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## More Gray, More Volatile? Aging and (Optimal) Monetary Policy

by Daniel Baksa and Zsuzsa Munkacsi

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I N T E R N A T I O N A L M O N E T A R Y F U N D

**International Monetary Fund**

Strategy, Policy and Review Department

**More Gray, More Volatile? Aging and (Optimal) Monetary Policy<sup>1</sup>**

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**Abstract**

The evidence on the inflation impact of aging is mixed, and there is no evidence regarding the volatility of inflation. Based on advanced economies' data and a DSGE-OLG model, we find that aging leads to downward pressure on inflation and higher inflation volatility. Our paper is also the first, using this framework, to discuss how aging affects the transmission channels of monetary policy. We are also the first to examine aging and optimal central bank policies. As aging redistributes wealth among generations and the labor force becomes more scarce, our model suggests that aging makes monetary policy less effective and in more gray societies central banks should react more strongly to nominal variables.

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*"Old age is like a plane flying through a storm. Once you're aboard there's nothing you can do."*

*Golda Meir (1898-1978)*

*"But there is something central banks can do."*

*The authors*

## 1 Introduction: Motivation and Literature

In his 2012 speech<sup>1</sup>, William C. Dudley, the President of the Federal Reserve Bank of New York claimed (Dudley 2012) that "the weaker than expected recovery [since the recent crisis] likely lies in the interplay between secular and cyclical factors", and "demographic factors have played a role in [it]." This means that in addition to stressing the more straightforward fiscal consequences of aging<sup>2</sup>, President Dudley highlighted the interaction between demographics, nominal variables, and central bank policies. Specifically, he pointed out that the spending decisions of older-age cohorts are less likely to be easily stimulated by monetary policy as such age groups tend to spend less of their income on consumer durables and housing. Governor Shirakawa of the Bank of Japan, also in 2012<sup>3</sup>, argued that population aging can lead to disinflationary pressures by lowering expectations on future economic growth (Shirakawa 2012). He referred to a cross-country comparison of 24 advanced economies which revealed that population growth and inflation rates correlated positively over the decade of the 2000s. Further, President Bullard of the St. Louis Federal Reserve Bank in Bullard, Garriga, and Walker (2012)<sup>4</sup> said that the old might prefer lower inflation than the young due to the redistributive effects of inflation.

They were all preceded by Charlie Bean, Chief Economist of the Bank of England in 2004 (the Bank of England's Deputy Governor for Monetary Policy between 2008 and 2014) who suggested that both along the transition and in the long run aging may lead to a sharp decline in savings, labor supply, and the natural rate of interest (Bean 2004). In his speech Mr. Bean summarized the previous findings in this field and pointed out their implications for central bank(er)s: (i) demographic developments represent a macroeconomic shock, which may lead to abrupt movements in asset prices and sharp movements in saving behavior; (ii) the natural rate of interest falls both along the transition path and in the steady state; (iii) the natural rate

<sup>1</sup><https://www.newyorkfed.org/newsevents/speeches/2012/dud121015>

<sup>2</sup>See e.g. Clements, Dybczak, Gaspar, Gupta, and Soto (2015).

<sup>3</sup><https://www.bis.org/review/r120531e.pdf>

<sup>4</sup><https://files.stlouisfed.org/files/htdocs/publications/review/12/11/Bullard.pdf>

of unemployment may also be affected through the matching mechanism; (iv) the wealth channel is likely to become a more important transmission channel of monetary policy than intertemporal substitution; (v) the Phillips curve is flatter due to immigration and the increased participation of retired workers whose supply of labor is considered to be relatively elastic; (vi) the constituency for keeping inflation low will be larger thanks to higher average wealth accumulation; and (vii) societal aging may induce diversification and risk-shifting with a securitized market rather than bank-intermediated finance, which has implications for financial stability.<sup>5</sup>

The above examples highlight the steady interest of central bankers in population aging, a long way back. Relatedly, there are numerous empirical and theoretical papers which discuss the longer-term monetary (inflation rate, but not inflation volatility) and interest rate implications of aging, particularly, according to the secular stagnation literature slower economic growth is coupled with a fall in the natural rate of interest (examples include Summers (2014), Favero and Galasso (2015), Eggertsson, Mehrotra, and Robbins (2017), Ferrero, Gross, and Neri (2017) and Jones (2018)).<sup>6</sup> There are also many works which talk about the shorter-term behavior of the macroeconomy and monetary policy transmission (but not based on a multi-period general equilibrium framework with overlapping generations) which we will discuss as next, however, we are not aware of any papers on the interplay between aging and optimal monetary policies.

First, several empirical papers look into the impact of population aging and demographics on the rate of inflation. The findings are contradictory. Many report that aging puts a downward pressure on inflation. For example, Kim, Lee, and Yoon (2014), based on a panel dataset covering 30 OECD economies between 1960-2013, find that population growth affects the inflation rate positively. Lindh and Malmberg (1998) and McKibbin and Panton (2018) estimate the relation between inflation and aging also on OECD data between 1960-1994 for 20 countries, and claim that increases in the population of net savers dampen inflation, while net consumers have a positive effect on it (young retirees fan inflation as they start consuming out of accumulated pension claims). At the same time, Juselius and Takats (2015) and Juselius and Takats (2016) introduce the age-structure-inflation puzzle, i.e., they report that aging

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<sup>5</sup>Although it is beyond the scope of this paper to investigate all the topics he raised, we deal with several of them. First, we contribute to (i) and (ii). Second, we formally verify (iv). Last, regarding (ii) and (vi), we also look into inflation volatility: in addition to the rate of inflation, the volatility of inflation is crucial for monetary policy transmission and optimal monetary policies.

<sup>6</sup>There is also a related stream on the zero lower bound, however, reviewing it is beyond the scope of this paper. Additionally, regarding the cyclical volatility of real variables, Jaimovich and Siu (2009) report that the workforce age composition has a large and significant effect on the cyclical volatility of hours worked.

increases inflation. Specifically, based on a panel of 22 advanced economies over the period of 1955-2010 they show that both the young and old dependents are inflationary, whereas the working age population is disinflationary. In our paper we will present evidence that aging is associated with lower inflation, based on data of several advanced economies. Moreover, in a panel regression of OECD countries, we also provide evidence on the positive impact of aging on inflation volatility, which, according to our knowledge, was not done in the literature yet.

Second, from an empirical point of view several authors explored the effect of aging on monetary policy transmission. Imam (2013) and Imam (2014) report that in graying societies more aggressive monetary policy is needed because it becomes less effective with aging. The author, based on Bayesian estimation techniques for the U.S., Canada, Japan, the U.K., and Germany, confirms the weakening of monetary policy effectiveness over time, and provides evidence - using dynamic panel-OLS techniques - that this attributes to demographic changes. Relatedly, Deok Ryong and Dong-Eun (2017) perform a panel-VAR analysis using OECD data between 1995 and 2014, and reveal that monetary policies lose their effectiveness considerably as aging hits the economy.

Third, structural models are also considered, on the one hand, when examining the long-run inflationary, interest rate, and savings rate impacts of aging, and, on the other hand, when exploring the transition path between the steady states of less and more gray economies, or the impulse responses of monetary policy shocks in an aged society. The findings are somewhat mixed, although most authors claim that aging has a significant impact on nominal variables and monetary policy transmission: it increases inflation and reduces the effectiveness of monetary policy transmission. Nonetheless, we are not aware of any multi-period general equilibrium model with overlapping generations in this literature; the available models are either two-period, or partial equilibrium models, or lack important overlapping generation or pension aspects. Additionally, to the best of our knowledge, the interplay between aging and optimal monetary policies was not at all explored in the literature yet.

Fujiwara and Teranishi (2017), based on a closed-economy overlapping generations framework a la Gertler (1999), find that longevity lowers the natural interest rate, but aging does not significantly alter the impulse responses of monetary policy shocks. Nevertheless, as pointed out by Ripatti (2008), in the absence of aging-related fiscal policy and social security aspects, the study does not explore the links between demographics, pensions, and monetary policy. Hence, the results arise from the fact that the retirees' population share is not large enough, not even in an aged society. Similarly, Kara and von Thadden (2010) report that demographic changes, while contributing slowly over time to a decline in the equilibrium

interest rate, are not visible enough within the time horizon relevant for policy-making to require monetary policy reactions. They develop a small-scale DSGE model, calibrated for the euro area, which embeds a demographic structure within a monetary policy framework by extending Gertler (1999) such that the short-run dynamics are similar to the paradigm summarized in Woodford (2003). Nonetheless, they do not examine monetary policy transmission in their paper.

At the same time, Carvalho and Ferrero (2014), based on a calibrated model of a dynamic monetary economy with a life-cycle structure a la Gertler (1999) for Japan, point out that an increase in life expectancy puts a downward pressure on the effective real interest rate. Kantur (2013) in a two-period OLG New Keynesian model and Carvalho, Ferrero, and Nechio (2016) in a life-cycle model calibrated for developed economies also come to the conclusion that the natural rate of interest decreases as the old-age dependency ratio increases. Kantur (2013) also claims that the effectiveness of monetary policy decreases due to a decrease in interest rate sensitivity of the society as the population ages. Similarly, Auerbach, Kotlikoff, Hagemann, and Nicoletti (1989) and Auerbach, Cai, and Kotlikoff (1991), using three different types of models, among others a life-cycle model, and Rios-Rull (2001) - based on Spanish data - report that aging negatively influences the long-term savings rate. Anderson, Botman, and Hunt (2014), based on the IMF's Global Integrated Monetary and Fiscal (GIMF) model, find that aging causes deflation, mainly via slowing growth and falling land prices. Baesel and McMillan (1990) claim that as the Baby Boomers age, it is reasonable to expect that the unemployment rate and the real interest rates will become lower. Further, Miles (1999), in a general equilibrium model with overlapping generations for the UK and Europe, highlights that there will be a radical decline in saving rates as a result of an increase in the old-age dependency ratio. Other examples in this stream of the literature include Miles (2002), Katagiri and Ueda (2014), and Faruqee and Muhleisen (2003).

In this paper, in addition to providing empirical and theoretical evidence on the impact of aging on the rate of inflation and inflation volatility, we study the short-run cyclical behavior of the macroeconomy and the transmission mechanism of monetary policy as the population ages. Wong (2018) and Miyamoto and Yoshino (2017) are the closest to our framework, and make similar conclusions. Nevertheless, Wong (2018) presents a partial equilibrium life-cycle model, i.e., a household model of mortgages and housing.<sup>7</sup> Hence, she does not take all the general equilibrium channels and effects into account. At the same time, Miyamoto and

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<sup>7</sup>It is beyond the scope of this paper to review the literature on the role of household balance sheets (e.g., debt and housing) in detail. Examples include Hedlund, Karahan, Mitman, and Ozkan 2017 and Gelos, Grinberg, Mancini-Griffoli, Khan, Narita, and Rawat 2019.



Yoshino (2017) do not incorporate overlapping generations into their framework: the retirees simply are rule-of-thumb agents who consume all of their transfers (pensions). To the best of our knowledge, our model is the first multi-period dynamic general equilibrium model with overlapping generations to explore the short-term cyclical behavior of the macroeconomy and the monetary policy transmission mechanism in the presence of aging. In addition, our paper is also the first to look into optimal monetary policies in an aging society.

The model is a dynamic general equilibrium model with demographics and overlapping generations following Baksa and Munkacsi (2016). Population changes over time, and there are two cohorts: the young (workers) and the old (retired). We assume a simple production sector (one sector with Cobb-Douglas utility function), a simple labor market block (only labor supply, no unemployment), a simple fiscal block (only lump-sum taxes), and consider a closed-economy<sup>8</sup>. Nevertheless, six different monetary policy rules are investigated. The model is parametrized for a standard advanced economy.

Aging is a challenge: it changes the relative and absolute sizes of each cohort. An increase in longevity increases the number and share of the elderly. If the fertility rate (birth rate) also declines, in addition to the higher old-age dependency ratio, the number of the young declines as well. The most important channel is that as agents live longer and their planning horizon becomes longer, their savings position changes: the young are willing to borrow more, while the retired accumulate more savings to guarantee their consumption over a longer time horizon. Hence, when the interest rate changes, it has different implications for the young and the old: higher interest rates imply an extra cost for the young who are indebted, while the old generate higher income. Next, the young and the old also make different consumption-savings decisions. In particular, there are age-dependent elasticities of intertemporal substitution in the model as the old are less sensitive to the changes in monetary conditions than the young.<sup>9</sup> Last, there are labor market implications: when the labor force shrinks, the labor market becomes more tight, i.e., the labor supply is more inelastic (the Frisch-elasticity decreases as the old-age dependency ratio increases). This also affects real wages: the more scarce the

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<sup>8</sup>As stressed by Batini, Callen, and McKibbin (2006), Boersch-Supan, Ludwig, and Winter (2006), and Krueger and Ludwig (2007), open-economy channels matter in multi-country or global settings. In particular, closed-economy predictions for the decline in the interest rate tend to be overstated, i.e. capital mobility tends to moderate the pressure on factor price adjustments.

