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A Buffer-Stock Model for the Government: Balancing Stability and Sustainability

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A Buffer-Stock Model for the Government: Balancing Stability and Sustainability

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Abstract

A fiscal reaction function to debt and the cycle is built on a buffer-stock model for the government. This model inspired by the buffer-stock model of the consumer (Deaton 1991; Carroll 1997) includes a debt limit instead of the Intertemporal Budget Constraint (IBC). The IBC is weak (Bohn, 2007), a debt limit is more realistic as it reflects the risk of losing market access. This risk increases the welfare cost of fiscal stimulus at high debt. As a result, the higher the debt, the less governments should smooth the cycle. A larger reaction of interest rates to debt and higher hysteresis magnify this interaction between the debt level and the appropriate reaction to shocks. With very persistent shocks, the appropriate reaction to negative shocks in highly indebted countries can even be procyclical.

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I. INTRODUCTION

A. How the Fiscal Stance Should Change with the Debt Level

The global financial crisis highlighted the trade-off that fiscal policy faces between economic stabilization and debt sustainability. When the crisis started, policy focused on economic stabilization, with calls to use fiscal measures to stimulate domestic demand (G20 2008). But in the following years, several countries experienced sharp rises in risk premia, leading to large fiscal consolidations (IMF 2012). Ex post, this raises the question of whether this approach struck the right balance between the objective of economic stabilization and constraint of debt sustainability. For instance, was the large fiscal stimulus in 2009 appropriate for the most indebted countries? Or were the gains from short-term stabilization swamped by the cost of higher risks of fiscal stress in the future? Was this stimulus withdrawn too quickly?

I address this question by presenting a buffer-stock model of the government. This model includes a public debt limit—a debt level at which the government loses market access. The distance to this limit is the extent to which a government can adjust fiscal policy, analogous to the cash buffer that enables a consumer to offset shocks (Deaton 1991; Carroll 1997).

The main new insight of this model is that the fiscal stance should be less counter-cyclical at high debt levels than at low debt levels. Reaching the debt limit is costly because a government without market access cannot use fiscal policy to smooth shocks at all. The debt buffer (the difference between debt and its limit) therefore has an insurance value—it is the "reserve" of debt that the government can access to smooth shocks. When the buffer is small, the probability of breaching the debt limit is high, and so the marginal value of an extra unit of buffer is larger. This provides an incentive to preserve the buffer to guard against future shocks. As a result, when debt is high (and so the buffer is small), the government's optimal policy is to respond less to offset a given negative shock even if it could do so, thereby preserving the buffer. Conversely, if the buffer is large, the government should use the fiscal stance to smooth the cycle more. Thus, the debt limit creates an interaction between the debt level and the advisable reaction to shocks.

This buffer-stock model is in the spirit of Bohn's (2007) encouragement to rethink the budget constraint of the government. He shows the Intertemporal Budget Constraint (IBC) is weak because it can hold when the public debt has a unit root (i.e. public debt is explosive). Bohn suggests lenders may impose additional bounds on debt or deficits, and a new stream of literature provides model-based debt limits (Bi 2012; Ghosh et al. 2013; Fournier and Fall 2017). The risk of losing market access might also arise if liquidity risks constrain solvent governments (Cole and Kehoe 2000).

Yet work on optimal fiscal policy has not taken seriously the need for a constraint beyond the IBC. Following seminal works by Barro (1979), Lucas and Stockey (1983), or Chamley (1986), most cases in the optimal fiscal policy literature include only an IBC. Only a few papers have explored optimal fiscal policy with a public debt limit or a default risk (e.g., Aiyagari et al. 2002; Mendoza and Oviedo 2009; Cuadra et al. 2010). None of these discuss how the distance to debt limit

changes the assessment of the appropriate fiscal stance. This paper aims to fill this gap, with a benevolent government that wishes to smooth output shocks under a debt limit constraint.

I solve the model globally to cover any debt level, including high debt cases which should not be stable and hence are far away from the steady state.[2] Even near the steady state, the global approach is useful as a forward-looking government avoids fast debt increases precisely because it anticipates constraints on the future fiscal stance under such a path. In addition, when a boundary is binding, local approximations are not valid (see for instance Christiano et al. 2005). Lastly, with no linearization, the shape of the global solution can exhibit any form of interaction or non-linearity—a key feature that I aim to capture.

B. Modelling the Trade-off between Stabilization and Market Access

I embed the buffer-stock mechanism in a model with five key further features which govern the relationship of fiscal policies to the real economy.[3] First, the important role of automatic stabilizers is modeled as an effect of output on the primary balance. For instance, an increase in output will produce higher tax revenues. Second, long-term negative effects of downturns on potential output (hysteresis) can reflect permanent loss of skills following unemployment (Blanchard and Summers 1987; DeLong and Summers 2012). Third, I introduce an interest-elastic debt demand curve, allowing the level (and change) of debt to affect the interest rate on government debt. This captures either a risk premium or a price effect of bond supply. Fourth, I refine the fiscal multiplier with a sensitivity to the output gap, and the market access risk with an uncertain debt limit. Fifth, I make the fiscal stance setting process more realistic, reflecting lags and uncertainties in the policymaking process. Using the fiscal stance to stabilize the economy is not straightforward (IMF 2017). As fiscal policy is typically subject to an implementation lag, I consider a government that decides its fiscal stance one-year ahead. The government does not know the position in the cycle yet and can only forecast it. And it can be costly to change fiscal policy due to spending implementation costs, or tax uncertainty costs (e.g., Skinner 1988). I thus include a quadratic adjustment cost.

I present a model with a stochastic debt limit because such a limit cannot be known with great accuracy. The purpose of this paper is not to establish how a debt limit arises, but instead to examine its impact on optimal fiscal policy. I therefore do not model the determinants of this limit but take it as given, following an exogenous stochastic process.

To keep the focus on the fiscal stance trade-off between stabilization and sustainability, monetary policy is implicit in the model. This corresponds to the case where the central bank is autonomous and the fiscal stance optimization of the government takes monetary policy as given. The implicit assumptions on monetary policy are embedded in the calibration of

[2] Some papers consider a continuum of steady states to cover any debt level (e.g. Adam, 2011), Then, any debt level can be associated with a steady state and hence can be stable.

[3] My approach follows Blanchard's (2018) suggestion to start with a partial equilibrium and then to move to a reasonably simple equilibrium closure. Simplicity helps to describe mechanisms and to focus on the key feedback effects.

parameters. Monetary policy dampens shocks, and changes the fiscal multiplier (see Woodford, 2011, for didactic examples of different monetary policy assumptions leading to different fiscal multipliers). Furthermore, short-term simulations conditional on the medium-term interest rate forecast can illustrate the implications of temporarily lower interest rate on the fiscal stance (Fournier and Lieberknecht, 2019).

The debt limit assumption is less crucial when the level (and change) of debt affects the interest rate on government debt. Before an abrupt risk to market access materializes, the government can react to a more progressive risk of rising interest rates. The interest-elastic demand curve for debt reinforces the mechanism of the simple buffer-stock model as the marginal cost of debt is even more sensitive to public debt. The higher the debt level, the faster it grows, as interest compounds at a higher rate. This reduces the appropriate debt level and reduces the counter-cyclicality of fiscal policy at high debt levels. The buffer could be viewed here as the distance between the current debt level and a level at which interest rates make it undesirable to counter negative output shocks. As a government should react to rising interest rates, the debt limit is less likely to bind. Therefore, results presented here are not very sensitive to the debt limit assumption.

The presence of hysteresis reinforces the main finding of the simple model, namely that the higher the debt, the less governments should use a counter-cyclical fiscal stance. Delong and Summers (2012) have argued that the presence of hysteresis supports the case for more stimulus during downturns as the welfare cost of fiscal inaction is magnified by long-term output costs. The model presented here presents a more complex picture. With risk to market access, the appropriate stimulus depends on the debt level. At low debt, it is indeed the case that fiscal stimulus prevents long-term output costs while incurring only a small marginal cost of a debt increase. In contrast, at high debt levels, the appropriate reaction to a sharp downturn becomes ambiguous. During a downturn, hysteresis not only magnifies the cost of inaction, but also raises the future cost of current stimulus. At high debt, an economy with high hysteresis thus should not react to the cycle too much. As a result, the buffer is much more valuable at low debt, so that the marginal value of the buffer stock is higher, and the average optimal debt is lower. In other words, countries with higher hysteresis should be careful to preserve a higher buffer.

I also explore the sensitivity of results to the growth rate, the fiscal multiplier, the size of automatic stabilizers, the persistence and the size of shocks. It is worth highlighting that a higher growth rate gives more room for maneuver, especially at high debt levels. And the sensitivity to shocks persistence illustrates the importance of future risks. With high shock persistence and high debt, it may even be appropriate to react to a negative shock with a tighter fiscal stance. This is because a highly persistent negative shock will weigh on output for a long time. The risk of hitting the debt limit under a countercyclical policy is thus higher. With high debt, a countercyclical stimulus in reaction to a negative shock would create an intolerably high risk of losing market access in future.

The rest of the paper is organized as follows. Section 2 compares two simple models of a forward-looking government, with and without market access risk. Section 3 is a self-contained

presentation of a more realistic model calibrated for advanced economies with an uncertain debt limit, a reaction of interest rates to debt, shock persistence, automatic stabilizers, a government that decides plans before he observes the year's shock, a fiscal multiplier that depends on the output gap, hysteresis and an adjustment cost. Section 4 reports baseline results of this more realistic model and illustrates key mechanisms with a sensitivity analysis.

II. A Simple Model Shows Some Fiscal Implications of the Risk of Losing Market Access

In this section, I show with a simple partial equilibrium stochastic model of a forward-looking government that the introduction of a risk of losing market access creates an interaction between the debt level and the appropriate reaction to shocks. The model is stylized to focus on qualitative fiscal implications of the existence of a debt limit. For the sake of demonstration, two cases are compared. In the first case, there is a standard IBC with no market access risk. Under the reasonable assumption of a quadratic instantaneous utility function, the reaction to shocks should abstract from the debt level.[4] The buffer-stock model of the government is the second case, which features the same model with a debt limit to single out direct consequences of market access risk. A striking implication of this risk is that stabilization and sustainability need to be considered together. Even with no interest rate reaction to debt, the higher the debt, the less the government should use fiscal policy to smooth the cycle.

