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Sovereign Risk and Bank Risk-Taking

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Sovereign Risk and Bank Risk-Taking[1]

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Abstract

I propose a dynamic general equilibrium model in which strategic interactions between banks and depositors may lead to endogenous bank fragility and slow recovery from crises. When banks' investment decisions are not contractible, depositors form expectations about bank risk-taking and demand a return on deposits according to their risk. This creates strategic complementarities and possibly multiple equilibria: in response to an increase in funding costs, banks may optimally choose to pursue risky portfolios that undermine their solvency prospects. In a bad equilibrium, high funding costs hinder the accumulation of bank net worth, leading to a persistent drop in investment and output. I bring the model to bear on the European sovereign debt crisis, in the course of which under-capitalized banks in default-risky countries experienced an increase in funding costs and raised their holdings of domestic government debt. The model is quantified using Portuguese data and accounts for macroeconomic dynamics in Portugal in 2010-2016. Policy interventions face a trade-off between alleviating banks' funding conditions and strengthening risk-taking incentives. Liquidity provision to banks may eliminate the good equilibrium when not targeted. Targeted interventions have the capacity to eliminate adverse equilibria.

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Contents

1 Introduction

Evidence from recent financial and sovereign debt crises shows that in response to higher aggregate risk, under-capitalized banks increase their exposure to aggregate risky assets and experience a rise in their funding costs. This leads to rising bank fragility and default risk, and raises two important questions. First, what are the circumstances and mechanism that drive banks to become excessively exposed to aggregate risk? Second, what is the role of bank funding costs? At the same time, this recent evidence challenges current theoretical models that typically abstract from bank funding costs, assuming that banks have access to deposits at the risk-free rate (see e.g. Brunnermeier and Sannikov, 2014; Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010).

In this paper, I propose a framework where deposits are assets priced according to their risk, and banks can optimally choose to pursue risky portfolios (which may lead to default in equilibrium) under limited liability. This creates strategic complementarities: high required deposit interest rates in anticipation of risk-taking behaviour raise the costs of funding for banks and strengthen their incentives to take on more risk. Banks may then endogenously validate depositor expectations in equilibrium, raising the possibility of multiple equilibria.

I bring this theoretical model to bear on the European sovereign debt crisis, its transmission to economic activity and policy debates on interventions in support of the banking sector. In doing so, I bring forward two key empirical facts to motivate my model. In countries hit by the sovereign debt crisis, under-capitalized banks increased their exposure to sovereign risk by investing heavily in their own government's debt. In these countries, there is also significant co-movement between yield spreads on sovereign bonds and deposit interest rates.

I develop my analysis by specifying a dynamic small open economy model with households, firms, and a banking sector. Banks collect deposits from households and choose their portfolios of sovereign bonds and loans to firms; households lend to banks on terms that depend on bank solvency prospects; firms invest. The government issues default-risky bonds.

Modelling the equilibrium adjustment in bank risk-taking strategies in response to funding conditions has key macroeconomic and policy implications. The kernel intuition is that, when banks are well capitalized and/or market sentiment is "good", the resulting banking equilibrium can be described as safe. In a "safe equilibrium" banks keep their holdings of government debt low, reducing their exposure to sovereign risk. Since banks are safe, depositors accept low interest rates. When banks' portfolio exposures cannot be specified in a contract with depositors, however, another equilibrium may emerge depending on the conditions of the economy and the net worth of banks.[1] In this "gambling equilibrium", depositors expect banks to have a high

[1]Non-contractibility of portfolio exposures may arise due to (sufficiently) costly enforcement on behalf of depositors or information frictions such as opacity in bank balance sheets preventing depositors from observing bank portfolios in detail.

exposure to government debt and hence become risky. As depositors require a risk premium, banks find it optimal to gamble and buy risky sovereign debt.[2] The possibility of multiple equilibria depends on bank capitalization: the problem plagues countries where the banking sector is under-capitalized.

In the gambling equilibrium, shocks to sovereign risk simultaneously raise bank funding costs and drive banks to increase their purchases of government debt at the expense of credit to firms. This has significant consequences on the macroeconomy as high bank funding costs hinder the recovery of bank net worth and lead to a prolonged period of financial fragility and a persistent drop in output. Persistence here is endogenous, and absent in the safe equilibrium, where banks deleverage and all of the adjustment (in credit and output) is front-loaded and short-lived.

I bring the model to data by calibrating it to Portugal over 2010-2016 and simulating it under a series of sovereign risk shocks that emulate Portuguese sovereign bond yields. The simulation indicates that the Portuguese economy is vulnerable to multiple equilibria and shows that a sequence of bad sentiments (i.e. the gambling equilibrium) can account for dynamics of key macroeconomic and financial variables during the sovereign debt crisis.

The model naturally provides novel and important insights on the effectiveness of central banks' liquidity interventions in support of financial intermediaries. A key prerequisite for successful interventions is that they need to provide some risk-sharing with depositors. I show that when the repayment of official debt takes precedence over deposits, liquidity provision is completely ineffective. This is because depositors anticipate the dilution of their claims to bank revenues in the event of default and raise deposit rates accordingly.

The second requirement for a successful intervention is that it must be well-targeted. Non-targeted interventions that provide liquidity unconditionally face an adverse trade-off between their goal of alleviating banks' funding conditions and strengthening their incentives to gamble. When bank net worth is low, non-targeted liquidity provision eliminates the safe equilibrium and banks use the additional funding to increase their sovereign exposure until their funding costs return to their pre-intervention level. On the contrary, targeted interventions that provide liquidity conditional on bank leverage overcome the adverse trade-off and eliminate the gambling equilibrium.

These insights can be generalized to a large set of policy instruments. I show that, on its own, deposit insurance faces the same trade-off as non-targeted liquidity provision (with risk sharing). A wide range of macroprudential policy instruments can be used in conjunction with deposit insurance to overcome the trade-off, leading to a similar outcome as targeted liquidity

[2] Deposit insurance schemes typically guarantee deposits only up to a limit (Demirguc-Kunt et al., 2008). In real terms, depositor losses can take the form of a suspension of convertibility and a currency re-denomination as well as an explicit bail-in.

provision. Specifically, this outcome is implementable using regulatory constraints on bank liabilities or capital regulation with a positive risk-weight on domestic sovereign bond holdings.

This paper lies at the intersections of the literatures on bank risk-taking and macroeconomic dynamics under financial frictions. The insight that limited liability and non-contractibility of investment decisions may lead to risk-shifting can be traced back to Jensen and Meckling (1976). In the context of banking, Kareken and Wallace (1978) show that deposit insurance and bailout guarantees strengthen risk-taking incentives. Keeley (1990) and Hellmann et al. (2000) among many others[3] develop models where imperfect competition in the banking sector reduces risk-taking as rents associated with market power provide skin in the game.

Similar to these studies, I propose a model with imperfectly competitive banks but focus on depositors and their role in determining bank funding costs.[4] My contribution to the aforementioned literature is to show that depositor expectations about bank risk-taking may become self-fulfilling such that banks pursue risk-shifting strategies even in the absence of moral hazard arising from government guarantees on the banking sector. Non-contractibility of bank portfolio exposures has a key role in undermining market discipline and bringing about this result. Because of this friction, banks may not reduce their funding costs by committing to a safe portfolio.[5] Therefore, when depositors demand higher rates in anticipation of risk-taking, the resulting increase in bank funding costs drives banks to invest in risky assets with high yield. With sufficiently low bank net worth and high aggregate risk, depositor expectations become self-fulfilling and there are multiple equilibria.[6]

A recent literature focuses on the impact of bank balance sheet constraints on macroeconomic dynamics. In this literature, banks channel funds from households to productive investment opportunities and face an occasionally binding constraint on their leverage. For example, in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), the balance sheet constraint prevents bank managers from diverting funds to themselves. Brunnermeier and Sannikov (2014) consider a similar constraint in a highly non-linear environment with fire-sales. In the context of sovereign debt crises, Gennaioli et al. (2014) and Perez (2015) propose models where sovereign default tightens balance sheet constraints. Bolton and Jeanne (2011) and Bocola (2016) show

[3]See Carletti (2008) for an extensive review of the literature on bank competition and risk-taking.

[4]I rely on expected rents from imperfect competition to moderate banks' risk-taking incentives. When there is perfect competition in the banking sector, the combination of limited liability and non-contractibility of portfolio exposures always leads to a gambling equilibum.

[5]When banks' portfolio exposures are contractible, it is always optimal for banks to commit to a safe portfolio as by doing so they may reduce their funding costs to the risk-free rate.

[6]The multiplicity mechanism considered here differs from bank-runs à la Diamond and Dybvig (1983) in that it pertains to banks' ex-ante risk-taking decisions rather than ex-post withdrawals. Farhi and Tirole (2012) and Acharya et al. (2016) also propose models with multiplicity in bank risk-taking. In these studies, multiple equilibria arise due to strategic complementarities across banks as correlation in bank exposures makes it ex-post optimal for the government to provide support. This paper instead focuses on strategic complementarities between banks and depositors.

that anticipation of sovereign default may also tighten these constraints.[7]

A common feature of these studies is that constraints on bank balance sheets rule out banking default in equilibrium, thereby ensuring that banks have access to funds at the risk-free rate. As a result, adjustment in the banking sector takes place through the quantity of intermediation rather than bank funding costs and risk-taking: Following adverse shocks, the constraint tightens and banks are forced to deleverage. This leads to a decline in investment and output but also creates excess returns in intermediation that re-build bank net worth, paving the way to a recovery.

The safe equilibrium in this paper features a similar adjustment mechanism. Most importantly, however, this paper proposes an alternative adjustment mechanism which may be present in countries with high aggregate risk and under-capitalized banking sectors. In the gambling equilibrium, banks respond to adverse shocks by increasing their risk-taking rather than deleveraging and experience a rise in their funding costs. High funding costs in turn hinder the recovery in bank net worth, leading to endogenously slow recovery from crises. Macroeconomic dynamics generated by this mechanism are consistent with recent evidence from the European sovereign debt crisis and quantitatively account for the adjustment of key macroeconomic and financial variables in Portugal to the debt crisis.[8]

This paper also draws from a recent literature that analyzes bank-sovereign linkages. Merler and Pisani-Ferry (2012) document the repatriation of sovereign debt to domestic banks at European countries hit by the debt crisis.[9] To explain this, Broner et al. (2014) propose a model with creditor discrimination in favour of domestic banks during sovereign default episodes. Acharya et al. (2014a) and Livshits and Schoors (2009) develop models where anticipated bailouts and/or deposit insurance drive banks to risk-shift by purchasing risky domestic sovereign debt. De Marco and Macchiavelli (2016) and Ongena et al. (2016) provide evidence for moral suasion whereby governments in need of funding incentivize or coerce domestic banks to purchase their debt. Brunnermeier et al. (2016) and Farhi and Tirole (2017) analyze the macroeconomic consequences of financial linkages between banks and sovereigns.

The gambling equilibrium in this paper brings to attention an alternative reason for banks to purchase risky domestic sovereign debt: banks may gamble on these bonds because their payoff is positively correlated with their solvency prospects. Gambling of this form may be optimal for banks even in the absence of any anticipated government support or favorable treatment during default; the model indicates that market discipline is insufficient to offset risk-taking incentives

[7]Like Bocola (2016), I treat sovereign default risk as driven by some exogenous latent factor. Abstracting from the government's default decision allows me to focus sharply on the properties of the novel mechanism my model is about.

[8]Section 2 presents motivating evidence from the European sovereign debt crisis and Section 5.4 conducts a quantitative exercise with Portuguese data.

[9]See also Fact 1 in the next section.

when banks are under-capitalized and facing high funding costs due to their inability to credibly commit to a safe portfolio. This finding is based on a lack of regulation preventing banks from gambling on domestic sovereign debt which may be considered as a form of moral suasion. Uhlig (2014) and Crosignani (2015) discuss the optimality of this from the government's perspective.

Finally, the multiplicity mechanism in this paper indicates that policy interventions may have equilibrium-switching effects in addition to the within-equilibrium effects considered in the existing literature. For example, (non-targeted) liquidity provision may cause a switch to the gambling equilibrium, driving banks to increase their exposure to risky domestic sovereign debt. These results are consistent with recent empirical findings. Drechsler et al. (2016) find that lender of last resort loans taken by under-capitalized banks were used for purchases of risky sovereign debt. Crosignani et al. (2016) show that the longer-term refinancing operations (LTRO) conducted by the European Central Bank (ECB) induced Portuguese banks to increase their holdings of risky domestic sovereign bonds. Alter and Schüler (2012) document that sovereign credit default swap (CDS) spreads in Euro area countries became an important determinant of market perceptions about resident banks' solvency prospects during the sovereign debt crisis.

The paper is structured as follows: Section 2 presents motivating evidence from the European sovereign debt crisis. Section 3 demonstrates the main mechanism behind multiple equilibria in a simplified, two-period framework. Section 4 presents the fully dynamic model. Section 5 describes the propagation of sovereign risk shocks and examines the fit of the model to Portuguese data. Section 6 conducts policy analysis. Section 7 concludes.

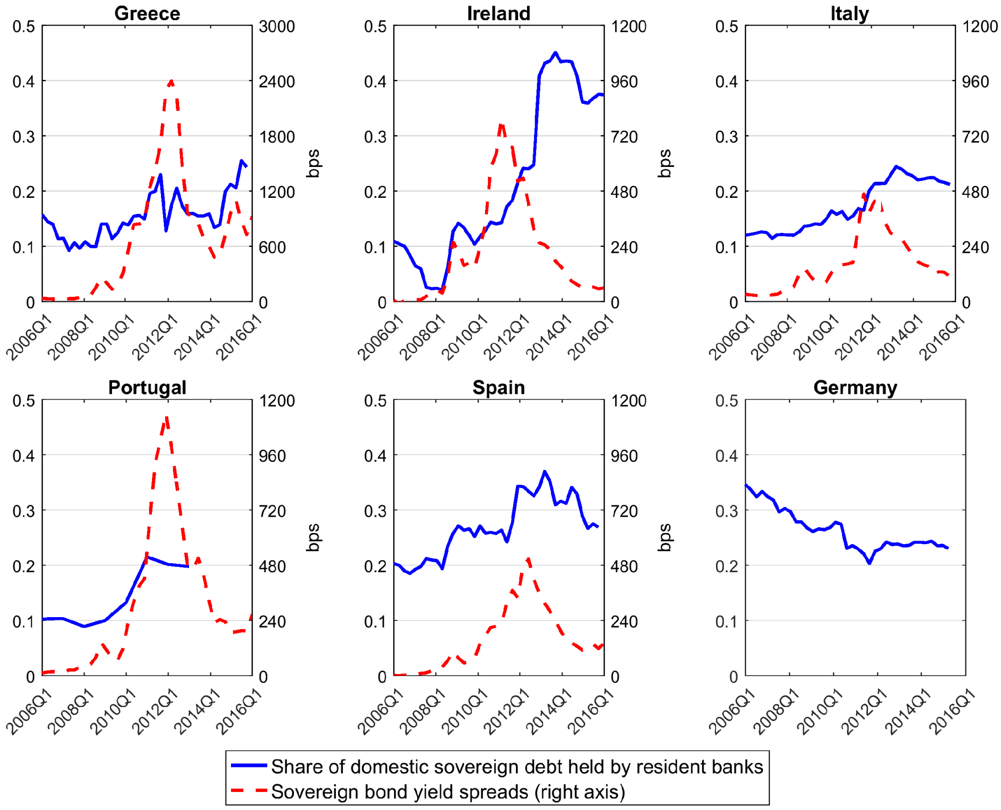
2 Motivating evidence

In this section, I present four key stylized facts about the European sovereign debt crisis. I focus on five countries that were hit by the crisis, Greece, Ireland, Italy, Portugal and Spain (periphery), and contrast them with Germany (core) as a benchmark.

Fact 1. *In the periphery, the share of domestic sovereign debt held by the national banking system has sharply increased.*

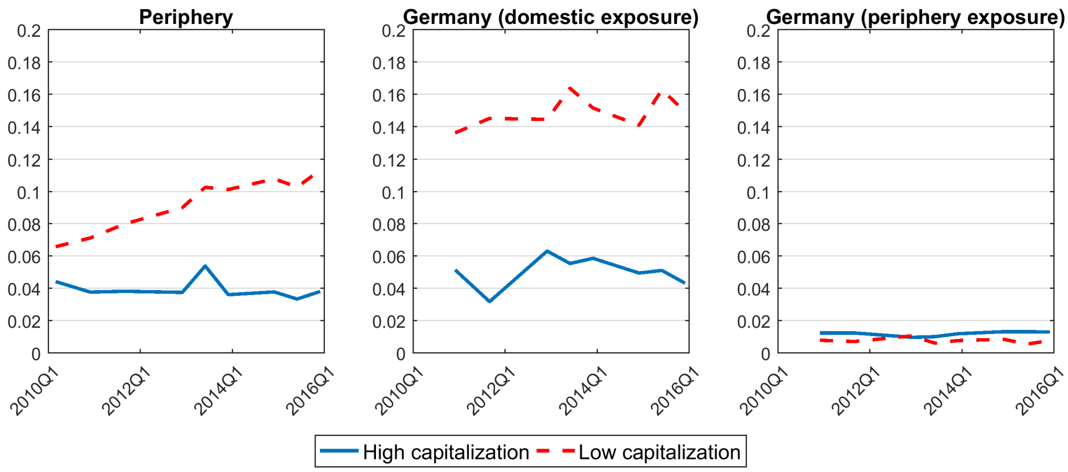
Figure 1 shows that spreads between sovereign bonds issued by the periphery countries and Germany (as a benchmark for safe assets) increase sharply after 2009 and peak in 2012. Thereafter, spreads decrease but settle at a higher level than before the crisis. At the same time, there is an increase in the share of domestic government debt held by banks resident in these countries. In contrast, there is a decrease in the share of German sovereign debt held by German banks.

Figure 1: **Sovereign bond holdings and yield spreads**



Note: Sovereign bond yields refer to bonds with 10 year maturity. Spreads are from German sovereign bond yields. Portuguese data on bond holdings is only available until 2012 and on an annual basis. All other data is quarterly. Source: OECD (MEI) and Merler and Pisani-Ferry (2012).

Figure 2: **Bank capitalization and sovereign exposures**



Note: Sovereign bond exposure refers to the share of sovereign bonds within total assets. No data is available for Greek banks. Low capitalization refers to banks with a Tier 1 Capital ratio below the first quartile in 2009. High capitalization refers to those above the third quartile. Source: Bloomberg and the European Banking Authority.

Fact 2. *Under-capitalized banks in the periphery have increased their exposure to domestic sovereign debt, while the exposures of well-capitalized banks in the periphery and German banks have remained nearly constant.*

The first panel of Figure 2 shows that the average domestic sovereign exposure of under-capitalized banks in the periphery has nearly doubled over 2010-2016, while that of capitalized banks remained near constant. This indicates a negative relationship between bank capitalization and the change in domestic sovereign debt exposures over the debt crisis.[10] The second panel shows that, in contrast to the periphery, domestic sovereign bond exposures of German banks with low and high capitalization do not follow a measurably different pattern over the crisis. This is also true for their exposure to bonds issued by peripheral countries as shown in the last panel. In other words, there is no apparent relationship between bank capitalization and changes in sovereign exposures for banks based in Germany.[11]

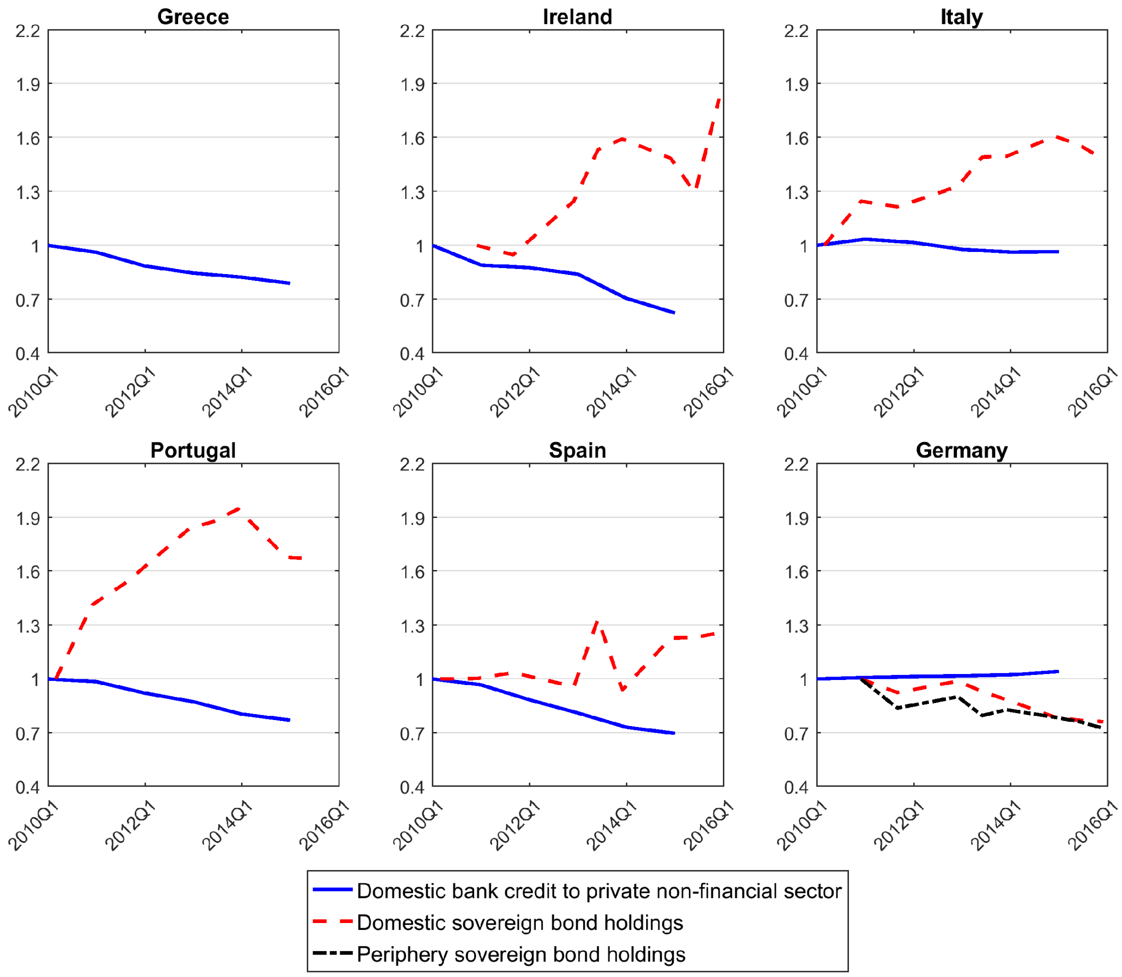
Together, these findings lend support to the view that under-capitalized banks in the periphery are gambling on domestic sovereign bonds. Banks have incentives to do this for three reasons: First, they are protected by limited liability. If the government does not default ex-post, sovereign bonds pay a high return driven by the default-risk premium; if the government imposes a haircut on bond holders, banks are shielded from the full consequences of the default by limited liability. Second, domestic sovereign bonds are aggregate-risky making their return positively correlated with banks' solvency prospects. During domestic default episodes, banks may anticipate default costs that may hit their profits independently of their holdings of sovereign bonds. By way of example, default usually leads to a deterioration in the value of illiquid assets, loss of access to foreign financing needed to roll over debt, and higher taxes. Third, domestic sovereign bonds receive favorable treatment in regulation relative to other risky assets.

