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UNCONVENTIONAL MONETARY POLICY IN A SMALL OPEN ECONOMY

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I N T E R N A T I O N A L M O N E T A R Y F U N D

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African Department

Unconventional Monetary Policy in a Small Open Economy¹

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Abstract

We investigate unconventional monetary policy in a small open economy using shadow interest rates to capture policy at the zero lower bound (ZLB) in a Bayesian structural vector autoregressive model for Canada - a useful case where foreign shocks can be proxied by the U.S. alone. We estimate that during the ZLB period, on average, Canadian unconventional monetary policy increased output by 0.13 percent, while US unconventional monetary policy increased Canadian output by 1.2 percent on average. Results demonstrate the effectiveness of domestic unconventional monetary policy and the strong spillovers from foreign unconventional monetary policy in a small open economy.

JEL Classification Numbers: E52, E58, F42.

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I. INTRODUCTION

Following the 2008 global financial crisis, many central banks quickly exhausted their ability to stimulate economic activity as policy rates reached the zero lower bound (ZLB).¹ To continue to encourage lending, they turned to unconventional measures, including direct market interventions through large scale asset purchases (LSAPs) and forward guidance, to influence expectations of the future short rate. Recent literature has quantified the effects of these policies on long term rates, asset prices, credit costs and (to a lesser extent) the real economy, and has generally focused on the domestic effects of such policies in large open economies, most notably the US.² In contrast, this paper analyzes the effects of unconventional monetary policy measures in a small open economy, including spillovers from foreign unconventional monetary policy, a topic which has hitherto received relatively little attention.³ Specifically, we look at the case of Canada, for reasons explained below, and measure the effects of the Bank of Canada and Federal Reserve's unconventional monetary policies at the ZLB on the real economy.

There are two main challenges that we face in conducting our analysis, both of which can be addressed by studying the Canadian case. First, small open economies often respond strongly to foreign shocks. Therefore, to gain meaningful insight into the role of domestic monetary policy we must control for international variables. International spillovers to Canada, among several candidate small open economies that engaged in unconventional monetary policy following the global financial crisis, can be mostly captured by US variables. This greatly simplifies our analysis. Moreover, since the Canadian economy is relatively small, we can assume that domestic shocks have little impact on the US. That is, there are no "spillback"

¹We define the ZLB as 25 basis points (bps), consistent with central bank definitions in 2008.

²See, for example, Hamilton and Wu (2012), Swanson and Williams (2014), D'Amico and King (2013), Krishnamurthy and Vissing-Jorgensen (2011), and Gagnon, Raskin, Remache, and Sack (2010), among others, for studies that look at the impact on interest rates, term spreads, asset prices, and credit costs, and Dahlhaus, Hess, and Reza (2014) and Gambacorta, Hofmann, and Peersman (2014) for studies that look at the impact on the real economy.

³Note that there is a large literature on Japanese unconventional monetary policy from the early 2000s, which is separate from the literature we refer to here that is focused on unconventional monetary policy during the global financial crisis. See Bernanke and Reinhart (2004) for an overview.

effects from Canada to the US, which allows us to isolate a single direction of international spillovers. In fact there is evidence that globally, monetary policy spillovers tend to be one-directional from the US to abroad, with limited evidence of spillbacks even from other large open economies.⁴ We impose this single-direction spillover assumption via block-exogeneity restrictions in a two-country Bayesian structural vector autoregressive (B-SVAR) model. Structural autoregression (SVAR) models with block-exogeneity restrictions have been used to study the impact of US monetary policy shocks on the Euro Area (Neri and Nobili, 2010), on Latin American countries (Canova, 2005), on a range of emerging markets (Maćkowiak, 2007), and on Canada starting with Cushman and Zha (1997) and later by others using alternative model characterizations and estimation methods (Bhuiyan (2012) and McNeil (2017)). Block-exogeneity assumptions can also be useful in non-monetary policy spillover models, such as models of global commodity shocks on small open economies (Charnavoki and Dolado, 2014).

Second, capturing the stance of monetary policy is complicated by the fact that there is no variation in the target interest rate — the standard choice for a policy variable — at the ZLB. To get around this problem, the literature provides two alternatives: event study methods or SVAR models with a different policy variable such as the central bank balance sheet (Dahlhaus, Hess, and Reza, 2014; Gambacorta, Hofmann, and Peersman, 2014; MacDonald, 2017), the term spread (Baumeister and Benati, 2013) or a shadow rate (Krippner, 2013; Wu and Xia, 2016). Event studies isolate monetary policy shocks that are not contaminated by any other important events by focusing on the response around a very short time interval. While insightful on the immediate response of financial variables, event studies are unable to capture the effects on real variables because their response is much slower. Additionally, they capture only changes in market expectations that occur within the specified time interval, which itself may be misspecified. Using the central bank balance sheet as a policy variable in an otherwise standard SVAR directly tackles this issue. However, the balance sheet only captures

⁴See Rogers, Scotti, and Wright (2014) and Ehrmann and Fratzscher (2005).

the implementation of LSAPs and does not reflect either their announcements, which have been shown to have a significant stimulating effect on the economy, nor other policies such as forward guidance.⁵ The term spread captures the effect of both announcements and LSAPs but by measuring these effects through the compression of the yield curve it cannot be constructed as an uninterrupted variable through conventional and unconventional episodes (as is also the case with the balance sheet variable). Moreover, any estimation of counterfactuals using the term spread rely on outside estimates of the effects of LSAPs on the spread. Given the limitations of these methods, we adopt the remaining alternative and define the policy variables using estimated shadow interest rates, which uses the term structure of interest rates to predict the level of the short-term rates as if they were not bound by zero.

Although the use of shadow rates comes at a cost of not being able to disentangle the effects of specific policies or identifying the specific channels of monetary policy spillovers, it has several distinct advantages. The shadow rate provides a consistent measure of monetary policy stance for *both* central banks in our empirical model. Since the Bank of Canada's unconventional monetary policy measures did not include LSAPs, the shadow rate is more appropriate than the balance sheet as a policy variable. Furthermore, its flexibility also makes it appropriate as a policy variable for the Federal Reserve (the Fed) because it captures the effects of both forward guidance and LSAPs. Moreover, since the shadow rate can be viewed as an extension of the traditional policy variable uninterrupted by the ZLB, we are able to begin our sample long before the ZLB episode and use the additional observations to improve the precision of our results.

We estimate our model for the period 1994–2016 and quantify the impact of unconventional monetary policies through counterfactual experiments that restrict the shadow rate to the

⁵ Krishnamurthy and Vissing-Jorgensen (2011) and Gagnon and others (2010) show unconventional monetary policy announcements had significant impacts on US financial markets. Neely (2015), Chen, Filardo, He, and Zhu (2015), and Bowman, Londono, and Sapriza (2015) show that unconventional monetary policy announcements had significant impacts on foreign country financial markets. Eichengreen and Gupta (2014), and Dahlhaus and Vasishtha (2014) show the same is true for unconventional monetary policy tapering announcements.

ZLB. Our main finding is that Canadian unconventional monetary policy had expansionary effects on the Canadian economy, boosting output—in level terms—by approximately 0.131 percent on average during the ZLB episode. This result is robust to several different specifications. Our setup also allows us to investigate the effects of US unconventional monetary policies on the Canadian economy, and we find that Canadian output (industrial production) would have been approximately 1.194 percent lower on average without US unconventional monetary policies.

Our paper is related to a strand of literature that measures US unconventional monetary policy spillovers: Fratzscher, Lo Duca, and Straub (2013) analyze the impact of 12 key quantitative easing (QE) announcements by the Fed and find that they lowered sovereign yields and raised equity prices, but that there was substantial heterogeneity in the effects on emerging market versus advanced economies. Fratzscher, Lo Duca, and Straub (2014) perform a similar analysis for European Central Bank QE announcements. Bauer and Neely (2014) use an event study to identify international transmission channels of unconventional US monetary policy. They find that the LSAPs decreased Canadian long-term yields significantly through both signaling and portfolio balance channels. Neely (2015) also uses an event-study and finds large effects on Canadian yields and on the exchange rate. Dahlhaus, Hess, and Reza (2014) use a factor-augmented VAR (FAVAR) model and the Fed's balance sheet as the policy instrument to evaluate spillovers from US QE on the Canadian economy. They find that QE had a positive impact on Canadian output and that Canadian asset prices and interest rates move in tandem with their US counterparts. Our analysis is distinct from these as we model domestic monetary policy in the US, and, by using the shadow rate, we incorporate both the unconventional monetary policy announcement and large scale asset purchase spillovers from the US to Canada.

There is also a growing literature that uses general equilibrium models to study the international impacts of unconventional monetary policies. Similar to our work, Alpanda and

Kabaca (2015) and Kolasa and Wesolowski (2018) look at spillovers from unconventional policy in a large open economy (the US's large-scale asset purchase program, in the case of Alpanda and Kabaca (2015)), and similar to our result find that it stimulates economic activity both inside the US and internationally.⁶ Justiniano and Preston (2010) build a small open dynamic stochastic general equilibrium of the Canadian economy to show that such models cannot account for substantial influence of foreign-sourced disturbances (of any kind, not only monetary policy shock) identified in reduced-form, empirical studies. Hohberger, Priftis, and Vogel (2019) and Mouabbi and Sahuc (2019) look at the impact from unconventional monetary policy conducted by the European Central Banking, finding that it contributed to GDP growth and inflation in both the EU and abroad.

Also related is a large literature on international conventional monetary policy spillovers. Among these, Dedola, Rivolta, and Stracca (2017) show that surprise US monetary tightening leads to US dollar appreciation and, in foreign countries, a fall in GDP and in inflation, and a rise in unemployment (albeit with significant heterogeneity across countries). Bruno and Shin (2015) show that a monetary policy loosening leads to dollar depreciation, which is transmitted through the leverage cycle (or the "risk-taking" channel of monetary policy). In the same vein, Cecchetti, Griffoli, and Narita (2017) show that leverage ratios and risk-taking rise in foreign countries when both their domestic monetary policy, and US monetary policy, eases. Nsafoah and Serletis (2019) show that US monetary policy spills over to policy rates in Canada (and other small open economies).

The effects of unconventional monetary policy is thus an ongoing area of active research (for a review see Kozicki, Santor, and Suchanek (2011), Krishnamurthy, Vissing-Jorgensen and others (2013), Santor and Suchanek (2016), Colciago, Samarina, and de Haan (2018), Driffill (2016) and references therein) and although most empirical evidence suggests that un-

⁶The mechanisms for the spillovers are very different in these two papers, with Alpanda and Kabaca (2015) finding the portfolio balance channel stimulates activity abroad, while Kolasa and Wesolowski (2018) find a boost to domestic demand but the competitiveness channel actually depresses output in the short run, which is different from our findings and much of the literature.

conventional policies had expansionary effects, the magnitude of these effects depends on many factors, particularly in a small open economy, and is difficult to generalize, highlighting the need for studying individual cases where it is possible to quantify these effects more precisely, such as the one considered in this paper.

Recent concerns about the large adverse cross-border spillovers of unconventional monetary policies have rekindled the discussion of international coordination of monetary policy (Engel, 2015; Taylor, 2013). It has been argued that countries who are responsible for substantial international monetary policy spillovers should acknowledge their role in influencing foreign economies, and should consider feasible remedies to limit negative spillovers (Blanchard, Ostry, and Ghosh, 2013; Mishra and Rajan, 2016). Our work contributes to an alternative perspective on unconventional monetary policy spillovers: that under certain circumstances, in particular when international monetary policy is coordinated, such spillovers can be favourable. This literature, and our contribution, is also linked to global financial cycles and their spillovers. This literature is reviewed by IMF (2017), who show that stronger financial linkages with the US increase the sensitivity of domestic financial conditions to global shocks, but that overall countries are still able to steer their domestic financial conditions. In related work, Rey (2015) demonstrates that US monetary policy has spillovers globally regardless of domestic macroeconomic conditions and Obstfeld (2015) shows that emerging markets are particularly impacted by foreign monetary policy, regardless of the exchange rate regime. In addition, Caceres, Carriere-Swallow, Demir, and Gruss (2016) show that the response of domestic interest rate globally depend on changes in US interest rates, how much autonomy the monetary authorities have, and the exchange rate regime in place.

Finally, our analysis is also related to studies quantifying the effects of unconventional monetary policy measures in Canada. He (2010) showed that the Bank of Canada's conditional commitment on interest rates effectively reduced yields (albeit not statistically significantly). Chang and Feunou (2013) found that forward guidance, the expansion of liquidity, and policy

rate cuts successfully reduce market volatility. Gambacorta, Hofmann, and Peersman (2014) used central bank balance sheets in seven countries to assess the effectiveness of monetary policy at the ZLB domestically, and found that a positive shock to the balance sheet increased economic activity and consumer prices but the effect on inflation was weaker and less persistent. Our paper is distinct from these cited papers in two important ways: first they do not control for potential spillovers from the US or elsewhere and second, they do not evaluate the total effect of all unconventional monetary policies during the ZLB, as our shadow rate does.

This paper is structured as follows. The next section describes our measure of unconventional monetary policy. Section III presents the data and method. Our main analysis, which studies the effect of Canadian and US monetary policy on the Canadian economy from 1994–2016, is discussed in Section IV. We conduct several robustness exercises, by considering alternative model specifications in Section V. Section VI concludes.

II. MEASURING UNCONVENTIONAL MONETARY POLICY

On April 21 2009 the Bank of Canada, in coordination with other central banks, lowered the target rate from 50 bps to 25 bps, considered at the time its effective lower bound, reduced the operating band from 50 bps to 25 bps, and thereby set the target rate to the lower bound (25 bps).⁷ Aiming to reduce uncertainty in Canadian financial markets, the Bank of Canada committed to keep the target rate unchanged until the end of the second quarter of 2010, conditional on inflation expectations. It further reinforced the upper bound on the operating band (50 bps) through its standing liquidity facility (SLF), which provided access to overdraft loans at the bank rate for the Bank’s Large Value Transfer System (LVTS) participants, and by prorating access to new standing purchase and resale agreements (PRAs) at the bank rate for Canadian primary dealers.⁸ The Bank of Canada reinforced the lower bound on the oper-

⁷The Bank of Canada operates under a 2 percent inflation target by adjusting (or not) the overnight rate (the “target rate”) at eight fixed announcement dates annually.

⁸The Bank of Canada targets the overnight rate in the LVTS, which allows transfers of large payments with a guarantee of settlement.

ating band by providing LVTS participants with access to a standing deposit facility on which they could earn the deposit rate and by conducting sale and repurchase agreements (SRAs) intra-day at 25 bps with primary dealers, if required. Finally, it targeted daily settlement balance levels at \$3 billion, a dramatic increase from the small positive balance target during conventional times (Bank of Canada, 2010). With market conditions improving, the Bank of Canada removed its statement of commitment on April 20 2010. On June 1 2010 the target rate was raised to 50 bps, and the standard operating framework was reestablished.

In the US, the federal funds rate reached the ZLB in November 2008 at which point the Fed began engaging in unconventional monetary policies, including LSAP purchases, a commitment to continue the LSAP program, and a commitment to keep the policy rate at the ZLB. LSAPs involved purchases of asset with medium- and long-term maturities, including US Treasuries, mortgage-backed securities, Federal agency debt, and currency swaps, and were meant to lower the cost of long-term private borrowing, or flatten yield curves. The Fed conducted several rounds of LSAPs: QE1 (November 2008–March 2010), QE2 (November 2010–June 2011), and QE3 (September 2012–October 2014).⁹ The Fed’s commitment to LSAPs and the 0-25 bps rate range was meant to reduce uncertainty in US financial markets and stimulate the economy. Importantly, the central bank target rate did not reflect any of these actions because it remained at the ZLB throughout. On December 16, 2015 the Fed moved away from the ZLB for the first time since 2008 by increasing the federal funds rate to 50 bps.

A. Shadow rates

We define the Canadian and US policy variables a using shadow rate term structure model (SRTSM), first proposed by Black (1995), which provides a consistent measure of monetary

⁹The Fed also conducted Operation Twist in September 2011, which involved purchases of bonds with long-term maturities and sales of bonds with short-term maturities. This operation left the overall size of the Fed’s balance sheet unchanged.

policy stance for both central banks at and away from the ZLB.¹⁰ The model defines the observed nominal rate, r_t , as the sum of an unobserved and unrestricted shadow rate (which can be less than zero), sr_t , and the option value of holding physical currency that is exercised at the effective lower bound, \underline{r} . Specifically,

$$r_t = \max\{sr_t, \underline{r}\} = sr_t + \underbrace{\max\{0, \underline{r} - sr_t\}}_{\text{option value of currency}} \quad (1)$$

Only when the shadow rate is below the ZLB is the currency option positive. Fitting this model to the data allows one to back out an estimate of the shadow rate process.

There are several benefits to using the shadow rate as a measure of unconventional monetary policy. First, the shadow rate captures both the signaling and portfolio balance channels of central banks actions. While the Fed's LSAPs were meant to flatten yield curves, Fed announcements were meant to reduce both future rate expectations and uncertainty in financial markets. Measuring monetary policy by either the balance sheet or Fed announcements alone would then only capture one of the two channels. By taking information from the entire yield curve, the shadow rate captures both channels as well as forward guidance and other actions at the ZLB. Second, the shadow rate provides us with an uninterrupted measure of monetary policy through conventional and unconventional ZLB episodes. This allows for more precise estimation as we can extend the time series beyond the ZLB period.

While Krippner (2012, 2013) and Bauer and Rudebusch (2013) use numerical simulation methods to solve for the rate, Wu and Xia (2016) take an alternative approach and derive an approximate solution to the shadow rate as a function of the yield curve and the probability that the shadow rate will fall below the effective lower bound. Mouabbi and Sahuc (2019) use several methods and extract a common factor from them. Since the method by Wu and Xia (2016) provides a tractable analytical approximation of the shadow rate, and does not require numerical simulation, it is our preferred approach. Moreover, Wu and Xia (2016)

¹⁰ See Appendix F for additional details on the SRTSM derivation.

estimate more conservative values for the US shadow rate relative to those from Krippner (2012, 2013), so that our results could be interpreted as a lower bound. We take the Wu and Xia (2016) estimate of the US shadow rate directly, and we adopt their method to estimate a Canadian shadow rate. To our knowledge, we are the first to report estimates for a Canadian shadow rate during the 2009-10 ZLB period.

For both countries we assume the ZLB is at 25 bps, consistent with the lower bound both central banks set interest rate to and with the prevailing assumption of what the ZLB threshold was during this period. While several European central banks later set interest rates below zero, in the case of Canada and the US assuming a ZLB below 25 bps would mis-attribute UMP actions taken during this period, and thereby captured by the shadow rate, to conventional policy. However, since once the policy rate enters positive territory the shadow rate no longer captures any unconventional monetary policy, our analysis may be underestimating the effects of unconventional monetary policy to the extent that long-term assets that the Fed continued to hold after December 2015 generated additional stimulus.¹¹

Figure 1 plots our estimated Canadian shadow rate and the US shadow rate, spliced with the Bank of Canada target rate and federal funds target rates when these policy rates were at the ZLB. The Canadian shadow rate reached a low of approximately -0.34 percent in November 2009, at which point the Bank of Canada was midway through its conditional commitment mandate and was actively engaging in liquidity provision activities. The US shadow rate was decreasing from 2008 through 2014, reaching a low of -2.99 percent. Throughout this period the Fed engaged in three rounds of QE and made numerous statements regarding continued policy easing.¹²

¹¹ We thank an anonymous referee for pointing out this caveat.

¹² We also note one potential issue with using the shadow rate as a continuous policy variable is the increased volatility during the financial crisis. One way to test for such effects is to allow for stochastic volatility in the model. However, due to the relatively large complexity of our baseline model, we leave this analysis for future research.

B. Event study

Since we are the first to estimate a shadow rate for Canada, we begin by assessing whether its movements are consistent with the actions taken by the Bank of Canada during the ZLB.¹³ Specifically, we conduct an event study around Bank of Canada policy announcements. Our shadow rate is estimated at a daily frequency, because the key input variable, the Bank of Canada's zero coupon bond yield curve data, is daily. As this may be considered an insufficiently high frequency to identify any effects from central bank announcements and because the ZLB episode was relatively short for Canada (and thus the number of monetary policy events is limited), we are cautious in interpreting our results and treat the event study as a consistency check rather than a careful empirical exercise.

The Bank of Canada made several announcements during the ZLB period, three of which were particularly important: the announcement that the short rate was being lowered to 25 bps and would remain there until the second quarter of 2010 (April 21, 2009), the announcement that the Bank of Canada's conditional commitment to the ZLB was being removed (April 20, 2010), and the announcement that the short rate was being raised to 50 bps (June 1, 2010). We include several other announcements in our event study that relate to the Bank of Canada's extraordinary liquidity operations during the ZLB, but we suspect, *a priori*, that these events may have a lesser impact on the shadow rate than the others as they were less widely reported or known outside of the traditional banking sector.

We look at the one-day changes in the shadow rate on announcement dates and identify whether they can be classified as tail events. We define a tail event as a statistically significant change in the shadow rate based on standard errors calculated assuming a normal distribution (following Bowman, Londono, and Sapriza, 2015). On several occasions the Bank of Canada made their announcements late in the day, either near to or after financial market closing. In these cases we look at the next-day change in the shadow rate for our analysis. The dates,

¹³ In their paper, Wu and Xia (2016) show that there was no substantive difference in the response of US macro-economic variables to the US shadow rate at and away from the ZLB. We do not repeat that exercise here.

event descriptions and one-day changes in the shadow short rate are reported in Table 1. We see that two of the three main ZLB announcements by the Bank of Canada were associated with tail events in changes in the shadow short rate — the 21 April 2009 announcement of the shift to the ZLB and the 20 April 2010 removal of conditional commitment. The announcement of exit from the ZLB on 1 June 2010 was not associated with a significant shift in the shadow rate, which is consistent with the further monetary tightening being widely expected after the removal of conditional commitment. Given the results are consistent with expected movements in the shadow rate, we proceed with our empirical analysis and use the shadow rate as our policy indicator during the ZLB.

