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A Generalized Framework for the Assessment
of Household Financial Vulnerability

by Mindaugas Leika and Daniela Marchettini

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I N T E R N A T I O N A L M O N E T A R Y F U N D

IMF Working Paper

African Department

A Generalized Framework for the Assessment of Household Financial Vulnerability¹**Prepared by Mindaugas Leika and Daniela Marchettini**

Authorized for distribution by Anne-Marie Gulde Wolf

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Abstract

Household financial fragility has received considerable attention following the global financial crisis, but substantial gaps remain in the analytical underpinnings of household financial vulnerability assessment, as well as in data availability. This paper aims at integrating the contributions in the literature in a coherent fashion. The study proposes also analytical and estimation extensions aimed at improving the quality of estimates and allowing the assessment of household financial vulnerability in presence of data limitations. The result of this effort is a comprehensive framework, that has wide applicability to both advanced and developing economies. For illustrative purposes the paper includes a detailed application to one developing country (Namibia).

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Glossary

AUC	Area Under the Curve
BIS	Bank for International Settlements
BON	Bank of Namibia
DTI	Debt-to-Income
DSTI	Debt-Service-to-Income
DSGE	Dynamic Stochastic General Equilibrium
ECB	European Central Bank
GDP	Gross Domestic Product
HFCS	Housing Finance and Consumption Survey
LTV	Loan to Value
NHIES	National Household Income and Expenditure Survey
NLPCA	Nonlinear Principal Components Analysis
NPLs	Nonperforming Loans (used interchangeably with distressed assets)
NSA	Namibia Statistics Agency
OECD	Organization for Economic Cooperation and Development
ROC	Receiver Operating Characteristic
WEO	IMF World Economic Outlook

I. BACKGROUND

The global financial crisis highlighted the risks that the household sector can pose to financial stability and to the broader economy. Strains in household balance sheets played in fact a crucial role in determining the intensity, duration and macroeconomic impact of the crisis. This experience has sparked considerable interest among researchers, particularly within the central bank community, who have stepped up efforts to assess household financial vulnerability and model the links between macroeconomic shocks and household distress.

Until now, however, there has been a deficit in defining a conceptual framework. Studies have been mainly “application-oriented”, focused on gaining insight into household vulnerability in specific countries or regions. Attempts to standardize and conceptualize elements of the methodology have been, so far, limited.² It follows that the analytical underpinnings of household financial vulnerability remain fragmented and several challenges are unaddressed.

Most notably, the definition of household vulnerability remains vague and there is a lack of consensus on operative definitions. In addition, prevailing estimation methods tend to overlook endogeneity caused by unobserved heterogeneity and measurement errors, which are frequent in household microdata. Despite these limitations, review and validation of estimates are rare in empirical applications on household vulnerability. Finally, existing studies provide limited solutions to address persistent data gaps, which restrain applications in emerging markets and developing countries.

In this paper, we aim to contribute to the existing literature by providing an integrated perspective on household financial vulnerability assessment. We first provide a critical review of existing studies and identify the major areas of progress and remaining challenges. Then, we consolidate the main contributions in literature in a coherent fashion, while suggesting avenues to address lingering gaps. The result of this effort is a comprehensive framework, that we propose as a blueprint for empirical research in this field.

The proposed framework helps to firm and operationalize the concept of household vulnerability and envisages different modelling and estimation options depending on the purpose of the analysis and data availability. In addition, the methodology illustrates how to integrate micro- and macro-data and merge different microdata sources and proposes

² Only a few works have conceptualized some elements of the assessment of household vulnerability. For instance, Andreoli et al. (2010), presenting an empirical application on the Italian case, focused on the conceptual definition of household financial fragility, while Ampudia et al. (2014), developing a framework to identify distressed households in the euro area, discussed how to bridge the concepts of liquidity and solvency of household financial positions.

estimation enhancements to reduce possible sources of estimation bias and allow implementation in presence of data limitations.

The framework's major strength lies in its wide applicability, making it a useful tool for policy analysis and formulation in countries at different levels of development. Whilst the methodology is expected to produce the most efficient estimates using granular and precise microdata, it allows to derive a meaningful assessment of household vulnerability from a relatively incomplete and fragmented information set. This partly addresses the issue of data limitations that has so far restricted the possibility to assess the vulnerability of the household sector to macroeconomic and policy shocks in emerging markets and developing countries.

