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The Effects of Economic Shocks on Heterogeneous Inflation Expectations

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Prepared by Yoosoon Chang, Fabio Gómez-Rodríguez and Gee Hee Hong*

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ABSTRACT: In this paper, we examine how economic shocks affect the distribution of household inflation expectations. We show that the dynamics of households' expected inflation distributions are driven by three distinctive functional shocks, which influence the expected inflation distribution through disagreement, level shift and ambiguity. Linking these functional shocks to economic shocks, we find that contractionary monetary shocks increase the average level of inflation expectation with anchoring effects, with a reduction in disagreement and an increase in the share of households expecting future inflation to be between 2 to 4 percent. Such anchoring effects are not observed when the high inflation periods prior to the Volcker disinflation are included. Expansionary government spending shocks have inflationary effects on both short and medium-run inflation expectations, while an increase in personal income tax shocks is inflationary for medium-run. A surprise increase in gasoline prices increases the level of inflation expectations, but lowers the share of households with 2 percent inflation expectations.

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WORKING PAPERS

The Effects of Economic Shocks on Heterogeneous Inflation Expectations

Prepared by Yoosoon Chang, Fabio Gómez-Rodríguez, and Gee Hee Hong[1]

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Glossary

EID	Expected Inflation Expectations
FAR	Functional Autoregression
VAR	Vector Autoregression
FPCA	Functional Principal Component Analysis
FR2	Functional R-squared

Executive summary

Inflation expectations are at the heart of macroeconomics and the conduct of monetary policy. In this paper, we argue that the average inflation expectations do not properly summarize heterogeneous beliefs about inflation expectations and that the entire distribution of inflation expectations should be considered. To this end, we apply a functional approach to a survey of household inflation expectations using the data from the University of Michigan's *Survey of Consumers* to document how economic shocks affect the distribution of household inflation expectations.

We show that the dynamics of households' expected inflation distributions are driven by three distinctive functional shocks, which collectively influence the expected inflation distribution through (i) disagreement, (ii) level shift and (iii) degree of inflationary ambiguity. Linking these functional shocks to externally identified economic shocks, we find that contractionary monetary shocks increase the average level of inflation expectation with *anchoring effects*, with a reduction in disagreement and an increase in the share of households expecting future inflation to be between 2 to 4 percent. Such *anchoring effects*, however, are not observed when the periods of high inflation prior to the Volcker disinflation are included. Expansionary government spending shocks have inflationary effects on both short and medium-run inflation expectations, while an increase in personal income tax shocks is inflationary only for medium-run. A surprise increase in gasoline prices increases the level of inflation expectations, but lowers the share of households with 2 percent inflation expectations.

Based on these findings, we highlight the role of economic policies in affecting inflation expectations and that each transmission channel should be studied carefully, individually and jointly. This conclusion invites us to reconsider the policy recommendations derived from previous findings in the literature, which show that, aside from gasoline price shocks, there is little room for economic policies to affect economic agents' inflation expectations. We emphasize the need to better understand how households perceive economic shocks and update their expectations for future inflation based on these shocks. This is related to the question of whether economic policies can be designed to ensure the anchoring of inflation expectations.

1 Introduction

Perhaps more importantly, we need to know more about the manner in which inflation expectations are formed and how monetary policy influences them. (Janet Yellen, 2016)

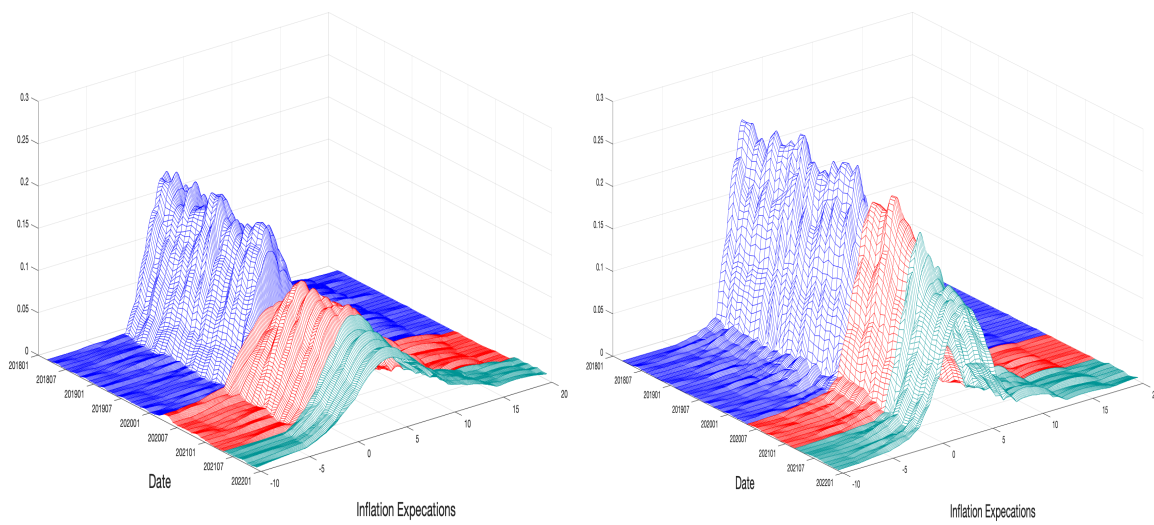
The seminal works of Friedman et al. (1968) and Phelps (1967) have placed inflation expectations at the heart of macroeconomics and the conduct of monetary policy. There is a well-established literature, showing that households' spending and savings decisions depend on their expectations about future inflation. Firms' wage and price-setting mechanisms also hinge crucially on their expectations on future prices. Empirically, there is a rich and burgeoning literature that draws on surveys to examine the behavior of households and firms' inflation expectations. In this literature, however, researchers tend to focus on specific statistics of survey responses, such as the mean and standard deviation. Furthermore, studies that use inflation expectations, for instance, the estimation of the Phillips Curve or expectations formation, use the average of heterogeneous survey responses at a given time as *the* inflation expectation.[1]

In this paper, we use a novel functional approach to examine the distribution of inflation expectations and how it is affected by economic shocks. Throughout the paper, the acronym 'EID' is used to denote the expected inflation distribution, as well as its corresponding (estimated) density. To motivate the discussion, Figure 1 plots the change in the distribution of household inflation expectations for the one-year ahead (left panel) and medium-run (right panel) inflation expectations from March 2018 to December 2021, using data from the University of Michigan's *Survey of Consumers*. The sample is separated into three sub-periods: pre-pandemic months (March 2018 to February 2020, in blue), early pandemic months before the increase in inflation (March 2020 to March 2021, in red), and the pandemic months with higher levels of inflation (April 2021 to December 2021, in green). The shape of the distribution for one-year ahead inflation expectations changed noticeably between the pre-pandemic periods (blue) and the early pandemic months (red), as the curve flattened and the concentration of survey responses at the modal value declined sharply over time. Despite this visible change in the shape of the distributions, the average inflation expectations of survey responses barely moved, from 2.7 percent in the pre-pandemic months

[1]Recent work by Reis (2021), which examines different moments and tail behavior of inflation expectations in several episodes of inflation in numerous countries, is a notable exception. Here again, higher-moments were examined, rather than the entire distribution.

(blue) to 2.8 percent in the early pandemic months (red). The same holds for medium-run inflation expectations, whose change in the average from 2.4 percent (blue) to 2.5 percent (red) undermines the significant change we see in the shape of the distributions.

Figure 1. *Evolution of the Distribution of Inflation Expectations: One-year-ahead vs. Medium-run*



Notes: The distributions of inflation expectations at monthly frequency are separated into three sub-periods: from March 2018 to February 2020 (blue), from March 2020 to March 2021 (red), and from April 2021 to December 2021 (green). Left panel plots one-year ahead household inflation expectations. Right panel plots medium-run household inflation expectations. Both series are from the University of Michigan's Survey of Consumers.

To understand the movements in the distribution of inflation expectations beyond specific moments and their potential economic implications, we use a functional approach based on the entire distribution of inflation expectations. Especially in a low-inflation environment, the conventional wisdom is that households are inattentive to economic shocks, causing inflation expectations to remain unresponsive to them. However, when we examine the behavior of the distribution of inflation expectations, we find that monetary and fiscal policy shocks affect household inflation expectations in their own distinct ways. The only shock that the literature shows, time and again, to affect household inflation expectations is gasoline prices. We confirm these findings, i.e. that the change in gasoline prices is an important driver of household inflation expectations. Our results, however, are more nuanced, as we see the distributional effects of gasoline price shocks on household inflation expectations.

Using survey data on household inflation expectations from the University of Michigan's *Survey of Consumers* from January 1983 to December 2021, we summarize the rich information presented in the cross-section of responses each month into a density function representing the underlying distribution of household inflation expectations. We then apply a novel functional approach to the time series of estimated densities. This process shows that the estimated EIDs are driven by three distinctive functional shocks, which collectively influence the expected inflation distribution through (i) disagreement, (ii) level shift, and (iii) degree of inflationary ambiguity. We then connect these functional shocks to externally identified economic shocks, such as monetary policy, government spending, personal income tax, and gasoline price, to demonstrate EID's response to them.

Starting with monetary policy, we find that a contractionary shock increases the average expected inflation for one-year ahead inflation expectations. At the same time, EID has the desired *anchoring effects*, as evidenced by a reduction in expected inflation disagreement and an increase of the share of households expecting inflation to be in the range of 2 to 4 percent. When we extend the sample to include the high inflation period, from the late 1970s to early 1980s, however, the distributional response of inflation expectations to monetary policy shocks shows a marked difference, as they lead to a fatter right tail (i.e. more households believing that inflation will be at high levels) and an increase in dispersion. This finding supports a shift in the conduct of monetary policy since 1979, which is well-documented in the literature (Judd, Rudebusch, et al., 1998; Clarida, Gali, and Gertler, 2000; Boivin and Giannoni, 2006). Compared to these previous studies, our paper provides additional evidence obtained from the analysis of the entire distribution of inflation expectations.

A surprise hike in gasoline prices increases the level of inflation expectations for both the one-year and the medium-run horizons, which is consistent with the findings in the literature highlighting the role of gasoline prices in the formation of household inflation expectations as documented in Coibion and Gorodnichenko (2015), among others. At the same time, measured by the standard deviation, gasoline price shocks decrease the disagreement on one-year ahead inflation expectations. However, for the medium-term horizon, it widens the disagreement among survey respondents, possibly reflecting uncertainty about the future path of gasoline prices. A shock to gasoline prices is found to lower the share of households with inflation expectations at 2 percent, suggesting that gasoline price hikes may affect the central bank's inflation targeting objective.

To our knowledge, with the notable exceptions of D'Acunto, Hoang, and Weber (2018) and Coibion, Gorodnichenko, and Weber (2021), the effects of fiscal policy shocks on inflation expectations are relatively under-studied in the literature.[2] We find that fiscal policy shocks also affect the distribution of household inflation expectations. Government spending shocks are inflationary for both one-year and medium-run inflation expectations, although there is a larger disagreement among households on the effects of government spending on medium-run inflation expectations. This may be attributable to households' varying perceptions of the multiplier of fiscal measures or future fiscal conditions. Shocks to personal income tax expectations, on the other hand, have no effect on one-year ahead inflation expectations but have long-term effects on medium-run inflation expectations.

Based on these findings, we are able to deduce that economic policies play a non-negligible role in affecting inflation expectations and that each transmission channel should be studied carefully, individually and jointly. This conclusion invites us to reconsider the policy recommendations derived from previous findings in the literature, which show that, aside from gasoline price shocks, there is little room for economic policies to affect economic agents' inflation expectations. What has been highlighted instead is the role of credibility played by the central banks in anchoring inflation expectations. Our findings do not contradict the importance of credible and clear communication by the central bank to anchor inflation expectations. Rather, we emphasize the need to better understand how households perceive economic shocks and update their expectations for future inflation based on these shocks. This is related to the question of economic policies can be designed to ensure the anchoring of inflation expectations.

The contribution of our work to the literature is three-fold. First, it contributes to the rich empirical literature using survey data on inflation expectations to understand the formation of inflation expectations. There have been significant recent efforts to understand how households and firms form their inflation expectations based on various survey data (see Coibion, Gorodnichenko, and Kamdar (2018) for a recent survey). A strand of this research field examines the disagreement of inflation expectations across survey respondents, which is a specific moment in the distribution of inflation expectations. These studies look at the disagreement among respondents to infer how well-anchored inflation expectations are and have established information rigidities as an

[2] D'Acunto, Hoang, and Weber (2018) discusses how to design a specific path of consumption tax to affect inflation expectations when monetary policy is constrained at zero lower bound. Our interest lies in more general fiscal policy shocks.

important source of cross-sectional variation among survey respondents on their views on future inflation (Mankiw, Reis, and Wolfers, 2004; Dovern, Fritsche, and Slacalek, 2012; Andrade et al., 2016; Coibion, Gorodnichenko, and Weber, 2019; Bems et al., 2021). Second, the paper contributes to the literature on the drivers of inflation expectations. Households are shown to be inattentive to monetary policy shocks and other policy shocks when inflation expectations are well-anchored (Coibion et al., 2020), while some studies find that monetary policy shocks increase the disagreement among survey respondents due to information rigidities (Grigoli, Gruss, and Lizarazo, 2020). The transmission of fiscal policy shocks to inflation expectations is less studied, except to understand how the fear of fiscal dominance potentially undermines the independence of a central bank in the context of emerging economies (Favero and Giavazzi, 2004). One notable exception is the recent work by Coibion, Gorodnichenko, and Weber (2021), which looks at the impact of fiscal news (deficits and public debt) on inflation expectations in the US. Finally, we analyze the dynamics of the distribution of inflation expectations fully and completely using a novel methodology developed by Chang, Park, and Pyun (2021) for functional autoregressions. The framework and methodology in the paper are widely applicable to study various functional dynamics in many different areas of economics. See Inoue and Rossi (2019), Chang, Hu, and Park (2020), Chang, Kim, and Park (2016), Chang, Chen, and Schorfheide (2018) for other related approaches and issues in analyzing economic models with functional variables.

Our paper is constructed as follows. Section 2 documents the characteristics of the distribution of inflation expectations to motivate our approach. Section 3 provides the description of the methodology that transforms a functional autoregression model (FAR) to a vector autoregression (VAR) model. Section 4 describes the implementation of the econometric methodology using the Survey of Consumers database. Section 5 presents the findings on the dynamic effects of economic shocks on EID. Finally, Section 6 concludes the paper.