<sup>9</sup>Wong (2018) provides empirical estimates of age-specific consumption elasticities to interest rate shocks. The consumption of younger people is twice as responsive to interest rate shocks than that of older people, and explains about two-third of the aggregate response. The consumption responses are driven by homeowners who refinance or enter new loans after the interest rate declines. This implies that under an older demographic structure aggregate consumption will response less to monetary policy shocks.

labor supply, the more profound the real wage reaction to shocks.<sup>10</sup>

As aging redistributes wealth among generations, the old are more patient than the young, and the labor force becomes more scarce with aging, our model suggests that aging makes inflation more volatile, monetary policy less effective, and aggregate demand less elastic to changes in the interest rate. Moreover, in more gray societies central banks should react more strongly to nominal variables, and in a very old society the nominal GDP targeting rule becomes the most effective monetary policy rule to compensate for higher inflation volatility.

In the next section we describe the equations of the model, which is a simplified version of Baksa and Munkacsi (2016). In Section 3 we talk about how to parametrize the framework. Section 4 demonstrates the steady states of the young and old societies. Then, in Section 5 we present the main impulse-response functions and, in particular, discuss the transmission mechanism of monetary policy. Next, Section 6 is devoted for studying the welfare consequences of aging and optimal monetary policy rules. Finally, in Section 7 we provide empirical support for our theoretical findings, and summarize the main policy conclusions.

## 2 The Model

Demographics and over-lapping generations are modeled following Baksa and Munkacsi (2016). We distinguish two cohorts, i.e. the 20+ generation is divided into working and retired agents. In each period the number of working-age population changes by an exogenous net fertility rate. In addition, there is a given probability of retirement, with no age-specific retirement probabilities within the cohort. The number of the retired increases because some of the young get retired, while we also assume a cohort-specific probability of death (Gertler 1999).

Workers decide on how much to consume and save, and supply labor, they receive labor income and dividends from the firms. Only the young cohort pays taxes to the government which is used to finance the pension system and other public expenditures. The retired agents receive pension benefits from the pay-as-you-go (PAYG) pension system: for each agent, when she gets retired, the government calculates a level of pension benefits that is assumed to be fixed in real terms (or, to put it otherwise, in nominal terms the government adjusts it with

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<sup>10</sup>Additionally, according to the literature, there are other channels as well: the old consume less durable goods and housing, and spend more on health than the young; and the old also tend to shift their portfolio towards safer assets (e.g., government bonds). These channels are not directly modeled in our framework.

the inflation rate every year). The retired households decide on how much of their previous income they wish to spend on consumption or save. Since state-contingent bonds are assumed, the young agents are able to fully insure themselves against any possible survival outcomes.

The production block follows standard neoclassical assumptions, and Calvo-style price rigidity. The firms are responsible for physical capital accumulation and investment activity, and they demand labor. In this model we do not assume productivity growth on the balanced-growth path.

The paper focuses on the role of monetary policy and short-run cyclical behavior. In the baseline scenario we consider a standard Taylor-type reaction function, while at a later stage we test the model's robustness using various policy rules as well. The rest of this section describes the main technical assumptions and the parametrization of the framework.

## 2.1 Demographics

First of all, we need to define the demographic structure of the economy. Total population ( $N_t$ ) is equal to the sum of the old (the retired) ( $N_t^O$ ) and the young (workers) ( $N_t^Y$ ):

$$N_t = N_t^O + N_t^Y \quad (1)$$

Agents become retired with  $\omega^Y$  probability and  $n$  is the net fertility rate, i.e. the share of the new-coming workers:

$$N_t^Y = (1 - \omega_{t-1}^Y)N_{t-1}^Y + n_t N_{t-1}^Y \quad (2)$$

Only  $1 - \omega^O$  of the retired survive and live in the next period:

$$N_t^O = (1 - \omega_{t-1}^O)N_{t-1}^O + \omega_{t-1}^Y N_{t-1}^Y \quad (3)$$

Similarly to other standard general equilibrium models with population growth, we focus on the relative shares of different cohorts, and not on their levels.  $s_t$  denotes the ratio of the number of old and young people (i.e., the old-age dependency ratio), while  $s_t^Y$  is the share of the young people in the whole population. Based on the assumptions above we can express

the ratios as a function of the survival probabilities and the fertility rate:

$$s_t = \frac{N_t^O}{N_t^Y} = \frac{(1 - \omega_{t-1}^O)}{(1 - \omega_{t-1}^Y + n_t)} s_{t-1} + \frac{\omega_{t-1}^Y}{(1 - \omega_{t-1}^Y + n_t)} \quad (4)$$

$$s_t^Y = \frac{N_t^Y}{N_t} = \frac{1}{1 + s_t} \quad (5)$$

The young, the old, and total population growth can then be expressed as functions of the relative ratios and survival probabilities:

$$1 + g_t^{N,Y} = \frac{N_t^Y}{N_{t-1}^Y} = 1 - \omega_{t-1}^Y + n_t \quad (6)$$

$$1 + g_t^{N,O} = \frac{N_t^O}{N_{t-1}^O} = (1 - \omega_{t-1}^O) + \frac{\omega_{t-1}^Y}{s_{t-1}} \quad (7)$$

$$1 + g_t^N = \frac{N_t}{N_{t-1}} = (1 + g_t^{N,Y}) \frac{1 + s_t}{1 + s_{t-1}} \quad (8)$$

## 2.2 Households

The households optimize their life-time utility, nevertheless, several individuals, i.e. overlapping generations live. In the next sections we present the individuals' decisions and their consumption functions, and calculate the cohort-level aggregate variables as well. The solution for the households' problem is based on backward induction which means that we start with the retired individuals' optimization, and, conditional on the expected behavior of retired agents, we can also solve the young households' optimization problem.

### 2.2.1 Retired Households

'Retired' agent  $i$  of retired cohort  $a$  is one individual who retired  $a$  years ago. Each agent maximizes the following Bellman equation:

$$V^O(B_{a-1,t-1}^O(i)) = \max \left\{ \frac{1}{1 - \gamma} \left\{ C_{a,t}^O(i) \right\}^{1-\gamma} + \beta^O E_t (1 - \omega_t^O) V^O(B_{a,t}^O(i)) \right\} \quad (9)$$

subject to this budget constraint:

$$C_{a,t}^O(i) + (1 - \omega_t^O) B_{a,t}^O(i) = (1 + r_{t-1}) B_{a-1,t-1}^O(i) + TR_{a,t}^{YO}(i) \quad (10)$$

where  $C^O(i)$  denotes the individual consumption of the retired agent,  $B^O(i)$  is individual savings,  $\gamma$  is the inverse of the intertemporal elasticity of substitution,  $\beta^O$  is the cohort-specific discount factor,  $r$  is the real interest rate, and  $TR^{YO}(i)$  is the level of retirement benefits which was calculated at the time of retirement.

The first order conditions imply the Euler-equation that describes the substitution between the current and future individual retired consumption levels:

$$\beta^O E_t \frac{\left(C_{a+1,t+1}^O(i)\right)^{-\gamma}}{\left(C_{a,t}^O(i)\right)^{-\gamma}} (1+r_t) = 1 \quad (11)$$

After some simplifications and introducing some additional variables, the final version of the consumption of agent  $i$  of cohort  $a$  at time  $t$  is as follows:

$$C_{a,t}^O(i) = MPC_t^O Inc_{a,t}^O(i) + MPC_t^O (1+r_{t-1}) B_{a-1,t-1}^O(i) \quad (12)$$

$$Inc_{a,t}^O(i) = TR_{a,t}^{YO}(i) \Omega_t^O \quad (13)$$

$$\Omega_t^O = 1 + E_t \frac{1 - \omega_t^O}{1+r_t} \Omega_{t+1}^O \quad (14)$$

$$\frac{1}{MPC_t^O} = 1 + E_t (1 - \omega_t^O) (1+r_t)^{\frac{1}{\gamma}-1} \left\{ \beta^O \right\}^{\frac{1}{\gamma}} \frac{1}{MPC_{t+1}^O} \quad (15)$$

Here,  $TR_{n,t+n}^{YO}(i) = TR_{0,t}^{YO}(i)$  for all  $n > 0$ . Following the conventions, we introduce the marginal propensity to consume of the retired cohort as  $MPC^O$ . The cohort-level consumption ( $C^O$ ) and savings ( $B^O$ ) are as follows (for the technical details see Baksa and Munkacsi (2016)):

$$C_t^O = MPC_t^O Inc_t^O + (1+r_{t-1}) MPC_t^O (\omega_{t-1}^Y B_{t-1}^Y + B_{t-1}^O) \quad (16)$$

where the present value of the expected retirement benefits is the following:

$$Inc_t^O = TR_t \Omega_t^O \quad (17)$$

The retired cohort's consumption depends on the present value of expected revenues and accumulated wealth from the previous periods, and the marginal propensity to consume shows the proportion of life-time income spent on consumption which is a function of the real interest rate and the survival probability.

### 2.2.2 Young Households

'Young' agent  $i$  of cohort  $b$  is one individual of its cohort who started to work (was born)  $b$  years ago. The dynamic optimization problem of the young can be described by the following Bellman-equation:

$$V_t^Y(B_{b-1,t-1}^Y(i)) = \max \left\{ \frac{1}{1-\gamma} \left\{ C_{b,t}^Y(i)^\sigma (1-L_{b,t}(i))^{1-\sigma} \right\}^{1-\gamma} \right. \quad (18)$$

$$\left. + E_t \left( (1-\omega_t^Y) \beta^Y V_{t+1}^Y(B_{b,t}^Y(i)) + \omega_t^Y \beta^O V_{t+1}^O(B_{b,t}^{YO}(i)) \right) \right\} \quad (19)$$

while the budget constraint is:

$$C_{b,t}^Y(i) + (1-\omega_t^Y) B_{b,t}^Y(i) + \omega_t^Y B_{b,t}^{YO}(i) = (1+r_{t-1}) B_{b-1,t-1}^Y(i) + w_t L_{b,t}(i) + Profit_{b,t}(i) - Tax_{b,t}(i) \quad (20)$$

Here,  $C^Y(i)$  denotes the young individual's consumption,  $L(i)$  is her labor supply,  $\sigma$  is the weight of consumption in the one-period utility function,  $\beta^Y$  is the cohort-specific discount factor,  $Tax(i)$  is the lump-sum tax,  $Profit(i)$  denotes the dividend from firms, and  $w$  is the real wage. A young agent saves for two possible future states: first, for staying young ( $B^Y(i)$ ); and second, for retiring ( $B^{YO}(i)$ ).

Due to the presence of state-contingent bonds, the optimization problem of the young household results in two Euler equations, i.e. the young households are able to insure themselves against the future retired status as well:

$$\beta^Y E_t \frac{\left( C_{b+1,t+1}^Y(i)^\sigma (1-L_{b+1,t+1}(i))^{1-\sigma} \right)^{-\gamma} C_{b+1,t+1}^Y(i)^{\sigma-1} (1-L_{b+1,t+1}(i))^{1-\sigma}}{\left( C_{b,t}^Y(i)^\sigma (1-L_{b,t}(i))^{1-\sigma} \right)^{-\gamma} C_{b,t}^Y(i)^{\sigma-1} (1-L_{b,t}(i))^{1-\sigma}} (1+r_t) = 1 \quad (21)$$

$$\beta^O E_t \frac{\left( C_{b+1,t+1}^O(i) \right)^{-\gamma}}{\left( C_{b,t}^Y(i)^\sigma (1-L_{b,t}(i))^{1-\sigma} \right)^{-\gamma} \sigma C_{b,t}^Y(i)^{\sigma-1} (1-L_{b,t}(i))^{1-\sigma}} (1+r_t) = 1 \quad (22)$$

In addition, the young decide about their labor supply:

$$\frac{C_{b,t}^Y(i)}{1-L_{b,t}(i)} = \frac{\sigma}{1-\sigma} w_t \quad (23)$$

After some simplifications and by introducing some additional variables we can write up the

young individual's consumption function as follows:

$$C_{b,t}^Y(i) = MPC_t^Y Inc_{b,t}^Y(i) + MPC_t^Y (1 + r_{t-1}) B_{b-1,t-1}^Y(i) \quad (24)$$

$$Inc_{b,t}^Y(i) = \mathcal{J}_{b,t}^Y(i) + \frac{\mathcal{J}_{b,t}^{YO}(i)}{1 + r_t} \quad (25)$$

$$\frac{1}{MPC_t^Y} = \frac{1}{\sigma} + E_t(1 + r_t)^{\frac{1}{\gamma}-1} \left[ (1 - \omega_t^Y) \Lambda_t^Y \frac{1}{MPC_{t+1}^Y} + \omega_t^Y \Lambda_t^{YO} \frac{1}{MPC_{t+1}^O} \right] \quad (26)$$

$$\Lambda_t^Y = E_t \{ \beta^Y \}^{\frac{1}{\gamma}} \left( \frac{w_{t+1}}{w_t} \right)^{(1-\sigma)(1-\frac{1}{\gamma})} \quad (27)$$

$$\Lambda_t^{YO} = E_t \left\{ \frac{\beta^O}{\sigma} \right\}^{\frac{1}{\gamma}} \left( \frac{1}{\frac{\sigma}{1-\sigma} w_t} \right)^{(1-\sigma)(1-\frac{1}{\gamma})} \quad (28)$$

Here,  $MPC^Y$  is the marginal propensity to consume of the young, and  $Inc^Y(i)$  is the discounted sum of current and expected incomes ( $\mathcal{J}^Y(i)$ ) and future pension benefits ( $\mathcal{J}^{YO}(i)$ ).