III. SMALL OPEN ECONOMY B-SVAR MODEL

We model the dynamic interaction of the variables in the two countries using the SVAR:

$$AY_t = C + \sum_{l=1}^p B_l Y_{t-l} + \varepsilon_t,$$

where Y_t is an $n \times 1$ vector of endogenous variables, A and B_l are $n \times n$ parameter matrices, ε_t is an $n \times 1$ vector of structural shocks with $E(\varepsilon_t | Y_1, \dots, Y_{t-1}) = 0$ and $E(\varepsilon_t \varepsilon_t' | Y_1, \dots, Y_{t-1}) = I_n$.

We impose the small open economy assumption that Canada cannot influence US variables via block exogeneity restrictions on the parameters A and B_l . This greatly reduces the number of parameters to be estimated despite the number of overall parameters increasing with the inclusion of US variables, which are meant to capture international influence on Canadian variables. We follow Cushman and Zha (1997) and assume that the endogenous variable vector Y_t comprises two blocks: a Canadian block Y_t^{CAN} and a US block Y_t^{US} . We allow the Canadian block to respond to the US variables both contemporaneously and with a lag, but restrict the US block to be self contained and not influenced by the dynamics in the Canadian

variables. Specifically, we impose the restrictions in the following way

$$\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} Y_t^{\text{CAN}} \\ Y_t^{\text{US}} \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{bmatrix} \begin{bmatrix} Z_t^{\text{CAN}} \\ Z_t^{\text{US}} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\text{CAN}} \\ \varepsilon_t^{\text{US}} \end{bmatrix},$$

where $F = [C, B_1, \dots, B_p]$ and $Z_t^i = [I, Y_{t-1}^i, \dots, Y_{t-p}^i]'$ for $i = \{\text{CAN}, \text{US}\}$.

For the Canadian block, our main specification includes the Canadian policy rate, r ; the Canadian/US dollar exchange rate, s ; seasonally-adjusted real Canadian industrial production, y ; the Canadian consumer price index, p ; seasonally adjusted Canadian real exports ex , and seasonally adjusted Canadian real imports, im . We calculate net exports post-estimation. For the US block, we include the US policy rate r^* ; the implied volatility of the S&P 500 index, VIX ; commodity export prices, wpx ; the US consumer price index, p^* ; and seasonally-adjusted real US industrial production, y^* . Detailed data definition and sources are provided in Table A.1 of Appendix A. All variables are in logs except for the two policy rates, which we construct using (4), *i.e.* splicing the Bank of Canada target rate and the federal funds rate with their respective shadow rates whenever the shadow rates are below 25 bps (the ZLB). The Canadian shadow rate is calculated as described in Section II.A, using zero coupon bond yield curve data published by the Bank of Canada.¹⁴ We take the US shadow rate directly from Wu and Xia (2016). Using the policy rate, as defined in (4), allows us to extend the data much farther and treat both central banks as if they have a constant reaction function but a varying set of policy instruments at the ZLB. It also allows us to conduct counterfactual exercises when the policy rate is at the ZLB. Including multiple business cycles gives our estimates more precision. The data is monthly from August 1994 to December 2016. The beginning of our sample coincides with a shift in the Bank of Canada's operating procedure as it adopted a 50 bps operating band around the target rate and shifted to targeting the overnight rate as its key monetary policy instrument. This change was part of a broader transition in the 1990s to targeting two percent inflation and improving both the clarity and efficiency of monetary policy.

¹⁴See Bolder, Johnson, and Metzler (2004). Data is available at <http://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/>.

Our identification is based on exclusion restrictions in the contemporaneous coefficient matrix A . We assume that the Bank of Canada reacts contemporaneously to the exchange rate, the US policy rate, the VIX and commodity prices. Including commodity prices is particularly important for decision making by the Bank of Canada for two reasons. First, commodity prices adjust very quickly to market conditions and hence they control for future price expectations, *i.e.* they help mitigate the *price puzzle* often found in similar SVAR analyses, and second, since Canada is a commodity exporter, commodity prices have a large impact on Canadian output as well as on the value of the Canadian dollar. Following Gambacorta, Hofmann, and Peersman (2014), we include the VIX as a proxy for financial turmoil, economic risk, and uncertainty, which played a critical role in the latter part of our sample. We control for imports and exports to capture real spillovers between Canada and the US. We let the exchange rate react contemporaneously to all variables, domestic and foreign, in the model and assume that the production sector of the Canadian economy takes an upper triangular form with the variables ordered (p, y, im, ex) . We assume the US block takes an upper triangular form, ordered $(r^*, VIX, wxp, p^*, y^*)$, so that industrial production and prices cannot respond to a monetary policy shock within the same month. Because we are interested only in responses to unexpected monetary policy shocks (and not shocks to other U.S. variables in the model) the ordering of the remaining variables in the US block is immaterial.

The total number of restrictions on the contemporaneous coefficient matrix is 111. However, since we need only $n(n - 1)/2 = 55$ restrictions for exact identification, the model is over-identified. Although over-identification permits a more sensible set of restrictions than a recursive ordering, it imposes restrictions on the reduced-form covariance matrix, $A^{-1}A^{-1'} = \Omega$, which complicates the estimation. The posterior distribution of Ω does not have a convenient form and regular Monte Carlo integration methods cannot be used. To overcome this, we follow the Waggoner and Zha (2003) Gibbs sampler method with 5000 draws, discarding the first 1000 to ensure that the initial values do not affect the posterior distribution. We use the Sims and Zha (1998) prior and set loose values for all hyper-parameters

except for the lag decay: $\mu_i = 10$ for $i = 1, \dots, 6$. We estimate the model with 12 lags and report 68 percent error bands as well as the means of the posterior distribution.

Prior to estimation, we check the validity of the over-identifying restrictions.¹⁵ Since the contemporaneous coefficient matrix is over-identified, we cannot directly test block exogeneity in the reduced form VAR. Similarly, we cannot directly test over-identifying restrictions because of block exogeneity imposed in the contemporaneous coefficient matrix. Instead, we follow Cushman and Zha (1995, 1997) and perform a joint likelihood ratio test for over-identification and block exogeneity in the contemporaneous coefficient matrix and block exogeneity in the lagged coefficient matrices (a total of 385 restrictions). This test fails to reject the null hypothesis of block exogeneity, and we proceed on the basis that our assumption of block exogeneity is valid.¹⁶

IV. RESULTS

A. Monetary Policy Shocks

We first consider the dynamic response of the variables in our system to both domestic and foreign expansionary monetary policy shocks and then use the estimated system to perform counterfactual experiments based on the historical decomposition. In all cases we report the mean, the 68 percent confidence bands (representing one standard deviation from the mean) and the 90 percent confidence bands (to impose more stringent criteria on our results).

The impulse response functions from a 25 bps expansionary shock to Canadian monetary policy are shown in Figure 2. Prices, industrial production, and the exchange rate respond to this shock in accordance with theoretical models and existing empirical evidence. That is, an expansionary monetary policy shock raises industrial production (Figure 2d) and prices (Figure

¹⁵ Due to the restriction of block exogeneity, the model is over-identified, which means that more restrictions are imposed than would be needed to identify the structural parameters. Excess restrictions allow us to test the validity of block exogeneity by comparing the likelihood with and without imposing these restrictions. Failing to reject block exogeneity implies that Canadian variables do not affect the US block in the model.

¹⁶The likelihood-ratio test statistic is 419.01 and the 5 percent critical value is 431.75

2c) significantly on impact, and for approximately 18 to 24 months after the initial shock. The currency depreciates on impact, and for about one year after the shock (statistically significant at the 68 percent level) (Figure 2b).¹⁷ Along with this slightly weaker Canadian dollar we see a significant, albeit temporary, rise in exports about 6 months after the initial shock (Figure 2f). We also see a significant (and temporary) rise in imports (Figure 2e), which implies an income effect stemming from the rise in industrial production. Post-estimation we calculate the impulse response of net exports, which shows that the income effect of the rise in industrial production outweighs the currency effect (albeit only marginally negative and significant at the 68 percent level) (Figure 2g).

The impulse response functions from a 25 bps expansionary shock to US monetary policy are shown in Figure 3. Though our primary focus is on the impact to Canadian variables, we also report the response of US variables in Figures 3a–3e, showing that these are generally consistent with theoretical and empirical literature.¹⁸ Following the US monetary policy shock industrial production rises significantly with a delay of approximately 1–2 years (Figure 3e). Though prices fall significantly on impact (this price puzzle is commonly found in the recursive SVAR literature), they rise slightly after about 6 months for a short period (Figure 3d). Commodity prices rise on impact, consistent with literature that has found expansionary policy raises these prices through expectations (of inflation and growth) channels and demand and supply channels based on the cost of carrying inventories and extracting goods (Anzuini, Lombardi, and Pagano (2013), Figure 3c). US equity market volatility falls significantly, also consistent with empirical literature that has shown loose monetary policy dampens risk aversion and uncertainty (Bekaert, Hoerova, and Lo Duca (2013) and Bruno and Shin (2015), Figure 3b).

¹⁷ We have also tested for uncovered interest parity (UIP), and find that, following an initial deviation, UIP begins to hold after a 12 months for a Canadian shock and 24 months for a US shock. Results are available upon request.

¹⁸ The impact on the US economy has been studied extensively, as discussed in section I. We have not attempted to model the US economy explicitly here, but rather included those US variables which have been showed to be important for Canadian spillovers.

Figure 3f shows a response from the Canadian policy rate both on impact and for over two years following the initial shock, demonstrating the substantive spillover from US policy rates to the Canadian economy. Following domestic monetary contraction, the Canadian dollar depreciates (Figure 3g), and both the policy rate and exchange rate reach a minimum after about 18 months. Canadian industrial production (Figure 3i) rises after about 6 months in response to the US monetary policy shock and then again following the same delay as US industrial production. As in the US, Canadian prices fall in the first month and then rise after about 5 months (Figure 3h). Both impulse responses reiterate the strong spillover from US monetary policy on real Canadian variables. On a net basis, exports rise initially then fall temporarily after about a year, which indicates alternating exchange rate and income effects on Canadian trade (Figure 3l).

We draw two conclusions from these impulse response functions: first, that a Canadian expansionary monetary policy shock boosts domestic prices and industrial production, and depreciates the Canadian dollar. This provides support for our model, and for the use of the shadow rate as our policy variable, as the impulse response functions are generally consistent with other pre-global financial crisis empirical studies and theoretical literature on the impact of monetary policy shocks. Second, the results show that while a US expansionary monetary policy shock increases US industrial production, as expected, it also has cross border effects on the Canadian economy. These spillovers operate through the exchange rate channel, through the endogenous response of the Bank of Canada's policy rate, and through spillovers to real variables. To explore the extent to which either of these results are driven by the conventional monetary policy regime during the pre-ZLB portion of our sample we also look at a subsample of the financial crisis and UMP regime, which shows robust dynamics.¹⁹

¹⁹ Results available upon request.

B. Effects of unconventional monetary policy

In order to quantify the effects of unconventional monetary policy we conduct a counterfactual experiment where we simulate a scenario in which the shadow rate remains at 25 bps (the ZLB) while the other variables remain unrestricted, using the method proposed by Wu and Xia (2016). We assume that, since the nominal rate is bounded by 25 bps, any movement in the policy rate below this bound is driven by unconventional monetary policies. As a result, we interpret our estimates as an upper bound on the effect of unconventional monetary policy. We begin with a historical decomposition of our model, which decomposes each variable in Y_t into the contributions from their initial value, their constant term, and their structural shocks:

$$Y_t = \underbrace{G^t Y_0}_{\text{initial condition}} + \underbrace{\sum_{i=1}^t G^i C}_{\text{constant}} + \underbrace{\sum_{i=1}^t \Psi_{i-1} \varepsilon_{t-i+1}}_{\text{structural shocks}}, \quad (2)$$

where $G = A^{-1}F$ and Ψ_i is the set of coefficients for the impulse response function in period i . We use this equation to calculate the paths of the variables under different scenarios by manipulating the set of contributing structural shocks. The scenarios we are interested in are those that restrict the Canadian and US shadow rate to 25 bps. That is, we want to see the counterfactual paths of variables in our model had the Bank of Canada and Fed not engaged in any unconventional policies.

For Canada, the counterfactual is implemented by replacing the realized monetary policy shock, $\varepsilon_t^{MP_{CAN}}$ in (2), with a counterfactual shock, $\tilde{\varepsilon}_t^{MP_{CAN}}$, such that the shadow rate respects the lower bound, $\tilde{s}_t = 0.25$ when the actual target rate was at 25 bps. Similarly for the US, the counterfactual is implemented by replacing replace $\varepsilon_t^{MP_{US}}$ in (2) with $\tilde{\varepsilon}_t^{MP_{US}}$, such that $\tilde{s}_t^* = 0.25$ when the federal funds rate was at the ZLB.²⁰ The path of each variable is then simulated under these two scenarios.

²⁰We have also conducted a third counterfactual experiment that replaces both the Canadian and US monetary policy shocks with counterfactual shocks so as to make both the Canadian and US shadow rates constrained at $\tilde{s}_t = \tilde{s}_t^* = 0.25$. Because the impact of US unconventional monetary policy is so large relative to Canadian unconventional policy (as reported in this section), the results from this experiment are essentially unchanged from the US counterfactual experiment. We thus do not report these results here, but note that they are available upon request.

1. Restricted monetary policy in Canada

Figure 4 reports the results for our first experiment which restricts Canadian unconventional monetary policy. We show each original series with its counterfactual path, which we interpret as (an upper bound of) what the state of the economy would have been had the Bank of Canada not provided any additional stimulus beyond lowering the target rate to 25 bps.

Figure 4a visually demonstrates how the counterfactual scenario is constructed: while the shadow rate was below 25 bps from 2009–2010, we restrict the policy rate in our experiment to be bound by 25 bps, the red line represents this counterfactual path. Figure 4e (which shows the ZLB period only) shows that under the counterfactual scenario Canadian industrial production would have been significantly lower than its observed value. Table 2 contains the average percentage difference between each observed series and its counterfactual paths during the ZLB period, along with the 68 percent confidence band of this estimate.²¹ Industrial production and prices—in level terms— would have been 0.13 percent and 0.10 percent lower on average during the Canadian ZLB period, respectively, had the Bank of Canada been unable to provide any additional stimulus (with both values significantly different than zero on average through the entire period, based on 68 percent confidence bands). Over the entire ZLB period, industrial production would have been cumulatively 1.828 percent lower.²² While the brevity of the ZLB period in Canada limits the extent to which unconventional monetary policy could affect the real economy, the counterfactual and observed paths are nonetheless significantly different from one another.

Importantly, although industrial production and prices are significantly lower under the counterfactual scenario, they would have returned to their observed path within two years, as can be seen at the end of their plots. This implies that unconventional monetary policy operations conducted by the Bank of Canada sped up the recovery of the Canadian economy significantly, but had relatively short term, or transitory, effects.

²¹ Results hold with 90 percent confidence bands, which are displayed in Figure B.1. and Table B.1. of Appendix B.

²² In annualized terms, the decline in output is 1.572 percent percent.

2. Restricted monetary policy in the US

Figure 5 reports the results from our second counterfactual experiment where we restrict the US shadow rate to 25 bps after November 2008, while the Federal Funds rate was at the ZLB.²³ US unconventional monetary was significantly more expansionary than Canadian unconventional monetary policy and the ZLB episode lasted much longer (Figure 5a). The measured magnitude of the effects of US unconventional monetary policy is also much larger. US industrial production is substantially, and almost persistently, lower throughout the counterfactual experiment (Figure 5e). The trajectory of industrial production reveals that without unconventional intervention from the Fed, US industrial production would have significantly diverted from its growth path. Table 2 reports the average percentage difference between the observed and counterfactual, and shows that US industrial production would have been 1.535 percent lower on average without the Fed's unconventional monetary policy (Table 2).²⁴ This estimate is only slightly more conservative than previous studies: Dahlhaus, Hess, and Reza (2014) find that US GDP would have been 2.3 percent lower on average over the period from 2008Q4 to 2013Q3 if the Fed's balance sheet continued to grow at pre-crisis levels; Chung, Laforte, Reifschneider, and Williams (2012), find that US GDP would have been 3 percent lower in 2012 without the first installment of quantitative easing (QE I); Baumeister and Benati (2013) find that QE I boosted US GDP growth by 2 percent in 2009; and Wu and Xia (2016) find that the US industrial production index would have been 0.79 percent lower in December 2013. In contrast to these studies we look at the entire sample period during which the US was at the ZLB, notably capturing the more recent years when the economy began recovering before the Fed raised rates, which may explain part of the difference. Cumulatively, the difference in industrial production from the observed path to the counterfactual is 130.5 percent lower over the ZLB period.

²³Figure B.2. in Appendix B presents results with 90 percent confidence bands.

²⁴ Or 18.42 percent lower in annualized terms.

Importantly, we find that US unconventional monetary policy had a significant impact on the Canadian economy. The Canadian policy rate would have been much higher under this counterfactual scenario, remaining at around 0.6 percent for much of the experiment (Figure 5f). The higher Canadian policy rate again highlights the sensitivity of the Bank of Canada’s reaction function to movements in the US policy rate. Canadian industrial production clearly benefited from US expansionary policies (Figure 5k). Not only is it significantly lower under the counterfactual scenario — about 1.2 percent lower on average (Table 2) and 102 percent lower cumulatively — but it would be on a different trajectory without US policy intervention. This estimate is also more conservative than the only other study that we are aware of that considers a similar question: Dahlhaus, Hess, and Reza (2014) find that US QE alone increased Canadian GDP by 2.2 percent on average over the period from 2008Q4 to 2013Q3. Notably, our results also shows that the large effect of US UMP (when compared to the effects of Canadian UMP) stems largely from the intensity of US UMP rather than the UMP multiplier, which is more muted relative to the Canadian UMP multiplier, particularly in the first two years following a monetary policy shock (Figure [2.a.] versus [3.i]).

The fundamental result from this US counterfactual experiment, which can be seen clearly in Figure 5, is that not only would Canadian industrial production have been lower on average during the US ZLB, but its trend would have been altered for a considerable length of time.

V. ROBUSTNESS

In order to determine whether our results are overestimated due to omitted variable bias, or misspecification, we conduct several robustness exercises. In Section V.A we control for government expenditure, which often plays a complimentary role to expansionary monetary policy. In Section V.B we use the current account as an alternative to trade, to capture real spillovers between the US and Canada that include income flows and transfers. Finally, in Section V.C we described other robust alternative specifications including an alternate restriction

specification and replacing the shadow rate with an estimated Taylor Rule rate. In all cases our results are robust.

A. Government expenditure

Government stimulus can play an important role in economic recoveries, and it has been argued that this was particularly true in the aftermath of the 2007-09 financial crisis. Since monetary policy easing often coincides with expansionary fiscal policy, excluding a control for this variable could lead to overestimating the effect of monetary policy on industrial production (Rossi and Zubairy, 2011). In this section we adjust our main specification to control for fiscal policy, measured by federal government expenditures. Following the literature on government shocks (see *e.g.* Ramey, 2011, 2016), we assume that discretionary fiscal policy does not respond contemporaneously to any variable because of legislative delays. Based on this assumption we add to our original model: Canadian government discretionary spending, ordered last in the production block of the vector of Canadian variables, Y_t^{CAN} ; and US government discretionary spending, also ordering it last in the US block, Y_t^{US} .²⁵

The results of the counterfactual experiments, reported in Table 3, are similar to the findings in the main specification.²⁶ We estimate that Canadian industrial production would have been 0.132 percent lower on average during the ZLB without Canadian unconventional monetary policy. US industrial production would have been 1.169 percent lower on average during the ZLB period, and Canadian industrial production 1.054 percent lower on average without US unconventional monetary policy. These estimates are within the confidence bands reported in Table 2.

²⁵Figures C.1-C.4 of Appendix C present the impulse response functions of this model for Canadian and US shocks, with 90 and 68 percent confidence bands, respectively.

²⁶The corresponding counterfactual figures are in Appendix C, Figures C.5 and C.6.

B. Current Account

As a second robustness exercise, we replace our import and export variables with estimates of the current account. The purpose is to capture both the trade balance and international receipts and payments of income. In Canada from 1994 to 2016, receipts of income from abroad accounted for about 9 percent of all current account receipts, and payments of income from abroad account for about 15 percent of current account payments, on average. Thus, by simply looking at the trade balance we may miss a large component of Canadian international borrowing and lending, and thereby missing important real spillovers from the US to the Canadian economy.

We replace export and import variables in the main model with a single current account variable, ordered first, in the vector of Canada variables, Y_t^{CAN} . We create a monthly series by linearly interpolating quarterly values (see Appendix A for details), and do not take the log of the current account, as it can take on negative values.²⁷

The counterfactual experiments are reported in Table 4, and further support the main model.²⁸ Under this specification, Canadian industrial production would have been about -0.147 percent lower on average during the Canadian ZLB without unconventional monetary policies from the Bank of Canada, and 1.054 percent lower on average had the Fed not enacted any unconventional monetary policies. These estimates are within the confidence bands reported in Table 2. In addition, we find that the current account rises marginally in response to Canadian and US expansionary policy, consistent with existing evidence that unconventional monetary policy is associated with modest positive increases in current account balances (see Gagnon and others (2017)).

²⁷The impulse response functions of this model are reported in Appendix D, Figures D.1. and D.2..

²⁸The corresponding counterfactual figures are reported in Appendix D, Figures D.3 and D.4.

C. Alternative Specifications

The results discussed above are also robust to two alternative specifications. First, we test the exclusion restrictions on our original contemporaneous coefficient matrix by leaving the Bank of Canada's contemporaneous reaction function completely unconstrained. Although this alternative specification differs from the literature on the Bank of Canada reaction function, it makes the assumptions about the Bank of Canada's reaction function consistent with those of the Fed's reaction function. Second, we use a derived Taylor Rule rates instead of US and Canadian shadow rates as proxies for monetary policy stance. The policy rate implied by a Taylor Rule is based on the output gap and inflation rate and is commonly used as a proxy for the policy stance. Moreover, like the shadow rate, it is not bounded below by zero. Both sets of results are presented in detail in Appendix E.

