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Robust Optimal Macroprudential Policy

by Giselle Montamat and Francisco Roch

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I N T E R N A T I O N A L M O N E T A R Y F U N D

IMF Working Paper

Research Department

Robust Optimal Macroprudential Policy¹

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Abstract

We consider how fear of model misspecification on the part of the planner and/or the households affects welfare gains from optimal macroprudential taxes in an economy with occasionally binding collateral constraints as in Bianchi (2011). On the one hand, there exist welfare gains from internalizing how borrowing decisions in good times affect the value of collateral during a crisis. On the other hand, interventions by a robust planner that has in mind a model far from the true underlying distribution of shocks, can result in negligible welfare gains, or even losses. This is because a policy that is robust to misspecification, as in Hansen and Sargent (2011), is optimal under a "worst-case" scenario but not under alternative distributions of the state. A robust planner introduces taxes that are 5 percentage points higher but does not achieve a significant increase in welfare gains compared to a non-robust planner when the true underlying model is not the worst-case. If households also make choices that are robust to model misspecification, the gains are significantly reduced and a highly-robust planner "underborrows" and induces welfare losses. If, however, the worst-case scenario is indeed realized, then welfare gains are the largest possible.

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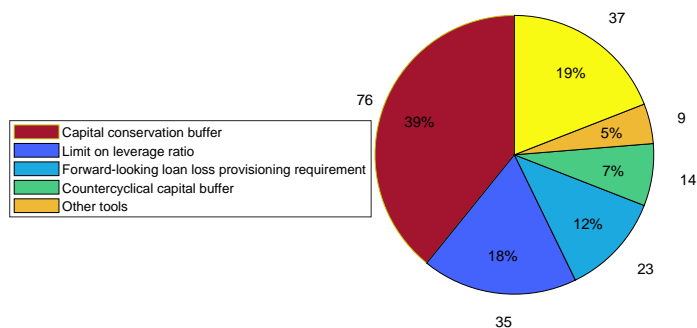
1 Introduction

Since the global financial crisis there has been a new push in favor of macroprudential regulation in order to safeguard financial stability both in advanced and emerging countries. Figure 1 provides information on macroprudential policy measures taken by IMF member countries from a survey conducted in 2018. It shows that most of these tools are aimed at managing liquidity and currency mismatches in the balance sheets of the banking sector, as well as restricting credit (e.g., capital buffers and loan to value caps).¹ The survey also reports that there are, on average, 9 macroprudential instruments in place per country (as of 2018). Following the seminal papers by Bianchi and Mendoza (2010) and Bianchi (2011), several studies have analyzed how different financial frictions can lead to inefficient overborrowing that justifies the implementation of these regulatory measures. Most of them report optimal macroprudential policies that are relatively small in size: the average tax on debt reported in the literature varies from 3.6 to 5 percent. Albeit small, these optimal taxes can almost eliminate the long-run probability of financial crises. However, the welfare gains are usually not too large and sometimes even ambiguous. Thus, much debate remains on whether prudential taxes should at all be imposed given the possibility that the model that regulators have in mind may not be an accurate characterization of the real world. Both in academic and policy circles, it is relevant to analyze how robust these optimal prudential taxes are to potential misspecifications of the model that governs the shocks to the economy.

This paper illustrates the importance of considering macroprudential policies that are robust to model misspecification (Hansen and Sargent, 2011). Specifically, we study how robust these policies are to the decision maker not being sure about the probabilities that are governing the evolution of shocks in the economy. Instead of considering just one probability distribution (one benchmark model), the planner will account for the fact that there is a set of distributions for shocks that could fit the data as well as the benchmark model, and should implement a policy that is optimal under the

¹BIS-FSB-IMF (2016) defines macroprudential policy as the use of primarily prudential tools to limit systemic risk which pursues three interlocking objectives: (i) contain build-up of cyclical vulnerabilities, (ii) limit structural vulnerabilities, and (iii) build resilience.

Figure 1: Number of Broad Based Tools



This figure shows the number of broad based tools reported by 141 countries in the Macprudential Policy Survey of 2018. Numbers denote frequency of measures reported; percentages denote the share among total measures reported.

worst case scenario (i.e., the policy that does not create vulnerabilities if one of the non-benchmark models turns out to be the true one²). In the words of Hansen and Sargent (VOX - 2019): “Prudent decision making should acknowledge what we do not know... Policy makers should strive to quantify the dimensions of their ignorance and adjust their decisions accordingly.”

We evaluate the robustness of macroprudential instruments by introducing fear of model misspecification into the canonical model of optimal macroprudential policies developed in Bianchi (2011). This paper rationalizes and justifies the need for macroprudential policy on the basis of a financial friction that generates sudden stops within the business cycle. This friction takes the form of an occasionally binding collateral constraint that generates a pecuniary externality. Specifically, the value of the collateral that constrains households’ borrowing depends on aggregate indebtedness. However, given that individuals are price takers, they fail to internalize how their borrowing decisions impact the price of the collateral. This externality leads to inefficient overborrowing. Instead, the planner internalizes the pecuniary externality and increases precautionary savings that can reduce both the magnitude and likelihood of a crisis when the collateral constraint becomes binding. Bianchi (2011) shows that the efficient allocation can be achieved through the implementation of

²The optimal policy under the worst-case scenario guarantees that one is not worse-off compared to the worst-case outcome if the actual probability distribution turns out to be another one. You are, however, worse-off than the outcome derived from the optimal policy under that true distribution.

macroprudential debt taxes.

Our contribution is to adapt this workhorse model to an environment in which there is a fear of model uncertainty from both the planner and/or households, following the framework in Hansen and Sargent (2011). We take the model in Bianchi (2011) as our benchmark model, in which both the planner and household make their optimal decisions under the true probability distribution. Under model misspecification, agents fear that the transition probabilities they are considering may not be the correct ones. Thus, they will now make their optimal decisions by considering a set of alternative probabilities that are not too far from the benchmark.³ Under robustness, agents make their optimal decisions as if an “evil” agent chooses the worst-case density among all those possible distributions surrounding the benchmark. Relative to previous studies that introduce model uncertainty, two subtleties arise in our application. First, the borrowing inefficiency generates a dislocation between the planner and households’ optimal policies. This implies that there are two dimensions of ambiguity that the planner faces: (i) uncertainty about the model that governs the evolution of the shocks, and (ii) uncertainty about what households believe about this distribution. Thus, we can distinguish between a “paternalistic” and “non-paternalistic” planner. The former will maximize welfare according to her own view of the world while the latter will find the optimal allocation by adopting the private sector’s beliefs. The second subtlety is related to how we measure welfare gains. The welfare analysis differs depending of what perspective we are focusing on (the planner’s, the household’s, the “true” underlying distribution) and whether the agents beliefs are aligned. When the views of the world differ between planner and household, or these are different than the true underlying model, intervention via macroprudential taxes can result in smaller welfare gains or even losses.

We find that introducing fear of misspecification leads the planner to increase savings, for precautionary motives, in every possible state of the world when the collateral constraint is not binding. The reason behind this result is that she is now assigning higher probabilities to worse shocks tomorrow relative to the case without robustness. We note, however, that these precautionary motives

³To discipline how far they can be from the benchmark distribution, Hansen and Sargent suggest using the notion of relative entropy.’

are weaker than those of a non-robust planner that instead features higher risk aversion. Decisions under robustness and under higher aversion are very similar close to the binding region because a robust planner is assigning a high probability to future negative shocks and thus behaves similarly to a high risk-averse agent. But the further away the economy is from this binding region, the worst-case scenario is a more favorable one and so a robust planner chooses levels of debt higher than those of a risk-averse agent who remains cautious.

In line with this, we also find that the optimal taxes over the region at which the collateral constraint is close to binding depend on the type of planner we consider. For instance, if we model a paternalistic planner who fears misspecification while households make their decisions under the benchmark model, then the difference between the planner and households' debt levels widens and leads to higher macroprudential taxes than in Bianchi (2011). For the purposes of correcting pecuniary externalities, taxes are needed when bond holdings are close to the region where the constraint is binding. In addition, for the purpose of saving as a precaution against the worst-case scenario in the future, this widening of the optimal bond holdings justifies the introduction of higher taxes to reduce households' borrowing over all of this non-binding region. In the long-run, a robust planner imposes taxes that are about 5 percentage points higher than a non-robust planner. These higher taxes, however, will not result in larger welfare gains if shocks are truly governed by the benchmark model. If households also make choices that are robust to model misspecification, though, the gains are significantly reduced. This is because the household already saves more as a precaution against future bad shocks, indirectly addressing the overborrowing problem due to the pecuniary externality. A highly-robust planner "underborrows" and induces welfare losses. However, if the worst-case scenario that the robust agents fear is the one that actually governs the evolution of shocks, then welfare gains are the largest possible. Finally, we find that the long run probability of a crisis is further reduced by a robust planner's allocation relative to what a non-robust planner can achieve because introducing model uncertainty leads to higher savings overall.

This paper relates mainly to two strands of the literature. First, following the seminal contribu-

tions of Mendoza (2010) and Bianchi (2011), a vast literature has emerged that rationalizes macroprudential policies as measures that address an externality that causes inefficient borrowing and quantifies the welfare gains from preemptive intervention. Relative to Bianchi (2011), Benigno et al (2013) analyze a production economy model in which the pecuniary externality arises because agents fail to internalize the effect of their decisions on the relative price of non-tradable goods. Schmitt-Grohe and Uribe (2017) characterize the optimal capital control policy in an economy with a collateral constraint in which both tradable and non-tradable goods have collateral value, and find that, contrary to the general wisdom, the optimal prudential policy is procyclical. Acosta et al (2019) extend the model of Bianchi (2011) by incorporating an Ss-type cost of implementing macroprudential taxes to generate “sticky” optimal capital controls. Andreasen et al (2019) study a general equilibrium model with heterogeneous firms and collateral constraints, and find that capital controls reduce aggregate production and investment while increasing exports, the share of exporters and TFP. Cuadra and Nuguer (2018) show that implementing macroprudential tools that target cross-border bank flows is welfare improving in emerging economies. Farhi and Werning (2016) build a small-scale model with nominal rigidities in which macroprudential policies correct for aggregate demand externalities. Basu et al (2020) develop a micro-founded model that characterizes optimal monetary policy, capital controls and foreign exchange intervention. While all these papers justify the use of macroprudential policies, they do not consider the robustness of the optimal policy (in the sense of Hansen and Sargent, 2011) which is the focus of our paper.

Second, our study relates to the literature on robust control methods pioneered by Hansen and Sargent (2011). A growing theoretical macro literature extends canonical models to the case in which the social planner and/or private agents fear model misspecification and search for robust policies under worst-case scenarios. Adam and Woodford (2012) introduce the robustness framework in a New Keynesian Model to analyze optimal robust monetary policy. Bidder and Smith (2012) develop an algorithm to apply robust control methods within nonlinear DSGE models, and show that the interaction between time-varying risk and robustness provides an amplification mechanism for

volatility shocks (which they interpret as animal spirits). Finally, Young (2012) is the paper closest to ours as they also study robust macroprudential policy. They consider a production economy with occasionally binding constraints as well. But we differ from them in two aspects: (i) they take into consideration policies that are not prudential in spirit, but rather they focus more on policies or interventions that occur at the time of a crisis (i.e., when the collateral constraint binds); and, (ii) we examine more carefully how to interpret results when we have planner or/and agents that actually disagree on the view of the world that they have. As discussed above, maybe the planner is a robust planner who fears misspecification and wants to obtain optimal policy taking into account that his model could be wrong, but agents do not or vice versa, or both do. We show that results depend on whose perspective we adopt when measuring welfare.

The rest of this paper proceeds as follows. First, Section 2 lays out the canonical model of pecuniary externalities developed by Bianchi (2011) and adds model misspecification following the approach of Hansen and Sargent (2011). Section 3 discusses how to obtain optimal taxes to decentralize the planner's allocation when household and planner fear misspecification and in the mixed cases when only one faces ambiguity. Section 4 presents the quantitative results. Finally, Section 5 concludes.

2 A model of occasionally binding constraints with robustness

Bianchi's 2011 seminal paper motivates macroprudential taxes as an optimal policy to address over-borrowing in the presence of collateral constraints. A representative agent of a small open endowment economy fails to internalize the effect of her debt choice on the price of the non-tradable good and therefore on the value of collateral (a pecuniary externality). As a consequence, the constraint on debt becomes binding and leads to a sudden stop with a current account reversal of the type that is observed in emerging countries. A planner, instead, is able to internalize the pecuniary externality and chooses a debt policy that significantly lowers the probability of the constraint binding, as well as the severity of a sudden stop when it does occur. Importantly, this policy can be decentralized by means of macroprudential taxes on debt.

Bianchi's model assumes that both planner and agent share a view of the world about the endowment processes and productivity shocks for tradable and non-tradable goods. In this paper, we explore the possibility that an agent (be that the planner and/or the representative household) may not have a unique prior over alternative distributions of the state variables implied by their benchmark model. In such a case, the agent might want to make a decision that would remain reasonable (though not necessarily optimal) under different possible scenarios for the evolution of the state variables. Hansen and Sargent (2011) provide a systematic framework to model this fear of model misspecification. In this framework, the robust agent considers the worst case within a set of possible distributions that are close enough to a benchmark model and finds the optimal policy under this scenario. While this robust policy would not be optimal if the benchmark model were indeed the true distribution, it is the optimal choice given the model ambiguity faced by the agent. That is, under a more favorable distribution of shocks within the set of plausible distributions, the welfare implied by the robust allocation would certainly improve; however, it would remain lower than welfare under the optimal allocation associated to that favorable distribution. The robust agent therefore ensures that her choice cannot leave her worse-off compared to the worst-case scenario if an alternative distribution were to be the true distribution governing the shocks (i.e., a robust allocation provides a

lower bound for welfare).

In this section, we explain how Bianchi's model of occasionally binding constraints can be extended to allow for robust choices using Hansen and Sargent's toolkit. We discuss how to interpret fear of misspecification and what it entails in terms of welfare gains when the household and the planner share different views of the world.

