WP/21/196

IMF Working Paper

Numerical Fiscal Rules for Economic Unions: the Role of Sovereign Spreads

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INTERNATIONAL MONETARY FUND

IMF Working Paper

Institute for Capacity and Development and Research Department

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July 2021

Abstract

We study gains from introducing a common numerical fiscal rule in a "Union" of model economies facing sovereign default risk. We show that among economies in the Union, there is significant disagreement about the common debt limit the Union should implement: the limit preferred by some economies can generate welfare losses in other economies. In contrast, a common sovereign spread limit results in higher welfare across economies in the Union.

JEL Classification Numbers: F34, F41

Keywords: Fiscal Rules, Sovereign Spread, Spread Limit, Debt Dilution, Debt Intolerance

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1 Introduction

Economic unions often implement fiscal rules with a common debt limit.[1] However, the complexity of finding an appropriate common debt limit led to proposals to abandon numerical fiscal rules for unions of countries, as debt levels that may be appropriate or sustainable for some countries in the union may not be so for others (Blanchard et al., 2021, Martin et al., 2021). The impossibility of finding an appropriate common debt limit for an economic union is not surprising. Fiscal rules intend to mitigate sovereign risk and the mapping from sovereign debt levels to sovereign risk, often referred to as debt intolerance, varies greatly both across countries and over time (Reinhart et al., 2003; Reinhart et al., 2015). In fact, the use of debt limits in fiscal rules has been criticized even for single countries. For example, Furman and Summers (2020) suggest focusing on interest payments instead.

This paper presents a stylized framework that allows for an analytical characterization of optimal fiscal rules for a "Union" of model economies that differ in their level of debt intolerance. Consistent with the arguments in policy discussions, the paper shows the impossibility of having a common debt limit that produces welfare gains across the Union. However, the paper also shows that, in contrast, a common limit on sovereign spreads (i.e., on the difference between a sovereign bond yield and a risk-free interest rate) can generate welfare gains across all economies in the Union. Our result also implies that a spread limit is a more robust rule than a debt limit for a single economy that faces uncertainty about its future debt intolerance. In a companion paper, Hatchondo et al. (2021), we show that the advantages of common spread fiscal rules are quantitatively important. While this paper is an illustration of the principle at hand, further research is needed to assess the practicality or the approach.[2]

Several issues need to be considered for the successful use of the sovereign spreads as fiscal anchors. Eyraud et al. (2018) argue that the design of fiscal rules should balance different properties: simplicity, flexibility, sustainability, operational guidance, and enforceability. In the past decade, countries have taken several steps to attain these objectives which could also

[1]In 2014, 48 of the 85 countries with fiscal rules had supranational rules (IMF Fiscal Rules dataset).

[2]Calvo (1988) discusses gains from introducing interest-rate limits for sovereign debt to eliminate bad equilibria in a multiple-equilibria framework. While we rule out multiple equilibria in our analysis, this could present additional gains from establishing spread limits.

be applied to spread rules. For instance, the establishment of independent fiscal councils and escape clauses has improved enforceability and flexibility, respectively. Buti and Gaspar (2021) argue that since bond prices are sometimes delinked from fundamentals, spreads could fail to signal that fiscal policy is unsustainable. They also argue that spreads can sometimes react too abruptly. However, the quantitative analysis in Hatchondo et al. (2021) indicates that spread rules can perform well even when spreads are not only affected by domestic fundamentals, and that having a spread rule at the time a large shock hits the economy moderates the spread increase triggered by the shock and produces substantial gains for both the government and bondholders, even when the rule was not playing any role before (as when spreads fail to signal sustainability problems). In a world in which sovereign spreads are zero and thus contain no information, the use of spreads as fiscal anchors could be problematic, but anchored expectations would still be useful when a negative shock increases spreads. Emphasizing the sovereign spread as a fiscal anchor would underscore the importance of having a sovereign interest rate freely determined in a liquid market that does not reflect the expectation of bailouts. Of course, not every country has such a rate. But in general, it is difficult to argue that there is no valuable information in spreads.

The rest of the article proceeds as follows. Section 2 presents the model. Section 3 presents the results. Section 4 concludes. Proofs of the propositions are presented in the Appendix.

