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Public Debt Dynamics and Intra-Year Exchange Rate Fluctuations

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Intra-Year Exchange Rate Fluctuations and Public Debt Dynamics

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Abstract

The public sector, in carrying out its operations, often incurs foreign currency denominated liabilities and, as such, is exposed to exchange rate fluctuations that could affect the value of public debt to GDP ratios over time. This paper shows that converting foreign currency denominated *flows* and *stocks* into local currency using the average and the end-of-period exchange rates, respectively, as envisaged in public finance manuals, gives rise to an identifiable stock-flow adjustment term—due to intra-year exchange rate fluctuations—that affects public debt accumulation. Importantly, the inclusion of this often-ignored stock-flow adjustment term is critical to accurately project public debt levels and any related indicator that could in turn inform about the risk of debt distress. Using a novel dataset covering 82 countries during 2008–19, the paper shows that this stock flow adjustment term is sizable in countries experiencing large exchange rate depreciations, namely above the 99th percentile of the full sample, reaching 1.2 percent of GDP. Interestingly, the measurement of policy-related concepts such as interest rate-growth differentials and debt stabilizing primary balances are also affected by intra-year exchange rate fluctuations, and in non-negligible ways.

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I. INTRODUCTION

Periods of heightened exchange rate volatility can lead to substantial revaluations of the public sector liabilities denominated in foreign currency through increases in the local currency value of *flows,* e.g., the interest payments on foreign currency denominated debt, and of *stocks*, e.g., the outstanding amount of foreign currency denominated debt, potentially undermining solvency and/or liquidity considerations in certain countries.[1]

Because of its prominent significance in many emerging and developing economies, where the public sector is often exposed to liabilities denominated in foreign currency, a careful assessment of how exchange rate fluctuations affect *flows* and *stocks* is critical for determining how public debt evolves over time. The Excel-based templates used by the IMF to assess public debt sustainability in low-income and market-access countries do this in a precise manner, for instance, by disentangling carefully the currency of denomination and the exchange rates used for the conversion into local currency of the *below-the-line* or financing components of the public sector over time.[2]

However, the existing literature that assesses *above-the-line* public debt dynamics with foreign currency denominated liabilities, namely by measuring the public sector debt path over time through the overall fiscal balance (or primary balance and interest bill), has abstracted away from the importance of using different exchange rates to convert foreign currency *flows* and *stocks* into local currency (e.g., Croce and Juan-Ramón, 2003; IMF, 2013; Escolano and others, 2017). Since the standard convention in public finance manuals is that debt-related stocks and flows should be converted into local currency using the *end-of-period* and *average* exchange rates, respectively (see IMF, 2011, pp. 26–27), ignoring this accounting principle could lead to measurement errors in the projection of public debt in countries exposed to foreign currency liabilities.

The objective of this paper is to lay out the basic *above-the-line* public sector debt accumulation equation when a portion of debt is denominated in foreign currency, but incorporating the fact that the average and the end-of-period exchange rates should be used simultaneously for the conversion of flows and stocks into the local currency. This gives rise to a stock-flow adjustment term stemming from intra-year exchange rate valuation effects, which becomes an additional and identifiable component that affects the evolution of public debt. Importantly, understanding carefully public debt dynamics *above-the-line*, permits to make public debt rules easily operationalizable by quantifying, for instance, what would be the relevant interest rate-growth differential that affects debt changes and/or the primary balance that would be needed to stabilize debt at the previous period level. Moreover, the inclusion of this often-ignored stock-flow adjustment term is critical to accurately project public debt levels and any associated indicator that could in turn inform about the risk of debt distress.

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[1] Similar considerations are relevant when assessing the foreign currency liabilities of the private sector. However, since the focus of the paper is only on public sector debt, these are not evaluated here.

[2] See IMF (2013, 2018) for details on the so-called MAC DSA and LIC DSF frameworks.

To evaluate in which cases intra-year exchange rate fluctuations have been relevant from a quantitative standpoint, a novel dataset covering 82 countries during the period 2008–19 is assembled. With this dataset in hand, the paper then evaluates how this type of exchange rate valuation effect affects the interest rate-growth differential, public debt projections and the debt-stabilizing primary balance.

The paper shows that, while not sizeable on average, the stock-flow adjustment term arising from intra-year exchange rate fluctuations was large in countries that experienced significant nominal depreciations. While the full sample mean of this term is 0.02 percent of GDP, it reaches 0.2 and 1.2 percent of GDP in those countries that went through large nominal depreciations, namely above the 90th and 99th percentile of the full sample, respectively.[3] Next, for the full sample, the interest rate-growth differential is reduced on average about 0.04 percentage points once intra-year exchange rate fluctuations are accounted for, but this differential is 0.6 percentage points lower in countries that experienced depreciations above the 99th percentile of the full sample. In those countries, estimations suggest that in the year of the large nominal depreciation debt projections could have been *underestimated* by about 0.9 percentage points of GDP while the debt-stabilizing primary balances could have been *overestimated* by about 1.5 percentage points of GDP, if intra-year exchange rate valuation effects were not properly accounted for.

The rest of the paper is organized as follows. Section II introduces the basic *above-the-line* public debt accumulation equation when intra-year exchange rate fluctuations are considered. Section III then turns into a discussion of the analytics to assess policy-relevant implications. Section IV introduces a novel dataset to quantify the relevance of intra-year exchange rate valuation effects, while Section V summarizes the main quantitative results. Finally, Section VI concludes.

II.   PUBLIC SECTOR DEBT DYNAMICS

Existing papers that assess public debt dynamics considering that public debt can either be denominated in local or in foreign currency, as it is observed in many emerging and developing countries, specify a dynamic equation for gross public debt in nominal terms as follows (e.g., Escolano and others, 2017; IMF, 2013):

$$D_t = (e_t^{eop} D_{t-1}^f + D_{t-1}^d) + e_t^{eop} i_t^f D_{t-1}^f + i_t^d D_{t-1}^d - PB_t + OF_t, \qquad (1)$$

where D_t ($\equiv e_t^{eop} D_t^f + D_t^d$) refers to the total stock of gross public debt measured in local currency at the end of period t; D_t^f and D_t^d are public debt denominated in foreign and in local currency, respectively, at the end of period t; e_t^{eop} indicates the end-of-period exchange rate (local currency per unit of the foreign currency) at period t; i_t^f and i_t^d are the effective

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[3] Currently, the IMF MAC DSA and IMF/WB LIC DSF frameworks allocate intra-year exchange rate valuation effects as part of a 'residual' term that is shown in output tables. Given that these are flows that can be identified, in principle these valuation effects can be stripped out from that 'residual' to be presented as a stand-alone component the same way as any other identifiable debt-creating flow.

nominal interest rates on foreign and local currency denominated debt during period t, respectively;[4,5] and PB_t and OF_t refer to the primary balance (i.e., primary government revenue minus primary government expenditure) and other net debt-creating flows (e.g., realization of contingent liabilities, privatizations, use/build-up of liquid assets) that take place during period t.[6,7]

This specification, however, abstracts away from the fact that the *flow* interest payment on foreign currency debt, $i_t^f D_{t-1}^f$, is *paid throughout the year*, and as such it should be converted to local currency using the *average*, instead of the *end-of-period,* exchange rate (see IMF, 2011, p. 26-27). Acknowledging this, (1) could be re-written as:

$$D_t = (e_t^{eop} D_{t-1}^f + D_{t-1}^d) + e_t^{avg} i_t^f D_{t-1}^f + i_t^d D_{t-1}^d - PB_t + OF_t + SF_t. \tag{2}$$

When comparing (1) and (2) it is apparent that besides the use of the average exchange rate for the foreign currency interest bill, an additional term, SF_t, is included. This is a stock-flow adjustment element that takes place during period t, whose role is to 'adjust' for the different exchange rates used to convert flows and stocks from the foreign and into the local currency, such that (2) holds. Although SF_t is taken as exogenous in (2), it should be endogenized to properly measure the evolution of public debt over time in local currency.