Based on the individual consumption function, we can express the aggregate consumption function and the aggregate income function as follows:

$$C_t^Y = MPC_t^Y Inc_t^Y + (1 + r_{t-1}) MPC_t^Y (1 - \omega_{t-1}^Y) B_{t-1}^Y \quad (29)$$

$$\mathcal{J}_t^Y = w_t N_t^Y + Profit_t - Tax_t + E_t \frac{1 - \omega_t^Y}{(1 + r_t)(1 + g_{t+1}^{N,Y})} \mathcal{J}_{t+1}^Y \quad (30)$$

$$\mathcal{J}_t^{YO} = E_t TR_{t+1}^{YO} \Omega_{t+1}^O + E_t \frac{1 - \omega_t^Y}{(1 + r_{t+1})(1 + g_{t+1}^{N,Y})} \mathcal{J}_{t+1}^{YO} \quad (31)$$

$$Inc_t^Y = \mathcal{J}_t^Y + \frac{\mathcal{J}_t^{YO}}{1 + r_t} \quad (32)$$

Since there are two distinct cohorts, and both can cumulate risk-free bonds one of the budget constraints closes the model; the aggregate young budget constraint is as follows:

$$C_t^Y + B_t^Y = w_t L_t + Profit_t - Tax_t + (1 + r_{t-1})(1 - \omega_{t-1}^Y) B_{t-1}^Y \quad (33)$$

## 2.3 Firms

We assume monopolistic competition and price stickiness a la Calvo: in every period a fraction of  $1 - \omega^P$  of firms have a chance to change the nominal price level; the rest of them

can only adjust the previously agreed prices by the previous-period's aggregate inflation rate (Calvo 1983; Milani 2005). The optimization problem is conditional on the expectation that firm  $j$  is not able to set the price in the next periods. As a result, all firms maximize the present value of current and expected future profit flows subject to the production function, the demand function, and the capital accumulation equation. I.e., they maximize

$$E_t \sum_{n=0}^{\infty} \omega^{Pn} \prod_{k=1}^n \frac{1 - \omega_{t+k-1}^Y}{(1 + i_{t+k-1})} \left( P_t^*(j) \left( \frac{P_{t+n-1}}{P_{t-1}} \right)^{\gamma_P} Y_{t+n}(j) - V_{t+n} L_{t+n}(j) - P_{t+n} \text{Inv}_{t+n}(j) \right) \quad (34)$$

subject to:

$$Y_t(j) = e^{-\varepsilon_t^P} A_t K_{t-1}(j)^\alpha L_t(j)^{1-\alpha} \quad (35)$$

$$Y_t(j) = \left( \frac{P_t^*(j)}{P_t} \right)^{-\varphi_P} Y_t \quad (36)$$

$$K_t(j) = \text{Inv}_t(j) \left( 1 - S \left( (1 + g_t^N) \frac{\text{Inv}_t(j)}{\text{Inv}_{t-1}(j)} \right) \right) + (1 - \delta) K_{t-1}(j) \quad (37)$$

Here,  $i$  denotes the nominal interest rate, and the young households are the owners of the firms, so their survival probability is also taken into account.  $P^*(j)$  denotes the individual optimal nominal prices, which are adjusted by the inflation rate. Further,  $\text{Inv}(j)$  is the real investment variable which is multiplied by the aggregate price index;  $\alpha$  is the capital share in the Cobb-Douglas production function;  $\delta$  is the depreciation rate of capital;  $\varphi_P$  is the price elasticity for  $Y(j)$  demanded individual products;  $\gamma_P$  is the degree of price indexation; and  $S(\cdot)$  is the convex investment adjustment cost function.  $\varepsilon_t^P$  denotes the supply shock which is only defined in a sticky price equilibrium. We also assume nominal wage stickiness a la Calvo, and  $V$  denotes the relevant nominal wage index.

Taking the first order conditions, we can derive the usual input demand functions and marginal cost functions:

$$\alpha \frac{Y_t(j)}{K_{t-1}(j)} mc_t = r_t^K \quad (38)$$

$$(1 - \alpha) \frac{Y_t(j)}{L_t(j)} mc_t = v_t \quad (39)$$

$$mc_t = e^{\varepsilon_t^P} \frac{1}{A_t} \left( \frac{r_t^K}{\alpha} \right)^\alpha \left( \frac{v_t}{1 - \alpha} \right)^{1-\alpha} \quad (40)$$

Because the firms are responsible for physical capital accumulation, the set of first order



conditions also contains the Tobin-Q equation and a no-arbitrage condition:

$$Q_t \left( 1 - S \left( (1 + g_t^N) \frac{Inv_t(j)}{Inv_{t-1}(j)} \right) - S' \left( (1 + g_t^N) \frac{Inv_t(j)}{Inv_{t-1}(j)} \right) \frac{Inv_t(j)}{Inv_{t-1}(j)} \right) = \quad (41)$$

$$= 1 - E_t \frac{1 - \omega_t^Y}{1 + r_t} Q_{t+1} S' \left( (1 + g_{t+1}^N) \frac{Inv_{t+1}(j)}{Inv_t(j)} \right) \left( \frac{Inv_{t+1}(j)}{Inv_t(j)} \right)^2 \quad (42)$$

$$E_t (1 - \omega_t^Y) (r_{t+1}^K + Q_{t+1} (1 - \delta)) = Q_t (1 + r_t) \quad (43)$$

Next, the monopolistic firm sets the optimal price as follows:

$$p_t^*(j) = \frac{\varphi_P}{\varphi_P - 1} \frac{\mathcal{Z}_t^1}{\mathcal{Z}_t^2} \quad (44)$$

$$\mathcal{Z}_t^1 = p_t(j)^{-\varphi_P} Y_t m c_t + E_t \left( \frac{p_t(j)}{p_{t+1}(j)} \frac{(1 + \pi_t)^{\gamma_P}}{1 + \pi_{t+1}} \right)^{-\varphi_P} \omega_P (1 - \omega_t^Y) \frac{1 + \pi_{t+1}}{1 + i_t} \mathcal{Z}_{t+1}^1 \quad (45)$$

$$\mathcal{Z}_t^2 = p_t(j)^{-\varphi_P} Y_t + E_t \left( \frac{p_t(j)}{p_{t+1}(j)} \frac{(1 + \pi_t)^{\gamma_P}}{1 + \pi_{t+1}} \right)^{-\varphi_P} \frac{(1 + \pi_t)^{\gamma_P}}{1 + i_t} \omega_P (1 - \omega_t^Y) \mathcal{Z}_{t+1}^2 \quad (46)$$

Based on the definition of the price index, we can express the optimal individual relative price as a function of actual and previous inflation rates:

$$1 = (1 - \omega_P) p_t^*(j)^{1 - \varphi_P} + \omega_P \left( \frac{(1 + \pi_{t-1})^{\gamma_P}}{1 + \pi_t} \right)^{1 - \varphi_P} \quad (47)$$

Next, the optimization problem of the labor union can be given as follows:

$$E_t \sum_{n=0}^{\infty} \omega_V^n \prod_{k=1}^n \frac{1 - \omega_{t+k-1}^Y}{(1 + i_{t+k-1})} \left( V_t^*(j) \left( \frac{V_{t+n-1}}{V_{t-1}} \right)^{\gamma_V} L_{t+n}(j) - W_{t+n} L_{t+n}(j) \right) \quad (48)$$

subject to

$$L_t(j) = \left( \frac{V_t^*(j)}{V_t} \right)^{-\varphi_W} L_t \quad (49)$$

where  $\omega_V$  is the fraction of unions that are not able to set their prices in a given period.  $V^*(j)$  denotes the individual optimal nominal wage adjusted by the wage inflation rate.  $\varphi_W$  is the wage elasticity for labor, and  $\gamma_V$  is the degree of price indexation. The monopolistic labor union sets the optimal nominal wage as follows:

$$v_t^*(j) = \frac{\varphi_V}{\varphi_V - 1} \frac{\mathcal{W}_t^1}{\mathcal{W}_t^2} \quad (50)$$

$$\mathcal{W}_t^1 = \left( \frac{v_t(j)}{v_t} \right)^{-\varphi_V} L_t w_t + E_t \left( \frac{v_t(j)}{v_{t+1}(j)} \frac{(1 + \pi_t^V)^{\gamma_V}}{1 + \pi_{t+1}^V} \right)^{-\varphi_V} \frac{\omega_V (1 - \omega_t^Y) (1 + \pi_{t+1})}{1 + i_t} \mathcal{W}_{t+1}^1 \quad (51)$$

$$\mathcal{W}_t^2 = \left( \frac{v_t(j)}{v_t} \right)^{-\varphi_V} L_t + E_t \left( \frac{v_t(j)}{v_{t+1}(j)} \frac{(1 + \pi_t^V)^{\gamma_V}}{1 + \pi_{t+1}^V} \right)^{-\varphi_V} \frac{\omega_V (1 - \omega_t^Y) (1 + \pi_t^V)^{\gamma_V}}{1 + i_t} \mathcal{W}_{t+1}^2 \quad (52)$$

Similarly, based on the definition of the price index, we can express the optimal individual relative wage as a function of actual and previous wage inflation and real aggregate wage indices:

$$1 = (1 - \omega_V) \left( \frac{v_t^*(j)}{v_t} \right)^{1-\varphi_V} + \omega_V \left( \frac{(1 + \pi_{t-1}^V)^{\gamma_V}}{1 + \pi_t^V} \right)^{1-\varphi_V} \quad (53)$$

where the  $\pi^V$  is the nominal wage inflation which can be given as:

$$1 + \pi_t^V = (1 + \pi_t) \frac{v_t}{v_{t-1}} \quad (54)$$

Here,  $v$  denotes the union wage in real terms. Last, the profits of firms and the labor union go to the young:

$$Profit_t = Y_t - w_t L_t - Inv_t \quad (55)$$

## 2.4 Fiscal Policy

The role of fiscal policy is limited in this paper: the government is responsible for providing pay-as-you-go pension benefits and finances its expenditures by taxing the young cohort. It can freely adjust the lump-sum taxes and does not accumulate any debt:

$$Tax_t = TR_t + Gov_t \quad (56)$$

In the PAYG-regime individual ( $i$ )'s pension benefits in the year of retirement  $t$  are based on the replacement rate  $v$  and the pre-retirement labor income:

$$TR_{0,t}^{YO}(i) = v w_{t-1} L_{b-1,t-1}(i) \quad (57)$$

The aggregated version of the pension rules are:

$$TR_t^{YO} = v\omega_{t-1}^Y w_{t-1} L_{t-1} \quad (58)$$

Furthermore, the total pension expenditure of all retired agents can be described as a function of pension benefits and survival probabilities:

$$TR_t = \sum_{n=0}^{\infty} \prod_{k=1}^n (1 - \omega_{t-k}^O) TR_{t-n}^{YO} \quad (59)$$

which can be simply rewritten in recursive form as:

$$TR_t = TR_t^{YO} + (1 - \omega_{t-1}^O) TR_{t-1} \quad (60)$$

## 2.5 Monetary Policy

Initially, the central bank follows a simple rule with inflation reaction and interest rate smoothing:

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left( (1 + \tilde{r}_t) (1 + \pi_t)^{\phi_\pi} \right)^{1 - \rho_i} e^{\varepsilon_t^i} \quad (61)$$

where  $\varepsilon_t^i$  is the monetary policy shock. By assumption, the central bank raises the interest rate if inflation exceeds its steady-state (SS) level. Since prices are sticky, the central bank is able to influence the real economy's performance via the real interest rate channel.  $\tilde{r}$  notes the central bank's long-term real interest rate:

$$1 + \tilde{r}_t = (1 + \tilde{r}_{t-1})^{\rho_{\tilde{r}}} (1 + r_t^n)^{1 - \rho_{\tilde{r}}} \quad (62)$$

where  $r^n$  is the natural interest rate that is consistent with the flexible-price version of the model. We assume that the monetary policy is not able to automatically adjust itself to the new natural rate, it can do only gradually (Kumhof, Laxton, Muir, and Mursula 2010). The Fisher-identity expresses the relationship between the nominal and real interest rates:

$$1 + i_t = E_t(1 + r_t)(1 + \pi_{t+1}) \quad (63)$$

## 2.6 Market Clearing Conditions

Finally, we need to clarify the market clearing conditions. Since there is no government debt, the sum of the two cohorts' savings should be zero, i.e. that the net savings/debt of young is equal to the net

debt/savings of the retired.

$$0 = B_t^Y + B_t^O \quad (64)$$

In the goods market all supplied goods are equal to the demanded consumption goods, investment, and government expenditure:

$$Y_t = C_t^Y + C_t^O + Inv_t + Gov_t \quad (65)$$

### 3 Parametrization

Because in this paper we focus on general patterns, we parametrize the model by choosing parameter values which are typical in the DSGE literature. Table 1 shows the chosen parameter values. As we express all variables in per capita values, and solve the steady state of the normalized model, we need a candidate for the steady-state value of the equilibrium real interest rate. Conditional on this assumed level of the real interest rate, we can calculate all the other endogenous variables. Then, as a final step, we also need to check whether the financial market equilibrium condition holds; if not, another starting value for the steady-state real interest rate needs to be picked, and the process starts again.<sup>11</sup>

In this paper the focus is on short-run cyclical fluctuations, where the sizes of the discount rates, the interest rate and labor supply elasticities have a crucial role. They reflect the relative importance of different channels for different instruments used in the model.