2.1 Modelling robustness

Bianchi's decentralized economy features the state variables $x_t = (b_t, B_t, y_{t-1}, \epsilon_t)$, where b_t is the household's level of debt, B_t is aggregate debt, $y_{t-1} = (y_{t-1}^T, y_{t-1}^N)$ is the endowment of tradable and non-tradable goods (all of these predetermined by x_{t-1} upon entering period t) and $\epsilon_t = (\epsilon_t^T, \epsilon_t^N)$ are the innovations (shocks to tradable and non-tradable endowments). An agent chooses consumption and debt holdings, $w_t = (c_t^T, c_t^N, b_{t+1})$, to maximize utility subject to a budget and a collateral constraint, and given the state at period t . The benchmark model of the household is characterized by all the equations that govern the evolution of these state variables. Lemma 1 summarizes these in a function $g(\cdot)$:

Lemma 1 *The equations that govern the evolution of state variables x_t are:*

- *The budget constraint:*

$$b_{t+1} + c_t^T + p_t^N c_t^N = b_t(1 + r) + y_t^T + p_t^N y_t^N \quad (1)$$

- *The collateral constraint:*

$$b_{t+1} \geq -(\kappa^N p_t^N y_t^N + \kappa^T y_t^T) \quad (2)$$

- *The evolution of aggregate bond holdings:*

$$B_{t+1} = \Gamma(B_t, y_t) \quad (3)$$

- *The price of non-tradables:*

$$p_t^N \text{ given } \forall t \quad (4)$$

- *The endowment process $y_t = (y_t^N, y_t^T)$:*

$$\log(y_t) = \rho \log(y_{t-1}) + \epsilon_t \quad \epsilon_{t+1} \sim N(0, V), \text{ i.i.d} \quad (5)$$

These contain all the information embedded in the benchmark model which can be summarized through the notation:

$$x_{t+1} = g(x_t, w_t, \epsilon_{t+1}) \quad (6)$$

$$\epsilon_{t+1} \sim N(0, V), \text{ i.i.d} \quad (7)$$

Together with the policy functions for consumption $c_t^T(x_t)$ and $c_t^N(x_t)$, and debt $b_{t+1}(x_t)$, the benchmark model implies a transition density for the state, $f(x_{t+1}|x_t)$. Under misspecification $\tilde{f}(x_{t+1}|x_t)$ can be any arbitrary conditional distribution that is not too far from the benchmark. Hansen and Sargent suggest using the conditional relative entropy as a measure of the distance between the benchmark transition density and the misspecification transition density:

$$I(f, \tilde{f}) = E_{\tilde{f}} \left[\log \left(\frac{\tilde{f}(x_{t+1}|x_t)}{f(x_{t+1}|x_t)} \right) \middle| x_t \right] \quad (8)$$

A robust household who fears misspecification will choose an optimal policy that maximizes utility given that shocks are governed by a worst-case transition density chosen by an “evil” agent.⁴

Proposition 1 formalizes this notion.

⁴A common metaphor is that agents choose their actions to maximize utility while a fictitious “evil” agent chooses a probability distribution to minimize that same utility.

Proposition 1 *A robust household maximizes utility under the “worst case” transition density for the state within the set of distorted models that an “evil” minimizing agent is allowed to consider:*

$$\max_{c_t^N, c_t^T, b_{t+1}} \min_{\tilde{f}} E_{0, \tilde{f}} \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right] \quad (9)$$

$$c_t \equiv [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{-1/\eta} ; \quad \eta > -1 \quad (10)$$

(b_0, B_0, y_0) given

$$(b_{t+1}, B_{t+1}, y_{t+1}) = g((b_t, B_t, y_t), (c_t^N, c_t^T, b_{t+1}), \epsilon_{t+1}) \quad (11)$$

$$\epsilon_{t+1} \text{ i.i.d} \sim N(0, V) \quad (12)$$

$$E_{0, \tilde{f}} \left[\sum_{t=0}^{\infty} \beta^{t+1} I(f, \tilde{f}) \right] \leq \omega \quad (13)$$

$$I(f, \tilde{f}) = E_{\tilde{f}} \left[\log \left(\frac{\tilde{f}(b_{t+1}, B_{t+1}, y_{t+1} | b_t, B_t, y_t)}{f(b_{t+1}, B_{t+1}, y_{t+1} | b_t, B_t, y_t)} \right) | b_t, B_t, y_t \right] \quad (14)$$

$$0 \leq \omega \leq \bar{\omega} \quad (15)$$

Condition 13 restricts the choice of possible worst-case \tilde{f} to be within a distance ω from the benchmark. An equivalent multiplier representation of this problem introduces the condition into the objective function:

$$\max_{c_t^N, c_t^T, b_{t+1}} \min_{\tilde{f}} E_{0, \tilde{f}} \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right] + \theta E_{0, \tilde{f}} \left[\sum_{t=0}^{\infty} \beta^{t+1} I(\tilde{f}, f) \right] \quad (16)$$

Where $\theta \in (\underline{\theta}, +\infty)$ regulates the degree to which the minimizing agent is penalized for looking at misspecifications that are too far from the benchmark model. The lower the θ , the lower the penalty, i.e., the more robustness to misspecification we allow. Below $\underline{\theta}$ the minimizing agent can achieve $-\infty$ no matter what the maximizing agent does, meaning that we allow for such a degree of misspecification that whatever the maximizing agent does, her utility is still $-\infty$. There is a direct mapping between $\underline{\theta}$ and $\bar{\omega}$.

To avoid having to compute expectations relative to a density \tilde{f} that we do not explicitly know,

Hansen and Sargent suggest using a martingale representation of the robust problem. We define a martingale $M_t = \frac{\tilde{f}(x^t)}{f(x^t)}$ that satisfies $E_f[M_{t+1}|x^t] = M_t$, where information at t , \mathbb{F}_t , is summed up in x^t . A corresponding martingale increment is defined by $m_{t+1} = \frac{M_{t+1}}{M_t}$ and satisfies $E_f[m_{t+1}|x^t] = 1$. Using these martingale and martingale increments, we are able to take expectations with respect to the misspecification model \tilde{f} by using the benchmark transition density f :

$$E_{\tilde{f}}[y_t|x_0] = E_f[M_t y_t|x_0] \quad (17)$$

$$E_{\tilde{f}}[y_{t+1}|x^t] = E_f[m_{t+1} y_{t+1}|x^t] \quad (18)$$

This martingale representation allows us to re-state the minimization problem of the “evil” agent as one in which she chooses m_{t+1} rather than \tilde{f} . The following proposition summarizes the problem that a robust agent solves, expressed in recursive form. We refer to the Appendix for the details of the derivations.

Proposition 2 *The problem of a robust household that fears misspecification can be expressed in recursive form as follows:*

$$V(b_t, B_t, y_t) = \max_{c_t^N, c_t^T, b_{t+1}} \frac{c_t^{1-\sigma}}{1-\sigma} - \beta \theta \log \left(E_{y_{t+1}|y_t} [e^{-V(b_{t+1}, B_{t+1}, y_{t+1})/\theta} | b_t, B_t, y_t] \right) \quad (19)$$

$$c_t \equiv [\omega(c_t^T)^{-\eta} + (1-\omega)(c_t^N)^{-\eta}]^{-1/\eta} ; \quad \eta > -1 \quad (20)$$

$$b_t, B_t, y_t \text{ given} \quad (21)$$

$$(b_{t+1}, B_{t+1}, y_{t+1}) = g((b_t, B_t, y_t), (c_t^N, c_t^T, b_{t+1}), \epsilon_{t+1}) \quad (22)$$

$$\epsilon_{t+1} \text{ i.i.d} \sim N(0, V) \quad (23)$$

$$\theta \in (\underline{\theta}, +\infty) \quad (24)$$

Notice that the difference between a household that does not fear misspecification and one that

does, boils down to the functional form of the Bellman equation. A non-robust agent considers:

$$V(B_t, y_t, b_t) = \max_{c_t^T, c_t^N, b_{t+1}} \frac{c(c_t)^{1-\sigma}}{1-\sigma} + \beta E_{y_{t+1}|y_t} [V(b_{t+1}, B_{t+1}, y_{t+1}) | b_t, B_t, y_t] \quad (25)$$

Once we have established the maximization problem of a robust household, a *recursive competitive equilibrium* is defined, as in Bianchi (2011), by: a price of non-tradables $p^N(B_t, y_t)$, a law of motion for aggregate bond holdings $B_{t+1} = \Gamma(B_t, y_t)$, and individual decision rules $b(B_t, y_t, b_t)$, $c^T(B_t, y_t, b_t)$, $c^N(B_t, y_t, b_t)$, with the associated value function $V(B_t, y_t, b_t)$, such that:

- Decision rules and value function solve the individual's recursive optimization problem 19.
- Markets clear:

$$y_t^N = c^N(B_t, y_t, b_t) \quad (26)$$

$$\Gamma(B_t, y_t) + c^T(B_t, y_t, b_t) = y_t^T + B_t(1+r) \quad (27)$$

- The perceived law of motion of aggregate bond holdings is the same as the actual law of motion induced by the agent's choice, and the price of non-tradables taken as given is the same as the one obtained from the optimal decisions:

$$B_{t+1} = \Gamma(B_t, y_t) = b(B_t, y_t, B_t) \quad (28)$$

$$p^N(B_t, y_t) = \frac{1-\omega}{\omega} \left(\frac{y_t^T + B_t(1+r) - \Gamma(B_t, y_t)}{y_t^N} \right)^{\eta+1} \quad (29)$$

A planner is constrained by the same credit limit of the households and it chooses the level of debt but lets the goods markets clear competitively (and this determines quantities and prices). The households receive the proceeds from the credit choices of the planner in a lump sum fashion. Formally, this means that the planner solves a utility maximization problem subject to the collateral constraint (2), resource constraints (i.e. market clearing (26)-(27)) and an implementability constraint (i.e. competitive prices (29)).

A *recursive constrained-efficient equilibrium* is defined by a price of non-tradables $p^N(B_t, y_t)$, the planner's law of motion for bond holdings $B(B_t, y_t)$ and its value function $V(B_t, y_t)$, and the household's decision rules $c^T(B_t, y_t)$, $c^N(B_t, y_t)$ -given the planner's decision for debt-, such that:

- Bond holdings, decision rules and value function solve the planner's recursive optimization problem. A robust planner considers:

$$V(B_t, y_t) = \max_{b_{t+1}, c_t^T} \frac{c(c_t^T, y^N(B_t, y_t))^{1-\sigma}}{1-\sigma} - \beta \theta \log \left(E_{y_{t+1}|y_t} [e^{-V(b_{t+1}, y_t)/\theta} | b_t, B_t, y_t] \right) \quad (30)$$

$$c(c_t^T, y_t^N) \equiv [\omega(c_t^T)^{-\eta} + (1-\omega)(y_t^N)^{-\eta}]^{-1/\eta} ; \quad \eta > -1 \quad (31)$$

$$B(B_t, y_t) + c^T(B_t, y_t) = B_t(1+r) + y_t^T \quad (32)$$

$$B(B_t, y_t) \geq -(\kappa^N p^N(B_t, y_t) y_t^N + \kappa^T y_t^T) \quad (33)$$

$$c_t \geq 0 \quad (34)$$

$$p^N(B_t, y_t) = \frac{1-\omega}{\omega} \left(\frac{y_t^T + B_t(1+r) - B_{t+1}}{y_t^N} \right)^{\eta+1} \quad (35)$$

Consumption of non-tradables is given by $c^N(B_t, y_t) = y_t^N$.

Bianchi's (2011) non-robust planner would instead feature the Bellman equation:

$$V(B_t, y_t) = \max_{B_{t+1}, c_t^T} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_{y_{t+1}|y_t} [V(B_{t+1}, y_{t+1})] \quad (36)$$

The key difference between the planner and the household is that the planner internalizes the effect that the choice of debt has on the price of non-tradables via the level of tradable consumption it permits. This implies that, for periods in which the constraint binds, the planner takes into account how an increasing level of debt (i.e., increasing consumption of tradables) impacts the value of collateral and, therefore, the degree to which the constraint binds.

This pecuniary externality, introduced by the presence of a credit constraint where the value of collateral depends on the household's debt decision, generates the key feature of a model with occasionally binding collateral constraints. Namely, an amplification mechanism that is triggered by

a negative shock when the constraint binds and that worsens the initial shock. For a given B_t , if there is a negative shock that determines a low realization of y_t^T and/or y_t^N , the collateral constraint might become binding. This means that, compared to a non-constrained case, borrowing will be lower and so tradable consumption will be lower; thus, the price of non-tradables will be lower, which tightens the constraint more and decreases borrowing further. At the time when the constraint binds, the collateral constraint can be relaxed by increasing tradable consumption because this increases the price of non-tradables.

This mechanism is what generates the result that, in bad times, borrowing decreases (i.e., capital inflows decrease and the current account increases). This effect is not present when the constraint does not bind: due to consumption smoothing, in bad times borrowing increases -capital inflows increase- and the current account decreases. A positive shock, on the other hand, simply relaxes the constraint further (in good times borrowing decreases -capital inflows decrease- and the current account increases). In this context, there is a role for macroprudential policies to reduce the probability and magnitude of sudden stops.

Introducing the possibility of either household and/or planner fearing model misspecification in the context of a market friction, adds a layer of decision in the modelling of their behavior. The fact that household and planner are different agents that arrive at different optimal choices implies that there is scope for modelling either one or both as fearing misspecification. Different cases entail different interpretations and we discuss these in the following section.

2.2 Interpreting robustness

In the absence of market inefficiencies, the welfare theorems apply and so the representative household's (robust) optimal allocation coincides with the (robust) social planner's. But when we introduce a friction like the collateral constraint in this problem, there is a dissociation between the social planner's and the private agent's optimal choices. In this context, the planner faces two dimensions of ambiguity: one with regard to the model that governs the evolution of the state; and another with

regard to the model that the households believe governs the evolution of the state. Whenever the two models of the world (the planner's and the household's) differ, we are left with a choice between two types of planners. A paternalistic one will maximize welfare according to her view of the world (she might fear misspecification or not). A non-paternalistic planner will find the optimal allocation from the perspective of the household. The planner's robust problem that we solve (37) -where all expectations are taken with respect to the misspecified model \tilde{f} - can either be interpreted as that of a paternalistic planner that fears misspecification, or of a non-paternalistic planner that believes the private sector fears misspecification and shares the same ambiguity⁵.

$$\max_{c_t^N, c_t^T, b_{t+1}} \min_{\tilde{f}} E_{0, \tilde{f}} \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right] + \theta E_{0, \tilde{f}} \left[\sum_{t=0}^{\infty} \beta^{t+1} E_{\tilde{f}} \left[\log \left(\frac{\tilde{f}}{f} \right) | b_t, B_t, y_t \right] \right] \quad (37)$$

A second thing to note is that, whenever the views of the world differ, we need to take a stance about what model (i.e., distribution of shocks) we will consider to measure welfare gains, whose computation involves expectations. This matters especially if we are trying to decide whether a planner's intervention is optimal or not; while the planner will address the externality that leads to overborrowing, she might solve for an optimal allocation under a view of the world that is incorrect and/or that the households do not share, and so it might be that under an alternative model, welfare does not improve or, at the very least, welfare gains are very small. We consider two possibilities:

1. We measure welfare from the perspective of the private agents. Table 1 considers the possible scenarios. Superscripts indicate whether the expected utility is evaluated at the decentralized competitive equilibrium (DE) or the planner's (SP) allocation, and the subscripts indicate whether it considers a worst-case distribution or not (note that this coincides with what the

⁵An alternative would be to consider, for example, a planner who operates under the benchmark (this can be interpreted as him knowing the "truth") but is concerned about distorted beliefs by the private sector (as in Adam and Woodford (2012)). Therefore, while expectations over discounted utility and penalties are taken with respect to the benchmark density, the measure of relative entropy takes into account the distorted density \tilde{f} .

$$\max_{c_t^N, c_t^T, b_{t+1}} \min_{\tilde{f}} E_{0, f} \left[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right] + \theta E_{0, f} \left[\sum_{t=0}^{\infty} \beta^{t+1} E_{\tilde{f}} \left[\log \left(\frac{\tilde{f}}{f} \right) | b_t, B_t, y_t \right] \right]$$

Additionally, one could allow for planner and household to have a different degree of ambiguity.

household believes). Specifically, the value functions must satisfy either the traditional or the modified Bellman equation, depending on what probabilities are used to compute expectations:

$$V_f^{(i;f^i)}(B, y, B) = \frac{\left(c^{(i;f^i)}(B, y, B)\right)^{1-\sigma}}{1-\sigma} + \beta E_{y'|y}[V_f^{(i;f^i)}(B', y', B')] \quad (38)$$

$$V_{\tilde{f}}^{(i;f^i)}(B, y, B) = \frac{\left(c^{(i;f^i)}(B, y, B)\right)^{1-\sigma}}{1-\sigma} - \beta \theta \log \left(E_{y'|y} \left[e^{-V_{\tilde{f}}^{(i;f^i)}(B', y', B')/\theta} \right] \right) \quad (39)$$

Where $f^i \in \{f, \tilde{f}\}$.