2 The model

The economy lasts for three periods, $t = 1, 2, 3$. The government receives a sequence of endowments, given by $y_1 = y_2 = 0$, and $y_3 \geq 0$. The only uncertainty in the model is about the value of y_3. Let F and f denote the c.d.f. and density functions of y_3, with $f(y_3) > 0$ for all $y_3 \geq 0$. Let σ_y denote the standard deviation of y_3. Let u denote the utility function that satisfies the Inada conditions, $c_t \geq 0$ denote period-t consumption, $\beta < 1$ denote the government's discount factor, and $\mathbb{E}$ denote the expectation operator.

The government can borrow to finance consumption in periods 1 and 2. A bond issued in period 1 promises to pay δ unit of the good in period 2 and $(1 - \delta)$ units in period 3. Thus, if $\delta = 1$, the government issues one-period bonds in period 1. If $\delta < 1$, the government issues long-term bonds in period 1. A bond issued in period 2 promises to pay one unit of the good in period 3.

The government may choose to default in period 3. If the government defaults, it does not pay its debt but looses a fraction $\phi > 0$ of the period-3 endowment y_3. Bonds are priced by competitive risk-neutral investors who discount future payments at a rate of 1.

3 Results

3.1 Optimal fiscal rules for a single economy

In this setup, it is optimal to borrow because the government has no income in the first two periods ($y_1 = y_2 = 0$) and the assumed preferences, and is impatient ($\beta < 1$). However, the borrowing choices available to the government are restricted by a limited commitment problem.

Let b_t denote the number of bonds issued by the government in period t. The equilibrium default decision of the period 3 government is given by

$$\hat{d}(b_1, b_2, y_3) = \begin{cases} 1 & \text{if } y_3 < \frac{b_1(1-\delta)+b_2}{\phi}, \\ 0 & \text{otherwise}, \end{cases} \tag{1}$$

where $\hat{d}(b_1, b_2, y_3) = 1$ $(= 0)$ if the government defaults (pays its debt). Given the above defaulting rule, the price of a bond issued in period 1 is given by

$$q_1(b_1, b_2) = \delta + (1-\delta)\left[1 - F\left(\frac{b_1(1-\delta)+b_2}{\phi}\right)\right]. \tag{2}$$

The price of a bond issued in period 2 is given by

$$q_2(b_1, b_2) = 1 - F\left(\frac{b_1(1-\delta)+b_2}{\phi}\right). \tag{3}$$

The equilibrium levels of consumption are

$$c_1(b_1, b_2) = b_1 q_1(b_1, b_2),$$

$$c_2(b_1, b_2) = b_2 q_2(b_1, b_2) - \delta b_1, \tag{4}$$

$$c_3(b_1, b_2, y_3) = y_3[1 - \hat{d}(b_1, b_2, y_3)\phi] - [1 - \hat{d}(b_1, b_2, y_3)][b_1(1 - \delta) + b_2].$$

Let $\{b_1^R, b_2^R\}$ denote the sequence of borrowing that maximizes the government's expected utility in period 1, given that the default rule of the period-3 government in equation (1). This is, $\{b_1^R, b_2^R\}$ solves

$$\begin{aligned} \underset{b_1, b_2}{Max} \quad & u\left(c_1(b_1, b_2)\right) + \beta u\left(c_2(b_1, b_2)\right) + \beta^2 \mathbb{E}[u\left(c_3(b_1, b_2, y_3)\right)] \\ & s.t. \; c_t \geq 0 \text{ for } t = 1, 2, 3. \end{aligned}$$

We refer to $\{b_1^R, b_2^R\}$ as Ramsey policies.[3]

Let $\{b_1^M, b_2^M(b_1^M)\}$ denote the sequence of borrowing chosen sequentially by the governments in periods 1 and 2. We refer to $\{b_1^M, b_2^M(b_1^M)\}$ as Markov policies. For any b_1, the period 2 Markov strategy b_2^M solves

$$\begin{aligned} \underset{b_2}{Max} \quad & u\left(c_2(b_1, b_2)\right) + \beta \mathbb{E}[u\left(c_3(b_1, b_2, y_3)\right)] \\ & s.t. \; c_t \geq 0 \text{ for } t = 2, 3. \end{aligned}$$