First, the discount rate of the old is higher than that of the young reflecting on the fact that the old are more patient than the young. Then, the interest rate elasticity (i.e. the intertemporal substitution) can be derived as follows:

$$\varepsilon = -\frac{U_{C(i)}}{U_{C(i),C(i)}C(i)} = \frac{1}{1 + \sigma(\gamma - 1)} \quad (66)$$

The retired agents do not supply labor ( $\sigma = 1$ ), which means that the  $\varepsilon$ -s are cohort-specific. The parametrized values are based on typical values of the DSGE-literature, and the dynamic responses are relatively close to those estimated by Wong (2018) for the old and the young, respectively.

Next, the Frisch labor supply elasticity can only be calculated for the young cohort:

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<sup>11</sup>The same approach was followed in Baksa and Munkacsi (2016).

$$\eta = \frac{U_{L(i)}}{L(i) \left( U_{L(i),L(i)} - \frac{U_{C(i),L(i)}^2}{U_{C(i),C(i)}} \right)} = \frac{s^Y - \tilde{L} 1 - \sigma(1 - \gamma)}{\tilde{L} \gamma} \quad (67)$$

where  $\tilde{L}$  is the normalized level of labor. Our specification is consistent with the available microeconomic estimates (such as that of Whalen and Reichling (2017)). Namely, the Frisch elasticity is a function of the steady-state demographic and labor market variables. Thus, aging directly changes the relative size of cohorts and the supply of labor: the labor force shrinks and the labor market becomes more tight with aging. Relatedly, labor supply becomes more inelastic (the Frisch-elasticity decreases as the old-age dependency ratio increases), and the volatility of real wages is intensified (the more scarce the labor supply, the more profound the real wage reaction to shocks) (Table 3).

Table 1: Model parameters

Name	Notation	Value	Comments
<b>Demography</b>			
Probability of retirement	$\omega^Y$	0.0050	Quarterly frequency
Probability of death	$\omega^O$	0.0200	Quarterly frequency
Fertility rate	$n$	0.0100	Quarterly frequency
<b>Households</b>			
Discount rate (old)	$\beta^O$	0.9990	Quarterly frequency
Discount rate (young)	$\beta^Y$	0.9950	Quarterly frequency
Weight of consumption utility	$\sigma$	0.6000	Only for youngs
Inverse of intertemporal elasticity	$\gamma$	2.0000	Same for both cohort
<b>Firms</b>			
Price elasticity	$\varphi_P$	6.0000	-
Wage elasticity	$\varphi_W$	6.0000	-
Capital share	$\alpha$	0.3300	-
Capital depreciation	$\delta$	0.0250	Quarterly frequency
Investment adjustment cost	$\phi_{Inv}$	2.0000	-
Share of price setters (Calvo)	$\omega_P$	0.8000	Quarterly frequency
Price indexation	$\gamma_P$	0.5000	-
Share of union wage setters (Calvo)	$\omega_V$	0.8000	Quarterly frequency
Nominal wage indexation	$\gamma_V$	0.5000	-
<b>Monetary policy</b>			
Interest rate smoothing	$\rho^i$	0.7500	Quarterly frequency
Reaction to inflation	$\phi_\pi$	1.5000	-
<b>Fiscal policy</b>			
Replacement rate	$v$	1.0746	Calibrated from $\frac{TR}{Y} = 0.1$
Gov. cons.	$Gov$	0.2777	Calibrated from $\frac{Gov}{Y} = 0.2$

## 4 A Permanent Demographic Shift

In this paper we mainly focus on (i) how aging affects the short-run cyclical behavior of the macroeconomy and (ii) how optimal monetary policies change with aging. Before doing so in Sections 5 and 6, though, in this section we demonstrate a transition from a young to an old society. Particularly, Table 2 shows the SS comparison of a young and an old society when the old-age dependency ratio increases from 20 to 80 percent over a 50-year horizon, while Figure 1<sup>12</sup> the transition path between the two SS-s. This exercise gives an opportunity to point out that the long-run properties of our model are consistent with those of the secular stagnation literature, such as Summers (2014), Favero and Galasso (2015), Eggertsson, Mehrotra, and Robbins (2017), Ferrero, Gross, and Neri (2017), or Jones (2018).

Table 2: Steady-state comparison: young and gray societies

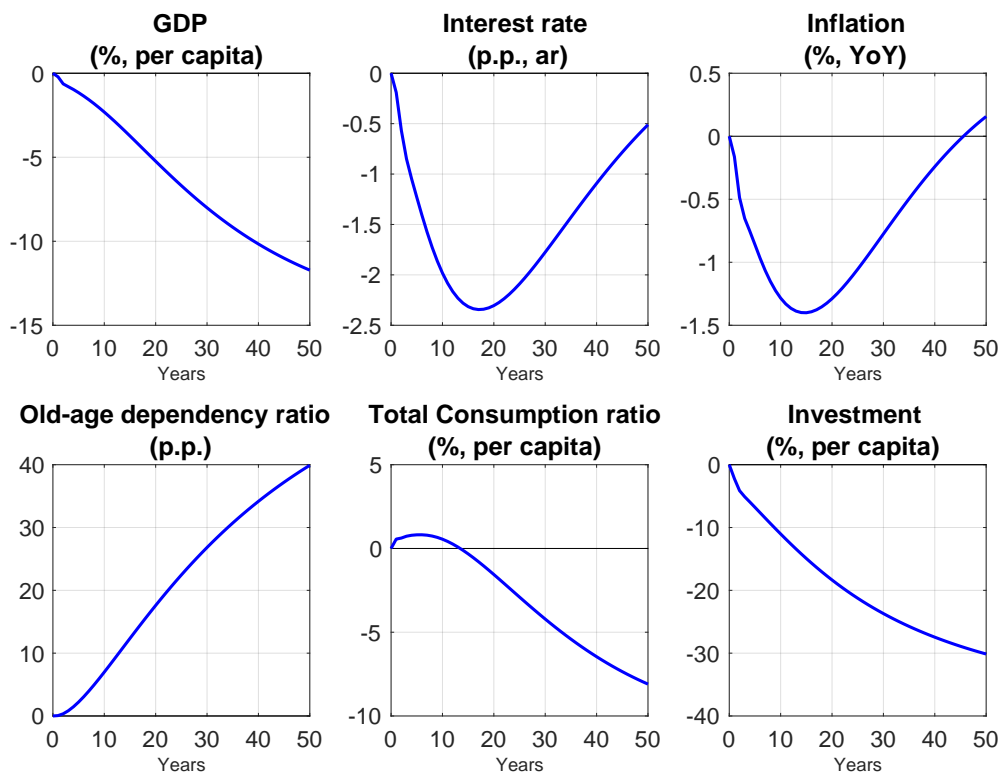
Name	Notation	Young society	Graying society
<b>Demography</b>			
Old-age dependency ratio	$s$	0.2000	0.8000
Share of the young	$s^Y$	0.8333	0.5556
<b>Households</b>			
Consumption to GDP: Total	$\frac{C}{\bar{Y}}$	0.5622	0.6013
Consumption to GDP: Young	$\frac{C^Y}{\bar{Y}}$	0.4635	0.1677
Consumption to GDP: Old	$\frac{C^O}{\bar{Y}}$	0.0986	0.4336
Bonds to GDP: Old	$\frac{B^O}{\bar{Y}}$	0.2579	8.4030
Interest rate elasticity: Young	$\varepsilon^Y$	0.6250	0.6250
Interest rate elasticity: Old	$\varepsilon^O$	0.5000	0.5000
Frisch elasticity	$\eta$	0.5313	0.1922
<b>Firms</b>			
GDP	$Y$	1.3883	1.2098
Investment to GDP	$\frac{Inv}{Y}$	0.2378	0.1691
Capital to GDP	$\frac{K}{Y}$	7.9677	7.4986
Profit to GDP	$\frac{Profit}{Y}$	0.2969	0.3656
<b>Monetary policy</b>			
Real interest rate	$r$	0.0046	0.0065
<b>Fiscal policy</b>			
Transfer to GDP	$\frac{TR}{Y}$	0.1000	0.4000
Public consumption to GDP	$\frac{Gov}{Y}$	0.2000	0.2295

Due to the longer lifetime horizon (decreasing  $\omega^O$ ), the retired households accumulate more savings. At the same time, the PAYG pension system is required to provide a higher amount of pension benefits which can be financed by taxes deducted from the young workers. So, the young agents face the increasing financing need from the pension system, and due to consumption smoothing (and to

<sup>12</sup>ar is an abbreviation for annualized rate.

minimize their own sacrifice) their indebtedness increases. Green, Fry, and Myerson (1994) point out a psychological fact: the old households are more patient which, in our framework, is modeled by a higher discount rate than that of the young households. This assumption helps magnify the level of indebtedness of the young cohort.

Figure 1: Transition from a young to an old society



Regarding the supply side of the economy, by lower fertility and shrinking labor force the firms accommodate to the new situation: they realize the decrease in demand, so they gradually disinvest and reduce prices (generate disinflation or deflation) to offset the loss in profits. In the medium term, instead of investing in physical capital, they hire relatively more labor. The decreasing free labor capacities make the real wage more volatile and more responsive to short-run shocks as the Frisch elasticity decreases. Due to the labor shortages, real wages rise, and inflation goes back to its steady-state level. The central bank first decreases the interest rate to compensate for the downward pressure on inflation. However, in the long run it gradually normalizes the policy rate since inflation goes back to its steady-state level. Because of the lower level of the physical capital stock, the increased marginal product of capital implies a slightly higher equilibrium interest rate in the new steady state.

## 5 Cyclical Behavior and Monetary Policy Transmission

Now we turn to presenting the dynamic properties of the model. Namely, in a young and an old society we compare the short-run cyclical reactions of the macroeconomy to a monetary policy shock, a government expenditure shock and a supply shock. We examine one-off, temporary shocks only, i.e., all the impulse response functions converge back to the demographic-specific steady state.<sup>13</sup>

### 5.1 Monetary Policy Shock

It is well-known that if the central bank deviates from its systematic Taylor-rule, i.e., it increases the nominal interest rate, then, because of price stickiness, it also changes the real interest rate, and so can influence the performance of the real economy (Figure 2). Tighter monetary policy conditions force firms to postpone investment, and households to start accumulating more savings or decreasing their credit stock. Due to the shrinking aggregate demand, share  $1 - \omega^P$  of the firms decrease their prices to avoid (reduce) the loss in profits. Due to price stickiness and price indexation, the nominal adjustment is gradual, and disinflation takes more than 4 quarters. The central bank observes the shrinking aggregate demand and decreases the nominal interest rate back to its steady-state level, and the whole economy stabilizes at its initial steady state.