In the unrestricted Bank of Canada reaction function model the estimated impulse response functions are robust for our main variables of interest (industrial production and prices) and the counterfactual exercises show that Canadian industrial production would have been significantly lower without US and Canadian unconventional monetary policies.²⁹ Results from the Taylor Rule model are also highly robust to our main model.³⁰

VI. CONCLUSION

This paper contributes to a growing literature on the effects of unconventional monetary policy by analyzing these policies in a small open economy. There are two main challenges for quantifying the effects of unconventional policy in this setting: controlling for the outside world and finding an appropriate policy variable. To deal with these issues, we use the recently proposed shadow rates (Wu and Xia, 2016) as a proxy for monetary policy at the ZLB and construct a block-exogenous B-SVAR model to allow for international policy spillovers. By studying Canada as a candidate small open economy we are able to both assume that the

²⁹The impulse response and counterfactual figures are reported in Appendix E.1 Figures E.1 - E.4.

³⁰The Taylor rules are plotted in Appendix E.2 Figure E.5, impulse response functions in Appendix E.2 Figures E.6 - E.7, and counterfactuals in Appendix E.2 Figures E.8-E.9 and Table E.1.

US adequately controls for the outside world and to explore the effects of unconventional monetary policy spillovers from the US.

We find that over the 1994–2016 period both Canadian and US expansionary monetary policy shocks were associated with increased industrial production, which we use as a proxy for total output, and prices in Canada. This result is consistent with theoretical and empirical literature based on pre-global financial crisis data. To quantify the magnitude of unconventional monetary policies in both countries, we conduct two counterfactual experiments that restrict the policy rate separately in each country to the ZLB. We find that without the Bank of Canada’s unconventional policies, industrial production would have been 0.131 percent lower on average during the Canadian ZLB period. Although these policies had significant expansionary effects, we also show that without them, the Canadian economy would have eventually recovered to the path observed at the end of our sample. The relatively small impact is consistent with the limited unconventional intervention by the Bank of Canada during the ZLB period (that is, no quantitative easing). In contrast, without the Fed’s unconventional monetary policies, both Canadian and US industrial production would have been on different paths at the end of 2016. On average, over the July 2007– December 2015 period, US policies boosted industrial production by approximately 1.535 percent in the US and 1.194 percent in Canada on average. Our results are robust to alternative specifications, including controlling for government spending, using an alternative definition of bilateral integration that accounts for cross-border income flows, using a Taylor Rule, and alternative exclusion restrictions on our main model.

Our findings reveal that unconventional monetary policy in a small open economy is effective, but underlines the importance of favorable foreign monetary policy spillovers. Recent concerns about the large adverse cross-border spillovers of unconventional monetary policies have rekindled the discussion of international coordination of monetary policy and the need for individual countries to acknowledge their role in spillovers. However, as we demon-

strate, in some cases these spillovers may have beneficial effects on economic recover. That the Bank of Canada ceased UMP operation long before the Fed yet still benefited from spillovers supports existing literature related to global financial cycles driven by center countries. Future research should, in a similar vein, quantify the costs and benefits of spillovers from monetary policy normalization or reversal, which will not necessarily have symmetric impacts on small open economies.

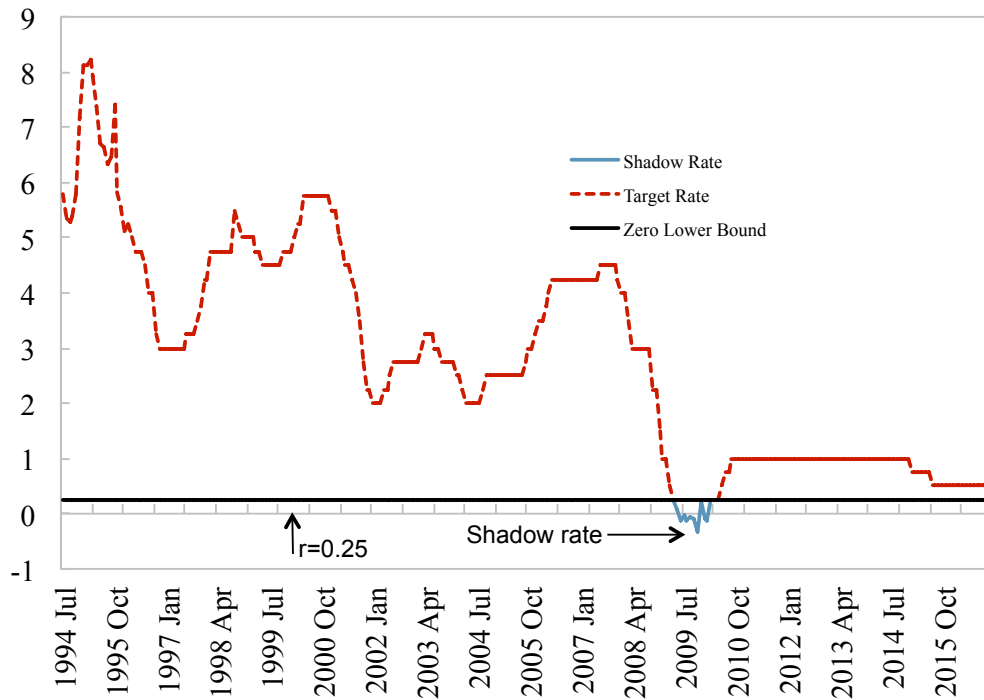
Table 1. Bank of Canada announcements and shadow short rate changes

Date	Δsr_t	Event Description
April 21, 2009	-0.0964**	BoC announces operating framework for the implementation of monetary policy at the effective lower bound for the target rate, lowers target rate target to 1/4 percent, introduces conditional commitment to hold current policy rate until the end of the second quarter of 2010 and announces term PRA transaction schedule.
April 24, 2009	-0.0408	BoC announces term PRA, term PRA for private sector instruments, and term loan facility (TLF) transaction schedules.
June 25, 2009	-0.0097	BoC announces extension of TLF and expanded swap facility with the Fed as well as temporary reciprocal currency arrangements (swap lines) between the Fed and other central banks extended to 1 February 2010.
July 21, 2009	-0.0014	BoC announces term PRA, term PRA for private sector instruments, and TLF transaction schedules.
September 23 2009 [†]	-0.0124	Reflecting the improved conditions in funding markets the BoC announced term PRA Facility for private sector instruments (after a final operation on October 27, 2009) and TLF (after a final operation on October 28, 2009) will expire at the end of October 2009.
October 20, 2009	-0.0914**	BoC announces another term PRA transaction schedule.
November 6, 2009 [†]	-0.0056	Given improved conditions in funding markets the BoC announces that the temporary measure allowing LVTS participants to assign their non-mortgage loan portfolios as eligible collateral for LVTS and SLF purposes will be gradually reduced from 100 per cent to 20 per cent of each participant's total pledged collateral starting February 2, 2010.
December 16, 2009 [†]	0.0282	Given improved conditions in funding markets the BoC announces term PRA operations will be held on a monthly basis rather than bi-weekly, and that only Canadian dollar securities eligible as collateral under the BoC's SLF will be eligible for term PRAs effective January 19, 2010. Affiliated-dealer bank-sponsored ABCP, and BBB corporate bonds will no longer be eligible.
April 20, 2010	0.1110**	BoC maintains target rate at 1/4 per cent and removes conditional commitment.
May 10, 2010 [†]	0.0873*	BoC and the Fed reestablish US\$30 billion swap facility (reciprocal currency arrangement) that had expired on February 1, 2010.
June 1, 2010	-0.0478	BoC increases target rate to 1/2 per cent and reestablishes normal functioning of the overnight market as well as the standard operating framework for the implementation of monetary policy. The target for the target rate is set to the midpoint of the operating band and the width of the operating band to 50 bps.

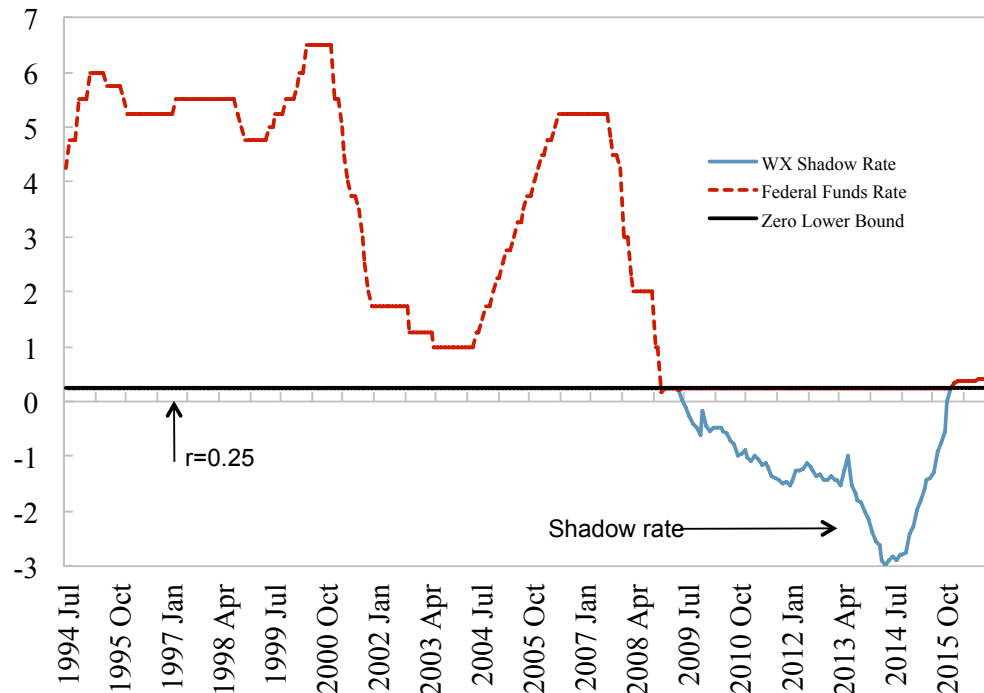
Note: This table contains a list of major policy events and the corresponding changes in the shadow rate. * and ** represent tail events with respect to a normal distribution at the standard 5, and 1 percent confidence levels, respectively. † marks announcement dates that were made the day before but after markets closed.

Figure 1. Canadian and US Policy Rates 1994–2016

(a) Canadian Policy Rate



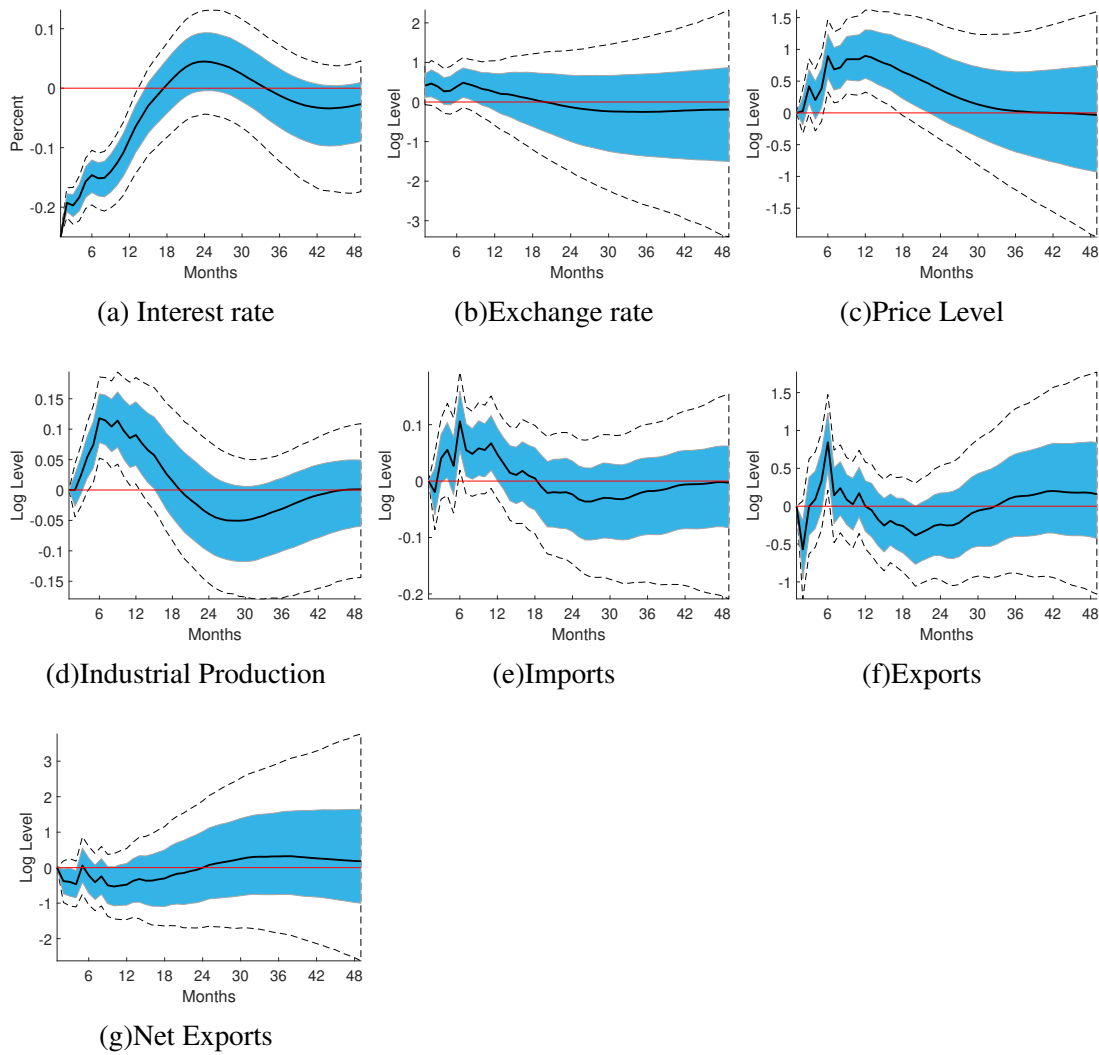
(b) US Policy Rate



Note: The federal funds rate is the target federal funds rate before November 2008 and the upper bound of the federal funds rate from November 2008 onwards.

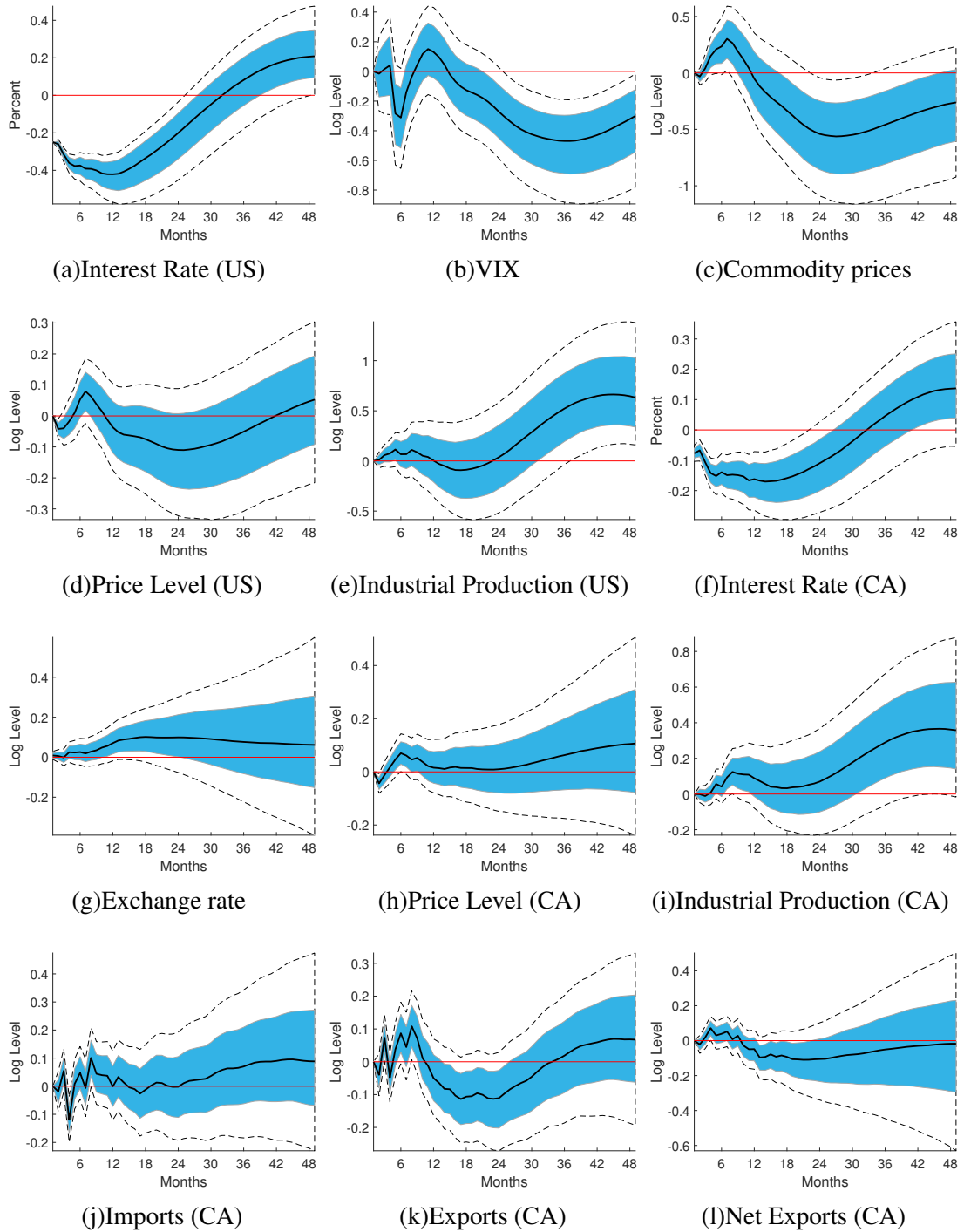
Source: FRED, Statistics Canada CANSIM, Bank of Canada, Wu and Xia (2016), and authors' own calculations.

Figure 2. Impulse Response Functions: Canadian Monetary Policy Shock



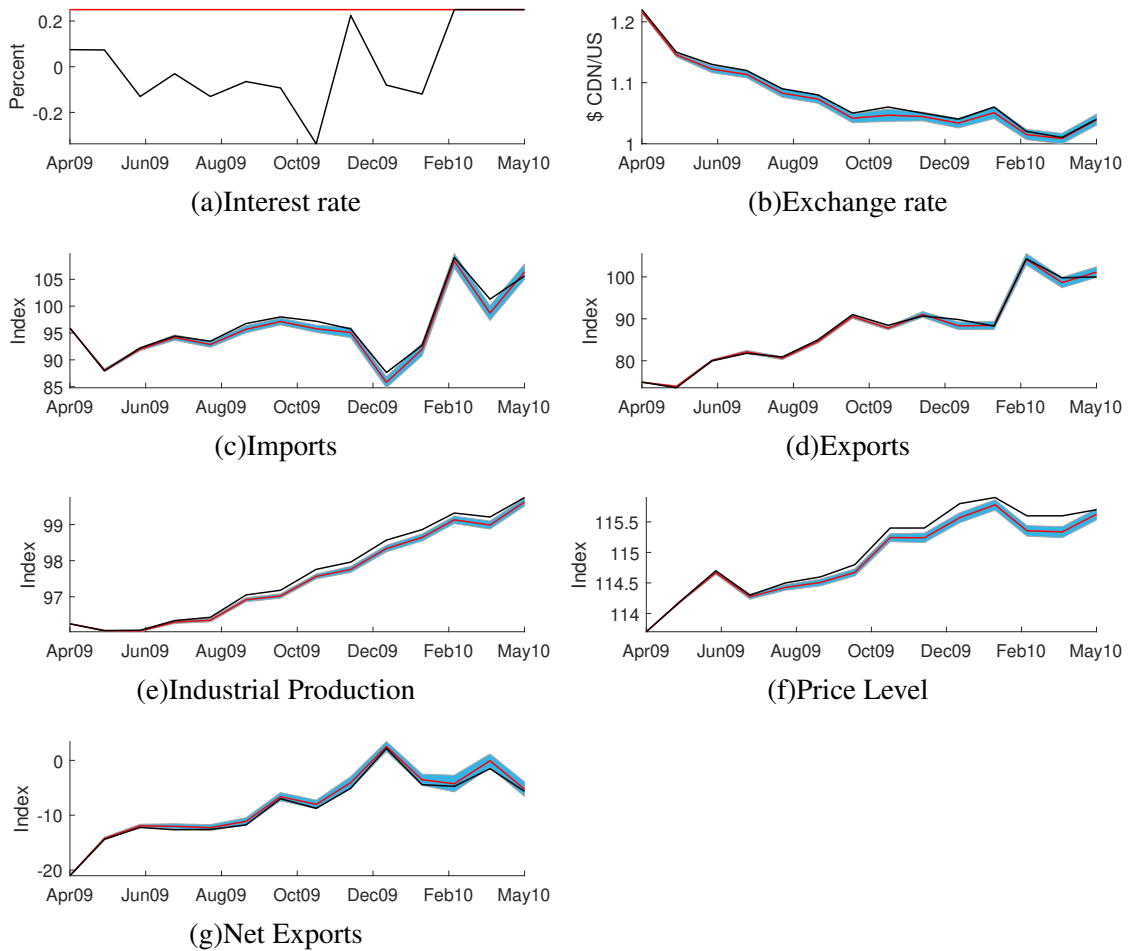
Note: Impulse response functions for the Canadian variables in response to a 25 bps expansionary Canadian monetary policy shock. Error bands are constructed based on 5000 Gibbs sampling draws from the posterior distribution. The blue shaded area represents the 68 percent error band, the dashed line represents the 90 percent error band. Net exports are calculated *ex-post* estimation, not included in the estimated model.