A. A Stochastic Model of the Government without a Debt Limit

The model considers a forward-looking government subject to the IBC that can offset output shocks (Problem A). At each discrete period t, the output is determined by an output shock e_t and the primary balance pb_t with a fiscal multiplier m.[5] This is consistent with the literature, which either defines the fiscal multiplier as the effect of level of primary balance (or tax, spending level) with a level of output (or consumption, investment) as in Blanchard and Perotti (2002), or matches first differences on both sides (e.g., Alesina et al. 2015 in the empirical literature or Zubairy 2014 in the modeling literature). The government observes the shock before it decides the primary balance. The interest rate is adjusted for the potential growth rate. Accordingly, the debt dynamic is calculated with the debt to potential GDP ratio, which is relevant to assess sustainability.[6] This government is maximizing the intertemporal welfare—it strikes a balance between instantaneous utility $u(y_t)$ and future utility:

Problem A.

$$V_t(d_{t-1}, e_t) = \max_{pb_t} \left\{ u(y_t) + \beta E_t \left[V_{t+1}(d_t, e_{t+1}) \right] \right\} \quad (1)$$

[4] Without a quadratic utility function, in the absence of debt limit, the government would already have a precautionary motive that creates a third order interaction between debt and shocks, and the existence of a debt limit sharpens this interaction. The quadratic utility function is used to single out the role of the debt limit.

[5] In this set up, the cost of future consolidation is captured by this fiscal multiplier.

[6] Eyraud and Weber, 2013, recommend monitoring debt targets in cyclically-adjusted terms.

s.t.

$$d_t = (1+r)\, d_{t-1} - pb_t \quad (2)$$

$$y_t = 1 + e_t - m.pb_t \quad (3)$$

$$\lim_{t\to\infty} \beta^t E(d_t) \leq 0 \quad (4)$$

with $E_{t-1}e_t = 0$, $0 < \beta < 1$, $r > 0$, $u'' < 0$, and the assumption that there can be adverse shocks: $P(u(e_t) < u(1)) > 0$.

With an optimal fiscal policy, the people are indifferent between the government widening deficits more today or tomorrow. The Euler equation equates the corresponding marginal utilities:

$$-m\, u'(y_t) = -m\, \beta\, (1+r)\, E_t[u'(y_{t+1})]$$

The solution of the model is a function governing the primary balance for any state of the economy. The state of the economy is summarized by two variables observed by the government: the previous debt level d_{t-1} and the current shock e_t. The solution of the problem is thus a function. For the simplicity of the exposure, the notation pb_t is used in place of $pb_t(e_t, d_{t-1})$. I assume that the government is time consistent; therefore, this function is the same at any period t. It is a normative prior that a consistent government is preferable to anchor agents' decisions and to build or preserve credibility. Such a solution provides both the reaction to debt and the reaction to shocks: both are considered jointly *a priori*.

The solution is separable if it is the sum of a function of the shock and a function of past debt. This definition is convenient because separability is a formal way to say that the appropriate primary balance response to a change in the shock does not depend on the debt level. It is worth noting that this does not mean that the fiscal behavior is independent to debt. It rather means that the reaction to debt does not interact with the reaction to shocks. If there is separability, it can be appropriate for a highly indebted government to have higher primary surpluses on average over the cycle. But at the same time, the primary balance should fluctuate around this prudent average as much as it would fluctuate if debt were low.

Without any debt limit, the appropriate reaction to shocks should ignore the debt level in the case of a quadratic instantaneous utility function (Proposition 1). Indeed, there is no interaction between the shock and the debt level in this solution that is linear in the shock and in the debt level. The reaction to shocks is positive: the government should counteract them. If the government is impatient ($\beta(1+r) < 1$), the optimal policy is to offset shocks perfectly. If $\beta(1+r) \geq 1$, the reaction to the shock is only slightly below the inverse of the multiplier: the observed shock should be offset almost completely in this stylized case.[7]

[7] In particular, this stylized case assumes the shock is perfectly observed and no adjustment cost. In the next section, these assumptions will be relaxed, that will reduce the appropriate reaction of the government.

Proposition 1. With a quadratic utility function $u(y_t) = -(1/2)y_t^2 + \omega y_t$, the optimal fiscal reaction to problem A is separable. This fiscal reaction function is:

$$\begin{cases} pb_t = \dfrac{e_t + 1 - \omega}{m} & if \quad \beta(1+r) < 1 \\ pb_t = \dfrac{1}{m\beta(1+r)^2} e_t + rd_{t-1} + \dfrac{\beta(1+r)-1}{\beta(1+r)}\left(d_{t-1} - d_{eq}\right); d_{eq} = \dfrac{\omega - 1}{mr} & if \quad \beta(1+r) \geq 1 \end{cases} \tag{5}$$

Proof. Annex A1.1.

A constraint on the discount factor β is necessary to avoid an explosive debt path. Let us discuss the cases, from the most prudent to the less prudent. If $\beta(1+r)>1$, then the reaction to debt more than offsets a rise in interest payments rd_{t-1}, so that debt is stationary around the equilibrium level d_{eq}. In the limit case $\beta(1+r)=1$, the primary balance offsets interest payments, so that the total balance is proportional to the output shock: $pb_t - rd_{t-1} = e_t / \left(m(1+r)\right)$. In this limit case, the debt ratio follows a random walk.[8] If $\beta(1+r)<1$, people are too impatient and the government perfectly offsets shocks to reach the instantaneous optimum, ignoring the debt level. This does not respect Bohn's (2007, 2008) sustainability condition which is precisely that the primary balance should increase in debt. And even if one adds the constraint that the government need to respect this condition, public debt would remain explosive. This last case holds for any discount factor in the (excluded) case of an interest rate below growth rate (here the growth-adjusted interest rate r is negative). In this case, even a prudent government would not care about debt.[9]

In the case with stationarity of public debt ($\beta(1+r)>1$), the peak of the utility function, the fiscal multiplier and the interest rate are driving the equilibrium debt level d_{eq}. When a primary balance at equilibrium corresponds to the peak of the utility function in the absence of shock ($\omega = 1$), the optimal debt level is zero. When the peak corresponds to a higher output ($\omega > 1$), there is a preference for a stimulus, so that the optimal debt level is positive. If the multiplier is larger, less deficit is necessary to reach this peak. As the government runs less deficits, the equilibrium debt level is lower. And if the interest rate is higher, the equilibrium debt level is lower as debt is costlier.

The quadratic utility is quite a general case as there is a quadratic approximation for any regular utility function. For instance, this can correspond to a standard utility function that increases with consumption and decreases with labor that will be used in the third section. In such a case, utility peaks at the production level for which the marginal gain of consumption equates the marginal cost of labor, and the approximation could be considered around this peak.

[8] In this case, the level of debt becomes irrelevant for new debt issuance, as in Barro (1979).

[9] See Barrett (2018) for a thorough investigation of implications of a negative growth-adjusted interest rate, showing in particular that maximum sustainable debts become unbounded.

B. The Buffer-stock Model of the Government with a Debt Limit

The buffer-stock model of the government is an analogy with the buffer-stock model of the consumer (Deaton 1991; Carroll 1992, 1997). In the buffer-stock model, a risk-adverse forward-looking consumer faces a borrowing limit. He saves in good times for a precautionary purpose, and hence can dissave in bad times. This departs from the permanent income hypothesis with perfect markets in which the consumer does not need a buffer. In a government analogy, the forward-looking government faces a risk of losing market access beyond a debt limit. The difference between current debt and its limit is equivalent to a buffer. The government may offset output shocks with counter-cyclical changes in primary balance financed by borrowing. This simple model is useful to understand how the shape of the debt constraint affects the appropriate reaction to shocks.

This buffer-stock model of the government is the same model as in the previous section with a debt limit $\bar{d}$ in terms of debt to potential GDP. When this is binding, the government is assumed to have no choice and hence to generate enough surpluses to keep debt below the limit. This excludes unbounded solutions among those of the model without debt limit. This buffer-stock model can be summarized as follows:

Problem B.

$$V_t(d_{t-1}, e_t) = \max_{pb_t} \left\{ u(y_t) + \beta E_t \left[V_{t+1}(d_t, e_{t+1}) \right] \right\} \quad (6)$$

s.t.

$$d_t = (1+r)\, d_{t-1} - pb_t \quad (7)$$

$$y_t = 1 + e_t - m.pb_t \quad (8)$$

$$d_t \leq \bar{d} \quad (9)$$

with $E_{t-1}e_t = 0$, $V(e_t) > 0$, $0 < \beta < 1$, $r > 0$ and $u'' < 0$.

The Euler equation is unchanged if the constraint is not binding, and reflects the debt constraint otherwise:

$$pb_t = \max\left(\frac{1}{m}\left[1 + e_t - u'^{-1}\left(\beta(1+r) E_t[u'(1 + e_{t+1} - m.pb_{t+1})] \right) \right], (1+r)d_{t-1} - \bar{d} \right) \quad (10)$$

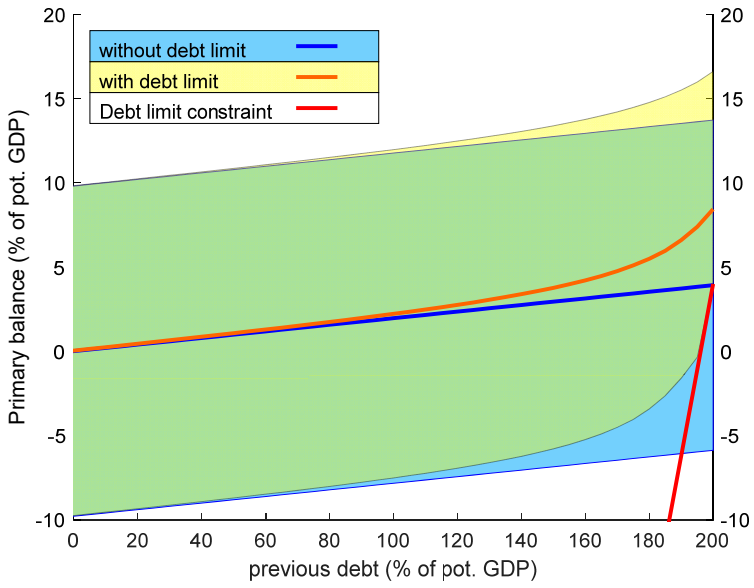
There are two cases here, depending on which of the two terms in brackets is the largest. If the first one is the largest, the government has market access and the constraint does not bind. Then, the primary balance is governed by the Euler equation of the case without debt limit. By contrast, the constraint binds when debt is too high (when the second term is larger), so that the government cannot smooth shocks as much as it wishes. I thus define the market access dummy as a dummy that is equal to one where the first term is larger than the second one:

$$MA_t = 1\left\{(1+r)d_{t-1} - \bar{d} \le \frac{1}{m}\left[1 + e_t - u'^{-1}\left(\beta(1+r)E_t[u'(1+e_{t+1} - m.pb_{t+1})]\right)\right]\right\} \quad (11)$$

The model with debt limit is solved backward, and the limit case of the finite horizon problem is the solution of the infinite horizon case (Annex A1.2), in the same manner as the buffer-stock model of the consumer solved by Deaton (1991) and Carroll (1997). This solution cannot be formulated explicitly. Still, one can calculate a numerical solution to observe patterns and explore the features of the reaction of the primary balance to shocks and to debt with its partial derivatives.