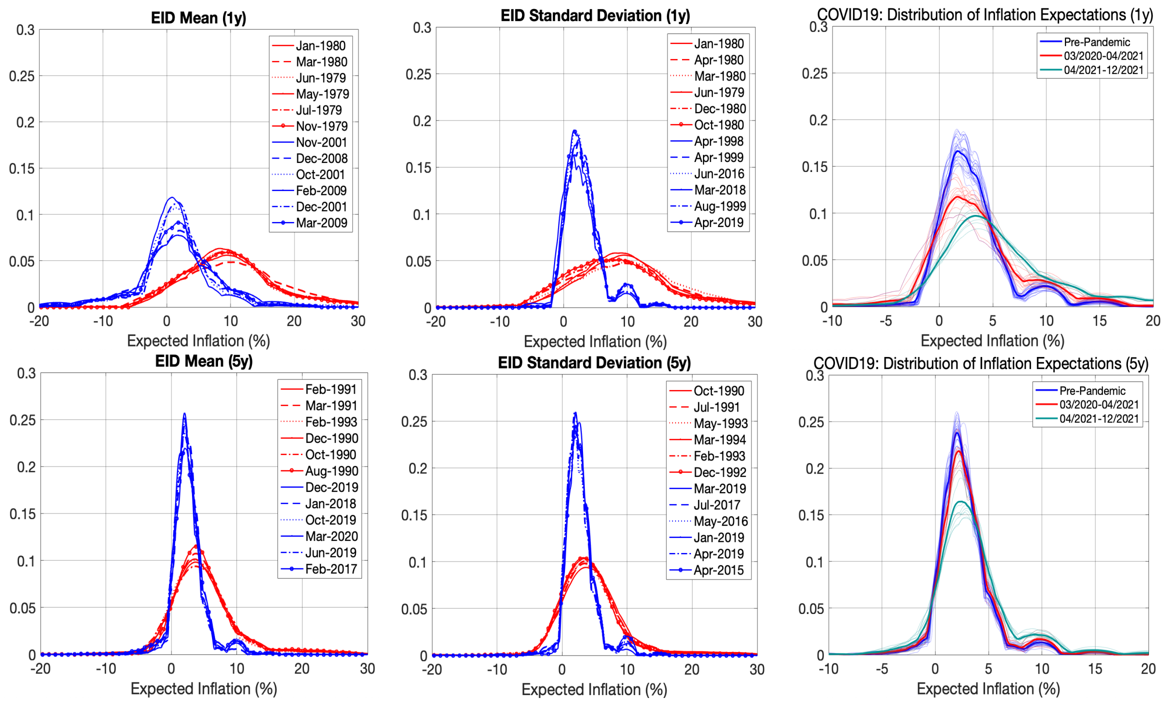
2 Characteristics of the Distribution of Inflation Expectations

To start, this section illustrates the dynamics of the distribution of inflation expectations. Then, we use simple correlations to show how different macroeconomic variables relate to different features of the distribution of inflation expectations.

Figure 2 presents selected observations of the distribution of inflation expectations starting in the late 1970s to document the substantial shift over time of the entire distribution of inflation expectations (as opposed to only its average). The top panels are for one-year inflation expectations, and the bottom panels show medium-run inflation expectations. In the left-most column, we find the months with the highest and lowest values of average inflation expectations. There, we observe that the months with the highest average inflation expectations were in 1979 and 1980, the years preceding the Volcker disinflation in the early 1980s. During the Great Moderation, the months with the lowest one-year ahead inflation expectations were observed. For medium-run inflation expectations, the months with the lowest means are from 2017 to 2019, a period known for its persistently low inflation below the target. In the center column, we select the months showing the highest and lowest values of standard deviation. There is a strong correlation between the months chosen for high average inflation expectations and those with the high standard deviations. Compared to the one-year ahead inflation expectations, the medium-run inflation expectations show lower disagreement, even for the months where the standard deviation is relatively high. The right-most column plots the average distributions of inflation expectations as dark solid lines from January 2018 to December 2021, split into three periods as in Figure 1, where the colors match the shaded region of Figure 1. We reiterate that the average value does not fully represent the change in the shape of distributions. For one-year ahead inflation expectations, the distribution of early pandemic months, represented by the red line, has fatter tails for inflation expectations above 5 percent, even though the average value for the blue line and the red line is very similar. The distribution of the medium-run inflation expectations has also shifted to the right over time, albeit more moderately.

Furthermore, we examine the correlation between different features of the distribution of inflation expectations and key macroeconomic variables to underscore the importance of considering the entire distribution. Table 1 displays the regression results of three key economic indicators on different aspects of the distribution. These aspects include the mean, standard deviation, skewness and kurtosis of households' one-year ahead inflation expectations for each month. In addition, we look at the share of households who expect future inflation to be at 0 percent (or 'Frequency at 0 percent') and the share of households who expect future inflation to be below zero (deflation expectations). The regressions reported here have been chosen among various combinations of the moments that result in the largest R^2 for each regression. We see that the chosen macroeconomic

Figure 2. Distributions of Inflation Expectations with Largest and Smallest Means and Standard Deviations



Notes: The top panel represents distributions for the one-year ahead inflation expectations. The bottom panel represents the distributions for the medium-run inflation expectations. The left column plots the densities of six observations with highest means and six observations with lowest means across the sample. The center column shows six observations with highest disagreements (measured by standard deviations) and six observations with lowest disagreements across the sample. The right column shows the change in the average distribution for three different periods: blue (from 2018/01 to 2020/02), red (from 2020/03 to 2021/03), and green (2021/04 to 2021/12).

indicators show varying degrees of correlation with different features of the distribution of inflation expectations. Surprisingly, the mean inflation expectation is not always selected in the group of EID properties explaining the largest portion of variance in macroeconomic variables, as is the case of real GDP growth.

Figure 3 reinforces this point by plotting the time series of macroeconomic variables and selected aspects of inflation expectations. We specifically examine the relationship between key macroeconomic variables (real GDP growth, CPI inflation, and unemployment rate) and various aspects of EID (frequency of inflation expectations at 2%, average mean, and deflation expectations). While average inflation expectations and some macroeconomic variables such as real GDP growth and CPI inflation seem to be relatively highly correlated, the correlation between average inflation expectations and the unemployment rate is weak, as suggested by the flattening of the Phillips Curve. On the other hand, the correlation between deflation expectations and the unemployment

Table 1. *Correlation between Macroeconomic Variables and Different Moments of Inflation Expectations*

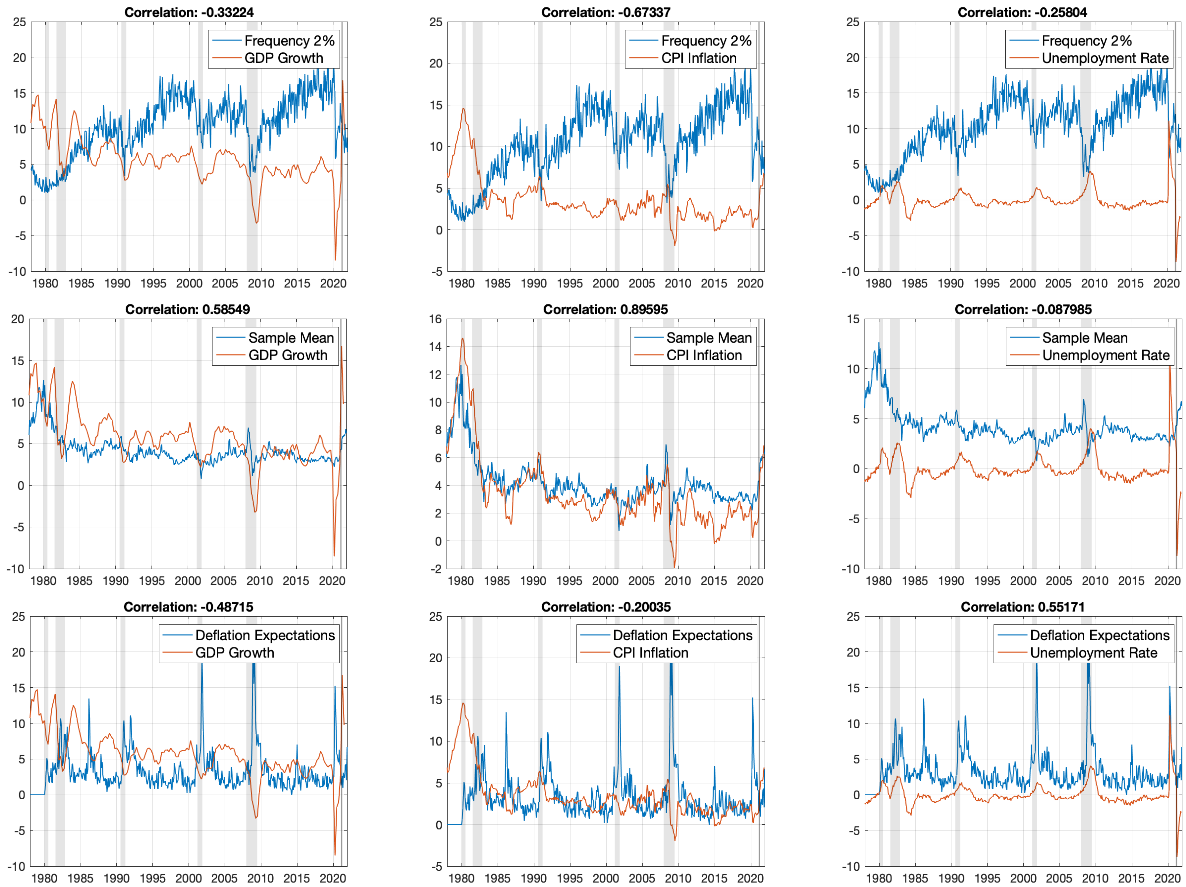
	GDP growth	Inflation	Δ Unemployment
	Estimate (p-Value)		
Intercept	2.1156 (0.000001)	−4.1311 (0.0000)	−2.1522 (0.0000)
Mean		1.5426 (0.0000)	0.4090 (0.0000)
Standard Deviation	1.0444 (0.0000)		−0.2822 (0.0000)
Skewness		−0.2044 (0.0026)	
Kurtosis		0.0329 (0.0002)	
Frequency at 0 percent		6.2283 (0.0000)	4.7667 (0.0000)
Frequency below 0 percent (Deflation)	−57.722 (0.0000)		33.4130 (0.0000)
R^2	0.502	0.828	0.359

Notes: The table reports the coefficients from regressions of *CPI inflation*, changes in *unemployment* and *GDP growth* to different aspects of the distribution of inflation expectations. Mean, standard deviation, skewness and kurtosis refer to the sample estimates from one-year ahead household inflation expectations using the University of Michigan's *Survey of Consumers*. Standard errors are reported in parentheses.

rate is relatively strong, suggesting that some aspects of household inflation expectations are sensitive to labor market conditions.

Finally, from an econometric perspective, we argue that using a set of pre-specified moments (such as mean or standard deviation) is inferior in explaining the variation in EID compared to using the basis from functional principal component analysis. Table 4 in Appendix Section A compares the functional R-squared (or FR2), which measures the extent to which variations in EIDs are explained by four different approaches of selecting a basis. For each column, the increase in the number of bases (m) leads to an increase in FR2. The first column reports the methodology adopted in this paper, which is to use functional principal component analysis. Using the basis derived from the functional principal component analysis, the first component alone explains about 74 percent of the variations in EIDs. In this paper, we choose the number of bases (m) to be three, which explains EID variations over 90 percent. Compared to this, we see that three other choices of basis barely explain EID variations. For instance, the moment basis approach in the fourth column, which is closest to the current literature by focusing on certain moments, fail to explain even 10 percent of EID variations, even when m equals 10.

Figure 3. Correlations between key macroeconomic variables and selected statistics of EID



Notes: The plots show the correlations between key macroeconomic variables (real GDP growth in the first column, CPI inflation in the second column, and unemployment rate in the third column) in orange and three statistics of the EID (frequency of inflation expectations at 2 percent in the first row, the average in the second row, and deflation inflation expectations in the final row) in blue. Correlations between the pairs are reported at the top of each figure.

3 Econometric Methodology

Our analysis of EID relies on a functional autoregression (FAR). In this section, we present a theoretical framework on how we formulate and implement the FAR to study the dynamics of EID. In what follows, we denote by (f_t) the density representing the EID, which we formally view as a functional time series taking values in the Hilbert space H of square integrable functions defined in R generated as

$$f_t = A_1 f_{t-1} + A_2 f_{t-2} + \varepsilon_t, \tag{1}$$

where A_1 and A_2 are bounded linear operators on H and (ε_t) is a functional white noise in H, which will be defined more precisely below. The order of FAR is set by an order selection criterion such as AIC and BIC, as will be discussed in more detail in a later section.

To explain our model and methodology more precisely, we need to introduce some basics of the Hilbert space. The functional time series (f_t) represents a time series of random elements taking values in H, where H is endowed with the inner product $\langle \cdot, \cdot \rangle$ and the norm $\|\cdot\|$ given by

$$\langle f, g \rangle = \int f(r)g(r)dr \quad \text{and} \quad \|h\|^2 = \int h^2(r)dr$$

for all f, g and h in H. The autoregressive operators A_k, $k = 1$ and 2, are the bounded linear operators on H taking f to $A_k f$ for any f in H, which we may also define as

$$(A_k f)(r) = \int A_k(r,s)f(s)ds$$

and interpret A_k as a kernel function, where we use the same notation for both the operator and kernel given by A_k.

In addition to the inner product $\langle f, g \rangle$ of f and g in H, we need to introduce the tensor product $f \otimes g$ of f and g in H. In particular, for the functional white noise (ε_t) introduced in (1), we assume that (ε_t) is serially uncorrelated with $\mathbb{E}(\varepsilon_t) = 0$ and $\mathbb{E}(\varepsilon_t \otimes \varepsilon_t) = \Sigma$ for all $t \geqslant 1$. The tensor product $f \otimes g$ with any given f and g in H is a linear operator on H defined as $(f \otimes g)v = \langle v, g \rangle f$ for all v in H. If H is finite dimensional and represented by R^n, $f \otimes g = fg'$, i.e., $f \otimes g$ reduces to the outer product, in contrast to the inner product $\langle f, g \rangle = f'g$, where f' and g' are the transposes of f and g. If f and g are random functions taking values in H, then their covariance operator $\mathbb{E}(f \otimes g)$ is generally defined as a linear operator satisfying

$$\big\langle u, [\mathbb{E}(f \otimes g)]v \big\rangle = \mathbb{E}\langle u, f \rangle \langle v, g \rangle$$

for all u and v in H.

For a given orthonormal basis (v_i) of H, we may write any f in H as

$$f = \sum_{i=1}^{\infty} \langle v_i, f \rangle v_i,$$

and approximate it as

$$f \approx \sum_{i=1}^{m} \langle v_i, f \rangle v_i, \tag{2}$$

where m is the truncation number which should be chosen appropriately. We let V be the subspace of H spanned by a sub-basis $(v_i)_{i=1}^m$, and denote by P the Hilbert space projection on the m-dimensional subspace V. Then the approximation in (2) may simply be regarded as a projection of f on V yielding $Pf = \sum_{i=1}^m \langle v_i, f \rangle v_i$. Once f is approximated by an m-dimensional element Pf in V, we may represent it as an m-dimensional vector. Consequently, it is well expected that the FAR in (1) can be represented by a VAR upon this approximation. This will be explained in detail below.

We approximate the FAR in (1) as

$$\begin{aligned} f_t &= A_1 P f_{t-1} + A_2 P f_{t-2} + A_1(1-P) f_{t-1} + A_2(1-P) f_{t-2} + \varepsilon_t \\ &\approx A_1 P f_{t-1} + A_2 P f_{t-2} + \varepsilon_t, \end{aligned} \tag{3}$$

where $1 - P$ is an operator on H defined as $(1-P)f = f - Pf$ for all f in H. The approximation error terms $\big(A_k(1-P)f_{t-k}\big)$ for $k = 1$ and 2 are asymptotically negligible under suitable regularity conditions, if we set $m \to \infty$ as $T \to \infty$ at an appropriate rate. The required conditions are not very stringent and they are expected to hold generally. Our empirical analysis is based on the approximate FAR in (3). Subsequently, we will explain how we may represent this approximate FAR as a finite dimensional VAR.