The period 1 Markov policy b_1^M solves

$$\begin{aligned} \underset{b_1}{Max} \quad & u\left(c_1(b_1, b_2^M(b_1))\right) + \beta u\left(c_2(b_1, b_2^M(b_1))\right) + \beta^2 \mathbb{E}[u\left(c_3(b_1, b_2^M(b_1), y_3)\right)] \\ & s.t. \; c_t \geq 0 \text{ for } t = 1, 2, 3. \end{aligned}$$

Proposition 1 shows that when the government issues one-period debt, Ramsey policies coincide with Markov policies and, therefore, there is no role for fiscal rules.

Proposition 1 *When $\delta = 1$, Markov policies coincide with Ramsey policies (and thus there is no need for fiscal rules).*

[3] Note that b_2^R is the level of borrowing in period 2 that the government in period 1 would like to commit to, if it cannot commit to a period 3 default decision. In Hatchondo et al. (2020), we characterize such policies in a quantitative default model.

Proposition 2 shows that when the government issues long-term debt, Ramsey policies do not coincide with Markov policies.[4] Because there is default risk in equilibrium, long-term debt creates a time inconsistency problem, often referred to as debt dilution (Hatchondo et al., 2016).

Proposition 2 *When $\delta < 1$, Markov policies and Ramsey policies do not coincide.*

We study two ways of imposing a limit on government's choices in period 2.[5] First, using a debt-limit rule that imposes a ceiling on the debt level, $(1-\delta)b_1 + b_2 \leq \bar{b}$. Second, using a spread-limit rule that imposes a ceiling on the spread paid by the government and thus a floor on the price at which the government sells bonds, $q_2(b_1, b_2) \geq \underline{q}$.

Proposition 3 *If the government's choices in period 2 are limited with either a debt limit with threshold $\bar{b}^* = (1-\delta)b_1^R + b_2^R$ or a spread limit with threshold $\underline{q}^* = q_2(b_1^R, b_2^R)$, Markov policies coincide with Ramsey policies.*

Proposition 3 states that for a single economy, committing to any of these two fiscal rules is sufficient for making the sequential government choose Ramsey policies. It implies that having a debt or an interest rate instrument is inconsequential for a Ramsey planner. This is the standard result of equivalency of between prices and quantities as planning instruments without heterogeneity or uncertainty (Weitzman, 1974), including the use of the interest rate or the money stock as the monetary policy instrument (Poole, 1970). In contrast, the next subsection shows that, as the common rule for a Union of heterogeneous economies, a spread limit performs better than a debt limit.

3.2 Optimal common fiscal rule for a Union of heterogeneous economies

We focus on a Union of heterogenous economies that differ only in the level of debt intolerance, as given by the cost of defaulting ϕ (economies with a lower cost of defaulting display more debt intolerance because they pay a higher spread for the same level of debt; see equations 2

[4]While in this stylized model it would be optimal to issue one-period debt, this is not true in models with a plausible calibration of the volatility of the borrowing cost (e.g., Hatchondo et al., 2020).

[5]A third-party international organization could work as a commitment device as discussed in Goncalves and Guimaraes (2015).

and 3), and a constrained Ramsey planner that must apply the same policy to every economy in the Union. There are two interpretations of the constrained Ramsey policy. First, political constraints may force a supranational authority to impose the same fiscal rule to all countries in the Union. Second, the planner must commit to a constrained Ramsey policy for a single economy but without knowing the value of the parameter ϕ. One could think that the policy could be changed after learning ϕ. However, an essential characteristic of effective fiscal rules is that they cannot be frequently changed.

3.2.1 Ramsey borrowing and debt intolerance

The next assumption is a sufficient condition for the existence of a unique Ramsey policy for each level of ϕ.

Assumption 1: The function

$$\zeta_q(b) = \frac{b}{\phi}\frac{f\left(\frac{b}{\phi}\right)}{1-F\left(\frac{b}{\phi}\right)}$$

is increasing with respect to b, $\lim_{b\to 0}\zeta_q(b) = 0$, and $\lim_{b\to\infty}\zeta_q(b) \geq 1$. The function ζ_q is the absolute value of the elasticity of the (period 2) bond price with respect to the debt level b. Thus, Assumption 1 states that the bond price is more responsive to changes in the debt level for higher debt levels.[6]

Proposition 4 presents sufficient conditions under which the borrowing level under Ramsey policies is proportional to ϕ, which will allow us to present an analytical characterization of optimal fiscal rules.