Table 1: Welfare measured from the household's perspective

	SP: Non-robust $\theta = \infty$	SP: Robust $\theta = \xi$
HH: Non-robust $\theta = \infty$	$V_f^{(DE;f)} < V_f^{(SP;f)}$	$V_f^{(DE;f)} \leq V_f^{(SP;\tilde{f})}$
HH: Robust $\theta = \xi$	$V_{\tilde{f}}^{(DE;\tilde{f})} \leq V_{\tilde{f}}^{(SP;f)}$	$V_{\tilde{f}}^{(DE;\tilde{f})} < V_{\tilde{f}}^{(SP;\tilde{f})}$

Note: this table considers welfare under the four possible combinations of robust/non-robust planner and household allocations. V measures welfare under the optimal allocation chosen by either the household (the decentralized equilibrium with no government intervention, DE) or the social planner (SP). The subscript indicates whether welfare is computed based on a series of shocks that follow the benchmark distribution f , or the worst-case distribution \tilde{f} . In this table, this always coincides with the household's view of the world. Welfare is maximized by the optimal allocation of a social planner that considers that same distribution; for any other case, it is not obvious whether a planner can increase welfare by internalizing the effect that debt has on the value of collateral. We assume that a robust household and a robust planner share the same degree of misspecification, $\theta = \xi$.

Under this paradigm, we can think of different reasons that justify looking for a planner's decision that is robust to misspecification:

- (a) We want to depart from the rational expectations theory. In a rational expectations model, agents do not have to worry about model misspecification. They can trust their model because subjective and objective probability distributions coincide. Each agent's model (her subjective probability distribution over exogenous and endogenous variables) is an equilibrium outcome, not something to be specified by the model builder. But, just as

econometricians face specification doubts, agents inside the model might too. A planner that faces fear of misspecification is therefore assuming that agents do not have rational expectations.

(b) We still trust rational expectations, but we make a distinction between the artificial agents within a rational expectations model that trust their model, and the model’s author that doubts it. This distinguishes the economist that writes the model from the agents that are portrayed in it. The planner is then the model writer that cannot be sure about the agents’ behavior.

2. We take the benchmark model to be the true distribution, which agent and planner might know or not. In this case, welfare gains are always measured taking into account an expected utility that is based on the benchmark model, regardless of the information that agents and planner have. We may think of this as there being a “super” planner that knows the truth, but both agents and planner can make mistakes when making their decisions because they fear misspecification. When the government knows more than the agents, it can improve welfare, but it can cause harm if it knows less. Table 2 shows the possible combinations (note that the upper row is identical to Table 1).

Table 2: Welfare measured based on benchmark f (“truth”)

	SP: Non-robust $\theta = \infty$	SP: Robust $\theta = \xi$
HH: Non-robust $\theta = \infty$	$V_f^{(DE;f)} < V_f^{(SP;f)}$	$V_f^{(DE;f)} \leq V_f^{(SP;\tilde{f})}$
HH: Robust $\theta = \xi$	$V_f^{(DE;\tilde{f})} < V_f^{(SP;f)}$	$V_f^{(DE;\tilde{f})} \leq V_f^{(SP;\tilde{f})}$

Note: this table considers welfare under the four possible combinations of robust/non-robust planner and household allocations. V measures welfare under the optimal allocation chosen by either the household (the decentralized equilibrium with no government intervention, DE) or the social planner (SP). The subscript indicates whether welfare is computed based on a series of shocks that follow the benchmark distribution f , or the worst-case distribution \tilde{f} . In this table, this is always the benchmark model. Welfare is maximized by the optimal allocation of a social planner that doesn’t fear misspecification; if she does, it is not obvious whether her policy can increase welfare by internalizing the effect that debt has on the value of collateral. We assume that a robust household and a robust planner share the same degree of misspecification, $\theta = \xi$.

Under this interpretation, the stress is put on the level of information that planner and agents have about the evolution of state variables or, in other words, the quality of their forecasting methods. If both take into account all information available in each period, then neither should fear misspecification.

From this analysis, we observe that the presence of externalities combined with the possibility that planner and/or agents fear misspecification, implies that welfare under the planner's allocation might not improve or that welfare gains are subdued. The planner's allocation guarantees an improvement in welfare only when the planner's view of the world is used to compute expected utility. When planner and household disagree on the model and we are measuring welfare from the household's perspective (Table 1), then there are two forces to consider: on the one hand, welfare might decrease because the planner is solving for an allocation under a different model but, on the other hand, welfare might be improved because she is correcting the market failure. These are the cases outside of the main diagonal. In none of these cases is maximum welfare achieved because, while the planner is internalizing the externality, she is doing so under a model different than the household's. If, however, welfare is measured according to the benchmark model and the planner shares this view of the world (bottom left corner of Table 2), then the planner improves social welfare and achieves the maximum possible utility.

We use a compensating variation approach as in Lucas (1987) to obtain a quantitative measure of welfare gains and losses in terms of units of consumption. We compute the constant proportional increase in consumption (the percent subsidy to consumption) that would make agents indifferent between remaining in a decentralized equilibrium without government intervention and adopting the planner's socially optimal allocation. This consumption subsidy, which we call $\gamma(B, y)$, is state dependent (that is, while the proportional increase in consumption is constant over time, the γ that can achieve the welfare from the planner's allocation will depend on what initial state we compute it at). If we compute welfare using benchmark probabilities (upper row of Table 1, and all cases of Table

2), γ must satisfy condition (40)⁶. If we assume that worst-case probabilities govern the evolution of shocks (bottom row of Table 1), it satisfies condition (41). In these expressions too, subscript f indicates the probabilities that are used to compute expectations; f is the benchmark model while \tilde{f} is the worst-state distribution. Superscript (i, f^i) indicates the allocation at which utility is being evaluated and the density that agent i has in mind when making the optimal choice.

$$\frac{\left((1 + \gamma)c^{(DE;f^{DE})}(B, y, B)\right)^{1-\sigma}}{1 - \sigma} + \beta E_{y'|y}[V_f^{(SP;f^{SP})}(B', y', B')] = V_f^{(SP;f^{SP})}(B, y, B) \quad (40)$$

$$\frac{\left((1 + \gamma)c^{(DE;f^{DE})}(B, y, B)\right)^{1-\sigma}}{1 - \sigma} - \beta \theta \log \left(E_{y'|y} \left[e^{-V_{\tilde{f}}^{(SP;f^{SP})}(B', y', B')/\theta} \right] \right) = V_{\tilde{f}}^{(SP;f^{SP})}(B, y, B) \quad (41)$$

The cases that assume benchmark probabilities for computing expectations differ in terms of what allocations utility is evaluated at; namely, the planner's ($SP; f^{SP}$), or the household's ($DE; f^{DE}$). In particular, Bianchi (2011) corresponds to the upper left case of Tables 1 and 2, that consider allocations obtained under the benchmark view of the world. Note that the homotheticity of the utility function allows one to simplify the computation of γ , which can be derived from the following expression (a proof is provided in the Appendix):

$$(1 + \gamma(B, y))^{1-\sigma} V_f^{(DE;f^{DE})}(B, y, B) = V_f^{(SP;f^{SP})}(B, y, B) \quad (42)$$

Where:

$$V_f^{(DE;f^{DE})}(B, y, B) = \frac{\left(c^{(DE;f^{DE})}(B, y, B)\right)^{1-\sigma}}{1 - \sigma} + \beta E_{y'|y}[V_f^{(DE;f^{DE})}(B', y', B')] \quad (43)$$

We cannot leverage on the homotheticity of the utility function when there is fear of misspecification since the modified Bellman equation is no longer linear in the expectation of the future value

⁶Note that while we compute a γ for each state (B, y) , we change the notation here from $\gamma(B, y)$ to γ to make it clear that the proportional increment in consumption is meant to be fixed over time.

function. Thus, we compute welfare gains numerically.

3 Macprudential taxes

For periods when the collateral constraint is not (currently) binding, the tax on debt that implements the planner's allocation is such that both the household's and the planner's Euler equations are satisfied when evaluated at the planner's allocation. For non-robust agents as in Bianchi (2011), these Euler equations are:

$$u_{T,t} = \beta(1+r)(1+\tau_t)E_{y_{t+1}|y_t}[u_{T,t+1}] \quad (44)$$

$$u_{T,t} = \beta(1+r)E_{y_{t+1}|y_t}[u_{T,t+1} + \mu_{t+1}\psi_{t+1}] \quad (45)$$

where $u_{T,t}$ is the marginal utility with respect to tradables, μ_t is the shadow value of relaxing the collateral constraint and ψ_t captures how much the value of collateral changes when tradable consumption increases.

If household and planner make robust decisions, however, expectations have to be taken using the martingale increment that allows one to compute these under the worst-case distribution, so that the planner's allocation must satisfy:

$$u_{T,t} = \beta(1+r)(1+\tau)E_{y_{t+1}|y_t}[m_{t+1}u_{T,t+1}] \quad (46)$$

$$u_{T,t} = \beta(1+r)E_{y_{t+1}|y_t}[m_{t+1}(u_{T,t+1} + \mu_{t+1}\psi_{t+1})] \quad (47)$$

$$m_{t+1} = \frac{e^{-V_{t+1}/\theta}}{E_{y_{t+1}|y_t}[e^{-V_{t+1}/\theta}]} \quad (48)$$

Note that, when trying to decentralize the planner's allocation, it is important to consider the distribution of shocks that the household has in mind (when computing expectations), irrespective of whether that is the true distribution or not. We wish to emphasize that this is different from the exercise of computing welfare gains from Section 2.2 - in that case, we must take a stance on what distribution of shocks we will consider (the household's, the planner's, the true one) when computing the expected discounted utility under the different allocations.

There are four possible combinations of robust/non-robust allocations for the planner and house-

hold, and the following expressions indicate what taxes (or subsidies) can implement the planner's allocation in each case:

1. If neither household nor planner make a robust choice (this is Bianchi (2011)):

$$\tau = \frac{E_{y_{t+1}|y_t}[\mu_{t+1}\psi_{t+1}]}{E_{y_{t+1}|y_t}[u_{T,t+1}]} \quad (49)$$

2. If the planner makes robust decisions and the household doesn't:

$$1 + \tau = \frac{E_{y_{t+1}|y_t}[m_{t+1}(u_{T,t+1} + \mu_{t+1}\psi_{t+1})]}{E_{y_{t+1}|y_t}[u_{T,t+1}]} \quad (50)$$

3. If planner and household both make robust choices:

$$\tau = \frac{E_{y_{t+1}|y_t}[m_{t+1}\mu_{t+1}\psi_{t+1}]}{E_{y_{t+1}|y_t}[m_{t+1}u_{T,t+1}]} \quad (51)$$

4. If the household makes a robust choice but the planner doesn't:

$$1 + \tau = \frac{E_{y_{t+1}|y_t}[u_{T,t+1} + \mu_{t+1}\psi_{t+1}]}{E_{y_{t+1}|y_t}[m_{t+1}u_{T,t+1}]} \quad (52)$$

The last case is the only one in which the constraint can bind for the planner for certain values of current debt but not for the household, given that a robust household chooses higher levels of precautionary savings. In this case, a subsidy rather than a tax is what is required to implement the planner's allocation (i.e., τ will be negative).

4 Quantitative Results

We follow closely the calibration from Bianchi (2011) that uses data from Argentina, summarized in Table 3. We re-calibrate only the discount factor and the share of collateralized tradable to achieve a probability of a sudden stop of 13% and an average debt-to-GDP ratio of 30% under the decentralized equilibrium without government intervention. While Bianchi targets a probability of 5%, we focus on a higher value following evidence from Jeanne and Ranciere (2011) and Furceri et al (2011) that sudden stops are twice as frequent, with probabilities of around 10% and as high as 20%. This probability justifies the implementation of higher macroprudential taxes, consistent with the empirical evidence. We also present the results from Bianchi’s exact calibration for comparability.

In terms of risk aversion and the degree of misspecification, we consider a low and a high value both to assess the sensitivity of the results to the different parameterizations and to discuss the differences between introducing robustness in the decision-making process versus simply increasing an agent’s risk aversion. Our baseline calibration considers a standard risk-aversion of 2, and we later double this to compare results relative to those of a robust agent. In order to discipline the choice of θ , we compute detection error probabilities. These capture the probability with which an agent is able to distinguish between the worst-case and the benchmark densities. We search for a level of robustness (i.e., a value of θ) such that an agent would not “easily” distinguish between the two, so that it remains plausible for her to want to make a robust choice against the possibility that the model is not the benchmark. Based on this analysis, we take $\theta = 0.35$ as a baseline value in our calibration, and later compare with results from increasing the level of robustness to $\theta = 0.15$.

Table 3: Calibration

Parameter	Value	Source
Utility function		
Risk aversion σ	2	Standard, Bianchi (2011)
Intratemp elast of substitution $1/(1 + \eta)$	0.83	Bianchi (2011), conservative
Discount factor β	0.8	To match data moments - Bianchi (2011): 0.91
Share of tradables in CES ω	0.31	Bianchi (2011), to match data moments
Interest rate		
World risk-free interest rate r	0.04	Standard, Bianchi (2011)
Collateral constraint		
Relative quality of collateral κ^N/κ^T	1	Bianchi (2011)
Share of collateralized tradable κ^T	0.31	To match data moments - Bianchi (2011): 0.32
Robustness		
Degree of misspecification θ	0.35	To target an error detection probability of $\approx 20\%$

Note: Our main calibration follows closely that of Bianchi (2011), with two differences: we consider a lower discount factor and a slightly lower share of collateralized tradable to match a probability of sudden stop of about 13% and a debt-to-GDP ratio of about 30%. To analyze the sensitivity of results to a higher risk aversion and degree of robustness, we also present results computed with a risk aversion of 4 and a $\theta = 0.15$.