Aggregate risk is a key ingredient of this mechanism. Under the regulatory framework present in the Euro area, sovereign bonds issued by all European Union member states carry zero risk-weight in capital regulation (Bank for International Settlements, 2013). Therefore, if limited liability and favorable regulatory treatment were the sole driving factors, under-capitalized German banks would also have an incentive to purchase periphery sovereign debt. This would lead to a negative relationship between bank capitalization and periphery exposure in Germany, which is not observed in Figure 2. In a similar vein, if the increase in domestic sovereign bond holdings were driven by expectation of selective default in favour domestic bond

[10] See Acharya and Steffen (2015) for an empirical analysis. They reach the same conclusion with a regression that controls for bank and country characteristics.

[11] While under-capitalized banks in Germany have a higher exposure to domestic sovereign bonds than their well-capitalized counterparts, their holdings do not increase over the crisis. Since German government bonds were widely considered as a safe asset throughout the sovereign debt crisis, these holdings may be due to their use as collateral or regulatory requirements.

Figure 3: **Bank lending**



Note: Sovereign bond holdings are attained using data from EU-wide stress tests and transparency exercises. There is no data available for Greek banks. Domestic bank credit to private non-financial sector refers to financial resources provided to the private non-financial sector by domestic banks that establish a claim for repayment. Source: World Bank and the European Banking Authority.

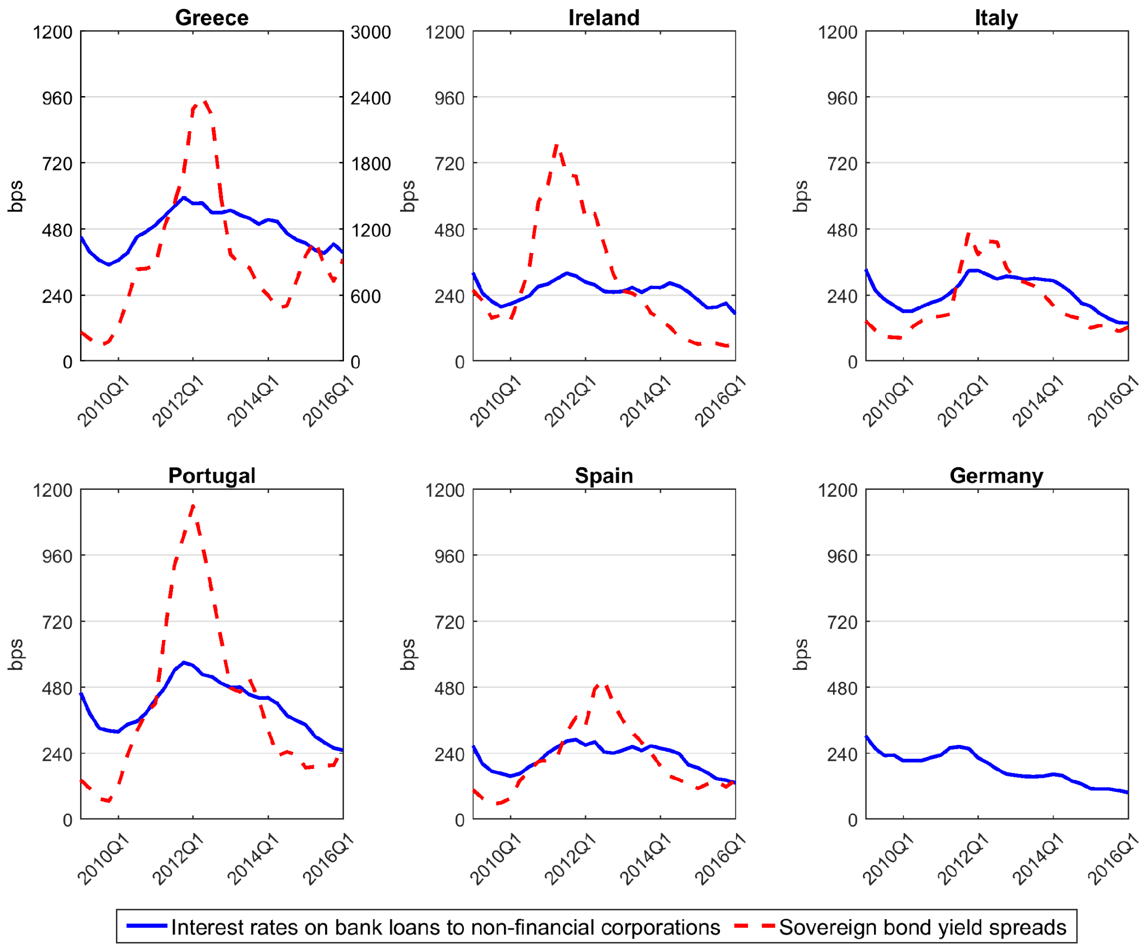
holders, we would see an increase in domestic sovereign exposure of periphery banks regardless of their capitalization. This is also not observed in Figure 2.[12]

Fact 3. *In the periphery, banks reduced their lending to the private non-financial sector while increasing their domestic sovereign bond holdings. At the same time, there was a rise in private borrowing costs.*

Figure 3 shows that the volume of domestic sovereign bonds held by the national banking sector has increased by varying degrees in the periphery, ranging from about 30% in Spain to

[12]The patterns in Figure 2 are compatible with moral suasion under the condition that risky governments can exert greater pressure on under-capitalized banks to purchase domestic sovereign debt. Note, however, that the gambling and moral suasion channels are not mutually exclusive. In fact, gambling relies on moral suasion in the sense that the government neglects to regulate against the domestic sovereign exposure of local banks.

Figure 4: **Loan interest rates**



Note: Loan interest rates refer to loans of all amounts by domestic banks to non-financial corporations (new business). Source: ECB.

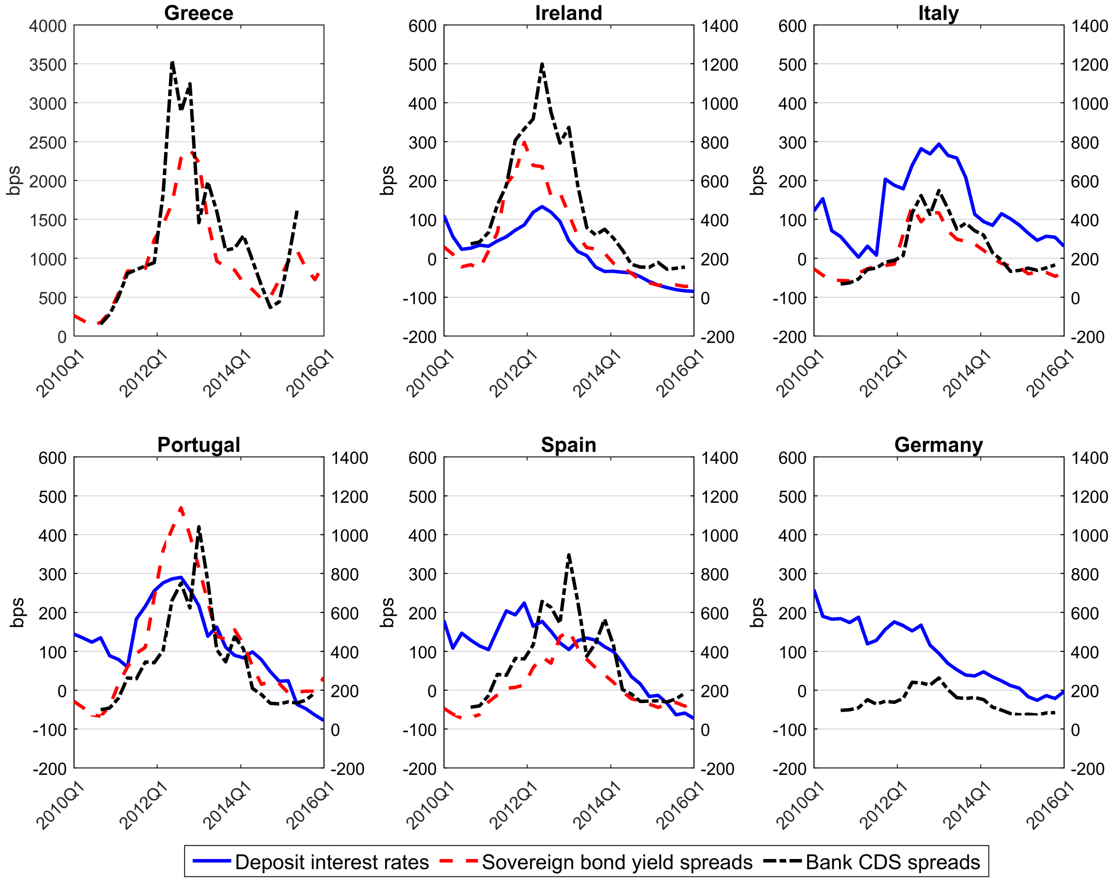
nearly double its initial amount in Ireland and Portugal. At the same time, credit to the private sector by domestic banks decreased by up to 30% in each periphery country except for Italy where it was stagnant. Figure 4 shows that interest rates on loans to non-financial corporations also increased at the peak of the debt crisis in 2011-2012, especially in Portugal and Greece.

In Germany, on the other hand, banks reduced their holdings of both domestic and periphery sovereign bonds, and slightly increased their lending to the private sector. At the same time, there was a significant improvement in borrowing conditions faced by private non-financial corporations, with a decline of over 200 basis points in loan interest rates between 2010-2016.

The patterns in Figures 3 and 4 are consistent with the crowding out of bank lending by domestic sovereign bond purchases.[13]

[13]For further empirical evidence on the effects of the sovereign debt crisis on credit to the private sector, see Acharya et al. (2014b), Becker and Ivashina (2014), De Marco (2017) and Popov and Van Horen (2015).

Figure 5: **Bank funding costs**



Note: The left axis represents deposit interest rates and the right axis represents bank CDS and sovereign bond yield spreads. Both axes are in basis points. Deposit interest rates refer to time deposits of all agreed maturities and amounts (new business). Bank CDS spreads refer to the implied CDS spread measure in Bloomberg. There is no available deposit interest rate data for Greece. Source: Bloomberg, ECB, OECD.

Fact 4. *There is substantial co-movement between sovereign bond yield spreads and bank funding costs in the periphery.*

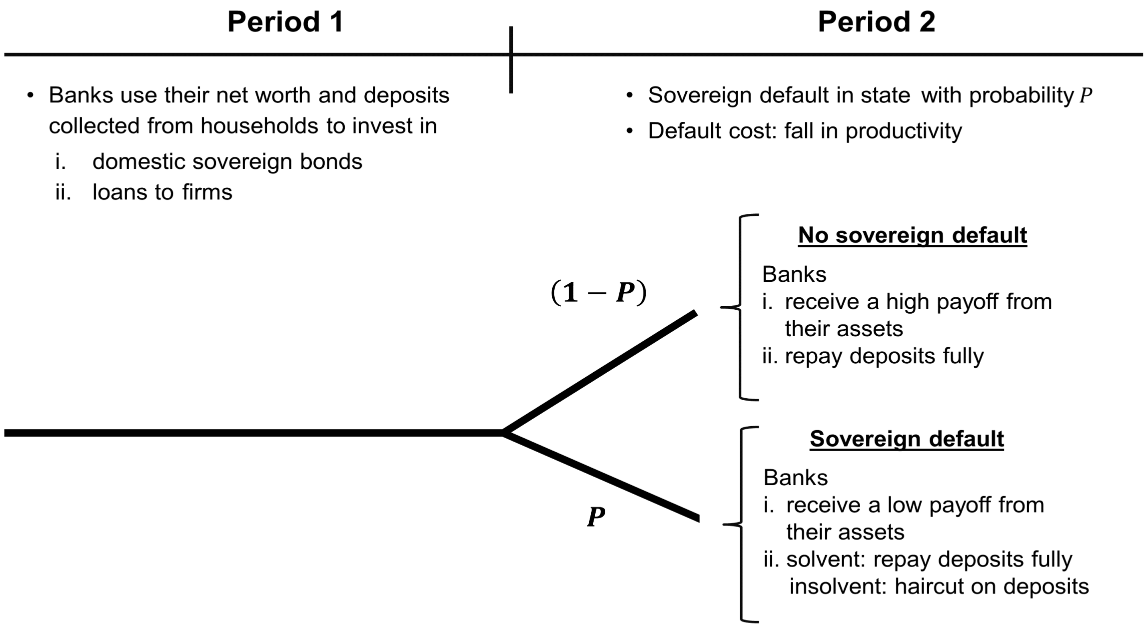
Figure 5 plots bank CDS spreads and deposit interest rates against sovereign bond yield spreads and Table 1 reports the corresponding correlation coefficients. The CDS spreads co-move significantly with sovereign spreads in the periphery, consistent with the notion that solvency prospects of the government and the banking sector are intertwined.[14] To a lesser extent, deposit interest rates also move with yield spreads, especially during the peak of the crisis in 2011-2012. A potential explanation for this is that depositors expect a decline in the real value of their deposits in the event that the banking sector and government are both in default.

[14] Acharya et al. (2014a) show that changes in sovereign CDS explain changes in bank CDS even after controlling for aggregate and bank-level determinants of credit spreads.

Table 1: **Correlation with sovereign bond yield spreads over 2010-2015**

	Greece	Ireland	Italy	Portugal	Spain
Bank CDS spreads	0.85	0.93	0.93	0.85	0.93
Deposit interest rates	–	0.84	0.84	0.74	0.37

Figure 6: **Timeline**



In the next section, I present a simple, two-period model where gambling on aggregate-risky domestic sovereign debt may arise as an equilibrium outcome when banks are under-capitalized. In this gambling equilibrium, bank lending is crowded out by domestic sovereign bond purchases and bank funding costs co-move with domestic sovereign bond yields, consistent with the stylized facts described here.

3 A two period model

I consider a stylized model of small open financial economy with three private agents: households, banks and firms, and a government issuing default-risky debt. Events unfold over two time periods (see Figure 6 for a graphical timeline). In the first period, banks collect deposits from households and use these funds, along with their own net worth, for domestic sovereign bonds purchases and working capital lending to firms, which in turn produce the consumption good. In the second period, sovereign default occurs if fundamentals turn out to be weak with (exogenous) probability P.

Borensztein and Panizza (2009) and Reinhart and Rogoff (2009) find that sovereign defaults are often accompanied with banking crises while Yeyati and Panizza (2011) attribute a large portion of the output costs of default to anticipation effects that precede the default event itself. Motivated by this empirical evidence, I focus on the financial interactions that take place under sovereign default risk and abstain from an explicit treatment of the processes that drive governments to default on their debt, which may include a range of economic and political factors.[15]

Sovereign default reduces the productivity of firms. As a result, banks receive a low return from their lending to firms as well as their domestic sovereign bond holdings under sovereign default. This reflects the costs of domestic sovereign default on bank balance sheets, which hit them independently of their sovereign bond holdings.[16] If banks are left with insufficient funds to pay the promised return to their depositors, they become insolvent under limited liability and a haircut proportionate to their funding shortfall is imposed on deposits.[17]

Banks' solvency prospects in the event of sovereign default are determined by the strategy their managers adopt in the first period. The 'safe strategy' consists of investing in a precautionary manner that leaves them solvent after sovereign default, whereas the 'gambling strategy' leads to insolvency. Bank managers find it optimal to follow the strategy that maximizes their expected payoff.

A key friction in the model is the non-contractibility of banks' investment decisions. Specifically, households may not make their deposits contingent on banks' exposures to domestic sovereign bonds.[18] This may be due to information asymmetries whereby banks may obscure their portfolio exposures through the use of shell corporations and complex financial instruments, or limited enforcement on behalf of households combined with bank managers' ability to change portfolio allocations ex-post. In either case, non-contractibility brings about a two-way relationship between the optimal strategies of bank managers and households. When households anticipate that banks follow a gambling strategy, their optimal deposit schedule changes in a manner that increases banks' incentives to gamble. Household expectations about bank risk-taking may then become self-fulfilling.

Finally, before I explain these activities in more detail, it is convenient to describe some notational conventions. Table 2 provides a list of variables and parameters. Deposits, sovereign bonds, loans and safe assets are respectively labelled as (d, b, l, d^*) and take the form of discount

[15] See also Broner et al. (2014), Bocola (2016) and Brunnermeier et al. (2016) for other studies which analyse the financial effects of sovereign default without explicitly modelling the causes thereof.

[16] For other studies which rely on output costs of default, see e.g. Cole and Kehoe (2000), Arellano (2008) and Aguiar et al. (2015).

[17] The absence of risk-free assets among banks' investment opportunities serves only to simplify the exposition. Their inclusion would be completely inconsequential in this set up as purchasing a safe asset is either equivalent to or less profitable than a reduction in deposits by the same amount.

[18] In other words, the contracting space between households and banks is limited to time deposits.

bonds with prices (q, q^b, q^l, q^*).[19] The recovery rates of (d, b, l) under sovereign default are $(\theta, \theta^b, \theta^l)$. An underbar denotes variables at the state with sovereign default such that $\underline{A}$ is productivity under sovereign default. Aggregate quantities, such as aggregate loans L, are in the upper case while lower case variables pertain to an individual bank.

Table 2: **Notation**

Variables

Label	Description
d	Deposits
b	Domestic sovereign bonds
l	Loans to firms
d^*	Safe assets
q, q^l, q^b, q^*	Asset prices
$\theta, \theta^l, \theta^b$	Recovery rates
H	Labour supply
w	Wages
K	Working capital
Y	Output
n	Bank net worth
π	Bank profits
v	Bank expected payoff
γ	Sovereign bond exposure
c	Consumption
μ_l	Loans market mark-up
μ_d	Deposit market mark-up

Parameters

Label	Description
P	Probability of sovereign default
ν	Market share of banks
A	Productivity
α	Cobb-Douglas elasticity
β	Discount factor
$\bar{E}$	Household endowment

3.1 Agents and their optimal strategies

3.1.1 Government

In the first period, the government issues discount bonds b at a price q^b. Sovereign bonds are internationally traded and their marginal buyers are deep pocketed foreign investors. As such, they are priced at their expected return

$$q^b = \left(1 - P + P\theta^b\right) q^* \tag{1}$$

[19] This helps simplify the exposition without any actual impact on the model mechanisms.

where $\theta^b \in (0,1)$ is their recovery rate and q^* is the price of an international safe asset d^* with perfectly elastic supply. In a monetary union setting, $1/q^*$ can be interpreted as the interest rate set by the common central bank.

3.1.2 Firms

Firms are perfectly competitive. In order to produce the consumption good Y, they hire labour H from households at a wage w and borrow working capital

$$K = q^l L \tag{2}$$

from the domestic banking sector. In the interest of a clear exposition, loans to firms take the form of discount bonds L sold at a price q^l. Under a standard Cobb-Douglas production function, the representative firm's profit maximization problem is

$$\max_{K,L,H,\underline{H}} \quad (1-P)\left[AK^\alpha H^{1-\alpha} - L - wH\right] + P\left[\underline{A}K^\alpha \underline{H}^{1-\alpha} - \theta^l L - \underline{wH}\right]$$

subject to (2), where A is productivity and θ^l is the recovery rate of loans. Crucially, $\left(q^l, L, K\right)$ are not state contingent as firms borrow in advance. When the government defaults, loans become non-performing due to the productivity decline $\underline{A} < A$ and banks claim the firm's revenues net of salary payments such that[20]

$$\theta^l = \frac{\underline{A}K^\alpha \underline{H}^{1-\alpha} - \underline{wH}}{L}$$

Combining this with the first order conditions of the firm's problem yields the expressions

$$\begin{aligned} w &= (1-\alpha)\, AK^\alpha \\ \underline{w} &= (1-\alpha)\, \underline{A}K^\alpha \\ q^l &= \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} L^{\frac{1-\alpha}{\alpha}} \\ \theta^l &= \frac{\underline{A}}{A} \end{aligned} \tag{3}$$

[20]This is the reduced-form outcome of a re-negotiation game between firms and banks after loans become non-performing. As firms are perfectly competitive and banks have market power, the latter extracts all of the remaining revenues after salary payments. Implicitly, this relies on the absence of information asymmetries, which can be motivated by relationship banking. This also makes it prohibitively costly for households and foreign entities to lend directly to firms. The domestic banking sector thus acts as a financial intermediary that channels funds to firms. Note that the outcome here is equivalent to the issuance of state-contingent debt by firms.

where labour supply is perfectly inelastic and normalized to $H = \underline{H} = 1$. Of particular importance are the last two expressions, which respectively establish an upward-sloping loan supply schedule and pin down the recovery rate.

3.1.3 Households

There is a unit continuum of risk neutral households with an initial endowment $\bar{E}$. They save by purchasing risk-free assets D^* at a price q^* or deposits D from domestic banks at a price q.[21,22] The representative household's utility maximization problem can be described as follows

$$\max_{c_1, c_2, \underline{c}_2, D, D^*} u(c_1) + \beta\left[(1-P)\, u(c_2) + P u(\underline{c}_2)\right]$$

subject to the period budget constraints

$$\begin{aligned} c_1 + qD + q^* D^* &= \bar{E} \\ c_2 &= D + D^* + w \\ \underline{c}_2 &= \theta D + D^* + \underline{w} \end{aligned}$$

where β is the rate at which households discount future consumption and θ is the recovery rate of domestic bank deposits under sovereign default. This yields the first order conditions

$$q^* = \beta \tag{4}$$

$$q = (1 - P + P\theta)\, q^* \tag{5}$$

which indicate that domestic deposits are priced at their expected return relative to the safe asset. Observe that households' valuation of domestic deposits increases in recovery rate θ. I provide an expression for θ in the next section before deriving the optimal deposit demand schedule of households in section 3.1.5.

3.1.4 Banks

The domestic banking sector is imperfectly competitive in the manner of Cournot. Each bank is risk neutral with a market share $\nu \in (0, 1]$. The representative bank finances its domestic sovereign bond purchases and lending to firms with deposits collected from households as well

[21] The assumption of risk neutrality only serves to attain a tractable expression for the deposit demand schedule. The results presented below retain their validity under risk aversion, which is introduced in section 4.