Proposition 4 *Suppose $u(c) = c$, $\delta = 0$, and Assumption 1 holds. Then, Ramsey policies are given by $\{b_1^R = \eta\phi, b_2^R = 0\}$, where $\eta \in \mathbb{R}_{++}$ satisfies*

$$1 - \eta\frac{f(\eta)}{1-F(\eta)} = \beta^2. \tag{5}$$

[6] This is a standard feature of default models and is also consistent with existing empirical evidence (Jaramillo and Tejada, 2011; David et al., 2019; Hatchondo et al., 2020).

Proposition 4 states that an impatient risk-neutral borrower should only borrow in period 1. Intuitively, the Ramsey government borrows more for lower bond price elasticity functions (lower marginal cost of borrowing). The Ramsey borrowing also increases with the cost of defaulting. Given that $\beta < 1$, it is optimal for the borrower to front-load as much consumption as it can, and the cost of defaulting determines how much consumption is willing to sacrifice in $t = 3$ to repay debt.

3.2.2 Optimal common fiscal rule and debt intolerance

We define a common debt limit as a rule imposing to all economies in the Union a common debt ceiling $\bar{B}$ such that $(1-\delta)b_1 + b_2 \leq \bar{B}$. A common spread limit imposes to all economies in the Union a common ceiling on the spread paid by the government in the second period, and thus it imposes a floor $\underline{Q}$ on the sovereign bond price, such that $q_2(b_1, b_2) \geq \underline{Q}$.

Note that since the Ramsey debt level $b_1^R = \eta\phi$ is increasing in the cost of defaulting ϕ, a common debt-limit threshold $\bar{B}$ cannot achieve the Ramsey allocation in every economy in the Union. Intuitively, economies with less debt intolerance should be allowed to borrow more. A common debt limit cannot achieve this. Thus, our stylized model captures the often-cited difficulties of using common debt thresholds to guide fiscal policy (Blanchard et al., 2021; Eberhardt and Presbitero, 2015; Furman and Summers, 2020).

A common spread limit performs better: the same spread limit allows economies with less debt intolerance to borrow more while forcing economies suffering more debt intolerance to borrow less. Under the assumptions in Proposition 4, the bond price implied by the Ramsey debt level is the same in all economies $(1 - F(\eta))$. Therefore, the optimal common spread-limit threshold is given by this bond price ($\underline{Q}^* = 1 - F(\eta)$) and delivers the Ramsey allocation for every economy in the Union. Thus, the optimal common spread limit delivers larger welfare gains than any common debt limit. This is summarized in the following proposition.

Proposition 5 *Suppose $u(c) = c$, $\delta = 0$, and Assumption 1 holds. Consider any Union of economies that are different only in the value of the cost of defaulting ϕ. The optimal common spread-limit threshold for any such Union is $\underline{Q}^* = 1 - F(\eta)$ and achieves the Ramsey allocation* *in every economy of the Union, with η given by equation (5). Furthermore, $\underline{Q}^*$ generates larger welfare gains than any common debt limit $\bar{B}$.*

Proposition 5 brings to fiscal policy a standard result in the prices vs. quantities debate. When there is heterogeneity in costs (in this case in the cost of borrowing), using a price (the bond price) allows the planner to encourage low-cost agents to choose larger quantities (more borrowing) while forcing high-cost agents to choose lower quantities (Weitzman, 1974). Similarly, when costs are not known, prices are a superior instrument (Weitzman, 1974).

Note also that the optimal common spread limit with threshold $\underline{Q}^* = 1 - F(\eta)$ is the same for any distribution of welfare weights across economies in the Union. Therefore, the optimal common spread limit can be found without knowledge of this distribution and is a robust policy as described by Hansen and Sargent (2008). However, since the Ramsey debt level $b_1^R = \eta\phi$ is a function of ϕ, the optimal common debt limit depends on the distribution of welfare weights.

