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Equilibrium Foreign Currency Mortgages*

by Marcin Kolasa

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Equilibrium Foreign Currency Mortgages*

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Abstract

This paper proposes a novel explanation for why foreign currency denominated loans to households have become so popular in some emerging economies. Our argument is based on what we call the debt limit channel, which arises when multi-period contracts are offered to financially constrained borrowers against collateral that is established on newly acquired assets. Whenever the difference between domestic and foreign interest rates is positive, this effect biases borrowers' choices towards foreign currency, even if the exchange rate is known to depreciate as implied by the interest parity condition. We demonstrate in a structural macroeconomic framework that the debt limit channel is quantitatively important and can result in dollarization of debt also in the presence of realistic exchange rate risk. Comparing this outcome to allocations under constrained-optimal time-consistent policy reveals that a substantial part of the identified bias towards foreign currency is due to a pecuniary externality, i.e. borrowers' failure to internalize how their currency choice affects collateral prices.

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1 Introduction

Despite ongoing development of local financial markets and progress in inflation control achieved during the last decades, foreign currency lending remains an important feature of the financial structure in many countries, raising concerns among policy makers. According to the Financial Soundness Indicators published by the IMF, the (unweighted) average share of foreign currency and foreign-currency-linked borrowing in total gross outstanding loans amounted in 2014 to nearly 30% in the 66 surveyed countries (see Table 1). While this proportion is non-negligible even among developed euro area economies, it is particularly high in Central and Eastern Europe (CEE), where it comes close to a half. As Table 2 reveals, the currency substitution in this region is not restricted to the sector of non-financial firms, but also prevalent among households, and in particular related to financing house purchases. While the euro adoption by Slovenia, Slovakia and the three Baltic states effectively solved this problem in these economies, it took strong actions by the government or financial supervision in other countries to curb lending in foreign currency to households.[1]

A number of papers have tried to rationalize the observed widespread and persistent dollarization of lending. Most of this literature focuses on the supply side factors related to the functioning of international financial markets and banks.[2] While all these channels might help understand the currency composition of loans to firms, their relevance for borrowing by households is limited. Household debt is predominantly domestic and not international, which means that preferences of foreign lenders should not be relevant in this context. Indeed, based on the bank-level survey data, Brown and Haas (2012) argue that foreign currency lending to households in emerging Europe has not been driven by foreign banks due to their easier access to foreign wholesale funding. If anything, lending in foreign currency should be rather less attractive to banks because of higher credit risk. However, the demand-side rationalizations existing in the literature are also not very appealing in this context. In

[1] See Rosenberg and Tirpak (2008) for early evidence of regulation aimed at restricting foreign currency lending in CEE countries. More recently, some tougher measures have been taken. In 2013, the financial supervision authority in Poland issued a recommendation to banks against offering foreign currency mortgage loans to households that do not earn in that currency, which effectively shut down new originations of this type of lending. In Hungary, foreign currency housing loans were converted to domestic currency in late 2014. A couple of months later, a controversial law was passed in Croatia that facilitated the conversion of loans denominated in Swiss franks into loans denominated in euro.

[2] In the context of international debt, the literature has stressed the so-called original sin, i.e. the inability of emerging economies to borrow internationally in its own currency, see e.g. Terrones and Catao (2000), Calvo (2001), Ize and Yeyati (2003) or Eichengreen et al. (2004). Several demand-side explanations have also been offered, usually focusing on borrowing by firms, see e.g. Goswami and Shrikhande (2001), Jeanne (2003) or Ranciere et al. (2003).

particular, households usually earn in domestic currency and hence natural hedging cannot be the major motive in choosing the foreign currency. As a result, popular explanations for dollarization of household debt are based on some irrational behavior, borrowers' limited ability to assess exchange rate risk or speculative motives, like those created by the empirically observed violation of the uncovered interest rate parity (UIP) or implicit government bailout guarantees in case of severe exchange rate depreciation (Ranciere et al., 2010).

This paper offers an alternative and novel explanation for why foreign currency loans, and foreign currency mortgages in particular, have become so popular in some countries whenever banks made them available at non-discriminatory terms, as it was the case in the CEE region. At the heart of our argument is what we call the debt limit channel, which, whenever the exchange rate is expected to depreciate, allows credit constrained agents to backload the real payments on their loans (i.e. effectively hold more debt over the loan duration) if they choose to borrow in foreign currency.

To illustrate how this backloading works, let us use a simple deterministic example. Consider a financially constrained agent who at period 0 can take a 2-period adjustable-rate loan denominated either in local or foreign currency.[3] The amount that can be borrowed is independent of denomination and equal to 100 units of either domestic or foreign currency, assuming that the nominal exchange rate at time 0 equals unity. Loans are repaid in equal principal payments of 50 units of either local or foreign currency, depending on the chosen denomination. The real interest rate is constant and normalized to zero. The nominal interest rate charged on outstanding debt in the domestic currency is 1% for period 1 and 0% for period 2, while in the case of a foreign currency loan it equals zero in both periods. Financial markets, at which the agent cannot directly participate, ensure no arbitrage between one-period holdings of either of the two currencies, which implies that the nominal exchange rate depreciates by 1% in period 1 and then remains constant. With these assumptions, the cash flows related to loan repayment, expressed in local currency units, are $50+0.01\times 100 = 51$ in period 1 and $50+0\times 50 = 50$ in period 2 for domestic currency borrowing, and $50\times 1.01 = 50.5$ in both period 1 and 2 for foreign currency borrowing. Note that the present discounted value of both streams of payments is the same and equal to $101/1.01 = 100$, so what makes them distinct is their different distribution over time: exchange rate depreciation effectively backloads real loan payments.[4]

[3]In the CEE countries, vast majority of housing loans are adjustable rate mortgages.

[4]In this respect, the debt limit channel has some resemblance to the effect of inflation on mortgages studied in the 1970s, see e.g. Modigliani (1974). As that literature pointed out, when inflation (and hence

Naturally, if the agent in our example was not financially constrained, she would be indifferent between the alternative repayment schedules. For a borrower that is constrained over the loan duration, it does matter how much debt she can effectively hold at every point in time. Hence, another way of looking at our mechanism is by noting that a multi-period adjustable rate loan is equivalent to a sequence of one-period loans, but with a special nominal rollover commitment. This commitment is different for local and foreign currency borrowing because exchange rate movements affect the local currency value of the latter. As a result, exchange rate depreciation acts as if the agent has a higher debt limit for the debt rollover if she borrows in foreign currency. This can be also seen in our illustrative example, in which outstanding debt at the end of period 1, expressed in local currency units, is equal to 50 in the case of local currency borrowing, and 50.5 if borrowing is denominated in the foreign currency.

This simple example can be extended to a more realistic one, in which a mortgage contract is taken for more than two periods, and where additionally agents are allowed to borrow short term subject to some standard constraint, e.g. up to a fraction of their labor income. The logic presented above survives as long as the nominal exchange rate is expected to depreciate (the interest rate differential is positive) and the short-term borrowing constraint is binding at some time in the future. A foreign currency loan is more attractive as it essentially allows to hold more debt. One of the insights from this discussion is that foreign and domestic currency loans granted at apparently equal terms (loan-to-value ratio, repayment schedule) are not perfect substitutes to credit constrained agents, even if there is no risk and the interest rate parity holds exactly, i.e. the exchange rate depreciates whenever there is a positive difference between domestic and foreign interest rates. Hence, our argument does not rely on some speculative behavior of (usually considered risk-averse) households, like betting on an appreciation of the local currency, or their misperception of exchange rate risk. Instead, for the debt limit channel to bias household mortgage choices towards foreign currency, the following conditions must hold: (i) the difference between domestic and foreign interest rates is positive, (ii) borrowers are credit constrained, (iii) mortgage debt is long-

nominal interest rates) are low, mortgages repaid in equal nominal installments imply repayment schedules that are effectively backloaded compared to the periods when inflation is high. This is because the standard annuity formula implies a positive relationship between the nominal installment and the interest rate, and the real value of future repayments (fixed in nominal terms) declines at a slower pace when inflation is low. Our mechanism is a bit different in that, when loans are taken in foreign currency, exchange rate depreciation increases the nominal value of future repayments expressed in local currency, and hence their real value, for given inflation.

term, (iv) collateral constraint applies to new borrowing rather than outstanding debt, i.e. it is established on newly purchased assets. We argue that the available evidence indicates that these conditions fit very well to the mortgage markets in the CEE region.

If we leave a deterministic world and allow for uncertainty, borrowers' portfolio choices additionally reflect risk considerations. If households earn in local currency, exchange rate fluctuations can make foreign currency borrowing relatively risky. In particular, unexpected exchange rate depreciation negatively affects the balance sheets of those who hold foreign currency debt. For sufficiently high risk, the benefits of backloading debt repayment may turn out to be too weak to justify mortgage debt dollarization. Hence, any evaluation of the empirical relevance of the debt limit channel must confront its strength with a realistic description of risk.

To this end, we formalize the simple example and discussion offered above by embedding it into a quantitative general equilibrium framework. In this model, agents can borrow from abroad either in domestic or foreign currency, face uncertainty about their income and nominal exchange rate movements, and their borrowing is subject to a collateral constraint that depends on the value of newly purchased housing, hence resembling new mortgages as e.g. in Garriga et al. (2017). When calibrated to Poland, which can be considered a representative CEE economy, the model produces a sizable share of foreign currency mortgage debt. It also generates a number of predictions consistent with the empirical evidence from the CEE region.

Because of the presence of house prices in the collateral constraint, the model features a pecuniary externality: households do not internalize how their total borrowing and its currency composition affects house prices. Comparing the obtained decentralized outcome to allocations under constrained-optimal time-consistent policy as in Bianchi and Mendoza (2018) reveals that the former is characterized by overborrowing and bias towards foreign currency. To our knowledge, the latter result is new in the literature and suggests that a substantial part of the observed dollarization of mortgages in the CEE region was inefficient and warranted at least some intervention by a financial regulator.

Apart from the papers already mentioned, this work is related to several strands of the literature, of which we list only a couple of representative contributions. The determinants of mortgage dollarization in the CEE region was investigated empirically by e.g. Barajas and Morales (2003), Luca and Petrova (2008), Cuaresma et al. (2011) or Fidrmuc et al. (2013). General equilibrium analysis of the currency composition of borrowing by emerging market economies is offered by Korinek (2011), while Brzoza-Brzezina et al. (2017) use a DSGE model to examine the effect of foreign currency borrowing on monetary and macroprudential policy transmission, treating the composition of household debt as exogenous. However, both of these papers have contracts lasting only one period. Modeling collateral constraints in housing was pioneered by Iacoviello (2005), and the multi-period extensions can be found e.g. in Garriga et al. (2017) or Bluwstein et al. (2019). Pecuniary externalities associated with collateral prices are studied by e.g. Mendoza (2010), Jeanne and Korinek (2010), Bianchi (2011) or Benigno et al. (2013). However, these papers deal with total overborrowing and not with (possibly inefficient) currency composition of debt.