[22] D^* can be interpreted as deposits in a safe foreign bank or simply as a safe real asset. As there is a unit continuum of homogenous households, individual households' deposits are identical to the aggregate quantities. I abuse notation by using the aggregate terms (D, D^*) to describe the household's problem.

as its own net worth $n \geq 0$. Its budget constraint can be written as

$$n + qd = q^b b + q^l l \tag{6}$$

where $l = \nu L$, $d = \nu D$ represent lending and deposits at individual bank level. Profits are contingent on sovereign default as follows

$$\begin{aligned} \pi &= \max\{0, l + b - d\} && (7) \\ \underline{\pi} &= \max\left\{0, \theta^l l + \theta^b b - d\right\} && (8) \end{aligned}$$

where $\underline{\pi}$ represents profits in the event of sovereign default, and the maximum operators reflect limited liability. Banks always make a strictly positive profit under strong fundamentals ($\pi > 0$) but may become reliant on limited liability after sovereign default. This leads to insolvency, with losses passed on to depositors through a haircut on deposits. The recovery rate of deposits reflects the bank's shortfall of funds[23]

$$\theta = \min\left\{1, \frac{\theta^l l + \theta^b b}{d}\right\} \tag{9}$$

with $\theta < 1$ indicating that limited liability binds.

The representative bank chooses its deposits d, domestic sovereign bond purchases b and loans l in order to maximize its expected payoff

$$v = (1 - P)\,\pi + P\underline{\pi}$$

subject to the budget constraint. Note that choosing (b, l) is equivalent to selecting the share of funds $\gamma \in [0, 1]$ spent on domestic sovereign bonds purchases. Using (6), (b, l) can be defined in terms of γ as

$$\begin{aligned} b &= \gamma\left(\frac{n + qd}{q^b}\right) && (10) \\ l &= (1 - \gamma)\left(\frac{n + qd}{q^l}\right) && (11) \end{aligned}$$

It is convenient for the remainder of the text to express the recovery rate θ in terms of

[23] There is no deposit insurance or bailot guarantees in the baseline model. These are evaluated as policy interventions in section 6.

sovereign exposure γ

$$\theta = \begin{cases} 1 & \text{for } d \leq \bar{d}(\gamma) \\ \left(\gamma \frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right)\left(\frac{n}{d} + q\right) & \text{for } d > \bar{d}(\gamma) \end{cases} \tag{12}$$

$$\bar{d}(\gamma) = \frac{\left(\gamma \frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right) n}{1 - q^* \left(\gamma \frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right)} \tag{13}$$

where $\bar{d}(\gamma)$ represents the threshold of deposits above which the bank becomes insolvent following sovereign default.[24] Observe that $\bar{d}(\gamma)$ and θ are positively related to bank net worth n and the rate of return $\gamma \frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}$ on bank funds.

Recall from the previous section that the price of deposits q increases in θ. Under imperfect competition, banks internalize the effects of their actions on θ and hence q. As such, it is necessary to determine the household's optimal deposit demand schedule in the next section before evaluating bank strategies in section 3.1.6.

3.1.5 Deposit demand schedule

Combining (5) with (12) yields the household's optimal deposit demand schedule contingent on γ

$$q(\gamma, d) = \begin{cases} q^* & \text{for } d \leq \bar{d}(\gamma) \\ q^* \frac{1 - P + P\left(\gamma \frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right)\frac{n}{d}}{1 - q^* P\left(\gamma \frac{\theta^b}{q^b} + (1-\gamma)\frac{\theta^l}{q^l}\right)} & \text{for } d > \bar{d}(\gamma) \end{cases} \tag{14}$$

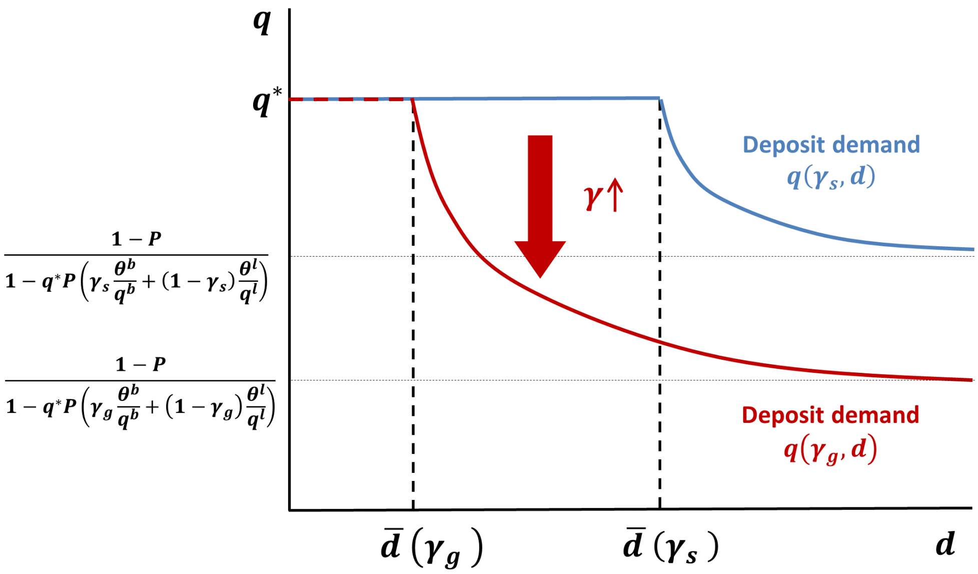
where $\bar{d}(\gamma)$ is defined by (13). The deposit demand schedule is downward sloping and negatively related to γ under the parameter restrictions

$$\frac{\alpha(1-P)}{\alpha(1-P) + \nu(1-\alpha)} > \frac{\underline{A}}{A} > \frac{\alpha\theta^b}{\alpha + \nu(1-\alpha)} \tag{15}$$

These restrictions ensure that in the event of sovereign default, the rate of return from lending to firms falls short of the promised return on deposits but exceeds that of domestic sovereign bond purchases. When the first inequality is satisfied, the bank becomes insolvent after sovereign default when $d > \bar{d}(\gamma)$ and the deposit demand schedule is downward sloping in this region. Therefore, I refer to $d > \bar{d}(\gamma)$ as the 'risky' region of the deposit demand schedule and $d \leq \bar{d}(\gamma)$ as the 'safe region'. In the safe region, deposits are deemed to be risk-free with $\theta = 1$ by households and priced on par with safe assets $q = q^*$. Conversely, in the risky region, households price deposits at a discount $q < q^*$ in anticipation of a haircut following sovereign

[24] This can also be interpreted as a leverage threshold $\bar{d}(\gamma)/n$. The claim that $\theta < 1$ for $d > \bar{d}(\gamma)$ is valid under the parameter restrictions discussed in the next section.

Figure 7: **Deposit demand schedule**



default ($\theta < 1$). At the limit $d \to \infty$, the recovery rate tends to the rate of return on bank funds and the value of deposits approaches the lower bound

$$\lim_{d\to\infty} q(\gamma, d) = q^* \frac{1-P}{1-q^*P\left(\gamma\frac{\theta^b}{q^b}+(1-\gamma)\frac{\theta^l}{q^l}\right)}$$

The second inequality in (15) establishes a negative relationship between the sovereign bond exposure γ and the rate of return on bank funds. This ensures that the deposit threshold $\bar{d}(\gamma)$ shifts inwards in response to a rise in γ, while the risky region of the deposit demand schedule pivots downward. Figure 7 shows the effect of a rise in sovereign exposure from an arbitrary level γ_s to $\gamma_g > \gamma_s$ on the deposit demand schedule.

Along with the parameter restrictions, a necessary assumption to attain the results described below is that sovereign exposure γ is non-contractible such that banks may not commit to a certain level of exposure. When γ is contractible, bank managers internalize the negative relationship between sovereign exposures and their funding conditions. Lemma 1 shows that this imposes market discipline and deters banks from gambling on domestic sovereign bonds.

Lemma 1 *When households and banks can specify γ in a contract for deposits, limited liability has no impact on banks' optimal strategy.*

Proof. Provided in the Technical Appendix.[25] ■

I elaborate further on the formation of household expectations on γ in section 3.2.2. This discussion builds upon optimal bank strategies, however, which necessitates their explanation in

[25] The Technical Appendix is available online at https://sites.google.com/site/anlari/files/Technical_Appendix

advance. In the meantime, both the deposit demand schedule and the bank strategies described in the next section should be taken to be contingent on household expectations about sovereign exposure, which I label as $\tilde{\gamma}$. Lacking commitment, banks take $\tilde{\gamma}$ as given and do not internalize the impact of their sovereign exposure on the deposit demand schedule $q(\tilde{\gamma}, d)$ facing them.

3.1.6 Bank strategies

Limited liability creates a discontinuity in the representative bank's optimal strategy such that it can be evaluated as a choice between two distinct strategies. Under a 'safe strategy' (labelled as 's'), the bank satisfies a solvency constraint

$$d \leq \theta^l l + \theta^b b \tag{16}$$

which ensures that it does not rely on limited liability after sovereign default. The 'gambling strategy' (labelled as 'g'), on the other hand, results in the bank's insolvency and the imposition of a haircut on deposits after sovereign default.

In the first period, the representative bank adopts the strategy that maximizes its expected payoff such that the safe strategy is preferred when

$$v_s \geq v_g$$

where (v_s, v_g) are respectively the expected payoffs associated with safe and gambling strategies.

Gambling strategy When the bank follows the gambling strategy, it solves the problem

$$\begin{aligned} v_g &= \max_{d,\gamma\in[0,1]} (1-P)(l+b-d) \\ &\quad \text{s.t.} \\ n + qd &= q^b b + q^l l \end{aligned} \tag{17}$$

where (10) and (11) map the choice of γ into (l, b). Since limited liability binds after sovereign default, the bank only internalizes the payoff in the state with strong fundamentals. It also internalizes the deposit demand and loan supply schedules

$$q \equiv q(\tilde{\gamma}, d) \tag{18}$$

$$q^l = \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} (l + (1-\nu) L)^{\frac{1-\alpha}{\alpha}} \tag{19}$$

given by (14) and (3) due to imperfect competition.[26]

The first order conditions can then be written as

$$q^b = (1 - \mu_d(\tilde{\gamma}, d))\, q \tag{20}$$
$$q^l = (1 - \mu_l)\, q^b \tag{21}$$

where $\mu_d(\tilde{\gamma}, d)$ and μ_l are the mark-ups the bank enjoys in the deposit and loan markets due to its market power. They are defined as[27]

$$\mu_d(\tilde{\gamma}, d) \equiv -\frac{\partial q(\tilde{\gamma}, d)}{\partial d}\frac{d}{q} = \begin{cases} 0 & \text{for } d \leq \bar{d}(\tilde{\gamma}) \\ \frac{P\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)\frac{n}{d}}{1-P+P\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)\frac{n}{d}} & \text{for } d > \bar{d}(\tilde{\gamma}) \end{cases} \tag{22}$$

$$\mu_l \equiv \frac{\nu(1-\alpha)}{\alpha + \nu(1-\alpha)} \tag{23}$$

Observe that the recovery rates (θ^b, θ^l) do not feature in the first order conditions, since the bank does not internalize its payoff under sovereign default. I elaborate further on the consequences of this while considering the gambling equilibrium in section 3.2.1.

Safe strategy Under the safe strategy, the bank's problem differs from its gambling counterpart in two respects. First, as the bank does not rely on limited liability, the objective function internalizes the payoff in both states of nature such that

$$\begin{aligned} v_s &= \max_{d,\gamma\in[0,1]} (1-P)\,\pi + P\underline{\pi} \\ &= \max_{d,\gamma\in[0,1]} (1-P)(l+b) + P\left(\theta^l l + \theta^b b\right) - d \end{aligned}$$

Second, this is subject to an occasionally binding solvency constraint given by (16) in addition to the budget constraint. The first order conditions for the safe strategy can then be written

[26] (19) differs slightly from (3) as it is from the perspective of an individual bank. L represents aggregate bank lending which is taken as given by the representative bank.

[27] Observe that there is no deposit market mark-up in the safe region of the deposit demand schedule. This is because banks face a horizontal deposit demand schedule in this region as their deposits become perfectly substitutable with safe assets.

as

$$\left(\theta^l l + \theta^b b - d\right)\lambda = 0 \ , \ \lambda \geq 0 \ , \ d \leq \theta^l l + \theta^b b \tag{24}$$

$$q^b \geq \frac{\left(1 - P + P\theta^b\right) + \lambda\theta^b}{1+\lambda}\left(1 - \mu_d\left(\tilde{\gamma}, d\right)\right) q \tag{25}$$

$$q^l = \frac{\left(1 - P + P\theta^l\right) + \lambda\theta^l}{1+\lambda}\left(1 - \mu_l\right)\left(1 - \mu_d\left(\tilde{\gamma}, d\right)\right) q \tag{26}$$

where λ is the Lagrange multiplier for the solvency constraint and (24) is the corresponding complementary slackness condition. Compared to the gambling case, the bank has a lower valuation for both b and l since it internalizes the low payoff from these assets in the state with sovereign default. When $\theta^l > \theta^b$, however, greater value is placed on loans compared to domestic sovereign bonds relative to the gambling case. Both of these effects are amplified when the solvency constraint is binding such that $\lambda > 0$.

The weak inequality in (25) reflects the possibility that the bank may prefer not to purchase any domestic sovereign bonds ($\gamma = 0$), since the sovereign bond price is fixed at $q^b = \left(1 - P + P\theta^b\right) q^*$ as explained in section 3.1.1.[28] Lemma 2 describes the conditions under which (25) holds with equality.

Lemma 2 *When $\lambda = 0$ and $q = q^*$, condition (25) holds with equality and reduces to*

$$q^b = \left(1 - P + P\theta^b\right) q^* \tag{27}$$

and there is an interior solution for b within the range

$$b \in \left[0, \frac{q^* \bar{d}\left(\tilde{\gamma}\right) + n - q^l l}{q^b}\right] \tag{28}$$

Otherwise, there is a strict inequality and a corner solution

$$q^b > \frac{\left(1 - P + P\theta^b\right) + \lambda\theta^b}{1+\lambda}\left(1 - \mu_d\left(\tilde{\gamma}, d\right)\right) q$$
$$b = 0$$

Proof. Provided in the Technical Appendix. ■

This indicates that the bank only purchases a positive amount of sovereign bonds $b > 0$ when the solvency constraint is slack with $\lambda = 0$ and bank deposits are at the safe region of the

[28] Implicitly, this is a complementary slackness condition for an occassionally binding non-negativity constraint $b \geq 0$. This constraint never binds under the gambling strategy due to the higher valuation of domestic sovereign bonds. An equivalent constraint for lending ($l \geq 0$) is also slack at all times since q^l declines in response to a fall in l.

deposit demand schedule such that $q = q^*$. In this case, (27) shows that the bank's valuation of sovereign bonds is at their expected payoff, which is equivalent to their market price given by (1). The bank is thus indifferent to the amount of its domestic sovereign bond purchases within the range (28). When the solvency constraint binds ($\lambda > 0$) and/or bank deposits are considered to be risky ($q < q^*$), on the other hand, the bank does not purchase any domestic sovereign bonds.

In the next section, I characterize two candidate equilibria and determine the conditions under which they are self-confirming.

3.2 Equilibrium

I solve for a symmetric rational expectations equilibrium which requires that all optimality conditions and constraints of banks, firms and households are satisfied, and household expectations on sovereign exposure $\tilde{\gamma}$ are confirmed in the equilibrium.[29] Section 3.2.1 characterizes the candidate equilibria. Section 3.2.2 describes how households formulate their expectations $\tilde{\gamma}$. Section 3.2.3 provides the equilibrium conditions as well as an intuitive demonstration of the mechanism behind multiple equilibria. Finally, section 3.2.4 formally characterizes the equilibrium regions.

3.2.1 Candidate equilibria

In a rational expectations framework, two candidate equilibria emerge: a 'gambling equilibrium' where household expectations of high exposure to domestic sovereign bonds in the banking sector is confirmed by the adoption of a gambling strategy by banks, and a 'safe equilibrium' where the opposite is true. With a slight abuse of notation, I use the labels 'g' and 's' to refer to variables pertaining to the gambling and safe equilibria.

Gambling equilibrium Under the gambling equilibrium, banks follow the first order conditions (20) and (21). The sovereign exposure γ_g, which must be consistent with household expectations $\tilde{\gamma}$, is determined by combining (20) with the deposit demand schedule (14). This yields

$$\begin{aligned} \gamma_g &\rightarrow 1 \\ q_g &= q^b \end{aligned} \tag{29}$$

[29] I abstain from mixed equilibria, as this would complicate the model solution significantly without yielding any interesting insights in addition to those provided by analyzing symmetric equilibria. Note also that the candidate equilibria described here, and the conditions under which they are valid, would remain unchanged even when mixed equilibria are taken into account.

where the main takeaway is the co-movement between the value of deposits q_g and sovereign bond prices q^b. Note that the corner solution is due to the risk neutrality of households. In section 4, I show that risk aversion leads to an interior solution $\gamma_g \in (0,1)$, $q_g \in \left(q^b, q^*\right)$ while preserving the co-movement property.[30]

The second condition (21) pins down the price and quantity of loans purchased by the representative bank as

$$
\begin{aligned}
q_g^l &= (1-\mu_l)\, q^b && (30)\\
l_g &= \upsilon\, (\alpha A)^{\frac{1}{1-\alpha}}\, q_g^{l\,\frac{\alpha}{1-\alpha}} && (31)
\end{aligned}
$$

where aggregate loans is given by $L_g = l_g/\nu$. Since the bank only internalizes asset payoffs in the state with no sovereign default, a rise in sovereign default probability P (which reduces q^b) leads to a decline in bank lending. This reflects the crowding out of bank lending by domestic sovereign bond purchases.

Finally, the expected payoff of banks under the gambling equilibrium is given by

$$
v_g = (1-P)\,\mu_l l_g + \frac{n}{q^*} \tag{32}
$$

where the first term reflects the mark-up from lending and the second term is the expected return on the banks' initial net worth.

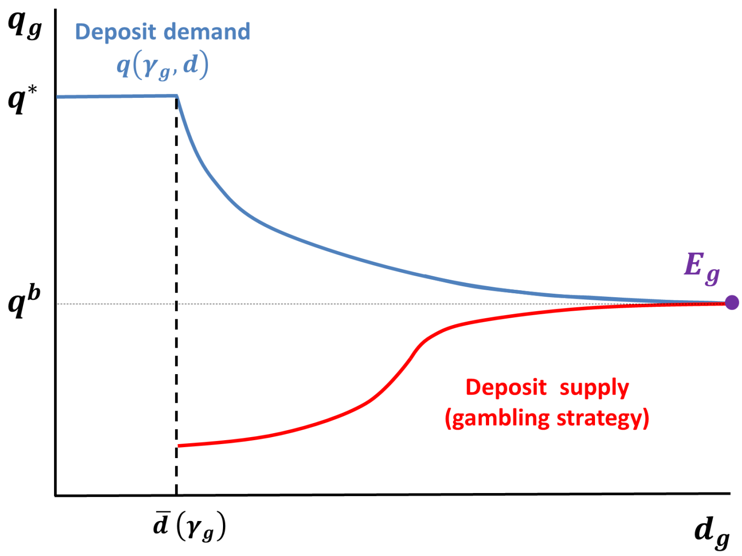
Figure 8 provides a graphical depiction of the gambling equilibrium, where the red line represents the bank's optimal deposit supply schedule under a gambling strategy and E_g marks the equilibrium allocation.[31]

Safe Equilibrium Under the safe equilibrium, the deposit threshold $\bar{d}\,(\gamma_s)$ coincides with the solvency constraint (16) such that banks always remain within the safe region of the deposit

[30]Under risk neutrality, bank deposits are priced at their expected value and the curvature of the deposit demand schedule is such that the mark-up $\mu_d\,(\tilde{\gamma}, d)$ tends to zero as deposits increase. Therefore, under a gambling strategy, banks find it profitable to issue more deposits and use the funds to purchase domestic sovereign bonds until their anticipated exposure is $\gamma_g = 1$.

[31]Observe that the rate of change in the deposit supply schedule changes direction. This occurs at $q_g = q_g^l / \left[(1-\mu_l)\,(1-\mu_d\,(\tilde{\gamma}, d))\right]$. Until this point, the bank invests only in lending to firms. By virtue of diminishing returns to scale in the production function, q^l increases at an increasing rate and so does the deposit supply schedule. Beyond this point, however, the bank invests additional funds in domestic sovereign bonds and the deposit supply schedule is guided by (20). The relationship between $\mu_d\,(\tilde{\gamma}, d)$ and d then gives the schedule a positive, but decreasing rate of change that tends to zero at $q_g = q^b$.

Figure 8: **Gambling equilibrium**



Note: The deposit supply curve is attained by combining (19)-(22). Deposit demand stems from the combination of (14) and (29).

demand schedule with $q_s = q^*$. The first order conditions can then be written as

$$q^b \geq \frac{\left(1 - P + P\theta^b\right) + \lambda\theta^b}{1+\lambda} q^* \tag{33}$$

$$q^l = \frac{\left(1 - P + P\theta^l\right) + \lambda\theta^l}{1+\lambda} \left(1 - \mu_l\right) q^* \tag{34}$$

It follows from Lemma 2 that there are two possible cases of the safe equilibrium, one where the solvency constraint is slack and another where it binds. Lemma 3 characterizes the safe equilibrium under both of these cases.