Numerical solutions provide a qualitative illustration of implications of a debt limit. Figure 1 compares the appropriate primary balance without (in blue) and with (in orange) a debt limit. Solid lines indicate the appropriate the primary balance if the shock is equal to zero, and the bands are delimited by the appropriate primary balance in case of a positive and negative output shock of 5%. The width of each band shows the extent to which governments should react to shocks. At low debt, when reaching the debt limit is unlikely in the near future, results with and without debt limits are almost the same. As shown above, the reaction of primary balance to debt is linear and the reaction to shocks does not depend on the debt level in the absence of a debt limit. With a debt limit, the solid line becomes convex: the reaction of primary surplus to debt is getting sharper and sharper as debt gets closer to its limit. The band narrows when debt is getting close to its limit; with higher debt, the reaction to shocks should become smaller. This is because negative shocks at high debt increase the risk to reach the debt limit in the future, and the forward-looking government is willing to reduce this risk. The red lines show the debt constraint, the government facing a debt limit cannot run a primary balance below this line. The narrowing

Figure 1. Primary Balance with and without Debt Limit



Note: The blue and orange solid lines indicate the appropriate fiscal stance if the shock is zero without and with debt limit, respectively. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the blue and yellow bands thus indicates the extent to which a government should use fiscal policy to counteract output shocks. This figure is provided for qualitative discussion of the difference between problems A and B, which are too stylized to be calibrated to match a particular country.

of the yellow band appears before the debt limit binds: even when the government can increase deficit, it should react less to shocks to reduce the risk to reach the debt limit in the future.

The analysis of derivatives will provide more general evidence of the patterns observed with Figure 1, beyond numerical results calculated with a given calibration.

Without market access, the financially constrained government cannot offset shocks. Its primary balance reacts to the debt level only:

$$\frac{\partial pb_t}{\partial d_{t-1}} = 1 + r \qquad (12)$$

$$\frac{\partial pb_t}{\partial e_t} = 0 \qquad (13)$$

With market access, the unconstrained government should react to both debt and shocks. The partial derivatives are calculated with the usual Euler equation. This describes how the sensitivity of fiscal policy to debt and to the shock depends on the sensitivity of fiscal policy to debt in the next period:

$$\frac{\partial pb_t}{\partial d_{t-1}} = \frac{(1+r)\,E_t\left[u''(y_{t+1})\frac{\partial\_pb_{t+1}}{\partial d_t}\right]}{u''(y_t) + \beta(1+r)E_t\left[u''(y_{t+1})\frac{\partial\_pb_{t+1}}{\partial d_t}\right]} \qquad (14)$$

$$\frac{\partial pb_t}{\partial e_t} = \frac{1}{m\left(1 + \beta(1+r)E_t\left[\frac{u''(y_{t+1})}{u''(y_t)}\frac{\partial\_pb_{t+1}}{\partial d_t}\right]\right)} \qquad (15)$$

The presence of a risk to reach the debt limit leads the government to react more to debt. When the government loses market access the reaction to debt is linear with a coefficient (1+r) that is higher than the one that was found in the previous case without debt limit (equations 12). Results also differ as soon as there is a risk of losing market access later as the forward-looking government foresees the lower welfare associated with the constraint. The sensitivity to debt increases with the sensitivity to debt in the next period (equation 14). This next period, or further periods in the future, can include cases with a loss of market access and hence a higher sensitivity to debt. In other words, by backward induction, the sharp reaction to debt at the limit affects the reaction to debt below the limit. In addition, the sensitivity to shock decreases with the sensitivity to debt in the next period (equation 15). The government acts preventively to reduce future risks. Last, whatever the discount factor, the backward iteration implies that the government is always reacting positively to debt, and that this reaction is increasing in debt (annex A1.2).

For the simplicity of the exposure, I will consider a quadratic utility in the next steps, similarly to the case without debt limit.

A key difference induced by the debt limit is that even in a period when the constraint is not binding, governments facing uncertainty on future market access should react less to shocks when debt is higher (Proposition 2). This is a direct consequence of the sharper fiscal reaction to debt at higher debt levels. This sharper reaction is because the marginal cost of issuing new debt increases in debt. As a result, the forward-looking government offsets shocks less. Even if highly indebted governments enjoy market access and can offset current shocks, they should preserve part of their limited fiscal space in case of future negative shocks.

Proposition 2. For any government facing uncertainty on market access in the future, the optimal primary balance function is not separable: for any shock e_t and any pair of debt levels such that $d_1 < d_2$ and the government has market access, the sensitivity to shocks is higher at the higher debt level d_2:

$$\frac{\partial pb_t}{\partial e_t}(d_1, e_t) > \underbrace{\frac{\partial pb_t}{\partial e_t}(d_2, e_t)}_{\text{less sensitive to shocks at higher debt}}$$

Proof. Annex 1.3.

The general finding in proposition 2 excludes the case of a government that wants and can ensure it will never lose market access at any debt below the limit. In the presence of a debt limit, this is a peculiar case with bounded shocks and sufficiently high discount factor (Proposition 3). Even with a very prudent government (β = 0.99) encouraged to generate surpluses by a high growth-adjusted interest rate of 2%, with a fiscal multiplier of one, a debt limit equal to 200% of GDP and with no preference for deficits (equilibrium debt level d_{eq} = 0%), negative shocks would need not to exceed 2% of GDP for this to happen.

Proposition 3. A government for which negative shocks can be large so that $P\left(e_t < m(1+r)\left(1-\beta(1+r)\right)\left(\overline{d}-d_{eq}\right)\right) > 0$ is always at risk to lose market access at some time in the future.

Proof. Annex 1.4.

III. A MORE REALISTIC MODEL WITH FEEDBACK EFFECTS

This section augments the stylized buffer-stock model to include more realistic settings for the government with key feedback effects, such as automatic stabilizers—an effect of output on the primary balance—and an effect of debt on the interest rate. The government decides to change the structural component of the primary balance before observing the current year output shock, as budget plans are set up before a year starts. It rather observes the previous year's shock and knows the extent of its persistence. The fiscal stance is defined as the change in the structural primary balance, singling out discretionary decisions. The remainder of the primary balance is a cyclical component proportional to the current output gap. At the same time, the output gap depends on fiscal policy with a multiplier that depends on the gap itself. These two-way links are

solved to calculate the equilibrium output gap as a function of the structural primary balance and the output shock. The model also features hysteresis that makes inaction in recessions costlier, an uncertain debt limit, a reaction of interest rates to debt that make stimulus costlier in times of high debt, and an adjustment cost reflecting some inertia in fiscal policy.

A. The maximization problem

The government maximizes household welfare by choosing a change in structural primary balance to stabilize output fluctuations intertemporally under constraints. The value function of the government is

$$V_t(d_{t-1}, gap_{t-1}, pb^{st}_{t-1}) = \max_{\Delta pb^{st}_t} E_t\left[u(c_t, L_t) + \beta V_{t+1}(d_t, gap_t, pb^{st}_t)\right]$$

where *t* is the year, d_t is the gross government debt to potential GDP ratio, gap_t is the output gap, pb^{st}_t is the structural primary balance, c_t is aggregate consumption[10], L_t is labor, $u(.,.)$ is the instantaneous utility function, and β is the discount factor. The state of the economy is summarized by three variables: government debt, the output gap, and the structural primary balance. The optimization is subject to the structure of the economy and the government budget constraint that takes the form of market access risk that is rising in debt (see below).

The value function consists of the per-period utility function u(.) and the expected continuation value discounted by β. The per-period utility function is

$$u(c_t, L_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \xi y_t^{*1-\sigma} \frac{L_t^{1+\eta}}{1+\eta}$$

which is a standard constant relative risk aversion utility function in consumption and labor where ρ is the parameter of risk aversion. Households enjoy consumption, but also face labor disutility. Utility peaks at an equilibrium output for which the marginal income gain of work equates the marginal loss of utility due to labor. The labor weight $\xi y_t^{*1-\sigma}$ evolves with potential output per capita, so that the optimal output level also grows with the economy, and ξ can be calibrated so that utility peaks when output is equal to its potential. In other words, utility declines not only if output decreases below its potential, but also if output increases above potential, consistent with the view that a positive output gap can be associated with costly distortions. This gives the government a motive to counter output deviations from this potential.

The model features rising market pressure when debt is rising. First, the interest rate increases in public debt, in line with empirical evidence (Gruber and Kamin 2012; Poghosyan 2012; D'Agostino and Ehrmann 2014; Fall and Fournier 2015; Henao-Arbelaez and Sobrinho 2017). This sensitivity of the interest rate to debt reflects a higher risk premium; it can be regarded as the consequence of an excess of supply of government bonds. Furthermore, the risk premium

[10] Public and private consumption are not distinguished, and hence assumed to provide the same utility.

increases in the change in debt; investors are more likely to be concerned if debt is rising. Symmetrically, even at high debt levels, risk premium may be moderate if the government shows its capacity to reduce it. Second, a risk of losing market access rules out unbounded debt paths. The probability to lose market access also depends on the level and the change of public debt:

$$P(lma) = [1 + \exp(d_1(1 - d_t / \overline{d} - d_2(d_t - d_{t-1})))]^{-1}$$

where d_1 governs the debt limit uncertainty, d_2 governs the effect of a debt change on the risk to lose market access, and $\overline{d}$ is the debt level at which the probability to lose market access is 50% (given no change in the debt level). If the government loses market access, the government has to keep debt constant under an adverse scenario of a shock of $d_3\sigma$, where σ is the standard deviation of economic shocks, to be explained below, and d_3 is the size of this adverse shock in terms of number of standard deviations.[11]

The budget constraint of the government is governed by a standard debt accumulation dynamic, with a deterministic stock-flow adjustment sf_t that can capture planned one-offs:

$$d_t = \left(\frac{1 + r(d_{t-1}, \Delta d_t)}{1 + g_t^*} \right) d_{t-1} - pb_t + sf_t$$

In a simple baseline approach, the risk premium is linear in past debt and in debt change, so that this implicit definition of the new debt level can be solved directly. In alternative cases with a non-linear effect of debt and debt change on the risk premium, this can be solved with a linear approximation of the effect of debt change on the risk premium.