Define a mapping

$$\pi : f \mapsto \llparenthesis f \rrparenthesis \equiv \begin{pmatrix} \langle v_1, f \rangle \\ \vdots \\ \langle v_m, f \rangle \end{pmatrix} \tag{4}$$

for any f in H, and

$$\pi : A \mapsto \llparenthesis A \rrparenthesis \equiv \begin{pmatrix} \langle v_1, A v_1 \rangle & \cdots & \langle v_1, A v_m \rangle \\ \vdots & \vdots & \vdots \\ \langle v_m, A v_1 \rangle & \cdots & \langle v_m, A v_m \rangle \end{pmatrix} \tag{5}$$

for any operator A on H. Then we may represent the approximate FAR in (3) as

$$\llparenthesis f_t \rrparenthesis \approx \llparenthesis A_1 \rrparenthesis \llparenthesis f_{t-1} \rrparenthesis + \llparenthesis A_2 \rrparenthesis \llparenthesis f_{t-2} \rrparenthesis + \llparenthesis \varepsilon_t \rrparenthesis, \tag{6}$$

a conventional m-dimensional VAR, which is referred to as the approximate VAR of our FAR. Note that $\big(\llparenthesis f_t \rrparenthesis\big)$ and $\big(\llparenthesis \varepsilon_t \rrparenthesis\big)$ are m-dimensional time series and $\llparenthesis A_1 \rrparenthesis$ and $\llparenthesis A_2 \rrparenthesis$ are $m \times m$ matrices. The approximate VAR in (6) is readily derived from the approximate FAR in (3), since we have $\llparenthesis APf \rrparenthesis = \llparenthesis A \rrparenthesis \llparenthesis f \rrparenthesis$ for any f in H and any operator A on H, and $\llparenthesis f + g \rrparenthesis = \llparenthesis f \rrparenthesis + \llparenthesis g \rrparenthesis$ for all f and g in H. The approximate FAR in (3) is therefore equivalent to the approximate VAR in (6), which implies that the original FAR in (1) may be analyzed by the approximate VAR in (6) if we let $m \to \infty$ as $T \to \infty$ as mentioned earlier. Indeed, Chang, Park, and Pyun (2021) show that the use of the VAR in (6) is valid for the general structural analysis of the FAR in (1) relying on the general sample and bootstrap asymptotic theories under mild conditions.

Although π's in (4) and (5) are defined for any f in H and for any linear operator A on H, we interpret them as their restricted versions on the linear subspace V spanned by the sub-basis $(v_i)_{i=1}^m$ whenever necessary. The restricted versions of π's are one-to-one, so that their inverses exist and are well defined. We may indeed easily show that

$$\pi^{-1}\big(\llparenthesis f \rrparenthesis\big) = Pf \quad \text{and} \quad \pi^{-1}\big(\llparenthesis A \rrparenthesis\big) = PAP.$$

Consequently, from the estimate $\widehat{\llparenthesis A_1 \rrparenthesis}$ and $\widehat{\llparenthesis A_2 \rrparenthesis}$ of the autoregressive coefficient matrices $\llparenthesis A_1 \rrparenthesis$ and $\llparenthesis A_2 \rrparenthesis$ and the fitted values $\big(\widehat{\llparenthesis \varepsilon_t \rrparenthesis}\big)$ of the residuals $\big(\llparenthesis \varepsilon_t \rrparenthesis\big)$ in (6), we may easily obtain the corresponding estimates $\widehat{A}_1$ and $\widehat{A}_2$ as linear operators on V and the fitted functional residuals $(\widehat{\varepsilon}_t)$ as a time series taking values in V.

The VAR representation in (6) may be obtained for any choice of an orthonormal basis (v_i) of H. The effectiveness of the resulting approximation, however, depends crucially on the choice of basis. Following Bosq (2000), Ramsay and Silverman (2005), Hall and Horowitz (2007), Park and Qian (2012) and Chang, Park, and Pyun (2021), among others, we use the functional principal component basis (v_i^*). For $i = 1, \ldots, T$, we define v_i^* as the eigen-function of the sample variance operator of EID given by

$$\Gamma = \frac{1}{T}\sum_{t=1}^{T}(f_t \otimes f_t) \tag{7}$$

associated with the i-th largest eigenvalue. The functional principal component basis (v_i^*) is known to most effectively approximate the temporal variations of functional time series, although other bases can also be used. In Appendix A, we demonstrate that the use of other bases such as the moment and quantile bases is much less effective. They not only explain much less of the variation in the EID, but also yield the estimators of autoregressive operators A_1 and A_2 in (1) with unacceptably large variances.

4 Functional Representations of Heterogeneous Beliefs on Inflation Expectations

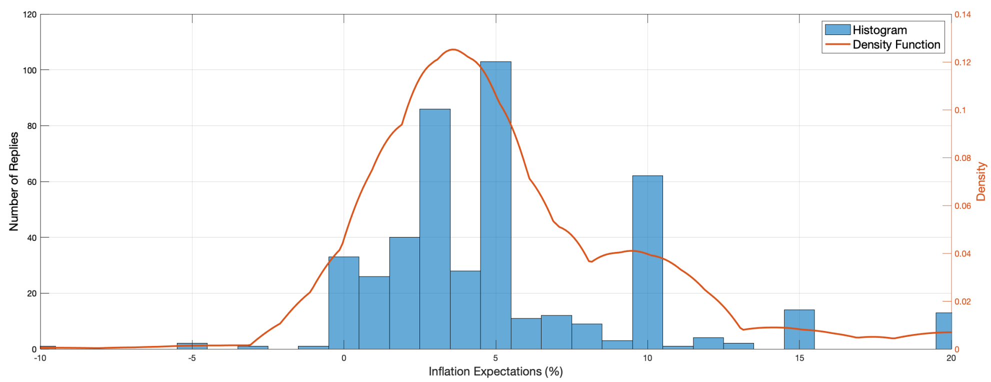
This section describes the implementation of the novel functional approach presented in Section 3 to analyze the effects of economic shocks on the functional variable EID. Based on the responses from the Survey of Consumers, we estimate the monthly time series of density functions representing the EID time series. Then, as described in the previous section, we consider the basis of functional principal components to obtain a precise finite-dimensional representation of EID. By applying a recursive identification to the corresponding vector autoregression (VAR), we obtain so-called *recursive functional shocks* (or simply functional shocks) and their respective functional impulse responses, which we call *baseline EID responses*. Finally, using the correlations between the functional shocks and external shocks as weights, we use the baseline EID responses to construct the response of the distribution of inflation expectations to economic shocks such as monetary, fiscal, and gasoline price.

We use the monthly data from January 1983 to December 2021 from the University of Michigan's *Survey of Consumers*, a *pseudo-panel* providing a time series of large scale repeated cross-sections.[3] We focus on the following two questions related to households' inflation expectations: (1) "About what percent do you expect prices to go (up/down) on the average, during the next twelve months?"; and (2) "About what percent do you expect prices to go (up/down) on the average, during the next five to ten years?" The responses to the first question are households' one-year ahead inflation expectations, and those to the second question are households' medium-run inflation expectations. The number of recorded responses for each month varies from 408 to 1205. During the first half of 1980s, there were about 1200 valid values on average, which decreased to about 600 during the second half of 1980s and around 450 afterwards.

[3]The original data starts from January 1978. As we will describe in Section 5, the longer sample starting from 1978 is used as a robustness check.

We summarize the monthly survey responses to a density function using a standard kernel density estimation method.[4] The estimated densities represent the underlying distribution of heterogeneous inflation expectations. In Figure 4, for a particular month, April 2011, we present an example of the estimated density of the one-year ahead expected inflation (red line) and the distribution based on actual responses (blue bar).

Figure 4. *Actual Survey Responses and the Estimated Density: An Example*



Notes: Survey responses of one-year ahead inflation expectations in April 2011 was used as an example. Y-axis on the left represents the frequency of survey responses based on the actual survey responses. Y-axis on the right shows the density of inflation expectations based on the estimated density.

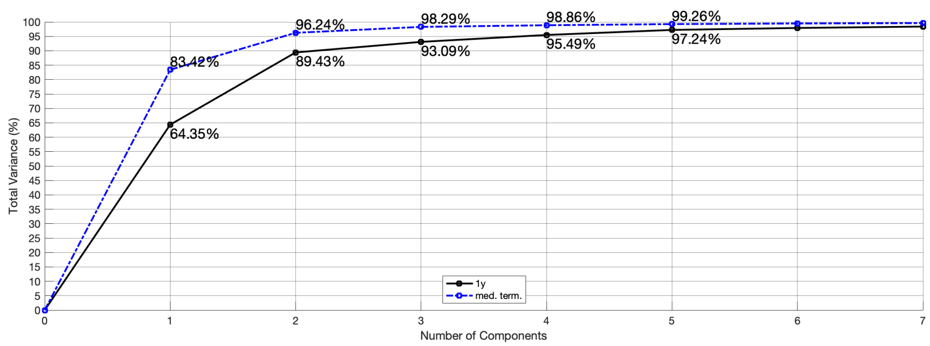
Functional Principal Component Analysis

We apply the functional principal component analysis (FPCA) to these estimated densities to extract the leading components of one-year ahead and medium-run expected inflation distributions. The scree-plot presented in Figure 5 shows the relationship between the number of functional principal components and the cumulative share of the total variance of the EID explained. The first principal component explains more than 64 percent of the total variance in one-year inflation expectations. In turn, the first three components account for more than 93 percent of the total variance in one-year inflation expectations and more than 98 percent of the variance in medium-run inflation expectations. Based on this evidence, in this paper, we choose the first three elements

[4]Density functions are estimated using the Epanechnikov's kernel function with the rule-of-thumb time-varying bandwidth.

($m = 3$) to implement our functional approach described in the previous section, which seems sufficient since they capture most of the variation of EIDs over time.[5]

Figure 5. *Percent of Variance Explained by the Number of Functional Principal Components*



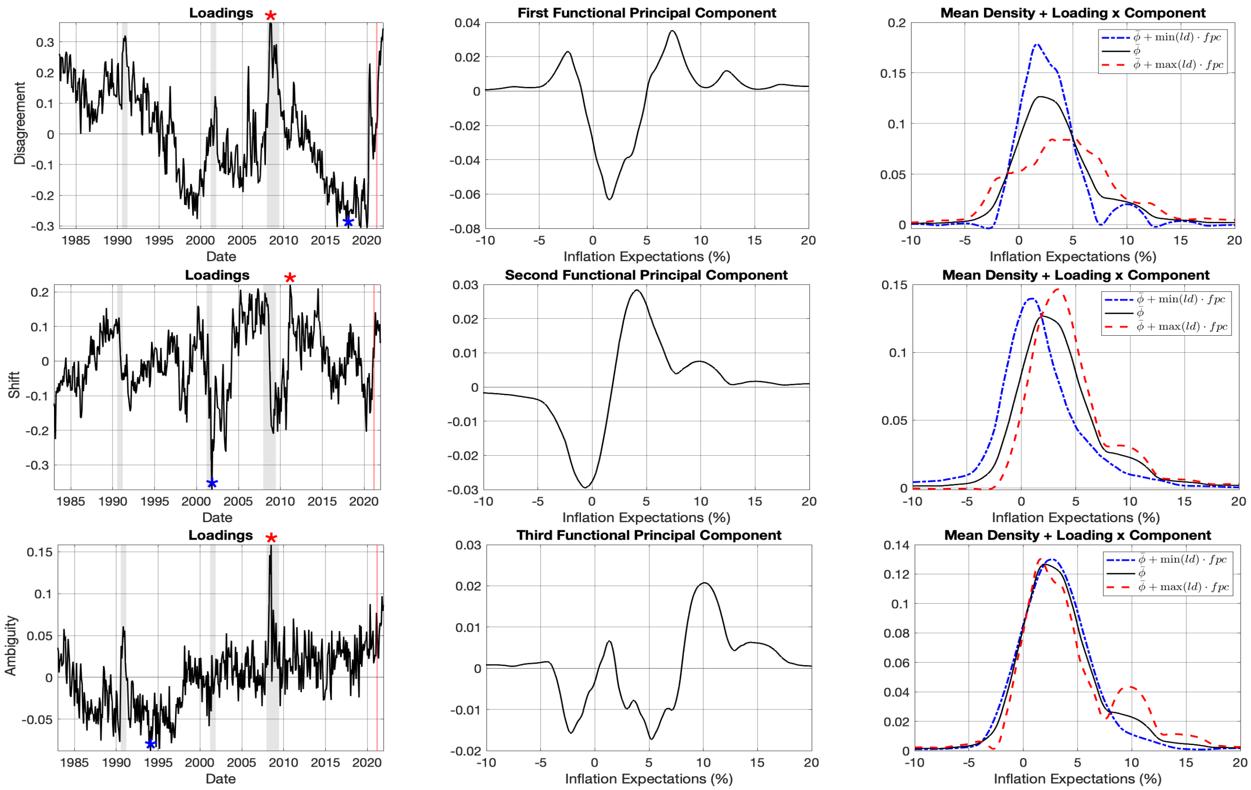
Notes: X-axis shows the number of components derived from the functional principal component. Y-axis shows the cumulative total variations explained by the corresponding number of components. For example, 93.09% of total variance of EIDs of one-year ahead inflation expectations is explained by the first three components.

Figure 6 presents the results of FPCA for one-year ahead expected inflation distributions. In Appendix D, the results for medium-run inflation expectations are presented, where the results are qualitatively similar. In Figure 6, loadings and functional principal components (FPC) are shown in the left and the center columns, respectively. Specifically, in the center column, each row corresponds to the first (v_1^*), second (v_2^*), and third (v_3^*) functional principal component. In the left column, the corresponding loadings for each principal components, $(\alpha_{kt}) = (\langle f_t, v_k^* \rangle)$, are plotted, where the blue and red stars in the figures indicate the minimum and maximum values of loadings, or $\min(\alpha_{kt})$ and $\max(\alpha_{kt})$, over time $t = 1, \ldots, T$ for each $k = 1, 2, 3$.

In the right column of Figure 6, we plot the combined effects of FPC and loadings on the shape of EID. To do so, we multiply each FPC by its minimum and maximum loadings and add them to the average EID density over time, which is calculated as $\bar{f} = (1/T) \sum_{t=1}^{T} f_t$. The black solid, blue dotted and red dashed lines represent the average density ($\bar{f}$), average density plus the FPC scaled by the minimum loading ($\bar{f} + \min(\alpha_{kt}) v_k^*$) and the average density plus the FPC scaled by the maximum loading ($\bar{f} + \max(\alpha_{kt}) v_k^*$) for $k = 1, 2, 3$, respectively. The red-dashed (blue solid) line, therefore, illustrates the case in which the EID is affected most positively (negatively) by the FPC.

[5]With a larger choice of m, we may explain more variation in the EID, but it comes with a cost. The variances of functional coefficients often sharply increase with m, and become unacceptably large for a modest choice of m.

Figure 6. Functional principal components and loadings: One-year-ahead inflation expectations



Notes: Left panels shows the value of the loadings over-time for the three principal components (top: disagreement, middle: shift and bottom: uncertainty). The center panels show the principal component as a function (which integrates to 0). The right panel shows how the sample mean density function is modified with the maximum and minimum value from the loadings shown in the left panel.

The first principal component, shown in the first row of Figure 6, explains more than 64 percent of total variance of EID, alone. We refer to this component as the *disagreement component* for the following reasons. This FPC accentuates the bi-modal distribution, with an increase in the densities of negative inflation expectations between -5 and -1 percent, as well as an increase in the densities of high (above 5 percent) inflation expectations. At the same time, the component decreases the densities in the middle range, with inflation expectations between -1 and 5 percent. As shown in the top right figure, the dispersion of the EID increases with the maximum loading (red dashed line), while the EID becomes more concentrated with the minimum loading (blue dashed line). Interestingly, the loading for the *disagreement component* demonstrates some cyclical features, as the periods when the value of loading increases sharply overlap with the periods of economic recessions, marked as gray shaded areas. That is, disagreement among households regarding future inflation increases during economic downturns. High positive values of factor loading were observed in the early 1980s as well, suggesting that disagreements are high when the level of inflation is high, often interpreted as poorly anchored inflation expectations.

The second component, shown in the middle row of Figure 6, explains around 25 percent of the variations in the inflation expectation distributions over time, alone. We call this component the *shift component*, as the EID shifts to the right with positive loading and to the left with negative loading, as shown by the red dashed and blue dotted lines in the middle figure in the right column.