Lemma 3 *There are two cases of the safe equilibrium*

Case 1 *When $n \geq n_c \equiv \left(q_s^l - q^*\theta^l\right) l_s$, the solvency constraint is slack $(\lambda = 0)$ and (33) holds* *with equality. The safe equilibrium is then characterized by*[32]

$$
\begin{aligned}
q_s^l &= \left(1 - P + P\theta^l\right)\left(1 - \mu_l\right) q^* && (35) \\
l_s &= \nu \left(\alpha A\right)^{\frac{1}{1-\alpha}} q_s^{l\,\frac{\alpha}{1-\alpha}} && (36) \\
b_s &\in \left[0, \frac{n - \left(q_s^l - q^*\theta^l\right) l_s}{q^b - q^*\theta^b}\right] && (37) \\
d_s &= \frac{q^b b_s + q^l l_s - n}{q^*} \\
\gamma_s &= \frac{q^b b_s}{q^* d_s + n} && (38) \\
v_s &= \left(1 - P + P\theta^l\right)\mu_l l_s + \frac{n}{q^*} && (39)
\end{aligned}
$$

Case 2 *When $n < n_c$, the solvency constraint binds $(\lambda > 0)$ and the safe equilibrium is characterized by*

$$
\begin{aligned}
q^*\theta^l l_s &= \left(\frac{1}{\nu}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{l_s}{\alpha A}\right)^{\frac{1}{\alpha}} - n && (40) \\
q_s^l &= \left(\frac{1}{\alpha A}\right)^{\frac{1}{\alpha}} \left(\frac{l_s}{\nu}\right)^{\frac{1-\alpha}{\alpha}} \\
b_s &= \gamma_s = 0 && (41) \\
d_s &= \theta^l l_s \\
v_s &= (1 - P)\left(1 - \theta^l\right) l_s && (42)
\end{aligned}
$$

where the parameter restrictions (15) are sufficient to show that

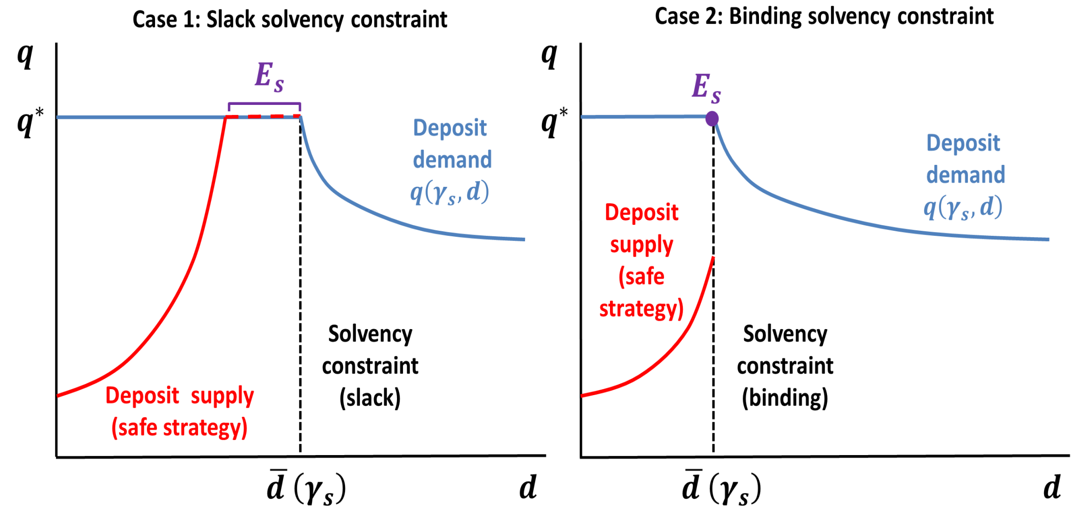
$$\frac{\partial l_s}{\partial n} > 0 \ \forall \ n < n_c$$

Proof. Provided in the Technical Appendix. ■

Figure 9 represents the two cases graphically. In the first case, banks value assets according to their expected return since they do not face a binding constraint or expect to rely on limited liability. The equilibrium price of loans is then given by (35). As explained in section 3.1.6, banks are indifferent to the amount of their sovereign bond purchases within a range given by (37), because their valuation of these bonds coincides with their market price. Consistent with this, there is also a range of admittable equilibrium values for (d_s, γ_s). In Figure 9, this is depicted by the overlapping region E_s between the deposit demand and supply curves. In order to pin down these variables in equilibrium, I select the upper bound of (37) as the equilibrium

[32] In the definiton for n_c, $\left(q_s^l, l_s\right)$ correspond to (35), (36)

Figure 9: **Safe equilbrium**



Note: The deposit supply curve is attained by combining (3) with (34) and (34). Deposit demand stems from the combination of (14) and (38).

value for b_s. This amounts to eliminating a range of safe equilibria with lower (b_s, γ_s) values without any impact on the characteristics of the equilibrium outcome.[33]

In the second case, the binding solvency constraint creates a wedge between the demand and supply of deposits. Therefore, banks do not find it optimal to purchase any domestic sovereign bonds and the equilibrium quantity of loans is implicitly defined by (40). A rise in net worth n relaxes the solvency constraint, leading to a rise in the price and quantity of loans.

Finally, it is worth discussing bank lending in the context of safe and gambling equilibria. Proposition 1 outlines the conditions under which a gambling equilibrium is associated with lower bank lending.

Proposition 1 *Bank lending is lower in a gambling equilibrium under the conditions*

$$\theta^l > \theta^b$$

$$n > \left(\frac{1}{\nu}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{l_g}{\alpha A}\right)^{\frac{1}{\alpha}} - q^* \theta^l l_g$$

Proof. Provided in the Technical Appendix. ■

The first condition pertains to banks' risk-taking incentives. In a gambling equilibrium, sovereign default drives the banking sector into insolvency. Because of limited liability, banks then cease to internalize the payoff of assets in the state with sovereign default. When the

[33]The parameter regions under which the safe equilibrium with the selected b_s value exists fully encompasses that of safe equilibria with lower b_s values. In other words, whenever the safe equilibria with lower b_s values exist, so does the selected equilibrium, which is identical to them in all other aspects.

recovery rate of loans exceeds that of domestic sovereign bonds, this leads to the crowding out of bank lending by domestic sovereign bond purchases.

In spite of this, bank lending is higher under the gambling equilibrium when net worth falls short of the level required to satisfy the second condition. In this case, a tight solvency constraint forces banks to reduce their lending below the gambling level in order to ensure their solvency following sovereign default. Note that as the recovery rate θ^l of loans increases, the second condition is satisfied at a wider range of net worth, while crowding out effects get stronger.

3.2.2 Sentiments

Recall from section 3.1.5 that banks' sovereign exposure γ is not contractible. Nevertheless, it is a key determinant of their solvency prospects and hence the optimal deposit demand schedule $q(\gamma, d)$. In this section, I describe how households formulate their expectations $\tilde{\gamma}$ about banks' sovereign exposures. This is equivalent to forming an expectation about bank strategy since (29), (38), (41) establish a one-to-one mapping between the two conditional on (n, d).

Figure 7 shows the deposit demand schedules associated with the expectation of safe ($\tilde{\gamma} = \gamma_s$) and gambling ($\tilde{\gamma} = \gamma_g$) strategies. Observe that households may infer the bank strategy from the level of deposits d when it lies outside the range $d \in (\bar{d}(\gamma_g), \bar{d}(\gamma_s)]$. When $d \leq \bar{d}(\gamma_g)$, banks remain solvent after sovereign default even when their exposure is at a level associated with the gambling strategy. As such, banks cannot possibly follow a gambling strategy when their deposits remain within this region. Similarly, even the low exposure γ_s associated with the safe strategy leads to insolvency when deposits exceed $\bar{d}(\gamma_s)$ such that $d > \bar{d}(\gamma_s)$ is not consistent with a safe strategy.

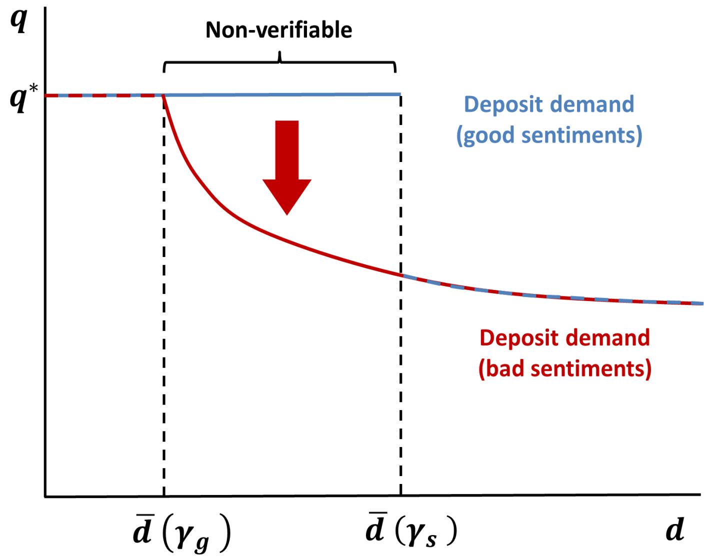
In contrast, within the 'non-verifiable' region $d \in (\bar{d}(\gamma_g), \bar{d}(\gamma_s)]$, it is not possible to deduce the bank strategy. Expectations about the sovereign exposure $\tilde{\gamma}$ are instead determined by household sentiments such that 'good sentiments' refer to the expectation of a safe strategy and 'bad sentiments' refer to that of a gambling strategy. Figure 10 displays the deposit demand schedule under each type of sentiments. As I solve for a rational expectations equilibrium, sentiments can only exist when they are self-confirming in equilibrium.

3.2.3 Equilibrium conditions

Under the rational expectations equilibrium framework described in section 3.2.1, the safe equilibrium exists when the representative bank finds it optimal to follow a safe strategy provided that there are good sentiments and other banks also follow a safe strategy. This leads to the equilibrium condition

$$v_s \geq v_{g|s} \tag{43}$$

Figure 10: **Sentiments**



where v_s is the representative bank's expected payoff in the safe equilibrium given in Lemma 3 and $v_{g|s}$ is the expected payoff from a 'deviation to the gambling strategy'. I refer to $v_{g|s}$ as a deviation payoff since it describes the expected payoff from adopting a gambling strategy when sentiments and other banks' strategies are consistent with a safe equilibrium.

Similarly, the gambling equilibrium exists under the equilibrium condition

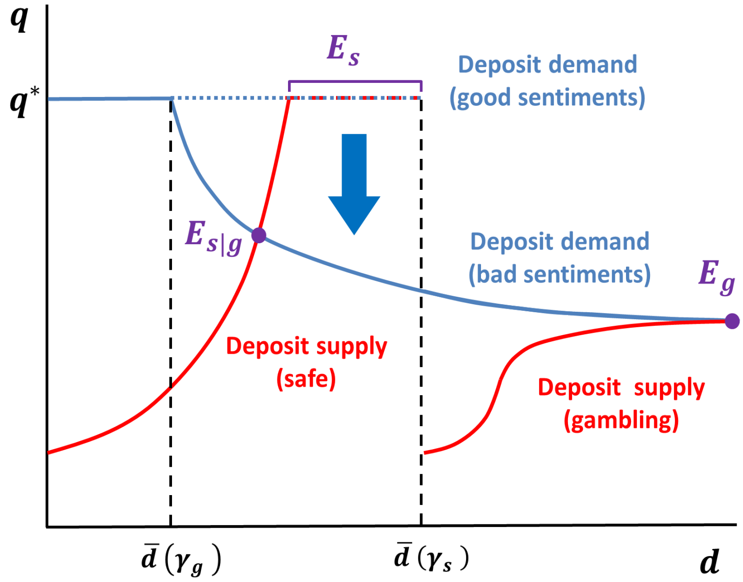
$$v_g \geq v_{s|g} \tag{44}$$

where v_g is the expected payoff under the gambling equilibrium given by (32) and $v_{s|g}$ is the expected payoff from a 'deviation to the safe strategy'. I elaborate further on these deviations below.

There are three possible equilibrium outcomes. First, when (43) is satisfied and (44) is not, banks follow a safe strategy regardless of household sentiments and there is a unique safe equilibrium. In this case, bad sentiments are not self-confirming and thus may not exist. In contrast, when (44) is satisfied and (43) is violated, there is a unique gambling equilibrium and only bad sentiments exist. Finally, when both conditions are satisfied, banks follow a safe strategy under good sentiments and gamble under bad sentiments such that there are multiple equilibria.

I use Figure 11 as an informal example to provide further intuition about the mechanism behind multiple equilibria. In the interest of a clear exposition, I focus on a case where the

Figure 11: **Example with multiple equilibria**



solvency constraint remains slack regardless of household sentiments.[34] Under good sentiments, the representative bank faces the deposit demand schedule depicted by the dotted line, where the deposit threshold $\bar{d}(\gamma_s)$ is consistent with a safe strategy. This permits the bank to raise sufficient deposits to satisfy its optimality condition for lending (35) without reducing the price of its deposits below the risk-free level q^* under a safe strategy. It then finds it optimal to adopt a safe strategy such that there is a safe equilibrium E_s and good sentiments are confirmed.

When there is a shift to bad sentiments, the expectation of a high sovereign exposure $\gamma_g > \gamma_s$ leads to an inward shift of the deposit threshold to $\bar{d}(\gamma_g) < \bar{d}(\gamma_s)$. The deposit demand schedule then pivots downward in the non-verifiable region $d \in (\bar{d}(\gamma_g), \bar{d}(\gamma_s)]$. Because of this deterioration in the bank's borrowing conditions, the quantity and price of deposits fall to $E_{s|g}$ under the safe strategy. This leads to a decline in the expected payoff associated with this strategy. If the bank finds it optimal to deviate to a gambling strategy that leads to the outcome E_g, bad sentiments are also confirmed and there are multiple equilibria.

Below, I briefly describe the deviations to gambling and safe strategies before characterizing the parameter boundaries for the three equilibrium regions (with a unique safe equilibrium, a unique gambling equilibrium, and multiplicity) in section 3.2.4.

Deviation to the gambling strategy Consider a deviation to the gambling strategy when sentiments and other banks' strategies correspond to the safe equilibrium in section 3.2.1.

[34]This mechanism becomes even stronger when the solvency constraint binds, since the downward pivot in the deposit demand schedule under bad sentiments leads to a tightening of the solvency constraint as shown in the third panel of Figure 12.

Under such a deviation, the bank's strategy is guided by the first order conditions (20) and (21), yielding valuations for deposits and loans that are consistent with a gambling equilibrium.

However, the quantity of loans purchased by the deviating bank

$$l_{g|s} = \left(q_g^l\right)^{\frac{\alpha}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} - \frac{1-\nu}{\nu} l_s \tag{45}$$

differs from its gambling equilibrium counterpart, which is given by (31). This is because the remaining banks each purchase an amount l_s consistent with the safe equilibrium, thus driving up loan prices. The negative relationship between $l_{g|s}$ and l_s follows directly from the upward-sloping loan supply schedule. As other banks provide more loans, the scope for lending by the deviating bank diminishes. This also reduces the expected payoff from deviation which is increasing in bank lending as in the gambling equilibrium

$$v_{g|s} = (1-P)\,\mu_l l_{g|s} + \frac{n}{q^*} \tag{46}$$

Lemma 4 builds upon this intuition to show that the safe equilibrium is always satisfied when the solvency constraint is slack.

Lemma 4 *The parameter restrictions given by (15) are sufficient to show that*

$$v_s > v_{g|s} \;\forall\; n \geq n_c$$

Proof. Provided in the Technical Appendix. ■

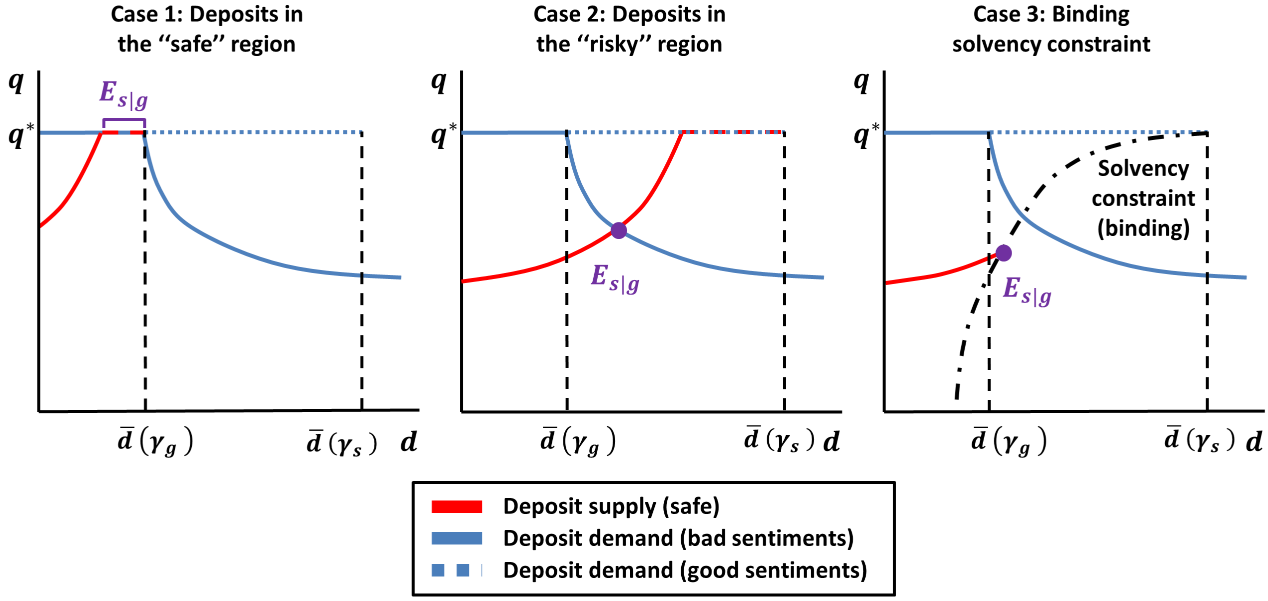
Recall from Lemma 3 that l_s is increasing in net worth n when the solvency constraint binds. It is thus possible for (43) to be violated at a level of net worth below n_c such that there is a unique gambling equilibrium. I elaborate further on this in section 3.2.4 after describing deviations to the safe strategy.

Deviation to the safe strategy Under a deviation to the safe strategy, the bank follows the first order conditions (24)-(26) but faces a deposit demand schedule

$$q\left(\gamma_g, d\right) = \begin{cases} q^* & \text{for } d \leq \bar{d}\left(\gamma_g\right) \\ q^b + \frac{P\theta^b}{1-P}\frac{n}{d} & \text{for } d > \bar{d}\left(\gamma_g\right) \end{cases} \tag{47}$$
$$\bar{d}\left(\gamma_g\right) = \frac{\theta^b}{q^b - \theta^b q^*} n$$

consistent with bad sentiments. As the bank's actual sovereign exposure diverges from household expectations, the solvency constraint no longer corresponds to the deposit threshold $\bar{d}\left(\gamma_g\right)$. This opens up the possibility that the bank may move to the risky region of the deposit demand

Figure 12: **Deviation to the safe Strategy**



Note: Deposit demand is attained by combining (14) with (38) under good sentiments and (29) under bad sentiments. The deposit supply curve stems from the combination of (3), (25), (26) and (47). The solvency constraint is given by (48).

schedule despite satisfying the solvency constraint.

There are thus three possible cases of the deviation to the safe strategy which are valid at different regions of bank net worth n. In the interest of brevity, I relegate the characterization of these cases to Appendix A and instead provide a brief description of each case with the aid of Figure 12. In the first case, the deviating bank has a slack solvency constraint and remains in the safe region of the deposit threshold $d_{s|g} \leq \bar{d}\left(\gamma_g\right)$. This case is nearly identical to case 1 of the safe equilibrium, except for a rise in the boundary level of net worth required for this case to be valid to $n_{r|g} > n_c$ due to the inwards shift of the deposit threshold under bad sentiments.[35]

In the second case, the shift to bad sentiments leaves the optimal level of deposits in the "risky" region of the deposit demand schedule, while the actual solvency constraint remains slack. The decline in the value of deposits to $q_{s|g} < q^*$ leads to a fall in bank lending and expected payoff. Finally, in the third case, the solvency constraint binds, creating a wedge between deposit demand and deposit supply and further reducing lending and expected payoff. Note that the solvency constraint, which is given by

$$\left(q^l_{s|g} - q\left(\gamma_g, d_{s|g}\right)\theta^l\right) l_{s|g} = n \tag{48}$$

tightens in response to a decline in the price of deposits.

[35] See Appendix A for a definition for $n_{r|g}$.

3.2.4 Regions of equilibria

There are three possible equilibrium outcomes to the model. First, there is a unique gambling equilibrium when banks follow a gambling strategy regardless of household sentiments. Second, there are multiple equilibria if banks adopt a safe strategy under good sentiments and a gambling strategy under bad sentiments such that both good and bad sentiments are self-fulfilling. Third, there is a unique safe equilibrium when banks follow a safe strategy regardless of household sentiments. I denote the regions of parameters where these outcomes are prevalent as $\mathcal{G}$, $\mathcal{M}$ and $\mathcal{S}$ respectively.

Proposition 2 expresses the equilibrium conditions (43), (44) as parameter boundaries for these regions.

Proposition 2 *Under the parameter restrictions given by (15), the mapping of equilibrium regions across net worth n is given by*

$$\mathcal{E}(n) = \begin{cases} \mathcal{G} & \text{if } n \leq \underline{n} \\ \mathcal{M} & \text{if } \underline{n} < n < \bar{n} \\ \mathcal{S} & \text{if } n > \underline{n} \end{cases} \tag{49}$$

where $\underline{n} < n_c$ is implicitly defined by the expression

$$\begin{aligned} \underline{n} \;=\; & \left(\frac{1}{\nu}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{1}{A\alpha} \frac{q^* (1-P) \mu_l \left(q_g^l\right)^{\frac{\alpha}{1-\alpha}} (A\alpha)^{\frac{1}{1-\alpha}} + \underline{n}}{q^* (1-P) \left[1 - \theta^l + \mu_l \frac{1-\nu}{\nu}\right]}\right)^{\frac{1}{\alpha}} \\ & - \theta^l \frac{q^* (1-P) \mu_l \left(q_g^l\right)^{\frac{\alpha}{1-\alpha}} (A\alpha)^{\frac{1}{1-\alpha}} + \underline{n}}{(1-P) \left[\left(1-\theta^l\right) + \mu_l \frac{1-\nu}{\nu}\right]} \end{aligned} \tag{50}$$

and $\bar{n}$ is given by

$$\bar{n} \equiv \frac{(1-P) q^*}{P} \left[(1-P) + P\theta^l (1-\nu) - \left(1 - P + P\theta^l\right)^{\frac{1}{1-\alpha}}\right] \left((1-\mu_l) q^b\right)^{\frac{\alpha}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} \mu_l \tag{51}$$

under the sufficient conditions $\alpha \in (0, \frac{1}{2}]$, $\nu \in (0, \frac{1}{2}]$.

Proof. Provided in the Technical Appendix. ■

Note that (49) indicates a monotonic ordering of equilibria across bank net worth n. Since $\underline{n} < n_c$, there is no overlap between $\mathcal{M}$ and the case of the safe equilibrium with a slack solvency constraint. Without an upper bound to bank net worth n, this is sufficient to show that $\mathcal{S}$ is non-empty. Proposition 3 describes the conditions under which $\{\mathcal{G}, \mathcal{M}\}$ are also non-empty.

Proposition 3 *Under the parameter restrictions given by (15), the non-emptiness of regions $\{\mathcal{G}, \mathcal{M}\}$ depends on where θ^l stands with respect to the boundary $\underline{\theta}^l$, which is implicitly defined by the expression*

$$(1-\nu)+\nu\frac{1-\underline{\theta}^l}{\mu_l} = \left(\frac{(1-\mu_l)\left(1-P+P\theta^b\right)}{\underline{\theta}^l}\right)^{\frac{\alpha}{1-\alpha}} \tag{52}$$

There are two possible cases.

Case 1 *If $\theta^l \geq \underline{\theta}^l$, $\mathcal{G}$ is empty and $\mathcal{M}$ is always non-empty.*

Case 2 *If $\theta^l < \underline{\theta}^l$, $\mathcal{G}$ is non-empty and a sufficient condition for $\mathcal{M}$ to be non-empty is*

$$\frac{\theta^b}{\alpha+(1-\alpha)\,\nu} > 1-P+P\theta^b \tag{53}$$

Proof. Provided in the Technical Appendix. ■

4 The dynamic model

In this section, I extend the two-period model to a recursive-dynamic setting with risk averse households and sovereign risk shocks. Figure 13 shows the recursive timeline. The vector $\mathbf{S}$ collects the value of aggregate state variables (to be defined explicitly later on) in the current period and $\mathbf{S}'$ denotes the state vector for the next period. Sovereign default is incorporated into the model as an absorbing state. In each period, the government defaults with probability $P(\mathbf{S})$. Once the government defaults, there is no more sovereign default risk in future periods and the model economy moves to a steady state $\underline{S}$ where the continuation values $\left(\underline{v}^h, \underline{v}^b\right)$ of banks and households depend on $\mathbf{S}$.