B. Output and fiscal policy

Output is driven by a long-term exogenous potential growth and hysteresis costs in the long-run. Output is produced by a standard linear production function in labor:

$$Y_t = A_t L_t$$

where A_t is productivity and L is labor. Potential output $\overline{Y_t}$ is the output that would prevail if labor is at its equilibrium level $\overline{L}$:

$$\overline{Y_t} = A_t \overline{L}$$

Productivity is affected by a permanent hysteresis effect of downturns. In a depressed economy, unemployed workers can see their skills, their network, and their morale all decay (Blanchard and Summers 1987; DeLong and Summers 2012).

[11] In practice, simulations show that governments should generate surpluses at high debt levels beyond those required to make sure debt is expected to stabilize when the interest rate reacts to debt. Thus results are not much sensitive to the parameters governing the risk of losing market access, while they are strongly sensitive to the interest rate reaction to debt at very high debt levels.

$$A_t = A\prod_{\tau=1}^{t}\left((1+g_\tau^*)\left(1+h.\left(min(gap_{\tau-1},h^{th})-h^{th}\right)\right)\right)$$

where $A=\overline{L}=1$ and g_t^* is the potential growth rate that would prevail in the absence of hysteresis.

The parameter $h \geq 0$ governs the size of hysteresis, and h^{th} is the output gap threshold below which hysteresis affects potential output. Hysteresis is a permanent loss of potential output level if output gap is below a given threshold. Such an effect on output level is in line with Mourougane (2017), who finds large hysteresis effects on potential GDP level but no effect on long-run potential growth. The threshold enables to associate hysteresis with large negative output, in line with the literature on hysteresis that focuses on deep recession (e.g. Cerra and Sexana).

The output deviates from its potential because of an underlying process v_t driven by exogenous white noise shocks ε_t and because of the primary balance. This gap is defined as the percentage deviation of output from potential output:

$$gap_t(pb_t, v_t) = \frac{Y_t - \overline{Y}_t}{\overline{Y}_t}$$

The shock process v_t is normalized to be equal to the gap if the primary balance is zero. In the absence of hysteresis, this would simply be an autoregressive process, as effects of shocks persist for some time. This needs to be adjusted in the presence of hysteresis as a part of output shocks becomes permanent and hence needs to be subtracted from this cyclical component:

$$v_t = \rho v_{t-1} - h.\left(min(gap_{\tau-1},h^{th})-h^{th}\right)+\varepsilon_t$$
$$\varepsilon_t \sim N(0,\sigma^2)$$

The fiscal multiplier depends on the output gap itself, reflecting recent empirical literature on larger multipliers in downturns (Baum et al 2012; Auerbach and Gorodnichenko 2013), corroborated by modeling with financial frictions (Canzoneri et al. 2016). Indeed, when slack is large, a demand stimulus is more likely to boost output as there is spare production capacity. The equation that captures this mechanism follows the usual definition of state-contingent fiscal multipliers in the empirical literature—the effect of a change in primary balance on a change in the output gap is governed by a multiplier that declines with the output gap itself. m_1 is the fiscal multiplier when output is at equilibrium, and the additional term governed by coefficient m_2 magnifies the multiplier in downturns and attenuates effects in upturns. The limit case with an infinitesimal period provides a differential equation—the derivative of the output gap with respect to the primary balance depends on the output gap itself.

$$\Delta gap(pb_t, v_t) = -\underbrace{m_1(1-m_2 gap(pb_t, v_t))}_{\text{multiplier decreases in gap}}\Delta pb_t$$

$$\frac{\partial gap(pb_t, v_t)}{\partial pb_t} = -m_1(1 - m_2 gap(pb_t, v_t))$$

With the assumption that the output gap is equal to the transitory shock process v_t if the primary balance is null, this differential equation is solved:

$$gap_t(pb_t, v_t) = \frac{1}{m_2} + (v_t - \frac{1}{m_2})e^{m_1 m_2 pb_t}$$

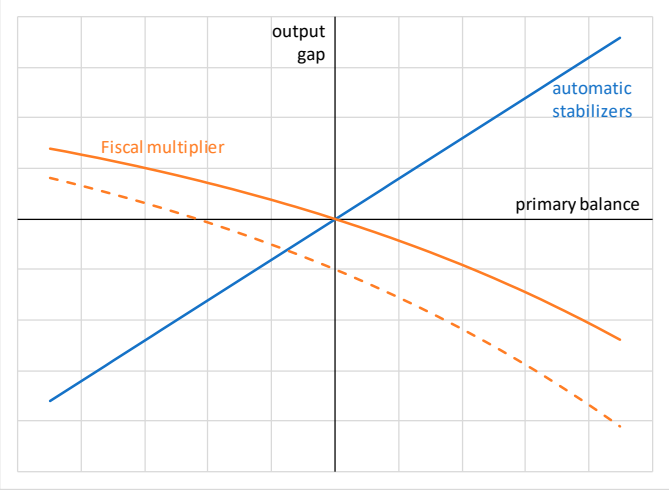
At the same time, the primary balance is the sum of a cyclical component and of a structural component that is decided by the government:

$$pb_t = pb_t^{st} + a.gap_t$$

where *a* is an automatic stabilizer coefficient.

This is the semi elasticity of the primary balance to the output gap as defined in Price et al. (2015). This set up is adopted to match empirical data directly. This defines a two-way relationship between the output gap and the primary balance. An increase in the primary balance is a fiscal tightening, this implies a decrease in the output gap (Figure 2, orange curve). This effect is sharper when the output gap is negative (on the right side of Figure 2). At the same time, a decrease in the output gap reduces tax revenue or increases means-tested transfers, and this implies a decrease in the primary balance (Figure 2, blue curve). Figure 2 reports a baseline case in which the structural primary balance and the exogenous shock are both set to zero (solid lines), and an alternative case with a negative output shock (dashed orange line). The size of the shock is the level at which this dashed line crosses the vertical axis, and the equilibrium output gap where the dashed line crosses the blue line is less negative thanks to the automatic stabilizers.

Figure 2. The Equilibrium between the Primary Balance and the Output Gap



The equilibrium is solved analytically, and the output gap is a function of the shock and the structural balance:

$$gap_t = -\frac{W\left(am_1(1-m_2v_t)e^{m_1(a+m_2pb_t^{st})}\right)}{am_1m_2} + \frac{1}{m_2} \quad (16)$$

where W(.) is the Lambert W function (see Annex 2 for more details).

Since the formulas used in the model are difficult to read, a few linearized formula valid for small deviations are thus useful to understand the linkages. The gap is a shock offset by fiscal policy:

$$gap_t \approx v_t - m_1(1-v_t m_2)pb_t$$

This approximation can be further simplified if the shock is small as the product of the shock and the primary balance is a second order term:

$$gap_t \approx v_t - m_1 pb_t$$

In this set up, the gap reflects a mismatch between the underlying shock and the fiscal policy. It can be reformulated with the structural balance:

$$gap_t \approx v_t - m_1 pb_t = v_t - m_1 pb_t^{st} - m_1 a.gap_t$$

$$gap_t \approx \frac{v_t - m_1 pb_t^{st}}{1+m_1 a}$$

The structural balance that offsets the underlying shock process v_t/m_1 is larger when the fiscal multiplier is lower. And as expected, automatic stabilizers reduce the size of the output gap. It is worth noting that the parameter m_1 captures a causal effect of the primary balance on the output gap. Many authors regard the fiscal multiplier as the causal effect of a change in the structural primary balance on output, encompassing the mitigating effect of automatic stabilisers (as in Batini et al. 2014). The multiplier considered in such papers corresponds to $m_1/(1+m_1.a)$.

Finally, the aggregate resource constraint is:

$$c_t = y_t\left(1-\chi(\Delta pb_t^{st})^2\right)$$

where c_t denotes aggregate consumption (both private and public), and the last term represents some fiscal adjustment costs, which I model as direct resource costs. These adjustment costs can reflect implementation costs of changes in spending plans or costs associated with tax uncertainty (e.g., Skinner 1988). This can also reflect the difficulty in reversing fiscal decisions (IMF, 2017). This adjustment cost is relative to output.

To summarize, the model is rich on the demand side and parsimonious on the supply side. I include key relevant channels that affect the appropriate fiscal stance, advocated from both dovish and hawkish perspectives. The dovish channels include hysteresis and cycle-dependent fiscal multipliers that magnify the cost of inaction in bad times, whereas the hawkish channels are

given by the risk premium and the risk to lose market access that constitute the costs of debt. Some channels are implicit. This is the case of monetary policy reacting to fiscal stance change, reducing its effect on output and dampening shocks. The calibration of the fiscal multiplier and of the output shocks should capture these effects.

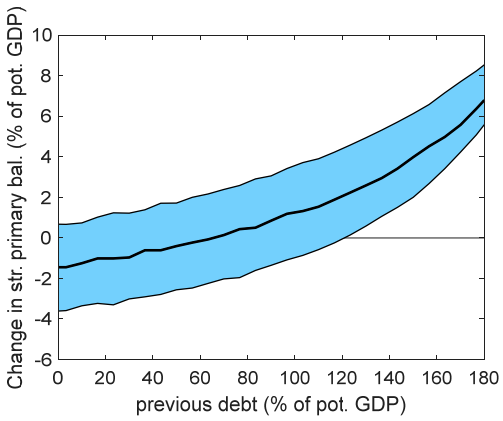
IV. The Appropriate Fiscal Stance: Results

A. Baseline Results

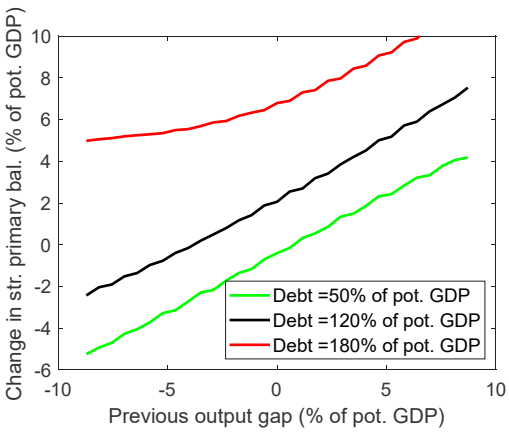
Baseline results with the augmented framework presented in the previous section confirm the qualitative results singled out in the discussion of the simple buffer stock model. The calibration follows Fournier and Lieberknecht, 2019, representing an average advanced economy (Annex 3).