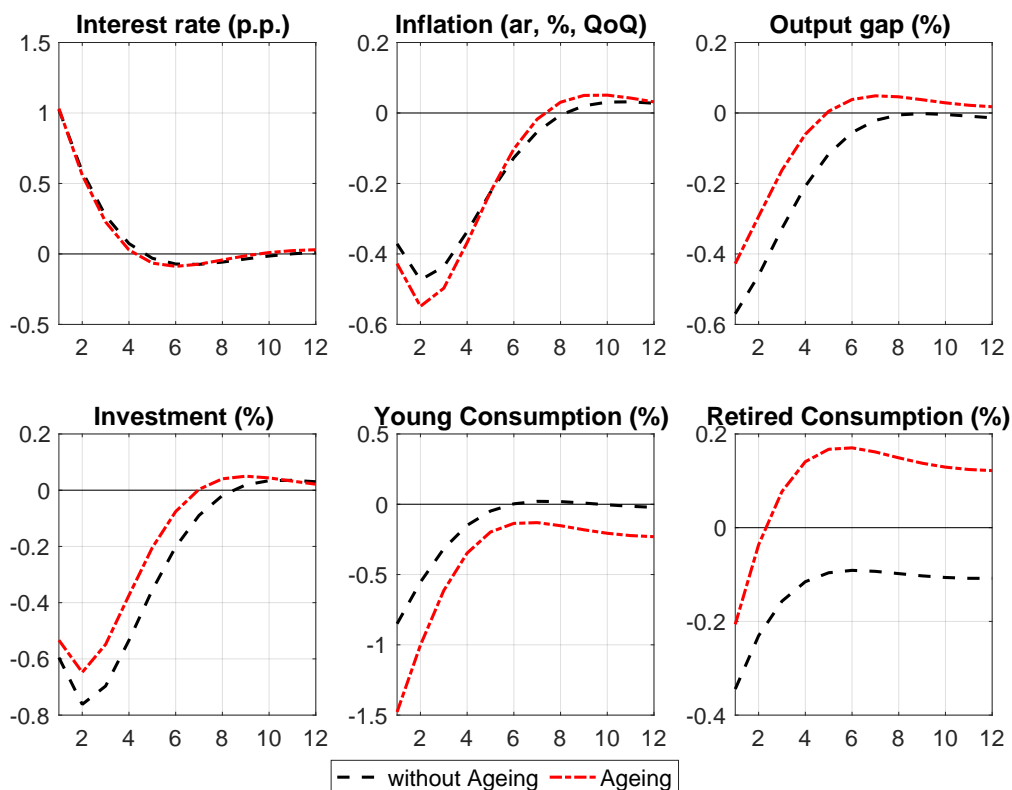
Aging generates significant differences in the output gap reaction (the deviation of GDP from its flexible-price equilibrium level). This is also reflected in the fact that the young and the old respond differently. Particularly, in an old society the retired agents hold more savings, while the workers have more debt during their longer life-time. Additionally, in a young society the monetary restriction creates an incentive for postponing consumption and increasing savings. However, in an old society the interest rate hike has an even more asymmetric effect: the old can interpret the shock as additional income, thus, they increase consumption by a larger amount, while the young face higher credit costs and decrease consumption more. Because aging also changes the relative size of cohorts, aggregate consumption and output gap fall less in grey societies. Hence, we find that aging reallocates the asset position among generations, which makes monetary policy less effective - defined as a larger drop in the inflation rate to the same monetary tightening -, and aggregate demand less elastic to interest rate changes. Our finding is consistent with that of Wong (2018) who, based on micro-level cohort-specific US data, shows that the old react less to expansionary monetary policy shocks.

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<sup>13</sup>In all simulations we assume that in young societies (on Figures 2-4 labeled *without Aging*) the old-age dependency ratio is 20%, in old societies (on Figures 2-4 labeled *Aging*) 80%. On the same figures there is an abbreviation for annualized rate.



Figure 2: Impulse responses of a monetary policy shock



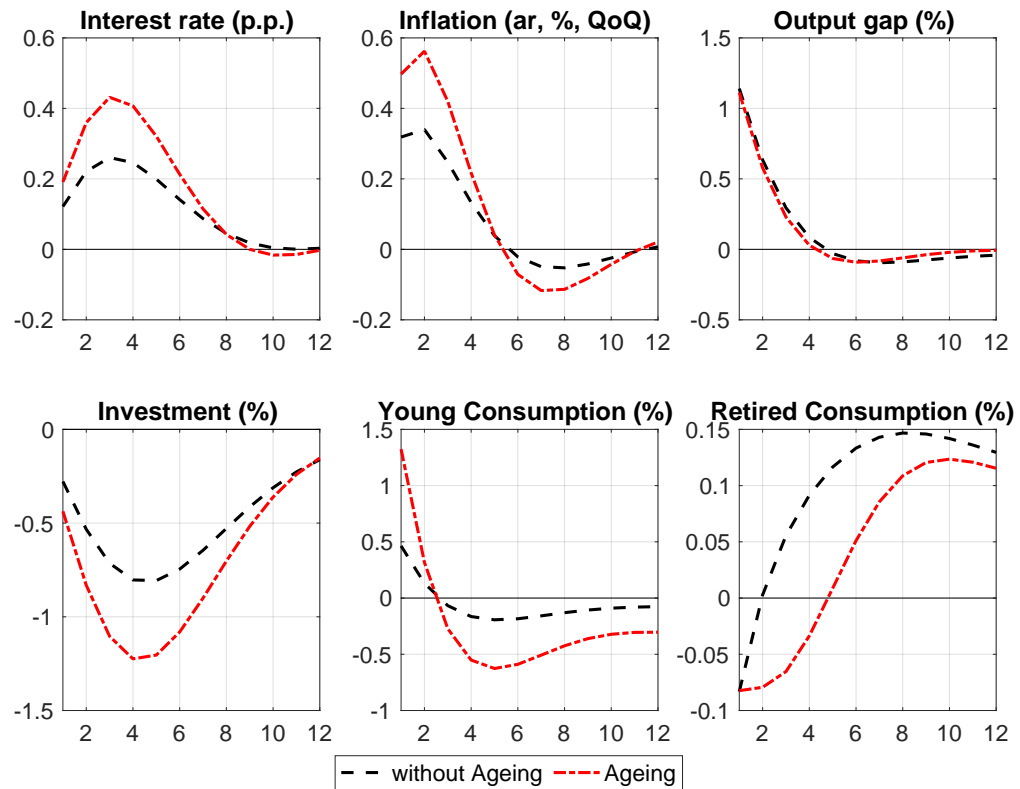
## 5.2 Government Expenditure Shock

The government increases public expenditure by one percent of steady-state GDP (Figure 3). As a result, firms increase production to satisfy the extra demand. A higher level of production requires more labor, so firms increase wages to attract more workers. To offset the increase in production costs and the loss in profits, price-setting firms increase their prices. The central bank launches a tightening cycle, and holds the interest rate elevated until the demand-side inflationary pressure disappears, and inflation goes back to its original steady-state level. Due to the increase in wages, young households consume more. Old households consume less because of the increasing interest rate and credit demand (from the young households). Later on, though, the higher interest rates push young consumption down and old consumption up.

Ageing also changes the relative size of cohorts, and decreases the available labor force in the economy. In an old society the labor supply is more inelastic (the Frisch-elasticity is lower). Hence, firms are forced to increase wages more than in a young society to attract the required labor force. This

additional increase in wages amplifies the increase in marginal costs and inflation, too. As a consequence, in an old society the central bank needs to raise the policy rate by a larger amount than in a young society. A stronger monetary policy reaction in a more gray society forces the young to give up more consumption.

Figure 3: Impulse responses of a government expenditure shock

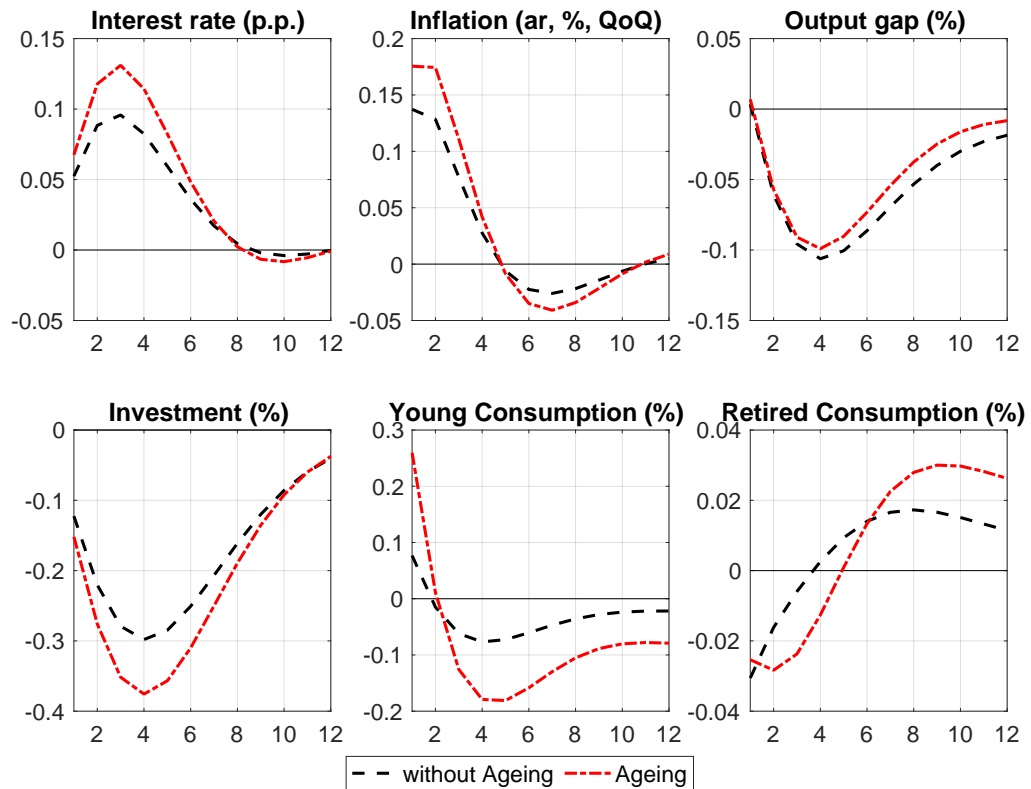


### 5.3 Supply Shock

The main channel of the supply shock is that price-setting firms decide to increase prices to improve profitability, or offset profit losses (Figure 4). Due to higher inflation, the central bank increases the nominal interest rate which forces households and firms to postpone expenditure items (consumption and investment, respectively). Observing the fall in domestic demand, firms decide to give up on

further price increases, thus, inflation starts to normalize. As a result, the central bank reduces the policy rate, and the economy stabilizes around its original steady state.

Figure 4: Impulse responses of a supply shock



Even though the size of the shocks in the old and young societies are the same, the immediate impact on nominal and real variables differs. The central bank reacts to offset the positive inflationary expectations (and more negative real interest rate), and discourages households from reallocating future consumption to present consumption. In a young society both households and firms are aware of a possible monetary tightening in the near future, and due to the expected decline in aggregate demand the overall increase in inflation is lower than what the shock would imply. In an old society retired households hold more savings, and their reaction to monetary tightening quickly vanishes, so aggregate consumption does not decrease as much as in a young society. At the same time, the young hold more debt, and at the time of the shock the temporary negative real interest rate depreciates the young

households' credit stock, thus they initially increase their consumption. This effect in an old society is larger (in an old society the young are more indebted), hence, the jump in young consumption is more significant at the beginning. Due to the relatively higher demand in the old society, firms are able to increase their prices by a larger amount. In the old society the overall inflationary pressure is also higher, therefore, the central bank needs to be more responsive, which later requires a higher sacrifice (in terms of consumption goods) from the young.

## 6 Welfare and Optimal Monetary Policy

Aging also influences the size of welfare losses. To demonstrate that, first we need to calculate the conditional and unconditional variances of the endogenous variables which are used as inputs in the welfare loss function. The optimal monetary policy rule is the rule which minimizes the loss in social welfare.<sup>14</sup>

### 6.1 The Model's State-Space Form

Based on the log-linear version of the model one can express the following forward looking system of equations which are conditional on the model's steady state, in particular on the demographic structure:

$$A(\Theta)\xi_t = B(\Theta)\xi_{t-1} + C(\Theta)E_t\xi_{t+1} + D(\Theta)\varepsilon_t \quad (68)$$

where  $\xi$  is the vector of endogenous variables,  $\varepsilon$  is the vector of structural shocks with given variances, and  $\Theta$  is the set of deep parameters including the steady-state levels of the endogenous variables.  $A$ ,  $B$ ,  $C$ , and  $D$  matrices consist of the linearized equations. Using the method of undetermined coefficients one can express the state-space form of the forward-looking model as follows:

$$\xi_t = \Phi(\Theta)\xi_{t-1} + \Gamma(\Theta)\varepsilon_t \quad (69)$$

Here,  $\Phi$  and  $\Gamma$  are matrices which are combinations of  $A$ ,  $B$ ,  $C$ , and  $D$ . Using the state-space form of the model we can express the conditional covariances as follows:

$$\Xi_t = \Phi(\Theta)\Xi_{t-1}\Phi(\Theta)' + \Gamma(\Theta)\Omega\Gamma(\Theta)' \quad (70)$$

Here,  $\xi_t\xi_t' = \Xi_t$  are conditional covariances of the endogenous variables, while  $\varepsilon_t\varepsilon_t' = \Omega$  is a diagonal covariance matrix of the structural shocks. In the optimal policy exercise we assumed a standard

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<sup>14</sup>Calculations are done by the Optimal Simple Rule toolbox of Dynare.

deviation of 0.3 for the government expenditure, mark-up and technology shocks, while the standard deviation of the monetary policy and other shocks is set to zero. Iterating this equation forward ( $t \rightarrow \infty$ ), we can express the unconditional covariances as follows:

$$\text{vec}(\Xi) = (I - \Phi(\Theta) \otimes \Phi(\Theta))^{-1} \text{vec}(\Gamma(\Theta)\Omega\Gamma(\Theta)') \quad (71)$$

where  $I$  is an identity matrix which has the same size as  $\Phi(\Theta) \otimes \Phi(\Theta)$ .