Defining $SF_t = \left(e_t^{eop} - e_t^{avg}\right)\left(D_t^f - D_{t-1}^f\right)$, it is now possible to obtain the analytical term that *revalues* foreign currency *flows* from the *average* and to the *end-of period* exchange rate, to ensure that public debt measured in local currency is properly accounted for. Then, (2) takes the form,

$$D_t = (e_t^{eop} D_{t-1}^f + D_{t-1}^d) + e_t^{avg} i_t^f D_{t-1}^f + i_t^d D_{t-1}^d - PB_t + OF_t \tag{3}$$
$$+ \left(e_t^{eop} - e_t^{avg}\right)\left(D_t^f - D_{t-1}^f\right).$$

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[4] Note that the effective interest rates i_t^f and i_t^d may subsume the effects of interest payments on *existing* debt and on *new* debt issuances that accrue interest during period t.

[5] Observe that the effective interest rate on foreign currency debt is given by $i_t^f = \frac{I_t^f}{D_{t-1}^f}$, where I_t^f denote the interest payment on foreign currency debt accruing during period t. Since both I_t^f and D_{t-1}^f are expressed in the foreign currency, i_t^f is not affected by exchange rate fluctuations.

[6] Strictly speaking, other net debt-creating flows likely involve financing components of the budget. In that sense, neither (1) nor (2) is a 'pure' above-the-line relation. However, including this term is relevant to determine accurately the key drivers of gross public debt over time (see, e.g., IMF, 2013).

[7] IMF (2013) also includes a residual term in (1) to ensure that the accounting identity that determines public debt holds. However, since there is no guidance provided on how such residual can be decomposed into identifiable terms to project public debt, this term is excluded from (1).

Note that (1) and (3) are identical if $e_t^{avg}/e_t^{eop}=1$ and/or if the change in foreign currency borrowing is such that $D_t^f - D_{t-1}^f = i_t^f D_{t-1}^f$. However, the two equations may differ if intra-year exchange rate fluctuations are present, namely if $e_t^{avg}/e_t^{eop} \neq 1$, as it would occur in countries with foreign currency debt experiencing exchange rate appreciations or depreciations throughout the year.[8]

To better understand the implications of intra-year exchange rate valuation effects, it is useful to define the share of foreign currency denominated debt in total debt as $\alpha_t = e_t^{eop} D_t^f / D_t$ and the rate of change of the end-of-period exchange rate as $\varepsilon_t^{eop} = e_t^{eop}/e_{t-1}^{eop} - 1$. In addition, it is also useful to introduce the valuation-adjusted average nominal effective interest rate (that is, the overall effective interest rate paid by the government including capital gains/losses on the principal due to exchange rate fluctuations) as:

$$i_t^{iyxr} = \left[\alpha_{t-1}(1 + i_t^f)(1 + \varepsilon_t^{eop})e_t^{avg}/e_t^{eop} + (1 - \alpha_{t-1})(1 + i_t^d)\right](1 - \alpha_t \gamma_t)^{-1} - 1, \quad (4)$$

where $\gamma_t \equiv (1 - e_t^{avg}/e_t^{eop})$ is a wedge between the end-of-period and the average exchange rates measured as a fraction of the former. Note that (4) simplifies, when assuming $e_t^{avg}/e_t^{eop}=1$, to

$$i_t^{no\_iyxr} = \alpha_{t-1}(1 + i_t^f)(1 + \varepsilon_t^{eop}) + (1 - \alpha_{t-1})(1 + i_t^d) - 1. \quad (5)$$

From here onwards, variables with superscripts *iyxr* and *no_iyxr* indicate that intra-year exchange rate fluctuations are and are not included, respectively. Note that (4) is the relevant net effective cost of public debt when $e_t^{avg}/e_t^{eop} \neq 1$.[9] After some rearrangement and using

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[8] To better understand why (3) properly computes debt levels, and why this equation also requires a stock-flow adjustment term, consider a simple case where $D_t^d = D_{t-1}^d = OF_t = PB_t = 0$. Since there is only foreign currency borrowing, it follows that the total stock of public debt measured in foreign currency is $D_t^f = D_{t-1}^f + i_t^f D_{t-1}^f$. Assuming that $D_{t-1}^f = US\$100$ and that $i_t^f = 0.1$, it follows that $D_t^f = US\$110$ at the end of period t. If the end-of-period and the average exchange rates are say 3 LC\$/US\$ and 2 LC\$/US\$, respectively, then the total stock of debt at the end of period t *expressed in local currency* is $D_t = e_t^{eop} D_t^f = LC\$330$. It can be easily checked that this total can be equivalently obtained from (3) as $D_t = e_t^{eop} D_{t-1}^f + e_t^{avg} i_t^f D_{t-1}^f + \left(e_t^{eop} - e_t^{avg}\right)\left(D_t^f - D_{t-1}^f\right)$. Moreover, suppose now that $PB_t = LC\$20$ (instead of 0), namely a primary balance surplus that exactly covers the flow interest payments on foreign currency debt. The total stock of debt D_t at the end of period t *expressed in local currency* now becomes $D_t = LC\$300$ (i.e., the total stock of foreign currency debt does not change between periods), an amount which can be exactly obtained again using (3). However, when using (1) instead, the stock of debt would have been *overestimated* at $LC\$310$.

[9] It is useful to briefly outline how to 'match' the data obtained from fiscal accounts into this valuation-adjusted nominal effective interest rate. From fiscal statements one might be able to obtain the total interest bill paid for local and foreign currency debt altogether, but likely already converted into local currency (i.e., Int_t). The total stock of debt (i.e., D_t), is also likely to be reported in local currency. Thus, the effective nominal interest rate from fiscal accounts is equal to $i_t^{fisc} = \frac{Int_t}{D_{t-1}} = \alpha_{t-1} i_t^f (1 + \varepsilon_t^{eop})e_t^{avg}/e_t^{eop} + (1 - \alpha_{t-1})i_t^d$. It then follows that $i_t^{iyxr} = \left[i_t^{fisc} + \alpha_{t-1}(1 + \varepsilon_t^{eop})e_t^{avg}/e_t^{eo} + (1 - \alpha_{t-1})\right](1 - \alpha_t \gamma_t)^{-1} - 1$.