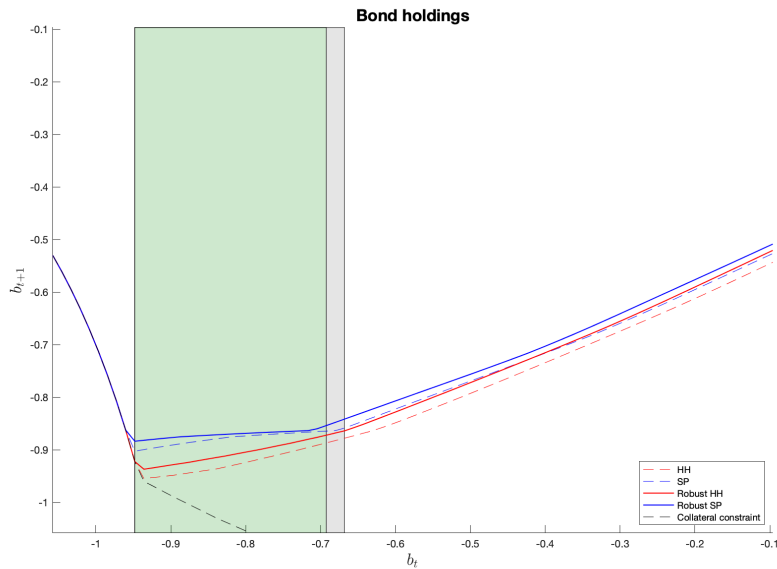
We also need to specify the distribution of shocks to the endowment process, as described in 5. Following Bianchi (2011), we discretize the vector of shocks and consider four grid points for each type, shocks to tradable and non-tradable goods, for a total of 16 possible states. We consider the same calibration for the first-order bivariate autoregressive process of endowment as in Bianchi (2011), and the same set of grid values and transition probabilities that discretize this process. We will refer to these transition probabilities as the “benchmark” distribution. The benchmark transition density for the state, $f(x_{t+1}|x_t)$, encompasses these transition probabilities.

4.1 Debt policy: household vs planner

Figure 2 shows the optimal borrowing decisions of a planner and a household when both tradable and non-tradable endowments are about one standard deviation below mean (see Appendix for other states). The dashed lines show the debt policies for a household and a planner that do not fear misspecification and replicate the findings in Bianchi (2011) with the noted change in the calibration. The solid lines depict the debt policies for these agents under robustness. The green area that overlaps

a larger grey area indicates the region over which macroprudential taxes are required to decentralize the allocation of a robust planner in a context in which households also make robust decisions. The grey area is the corresponding tax region for a planner and a household that are not robust. In Section 4.3 we discuss how these regions compare, we present the optimal taxes and also extend this analysis for the mixed cases (when one agent makes robust choices but the other does not).

Figure 2: Debt policies of (robust) household and (robust) planner



Note: This plot depicts the optimal debt policy of a planner and an agent as a function of current debt holdings, for a state of nature characterized by a one standard deviation negative shock for tradables and non-tradables. For the robust agents, the degree of misspecification considered is characterized by $\theta = 0.35$. The green area is the region of current debt holdings for which a tax is needed to implement a robust planner's allocation in a decentralized economy that features a robust household. It overlaps a wider grey area that corresponds to the tax region for non-robust agents. Parameters are calibrated to generate a probability of crisis of 13% under the debt policy of a non-robust household and the benchmark distribution of shocks.

The decisions of both household and planner share in common a U-shape pattern. Without a borrowing constraint, consumption smoothing would imply a monotonically increasing policy for bond holdings: the higher the current debt level (lower current bond holdings), the higher the debt that is optimally chosen. When there is a collateral constraint and it binds, however, the choice of bond holdings increases with a higher current debt. This is because a higher debt today implies a

lower tradable consumption given a choice of next-period bond holdings. This lowers the price of non-tradables, tightening the collateral constraint and leading to a lower level of borrowing. This binding region is smaller for robust agents.

In the non-binding area, a planner accumulates larger bond holdings compared to a household, to serve as precautionary savings against the possibility of the constraint binding in the future. Robust agents will also save more compared to their non-robust versions, over the entire non-binding region, in a precautionary attitude against a higher probability of negative shocks in the future. Notably, a robust household at some point overtakes the social planner and chooses higher levels of savings given current levels of debt that are far from the binding region. Closer to the binding area, a non-robust planner still chooses higher savings to account for the externalities on the value of collateral.

The tax areas are regions in which the constraint for the private agents will bind tomorrow with positive probability (i.e. given the household's choice of debt, there will be some states y' tomorrow for which the constraint will bind). We note that a robust planner will tax a robust household over a slightly narrower region of debt (the green shaded area) than a non-robust planner with respect to a non-robust household (the grey area). While it is not entirely obvious whether taxes are larger when both agents are robust relative to when they are not (note that the gap between debt choices is very similar), the difference in debt choices of a robust planner compared to a non-robust household is significantly larger. A robust planner borrows considerably less in the unconstrained region close to the binding point compared to a non-robust household, and this would suggest that much larger taxes are needed to implement the socially optimal allocation, and over a much larger tax region. We explore this in Section 4.3.

4.2 Simulations

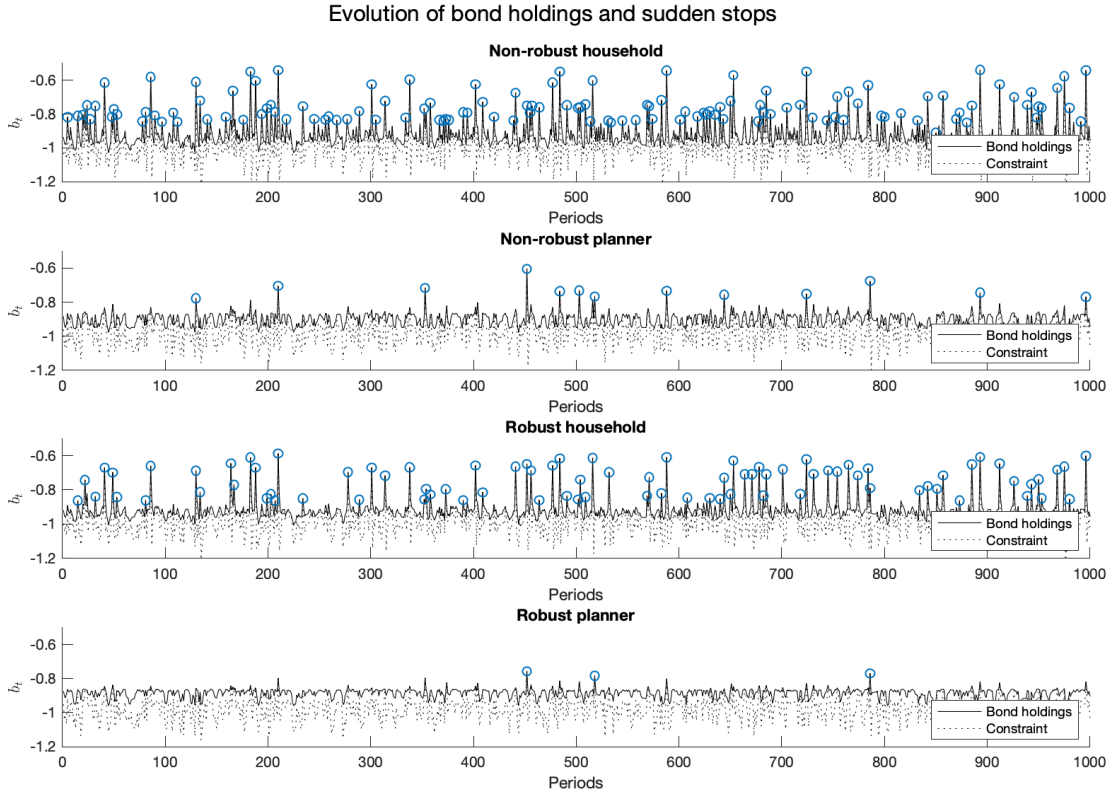
Based on the allocations that result from the optimal debt policies, Bianchi (2011) simulates the centralized and decentralized economies to analyze how the difference in bond accumulation of households and planner affect the long-run distribution of debt, and the probability and magnitude of

sudden stops. We follow this approach to compare the results from policies that respond to non-robust and robust choices. In line with Bianchi (2011), we obtain 80,000 period allocations given some history of realized shocks for the endowment process. These shocks are obtained from the benchmark distribution but, when analyzing welfare gains, we also report the results from simulations that use worst-case probabilities viewed from the perspective of the household⁷. Based on a history of shocks, we obtain the simulated series for all relevant macroeconomic variables when the debt policy follows the optimal choices of: a non-robust household, a non-robust planner, a robust household or a robust planner. With respect to robust policies, we consider both a value for θ of 0.15 (high degree of misspecification) and of 0.35 (low degree of misspecification). For all the other parameters, we use the calibration that renders a probability of crisis of 13% under the household's non-robust allocation and the benchmark distribution of shocks (but we also report the results that replicate Bianchi (2011) for comparability).

A sudden stop happens when the collateral constraint binds and this leads to an increase in net capital outflows (i.e. an increase in bond holdings) that exceeds *one standard deviation* of net capital outflows according to the long-run distribution of bond holdings of the decentralized economy (i.e., under the non-robust household's allocation). Figure 3 shows the evolution of the simulated time series of bond holdings for an interval of 1000 periods, based on the (robust) household's and the (robust) planner's chosen allocations. The shocks are realizations from the benchmark distribution and the robust allocations are based on a degree of misspecification of $\theta = 0.35$. Figure 9 in the Appendix shows the corresponding results when shocks are derived from a worst-case distribution. As expected, a planner is able to reduce the number of periods in which a sudden stop occurs, and robust agents can also achieve this relative to their non-robust counterparts.

⁷There is a different matrix of "worst-case" transition probabilities for each possible current state. This is because what constitutes a worst-case scenario for a household, based on her optimal debt policy, depends on the current level of bond holdings and how close the economy is to the binding region for the collateral constraint.

Figure 3: Evolution of bond holdings based on benchmark distribution of shocks



Note: These plots show the evolution of bond holdings from simulations that consider a specific history of shocks to tradable and non-tradable endowments based on the benchmark distribution. The same series of shocks is fed into the four cases depicted here. The first plot considers the optimal debt choice of a non-robust household, the second that of a non-robust planner, the third shows the choices from a robust household and the fourth from a robust planner. The blue dots indicate a sudden stop (an increase in bond holdings that exceeds one standard deviation of net capital outflows according to the long-run distribution of bond holdings of the decentralized economy with no government intervention and a non-robust household).

Table 4 compares the simulation results for the two economies and different parameterizations. Using the generated data, we take the time series for debt (bond holdings) and we count the number of periods with a sudden stop. To measure the severity of a crisis, we follow Bianchi (2011) and consider the following experiment: we find the median financial crisis in the decentralized equilibrium (i.e., under the non-robust household’s allocation), defined as that which features the median current account reversal. We then backtrack the level of debt two periods before and simulate the response of a robust household and of a planner (both a robust one and a non-robust one) that are faced with

that same initial level of debt and the same set of shocks over the next three periods. Note that this approach takes as the baseline case, used to define the median crisis, the policy of a non-robust household with a parameterization for the discount factor and a share of tradables that leads to a probability of a sudden stop of 13% (column 3 of Table 4). For comparability, the first two columns replicate the results in Bianchi (2011) (in this case the results on the relative magnitude of a crisis is based on the decentralized equilibrium result that leads to a probability of crisis of 5%). Finally, we report a set of standard deviations and correlations with GDP.

Table 4: Probability and magnitude of a sudden stop

	HH Bianchi (2011)	SP Bianchi (2011)	HH	SP	HH Robust $\theta = 0.35$	SP Robust $\theta = 0.35$
Long-run prob of crisis* (%):	5.156	0.339	12.696	1.503	8.577	0.653
Long-run prob of constraint binding (%):	5.934	0.529	19.348	2.862	11.857	1.725
Average debt as % of GDP:	29.349	28.633	29.181	28.564	29.035	28.075
Decrease in consumption in median crisis:	-16.685	-10.342	-8.810	-3.717	-4.912	-2.764
Depreciation RER in median crisis:	-19.174	-2.027	-20.055	-7.930	-10.714	-5.280
CA reversal in median crisis:	7.832	0.226	7.940	2.246	7.940	1.193
Std dev of consumption:	0.058	0.052	0.071	0.057	0.065	0.054
Std dev of RER:	0.233	0.109	0.374	0.200	0.304	0.151
Std dev of CA (as % of GDP):	0.027	0.006	0.047	0.019	0.036	0.010
Corr consumption with GDP:	0.833	0.645	0.866	0.643	0.757	0.576
Corr RER with GDP:	0.785	0.533	0.907	0.692	0.792	0.632
Corr CA (as % of GDP) with GDP:	-0.752	-0.287	-0.892	-0.656	-0.772	-0.608

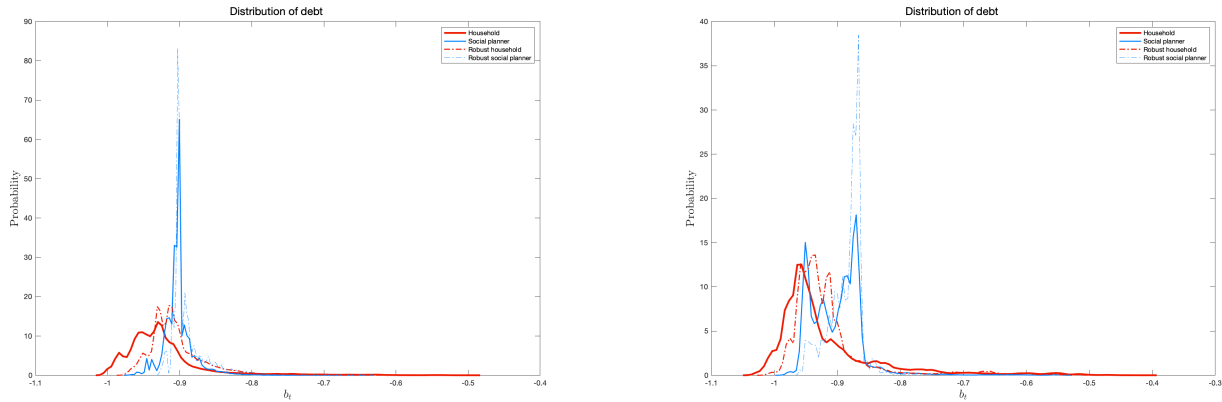
Note: This table reports the results from the simulation exercise that considers a history of shocks to tradable and non-tradable endowments based on the benchmark distribution. A sudden stop happens when the constraint binds *and* there is a one standard deviation reversal of the current account. The first two columns replicate Bianchi (2011). Columns 3 and 4 report the results from economies that feature non-robust agents, while columns 5 and 6 are based on the optimal choices of a robust household and planner.