Figure 3. Baseline Fiscal Reaction Function to Debt

Panel A. Structural primary balance change in percent of potential GDP



Panel B. Reaction to output shocks at different debt levels



Note to Panel A: The large solid line indicates the appropriate fiscal stance if the previous output gap is zero. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the blue band thus indicates the extent to which a government should use fiscal policy to counteract output shocks.

I report charts illustrating the effect of debt and of the output gap on the appropriate fiscal stance (Figure 3, panels A and B). The first panel illustrates the reaction to debt, reporting the appropriate fiscal stance as a function of the past debt level, holding previous output gap and structural balance at zero (solid line in Figure 3, Panel A). The appropriate fiscal stance is increasing in the debt level. The government should run more deficits at low debt and more surplus at high debt, converging toward the appropriate debt level of the model. I also report a similar reaction to debt if the lagged output gap corresponds to a one-standard deviation positive or negative shock (Thin solid lines in Figure 3, Panel A). The width of the band delineated by these two lines shows the extent to which governments should react to shocks. In a second chart, I also report the appropriate fiscal stance as a function of the past output gap, holding the

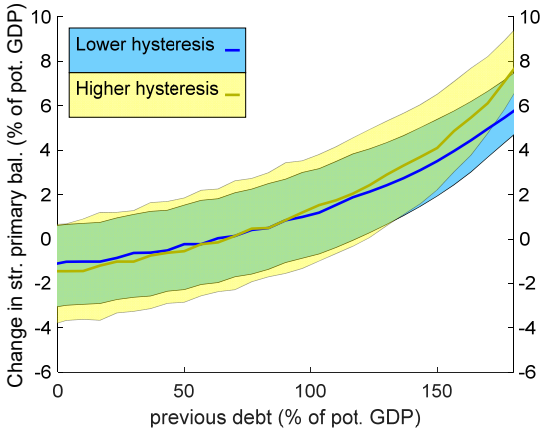
debt level and previous primary balance constant (Figure 3, Panel B). This second chart suggests that the reaction to the output gap is roughly linear, so that the information on reaction to shocks is well summarized with the width of the band in the first chart.

As debt increases, the reaction to debt is sharper and the reaction to shocks is weaker, as in the simple model discussed in section 2. The solid line in figure 3, Panel A is convex—the slope is steeper at high debt—as there are rising costs of high debt, the reaction to debt should become sharper and sharper as debt increases. This reaction to debt is consistent with the assumption that governments do not default. They rather generate the necessary surpluses to repay debt. Results indicate a substantial tightening at very high debt, reflecting a sizeable interest rate risk premia and low adjustment cost in the set of parameters used here. At low debt, the wide blue band indicates that governments should make a wide use of fiscal policy to offset shocks. This is less the case at high debt – the blue band narrows and the line is flatter in Figure 3, Panel B, as discussed with the simpler buffer-stock model. Because high debt reduces the capacity to offset shock, such a consolidation policy increases future welfare, offsetting the instantaneous welfare cost.

B. Hysteresis Can Magnify the Stabilization-debt Nexus

The presence of higher hysteresis should encourage policy makers to offset shocks more at low debt level only. This can be seen with a comparison of fiscal reaction function with different sizes of hysteresis, one that is lower than the baseline case, and one that is higher (Figure 4).

Figure 4. Fiscal Reaction Function and Hysteresis
Structural primary balance in percent of potential GDP



Note. The large solid line indicates the appropriate fiscal stance if the previous output gap is zero. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the bands thus indicates the extent to which a government should use fiscal policy to counteract output shocks.

The effect of hysteresis at low debt level is straightforward; the recession is costly in the long run, so the governments should do more to avoid it. When debt is low, the blue band is thus wider. As debt is not a concern, the stimulus can be debt financed. At high debt level, this is ambiguous. A highly indebted country facing a negative shock is in a difficult situation, as all options are affected by hysteresis. The adverse effect of the absence of stimulus during a downturn is magnified. But a stimulus during the shock increases the risk of a debt crisis later, and that is also costlier because of hysteresis. At high debt level, the appropriate reaction to shocks is roughly the same with low or high hysteresis in the numerical solution; these two

effects offset each other. This can be seen with the similar bandwidths at high debt in Figure 4. Overall, the difference between low and high debt reaction to shocks is magnified by hysteresis.

With high hysteresis, the model also prescribes a lower debt target. The buffer has a higher value because shocks can have a permanent negative cost. Accordingly, balance change increases more steeply with debt at high debt level, to avoid falling into a high-debt situation which restricts the capacity to avoid the long-term effect of a recession. Similarly, Cherif and Hasanov (2018) find with a buffer-stock model that larger permanent income shocks lead to higher precautionary saving by cash-constrained farmers.

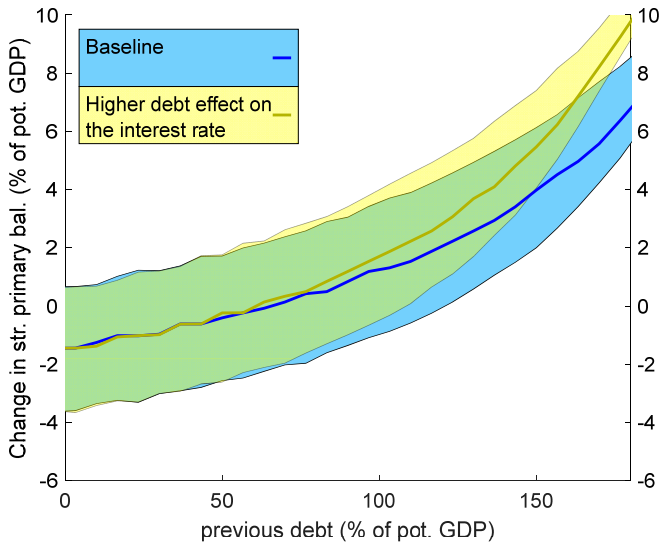
C. The Magnifying Role of Interest Rate Reaction to Debt

At a low debt level, the small interest payments do not depend much on this parameter, and hence the appropriate fiscal stance hardly depends on this parameter either. By contrast, at high debt level, this increases the marginal cost of debt, so that governments should refrain from using fiscal policy to offset shocks too much. In addition, the higher sensitivity of interest rate to debt also increase the convexity of the reaction to debt; at high debt, the reaction to debt is steeper to avoid excessive interest payments in the future.

A higher reaction of the risk premium to debt also magnifies the difference between the reaction to shocks at low and at high debt (Figure 5).

Figure 5. Fiscal Reaction Function and the Effect of Debt on the Risk Premium

Structural primary balance in percent of potential GDP



Note. The large solid line indicates the appropriate fiscal stance if the previous output gap is zero. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the bands thus indicates the extent to which a government should use fiscal policy to counteract output shocks.

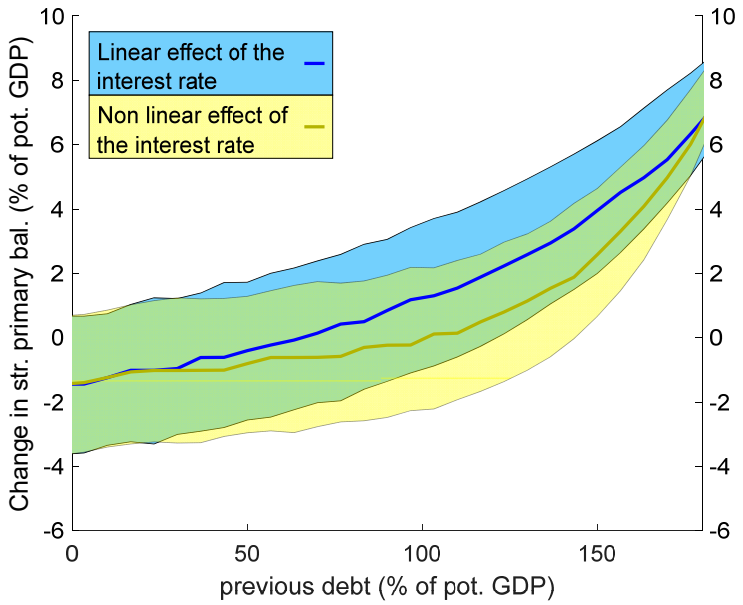
Further simulations with a non-linear reaction of the risk premium to debt

$$rp(d_{t-1}) = \lambda_{rp} e^{-\alpha_{rp} \frac{\overline{d} - d_{t-1}}{d_{t-1} - \underline{d}}} \quad (17)$$

confirm this pattern. With this non-linear hypothesis, the interest rate does not react much to debt at low debt, while it reacts more at high debt. The fiscal stance reaction to debt reflects this pattern: it reacts less to debt at low debt and more at high debt (Figure 6). All this suggests that countries that are more subject to market pressure (e.g., countries in a currency union that may not enjoy a lender of last resort, as found in Fall and Fournier 2015) should offset shocks less at high debt level.

Figure 6. Fiscal Reaction Function and a Non-linear Effect of Debt on the Risk Premium

Structural primary balance in percent of potential GDP



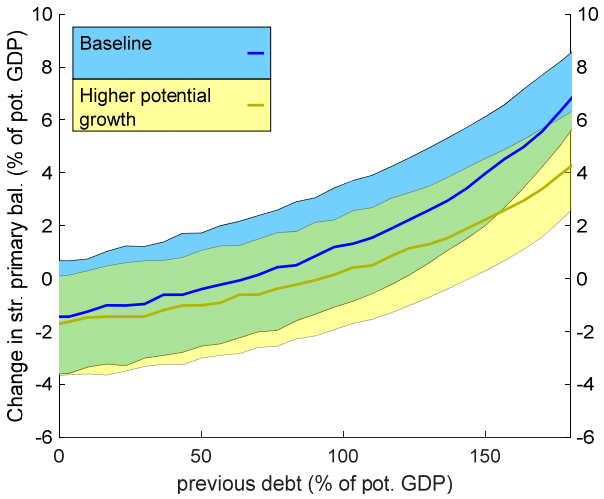
Note. The large solid line indicates the appropriate fiscal stance if the previous output gap is zero. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the bands thus indicates the extent to which a government should use fiscal policy to counteract output shocks. For this figure, there is no effect of debt change on the risk premium in both cases.