The rest of the paper is organized as follows. Section II reviews the major contributions in literature and identifies progress and remaining analytical challenges and gaps. Based on this analysis, Section III introduces a comprehensive framework for the assessment of household financial vulnerability, based on a three-step procedure. Section IV presents an application of the methodology to a country with data limitations (Namibia). Lastly, Section V presents the conclusions, and illustrates possible uses of the methodology for policy analysis and formulation.

II. ASSESSMENT OF HOUSEHOLD FINANCIAL VULNERABILITY: WHERE DO WE STAND?

Empirical research on household vulnerability has proliferated over the past decade and has contributed to distil several lessons and best practices. Nevertheless, a coherent structure to organize the knowledge of the field is still missing and several gaps and challenges remain.

Overall, there is broad consensus in the literature on the *limitations of aggregate data* to exhaustively analyze household financial vulnerability.³ National indicators, such as the average household debt-to-income (DTI) and debt-service-to-income (DSTI) ratios, are helpful to detect the build-up of vulnerabilities over time and across countries,⁴ but can mask large variation among households, which is critical in the analysis of risks. Consequently, empirical research on household financial vulnerability has increasingly used microdata from surveys or other sources to conduct sensitivity analysis on individual household financial

³ Many studies have highlighted the limitations of aggregate data, including: Albacete and Fessler (2010); Albacete and Linder (2013); Arins et al. (2014); Bilston et al. (2015); Dey et al. (2008); Djoudad, (2010); Herrala and Kauko (2007); Holló and Papp (2007); Karasulu (2008); Ruiz-Arranz (2014).

⁴ Three household-related economy-wide indicators are included in the revised IMF list of "Core and Additional Financial Soundness Indicators": the household debt to gross domestic product ratio, the household debt service and principal payments to income ratio and the household debt to household disposable income ratio (see IMF, 2013).

positions and capture the impact of changes in the macroeconomic and policy environment on different groups of the population.⁵

Another feature common to most studies is the adoption of a *methodological structure based on a three-step procedure*. The first step entails choosing one or more indicators to measure household financial vulnerability. The second step defines the criteria to evaluate whether a household is at risk of financial distress based on its vulnerability measures. The final step simulates the impact of alternative macro and policy scenarios on vulnerability indicators to quantify the changes in the share of households at risk.

Besides these points, however, the analytical framework for household vulnerability assessment remains fragmented and several gaps are unaddressed.

To begin with, there is no uniform standard for the *concept of household financial vulnerability* in literature. Some authors (Worthington, 2006; Bridges and Disney, 2004; Anderloni et al., 2011) have used a comprehensive notion, that considers not only the fragility associated with debt commitments, but also with other outlays, such as utility bills, grocery purchases or rent payments.⁶ Most of the studies, however, have restricted the analysis to indebted households because their fragility can entail risks to financial stability. Nonetheless, even with this restricted focus, the proposed concept of financial vulnerability is not uniform.

Studies on indebted households propose indeed two competing paradigms, one related to the solvency of the financial position of individual households, the other to their liquidity. Based on the *solvency definition*, authors have associated household financial fragility with the level of gross or net indebtedness, as measured by the debt to asset ratio (Albacete and Linder, 2013) or the debt-to-income ratio (Bańbuła et al., 2016). Indicators of financial vulnerability based on the *liquidity definition*, instead, are associated with current debt repayments or with budget constraints, such as DSTI ratios and so-called “financial margins”, the latter defined as the difference between household income and the estimated minimum expenses and debt payments.⁷

On their own, however, solvency and liquidity indicators provide an incomplete picture of an indebted household’s financial vulnerability. Metrics based on debt levels offer a limited

⁵ In several recent Financial Sector Assessment Program updates and Article IV consultations the IMF has conducted detailed analysis of household risks using micro-data. Countries covered in these applications include, among the others, United Kingdom (IMF, 2011), Spain (IMF, 2012), Italy (IMF, 2013), Norway (IMF, 2015), Namibia (IMF, 2016), Finland (IMF, 2017), and Luxemburg (IMF, 2017b).

⁶ In this literature, a household may be qualified as vulnerable also if it is not indebted but accumulates arrears on its non-debt-related payment obligations.