The rest of this paper is organized as follows. Section two presents the model and defines its decentralized and constrained-efficient equilibrium. Section three conducts a quantitative and qualitative analysis with the model. Section four concludes. Additional details on the model solution and data are delegated to the Appendix.

2 Model

To demonstrate the working of the debt limit channel in a more formal way, and to prepare ground for an evaluation of its empirical relevance, we embed it into a general equilibrium model. We characterize both its decentralized competitive equilibrium and the outcomes chosen by a constrained-efficient social planner. Throughout, we apply a convention to use smaller case letters to denote real allocations and real prices, and upper case letters to indicate variables expressed in nominal terms.

2.1 Environment

Let us consider a small open economy (referred to as home) with stochastic endowment, nominal long-term debt collateralized on housing, and nominal exchange rate risk. Below we describe the key building blocks of our setup.

International investors Risk-neutral international investors can take positions in one-period bonds denominated in home and foreign currency, which pay nominally risk-free gross interest R_t and R_t^*. They can also issue home and domestic currency mortgage contracts, or hold them for financial purposes. As in Greenwald (2018), a mortgage contract is a nominal perpetuity with principal payments that decline geometrically at rate $0 < \delta \leq 1$. We also assume that it is of a variable-rate type, i.e. interest paid on outstanding debt is adjusted every period. More specifically, for one unit of currency received at time t from a mortgagee (lender), a mortgagor (borrower) commits to pay at time $t+k$ (for $k = 1, 2, ...$) principal $\delta(1-\delta)^{k-1}$ and interest $(1-\delta)^{k-1}q_{t+k}$, and both payments are denominated in the currency of origination, .

As international investors are unconstrained in their financial decisions, i.e. they can take any position in bonds and mortgages, arbitrage between these two assets implies $q_t = R_{t-1} - 1$ for mortgages denominated in domestic currency and $q_t = R^*_{t-1} - 1$ for those denominated in foreign currency. Moreover, risk-neutrality of these agents postulates equalization of the (ex ante) one-period returns on both currencies

$$R_t = R^*_t \mathbb{E}_t \left\{ \frac{S_{t+1}}{S_t} \right\} \tag{1}$$

where S_t is the nominal exchange rate expressed as the home currency price of one unit of foreign currency. Equation (1) is a standard uncovered interest rate parity (UIP) condition.

Domestic households A representative household in the home economy maximizes the expected value of her discounted period utility flows

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right\} \tag{2}$$

where c_t is consumption, h_t is the housing stock, u is a continuously differentiable, strictly increasing and concave function in both arguments, and $0 < \beta < 1$. We assume that agents in the home economy are impatient relative to the rest of the world so that $\beta r^* < 1$, with r^* denoting the world (gross) real interest rate that we assume to be constant.

Every period each household receives a stochastic real endowment y_t. This endowment can be freely traded, also internationally, so that $P_t = S_t P^*_t$, where P_t and P^*_t are the home and foreign price levels. Households cannot fully participate in international financial markets, and can only borrow from international investors using mortgages, denominated either in domestic or foreign currency. Households can spend their available financial resources on consumption c_t or new house purchases h^{new}_t, the latter traded at price $P_{h,t}$.

Using the characterization of a mortgage contract described before, the representative household's budget constraint can be written as

$$P_t c_t + P_{h,t} h_t^{new} + (R_{t-1} - 1 + \delta) L_{H,t-1} + (R^*_{t-1} - 1 + \delta) S_t L_{F,t-1} \leq P_t y_t + L_{H,t}^{new} + S_t L_{F,t}^{new} \quad (3)$$

where $L_{H,t}^{new}$ is new borrowing in domestic currency (net of prepayment of past loans), $L_{F,t}^{new}$ is net new borrowing in foreign currency, and debt in both currencies evolves according to

$$L_{H,t} = L_{H,t}^{new} + (1 - \delta) L_{H,t-1} \quad (4)$$

$$L_{F,t} = L_{F,t}^{new} + (1 - \delta) L_{F,t-1} \quad (5)$$

We restrict $L_{H,t}$ and $L_{F,t}$ to be non-negative, but do not impose such restrictions on $L_{H,t}^{new}$ and $L_{F,t}^{new}$. This means that each type of loans can be prepaid or converted into another currency at no additional cost, but cannot turn into a deposit.

Note that, since interest payments depend on the current interest rates (as it is the case of standard adjustable-rate mortgages), the budget constraint (3) can be rewritten using (4) and (5) in a compact form as

$$P_t c_t + P_{h,t} h_t^{new} + R_{t-1} L_{H,t-1} + R^*_{t-1} S_t L_{F,t-1} \leq P_t y + L_{H,t} + S_t L_{F,t} \quad (6)$$

so that it does not depend on δ, and hence is identical to the case of one-period debt.

Additionally, households face the following borrowing limit

$$L_{H,t}^{new} + S_t L_{F,t}^{new} \leq m P_{h,t} h_t^{new} \quad (7)$$

which can be interpreted as a collateral constraint, with $0 < m < 1$ denoting the loan-to-value (LTV) ratio.

The housing stock that enters household utility is accumulated according to

$$h_t = h_t^{new} + (1 - \delta_h) h_{t-1} \quad (8)$$

where $0 < \delta_h \leq 1$ is the housing depreciation rate.

Price level determination The nominal interest rates R_t and R^*_t are set by the domestic and foreign monetary authorities, respectively, according to the following simple feedback rules

$$R_t = r^* \bar{\pi} \left(\frac{\pi_t}{\bar{\pi}_t} \right)^{\nu} \tag{9}$$

$$R_t^* = r^* \bar{\pi}^* \left(\frac{\pi_t^*}{\bar{\pi}_t^*} \right)^{\nu} \tag{10}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ and $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$, while $\bar{\pi}_t$ and $\bar{\pi}_t^*$ are exogenous stochastic processes with means $\bar{\pi}$ and $\bar{\pi}^*$, respectively. We assume that $\nu > 1$ so that the price levels in both regions are determinate, see Woodford (2003). While a literal interpretation of $\bar{\pi}_t$ and $\bar{\pi}_t^*$ would be to call them inflation targets, their role in our model is to represent nominal risk associated with, respectively, home and foreign currency holdings. In particular, we will use $\bar{\pi}_t^*$ to generate unexpected fluctuations in the nominal exchange rate. Note that this formulation makes this variable basically exogenous to the model outcomes, an issue we discuss in more detail in Section 3.6.

Finally, given our assumption that the world real interest rate r^* is constant, the Fisher relationship can be written as

$$\mathbb{E}_t \left\{ \frac{R_t^*}{\pi_{t+1}^*} \right\} = r^* \tag{11}$$

.

Market clearing The model is closed with the following housing market clearing condition

$$h_t = \bar{h} \tag{12}$$

where $\bar{h} > 0$ is aggregate housing stock that we assume to be fixed.

2.2 Decentralized competitive equilibrium

An equilibrium in this economy is a sequence $\{c_t, h_t, L_{H,t}, L_{F,t}, L_{H,t}^{new}, L_{F,t}^{new}, R_t, R_t^*, P_t, P_t^*, S_t, P_{h,t}\}_{t=1}^{\infty}$, for given sequence $\{z_t, \bar{\pi}_t, \bar{\pi}_t^*\}_{t=1}^{\infty}$, initial debt $L_{H,0}$ and $L_{F,0}$, and initial price levels P_0 and P_0^*, such that households maximize their utility (2) subject to constraints (3)-(5) and (7) and taking all prices as given, the monetary authorities follow their feedback rules (9) and (10), the UIP relationship (1) holds, the real interest rate is given by (11), $P_t = S_t P_t^*$ for every t, and all markets clear.

In the Appendix, we show how this equilibrium can be recast in a recursive form, which we will use to solve the model with numerical methods, also described in the Appendix.

2.3 Debt limit channel

Let us now present how the debt limit channel arises in this general equilibrium framework, where agents can borrow against new house purchases every period. As we have argued in the introduction, one way of thinking about why foreign and domestic currency debt may not be perfect substitutes to financially constrained agents is that the level of total debt they hold on average depends on its currency composition.

To see it, let us use the housing and debt accumulation formulas (8)-(5) and housing market clearing condition (12) to rewrite collateral constraint (7) as follows

$$l_t \leq m\delta_h p_{h,t}\bar{h} + (1-\delta)\frac{l_{t-1}}{\pi_t}\left(1+\gamma_t\omega_{t-1}\right) \tag{13}$$

where smaller case letters are used to indicate the real values of variables previously defined in nominal terms, $l_t = l_{H,t} + l_{F,t}$ is total real debt, $\omega_t = \frac{l_{F,t}}{l_t}$ is its foreign currency share, and $\gamma_t = \frac{S_t - S_{t-1}}{S_{t-1}} = \frac{\pi_t}{\pi_t^*} - 1$ is the rate of nominal exchange rate depreciation. It is clear from this equation that past debt composition matters for the level of current debt (and consequently for current consumption if the collateral constraint is binding), unless loans are one period ($\delta = 1$).

As we mentioned in the introduction, the debt limit channel does not arise under all types of long-term contracts. In addition to multi-periodicity, the key assumption is that, like in equation (7), the collateral constraint restricts new borrowing rather than total debt. This formulation has become quite common in the recent literature that incorporates mortgages into a general equilibrium setup (see e.g. Garriga et al., 2017; Gelain et al., 2018; Landvoigt et al., 2020). It essentially assumes that each mortgage origination occurs simultaneously with a house purchase and that multiple mortgages on a single property are not allowed. As a consequence, the credit constraint does not depend on how many mortgages a household has taken in the past. One way of interpreting this financial arrangement is that, from a lender's perspective, the required down payment (i.e. fraction $1-m$ of the house purchase value) is perceived as sufficient to secure repayment of a loan, and hence other considerations like other debt do not matter.