In line with the results from Bianchi (2011), a social planner that internalizes the effect of borrowing on the value of collateral is able to significantly reduce the occurrence of sudden stops, from 5% to 0.3% for Bianchi's calibration, and from 13% to 1.5% for our preferred calibration. The severity of the crisis is also greatly reduced, as evidenced by a much lower reduction in consumption (4% versus 9%, where percentages are relative to the long-run mean), a lower depreciation of the real exchange rate (8% versus 20%), and a smaller current account reversal (2% versus 8%). A robust household's

allocation is able to improve on the results of a non-robust household for the simple reason that due to fear of misspecification the robust agent will always save more for precautionary motives. Robust choices, even if they still do not account for the pecuniary externalities, can decrease the probability of a sudden stop to 9% and halve the reduction in consumption and in the depreciation of the real exchange rate (though there are no significant improvements in terms of median current account reversal). A robust planner is able to improve on the results of a non-robust planner, bringing the probability of a crisis to 0.7% and reducing the severity of these episodes. Whether these improvements imply gains in welfare is still something that needs to be assessed and we address this in Section 4.4.

The precautionary savings of the planner imply that the long-run (ergodic) distribution of bond holdings for the planner assigns a higher probability to lower levels of debt (high bond holdings) compared to the private agents (Figure 4). Notably, on average, both planner and private agents -whether robust or not- tend to hold the same level of debt (30%). However, the planner reduces exposure to very high levels of debt (low bond holdings) that would make the collateral constraint bind and trigger a sudden stop when there is a negative shock to output. This same observation holds when comparing a robust agent against her non-robust counterpart.

Figure 4: Distribution of debt holdings

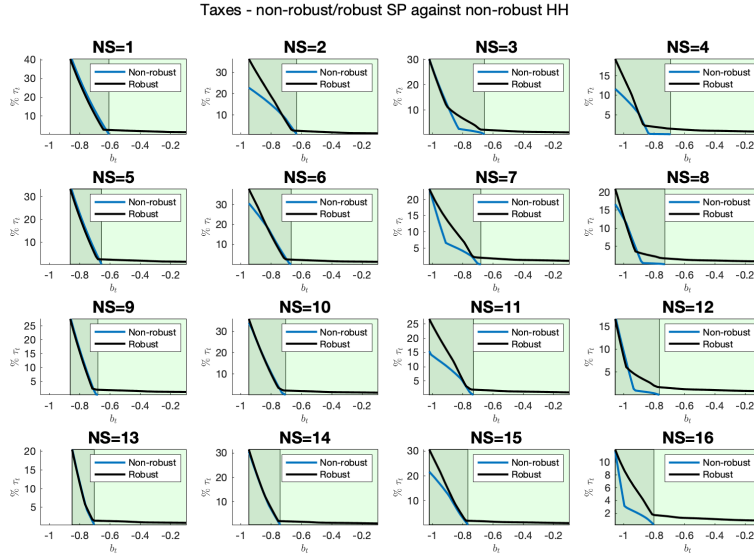


Note: Distribution of debt holdings based on robust and non-robust allocations by household and planner. The left plot replicates Bianchi (2011), with a parameterization that leads to a 5% probability of crisis under the decentralized equilibrium with no intervention. The right plot assumes a parameterization that leads to a probability of crisis of 13%.

4.3 Macprudential taxes

The fact that the difference in optimal bond holding decisions between a non-robust household and a robust social planner widens over the non-binding region, suggests that in these cases the macroprudential taxes that would be required for decentralizing the planner’s allocation are higher compared to a situation in which the planner is non-robust. Figure 5 confirms this through plots that compare the optimal taxes and the tax region imposed by a non-robust planner against those set by a planner that fears misspecification. In all states of nature and across the entire bond grid, the macroprudential taxes that are required to decentralize the allocation of a robust planner are larger, and are set over the entire set of possible current bond holdings such that the constraint is not yet binding.

Figure 5: Tax regions



Note: These plots show the macroprudential taxes that are required to implement a social planner’s allocation in a decentralized economy with non-robust households. Taxes are functions of the current level of debt (the x-axis of the plots), and of the shocks to non-tradable and tradable goods (there are 16 possible states based on these shocks) The black lines are taxes that implement a robust planner’s allocation (with $\theta = 0.35$ and the blue lines refer to a non-robust planner. The light green area is the tax region for taxes from a robust-planner, and it overlaps with the narrower dark green area that indicates the tax region for a non-robust planner. The calibration used for computing the optimal debt policies underlying these taxes is our baseline specification that delivers a 13% probability of crisis and an average debt over GDP of 30%.

Figures 10 to 13 in the Appendix depict the taxes (subsidies) for each of the four possible combinations of robust and non-robust household and planner, across all states of nature. The grey areas in those plots indicate the values of current debt for which a tax is required to implement the planner’s allocation. Once again, we consider a value of $\theta = 0.35$ for the robust decision-making agents. Table 10 in the Appendix presents the average tax over the tax region in these four cases, where the mean is taken for each state of nature individually. These averages can be lower for the case of a robust planner compared to that of a non-robust planner only because the mean is taken over a more extended tax region that includes ranges of current debt that lead to relatively low taxes. When the household is robust but the planner is not, then a subsidy is required to implement the planner’s allocation because a household that fears misspecification and is optimizing against worst-case probabilities will tend to underborrow and hold more savings than are required to internalize

the pecuniary externality.

In order to compare the average tax over a long-run horizon that is required to decentralize a planner's allocation across different cases, we compute the mean tax over the simulations from the previous section. Table 5 summarizes the average macroprudential tax that implements the planner's allocation when the economy is subject to shocks from the baseline distribution. Table 6 indicates the probability that a tax is implemented. The mere change in calibration that renders a probability of crisis of 13% already justifies macroprudential taxes that are more than twice as large as those in Bianchi (2011) - 16% against 7% -. A robust planner will further induce an increase of almost 5 percentage points, to 20%; and even more so the higher the degree of robustness (i.e., lower θ). Moreover, the probability that a tax is needed to implement the social planner's allocation is much higher (88% compared to 78%). This was to be expected, as we noted before that the region of current debt over which a tax is imposed widens. Taxes remain fairly stable though if both agents fear misspecification, compared to the case when neither does, although the probability that one is implemented increases.

Table 5: Average taxes (simulations)

	Average tax (%) Bianchi (2011)	Average tax (%) $\theta = 0.35$	Average tax (%) $\theta = 0.15$
1) non-robust HH v non-robust SP	7.274	15.971	15.971
2) non-robust HH v robust SP	6.333	20.113	22.260
3) robust HH v robust SP	5.655	16.999	15.719
4) robust HH v non-robust SP	-1.372	-1.637	-6.247

The first column replicates Bianchi (2011) by considering a parameterization that leads to a probability of crisis of 5% in the decentralized equilibrium with no intervention. The second and third columns consider a parameterization to match a probability of crisis of 13%. The numbers represent the average macroprudential tax that implements the planner's allocation over the 80,000 periods of the simulation. The realization of shocks to tradable and non-tradable goods is based on the baseline distribution.

Table 6: Probability of a tax (simulations)

	Prob of tax (%) Bianchi (2011)	Prob of tax (%) $\theta = 0.35$	Prob of tax (%) $\theta = 0.15$
1) non-robust HH v non-robust SP	80.111	77.705	77.705
2) non-robust HH v robust SP	94.310	87.911	93.010
3) robust HH v robust SP	70.058	84.816	90.689
4) robust HH v non-robust SP	27.022	6.461	43.191

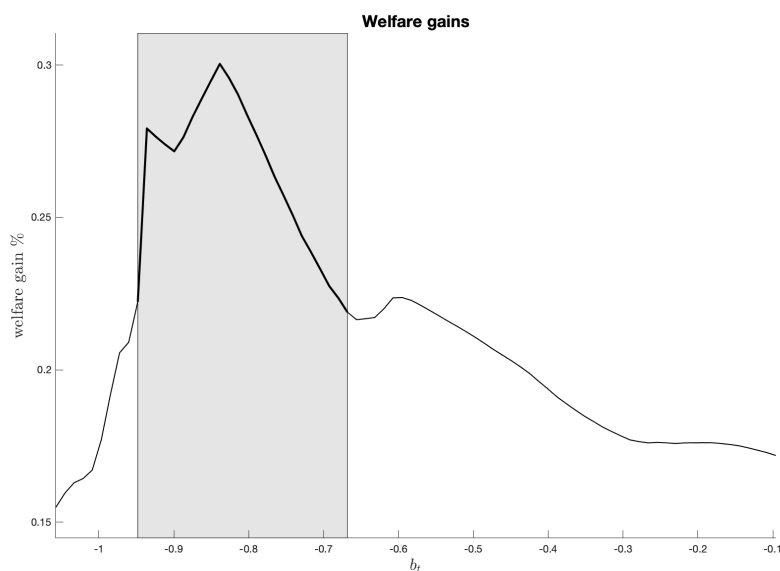
The first column replicates Bianchi (2011) by considering a parameterization that leads to a probability of crisis of 5% in the decentralized equilibrium with no intervention. The second and third columns consider a parameterization to match a probability of crisis of 13%. The numbers represent the probability that a tax is implemented by counting the periods in the simulation in which one is imposed. A negative value indicates a subsidy. The realization of shocks to tradable and non-tradable goods is based on the baseline distribution.

4.4 Welfare gains and losses

To assess the gains from implementing the planner’s allocation through macroprudential taxes, we compute welfare from each of the cases in Tables 1 and 2 (i.e., assuming that shocks are distributed according to the density that the household has in mind, or assuming the benchmark density is the “true” underlying distribution). In order to obtain a quantitative measure of welfare gains and losses, we use the compensating variation approach described in Section 2.2. This requires that we first derive the value function for each of the possible combinations of allocations and of probabilities used to compute expectations (benchmark or worst-case densities). Figures 14 to 17 in the Appendix depict the value functions under different combinations of robust and non-robust allocations and probabilities.

As noted before, the measure $\gamma(B, y)$ of welfare gains is state dependent. Figure 6 shows the gains at a given state of nature that corresponds to a negative one-standard deviation shock of tradables and non-tradables, for the case of a non-robust planner and a non-robust household when expectations are taken assuming a benchmark distribution of shocks. The grey area indicates the region of current debt over which a macroprudential tax is needed to implement the planner’s allocation. Figures 18 and 23 in the Appendix depict what gains (and losses) look like in each state of nature for the six possible combinations of Tables 1 and 2.

Figure 6: Welfare gains



Note: This plot shows the welfare gains from implementing a non-robust social planner’s allocation in a decentralized economy with non-robust households, assuming that the benchmark distribution governs the evolution of shocks. Welfare gains are functions of the current level of debt (the x-axis of the plot), and of the shocks to non-tradable and tradable goods. This case considers a one-standard negative shock to tradable and non-tradable goods. The grey area is the tax region. The calibration used for computing the optimal debt policies underlying these taxes is our baseline specification that delivers a 13% probability of crisis.

To summarize these gains in a single number, we calculate the average over the simulations from Section 4.2. The mean is taken over the periods in which a macroprudential tax is required to implement the planner’s allocation. Note that it is possible to compute, based on the expressions derived in Section 2.2, the welfare gains $\gamma(B, y)$ for all possible combinations of robust versus non-robust allocations and of benchmark versus worst-case probabilities (to compute expectations under the worst case, we use the martingale representation from Sections 2.1 and 3). However, in order to obtain one average measure of gains in the long-run based on simulations, we require that these simulations be run based on shocks that are either realizations from the benchmark distribution or from the worst-case probabilities, corresponding to each case. For example, if we want to find an average welfare gain from implementing a robust planner’s allocation compared to the allocation from a robust household, and we want to compute utility expectations from the perspective of the

household, then we use the simulation of shocks that are derived from the worst-case distribution that the household has in mind⁸.

Table 7: Average welfare gains (simulations)

	Bianchi (2011) $\theta = 0.35$	Higher prob crisis $\theta = 0.35$	Higher prob crisis $\theta = 0.15$
Benchmark probabilities			
1) non-robust HH v non-robust SP	0.137	0.270	0.270
2) non-robust HH v robust SP	0.129	0.241	0.196
3) robust HH v robust SP	0.030	0.090	-0.022
4) robust HH v non-robust SP	0.028	0.096	0.035
Worst-case probabilities			
5) robust HH v robust SP	0.108	0.31	0.339
6) robust HH v non-robust SP	0.085	0.227	0.092

Note: The numbers represent the mean welfare gain over the periods in the simulation when a tax is needed to decentralize the planner’s allocation. For the first set of results (“Benchmark probabilities”), the realization of shocks to tradable and non-tradable goods in the simulations is based on the baseline distribution. The second block of results (“Worst-case probabilities”) considers worst-case probabilities. The first column replicates Bianchi (2011) by considering a parameterization that leads to a probability of crisis of 5% in the decentralized equilibrium with no intervention. The second and third columns consider a parameterization to match a probability of crisis of 13%.

For a calibration that renders a probability of sudden stops of 13% in the decentralized economy with no government intervention, welfare gains from implementing the planner’s allocation are twice as large as those obtained by Bianchi (2011) - 0.27% of permanent consumption. These numbers are still rather small, but consistent with the literature’s finding, as noted in Bianchi (2011), that the welfare cost of business cycles is small. Importantly, we find that a robust planner’s allocation will still increase welfare even if this is measured assuming shocks are realized according to the benchmark model that the households have in mind. This was not evident a priori: a robust planner will choose an allocation that generates a lower bound of lifetime discounted utility, in the sense that implementing that allocation will result in larger welfare if shocks are actually drawn from the benchmark model rather than the worst case distribution. However, if indeed shocks follow the

⁸One caveat in our computations is that, in a case like this, the value function associated to the robust planner’s allocation was computed using expectations that reflect the planner’s worst-case scenario, which differs slightly from the household’s worst-case scenario - recall that these worst-case transition probabilities are dependent on the level of current debt and take into account an agent’s optimal debt choice. Ideally, we should re-compute these value functions under expectations that consider a household’s worst-case distribution. But the difference between these is sufficiently small that we can be confident it doesn’t significantly change the results.

benchmark model, then it would have been optimal to pick an allocation based on this benchmark view of the world. We know that a robust planner will correct the externality, and so this should increase welfare compared to a competitive equilibrium, but we do not know how much welfare is lost from not picking the optimal allocation based on the correct (benchmark) view of the world. Results indicate the gains are larger than the costs. A robust planner that fears misspecification with $\theta = 0.35$ may still achieve a welfare gain almost as large as a non-robust planner when shocks follow the benchmark distribution (0.24% of permanent consumption compared to the 0.27% for a non-robust planner). The higher the degree of robustness, however, the lower the welfare gain (0.20% for a $\theta = 0.15$).

If both agents fear misspecification but shocks follow a benchmark distribution, then the gains from implementing a planner's allocation are significantly reduced. This is because the household already saves more as a precaution against future bad shocks, and so this indirectly addresses the problem of overborrowing due to pecuniary externalities. Any further gains from a planner that internalizes this externality doesn't increase welfare by much (0.096%). In fact, if fear of misspecification is sufficiently high (for example, $\theta = 0.15$), then a planner's allocation decreases welfare. A robust planner "overshoots" in saving.

