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# Fix vs. Float

## Evaluating the Transition to a Sustainable Equilibrium in Bolivia

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**Fix vs. Float: Evaluating the Transition to a Sustainable Equilibrium in Bolivia**
**Prepared by Andrés Gonzalez, Etibar Jafarov, Diego Rodriguez Guzman, and Chris Walker***

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**ABSTRACT:** Bolivia has achieved noteworthy success over the past 15 years in raising incomes, reducing poverty, and maintaining macroeconomic stability by deploying commodity revenues to finance transfers, public investment, and state-led development, using an exchange rate peg as a policy anchor. However, with the end of the commodity boom in 2014, fiscal deficits have grown and reserves have fallen. One route to restoring long-run sustainability would be to combine fiscal consolidation with a switch to a floating exchange rate. However, a preference for maintaining the peg could be accommodated with adjustments elsewhere in the policy framework. Employing a detailed dynamic stochastic general equilibrium model of the Bolivian economy, this study assesses the long-run sustainability and relative benefits of alternative policy combinations, and calculates optimal adjustment paths for the transition from the present situation to the steady state. It concludes that continued adherence to a fixed-rate regime, while not optimal, is feasible, if supported by a larger fiscal effort.



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# Contents

# I. Bolivia's Current Trajectory

Since 2014 Bolivia's fiscal deficit has increased from 3.4 percent of GDP to 8.1 percent of GDP in 2018, falling slightly to 7.2 percent of GDP in 2019 before rising again with Covid-19-related fiscal measures. Over the same period, the real effective exchange rate appreciated by 34 percent, as the boliviano's peg to the U.S. dollar entailed substantial appreciation against Bolivia's trading partners, resulting in a moderate overvaluation under various IMF metrics, and helping to push the external current account deficit to an average of 4 percent of GDP in 2015-2019. At the same time, international reserves fell from $US 15.1 billion (45.5 % of GDP) in 2014 to $US 4.7 billion (12.2% of GDP) in mid-August 2021. While concerns over declining reserves have been associated with occasional episodes of deposit withdrawals, particularly during periods of political volatility, these have been moderate, as administrative guidance and macroprudential measures have limited pressures on the exchange rate peg.

Under these circumstances, a reduction in the fiscal imbalance can lower debt, sustain reserves, and help to avoid an eventual disorderly adjustment. The elimination of the fiscal deficit, or its reduction to a lower level, will reduce the drain on national savings and protect reserves. Conversely, while it may be possible to continue funding the fiscal deficit with foreign or private domestic savings, doing so reduces net national savings and crowds out private investment.

Along with the fiscal adjustment, a further option would be to move to a floating exchange rate. Shifting from a peg to a float can lessen the drain on international reserves, as the monetary authority is no longer obliged to sell dollars to resist depreciation pressures. If the shift also leads to a real depreciation, as is typically the case, the positive effect on the current account would strengthen the balance of payments. If domestic prices and wages display nominal rigidities, as is normally the case, the shift to a float dampens the impact of external shocks and helps smooth consumption. In addition, the shift to a float offers the country the ability to calibrate monetary policy to its own specific circumstances.

Weighed against these positive aspects of a flexible exchange rate, a pegged exchange rate also offers significant benefits to a developing economy, particularly one such as Bolivia with a history of hyperinflation. As a strong and highly visible policy anchor, the exchange rate peg anchors inflationary expectations. The peg can, at least in principle, help to discipline fiscal policy. To the extent that the unhedged liabilities of domestic financial institutions are in dollars, a fixed exchange rate may also increase financial stability, although this effect can be reversed if the credibility of the peg comes into question.

# II. Theoretical Approaches to Setting Exchange Rate Policy

What considerations should be paramount in deciding between a fixed and floating regime, or in determining to move from one to the other? The basic open economy Mundell-Fleming model provides no specific grounds for choosing between the two. Based on the joint assumptions of sticky prices, uncovered interest parity, and perfect substitutability of domestic and foreign assets, it implies that, in response to a negative shock (e.g., a negative productivity shock), a fiscal stimulus is effective under a fixed exchange rate, whereas monetary stimulus is ruled out by the requirement to maintain the peg. Under a float (e.g., with an inflation target), fiscal policy becomes a less effective demand management tool, as the positive effect of a fiscal stimulus is partially dispersed through a currency appreciation. However, as compared with a peg, the authorities are able to deploy monetary policy to raise output, which will be accompanied by a depreciation.

Other relative advantages and disadvantages of the regimes, such as stability, credibility, and time consistency, are not included in the model.

Speculative attack models identified with Krugman (1979), Flood and Garber (1984), and others focus on the sustainability of fixed exchange rate regimes. Unsustainable regimes are likely to impose high welfare costs if a speculative attack leads to a disorderly adjustment. Krugman shows that under specific conditions, which include zero growth and uncovered interest parity, any recurring fiscal deficit with a fixed regime leads eventually to a speculative attack and uncontrolled float, as the central bank balance sheet is increasingly dominated by domestic debt. The model of Flood and Garber, which assumes growth and external inflation, yields a similar result, but allows that the fiscal deficit may match seignorage gains of the central bank (or, equivalently, that the gains are transferred to the government). These models are deterministic; the inclusion of stochastic shocks may increase the likelihood of a speculative attack or disorderly adjustment. Insofar as a floating exchange rate is not susceptible to such attacks, the risk of disorderly adjustment should be counted as a relative disadvantage of fixed regimes.

In a family of neo-Keynesian models exemplified by Schmitt-Grohé and Uribe (2017), a fixed exchange rate regime introduces a welfare-reducing inefficiency that limits policymakers' capacity to respond to adverse shocks. The combination of nominal wage rigidity (which is well-documented across a wide range of economies) and a fixed exchange rate results in involuntary unemployment in the event of a negative shock, as will inevitably occur if shocks have zero mean and nonzero variance. Such shocks could originate from the terms of trade, world interest rate, asset preferences, or domestic productivity, among other sources. In contrast, exchange rate flexibility allows real wages to adjust in response to a shock, permitting the labor market to clear, and maintaining full employment through the business cycle.

Some interesting implications for fiscal policy are spelled out in work by Bianchi and Sosa-Padilla (2020), who note that reserves levels in countries with pegged exchange rates average 16 percent of GDP, more than twice the average rate for floaters of 7 percent of GDP. This is explained within their model by the need for fixed-rate countries to maintain large reserves to help smooth consumption in the event of a negative shock, to compensate in part for their inability to sustain employment via a real depreciation. It follows that countries with fixed-rate regimes would need to run higher primary surpluses for some time in order to build reserves to the required levels.

To achieve a realistic evaluation of the policy choices facing Bolivia, a dynamic stochastic model that incorporates these price rigidities and credibility factors is needed. The model should provide evaluations of alternative steady states with either a fixed or floating exchange rate, including assessment of the feasibility, stability, and welfare implications of the transition paths needed to reach the steady state. Section III presents a dynamic model fulfilling those criteria, with parameters calibrated to match salient features of the Bolivian economy.

## III. A Calibrated Model of the Bolivian Economy

The analysis of optimal policies is done using a dynamic stochastic general equilibrium model for a small open economy dependent on commodity exports. Production is characterized by standard upward-sloping Phillips curves that reflect the presence of short-term price rigidities. There are two basic goods, a home good and a foreign good. Demand for output of the home good comes from government consumption, private consumption, public and private investment, and exports. Demand for the foreign good comes from domestic private investment and consumption.

Production of the home good is by a perfectly competitive firm that sources a continuum of intermediate goods from monopolistically competitive domestic suppliers. Price stickiness in the goods market comes from Calvo price adjustment for intermediate goods. Similarly, wage stickiness arises from an *ex-ante* wage bargaining arrangement conducted by unions, which can result in underemployment.

The government receives income from taxes on consumption, labor, and capital, from commodity revenues, and from the quasi-fiscal balance from the central bank (i.e. seignorage). Government expenditures consist of transfers to households, public investment, government consumption, and interest payments on government debt. The government has access to the financial market, where the central bank and the private sector buy public debt.

The model incorporates several neo-Keynesian features that enable transmission of fiscal policy inputs to the real economy. First, nominal price and wage rigidities enable aggregate demand to drive output and employment (because workers are not necessarily able to work as much as they want at the prevailing wage). Second, there are Ricardian and non-Ricardian "hand-to-mouth" households. Third, the structure of the labor market limits the income effect of taxes. And fourth, there are adjustment costs of investment. This last feature limits the degree to which government spending crowds out private investment on impact. Overall, the fiscal multiplier in the model averages 0.4 for expenditure increases.

The balance sheet of the central bank characterizes the monetary sector. The central bank holds government bonds and foreign reserves on the asset side of its balance sheet and monetary aggregates on the liability side. The central bank transfers quasi-fiscal (or seignorage) revenues to the central government. In the central bank flow of funds, the quasi-fiscal revenues accrue from differences between the rates on return of assets and liabilities.

On the external side, the model diverges from uncovered interest parity with the inclusion of a financial accelerator-style wedge between domestic and foreign interest rates, the size of which varies inversely with Bolivia's net foreign asset position. Consequently, foreign and domestic bonds are imperfect substitutes, and sterilized intervention is effective.

The monetary authority can follow different strategies affecting the balance sheet of the central bank and the response of the economy to different macroeconomic shocks. For example, if it pegs the exchange rate, the central bank will intervene in the currency market, buying and selling foreign reserves and adjusting the money supply accordingly. However, if it targets the inflation rate, foreign reserves remain unchanged, and the central bank supplies money to keep the nominal interest rate equal to the policy rate.

The model structure is presented in detail in Appendix I.