(4) and (5), it is then possible to express the *change* in public debt as a share of nominal GDP at time $t\ (=P_t Y_t)$ with and without the inclusion of intra-year exchange rate effects as follows,

$$d_t^{iyxr} - d_{t-1}^{iyxr} = \left[\frac{1 + i_t^{iyxr}}{(1 + g_t)(1 + \pi_t)} - 1\right] d_{t-1}^{iyxr} + \frac{1}{1 - \alpha_t \gamma_t}(-pb_t + of_t), \tag{6}$$

and

$$d_t^{no\_iyxr} - d_{t-1}^{no\_iyxr} = \left[\frac{1 + i_t^{no\_iyxr}}{(1 + g_t)(1 + \pi_t)} - 1\right] d_{t-1}^{no\_iyxr} - pb_t + of_t. \tag{7}$$

As usual, lower-case variables refer to shares of GDP; d_t^{iyxr} is the debt-to-GDP ratio when intra-year exchange rate fluctuations are considered (i.e., derived from (3)); $d_t^{no\_iyxr}$ refers to the debt-to-GDP ratio when intra-year exchange rate fluctuations are not considered (i.e., derived from (1)); g_t is the real GDP growth rate at time t; and π_t is domestic inflation at time t as measured by the GDP deflator.[10]

It follows that (6) and (7) are identical when $e_t^{avg}/e_t^{eop} = 1$, yet they could differ if $e_t^{avg}/e_t^{eop} \neq 1$. Before turning to a quantitative assessment, it is useful to discuss the analytics of two policy-related implications.

III.   POLICY-RELATED IMPLICATIONS

The interest rate-growth differential (IRGD) and the debt-stabilizing primary balance (DSPB) at the previous-period debt level can now be easily computed from either (6) or (7). The IRGD provides an indicator of how public debt would change over time (as a share of GDP) absent policy actions, and is a key determinant of the 'automatic-debt-dynamics' portion of debt changes as identified in (6) and (7). The DSPB summarizes what would be the appropriate primary balance to stabilize public debt at its previous-period level. From (6) and (7) the IRGD will also depend on whether intra-year exchange rate fluctuations are considered. Formally, the IRGD is given by

$$IRGD_t^{iyxr} = \frac{1 + i_t^{iyxr}}{(1 + g_t)(1 + \pi_t)} - 1, \tag{8}$$

and

$$IRGD_t^{no\_iyxr} = \frac{1 + i_t^{no\_iyxr}}{(1 + g_t)(1 + \pi_t)} - 1. \tag{9}$$

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[10] To obtain (6) it is helpful to realize that the stock-flow adjustment term as a share of GDP is: $sf_t = \frac{SF_t}{Y_t P_t} = \frac{(e_t^{eop} - e_t^{avg})(D_t^f - D_{t-1}^f)}{Y_t P_t} = \frac{(e_t^{eop} - e_t^{avg})}{e_t^{eop}}\left(\alpha_t d_t - \alpha_{t-1} d_{t-1} \frac{(1 + \varepsilon_t^{eop})}{(1 + g_t)(1 + \pi_t)}\right).$

The expression for $IRGD_t^{no\_iyxr}$ in (9) is the same as that of Escolano and others (2017, p. 183). However, this does not fully account for exchange rate valuation effects the same way as $IRGD_t^{iyxr}$ in (8) does. Next, by setting $d_t^{iyxr} - d_{t-1}^{iyxr} = 0$ and $d_t^{no\_iyxr} - d_{t-1}^{no\_iyxr} = 0$ in (6) and (7), the debt-stabilizing primary balances (at the previous-period debt levels) are given by,

$$DSPB_t^{iyxr} = \left[\frac{1 + i_t^{iyxr}}{(1 + g_t)(1 + \pi_t)} - 1 \right] (1 - \alpha_t \gamma_t) d_{t-1}^{iyxr} + of_t, \tag{10}$$

and

$$DSPB_t^{no\_iyxr} = \left[\frac{1 + i_t^{no_{iyxr}}}{(1 + g_t)(1 + \pi_t)} - 1 \right] d_{t-1}^{no\_iyxr} + of_t. \tag{11}$$

As with the IRGD, (10) and (11) are identical if $e_t^{avg}/e_t^{eop} = 1$, yet they would differ if $e_t^{avg}/e_t^{eop} \neq 1$.

To quantitatively illustrate the implications of these relations, consider an economy where in a given year t $\alpha_t = 0.6$, $i_t^f = i_t^d = 0.05$, $g_t = \pi_t = 0.03$, $pb_t = -0.1$ and $of_t = 0.05$, and that these figures are expected to remain the same in year $t+1$, with the exception of α_{t+1}, which is expected to increase to $\alpha_{t+1} = 0.7$. Assume also that in period t $e_t^{avg} = 1.25$ LC\$/US\$ and $e_t^{eop} = 1.50$ LC\$/US\$, but that in period $t+1$ it is expected that $e_{t+1}^{avg} = 2.25$ LC\$/US\$ and $e_{t+1}^{eop} = 3.00$ LC\$/US\$. That is, a large nominal exchange rate depreciation is anticipated for the coming year, leading to an increase in the gap between the average and the end-of-period exchange rates.

The valuation-adjusted nominal effective interest rates can be calculated using (4) and (5), yielding $i_{t+1}^{iyxr} = 65.5$ and $i_{t+1}^{no\_iyxr} = 68.0$ percent. That is, the relevant effective rate on debt would be *overestimated* if intra-year exchange rate fluctuations were not properly accounted for. A similar pattern can be observed for the IRGD, yielding $IRGD_{t+1}^{iyxr} = 56.0$ and $IRGD_{t+1}^{no\_iyxr} = 58.4$ percent. Further, if the debt-to-GDP ratio in period t is assumed to be 50 percent (i.e., $d_{t-1}^{iyxr} = d_{t-1}^{no\_iyxr} = 0.5$), then from (6) and (7) debt projections would be $d_t^{iyxr} = 96.2$ and $d_t^{no\_iyxr} = 94.2$ percent of GDP. That is, the lower overall cost of debt assumed *with* intra-year exchange rate fluctuations is more than compensated by the relatively larger contribution of the primary deficit and other net debt-creating flows to the evolution of public debt, leading to an overall higher public debt projection.[11] The debt-stabilizing primary balance needed to keep the debt ratio at 50 percent of GDP can be obtained from (10) and (11), yielding $DSPB_{t+1}^{iyxr} = 28.1$ and $DSPB_{t+1}^{no\_iyxr} = 34.2$ percent of GDP. Interestingly, ignoring intra-year valuation effects would have led to an

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[11] Observe from (6) the contribution of the primary balance and other net debt-creating flows in explaining public debt changes, when including intra-year valuation effects, is augmented by the term $\frac{1}{1 - \alpha_t \gamma_t} = 1.2$ under this parameterization.

underestimation of debt levels and an *overestimation* of the primary balance needed to stabilize debt at the previous-period level. The direction of results in the simulation can change significantly depending on whether the exchange rate is expected to appreciate or depreciate, whether the share of foreign currency debt in total debt, α_t, is expected to change between periods, and on the evolution of the primary balance and other net debt-creating flows. But under plausible parameterizations differences between the average and the end-of-period exchange rates would have relevant policy implications.

The remaining of the paper evaluates the relevance of intra-year exchange rate fluctuations in the observed data, and whether public debt projections and the associated concepts of IRGD and DSPB are affected by differences between the average and the end-of-period exchange rates, namely if $e_t^{avg}/e_t^{eop} \neq 1$.