The third component, shown in the bottom row of Figure 6, explains about 3 percent of the variation in the inflation expectation distributions over time, and it is referred to as the *ambiguity component*. This component increases the frequency of moderate inflation expectations at around 2 percent, the Fed's current (average) inflation target, but also very high expectations above 7 percent. That is, some households have more anchored inflation expectations at 2 percent, while others expect very high levels of inflation expectations. When this ambiguity factor contributes positively, or when its loading takes a positive value, it generates fat tails on the right-side of the distribution, generating several bumps in very high levels of inflation expectations, as shown by the red dashed line in the bottom panel in the last column of Figure 6. In contrast, when the ambiguity component is interacted with with negative loadings, it leads to a smoother tail, as demonstrated by the blue dotted line in the same panel. Interestingly, we observe that loadings increased between 2010 and 2019, suggesting that in addition to the increase in the share of households with well-anchored inflation expectations, the fraction of households projecting very high levels of inflation in the future was also increasing.

To substantiate our interpretation and labeling of the three functional components, Appendix B provides detailed analyses on how the moments of EID change with respect to each functional principal component. Figure 20 in the Appendix plots the change in the four moments of EID (mean, standard deviation, skewness, and kurtosis) with respect to factor loadings. For the first functional component, we see a clear increase in the standard deviation in line with factor loadings, leading to a cross-sectional dispersion of inflation expectations. For the second functional component, we see a clear increase in the first moment (mean), which is expected from the shape of this FPC. Finally, for the third functional component, the first three moments (mean, standard deviation, and skewness) remain relatively flat, while the fourth moment changes most significantly.

From Functional Principal Components to Recursive Functional Shocks

As discussed earlier, based on the scree-plot in Figure 5, we set the truncation number used for the approximation of the densities (f_t) of EID in (2) to be three, i.e., $m = 3$. This implies that we approximate a FAR by a three-dimensional VAR. In our analysis of EID, the standard lag selection procedures (AIC and BIC) suggest the optimal lag to be two, and therefore, we use a second-order FAR in Equation (1) approximated by a three-dimensional VAR in Equation (6).

To achieve the structural interpretation, we first define our estimate $\widehat{(\Sigma)}$ of the reduced form errors as follows:

$$\widehat{(\Sigma)} = \frac{1}{T}\sum_{t=1}^{T} \widehat{(\varepsilon_t)}\widehat{(\varepsilon_t)}',$$

which is obtained from the fitted reduced form errors $\left(\widehat{(\varepsilon_t)}\right)$ in VAR in Equation (6). Then we identify recursive shocks (e_t), or *recursive functional shocks*, using the Cholesky decomposition of $\widehat{(\Sigma)}$

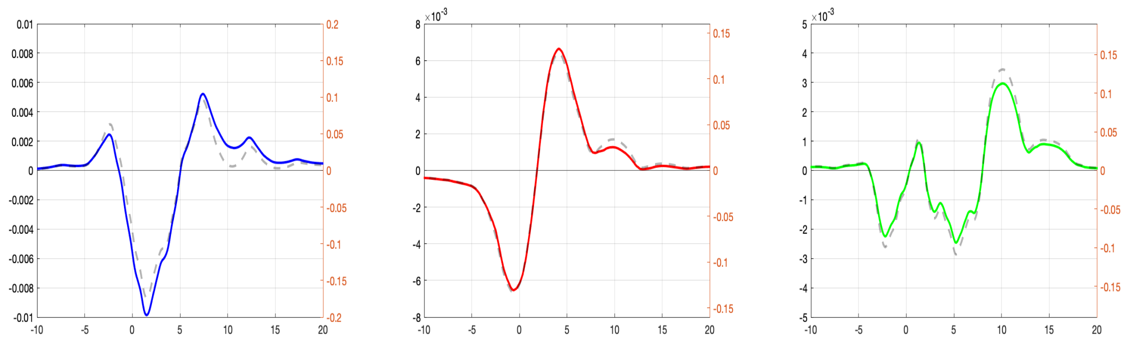
$$\widehat{(\Sigma)} = LL',$$

with a lower triangular matrix L, so that $\widehat{(\varepsilon_t)} = Le_t$. Each column of L contains the response of EID to the corresponding shock. Define h_i to be the response of EID to the shock represented in the i^{th} column of $L = [\ell_{ij}],\ i, j = 1, 2, 3$, namely,

$$h_i = \pi^{-1}\begin{pmatrix} \ell_{1i} \\ \ell_{2i} \\ \ell_{3i} \end{pmatrix}$$

which we call the *baseline* EID response to functional shock i. Recall from Section 3 that π is a one-to-one mapping from a three-dimensional subspace of the function space H to R^3. Figure 7 presents the baseline EID responses h_i's to each of the three functional shocks.

Figure 7. At-impact baseline EID responses to functional shocks



Notes: At-impact baseline impulse responses of the expected inflation distribution to the three recursively identified functional shocks. For comparison, each baseline response is plotted together with the corresponding functional principal components.

Representation of External Shocks using Functional Shocks

Finally, we describe how we represent externally identified economic shocks using the three functional shocks recursively identified from the time series of the expected inflation distributions. Let (e_{1t}), (e_{2t}) and (e_{3t}) denote the three functional shocks and (x_t), an externally identified economic shock. In the following section, we introduce monetary policy (mp), government spending (gs), personal income tax (it), and gasoline price change (gp) shocks as externally identified economic shocks.[6]

We search for the association between (x_t) and (e_t) as a linear combination of the three functional shocks, (e_{1t}), (e_{2t}) and (e_{3t}), that gives the maximum correlation between (e_t) and (x_t). This amounts to estimating the coefficient vector ρ_x in the regression given by

$$x_t = \rho_x' e_t + \eta_{xt}, \tag{8}$$

where η_{xt} is an i.i.d. innovation specific to the external shock (x_t) under consideration. Since the individual functional shocks (e_{kt}) for $k = 1, 2, 3$ are uncorrelated with each other, we may interpret the norm $\|\rho_x\|$ of the correlation vector $\rho_x = (\rho_{x1}, \rho_{x2}, \rho_{x3})'$, with $\rho_{xk} = \text{corr}(e_{kt}, x_t)$, as the percentage of the economic shock (x_t) that is transmitted to the distribution of expected inflation. We use the

[6]When the external shock is available at quarterly frequency and not at monthly frequency, which is the case for the two fiscal policy shocks we consider, we use the cumulative sum of functional shocks at monthly frequency to obtain the quarterly series.

estimated values of correlation coefficients in $\rho_x = (\rho_{x1}, \rho_{x2}, \rho_{x3})'$ as weights to define the response of EID to an external shock x_t as

$$h_x = \rho_{x1} h_1 + \rho_{x2} h_2 + \rho_{x3} h_3,$$

where h_k is the baseline response of the EID to the k-th functional shock for $k = 1, 2, 3$. We note that the external shock x_t has been normalized to have unit variance so that we may interpret h_x as the response of the EID to a one standard deviation external shock (x_t).[7]

5 The Effects of Economic Shocks on EID

This section presents the effects of economic shocks on the distribution of one-year and medium-run inflation expectations. To estimate those effects, we used external shocks, which we first describe. Then, we present the responses of both distributions (one-year and medium-run) to these shocks. Our baseline sample is from January 1983 to December 2021 for the one-year horizon and from April 1990 to December 2021 for the medium-run. In Appendix C, we present the comparison of results during the recession and non-recession periods.

5.1 Description of Economic Shocks

The four external shocks we used for our main results are monetary policy, government spending, personal income tax and gasoline price.[8]

[7] When a different shock size is desired, we may adjust the EID response accordingly as

$$\theta_x h_x = \theta_x \rho_{x1} h_1 + \theta_x \rho_{x2} h_2 + \theta_x \rho_{x3} h_3,$$

where θ_x is an appropriately chosen factor. For example, when a one-standard deviation monetary policy shock represents an increase in 30 bp of the federal funds rate, we may compute the response of EID to a 25 bp increase in the policy rate simply by setting the factor $\theta_{mp} = 25/30 = 5/6$.

[8] Except for the gasoline price shock, these shocks and the estimation codes are made available by the authors.

For monetary policy, we use the series constructed by Miranda-Agrippino and Ricco (2021).[9] This series is available monthly and quarterly from March 1963 to December 2007.[10] As discussed in Falck, Hoffmann, and Hürtgen (2019), how contractionary monetary policy shock affect inflation expectations can be ambiguous, *a priori*. On the one hand, an increase in interest rates can hold back aggregate demand, lowering inflation expectations to reflect lower output in the future. At the same time, an increase in interest rates can be interpreted as a signal of the central bank anticipating an increase in inflation (Nakamura and Steinsson, 2018) and can be inflationary.

For government spending, we use our own estimation of the linear model described in Auerbach and Gorodnichenko (2012). These recursively identified shocks originate in a VAR model with real government spending, real government receipts, direct and indirect taxes, net transfers to businesses and individuals, and real gross domestic product (GDP) in chained 2000 dollars. With a positive government spending shock, we expect an increase in aggregate demand to boost inflation expectations. On the other hand, this shock also generates uncertainty about how the government will finance spending. At the same time, households may have different perceptions of the fiscal multipliers of government spending, leading to an increase in disagreement on future inflation.

We use the personal income tax shock constructed by Mertens and Ravn (2013) based on a narrative approach. An increase in taxes, in general, is shown to have clear contractionary effects, accompanied by a significant drop in inflation Alesina and Ardagna (2010); Romer and Romer (2010). However, as documented in Mertens and Ravn (2013), the evidence on how the change in specific tax items, such as personal income tax or corporate income tax, affect inflation is less clear. The impact of these specific tax changes on household inflation expectations is even less understood. To the extent that any change in personal income tax directly affects household disposable income, one can conjecture that household inflation expectations will be adjusted in response to a change in personal income tax. However, *a priori*, the direction of the adjustment is ambiguous, as there are various channels through which households may perceive the nature of personal income tax changes and ultimately revise their expectations of future inflation. Specifically, how households interpret a decline in disposable income and link it with future inflation is unclear.

[9] In the paper, they explain that "*the construction of the instrument closely follows Romer and Romer (2004), where the indicator variable for monetary policy is constructed as the intended changes in the Federal Fund Rates that are independent from monetary policy actions taken in response to information about future economic developments (in the form of forecasts of information and real activity that are available to policymakers at the time of the FOMC decision)*".

[10] `http://silviamirandaagrippino.com/code-data`

Finally, for the gasoline price shock, we look at the relationship between inflation expectations and the retail gasoline price, as in Kilian and Zhou (2021), which shows that the gasoline price, rather than the oil price, is what consumers pay attention to. We also take the standard approach of taking the gasoline price as exogenously given from the point of view of the consumer, so the identification problem does not arise. Anderson, Kellogg, and Sallee (2013) show that households participating in the University of Michigan's survey treat the real price of gasoline approximately as a random walk. As the percent change in the real price of gasoline is about the same as the percent change in the nominal price of gasoline at monthly frequency, we use the percentage change in the nominal price of gasoline as our gasoline price shock, and the FRED's Consumer Price Index for All Urban Consumers: Gasoline (All Types) in the U.S. City Average as the gasoline price shocks.

Because the individual functional shocks, $(e_{kt}),\ k = 1,2,3$, are uncorrelated with each other, the norm of the correlation vector $\rho_x = (\rho_{x1}, \rho_{x2}, \rho_{x3})'$ can be interpreted as percentage of the economic shock x that is transmitted to expected inflation distribution. The correlation between functional shocks and economic shocks ranges from about 5 (personal income tax shocks) to more than 56 percent (gasoline price shocks).[11] The correlation of each functional shock with each economic shock for the one-year period preceding EID is reported in Table 2. The effects of monetary policy are mainly focused on the shifting component. The correlation between functional shocks and fiscal policy shocks, both government spending and personal income tax shocks, is relatively weak for one-year ahead inflation expectations. Changes in gasoline prices, on the other hand, have a positive relationship with both shifting and disagreement shocks.

In Table 3, we present the correlations between each economic shock and functional shock, as well as the norm of the vector of correlation coefficients for medium-run inflation expectations, in a manner similar to how we presented the results for one-year ahead expectations.The norm for government spending shocks increases from 20 percent for one-year ahead inflation expectations to 23 percent for medium-run inflation expectations. For personal income tax shocks, the norm increases from 5 percent for one-year ahead inflation expectations to 34 percent for medium-run inflation expectations.On the other hand, based on the norms, the correlations between EIDs and monetary and gasoline price shocks are lower for medium-run inflation expectations than for

[11]Since the shocks we have used here come from different sources, it is important to interpret these results individually. We do not attempt or claim to identify the effect of these shocks simultaneously. Future work relating the EID to other economic aggregates such as inflation, output or unemployment, for instance, will address the issue of simultaneous identification.

Table 2. *Correlations between functional shocks and economic shocks: One-year-ahead inflation expectations*

	Disagreement	Shifting	Ambiguity	Norm
Monetary Policy: ρ_{mp}	−0.029 (0.097)	0.244 (0.082)	−0.040 (0.092)	0.249 (0.075)
Government Spending: ρ_{fp}	0.019 (0.114)	0.166 (0.109)	0.110 (0.106)	0.200 (.089)
Personal Income Tax: ρ_{it}	−0.021 (0.092)	−0.043 (0.112)	0.003 (0.094)	0.048 (0.070)
Gasoline Prices: ρ_{gp}	0.107 (0.119)	0.463 (0.082)	0.310 (0.077)	0.567 (0.063)

Notes: Correlation between three EID functional shocks and each of the external shocks considered: (i) expansionary monetary policy shock (Miranda-Agrippino and Ricco (2021)); (ii) government spending (Auerbach and Gorodnichenko (2012)); (iii) average personal income tax increase (Mertens and Ravn (2013)); and (iv) gasoline price changes (FRED). The numbers in parenthesis below each estimate are the 68% bootstrap confidence bands.

one-year ahead inflation expectations. Correlations between monetary shocks and all the functional shocks are shown to be weak. At the same time, government spending shocks are correlated with the disagreement functional shock, while personal income tax shocks are correlated with the shifting functional shock with statistical significance. Compared with one-year ahead inflation expectations, the correlation between gasoline price shocks and shifting functional shocks weakened, while the correlation with disagreement shocks became more significant.

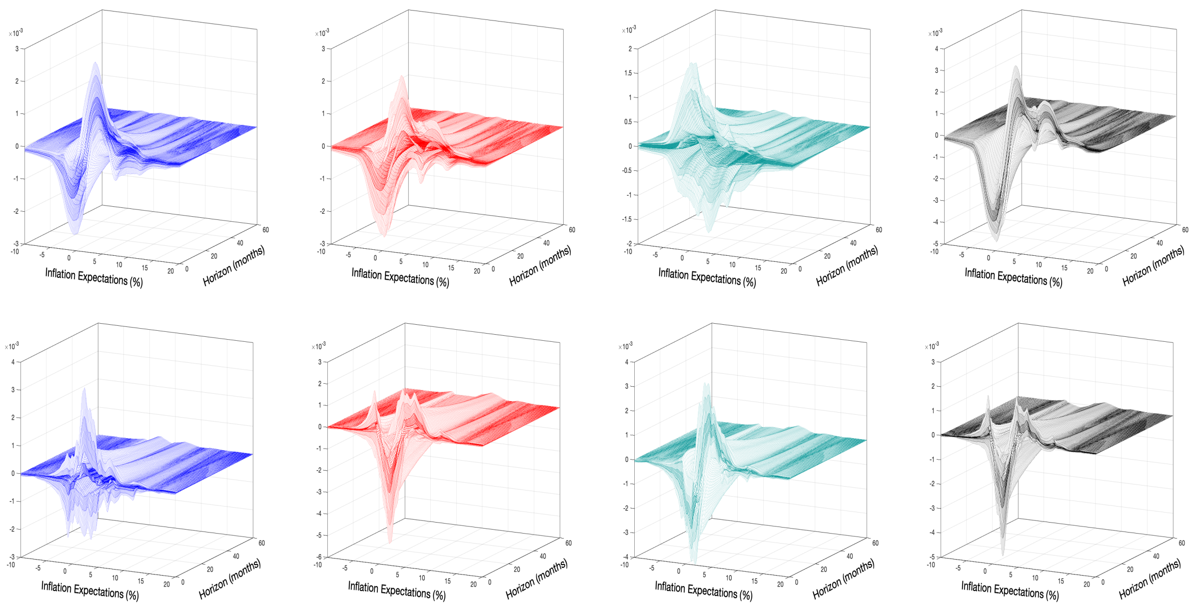
Table 3. *Correlations between functional shocks and economic shocks: Medium-run inflation expectations*

	Disagreement	Shifting	Ambiguity	Norm
Monetary Policy: ρ_{mp}	−0.035 (0.116)	0.028 (0.139)	−0.089 (0.102)	0.100 (0.094)
Government Spending: ρ_{fp}	0.218 (0.123)	0.025 (0.153)	−0.075 (0.118)	0.231 (0.107)
Personal Income Tax: ρ_{it}	0.110 (0.125)	0.294 (0.102)	−0.130 (0.096)	0.340 (0.083)
Gasoline Prices: ρ_{gp}	0.210 (0.087)	0.043 (0.095)	−0.018 (0.124)	0.216 (0.078)

Notes: Correlation between three EID functional shocks and each of the external shocks considered: (i) monetary policy shock (Miranda-Agrippino and Ricco (2021)); (ii) government spending (Auerbach and Gorodnichenko (2012)); (iii) personal income tax increase (Mertens and Ravn (2013)); and (iv) gasoline price changes (FRED). The numbers in parenthesis below each estimate are the 90% bootstrap standard errors.