Figure 3. Impulse Response Functions: US Monetary Policy Shock



Note: This figure contains the impulse response functions for the all variables in response to a 25 bps expansionary US monetary policy shock. Error bands are constructed based on 5000 Gibbs sampling draws from the posterior distribution. The blue shaded area represents the 68 percent error band, the dashed line represents the 90 percent error band. Net exports are calculated *ex-post* estimation, not included in the estimated model.

Figure 4. Counterfactual Paths: Canadian ZLB Imposed



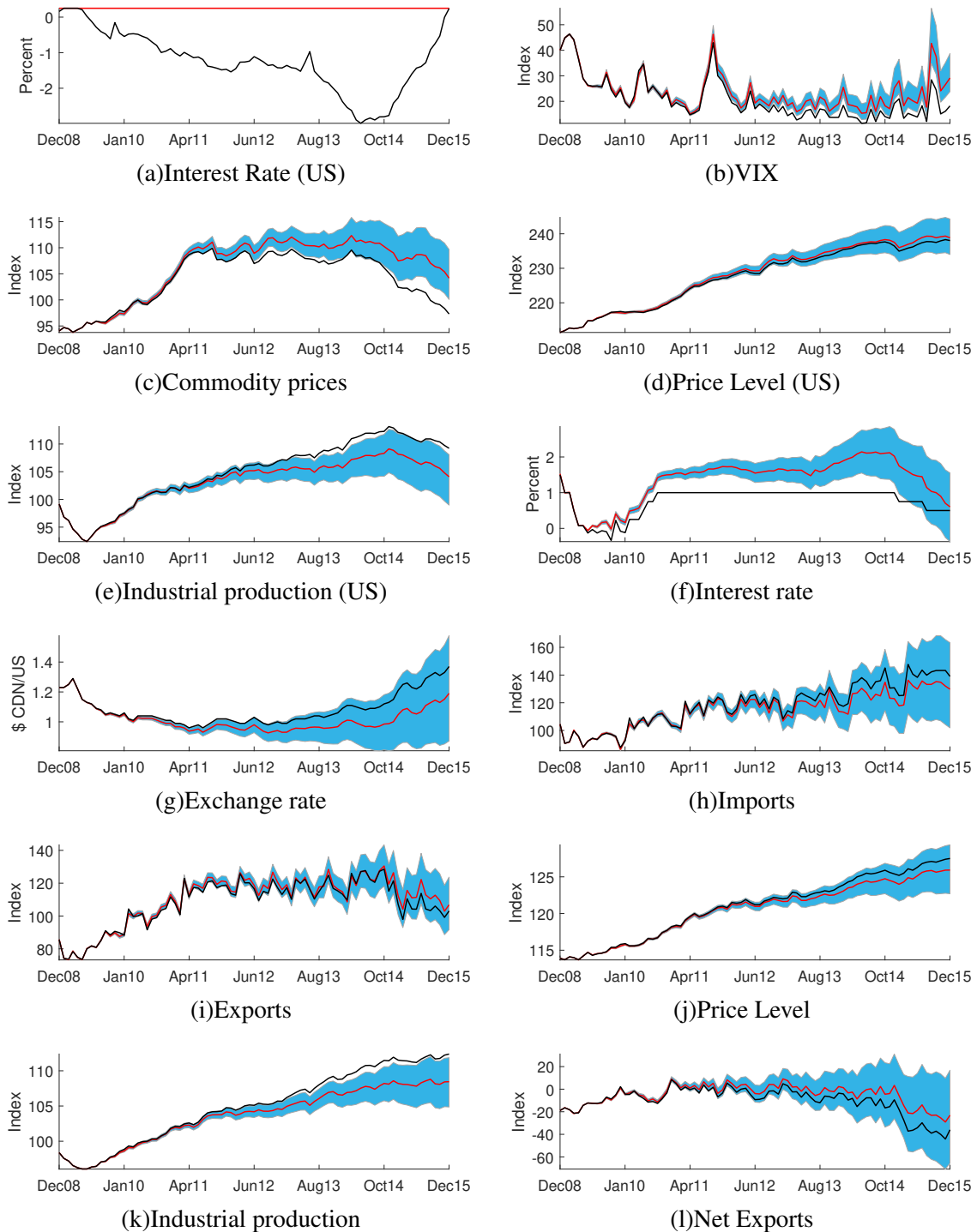
Note: This figure plots the observed value of each series in our model along with an estimate of its counterfactual path, which is constructed by generating a set of structural shocks for the policy rate such that the policy rate is forced to respect the ZLB (25 bps). Error bands are constructed based on 5000 Gibbs sampling draws from the posterior distribution and reported at the 68 percent level. The red line is the counterfactual value, blue band is the error band, and the black line is the real observed value. Net exports are calculated *ex-post* estimation, not included in the estimated model.

Table 2. Average Counterfactual Path for ZLB

Variable	Canadian ZLB Imposed			US ZLB Imposed		
	Mean	68% Conf. Bound		Mean	68% Conf. Bound	
		Upper bound	Lower bound		Upper bound	Lower bound
	Percent Difference					
Canadian interest rate	0.24	0.24	0.24	0.57	0.917	0.219
Canadian CPI	-0.099	-0.05	-0.152	-0.375	0.399	-1.134
Canadian industrial production	-0.131	-0.064	-0.198	-1.194	-0.215	-2.283
US interest rate				1.481	1.481	1.481
US CPI				0.256	1.053	-0.497
US industrial production				-1.535	-0.009	-3.337
	Cumulative Percent Difference					
Canadian industrial production	-1.828	-0.902	-2.772	-101.51	-18.278	-194.095
US industrial production				-130.517	-0.773	-283.639
	Unit Difference					
Canadian CPI	-1.145	-0.578	-1.75	-4.671	4.978	-14.137
Canadian industrial production	-1.283	-0.633	-1.946	-13.068	-2.379	-24.967
US CPI				5.959	24.621	-11.668
US industrial production				-16.86	-0.24	-36.498
	Cumulative Unit Difference					
Canadian industrial production	-17.96	-8.869	-27.246	-1110.809	-202.249	-2122.169
US industrial production				-1433.141	-20.381	-3102.297

Note: This table contains the average and cumulative percent (top panel) and average and cumulative unit (bottom panel) differences between the observed values and the counterfactual mean as well as 68% bounds. The average difference is calculated as $\frac{1}{\tilde{T}} \sum_{t \in ZLB} \frac{y_t - y_t^{CF}}{y_t}$ (percent difference) and $\frac{1}{\tilde{T}} \sum_{t \in ZLB} y_t - y_t^{CF}$ (unit difference), where y_t is the observed value at time t and y_t^{CF} is the value under the counterfactual scenario, ZLB denotes the time period for the experiment and \tilde{T} the number of observations for the experiment during the ZLB. Note that in those cases when the max and min bounds include 0, the possibility that the paths were significantly different from zero for individual months during the experiment period is not excluded. The cumulative difference is calculated as $\sum_{t \in ZLB} \frac{y_t - y_t^{CF}}{y_t}$ (percent difference) and $\sum_{t \in ZLB} y_t - y_t^{CF}$ (unit difference). Note that for interest rates, the percent and unit difference are identical.

Figure 5. Counterfactual Paths: US ZLB Imposed



Note: This figure plots the observe value of each series in our model along with an estimate of its counterfactual path, which is constructed by generating a set of structural shocks for the policy rate such that the policy rate is forced to respect the ZLB (25 bps). Error bands are constructed based on 5000 Gibbs sampling draws from the posterior distribution and reported at the 68 percent level. The red line is the counterfactual value, blue band is the error band, and the black line is the real observed value. Net exports are calculated *ex-post* estimation, not included in the estimated model.

Table 3. Average Counterfactual Path for ZLB, Government Expenditure Model

Variable	Canadian ZLB Imposed			US ZLB Imposed		
	Mean	68% Conf. Bound		Mean	68% Conf. Bound	
		Upper bound	Lower bound		Upper bound	Lower bound
	Percent Difference					
Canadian Interest rate	0.24	0.24	0.24	0.692	1.014	0.432
Canadian CPI	-0.122	-0.067	-0.18	-0.577	-0.057	-1.181
Canadian industrial production	-0.132	-0.062	-0.203	-1.054	-0.167	-1.993
US interest rate				1.481	1.481	1.481
US CPI				0.081	0.717	-0.556
US industrial production				-1.169	0.255	-2.774
	Cumulative Percent Difference					
Canadian industrial production	-1.843	-0.866	-2.848	-89.584	-14.174	-169.416
US industrial production				-99.334	21.693	-235.796
	Unit Difference					
Canadian CPI	-1.413	-0.775	-2.075	-7.16	-0.711	-14.661
Canadian industrial production	-1.294	-0.608	-2	-11.51	-1.839	-21.747
US CPI				1.863	16.754	-13.018
US industrial production				-12.847	2.645	-30.325
	Unit Percent Difference					
Canadian industrial production	-18.11	-8.505	-27.994	-978.34	-156.321	-1848.467
US industrial production				-1091.964	224.824	-2577.616

Note: See Table 2.

Table 4. Average Counterfactual Path for ZLB, Current Account Model

Variable	Canadian ZLB Imposed			US ZLB Imposed		
	Mean	68% Conf. Bound		Mean	68% Conf. Bound	
		Upper bound	Lower bound		Upper bound	Lower bound
	Percent Difference					
Canadian Interest rate	0.24	0.24	0.24	0.655	0.928	0.387
Canadian CPI	-0.105	-0.047	-0.166	-0.239	0.350	-0.787
Canadian industrial production	-0.147	-0.077	-0.219	-1.054	-0.184	-2.02
US interest rate				1.481	1.481	1.481
US CPI				0.216	1.037	-0.544
US industrial production				-1.591	0.003	-3.432
	Cumulative Percent Difference					
Canadian industrial production	-1.474	-0.665	-2.330	-20.326	29.789	-66.862
US industrial production				-135.195	0.272	-291.737
	Unit Difference					
Canadian CPI	-1.214	-0.548	-1.920	-2.972	4.368	-9.783
Canadian industrial production	-1.449	-0.758	-2.151	-11.548	-2.051	-22.098
US CPI				5.029	24.243	-12.760
US industrial production				-17.461	-0.105	-37.537
	Cumulative Unit Difference					
Canadian industrial production	-20.281	-10.606	-30.117	-981.593	-174.328	-1878.310
US industrial production				-1484.171	-8.913	-3190.650

Note: See Table 2.

REFERENCES

- Alpanda, Sami, and Serdar Kabaca, 2015, "International spillovers of large-scale asset purchases," *Journal of the European Economic Association*.
- Anzuini, Alessio, Marco J Lombardi, and Patrizio Pagano, 2013, "The impact of monetary policy shocks on commodity prices," *International Journal of Central Banking*, Vol. 9, No. 3, pp. 119–144.
- Bank of Canada, 2010, *Operating framework for the implementation of monetary policy at the effective lower bound for the overnight interest rate*, available at http://www.bankofcanada.ca/wp-content/uploads/2010/04/rate_210409_1.pdf.
- Bauer, Michael D, and Christopher J Neely, 2014, "International channels of the Fed's unconventional monetary policy," *Journal of International Money and Finance*, Vol. 44, pp. 24–46.
- Bauer, Michael D, and Glenn D Rudebusch, 2013, "Monetary policy expectations at the zero lower bound," Federal Reserve Bank of San Francisco Working Paper 2013-18.
- Baumeister, Christiane, and Luca Benati, 2013, "Unconventional monetary policy and the Great Recession: estimating the macroeconomic effects of a spread compression at the zero lower bound," *International Journal of Central Banking*, Vol. 9, No. 2, pp. 165–212.
- Bekaert, Geert, Marie Hoerova, and Marco Lo Duca, 2013, "Risk, uncertainty and monetary policy," *Journal of Monetary Economics*, Vol. 60, No. 7, pp. 771–788.
- Bernanke, Ben S, and Vincent R Reinhart, 2004, "Conducting monetary policy at very low short-term interest rates," *The American Economic Review*, Vol. 94, No. 2, p. 85.
- Bhuiyan, Rokon, 2012, "Monetary transmission mechanisms in a small open economy: a Bayesian structural VAR approach," *Canadian Journal of Economics/Revue canadienne d'économique*, Vol. 45, No. 3, pp. 1037–1061.
- Black, Fischer, 1995, "Interest rates as options," *Journal of Finance*, Vol. 50, No. 5, pp. 1371–1376.
- Blanchard, O, JD Ostry, and AR Ghosh, 2013, "International policy coordination: The Loch Ness monster," Available at <https://blog-imfdirect.imf.org/2013/12/15/international-policy-coordination-the-loch-ness-monster/>.
- Bolder, David Jamieson, Grahame Johnson, and Adam Metzler, 2004, "An empirical analysis of the Canadian term structure of zero-coupon interest rates," Bank of Canada Staff Working Paper 2004-48.
- Bowman, David, Juan M Londono, and Horacio Sapriza, 2015, "U.S. unconventional monetary policy and transmission to emerging market economies," *Journal of International Money and Finance*, Vol. 55, pp. 27–59.
- Bruno, Valentina, and Hyun Song Shin, 2015, "Capital flows and the risk-taking channel of monetary policy," *Journal of Monetary Economics*, Vol. 71, pp. 119–132.

- Caceres, Carlos, Yan Carriere-Swallow, Ishak Demir, and Bertrand Gruss, 2016, “US Monetary Policy Normalization and Global Interest Rates,” International Monetary Fund Working Paper 16/195.
- Canova, Fabio, 2005, “The transmission of U.S. shocks to Latin America,” *Journal of Applied Econometrics*, Vol. 20, No. 2, pp. 229–251.
- Cecchetti, Stephen, Tommaso Mancini Griffoli, and Machiko Narita, 2017, “Does Prolonged Monetary Policy Easing Increase Financial Vulnerability?” International Monetary Fund Working Paper 17/65.
- Chang, Bo Young, and Bruno Feunou, 2013, “Measuring uncertainty in monetary policy using implied volatility and realized volatility,” Bank of Canada Staff Working Paper 2013-37.
- Charnavoki, Valery, and Juan J Dolado, 2014, “The effects of global shocks on small commodity-exporting economies: Lessons from Canada,” *American Economic Journal: Macroeconomics*, Vol. 6, No. 2, pp. 207–237.
- Chen, Qianying, Andrew Filardo, Dong He, and Feng Zhu, 2015, “Financial crisis, US unconventional monetary policy and international spillovers,” IMF Working Paper 85.
- Christensen, Jens HE, and Glenn D Rudebusch, 2014, “Estimating shadow-rate term structure models with near-zero yields,” *Journal of Financial Econometrics*, pp. 1–34.
- Chung, Hess, Jean-Philippe Laforte, David Reifschneider, and John C Williams, 2012, “Have we underestimated the likelihood and severity of zero lower bound events?” *Journal of Money, Credit and Banking*, Vol. 44, No. s1, pp. 47–82.
- Colciago, Andrea, Anna Samarina, and Jakob de Haan, 2018, “Central bank policies and income and wealth inequality: A survey,” *Journal of Economic Surveys*, Vol. 33, No. 4, pp. 1199–1231.
- Cushman, David O, and Tao Zha, 1995, “Identifying monetary policy in a small open economy under flexible exchange rates,” Working Paper, Federal Reserve Bank of Atlanta 95-7.
- , 1997, “Identifying monetary policy in a small open economy under flexible exchange rates,” *Journal of Monetary economics*, Vol. 39, No. 3, pp. 433–448.
- Dahlhaus, Tatjana, Kristina Hess, and Abeer Reza, 2014, “International transmission channels of US quantitative easing: evidence from Canada,” Bank of Canada Working Paper 2014-43.
- Dahlhaus, Tatjana, and Garima Vasishtha, 2014, “The impact of US monetary policy normalization on capital flows to emerging-market economies,” Bank of Canada Working Paper 2014-53.
- D’Amico, Stefania, and Thomas B King, 2013, “Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply,” *Journal of Financial Economics*, Vol. 108, No. 2, pp. 425–448.

- Dedola, Luca, Giulia Rivolta, and Livio Stracca, 2017, "If the Fed sneezes, who catches a cold?" *Journal of International Economics*.
- Driffill, John, 2016, "Unconventional monetary policy in the euro zone," *Open Economies Review*, Vol. 27, No. 2, pp. 387–404.
- Ehrmann, Michael, and Marcel Fratzscher, 2005, "Equal size, equal role? Interest rate interdependence between the Euro Area and the United States," *The Economic Journal*, Vol. 115, No. 506, pp. 928–948.
- Eichengreen, Barry, and Poonam Gupta, 2014, "Tapering talk: the impact of expectations of reduced Federal Reserve security purchases on emerging markets," World Bank Policy Research Working Paper 6754.
- Engel, Charles, 2015, "International coordination of central bank policy," NBER Working Paper 20952.
- Fratzscher, Marcel, Marco Lo Duca, and Roland Straub, 2013, "On the international spillovers of US quantitative easing," DIW Berlin Discussion Paper 1304.
- , 2014, "ECB unconventional monetary policy actions: market impact, international spillovers and transmission channels," Presented at the 15th Jacques Polak Annual Research Conference, International Monetary Fund.
- Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian P Sack, 2010, "Large-scale asset purchases by the Federal Reserve: did they work?" Federal Reserve Board of New York Staff Report 441.
- Gagnon, Joseph E, Tamim Bayoumi, Juan M Londono, Christian Saborowski, and Horacio Sapriza, 2017, "Direct and spillover effects of unconventional monetary and exchange rate policies," *Open Economies Review*, Vol. 28, No. 2, pp. 191–232.
- Gambacorta, Leonardo, Boris Hofmann, and Gert Peersman, 2014, "The effectiveness of unconventional monetary policy at the zero lower bound: A cross-country analysis," *Journal of Money, Credit and Banking*, Vol. 46, No. 4, pp. 615–642.
- Gürkaynak, Refet S, Brian Sack, and Jonathan H Wright, 2007, "The US Treasury yield curve: 1961 to the present," *Journal of Monetary Economics*, Vol. 54, No. 8, pp. 2291–2304.
- Hamilton, James D, and Jing Cynthia Wu, 2012, "The effectiveness of alternative monetary policy tools in a zero lower bound environment," *Journal of Money, Credit and Banking*, Vol. 44, No. s1, pp. 3–46.
- He, Zhongfang, 2010, "Evaluating the effect of the Bank of Canada's conditional commitment policy," Bank of Canada Discussion Paper 2010-11.
- Hohberger, Stefan, Romanos Priftis, and Lukas Vogel, 2019, "The macroeconomic effects of quantitative easing in the Euro area: evidence from an estimated DSGE model," *Journal of Economic Dynamics and Control*, p. 103756.

- Holston, Kathryn, Thomas Laubach, and John C Williams, 2017, “Measuring the natural rate of interest: International trends and determinants,” *Journal of International Economics*.
- IMF, 2017, *Global Financial Stability Report: Are Countries Losing Control of Domestic Financial Conditions?*, available at <https://www.imf.org/en/Publications/GFSR/Issues/2017/03/30/global-financial-stability-report-april-2017#Chapter3>.
- Justiniano, Alejandro, and Bruce Preston, 2010, “Can structural small open-economy models account for the influence of foreign disturbances?” *Journal of International Economics*, Vol. 81, No. 1, pp. 61–74.
- Kolasa, Marcin, and Grzegorz Wesolowski, 2018, “International spillovers of quantitative easing,” .
- Kozicki, Sharon, Eric Santor, and Lena Suchanek, 2011, “Unconventional Monetary Policy: the international experience with central bank asset purchases,” *Bank of Canada review*, Vol. 2011, No. Spring, pp. 13–25.
- Krippner, Leo, 2012, “Modifying Gaussian term structure models when interest rates are near the zero lower bound,” .
- , 2013, “A tractable framework for zero lower bound Gaussian term structure models,” Reserve Bank of New Zealand Discussion Paper Series DP2013/02.
- Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, 2011, “The effects of quantitative easing on interest rates: channels and implications for policy,” NBER Working Paper 17555.
- Krishnamurthy, Arvind, Annette Vissing-Jorgensen, and others, 2013, “The ins and outs of LSAPs,” in *Kansas City Federal Reserve Symposium on Global Dimensions of Unconventional Monetary Policy*, pp. 57–111.
- MacDonald, Margaux, 2017, “International capital market frictions and spillovers from quantitative easing,” *Journal of International Money and Finance*, Vol. 70, pp. 135–156.
- Maćkowiak, Bartosz, 2007, “External shocks, U.S. monetary policy and macroeconomic fluctuations in emerging markets,” *Journal of Monetary Economics*, Vol. 54, No. 8, pp. 2512–2520.
- McNeil, James, 2017, “New measures of Canadian monetary policy shocks and their effects,” Unpublished Working Paper.
- Mishra, Prachi, and Raghuram Rajan, 2016, “Rules of the monetary game,” Reserve Bank of India Working Paper Series 4.
- Mouabbi, Sarah, and Jean-guillaume Sahuc, 2019, “Evaluating the Macroeconomic Effects of the ECB’s Unconventional Monetary Policies,” *Journal of Money, Credit and Banking*, Vol. 51, No. 4, pp. 831–858.
- Neely, Christopher J, 2015, “Unconventional monetary policy had large international effects,” *Journal of Banking & Finance*, Vol. 52, pp. 101–111.