D. Sensitivity to Higher Growth Rate

A higher potential growth rate gives leeway for larger debt and allows for less fiscal tightening at high debt level (Figure 7). First, this eases the debt dynamic as the interest rate growth rate differential is lower. And with concave utility function in consumption, a rosier future is discounted more by households. Reactions to shock can be a bit more pronounced at high debt as higher growth reduces debt concerns. Alternatively, a country with a low potential growth rate needs a tighter fiscal policy. These alternative scenarios can be used to think about structural reforms that can increase potential growth in the long-run. Also, the fiscal stance implication of reforms that increase output level, such as an improvement in the spending mix, could be considered with two scenarios. Comparing a baseline scenario without the reform and an alternative with a temporary boost in growth would go the extent to which that temporarily eases the debt pressure (see Fournier and Lieberknecht, 2019, for an example).

Figure 7. Fiscal Reaction Function and Potential Growth

Structural primary balance in percent of potential GDP



Note. The large solid line indicates the appropriate fiscal stance if the previous output gap is zero. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the blue band thus indicates the extent to which a government should use fiscal policy to counteract output shocks.

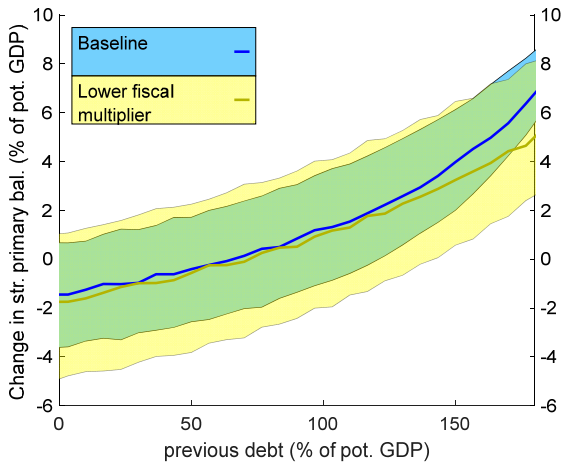
E. The Fiscal Multiplier

Countries with lower fiscal multipliers need slightly more changes in their primary balance to achieve the stabilization objective (Figure 8). This holds both at low and high debt. In addition, fiscal tightening at high debt has a lower welfare cost. As a result, the government is less concerned by debt and hence the primary balance is less sensitive to the debt level. The assumption that the fiscal multiplier remains above zero is critical; if it were ineffective to offset shocks, then only the debt concern would prevail, and fiscal policy should ignore shocks. Accordingly, if fiscal multipliers are very low, then governments should not react to the cycle too much (Figure 9). As the composition of fiscal stance can affect the fiscal multiplier, this sensitivity exercise can be used to distinguish the fiscal stance implication of a different fiscal mix strategy.

For instance, Alesina et al. (2015) find that spending multipliers are much lower than tax multipliers.[12] Alternative results with lower multipliers would be more suitable in this case.

Figure 8. Fiscal Reaction Function and the Fiscal Multiplier

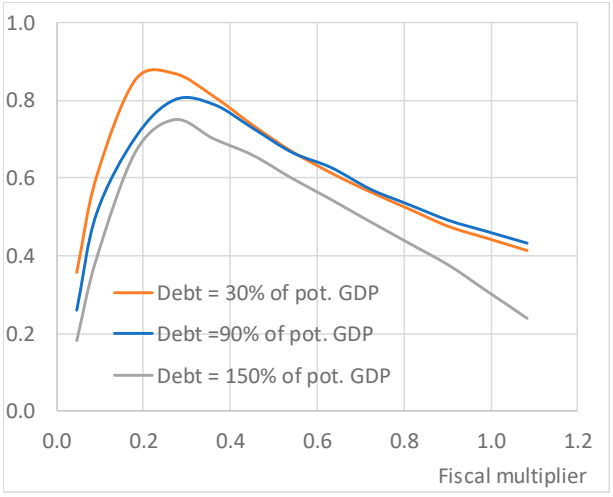
Structural primary balance in percent of potential GDP



Note. The large solid line indicates the appropriate fiscal stance if the previous output gap is zero. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the blue band thus indicates the extent to which a government should use fiscal policy to counteract output shocks.

Figure 9. Reaction to Output Shocks and the Fiscal Multiplier

Change in structural primary balance in reaction to a negative output shock



Note. The vertical axis reports the difference between the appropriate fiscal stance when the output shock in null and the appropriate fiscal stance when the exogenous shock is minus one percent of GDP.

F. Automatic Stabilizers

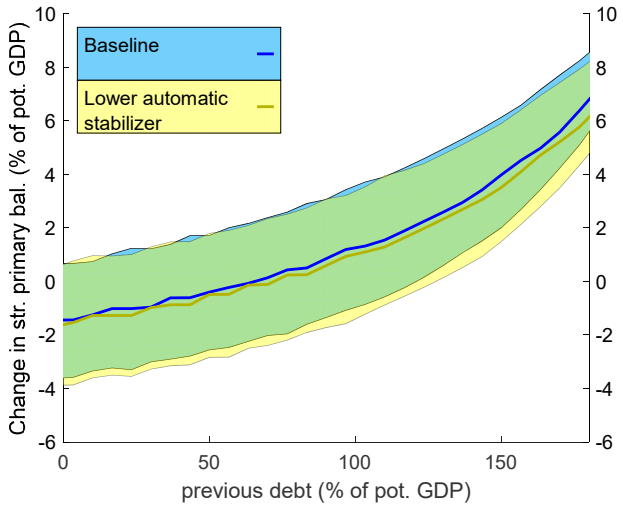
Results do not depend much on the size of automatic stabilizers (Figure 10). At low debt, countries should react slightly more to shocks, as smaller automatic stabilizers imply that the government needs more discretionary actions to offset shocks.

[12] See Batini et al. (2014) for a summary of the literature that investigates differences between taxes or spending instruments. In particular, macroeconomic models suggest higher multipliers for investment (e.g. Coenen et al., 2012), although it is difficult to identify a robust difference empirically (e.g. Perotti, 2004, finds no difference).

At the same time, each discretionary primary balance change has a larger effect on primary balance and growth as its impact is not cushioned by the stabilizing effect of multipliers. In sum, it does not need to undertake much more discretionary action, relative to the baseline case with higher automatic stabilizers. Countries with lower automatic stabilizers also need less discretionary action to reduce debt, as these discretionary actions have a larger effect on primary balances. The reaction of the structural primary balance to debt is thus less steep with lower automatic stabilizers.

Figure 10. Fiscal Reaction Function and the Size of Automatic Stabilizers

Structural primary balance in percent of potential GDP



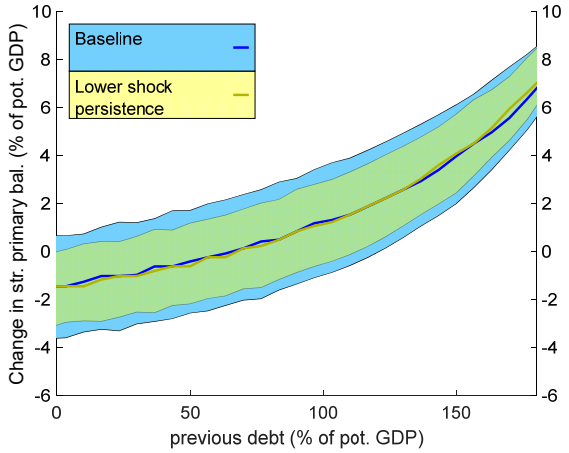
Note. The large solid line indicates the appropriate fiscal stance if the previous output gap is zero. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the blue band thus indicates the extent to which a government should use fiscal policy to counteract output shocks.

G. The Persistence of Shocks

The reaction to the previous year output gap should be lower when it is less persistent (Figure 11). This is because the government reacts with delay: the government should only react to the fraction of the shock that will persist in the next year. If the government reacts too much to the observed shock, that is likely not to match with the updated position in the cycle when the decision materializes and hence this may reduce welfare. However, with very persistent negative shock and high debt, the effect becomes ambiguous. On one side, the high shock persistence means the government knows with a high degree of certainty the position in the cycle in the next year, so the risk to act too late is limited. On the other side, the government also knows that this protracted shock will have a long-lasting impact on debt. If the debt concern dominates, a highly indebted government may even need to react to a highly persistent shock with a fiscal tightening to avoid facing a debt crisis in the future (Figure 12).

Figure 11. Fiscal Reaction Function and Lower Shock Persistence

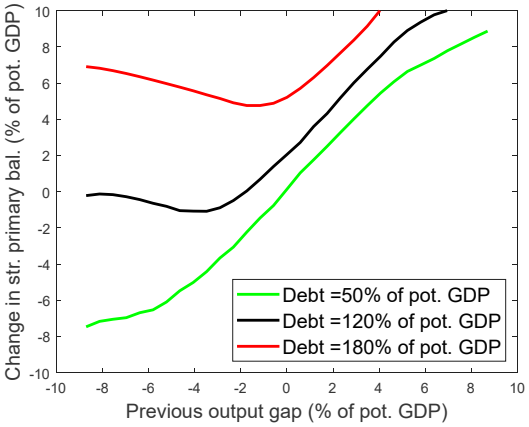
Structural primary balance in percent of potential GDP



Note. The large solid line indicates the appropriate fiscal stance if the previous output gap is zero. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the blue band thus indicates the extent to which a government should use fiscal policy to counteract output shocks.

Figure 12. Fiscal Reaction Function and Higher Shock Persistence

Structural primary balance in percent of potential GDP



Note. The calibration used for this figure is the same as the one used for the higher shock persistence case in Figure 10.