Finally, using the correlations of the economic shocks and the three functional shocks, we present the impulse response functions of EID. Figure 8 shows three-dimensional IRFs of one-year ahead (top row) and medium-run EIDs (bottom row) to monetary (blue), government spending (red), personal income tax (green), and gasoline price (black) shocks. Broadly, we see that the responses phase out over time. To examine the effects more closely, we focus on the responses of EID at-impact from now on, and only look at multi-horizon effects for single statistics.

Figure 8. Three-dimensional Impulse Response Function Surfaces of EIDs to Economic Shocks



Notes: Top panel shows IRFs of EIDs of one-year inflation expectations. Bottom panel shows IRFs of EIDs of medium-run inflation expectations. IRFs to monetary policy shocks, government spending shocks, personal income tax shocks, and gasoline price shocks are represented in blue, red, green, and black, respectively. Horizon is in months.

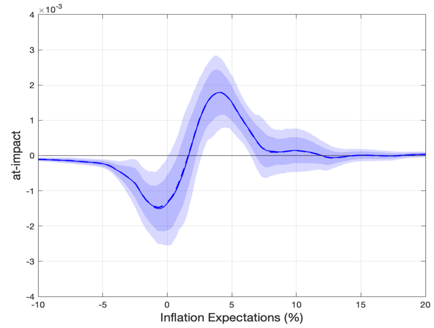
5.2 Monetary Policy Shocks and EIDs

Here, we examine how monetary policy shocks affect household inflation expectations. In sum, our findings suggest that the early 1980s were indeed a watershed moment in terms of how household inflation expectations respond to monetary policy shocks.

The at-impact response of one-year ahead inflation expectations to contractionary monetary policy shocks using the baseline sample from January 1983 to December 2021 is depicted in Figure 9. Monetary policy shocks raise the level of average inflation expectations by decreasing the frequency of negative inflation expectations and increasing the frequency of positive inflation expectations by 2 to 7 percent.

Figure 10 compares the at-impact response of one-year ahead EID to monetary policy shocks, using different sample periods to observe the evolution of the effects of monetary policy shocks on EID. The top left chart is the IRF using the longest sample, from January 1978 to December 2021, including the periods before the Volcker disinflation. The top right chart uses the baseline sample, running from January 1983. Figures in the second row use increasingly shorter samples,

Figure 9. At-Impact Response of One-year ahead EID to Monetary Policy Shocks: Baseline

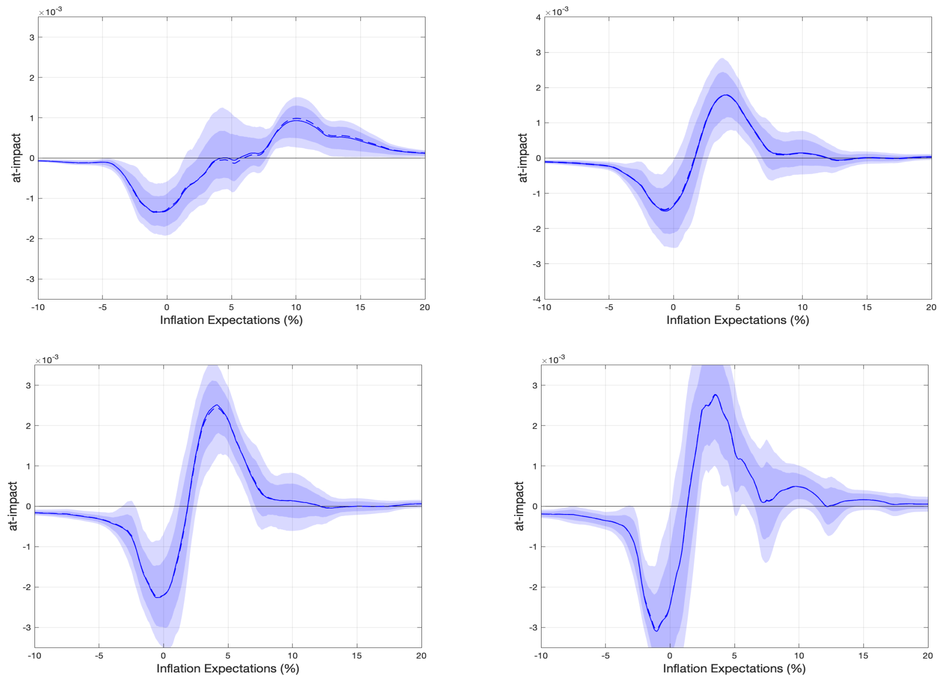


Notes: This uses the sample from January 1983 to December 2021.

each starting from April 1990 (lower left) and January 1997 (lower right). The IRFs show that for all four samples, contractionary monetary policy shocks have inflationary effects, by decreasing the frequencies of small negative values while increasing the frequencies of positive values. There is a visible difference, even without a formal analysis, between the shapes of the IRFs using the last three samples and the IRF using the longest sample (top left). Using the longest sample, the response increases the density of positive inflation expectations from 7% and above, contributing to a fatter tail of EID. This contrasts with other responses whose shapes take a more bell-shaped form. To put it differently, the range of positive values where the densities increase in response to monetary policy shocks is bounded for these IRFs and the densities around the peak are more concentrated than the first IRF. Furthermore, we observe a visible decline in the level of inflation expectations corresponding to the peak of the distribution, from around 10 percent for the baseline sample (top right) to a range of 2 to 3 percent for the most recent sample running from January 1997.

To examine the distributional effects of monetary policy shocks on inflation expectations more rigorously, we select several features of EID and see how they respond to monetary policy shocks. Figure 11 presents the results of the impact of monetary policy shocks on the first four moments: mean, standard deviation, skewness, and kurtosis. Figure 12 presents the results using other features of the distribution, which are less explored in the literature compared to moments, but nevertheless crucial with regard to the conduct of monetary policy. These characteristics include the share of households with inflation expectations above 5 percent, the share of households with inflation expectations at 2 percent and the share of households with inflation expectations between 2 to 4 percent. The results are shown in two rows in both Figures 11 and 12, the results are represented

Figure 10. *At-Impact Responses of One-year ahead EIDs to Monetary Policy Shocks: Different Samples*

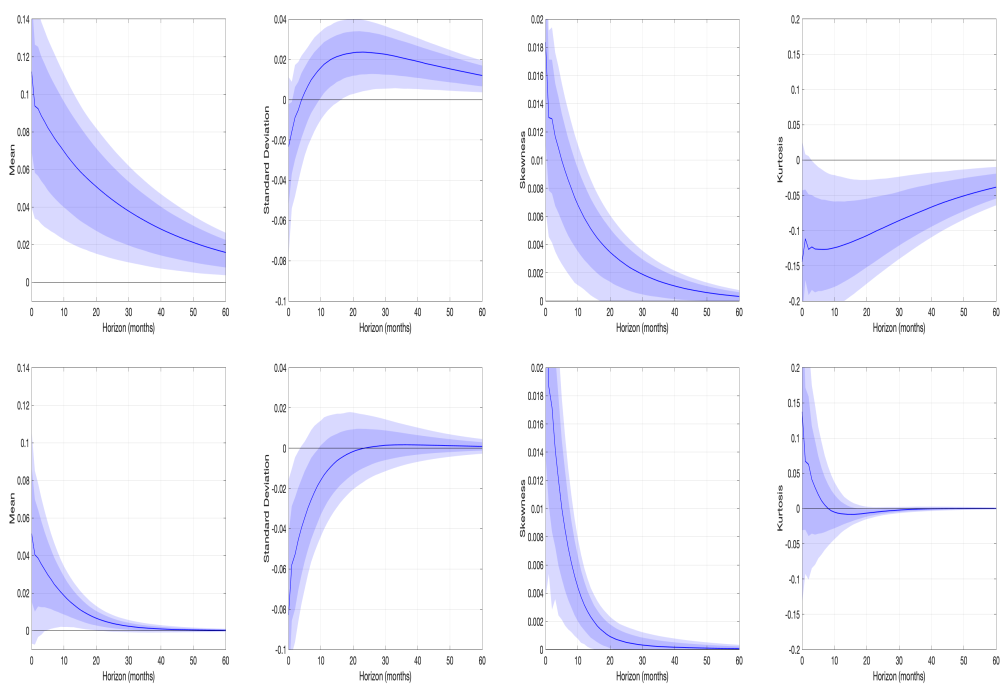


Notes: In the top row, the left chart uses the sample from January 1978 and the right chart uses the sample from January 1983. In the bottom row, the left chart uses the sample from April 1990, which is the first available month of medium-run inflation expectations, and the right chart uses the sample from January 1997.

in two rows. The only difference between the top row and the bottom row is the sample coverage. The top row corresponds to the sample starting from January 1978 to December 2021 (including the high inflation periods before the Volcker disinflation periods), while the bottom row is based on a shorter sample (baseline) from January 1983 to December 2021. Both rows use one-year ahead inflation expectations.

Starting from Figure 11, the left column shows the responses of the average household inflation expectations to monetary policy shocks in a given month. We see that the average inflation expectations increase in response to monetary policy shocks for both top and bottom figures, and the coefficient is statistically significant. However, there are important differences between the two figures that are worth mentioning. First, in terms of the magnitude of the coefficient, the coefficient using the longer-sample (top row) is at least two times larger than what is observed for the shorter sample (bottom row). Second, the impact of monetary policy shocks is more persistent in the longer sample (top row) than in the shorter-sample (bottom row). In the second column, we look at standard deviation.[12] We find that during the high inflation periods, monetary policy shocks increased disagreement on inflation expectations among households (top) and such effects were persistent. On the other hand, monetary policy shocks, excluding the high inflation periods (bottom), reduce disagreement and have less persistent effects, with the effects disappearing within one year. In the third column, we look at how skewness is affected by monetary policy shocks. Both figures in the top and bottom rows show that monetary policy shocks lead to a more asymmetric shape of inflation expectations, while the at-impact magnitudes are much larger for the result using the longer sample (top). Finally, we look at how kurtosis changes in response to monetary policy shocks in the fourth column. A decrease in kurtosis observed for the longer sample, but not for the shorter sample, can be interpreted as a reduction in the tail behavior.

Figure 11. *Impacts of Monetary Policy Shocks on Four Moments of One-year-ahead EID: 1978:1-2021:12 vs. 1983:1-2021:12*



Notes: The selected moments are the mean (first column), standard deviation (second column), skewness (third column), and kurtosis (fourth column). The top row uses the sample from January 1978 to December 2021. The second row uses the sample from January 1983 to December 2021. Both rows use one-year ahead inflation expectations.

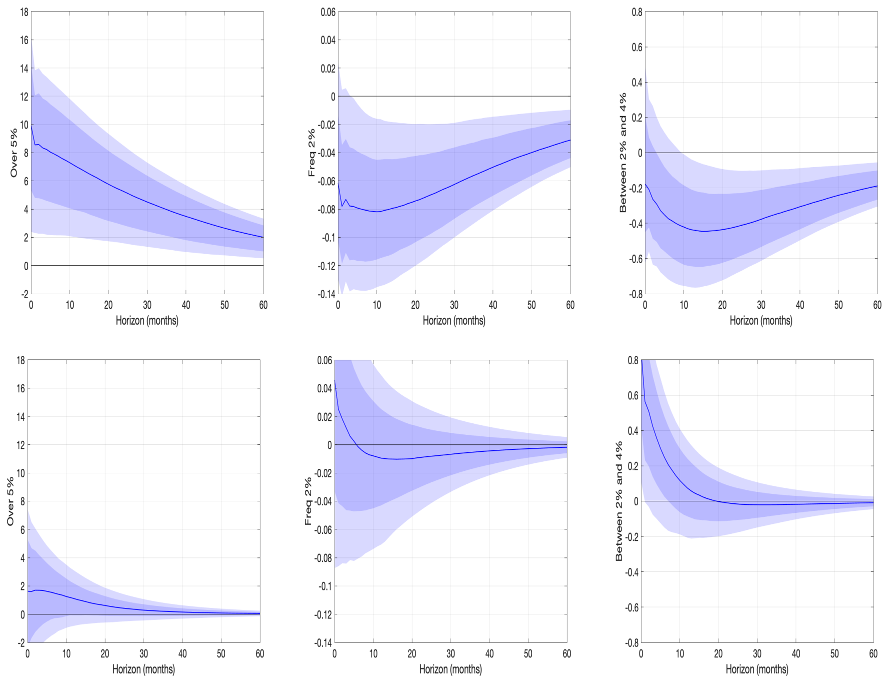
[12]There is a rich literature that exists that study the disagreement among forecasters, since the seminal paper by Mankiw, Reis, and Wolfers (2003) to more recent works by Andrade et al. (2016) and Dovern, Fritsche, and Slacalek (2012), for instance.

Figure 12 focuses on three features of the distribution of inflation expectations (beyond moments) and how they react to monetary policy shocks. Namely, we look at (i) the share of households with inflation expectations above 5 percent, which captures the share of un-anchored inflation expectations (right tail); (ii) the share of households with inflation expectations at 2 percent, which summarizes the share of households whose inflation expectations are in line with the central bank's inflation target; and (iii) the share of households with inflation expectations between 2 and 4 percent.

The left column of Figure 12 depicts the effects of monetary policy shocks on the share of households with inflation expectations above 5 percent. A positive and persistent increase is observed for the longer sample (top row), while the effects become statistically insignificant for the shorter sample (bottom row). Monetary policy shocks previously generated fatter tails on the right-side of the distribution, but this is no longer the case here. An economic interpretation is that monetary policy shocks used to un-anchor inflation expectations and led households to expect high levels of future inflation. Such effects have disappeared since the early 1980s. To examine the anchoring effects more closely, we examine the share of households whose inflation expectations are exactly 2 percent, in line with the central bank's inflation target (center column). Monetary policy shocks reduce the share of households with 2 percent inflation expectations in the larger sample (top center figure). For the shorter sample (bottom center figure), we see a positive coefficient at impact, although this coefficient is not statistically significant. The third column examines the impact of monetary policy shocks on the share of households with inflation expectations between 2 and 4 percent by relaxing the responses to a range in the neighborhood of 2 percent rather than responses exactly at 2 percent. Unlike the persistent and negative impact on the share for the longer sample (top left figure), there is a clear, albeit temporary (lasting less than one year), increase in response to monetary policy shocks.