⁷ Studies mostly employ DSTI ratios, including: Dey et al. (2008); Djoudad, (2010); ECB (2014); Fuenzalida and Ruiz-Tagle (2011); IMF (2012); Karasulu (2008); Michelangeli and Pietruni (2014); and Ruiz-Arranz

measure of short-term financial fragility, as highly indebted households might still be able to pay their short-term obligations if current liabilities and expenses are lower than income and liquid assets. A number of studies even find that in some countries default rates are lower among the most indebted households (Costa and Farinha, 2012; Dietsch and Welter-Nicol, 2014; Worthington, 2006).⁸ Metrics associated with households' current budget constraints have also limitations, as they ignore that, on the one hand, households may still draw down on their assets or borrow against suitable collateral to avoid default (Ampudia et al., 2014), on the other, high debt levels may be associated with higher fragility to prolonged shocks.

Nevertheless, only a few studies have proposed measures aimed at bridging the two concepts. For example, Herrala and Kauko (2007), Karasulu (2008), and Gross and Población (2017) included a measure of assets directly into each household's financial margin, while Ampudia et al. (2014) and Meriküll and Rõõm (2017) integrated the condition on financial margins with one on the use of liquid assets. Other authors have analyzed the long-term impact of shocks on household vulnerability by deriving forward paths of budget constraints through pseudo-panel techniques (Djoudad, 2010, 2012) or reproducing the projected development of macro (i.e. country-level) variables (Gross and Población, 2017).

Another challenge is associated with the fact that prevailing estimation methods are vulnerable to *endogeneity issues*. This is particularly the case for the most widely used technique that estimates distressed households non-parametrically based on a binary classifier, valued positively if a vulnerability measure breaches a specified threshold equal for all households.⁹ The assumption that all households breaching a certain edge will default may however lead to inconsistent estimates in the presence of unobserved heterogeneity and/or measurement errors, which are frequent in analyses based on household microdata.

For instance, disregarding the possibility that households may use their assets when facing liquidity constraints induces unobserved heterogeneity, which may lead to an overestimation of the number of wealthy households that are expected to default. A source of measurement error can instead derive from the fact that income is frequently under-reported in survey data (Deaton, 1997; Meyer et al., 2009, Hurst et al., 2014), particularly asset-related income (Moore *et al.*, 2000) which may lead to an underestimation of wealthy households' income and thus, again, in an overestimation of their vulnerability.

(2014). Examples of analyses that use financial margins can be found in Albacete and Fessler (2010); Ampudia et al. (2014); Bilston et al. (2015); Hlaváč et al. (2014), Gross and Población (2017) and Meriküll and Rõõm (2017).

⁸ This is notably the case when the highest debt levels are recorded by households with high income and wealth.

⁹ The threshold for debt-servicing costs is usually set between 20 and 40 percent of income, depending on historical values, and the edge for financial margins is set equal to zero.

Despite the sensitivity of estimation techniques to unobserved heterogeneity and measurement errors, most of the existing studies lack a proper assessment of the precision of estimates. An accurate *review and validation process* should, however, be integral part of the analysis, particularly if the output of the household vulnerability assessment must inform policy decisions. For instance, if the assessment is used to calibrate macro-prudential tools, the lack of accuracy in estimates may lead to either leaving systemic risk undetected or to impose undue restrictions on household credit. Unfortunately, only a few studies address this issue in an appropriate way¹⁰ by either providing a statistical assessment of the quality of estimates (Hallo and Papp, 2007; Herrala and Kauko, 2007; Arins et al., 2014; Michelangeli and Pietruti, 2014) or calibrating vulnerability measures to reduce fitting errors (Ampudia et al., 2016; Meriküll and Rõõm, 2017).

Finally, a relevant challenge in household vulnerability assessment is associated with the issue of *data availability, quality, and timeliness*. This is critical not only for the accuracy of estimates, as illustrated above, but, also, for the feasibility of the analysis *tout court*. Assessing household financial vulnerability requires indeed detailed financial information at the household level but these data are scarce, subject to non-response bias and measurement errors (Daniel Kasprzyk, 2010; Meyer et al., 2015), and have typically low frequency.