Table 8: Average welfare gains and losses

	SP: Non-robust $\theta = \infty$	SP: Robust $\theta = 0.35$
Benchmark probabilities		
HH: Non-robust $\theta = \infty$	$V_f^{(HH;f)} < V_f^{(SP;f)}$ Average gain: 0.270%	$V_f^{(HH;f)} \leq V_f^{(SP;\tilde{f})}$ Average gain: 0.241%
HH: Robust $\theta = 0.35$	$V_f^{(HH;\tilde{f})} < V_f^{(SP;f)}$ Average gain: 0.096%	$V_f^{(HH;\tilde{f})} \leq V_f^{(SP;\tilde{f})}$ Average gain: 0.090%
Worst-case probabilities		
HH: Robust $\theta = 0.35$	$V_{\tilde{f}}^{(HH;\tilde{f})} \leq V_{\tilde{f}}^{(SP;f)}$ Average gain: 0.227%	$V_{\tilde{f}}^{(HH;\tilde{f})} < V_{\tilde{f}}^{(SP;\tilde{f})}$ Average gain: 0.31%

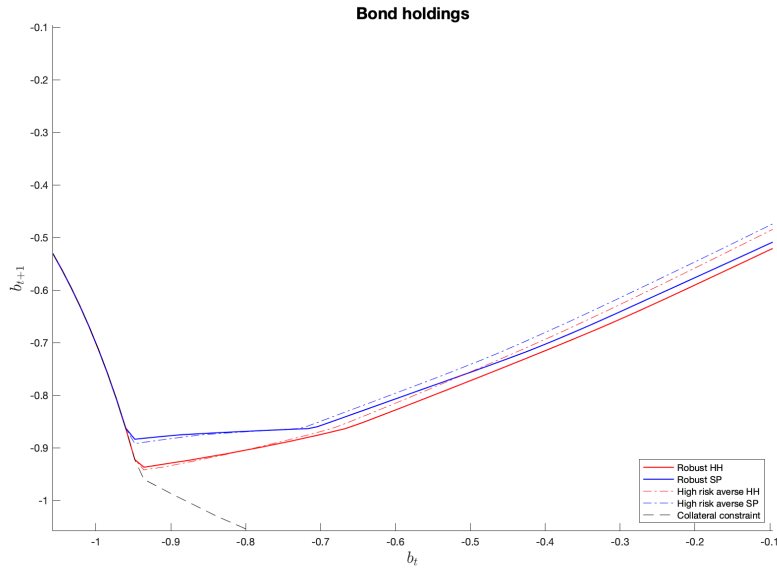
Note: This table summarizes the welfare results from the simulations with regard to Tables 1 and 2. The first block considers welfare that is computed based on benchmark probabilities (i.e., they correspond to Table 1). The second block calculates welfare based on worst-case probabilities - it corresponds to the second row of Table 2). Average gains are computed as means over the simulated periods.

4.5 Comparing robustness with higher risk aversion

One interesting question is how the decisions of a robust agent compare to those of an agent that is simply more risk averse. Figure 7 shows that they tend to make very similar decisions when debt is high and the collateral constraint is close to binding. However, for levels of debt further away from the binding region, a highly risk averse agent continues to be conservative in terms of savings, while the robust agent no longer fears very negative shocks in the following period and thus accumulates larger amounts of debt. Recall that a robust agent's worst-case scenario of probabilities for the following period depends on the current state; if the economy is far from the binding region, then there is less concern about negative future shocks. Table 9 compares the results from the simulations. Notably, both a robust and a highly risk averse household and planner arrive at similar probabilities of a crisis (around 8.5% based on a household's debt policy, and this is reduced to 0.7% for a planner). This is not surprising given that their debt choices are extremely similar close to the binding region. In terms of the severity of a median crisis, both types of planners are able to

halve the reduction in consumption, although the median crisis in the context of highly risk averse agents renders a median financial crisis that is more severe compared to that which arises when agents are robust.

Figure 7: Debt policies of household and planner - comparing robustness with high risk aversion



Note: This plot depicts the optimal debt policy of a planner and an agent as a function of current debt holdings, for a state of nature characterized by a one standard deviation negative shock for tradables and non-tradables. For the robust agents, the degree of misspecification considered is characterized by $\theta = 0.35$. Non-robust agents in this plot have a high risk aversion of $\sigma = 4$.

Table 9: Probability and magnitude of a sudden stop

	HH	SP	HH	SP	HH	SP
	$\theta = 0.35$	$\theta = 0.35$	$\theta = 0.15$	$\theta = 0.15$	$\sigma = 4$	$\sigma = 4$
Long-run prob of crisis* (%):	8.577	0.653	5.900	0.172	8.738	0.776
Long-run prob of constraint binding (%):	11.857	1.725	6.259	0.594	11.634	2.094
Average debt as % of GDP:	29.035	28.075	28.555	27.663	28.884	28.060
Decrease in consumption in median crisis:	-4.912	-2.764	-2.574	-2.110	-13.898	-7.904
Depreciation RER in median crisis:	-10.714	-5.280	-4.619	-3.400	-19.893	-4.504
CA reversal in median crisis:	7.940	1.193	7.940	0.458	7.054	0.119
Std dev of consumption:	0.065	0.054	0.059	0.053	0.063	0.054
Std dev of RER:	0.304	0.151	0.228	0.113	0.296	0.155
Std dev of CA (as % of GDP):	0.036	0.010	0.023	0.005	0.035	0.012
Corr consumption with GDP:	0.757	0.576	0.649	0.518	0.842	0.644
Corr RER with GDP:	0.792	0.632	0.674	0.560	0.860	0.636
Corr CA (as % of GDP) with GDP:	-0.772	-0.608	-0.633	-0.506	-0.845	-0.562

Note: This table reports the results from the simulation exercise that considers a history of shocks to tradable and non-tradable endowments based on the benchmark distribution. A sudden stop happens when the constraint binds *and* there is a one standard deviation reversal of the current account. The first two columns report the results from economies that feature a robust household and planner with $\theta = 0.35$, columns 3 and 4 refer to agents with a higher degree of robustness ($\theta = 0.15$) and the final two columns concern non-robust agents with a higher risk aversion ($\sigma = 4$, double the value of our baseline calibration).

5 Conclusion

Following the 2008 global financial crisis, macroprudential regulation has become a well-established policy to contain credit booms and reduce financial fragility in both advanced and emerging countries. Against this background, starting with the seminal contributions of Bianchi and Mendoza (2010) and Bianchi (2011), an increasing number of studies have focused on understanding how these policies affect the transmission mechanism driving financial crises. While this literature shows the optimality of adopting macroprudential policies, the prescribed taxes are usually small in size as well as the associated welfare gains. From a policy perspective, this raises the question of whether such policies would still remain optimal once the decision maker acknowledges that there may exist a vast set of alternative statistical models with unknown forms that could fit the data nearly as well as the benchmark model. This paper attempts to answer this question and, thus, characterizes robust optimal macroprudential policy in the context of Bianchi (2011)'s small open economy with occasionally binding collateral constraints.

Without model uncertainty, there exist welfare gains from internalizing how borrowing decisions in good times affect the value of collateral during a crisis. When there is fear of model misspecification by the planner, however, the desirability of macroprudential taxes also depends on the distribution of shocks that we consider for computing welfare gains. This is because a policy that is robust to misspecification, as in Hansen and Sargent (2011), is optimal under a “worst-case” scenario but not under alternative distributions of the state. We find that a robust planner may still achieve a welfare gain almost as large as a non-robust planner when shocks follow the benchmark distribution. If households also make choices that are robust to model misspecification, however, those gains are significantly reduced. This is because the household already saves more as a precaution against future bad shocks, indirectly addressing the overborrowing problem from ignoring the effect of the level of debt on the value of collateral. A highly-robust planner can actually “overshoot” its savings and induce welfare losses. However, if the worst-case scenario that the robust agents fear is the one that actually governs the evolution of shocks, then welfare gains are the largest possible.

The taxes that are required to implement a robust planner's allocation are 5 percentage points higher, and the probability that they need to be implemented is also larger. We also find that a robust planner is able to improve on the results of a non-robust planner, bringing the probability of a crisis from 1.5% to 0.7% and reducing the severity of these episodes. The findings of this paper illustrate the importance of acknowledging model misspecification in the design and evaluation of macroprudential policies.

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6 Appendix

6.1 Figures

Figure 8: Optimal debt policy - all states of nature

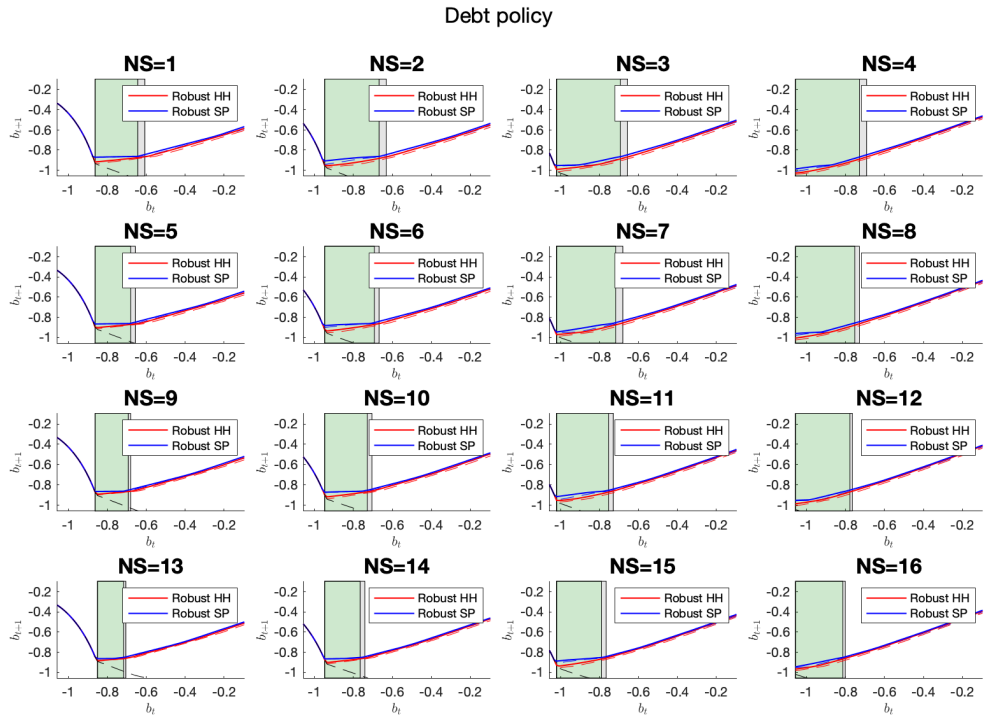
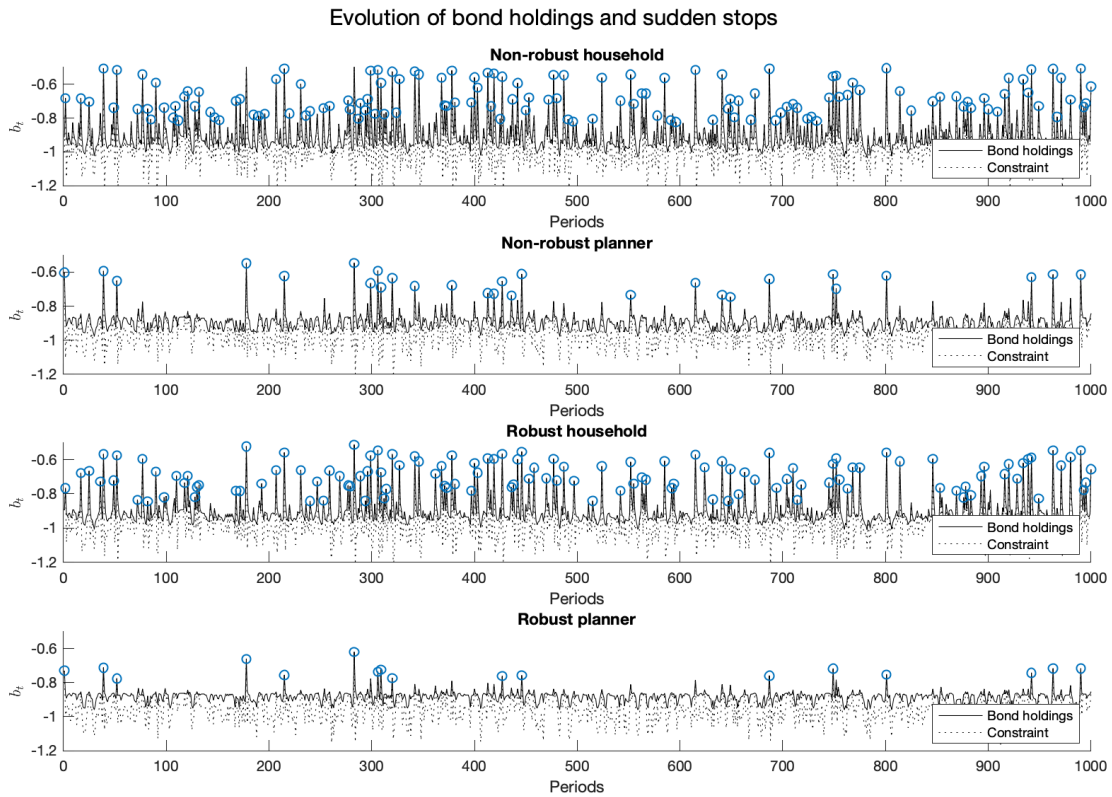


Figure 9: Evolution of bond holdings based on worst-case distribution of shocks



Note: These plots show the evolution of bond holdings from simulations that consider a specific history of shocks to tradable and non-tradable endowments based on the worst-case distribution of shocks that a robust household considers. The same series of shocks is fed into the four cases depicted here. The first plot considers the optimal debt choice of a non-robust household, the second that of a non-robust planner, the third shows the choices from a robust household and the fourth from a robust planner. The blue dots indicate a sudden stop (an increase in bond holdings that exceeds one standard deviation of net capital outflows according to the long-run distribution of bond holdings of the decentralized economy with no government intervention and a non-robust household).