Overall, our borrowing contract is valid for new mortgages (i.e. loans for house purchase), and perhaps also for credit-financed purchase of fixed assets by firms, so that the collateral is effectively established on newly acquired assets. The framework does not necessarily fit to other types of lending, including those resembling home equity lines of credit as considered in a DSGE framework by Bluwstein et al. (2019), and for which a more valid collateral constraint would read $L_{H,t} + S_t L_{F,t} \leq m P_{h,t} h_t$ instead of (7). Under this alternative constraint and absent risk considerations, households would be indifferent between domestic and foreign currency denomination of their debt. However, it needs to be stressed that home equity lines and multiple mortgages on a single property are not the dominant source of housing finance in the CEE region, hence our modeling choices adequately describe the mortgage market structure in the countries we focus on.

The steady state version of equation (13) is

$$l = \frac{m\delta_h \bar{\pi} p_h \bar{h}}{\bar{\pi} - (1-\delta)(1+\gamma\omega)} \tag{14}$$

where variables without a time subscript indicate the steady state values and the equality in the formula above results from our assumption that agents in the home economy are relatively impatient ($\beta r^* < 1$). Consider the empirically relevant case $R > R^*$ (nominal interest rates in emerging economies are usually higher than in developed countries), which implies that $\gamma > 0$. Then, for given real value of the collateral, the higher is the share of foreign currency in borrowers' portfolio ω, the larger is the total amount of debt l they can hold. Since households are financially constrained, their choice will be to borrow only in foreign currency, which means that the portfolio problem is degenerate. This is because, if the nominal exchange rate is known to depreciate, borrowing in foreign currency increases the effective debt limit.

The working of the debt limit channel can be also intuitively explained as follows. Note that a nominal multi-period adjustable rate loan can be thought of as a sequence of one-period loans, but with a special nominal rollover commitment that is different for local and foreign currency borrowing because exchange rate movements affect the local currency value of the latter. For one unit worth of local currency initially borrowed at time $t = 0$, these rollover commitments are $1-\delta, (1-\delta)^2, (1-\delta)^3, \ldots$ for a domestic currency loan, and $(1-\delta)\frac{S_1}{S_0}, (1-\delta)^2\frac{S_2}{S_0}, (1-\delta)^3\frac{S_3}{S_0}, \ldots$ if a loan is taken in the foreign currency.[5] No-arbitrage pricing implies that the interest cost per unit borrowed at every refinancing stage is the same

[5] Note that δ, and hence effectively debt maturity, is treated as a (constant) contractual constraint rather than a choice parameter. Otherwise, since households are impatient, they would choose the lowest possible δ. The constant value of this parameter and its independence on currency denomination can be motivated by the fact that foreign currency mortgage contracts in CEE countries, at least during the period of their expansion and until the regulators stepped in, were granted at very similar terms as those denominated in domestic currency.

for domestic and foreign currency loans. However, if the exchange rate depreciates after the loan is taken so that $S_t > S_0$ for $i > 0$, the rollovers are higher for foreign currency loans. This matters when households are financially constrained and hence, in contrast to more patient financial markets, attach a positive value to an additional unit of debt.[6]

2.4 Risk considerations

We now explain how the bias towards foreign currency can be modified by the presence of risk. To this end, let us first define the one-period gross real rates of return on local and foreign currency from t to $t+1$

$$r_{H,t+1} = \frac{R_t}{\pi_{t+1}} \tag{15}$$

$$r_{F,t+1} = \frac{R_t^*}{\pi_{t+1}^*} = \frac{R_t^*}{\pi_{t+1}}(1+\gamma_{t+1}) \tag{16}$$

and the excess return on the latter as

$$r_{x,t+1} = r_{F,t+1} - r_{H,t+1} \tag{17}$$

The two Euler equations associated with, respectively, local and foreign currency borrowing can then be written as

$$u_{c,t} - \Theta_t = \beta \mathbb{E}_t \left\{ u_{c,t+1} r_{H,t+1} \right\} - \beta(1-\delta)\mathbb{E}_t \left\{ \frac{\Theta_{t+1}}{\pi_{t+1}} \right\} - \mu_{H,t} \tag{18}$$

$$u_{c,t} - \Theta_t = \beta \mathbb{E}_t \left\{ u_{c,t+1} r_{F,t+1} \right\} - \beta(1-\delta)\mathbb{E}_t \left\{ \frac{\Theta_{t+1}}{\pi_{t+1}}(1+\gamma_{t+1}) \right\} - \mu_{F,t} \tag{19}$$

where $u_{c,t}$ is the marginal utility of consumption, Θ_t denotes the Lagrange multiplier on the collateral constraint (expressed in real terms), while $\mu_{H,t}$ and $\mu_{F,t}$ denote the Lagrange multipliers on the non-negativity constraints $L_{H,t} \geq 0$ and $L_{F,t} \geq 0$.

The left-hand sides of each of these equilibrium conditions are the same for both types of loans and can be interpreted as the current period net benefit of one additional unit of borrowing. Ignoring the terms related to the non-negativity constraints, the right-hand sides describe the expected net cost, and each is a difference between two components. The first is

[6] Note that the debt limit channel described above is independent of the repayment plan, as long as it is the same for local and foreign currency loans. For example, the latter will be strictly preferred by credit constrained borrowers if the exchange rate is expected to depreciate and repayment can be described by any sequence of positive principal payments $\{\delta_{t+s}\}_{s=1}^{n}$, where n is loan maturity. Naturally, whenever $R_t < R_t^*$, the debt limit channel will work in a way that favors domestic rather than foreign currency borrowing.

related to the expected financial cost of repaying a loan, and we will call it the balance sheet effect. The second term corresponds to the debt limit channel, and can be interpreted as the expected value of the rollover commitment that the loan contract guarantees. The latter effect shows up only if loans are multi-period ($\delta < 1$), and the gain it describes is strictly positive unless the collateral constraint is known to be slack in the future ($\Theta_{t+1} = 0$). It can be also easily verified that the rollover commitment effect would vanish if the collateral constraint was defined for the stock of debt rather than new borrowing, highlighting again the importance of this modeling assumption.

To better understand the portfolio choice faced by borrowers, let us rewrite the two associated equilibrium conditions as follows

$$\begin{aligned} u_{c,t} - \Theta_t = {} & \beta \mathbb{E}_t \left\{ u_{c,t+1} \right\} \mathbb{E}_t \left\{ r_{H,t+1} \right\} + \beta \mathbb{C}\mathrm{ov}_t \left\{ u_{c,t+1}, r_{H,t+1} \right\} \\ & - \beta(1-\delta) \mathbb{E}_t \left\{ \frac{\Theta_{t+1}}{\pi_{t+1}} \right\} - \mu_{H,t} \end{aligned} \tag{20}$$

$$\begin{aligned} u_{c,t} - \Theta_t = {} & \beta \mathbb{E}_t \left\{ u_{c,t+1} \right\} \mathbb{E}_t \left\{ r_{F,t+1} \right\} + \beta \mathbb{C}\mathrm{ov}_t \left\{ u_{c,t+1}, r_{F,t+1} \right\} \\ & - \beta(1-\delta) \mathbb{E}_t \left\{ \frac{\Theta_{t+1}}{\pi_{t+1}} \right\} \mathbb{E}_t \left\{ 1 + \gamma_{t+1} \right\} - \beta(1-\delta) \mathbb{C}\mathrm{ov}_t \left\{ \frac{\Theta_{t+1}}{\pi_{t+1}}, \gamma_{t+1} \right\} - \mu_{F,t} \end{aligned} \tag{21}$$

where, in each formula, the first line of the right-hand side unpacks the balance sheet effect while the second decomposes the debt limit channel. Note that the Fisher relationship (11) implies $\mathbb{E}_t \left\{ r_{H,t+1} \right\} = \mathbb{E}_t \left\{ r_{F,t+1} \right\} = r^*$ so that the balance sheet effect is essentially all about risk, making the currency generating more risky returns less attractive for borrowing purposes. The debt limit channel has two components. The first one concerns expected direction of exchange rate movements, and represents its impact on effective rollover commitment that we described above under certainty. This effect clearly favors foreign (domestic) currency borrowing when the exchange rate is expected to depreciate (appreciate). The second component is about risk, defined as the expected comovement between the tightness of the collateral constraint and the exchange rate. This comovement is positive as long as households hold some foreign currency debt. This is because unexpected exchange rate depreciation hurts households' balance sheets, making their borrowing needs larger, and additionally depresses house prices due to fire sales. As a result, the tightness of the collateral constraint is positively correlated with exchange rate depreciation. Hence, both terms associated with the debt limit channel bias household choices towards foreign currency if it holds lower nominal interest rate.

All in all, the solution to the portfolio problem faced by borrowers crucially depends on two factors. One is the nominal interest rate spread $R_t - R_t^*$, which determines the strength of the debt limit channel, and whose higher values favor borrowing in foreign currency. The other is the nominal exchange rate volatility, which is key for the balance sheet channel, and whose increase discourages debt dollarization, at least for the typical case when household income is denominated in local currency. The relative strength of these channels may be additionally affected by other exogenous forces, like the stochastic endowment process z_t, through their impact on household marginal utility and tightness of the collateral constraint. As the presence of the Lagrange multipliers on the loan non-negativity constraints suggests, the solution to this portfolio problem does not need to be internal, and households may choose to denominate all of their debt in one currency. In particular, if risk goes to zero, borrowers' choice will depend only on the sign of the interest rate spread, leading to full dollarization for $R > R^*$ as we have seen in our steady state analysis. In this case $\mu_F = 0$, but $\mu_H > 0$ so that both equilibrium conditions (18) and (19) can simultaneously hold.

2.5 Constrained-efficient equilibrium

Apart from a characterization of a decentralized equilibrium, we also offer a normative analysis by studying constrained-efficient allocations chosen by a social planner. While formulating her problem, we follow closely Bianchi and Mendoza (2018), who also consider a model with borrowing against a collateral, but do not allow for currency choice of debt nor its multi-periodicity, i.e. the two features that are at the heart of our analysis. Still, in the key general aspects, our definition of the planner's problem is very similar, so we refer the reader to that paper for conceptual details.

The social planner chooses debt and its currency composition on behalf of households, but lets house prices remain determined by the market. Consistently with the definition of constrained efficiency of Kehoe and Levine (1993), the Euler equation describing housing decisions by households enters the planner's problem as an implementability constraint, but she does internalize the impact of her borrowing and portfolio choices on house prices. The planner also does not interfere with the nominal side of the economy, defined by the feedback rules of the monetary authorities and Fisher equations, treating them all as exogenous. Importantly, the social planner cannot commit to her future decisions, and hence takes them as given while choosing current allocations. As in Bianchi and Mendoza (2018), the time-consistency of this optimization is ensured by describing the planner's decisions with policy rules that depend on state variables ($l_{H,t}$, $l_{F,t}$ and current realization of stochastic shocks), and defining their fixed point as the (Markov-stationary) outcome.