Advanced economies have been gradually circumventing data problems, under the impulse of several high-level initiatives on data gaps (IMF/FSB, 2009; Stiglitz et al., 2009). For instance, in 2013 the OECD issued guidelines for the compilation of micro statistics on household wealth (OECD, 2013), while the euro area has introduced the Household Finance and Consumption Survey (HFCS), carried out every three years starting with 2011, which collects information on socio-demographic variables, assets, liabilities, income and consumption for individual countries and the overall region. In addition, several authors have tackled the difficulties associated with the low frequency of household microdata by extrapolating available microdata to the period of interest by replicating the developments of aggregate country-level counterparts (Ampudia *et al.*, 2014; ECB, 2014; Hlaváč et al., 2014; IMF, 2012; Michelangeli and Pietruti, 2014).

Progress in enhancing household data collection has been slower in emerging economies and developing countries. At the same time, analytical and estimation solutions to address data limitations have yet to be developed in literature, which explains why there are only a few household-level studies in countries other than advanced economies (Fuenzalida and Ruiz-Tagle, 2011; Tiongson *et al.*, 2010).

¹⁰ Some studies compare their results with similar results in literature (Albacete and Fessler, 2010; Bilston et al., 2015). This approach, however, has limitations as the distribution of distressed households is likely to be different in different countries or in different periods.

III. ASSESSMENT OF HOUSEHOLD FINANCIAL VULNERABILITY: THE WAY FORWARD

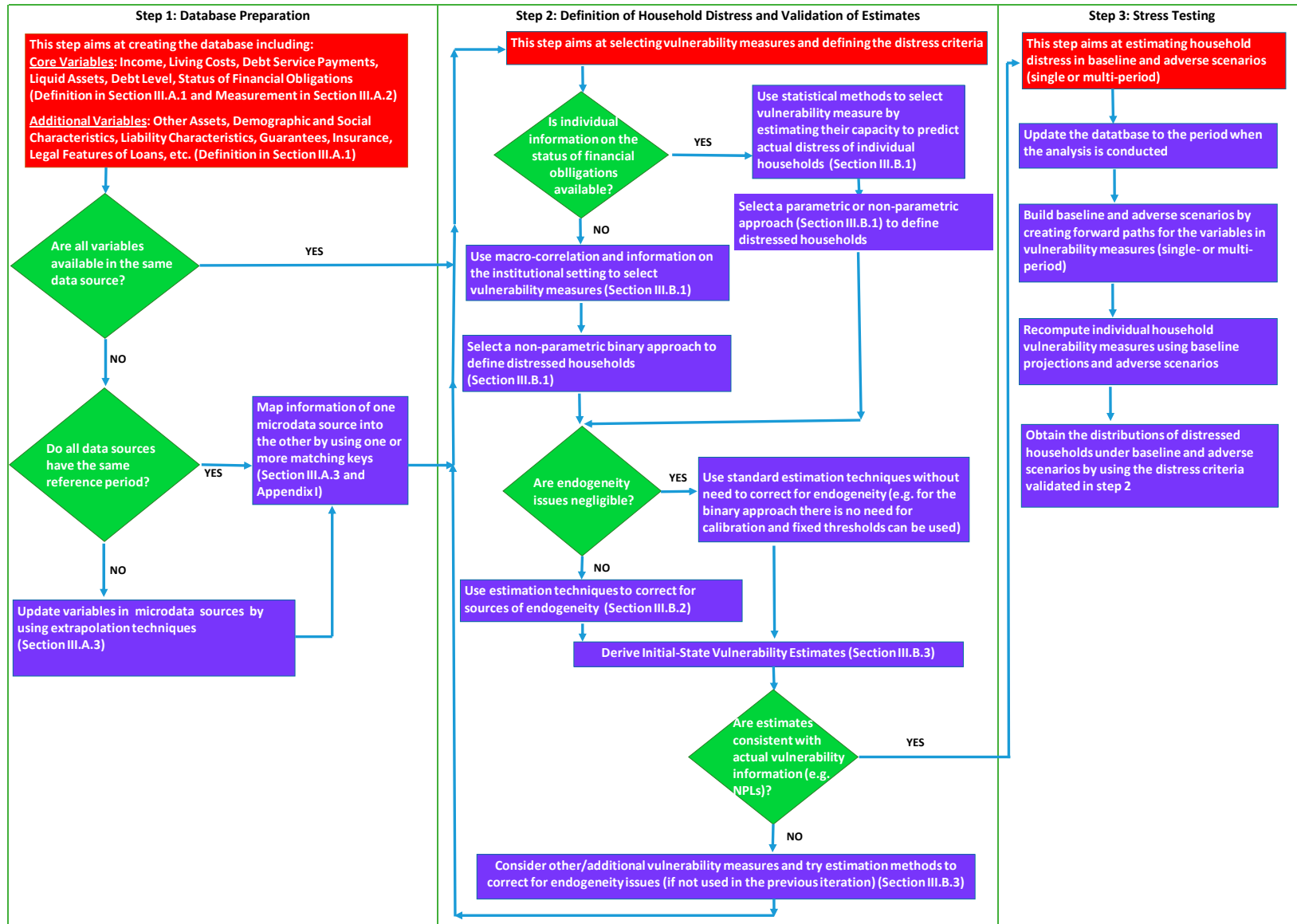
In this section, we introduce a conceptual framework for the assessment of household financial vulnerability. The framework, based on a three-step procedure, should be interpreted as an organizing device for empirical research in this field. It lays out key concepts and proposes a generalized methodology with several variations and options to deal with data limitations. Figure 1 illustrates the structure and provides guidance on how to use the framework.