# IV. Two Steady States

The model is calibrated for Bolivia using the past twenty years of Bolivian macroeconomic data to capture the main structural parameters of the economy. The initial state of the model, calculated to reflect Bolivia's situation at the end of 2019, incorporates an external deficit of 5 percent of GDP, a fiscal deficit of 7 percent of GDP, output growth of 3 percent, potential output growth of 3.8 percent, an inflation rate of 1.8 percent, and foreign reserves at 25 percent of GDP. The central bank pegs the exchange rate to the US dollar with a target devaluation rate of zero percent.

To facilitate policy comparisons, two steady states are obtained - one with a fixed exchange rate and the other with a floating exchange rate and inflation target. The two steady states share the same basic ratios, such as the current account to GDP, the real exchange rate, and the share of debt in GDP. However, in the second (IT) steady state, the inflation rate is 2 percent higher. Other nominal variables such as the nominal devaluation rate and the nominal interest rate are adjusted to reflect this difference.

| | FIX | IT | IT Alternative |
|---|---|---|---|
| Inflation Target | 2.0% | 4.0% | 4.0% |
| Nominal devaluation | 0.0% | 0.0% | 0.0% |
| Nominal Interest Rate | 5.1% | 7.1% | 7.1% |
| Foreign Inflation | 2.0% | 2.0% | 2.0% |
| **Central Bank** | | | |
| Money / GDP | 25.0% | 25.0% | 21.4% |
| Foreign Reserves / GDP | 21.0% | 21.0% | 17.4% |
| **Central Government** | | | |
| Long run Government Debt / GDP | 40.0% | 40.0% | 40.0% |
| Quasifiscal gain / GDP | 1.2% | 1.7% | 1.4% |
| Government Expenditure / GDP | 14.0% | 14.4% | 14.2% |
| Government Investment / GDP | 10.0% | 10.0% | 10.0% |
| Long run sustainable primary deficit / GDP | -1.5% | -1.9% | -1.7% |
| **External Sector** | | | |
| NFA / GDP | 15.1% | 15.1% | 15.1% |
| Private external debt / GDP | 5.9% | 5.9% | 2.3% |
| Foreign Reserves / GDP | 21.0% | 21.0% | 17.4% |
| NX / GDP | 0.1% | 0.1% | 0.1% |
| CA / GDP | 0.8% | 0.8% | 0.8% |
| **National Accounts** | | | |
| Private Consumption / GDP | 66.9% | 66.9% | 66.9% |
| Private Investment / GDP | 8.0% | 8.0% | 8.0% |

Under inflation targeting, which is assumed to allow a rate of inflation two percent higher than the average inflation rate under the peg, the quasi-fiscal balance (i.e., seignorage gain) is larger, allowing the non-seignorage primary deficit to be somewhat wider. However, because it abstracts from stochastic shocks, this comparison between fixed and floating steady states leaves out important factors, including the greater need under a pegged regime for self-insurance in the form of higher reserves, as analyzed in Sosa and Padilla.

In addition, it does not reflect the benefits of monetary policy autonomy if business cycles are not fully synchronized.

Consequently, a direct comparison between the fixed and floating steady states shows limited differences (see table) in key variables. While the sustainable primary fiscal deficit in the floating steady state is 0.5 percent of GDP larger than with the fix, many other quantities are identical. The debt-to-GDP ratio remains unchanged, but government expenditure is higher by 0.5 percent of GDP. This difference shrinks but does not disappear under the assumption that money demand becomes more inflation-elastic under IT (alternative steady state in the table). However, as indicated above, if values were computed in a stochastic steady state setting,[1] it is expected that there would be greater differences between the fiscal balances in the fixed and floating steady states, given the greater need to self-insure in the case of a fixed exchange rate.

# V. Transition Paths to the Steady State

As Bolivia is far from a sustainable steady state – whether with a fix or a float – the question of how to transition from the current juncture to long-run sustainability, identified here with the steady state, becomes a central concern, potentially determining policy over several years. To identify an optimal path, an objective function for the policymaker is introduced that incorporates the household utility function aggregated over the two types of households (Ricardian and non-Ricardian), an adjustment cost of altering tax policy, and a term that penalizes the policymaker for divergences from the steady state public debt stock (this helps to ensure that policymakers do not behave in a non-credible fashion, such as by running a large deficit in the initial period while promising to tighten policy sharply in subsequent periods)[2].

$$\max_{\{policy\ variables\}} U(C, B^G) = U(C_t, N_t, m_t) + \omega_1 \left(B_t^g - \overline{B}^g\right)^2 + \omega_2 (Tr_t - \overline{Tr})^2$$

The model incorporates stochastic shocks over the transition path. In this version of the model policymakers are able to credibly commit to a trajectory for the exchange rate regime and other control variables. Private sector expectations correspond to the authorities' policy commitment. Although the policymaker is trying to maximize expected utility and the period utility function is concave, the fact that the model is linearized entails that solution paths are certainty-equivalent.

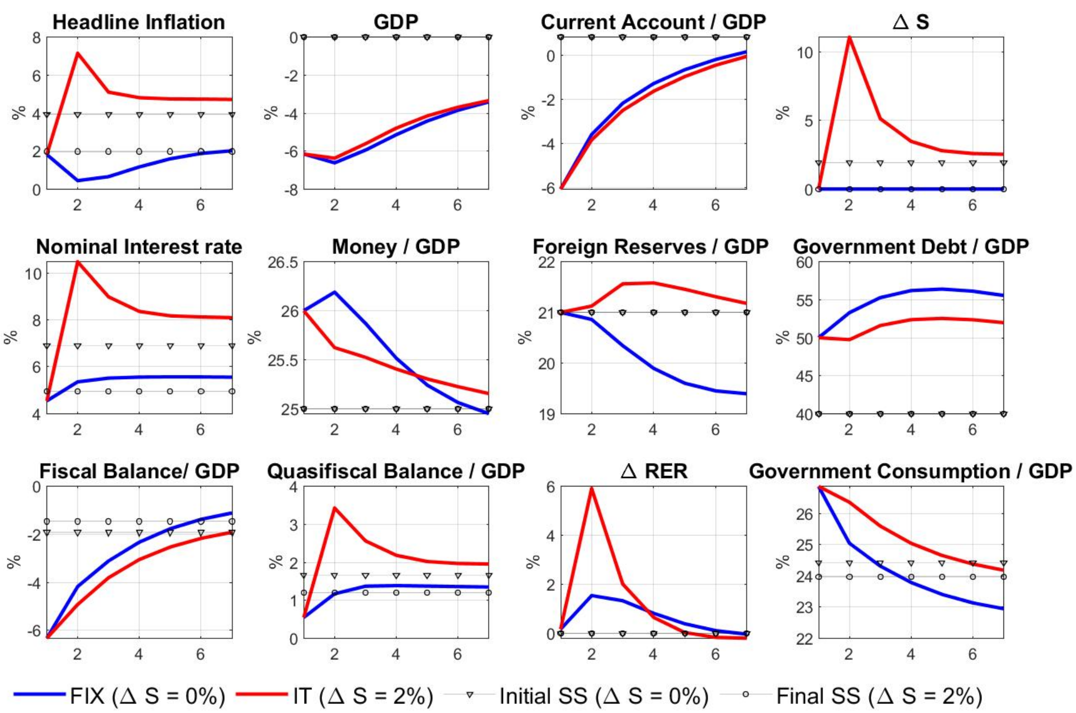
Subject to the *ex-ante* determination of either a fixed or floating exchange rate in the steady state, the policymaker selects the optimal trajectory for expenditure and thus the trajectory for the fiscal deficit. If the transition is to a floating exchange rate, the central bank shifts immediately to an inflation target corresponding to the inflation rate in the floating steady state (4 percent), and refrains from intervening in the foreign exchange market. If the transition is to a peg, the nominal exchange rate either remains fixed or depreciates at a steady 2 percent. Domestic interest rates, which affect both demand and capital flows, are adjusted following a standard Taylor rule, with reserves meeting any financing gap in the balance of payments.

[1] This possibility is ruled out by the calibration strategy employed in finding the steady state.

[2] This term approximates the effect of requiring a time consistent policy trajectory on the part of the policymaker. Under certain conditions, full time consistency can be imposed by requiring the policymaker in the initial period to incorporate as constraints his or her own first-order conditions in subsequent periods. However, the present modeling framework does not permit this.

Optimal paths for fiscal policy are obtained in each case. As shown in Figure 1, the optimal fiscal deficit under inflation targeting is wider than under the peg, with the difference attributable mainly to the larger quasi-fiscal balance associated with the higher rate of inflation. However, under the IT regime consumers also enjoy a higher average level of utility than they do under the pegged regime, due to the advantage of a flexible exchange rate in absorbing external shocks, as there is less underemployment and a smaller decline in consumption in the event of a negative shock.

**Figure 1. Transition Paths**




Under inflation targeting, the currency undergoes an 11 percent nominal depreciation and 6 percent real depreciation on impact, with a cumulative nominal depreciation of 21 percent and real depreciation of 10 percent after five years. Inflation rises to 7 percent in the first year of inflation targeting, then converges quickly to the 4 percent target as interest rates are increased under IT. There is of course no change in the nominal exchange rate under the peg, while the real exchange rate depreciates modestly with a decline in domestic inflation below its 2 percent steady state level. Without the possibility of achieving relative price adjustment through a movement in the nominal exchange rate, the economy adjusts through domestic disinflation, with adverse impacts on GDP and domestic welfare due to nominal rigidities.

Overall, the weight placed on public debt, $\omega_1$, makes a large difference to the fiscal consolidation path under either monetary regime. If the weight on public debt is high, raising the cost of maintaining debt above the target level, the fiscal balance reaches the steady state level after three years under IT and after four under the peg. Conversely, when the weight on public debt is low, the foreign reserves level declines rapidly under the peg. This scenario corresponds to a case where the government's commitment to an announced fiscal reform trajectory is likely to be questioned, and is unlikely to be time consistent or credible.