While solving her problem, the planner hence faces the same constraints as households, but additionally takes into account that housing is fixed as implied by equation (12), and observes the following dynamic house price equilibrium condition

$$p_{h,t}\left(u_{c,t} - m\Theta_t\right) = u_{h,t} + \beta(1-\delta_h)\mathbb{E}_t\left\{p_{h,t+1}\left(u_{c,t+1} - m\Theta_{t+1}\right)\right\} \tag{22}$$

which can be derived by solving for the decentralized equilibrium, and in which $u_{h,t}$ denotes marginal utility of housing.

A constrained-efficient equilibrium in this economy can hence be defined as a sequence $\{c_t, h_t, L_{H,t}, L_{F,t}, L^{new}_{H,t}, L^{new}_{F,t}, R_t, R^*_t, P_t, P^*_t, S_t, P_{h,t}\}_{t=1}^{\infty}$, for given sequence $\{z_t, \bar{\pi}_t, \bar{\pi}^*_t\}_{t=1}^{\infty}$, initial debt $L_{H,0}$ and $L_{F,0}$, and initial price levels P_0 and P^*_0, such that the social planner maximizes household utility (2) subject to constraints (3)-(5), (7), (12) and (22), and taking the future planners' decisions as given, the monetary authorities follow their feedback rules (9) and (10), the Fisher relationship (11) holds, $P_t = S_t P^*_t$ for every t, and the state-contingent rules characterizing the current planner's choices coincide with the conjectured functions describing the choices made by future planners.

In the Appendix we show how this equilibrium can be recast in a recursive form and solved using numerical methods.

3 Quantitative analysis

In this section we study the model's quantitative implications using numerical simulations. Our major goal is to check whether the debt limit channel is quantitatively important, i.e. if loans in foreign currency can emerge in the model as an equilibrium outcome, for a realistic calibration that takes into account relatively high exchange rate volatility in CEE economies. We also use the simulations to highlight some qualitative predictions of the model, which can be tested either directly against the data, or against the existing empirical evidence documented in the previous literature. Finally, we discuss the possible impact of some simplifying assumptions that we made to keep the model small so that it can be solved using global methods.

3.1 The rise and fall of foreign currency mortgages in CEE

Before presenting the model simulations and comparing them to the data, it is instructive to discuss how foreign currency mortgages emerged and evolved in the CEE region over time. To this end, in the first panel of Figure 1 we present the data on their share in total loans extended to households for house purchases in three economies: Czechia, Hungary and Poland.[7] Apart from data availability considerations, the reason for picking these three countries is that, while similar in many structural aspects, they differ significantly in the average level of the nominal interest rate differential with the euro area, which we plot in the second panel. These economies also represent interestingly different cases of regulatory approaches to foreign currency lending.

In Czechia, foreign currency mortgages have never been really popular, and their share in mortgage debt exceeded 2% only in the 1990s, when Czech interest rates were much higher than in more developed European countries. Since 2001, the interest rate differential in this economy has been very close to zero on average, and foreign currency borrowing for housing purposes virtually non-existent.

In contrast, nominal interest rates in the other two CEE countries have been consistently higher than in the euro area (and even more so than in Switzerland), and dollarization of mortgages proceeded quickly when borrowing in foreign currency loans became easily available. In Poland, foreign currency denomination accounted for about 10% of mortgage debt until 2000, when their popularity started to grow dramatically, reaching 60% in just three years. The foreign currency share fluctuated around this value until the global financial crisis in 2008, when the supply of new loans denominated in Swiss francs dried up. This, together with regulatory steps taken by the Polish financial supervision (and in particular Recommendation S issued in 2013), effectively eliminated foreign currency mortgage originations. As a result, housing debt dollarization fell to about 30% by 2018, and is projected to continue the downward trend as households keep repaying foreign currency loans taken when they were easily available.

The boom in foreign currency borrowing in Hungary started about four years later than in

[7]For now, we use the data on outstanding loans rather than originations as the former are available for a longer span of time. Obviously, exchange rate movements directly affect the thus measured degree of dollarization, but they do not have a significant effect on trends, which are the main focus in this section.

Poland, but also very quickly elevated the foreign currency share in total loans to about 60%. However, the demise of this type of debt after the global financial crisis was much faster as the Hungarian authorities targeted not just mortgage originations, but also facilitated faster debt repayment in 2011-2012, and eventually obliged all banks to convert foreign currency mortgage loans into the forint in 2015.

It is important to note that all three economies, and Poland and Hungary in particular, follow independent monetary policy with freely floating exchange rates. All this narrative evidence hence suggests the existence of a powerful mechanism, inclining households to accept a riskier contract denominated in foreign currency whenever the local nominal interest rate is significantly higher than abroad. Another important observation is that, during the period of fast accumulation of foreign currency mortgages in Poland, the Polish zloty was expected to depreciate. According to the Reuters FX polls, conducted monthly from January 2000 to March 2003, the average 1-year ahead expected depreciation of the Polish zloty against the euro was 8%. During the same period, the average interest rate differential vis-a-vis the euro area was 10 percentage points, and the share of foreign currency mortgages increased by 50.5 percentage points. These facts are consistent with the debt limit channel being the primary driver of household debt dollarization in Poland.[8]

3.2 Calibration

The goal of our quantitative evaluation is to examine not only the degree of mortgage debt dollarization, but also its fluctuations over time. With this objective, Poland emerges as the only CEE economy that can serve as a reasonable empirical benchmark as only for this country foreign currency borrowing for housing purposes was popular and available at non-discriminatory terms for a decently long period of time. Given the discussion in the previous section, we calibrate our model to Poland, using data for the period 1998q1-2008q2. The unit of time is one quarter. We follow the standard practice of picking the structural parameter values either from the literature or to match some key proportions observed in the data. The shock processes are calibrated more directly using time series for the relevant variables.

The calibrated parameter values are reported in Table 3. We set r^* to 1.005, which implies the steady state value of the world real interest rate of 2% annually. The discount

[8]Similar FX polls for Hungary are available only for the final phase of FCL accumulation, i.e. March-December 2008, over which its share increased by 9.5 percentage points, the average interest rate differential was 4.2 percentage points, and average expected 1-year ahead depreciation of the Hungarian forint against the euro was 10.2%.

parameter β is calibrated at 0.99, in line with what is typically assumed in the context of mortgage debt (Campbell and Hercowitz, 2009), and implying that the collateral constraint is binding (and the economy is net borrower) in the non-stochastic steady state. We calibrate the mortgage contract parameters to reflect the properties of a typical housing loan taken in Poland. This gives an LTV ratio $m = 0.85$ and the principal decay factor δ equal to 0.015. The latter is chosen such that it implies the Macaulay duration of a housing loan in our model's steady state of 10 years, as for a standard 25-year mortgage.

As in most of the business cycle literature with housing, we opt for a separable form of household utility function

$$u(c_t, h_t) = \log c_t + A_h \log h_t \tag{23}$$

where A_h is set to 0.09. This value, together with housing depreciation rate δ_h of 0.009 (3.5% annually), are chosen such that in the steady state the housing stock is equal to 1.3 of annual output, and the debt-to-output ratio is 0.5 (annualized). These two numbers correspond to the relevant long-term averages observed in the Polish economy. The fixed supply of housing $\bar{h}$ is used to normalize the steady state real house prices to unity.

For reasons explained before, one of the key steps in calibration of our model is to choose the steady state nominal interest rate differential, which by the Fisher equation (11) is determined by the steady state inflation differential or, for given foreign average inflation $\bar{\pi}^*$ that we set to a conventional value of 1.005 (2% annually), by domestic average inflation $\bar{\pi}$. We calibrate the latter at 1.012, which implies $R - R^* = 0.029$. This value is the average difference between the short-term (3-month money market) interest rates in Poland and the euro area over the period 2002q1-2008q2. The rationale for dropping the first four years from our sample while calculating this statistics is that they cover the period during which the nominal interest rates were exceptionally high as the Polish central bank followed a restrictive monetary policy aimed at completing the disinflation process and bringing inflation below 4%.[9]

Our model economy is driven by three stochastic shocks. Since the model is solved globally, we approximate them with discrete Markov processes using the Tauchen and Hussey (1991) method. For the endowment process y_t, we normalize its mean to unity and feed the discretization algorithm with the volatility and autocorrelation of the cyclical component of Polish GDP, obtained with the Hodrick-Prescott filter. To ease the computational burden

[9]Note that by dropping these initial years of high nominal interest rates in Poland acts against predicting a bias towards foreign currency in our model.

while solving the model, we allow for exogenous time variation in $\bar{\pi}^*_t$, but keep $\bar{\pi}_t$ fixed at $\bar{\pi}$. We also assume that $\bar{\pi}^*_t$ is i.i.d. as this shock is introduced to generate swings in the nominal exchange rate (or, as we argue below, in excess return on foreign currency), and these exhibit no inertia in the data.[10] As regards the assumption on fixed domestic inflation, it is motivated by the empirical observation that the return on domestic currency is much less volatile than that on foreign currency, and hence the excess return $r_{x,t}$ (which is the key determinant of the equilibrium portfolio) is dominated by the latter. Consistently with this logic, our discretization of $\bar{\pi}^*_t$ is based on the time series of excess return on the euro relative to the Polish zloty, using the same sample of data as while calculating the average nominal interest rate differential. We have also verified numerically in a simplified version of our model that allowing for fluctuations in both $\bar{\pi}_t$ and $\bar{\pi}^*_t$, and calibrating their volatility using the data on individual returns on the Polish zloty and the euro, leads to very similar conclusions on equilibrium portfolio choices as our specification using only $\bar{\pi}^*_t$ and data on excess returns. Finally, as in our dataset excess return on the foreign currency is only very weakly correlated with GDP, we model y_t and $\bar{\pi}^*_t$ as independent processes.

3.3 Equilibrium currency composition of loans

We solve for the decentralized and constrained-efficient equilibrium of the model using global numerical methods that we present in the Appendix. The obtained policy functions are next used to generate stochastic simulations, based on which we calculate the first and second moments of key endogenous variables. We simulate the model for 1000,000 periods, after running it for the initial 1,000 periods to make the moment estimates independent of the initial conditions. Table 4 presents the outcomes, which we use to explain some of the model mechanisms, but also later to test them against the data. The definitions of variables used to calculate data-based moments are presented in the Appendix.