The first step, illustrated in Section A, sets the stage for the analysis. It proposes a working definition of household financial vulnerability and illustrates how to operationalize it by defining the variables relevant to conduct the analysis, classified in two sets: a core set, critical for the assessment, and a complementary set, that may either enhance the precision of vulnerability estimates or help to assess the impact of household vulnerability on macro-financial stability (Section III.A.1). The text addresses also the measurement issue by identifying, for each of the variables in the core set, the most precise indicator to estimate the underlying economic concept and proposing proxies in case of data limitation (Section III.A.2). Finally, this step illustrates how to update micro-data sources and how to aggregate information from different microdata sources (Section III.A.3).

The second step, described in section B, provides guidance on how to define the criteria to identify distressed households (Section III.B.1) and proposes some techniques to correct estimation bias stemming from unobserved heterogeneity and measurement errors (Section III.B.2). This steps illustrates also methods to validate initial-state vulnerability estimates (Section III.B.3).

Finally, the last step, described in section C, explains how to simulate the impact of alternative macro and policy scenarios on household vulnerability measures to assess the changes in the distribution of distressed households.

Figure 1: Outline of the Framework for Household Vulnerability Assessment



A. STEP 1: DATABASE PREPARATION

1. OPERATIONALIZING THE CONCEPT OF HOUSEHOLD FINANCIAL VULNERABILITY

Household financial vulnerability can be defined as a situation where a household is exposed to the risk of failure to meet its financial obligations timely and completely,¹¹ thus incurring financial distress. This notion encompasses both a *liquidity* and a *solvency* dimension, depending on whether the status of financial distress is temporary, associated with transitory issues, or long-lasting, related to structural imbalances in the household's balance sheets.¹²

More concretely, the concept of financial vulnerability needs to be operationalized in terms of measurable variables. These can be grouped in two sets: a *core set*, critical for the assessment of financial vulnerability, and a *complementary set*, that may either enhance the precision of vulnerability estimates or help to assess the impact of household vulnerability on macro-financial stability.

Core Variables

While solvency and liquidity are two sides of the same coin and a comprehensive assessment of household vulnerability entails evaluating these two aspects together, for the sake of clarity, it is helpful to discuss separately the variables needed for the assessment of liquidity and those required for the assessment of solvency.

- Household liquidity is a short-term concept and refers to the ability to cover financial obligations when they come due. At any point in time, this is based on the household's budget constraint—which in turn depends on income, living costs and debt payments—and the availability of liquid assets. These variables allow to define for any household h and time t , a main vulnerability index $FM_{h,t}^A$, which is an extended version of financial margins, accounting also for the possible use of liquid assets:

$$FM_{h,t}^A = Y_{h,t} + LA_{h,t} - LC_{h,t} - DP_{h,t} \quad \text{Absolute Financial Margin (i.1)}$$

¹¹ A financial obligation is a commitment to pay money to another party at a specific time. It may arise from borrowing funds or from other commitments (paying taxes, rent, utility bills, etc.).