The higher the weight on public debt, the more likely it is that the optimal policy path will be self-reinforcing and, as a result, fully credible to private agents. However, even in this case, the commitment to the chosen exchange rate regime is assumed to be fully credible, ruling out *ex ante* the possibility of switching regimes under pressure.

## VI. Implications of the Exchange Rate Regime for Fiscal Policy

The preceding considerations suggest that a crucial difference between a fixed and a floating/inflation-targeting exchange rate regime lies in the implications for fiscal policy. In the hypothetical steady state, these are limited to the seignorage, or quasi-fiscal, gains associated with the higher average inflation rate in the IT regime. However, the inclusion of stochastic shocks in the transition path to the steady state introduces an additional element that works in favor of the floating rate, as the combination of a fixed exchange rate with labor market rigidity implies a lower rate of employment in the event of a negative shock, and thus a greater need for countercyclical fiscal support to households. In addition, the ability of a floating rate to accommodate the monetary policy adjustments needed to address the domestic business cycle is reflected in the model. As a result, in the pegged regime there is a need to adhere to a more conservative fiscal policy to provide fiscal space for future countercyclical expenditure. It also follows from the capacity of the floating/IT regime to accommodate shocks that, as the variance of such shocks increases, the relative advantage of the float over the currency peg rises.

## VII. A Consistent Model with Regime Switching

To present a more realistic range of policy option, the model needs to accommodate the possibility of a change in the exchange rate regime, either expected or unexpected, at some point after the initial period. If private agents are aware that the regime will change in the future, they will adjust their behavior to reflect this expectation. The exercise demonstrates the feasibility of an announced future shift away from a pegged exchange rate[3].

In the case in which the transition is expected, the authorities announce the eventual adoption of an inflation targeting regime and indicate when the transition will take place. All agents know the transition date. The authorities adopt an optimal fiscal path. At the crossover point (three years after the initial period in the model), the exchange rate floats, and the monetary authority switches to targeting a four percent inflation rate. In the case of an unexpected transition, private agents do not take into account the possibility that the authorities may switch to an inflation target.

[3] Appendix 2 describes the solution method.

**Figure 2. Expected Transition to IT Regime**

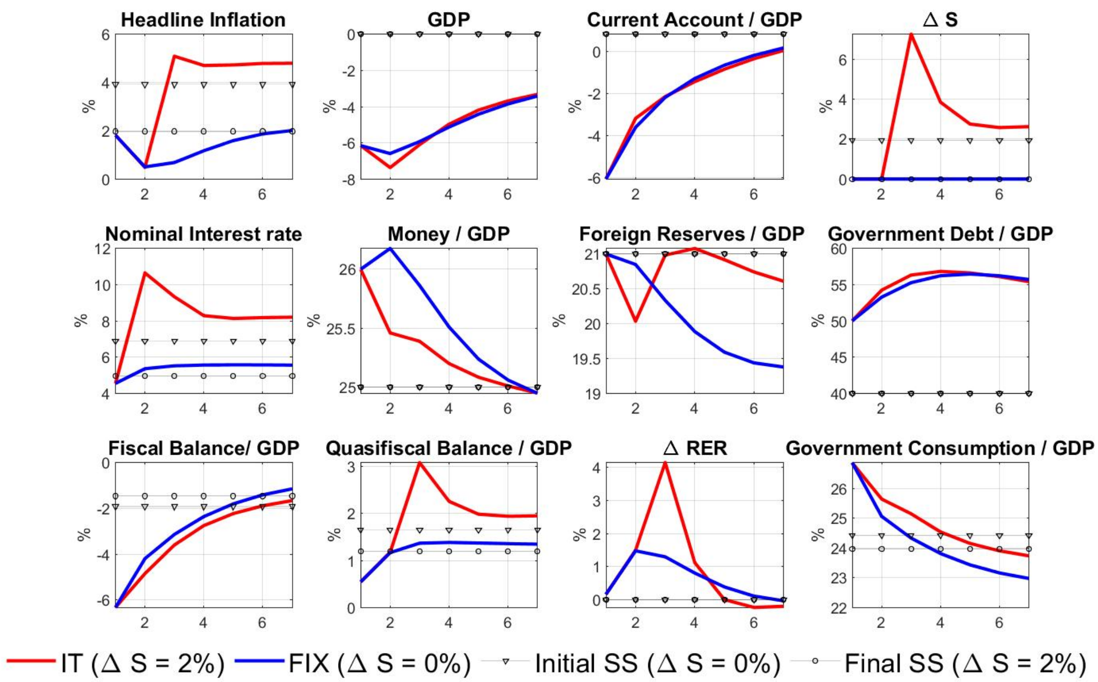
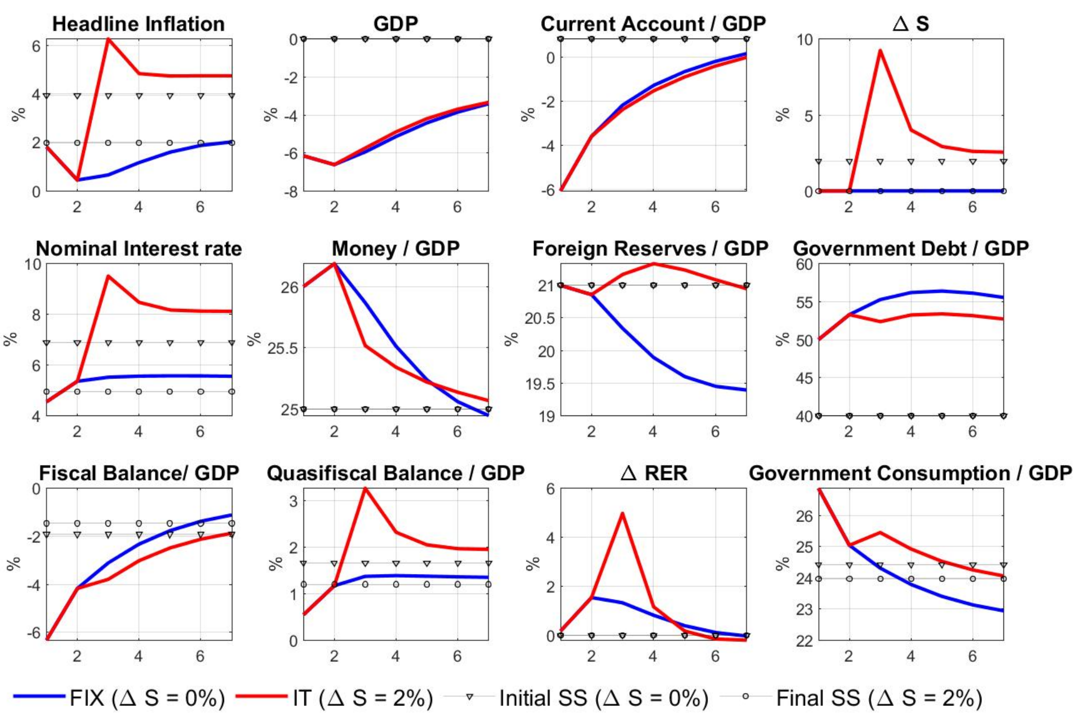



**Figure 3. Unexpected Transition to IT Regime**

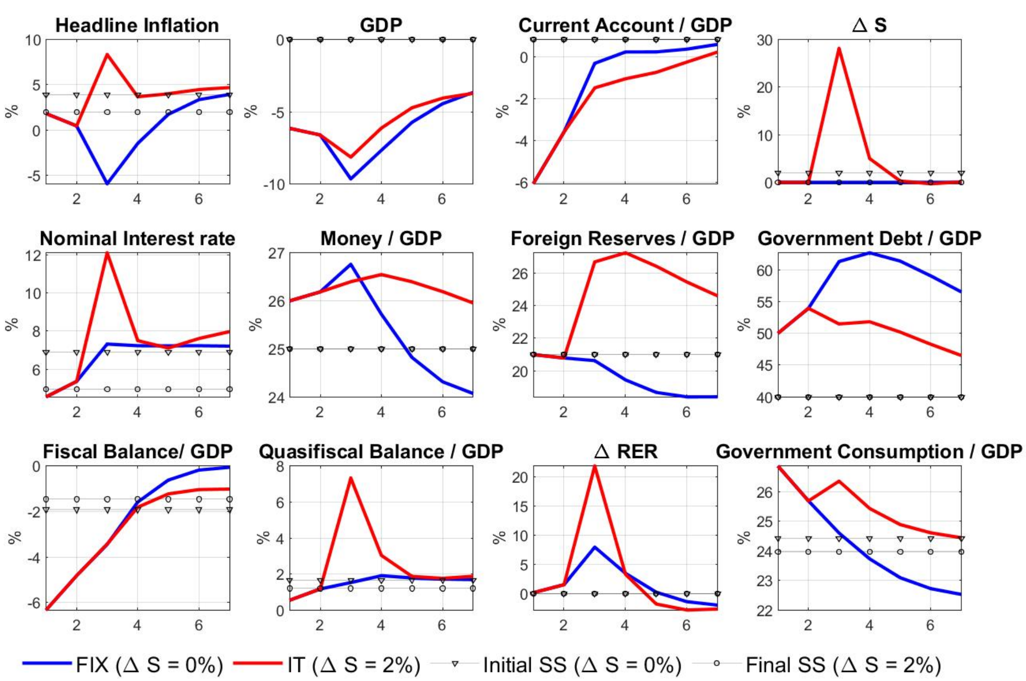


In either case, the policy path is optimal in the sense that the social planner (or the authorities) maximizes the intertemporal objective function and proceeds to execute the indicated policies. In the case of an expected transition, it is assumed that full commitment is possible, so that private sector agents know the policy trajectory and expect it to be implemented, even if at some point along the way there is an incentive for the planner to diverge from it. Under these conditions, the transition takes place smoothly with a steady reduction in the fiscal deficit, and a gradual depreciation of the currency once the transition to the IT regime takes place. This trajectory yields a level of intertemporal welfare, comparable to, though slightly below, that resulting from immediate adoption.