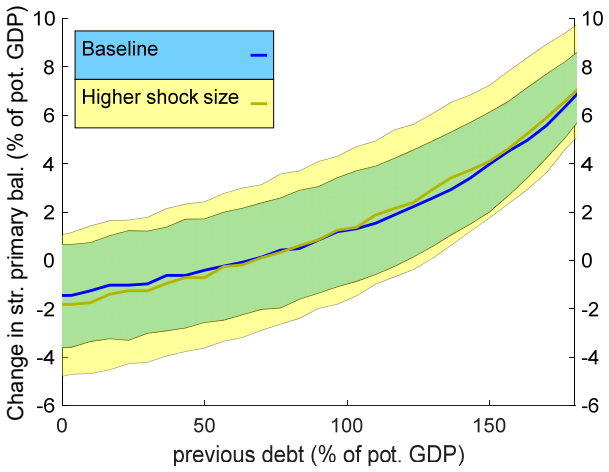
H. The Size of Shocks

Short-term changes in the primary balance should be larger for governments facing larger shocks (Figure 13), which is an effect of size (or scale). If one rescales the reaction function to observe the effect to, say, one percent of GDP shock instead of a one standard deviation shock, reactions in both cases would be quite similar as the government targets the same objective. In sum, this reflects the higher shock standard deviation almost directly, and this holds even at high debt levels.

With larger shocks, the appropriate primary balance when the shock is null is also very similar, reflecting two offsetting forces. On one side, one may want a lower debt level to preserve the capacity to react to larger shocks, which should lead to a higher primary balance on a systematic basis. On the other hand, one may want to avoid the higher risk of large recession which entail long-term costs. The government thus has an incentive to plan a lower structural primary balance to reduce this risk. These two channels offset each other's in the calibration considered here.

Figure 13. Fiscal Reaction Function and Shock Size

Structural primary balance in percent of potential GDP



Note. The large solid line indicates the appropriate fiscal stance if the previous output gap is zero. The thin solid lines indicate the appropriate fiscal stance in case of a one-standard deviation positive or negative shock. The width of the blue band thus indicates the extent to which a government should use fiscal policy to counteract output shocks.

V. Conclusion

This paper lays the foundation for fiscal stance advice tailored for each country. Beyond the recent applied guidance for policy makers (Kanda 2011; Carnot 2014; Bankowsky and Ferdinandusse 2017), a tool built with this framework can embed many country-specific features. The quantitative recommendation would thus consider fundamental characteristics such as the growth rate, the interest rate and its reaction to debt, hysteresis, the fiscal multiplier and its sensitivity to the output gap, automatic stabilizers, output shock persistence, and their size. Furthermore, the optimal fiscal stance can be conditional on a short-term forecast, and the model can then be simulated to produce a medium-run appropriate path (Fournier and Lieberknecht, 2019). Parameters can be adjusted. For instance, should a government introduce new fiscal institutions that reinforce government's credibility, an alternative simulation with a lower elasticity of interest rate to debt and a higher debt limit could illustrate the gains. Also, an explicit fiscal multiplier parameter, rather than multipliers that are the result of deeper preference parameters, makes the model easier to calibrate for practitioners. Furthermore, sensitivity analysis to different assumptions can help to discuss the key features underpinning the advice and the inherent uncertainties.

The tool would be particularly suited to tailor recommendations to different debt levels as it allows an interaction between the debt level and the shocks. This can be particularly useful to provide an evenhanded approach over a wide range of debt levels.

The interaction between the debt and the appropriate reaction to output shocks is not commonly used in the empirical literature on fiscal reaction functions thus far. The pioneering estimates by Bohn (1998) focus on the reaction to the debt level to test for sustainability, while the interaction between debt and the position in the cycle is not included. And in the rare cases where the interaction is tested, it can be regarded as an undesirable behavior as in Combes et al (2017) which places an emphasis on growth stabilization, thus recommending fiscal rules that mitigate fiscal policy procyclicality in a high-debt context. By contrast, the buffer-stock model shows that the lower propensity to offset shocks at times of high debt is appropriate. Another insight for the empirical estimates of fiscal reaction functions is that one should expect a non-linear reaction to debt. Ghosh et al. (2013) and Fournier and Fall (2017) indeed consider non-linear estimates which point to a convex reaction to debt on a debt level range that covers most of the observations.

Following the approach suggested by Blanchard (2018), the partial equilibrium buffer-stock model of the government presented here can be a building block in a dynamic stochastic general equilibrium (DSGE) model. For instance, an avenue for research could be to include this buffer-stock model in a DSGE with several fiscal instruments (government spending, tax rates, etc.) and relate these instruments with private agent decisions. Then, different instrument choices may lead to different output effects in the short and long run and hence to different implications for the fiscal stance. Another avenue for research is to consider a long-run learning process that makes the shape of market reaction endogenous. If a government facing a sharp reaction of the interest rate to debt follows the appropriate fiscal stance discussed here, this should address the market concerns. Then, the market reaction should become more moderate. In the short- to medium-run, the setting used here is plausible because this learning process may take time, and markets can be skeptical on the government capacity to commit to such a strategy. In the long run, if the government builds credibility regarding its capacity to follow an appropriate fiscal stance, this learning process can have large implications.

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Annex 1. Analytic Results with the Simple Model

A1.1. Analytic solution with a quadratic utility function and no debt limit (Proposition 1)

With a quadratic utility function $u(y_t) = -(1/2)y_t^2 + \omega y_t$, no debt constraint and the conjecture that the primary balance reaction is linear $pb_t = \beta_0 + \beta_1 e_t + \beta_2 d_{t-1}$:

$u'(y_t) = \beta(1+r)E_t[u'(y_{t+1})]$ becomes

$$(1 + e_t - m.pb_t) - \omega = \beta(1+r)E_t[1 + e_{t+1} - m.pb_{t+1} - \omega]$$

$$(1+e_t - m\beta_0 - m\beta_1 e_t - m\beta_2 d_{t-1}) - \omega = \beta(1+r)\left(1 - m\beta_0 - m\beta_2\left((1+r)d_{t-1} - \beta_0 - \beta_1 e_t - \beta_2 d_{t-1}\right) - \omega\right)$$

We extract terms with d_{t-1} to calculate β_2:

$$m\beta_2 d_{t-1} = \beta(1+r)\left(m\beta_2\left((1+r)d_{t-1} - m\beta_2 d_{t-1}\right)\right)$$

There are two solutions:

$\beta_2 = 0$ or

$1 = \beta(1+r)^2 - \beta(1+r)\beta_2$

If $\beta(1+r) < 1$, then the first solution respects the (weak) transversality condition and corresponds to a government that offsets shocks perfectly to reach the global maximum of the utility function $y_t = \omega = e_t + 1 - m\,p\,b_t$.

If $\beta(1+r) \geq 1$, then only the second solution respects the transversality condition. We extract terms with e_t to calculate β_1:

$e_t - m\beta_1 e_t = m\beta_2\beta(1+r)\beta_1 e_t$

$$\boxed{\beta_1 = \frac{1}{m(1+\beta_2\beta(1+r))} = \frac{1}{m\beta(1+r)^2}}$$

We extract the remaining terms without e_t and d_{t-1} to calculate β_0:

$1 - m\beta_0 - \omega = \beta(1+r)(1 - m\beta_0 + m\beta_2\beta_0 - \omega)$

$$\beta_0 = \frac{(1-\omega)(\beta(1+r)-1)}{m(1+\beta(1+r)\beta_2 - \beta(1+r))} = \frac{(1-\omega)(\beta(1+r)-1)}{m(\beta(1+r)^2 - \beta(1+r))} = \frac{(1-\omega)(\beta(1+r)-1)}{mr\beta(1+r)}$$

$$\boxed{\beta_2 = \frac{\beta(1+r)^2 - 1}{\beta(1+r)} = r + \frac{\beta(1+r)-1}{\beta(1+r)}}$$

In sum, the optimal primary balance is:

$$\begin{cases} pb_t = \dfrac{e_t + 1 - \omega}{m} & if \quad \beta(1+r) < 1 \\ pb_t = \dfrac{1}{m\beta(1+r)^2} e_t + r d_{t-1} + \dfrac{\beta(1+r)-1}{\beta(1+r)} \left(d_{t-1} - d_{eq} \right); d_{eq} = \dfrac{\omega - 1}{mr} & if \quad \beta(1+r) \geq 1 \end{cases}$$

A1.2. Some properties of the solution in the presence of a debt limit

Initialization and backward iteration

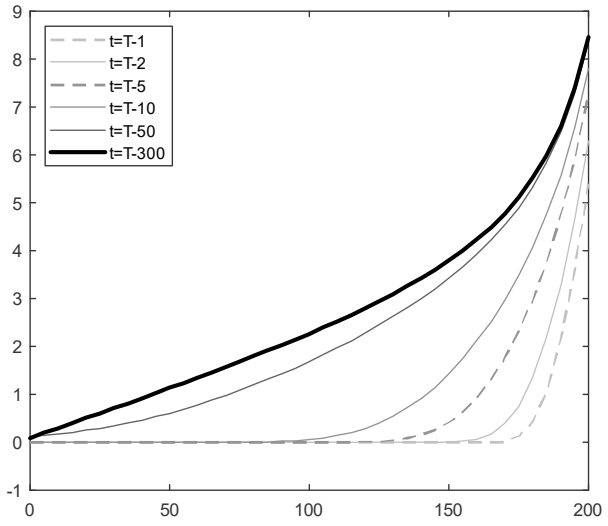
With a finite horizon, solution at the final period T is straightforward. Either the government can reach the maximum of the instantaneous utility function, or it reaches the debt limit:

$$pb_T = \max\left(\frac{e_T + 1 - \omega}{m}, (1+r) d_{T-1} - \bar{d} \right)$$

This is a stylized representation of a government that does not care about the future. This is the initialization for the backward iterations which converge to the fixed-point infinite horizon solution (Figure A1.1).

Figure A1.1. Backward convergence toward the infinite horizon solution

Appropriate primary balance as a function of the past debt level



Derivative with respect to debt

I first show that the derivative with respect to debt is strictly increasing for a government at risk of losing market access.