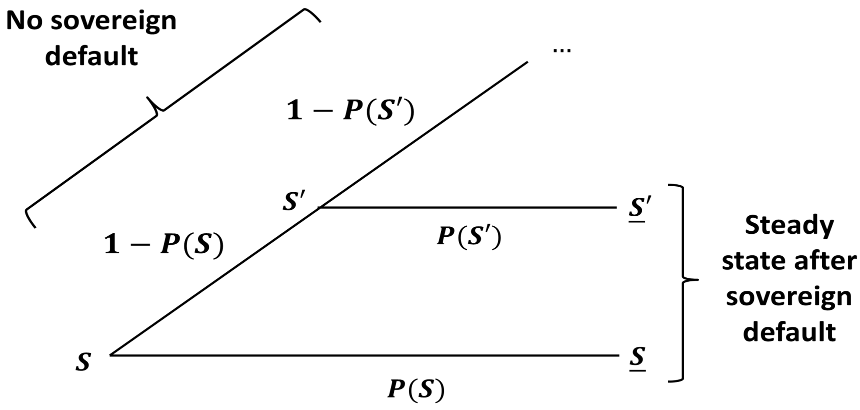
In the interest of brevity, I only describe the aspects of the dynamic model that differ from section 3.[36] The remainder of the section is organized as follows: First, I describe the process for sovereign risk, the deposit demand schedule under risk aversion, and the bank's recursive optimization problem. Then I discuss the formulation of household sentiments, define the equilibrium concept and characterize the steady state after sovereign default. Finally, I provide a sketch of the algorithm used for the numerical solution.

4.1 Government

Sovereign bonds are priced at their expected return by deep pocketed foreign investors as in section 3.1.1. Instead of taking a constant value, however, the sovereign default probability

[36] See the Technical Appendix for a complete specification of the dynamic model

Figure 13: **Recursive timeline**



$P(\mathbf{S})$ is determined by a stochastic fiscal limit. Let $\Upsilon(\mathbf{S})$ denote the fiscal stress faced by the government. At the beginning of each period, an i.i.d. shock ε that follows a standard logistic distribution determines the government's resolve to avoid default. Sovereign default occurs when $\varepsilon \leq \Upsilon(\mathbf{S})$. The default probability is then given by the logistic function

$$P(\mathbf{S}) = \frac{\exp(\Upsilon(\mathbf{S}))}{1 + \exp(\Upsilon(\mathbf{S}))} \tag{54}$$

Note that the stock of government debt B, output Y and the sovereign bond yield $1/q^b(\mathbf{S})$ may easily be incorporated into the fiscal stress function $\Upsilon(\mathbf{S})$ as determinants of sovereign risk. For the dynamic solution, however, I adopt a simple specification $\Upsilon(\mathbf{S}) = \delta$ where δ follows the AR(1) process

$$\delta' = \delta_{ss} + \rho_\delta(\delta - \delta_{ss}) + \sigma_\delta \varepsilon'_\delta, \quad \varepsilon'_\delta \sim N(0, 1) \tag{55}$$

and ε'_δ is a sovereign risk shock.

My reasons for adopting this specification are threefold. First, recent empirical studies show that a substantial portion of the movements in sovereign risk premia during the recent sovereign debt crisis were unrelated to country fundamentals (see e.g. Bahaj, 2014; De Grauwe and Ji, 2012). In line with these findings, the sovereign risk shock (55) reflects non-fundamental factors such as contagion and self-fulfilling sentiments in sovereign bond markets.

Second, by adopting this specification I abstain from the feedback loops between sovereign default risk and domestic fundamentals such as the stock of debt and sovereign bond yields. Although these feedback loops play a potentially important role in the transmission of sovereign risk, they have been studied extensively in recent literature (see e.g. Corsetti et al., 2013). Abstaining from them permits me to isolate the propagation channel of sovereign risk through bank-depositor interactions. Third, from a computational perspective, abstaining from these feedback loops reduces the number of state variables.

The law of motion for government debt is given by the government's budget constraint

$$q^b(\mathbf{S})B' = B + G(\mathbf{S}) - T(\mathbf{S})$$

where $T(\mathbf{S})$ is lump-sum taxation on households and $G(\mathbf{S})$ is government spending. Since B has no effect on the non-government sector under this specification, the only restriction I place on the primary surplus $G(\mathbf{S}) - T(\mathbf{S})$ is that it follows a fiscal rule that precludes Ponzi games.

4.2 Deposit demand schedule

Households are risk averse with their flow utility $u(c)$ given by a standard CRRA specification. I relegate the household's recursive optimization problem to Appendix B and discuss the implications of risk aversion for the deposit demand schedule

$$q(d', n, \mathbf{S}) = \left\{ \begin{array}{ll} q^* & \text{for } d' \leq \bar{d}(n, \mathbf{S}) \\ q^* \frac{1-P(\mathbf{S})+P(\mathbf{S})\frac{u_c(\underline{c})}{u_c(c')}\left(\tilde{\gamma}(n,\mathbf{S})\frac{\theta^b}{q^b(\mathbf{S})}+(1-\tilde{\gamma}(n,\mathbf{S}))\frac{\theta^l}{q^l(\mathbf{S})}\right)\frac{n}{d'}}{1-q^*P(\mathbf{S})\frac{u_c(\underline{c})}{u_c(c')}\left(\tilde{\gamma}(n,\mathbf{S})\frac{\theta^b}{q^b(\mathbf{S})}+(1-\tilde{\gamma}(n,\mathbf{S}))\frac{\theta^l}{q^l(\mathbf{S})}\right)} & \text{for } d' > \bar{d}(n, \mathbf{S}) \end{array} \right\}, \tag{56}$$

where d' is deposits at bank level, $u_c(.)$ is marginal utility and $(\underline{c}, c')$ are respectively consumption in future states with and without sovereign default. The sovereign exposure anticipated by households is denoted by $\tilde{\gamma}(n, \mathbf{S})$ and the deposit threshold $\bar{d}(n, \mathbf{S})$ is identical to its counterpart in section 3.

Under risk aversion, the marginal utility wedge $\frac{u_c(\underline{c})}{u_c(c')}$ exceeds unity and increases in d'. Compared to the case with risk neutrality, this leads to a small discontinuity in $q(d', n, \mathbf{S})$ around the deposit threshold and increases the curvature of the schedule in the risky region $d' > \bar{d}(n, \mathbf{S})$. As a result, there is an interior solution $\gamma_g \in (0, 1)$ for sovereign exposure under the gambling strategy.

4.3 Banks

Each bank is managed by a unit continuum of risk-neutral bankers. From a representative bank's perspective, the timeline of events within a period is as follows. At the beginning of each period, the bank observes the realization of $\mathbf{S}$ and collects deposits d' from households at a price $q(d', n, \mathbf{S})$. It uses these deposits, along with its accumulated net worth n to purchase domestic sovereign bonds b and loans l from firms at prices $q^b(\mathbf{S})$ and $q^l(l, \mathbf{S})$, thereby selecting its sovereign exposure γ.

Next, the bank learns whether the government is in default. The payoff from (b, l) and hence the bank's profits are contingent on the sovereign default realization

$$\pi = l + b - d' \tag{57}$$
$$\underline{\pi} = \max\left(\theta^l l + \theta^b b - d', 0\right) \tag{58}$$

such that the bank may be rendered insolvent by sovereign default. Bankers have limited liability, so when the bank becomes insolvent, all of its bankers exit the economy with zero payoff. When the bank is solvent, on the other hand, a randomly determined but constant portion $(1-\psi)$ of its bankers exit and consume their share of the profits.[37] The remaining profits are accumulated as net worth in the next period, according to the law of motion

$$n' = \psi\left(\pi - \omega\right) \tag{59}$$
$$\underline{n}' = \psi\left(\underline{\pi} - \omega\right) \tag{60}$$

where ω represents overhead costs.[38]

Limited liability creates a discontinuity in the representative bank's policy function such that its decision problem can be written as a choice between two alternative strategies, a 'safe strategy' where the bank satisfies an occasionally binding solvency constraint

$$d' \leq \theta^l l + \theta^b b \tag{61}$$

and limited liability never binds, and a 'gambling strategy' which leaves the bank reliant on limited liability in the event of sovereign default. I denote these with the subscripts s and g.

The representative bank's problem can then be written as

$$\begin{aligned} v^b\left(n;\mathbf{S}\right) &= \max\left\{v_s^b\left(n;\mathbf{S}\right), v_g^b\left(n;\mathbf{S}\right)\right\}\ , \\ v_s^b\left(n;\mathbf{S}\right) &= \max_{d',\gamma\in[0,1]}\left\{\begin{array}{c}\left(1-P\left(\mathbf{S}\right)\right)\left(\left(1-\psi\right)\pi + \psi\mathbb{E}_{\mathbf{S}}\left[v^b\left(n';\mathbf{S}'\right)\right]\right) \\ +P\left(\mathbf{S}\right)\left(\left(1-\psi\right)\underline{\pi} + \psi\underline{v}^b\left(\underline{n}';\mathbf{S}\right)\right)\end{array}\right\}, \\ v_g^b\left(n;\mathbf{S}\right) &= \max_{d',\gamma\in[0,1]}\left\{\left(1-P\left(\mathbf{S}\right)\right)\left(\left(1-\psi\right)\pi + \psi\mathbb{E}_{\mathbf{S}}\left[v^b\left(n';\mathbf{S}'\right)\right]\right)\right\}\end{aligned} \tag{62}$$

[37] The number of banks, and the bankers that manage them are constant over time. Insolvent banks are replaced with a new bank that has zero net worth. Bankers that exit from solvent banks are replaced with new bankers which do not contribute to net worth.

[38] The consumption of portion $(1-\psi)$ of profits and overhead costs ω serve to prevent the accumulation of infinite net worth by banks in the steady state after sovereign default. The former aspect is standard in dynamic financial models while the latter is necessitated by the excess profits banks make due to imperfect competition. Overhead costs are waived when $\underline{\pi} < \omega$ so as to ensure that they never drive the bank into insolvency or affect the recovery rate θ on deposits.

subject to (57)-(60) and

$$\begin{aligned} q^b(\mathbf{S})\,b + q^l(l,\mathbf{S})\,l &= q(d',n,\mathbf{S})\,d' + n \\ \mathbf{S}' &= \mathbf{\Gamma}(\mathbf{S}) \end{aligned} \tag{63}$$

for both strategies, as well as the solvency constraint (61) for the safe strategy. $\mathbf{\Gamma}(\mathbf{S})$ is the law of motion for aggregate state variables, (63) represents the bank's budget constraint and $\underline{v}^b(.)$ is the bank's continuation value under sovereign default. Lemma 5 provides an expression for $\underline{v}^b(.)$.

Lemma 5 *The continuation value for solvent banks in the steady state $\underline{S}$ is*

$$\underline{v}^b(\underline{n}';S) = \underline{\pi} \tag{64}$$

Proof. Provided in the Technical Appendix. ■

The bank's first order conditions under the safe strategy are

$$\left(\theta^l l + \theta^b b - d'\right)\lambda(n,\mathbf{S}) = 0 \;,\; \lambda(n,\mathbf{S}) \geq 0 \;,\; d' \leq \theta^l l + \theta^b b \tag{65}$$

$$q^b(\mathbf{S}) \geq \frac{(1-P(\mathbf{S}))\left(1-\psi+\psi\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n',\mathbf{S}')\right]}{\partial \pi}\right)+(P(\mathbf{S})+\lambda(n,\mathbf{S}))\,\theta^b}{1+\lambda(n,\mathbf{S})}(1-\mu_d(d',n,\mathbf{S}))\,q(d',n,\mathbf{S}) \tag{66}$$

$$\frac{q^l(l,\mathbf{S})}{1-\mu_l} = \frac{(1-P(\mathbf{S}))\left(1-\psi+\psi\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n',\mathbf{S}')\right]}{\partial \pi}\right)+(P(\mathbf{S})+\lambda(n,\mathbf{S}))\,\theta^l}{1+\lambda(n,\mathbf{S})}(1-\mu_d(d',n,\mathbf{S}))\,q(d',n,\mathbf{S}) \tag{67}$$

where $(\mu_l, \mu_d(d',n,\mathbf{S}))$ are the mark-ups in the loan and deposit markets and $\lambda(n,\mathbf{S})$ is the Lagrange multiplier associated with the solvency constraint. The interpretation of these conditions is similar to their counterparts (24)-(26) in section 3.1.6. The two sets of FOCs differ only due to the term $\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n',\mathbf{S}')\right]}{\partial \pi}$ which is the expected value of a marginal increase in profits in the state without sovereign default. In a two-period setting, this term is fixed at unity by the bank's risk neutrality. In a dynamic environment, on the other hand, it depends on the marginal value of net worth in future state realizations $\mathbf{S}'$ via (59). Proposition 4 shows that the FOCs in section 3.1.6 constitute a special case of the dynamic FOCs.

Proposition 4 *Let $\mathfrak{g}$ be the subset of state realizations where the bank follows a gambling strategy. If for all possible future aggregate state realizations $\mathbf{S}'$, either $(n';\mathbf{S}') \in \mathfrak{g}$ or $(n';\mathbf{S}') \notin$*

$\mathfrak{g}$ *and* $\lambda(n', \mathbf{S}') = 0$, $q(d', n', \mathbf{S}') = q^*$, *then*

$$\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n', \mathbf{S}')\right]}{\partial \pi} = 1$$

Otherwise

$$\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n', \mathbf{S}')\right]}{\partial \pi} > 1$$

Proof. Provided in the Technical Appendix. ■

The proposition states that the bank attaches a higher value to future net worth if there is a positive probability of visiting a future state realization where it follows a safe strategy with a binding solvency constraint and/or its deposits are perceived to be risky. This increase in the value attached to π relative to $\underline{\pi}$ increases the risk-taking incentives of the bank, leading to a rise in (b, d') under the safe strategy when the solvency constraint is slack, as well as stronger incentives to adopt the gambling strategy.

In contrast, the FOCs under the gambling strategy are identical to their counterparts in section 3.1.6.

$$q^b(\mathbf{S}) = (1 - \mu_d(d', n, \mathbf{S}))\, q(d', n, \mathbf{S}) \tag{68}$$

$$q^l(l, \mathbf{S}) = (1 - \mu_l)\, q^b(\mathbf{S}) \tag{69}$$

This is due to the bank's reliance on limited liability under sovereign default. Because of this, the bank only internalizes its profits π in the absence of sovereign default. Since the relative valuation of $(\pi, \underline{\pi})$ does not matter, the term $\frac{\partial \mathbb{E}_{\mathbf{S}}\left[v^b(n', \mathbf{S}')\right]}{\partial \pi}$ drops out of the gambling FOCs. In other words, when a bank follows the gambling strategy, its optimal set of actions are those that maximize π regardless of its time horizon.

4.4 Sentiments and sunspots

In this section, I describe how households formulate their expectations $\tilde{\gamma}(n, \mathbf{S})$ about a bank's domestic sovereign bond exposure. Conditional on $(n, \mathbf{S})$, the bank's first order conditions (65)-(69) provide a unique mapping between the strategy followed by a bank and its sovereign exposure.

Using (62), the optimality condition for the bank to adopt a gambling strategy can be written as

$$v^b_g(n; \mathbf{S}) \geq v^b_s(n; \mathbf{S}) \tag{70}$$

When this condition is satisfied, the bank's optimal exposure γ_g is given by (68), (69). Otherwise, the bank adopts a safe strategy and its exposure γ_s is pinned down by (65)-(67).

Sentiments may become self-fulfilling due to the dependence of both sides of the inequality in (70) on $\tilde{\gamma}(n, \mathbf{S})$.

The state space for $(n, \mathbf{S})$ can be segmented into three non-intersecting subsets according to the interaction between (70) and $\tilde{\gamma}(n, \mathbf{S})$. Let $\mathcal{G}$ denote a subset where (70) is satisfied for $\tilde{\gamma}(n, \mathbf{S}) = \{\gamma_g, \gamma_s\}$, $\mathcal{S}$ denote a second subset where (70) is violated for $\tilde{\gamma}(n, \mathbf{S}) = \{\gamma_g, \gamma_s\}$ and $\mathcal{M}$ denote a third subset where (70) is satisfied for $\tilde{\gamma}(n, \mathbf{S}) = \gamma_g$ and violated for $\tilde{\gamma}(n, \mathbf{S}) = \gamma_s$. In the first two subsets $\{\mathcal{G}, \mathcal{S}\}$, γ is uniquely determined regardless of $\tilde{\gamma}(n, \mathbf{S})$ while household sentiments become self-fulfilling when $(n, \mathbf{S}) \in \mathcal{M}$.

I resolve the multiplicity in $\mathcal{M}$ with the use of sunspots. Specifically, let ζ be a random variable drawn from a uniform distribution on the unit interval at the beginning of each period and $\bar{\zeta} \in [0, 1]$ a constant threshold. When $\zeta > \bar{\zeta}$ household expectations coordinate on $\tilde{\gamma}(n, \mathbf{S}) = \gamma_s$ consistent with the safe strategy. I refer to this as 'good sentiments'. When $\zeta \leq \bar{\zeta}$, on the other hand, expectations coordinate on $\tilde{\gamma}(n, \mathbf{S}) = \gamma_g$ in line with the gambling strategy and there are 'bad sentiments'. To provide a formal definition for $\tilde{\gamma}(n, \mathbf{S})$, $\mathcal{M}$ is further segmented into two subsets $\mathcal{M}^+$ and $\mathcal{M}^-$ which respectively denote good and bad sentiments such that

$$\tilde{\gamma}(n, \mathbf{S}) = \begin{cases} \gamma_g \text{ if } (n, \mathbf{S}) \in \{\mathcal{G}, \mathcal{M}^-\} \\ \gamma_s \text{ if } (n, \mathbf{S}) \in \{\mathcal{S}, \mathcal{M}^+\} \end{cases}$$

Since ζ is uniformly distributed on a unit interval, the probability of good and bad sentiments in the next period are simply given by $\left(1 - \bar{\zeta}\right)$ and $\bar{\zeta}$ respectively. Note that it is straightforward to introduce a more sophisticated specification for sunspots by replacing $\bar{\zeta}$ with an AR(1) shock process or a function of fundamentals such as the recovery rate θ of domestic deposits or government debt B. I opt for this simple specification as it permits me to isolate the role of sovereign risk and other relevant fundamentals in making household sentiments self-fulfilling in the first place. The subset $\mathcal{M}$ which provides a mapping of states with multiplicity is endogenously determined by the optimal strategies of households and banks, which in turn depend on these fundamentals.[39]

[39] Global games constitutes an alternative approach to sunspots in resolving multiple equilibria that creates an endogenous relationship between economic fundamentals and equilibrium selection. This approach, however, is not implementable in the context of the multiplicity considered in this paper since the strategic complementary is between banks and households rather, and takes place through a market mechanism that is capable of aggregating diverse beliefs. To see this, consider the introduction of a private signal to households about $\tilde{\gamma}(n, \mathbf{S})$. Provided households are not extremely risk averse, the solvency calculus of a household is not affected by the signal received by other households. Banks then find it optimal to borrow solely from the household with the lowest $\tilde{\gamma}(n, \mathbf{S})$ signal, which determines the price $q(d', n, \mathbf{S})$ in deposit markets. The model collapses to a sunspot solution where the lowest $\tilde{\gamma}(n, \mathbf{S})$ signal becomes the de facto sunspot.

4.5 Steady state after sovereign default

When the government defaults, sovereign bond holders receive a recovery rate $\theta^b < 1$. Productivity also declines to $\underline{A} < A$ which leads to a reduction in wages and a partial payment from loans. If the banks followed a gambling strategy before sovereign default, they become insolvent such that households receive a recovery rate θ from their deposits and the banking sector is replaced with a new set of banks with zero net worth. Otherwise, deposits are repaid fully and bank net worth is determined by (60).

In the following period, the economy immediately moves to a steady state $\underline{S}$ where productivity recovers back to A and there is no further sovereign default risk.[40] In the absence of bank insolvency risk, domestic deposits become perfectly substitutable with risk-free assets such that $\underline{q} = q^*$.[41] The steady state price and quantity of loans is then given by

$$\begin{aligned} \underline{q}^k &= (1-\mu_l)\, q^* \\ \underline{L} &= (\alpha A)^{\frac{1}{1-\alpha}} \left(\underline{q}^k\right)^{\frac{\alpha}{1-\alpha}} \end{aligned} \tag{71}$$

The following parameterization for (ψ, ω, q^*) is necessary to ensure this

$$\begin{aligned} \psi &= q^* = \beta \\ \omega &= \nu \mu_l \underline{L} \end{aligned}$$

The parameterization for (ψ, ω) ensures that bank net worth remains constant while equating the risk-free asset price to the household discount factor drives households to completely smooth their consumption after sovereign default.[42]

4.6 Equilibrium

Let $\mathbf{S} = [N, \delta, \zeta, \varkappa]$ be the aggregate state sector, where $N = n/\nu$ is aggregate bank net worth in equilibrium and $\varkappa = D + D^* + w(\mathbf{S}) - T(\mathbf{S})$ is disposable household wealth. A recursive rational expectations equilibrium is given by value functions for households and banks $\left\{v^h, v^b\right\}$ and policy functions for households $\{c, D', D^{*\prime}\}$ and for banks $\{\gamma, d'\}$ such that, given prices $\{w, \underline{w}, q^*\}$ and price schedules $\left\{q^l, q\right\}$: **(i)** households' and banks' value and policy functions

[40] The immediate recovery in productivity only serves to simplify the exposition. This can be replaced with any continuation path for productivity as long as there is perfect foresight about it.

[41] There is no need take a stance on when and whether the government returns to sovereign bond markets as long as there is no further default risk. If the government is able to issue bonds, they are priced at $\underline{q}^b = q^*$ and banks are indifferent to holding them.

[42] Solving the household's problem when q^* differs from the discount factor β is trivial but leads to a balanced growth path for consumption rather than a steady state value. I abstain from this since it leads to additional complication without yielding any insights of interest.

solve their optimization problems; **(ii)** the market for domestic deposits clears, $D' = d'/\nu$ **(iii)** the market for loans clears $L(\mathbf{S}) = l/\nu$; **(iv)** the government budget constraint is satisfied; **(v)** $\mathbf{\Gamma}(.)$ and $\{\mathcal{G}, \mathcal{M}, \mathcal{S}\}$ are consistent with agents' optimal strategies.[43]

4.7 Numerical solution

The solution for the recursive equilibrium is attained using global numerical methods. In this section, I sketch the main steps in the algorithm and relegate the remaining details to the Technical Appendix.

Note that the decentralized, imperfectly competitive nature of banks requires the inclusion of individual bank net worth n along with $\mathbf{S}$ as a state variable. Specifically, although banks are symmetric with net worth $n = \nu N$ on the equilibrium path, determining their optimal strategy as per section 4.3 requires considering off-equilibrium strategies (deviations) which lead to a different path of n for the specific bank than the remainder of the banking sector. The bank's value function $v^b(n; \mathbf{S})$ and the equilibrium regions $\{\mathcal{G}, \mathcal{M}, \mathcal{S}\}$ are thus defined over $(n, \mathbf{S})$.