- Neri, Stefano, and Andrea Nobili, 2010, "The transmission of U.S. monetary policy to the Euro Area," *International Finance*, Vol. 13, No. 1, pp. 55–78.
- Nsafoah, Dennis, and Apostolos Serletis, 2019, "International monetary policy spillovers," *Open Economies Review*, Vol. 30, No. 1, pp. 87–104.
- Obstfeld, Maurice, 2015, "Trilemmas and Tradeoffs: Living with Financial Globalization," in Claudio Raddatz, Diego Saravia, and Jaume Ventura (eds.), *Global Liquidity, Spillovers to Emerging Markets and Policy Responses*, Vol. 20, pp. 013–078 (Central Bank of Chile).
- Ramey, Valerie A, 2011, "Can government purchases stimulate the economy?" *Journal of Economic Literature*, Vol. 49, No. 3, pp. 673–685.
- , 2016, "Macroeconomic shocks and their propagation," *Handbook of Macroeconomics*, Vol. 2, pp. 71–162.
- Rey, H el ene, 2015, "Dilemma not trilemma: the global financial cycle and monetary policy independence," .
- Rogers, John H, Chiara Scotti, and Jonathan H Wright, 2014, "Evaluating asset-market effects of unconventional monetary policy: a multi-country review," *Economic Policy*, Vol. 29, No. 80, pp. 749–799.
- Rossi, Barbara, and Sarah Zubairy, 2011, "What is the importance of monetary and fiscal shocks in explaining US macroeconomic fluctuations?" *Journal of Money, Credit and Banking*, Vol. 43, No. 6, pp. 1247–1270.
- Santor, Eric, and Lena Suchanek, 2016, "A new era of Central Banking: Unconventional monetary policies," *Bank of Canada Review*, Vol. 2016, No. Spring, pp. 29–42.
- Sims, Christopher A, and Tao Zha, 1998, "Bayesian methods for dynamic multivariate models," *International Economic Review*, Vol. 39, No. 4, pp. 949–968.
- , 1999, "Error bands for impulse responses," *Econometrica*, Vol. 67, No. 5, pp. 1113–1155.
- Swanson, Eric T, and John C Williams, 2014, "Measuring the effect of the zero lower bound on medium-and longer-term interest rates," *The American Economic Review*, Vol. 104, No. 10, pp. 3154–3185.
- Taylor, John B, 1993, "Discretion versus policy rules in practice," in *Carnegie-Rochester conference series on public policy*, Vol. 39, pp. 195–214 (Elsevier).
- , 1999, "A historical analysis of monetary policy rules," in *Monetary policy rules*, pp. 319–348 (University of Chicago Press).
- , 2013, "International monetary policy coordination: past, present and future," Stanford Institute for Economic Policy Research Discussion Papers 12-034.
- Waggoner, Daniel F, and Tao Zha, 2003, "A Gibbs sampler for structural vector autoregressions," *Journal of Economic Dynamics and Control*, Vol. 28, No. 2, pp. 349–366.

Wu, Jing Cynthia, and Fan Dora Xia, 2016, “Measuring the macroeconomic impact of monetary policy at the zero lower bound,” *Journal of Money, Credit, and Banking*, Vol. 48, No. 2-3, pp. 253–291.

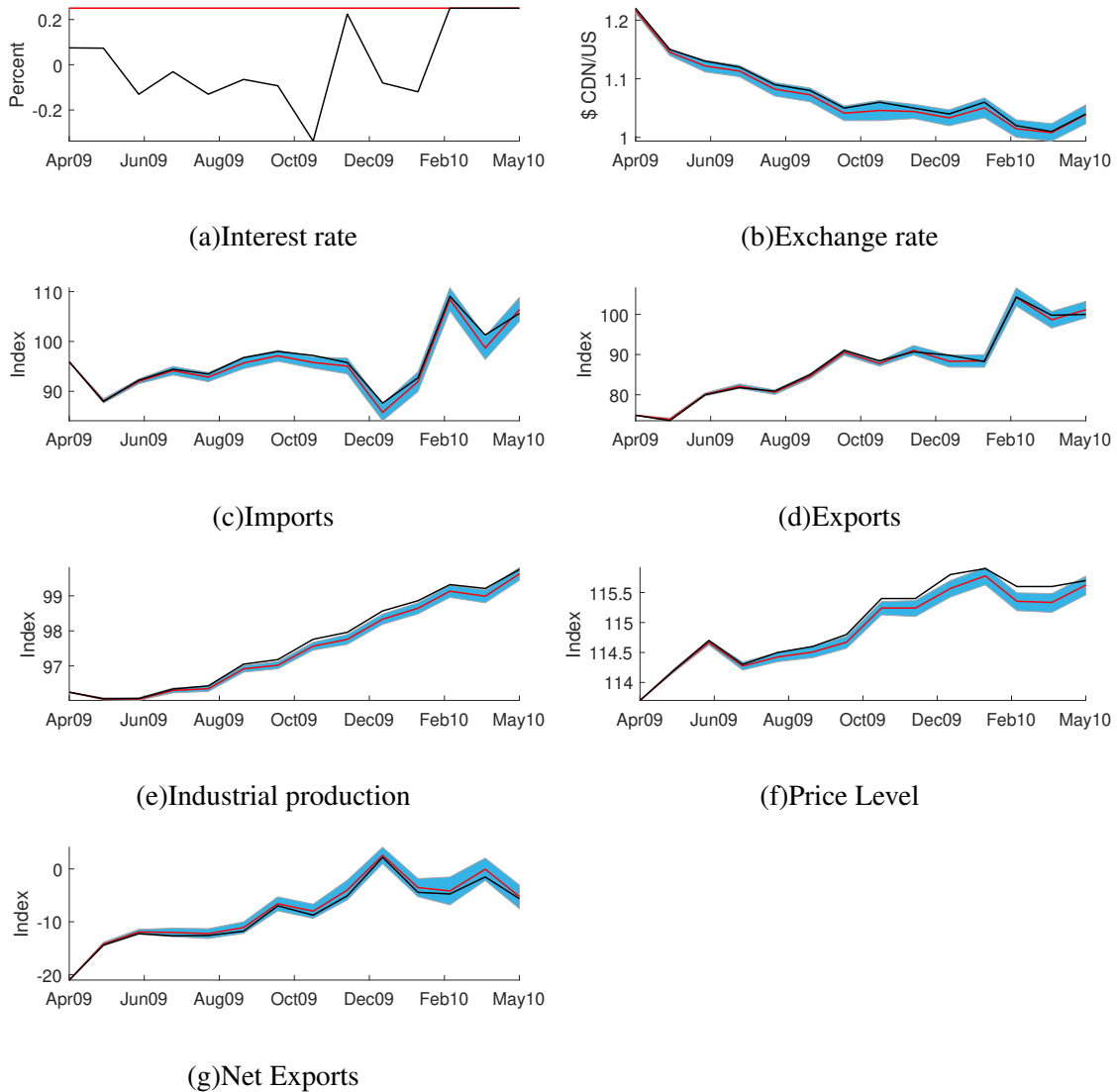
APPENDIX A. DATA

Table A.1. Data Sources and Definitions

Variable	Source	Definition
<i>ca</i>	CANSIM, author's calculation	Current account balance, seasonally adjusted. Quarterly series linearly interpolated to monthly frequency.
<i>g</i>	CANSIM, author's calculations	General federal governments final consumption expenditure, seasonally adjusted at annual rates, chained 2007 dollars. Quarterly series linearly interpolated to monthly frequency.
<i>g_{US}</i>	Federal Reserve of St. Louis Economic Data (FRED)	Federal government final consumption expenditures, seasonally adjusted at annual rates, chained 2009 dollars. Quarterly series linearly interpolated to monthly frequency.
<i>p</i>	CANSIM	Consumer Price Index, All items, Index, Monthly, seasonally adjusted..
<i>p_{US}</i>	FRED	Consumer Price Index, All items, Index, Monthly, seasonally adjusted.
<i>r</i>	Bank of Canada, author's calculations	Bank of Canada target rate spliced with shadow rate at ZLB when shadow rate < target rate = 25 bps, Monthly.
<i>r_{US}</i>	FRED, Wu and Xia (2016)	Federal funds target rate spliced with Wu and Xia (2016) shadow rate at ZLB when SSR < FFR = 25bp, Monthly.
<i>s</i>	IFS	National Currency per US Dollar, National Currency per US Dollar, Rate, Monthly average.
<i>vix</i>	CBOE	Chicago Board Options Exchange (CBOE) VIX index measuring market's expectation of 30-day volatility. Constructed using the implied volatilities of a range of S&P 500 index options, Monthly average.
<i>wxp</i>	IFS	Export Price, All Commodities, Index, Monthly average.
<i>y</i>	IFS	Industrial Production, Seasonally adjusted, Index, Monthly.
<i>y_{US}</i>	IFS	Industrial Production, Seasonally adjusted, Index, Monthly.
<i>ex, im</i>	CANSIM	International merchandise trade, by Standard International Trade Classification, customs-based price index, monthly.

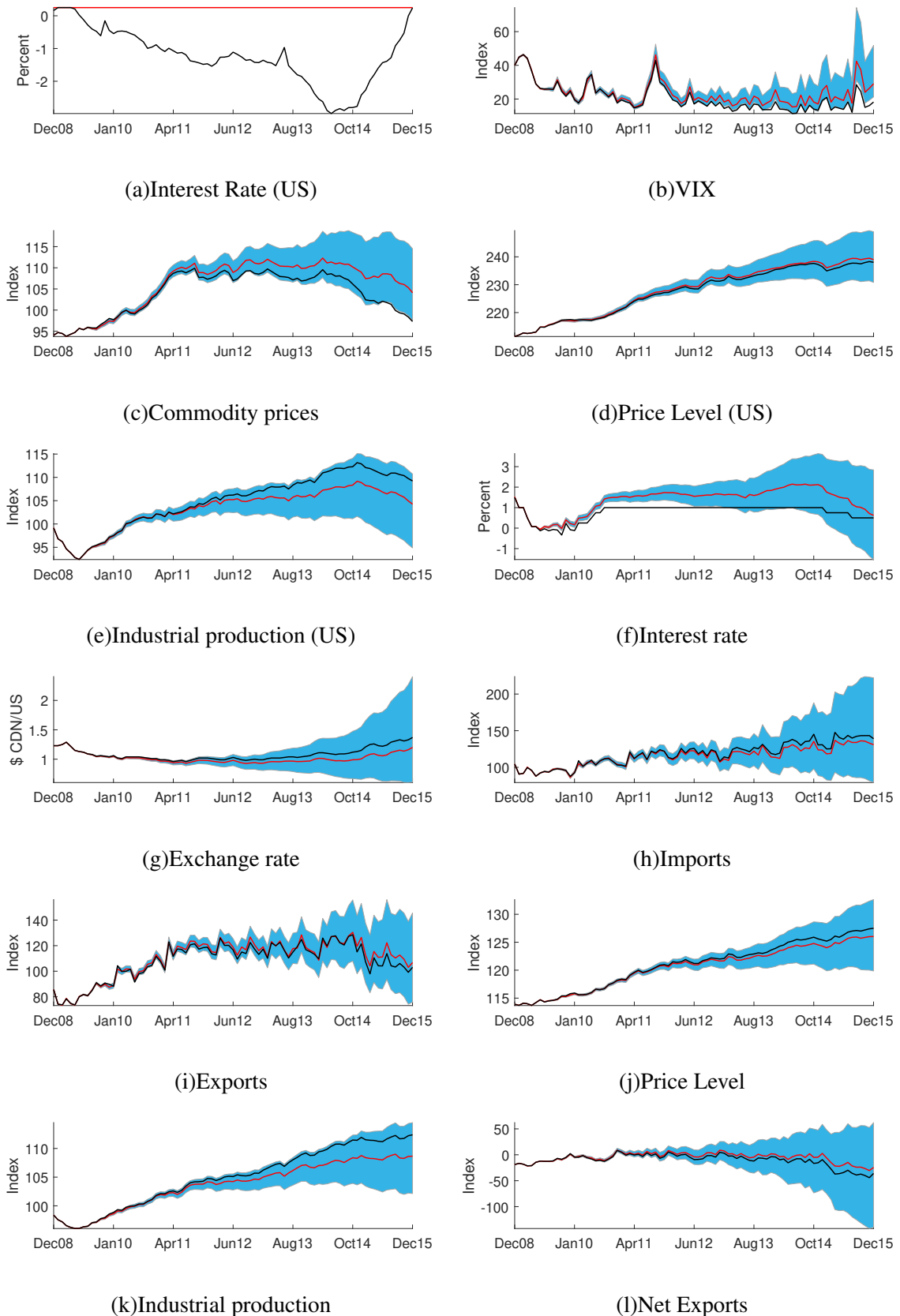
APPENDIX B. ALTERNATIVE CONFIDENCE BANDS

Figure B.1. Counterfactual Paths: Canadian ZLB Imposed (90 % confidence band)



Note: This figure plots the observe value of each series in our model along with an estimate of its counterfactual path, which is constructed by generating a set of structural shocks for the policy rate such that the policy rate is forced to respect the ZLB (25 bps). Error bands are constructed based on 5000 Gibbs sampling draws from the posterior distribution at the 90 percent confidence level. The red line is the counterfactual value, blue band is the error band, and the black line is the real observed value. Net exports are calculated *ex-post* estimation, not included in the estimated model.

Figure B.2. Counterfactual Paths: US ZLB Imposed (90 % confidence band)



Note: This figure plots the observe value of each series in our model along with an estimate of its counterfactual path, which is constructed by generating a set of structural shocks for the policy rate such that the policy rate is forced to respect the ZLB (25 bps). Error bands are constructed based on 5000 Gibbs sampling draws from the posterior distribution at 90 percent confidence level. The red line is the counterfactual value, blue band is the error band, and the black line is the real observed value. Net exports are calculated *ex-post* estimation, not included in the estimated model.

Table B.1. Average Counterfactual Path for ZLB (90 % confidence band)

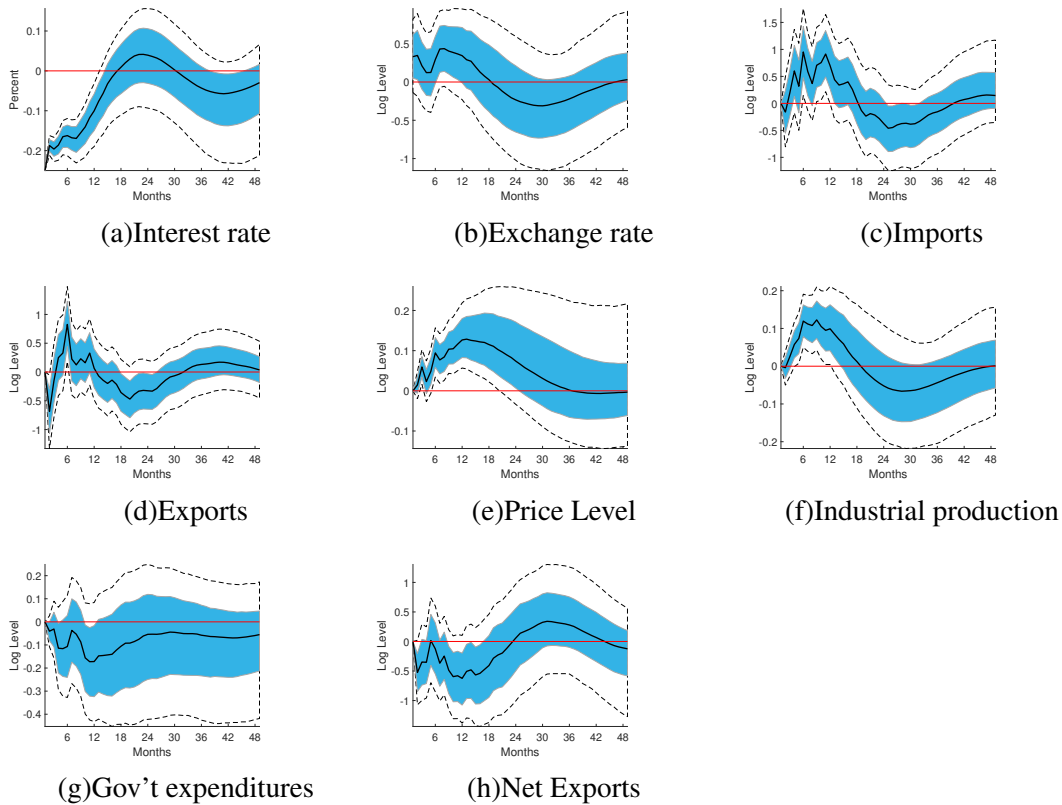
Variable	Canadian ZLB Imposed			US ZLB Imposed		
	Mean	90% Conf. Bound Upper bound	90% Conf. Bound Lower bound	Mean	90% Conf. Bound Upper bound	90% Conf. Bound Lower bound
	Percent Difference					
Canadian interest rate	0.24	0.24	0.24	0.574	1.282	-0.113
Canadian CPI	-0.1	-0.017	-0.19	-0.367	1.05	-1.725
Canadian industrial production	-0.132	-0.024	-0.243	-1.184	0.534	-3.099
US interest rate				1.481	1.481	1.481
US CPI				0.257	1.726	-1.004
US industrial production				-1.531	0.983	-4.727
	Unit Difference					
Canadian CPI	-1.156	-0.201	-2.190	-4.574	13.106	-21.508
Canadian industrial production	-1.293	-0.235	-2.386	-12.958	5.805	-33.882
US CPI				5.989	40.396	-23.526
US industrial production				-16.810	10.567	-51.668

Note: This table contains the average percentage (top panel) and average unit (bottom panel) differences between the observed values and the counterfactual Mean as well as 90% bounds. The calculation uses the formula $\frac{1}{\bar{T}} \sum_{t \in ZLB} \frac{y_t - y_t^{CF}}{y_t}$ (percent difference) and $\frac{1}{\bar{T}} \sum_{t \in ZLB} y_t - y_t^{CF}$ (unit difference), where y_t is the observed value at time t and y_t^{CF} is the value under the counterfactual scenario, ZLB denotes the time period for the experiment and \bar{T} the number of observations for the experiment during the ZLB. Note that in those cases when the max and min bounds include 0, the possibility that the paths were significantly different from zero for individual months during the experiment period is not excluded.

APPENDIX C. GOVERNMENT EXPENDITURE MODEL

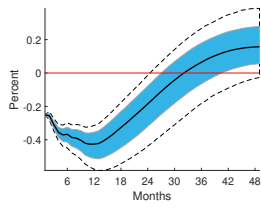
Figures C.1 and C.2 contain the impulse response functions from the government expenditure model. They are consistent with the main specification. Figures C.3 and C.4 present the same results showing only 68 percent confidence bands, which gives a better understanding of the shape of the impulses responses (see Sims and Zha (1999) for detailed explanation of this point), which are somewhat obscured with 90 percent confidence bands as they become very large after 2 years. Following a Canadian expansionary shock, industrial production and prices rise, the Canadian dollar depreciates on impact, and government shows no significant change. In response to a US expansionary shock, Canadian and US industrial production rise significantly after a short delay. Canadian government spending doesn't change significantly. US government spending, on the other hand, shows a significant temporary drop as monetary policy expands.

Figure C.1. Impulse Response Functions: Canadian Monetary Policy Shock, Government Expenditure Model

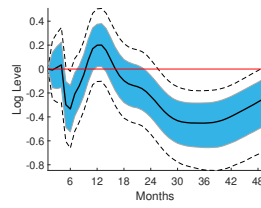


Note: See Figure 2 of paper.

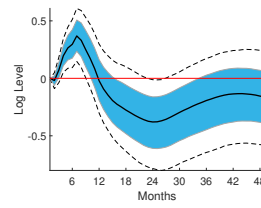
Figure C.2. Impulse Response Functions: US Monetary Policy Shock, Government Expenditure Model



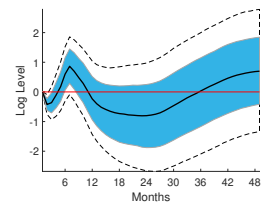
(a) Interest Rate (US)



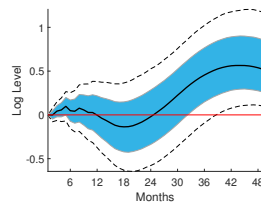
(b) VIX



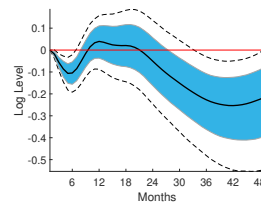
(c) Commodity prices



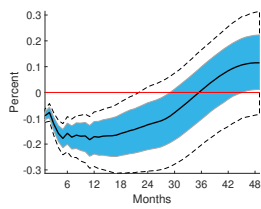
(d) Price Level (US)



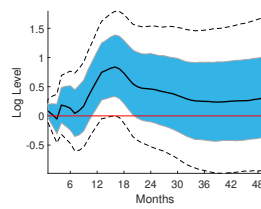
(e) Industrial production (US)



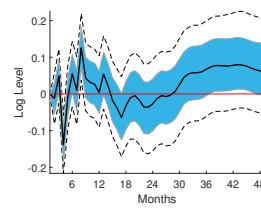
(f) Gov't expenditure (US)



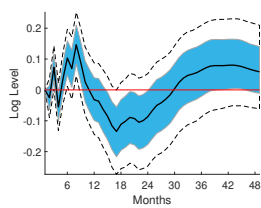
(g) Interest rate (CA)



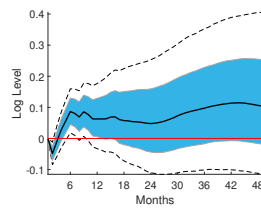
(h) Exchange rate



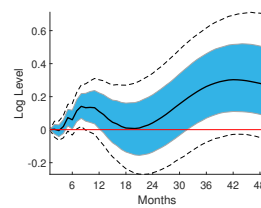
(i) Imports (CA)



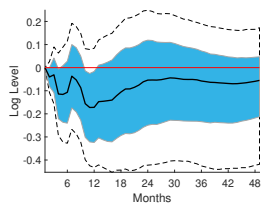
(j) Exports (CA)



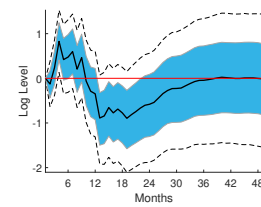
(k) Price Level (CA)



(l) Industrial production (CA)



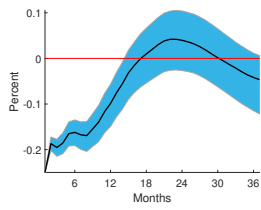
(m) Gov't expenditure (CA)



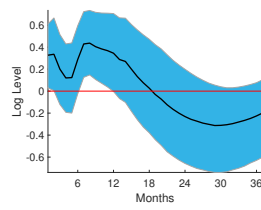
(n) Net Exports (CA)

Note: See Figure 3 of paper.