Figure 10: Taxes based on non-robust agents

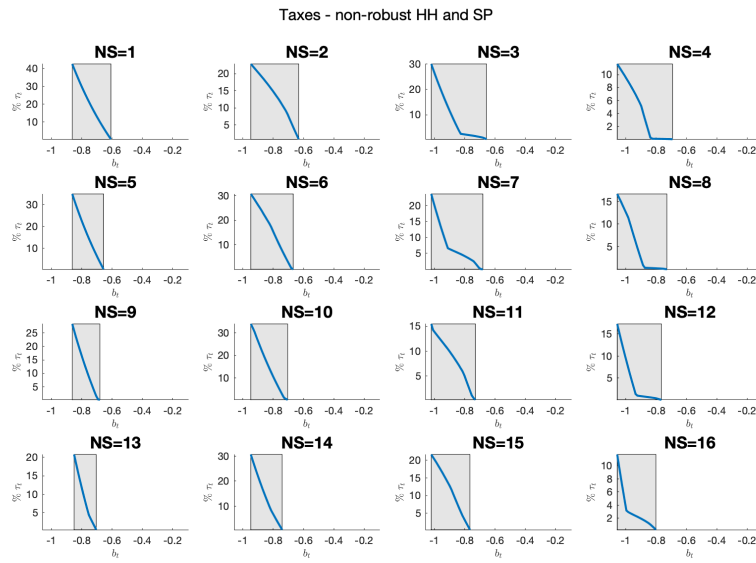


Figure 11: Taxes based on robust agents

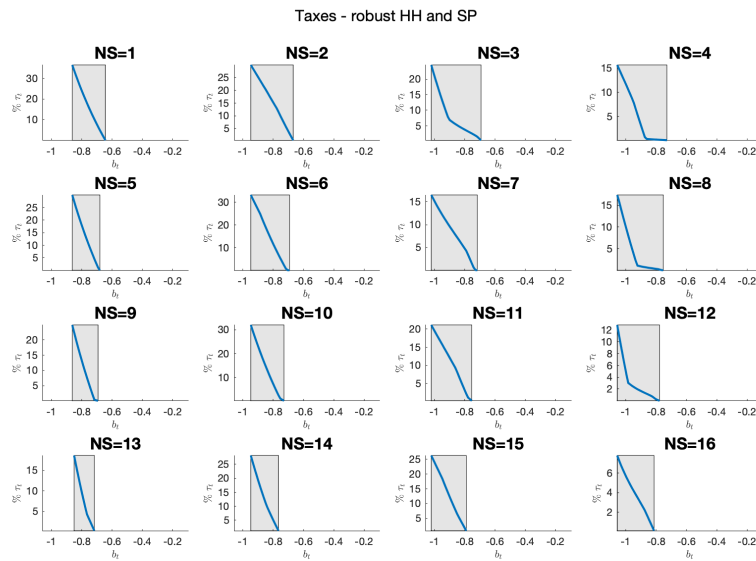


Figure 12: Taxes based on a robust planner and a non-robust household

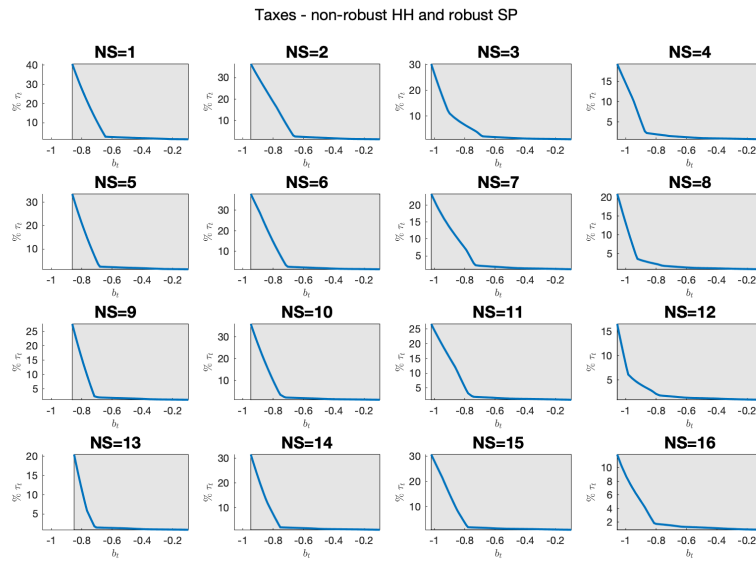


Figure 13: Subsidies based on a non-robust planner and a robust household

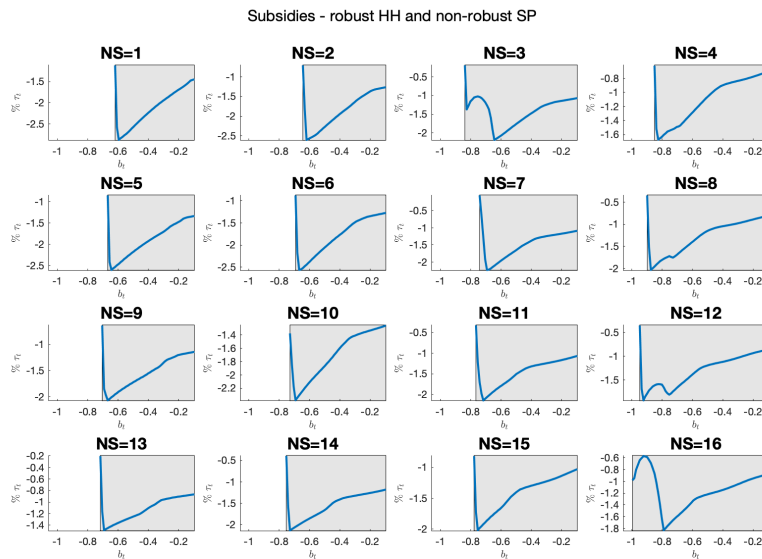


Figure 14: Benchmark allocations and probabilities (Bianchi 2011)

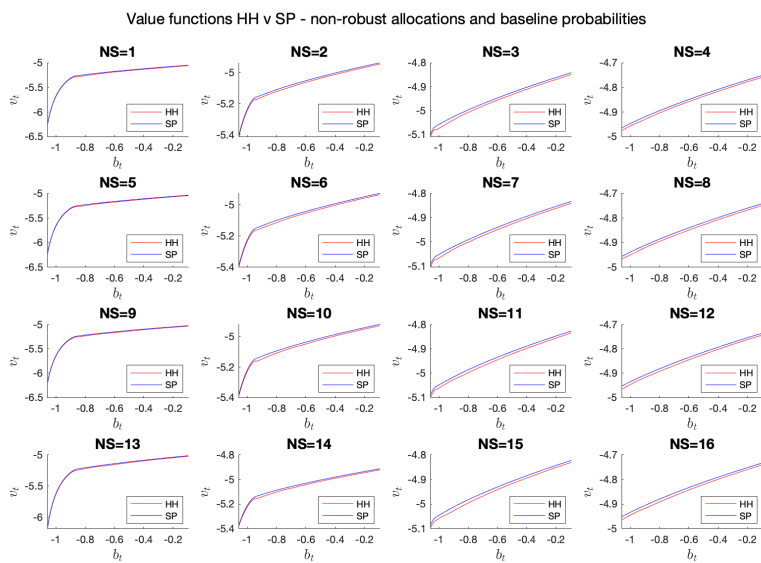


Figure 15: Robust allocations and probabilities

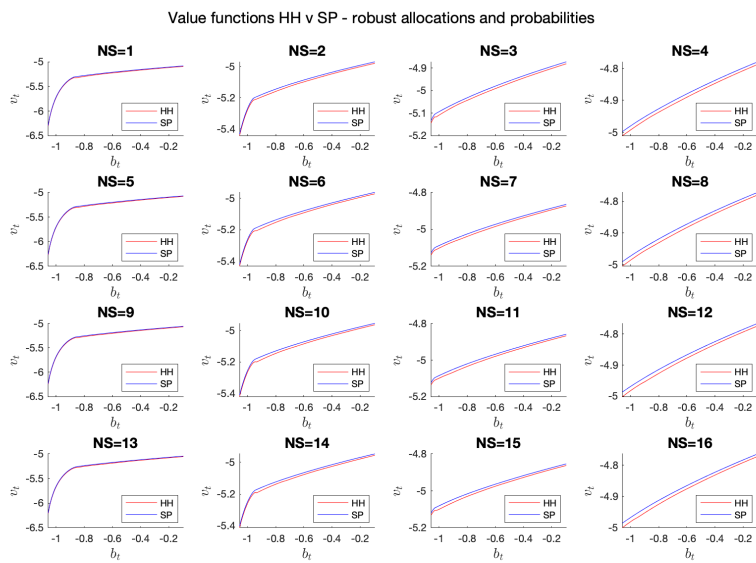


Figure 16: Robust allocations, baseline probabilities

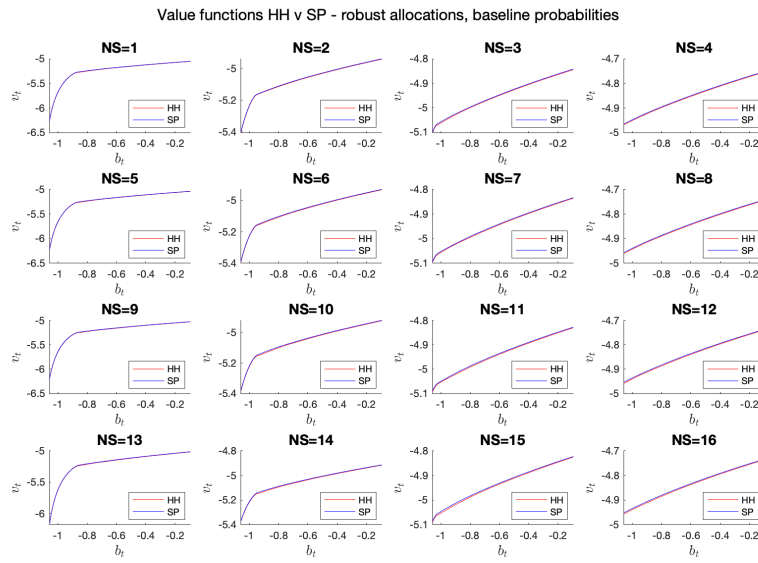


Figure 17: Baseline allocations, robust probabilities

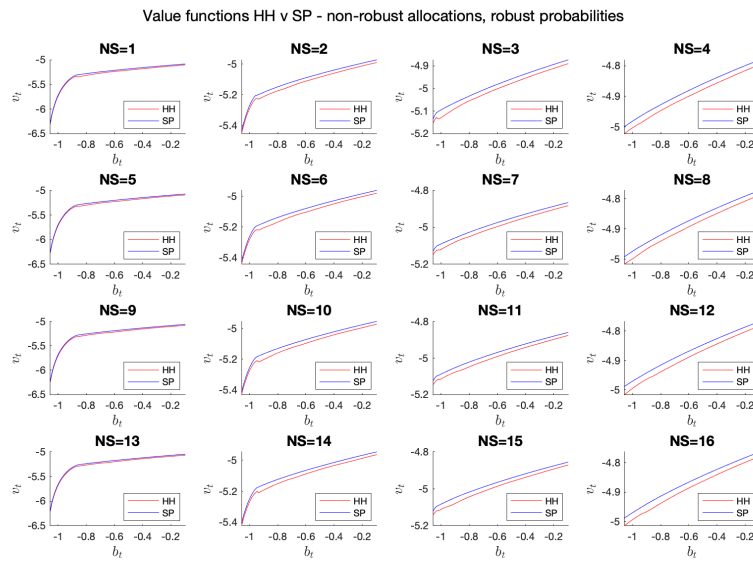


Figure 18: Baseline probabilities, non-robust HH and non-robust SP

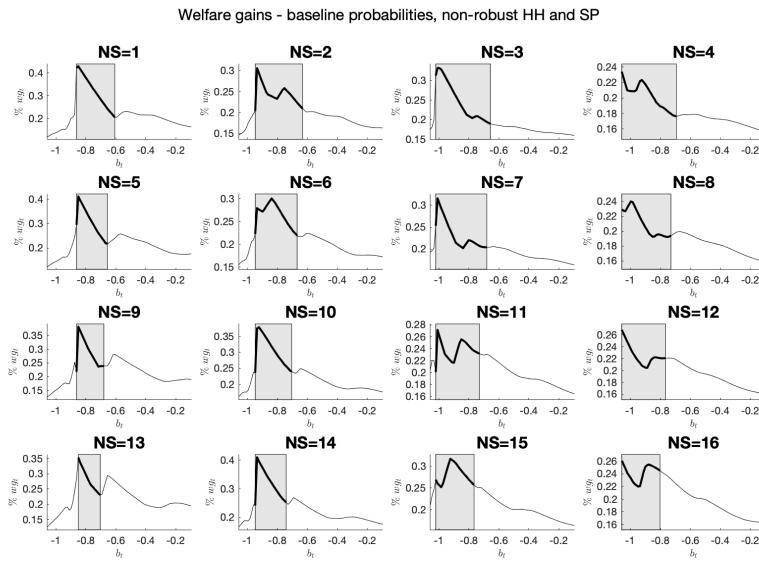


Table 1, [1,1] (Bianchi 2011)

Figure 19: Baseline probabilities, non-robust HH and robust SP

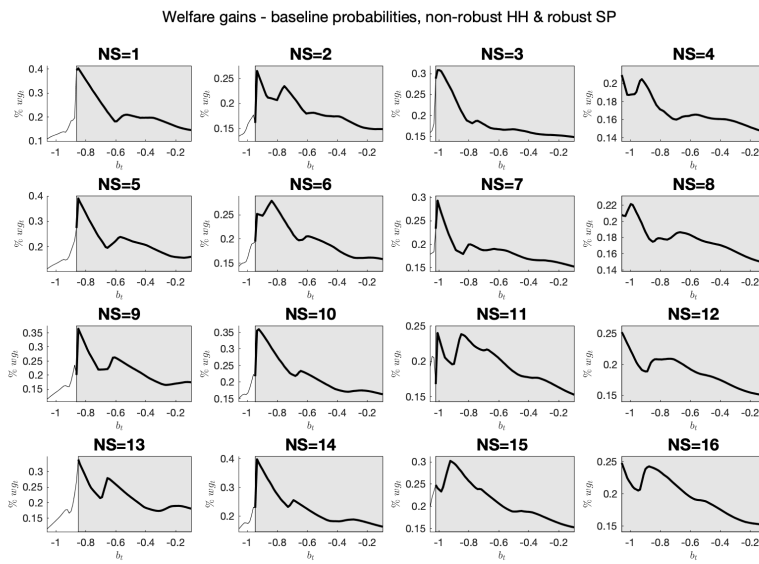


Table 1, [1,2]

Figure 20: Robust probabilities, robust HH and non-robust SP

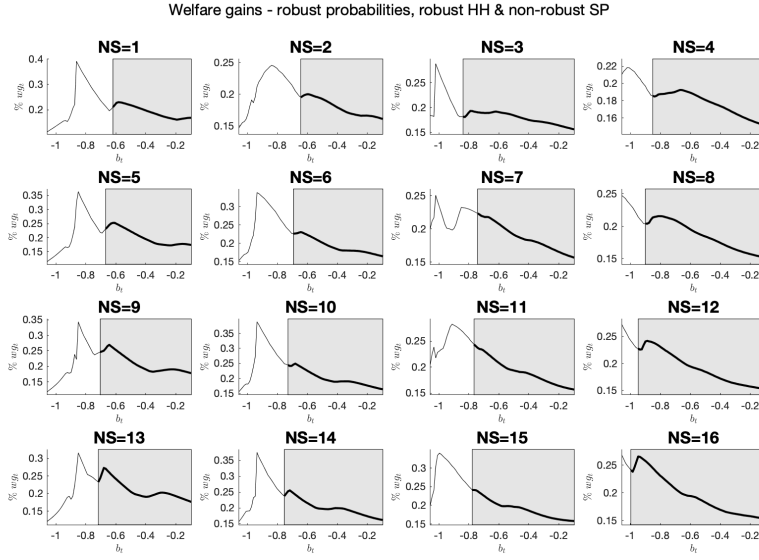


Table 1, [2,1]