## VIII. A Risk-Off Shock

Although the scenarios considered so far are stochastic, they have not extended to the relative responses of pegged and IT regimes to large, persistent negative external shocks, or to sudden changes in the preferences of domestic investors. A common shock, seen in "risk-off" scenarios in emerging markets, is an adverse shift in asset preferences, prompting investors to demand a higher risk premium to hold domestic assets (e.g., the shock that afflicted many emerging markets in the "taper tantrum" of 2013). Within the model, this results in a higher interest rate charged to Bolivians borrowing in dollars, and, through the imperfect substitution between domestic and foreign assets, a higher domestic interest rate. If the shock is large, the effect is similar to a sudden stop. As implemented in the model, this preference shift takes the form of a one-off increase in $\Omega_u$, the coefficient on debt in the interest rate specification, which effectively increases the interest rate that investors charge to hold domestic assets.[4]

**Figure 4. Response to One-off Increase in Ωu**




[4] Equation 4 in Appendix 1.

A sudden shift in asset preferences may also arise as a result of loss of confidence on the part of domestic agents. This might occur if private agents come to doubt the authorities' commitment to a fiscal or exchange rate target. Such doubts, as noted above, are more likely if the authorities' announced plans for bringing the economy to a sustainable steady state equilibrium depend on a sequence of increasingly difficult policy adjustments. Agents may doubt the willingness of the authorities to follow through on a plan that will become more painful over time. Plans that are not credible, in the sense of not being time consistent, may lead to a loss of confidence and deposit or capital flight. Such shifts can occur quite quickly.

Under either the peg or IT regime, a large unexpected risk-off shock forces a sudden switch in the current account, from a 5 percent of GDP deficit to a surplus of 5 percent of GDP, in just two periods. However, under the pegged regime (even allowing for a two percent nominal devaluation) there is a sudden large loss of reserves and increase in the real interest rate as the monetary authorities struggle to maintain the designated exchange rate against depreciation pressure. Prices adjust downward under the peg, but due to price stickiness, unemployment increases. In contrast, under inflation targeting, the nominal exchange rate depreciates 19 percent on impact, and another 5 percent in the following period. Domestic prices rise in local currency terms but decline in dollar terms. The loss in output and employment is significantly less than with the peg. The differential response to a sudden unexpected shock highlights the advantages of exchange rate flexibility in response to shocks to investor preferences.

The risk-off shock scenario illustrates the pressures on prices, employment, reserves, and output that may become too great to bear under continued adherence to a fixed exchange rate regime after a large shock. Under this interpretation, it shows circumstances under which the authorities may determine that they have little choice but to switch to a floating exchange rate. Future iterations of the model will expand on this approach, in order to quantify the risk of a disorderly adjustment during the transition to a steady state, or even while in the steady state.

# IX. Comparison with Other Exchange Rate Assessments

The model points to a cumulative real depreciation of 10 percent to bring Bolivia into a sustainable equilibrium, corresponding to the change between the initial conjuncture and the steady state. The adjustment is the same whether Bolivia adopts inflation targeting or a fixed exchange rate regime; the difference is in the speed of convergence. However, this degree of adjustment is smaller than the real exchange rate appreciation of 34 percent that Bolivia has experienced over the past 7 years, and is also of a lower order of magnitude than what is suggested by some other models. Some of the discrepancy between the real exchange rate depreciation shown in the model and the larger depreciations indicated elsewhere is attributable to the large fiscal adjustment implemented to reach the steady state in the model.[5] Assumptions about changes in other parameters may also explain part of the discrepancy. To the extent that alternative models presuppose, either explicitly or implicitly, an immediate shutdown of foreign financing, the situation may be better compared to the 20 percent real exchange rate depreciation realized on impact in the omega shock scenario, in which the risk premium on Bolivian assets rises suddenly.

[5] See IMF (2021)

# X. Experience with Transitions and Areas for Further Development

It is not possible for a theoretical model to incorporate every aspect of the complex transition from a fixed to a floating exchange rate regime, and credibility factors are often difficult to model. The framework employed in the present study recognizes these factors in the attribution of rational expectations to private agents in the model, in the restriction that fiscal policies be optimal given the exchange rate regime, and in the inclusion of a penalty for divergence from the steady-state debt level in the policymaker's objective function (which tends to force the policymaker towards a time consistent path with respect to private expectations). However, the time consistency requirement does not extend to the selection of an exchange rate regime, and it may be the case under a pegged regime that the social planner would be better off switching to inflation targeting. In addition, by imposing a first-order linear approximation of the economy's response to stochastic shocks, the model rules out some disorderly adjustment scenarios that might obtain in response to a series of negative shocks. The risk-off scenario approximates the impact of a shock strong enough to prompt a disorderly adjustment.

In general, individual country experience with transition demonstrates the importance of guiding expectations and maintaining a strong and credible fiscal trajectory, both of which are features of regime transitions in the model. A number of countries, including Chile (1984-99), Israel (1995-2005), Poland (1990-2000), and Russia (2005-14), have successfully managed transitions to greater exchange rate flexibility while avoiding disorderly market adjustments. Other countries, including Brazil (1999), the Czech Republic (1997), Turkey (2001), and Uruguay (2002), experienced more stressful transitions, although in the cases of Brazil and the Czech Republic, large output losses were avoided, and the transitions were ultimately effective. The successful cases typically took place under fairly tranquil market conditions with adequate reserve coverage, and involved careful communication from the authorities and parallel development of complementary institutions such as derivatives markets. Most were implemented gradually, often with a one-time step devaluation, followed by a sliding peg, and then a widening of a currency trading band. In the case of Bolivia, many of these factors are in place, notably a relatively benign external environment, and some institutions, such as a foreign currency auction mechanism that could be further developed in tandem with the regime shifts. Reserve coverage is low, in contrast with the successful cases, and the fiscal deficit is large, but an up-front shift in fiscal policy together with an increase of available financing from international financial institutions may serve to make up this gap.

In future versions of the model, credibility issues may be accommodated more fully by introducing a requirement of time consistency between exchange rate regimes, and by doing second-order linearization of the model. Nominal money demand in the model is assumed to be relatively inelastic with respect to inflation; a stronger negative correlation between the two would yield a lower quasi-fiscal gain from the IT regime. An additional area for exploration is that, whereas the model assumes free capital flows, Bolivia's capital account, like those of many other emerging markets, might best be characterized as possessing some informal limits and aspects of moral suasion to discourage large outflows.

# XI. Conclusion

The small country open economy DSGE model developed in this paper provides a detailed elucidation of the tradeoffs and differences entailed in the choice between a fixed and floating/IT exchange rate regime in Bolivia. Incorporating many, although not all, of the conditions that could provoke a loss of credibility and/or speculative attack under a fixed exchange rate, the model generates stable transition paths to a steady state under either a fixed or floating exchange rate regime.

If fiscal policy is time consistent and optimized for the chosen exchange rate path, the transition will be viewed as credible by market participants. There are welfare benefits from adopting a floating regime. However, these welfare gains are relatively modest, leaving continued adherence to a fixed exchange rate as a plausible policy alternative, albeit one that requires a larger fiscal adjustment and a higher path for the fiscal balance. In a risk-off scenario that replicates some of the conditions of the 2013 emerging market “taper tantrum,” the relative welfare advantage from exchange rate flexibility increases significantly.

# References

International Monetary Fund, Bolivia Article IV Staff Report, 2021.

Bianchi, Javier, and César Sosa-Padilla, 2020. "Reserve Accumulation, Macroeconomic Stabilization, and Sovereign Risk," NBER Working Paper No. 27323.

Flood, Robert P., and Peter M. Garber, 1984. "Collapsing Exchange-Rate Regimes: Some Linear Examples," *Journal of International Economics*, Volume 17, Issues 1–2: 1-13.

Galí, Jordi, J., David López-Salido, and J. Vallés, 2003. "Understanding the effects of government spending on consumption," *Proceedings, Board of Governors of the Federal Reserve System (U.S.).*

Galí, Jordi, J. David López-Salido, and J. Vallés, 2007. "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, MIT Press, Vol. 5(1): 227-270.

Krugman, Paul. "A Model of Balance-of-Payments Crises." *Journal of Money, Credit and Banking,*' 11, No. 3 (1979): 311-25.

Rendahl, Pontus, 2017. Linear Time Iteration, IHS Economics Series, No. 330, Institute for Advanced Studies (IHS), Vienna.

Schmitt-Grohé, Stephanie, and Martín Uribe, 2005. "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model." *NBER Macroeconomics Annual* 20: 383-425.

Schmitt-Grohé, Stephanie, and Martín Uribe, 2017. Open Economy Economics. Princeton University Press.

# Appendix I. Model Structure

The model described in this Appendix incorporates features from the canonical Pillar IV DSGE macroeconomic model of the financial programming initiative of the 2.0 of ICD.