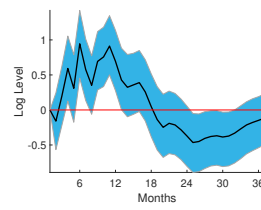
Figure C.3. Impulse Response Functions: Canadian Monetary Policy Shock, Government Expenditure Model, 68 % Confidence Bands



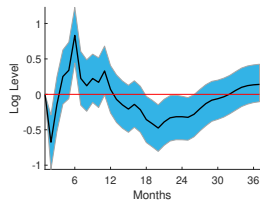
(a)Interest rate



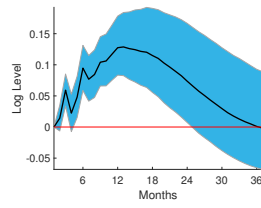
(b)Exchange rate



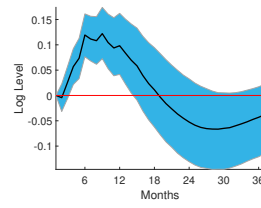
(c)Imports



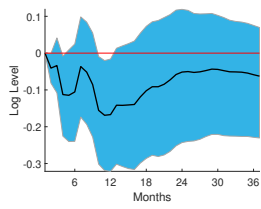
(d)Exports



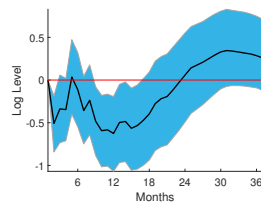
(e)Price Level



(f)Industrial production



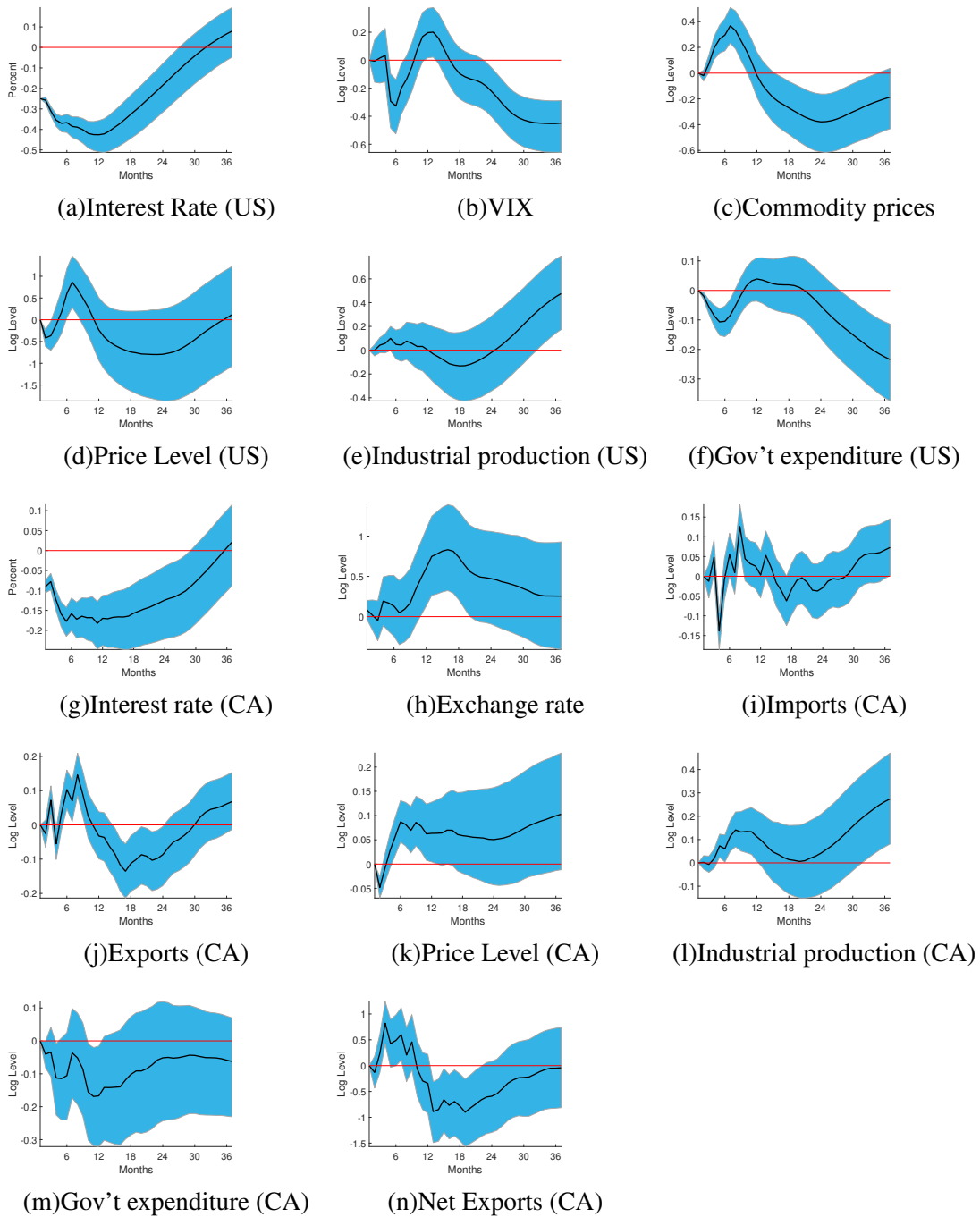
(g)Gov't expenditures



(h)Net Exports

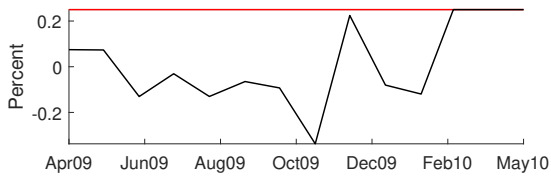
Note: See Figure 2 of paper. 68 % confidence bands reported.

Figure C.4. Impulse Response Functions: US Monetary Policy Shock, Government Expenditure Model, 68 % Confidence Bands

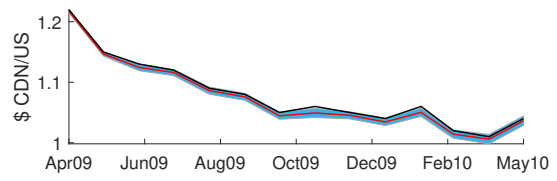


Note: See Figure 3 of paper. 68 % confidence bands reported.

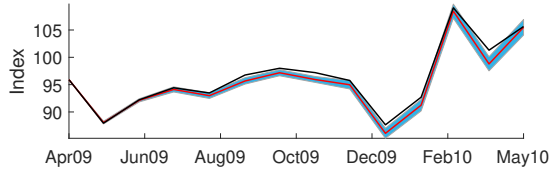
Figure C.5. Counterfactual Paths: Canadian ZLB Imposed, Government Expenditure Model



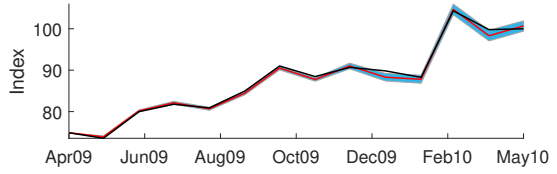
(a) Interest rate



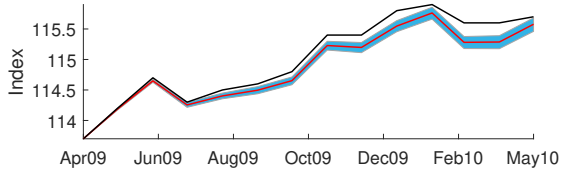
(b) Exchange rate



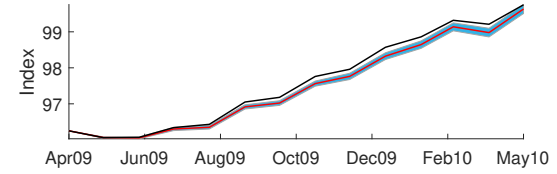
(c) Imports



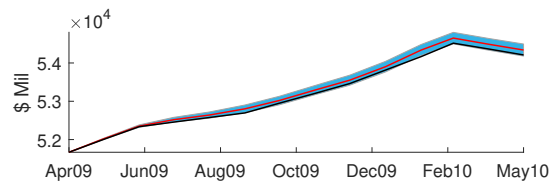
(d) Exports



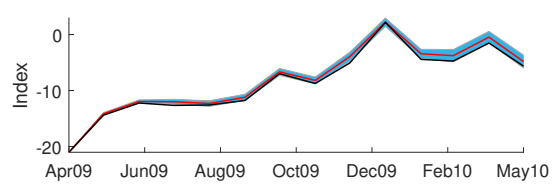
(e) Price Level



(f) Industrial production



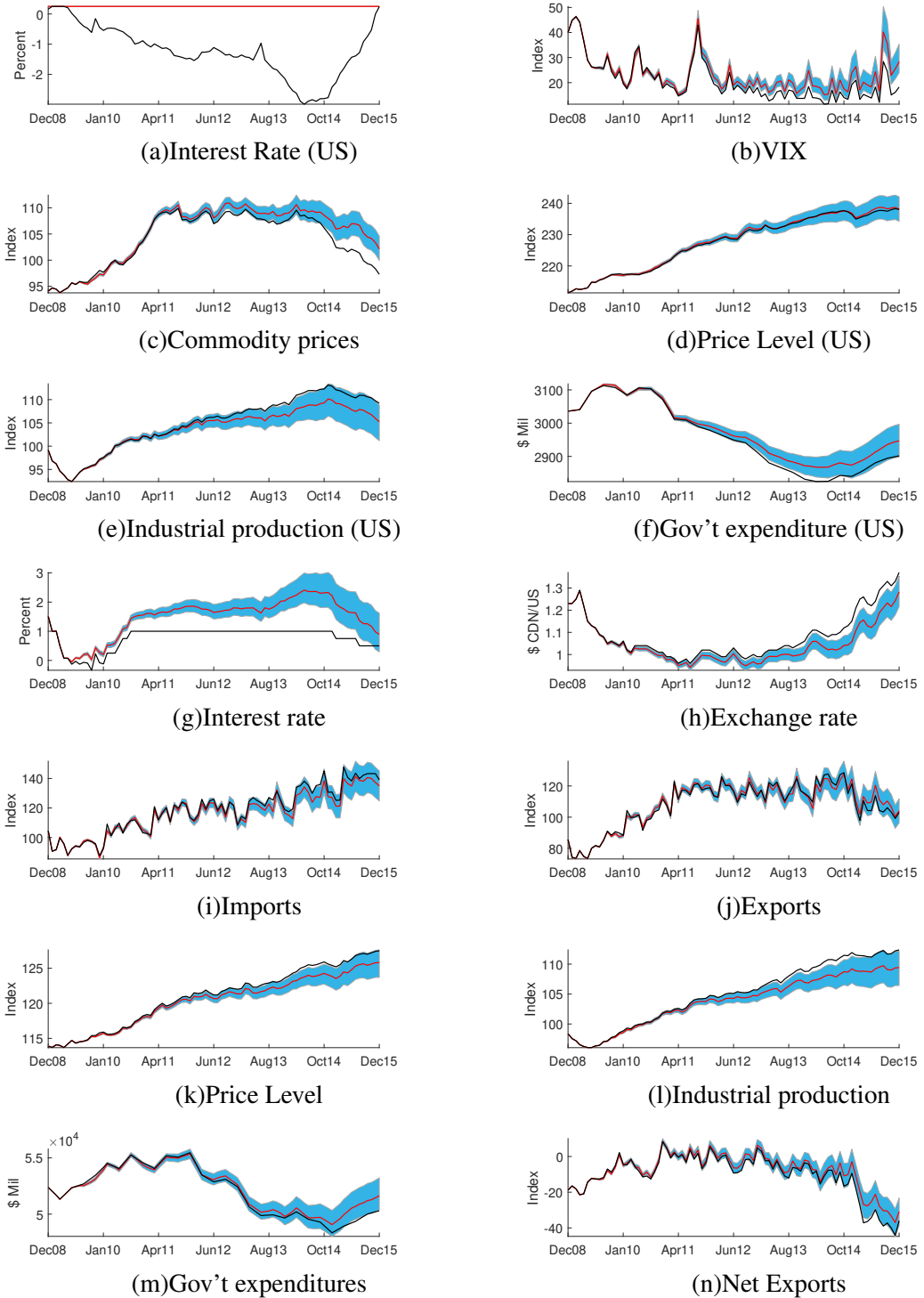
(g) Gov't expenditures



(h) Net Exports

Note: See Figure 4 of paper.

Figure C.6. Counterfactual Paths: US ZLB Imposed, Government Expenditure Model

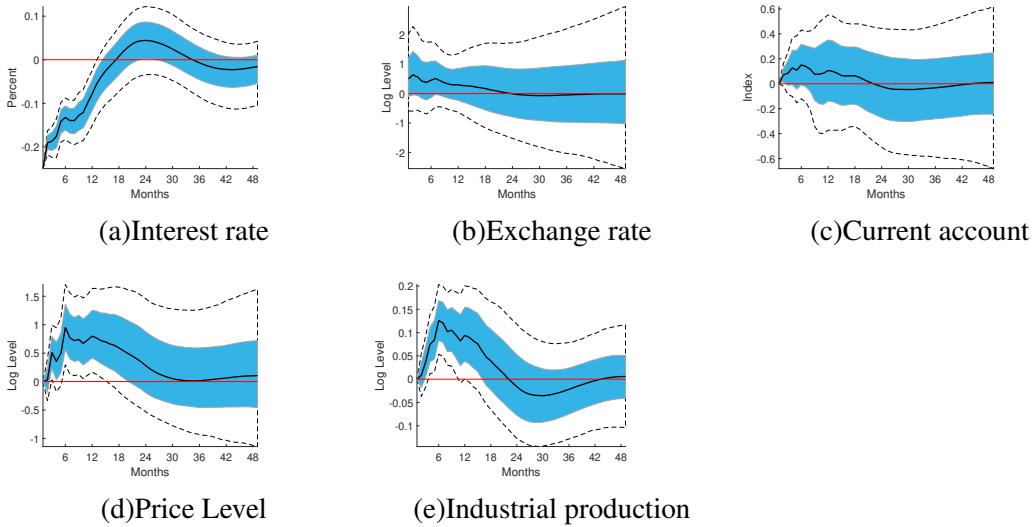


Note: Note: See Figure 5 of paper.

APPENDIX D. CURRENT ACCOUNT MODEL

Figure D.1 reports the impulse response functions for a 25 bps expansionary monetary policy shock in Canada. As in Section IV of the paper, both the price level and industrial production rise, but the initial exchange rate depreciation is no longer statistically significant. We observe no significant impact on the Canadian current account. The Canadian impulse response functions for a 25 bps expansionary monetary policy shock in the US are reported in Figure D.2.³¹ The results are also largely consistent with those in Section IV of the paper. Prices fall significantly on impact, albeit very temporarily, and rise after a lag of about 5 months. Industrial production rises after approximately a two year lag, following the path of US industrial production. As in our main results, the currency depreciates as the Canadian policy rate falls, albeit not significantly. Finally, we see a significant rise in the Canadian current account balance, which along with our main trade results, suggests that rising income in the US has a substantial spillover to Canada via an income effect (that is, rising US income receipts flowing to Canada).

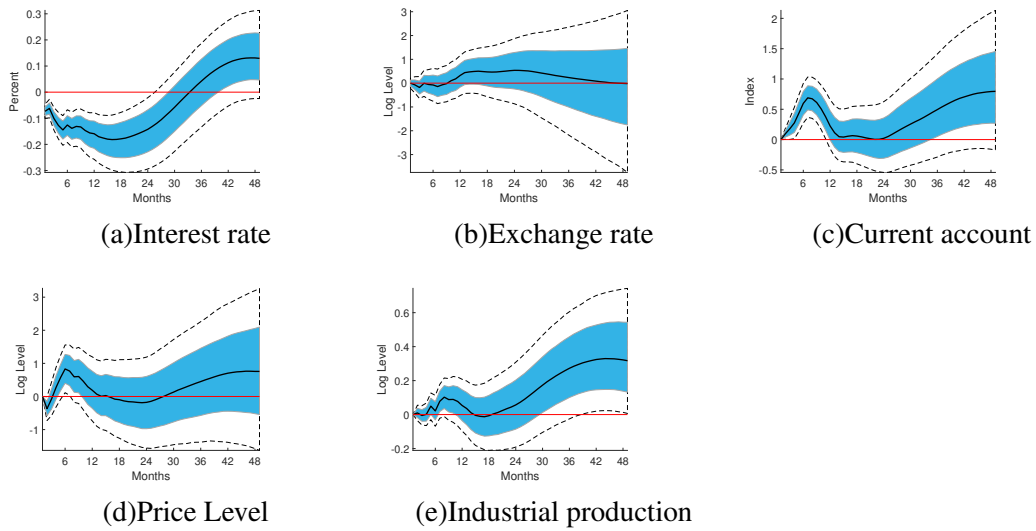
Figure D.1. Impulse Response Functions: Canadian Monetary Policy Shock, Current Account Model



Note: See Figure 2 of the paper.

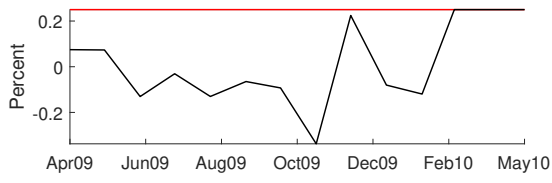
³¹Because of our block exogeneity assumption, the effect on US variables is identical to the main specification. We do not show them again here.

Figure D.2. Impulse Response Functions: US Monetary Policy Shock, Current Account Model

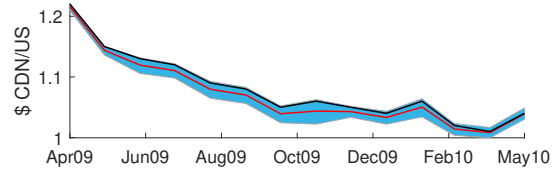


Note: See Figure 3 of the paper. US impulse response functions are unchanged from Figure 3 of the paper when we replace exports and imports with the current account balance.

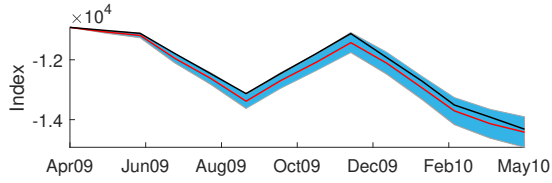
Figure D.3. Counterfactual Paths: Canadian ZLB Imposed, Current Account Model



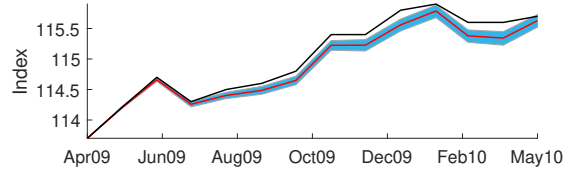
(a)Interest rate



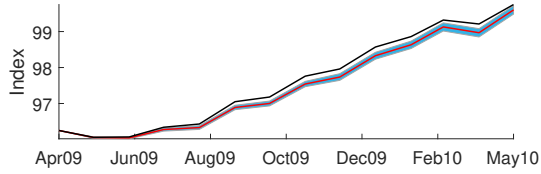
(b)Exchange rate



(c)Current account



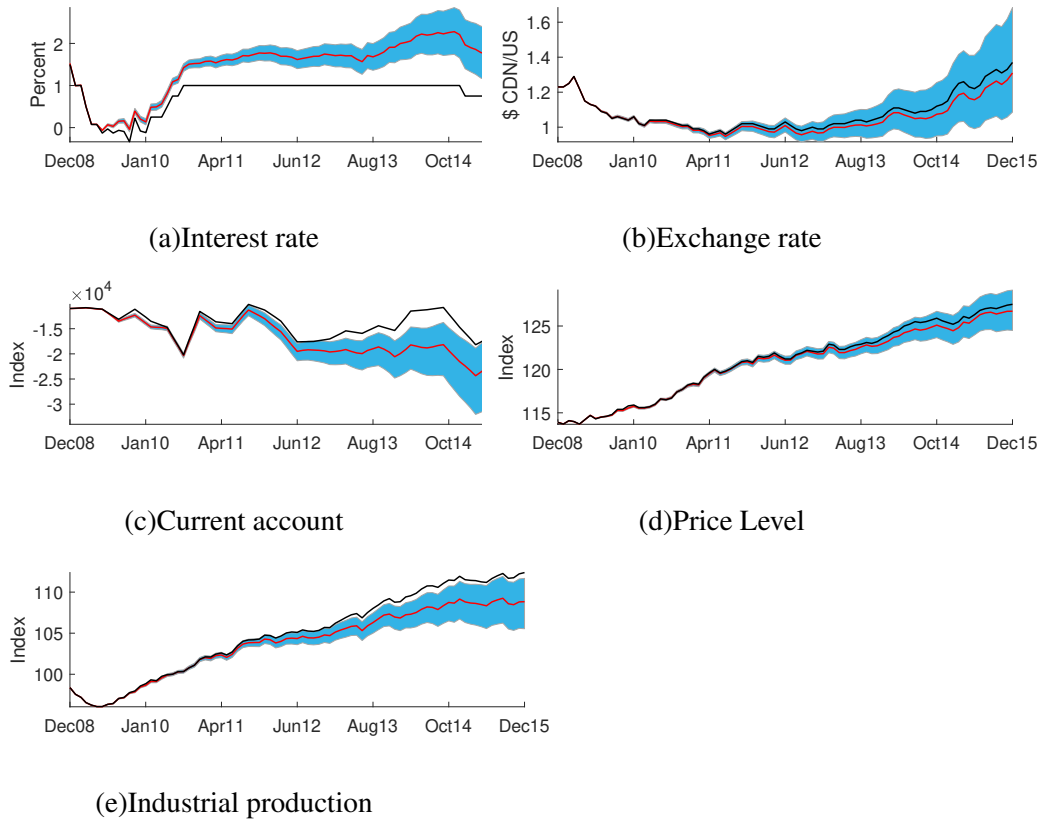
(d)Price Level



(e)Industrial production

Note: See Figure 4 of paper.