## 6.2 Welfare Loss Function and Monetary Policy Rules

In order to determine the optimal monetary policy rule, i.e. the rule which minimizes the weighted sum of unconditional variances, first we need to specify the social welfare loss function. The presence of overlapping generations and that of the non-representative features of the model make the model very complicated and hard to interpret welfare based on the agents' utility function. Therefore, we assume a simple welfare loss function which assigns a particular weight to the unconditional variances of inflation, output gap, and the difference between today's and yesterday's nominal interest rates. The latter is particularly relevant as it reflects the central bank's concern about financial stability: the central bank does not want to choose a policy which increases the volatility in the financial markets or discourages savings/credits. The weights taken from Adolfson, Laseen, Linde, and Svensson (2011) and represent the importance of each objective in the central bank's decision making:

$$\mathbf{L} = \text{var}(\pi_t) + 0.5 \cdot \text{var}(\hat{Y}_t) + 0.2 \cdot \text{var}(i_t - i_{t-1}) \quad (72)$$

Here,  $\hat{Y}$  is the output gap defined as the deviation of GDP from its flexible price level.

Several monetary policy rules are examined: (i) pure inflation targeting rule (including a forward looking IT), (ii) flexible inflation targeting with output reaction (including a forward looking version as well), (iii) price level targeting, and (iv) nominal GDP targeting. The rules are defined as follows:

$$i_t = \rho i_{t-1} + (1 - \rho) \tilde{r}_t^n + (1 - \rho) \left\{ \begin{array}{ll} \phi_{\pi_t} \pi_t & \text{Pure IT} \\ \phi_{E_t \pi_{t+1}} E_t \pi_{t+1} & \text{Pure IT \& Fwd} \\ \phi_{\pi_t} \pi_t + \phi_{\hat{Y}_t} \hat{Y}_t & \text{Flex. IT} \\ \phi_{E_t \pi_{t+1}} \pi_{t+1} + \phi_{\hat{Y}_t} \hat{Y}_t & \text{Flex. IT \& Fwd} \\ \phi_{\hat{P}_t} \hat{P}_t & \text{Price level targeting} \\ \phi_{\widehat{P}_t \widehat{Y}_t} \widehat{P}_t \widehat{Y}_t & \text{Nominal GDP targeting} \end{array} \right. \quad (73)$$

Conditional on the selected rules and the demographic structure, we look for the reaction parameters which minimize the total welfare loss described above.

### 6.3 Optimal Monetary Policy

First of all, we compare welfare losses with different demographic structures but with the same non-optimal monetary policy rule (same non-optimal parameters), and show how the increase in the old-age dependency ratio influences them (Table 3). The old-age dependency ratio is a function of the retirement probability, the survival probability, and the net fertility rate. During the aging process the latter two are assumed to change; in the table below we present the cases where the two probabilities change at the same time (alternative combinations of the two rates and their welfare values are available in the Appendix). As aging becomes more prevalent, i.e., the old-age dependency ratio increases, the volatility of the inflation rate and that of the interest rate increase. Initial losses are normalized to  $s = 0.2$  where  $s$  denotes the old-age dependency ratio, and the optimal values are always normalized by their own initial loss values. In the case of very high old-age dependency ratios the output gap volatility dominates, due to the increasing volatility of labor market variables.

Table 3: Old-age dependency ratios and welfare levels with the three baseline monetary policy rules

	<b>Old-age dependency ratios</b>			
	$s = 0.20$	$s = 0.40$	$s = 0.60$	$s = 0.80$
Initial losses	1.000	0.959	0.947	1.154
Inflation:	0.111	0.126	0.147	0.191
Output gap:	0.887	0.831	0.797	0.959
Interest rate:	0.002	0.002	0.003	0.003
Frisch elasticity:	0.531	0.441	0.325	0.192

Next, optimal monetary policy reaction functions (Table 4) are explored, i.e., for any given OADR we calculate the minimum social welfare loss with the different monetary policy regimes and the related monetary policy rule parameters. Our main finding is that in a younger society the flexible inflation targeting regime, with a strong reaction to inflation and a somewhat weaker reaction to the output gap, is the most effective rule, i.e. the lost-minimizing option. In a more gray society, however, even though the flexible inflation targeting regime could be one of the favorable options, the central bank should mildly react to the output gap, while the reaction to the level of inflation should be stronger.

At the same time, we can also perceive that the nominal GDP targeting rule is also favorable, and becomes the most effective in a very old society. Additionally, price level targeting generates higher welfare losses than nominal GDP targeting does. Price level targeting is quite hawkish, so the central bank tolerates a larger economic sacrifice to achieve the stability of the price level, while nominal GDP

targeting is basically a combination of price level targeting and output gap stabilization, meaning that second-round effects on aggregate demand are also taken into account. This kind of a dual-mandate guarantees that the welfare loss is minimized when both the price level and the output gap vary as little as possible. However, this is only an optimal policy if the old-age dependency ratio is extreme high.<sup>15</sup>

Table 4: Old-age dependency ratios and optimal reaction parameters in six monetary policy regimes

		<b>Old-age dependency ratios</b>			
		$s = 0.20$	$s = 0.40$	$s = 0.60$	$s = 0.80$
Initial rule	Initial loss	1.000	0.959	0.947	1.154
	$\phi_{\pi_t}$	1.500	1.500	1.500	1.500
Pure IT	Optimized loss	0.924	0.885	0.832	0.760
	$\phi_{\pi_t}$	3.718	4.217	5.032	6.751
Pure IT & Fwd	Optimized loss	0.872	0.837	0.646	0.646
	$\phi_{E_t \pi_{t+1}}$	6.353	6.880	7.835	9.541
Flex. IT	Optimized loss	0.478	0.495	0.528	0.584
	$\phi_{\pi_t}$	5.756	6.105	6.966	8.744
	$\phi_{\hat{Y}_t}$	3.428	3.391	3.209	2.646
Flex. IT & Fwd	Optimized loss	0.492	0.513	0.550	0.612
	$\phi_{E_t \pi_{t+1}}$	6.102	6.537	7.468	9.026
	$\phi_{\hat{Y}_t}$	3.164	3.088	2.820	2.122
Price level targ.	Optimized loss	0.913	1.093	1.032	0.898
	$\phi_{\hat{P}_t}$	0.017	0.603	1.250	3.130
Nominal GDP targ.	Optimized loss	0.499	0.498	0.499	0.481
	$\phi_{\widehat{P_t Y_t}}$	5.920	6.063	6.457	7.862

## 7 Discussion and Conclusion

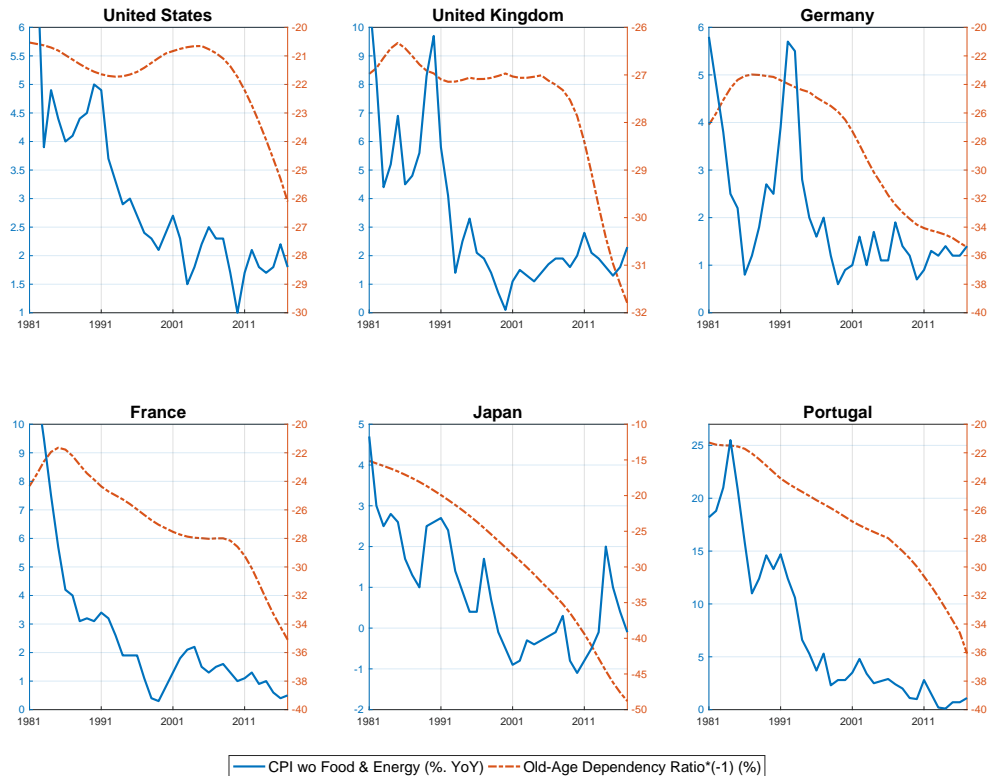
In the previous sections we explored the monetary consequences of aging in a multi-period DGE model with OLG agents: we examined how aging affects (i) inflation in the longer term, (ii) the short-run cyclical behavior of the economy, including monetary policy transmission, and (iii) optimal

<sup>15</sup>In the Appendix, we provide alternative simulations to check the robustness of our results, by using an output gap weight of 1.0 and an interest rate smoothing gap of 0.5. Qualitatively, we find very similar results. With a higher output gap weight, nominal GDP targeting becomes optimal only with a higher OADR. At the same time, with a higher weight on interest rate smoothing, nominal GDP targeting is the most effective rule already in a less aged society.

monetary policy rules. We reported that the rate of inflation decreases, while its volatility increases with aging, aging makes monetary policy less effective, and an IT regime with a high inflation reaction is optimal in the case of high, but still reasonable OADRs.

First, regarding the impact of aging on the rate of inflation, several papers listed in the introductory section claim that aging leads to a downward pressure on inflation, while some report the opposite. Using data on developed economies (the U.S., the U.K., Germany, France, Japan, and Portugal)<sup>16</sup> between 1981 and 2017, we find that the periods of disinflation correspond with the periods of increases in the old-age dependency ratio (Figure 5).<sup>17</sup> This is in line with what our model suggests when increasing the old-age dependency ratio in the steady state.

Figure 5: Core inflation and old-age dependency ratio



<sup>16</sup>Japan is already heavily hit by aging; Portugal will be among the countries with the highest old-age dependency ratio in Europe in a few decades; the US, the UK, Germany and France - although less affected by aging than Japan or Portugal - follow different monetary policy strategies (with France being less affected by aging than Germany).

<sup>17</sup>Inflation data comes from the OECD, while data on the old-age dependency ratio comes from the UN.



Second, regarding the impact of aging on the volatility of inflation, we also test the model’s hypothesis using an unbalanced panel dataset on developed OECD countries between 1993 and 2017. We run a fixed effect (FE) estimation with country and time fixed effects.<sup>18</sup> For this we need the time-variant volatility of inflation: first, we remove the trend component of non-food non-energy CPI by an HP-filter with  $\lambda = 150000$ , and then we calculate the yearly average of the standard deviation from the trend which is interpreted as the time-variant volatility of the cyclical inflation. Next, we run the following FE regression:

$$\sigma(cpi\_core)_{it} = \alpha_i + \delta_t + \beta OADR_{it} + \gamma X_{it} + u_{it} \quad (74)$$

where  $\sigma(cpi\_core)_{it}$  is the standard deviation of non-food non-energy CPI in country  $i$  and year  $t$ ,  $\alpha_i$  is the country fixed effect,  $\delta_t$  is the time fixed effect,  $OADR_{it}$  is the old-age dependency ratio,  $X$  contains other controls (lag and output gap volatility)<sup>19</sup>, and  $u_{it}$  denotes the error term.