**Households**

There is a continuum of households indexed by $i \in [0,1]$. As in Gali, Lopes, and Valles (2004,2007), households in the interval $[0, \omega)$ cannot access financial markets and do not have an initial capital endowment. Therefore, these households consume their disposable income in each period. The other households, in the interval $(\omega, 1]$, have access to the financial market and own physical capital. The utility function is common across agents and has the following functional form:

$$u_t(C_t,N_t) = z_{u,t}\ln(C_t - \text{hab}C_{t-1}) - \gamma_N \frac{N_t^{1+\sigma_L}}{1+\sigma_L} + \gamma_m + \frac{1}{1-\sigma_M}\left(\frac{M_t}{P_t}\right)^{1-\sigma_M}$$

where $C_t$ denotes consumption, $N_t$ labor, and $\frac{M_t}{P_t}$ the stock of real money balances. The parameters in the utility function are the inverse of Frisch elasticity, $\sigma_L$, the elasticity of money demand $\sigma_M$, and two scale parameters $\gamma_N$ and $\gamma_m$. This utility function allows for slowly changing consumer habits, where $hab$ is the parameter that controls the speed of habit adjustment. $z_{u,t}$ is the preference shock and follows and ARMA process.

There is a non-competitive labor market implying that there is a wedge between the marginal rate of substitution and the real wage. To incorporate the non-competitive labor market, we follow Schmitt-Grohé and Uribe (2005)

**Non-Ricardian Households**

Non-Ricardian households maximize their utility with respect to the following budget constraint:

$$(1+\tau_{c,t})P_{c,t}C_{r,t} + M_{r,t} = (1-\tau_{w,t})\int W_{jt}N_{r,t}^{j}dj + M_{r,t-1} + P_{c,t}T_{r,t}$$

where $\int W_{jt}N_{r,t}^{j}dj$ denotes labor income, $C_{r,t}$ per-capita non-Ricardian consumption, $P_{c,t}$ the consumer price index, and $\tau_{c,t}$ and $\tau_{w,t}$ the marginal tax rates on consumption expenditure and labor income. $T_{r,t}$ are transfers from the government.

The first-order conditions with respect to consumption and money demand are:

$$(1+\tau_{c,t})\Lambda_{r,t}P_{c,t} = \frac{z_{u,t}}{C_{r,t} - \text{hab}C_{r,t-1}} - \beta \text{ hab } \mathrm{E}_t\left(\frac{z_{u,t+1}}{C_{r,t+1} - \text{hab}C_{r,t}}\right)$$

$$\frac{\gamma_m}{(M_{r,t})^{\sigma_M}} + \beta\, \mathrm{E}_t\,(\Lambda_{r,t+1}) - \Lambda_{r,t} = 0$$

where $\Lambda_{r,t}$ is the Lagrange multiplier.

**Ricardian Households**

The resource constraint of these households is given by the following equation:

$$(1+\tau_{c,t})P_{c,t}C_{o,t}+P_{x,t}X_{o,t}+M_{o,t}+B_{o,t}+S_tB^*_{o,t}=$$
$$R_{t-1}B_{o,t-1}+S_tR^*_{t-1}B^*_{o,t-1}+(1-\tau_{w,t})\int W_{jt}N^j_{o,t}dj+[(1-\tau^k_t)R^k_t+\tau^k_t\delta Q_{t-1}]K_{o,t-1}$$
$$+\frac{\gamma_{com}}{(1-\omega)}S_t\bar{Co}P^*_{co,t}+T_{o,t}+\xi_{o,t}+M_{o,t-1}$$

where, $C_{o,t}$ denotes per-capital consumption by the Ricardian household, $X_{o,t}$ investment, $P_{x,t}$ nominal price of the investment good, $B_{o,t}$ a nominal government bond that pays a risk-free nominal interest rate $R_t$, $S_t$ the nominal exchange rate defined as domestic currency per unit of foreign currency, and $B^*_{o,t}$ nominal bond denominated in foreign currency.[6] $[(1-\tau_{k,t})R^k_t+\tau_{k,t}\delta Q_{t-1}]$ is the after tax capital income, where $R^k_t$ is the nominal rate of return of capital, $Q_t$ is the nominal price of a unit of installed capital, and $\tau_{k,t}$ is the marginal tax rate on capital income. $\frac{\gamma_{com}}{(1-\omega)}S_t\bar{Co}P^*_{co,t}$ is the per-capital revenue coming from the commodity export sector[7]. $P^*_{co,t}$ is the external nominal price of the commodity goods, and $\bar{Co}$ is a constant flow of commodity exports. Finally, $T_{o,t}$ are government transfers, and $\xi_{o,t}$ are benefits from the production firms.

We assume that rapid changes in investment are costly and the cost is modeled through the quadratic function given by:

$$f\left(\frac{X_{o,t}}{X_{o,t-1}}\right)=\frac{a}{2}\left(\frac{X_{o,t}}{X_{o,t-1}}-1\right)^2$$

Parameter $a$ controls the speed of the adjustment of investment.

The household's stock of capital evolves based on the following equation:

$$K_{o,t}=(1-\delta)K_{o,t-1}+z_{x,t}X_{o,t}\left(1-f\left(\frac{X_{o,t}}{X_{o,t-1}}\right)\right)$$

where $K_{o,t-1}$ is per-capital the stock of capital available at time $t$, and $\delta$ is the depreciation rate. $z_{x,t}$ is an investment-specific exogenous shock and follows an ARMA process.

The first-order conditions with respect to consumption, government bonds, foreign bonds, investment, capital, and money are as follow:

$$(1+\tau_{c,t})\Lambda_{o,t}P_{c,t}=\frac{z_{u,t}}{C_{o,t}-\text{hab}C_{o,t-1}}-\beta\ \text{hab}\ \mathrm{E}_t\left(\frac{z_{u,t+1}}{C_{o,t+1}-\text{hab}C_{o,t}}\right)$$

$$-\Lambda_{o,t}+\beta\mathrm{E}_t\Lambda_{o,t+1}R_t=0$$

$$-\Lambda_{o,t}S_t+\beta\mathrm{E}_t\Lambda_{o,t+1}S_{t+1}R^*_t=0$$

[6] In this notation, a negative number implies a debt.

[7] $(1-\gamma_{com})S_t\bar{Co}P^*_{co,t}$ is the share of the commodity revenue accrued to the government.

$$-\Lambda_{o,t}P_{x,t}+\mu_t z_{x,t}\left(1-f\left(\frac{X_{o,t}}{X_{o,t-1}}\right)-f'\left(\frac{X_{o,t}}{X_{o,t-1}}\right)X_{o,t}\right)+\beta\,\mathrm{E}_t\left(\mu_{t+1}z_{x,t+1}f'\left(\frac{X_{o,t+1}}{X_{o,t}}\right)X_{o,t+1}^2\right)=0$$

$$-\mu_t+\beta\mu_{t+1}(1-\delta)+\beta\,\mathrm{E}_t\left(\Lambda_{o,t+1}\left((1-\tau_{t+1}^k)R_{t+1}^k+\tau_{t+1}^k\delta Q_t\right)\right)=0$$

$$\frac{\gamma_m}{\left(M_{o,t}\right)^{\sigma_M}}+\beta\,\mathrm{E}_t\left(\Lambda_{o,t+1}\right)-\Lambda_{o,t}=0$$

where

$$f'\left(\frac{X_{o,t}}{X_{o,t-1}}\right)=a\left(\frac{X_{o,t}}{X_{o,t-1}}-1\right)\frac{1}{X_{o,t-1}} \text{ and } f'\left(\frac{X_{o,t+1}}{X_{o,t}}\right)=a\left(\frac{X_{o,t+1}}{X_{o,t}}-1\right)\frac{1}{X_{o,t}^2}.$$

### Labor Markets and Wage Setting

Households forgo labor and wage decisions and instead allow labor unions to make decisions for them. This introduces some rigidity into the labor market, allowing for the possibility of underemployment. We assume that there is a continuum of labor unions one for each labor type, and that labor types, $i$, are uniformly distributed across household. Labor unions will maximize profits, considering that their decision affects both Ricardian and non-Ricardian utilities. For each labor union $j$, the maximization is subject to two restrictions: A resource constraint

$$N_{j,t}=\int_0^1 N_t(j,i)di \qquad \text{Eq1}$$

that limits the total available labor for union $j$, and to the demand for labor type $j$ given by

$$N_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\epsilon_w}N_t^d \qquad \text{Eq2}$$

where $\epsilon_w$ denotes elasticity of substitution across labor type varieties, $N_t^d$ the aggregate labor demand, $W_t$ the aggregate nominal wage index, and $W_{j,t}$ the wage fixed by union $j$.[8]

When selecting the optimal wage, unions take into account that they cannot adjust wages freely and that there is an exogenous probability of not being able to adjust wages each period. In fact, each period there is a $1-\theta_w$ probability of setting wages optimally. When a union is able to adjust wages it does it by maximizing a weighted average of lifetime utility functions

$$\max_{W_{j,t}}\mathrm{E}_t\sum_{s=0}^{\infty}(\theta_w\beta)^s\left\{\left[\left((1-\omega)\ln\left(C_{o,t+s}-\mathrm{hab}C_{o,t+s-1}\right)+\omega\ln\left(C_{r,t+s}-\mathrm{hab}C_{r,t+s-1}\right)\right)-U(N_{t+s})\right]\right\}$$

subject to Eq1 and Eq2. In the above specification we have used the fact that labor types are uniformly distributed across household types. Hence, aggregate demand for labor type $j$ is spread uniformly across the households. When the union is not able to adjust wages optimally, it adjusts them accordingly to the indexation rule

$$W_t = W_{t-1}g_z\pi_{t-1}^{\chi_w}\bar{\pi}^{(1-\chi_w)}$$

[8] The section on firms contains the formal derivation of this demand equation.

where $\pi_t$ denotes the consumer price inflation and $\bar{\pi}$ the inflation target. This indexation rule implies that nominal wages are indexed to a weighted average of past inflation and the inflation target and to the long run productivity growth, $g_z$. $\chi_w$ is the wage indexation parameter. If $\chi_w = 1$, there is full indexation to past inflation.