Figure D.4. Counterfactual Paths: US ZLB Imposed, Current Account Model



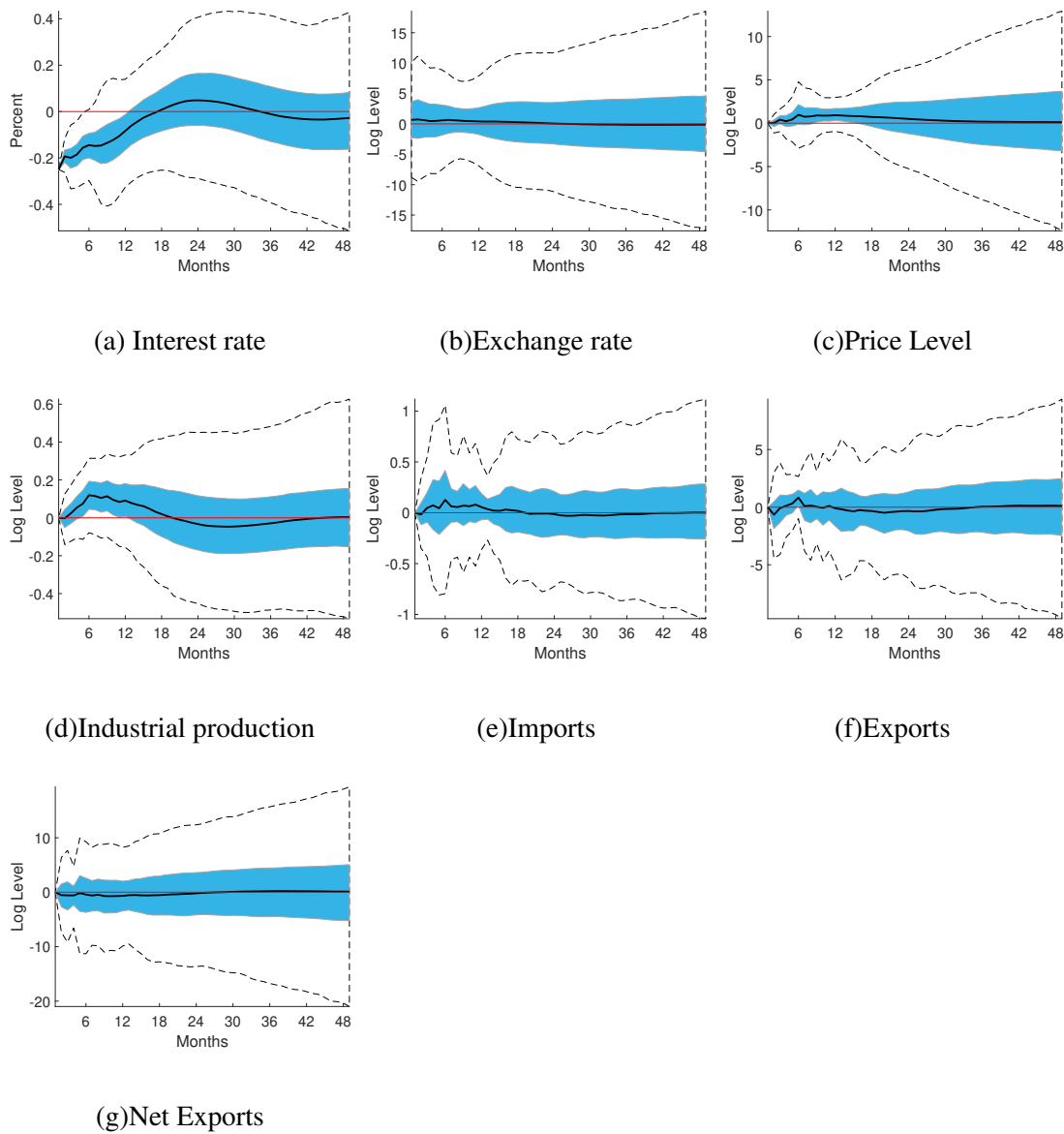
Note: See Figure 5 of paper. US impulse response functions are unchanged from figure 5 when we replace exports and imports with the current account balance.

APPENDIX E. ALTERNATIVE SPECIFICATIONS

E.1. Alternative Bank of Canada Reaction Function

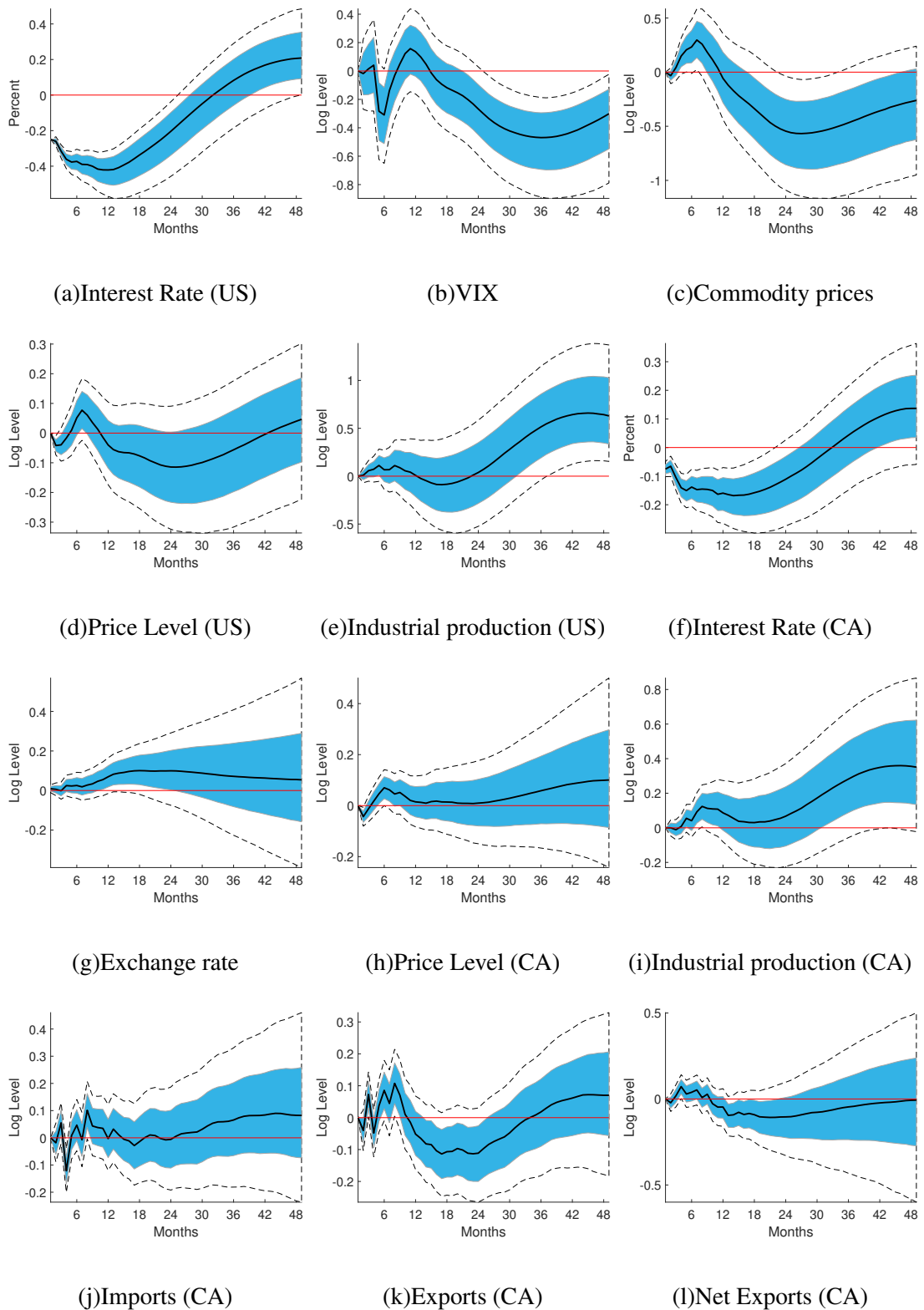
Figures E.1 - E.4 report the impulse response functions and counterfactual figures for the model with the Bank of Canada's reaction function completely unconstrained.

Figure E.1. Impulse Response Functions: Canadian Monetary Policy Shock, Alternative Exclusion Restrictions 1



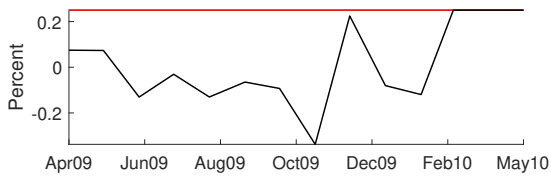
Note: See Figure 2 of paper.

Figure E.2. Impulse Response Functions: US Monetary Policy Shock, Alternative Exclusion Restrictions 1

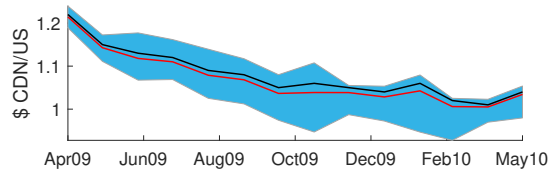


Note: See Figure 3 of paper.

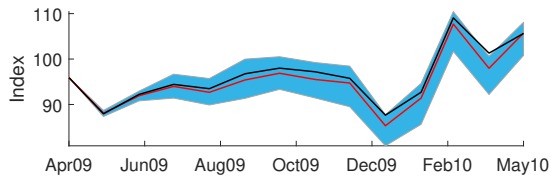
Figure E.3. Counterfactual Paths: Canadian ZLB Imposed, Alternative Exclusion Restrictions
1



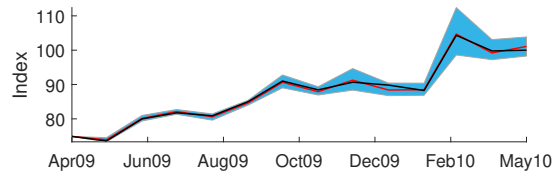
(a) Interest rate



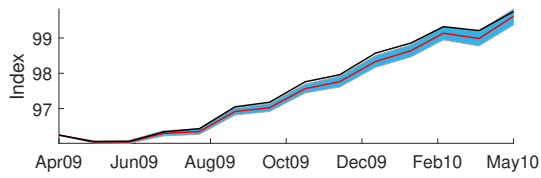
(b) Exchange rate



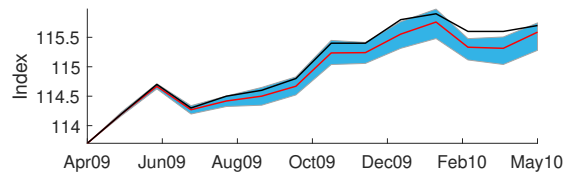
(c) Imports



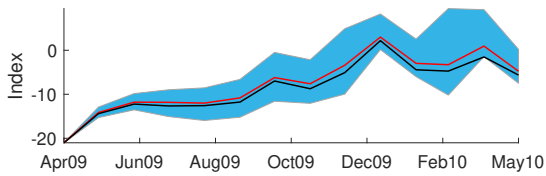
(d) Exports



(e) Industrial production



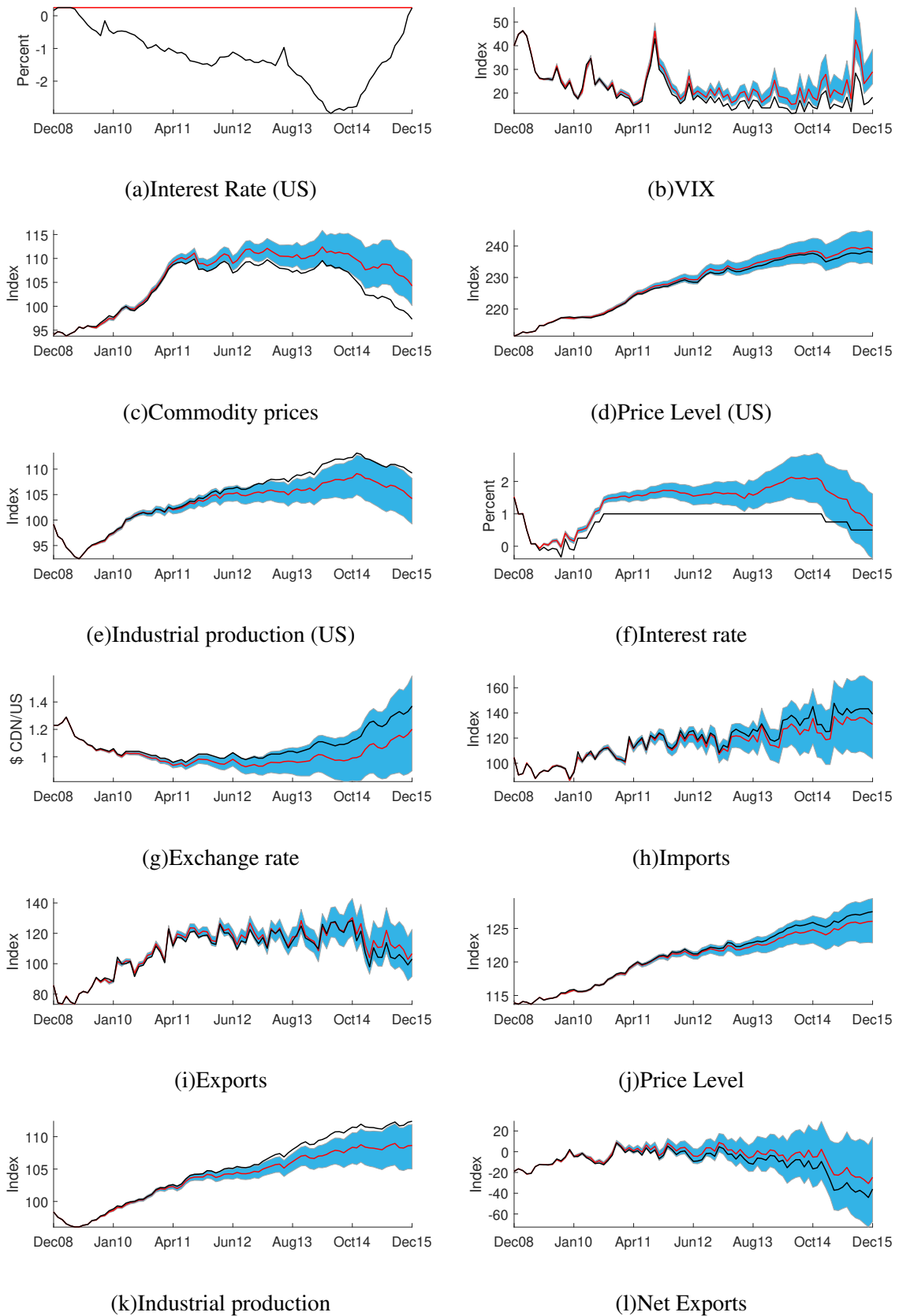
(f) Price Level



(g) Net Exports

Note: See Figure 4 of paper.

Figure E.4. Counterfactual Paths: US ZLB Imposed, Alternative Exclusion Restrictions 1



Note: See Figure 5 of paper.

E.2. Using a Taylor Rule

In this Section, we use a derived Taylor Rule as the measure of central bank policy stance in place of the shadow rate for both countries. The Taylor Rule, originally introduced in Taylor (1993) and Taylor (1999) is an alternative estimate of a policy rate which, like the shadow rate, is based on current economic conditions. Specifically, it is estimated using the output gap and the rate of inflation. We derive the Taylor Rule using the prescribed Taylor Rule for the US generated by the Federal Reserve Bank's of Atlanta's *Taylor Rule Utility*, which calculates the rate based on the generalization of Taylor's formula:

$$\widehat{FFR}_t = \rho FFR_{t-1} + (1 - \rho)[(r_t^* + \pi_t^*) + 1.5(\pi_t - \pi_t^*) + \beta gap_t] \quad (3)$$

where t denotes a quarter of a year and $t - 1$ denotes the previous quarter. FFR denotes the quarterly average of the effective federal funds rate while the hat denotes a prescribed value, π denotes the actual rate of inflation and π^* the target rate of inflation, which we set to 2%, gap is the estimated output gap and r_t^* the estimated natural (real) rate of interest. We use estimates from Holston, Laubach, and Williams (2017) for the natural rate of interest.³² We use the Congressional Budget Office's real GDP gap as the output gap. We assume the weight on the output gap, β , to be 0.5 and the weight on the interest rate smoothing parameter, ρ , to be zero, which are the generally accepted values in the literature.

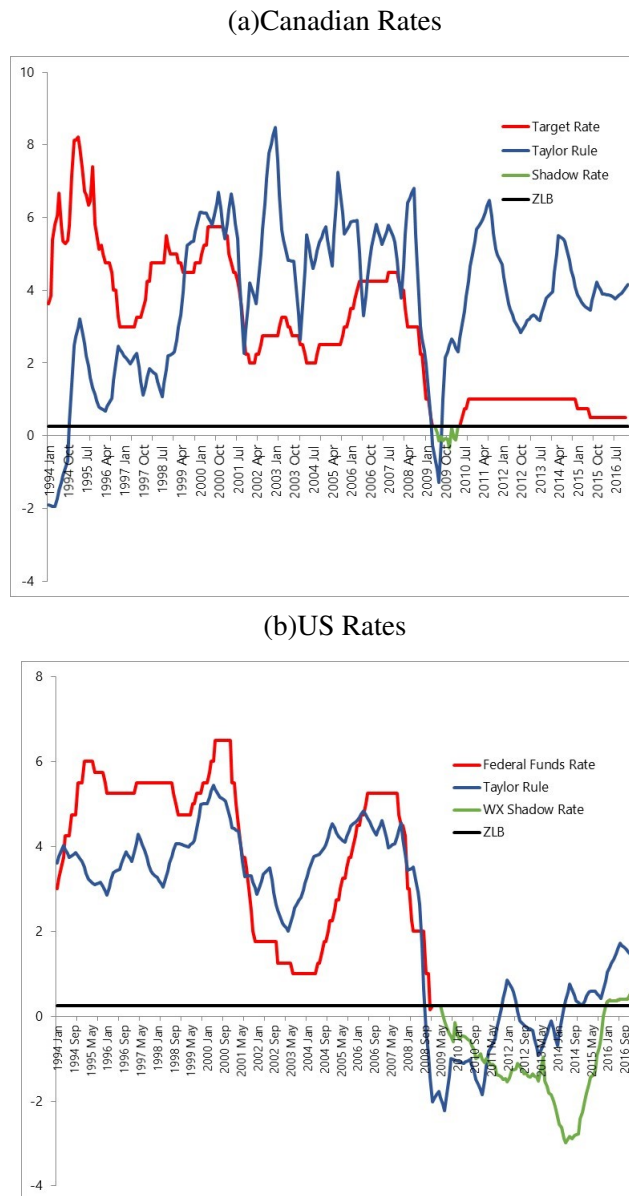
We take the same approach for calculating a Canadian Taylor Rule, replacing the FFR with the Bank of Canada's target interest rate. We use Holston, Laubach, and Williams (2017) estimated natural interest rate for Canada and the Bank of Canada's published output gap series. Finally, we assume the target interest rate for Canada is also 2%, which is consistent with the Bank of Canada's mandate, and the same weight on the output gap and interest rate smoothing parameters as for the US rate. For both the US and Canadian rates, the Taylor Rule rates are calculated at a quarterly frequency, matching the output gap and natural interest rate series, and then linearly interpolated to monthly series.

Figure E.5 plots the two Taylor Rule interest rates along with the actual policy rates for both countries. It's clear that the Taylor Rule rates do not follow the observed rate very closely, particularly for Canada. Despite this, in both cases the Taylor Rule rate does go below the ZLB during (at least part of) the unconventional monetary policy regimes in both countries.

We construct the policy rate according to the same definition as the shadow rate, (1), but using the prescribed Taylor Rule rate. That is, we use the prescribed rate when the Federal Funds Rate or the Bank of Canada's policy rate is at the ZLB and the prescribed Taylor Rule rate is below the ZLB. Unlike the shadow rate there are several months in both countries when the prescribed Taylor Rule rate was greater than 25 bps, while the official rates were 25 bps. In these cases, we set the policy rate to 25 bps (in accordance with the definition in (1)).

³²We choose this measure of the natural rate of interest because the authors estimate a rate for both Canada and the US, which provides us with the most consistent estimate of the Taylor Rule for both countries.