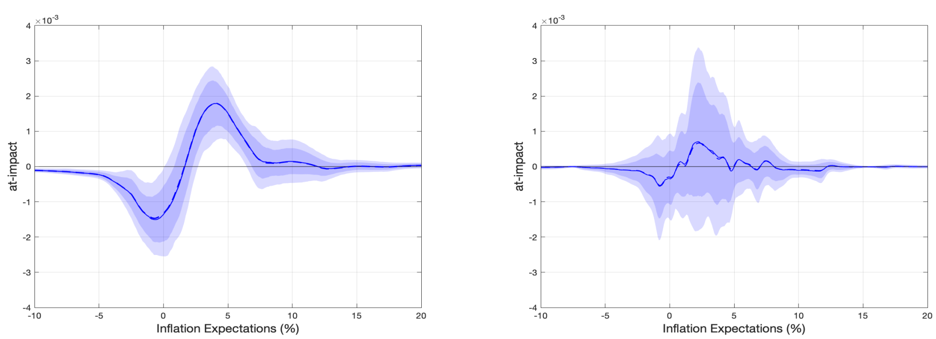
The at-impact responses for one-year ahead inflation expectations (left) and medium-run EIDs (right) are compared in Figure 13. For medium-run inflation expectations, we see that the densities increase for positive values, with the peak at around 2 percent. However, these results are not statistically significant.

Figure 12. *Impacts of Monetary Policy Shocks on Selected Moments of One-year-ahead Inflation Expectations: 1978:1-2021:12 vs. 1983:1-2021.12*



Notes: The selected moments are as follows: frequency of inflation expectations above 5 percent (left column), frequency at 2 percent inflation expectations (center column), and frequency of responses between 2 and 4 percent of inflation expectations (right column). The top row uses the sample from January 1978 to December 2021. The second row uses the sample from January 1983 to December 2021. Both rows use one-year ahead inflation expectations.

Figure 13. *At-Impact Responses of One-year-ahead and Medium-run EIDs to Monetary Policy Shocks*



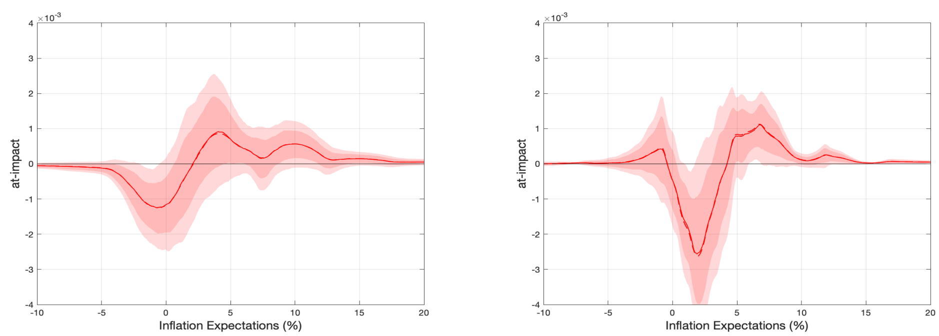
Notes: Left chart shows the at-impact response of one-year ahead inflation expectations using the sample from January 1983 to December 2021. Right chart shows the at-impact response of medium-run inflation expectations using the sample from April 1990 to December 2021.

5.3 Fiscal Policy Shocks and EIDs

To investigate the effects of fiscal shocks on EID, we introduce two commonly used fiscal shocks in the literature, a government spending shock (Auerbach and Gorodnichenko, 2012) and a personal income tax shock (Mertens and Ravn, 2013).

We start with government spending. Figure 14 presents the at-impact response for one-year ahead inflation expectations (left) and medium-run inflation expectations (right) to this shock.[13] Government spending shocks are inflationary for one-year ahead inflation expectations, by decreasing the frequencies of inflation expectations at below 1 percent and increasing the frequencies of high-level inflation expectations between 7 to 12 percent. The inflationary effect of government spending shocks, on the other hand, is more pronounced for medium-run inflation expectations, accompanied by increased frequencies of high inflation expectations above 5 percent.

Figure 14. *At-Impact Responses of One-year ahead and Medium-run EIDs to Government Spending Shocks*



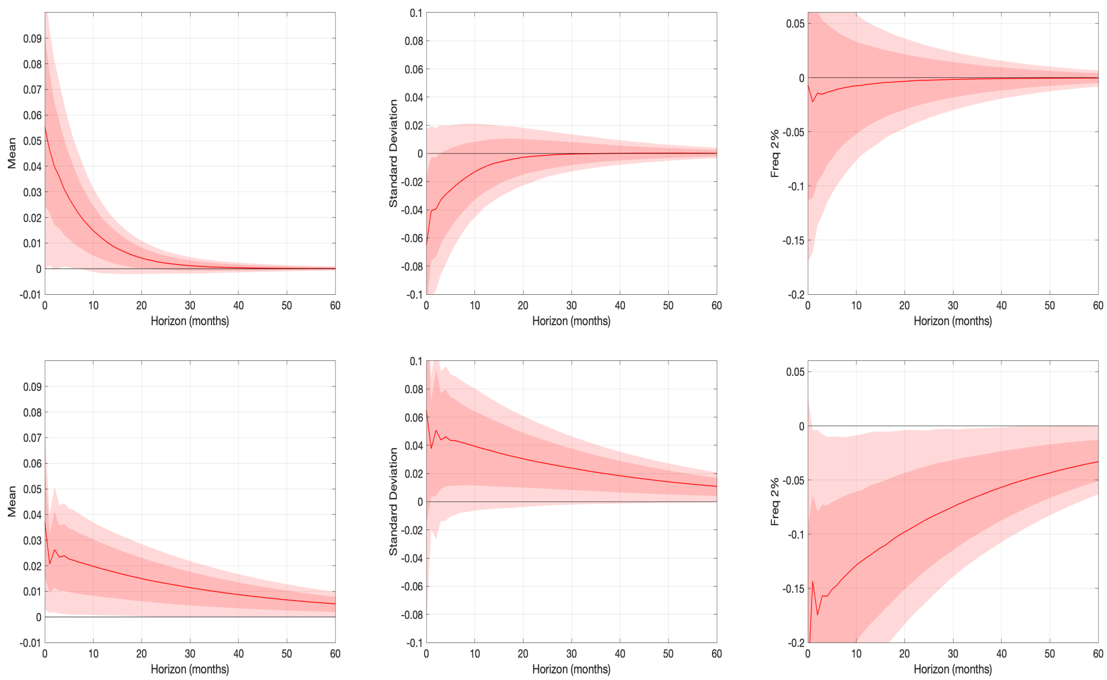
Notes: Left (right) chart shows the at-impact response of one-year ahead (medium-run) inflation expectations.

Looking at specific moments, Figure 15 compares how government spending shocks affect the mean (left column), standard deviations (center column), and the frequency of responses at 2 percent (right column), for one-year ahead inflation expectations (top row) and medium-run inflation expectations (bottom row). Government spending shocks are inflationary, shown in the increase in the average inflation expectations for both one-year ahead and medium-run inflation expectations. Interestingly, there is a stronger agreement among survey respondents to perceive government spending shocks as inflationary for the short-run rather than for the medium-run.

[13]The IRF results are broadly similar using different samples. The baseline sample runs from January 1983 to December 2021 for one-year ahead inflation expectations, and from April 1990 to December 2021 for medium-run inflation expectations.

In the center column of Figure 15, government spending shocks decrease the dispersion among survey respondents for one-year ahead inflation expectations, while increasing the dispersion for medium-run inflation expectations. The increase in dispersion is represented in the right chart of Figure 14, with an increase in the densities of negative values of inflation expectations. The increased dispersion is also shown in the negative impact on the frequency of responses at 2 percent inflation expectations (bottom right of Figure 15).

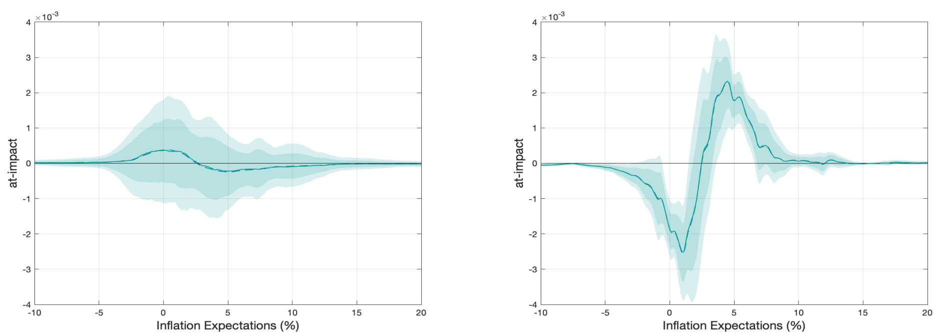
Figure 15. *Impacts of Government Spending Shocks on Specific Moments of EIDs*



Notes: Each column shows the impact of government spending shocks on the mean (left), standard deviation (center), and frequency at 2% (right). The top row reports the results for one-year ahead inflation expectations, using the sample from January 1983 to December 2021. The bottom row reports the results for medium-run inflation expectations, using the sample from April 1990 to December 2021.

On the contrary, the effects of personal income tax shocks are significant only for medium-run inflation expectations. Figure 16 compares the at-impact response for one-year ahead and medium-run inflation expectations. Personal income tax shocks appear to be even disinflationary for one-year ahead inflation expectations (left) by lowering the frequencies of high-level of inflation expectations. However, the results are statistically insignificant. On the other hand, for medium-run inflation expectations, personal income tax shocks are clearly inflationary, by lowering the frequencies of small positive inflation expectations ranging between 0 to 3 percent, while increasing the frequencies of expected inflations beyond 3 percent. Figure 17 presents the at-impact response of personal income tax shocks on selected moments of EIDs: mean (left), standard deviations (center), and frequency at 2 percent (right), for one-year ahead inflation (top row) and medium-run inflation expectations (bottom row). While the results presented on the top row are not statistically significant, personal income tax shocks lead to a persistent increase in the average inflation expectations at levels higher than 2 percent inflation expectations, therefore leading to a persistent decrease in the frequency of inflation expectations at 2 percent (bottom right chart).

Figure 16. *At-Impact Responses of One-year ahead and Medium-run EIDs to Personal Income Tax Shocks*

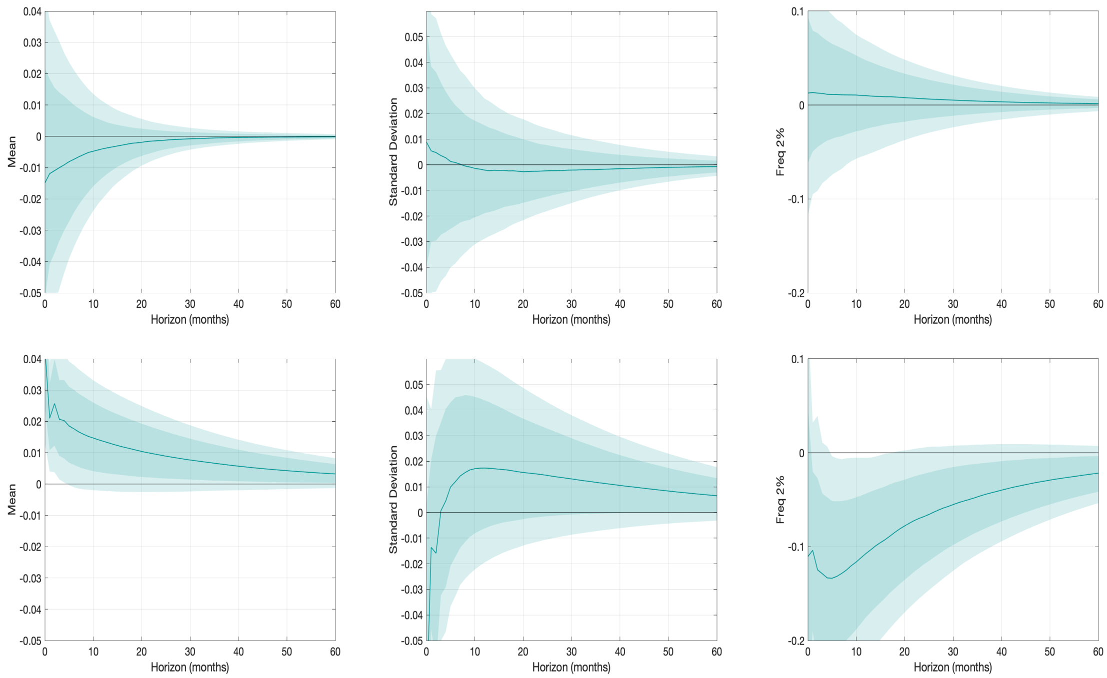


Notes: Left chart shows the at-impact response of one-year ahead inflation expectations using the sample from January 1983 to December 2021. Right chart shows the at-impact response of medium-run inflation expectations using the sample from April 1990 to December 2021.

5.4 Gasoline Price Shocks and EIDs

The literature has established a clear relationship between gasoline price and household inflation expectations formation (see, *inter alia*, Harris et al. (2009), Coibion and Gorodnichenko (2015)). We confirm that gasoline price shocks are indeed inflationary. Figure 18 compares the at-impact responses of EIDs to gasoline price shocks for one-year ahead and medium-run inflation expectations. For one-year ahead EIDs, a surprise hike in gasoline prices decreases the frequencies of inflation expectations below 3 percent, while at the same time, increasing the frequencies of inflation expectations higher than 3 percent. For medium-run inflation expectations, a surprise hike in gasoline prices again leads to an increase in densities for higher levels of inflation expectations. However, unlike the case for one-year ahead inflation expectations, this increase in densities in positive values is accompanied by an increase in densities for negative values of inflation expectations,

Figure 17. Impacts of Personal Income Tax Shocks on Specific Moments of EIDs

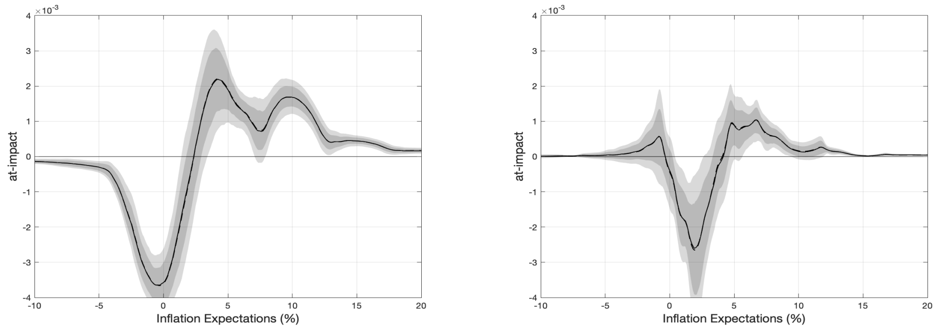


Notes: Each column shows the impact of government spending shocks on the mean (left), standard deviation (center), and frequency at 2% (right). The top row reports the results for one-year ahead inflation expectations, using the sample from January 1983 to December 2021. The bottom row reports the results for medium-run inflation expectations, using the sample from April 1990 to December 2021.

which was not observed in one-year ahead inflation expectations. This suggests that there is an increase in disagreement among households on how the current gasoline price increase will affect future inflation.