IV. DATASET

To evaluate the empirical relevance of intra-year exchange rate valuation effects, a new dataset is assembled using data from the IMF's World Economic Outlook (WEO), April 2020 release. In addition, the dataset is augmented with the IMF's MAC DSA dataset to include a decomposition of gross public debt and effective interest rates into their local and foreign currency components. Overall, the dataset covers a sample of 82 advanced and emerging economies that reported a positive share of public debt denominated in foreign currency out of total gross public debt during the period 2008–19 (Table 1).[12,13]

Table 1: Descriptive statistics
(sample 2008–19, in percent, unless otherwise noted)

Variables	Obs	Mean	Std Dev	Min	Max
Total gross public debt / GDP	574	51.54	27.55	7.60	172.18
Gross public debt in FX / total debt	574	38.78	24.12	0.03	97.98
Gross public debt in LC / total debt	574	61.22	24.12	2.02	99.97
Eff. nominal interest rate on FX debt	574	3.91	2.35	-1.04	30.91
Eff. nominal interest rate on LC debt	574	6.50	3.21	-1.93	19.17
Real GDP growth	574	3.13	2.80	-9.77	11.34
GDP deflator inflation	574	4.59	6.49	-20.19	51.51
Primary balance / GDP	574	-0.80	3.42	-16.10	16.38
Nominal exchange rate depreciation	574	4.42	12.40	-17.30	104.14
Avg to end-of-period XR (ratio)	574	0.98	0.05	0.68	1.11

Source: Author's calculations based on IMF's WEO and MAC DSA databases.

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[12] Unfortunately, a dataset on the currency of denomination of gross public debt and the associated effective nominal interest rates for low-income countries is not readily available, thereby restricting the sample of countries used in this analysis to market-access countries as defined by IMF (2013).

[13] The final sample used in the analysis is restricted relative to the original one that has been created, since only those country/year observations with non-missing values for effective interest rates in local and foreign currencies are considered. Importantly, however, there is no apparent bias in terms of which country/year observations are disregarded due to lack of available data on effective interest rates on local and foreign currency debt.

From Table 1, it follows that for the 82 countries of this dataset, on average, about 39 percent of total debt is denominated in foreign currency (roughly about 20 percent of GDP). As expected, effective interest rates in foreign currency are below effective rates in local currency, yet this difference is not fully accounted for *realized* exchange rate depreciations. Next, on average an annual primary deficit is observed at 0.8 percent of GDP for the full sample, and the average nominal GDP growth is about 7.7 percent annually. Finally, on average the ratio of the average to the end-of-period exchange rate is slightly below 1 at 0.98 (consistent with a trend depreciation over time), but with significant differences between the minimum (0.68) and the maximum (1.11) values of the sample. The next section evaluates the quantitative relevance of the previous analysis using this dataset.

V. RESULTS

A. Evidence on intra-year exchange rate fluctuations

To underscore the relevance of intra-year exchange rate fluctuations, Figure 1 plots the ratio between the average and the end-of-period exchange rates for the subset of the sample that experienced nominal depreciations below the 10th and above the 90th percentiles of the full sample. The objective is to assess if in those cases the ratio between the average and the end-of-period exchange rates can be substantially different from the full sample mean (0.98), a fact that may suggest the need for a careful assessment of the exchange rates used to convert flows and stocks to local currency at the time of undertaking public debt projections.

Figure 1: Ratio of average to end-of-period exchange rates
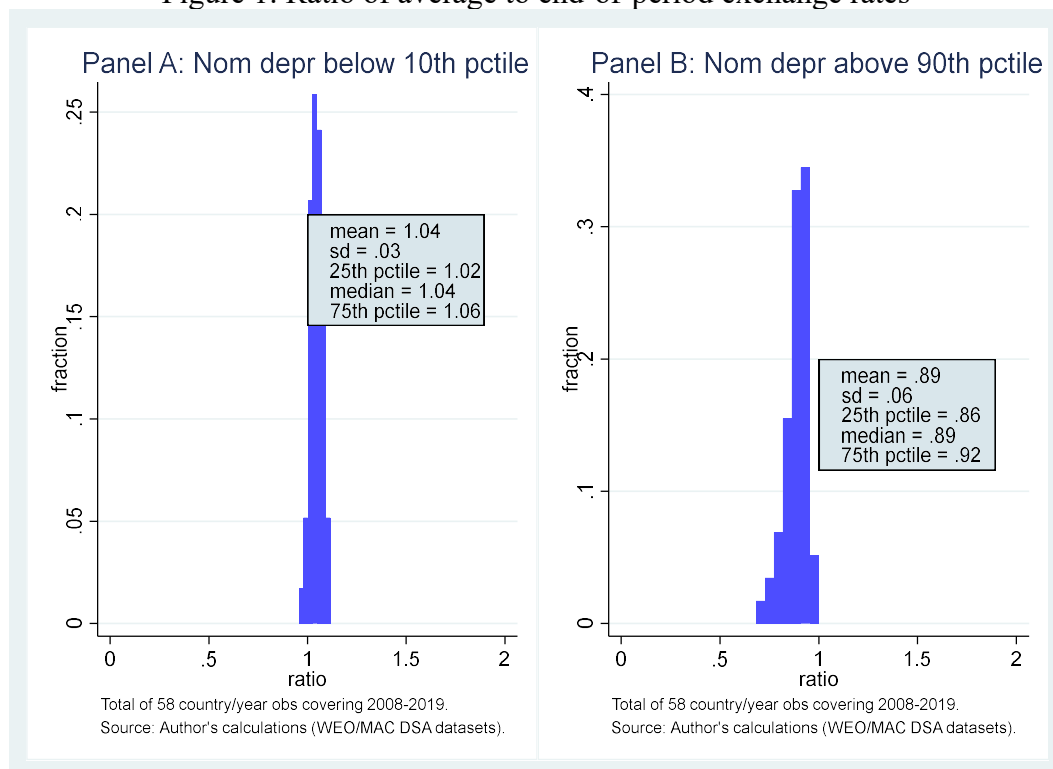
Panel A considers all cases where the exchange rate depreciated below the 10th percentile of the full sample, providing a total of 58 country/year observations. Panel B, instead, restricts

the sample to those country/year observations with nominal exchange rate depreciations above the 90th percentile of the full sample, bringing also a total of 58 observations. Although for the full sample the mean of the exchange rate ratio is 0.98, those countries experiencing 'significant' nominal appreciations/depreciations during the 2008–19 period had important deviations from that sample mean, with average ratios at 1.04 and 0.89, respectively.[14] This difference can lead to quantitatively relevant differences in debt projections over time, as discussed below.

B. Evidence on interest rate-growth differentials

From (8) and (9) the interest rate-growth differentials can now be computed for the full sample, and by country groups. To the extent there are differences between the average and the end-of-period exchange rates, the so-called 'automatic debt dynamics' portion of the debt accumulation equation will differ, and so will debt projections over time.