Let $X(\mathbf{S}) = \{\gamma, d', c, D', D^{*\prime}\}$ collect the policy functions of banks and households in the symmetric equilibrium with $n = \nu N$, and $\mathcal{E} = \{\mathcal{G}, \mathcal{M}, \mathcal{S}\}$ denote the equilibrium regions. The solution algorithm can then be sketched as follows

1. Begin with a set of guesses for $\{\mathcal{E}, \Gamma(\mathbf{S}), X(\mathbf{S})\}$.
2. Formulate future expectations according to $\{\mathcal{E}, \Gamma(\mathbf{S}), X(\mathbf{S})\}$. Then, use the deposit demand schedule in section 4.2, first order conditions in 4.3, and the market clearing conditions in section 4.6 to update $\{\Gamma(\mathbf{S}), X(\mathbf{S})\}$. Iterate until the solution for $\{\Gamma(\mathbf{S}), X(\mathbf{S})\}$ converges.
3. Guess the bank's value function $v^b(n; \mathbf{S})$.
4. Use the first order conditions in section (4.3), (70) and expectations formulated according to $\{\mathcal{E}, \Gamma(\mathbf{S}), X(\mathbf{S})\}$ to update $v^b(n; \mathbf{S})$. Iterate until the solution for $v^b(n; \mathbf{S})$ converges.
5. Update $\mathcal{E}$ according to the solution to step 4. Repeat from step 2 until convergence.

I follow three distinct approaches to alleviate the curse of dimensionality that arises from solving the model globally. First, I use a piecewise cubic Hermite spline to interpolate $\{\Gamma(\mathbf{S}), X(\mathbf{S}), v^b(n; \mathbf{S})\}$ between the pre-defined grid points. Second, I abstain from the household's wealth

[43] In the small open economy setting, the markets for goods and sovereign bonds are cleared through trade with foreign agents. Therefore, there is no need to explicitly include the clearing conditions for these markets in the equilibrium definition.

accumulation process by letting lump-sum taxes $T(\mathbf{S})$ adjust to ensure that

$$\varkappa = D + D^* + w(\mathbf{S}) - T(\mathbf{S}) = \bar{E}$$

as long as the government remains solvent, where $\bar{E}$ is a fixed wealth parameter. This does not affect households' incentives to save since they take $T(\mathbf{S})$ as given, but eliminates $\varkappa$ from the state vector, reducing the number of state variables to 4.

Third, I take advantage of a series of characteristics of the bank's first order conditions to reduce the computational burden in steps 2 and 4 significantly. Specifically, the FOCs (65) and (67) indicate that the optimal choices $\{\gamma_s, d'_s\}$ under a safe strategy are (i) independent of $\left\{\Gamma(\mathbf{S}), X(\mathbf{S}), v^b(n,\mathbf{S})\right\}$ when $\tilde{\gamma}(n,\mathbf{S}) = \gamma_s$ (ii) independent of $\left\{\Gamma(\mathbf{S}), v^b(n,\mathbf{S})\right\}$ when $\lambda(n,\mathbf{S}) > 0$. Similarly, the FOCs (68) and (69) indicate that the optimal choices $\left\{\gamma_g, d'_g\right\}$ under a gambling strategy are independent of $\left\{\Gamma(\mathbf{S}), v^b(n,\mathbf{S})\right\}$. The relevant proofs are provided in the Technical Appendix.

5 Numerical results

This section provides numerical results from the dynamic model under a calibration that targets Portugal. It proceeds in four steps. Section 5.1 describes the calibration. Section 5.2 discusses the relationship between sovereign risk and the equilibrium regions. Section 5.3 demonstrates the propagation of sovereign risk shocks with the use of impulse response functions to a sovereign risk shock.

Finally, in Section 5.4, I bring the model to data by comparing its fit to macroeconomic dynamics in Portugal over 2010-2016. Among countries hit by the European sovereign debt crisis, I choose Portugal for two reasons. First, unlike Greece and Cyprus, Portugal did not impose capital controls on its banking sector. This is important as the mechanism for the rise in bank funding costs in the model relies on households' ability to invest in a safe asset instead of domestic bank deposits. Second, the Portuguese economy did not undergo a major real estate bubble as in Ireland and Spain or face long-term economic stagnation like Italy. Therefore, the transmission mechanism captured by the model should be most prevalent in Portuguese data.

5.1 Calibration

The calibration targets Portugal over the crisis period of 2010-2016 with each period representing a quarter. Table 3 reports the calibrated parameters.

The recovery rate of sovereign bonds is set to $\theta^b = 0.6$ according to Cruces and Trebesch (2013). The calibration for the fiscal stress parameters $(\delta_{ss}, \rho_\delta, \sigma^2_\delta)$ matches $q^b(S)/q^*$ to the

Table 3: **Calibration**

Parameter	Value	Description	Source
θ^b	0.60	Sov. bond recovery rate	Cruces and Trebesch (2013)
δ_{ss}	-5.14	Fiscal stress (mean)	Bloomberg
ρ_δ	0.74	Fiscal stress (persistence)	Bloomberg
σ_δ^2	0.93	Fiscal stress (variance)	Bloomberg
β	$0.99^{1/4}$	Discount factor	-
$\bar{E}$	0.07E-9	Household wealth	OECD
σ	3.00	Coefficient of risk aversion	Thimme (2016)
α	0.33	Cobb-Douglas parameter	-
A	1.00	Productivity (no sov. default)	-
$\underline{A}$	0.90	Productivity (sov. default)	Hébert and Schreger (2016)
ν	0.005	Bank market share	ECB Statistical Data Warehouse
$\bar{\zeta}$	0.50	Probability of bad sentiments	-

yield spread between Portuguese and German bonds (which act as a benchmark for the safe rate).[44] Specifically, I use (1) and (54) to back out a time series of fiscal stress realizations $\hat{\delta}_t$ from the spread data under the calibrated recovery rate. The calibration for $(\delta_{ss}, \rho_\delta, \sigma_\delta^2)$ is then attained by fitting the AR(1) process given by (55) to $\hat{\delta}_t$.[45]

In the household sector, the discount factor is calibrated to $\beta = 0.99^{1/4}$ and the wealth parameter targets data on household net worth from OECD. The calibration for the coefficient of risk aversion $\sigma = 3$ lies within the range given by recent empirical estimates (Thimme, 2016).

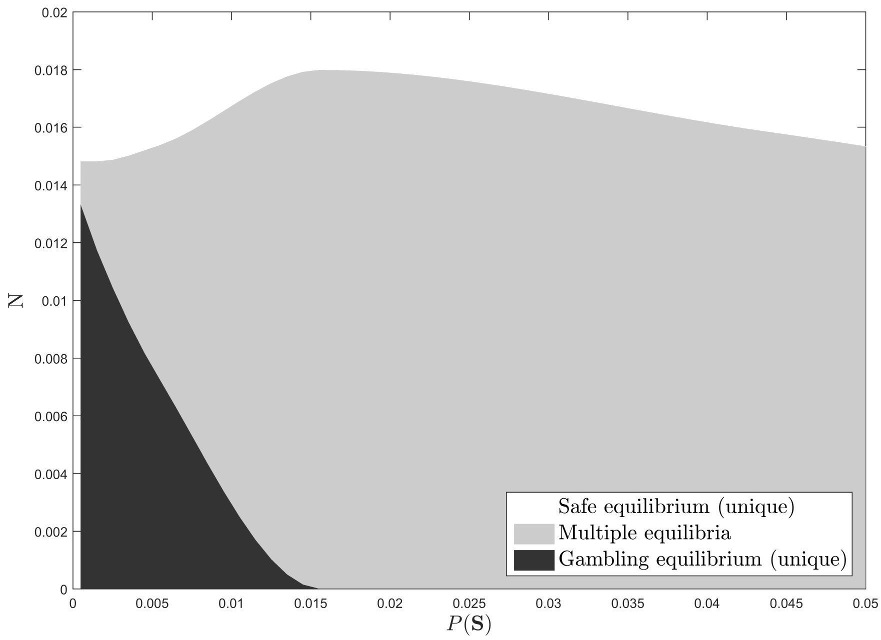
Regarding firms, I set the output elasticity of capital to the standard Cobb-Douglas value of $\alpha = 1/3$. In the absence of sovereign default, productivity is normalized to $A = 1$ such that $\underline{A}$ is equivalent to the recovery rate of loans θ^l. The calibration for $\underline{A}$ targets the recovery rate since sovereign default propagates through balance sheet costs to banks rather than the direct effects of productivity decline. Accordingly, I calibrate $\theta^l = 0.90$ in line with recent estimates on the effects of sovereign default on firm profitability (Hébert and Schreger, 2016).[46]

[44] I use bonds with a remaining maturity of 3 months due to the quarterly calibration of the model. While the standard benchmark for measuring sovereign default risk is the yield/CDS spreads on 10 year bonds, it is not possible to extract quarter-on-quarter default probabilities from these measures without imposing additional restrictions on the yield curve.

[45] See the Technical Appendix for further details.

[46] This implies a relatively high output cost of default compared to the previous literature. It is worth noting, however, that the calibration for θ^l can be reconciled with lower output costs with the introduction of bankruptcy costs or real frictions that limit the ability of firms to decrease salary costs following sovereign default. Note also that, under the baseline calibration, the parameter restrictions in (15) are satisfied for a wide range of recovery rates $\theta^l \in [0.59, 0.99]$. The qualitative results presented throughout the paper, including the non-emptiness of the multiple equilibria region, remain valid at all points within this range.

Figure 14: **Equilibrium Mapping**



The bank market share parameter ν is calibrated to match the mark-up μ_l in the loans market to the average interest margin on domestic bank lending to non-financial corporations during the pre-crisis period of 2003-2007.[47] Finally, I calibrate the sunspot threshold to $\bar{\zeta} = 0.5$ such that good and bad sentiments are equally likely.

5.2 Sovereign risk and equilibrium regions

I begin analyzing the numerical results by examining the implications of sovereign risk for the equilibrium regions. Figure 14 provides a mapping of the prevalent equilibrium type across a range of sovereign default probabilities $P(\mathbf{S})$ and aggregate bank net worth N. As with the two period model in section 3, the three equilibrium regions are ordered monotonically across net worth: First, the gambling equilibrium is unique when net worth falls short of a boundary $\underline{N}(\mathbf{S})$. Second, there is an intermediate multiplicity region $\underline{N}(\mathbf{S}) \leq N \leq \bar{N}(\mathbf{S})$. Finally, the safe equilibrium is unique when net worth exceeds $\overline{N}(\mathbf{S})$.

These boundaries are contingent on sovereign default risk. Only a safe equilibrium is possible

[47]The relationship between the mark-up and the steady state price of loans is given by (71). I match this with pre-crisis interest rates in order to isolate the excess return due to market power.

when sovereign bonds are completely safe[48] but the emergence of sovereign risk creates a large region with a unique gambling equilibrium. Further increases in sovereign risk have a non-linear effect on banks' incentives to gamble. As $P(\mathbf{S})$ rises, $\overline{N}(\mathbf{S})$ first increases, and then decreases while $\underline{N}(\mathbf{S})$ decreases monotonically, leading to an expanding multiplicity region.

To understand these findings, consider the implications of sovereign risk for bank payoffs under each strategy. When a bank follows the gambling strategy, a rise in sovereign risk has two opposing effects on its profits. First, it increases sovereign bond yields which raises profits from gambling. Second, it leads to a rise in bank funding costs which reduces profits. Since households are risk averse, the latter effect dominates.

In contrast, the impact of sovereign risk on the safe strategy payoff is contingent on household sentiments. Recall that the bank's solvency constraint coincides with its deposit threshold under good sentiments. This ensures that the bank borrows at the risk-free rate regardless of the sovereign default probability. As a result, the safe strategy payoff is largely independent of $P(\mathbf{S})$ when there are good sentiments.[49] Under bad sentiments, the deposit threshold is tighter than the solvency constraint due to the expectation of high sovereign exposure. Despite following a safe strategy, banks optimally breach the deposit threshold and households anticipate their insolvency under sovereign default. Therefore, as $P(\mathbf{S})$ increases, bank funding costs rise and safe strategy payoff decreases.

$\underline{N}(\mathbf{S})$ traces the levels of net worth where banks are indifferent between the two strategies under good sentiments. Since the payoff from gambling falls in $P(\mathbf{S})$ while that of adopting the safe strategy remains constant, $\underline{N}(\mathbf{S})$ declines sharply as sovereign risk increases. By the same logic, $\overline{N}(\mathbf{S})$ traces the points of indifference under bad sentiments where payoffs under both strategies decrease in $P(\mathbf{S})$. The non-monotonicity in $\overline{N}(\mathbf{S})$ stems from risk aversion. At low levels of $P(\mathbf{S})$, safe strategy payoffs decrease more rapidly such that $\overline{N}(\mathbf{S})$ increases in $P(\mathbf{S})$. As $P(\mathbf{S})$ rises, however, gambling profits decrease at an increasingly faster pace as the increase in bank funding costs becomes more prominent compared to higher sovereign bond yields. The peak of $\overline{N}(\mathbf{S})$ marks a turning point at which a rise in $P(\mathbf{S})$ makes the safe strategy relatively more profitable and $\overline{N}(\mathbf{S})$ falls in response to further increases in $P(\mathbf{S})$.

[48] This stems from the lack of other types of aggregate risk within the model environment. It can, however, be interpreted as the reduced form outcome of a richer environment with capital regulation based on risk-weighted assets. In this environment, capital requirements faced by a bank depend on the risk-weight attached to its portfolio. For assets with non-sovereign risk, positive risk weights align the bank's incentives towards following a safe strategy. If sovereign bonds have zero risk-weight, Sovereign bonds, on the other hand, have a zero risk-weight, then gambling is only possible in the presence of sovereign default risk. The preferential treatment for sovereign bonds described here approximately reflects the regulatory framework in the Euro area (Bank for International Settlements, 2013).

[49] To be precise, the payoff is independent of $P(S)$ when the solvency constraint is binding, which is the case at the boundary of net worth $\overline{N}(\mathbf{S})$. When the solvency constraint is slack, the expected payoff falls slightly as $P(\mathbf{S})$ increases due to a decline in bank lending.

5.3 Propagation of sovereign risk shocks

The next step is to evaluate the propagation of sovereign risk shocks. Figure 15 plots the response of key variables to an increase in fiscal stress by 1.5 standard deviations. The top left panel indicates that the shock increases the probability of sovereign default by the next quarter from 0.6% to 2.3%.[50]

The second panel shows the evolution of aggregate bank net worth and the equilibrium regions. The multiplicity region is depicted by the shaded area. Within this region, the prevalent equilibrium type is determined by household sentiments. Good sentiments (i.e. a high sunspot realization) lead to a safe equilibrium and bad sentiments result in a gambling equilibrium. The equilibrium is unique outside the multiplicity region with a safe equilibrium above it (and a gambling equilibrium below). For exposition's sake, I select an initial level of net worth that lies in this region and consider two specific scenarios. In the first scenario, sentiments come out to be good in each successive period such that there is always a safe equilibrium in the multiplicity region. In the second scenario, successive bad sentiments lead to a gambling equilibrium within the same region.

In the scenario with good sentiments, bank net worth increases rapidly and brings about an early exit from the multiplicity region. With bad sentiments, on the other hand, the economy remains "trapped" in the multiplicity region for a prolonged length of time. Since net worth is retained from bank profits, the implication is that profits are lower in the gambling equilibrium compared to the safe equilibrium. This finding is surprising since, in the absence of a sovereign default event, Figure 15 corresponds to a timeline where a gamble on domestic sovereign bonds is successful. In other words, despite collecting a high yield from their risky bond purchases, banks make lower profits under the gambling equilibrium than the safe equilibrium.[51]

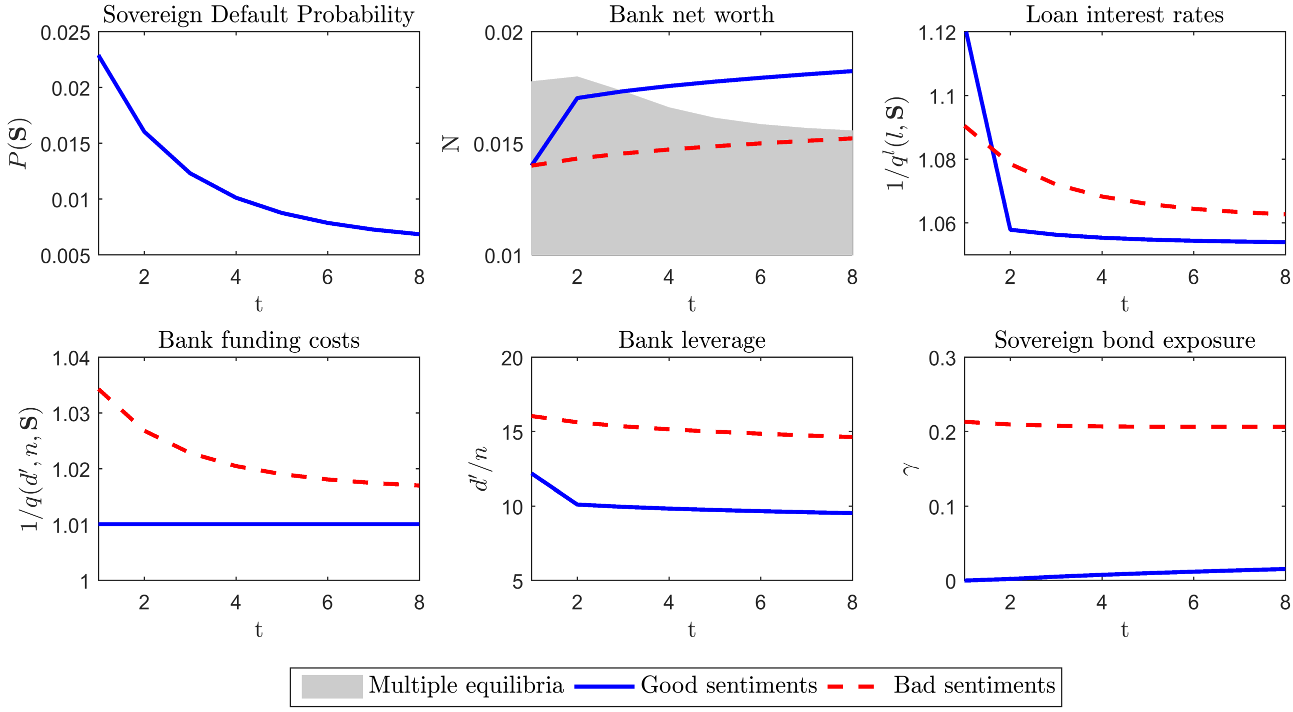
The explanation lies in the impulse responses for bank funding costs and lending. The panels in the second row of Figure 15 show that the gambling equilibrium entails high leverage and exposure to domestic sovereign bonds. This creates the prospect of insolvency in the case of sovereign default, which in turn increases bank funding costs to the detriment of profits.

In contrast, under the safe equilibrium, banks satisfy a solvency constraint that ensures their solvency following sovereign default. This leads to low leverage and sovereign bond exposure such that bank funding costs remain at the risk-free rate. Moreover, the solvency constraint binds in the multiplicity region such that banks reduce their lending to firms. The top right panel shows the rise in loan interest rates caused by this. Together with relatively low funding costs, the excess returns created by the rise in loan interest rates explains the rapid rise in net

[50] Recall that the economy immediately moves to the steady state following sovereign default. The impulse responses in Figure 15 correspond to a timeline where, in each successive period, it is revealed that the government remains solvent.

[51] Recall that banks take household sentiments as given when deciding on their optimal strategies.

Figure 15: **Impulse responses to a sovereign risk shock**



Note: All interest rates are annualized.

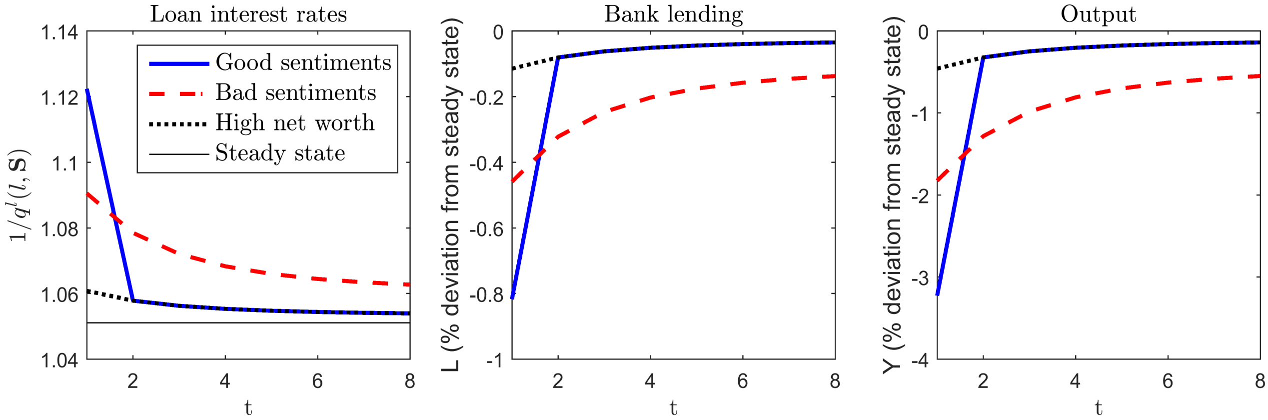
worth under good sentiments.

It is instructive to decompose the increase in loan interest rates, which is proportionate to the decline in bank lending and output. Figure 16 shows impulse responses for loan interest rates, aggregate bank lending and output under the same sovereign risk shock as Figure 15. In addition to the two scenarios above, it plots a third scenario with high initial net worth such that the safe equilibrium is unique and the solvency constraint is slack.

Compared to the (risk-free) steady state, the interest rates on loans increase and bank lending declines even in the high net worth case. This constitutes an 'efficient' decline in bank lending in view of the risk that loans become non-performing under sovereign default. In the scenario with good sentiments (and low initial net worth), bank lending initially declines significantly below the efficient level due to the deleveraging process described above, but returns back to the efficient level from the second period onwards as net worth increases. When there are bad sentiments, on the other hand, bank lending is crowded out by domestic sovereign bond purchases. This leads to a relatively mild, but still significant decline below the efficient level compared to the good sentiments case, with crowding out effects accounting for roughly 75% of the total decline in bank lending (and output). The decline is persistent, however, due to the slow increase in bank net worth.