Table 5: Volatility of inflation and old-dependency ratio

Models	FE (1)	FE (2)	FE (3)	FE (4)
$\sigma(cpi\_core)_{i,t-1}$			0.087*	0.083·
			(0.043)	(0.043)
$OADR_{it}$	0.021*	0.020*	0.019*	0.018*
	(0.011)	(0.010)	(0.009)	(0.008)
$\sigma(Y)_{it}$		0.015*		0.017*
		(0.008)		(0.008)
Country FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
N.obs	793	793	759	759
$R^2$	0.250	0.254	0.307	0.318

\*\*\* is  $p < 0.1\%$ , \*\* is  $p < 1\%$ , \* is  $p < 5\%$ , · is  $p < 10\%$

The estimation results are in line with the model’s intuitions (Table 5). Namely, all of the estimations suggest a significant and positive relationship between the volatility of inflation and the level of the old-age dependency ratio<sup>20</sup>. Even more, the impact is sizeable: in our sample the median volatility is 0.19, while the FE regressions suggest that a 10 percentage point increase in the old-age dependency ratio increases inflation volatility by around 0.18-0.20.

In Section 5 we reported that the same size of shock causes higher inflation volatility in an older society. Not only inflation, but the macroeconomy in general reacts differently to shocks, including to a

<sup>18</sup>In R with plm package (Croissant and Millo 2008).

<sup>19</sup>Data on the output gap was downloaded from the OECD.

<sup>20</sup>We calculated and report robust standard errors

monetary policy shock. The transmission of monetary policy changes, and monetary policy becomes less effective. The most important channel is that as agents live longer and their planning horizon becomes longer, their savings position changes: the young are willing to borrow more, while the retired accumulate more savings to guarantee their consumption over a longer time horizon. Hence, when the interest rate changes, it has different implications for the young and the old: higher interest rates imply an extra cost for the young who are indebted, while the old generate more income. Additionally, the young and the old also make different consumption-savings decisions (as the old are more patient than the young), and there are labor market implications as well (the labor market becomes more tight and real wages react more as the labor force shrinks).

Then in Section 6 we discussed that aging, via higher inflation, also increases social welfare loss. To avoid that central banks with inflation targeting regimes should more strongly react to nominal variables: they should increase the nominal interest rate by a larger amount - given the same size of shock. Thus, aging is clearly a concern for fiscal economists, nevertheless, it should also be a concern for central bankers. In a very old society, under some circumstances, even the most effective monetary policy regime could be different: nominal GDP targeting becomes the most effective rule instead of the common practice inflation targeting when the OADR reaches extreme high levels. Nominal GDP targeting as a possible option was recently discussed by former Board Member of Bank of Japan, Sayuri Shirai (Shirai 2018). Summers (2018) also proposed nominal GDP targeting of about 5-6 percent in the U.S., while McKibbin and Panton (2018) discussed several advantages of nominal GDP targeting over flexible inflation targeting and/or price-level targeting.

To the best of our knowledge, our paper is the first (i) which estimates the impact of aging on the volatility of inflation, (ii) which explores the impact of aging on the short-run cyclical behavior of the macroeconomy, including monetary policy transmission, in an aged society, using a multi-period dynamic general equilibrium model with overlapping generations, and (iii) which examines optimal monetary policy strategies in the presence of aging. Given the scarce literature in this field, we urge for more research to better understand the implications of aging for central banks, central bankers, and the elderly.

## A.1 Appendix: List of Non-Linear Equations<sup>21</sup>

Demography:

$$s_t = \frac{(1 - \omega_{t-1}^O)}{(1 - \omega_{t-1}^Y + n_t)} s_{t-1} + \frac{\omega_{t-1}^Y}{(1 - \omega_{t-1}^Y + n_t)} \quad (75)$$

$$s_t^Y = \frac{1}{1 + s_t} \quad (76)$$

$$1 + g_t^{N,Y} = 1 - \omega_{t-1}^Y + n_t \quad (77)$$

$$1 + g_t^{N,O} = (1 - \omega_{t-1}^O) + \frac{\omega_{t-1}^Y}{s_{t-1}} \quad (78)$$

$$1 + g_t^N = (1 + g_t^{N,Y}) \frac{1 + s_t}{1 + s_{t-1}} \quad (79)$$

Overlapping generations:

$$\tilde{C}_t^O = MPC_t^O Inc_t^O + MPC_t^O \frac{(1 + r_{t-1})}{1 + g_t^N} \left( \omega_{t-1}^Y B_{t-1}^{\tilde{Y}} + B_{t-1}^{\tilde{O}} \right) \quad (80)$$

$$\frac{1}{MPC_t^O} = 1 + (1 - \omega_t^O)(1 + r_t)^{\frac{1}{\gamma} - 1} E_t \{ \beta^O \}^{\frac{1}{\gamma}} \frac{1}{MPC_{t+1}^O} \quad (81)$$

$$\tilde{Inc}_t^O = T \tilde{R}_t \Omega_t^O \quad (82)$$

$$\Omega_t^O = 1 + E_t \frac{1 - \omega_t^O}{1 + r_t} \Omega_{t+1}^O \quad (83)$$

$$\tilde{C}_t^Y = MPC_t^Y Inc_t^Y + MPC_t^Y \frac{(1 + r_{t-1})(1 - \omega_{t-1}^Y)}{1 + g_t^N} B_{t-1}^{\tilde{Y}} \quad (84)$$

$$\frac{\tilde{C}_t^Y}{s_t^Y - \tilde{L}_t} = \frac{\sigma}{1 - \sigma} w_t \quad (85)$$

$$Inc_t^Y = \tilde{\mathcal{F}}_t^Y + \frac{\tilde{\mathcal{F}}_t^{YO}}{1 + r_t} \quad (86)$$

$$\Lambda_t^Y = E_t (\beta^Y)^{\frac{1}{\gamma}} \left( \frac{w_{t+1}}{w_t} \right)^{(1-\sigma)(1-\frac{1}{\gamma})} \quad (87)$$

$$\Lambda_t^{YO} = E_t \left( \frac{\beta^O}{\sigma} \right)^{\frac{1}{\gamma}} \left( \frac{1}{\frac{\sigma}{1-\sigma} w_t} \right)^{(1-\sigma)(1-\frac{1}{\gamma})} \quad (88)$$

$$\frac{1}{MPC_t^Y} = \frac{1}{\sigma} + (1 + r_t)^{\frac{1}{\gamma} - 1} E_t \left( (1 - \omega_t^Y) \Lambda_t^Y \frac{1}{MPC_{t+1}^Y} + \omega_t^Y \Lambda_t^{YO} \frac{1}{MPC_{t+1}^O} \right) \quad (89)$$

<sup>21</sup>The quantities are normalized by the total population number  $N_t$ .

$$\tilde{\mathcal{J}}_t^Y = w_t s_t^Y + \text{Profit}_t - T\tilde{a}x_t + E_t \frac{(1 - \omega_t^Y)(1 + g_{t+1}^N)}{(1 + r_t)(1 + g_{t+1}^{N,Y})} \tilde{\mathcal{J}}_{t+1}^Y \quad (90)$$

$$\tilde{\mathcal{J}}_t^{YO} = E_t(1 + g_{t+1}) T\tilde{R}_{t+1}^{YO} \Omega_{t+1}^O + E_t \frac{(1 - \omega_{t+1}^Y)(1 + g_{t+1}^N)}{(1 + r_{t+1})(1 + g_{t+1}^{N,Y})} \tilde{\mathcal{J}}_{t+1}^{YO} \quad (91)$$

$$\tilde{C}_t^Y + \tilde{B}_t^Y = w_t \tilde{L}_t + \text{Profit}_t - T\tilde{a}x_t + \frac{(1 + r_{t-1})}{1 + g_t^N} (1 - \omega_{t-1}^Y) \tilde{B}_{t-1}^Y \quad (92)$$

Firms:

$$p_t(i) = \frac{\varphi}{\varphi - 1} \frac{\mathcal{Z}_t^1}{\mathcal{Z}_t^2} \quad (93)$$

$$\mathcal{Z}_t^1 = p_t(i)^{-\varphi} Y_t mc_t + E_t \left( \frac{p_t(i)}{p_{t+1}(i)} \frac{(1 + \pi_t)^{\gamma_p}}{1 + \pi_{t+1}} \right)^{-\varphi} \omega^P (1 - \omega_t^Y) \frac{1 + \pi_{t+1}}{1 + i_t} \mathcal{Z}_{t+1}^1 \quad (94)$$

$$\mathcal{Z}_t^2 = p_t(i)^{-\varphi} Y_t + E_t \left( \frac{p_t(i)}{p_{t+1}(i)} \frac{(1 + \pi_t)^{\gamma_p}}{1 + \pi_{t+1}} \right)^{-\varphi} (1 + \pi_t)^{\gamma_p} \omega^P (1 - \omega_t^Y) \frac{1}{1 + i_t} \mathcal{Z}_{t+1}^2 \quad (95)$$

$$1 = \left( (1 - \omega_P) p_t(i)^{1-\varphi} + \omega_P \left( \frac{(1 + \pi_{t-1})^{\gamma_p}}{1 + \pi_t} \right)^{1-\varphi} \right)^{\frac{1}{1-\varphi}} \quad (96)$$

$$mc_t = \frac{e^{\varepsilon_t^p}}{A_t} \left( \frac{r_t^K}{\alpha} \right)^\alpha \left( \frac{w_t}{1 - \alpha} \right)^{1-\alpha} \quad (97)$$

$$\tilde{K}_{t-1} \frac{1}{1 + g_t^N} = \alpha \frac{mc_t}{r_t^K} \tilde{Y}_t \quad (98)$$

$$\tilde{L}_t = (1 - \alpha) \frac{mc_t}{w_t} \tilde{Y}_t \quad (99)$$

$$\text{profit}_t = \tilde{Y}_t - \tilde{I}nv_t - w_t \tilde{L}_t \quad (100)$$

$$(1 - \omega_t^Y) E_t (r_{t+1}^K + Q_{t+1}(1 - \delta)) = Q_t(1 + r_t) \quad (101)$$

$$1 = Q_t \left( 1 - S \left( \frac{\tilde{I}nv_t}{\tilde{I}nv_{t-1}} \right) - S' \left( \frac{\tilde{I}nv_t}{\tilde{I}nv_{t-1}} \right) \frac{\tilde{I}nv_t}{\tilde{I}nv_{t-1}} \right) + \frac{1}{1 + r_t} Q_{t+1} S' \left( \frac{\tilde{I}nv_{t+1}}{\tilde{I}nv_t} \right) \left( \frac{\tilde{I}nv_{t+1}}{\tilde{I}nv_t} \right)^2 \quad (102)$$

$$\tilde{I}nv_t \left( 1 - S \left( \frac{\tilde{I}nv_t}{\tilde{I}nv_{t-1}} \right) \right) = \tilde{K}_t - (1 - \delta) \frac{\tilde{K}_{t-1}}{1 + g_t^N} \quad (103)$$

Tax system:

$$T\tilde{a}x_t = T\tilde{R}_t + \tilde{G}ov_t \quad (104)$$

PAYG pension system:<sup>22</sup>

$$T\tilde{R}_t^{YO} = v_t \frac{\omega_{t-1}^Y}{1 + g_t^N} w_{t-1} L_{t-1} \tilde{\phantom{R}} \quad (105)$$

$$T\tilde{R}_t = T\tilde{R}_t^{YO} + \frac{(1 - \omega_{t-1}^O)}{1 + g_t^N} T\tilde{R}_{t-1} \quad (106)$$

Monetary policy:

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left( (1 + \tilde{r}_t) (1 + \pi_t)^{\phi_\pi} \right)^{1 - \rho_i} e^{\varepsilon_t^i} \quad (107)$$

$$1 + i_t = E_t(1 + r_t)(1 + \pi_{t+1}) \quad (108)$$

$$1 + \tilde{r}_t = (1 + \tilde{r}_{t-1})^{\rho_r} (1 + r_t^n)^{1 - \rho_r} \quad (109)$$

Market clearing:

$$0 = \tilde{B}_t^Y + \tilde{B}_t^O \quad (110)$$

$$\tilde{Y}_t = \tilde{C}_t^Y + \tilde{C}_t^O + I\tilde{m}_t + G\tilde{v}_t \quad (111)$$

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<sup>22</sup> $IB_t^Y$  is normalized by  $N_{t-1}$ .