The most important result is that the model can generate high debt dollarization in the decentralized equilibrium, thus confirming that the debt limit channel is quantitatively important and can bias borrowing by households towards foreign currency even under realistic exchange rate risk. The model also generates substantial variability of the composition of the loan portfolio. To understand the mechanisms underlying these choices, we plot in Figure 2 the policy function for the share of foreign currency loans, for three different realizations

[10] In our sample of data, the autocorrelation coefficient for the excess return on foreign currency is -0.15, which is statistically not different from zero.

of exogenous shocks. The first panel depicts the normal times, when both endowment and foreign inflation are at their mean levels, which also means that the nominal exchange rate has just depreciated as expected given the UIP. The second panel shows the policy function when foreign inflation turned out to be low, and hence the exchange rate depreciated by more than households had expected. The third panel considers the case of a negative endowment shock for average exchange rate depreciation.

The following observations can be made. Households' bias towards foreign currency is increasing in the level of previously accumulated debt. In particular, for very low (high) levels of indebtedness, households choose to borrow only in domestic (foreign) currency. This is because the higher the level of debt, the more resources need to be spent on servicing it, and hence the more financially constrained households are. The severity of the constraint is additionally magnified by relatively low house prices as households would like to reduce their housing stock to finance additional consumption, which turns out impossible as aggregate house supply is rigid. As explained before, tighter credit constraints make the debt limit channel stronger as borrowing more in foreign versus domestic currency can effectively relax these constraints, allowing households to keep more debt. The same considerations explain why agents tend to favor foreign currency more when hit by an adverse endowment or exchange rate shock. Because of no persistence, the effect of the latter is smaller and, obviously, present only if at least some debt was dollarized when the shock hit.

Turning to the degree of mortgage dollarization under constrained-efficient allocations chosen by the central planner, it turns out to be much lower than in the decentralized equilibrium. To understand this outcome, note that our model features a pecuniary externality associated with the presence of house prices in the collateral constraint. Agents do not internalize how their borrowing choices affect prices, and hence macroeconomic vulnerability. As it is known from the previous literature, this leads to overborrowing and, indeed, our model implies that the socially optimal level of debt is below the decentralized equilibrium. However, in our richer framework where households can choose not only the level of total debt, but also its currency composition, we additionally have a socially inefficient bias towards the foreign currency as agents fail to internalize how their portfolio choices make the economy more exposed to exchange rate risk. Therefore, the planner, who internalizes these general equilibrium effects, chooses not only lower total debt, but also its safer composition, i.e. a lower share of foreign currency. This positively affects household welfare as it increases the average level of consumption by 0.08% and reduces its standard deviation by 0.25%.

3.4 Model fit

An important element of establishing the empirical relevance of our theoretical framework is to compare its predictions directly to the data. To this end, we now discuss to what extent the decentralized equilibrium in our model produces realistic fluctuations in the foreign currency share and its comovement with other macrovariables. This discussion uses the second moments of Polish time series over the period when foreign currency mortgages were available at non-discriminatory terms. The definitions and sources of the data are described in the Appendix.

Comparing the first two columns in Table 4 leads to the following observations. The model does a very good job at matching the volatility of the foreign currency share in mortgages. It also implies substantial persistence of this proportion, even though it falls short of that observed in the data. As regards the comovement with key endogenous variables, the model correctly replicates all signs and correctly predicts nearly zero comovement of foreign currency loans share with excess return, but overemphasizes its negative correlation with output, consumption and house prices. One of the possible reasons reasons for this unsatisfactory fit is that our model generates too little variation in house prices, which is a common feature in this class of models unless one includes additional exogenous source of variation in this endogenous variable, see e.g. Iacoviello and Neri (2010) or Justiniano et al. (2015). Indeed, when we experimented with a model variant with stochastic housing weight in utility A_h, calibrating its process such that the model gets closer to replicating the inertia and volatility of house prices observed in the data, we could obtain a very good fit for the correlation of the foreign currency loan share with consumption and house prices, and also some improvement for that with output, without sacrificing the fit in other dimensions. However, such a model takes longer to solve, and its predictions about the degree of dollarization under both decentralized and constrained-efficient equilibrium barely differ from our baseline without housing preference shocks.

Overall, we conclude that our theoretical framework is consistent with the time series evidence on how the foreign currency share of mortgages behaves over the business cycle. Naturally, this conclusion is subject to a caveat that, due to data availability, our inference is based on a rather short sample of data for just one CEE economy.

3.5 Qualitative predictions

We complement the discussion from the previous section by comparing the model qualitative predictions with evidence documented in the previous empirical studies looking at mortgage debt dollarization in CEE economies. To this end, we rely on a meta-analysis on foreign currency loan determinants by Cuaresma et al. (2011), evidence based on household-level survey data discussed by Fidrmuc et al. (2013), and a more recent study based on country-level data by Skibinska (2018). The model predictions are constructed by comparing the equilibrium average share of foreign currency loans across different model parametrizations.

Table 5 summarizes those empirical findings that can be confronted with qualitative implications of our model. According to the data, debt dollarization is positively correlated with the interest rate differential, and, as explained before, this is the key factor biasing households' choices towards foreign currency in our framework. The empirically observed positive correlation with the level of inflation is also consistent with our model as the latter directly affects the average interest rate differential. Another key factor in our analysis was exchange rate volatility, which discouraged agents from taking foreign currency denominated debt, and this relationship can also be found in the data. For reasons explained before, in our baseline model we do not allow for domestic inflation volatility, but when we do by allowing for domestic inflation target shocks, we obtain an increase in dollarization of debt, consistently with the correlation found in the data. The intuition is that volatile home inflation increases riskiness of loans denominated in domestic currency – the covariance term in equation (20) becomes larger. Our model does not include remittances, but they are fairly easy to accommodate in our framework, at least if one treats them as exogenous.[11] When we add them to households' budget constraint as an additional endowment that is fixed in the foreign currency, the equilibrium foreign currency share in mortgage debt goes up, in line with the empirical relationship seen in the data. This outcome can be easily understood by referring to the natural hedging argument: foreign currency loans help insure against fluctuations in the home currency value of remittances.

The only empirical relationship documented in Table 5 that we do not qualitatively match with our model is the positive correlation between nominal interest rate volatility and foreign currency denomination of debt.[12] In our framework, this correlation is zero as

[11]Similarly, one could assume that an exogenous amount of household savings is held in foreign currency, thus capturing distrust in local currency as a savings vehicle. See Geng et al. (2018) for an analysis of this motive in CEE countries.

[12]It has to be noted, however, that this correlation does not appear to be strong in the data. According

the nominal interest rate is constant in equilibrium. Moreover, even if we did allow for movements in this variable, it would only affect households' portfolio choices through the impact on the nominal exchange rate via the UIP condition. This is because, in line with how adjustable rate mortgages are constructed in reality, the nominal interest due at the end of each period is known to agents at the period beginning and, not necessarily in line with typical contractual arrangements, households can freely convert their mortgages from one currency to the other. As a result, what matters to them while assessing the attractiveness of a domestic currency loan is the level of the nominal interest rate and inflation risk, the latter affecting the volatility of ex post return.

Finally, let us comment on the empirical relevance of the key prerequisites making the debt limit channel work. As we stated in the introduction, for this channel to bias household choices towards foreign currency, the following conditions must hold: (i) the nominal interest rate differential is positive, (ii) borrowers are credit constrained, (iii) mortgage debt is long-term, (iv) collateral constraint is established on newly purchased assets. We argue that all of them are easily met in those countries where foreign currency mortgages became popular. The first condition holds in all CEE economies, except for Czechia (as we have seen in Figure 1) and Slovakia. These two are the only countries in the region where foreign currency mortgages are virtually non-existent, see Table 2. As regards the second condition, Fidrmuc et al. (2013) document that foreign currency loans in the region were chosen mainly by young households, who are known to be relatively impatient (Read and Read, 2004). From a macroeconomic perspective, Falk et al. (2018) find that agents in the CEE economies are less patient than in more developed countries, whose currencies dominate in the world financial markets. Turning to the last two conditions, it is clear that mortgages are usually issued at longer maturities than other loans to households. It also has to be noted that home equity lines and multiple mortgages on a single property are not the dominant types of house financing in the CEE region, and hence mortgage loans are typically taken when a property is purchased. Therefore, according to our analysis, these last two features of mortgages may help explain why foreign currency became such a popular choice for this type of contracts, but not necessarily for consumer credit, see Table 2.

to Skibinska (2018), it amounts to just 0.21, which is only a half of the correlation coefficient between the share of foreign currency loans and inflation volatility.

3.6 Additional discussion of the model assumptions

Due to endogenous portfolio choice and occasionally binding constraints, the model needs to be solved with global methods, which imposes natural restrictions on its complexity. We now discuss the impact of several particular simplifying assumptions for the key model predictions. The presented arguments are additionally supported by the findings documented in the older version of this paper (Kolasa, 2018), which featured a fully-fledged DSGE extension of the model in Section 2. That richer model was solved with perturbation methods and required some simplifying assumptions for it to be feasible,[13] but can still serve as a useful reference for a discussion offered here.

One of the important model assumptions is that mortgage contracts are offered by risk-neutral foreign agents, which implies that the equilibrium share of foreign currency mortgages reflects risk considerations of only domestic households. In a richer setup, where foreign investors are risk averse, they would perceive mortgages denominated in home country's currency as more risky, especially if one takes into account that the emerging market exchange rates tend to depreciate in bad times for the global economy, i.e. when marginal utility is high. This means that our assumption of risk-neutrality of international investors actually acts against obtaining a bias towards foreign currency in the model.

A related concern is related to the UIP relationship holding exactly in our model while it is well known to fail in the data. A natural extension could be then to allow for time-varying risk premia in this condition. However, our experiments with a richer model reveal that, for the determination of the equilibrium share of foreign currency mortgages, UIP shocks essentially play the same role as foreign inflation shocks included in the current model. Hence, it is the exchange rate volatility that is key for the model predictions, not whether it is driven by foreign inflation or UIP disturbances, and we capture this volatility in our baseline setup by calibrating the variance of foreign inflation shocks.

The final point that we discuss here is the determination of the nominal interest rate and its impact on allocations and prices, including the nominal exchange rate. The asset market segmentation and a simple monetary feedback rule assumed in our model essentially rule out the response of the home country's nominal interest rate to domestic conditions. As a result, the nominal exchange rate is exogenous to the decisions made by home households.

[13] We used the perturbation methods to characterize the stationary solution to the portfolio problem as in Devereux and Sutherland (2011). To do it, we had to assume that the collateral constraint is always binding and that the certainty equivalence effects of the debt limit channel are eliminated.