To find the optimality condition for the unions that can adjust wages, it is useful to find the value of the nominal wage $s$ periods after the last re-optimization. Using the indexation rule, we can show that the value of nominal wage after $s$ periods is

$$W_{j,t+s} = W_t^* \prod_{k=1}^{s} \left( g_z \bar{\pi}^{(1-\chi_w)} \pi_{t+k-1}^{\chi_w} \right)$$

and, in real terms, it is

$$w_{j,t+s} = w_t^* X_{t,s}^w$$

where

$$X_{t,s}^w = \prod_{k=1}^{s} \left( \frac{g_z \bar{\pi}^{(1-\chi_w)} \pi_{t+k-1}^{\chi_w}}{\pi_{t+k}} \right)$$

where $w_t = \frac{W_t}{P_{c,t}}$ is the real wage.

In every period, a union chooses the optimal level of labor $N_t(j)$, employing a weighted average of utilities of Ricardian and non-Ricardian households to obtain the following optimality condition:

$$U_N\left(N_{j,t}\right) = \left(1 - \tau_{w,t}\right) w_t \left(\omega \lambda_{r,t} + (1-\omega)\lambda_{o,t}\right) mct_t^w$$

where $mct_{t+s}^w$ is the co-state variable of the restriction Eq2. Unions that are able to select wages will select it such that $w_t^*$ is

$$E_t \sum_{s=0}^{\infty} (\theta_w \beta)^s U_N(N_{t+s}) \left(X_{t,s}^w\right)^{-\epsilon_w} w_{t+s}^{\epsilon_w} h_{t+s}^d \left\{ \left[ (1-\omega) \frac{1}{MRS_{t+s}^o} + \omega \frac{1}{MRS_{t+s}^r} \right] X_{t,s}^w w_t^* - \frac{\epsilon_w}{(\epsilon_w - 1)} \right\} = 0$$

where

$$MRS_{t+s}^o = \frac{\left(1 + \tau_{c,t+s}\right)}{\left(1 - \tau_{w,t+s}\right)} \frac{U_{N,t+s}}{U_{c,t+s}^o}$$

$$MRS_{t+s}^r = \frac{\left(1 + \tau_{c,t+s}\right)}{\left(1 - \tau_{w,t+s}\right)} \frac{U_{N,t+s}}{U_{c,t+s}^r}$$

are the marginal rates of substitution (MRS) between consumption and labor. $U_{c,t}^j$ is the marginal utility of consumption of Ricardian and non-Ricardian agents.

Note that, if wages are flexible, the first order condition simplifies to

$$\left[ (1-\omega) \frac{1}{MRS_{t+s}^o} + \omega \frac{1}{MRS_{t+s}^r} \right] w_t^* = \frac{\epsilon_w}{(\epsilon_w - 1)}$$

This implies that there is a constant mark-up between the MRS and the real wage. Hence households of both types will always be willing to supply more labor when real wage increases (see Gali, Lopes and Valles, 2007, for more details).

The negotiated wage in all unions are identical, and $(1-\theta_w)$ of unions are able to negotiate wages in every period. Then, we have the following equilibrium condition

$$N_t = \nu_{w,t} N_t^d$$

where $\nu_t^w$ is a number bonded above one and measures the inefficiency created by the wage dispersion. Since, it is larger than one, it implies that the labor supply is larger than what the firms use effectively in production, $N_t^d$. $\nu_t^w$ may be expressed recursively as follows:

$$\nu_{w,t} = \theta_w \left( \frac{w_{t-1}}{w_t} g_z \frac{\pi_{t-1}^{\chi_w} \bar{\pi}^{1-\chi_w}}{\pi_t} \right)^{-\epsilon_w} \nu_{w,t-1} + (1-\theta_w) \left( \frac{w_t^*}{w_t} \right)^{-\epsilon_w}$$

Note that when wages are fully flexible, the wage dispersion disappears, that is $\nu_{w,t} = 1$.

The aggregate real wage index evolves as in the following equation:

$$w_t = \left( \theta_w \left( w_{t-1} g_z \frac{\pi_{t-1}^{\chi_w} \bar{\pi}^{1-\chi_w}}{\pi_t} \right)^{1-\epsilon_w} + (1-\theta_w)(w_t^*)^{1-\epsilon_w} \right)^{\frac{1}{1-\epsilon_w}}$$

**Firms**

There are three types of goods producers in the economy: producers of final goods, producers of intermediate goods, and producers of domestic goods. Final goods are for consumption and investment. These goods are produced by combining imported and domestic inputs. Intermediate goods producers use labor and capital to produce inputs for the domestic producer. The domestic producer produces a homogenous good used as input in the production of the final goods and it is also exported.

**Producers of Final Goods**

**Consumption Goods**

The final consumption good is produced using domestic, $C_{H,t}$, and foreign goods, $C_{F,t}$ as inputs. The producer of this good minimizes cost subject to the production technology

$$C_t = \left[ (1-\alpha_c)^{\frac{1}{\eta_c}} \left( C_{H,t} \right)^{\frac{\eta_c - 1}{\eta_c}} + (\alpha_c)^{\frac{1}{\eta_x}} \left( C_{F,t} \right)^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}}$$

where $\eta_c$ is the elasticity of substitution between home and foreign goods and $\alpha_c$ is the share of foreign goods. The optimality conditions for this problem are:

$$C_{H,t} = (1-\alpha_c) \left( p_{H,t} \right)^{-\eta_c} C_t$$

$$C_{F,t} = (\alpha_c) \left( p_{F,t} \right)^{-\eta_c} C_t$$

These conditions represent the demand for domestic and foreign goods and depend negatively on domestic relative prices $p_{H,t} = \frac{P_{H,t}}{P_{c,t}}$, and foreign relative prices $p_{F,t} = \frac{P_{F,t}}{P_{c,t}} = rer_t$ and positively on aggregate consumption, $C_t$.

**Investment Good**

The producer of the investment good solves a similar problem. That is, it minimizes costs subject to the following production technology:

$$X_t = \left[(1-\alpha_x)^{\frac{1}{\eta_x}}\left(X_{H,t}\right)^{\frac{\eta_x-1}{\eta_x}} + (\alpha_x)^{\frac{1}{\eta_x}}\left(X_{F,t}\right)^{\frac{\eta_x-1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x-1}}$$

where $\eta_x$ is the elasticity of substitution between home and foreign investment goods and $\alpha_x$ is the share of foreign investment in the production technology. The first-order conditions are

$$X_{H,t} = (1-\alpha_x)\left(\frac{p_{H,t}}{p_{x,t}}\right)^{-\eta_i} X_t$$

$$X_{F,t} = (\alpha_x)\left(\frac{p_{F,t}}{p_{x,t}}\right)^{-\eta_x} X_t$$

where $p_{x,t} = \frac{P_{x,t}}{P_{c,t}}$ is the relative price of investment. This relative price is function of the domestic good price and the price of the imported good.

**Producers of Domestic Good**

In each period $t$, a the domestic good $Y_{H,t}$ is produced by a perfectly competitive firm combining intermediate goods according to the following production function

$$Y_{H,t} = \left(\int_0^1 Y_{j,t}^{1-\frac{1}{\epsilon_H}} dj\right)^{\frac{\epsilon_H}{\epsilon_H-1}}$$

where $\epsilon_H$ is the elasticity of substitution between goods varieties, $j$. Producers of the domestic good takes prices as given and choose the quantities of intermediate goods that maximize their profits. This generates the demand for the intermediate good $j$ and the price of the domestic good as represented below:

$$Y_{H,j,t} = \left(\frac{P_{H,j,t}}{P_{H,t}}\right)^{-\epsilon_H} Y_{H,t}^d$$

and $P_{H,t} = \left(\int_0^1 P_{H,j,t}^{1-\epsilon_H} dj\right)^{\frac{1}{1-\epsilon_H}}$. The demand for the final domestic goods is

$$Y_{H,t}^d = C_{H,t}^* + C_{g,t} + X_{g,t} + C_{H,t} + X_{H,t}$$

where $C_{H,t}^*$ is the foreign demand for domestic output (exports) and $X_{g,t}$ is public investment.

**Intermediate Goods**

Intermediate goods are produced by a continuum of monopolistic firms indexed by $l$. These firms use capital and labor to produce $y_{H,l,t}$. The production function is

$$Y_{H,l,t} = z_{y,t} K_{l,t-1}^{\alpha} Z_t^{1-\alpha-\alpha_g} \left(N_{l,t}\right)^{1-\alpha} \left(K_{t-1}^{g}\right)^{\alpha_g}$$

where $\alpha \in (0,1)$ is the capital share of total output, $Z_t$ is a permanent productivity shock such that

$$\frac{Z_{t+1}}{Z_t} = g_{z,t}$$

$$g_{z,t} = \left(1-\rho_{gz}\right) g_z + \rho_{gz} g_{zt-1} + \epsilon_{z,t}$$

and $z_{y,t}$ is an exogenous transitory productivity shock, $g_{z,t}$ is a transitory shock to the growth rate of productivity and $K_{t-1}^{g}$ is public capital. Note that each intermediate-good firm $l$ has access to the same public capital stock and that the latter grows along the balanced growth path.

Following Schmitt-Grohe and Uribe, 2007, we assume that the labor input used by firm $l$ is a composite made of a continuum of differentiated labor services. Formally, the labor input is provided as follows:

$$N_{l,t} = \left[\int_o^1 N_{j,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \qquad \text{Eq3}$$

Firms select the optimal combination of labor varieties by min $\int_0^1 W_{j,t} N_{jt} dj$ subject to Eq3. The optimal demand for labor services $j$ by firm $l$ is

$$N_{l,j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\epsilon_W} N_{l,t}^{d}$$

where $W_t$ is the nominal wage index $W_t = \left(\int_0^1 W_{j,t}^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}} dj$. The total demand for labor services $j$ is, $N_{j,t} = \int_0^1 N_{l,j,t}\, dl$ and equals

$$N_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\epsilon_W} N_t^{d}$$

where $N_t^d = \int_0^1 N_{lt}\, dl$. This last expression is the labor demand used in the household optimization problem.