Table 2: Estimations of interest rate-growth differentials
(sample 2008–19, in percentage points)

	No of countries	Obs	IRGD_iyxr	IRGD_no_iyxr	Nom growth
			Mean	Mean	Mean
HICs	28	161	0.80	0.80	4.66
UMICs	35	231	0.03	0.09	8.09
LMICs	19	182	-2.04	-2.00	10.30
Full sample	82	574	-0.41	-0.38	7.83
Nom depr *below* 1st pctile	5	5	-6.22	-6.22	6.23
Nom depr *below* 10th pctile	35	57	-5.30	-5.35	8.20
Nom depr *above* 90th pctile	29	57	6.08	6.28	12.84
Nom depr *above* 99th pctile	5	5	15.28	15.84	24.98

Note: Countries' group income classification based on that of World Bank as of 2017. IRGD computed using (8) and (9) on a country/year basis.
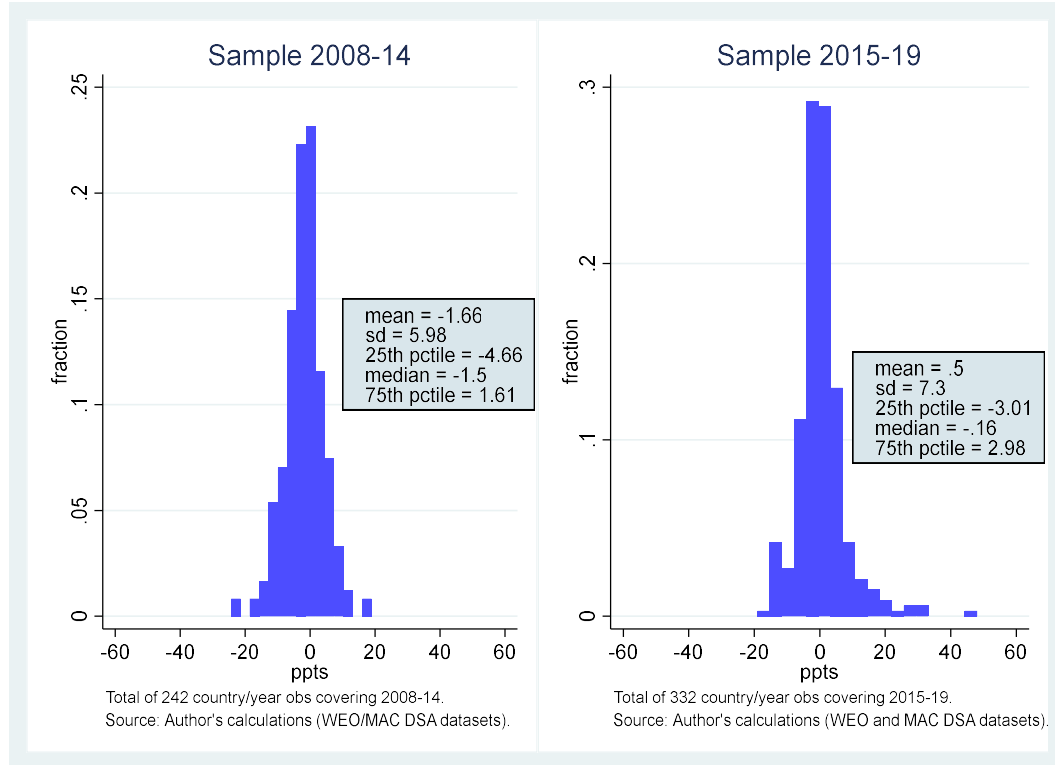Source: Author's calculations based on IMF's WEO and MAC DSA datasets.

From Table 2, intra-year exchange rate fluctuations are unlikely to be a major source of discrepancies in the estimation of the IRGD for the 'average' country. In fact, the IRGD seems to be only around 0.04 percentage points *lower* on average once intra-year exchange rate fluctuations are considered. However, there is significant heterogeneity across country groups. For instance, the IRGD is about 0.20 or even 0.57 percentage points lower once intra-year exchange rate fluctuations are accounted for in those countries that experienced episodes of significant nominal exchange depreciations (Table 2). Consistent with this finding, Appendix A shows that valuation-adjusted nominal effective interest rates tend to be lower once intra-year exchange rate differences are accounted for, thereby explaining this observed pattern with the IRGD. Overall, accounting for intra-year exchange rate

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[14] The mean exchange rate ratio is 1.09 and 0.78 if the sample is instead restricted to those country/year observations with nominal exchange rate depreciations below the 1st and above the 99th percentiles of the full sample, respectively.

fluctuations appears to *reduce* the IRGD relative to a case in which these valuation effects are ignored.[15]

Although the IRGD for the full sample is negative, there seems to be an upward trend over time once the sample is divided between the 2010-14 and 2015-19 subperiods. To see this, the IRGD is estimated using (8) and shown in Figure 2. A similar pattern is noted in Mauro and Zhou (2020) by comparing the post-1980 and post-Global Financial Crisis periods.

Figure 2: Interest rate-growth differentials including intra-year exchange rate fluctuations

Sample 2008-14

mean = -1.66
sd = 5.98
25th pctile = -4.66
median = -1.5
75th pctile = 1.61

Total of 242 country/year obs covering 2008-14.
Source: Author's calculations (WEO/MAC DSA datasets).

Sample 2015-19

mean = .5
sd = 7.3
25th pctile = -3.01
median = -.16
75th pctile = 2.98

Total of 332 country/year obs covering 2015-19.
Source: Author's calculations (WEO and MAC DSA datasets).

More stable average inflation rates, and possibly better fiscal reporting in a context of deeper financial integration and fewer sources of financial repression, could be contributing to this trend. In the case of advanced countries, efforts to normalize monetary policy amid improved growth prospects in the aftermath of the global financial crisis, may have also contributed to this observed tendency.[16] In any event, given that intra-year exchange rate fluctuations could affect debt projections in a quantitatively relevant way through interest rate-growth

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[15] It is worth observing that there is a tendency for $i_t^{iyxr} < i_t^{no\_iyxr}$ if the exchange rate depreciates over time. To see this compare (4) and (5) assuming α_t is equal to 0 yet allowing for all other variables including α_{t-1} to be different from 0. Thus, if $\varepsilon_t^{eop} > 0$, namely if the currency depreciates over time, it must be the case that $e_t^{avg}/e_t^{eop} < 1$ and so $i_t^{iyxr} < i_t^{no\_iyxr}$. It is the fact that the ratio e_t^{avg}/e_t^{eop} moves inversely with ε_t^{eop} that brings this relation between the valuation-adjusted effective rates. However, for "sufficiently large" values of α_t, this relation between these effective rates may revert.

[16] See Escolano and others (2017) and Mauro and Zhou (2020) for discussions on the rationale behind negative interest rate-growth differentials.

differentials, the primary balance and other net debt-creating flows, the next subsections discuss how these valuation effects affected public debt dynamics over time using this dataset.

C. Evidence on stock-flow adjustment, debt projections and debt-stabilizing primary balances

The stock-flow adjustment term due to intra-year exchange rate fluctuations as percent of GDP can be computed using $sf_t = \frac{(e_t^{eop}-e_t^{avg})}{e_t^{eop}}\left(\alpha_t d_t - \alpha_{t-1}d_{t-1}\frac{(1+\varepsilon_t^{eop})}{(1+g_t)(1+\pi_t)}\right)$ (see footnote 10), where d_t and d_{t-1} reflect the stock of debt as reported in WEO for the country/year under consideration. Table 3 does so, showing that the average stock-flow adjustment term is 0.02 percent of GDP for the full sample, with slightly larger figures for the UMICs and LMICs country groups (0.03 and 0.02 percent of GDP, respectively). This term, however, could take sizeable figures in some specific cases. In fact, for the sub-sample of country/year observations with large nominal exchange rate depreciations, namely above the 90th and 99th percentile of the full sample, the average stock-flow adjustment term reached 0.2 and 1.2 percent of GDP, respectively (Table 3). Ignoring this term could then lead to inaccuracies at the time of projecting public debt, and the associated policy-relevant indicators.