Again, our experiments with a richer model that features a Taylor-like monetary policy rule and sticky prices reveal that allowing for these New Keynesian features might have some impact on the equilibrium share of foreign currency loans. For instance, it becomes larger when responsiveness of the central bank to inflation is less aggressive. However, again, what is key for the outcomes is the exchange rate volatility. Keeping its magnitude constant by appropriate adjustments in the variance of UIP or foreign inflation shocks, the equilibrium bias towards foreign currency is not largely affected by reasonable modifications to the monetary policy rule.

4 Conclusions

In this paper we have offered a novel explanation of why foreign currency loans to households have become so popular in some CEE countries. Our argument was based on what we called the debt limit channel, which arises when loans are multi-period and collateralized on newly purchased assets. This channel makes borrowing in domestic and foreign currency imperfect substitutes even under certainty equivalence. We showed that offering these two types of loans at apparently equal terms (i.e. at the same LTV ratio and with the same distribution of principal repayments) biases borrowers' portfolio choices towards foreign currency if domestic interest rates are higher than abroad as doing so allows them to backload the real repayments and hence effectively increase the limit of debt imposed by the collateral constraint. Using a quantitative structural framework, we verified that this bias is empirically relevant and survives also in the presence of realistic exchange rate risk, which acts in the opposite direction.

We also showed that, if credit conditions depend on collateral prices, dollarization arising from the debt limit channel in a decentralized equilibrium is socially inefficient, and a social planner who internalizes this pecuniary externality would choose a much lower share of foreign currency debt in households' portfolio. As our model's quantitative predictions suggest, the reduction in dollarization resulting from implementing the socially optimal policy can be sizable, to the extent suggesting that banning foreign currency mortgages completely, as it was done by the authorities in several CEE countries, can be a fairly reasonable substitute for more sophisticated regulation that takes the debt limit channel explicitly into account.

To focus on the working of the debt limit channel, we abstracted away from several potentially relevant features of the mortgage market, including the possibility of default.[14] This choice is additionally motivated by the fact that, at least in Poland, debt write-offs due to default on housing loans are very low, averaging to merely 0.35% of outstanding mortgage debt over the period 2009-2014 (NBP, 2015). Importantly, this ratio has never exceeded 0.5%, despite massive depreciation of the Polish zloty against the Swiss frank (in which most foreign currency mortgages in Poland are denominated) during this period. This is not so surprising given that, unlike in the US, mortgages in the CEE region are recourse loans. However, credit risk may be more important in other countries, with different contractual arrangements, so extending our analysis in this direction is a promising research avenue. In particular, some interesting outcomes could arise from using shadow debt prices that capture the risk of default in the collateral constraint as a form of discrimination between domestic and foreign currency mortgages. This extension may be particularly important if one tries to adapt our analysis to firms, where long-term loans for purchases of new assets are prevalent and their currency composition has not yet been fully understood, especially from the debt limit channel perspective.

[14] See Elenev et al. (2016) for a recent general equilibrium model that allows households to default on their mortgage debt.

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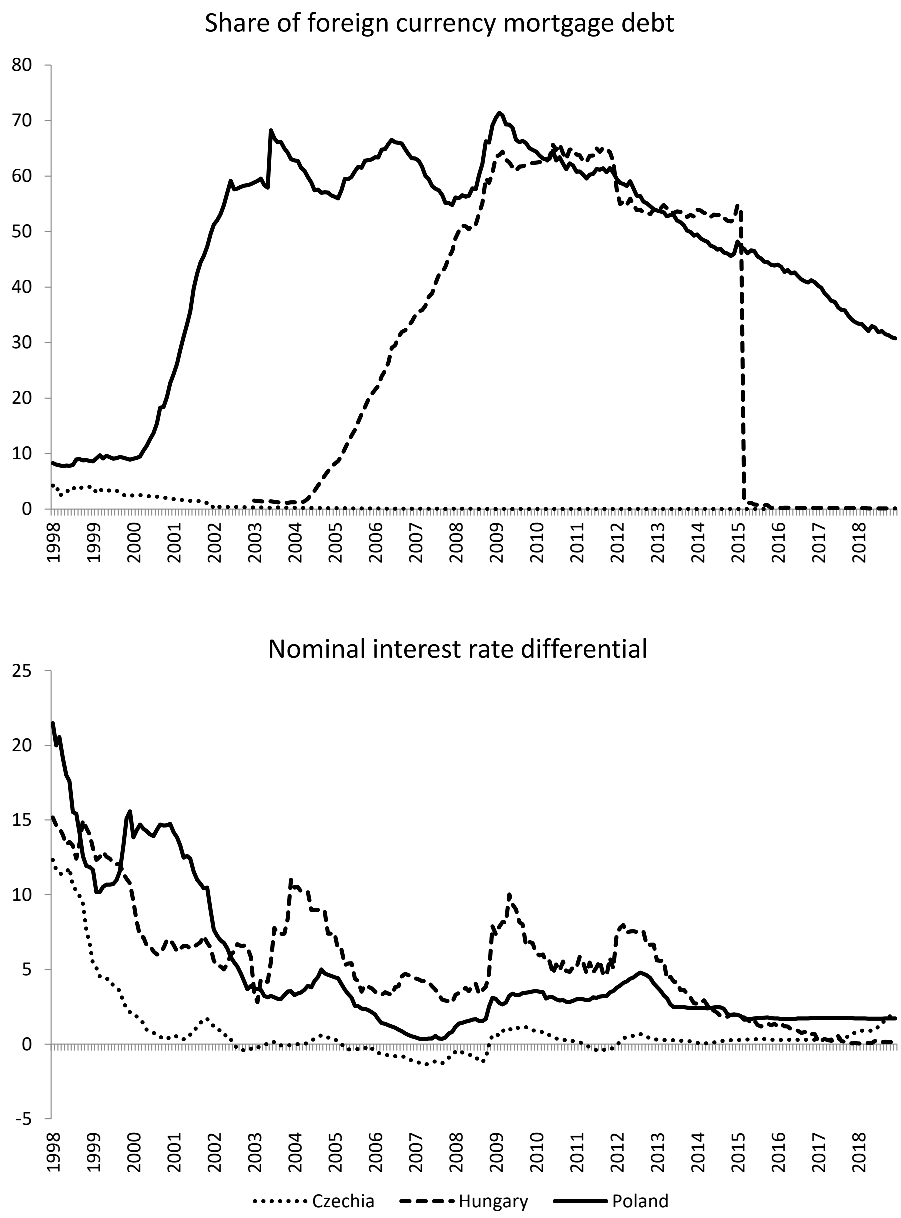
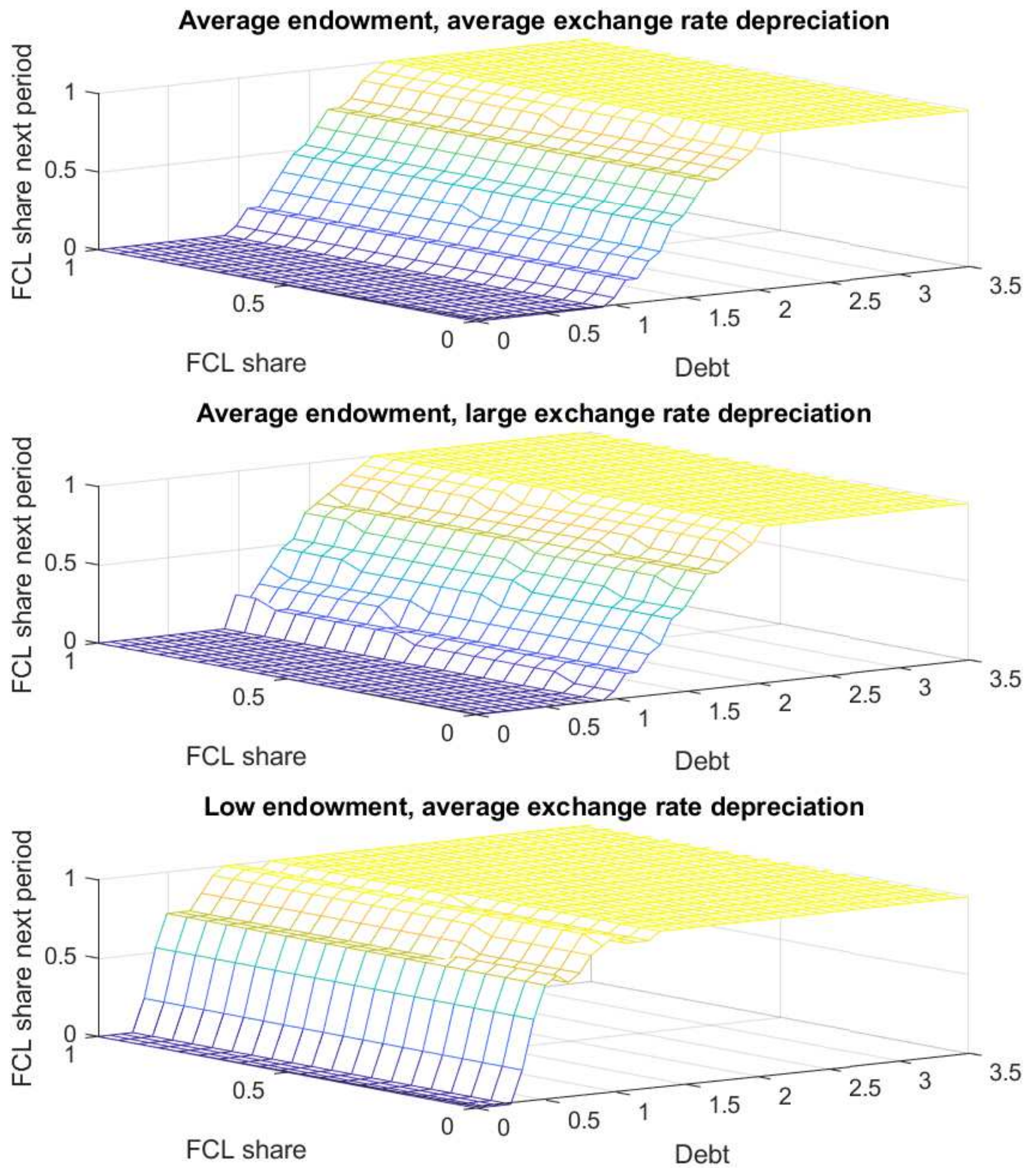
Tables and figures

Figure 1: Foreign currency loans and interest rate differential in CEE



Notes: The first panel presents the share of foreign currency loans in total debt for housing purchase (source: central banks of Czechia, Hungary and Poland). The second panel plots the difference between the 3-month money market rate in the three CEE countries and the corresponding interest rate in the euro area (source: Eurostat).