Figure E.5. Canadian and US Policy Rates and Taylor Rule Rates, 1994–2016



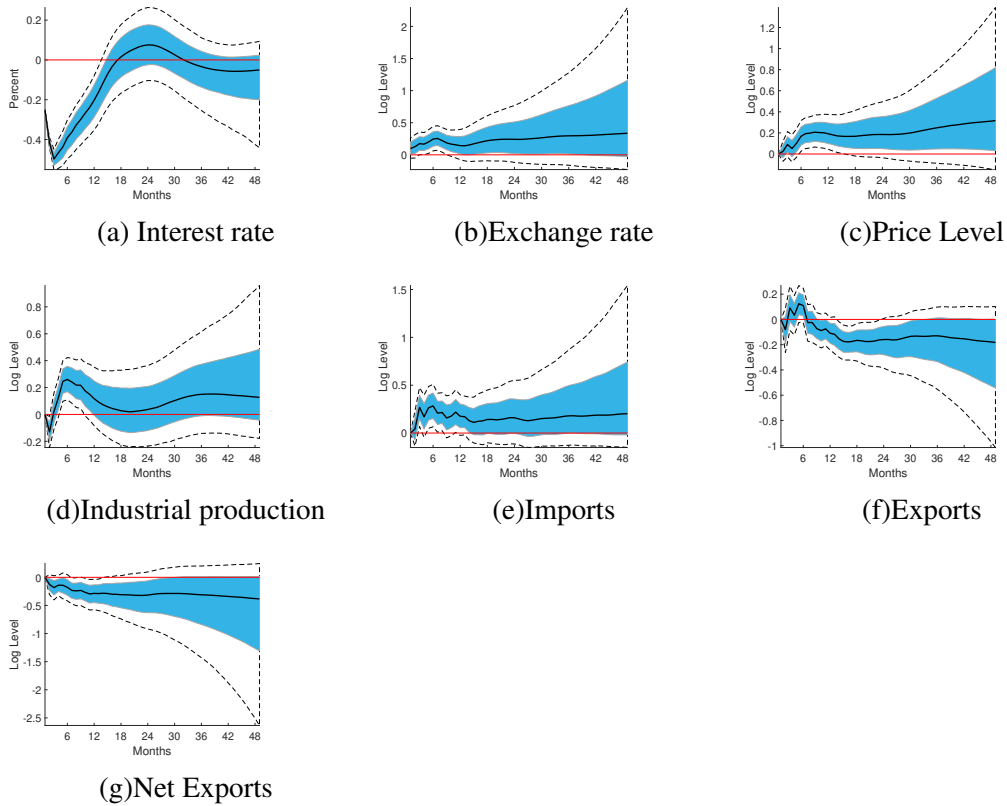
Note: The federal funds rate is the target federal funds rate before November 2008 and the upper bound of the federal funds rate from November 2008 onwards.

Source: FRED, Statistics Canada CANSIM, Bank of Canada, Holston, Laubach, and Williams (2017), Federal Reserve Bank of Atlanta, Taylor (1993), Taylor (1999), and authors' own calculations.

Impulse response results are presented in Figures E.6 and E.7. As in the main model, we see that a Canadian monetary policy shock is associated with a significant rise in the price level and industrial production, a depreciation of the currency and a slight fall in net exports. The US monetary policy shock results in strong spillovers to Canada, with Canadian prices rising after about six months and industrial production after about 25 months.

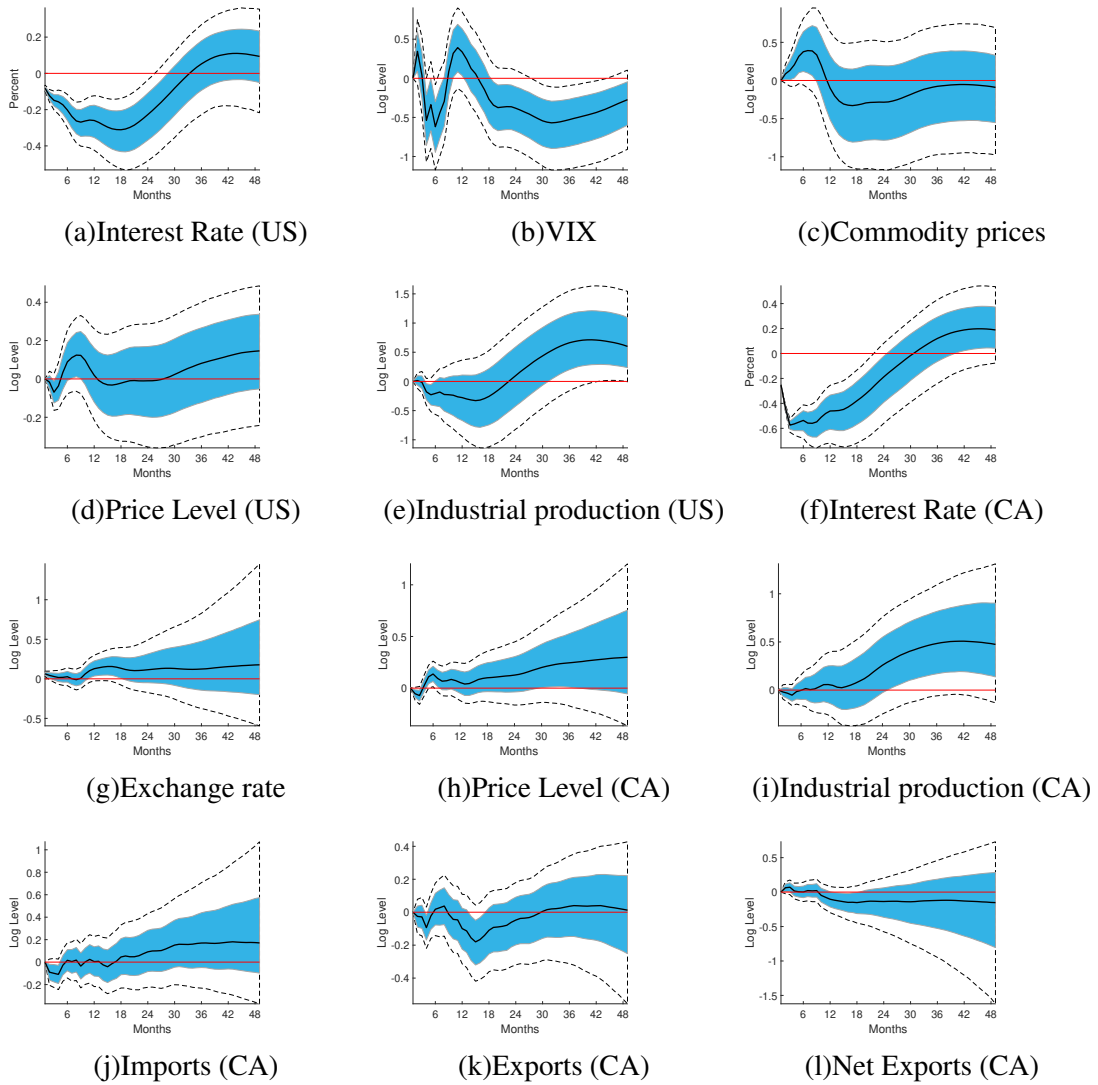
The counterfactual estimates are plotted in Figures E.8 and E.9, and the values reported in Table E.1. The Taylor Rule implied policy rate was more negative but stayed below the ZLB for a shorter time than the shadow rate. Nevertheless, the results are consistent with those from our main model, showing that US and Canadian unconventional monetary policies had a significant positive effect on output and prices.

Figure E.6. Impulse Response Functions: Canadian Monetary Policy Shock, Taylor Rule



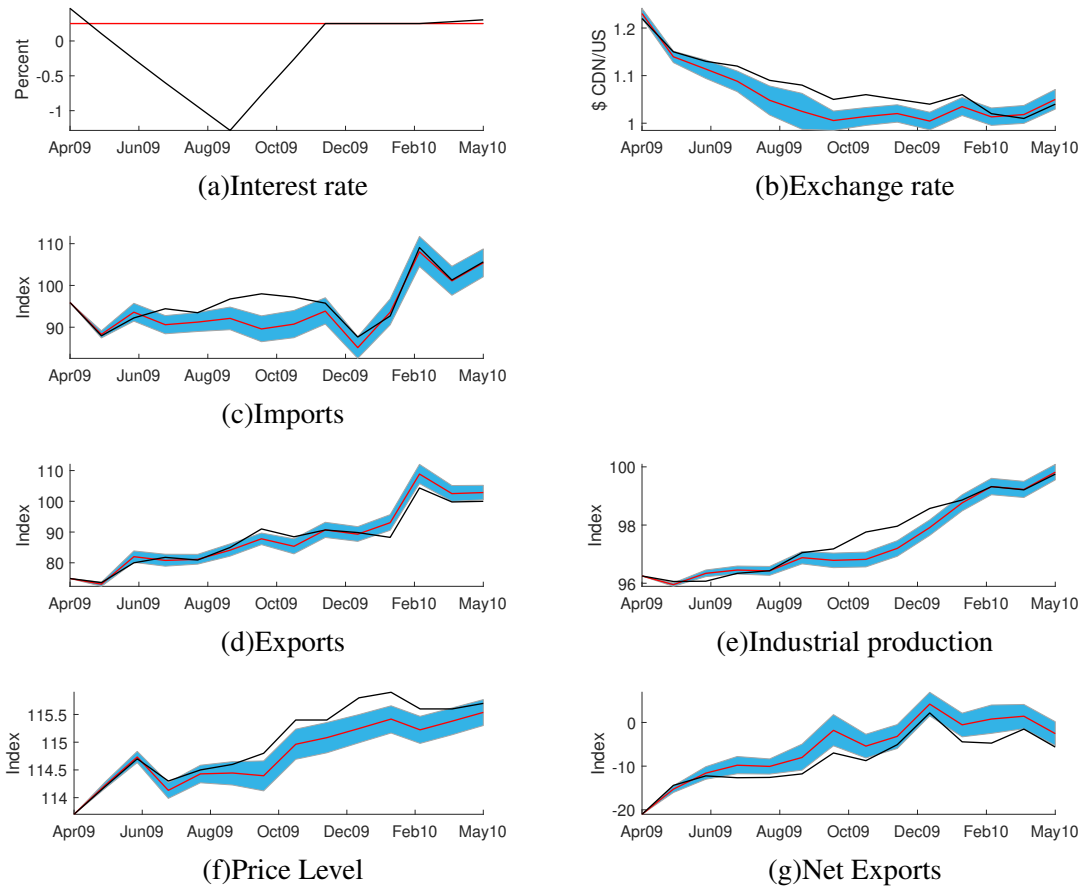
Note: See Figure 2 of paper.

Figure E.7. Impulse Response Functions: US Monetary Policy Shock, Taylor Rule



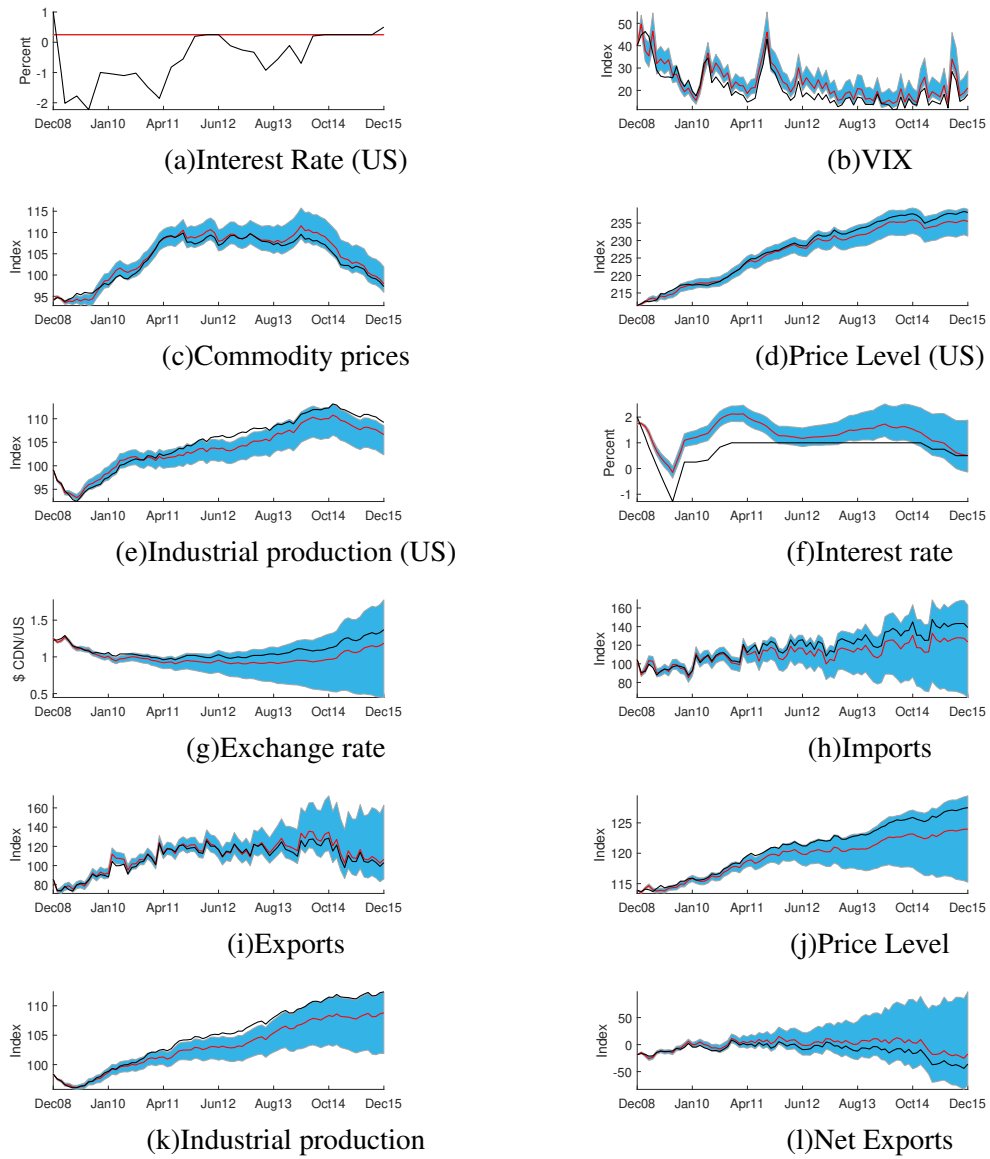
Note: See Figure 3 of paper.

Figure E.8. Counterfactual Paths: Canadian ZLB Imposed, Taylor Rule



Note: See Figure 4 of paper.

Figure E.9. Counterfactual Paths: US ZLB Imposed, Taylor Rule



Note: See Figure 5 of paper.

Table E.1. Average Counterfactual Path for ZLB, Taylor Rule

Variable	Canadian ZLB Imposed			US ZLB Imposed		
	Mean	68% Conf. Bound Max	Bound Min	Mean	68% Conf. Bound Max	Bound Min
	Percent Difference					
Canadian Interest rate	0.390	0.390	0.390	0.563	1.103	0.196
Canadian CPI	-0.204	-0.038	-0.373	-1.238	0.382	-3.447
Canadian industrial production	-0.194	0.011	-0.400	-1.712	-0.073	-4.161
US interest rate				0.771	0.771	0.771
US CPI				-0.420	0.532	-1.487
US industrial production				-1.193	0.424	-3.709
	Cumulative Percent Difference					
Canadian industrial production	-2.721	0.147	-5.602	-145.558	-6.205	-353.717
US industrial production				-101.376	36.073	-315.301
	Unit Difference					
Canadian CPI	-2.353	-0.440	-4.306	-15.313	4.753	-42.749
Canadian industrial production	-1.905	0.104	-3.924	-18.529	-0.883	-45.052
US CPI				-9.828	12.156	-34.499
US industrial production				-13.140	4.040	-40.123
	Cumulative Unit Difference					
Canadian industrial production	-26.672	1.461	-54.934	-1575.004	-75.035	-3829.440
US industrial production				-1116.936	343.413	-3410.482

Note: See Table 3 of paper.

APPENDIX F. SHADOW RATES

The shadow rate term structure model (SRTSM), first proposed by Black (1995), confronts the issue of modelling interest rates at the ZLB. Since economic agents have the option to hold physical currency for a rate of return of zero, deposit rates are bounded below by this constraint. However, standard term-structure (Gaussian affine term-structure or GATSM) models are linear in factors and thus allow for the possibility of estimating negative yields.³³ Black (1995) proposed thinking of the observed nominal rate, r_t , as the sum of an unobserved and unrestricted shadow rate, sr_t , which can go negative and the option value of holding physical currency that is exercised at the effective lower bound, \underline{r} . Specifically,

$$r_t = \max\{sr_t, \underline{r}\} = sr_t + \underbrace{\max\{0, \underline{r} - sr_t\}}_{\text{option value of currency}} \quad (4)$$

Note that when the shadow rate is above the ZLB, the option value of holding currency is zero. However, when the shadow rate begins to dip below the ZLB, the currency option begins to have an effect. Fitting this model to the data allows us to back out an estimate of the shadow rate process.

Despite its many advantages, estimating the SRTSM is significantly more involved than estimating a short rate in a standard term structure model. The main difference between the shadow rate defined by a SRTSM and a traditional short rate defined by a GATSM model is the non-linearity that the max operator introduces. This non-linearity implies that there is no analytical solution to the model. While Krippner (2012, 2013) and Bauer and Rudebusch (2013) use numerical simulation methods to solve for the rate, Wu and Xia (2016) take an alternative approach and derive an approximate solution to the shadow rate as a function of the yield curve and the probability that the shadow rate will fall below the effective lower bound.³⁴ This method provides a tractable analytical approximation of the shadow rate, and does not require numerical simulation.

We take the estimate of the US shadow rate directly from Wu and Xia (2016), and we adopt their method to estimate a Canadian shadow rate which is briefly outlined here.³⁵ The Canadian SRTSM is constructed as a nonlinear state space model estimated with the extended Kalman filter. The Canadian shadow rate is assumed to be an affine function of state vari-

³³Some models specify the short-rate diffusion process as quadratic or square-root to avoid negative rates. However, these specifications do not have theoretically consistent assumptions as they treat the ZLB as a reflecting barrier rather than an absorbing one. See Christensen and Rudebusch (2014) for a detailed discussion.

³⁴Specifically, they solve the shadow rate by defining it in a nonlinear state space model, where the shadow forward rate depends on the probability of the short rate being at its ZLB and a vector of state variables. The model is solved using the extended Kalman filter. The observed state variables are the forward rates associated with the 3 month, 6 month, 1, 2, 5, and 10 year yields on zero coupon bonds from the Gürkaynak, Sack, and Wright (2007) dataset.

³⁵See Wu and Xia (2016) for the complete derivation of the state space model for the SRTSM.

ables, X_t :

$$sr_t = \delta_0 + \delta_1' X_t. \quad (5)$$

Our state variables are the forward rates derived from the Bank of Canada's zero-coupon bond yield data for 3 month, 6 month, 1, 2, 5, 7, and 10 year maturities (Bolder, Johnson, and Metzler, 2004). We assume the state variables follow a first order vector autoregressive (VAR(1)) process under the physical measure \mathbb{P} ,

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I), \quad (6)$$

which is the transition equation. The log stochastic discount factor is essentially affine

$$M_{t+1} = \exp \left(-r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \right), \quad (7)$$

where the price of risk λ_t is linear in the factors: $\lambda_t = \lambda_0 + \lambda_1 X_t$. The dynamics under the risk neutral measure \mathbb{Q} also follow a VAR(1)

$$X_{t+1} = \mu^{\mathbb{Q}} + \rho^{\mathbb{Q}} X_t + \Sigma \varepsilon_{t+1}^{\mathbb{Q}}, \quad \varepsilon_{t+1}^{\mathbb{Q}} \sim^{\mathbb{Q}} N(0, I). \quad (8)$$

The parameters under the \mathbb{P} and \mathbb{Q} measures are related as follows: $\mu - \mu^{\mathbb{Q}} = \Sigma \lambda_0$ and $\rho - \rho^{\mathbb{Q}} = \Sigma \lambda_1$.

An analytical approximation of the forward rate in the SRTSM is generated by defining a forward rate, $f_{n,n+t,t}$, as the rate at time t for a loan starting at $t+n$ and maturing at $t+n+1$. The forward rate is a linear function of the yields on risk-free n and $n+1$ period pure discount bonds q

$$f_{n,n+t,t} = (n+1)q_{n+t,t} - nq_{nt}. \quad (9)$$

The forward rate in the SRTSM described in equations ((5))–((8)) is approximated by

$$f_{n,n+t,t}^{SRTSM} = \underline{r} + \sigma_n^{\mathbb{Q}} g \left(\frac{a_n + b_n' X_t - \underline{r}}{\sigma_n^{\mathbb{Q}}} \right) \quad (10)$$

where

$$\begin{aligned} a_n &= \delta_0 + \delta_1' + \left(\sum_{j=0}^{n-1} (\rho^{\mathbb{Q}})^j \right) \mu^{\mathbb{Q}} - \frac{1}{2} \delta_1' \left(\sum_{j=0}^{n-1} (\rho^{\mathbb{Q}})^j \right) \Sigma \Sigma' \left(\sum_{j=0}^{n-1} (\rho^{\mathbb{Q}})^j \right)' \delta_1, \\ b_n' &= \delta_1' (\rho^{\mathbb{Q}})^n, \text{ and} \\ \text{Var}_t^{\mathbb{Q}}(sr_{t+n}) &\equiv (\sigma_n^{\mathbb{Q}})^2 = \sum_{j=0}^{n-1} \delta_1' (\rho^{\mathbb{Q}})^j \Sigma \Sigma' (\rho^{\mathbb{Q}})^j \delta_1. \end{aligned}$$

The function $g(z) \equiv z\Phi(z) + \phi(z)$ consists of a normal cumulative distribution function $\Phi(\cdot)$ and a normal probability density function $\phi(\cdot)$. Its non-linearity comes from the moments of the truncated normal distribution.³⁶

We define a measurement equation that relates the observed forward rate, $f_{n,n+t,t}^o$, to the factors, based on equation ((10)), as follows

$$f_{n,n+t,t}^o = \underline{r} + \sigma_n^{\mathbb{Q}} g \left(\frac{a_n + b_n' X_t - \underline{r}}{\sigma_n^{\mathbb{Q}}} \right) + \eta_{nt}, \quad (11)$$

where the measurement error η_{nt} is i.i.d. normal, $\eta_{nt} \sim N(0, \omega)$. Finally, using the same identification assumptions as Wu and Xia (2016), we assume a three factor model and impose normalizing restrictions on the \mathbb{Q} parameters: $\delta = [1, 1, 0]'$; $\mu^{\mathbb{Q}} = 0$; and Σ is lower triangular. We also assume $\rho^{\mathbb{Q}}$ is real Jordan form with eigenvalues in descending order: $\rho^{\mathbb{Q}} = [\rho_1^{\mathbb{Q}} \ 0 \ 0; 0 \ \rho_2^{\mathbb{Q}} \ 0; 0 \ 0 \ \rho_3^{\mathbb{Q}}]$. The model is estimated using the extended Kalman filter with equation ((11)).

³⁶For details of the derivation of ((10)), see the appendix in Wu and Xia (2016).