Overall, the scenarios with good and bad sentiments highlight two alternative paths of

Figure 16: **Decomposition of bank lending**



Note: All values are annualized. Bank lending and output are in percentage points.

adjustment to a sovereign risk shock when the banking sector is under-capitalized. Under the safe equilibrium, the financial soundness of the banking sector is preserved by aggressive deleveraging and there is a sharp but short-lived recession. As banks remain solvent even in the event of sovereign default, bank funding costs remain at the risk-free rate. In contrast, under bad sentiments, the economy becomes stuck in a 'gambling trap' characterized by a banking sector with high domestic sovereign bond exposure and persistent crowding out of bank lending. There is also considerable financial fragility due to the sovereign-bank nexus. If the government defaults at any point before the exit from the multiplicity region, this causes a banking crisis. As such, bank funding costs become highly correlated with sovereign bond yields.[52]

5.4 Comparison with Portuguese data

This section compares the model's fit to Portuguese data. The comparative exercise is conducted by simulating the model economy under a series of sovereign risk shocks ε_d' that exactly match $q^b(\mathbf{S})^{-1}$ to quarterly Portuguese sovereign bond yields over 2010Q1-2016Q1. I also calibrate initial bank net worth to match the Tier 1 capital of the Portuguese banking sector in 2009, while the remainder of the parameters are calibrated as in section 5.1.

Figure 17 contrasts the simulated series under good and bad sentiments (which are taken to

[52] Household and bank values are higher under the safe equilibrium at all times despite the sharp decline in output. With regard to bank values, this finding is due to the increase in net worth under the safe equilibrium. Households place a higher value on the safe equilibrium due to risk aversion. Although the deleveraging by banks causes an initial decline in wages, it precludes a haircut on deposits in the event of sovereign default. Since consumption is already low following sovereign default due to the decline in productivity, risk averse households place a high value on avoiding the haircut. Note that this is true regardless of bank net worth as the size of a haircut on deposits under the gambling equilibrium is proportionate to the decline in bank lending in the safe equilibrium. Furthermore, due to the slow increase in net worth under the gambling equilibrium, this calculus is not affected by the extent of sovereign risk in the current period, but rather the cumulative probability of default until exit from the multiplicity region, which is significantly higher.

be persistent as in the previous section) with data on the Portuguese economy. The first panel displays the sovereign default probabilities implied by the match with Portuguese sovereign bond yields. The probability of government default by the next quarter peaks at 2.78% in the final quarter of 2011.

The second panel shows the simulated series for bank net worth and the multiplicity region which evolves according to changes in sovereign default probabilities as explained in section 5.2. The simulation places the Portuguese economy in the region with a unique gambling equilibrium in 2010Q1, after which it enters the multiplicity region. Thereafter, bank net worth follows different paths under good and bad sentiments. As in the previous section, good sentiments result in a safe equilibrium and a rapid increase in net worth that moves the economy into the region with a unique safe equilibrium. Bad sentiments, on the other hand, lead to a gambling equilibrium with stagnating net worth such that the economy remains in the multiplicity region.

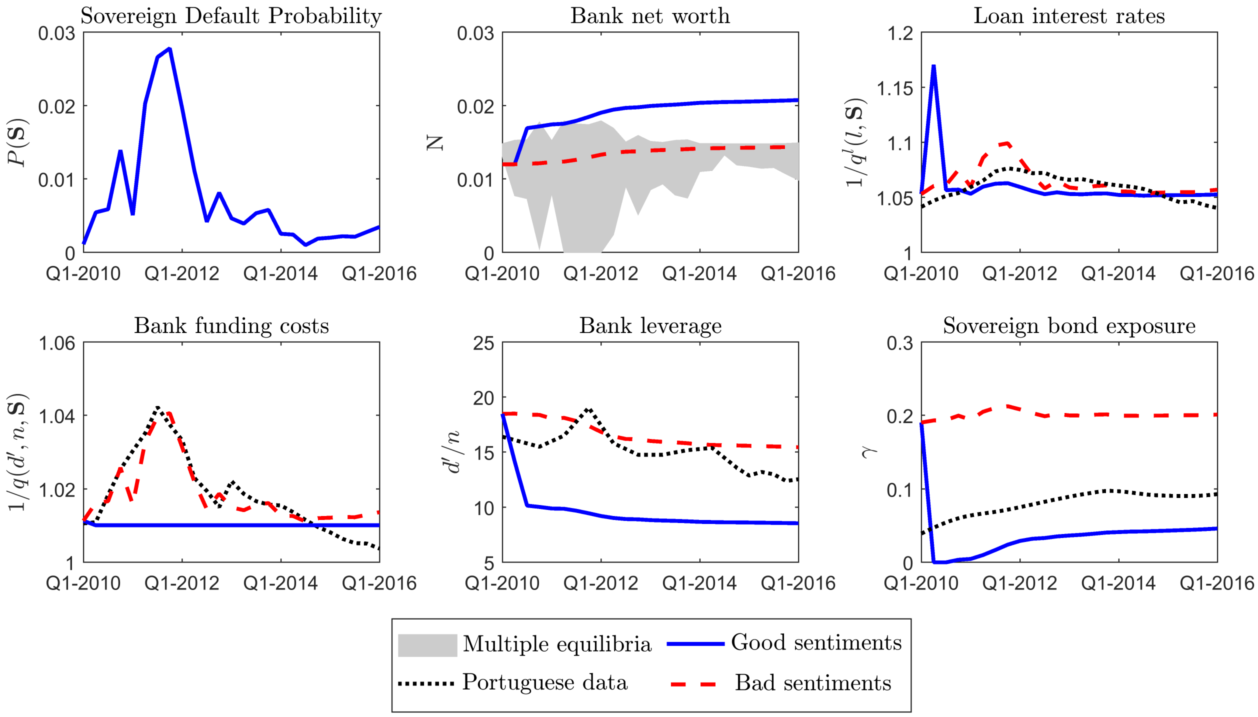
The model has partial success in emulating changes in loan interest rates. The third panel shows that the simulated series under bad sentiments captures the initial increase in loan interest rates but overshoots at the peak of the sovereign debt crisis in 2011-2012, and slightly undershoots thereafter. The simulated series under good sentiments suggests a large increase in interest rates in 2011, which is not reflected in the data. This is due to the binding solvency constraint prior to the exit from the multiplicity region, which leads to a significant decline in bank lending as in the previous section.

The model's main success is in replicating the evolution of bank funding costs. As shown in the fourth panel, the simulated series under bad sentiments provides a very close match to deposit interest rates in Portugal. Both of these series correlate highly with sovereign default probabilities. Under good sentiments, on the other hand, interest rates remain at the risk-free rate from 2010Q2 onwards. The fifth panel compares the simulated series for bank leverage with the leverage ratio of the Portuguese banking sector.[53] The simulated series under bad sentiments somewhat overshoots its counterpart in data, but captures the slow decline in bank leverage over the crisis period.

Finally, the last panel contrasts the share of funds spent on domestic sovereign bond purchases. The series under bad sentiments reflects the gradual increase in the exposure to domestic sovereign debt, but indicates a higher exposure than is observed in the data. A potential explanation for this is that the data accounts only for direct exposure via sovereign bond holdings, whereas a bank's actual exposure to domestic sovereign risk also involves indirect exposure through holdings of assets with correlated risk, such as government bonds of other risky European countries and securities issued by banks with a high exposure to these. The simulation

[53]Although the latter containts non-depository liabilities which are not directly present in the model, the nature of deposits as a choice variable captures the optimal leverage decision of banks.

Figure 17: **Comparison with Portuguese data**



Note: All interest rates are annualized. Sovereign default probabilities are extracted from the yield spread between Portuguese and German bonds with 3 month maturity remaining as described in section 5.1. The domestic sovereign bond exposure series is constructed using data from stress tests and transparency exercises conducted between 2009-2016 as well as individual bank balance sheets. See the Technical Appendix for further details about data. Source: OECD, ECB, EBA.

under good sentiments indicates a large drop in exposure which is not present in the data.

Overall, the gambling equilibrium, which is consistent with bad sentiments, has more success in replicating Portuguese data than the scenario with good sentiments.

6 Policy analysis

This section evaluates policy interventions aimed at strengthening the banking sector and reinvigorating bank lending. It is clear from the numerical results in section 5 that both of these aims can be achieved with a capital injection to the banking sector that directly increases bank net worth N. However, this requires a significant transfer of resources at a time when the government is cash-struck.

Instead, I focus on unconventional interventions that can be implemented by the central bank and macroprudential policy measures. Section 6.1 considers (non-targeted) liquidity provision to the banking sector by the central bank, which is comparable to the ECB's longer-term refinancing operations (LTRO) in the stylized environment of the model. Section 6.2 proposes an alternative measure, *targeted* liquidity provision, where the central bank provides liquidity conditional on bank leverage. Finally, section 6.3 shows that the findings from sections 6.1 and 6.2 can be generalized to provide insights for deposit insurance and a range of macroprudential policies.

6.1 Liquidity provision

I incorporate liquidity provision into the model by allowing each bank to issue debt $d^c \leq \bar{d}^c$ to the central bank at a risk-free price q^*.[54] It is instructive to first evaluate this intervention in the two-period environment described in section 3 before transitioning to a dynamic setting. With access to central bank liquidity, the representative bank's budget constraint and profits become

$$\begin{aligned} n + qd + q^* d^c &= q^b b + q^l l \\ \pi &= \max\{0, l + b - d - d^c\} \\ \underline{\pi} &= \max\left\{0, \theta^l l + \theta^b b - d - d^c\right\} \end{aligned}$$

Crucially, the effects of central bank liquidity hinge on whether it leads to a transfer of bank insolvency risk from depositors to the central bank.

Liquidity provision with no risk transfer Consider first the case with no risk transfer such that liabilities to the central bank have greater seniority than deposits. In other words, debt repayments to the central bank take priority over deposits in the event that the bank becomes insolvent. This ensures that the central bank is not exposed to any potential losses at the expense of diluting depositors' claim to bank revenues.[55]

The dilution of deposits proves to be crucial in undermining the policy intervention. It creates a negative relationship between the amount of central bank liquidity d^c held by the bank and the recovery rate of deposits θ. This is reflected in the deposit demand schedule,

[54] I abstain from collateral requirements on debt issued to the central bank. In practice, collateral requirements do not preclude the form of gambling considered here as long as risky domestic sovereign debt is eligible as collateral. This is the case with LTROs since the ECB's decision to suspend collateral eligibility requirements for sovereign debt issued by distressed Euro area countries (European Central Bank, 2012). In this context, placing a haircut on sovereign debt pledged as collateral is equivalent to a reduction in $\bar{d}_c$.

[55] This is true unless the liquidity provided by the central bank exceeds total bank revenues under sovereign default. The restriction $\bar{d}^c \leq \frac{\theta^b}{1-\theta^b} d$ is sufficient to preclude this, and is satisfied under plausible values for $\bar{d}^c$.

which is now given by

$$q^c(\tilde{\gamma}, d, d^c) = \begin{cases} q^* & \text{for } d + d^c \leq \bar{d}(\tilde{\gamma}) \\ q^* \frac{1-P+P\left(\left(\tilde{\gamma}\frac{\theta^b}{q^b}+(1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)\left(\frac{n+q^* d^c}{d}\right)-\frac{d^c}{d}\right)}{1-q^* P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+(1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)} & \text{for } d + d^c > \bar{d}(\tilde{\gamma}) \end{cases} \tag{72}$$

where the deposit threshold $\bar{d}(\tilde{\gamma})$ remains unchanged. When the parameter restrictions in (15) are satisfied, a rise in central bank liquidity d^c leads to an inward shift in the deposit demand schedule. Using (14) and (72), it is easy to show that the bank's ability to raise funds is independent of d^c such that

$$q^c(\tilde{\gamma}, d, d^c)\, d + d^c = q(\tilde{\gamma}, d)\, d \quad \forall\ d^c \leq \bar{d}^c$$

where $q(\tilde{\gamma}, d)$ is the deposit demand schedule in the absence of liquidity provision. This indicates that the deterioration in bank borrowing conditions due to dilution exactly offsets the gains from central bank liquidity. Consequently, liquidity provision is completely ineffective without a risk transfer to the central bank.

Liquidity provision with risk transfer Now consider the case where the repayment of deposits takes priority over obligations to the central bank. This constitutes an implicit transfer of bank insolvency risk from depositors to the central bank as the recovery rate of deposits increases at the expense of central bank losses. The deposit demand schedule is then given by the expressions

$$q^c(\tilde{\gamma}, d, d^c) = \left\{ \begin{array}{ll} q^* & \text{for } d \leq \bar{d}^c(\tilde{\gamma}, d^c) \\ q^* \frac{1-P+P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+(1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)\left(\frac{n+q^* d^c}{d}\right)}{1-q^* P\left(\tilde{\gamma}\frac{\theta^b}{q^b}+(1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)} & \text{for } d > \bar{d}^c(\tilde{\gamma}, d^c) \end{array} \right\}, \tag{73}$$

$$\bar{d}^c(\tilde{\gamma}, d^c) = \frac{\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)(n + q^* d^c)}{1 - q^*\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)}$$

Rather than being diluted, the expected value of deposits increases in central bank liquidity d^c, causing an outwards shift in the deposit demand schedule as shown in Figure 18.

To evaluate the implications of this, consider first the case under good sentiments. Suppose the representative bank adopts the safe strategy. Liquidity provision will then have no impact on its expected payoff and the bank will be indifferent to central bank liquidity. This is due to two reasons. First, the safe strategy requires that the bank remains solvent under sovereign default. This precludes the bank from taking advantage of the risk transfer. Second, recall from section 3.2.1 that banks which follow a safe strategy borrow from depositors at the risk-free rate under good sentiments. As such, the provision of cheap liquidity by the central bank does not lead to a reduction in bank funding costs.

Suppose instead that the bank deviates to a gambling strategy. It will optimally borrow the maximum amount $\bar{d}^c$ from the central bank, both to directly benefit from low interest rates attached to central bank liquidity and to attain a more favorable deposit demand schedule by facilitating the transfer of risk away from depositors.

Note that the first order conditions (20), (21) for the gambling strategy remain unchanged. Therefore, the bank does not change its lending to firms in response to liquidity provision. Instead, it takes advantage of the outward shift in its deposit demand schedule to increase its deposits and domestic sovereign bond purchases until its borrowing costs return to their level prior to the intervention. Therefore, the recovery rate of deposits θ also remains at its pre-intervention level such that depositors face the same amount of insolvency risk. In other words, the risk transfer simply provides the bank with an opportunity to increase the extent of its gamble on domestic sovereign bonds at the expense of the central bank. Accordingly, the expected payoff associated with a deviation to gambling increases to

$$v_{g|s} = (1-P)\,\mu_l l_{g|s} + \frac{n}{q^*} + P\bar{d}^c \tag{74}$$

Using (74), Proposition 5 shows that liquidity provision (with risk transfer) backfires by eliminating the safe equilibrium.

Proposition 5 *The gambling equilibrium is unique for all n when*

$$\bar{d}^c > \tilde{d}^c \equiv \frac{\mu_l\left(\alpha A\left(q_g^l\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}}{P}\left[\left(1-P+\nu P\theta^l\right)\left(\frac{1-P+P\theta^l}{1-P+P\theta^b}\right)^{\frac{\alpha}{1-\alpha}} - (1-P)\right]$$

When $\bar{d}^c \leq \tilde{d}^c$, the gambling equilibrium is unique for $n \leq \underline{n}$ where $\underline{n}$ is implicitly defined by the expression

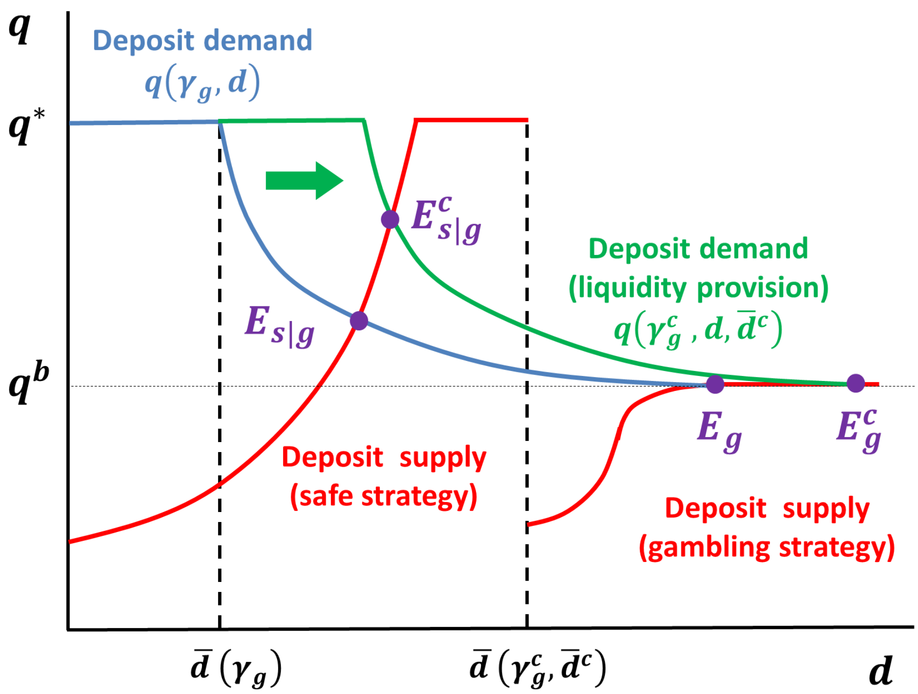
$$\begin{aligned}\underline{n} \;=\; & \left(\frac{1}{\nu}\right)^{\frac{1-\alpha}{\alpha}}\left(\frac{1}{A\alpha}\frac{q^*(1-P)\,\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}(A\alpha)^{\frac{1}{1-\alpha}} + \underline{n} + q^*\bar{d}^c}{q^*(1-P)\left[1-\theta^l+\mu_l\frac{1-\upsilon}{\upsilon}\right]}\right)^{\frac{1}{\alpha}} \\ & -\theta^l\frac{q^*(1-P)\,\mu_l\left(q_g^l\right)^{\frac{\alpha}{1-\alpha}}(A\alpha)^{\frac{1}{1-\alpha}} + \underline{n} + q^*\bar{d}^c}{(1-P)\left[\left(1-\theta^l\right)+\mu_l\frac{1-\nu}{\nu}\right]}\end{aligned}$$

and

$$\frac{\partial\underline{n}}{\partial\bar{d}^c} > 0$$

Proof. Provided in the Technical Appendix. ■

Figure 18: **Liquidity provision with risk transfer**



Note: The deposit demand schedule is attained by combining (73) with (29). The deposit supply curve stems from the combination of (3), (25), (26) and (47). Only the case with a slack solvency constraint is included. For the remaining cases see Figure 12.

The first part of the proposition shows that when banks have access to central bank liquidity in excess of an upper bound $\tilde{\bar{d}}^c$, they find it optimal to gamble even when the solvency constraint is slack. Gambling then becomes the unique equilibrium regardless of bank net worth. The second part shows that even for $\bar{d}^c \leq \tilde{\bar{d}}^c$, the intervention shifts up the boundary of net worth $\underline{n}$ below which there is a unique gambling equilibrium.

The case under bad sentiments is depicted in Figure 18. The outcome under the gambling strategy is similar to the deviation to gambling considered above. In contrast to the case with good sentiments, however, it is possible for banks deviating to the safe strategy to face borrowing costs above the risk-free rate.[56] Liquidity provision may then increase the expected payoff associated with the safe strategy by reducing bank funding costs. Therefore, the two period environment is ambiguous as to whether the upper boundary of the multiplicity region $\bar{n}$ increases or decreases in response to liquidity provision.

[56]Figure 18 provides an example of this where the solvency constraint remains slack. It is also possible for the solvency constraint to become binding as shown in the third panel of Figure 12. In this case, liquidity provision leads to a relaxation of the solvency constraint.

In order to analyze this, I conduct a policy experiment based on an extension of the dynamic model in section 4. Specifically, I extend the dynamic model to include liquidity provision (with risk transfer) as a pre-determined state variable $\bar{d}^c$.[57] For T periods, this variable follows a pre-determined path $\left\{\bar{d}_t^c\right\}_{t=0}^T$ before returning to zero permanently.[58]

I opt for this set up for two reasons. First, in the absence of debt with long-term maturity, giving banks the option to rollover their debt for T periods approximates the maturity structure of the LTROs.[59] Second, this set up allows for the solution of the extended model by iterating backwards from the end date T. This makes the additional computational burden from including the policy intervention negligible.[60]

Figure 19 plots the impulse responses to the same sovereign risk shock as in section 5.3.[61] The first panel shows that the multiplicity region shifts upwards and expands significantly due to the policy intervention. As a result of this, the economy remains in the multiplicity region even after the deleveraging process under good sentiments. Under bad sentiments, on the other hand, net worth increases slightly faster relative to the baseline case due to the increase in gambling profits. The lower boundary of the multiplicity region also shifts up, however, and entry into the region with a unique gambling equilibrium is only narrowly avoided.

The remaining panels highlight the changes in the gambling equilibrium under the policy intervention.[62] As in the two period environment, banks respond to liquidity provision by increasing their sovereign exposure until their funding costs return to their pre-intervention level. The top right panel shows that leverage initially increases due to the rise in borrowing by banks (both from the central bank and depositors) but falls below the baseline level over time as net worth increases more rapidly.

Overall, it appears that when liquidity provision transfers insolvency risk from depositors to the central bank, it backfires not only by eliminating the safe equilibrium at low levels of

[57]The changes in the deposit demand schedule and the bank's problem are similar to the two period model. I relegate the relevant expressions to Appendix C in the interest of brevity.

[58]The equilibrium allocation in the steady state after sovereign default is independent of $\bar{d}^c$. Therefore, there is no need to take a stance on the evolution of $\bar{d}_t^c$ following sovereign default.

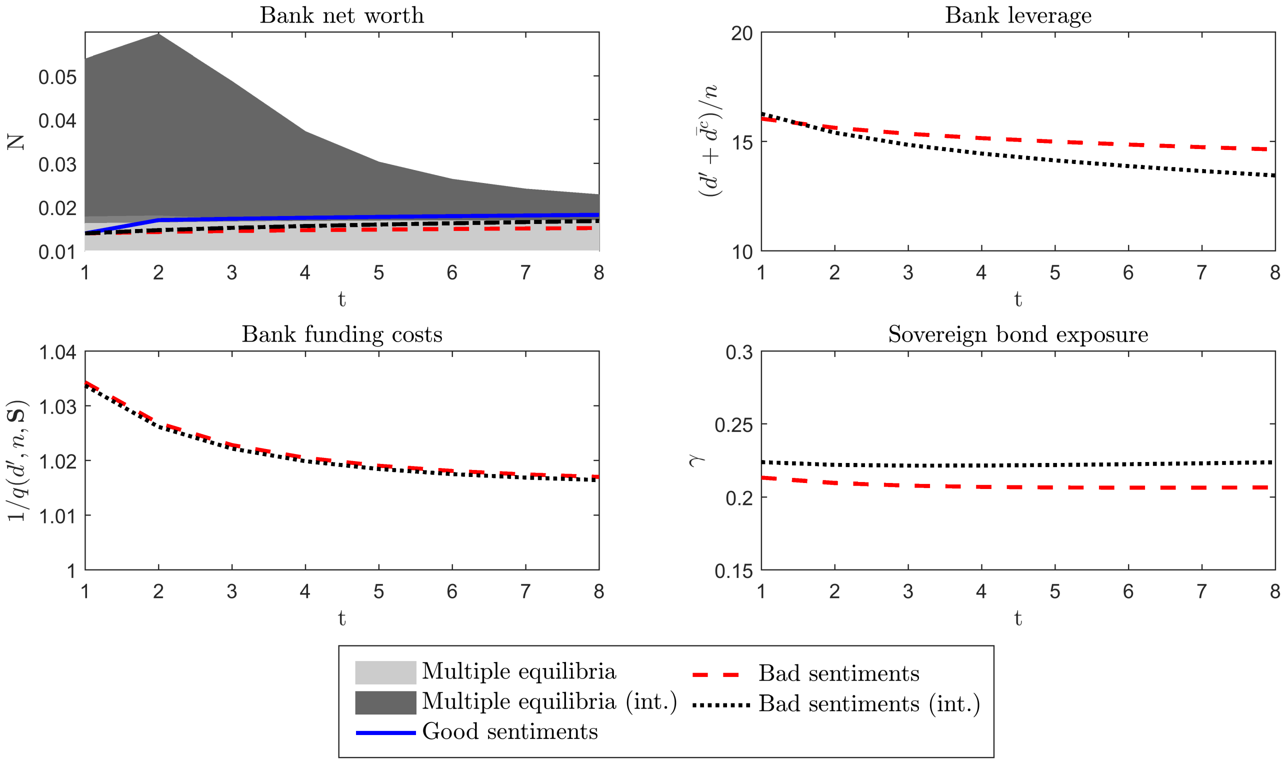
[59]The LTROs had a 3 year maturity with an early repayment option after 1 year (European Central Bank, 2011). In the context of the model, exercising the early repayment option is equivalent to choosing $d_t^c = 0$ for the remaining periods. Although this does not exactly correspond to the single window for repayment in LTROs, it emerges as a result that banks either strictly prefer to take the maximum amount of funding in each period or are indifferent to the amount of central bank liquidity they receive. Therefore, the frequency and timing of the early repayment option has no impact on the numerical results.