Table 3: Estimations of stock-flow adjustment due to intra-year exchange rate changes (sample 2008–19, in percent of GDP)

	No of countries	Obs	Stock-flow adjustor			
			Mean	Std Dev	Min	Max
HICs	28	161	0.00	0.07	-0.28	0.47
UMICs	35	231	0.03	0.31	-0.99	3.53
LMICs	19	182	0.02	0.11	-0.25	0.81
Full sample	82	574	0.02	0.21	-0.99	3.53
Nom depr *below* 1st pctile	5	5	-0.03	0.09	-0.14	0.09
Nom depr *below* 10th pctile	35	57	-0.03	0.07	-0.33	0.19
Nom depr *above* 90th pctile	29	57	0.20	0.62	-0.99	3.53
Nom depr *above* 99th pctile	5	5	1.22	1.59	-0.13	3.53

Note: Countries' group income classification based on that of World Bank as of 2017. Stock-flow adjustment computed on a country/year basis with the formula indicated in footnote 10.
Source: Author's calculations based on IMF's WEO and MAC DSA datasets.

To fully understand the implications of intra-year exchange rate valuation effects on public debt projections, it is useful to undertake the following counter-factual exercise. Let us assume that initially public gross debt levels are exactly as reported in WEO, April 2020 release, regardless of whether intra-year exchange rate fluctuations are considered, i.e., d_{t-1}^{iyxr} and $d_{t-1}^{no\_iyxr}$ as defined in (6) and (7) are initially the same, say in 2007. However, for the period 2008–19, it is assumed that observed gross public debt levels as reported in WEO followed the data generating process with intra-year valuation effects as defined in (6). This

allows us to generate an implicit time series for other net debt-creating flows, of_t, for the period 2008–19.[17]

With of_t at hand, it is then possible to compare the difference in public debt stocks projected one year ahead using (6) or (7) starting from an *initial* stock of debt such that $d_{t-1}^{iyxr} = d_{t-1}^{no\_iyxr}$. By construction, d_t^{iyxr} projected using (6) will always coincide with the debt stocks reported in WEO. That way the difference $d_t^{iyxr} - d_t^{no\_iyxr}$ would fully reflect the relevance of intra-year exchange rate fluctuations for projecting debt one year ahead. Similarly, the difference $DSPB_t^{iyxr} - DSPB_t^{no\_iyxr}$ would reflect the full impact of intra-year exchange rate valuation effects in assessing the primary balance needed to stabilize debt at the previous year debt level.

Results of this counter-factual exercise are shown in Figures 3 and 4, below, focusing only on the sub-samples with small (below 1st percentile of the full sample) and large (above 99th percentile of the full sample) nominal exchange rate depreciations. The scatter plots show the estimated values of the stock-flow adjustment and the difference in either debt levels, $d_t^{iyxr} - d_t^{no\_iyxr}$ (Figure 3), or debt stabilizing primary balances, $DSPB_t^{iyxr} - DSPB_t^{no\_iyxr}$ (Figure 4). Observe that it is assumed $d_{t-1}^{iyxr} = d_{t-1}^{no\_iyxr}$ one year before the country/year episodes identified in Figures 3 and 4. Also, and to further clarify the sources of the differences in debt projections, Appendix B shows the full list of the countries shown in Figures 3 and 4, identifying in full the contributors of their respective debt changes.

Figure 3, Panel B and Appendix B show that for those countries that experienced large nominal depreciations during 2008–19, the stock-flow adjustment term was on average significant at around 1.2 percent of GDP. Thus, ignoring these intra-year valuation effects would have led to an *underestimation* in debt projections by about 0.9 percentage points (ppts) of GDP. In some cases, differences in debt projections could have been even larger, such as Argentina in 2018 (1.6 ppts of GDP) or Surinam in 2016 (3.3 ppts of GDP). On the contrary, in those countries that experienced nominal depreciations below the 1st percentile, the stock-flow adjustment term was generally low or even negative, involving only more minor miscalculations in debt projections (Figure 3, Panel A and Appendix B).

Sizable figures also arise when assessing differences in the primary balance needed to stabilize debt at the previous-year level (DSPB). Again, for those countries that experienced large nominal exchange rate depreciations, the stock-flow adjustment term is relevant, and thus the inaccuracies in estimating the DSPB would have led to an *overestimation* of about 1.5 ppts of GDP on average (Figure 4, Panel B and Appendix B), with differences that are

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[17] That is, from (6) it is possible to derive for every country/year observation $of_t = d_t^{iyxr}(1 - \alpha_t \gamma_t) - \left(\frac{1+i_t^{iyxr}}{(1+g_t)(1+\pi_t)}\right)(1 - \alpha_t \gamma_t)d_{t-1}^{iyxr} + pb_t$, where d_t^{iyxr} is exactly the gross debt level reported by WEO for every country during 2007–19. Note that when assessing observed data, the term of_t would subsume a pure residual, capturing also non-identifiable debt-creating flows arising from accounting issues (e.g., cash vs accrual accounting), cross-exchange rate fluctuations when more than one foreign currency is involved, intra-year exchange rate fluctuations that occur at a lower frequency not properly accounted for in the annual frequency considered here, etc.

even larger for say Argentina in 2018 (-4 ppts of GDP) or Angola in 2018 (-2.2 ppts of GDP).

Figure 3: Public debt projections and stock-flow adjustment
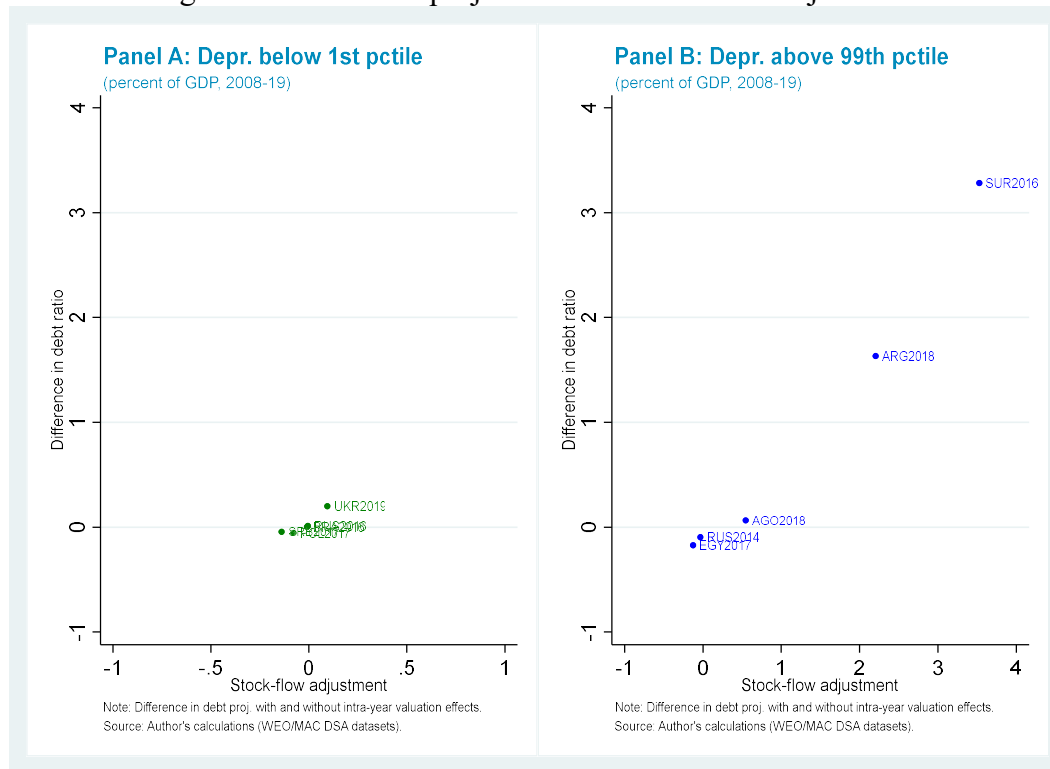
The intuition as of why public debt could be simultaneously *underestimated* while the debt-stabilizing primary balance *overestimated* when intra-year exchange rate fluctuations are ignored, is as follows. Consider the case of Argentina in 2018. Since the country experienced a large currency depreciation in 2018, it followed that $e_t^{avg}/e_t^{eop}<1$. Typically, $e_t^{avg}/e_t^{eop} < 1$ in countries experiencing nominal exchange rate depreciations. From Tables 5 and 6, the automatic debt dynamics portion of the change in debt is *lower* in Argentina when intra-year valuation effects are included (recall that it is assumed that in 2017 debt levels are the same), due to the *lower* interest rate-growth differential. However, the contribution of the primary deficit and other net debt-creating flows to the change in debt is *higher,* since these flows add new debt, including that denominated in foreign currency, at the lower exchange rate e_t^{avg} relative to e_t^{eop}. The overall net effect is a *higher* change in the debt stock projected in Argentina in 2018 if intra-year valuation effects are included. Now, if one is asking what would have been the primary balance needed in 2018 to stabilize debt at the 2017 level, the answer, for the reason explained before, is that it should have been *lower* relative to a case where intra-year exchange rate fluctuations are ignored. This occurs since the flow primary surplus would have reduced a larger amount of debt through its conversion to the foreign currency at the lower exchange rate e_t^{avg} relative to e_t^{eop}.