First, induction with backward iteration immediately shows that at any period

$$0 \leq \frac{\partial\_pb_t}{\partial d_{t-1}} \leq 1+r \quad \text{and equivalently} \quad 0 \leq \frac{\partial\_d_t}{\partial d_{t-1}} = 1+r-\frac{\partial\_pb_t}{\partial d_{t-1}} \leq 1+r$$

This thus holds in the limit case with infinite horizon. And when the government has market access, the derivative with respect to debt is also always below a bound strictly below 1+r:

$$0 \leq \frac{\partial\_pb_t}{\partial d_{t-1}} \leq \frac{1+r}{\frac{1}{\beta(1+r)^2}+1} = 1+r_m < 1+r$$

If the government always has market access in the next period, then:

$$\frac{\partial^2\_pb_t}{\partial d_{t-1}^2} = (1+r)\frac{E_t\left[\frac{\partial^2\_pb_{t+1}}{\partial d_t^2}(1+r-\frac{\partial\_pb_{t+1}}{\partial d_t})\right]}{\left(1+\beta(1+r)\left(E_t\left[\frac{\partial\_pb_{t+1}}{\partial d_t}\right]\right)\right)^2}$$

Induction with backward iteration also immediately shows that at any period, $\frac{\partial\_pb_t}{\partial d_{t-1}}$ is increasing in debt, whatever if government has market access or not.

A1.3. Proposition 2 proof

Lemma. $\frac{\partial^2\_pb_t}{\partial d_{t-1}^2} > 0$ **for a government that has market access and is facing uncertainty on future market access.**

By contraposition, assume d_a is the highest debt level for which there is a shock for which the government has market access and $\frac{\partial^2 pb_t}{\partial d_{t-1}^2} = 0$. The government cannot be at risk of losing market access in the next period (otherwise, this probability would depend on debt, implying that $E_t\left[u''(y_{t+1})\frac{\partial\_pb_{t+1}}{\partial d_t}\right]$ depends on *d*). Then,

$E_t\left[\frac{\partial^2\_pb_{t+1}}{\partial d_t^2}((1+r)d_a - pb(d_a,e_t),e_{t+1})(1+r-\frac{\partial\_pb_{t+1}}{\partial d_t})\right] = 0$. As $1+r-\frac{\partial\_pb_{t+1}}{\partial d_t} > 0$ and

$\frac{\partial^2_- pb_{t+1}}{\partial d_t^2}$ is positive, this implies that for any future shock e_{t+1},

$\frac{\partial^2_- pb_{t+1}}{\partial d_t^2}((1+r)d_a - pb(d_a, e_t), e_{t+1}) = 0$, and hence debt in the next period is always below d_a:

this is a bound below the limit: the government never loses market access.

Proof.

Proposition 2 follows with equation (15): as soon as the derivative with respect to debt increases in debt, the derivative with respect to the shock decreases in debt.

A1.4. Proposition 3 proof

Let us consider a case in which the government never loses market access. Then, the solution of the model without debt limit is an obvious solution of the problem with debt limit, and this can only happen with $\beta(1+r) > 1$. And for any shock and any debt level, the government is not affected by the debt limit constraint:

$$pb_t = \frac{1}{m}\left[1 + e_t - u'^{-1}\left(\beta(1+r)E_t[u'(1+e_{t+1} - m.pb_{t+1})]\right)\right] \geq \left((1+r)d_{t-1} - \bar{d}\right)$$

With the solution of the model without debt limit, and in the worst case in which past debt is at the limit:

$$\frac{1}{m\beta(1+r)^2}e_t + r\bar{d} + \frac{\beta(1+r)-1}{\beta(1+r)}\left(\bar{d} - d_{eq}\right) \geq r\bar{d}$$

This is always the case with the following condition:

$e_t \geq -m(1+r)\left(\beta(1+r)-1\right)\left(\bar{d} - d_{eq}\right)$ or equivalently:

$$P\left(e_t < m(1+r)\left(1-\beta(1+r)\right)\left(\bar{d} - d_{eq}\right)\right) > 0$$

Annex 2. The gap as a function of the underlying shock and the structural primary balance

Output gap as a function of the underlying shock and the current balance:

$$gap_t = \frac{1}{m_2} + (v_t - \frac{1}{m_2})e^{m_1 m_2 pb_t}$$

This leads to an identity in which the gap is regarded as the unknown:

$$(m_2.gap_t - 1) = (m_2 v_t - 1)e^{m_1 m_2 (pb_t^{st} + a.gap_t)}$$

I reformulate to have a similar formula $am_1\left(m_2.gap_t - 1\right)$ on both sides:

$$am_1\left(m_2.gap_t - 1\right) = a(m_1 m_2 v_t - m_1)e^{am_1(m_2.gap_t - 1)}e^{am_1 + m_1 m_2 pb_t^{st}}$$

$$am_1\left(-m_2.gap_t+1\right)e^{am_1(-m_2.gap_t+1)}=am_1(-m_2v_t+1)e^{m_1(a+m_2pb_t^{st})}$$

I use the Lambert W function, which is the principal inverse of the function f(x)=x.e^x.[13]

$$am_1\left(-m_2.gap_t+1\right)=W\left(am_1(-m_2v_t+1)e^{m_1(a+m_2pb_t^{st})}\right)$$

The output gap is thus a function of the underlying shock and the structural balance:

$$gap_t=-\frac{W\left(am_1(1-m_2v_t)e^{m_1(a+m_2pb_t^{st})}\right)}{am_1m_2}+\frac{1}{m_2}$$

Annex 3. Calibration

The baseline calibration presented here follows directly Fournier and Lieberknecht, 2019. It is built on standard values and averages across advanced economies to consider a general case. This calibration is a positive assessment of the economy reflecting empirical evidence in the literature and can easily be tailored for country-specific applications.[14] There are four sets of parameters: a) welfare function parameters, b) fiscal parameters, c) parameters governing the cost of debt, and d) economy parameters (Table 1).

13 In general, this inversion problem is multivalued. Here, m_2.gap is assumed to be below 1 to remain in the range for which the fiscal multiplier remains positive, so I can consider the principal Lambert function that is the only one real solution above -1.

14 As government choices can be suboptimal for political economy reasons (Alesina and Passalacqua 2016), past behavior can differ from the prescription of normative analysis. As a result, the calibration does not aim at replicating observed fiscal behavior.

Table 1. Baseline Calibration

Welfare function	
Discount factor β	0.99
Risk aversion σ	2
Labor elasticity η	1/0.3
Weight of labor ξ	1
Fiscal parameters	
Fiscal multiplier when the gap is null m_1	0.8
Fiscal multiplier sensitivity to shocks m_2	3
Automatic stabilizers (primary balance semi-elasticity to the gap) a	0.52
Adjustment cost χ	3
Interest rate and debt parameters	
Growth-adjusted interest rate at current debt level	0.49%
Effect of debt level on the risk premium α	1.5%
Effect of debt change on the risk premium α_2	0.5%
Debt level at which the risk to lose market access is 50% $\bar{d}$	150%
Debt limit accuracy d_1	3
Effect of debt change on the risk to lose market access is d_2	1
Adverse scenario coefficient in case of loss of market access d_3	0
Economy parameters	
Potential GDP per capita growth	1.6%
Shock persistence ρ	0.60
Shock size σ	3.2%
Hysteresis	10%
Hysteresis threshold	-1%

The welfare function parameters β, σ, η and ξ are standard in the literature. As a benevolent government should maximize household welfare, the government objective function can be interpreted as household utility. The parameters follow the rich literature on quantitative DSGE models. The discount factor is on the conservative side, but at the same time future utility is discounted via long-term growth in this model, so that with the per capita growth rate at 1.6%,

the overall discount rate is about 0.975. This is comparable with most DSGE models using detrended consumption (e.g., Smets and Wouters 2007). With a weight for labor set to one, the instantaneous utility peaks when the output gap is null. The labor elasticity is consistent with the inverse of usual values for labor supply elasticity.

The set of fiscal parameters includes fiscal multipliers and automatic stabilizers that can be directly calibrated. The fiscal multiplier if the output gap is zero m_1 is the cross-country average of a continuous version of the fiscal multiplier bucket approach following Batini et al. (2014) in which the fiscal multiplier is a function of a country's exchange rate regime (which also covers monetary policy types[15]), labor market rigidity, and trade openness. With m_2 = 3, a negative output gap of five percent lowers the standard multiplier by 0.15. This lies in between empirical estimates of such a link with the position in the cycle (Baum et al 2012; Auerbach and Gorodnichenko 2013) and other studies that find that the fiscal multiplier is independent from the state of the economy (Ramey and Zubairy 2018).[16] The size of automatic stabilizers a is the cross-country average of primary balance semi-elasticities to GDP estimated by Price et al. (2015). The adjustment cost parameter is set at $\chi = 3$. This is a moderate adjustment cost as the model can recommend a sizeable adjustment if the previous primary balance level was far away from an appropriate one. One can hardly infer this parameter from historical data as one cannot exclude that the observed degree of fiscal inertia reflects suboptimal political constraints. Higher adjustment cost values can create convergence difficulties – the government needs to be able to adjust to preserve long-run sustainability. For this reason, a moderate adjustment cost is regarded as appropriate here.

The debt parameters govern the cost of government debt via the risk premium and the risk to lose market access. The constant in the risk premium is calibrated with the current debt level and the current interest rate level. The risk premium is a linear function of government debt, where the slope is $\alpha = 1.5$, which is in the middle of empirical estimates (Gruber and Kamin 2012; Poghosyan 2012; D'Agostino and Ehrmann 2014; Fall and Fournier 2015; Henao-Arbelaez and Sobrinho 2017). The risk to lose market access is d_1 = 3, d_2 = 1, d_3 = 0 and $\bar{d}$ = 150%. The latter reflects debt levels at which fiscal stress has become more frequent over the last twenty years among the scarce cases of advanced economies with fiscal distress.

The potential growth assumption is an average of WEO potential growth over 2017-21. The growth interest rate differential is calibrated with 20-year averages across advanced economies of IMF World Economic Outlook (WEO) April 2018 data. Shock parameters (size σ and persistence ρ) are estimated with past shocks reflecting the output gap and the primary balance:

[15] Monetary is implicit here and needs to be embedded in the calibration of the fiscal multiplier. For instance, the comparison of fiscal multipliers when the central is and is not at the zero lower bound (Christiano et al., 2011) provides an indication of the extent to which fiscal multipliers are affected by monetary policy.

[16] Numerical results indicate, however, that results do not depend much on this parameter.

$$v_t = \left(gap_t - \frac{1}{m_2} \right) e^{-m_1 m_2 pb_t} + \frac{1}{m_2}$$

I calibrate h = 0.1 and h^{th} = -1%, such that the long-run effect is in line with Blanchard and Summers (1987), Delong and Summers (2012) and Ball (2014) who postulate or find long-term effects of around 0–20%.