¹² The concept of financial vulnerability extends to both debt-free and indebted households, the difference being that financial obligations for the former do not arise from debt commitments. In the paper, we will focus on indebted households, given that the vulnerability of this household group may have significant implications on macro-financial stability. We will provide some details on how to adjust model specification in the case of debt-free households. These adjustments will be illustrated mainly in footnote to avoid disrupting the reading flow.

where $Y_{h,t}$, $LC_{h,t}$ and $DP_{h,t}$ are flow variables corresponding, respectively, to household h 's (monthly, quarterly, or yearly) income, living costs, and debt payments, while $LA_{h,t}$ is the available stock of liquid assets at time t . It is also possible to define a relative version of the same index by normalizing equation (i.1) by income:¹³

$$FM_{h,t}^R = \frac{Y_{h,t} + LA_{h,t} - LC_{h,t} - DP_{h,t}}{Y_{h,t}} \quad \text{Relative Financial Margin (i.2)}$$

The extended financial margins are a straightforward and comprehensive measure of financial strength at time t , measuring the surplus left to the household after satisfying its expenses and financial commitments. Forward projections of (i.1) [or (i.2)] can be used to assess whether individual households will incur liquidity problems at any point in time along the simulation horizon:

$$PD_{h,\tau} = f(FM_{h,\tau}^X), \text{ (with } X \in \{A, R\} \text{ and } t + 1 \leq \tau \leq t + T)$$

where τ is a generic point along the (monthly, quarterly, or yearly) projection period $[t + 1, t + T]$, and $PD_{h,\tau}$ is the probability of incurring distress associated with liquidity problems at that time.

- Household solvency is a long-term concept that academic literature associates with the fulfillment of the household's intertemporal budget constraint, which requires that the present discounted value of the household's income stream is not lower than the present discounted value of its expenditure stream net of its initial debt stock less the initial stock of assets. For the purpose of household vulnerability assessment, however, this definition of solvency presents several problems. First, it imposes excessively mild restrictions on the future path of deficits that the household can accumulate, requiring only that the infinite sum of all income-expenditure balances is not lower than the initial stock of net debt. This implies that there are no borrowing restrictions other than a no-Ponzi game condition, which may be inconsistent with imperfections in the capital market that affect the ability to transfer resources across time periods. Second, this definition of solvency requires distant-future projections of income and expenditures, which may be empirically impracticable given that the uncertainty of projections grows significantly with the forecast horizon. A stricter and more pragmatic approach entails using a truncated version of the intertemporal

¹³ Relative financial margins should be used in applications which require calibration of distress thresholds because they are expressed as ratios and thus unit-less, allowing cross-sectional (i.e. across income groups) and time (i.e. across periods) comparisons (see Section III.B.3).

budget constraint—that restricts the analysis to shorter projection horizons¹⁴—and requiring that, at the end of the forecast period, the projected net-debt either stabilizes or declines or does not exceed a benchmark level considered “risky”. Based on this definition, the probability that the household will incur solvency problems would be associated with its initial level of net debt and its (discounted) stream of income-expenditure balances over the projection period:

$$PD_{h,T} = f\left(D_{h,t}^X, A_{h,t}^X, Y_{h,t+1}^X, Y_{h,t+2}^X, \dots, Y_{h,t+T}^X, LC_{h,t+1}^X, LC_{h,t+2}^X, \dots, LC_{h,t+T}^X\right), \text{ (with } X \in \{A, R\} \text{)}$$

where T is the projection horizon (in months, quarters, or years), $PD_{h,T}$ is the probability household h will incur distress associated with solvency, and $D_{h,t}^X$ and $A_{h,t}^X$ are, respectively, household h 's initial debt and asset levels (which, as the other variables, may be expressed either in nominal terms or as ratios of household h 's income).

Given their relevance in determining the liquidity and/or the solvency of household financial positions, variables that measure income, living costs, debt payments, debt level¹⁵ and liquid assets are part of the *core set* that is germane to the analysis of household financial vulnerability.

In addition, details on the status of the household's financial obligations (paid in full/overdue) should be part of the core set. This information does not contribute to the measurement of household financial vulnerability but is relevant to calibrate and validate vulnerability estimates (see Section III.B.2).

¹⁴ To date, empirical works have used projections periods inferior to 5 years. For instance, Djoudad (2010, 2012) simulated the path of the variables included in the DSTI ratios under different scenarios and for different groups of households over a 3-year period by using pseudo-panel techniques. Gross and Población (2017) used a GVAR and a logistic model to derive the forward paths of household balance sheets up to 16 quarters ahead.