## A.2 Appendix: Welfare Functions and Optimal Reactions

Figure A1: Welfare functions with the baseline monetary policy rules

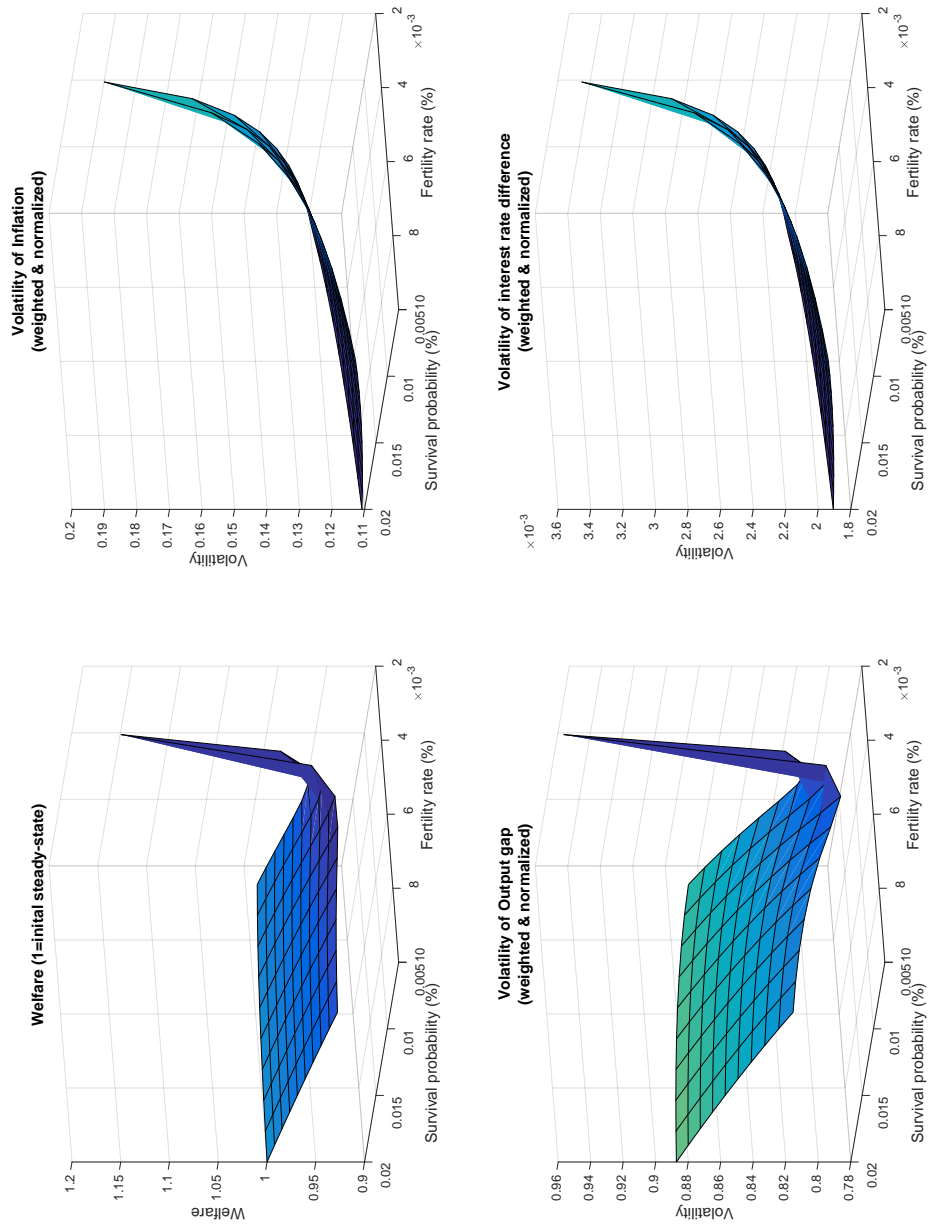


Figure A2: Optimal reactions (1)

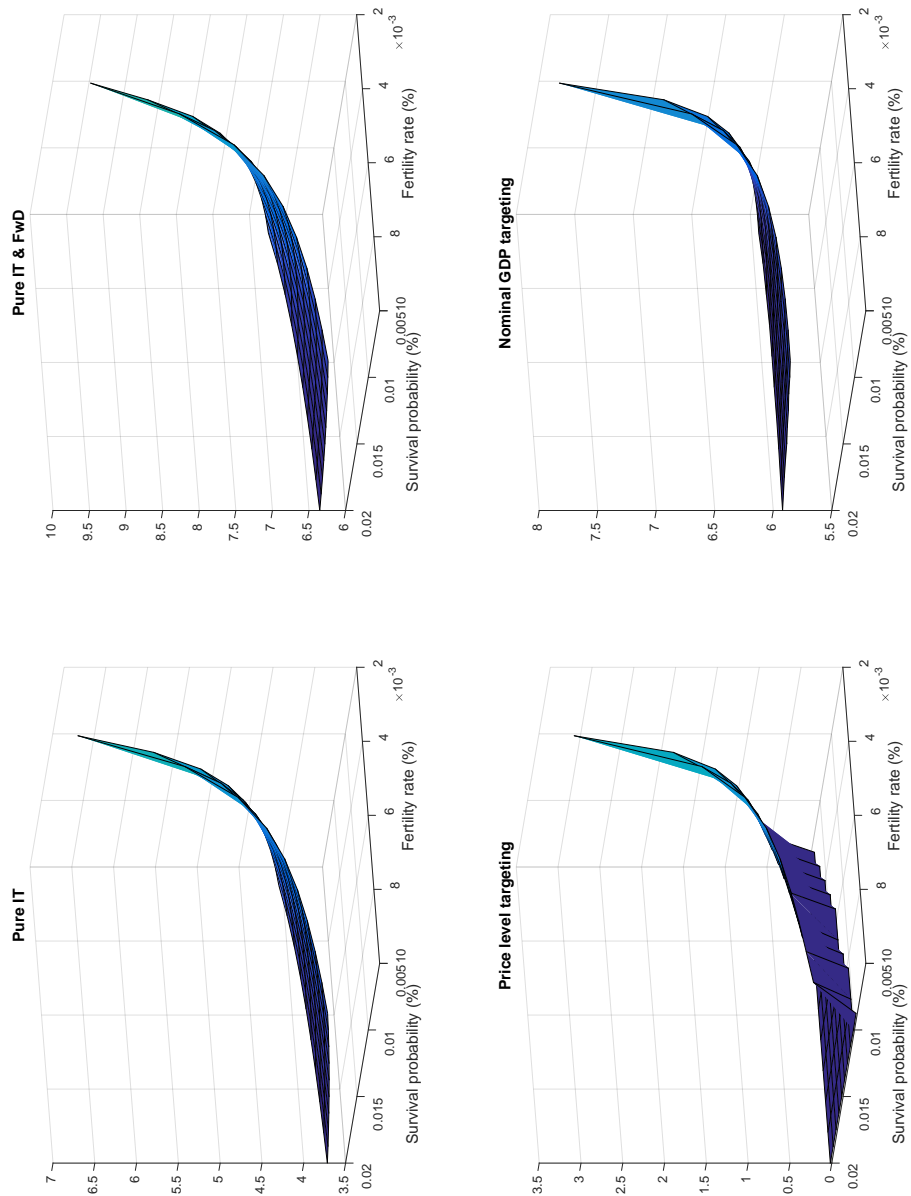


Figure A3: Optimal reactions (2)

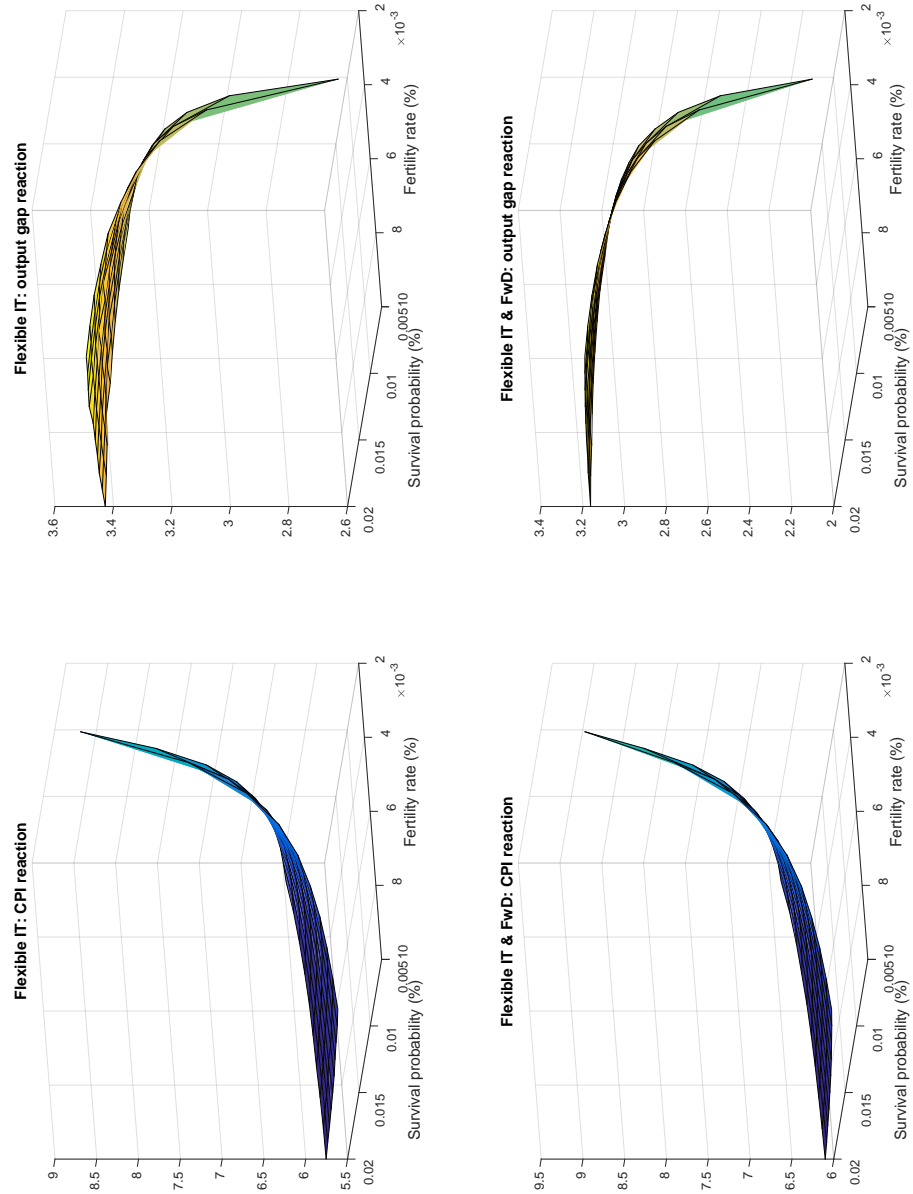




Table A1: Optimal monetary policy with higher weight (1.0) on output gap

		<b>Old-age dependency ratio</b>			
		$s = 0.20$	$s = 0.40$	$s = 0.60$	$s = 0.80$
Initial rule	Initial loss	1.000	0.948	0.924	1.120
	$\phi_{\pi_t}$	1.500	1.500	1.500	1.500
Pure IT	Optimized loss	0.953	0.919	0.873	0.801
	$\phi_{\pi_t}$	3.504	3.980	4.760	6.531
Pure IT & Fwd	Optimized loss	0.891	0.859	0.671	0.671
	$\phi_{E_t \pi_{t+1}}$	6.084	6.566	7.460	9.167
Flex. IT	Optimized loss	0.333	0.353	0.405	0.515
	$\phi_{\pi_t}$	5.861	6.111	7.265	9.112
	$\phi_{\hat{Y}_t}$	6.856	6.877	6.333	4.688
Flex. IT & Fwd	Optimized loss	0.340	0.364	0.420	0.541
	$\phi_{E_t \pi_{t+1}}$	5.935	6.223	7.266	8.526
	$\phi_{\hat{Y}_t}$	6.271	6.215	5.483	3.681
Price level targ.	Optimized loss	0.914	0.907	1.098	0.964
	$\phi_{\hat{P}_t}$	0.015	0.014	1.050	2.973
Nominal GDP targ.	Optimized loss	0.471	0.471	0.471	0.454
	$\phi_{\widehat{P_t Y_t}}$	9.376	9.647	10.437	12.162

Table A2: Optimal monetary policy with higher weight (0.5) on interest rate smoothing

		<b>Old-age dependency ratio</b>			
		$s = 0.20$	$s = 0.40$	$s = 0.60$	$s = 0.80$
Initial rule	Initial loss	1.000	0.959	0.948	1.156
	$\phi_{\pi_t}$	1.500	1.500	1.500	1.500
Pure IT	Optimized loss	0.931	0.896	0.848	0.782
	$\phi_{\pi_t}$	3.505	3.923	4.552	5.842
Pure IT & Fwd	Optimized loss	0.889	0.858	0.669	0.669
	$\phi_{E_t \pi_{t+1}}$	5.844	6.221	6.848	8.006
Flex. IT	Optimized loss	0.537	0.558	0.595	0.643
	$\phi_{\pi_t}$	4.096	4.322	4.841	6.226
	$\phi_{\hat{Y}_t}$	2.036	1.993	1.846	1.587
Flex. IT & Fwd	Optimized loss	0.557	0.584	0.626	0.678
	$\phi_{E_t \pi_{t+1}}$	4.475	4.788	5.428	6.810
	$\phi_{\hat{Y}_t}$	1.941	1.877	1.682	1.313
Price level targ.	Optimized loss	0.911	1.090	1.031	0.904
	$\phi_{\hat{P}_t}$	0.017	0.559	1.145	2.657
Nominal GDP targ.	Optimized loss	0.553	0.555	0.563	0.548
	$\phi_{\widehat{P_t Y_t}}$	3.201	3.252	3.394	4.187

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