The optimality conditions of the cost minimization problems are

$$W_t = (1-\alpha) MC_{H,t} \frac{Y_{lt}}{N_{lt}}$$

$$R_t^k = \alpha MC_{H,t} \frac{Y_{lt}}{K_{lt-1}}$$

where $MC_t^H$ is the marginal cost, which is determined as follows:

$$MC_{H,t} = \frac{1}{Z_t^{1-\alpha-\alpha_G} K_{g,t-1}^{\alpha_G}} \left(\frac{R_{k,t}}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}$$

Note that, we dropped the index $l$ since $MC_{H,l,t} = MC_{H,t}$ for $l$.

As in the tradition of Calvo pricing, firms will not adjust prices frequently. Instead, in each period $(1-\theta_H)$ firms will adjust prices optimally and the remaining $\theta_H$ firms will adjust their prices following a simple rule. Consequently, when choosing its optimal price, a firm will maximize the expected profit taking into account that there is a probability that it won't be able to adjust prices in the future. Formally, the profit maximization problem, in nominal terms, can be written as follows:

$$\max_{P_{H,l,t}} \sum_{s=0}^{\infty} (\beta\theta_H)^s E_t \left[\frac{\lambda_{o,t+s} P_{c,t} Z_t}{\lambda_{o,t} P_{c,t+s} Z_{t+s}} \left[P_{H,l,t+s} Y_{H,l,t+s} - MC_{H,t+s} Y_{H,l,t+s}\right]\right]$$

subject to $Y_{l,t} = \left(\frac{P_{H,l,t}}{P_{H,t}}\right)^{-\epsilon_H} Y_{H,t}^d$. Here, $MC_{H,t}$ denotes the nominal marginal cost, and $P_{H,t}$ the nominal price of the domestic goods.

We allow for price indexation. That is, firms that cannot adjust prices optimally change their prices following the indexation rule: $\pi_t^{rH} = \pi_t^{\iota_H} \bar{\pi}_t^{1-\iota_H}$. Hence, when a price at time $t$ is not adjusted optimally, the price next period nominal price is determined as follows:

$$P_{H,l,t} = P_{H,l,t-1} \pi_{t-1}^{\iota_H} \bar{\pi}^{1-\iota_H}$$

where $\iota_H$ is a parameter that controls the degree of price indexation. When $\iota_H = 1$, there is full indexation to past inflation. If $\iota_H = 0$, price changes follow the inflation target.

The first order condition of this problem is:

$$\sum (\beta\theta_H)^s E_t \left[\frac{\lambda_{o,t+s}}{\lambda_{o,t}} \frac{P_{H,t+s}}{P_{c,t+s}} X_{H,t,s}{}^{-\epsilon_H} \left[X_{H,t,s} \frac{P_{H,t}^*}{P_{H,t}} + \frac{\epsilon_H}{(1-\epsilon_H)} \frac{MC_{H,t+s}}{P_{H,t+s}}\right] \frac{Y_{H,t+s}}{Z_{t+s}}\right] = 0$$

where $P_t^{H*}$ denotes the optimal price in period $t$, and

$$X_{H,t,s} = \prod_{k=1}^{s} \frac{\pi_{t+k-1}^{rH}}{\pi_{t+k}^{H}}$$

Writing the first order condition in stationary variables, we get

$$\sum_{s=0}^{\infty} (\beta\theta_H)^s E_t \left[\frac{\lambda_{o,t+s}}{\lambda_{o,t}} p_{H,t+s} X_{H,t,s}{}^{1-\epsilon_H} \frac{p_{H,t}^*}{p_{H,t}} y_{H,t+s}\right] = \sum_{s=0}^{\infty} (\beta\theta_H)^s E_t \left[\frac{\lambda_{o,t+s}}{\lambda_{o,t}} p_{H,t+s} (X_{H,t,s})^{-\epsilon_H} \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{mc_{H,t+s}}{p_{H,t+s}} y_{H,t+s}\right]$$

Denoting the sum on the left-hand side and on the right-hand side $f_t^H$ the optimality condition can be written in a recursive form by the two equations

$$
\begin{aligned}
f_t^H &= p_{H,t}^* y_{H,t} + \theta_H E_t \left[ \frac{g_{t+1}^Z \pi_{c,t+1}}{\mathcal{R}_t} \left( \frac{\pi_t^{rH}}{\pi_{t+1}^H} \right)^{1-\epsilon_H} \frac{\pi_{t+1}^H}{\pi_{t+1}^{H*}} f_{t+1}^H \right] \\
f_t^H &= \frac{\epsilon_H}{\epsilon_H - 1} mc_{H,t} y_{H,t} + \theta_H E_t \left[ \frac{g_{t+1}^Z \pi_{c,t+1}}{\mathcal{R}_t} \left( \frac{\pi_t^{rH}}{\pi_{t+1}^H} \right)^{-\epsilon_H} f_{t+1}^H \right]
\end{aligned}
$$

To complete the model, we need the nominal price index

$$
P_{H,t} = \left[ \int_0^1 P_{H,l,t}^{1-\epsilon_H} dl \right]^{\frac{1}{1-\epsilon_H}}
$$

which can be written as

$$
\pi_t^H = \left[ \theta_H \left( \pi_{t-1}^{\iota_H} \bar{\pi}^{1-\iota_H} \right)^{1-\epsilon_H} + (1-\theta_H) \left( p_{H,t}^* \pi_t^H \right)^{1-\iota_H} \right]^{\frac{1}{1-\epsilon_H}}
$$

While we have found the optimality conditions at the firm level, we need to aggregate them. Under the current assumptions, aggregation is straightforward since the production technology is the same across firms, and the marginal cost is the same. The main difficulty is the price dispersion that creates a wedge between the output demanded and its supply. Formally,

$$
\int_0^1 Y_{H,l,t} \, dl = \int_0^1 \left( \frac{P_{H,l,t}}{P_{H,t}} \right)^{-\epsilon_H} Y_{H,t} dl
$$

which we write as

$$
Y_{H,t} = \nu_{H,t} Y_{H,t}^d
$$

where $\nu_{H,t} = \int_0^1 \left( \frac{P_{H,l,t}}{P_{H,t}} \right)^{-\epsilon_H} dl. \nu_{H.t}$ captures the price distortion, which is related to the welfare costs of inflation. $\nu_{H,t}^p$ can also be written recursively as

$$
\nu_{H,t} = \theta_H \left( \frac{\pi_{t-1}^{rH}}{\pi_t^H} \right)^{-\epsilon_H} \nu_{H,t-1} + (1-\theta_H) \left( P_{H,t}^* \right)^{-\epsilon_H}
$$

**Monetary policy**

We model two monetary policy regimes. An inflation targeting regime with flexible exchange rates and a peg regime. In the inflation targeting regime, the central bank controls the short-term nominal interest rate and sets it following a rule that responds to deviations of inflation from the target. In particular, the monetary policy rule is

$$
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left( \left( \frac{\pi_t}{\bar{\pi}} \right)^{\varphi_\pi} \right)^{1-\rho_R} \exp(z_t^m)
$$

where $\rho_R$ is the smoothing parameter, $\varphi_\pi$ measure the sensibility of the policy rule to deviations of inflation from the target, and $z_t^m$ is the monetary policy shock. This shock is exogenous and follows an ARMA model.

In the peg regime, the nominal devaluation rate is constant

$$d_t = \frac{S_t}{S_{t-1}} = \bar{d}$$

To completely characterize the policy regime, we write down the balance sheet of the central bank. The bank issues money, $M_t$, holds foreign reserves, $B^*_{cb,t}$, and net domestic assets comprising government and central bank bonds, $B_{cb,t}$ . Hence, the balance sheet of the central bank is

$$M_t = B_{cb,t} + S_t B^*_{cb,t}$$

The central bank flow of funds is

$$M_t - M_{t-1} + R_{t-1}B_{cb,t-1} + R^*_{t-1}S_t B^*_{cb,t-1} = B_{cb,t} + S_t B^*_{cb,t} + P_{c,t}qfb_t$$

Accordingly, the quasi-fiscal balance ($qf\, b_t$) is a function of the return on external and domestic assets, the domestic inflation rate, and the real exchange rate.

The adjustment of the balance sheet of the central bank depends on the policy regime. In the inflation targeting regime, the central bank adjusts the money supply is such that the short-term interest rate aligns with the policy rate. The holding of external assets is constant, and net domestic assets adjust endogenously. In the pegged regime, the central bank adjusts external asset holdings to maintain the exchange rate aligned with the target. The bank accommodates the changes in the holdings of external assets with changes in the supply of money. In the peg regime, changes in the government assets at the central bank lead to changes in holdings of foreign assets.

**Fiscal policy**

The government collects taxes on consumption, capital, and labor, receives the quasi-fiscal balance from the central bank and revenues from the commodity sector. It issues public debt to finance its overall balance. The central bank holds a fraction $\alpha_g$ of the government debt and households the remaining part. The government spends on consumption, investment, transfers to households, and interest payments on its debt.

In real terms, the government budget constraint is

$$rer_t p^*_{co,t}(1-\gamma_{com})\bar{Co} + Tax^c_t + Tax^l_t + Tax^k_t + b_t + qfb_t = p_{H,t}\left(C_{gt} + X_{gt}\right) + \frac{R_{t-1}}{\pi_t} b_{t-1} + Tr_t$$

where $Tax^k_t = \tau^k_t \left[r^k_t - \delta \frac{q_{t-1}}{\pi_t}\right] K_{t-1}$, $Tax^l_t = \tau^l_t w_t N_t$ and $Tax^c_t = \tau^c_t C_t$, $p_{H,t}C_{gt}$ and $p_{H,t}X_{gt}$ are government expenditures on consumption and investment goods. In the current setup, marginal tax rates, government consumption, and government investment are constant. An alternative to this assumption is to include a fiscal rule for each instrument.