3.2.3 Optimal common fiscal rule and debt intolerance: numerical examples

We next present numerical examples that relax some of the assumptions in propositions 4 and 5. Suppose $u(c) = -c^{-1}$, $log(y_3) \sim N(0, 0.1)$, $\delta = 0$, and that consumer discount future utility flows with a factor $\beta^C = 1$ while the discount factor of the government when making decision is $\beta = 0.8$. This allows us to study rules that do not only correct for the time inconsistency problem presented before, but also for political myopia, which is captured by the difference between these discount factors (as in Aguiar et al., 2020). We consider a Union of economies with $\phi \sim U[0.05, 0.2]$. Without a fiscal rule, at the end of period 2, these economies display debt levels between 3 and 12 percent of average period 3 income, and spreads between 3 and 5 percent.

We find the constrained Ramsey policy that maximizes the average period 1 expected utility assigning weight $h(\phi)$ to the economy with parameter value ϕ. We focus on three common fiscal rules that maximize the Union's welfare assigning different weights across economies. One rule assigns the same weight to all economies in the Union. To convey the effect of having an assymetric distribution of weights across the Union (either because some economies have more political power or the level of debt intolerance is not evenly distributed), we consider two additional rules that assign all the weight to the economy with either the least or the most debt intolerance.

Figure 1 shows that common spread limits outperforms common debt limits. Common debt limits that prioritize economies with less debt intolerance (i.e., with a higher cost of defaulting) fail to generate welfare gains in the majority of the Union economies (where the higher debt limit preferred by low-debt-intolerance economies is not binding). In contrast, lower common debt limits that would be chosen prioritizing hight-debt-intolerance economies (and are needed to lower sovereign risk across the Union) impose unnecessarily low debt levels and thus welfare losses to low-debt-intolerance economies.

In contrast, common spread limits produce welfare gains for all economies in the Union, with the rule that maximizes average welfare achieving more than 80 percent of the welfare gains obtained with the best rule for each economy. This occurs because the same spread limit allows economies with less debt intolerance to borrow more while forcing economies with more debt intolerance to borrow less.