Figure 21: Robust probabilities, robust HH and robust SP

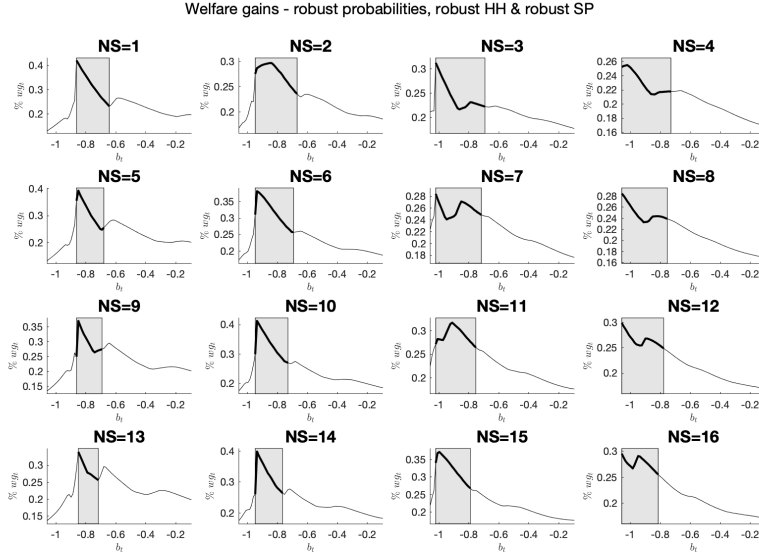


Table 1, [2,2]

Figure 22: Baseline probabilities, robust HH and non-robust SP

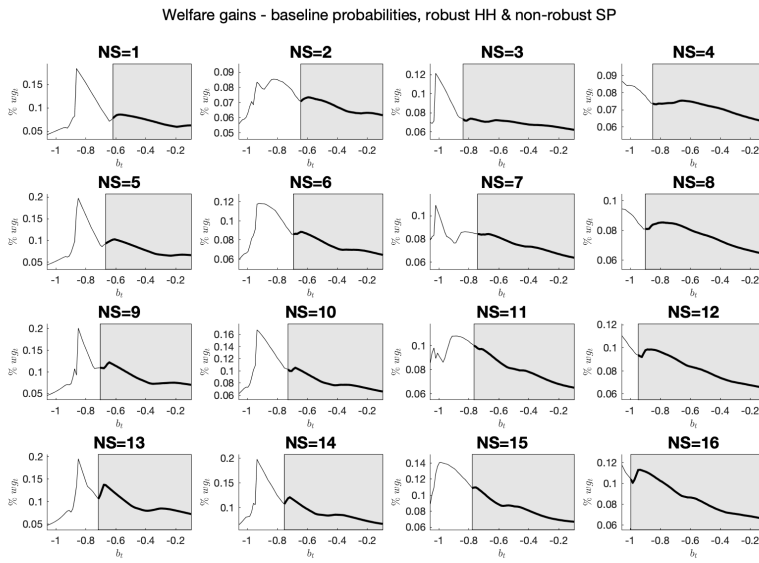


Table 2, [2,1]

Figure 23: Baseline probabilities, robust HH and robust SP

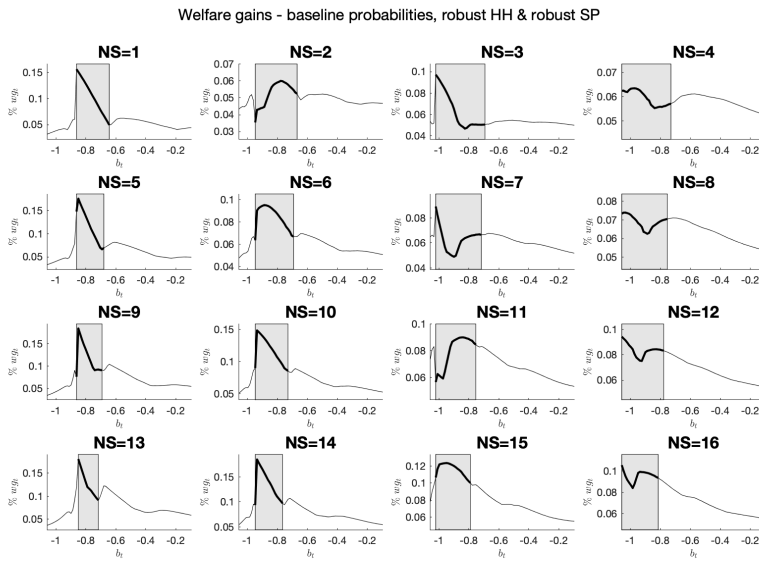


Table 2, [2,2]

Table 10: Average taxes (%)

	Both non-robust	Robust SP	Both robust	Robust HH
NS=1	19.504	7.354	16.753	-2.100
NS=2	13.317	7.781	15.596	-1.828
NS=3	9.127	5.190	8.039	-1.408
NS=4	4.257	3.183	5.059	-1.092
NS=5	16.274	5.484	13.691	-1.876
NS=6	15.676	6.869	15.117	-1.778
NS=7	7.411	4.934	8.085	-1.464
NS=8	5.173	2.978	4.408	-1.263
NS=9	12.410	4.091	10.358	-1.527
NS=10	15.277	5.702	13.887	-1.687
NS=11	8.245	5.169	10.701	-1.444
NS=12	4.324	2.556	3.231	-1.293
NS=13	9.048	2.737	8.072	-1.106
NS=14	13.890	4.751	13.253	-1.528
NS=15	11.506	5.199	13.231	-1.405
NS=16	3.279	2.589	3.863	-1.145

Note: Average taxes over the tax region for each state of nature. These correspond to a calibration that results in a 13% probability of a sudden stop. For robust choices we consider $\theta = 0.35$.

6.2 Proof of Section 2.2

Replace $V_f^{(SP;f^{SP})}(B, y, B)$ according to 42, into 40:

$$\begin{aligned}
& \frac{\left((1 + \gamma)c^{(DE;f^{DE})}(B, y, B) \right)^{1-\sigma}}{1 - \sigma} + \beta E_{y'|y} \left[(1 + \gamma)^{1-\sigma} V_f^{(DE;f^{DE})}(B', y', B') \right] = \\
& = (1 + \gamma)^{1-\sigma} V_f^{(DE;f^{DE})}(B, y, B) \\
& (1 + \gamma)^{1-\sigma} \left\{ \frac{\left(c^{(DE;f^{DE})}(B, y, B) \right)^{1-\sigma}}{1 - \sigma} + \beta E_{y'|y} \left[V_f^{(DE;f^{DE})}(B', y', B') \right] \right\} = \\
& = (1 + \gamma)^{1-\sigma} V_f^{(DE;f^{DE})}(B, y, B)
\end{aligned}$$

Remember that $V_f^{(DE;f^{DE})}(B, y, B)$ satisfies:

$$V_f^{(DE;f^{DE})}(B, y, B) = \frac{\left(c^{(DE;f^{DE})}(B, y, B) \right)^{1-\sigma}}{1 - \sigma} + \beta E_{y'|y} \left[V_f^{(DE;f^{DE})}(B', y', B') \right]$$

Thus, we confirm that 42 is correct.

6.3 Decentralized economy

To find the solution to the decentralized and centralized economies, we use a time-iteration projection method. In this section we describe how to solve for the equilibrium both with and without model misspecification.

This method requires one to work with the optimality conditions written in recursive form. One advantage is we don't need to keep track of aggregate and individual debt as separate state variables in order to capture the fact that the individual isn't internalizing the effect of his debt decision on the current price of non-tradables (the real exchange rate) and on future prices, which affect the agent's expectations. Using the FOCs allows one to impose that, in equilibrium, aggregate and individual debt are equal, while at the same time accounting for the fact that the agent doesn't internalize the pecuniary externality (as captured by the Euler equation). So we impose that $B = b$ and our state space is given by (B, y) .

1. Generate a discrete grid for bonds and output, and define the transition probability matrix for output.
 - Grid for the endogenous state variable, B (aggregate bond position): $G_B = \{B_1, \dots, B_M\}$, $M = 200$.
 - Grid for the exogenous state variables, $y \equiv (y^N, y^T)$ (tradable and non-tradable endowments): $G_y = \{y_1, \dots, y_S\}$, $S = N_T \times N_N$. We'll consider $N_T = N_S = 4$.
 - $\ln(y^T)$ and $\ln(y^N)$ follow a VAR(1). We discretize shocks to these variables using the Tauchen method. This method finds a Markov chain whose sample paths approximate those of the VAR(1) process.

$$\ln(y_{t+1}) = (1 - \rho)\mu + \rho \ln(y_t) + \epsilon_{t+1}$$

$$\epsilon_{t+1} \sim N(0, \sigma_\epsilon)$$

2. Start iteration $K = 1$ with a guess for the individual policy functions for bonds and consumption, and for the equilibrium price of non-tradables. Under robustness, include as well a guess for the value function.

$$c_K^T(B, y)$$

$$b_K'(B, y)$$

$$p_K^N(B, y)$$

$$V_K(B, y)$$

This implies a given expected marginal utility of tradables and, under robustness, a martingale (that allows to compute the expectation with respect to worst-case probabilities), both of which enter the right-hand side of the Euler Equation:

$$\tilde{c}' \equiv \omega(c_K^T(b_K'(B, y), y'))^{-\eta} + (1 - \omega)((y^N)')^{-\eta}$$

$$u_T' \equiv (\tilde{c}')^{\frac{\sigma}{\eta}} (\tilde{c}')^{-\frac{1}{\eta}-1} \omega(c_K^T(b_K'(B, y), y'))^{-\eta-1}$$

$$\Rightarrow RHS \equiv \beta(1 + r)E_{y'|y} [u_T'] \quad \text{if no robustness}$$

$$\Rightarrow RHS \equiv \beta(1 + r)E_{y'|y} \left[\frac{e^{-\frac{V(b_K'(B, y), y')}{\theta}}}{E_{y'|y} \left[e^{-\frac{V(b_K'(B, y), y')}{\theta}} \right]} u_T' \right] \quad \text{if robustness,}$$

where in order to find the expectation, we use the consumption policy, the debt policy and, under robustness, the value function that we started with in this iteration and apply interpolation over the grid.

Notice we have already incorporated market clearing for non-tradable goods.

3. Suppose the constraint binds. Compute the binding values of consumption and debt:

$$\underline{b}(B, y) = -(\kappa^N p_K^N(B, y)y^N + \kappa^T y^T)$$

$$\bar{c}(B, y) = B(1 + r) + y^T - \underline{b}(B, y)$$

The associated current marginal utility with this level of consumption is:

$$\tilde{c} \equiv \omega(\bar{c})^{-\eta} + (1 - \omega)(y^N)^{-\eta}$$

$$u_T \equiv \tilde{c}^{\frac{\sigma}{\eta}} \tilde{c}^{-\frac{1}{\eta}-1} \omega(\bar{c})^{-\eta-1}$$

4. The Euler Equation is:

$$u_T = RHS \text{ if constraint not binding today}$$

$$u_T = RHS + \mu \text{ if constraint binding today}$$

Where μ is the multiplier from the collateral constraint.

If under the consumption levels derived from the constraint, $u_T > RHS$, i.e., $\mu > 0$, then the agent would want to consume more tradables today but she can't because she has hit the maximum level of consumption according to the collateral constraint. So the collateral constraint is indeed binding:

$$c_{K+1}^T(B, y) = \bar{c}(B, y)$$

$$b'_{K+1}(B, y) = \underline{b}(B, y)$$

If under the consumption levels derived from the constraint $u_T < RHS$, i.e. $\mu < 0$, then the agent would want to reduce consumption today, so the collateral constraint is not binding.

Use *fminbnd* to find the level of current consumption that makes $u_T = RHS$.

5. Find the price of non-tradables:

$$p_{K+1}^N(B, y) = \frac{1 - \omega}{\omega} \left(\frac{y^T + B(1 + r) - b'_{K+1}(B, y)}{y^N} \right)^{\eta+1}$$

Note: the debt limit $b'_{K+1}(B, y) = \underline{b}(B, y)$ was obtained assuming the price of non-tradables is $p_K^N(B, y)$. The new price for non-tradables is obtained from $c_{K+1}^T(B, y)$ and will imply a

different debt limit. When plotting, use $p_K^N(B, y)$, the debt limit associated to this price and $c_{K+1}^T(B, y)$.

6.4 Centralized economy

1. Same as decentralized economy.
2. Same as decentralized economy but with the following distinction:

$$\Rightarrow RHS \equiv \beta(1+r)E_{y'|y}[u'_T] \quad \text{if not robust + constraint not binding tomorrow}$$

$$\Rightarrow RHS \equiv \beta(1+r)E_{y'|y} \left[\frac{e^{-\frac{V(b'_K(B,y),y')}{\theta}}}{E_{y'|y} \left[e^{-\frac{V(b'_K(B,y),y')}{\theta}} \right]} u'_T \right] \quad \text{if robust + constraint not binding tomorrow}$$

$$\Rightarrow RHS \equiv \beta(1+r)E_{y'|y}[u'_T + \phi' \mu'] \quad \text{if not robust + constraint binding tomorrow}$$

$$\Rightarrow RHS \equiv \beta(1+r)E_{y'|y} \left[\frac{e^{-\frac{V(b'_K(B,y),y')}{\theta}}}{E_{y'|y} \left[e^{-\frac{V(b'_K(B,y),y')}{\theta}} \right]} (u'_T + \phi' \mu') \right] \quad \text{if robust + constraint binding tomorrow,}$$

where ϕ' is the marginal effect on the future debt limit to changes in future consumption of tradables:

$$\phi' \equiv \kappa^N y^N \frac{\partial p^N}{\partial c^T} = \frac{1-\omega}{\omega} \left(\frac{c_K^T(b'_K(B,y),y')}{(y^N)'} \right)^\eta (\eta+1) \kappa^N$$

3. Same as decentralized economy.
4. Same as decentralized economy but with the following distinction:

The Euler Equation is:

$$u_T = RHS \quad \text{if constraint not binding today}$$

$$u_T + \mu\phi = RHS + \mu \quad \text{if constraint binding today}$$

Where ϕ is the marginal effect on the debt limit to changes in consumption of tradables and μ is the multiplier from the collateral constraint:

$$\phi \equiv \kappa^N y^N \frac{\partial p^N}{\partial c^T} = \frac{1 - \omega}{\omega} \left(\frac{c_K^T(B, y)}{y^N} \right)^\eta (\eta + 1) \kappa^N$$

$$\mu = \frac{u_T - RHS}{1 - \phi}$$