial \mathscr{H}(.)}{\partial k}) + y'_h(.)\frac{\partial \mathscr{H}(.)}{\partial k}\right\}.$$

Combining this expression with condition (21) and using the definition of $(I+L)^{**}$ then implies the above condition.