By looking at selected moments in Figure 19, these points are made clear. The top row plots the impact of a surprise increase in gasoline prices for selected moments using one-year ahead inflation expectations, and the bottom row for medium-run inflation expectations. As presented in the left column, a surprise increase in gasoline prices increases the average level of inflation expectations among households for both one-year ahead and medium-run inflation expectations. There is a difference in the magnitude, with the response of one-year ahead inflation expectations at-impact to be nearly three times larger than that of medium-run inflation expectations. The center column presents the response on standard deviation, a proxy for disagreement among survey respondents. The disagreement decreases for one-year ahead inflation expectations for nearly one-year ahead (top center), implying that there is less disagreement among households that

Figure 18. *At-Impact Responses of One-year ahead and Medium-run EIDs to Gasoline Price Shocks*



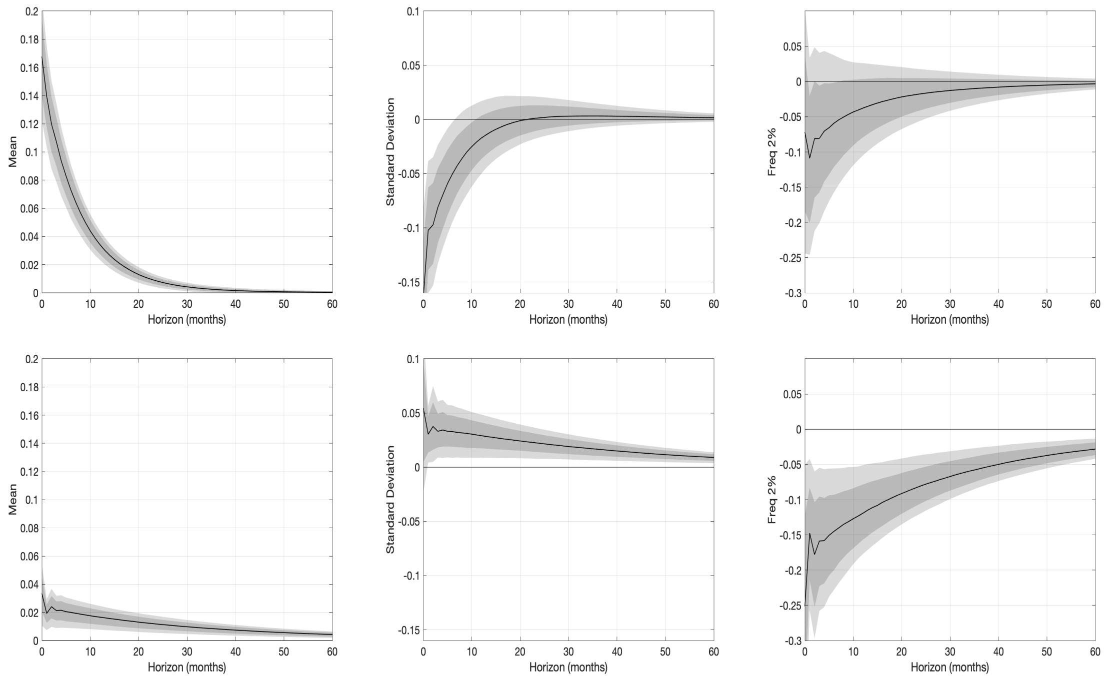
Notes: Left chart shows the at-impact response of one-year ahead inflation expectations using the sample from January 1983 to December 2021. Right chart shows the at-impact response of medium-run inflation expectations using the sample from April 1990 to December 2021.

gasoline price shocks will increase short-run inflation expectations. On the contrary, the standard deviation for medium-run inflation expectations (bottom center) increases persistently in response to gasoline price shocks, implying there is disagreement on how the current gasoline price shocks will affect medium-run inflation. Finally, there is evidence that gasoline price shocks do not help the central bank's inflation targeting, as it decreases the frequency at 2 percent for both one-year ahead and medium-run inflation expectations.

5.5 Economic Interpretation of Findings

In this section, we provide some plausible interpretations of our findings. Our findings can be summarized as follows: First, contractionary monetary policy shocks have inflationary effects on household inflation expectations, but only for one-year ahead inflation expectations. *Anchoring effects*, with a reduction in disagreement with an increased frequency of responses between 2 to 4 percent, are observed, but only for samples that exclude the period prior to Volcker disinflation. Second, government spending shocks are inflationary for both one-year ahead and medium-run inflation expectations, while increasing the disagreement for medium-run inflation expectations. Third, personal income tax shocks are inflationary, but only for medium-run inflation expectations. Finally, a surprise increase in gasoline prices increases the level of inflation expectations, for both one-year ahead and medium-run inflation expectations, while disagreement increases for medium-run inflation expectations. The last finding related to the impact of gasoline price hikes on

Figure 19. *Impacts of Gasoline Price Shocks on Specific Moments of EIDs*



Notes: Each column shows the impact of government spending shocks on the mean (left), standard deviation (center), and frequency at 2 percent (right). The top row reports the results for one-year ahead inflation expectations, using the sample from January 1983 to December 2021. The bottom row reports the results for medium-run inflation expectations, using the sample from April 1990 to December 2021.

household inflation expectations is relatively well understood in the literature. However, as for the effects of monetary and fiscal policy shocks on household inflation expectations, the literature is relatively thin. Below, we attempt to provide economic interpretations of our findings. A caveat needs to be drawn that these interpretations need to be validated with a more rigorous analysis.

Why do contractionary monetary policy shocks lead to an increase in household inflation expectations?[14] Two possible channels identified in the literature may provide plausible explanations: (i) information channel highlighted in Nakamura and Steinsson (2018) where monetary tightening provides information to the public that the central bank has superior information about the projection of inflation. If this channel dominates, households may interpret contractionary monetary policy as if the 'actual' underlying inflationary pressures must be higher than what households believed for the central bank to tighten monetary policy. As a result, households may

[14]In fact, empirical studies have shown that contractionary monetary policy shocks leading to an increase in inflation, through (Uribe, 2018) or price puzzles (Sims, 1992; Hanson, 2004; Rusnák, Havranek, and Horváth, 2013).

revise their inflation expectations upward with a contractionary monetary policy shock; and (ii) it has been shown that households consider inflation as negative news related to the underlying condition of the economy, as documented in Coibion and Gorodnichenko (2015). If households believe a slowdown of economic activities to be a result of tightening monetary policy, households may associate this projected economic downturn with higher future inflation.

The findings that fiscal policies have medium-run effects may be consistent with the predictions from well-established models. For instance, from the perspective of an inter-temporal government budget constraint where fiscal decisions in the current period affect the level of future prices (Leeper and Nason, 2010), households may already adjust their beliefs about future prices with contemporaneous fiscal shocks. Another plausible explanation could be that households may believe the expansionary effects of government spending today may be realized with a delay in implementation, as suggested in Leeper, Walker, and Yang (2010), lead to an upward revision of medium-run inflation expectations. The increase in disagreements of medium-run inflation expectations (or an increase in standard deviation) could be due to heterogeneous beliefs about the multiplier of government spending in the future. In addition, studies show that households, who may not be attentive to fiscal shocks per se, may adjust inflation expectations to future government debt as documented in Coibion, Gorodnichenko, and Weber (2021). In such case, households may associate an increase in government spending with an increase in future government debt, where households who are fearful of fiscal dominance may predict that future inflation will be high.

Finally, how personal income tax shocks affect the distribution of household inflation expectations needs to be understood better. Personal income tax shocks associated with an increase in inflation expectations may be consistent if households have perfect foresight, as households could perceive the current increase in personal income tax with higher government spending in the future, leading to an upward revision in medium-run inflation expectations is possible. However, previous studies show that perfect foresight is a very strong assumption and does not allow the agents to be uncertain about the beliefs and the response of others as documented in Angeletos and Lian (2018); García-Schmidt and Woodford (2019). Our findings should also be understood in the context of the findings in Mertens and Ravn (2013) that a cut in personal income tax does not influence the level of inflation. Taken together, there is a need to have a satisfying explanation to understand how households perceive the changes in personal income tax and incorporate this information in their inflation expectations.

6 Conclusion

In this paper, we use a functional approach to analyze the effects of several economic shocks on EID. Compared to the previous literature on inflation expectations that focus on specific moments such as the average or dispersion among survey respondents, we use the entire distribution of the inflation expectations from the University of Michigan's Survey of Consumers to understand the channels through which economic shocks affect inflation expectations.

In doing so, we document several novel findings on how economic shocks affect the distribution of household inflation expectations for one-year ahead and medium-run inflation expectations. Starting with monetary policy shocks, the channels through which monetary policy shocks affect the distribution of household inflation expectations underwent an important change over time, most notably with the commitment to fight inflation in early 1980s. Unlike one-year ahead inflation expectations, the impact on monetary policy shocks on medium-run inflation expectations is rather limited. Next, fiscal policy shocks also affect the distribution of inflation expectations. We find that households perceive expansionary government spending shocks as inflationary and both one-year ahead and medium-run inflation expectations respond to these spending shocks with statistical significance. This is different from monetary policy shocks that affect only one-year ahead inflation expectations. At the same time, we find that households perceive an increase in personal income tax shocks as inflationary, but only for the medium-run. While expansionary government spending shocks change the disagreement among households on their views on future inflation, the effects are less obvious with personal income tax shock. Finally, we confirm that the levels of household inflation expectations for both one-year ahead and medium-run inflation expectations respond to a change in gasoline price shocks, but gasoline price shocks also affect the distribution of inflation expectations.

Our findings contribute to the policy discussion how to anchor the inflation expectations of households. So far, the literature has documented that economic policies play a limited role, as monetary and fiscal policy shocks are shown to have little influence on household inflation expectations. However, our findings highlight that both fiscal and monetary policy shocks exert their influence on the distribution of household inflation expectations in their own ways, even at times in persistent ways. This calls for further investigation to better understand the exact channel through which these policy measures affect household inflation expectations, individually and jointly, which is a fruitful avenue for future research. Another important question for future research is to explore how the distribution of inflation expectations interacts with other macroeconomic variables, beyond the economic shocks that we studied in this paper.

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Appendices

A Choice of Basis

Here, we explain the approach adopted in this paper in order to select the basis. In fact, our approach based on the VAR representation in (6) may be implemented with any orthonormal basis (v_i) of H. However, the finite sample performance of the approach is critically dependent upon the choice of basis. In what follows, we denote by (v_i^*) the functional principal component basis used in the paper, and by V^* the subspace of H spanned by the sub-basis $(v_i^*)_{i=1}^m$ assuming $m < T$ and P^* to be the Hilbert space projection on V^*. In contrast, we let $(v_i)_{i=1}^m$ be an arbitrary sub-basis spanning the subspace V of H, and P be the Hilbert space projection on V in H.

As shown in Chang, Park, and Pyun (2021), the π's in (4) and (5) are *isometries*, not just one-to-one mappings between V and R^m. For π in (4), we have

$$\|f\|^2 = \|(f)\|^2$$

for any f in V, where we use the same notation $\|\cdot\|$ for the Hilbert space norm of f in V and the Euclidean norm of (f) in R^m. Similarly, for π in (5), we may show

$$|||A|||^2 = \text{trace}\,(A'A) = \text{trace}\,\big((A)'(A)\big) = |||(A)|||^2,$$

where A' is the adjoint of A, $(A)'$ is the transpose of (A), and $|||\cdot|||$ denotes both the Hilbert-Schmidt norm for a linear operator A on V whose trace$(A'A)$ is finite and the Frobenius norm for the $m \times m$ matrix (A).

Let

$$\text{FR}^2 = \frac{\sum_{t=1}^T \|Pf_t\|^2}{\sum_{t=1}^T \|f_t\|^2}$$

be the functional R-squared (FR-squared) of an arbitrary sub-basis $(v_i)_{i=1}^m$, which represents the proportion of the total variation of (f_t) explained by its projection (Pf_t) on the subspace V spanned by $(v_i)_{i=1}^m$, with FR_*^2 denoting the FR-squared of $(v_i^*)_{i=1}^m$. Then we have

$$\mathrm{FR}_*^2 \geqslant \mathrm{FR}^2,$$

which implies that $(P^* f_t)$ has the maximum temporal variation. For both $k = 1$ and 2, the approximation of A_k given by $P^* A_k P^*$ thus restricts A_k to the subspace of V^* of H, where (f_t) has the largest variation and thus A_k is most strongly identified. In this sense, the basis (v_i^*) provides the most effective approximation of A_k for $k = 1$ and 2 by its restriction on an m-dimensional subspace V^* of H spanned by $(v_i^*)_{i=1}^m$.

Given any basis (v_i), we may make the FR-squared as large as we want simply by increasing the truncation number m. However, this does not come without a cost. As m gets large, the variances of the estimators $\widehat{A}_k$ for the autoregressive operator A_k for $k = 1$ and 2 are expected to increase. They increase often very sharply in many practical applications we have observed so far, and therefore, we also need to examine how fast the variance of $\widehat{A}_k$ increases as m gets large.

Let $\widehat{A}_k$ for $k = 1$ or 2 be the estimator obtained from $\widehat{(A_k)}$ by $\widehat{A}_k = \pi^{-1}\big(\widehat{(A_k)}\big)$, which we may regard more explicitly as the estimator of $\overline{A}_k = \pi^{-1}\big((A_k)\big) = PA_kP$, and let

$$\widehat{A} = \begin{pmatrix} \widehat{A}_1 \\ \widehat{A}_2 \end{pmatrix} \quad \text{and} \quad \overline{A} = \begin{pmatrix} \overline{A}_1 \\ \overline{A}_2 \end{pmatrix},$$

which are operators from H to $H \times H$. Furthermore, we define

$$Q = \begin{pmatrix} P & 0 \\ 0 & P \end{pmatrix} \quad \text{and} \quad \Delta = \frac{1}{T} \sum_{t=1}^{T} \left[\begin{pmatrix} f_t \\ f_{t-1} \end{pmatrix} \otimes \begin{pmatrix} f_t \\ f_{t-1} \end{pmatrix} \right].$$

Then the mean-squared-error (MSE) of $\widehat{A}$ is given by

$$\mathbb{E}\big\|\widehat{A} - \overline{A}\big\|^2 = \mathbb{E}\big\|\widehat{A} - \mathbb{E}\widehat{A}\big\|^2 + \big\|\mathbb{E}\widehat{A} - \overline{A}\big\|^2,$$

where we decompose it into its variance and squared bias terms. The variance term of $\widehat{A}$ is approximately given by

$$(\operatorname{trace} \Sigma)\big(\operatorname{trace} (Q\Delta Q)^+\big),$$

where Σ is the variance operator of (ε_t) as defined earlier, and $(Q\Delta Q)^+$ is the inverse of the bounded linear operator $Q\Delta Q$ restricted to the subspace $V \times V$ of $H \times H$. The squared bias term of $\widehat{A}$ is approximately given by

$$\left\| A(1-Q)\Delta Q(Q\Delta Q)^+ \right\|^2,$$

where A is defined from A_1 and A_2 similarly as $\widehat{A}$ and $\overline{A}$.

In our subsequent discussions, we denote by $\widehat{A}^*$ and $\overline{A}^*$ the versions of $\widehat{A}$ and $\overline{A}$ obtained using our functional principal component basis (v_i^*), and show that the variance term $\mathbb{E}\left\|\widehat{A}^* - \mathbb{E}\widehat{A}^*\right\|^2$ of $\widehat{A}^*$ is significantly smaller than that of the estimator $\widehat{A}$ based on other bases in our empirical analysis. Although we cannot explicitly compute and compare them, the bias term $\left\|\mathbb{E}\widehat{A}^* - \overline{A}^*\right\|^2$ of $\widehat{A}^*$ is generally expected to be smaller than that of the other estimator $\widehat{A}$ since

$$P^*\Gamma(1-P^*) = 0,$$

whereas $P\Gamma(1-P) \neq 0$ for any other choice of basis (v_i). In fact, our methodology yields an unbiased estimator $\widehat{A}^*$ if the first order FAR, viz., $f_t = Af_{t-1} + \varepsilon_t$, is used in place of the second order FAR in (1).

To demonstrate the importance of the choice of basis in explaining the variance of EID, we compare the FR-squared's and the variance terms of the estimators of A based on our basis (v_i^*) and other bases. As an alternative to our basis (v_i^*), we consider three other bases given by the orthonormalized moments, histograms and quantiles, which will be referred to as the *moment basis*, *histogram basis* and *quantile basis*, respectively. The moment basis $(v_i)_{i=1}^m$ is obtained by the Gram-Schmidt orthogonalization procedure from the pre-basis defined as $u_i(r) = r^i$ for $i \geqslant 1$ over the interval $[p, q)$ with $p = -0.5$ and $q = 0.5$. We call (u_i) the moment basis, since

$$\langle u_i, f_t \rangle = \int r^i f_t(r) dr$$

and $(\langle u_i, f_t \rangle)$ represents the i-th moments of the EID given by the densities (f_t) for $i \geqslant 1$.