Figure 2: Policy functions in decentralized equilibrium



Notes: The figure presents household portfolio choices as a function of current level and composition of debt, for three different realizations of exogenous processes.

Table 1: Share of foreign currency debt in total outstanding debt

Region	Number of countries	Foreign currency loans (% of total)
All	66	29.4
Euro area	14	16.2
Non-euro area CEE	12	49.4
Latin America	5	26.0
Africa	13	20.7
Asia	14	34.4

Notes: This table presents foreign currency and foreign-currency-linked part of gross loans to residents and nonresidents as a percentage share of total gross loans, taken from the IMF Financial Soundness Indicators (FSIs) database. The numbers are for 2014, except for Uruguay and Zambia, for which 2013 data are used.

Table 2: Share of foreign currency debt in CEE countries by institutional sector

Country (year)	Total private non-financial	Non-financial corporations	Households	Households (for housing)
Non-euro area members (2014)				
Bulgaria	58.9	70.6	37.0	53.9
Croatia	71.4	68.8	73.2	92.6
Czech Republic	9.0	21.4	0.2	0.0
Hungary	51.9	50.1	53.6	52.9
Lithuania	72.2	75.1	69.3	.
Poland	28.8	26.1	30.3	46.3
Romania	58.3	53.6	63.3	85.1
Euro area members prior to euro adoption				
Estonia (2010)	88.5	92.0	85.2	93.9
Latvia (2013)	86.6	84.7	88.8	.
Slovakia (2008)	19.0	32.2	2.7	2.9
Slovenia (2006)	59.4	66.2	42.8	68.6

Notes: The percentage shares of foreign currency outstanding debt presented in the table are based on the official statistics published by each country's central bank. For non-euro area countries the shares are for 2014, while for the euro area members they cover the year prior to euro adoption.

Table 3: Model calibration

Parameter	Value	Description
		Structural parameters
r^*	1.005	World real interest rate
β	0.99	Discount factor
$\bar{\pi}^*$	1.005	Steady-state inflation abroad
$\bar{\pi}$	1.012	Steady-state inflation at home
δ	0.015	Loan decay parameter
m	0.85	LTV ratio on mortgage originations
A_h	0.09	Weight of housing in utility
δ_h	0.009	Housing depreciation rate
$\bar{h}$	5.24	Steady-state housing stock
		Shock processes for discretization
std(y)	0.007	Standard dev. of endowment process
corr(y,y_{-1})	0.87	Autocorrelation of endowment process
std($\bar{\pi}$)	0.035	Standard deviation of foreign inflation target

Table 4: Model and data moments

Moment	Data	Decentralized	Constrained-efficient
mean(ω)	57.86	71.9	7.1
std(ω)	29.44	32.8	14.1
corr(ω,ω_{-1})	0.95	0.79	0.80
corr(ω,l)	0.06	0.41	0.58
corr(ω,c)	-0.32	-0.76	-0.83
corr(ω,p_h)	-0.04	-0.64	-0.70
corr(ω,y)	-0.33	-0.82	-0.55
corr(ω,r_x)	-0.04	-0.05	-0.02

Notes: The mean and standard deviation of the share of foreign currency loans are expressed in percent. The data-based moments are calculated for the period 1998q1-2008q2. See the Appendix for data definitions. The model-based moments are calculated by simulating the model for 1000,000 periods.

Table 5: Model predictions versus cross-country evidence from empirical studies

FCL dependence on:	Empirical studies	Model
Interest rate differential	+	+
Interest rate volatility	+	0
Inflation	+	+
Inflation volatility	+	+
Exchange rate volatility	-	-
Remittances balance	-	-

Notes: Based on empirical studies of Cuaresma et al. (2011), Fidrmuc et al. (2013) and Skibinska (2018).

Appendix

A.1 Recursive competitive equilibrium

To solve for the competitive equilibrium in our model economy described in section 2, it is convenient to formulate it in a recursive form.

To this end, we first solve for the nominal block, that is exogenous to the rest of the model, and express the constraints in real terms. While doing it, we assume that inflation target shocks are i.i.d., as in our quantitative analysis presented in section 3. Note that, under this assumption, monetary policy rules (9) and (10) together with UIP equation (1) and constant real interest rate assumption (11) imply $\pi_t = \bar{\pi}_t$ and $\pi_t^* = \bar{\pi}_t^*$, which further leads to $R_t = r^*\bar{\pi}$ and $R_t^* = r^*\bar{\pi}^*$.

From now on, to suppress notation, we follow the standard convention in the recursive equilibrium literature of dropping time subscripts and using primes to denote next period realizations. We also collect shocks (exogenous states) in vector $s = \{y, \bar{\pi}, \bar{\pi}^*\}$ and separate debt positions l_H and l_F that are controlled by individual households from the corresponding economy-wide aggregates $\tilde{l}_H$ and $\tilde{l}_F$ that determine equilibrium house prices.

The recursive optimization problem faced by agents in our model economy can be written as

$$V(l_H, l_F, h, \tilde{l}_H, \tilde{l}_F, s) = \max_{c, l_H', l_F', h'} \left\{ u(c, h') + \beta \mathbb{E}_{s'|s} V(l_H', l_F', h', \tilde{l}_H', \tilde{l}_F', s') \right\} \tag{A.1}$$

subject to

$$c + p_h[h' - (1-\delta_h)h] + \frac{R}{\bar{\pi}} l_H + \frac{R^*}{\bar{\pi}^*} l_F = y + l_H' + l_F'$$

$$l_H' - (1-\delta)\frac{l_H}{\bar{\pi}} + l_F' - (1-\delta)\frac{l_F}{\bar{\pi}^*} \leq m p_h[h' - (1-\delta_h)h]$$

$$l_H' \geq 0$$

$$l_F' \geq 0$$

$$\tilde{l}_H' = \Gamma_H(\tilde{l}_H, \tilde{l}_F, s)$$

$$\tilde{l}_F' = \Gamma_F(\tilde{l}_H, \tilde{l}_F, s)$$

where Γ_H and Γ_F are functions describing agents' perceptions about how future aggregate endogenous states depend on current (endogenous and exogenous) aggregate states. Based on these perceptions, agents form expectations about house prices as they know how these depend on aggregate states. Note that housing is an individual state, but not an aggregate one as its supply is fixed.

The solution to this problem is a set of individual decision rules for c, $l_H^{'}$, $l_F^{'}$, $h^{'}$ that depend on individual and aggregate states. In general equilibrium, these individual choices and the perceptions that they are based on must coincide with economy-wide aggregates. Formally, and after collecting individual states into $z = \{l_H, l_F, h\}$ and aggregate endogenous states into $\tilde{z} = \{\tilde{l}_H, \tilde{l}_F\}$, we have the following definition.

Definition 1. A recursive competitive equilibrium is defined by value function $V(z, \tilde{z}, s)$, an associated set of individual decision rules $c(z, \tilde{z}, s)$, $l_H^{'}(z, \tilde{z}, s)$, $l_F^{'}(z, \tilde{z}, s)$, $h^{'}(z, \tilde{z}, s)$, the house pricing function $p_h(\tilde{z}, s)$, and the perceived laws of motion for domestic and foreign currency aggregate debt $\Gamma_H(\tilde{z}, s)$ and $\Gamma_H(\tilde{z}, s)$, such that:

1. $V(z, \tilde{z}, s)$, $c(z, \tilde{z}, s)$, $l_H^{'}(z, \tilde{z}, s)$, $l_F^{'}(z, \tilde{z}, s)$, $h^{'}(z, \tilde{z}, s)$ solve the recursive optimization problem (A.1) for given $p_h(\tilde{z}, s)$, $\tilde{l}_H^{'}(\tilde{z}, s)$ and $\tilde{l}_F^{'}(\tilde{z}, s)$.

2. Housing market clears: $h^{'}(z, \tilde{z}, s) = \bar{h}$.

3. The perceived aggregate laws of motion are consistent with individual choices: $\Gamma_H(\tilde{z}, s) = l_H^{'}(z, \tilde{z}, s)$ and $\Gamma_F(\tilde{z}, s) = l_F^{'}(z, \tilde{z}, s)$.

A.2 Recursive constrained-efficient equilibrium

While formulating the social planner's problem in a recursive form, unless indicated otherwise, we follow the notational convention introduced while defining the decentralized equilibrium and use the solution to the exogenous nominal block as in the previous section. As explained in section 2.5, in our formulation of constrained-efficient equilibrium, the social planner maximizes the utility of a representative household, subject to resource, collateral and implementability constraints, the last given by the housing Euler equation of the competitive equilibrium. While performing optimization, the planner chooses current allocations and prices, taking the decisions rules of future planners as given. The planner's decision rules depend on endogenous states $z = \{l_H, l_F\}$, which this time does not include housing as the social planner realizes it is in fixed supply, and exogenous shocks collected in s as before.

The recursive optimization problem faced by the planner can be written as

$$V(l_H, l_F, s) = \max_{c, l_H^{'}, l_F^{'}, p_h, \Theta} \left\{ u\left(c, \bar{h}\right) + \beta \mathbb{E}_{s^{'}|s} V(l_H^{'}, l_F^{'}, s^{'}) \right\} \tag{A.2}$$

subject to

$$c + \delta_h p_h \bar{h} + \frac{R}{\bar{\pi}} l_H + \frac{R^*}{\bar{\pi}^*} l_F = y + l_H^{'} + l_F^{'}$$

$$l_H^{'} - (1-\delta)\frac{l_H}{\bar{\pi}} + l_F^{'} - (1-\delta)\frac{l_F}{\bar{\pi}^*} \leq m\delta_h p_h \bar{h}$$

$$p_h \left(u_c(c,\bar{h}) - m\Theta\right) = u_h(c,\bar{h}) + \beta(1-\delta_h)\mathbb{E}_t\left\{\tilde{p}_h(z^{'},s^{'})\left(u_c\left(\tilde{c}(z^{'},s^{'}),\bar{h}\right) - m\tilde{\Theta}(z^{'},s^{'})\right)\right\}$$

$$l_H^{'} \geq 0$$

$$l_F^{'} \geq 0$$

$$\Theta \geq 0$$

$$\left(l_H^{'} - (1-\delta)\frac{l_H}{\bar{\pi}} + l_F^{'} - (1-\delta)\frac{l_F}{\bar{\pi}^*} - m\delta_h p_h \bar{h}\right)\Theta = 0$$

where the last two constraints are the complementary slackness conditions associated with the collateral constraint faced by households in the decentralized equilibrium,[15] and where the decision rules of future social planners (taken as given in optimization) are indicated with a tilde.