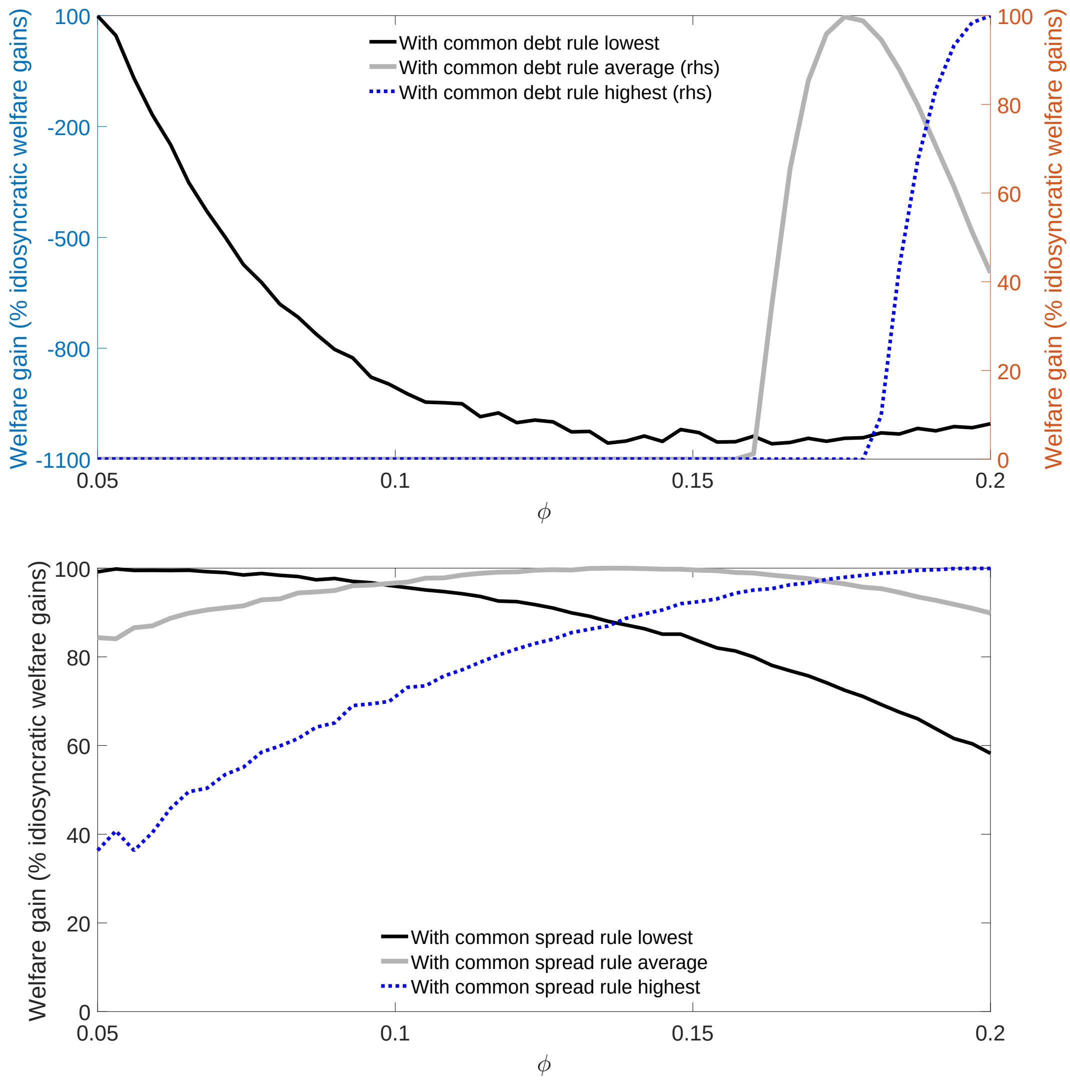

Figure 1: Ratio of welfare gain obtained with a common rule for the Union relative to the welfare gain delivered by the optimal idiosyncratic rules.

4 Conclusions

Since levels of debt intolerance are difficult to identify, and seem to vary greatly both across countries and over time, a spread limit is likely to be a more robust fiscal anchor than a debt limit. Economies should be allowed to issue more debt when they suffer less of a debt intolerance problem. A spread limit allows for this and a debt limit does not.

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A Proofs of propositions

A.1 Proof of Proposition 1

Ramsey policies satisfy

$$u'\left(c_1(b_1^R,b_2^R)\right)=\beta u'\left(c_2(b_1^R,b_2^R)\right)=\frac{\beta^2\mathbb{E}\left[u'\left(c_3(b_1^R,b_2^R,y_3)\right)\left[1-\hat{d}(b_1^R,b_2^R,y_3)\right]\right]}{q_2(b_1^R,b_2^R)+b_2^R\frac{\partial q_2(b_1^R,b_2^R)}{\partial b_2}}.$$

For any b_1, the period 2 Markov strategy satisfies

$$u'\left(c_2(b_1,b_2^M(b_1))\right)=\frac{\beta\mathbb{E}\left[u'\left(c_3(b_1,b_2^M(b_1),y_3)\right)\left[1-\hat{d}(b_1,b_2^M(b_1),y_3)\right]\right]}{q_2(b_1,b_2^M(b_1))+b_2^M(b_1)\frac{\partial q_2(b_1,b_2^M(b_1))}{\partial b_2}}.$$

Thus, if $b_1^M=b_1^R$, $b_2^M(b_1^M)=b_2^R$. Since Ramsey policies maximize the government's expected utility in period 1, they are the Markov policies (and there is no room for improving welfare with fiscal rules).

A.2 Proof of Proposition 2

Ramsey policies satisfy

$$\begin{aligned}u'\left(c_1(b_1^R,b_2^R)\right)\left[q_1(b_1^R,b_2^R)+b_1^R\frac{\partial q_1(b_1^R,b_2^R)}{\partial b_1}\right] \quad = \quad & \beta u'\left(c_2(b_1^R,b_2^R)\right)\left[\delta-b_2^R\frac{\partial q_2(b_1^R,b_2^R)}{\partial b_1}\right] \\ & +\beta^2(1-\delta)\mathbb{E}\left[u'\left(c_3(b_1^R,b_2^R,y_3)\right)\left[1-\hat{d}(b_1^R,b_2^R,y_3)\right]\right],\end{aligned} \tag{6}$$