The histogram basis $(v_i)_{i=1}^m$ is given by

$$v_i(r) = \frac{1}{\sqrt{q_i - p_i}} 1\{p_i \leqslant r < q_i\},$$

Table 4. *FR2 for Four Choices of Basis*

m	FPC Basis	Histogram Basis	Quantile Basis	Moment Basis
1	0.7419	0.0095	0.0322	0.0048
2	0.8829	0.0123	0.3707	0.0064
3	0.9513	0.0400	0.5573	0.0172
4	0.9676	0.1103	0.5611	0.0220
5	0.9785	0.0735	0.7181	0.0353
6	0.9861	0.2789	0.7309	0.0461
7	0.9898	0.1430	0.7580	0.0561
8	0.9921	0.3938	0.7731	0.0678
9	0.9936	0.2482	0.7737	0.0771
10	0.9946	0.4142	0.7849	0.0878

Notes: The FR2 are reported for four different choices of basis including the functional principal component (FPC) basis, histogram basis, quantile basis and moment basis. The FR2 is expected to strictly increase as m gets large. However, this is not the case for the histogram basis, since it is defined differently for different values of m.

Table 5. *trace $(Q\Lambda Q)^+$ for Four Choices of Basis ($\times 10^4$)*

m	FPC Basis	Histogram Basis	Quantile Basis	Moment Basis
1	0.613	16.084	7.990	69.714
2	1.475	2230.758	10.867	639.162
3	4.708	8905.797	27.642	3916.064
4	8.939	13743.106	52.248	11926.281
5	16.930	17148.747	57.780	16819.808
6	24.676	21877.429	209.583	21002.513
7	39.782	30720.005	161.306	27026.392
8	60.503	41978.159	487.056	35492.106
9	88.282	53705.816	666.356	42583.255
10	124.784	62145.588	1079.753	59256.713

Notes: The values of trace $(Q\Lambda Q)^+$, which are asymptotically proportional to the variances of the autoregressive operator estimators in the Hilbert-Schmidt norm relying on four different choices of basis including the functional principal component (FPC) basis, histogram basis, quantile basis and moment basis.

where $\big([p_i, q_i)\big)$ is a partition of the support $[p, q)$ of the densities (f_t). As before, we let $p = -0.5$ and $q = 0.5$ and obtain the $(m+1)$-number of sub-intervals $\big([p_i, q_i)\big)$ of equal length, from which we take only m indicators as a basis, ignoring the first sub-interval. This is because the $(m+1)$ indicators over the $(m+1)$-number of sub-intervals $\big([p_i, q_i)\big)$ are linearly dependent. The quantile basis $(v_i)_{i=1}^m$ is defined similarly as indicators over a different set of partition $\big([p_i, q_i)\big)$. The $(m+1)$-sub-intervals $\big([p_i, q_i)\big)$ in the partition are obtained with (q_i) defined as the $i/(m+1)$-th sample quantiles of entire observations for $i = 1, \ldots, m+1$. Similar to the histogram basis, we only include m indicators in a quantile basis.

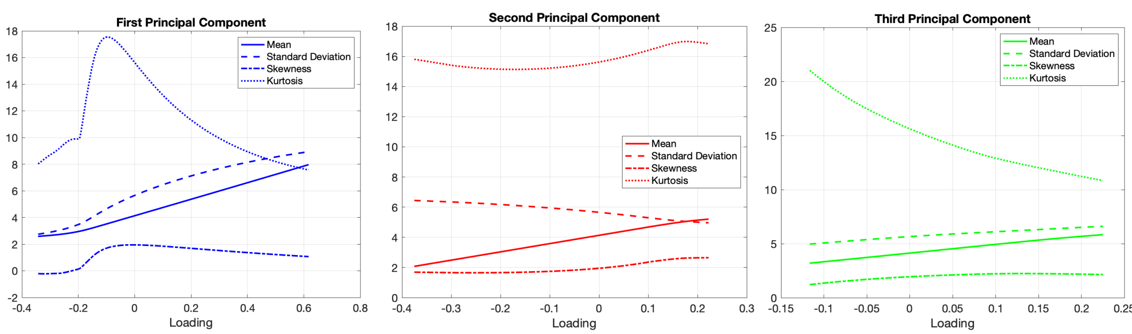
Tables 4 and 5 report the FR-squared's and the variances of $\widehat{A}$ in the Hilbert-Schmidt norm relying on four different choices of basis including the functional principal component (FPC) basis, histogram basis, quantile basis and moment basis. We may clearly see that the FPC basis effectively represents the temporal variation of the EID even with the trunction number m as small as $m = 1$ or 2. For $m = 3$, it captures more than 95% of the total temporal variation of the EID. Moreover, the variance of the autoregressive operator estimator based on the FPC basis increases as m gets large, but only at a moderate rate. In sharp contrast, all other bases obviously do not represent the temporal variation of the EID adequately. The moment basis is especially ineffective. It captures only 8.7 percent of the total variation of the EID over time even for $m = 10$. What is worse, the variances of the autoregressive operator estimator based on three other bases increase very rapidly as m increases. In particular, the use of the histogram basis or moment basis yields exploding variances of the autoregressive operator estimator even for a moderately large value of m.

As discussed, we use $m = 3$ in our empirical analysis of the EID using the FPC basis. Our choice of m with the use of the FPC basis explains 95.1 percent of the total EID variation and yields our variance measure 4.7 for the autoregressive operator estimator. If the moment basis is used, only 1.7 percent of the total EID variation is explained while we have the corresponding variance measure 3916 for the autoregressive operator estimator. The choice of basis is therefore critically important for our functional method in the paper.

B Interpretation of Functional Principal Components

Here, we examine how each of the three FPC affects the first four standardized moments, namely, the mean, standard deviaion, skewness, and kurtosis, computed from the EID. Each panel in Figure 20 shows the change in these four moments with respect to loadings for each FPC. Compared to the results we reported in Figure 6, where only the minimum and maximum values of loadings were chosen, here, we look at the movements of four moments for the entire range of historical values of loadings for each FPC.

Figure 20. *Range of variation of the mean, standard deviation, skewness and kurtosis due to the functional principal components*



Notes: Each figure reports the results for each FPC. The blue/red/green colors signify the first/second/third FPCs. The ordered loadings are on the x-axis and the values of the four standardized moments are shown on y-axis.

Overall this exercise confirms our labeling of the three FPCs. Below we describe the reasoning behind our conclusion:

(a) First FPC (blue): the second moment varies more than the first moment across all loadings. The variations in the third and fourth moments are implied by those in the first and the second. Therefore it makes sense to name the first FPC and call it as the disagreement component. While we observe that a significant variation in the fourth moment (kurtosis) with loadings, the movement in the second moment has a clear economic interpretation. Therefore, we label the functional component the 'dispersion' component to highlight the change in the second moment.

(b) Second FPC (red): It is the first moment (mean) that shows the strongest relationship with the loading. Therefore, we label the second functional component as the 'shift' component.

(c) Third FPC (green): the first three moments are pretty stable across all loadings, while the fourth moment shows a clear relationship with the loadings. As it is difficult to interpret the economic meaning of the fourth moment, we label this functional component as the 'ambiguity' component.

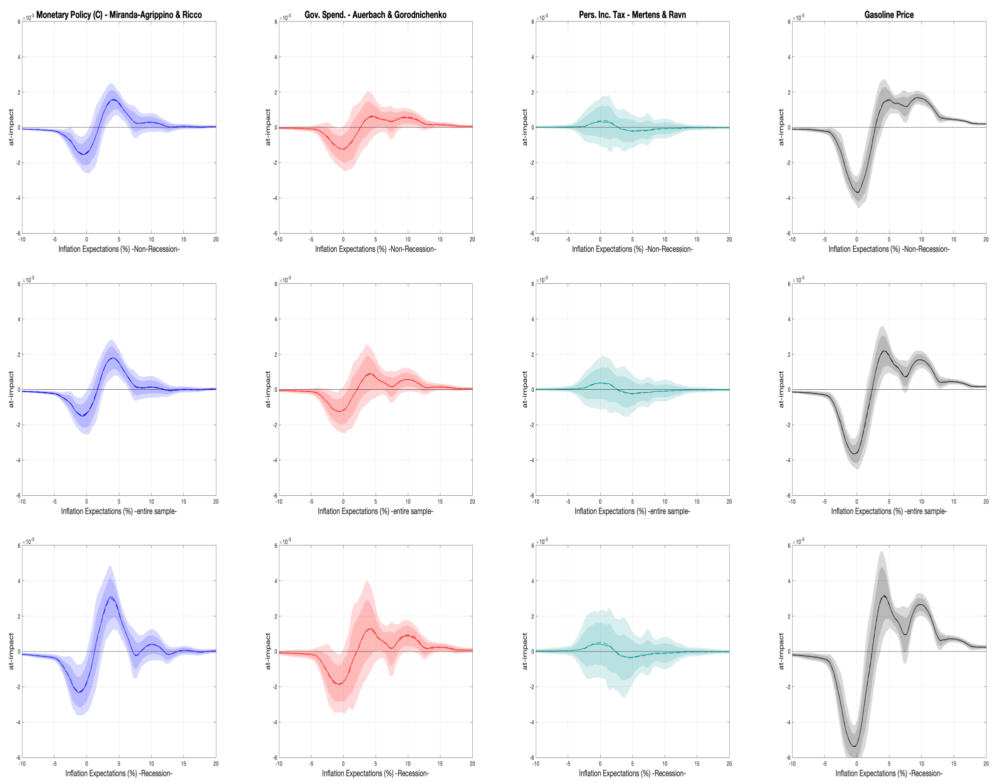
C State-Dependent IRFs: Recession vs. Non-Recession

In this Appendix, we conduct a state-dependent analysis by comparing the at-impact responses of one-year ahead inflation expectations, separately for the recession and non-recession periods, using the periods of recession identified by the NBER. We look at one-year ahead inflation expectations only in this exercise, as there are too few matched recession episodes for the medium-run inflation expectations. Figure 21 presents the at-impact responses of four economic shocks in three rows: the responses in the first row are obtained using non-recession periods, those in the middle row using the entire sample, and those in the last row using the recession periods. Based on the at-impact responses, we find that the effects of fiscal policy shocks on the distribution of household inflation expectations do not depend on the state of the economy, or whether an economy is in a recession or not. Confidence bands tend to widen during the recession periods, possibly due to a smaller number of observations. However, the coefficients and the shape of IRFs remain relatively unchanged across three rows.

On the other hand, for contractionary monetary policy shocks and an increase in gasoline prices, the inflationary effects on household inflation expectations are stronger during the recession compared to non-recession periods. For monetary policy, this is shown by the magnitude of coefficients corresponding to the peak during the recession, which is almost twice as large as the coefficient observed when the economy is not in a recession. This increase in the frequency around the peak is accompanied by a fall in the frequency of responses around the trough during recession periods. Similarly, for gasoline price shocks, a dramatic fall in the frequency of negative inflation expectations during recession periods is accompanied by an increase of similar size in the frequency of positive inflation expectations, which strengthens the inflationary effects during the recession periods.

Clearly, further research is warranted to identify the underlying causes of the state-dependent responses of household inflation expectations to monetary and gasoline price shocks. The current literature provides some guidance as to which channels may be at play. For instance, contractionary monetary policy shocks during a recession period may lead to a worsening of the economic situation, and households may translate these pessimistic views of the economy into higher inflation as discussed in Coibion, Gorodnichenko, and Kamdar (2018). Alternatively, households may not believe that the central bank will continue to adopt contractionary monetary policy to fight inflation during the recession as tightening monetary policy could further worsen the downturn. This may lead to an upward revision of future inflation expectations. In a similar spirit, households may view an increase in gasoline prices during economic recessions as a sign of forthcoming stagflation, and accordingly revise upward their inflation expectations. However, more work is needed to unravel the exact mechanism of these findings.

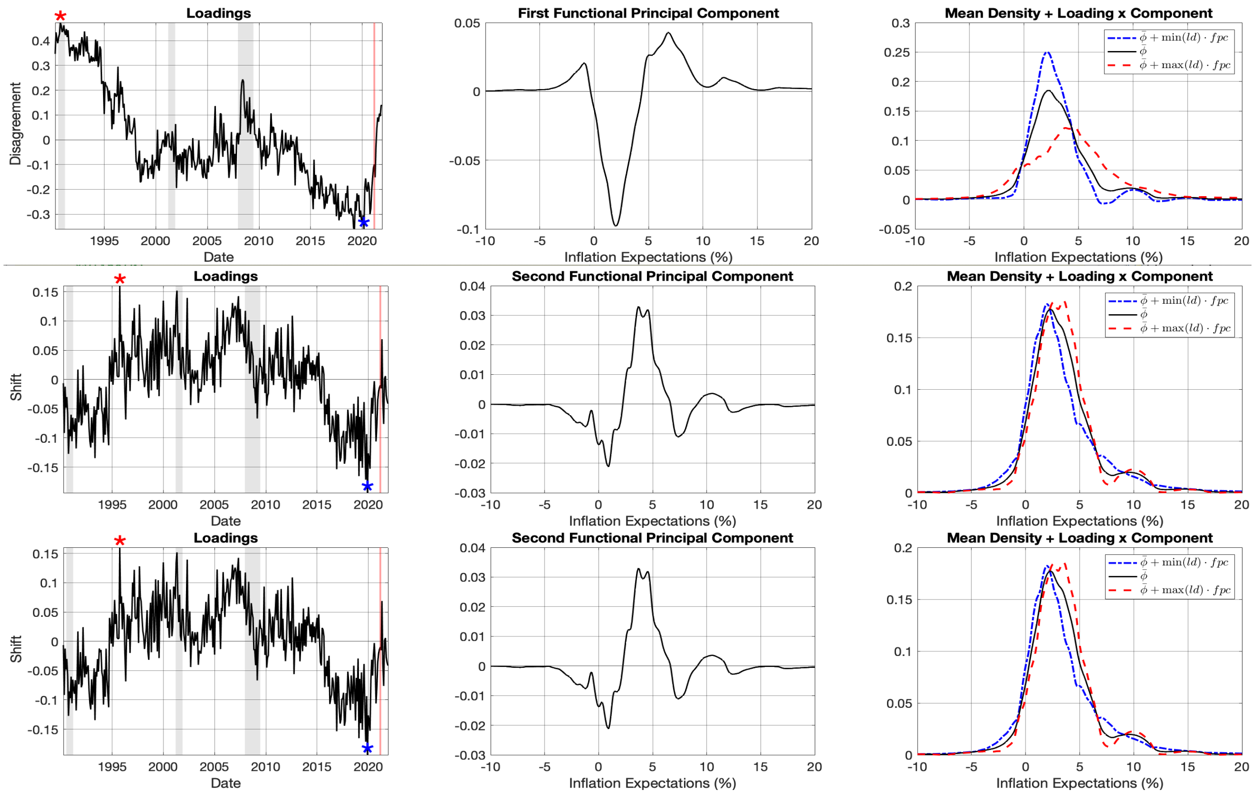
Figure 21. *At-Impact Responses: Recession vs. Non-Recession*



Notes: The top row presents the at-impact responses for economic shocks during non-recession periods. The middle row presents the at-impact responses for the entire sample from 01/1983 to 12/2021. The bottom row presents at-impact responses during the recession periods.

D Additional Figures

Figure 22. Functional principal components and loadings: Medium-run inflation expectations



Notes: Left panels shows the value of the loadings over time for the three principal components (top: disagreement, middle: shift and bottom: uncertainty). The center panels show the principal component as a function (which integrates to 0). The right panel shows how the sample mean density function is modified with the maximum and minimum value from the loadings shown in the left panel.

PUBLICATIONS

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