[60]When the policy expires at $T+1$, the extended model becomes identical to the baseline model. Therefore, future expectations at T for $\left\{\mathcal{E}_{T+1}, \Gamma_{T+1}(\mathbf{S}), X_{T+1}(\mathbf{S}), v_{T+1}^b(n, \mathbf{S})\right\}$ can be attained by taking expectations according to the solution to the baseline model. The solution to the model at period T is then attained by using the steps in section 4.7. Instead of iterating until convergence, the solution $\left\{\mathcal{E}_T, \Gamma_T(\mathbf{S}), X_T(\mathbf{S}), v_T^b(n, \mathbf{S})\right\}$ is used to take expectations for $T-1$. This process is repeated until $t=0$.

[61]I calibrate $T = 12$ in line with LTROs and set $\bar{d}_t^c = \bar{d}^c < \tilde{d}^c$. The remaining parameters follow the baseline calibration in section 5.1.

[62]The impulse responses under good sentiments, and those for loan interest rates are excluded as they remain identical to the baseline case in Figure 15.

Figure 19: **Liquidity provision in a dynamic environment**



Note: All interest rates are annualized.

net worth, but also by expanding multiplicity to higher levels of net worth. When combined with the irrelevance of liquidity provision–sans–risk transfer, this leads to the conclusion that the equilibrium outcome cannot be improved through the indiscriminate provision of liquidity to the banking sector.

This negative result stems from the inability of non-targeted interventions to distinguish between banking strategies, which in turn leads to a trade-off between alleviating funding conditions under the safe strategy and strengthening incentives to gamble. In the next section, I propose a targeted intervention that overcomes this trade-off.

6.2 Targeted liquidity provision

Under targeted liquidity provision, the central bank offers a liquidity schedule $\bar{d}^c(d,n)$ conditional on deposits and bank net worth. By offering a liquidity schedule

$$\bar{d}^c(n,d) = \frac{\left(\frac{\theta^l}{q^l_s} - \frac{\theta^b}{q^b}\right) q^l_s l_s + \frac{\theta^b}{q^b} n}{1 - \frac{\theta^b}{q^b} q^*} - d \tag{75}$$

which overlaps with the solvency constraint under good sentiments, the central bank can completely insulate the banking sector from shifts in depositor sentiments.

By design, the schedule has no impact on banks' funding conditions under good sentiments. When there is a shift to bad sentiments, however, it provides banks with low cost liquidity in a manner that artificially re-creates the funding conditions under good sentiments. It then follows directly from the equilibrium conditions (43), (44) that bad sentiments cease to be self-fulfilling throughout the multiplicity region. The intervention remains strictly off-equilibrium when it is successful, since banks are indifferent between central bank and deposit funding in the safe equilibrium.

The conditionalities on (n, d) are crucial for the success of the intervention. By placing an upper bound on participating banks' leverage, these conditionalities ensure that banks do not find it optimal to take up central bank liquidity under the gambling strategy. This overcomes the trade-off faced by non-targeted liquidity provision, allowing the intervention to improve banks' funding conditions under the safe strategy without increasing incentives to gamble.

Note that the results from section 6.1 with regard to the irrelevance of liquidity provision without a risk transfer remain valid. Therefore, at least in principle, the targeted intervention requires that the central bank becomes exposed to bank insolvency risk.[63] In practice, however, the central bank never faces losses under targeted liquidity provision. This is not just due to the fact that successful interventions are never implemented in equilibrium. Even if the liquidity schedule is offered in the region with a unique gambling equilibrium such that the intervention fails, the conditionalities ensure that banks do not take up central bank liquidity in a gambling equilibrium.

The role of the conditionalities is thus twofold. First, they drive a wedge between the safe and gambling strategies and allow the central bank to make the former more attractive, thereby eliminating multiplicity in favour of the safe equilibrium. Second, they ensure that the central bank is not subject to losses even when the intervention is unsuccessful.

Finally, note that the central bank does not need to make liquidity support contingent on the sovereign exposure γ in order to implement this intervention. This raises the question as to why the central bank is capable of carrying out this intervention while the households cannot. The answer lies in the ability of the central bank to internalize the equilibrium-switching effects of its behaviour, and thus commit to the liquidity schedule in (75). In contrast, for atomistic households that take sentiments as given, (75) is strictly sub-optimal to the deposit demand schedule. In other words, targeted liquidity provision resolves a coordination problem between

[63] This does not necessarily need to take the form of an explicit arrangement where depositors have greater seniority. When the central bank has priority in debt repayments, providing the liquidity schedule above under bad sentiments completely crowds out deposit funding. Without deposits to act as a buffer, bank insolvency results in losses for the central bank.

banks and depositors.[64]

6.3 Deposit insurance and macroprudential regulation

In this section, I generalize the findings from sections 6.1 and 6.2 to a wider set of policy instruments. To begin with, consider deposit insurance in the form of a limited amount of funds F/υ dedicated to increasing the recovery rate θ of deposits, which can then be written as

$$\theta = \min \left\{ 1, \left(\tilde{\gamma} \frac{\theta^b}{q^b} + (1 - \tilde{\gamma}) \frac{\theta^l}{q^l} \right) \left(\frac{n}{d} + q \right) + \frac{F}{d} \right\}$$

This leads to the following deposit demand schedule

$$q^F(\tilde{\gamma}, d, F) = \begin{cases} q^* & \text{for } d \leq \bar{d}^F(\tilde{\gamma}, F) \\ q^* \frac{1 - P + P\left(\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)n + F\right)\frac{1}{d}}{1 - q^* P\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)} & \text{for } d > \bar{d}^F(\tilde{\gamma}, F) \end{cases} \tag{76}$$

$$\bar{d}^F(\tilde{\gamma}, F) = \frac{\left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right) n + F}{1 - q^* \left(\tilde{\gamma}\frac{\theta^b}{q^b} + (1-\tilde{\gamma})\frac{\theta^l}{q^l}\right)}$$

which indicates that deposit insurance leads to an outward shift in the deposit demand schedule. Proposition 6 shows that, on its own, deposit insurance backfires in the same manner as non-targeted liquidity provision (with risk transfer).

Proposition 6 *For any arbitrary* $\varepsilon \geq 0$

$$q^F(\tilde{\gamma}, d, \varepsilon) d = q^c(\tilde{\gamma}, d, \varepsilon) d + q^* \varepsilon$$

Proof. Provided in the Technical Appendix. ■

As before, the negative result stems from the trade-off between alleviating funding conditions and strengthening incentives to gamble. This trade-off can be overcome with the use of macroprudential regulation. Specifically, the combination of deposit insurance with a regulatory constraint on bank liabilities can lead to a similar outcome to targeted liquidity provision. This is achieved by dedicating sufficient funds to deposit insurance to offset the effects of a shift

[64] Note that targeted liquidity provision differs from the targeted longer-term refinancing operations (TLTROs) implemented by the ECB in that the latter provide liquidity conditional on bank lending. In the setting here, liquidity provision conditional on l does not affect incentives to gamble since banks have the ability to further increase their leverage to purchase sovereign bonds after satisfying the lending conditionality. Therefore, it is largely similar to non-targeted liquidity provision, with the addition that it may lead to a rise in bank lending in the gambling equilibrium when sufficient liquidity is provided along with a risk transfer.

to bad sentiments on the deposit demand schedule

$$F = \left(\frac{\theta^l}{q_s^l} - \frac{\theta^b}{q^b}\right) q_s^l l_s$$

and imposing a regulatory constraint that overlaps with the solvency constraint in the safe equilibrium[65]

$$d \leq \frac{F + \frac{\theta^b}{q^b} n}{1 - \frac{\theta^b}{q^b} q^*}$$

Finally, note that the same outcome can be achieved with alternative forms of macroprudential regulation. For example, the liability constraint above is interchangeable with a constraint on asset holdings or capital requirements in a richer environment with equity issuance, provided that there is a positive risk-weight attached to domestic sovereign bond holdings.

7 Conclusion

This paper proposes a general equilibrium macroeconomic model with optimizing banks and depositors to analyze macroeconomic adjustment to financial and debt crises and draw insights for policy design. Two important findings emerge as a consequence. First, non-contractibility of banks' portfolio exposures leads to strategic complementarities between banks and depositors as depositors demand a return on deposits according to their expectations on bank risk-taking, and banks determine their risk-taking strategy according to their funding costs. This raises the possibility of multiple equilibria, where a safe equilibrium is characterized by low risk-taking and funding costs, and a gambling equilibrium is associated with bank insolvency risk and high funding costs.

Second, macroeconomic adjustment to crises differs substantially between the two equilibria. In a safe equilibrium, deleveraging by banks preserves the financial soundness of the banking sector at the expense of a sharp, brief drop in output. In a gambling equilibrium, banks respond to a crisis by increasing their exposure to aggregate risk. This leads to a rise in bank funding costs and the crowding out of bank lending to the private sector. The economy may then become stuck in a gambling trap with a prolonged period of financial fragility and a persistent drop in investment and output. The model is calibrated to Portugal over 2010-2016 and simulated dynamics under the gambling equilibrium account for macroeconomic dynamics in Portugal over the sovereign debt crisis. More generally, the model's implications are also consistent with

[65] If participation in the deposit insurance and macroprudential regulation scheme is non-voluntary, the failure of the policy may lead to the use of deposit insurance funds in equilibrium. In the region with a unique gambling equilibrium, banks respond to a non-voluntary scheme by following a gambling strategy despite satisfying the regulatory constraint.

stylized facts from the European sovereign debt crisis.

The model can also be used as a framework for policy analysis. As a novel insight, it indicates that a key prerequisite for successful liquidity interventions by central banks is that they provide some risk-sharing with depositors. Otherwise, liquidity interventions are completely ineffective as depositors raise bank funding costs in anticipation of the dilution of their claims to bank revenues. A second insight pertains to the targeting of interventions. Non-targeted interventions that provide liquidity (and risk sharing) unconditionally may eliminate the safe equilibrium and perpetuate gambling traps. This is because these interventions face a trade-off between alleviating funding constraints and strengthening incentives to gamble. It is possible to overcome this trade-off with a targeted intervention that provides liquidity conditional on bank leverage.

Finally, it is important to stress that the mechanisms considered in this paper can be interpreted in a broader context than a sovereign debt crisis. Incentives to gamble are strong whenever an asset's payoff is highly correlated to a bank's own insolvency risk. This would be the case, for example, with aggregate risky assets or illiquid assets that the bank has a large pre-existing exposure to. Self-fulfilling sentiments may then arise for creditors which are not covered by government guarantees, especially when regulation is perceived to be insufficiently strict to prevent excessive risk-taking. Nevertheless, these mechanisms are particularly strong in the case of domestic sovereign bonds due to the triple coincidence of high correlation between sovereign default risk and aggregate risk, zero risk-weight in regulation for domestic sovereign bonds and concerns about the credibility of government guarantees during a sovereign default episode.

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8 Appendix

A Deviation to the safe strategy

In the first case, the bank has sufficient net worth to satisfy the first order condition (26) while remaining within the deposit threshold $\bar{d}\left(\gamma_g\right)$. Its deposits are thus valued on par with safe assets $q_{s|g} = q^*$ and its valuation of loans is equivalent to the equilibrium counterpart such that $q^l_{s|g} = q^l_s$. There are, however, two notable differences. First, as with the deviation to gambling, the quantity of lending $l_{s|g}$ is conditional on lending provided by the remaining banks such that

$$l_{s|g} = \left(q^l_s\right)^{\frac{\alpha}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} - \frac{1-\nu}{\nu} l_g \tag{77}$$

Second, the inward shift in the deposit threshold $\bar{d}\left(\gamma_g\right)$ under bad sentiments increases the boundary of net worth

$$n_{r|g} \equiv \left(\frac{q^b - \theta^b q^*}{q^b}\right) q^l_s l_{s|g} > n_c \tag{78}$$

required for this case to be valid.

Note also that the deviating bank's expected payoff is given by the expression

$$v_{s|g} = \left(1 - P + P\theta^l\right) \mu_l l_{s|g} + \frac{n}{q^*} \tag{79}$$

which differs from (39) only in terms of $l_{s|g}$.[66] This reflects that a shift to bad sentiments has no impact on the bank's ability to borrow when its net worth lies above $n_{r|g}$.

In the second case, bank net worth falls short of $n_{r|g}$ such that it is not possible to satisfy (26) without breaching the deposit threshold $\bar{d}\left(\gamma_g\right)$. The optimal allocation, leaves the bank with a level of deposits $d_{s|g} > \bar{d}\left(\gamma_g\right)$ which is in the "risky" region of the deposit demand schedule with $q_{s|g} < q^*$, while the actual solvency constraint is slack. Proposition 2 indicates that there are no domestic sovereign bond purchases $\left(b_{s|g} = 0\right)$ in this case, while the price and quantity of loans are pinned down by the first order condition (26) as

$$q^l_{s|g} = \left(1 - P + P\theta^l\right) \left(1 - \mu_l\right) q^b \tag{80}$$

$$l_{s|g} = \left(q^l_{s|g}\right)^{\frac{\alpha}{1-\alpha}} (\alpha A)^{\frac{1}{1-\alpha}} - \frac{1-\nu}{\nu} l_g \tag{81}$$

Using the budget constraint, the price of deposits can also be written as

$$d_{s|g} = \frac{q^l_{s|g}}{q^b} l_{s|g} - \frac{n}{(1-P)\, q^*}$$

[66] As with the safe equilibrium, domestic sovereign bond purchases $b_{s|g}$ and deposits $d_{s|g}$ are indeterminate in this case but have no impact on expected payoff.

and the deviating bank's expected payoff is given by the expression

$$v_{s|g} = \left(1 - P + P\theta^l\right) \mu_l l_{s|g} + \frac{n}{(1-P)\, q^*} \tag{82}$$

which is lower than (79) due to the increase in bank funding costs.

The solvency constraint binds in the third case. The quantity of loans is determined implicitly by the expression

$$\left(\frac{\left(l_{s|g} + \frac{1-\nu}{\nu} l_g\right)^{\frac{1}{1-\alpha}}}{(\alpha A)^{\frac{1}{a}}} - q^b \theta^l \right) l_{s|g} = \frac{q^b}{1-P} \frac{n}{q^*}$$

attained by using (47) and (81) to substitute for $q\left(\gamma_g, d_{s|g}\right)$ and $q^l_{s|g}$ in (48). The expression for expected payoff in this case is identical to the constrained case of the safe equilibrium

$$v_{s|g} = (1-P)\left(1 - \theta^l\right) l_{s|g} \tag{83}$$

and this case is valid when net worth is below the boundary

$$n < n_{c|g} \equiv \left(\frac{q^l_{s|g}}{q^b} - \theta^l \right) (1-P)\, q^* l_{s|g} \tag{84}$$

where $\left(q^l_{s|g}, l_{s|g}\right)$ are defined according to (80) and (81).[67]

B Household's recursive problem

Households supply labour inelastically to firms and have risk averse preferences with their flow utility $u(c)$ given by a standard CRRA specification. The representative household's problem can be written as

$$v^h\left(D, D^*; \mathbf{S}\right) = \max_{c, D', D^{*\prime}} \left\{ \begin{array}{c} u(c) + \beta\left(1 - P(\mathbf{S})\right) \mathbb{E}_{\mathbf{S}}\left[v^h\left(D', D^{*\prime}; \mathbf{S}'\right)\right] \\ +\beta P(\mathbf{S})\, \underline{v}^h\left(D', D^{*\prime}; \mathbf{S}'\right) \end{array} \right\}$$

[67] The discontinuous jump in $\mu_d\left(\gamma_g, d\right)$ as deposits $d_{s|g}$ cross the threshold $d\left(\gamma_g\right)$ leads to the possibility of a fourth case. In this case, net worth is below $n_{r|g}$ but the first order condition (80) associated with the second case leads the bank to select a level of deposits within the threshold $d_{s|g} \leq d\left(\gamma_g\right)$. The optimal behaviour of the deviating bank, and the associated net worth boundaries can be then be determined by treating the deposit threshold as a binding constraint. I relegate this case to the Technical Appendix, as it does not have an impact on the mechanism or the outcome.

subject to

$$\begin{aligned} c + qD' + q^* D^{*\prime} &= D + D^* - T(\mathbf{S}) + w(\mathbf{S}) \\ \mathbf{S}' &= \mathbf{\Gamma}(\mathbf{S}) \end{aligned} \tag{85}$$

where $\Gamma(.)$ is the law of motion for the aggregate state variables and $\underline{v}^h(.)$ represents the household's continuation value under sovereign default. Lemma 6 provides an expression for $\underline{v}^h(.)$.

Lemma 6 *The continuation value for households in the steady state $\underline{S}$ is*

$$\underline{v}^h(D', D^{*\prime}; \mathbf{S}) = \frac{1}{1-\beta} u(\underline{c}) \ ,$$
$$\underline{c} = (1-q^*)\left(\theta D' + D^{*\prime} + \frac{1-\alpha}{\alpha}\frac{\underline{A}}{A} L(\mathbf{S})\right) + q^* \underline{w} - \underline{T}$$

where $\underline{w}$ is given by

$$\underline{w} = (1-\alpha)\,\underline{AK^\alpha}$$

Proof. Provided in the Technical Appendix. ■

Observe that consumption $\underline{c}$ in the steady state is positively related to household wealth after sovereign default, which is increasing in the recovery rate θ of domestic deposits. Using the above expressions, the first order conditions for risk-free assets D^* and domestic bank deposits D can be written as

$$\begin{aligned} q^* &= \beta \frac{(1-P(\mathbf{S}))\,u_c(c') + P(\mathbf{S})\,u_c(\underline{c})}{u_c(c)} \\ q &= \beta \frac{(1-P(\mathbf{S}))\,u_c(c') + P(\mathbf{S})\,\theta u_c(\underline{c})}{u_c(c)} \end{aligned}$$

where $u_c(.)$ is marginal utility.

As in section 3.1.5, the recovery rate anticipated by households depends on household expectations about the bank's domestic sovereign bond exposure $\tilde{\gamma}(n, \mathbf{S})$.

$$\theta = \min\left\{1, \left(\tilde{\gamma}(n,\mathbf{S})\frac{\theta^b}{q^b(\mathbf{S})} + (1-\tilde{\gamma}(n,\mathbf{S}))\frac{\theta^l}{q^l(\mathbf{S})}\right)\left(\frac{n}{d'} + q\right)\right\}$$

The deposit demand schedule is attained by combining this expression with the household's first order conditions.

C Liquidity Provision

In periods $t \leq T$, the model is characterized by

$$\bar{d}(n, \mathbf{S}) = \frac{\left(\tilde{\gamma}(n, \mathbf{S}) \frac{\theta^b}{q^b(\mathbf{S})} + (1 - \tilde{\gamma}(n, \mathbf{S})) \frac{\theta^l}{q^l(\mathbf{S})}\right)(n + q^* d_t^c)}{1 - q^* \left(\tilde{\gamma}(n, \mathbf{S}) \frac{\theta^b}{q^b(\mathbf{S})} + (1 - \tilde{\gamma}(n, \mathbf{S})) \frac{\theta^l}{q^l(\mathbf{S})}\right)}$$

$$q(d', n, \mathbf{S}) = \begin{cases} q^* & \text{for } d' \leq \bar{d}(n, \mathbf{S}) \\ q^* \frac{1 - P(\mathbf{S}) + P(\mathbf{S}) \frac{u_c(\underline{c})}{u_c(c')} \left(\gamma_g \frac{\theta^b}{q^b(\mathbf{S})} + (1 - \gamma_g) \frac{\theta^l}{q^l(\mathbf{S})}\right) \frac{n + q^* d_t^c}{d'}}{1 - q^* P(\mathbf{S}) \frac{u_c(\underline{c})}{u_c(c')} \left(\gamma_g \frac{\theta^b}{q^b(\mathbf{S})} + (1 - \gamma_g) \frac{\theta^l}{q^l(\mathbf{S})}\right)} & \text{for } d' > \bar{d}(n, \mathbf{S}) \end{cases}$$

$$\pi = l + b - d' - d_t^c$$

$$\underline{\pi} = \max\left(\theta^l l + \theta^b b - d' - d_t^c, 0\right)$$

$$d' + d_t^c \leq \theta^l l + \theta^b b$$

$$v_t^b(n; \mathbf{S}) = \max\left\{v_{s,t}^b(n; \mathbf{S}), v_{g,t}^b(n; \mathbf{S})\right\}$$

$$v_{s,t}^b(n; \mathbf{S}) = \max_{d', d_t^c \leq \bar{d}_t^c, \gamma \in [0,1]} \left\{ \begin{array}{c} (1 - P(\mathbf{S}))\left((1 - \psi)\pi + \psi E_{\mathbf{S}}\left[v_{t+1}^b(n'; \mathbf{S}')\right]\right) \\ + P(\mathbf{S})\left((1 - \psi)\underline{\pi} + \psi \underline{v}^b(\underline{n}'; \mathbf{S})\right) \end{array} \right\}$$

$$v_{g,t}^b(n; \mathbf{S}) = \max_{d', d_t^c \leq \bar{d}_t^c, \gamma \in [0,1]} \left\{(1 - P(\mathbf{S}))\left((1 - \psi)\pi + \psi E_{\mathbf{S}}\left[v_{t+1}^b(n'; \mathbf{S}')\right]\right)\right\}$$