Transfers to households are set optimally and the government maximizes the following objective function:

$$U_{p,t} = U(C_t, N_t, m_t) + \omega_1\left(b^g_t - \bar{b}^g\right)^2 + \omega_2(Tr_t - \overline{Tr})^2$$

where $U(C_t, N_t, m_t)$ is a weighted average of Ricardian and non-Ricardian utilities. The term $\omega_2(Tr_t - \overline{Tr}\ )^2$ captures the cost of adjusting the fiscal instrument. This term reflects the inability (or unwillingness) of the government to change the fiscal instrument abruptly. We added the public debt deviations with respect to the steady-state to the planner's objective function to capture the welfare effects of macroeconomic stability, and as a means of encouraging time consistency in fiscal policy. When $\omega_1$ is small, the impact of the public debt level on the planner's objective function is low, allowing the government to run larger deficits and deviations from the long-run debt level target.

Public investment is used to build public capital that enters with a lag in the production function of the intermediate good producers. Public capital is accumulated according to the following equation:

$$K_{gt} = (1-\delta)K_{gt-1} + z_{xg,t}A_{gt-L}$$

where $a_{gt-l}$ denotes authorized budget for government investment in period $t - L$. Government investment implemented at $t$ is

$$X_{gt} = \sum_{n=0}^{L-1} b_n A_{gt-n}$$

with $\sum_{n=0}^{L-1} b_n = 1$. This specification of the investment process assumes that it takes time to build public investment and that there are lags between the announcement of public investment and its implementation. $z_{xg,t}$ is a productivity shock in public investment.

**External sector and current account**

The external interest rate is the sum of an external risk-free rate $\bar{R}_t^*$ and an endogenous risk premium. That is,

$$R_t^* = \bar{R}_t^* - \Omega_u\left(\exp\left(\frac{rer_t\ nfa_t}{GDP_t} - \frac{\overline{rer}\ \bar{nfa}}{\overline{GDP}}\right) - 1\right) \qquad \text{Eq 4}$$

The country risk premium is a negative function of the ratio of NFA to GDP and $\Omega_u$ is the elasticity of the country risk to the NFA-to-GDP ratio[9]. With this parametrization, the risk premium reacts to domestic productivity and commodity price shocks. Accordingly, GDP in the model equals

$$GDP_t = p_t^H Y_t^H + rer_t\ \bar{Co}\ P_{co,t}^*$$

where $y_t^H$ is domestic output.

Non-commodity exports are modeled as

$$C_t^{h*} = \left(\frac{p_t^H}{rer_t}\right)^{-\epsilon_e} C_t^*$$

where $C_t^*$ is proportional to the external output and $\epsilon_e$ is the elasticity of exports to the exchange rate. The balance of payments equation is found by aggregating the household budget constraint, the government budget constraint, and the balance sheet of the central bank.

[9] Real net foreign assets are defined as $nfa_t = b_{cb,t-1}^* - b_{o,t-1}^{\star}$

$$rer_t\,(nfa_t - nfa_{t-1}) = \left[\left(p_t^H C_t^{H*} + \bar{Co}\,rer_t\,P_{co,t}^*\right) - rer_t(C_t^m + X_t^m)\right] + \left(\frac{R_{t-1}^*}{\pi_t^*} - 1\right) rer_t\,nfa_{t-1}^*$$

where the net foreign asset position in domestic currency is $nfa_t = b_t^* + b_{cb,t}^*$.

# Appendix II. Linear Time Iteration Algorithm

This appendix describes the linear time iteration (LTI) solution method proposed by Rendahl (2017). The LTI algorithm is a generalization of the traditional perturbation that allows simulation of models with regime changes at a known time in the future. Rendahl (2017) contains a general presentation.

The perturbation method finds an approximate solution of models written in the following way

$$E_t[F(x_t, x_{t+1}, x_{t+2})] = 0 \quad \text{(A0)}$$

where $x_t$ is a vector of endogenous and exogenous variables. The perturbation method finds an approximate solution around the non-stochastic steady state $x^*$ that satisfies

$$F(x^*, x^*, x^*) = 0.$$

In particular, the procedure approximates (A0) with a first-order Taylor expansion of

$$E_t[F(x_t, x_{t+1}, x_{t+2})] = 0$$

at around $x^* = x_t = x_{t+1} = x_{t+2}$. The approximation can be written as

$$\begin{aligned} E_t[F(x_t, x_{t+1}, x_{t+2})] &= E_t[F(x^*, x^*, x^*) + J_{x_t}(x_t - x^*) \\ &+ J_{x_{t+1}}(x_{t+1} - x^*) + J_{x_{t+2}}(x_{t+2} - x^*)] \end{aligned} \quad \text{(A1)}$$

where $J_i$ is a Jacobian matrix with respect to $x_t$, $x_{t+1}$ and $x_{t+2}$, evaluated at $x^*$. It can further be simplified as

$$0 = J_{x_t}(x_t - x^*) + J_{x_{t+1}}(x_{t+1} - x^*) + J_{x_{t+2}}(x_{t+2} - x^*)$$

and written conveniently as

$$Au_{t-1} + Bu_t + Cu_{t+1} = 0 \quad \text{(A2)}$$

$u_t = (x_{t+1} - x^*)$ and $A$, $B$ and $C$ represent the Jacobian matrices.

The solution of (A2) is

$$u_t = Fu_{t-1} \quad \text{(A3)}$$

where F is an unknown matrix.

There are many ways to find *F*. We briefly describe an iterative process that will be used later in the appendix. The iteration starts by replacing the solution (A3) into (A2) to get

$$0 = Au_{t-1} + Bu_t + CF_o u_t \quad \text{(A4)}$$

where $F_o$ denotes our initial guess for F. Solving (A4) for $u_t$ as a function of $u_{t-1}$, we get

$$u_t = -(B + CF_o)^{-1} Au_{t-1} = \mathrm{F}_1 u_{t-1}$$

where $F_1 = -(B + CF_o)^{-1}A$. This last expression provides a new approximation for F. The above procedure can be applied iteratively to find $F$.

The iterations can be summarized as follows:

1. Assume an initial guess for $F$, call it $F_o$, and compute $F_1$ as follows:

$$F_1 = -(B + CF_o)^{-1}A$$

2. Compute $F_2$ using F1:

$$F_2 = -(B + CF_1)^{-1}A$$

3. Continue the iteration until convergence is reached:

$$F_n \to F$$

The algorithm can be generalized to solve the system of equations (A1) around any point $\bar{x}$, not necessarily equal to the steady-state solutions. In this case,

$$F(\bar{x}, \bar{x}, \bar{x}) = D^*$$

and the linear approximation would be

$$A^* u_{t-1} + B^* u_t + C^* u_{t+1} + D^* = 0 \qquad \text{(A5)}$$

The solution of (A5) is in the form

$$u_t = E + F u_{t-1}$$

where the matrices $E$ and $F$ are unknowns.

The same iterative algorithm leads to

$$u_t = -(B^* + C^* F_0)^{-1} A^* u_{t-1} - (B^* + C^* F_0)^{-1}(C^* E_o + D^*)$$

and the iteration yields:

$$F_{n+1} = -(B^* + C^* F_n)^{-1} A^*$$

$$E_{n+1} = (B^* + C^* F_n)^{-1}(C^* E_n + D^*)$$

Again, the algorithm ends when

$$E_n \to E \quad F_n \to F$$

The above algorithm can be used to solve regime-switching models where the time of the regime switch is known. For simplicity, we assume two regimes M1 and regime M2. The regime $M_1$ is characterized by

$$Au_{t-1} + Bu_t + Cu_{t+1} = 0$$

and the regime $M_2$ by

$$A^* u_{t-1} + B^* u_t + C^* u_{t+1} + D^* = 0$$

To find the solution, we assume that regime M1 satisfies conditions similar to Blanchard and Kahn (1980) and that the system returns to M1. That is, M1 is an absorbing regime.

The solution method uses the fact that at some time $T$, known, the system will be in $M1$ and that it will not return to $M2$ after T. We know that at time T, the solution for M1 is:

$$u_t = F_T u_{t-1}$$

For t=T-1<T, a period before T, we are in regime $M2$ and we know that the solution for $T$ is $(u_T = F_T u_{T-1})$. We can use this solution to find $F$ and $E$ for period $(T-1)$ as follows:

$$A^* u_{t-1} + B^* u_t + C^* F_T u_t + D^* = 0$$

from where

$$u_t = -(B^* + C^* F_T)^{-1} A^* u_{t-1} - (B^* + C^* F_T)^{-1} D^*$$

Hence:

$$\begin{aligned} F_{T-1} &= -(B^* + C^* F_T)^{-1} A^* \\ E_{T-1} &= -(B^* + C^* F_T)^{-1} D^* \end{aligned}$$

For T-2, they are

$$\begin{aligned} F_{T-2} &= -(B^* + C^* F_{T-1})^{-1} A^* \\ E_{T-2} &= -(B^* + C^* F_{T-1})^{-1} D^* \end{aligned}$$

The solutions for the regime change model are a sequence of E's and F's. Thus, agents adjust their decision rules for every period before T, internalizing the deterministic convergence towards the stable and absorbing regimen. Consequently, this algorithm allows us to analyze the dynamics and transitions from potentially unstable regimens towards a stable regime.

PUBLICATIONS

**Fix vs. Float: Evaluating the Transition to a Sustainable Equilibrium in Bolivia**
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