$$\begin{aligned}\beta u'\left(c_2(b_1^R,b_2^R)\right)\left[q_2(b_1^R,b_2^R)+b_2^R\frac{\partial q_2(b_1^R,b_2^R)}{\partial b_2}\right] \quad = \quad & \beta^2\mathbb{E}\left[u'\left(c_3(b_1^R,b_2^R,y_3)\right)\left[1-\hat{d}(b_1^R,b_2^R,y_3)\right]\right] \\ & -\mathbf{u}'\left(\mathbf{c_1}(\mathbf{b_1^R},\mathbf{b_2^R})\right)\mathbf{b_1^R}\frac{\partial \mathbf{q_1}(\mathbf{b_1^R},\mathbf{b_2^R})}{\partial \mathbf{b_2}}.\end{aligned} \tag{7}$$

For any b_1, the period 2 Markov strategy satisfies

$$\begin{aligned}& u'\left(c_2(b_1,b_2^M(b_1))\right)\left[q_2(b_1,b_2^M(b_1))+b_2^M(b_1)\frac{\partial q_2(b_1,b_2^M(b_1))}{\partial b_2}\right] \\ & \quad =\beta\mathbb{E}\left[u'\left(c_3(b_1,b_2^M(b_1),y_3)\right)\left[1-\hat{d}(b_1,b_2^M(b_1),y_3)\right]\right].\end{aligned} \tag{8}$$

The second term in the right hand side of equation (7) represents the marginal cost that borrowing in period 2 imposes on consumption in period 1. Note that this term is positive (the marginal utility of period 1 consumption is positive, the period 1 bond price in decreasing in period 2 borrowing, and because period 1 income equals zero and the government is impatient, period 1 borrowing is positive). While a government choosing a borrowing sequence in period 1 would internalize the effect of borrowing in period 2 on consumption in period 1, this effect does not influence the decision of the government choosing in period 2 (equation 8). Therefore, Markov policies are different from Ramsey policies (if $b_1^M = b_1^R$, $b_2^M(b_1^M) \neq b_2^R$).

A.3 Proof of Proposition 3

Since $f(y_3) \geq 0$ for all $y_3 \geq 0$, q_2 is a strictly decreasing function of b_2. Therefore, if the period 1 government chooses b_1^R, imposing the spread brake threshold $\underline{q}^* = q_2(b_1^R, b_2^R)$ is equivalent to imposing the debt brake threshold $\bar{b}^* = (1-\delta)b_1^R + b_2^R$. Since

$$u'\left(c_2(b_1^R, b_2^R)\right)\left[q_2(b_1^R, b_2^R) + b_2^R \frac{\partial q_2(b_1^R, b_2^R)}{\partial b_2^R}\right] > \beta \mathbb{E}\left[u'\left(c_3(b_1^R, b_2^R, y_3)\right)\left[1 - \hat{d}(b_1^R, b_2^R, y_3)\right]\right],$$

with either a debt brake with threshold $\bar{b}^* = (1-\delta)b_1^R + b_2^R$ or a spread brake with threshold $\underline{q}^* = q_2(b_1^R, b_2^R)$, if $b_1^M = b_1^R$, $b_2^M(b_1^M) = b_2^R$, and Markov policies coincide with Ramsey policies.

A.4 Proof of Proposition 4

Note first that because $u(c) = c$ and $\beta < 1$, the government's expected utility in period 1 is maximized with period-2 consumption equal to zero (period-1 consumption is more valuable than period-2 consumption and without default risk in period 2, borrowing in period 1 is as costly as borrowing in period 2). Therefore, Ramsey policies are given by $\{b_1^R, 0\}$, where b_1^R satisfies

$$1 - \frac{b_1^R}{\phi} \frac{f\left(\frac{b_1^R}{\phi}\right)}{1 - F\left(\frac{b_1^R}{\phi}\right)} = \beta^2. \tag{9}$$

Assumption 1 guarantees that there is a unique level of $\frac{b_1^R}{\phi}$ that solves equation (9). Let η denote this level. Then, for any economy with cost of defaulting ϕ, $b_1^R = \eta\phi$.