Figure 4: Debt-stabilizing primary balance and stock-flow adjustment
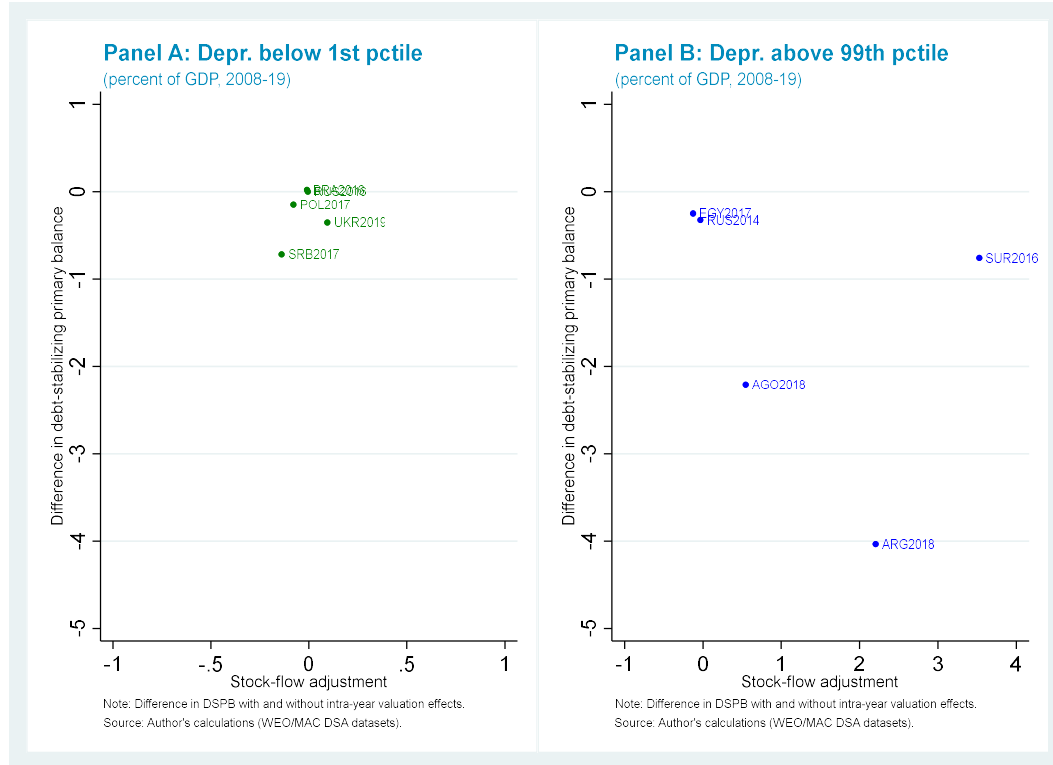
VI. CONCLUDING REMARKS

Since the public sector often incurs liabilities denominated in foreign currency, as observed in many emerging and developing countries, periods of heightened exchange rate volatility could lead to substantial revaluations of those liabilities when these are measured in local currency and/or as a share of GDP. Thus, a careful consideration of how exchange rate fluctuations affect foreign currency denominated flows and stocks is critical for assessing how public debt evolves over time.

This paper lays out the basic *above-the-line* equation to measure the evolution of public sector debt, considering that debt-related stocks and flows denominated in foreign currency should be converted to local currency using the end-of-period and the average exchange rates, respectively, as envisaged in public finance manuals. This accounting consideration, somewhat abstracted away in the related literature (e.g., Croce and Juan-Ramón, 2003; IMF, 2013; Escolano and others, 2017), gives rise to a stock-flow adjustment term stemming from intra-year exchange rate fluctuations. The paper then shows that this type of exchange rate valuation effect may have an impact on the computation of public debt over time, the interest rate-growth differential and the debt-stabilizing primary balance in non-negligible ways, particularly in cases with large exchange rate depreciations. Moreover, the inclusion of this often-ignored stock-flow adjustment term is critical to accurately project public debt levels and any associated policy indicator that could in turn inform about the risk of debt distress.

A relevant extension of the paper could be to increase the coverage of the empirical analysis to also include low-income countries to quantify, in a larger sample, the size of this stock-flow adjustment term and its impact on public debt projections. Another important strand of future work could involve the assessment of how intra-year exchange rate fluctuations might affect medium-term debt targets, particularly in those countries that have experienced large deviations between the average and the end-of-period exchange rates in their recent history. This could be particularly timely in those economies envisaging consolidation paths to be spread out over time, since assessing intra-year exchange rate valuation effects may help determine more accurately which are the fiscal paths that are consistent with a specific medium-term debt objective in these countries.

VII. APPENDIX

A. Valuation-adjusted nominal effective interest rates

This appendix summarizes the mean values of the effective nominal interest rates defined as i_t^{iyxr}, $i_t^{no\_iyxr}$ and i_t^{fisc} (the latter is derived according to the formula in footnote 9).

Table 4: Estimations of nominal effective interest rates
(average 2008–19, percent)

	No of countries	Obs	i_iyxr Mean	i_no_iyxr Mean	i_fisc Mean
HICs	28	161	5.30	5.31	4.81
UMICs	35	231	7.83	7.89	5.59
LMICs	19	182	7.83	7.88	5.53
Full sample	82	574	7.12	7.16	5.35
Nom depr *below* 1st pctile	5	5	-0.52	-0.52	6.40
Nom depr *below* 10th pctile	35	57	2.36	2.30	5.54
Nom depr *above* 90th pctile	29	57	19.42	19.66	6.67
Nom depr *above* 99th pctile	5	5	44.30	45.03	7.83

Note: Countries' group income classification based on that of World Bank as of 2017. Effective interest rates computed on a country-by-country basis using (4), (5) and footnote 9, and averaged across the different samples reported in the table.
Source: Author's calculations based on IMF's WEO and MAC DSA datasets.