Given this recursive formulation of the planner's problem, we can define the constrained-efficient Markov stationary equilibrium as follows.[16]

Definition 2. A recursive time-consistent constrained-efficient equilibrium is defined by value function $V(z,s)$, an associated set of policy functions $c(z,s)$, $l_H^{'}(z,s)$, $l_F^{'}(z,s)$, $p_h(z,s)$, $\Theta(z,s)$, as well as the corresponding policy functions of future planners $\tilde{c}(z,s)$, $\tilde{l}_H^{'}(z,s)$, $\tilde{l}_F^{'}(z,s)$, $\tilde{p}_h(z,s)$, $\tilde{\Theta}(z,s)$, such that:

1. $V(z,s)$, $c(z,s)$, $l_H^{'}(z,s)$, $l_F^{'}(z,s)$, $p_h(z,s)$, $\Theta(z,s)$ solve the planner's optimization problem (A.2) for given $\tilde{c}(z,s)$, $\tilde{l}_H^{'}(z,s)$, $\tilde{l}_F^{'}(z,s)$, $\tilde{p}_h(z,s)$, $\tilde{\Theta}(z,s)$.

2. The policy functions characterizing future planners' decisions match the decision rules solving the current planner's problem: $\tilde{c}(z,s) = c(z,s)$, $\tilde{l}_H^{'}(z,s) = l_H^{'}(z,s)$, $\tilde{l}_F^{'}(z,s) = l_F^{'}(z,s)$, $\tilde{p}_h(z,s) = p_h(z,s)$, $\tilde{\Theta}(z,s) = \Theta(z,s)$.

[15]The Lagrange multiplier on the collateral constraint from the decentralized equilibrium Θ, i.e. a shadow value of relaxing the constraint for individual households, should not be confused with the multiplier on the collateral constraint from the planner's perspective. See Bianchi and Mendoza (2018) for a formal analysis of the relationship linking the two multipliers in a simpler but related setup.

[16]For notational ease, we use the same letters to denote the policy functions solving the social planner's problem as we did while defining the decentralized equilibrium in the previous section. Naturally, these pairs of objects are not the same.

A.3 Computational algorithms

To compute the decentralized and constrained-efficient equilibrium, it is convenient to reformulate the households' and planner's problems such that they are defined in terms of total debt $l = l_H + l_F$ and its foreign currency share $\omega = l_F/l$ rather than in terms of domestic and foreign currency debt holdings l_H and l_F. Since there is a one-to-one relationship between the pairs $\{l_H, l_F\}$ and $\{l, \omega\}$, this reformulation is inconsequential for equilibrium characteristics.

The budget and collateral constraints can now be rewritten as

$$c + p_h[h^{'} - (1-\delta_h)h] + \left[\frac{R}{\bar{\pi}}(1-\omega) + \frac{R^*}{\bar{\pi}^*}\omega\right] l = y + l^{'}$$

$$l^{'} - (1-\delta)\left[\frac{1-\omega}{\bar{\pi}} + \frac{\omega}{\bar{\pi}^*}\right] l \leq m p_h[h^{'} - (1-\delta_h)h]$$

and the non-negativity constraints on debt holdings in individual currencies become

$$l \geq 0$$

$$0 \leq \omega \leq 1$$

A.3.1 Computing decentralized equilibrium

To compute the decentralized equilibrium, we solve households' problem using a mix of value and policy function iteration, iterating over house prices (that agents take as given) in the outer loop. The algorithm takes into account that the collateral constraint may be occasionally binding.[17]

To describe the solution method, we follow the convention established in section A.1, except that, to further ease notation, we will no longer distinguish explicitly between individual and aggregate states as they need to be the same in equilibrium, so that $z = \{l, \omega\}$. The numerical algorithm proceeds as follows.

1. Choose a grid for endogenous states z and the state space for exogenous states s together with the associated matrix describing the probability of transition from s to

[17] One of the complications in our model is that, due to multi-period debt, the equilibrium conditions characterizing decentralized allocations feature not only the current, but also the future Lagrange multiplier on the collateral constraint, see e.g. the house pricing equation (22). Together with the occasionally binding nature of the constraint, this precludes us from using the time iteration algorithm as in Bianchi (2011), which is normally much faster than methods based on value function iteration.

s'.

2. Guess the pricing function $p_h(z,s)$.

3. Solve the household problem (A.1) with value function iteration, constraining $h' = \bar{h}$ as implied by the housing market clearing condition.[18] This produces time-invariant policy functions $c(z,s)$, $l'(z,s)$ and $\omega'(z,s)$. Mark those states that result in $l' = 0$, as well as those resulting in either $\omega' = 0$ or $\omega' = 1$.

4. Calculate the tightness of the collateral constraint consistent with this solution

   (a) Set the initial guess of policy function $\Theta(z,\tilde{z},s)$ to zero for all states.

   (b) For all states but those implying $l' = 0$, iterate $\Theta(z,s)$ to convergence using Euler equation (18)

   $$\Theta(z,s) = u_c\left(c(z,s),\bar{h}\right) - \beta \mathbb{E}_{s'|s}\left\{u_c\left(c(z',s'),\bar{h}\right)\frac{R}{\bar{\pi}'}\right\} + \beta(1-\delta)\mathbb{E}_{s'|s}\left\{\frac{\Theta(z',s')}{\bar{\pi}'}\right\}$$

   if $\omega' < 1$ (as it implies $\mu_H = 0$) or using equation (19)

   $$\Theta(z,s) = u_c\left(c(z,s),\bar{h}\right) - \beta \mathbb{E}_{s'|s}\left\{u_c\left(c(z',s'),\bar{h}\right)\frac{R^*}{(\bar{\pi}^*)'}\right\} + \beta(1-\delta)\mathbb{E}_{s'|s}\left\{\frac{\Theta(z',s')}{(\bar{\pi}^*)'}\right\}$$

   if $\omega' > 0$ (and $\mu_F = 0$).

5. Proceed similarly to calculate the house pricing function $p_h(z,s)$ that is consistent with these allocations by iterating to convergence housing Euler equation (22)

$$p_h(z,s) = \frac{u_h\left(c(z,s),\bar{h}\right) + \beta(1-\delta_h)\mathbb{E}_{s'|s}\left\{p_h(z',s')\left[u_c\left(c(z',s'),\bar{h}\right) - m\Theta(z',s')\right]\right\}}{u_c\left(c(z,s),\bar{h}\right) - m\Theta(z,s)}$$

6. Update the guess of the pricing function $p_h(z,s)$ and repeat steps 3-5 until convergence.

To run the algorithm, we choose 50 points on the grid for l and 20 points for ω, which gives in total 1000 grid points. Increasing the number of grid points did not produce significantly different results. Optimization at each grid point during the value function iteration stage

[18] Note that, while housing is a choice variable from an individual household's perspective, fixing it at this stage is innocuous if policy functions converge as $h' = \bar{h}$ will become the optimal household choice given the prices.

uses Matlab function `fmincon` that can handle inequality constraints. We use linear interpolation while evaluating the value function for l' lying outside the grid for l, but force ω' to lie on the grid for ω. Allowing for interpolation for ω' resulted in numerically unstable results as the outcomes of optimization produced by `fmincon` was very sensitive to starting values. The shock state space and probability transition matrix is obtained using the Tauchen and Hussey (1991) algorithm, with 3 possible realizations for each shock, which gives 9 possible exogenous states in our baseline application with $s = \{y, \bar{\pi}^*\}$.

A.3.2 Computing constrained-efficient equilibrium

The constrained-efficient equilibrium is computed using value function iteration, with additional iteration over future planners' policy functions in the outer loop. The algorithm takes into account that the collateral constraint may be occasionally binding. To describe the solution method, we follow the convention established in section A.2, except that $z = \{l, \omega\}$. The numerical algorithm proceeds as follows.

1. Choose a grid for endogenous states z and the state space for exogenous states s together with the associated matrix describing the probability of transition from s to s'.

2. Guess the policy rules of future planners $\tilde{c}(z, s)$, $\tilde{l}'(z, s)$, $\tilde{\omega}'(z, s)$, $\tilde{p}_h(z, s)$, $\tilde{\Theta}(z, s)$.

3. Solve the social planner problem (A.2) with value function iteration. As a result, this step produces policy functions $c(z, s)$, $l'(z, s)$, $\omega'(z, s)$, $p_h(z, s)$, $\Theta(z, s)$.

4. Update the guess of future planners' policy functions $\tilde{c}(z, s)$, $\tilde{l}'(z, s)$, $\tilde{\omega}'(z, s)$, $\tilde{p}_h(z, s)$, $\tilde{\Theta}(z, s)$, and repeat step 3 until convergence.

We use the same discretization of the state space as while computing the decentralized equilibrium, applying linear interpolation only for l'. Optimization at each grid point during the value function iteration stage uses Matlab function `fmincon` that can handle inequality constraints.

A.4 Data

To calibrate and validate our model, we compiled the data from different sources. Their exact definitions and sources are:

- Interest rate -- short term (3-month money market) interest rate; Eurostat
- Nominal exchange rates -- bilateral exchange rates of the Polish zloty; Eurostat
- Output -- real gross domestic product at market prices, chain-linked volumes; Eurostat
- Consumption -- real household and NPISH final consumption expenditure, chain-linked volumes; Eurostat
- Inflation -- log-difference in all-items HICP
- Loans -- other monetary financial institutions (other MFIs) loans to households for house purchase, also by currency; Narodowy Bank Polski.
- House prices -- residential property prices of existing flats in big cities per square meter; Bank of International Settlements.

All series are quarterly. Except for the interest and exchange rates, all data are seasonally adjusted. When used in real terms, house prices and loans are deflated with HICP. Output, consumption, real loans and real house prices are logged and filtered with the Hodrick-Prescott filter, using 1600 as the smoothing parameter.

Our theoretical framework does not include any portfolio adjustment frictions, and in particular allows for currency conversion of loans at no cost. Therefore, the appropriate way of testing the model predictions about the currency composition of mortgages is to use data on originations rather than outstanding amounts. These can be obtained from the Polish Bank Association (ZBP), which publishes them in their AMRON-SARFiN reports, but the series go back only to 2006. Therefore, we additionally calculate the estimates of mortgage originations from stocks, using formulas (4), (5) and nominal exchange rate data. It turns out that the thus constructed series is very similar to the ZBP data over the period when the latter are available (the correlation coefficient is 0.95). Given this success, we use these estimates of the foreign currency loan share while testing our model against the data.