Table 4 shows that $i_t^{iyxr} < i_t^{no\_iyxr}$, with the only exception of those countries experiencing nominal depreciations below the 1st percentile, which is consistent with the fact that most countries experienced nominal exchange rate depreciations during the sample period or, put it differently, that the exchange rate ratio has been $e_t^{avg}/e_t^{eop} < 1$ for the average country (see Table 1). Table 4 further shows that i_t^{iyxr} tends to be particularly lower than $i_t^{no\_iyxr}$ for the sub-sample of countries experiencing depreciations above the 90th or the 99th percentile of the full sample. This table also illustrates that average nominal effective interest rates that can be retrieved from fiscal statements, i_t^{fisc}, cannot be used directly to assess public debt dynamics, unless these are properly *adjusted* for exchange rate valuation effects.

B. Decomposition of changes in the public debt to GDP ratio

This annex presents a full decomposition of the key drivers of public debt changes to better understand the implications of intra-year exchange rate valuation effects to project public debt. From (6), it is possible to identify three additive contributors of debt changes, namely: (i) the effect of the interest rate-growth differential on the previous period stock of debt (sometimes called automatic debt dynamics); (ii) the effect of the primary balance; and (iii) the effect of other net debt-creating flows. The underlying analytical expressions of each contributor are indicated below:

$$d_t^{iyxr} - d_{t-1}^{iyxr} = \left(\frac{1 + i_t^{iyxr}}{(1 + g_t)(1 + \pi_t)} - 1 \right) d_{t-1}^{iyxr} - \frac{1}{1 - \alpha_t \gamma_t} pb_t + \frac{1}{1 - \alpha_t \gamma_t} of_t. \tag{12}$$

| Automatic debt dynamics | Contribution of primary balance | Contribution of other debt-creating flows |

Instead, when intra-year valuation effects are abstracted away, these contributors to public debt changes, from (7), take the form:

$$d_t^{no\_iyxr} - d_{t-1}^{no\_iyxr} = \left(\frac{1 + i_t^{no\_iyxr}}{(1 + g_t)(1 + \pi_t)} - 1 \right) d_{t-1}^{no\_iyxr} - pb_t + of_t. \tag{13}$$

| Automatic debt dynamics | Contribution of primary balance | Contribution of other debt-creating flows |

Observe that the stock-flow adjustment term can also be estimated using the formula presented in footnote 10. Adding this stock-flow adjustment term to (13) provides an almost identical quantification of the annual change in debt ratios as identified in (12). However, to recover *exactly* the change in debt that could be obtained with (12), the nominal effective interest rate $i_t^{no\_iyxr}$ would need to be re-defined as,

$$i_t^{iyxr} = \alpha_{t-1}(1 + i_t^f e_t^{avg}/e_t^{eop})(1 + \varepsilon_t^{eop}) + (1 - \alpha_{t-1})(1 + i_t^d) - 1. \tag{14}$$

It stands out that the key difference between i_t^{iyxr} and $i_t^{no\_iyxr}$ is the fact that i_t^{iyxr} includes intra-year exchange rate differences, e_t^{avg}/e_t^{eop}, whereas $i_t^{no\_iyxr}$ does not (i.e., compare (5) and (14)).

Tables 5 and 6 summarize the debt levels projected with and without intra-year valuation effects, d_t^{iyxr} and $d_t^{no\_iyxr}$, changes in debt ratios, $d_t^{iyxr} - d_{t-1}^{iyxr}$ and $d_t^{no\_iyxr} - d_{t-1}^{no\_iyxr}$, and debt-stabilizing primary balances, $DSPB_t^{iyxr}$ and $DSPB_t^{no\_iyxr}$, for the countries/years shown in Figures 3 and 4. The contributors to annual debt changes with and without intra-year valuation effects (i.e., automatic debt dynamics, contribution of the primary balance and contribution of other net debt-creating flows) are defined as in (12) and (13) in Tables 5 and 6, respectively. Table 5 also presents the estimations of the stock-flow adjustment term. This confirms that intra-year exchange rate valuation effects could lead to sizable discrepancies in

terms of debt projections in countries exposed to large currency fluctuations and foreign currency debt (i.e., a large stock-flow adjustment term), with relevant implications at the time of identifying the contributors of debt changes and/or measuring policy relevant indicators such as the debt stabilizing primary balance.

Table 5: Debt projections drivers with intra-year exchange rate fluctuations
(percent of GDP)

	Stock of gross public debt	Change in gross public debt	Automatic debt dynamics	Contribution primary deficit	Contribution other flows	Stock-flow adjustment	DSPB 1/
Nom. Depr. < 1st pctile							
BRA2016	78.31	5.74	4.85	0.27	0.61	-0.01	5.48
POL2017	50.60	-3.63	-4.90	-0.10	1.36	-0.08	-3.62
RUS2016	14.85	-0.44	-0.36	2.69	-2.76	-0.01	-3.21
SRB2017	58.73	-10.15	-8.88	-3.68	2.41	-0.14	-6.90
UKR2019	50.12	-10.47	-8.20	-0.95	-1.32	0.09	-10.02
Nom. Depr. > 99th pctile							
AGO2018	89.00	19.74	12.42	-7.60	14.92	0.54	24.18
ARG2018	86.06	28.95	16.43	2.81	9.71	2.20	21.02
EGY2017	103.16	6.32	-9.40	2.25	13.47	-0.13	4.02
RUS2014	15.14	2.79	1.19	0.44	1.15	-0.03	2.15
SUR2016	76.07	32.72	12.90	7.38	12.44	3.53	22.21

1/ Debt-stabilizing primary balance, calculated using (10).
Note: Countries' group income classification based on that of World Bank as of 2017.
Source: Author's calculations based on IMF's WEO and MAC DSA datasets.

Table 6: Debt projections and drivers without intra-year exchange rate fluctuations
(percent of GDP)

	Stock of gross public debt	Change in gross public debt	Automatic debt dynamics	Contribution primary deficit	Contribution other flows	Stock-flow adjustment	DSPB 1/
Nom. Depr. < 1st pctile							
BRA2016	78.31	5.73	4.85	0.27	0.62	-	5.46
POL2017	50.66	-3.58	-4.88	-0.10	1.40	-	-3.48
RUS2016	14.84	-0.45	-0.37	2.76	-2.84	-	-3.21
SRB2017	58.77	-10.11	-8.76	-3.92	2.57	-	-6.19
UKR2019	49.91	-10.67	-8.28	-1.00	-1.39	-	-9.67
Nom. Depr. > 99th pctile							
AGO2018	88.93	19.67	13.20	-6.72	13.20	-	26.39
ARG2018	84.42	27.31	17.24	2.26	7.81	-	25.05
EGY2017	103.33	6.49	-9.03	2.22	13.30	-	4.27
RUS2014	15.23	2.88	1.42	0.41	1.06	-	2.48
SUR2016	72.79	29.43	12.06	6.47	10.91	-	22.97

1/ Debt-stabilizing primary balance, calculated using (11).
Note: Countries' group income classification based on that of World Bank as of 2017.
Source: Author's calculations based on IMF's WEO and MAC DSA datasets.

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