¹⁵ The debt level provides mainly information on solvency, because it affects households' vulnerability to persistent shocks on repayment capacity (e.g. permanent lower income, prolonged higher interest rates, etc.). In some jurisdictions, however, debt levels can provide insights also on households' capacity to face liquidity shocks (for instance in case of countries with interest-only loans requiring balloon payments). In addition, debt levels are key to assess the impact of household vulnerability on the banking sector, as they are closely related to the loss-given-default. Finally, information on debt levels may be used to estimate debt payments in case these are not available (Section III.A.2).

Complimentary Variables

Depending on data availability, modelling needs and the institutional setting, the core set may be complemented with additional variables that may: i) be used to construct additional vulnerability indexes; ii) contribute to build projections of the core variables; or iii) help assessing the impact of household vulnerability on macro-financial stability. The list which follows provides some examples of variables that could be part of the *complimentary set*.

- i. Information on physical and less-liquid financial assets provides insights on the capacity of households to withstand lengthy periods of financial stress, and may complement liquid assets in solvency estimates over long-projection periods. Information on non-liquid assets is also relevant for the assessment of the impact on the financial sector because it contributes to determine the loss-given-default. Despite its relevance, information on less-liquid assets is not included in the core set because these assets are susceptible to decline in prices and liquidity during crisis periods. This implies that even if households have relatively low debt-to-asset ratios, they may remain vulnerable to liquidity shocks, since they cannot liquidate or pledge non-liquid assets (for example, real estate) or those can be liquidated only with a substantial discount or delay.
- ii. Demographic and social characteristics can help determining future earning potential and financial fragility of specific groups of households (Anderloni et al., 2011; Arins et al., 2014; Holló and Papp, 2007, Dey et al., 2008), thus contributing to build projections of core variables (especially income).
- iii. The database may also include optional information that can contribute to assess the impact on macro-financial stability, including: value of collateral, presence of government or other guarantees, mortgage typology (fixed rates, ARM, interest-only, denominated in foreign currency, etc.), use of the mortgage (for primary or secondary residence), legal features of loans (full recourse or not), holding of an insurance policy, etc.

Figure 2 provides a synthetic illustration of the information to include in the database, distinguishing the core set of variables from the rest.

Figure 2. Relevant Information to Conduct Household Sensitivity Analysis and Assess the Macro-Financial Impact

Core Variables:

Income
Living Costs
Debt Payments
Liquid Assets
Debt Levels
Status of Financial Obligations

Additional Information on Household Vulnerability: Less Liquid Assets, Demographic and Social Characteristics, etc.

Information Helpful to Assess Macro-Financial Impact of Shocks: Collateral, Guarantees, Loans' Legal Features, etc.

2. DERIVING ESTIMATES OF CORE VARIABLES FROM AVAILABLE DATA SOURCES

Ideally, all the information in the core set should derive from granular microdata sources and each variable should be measured with the most precise indicator. However, limitations in data availability and quality prevent the use of the ideal set of data in several countries (particularly developing economies). Despite this, the methodology can still be applied to these countries, provided that a measure of income is available in a microdata source (e.g. household survey, credit registry, etc.), even if this is subject to measurement errors or it is incomplete or imprecise. The rest of information can be derived from macro-data sources.

The sources of estimation bias deriving from the use of an imperfect set of data can be reduced/corrected in the second step of the procedure through approaches aimed at addressing endogeneity issues (Section III.B.2).

For each variable in the core set, Table 1 illustrates synthetically which is the most precise and complete indicator and how this could be replaced with a proxy or a less precise and/or complete measure in case of data limitations.

- The *income* variable $Y_{h,t}$ should ideally measure the maximum amount a household h can spend (monthly, quarterly, or yearly) without reducing or pledging its wealth, which coincides with the definition of disposable income. This is derived from gross income by taking into account net current transfers, such as the payment of taxes on income and wealth and social security contributions, and the receipts of social benefits from government. In countries where household income surveys do not include information on net current transfers or this is incomplete, disposable income can be proxied by gross income, defined as the income that accrues to households because of their involvement in the production process or the ownership of assets. The availability of a microdata source which includes information on (disposable or gross) income is the basic requirement to proceed with the analysis.
- *Debt payments* $DP_{h,t}$ refer to the sum of household h 's (monthly, quarterly, or annual) payments for different forms of debt:

$$DP_{h,t} = MP_{h,t} + PP_{h,t} + CCP_{h,t}, \quad (i.)$$

where $MP_{h,t}$, $PP_{h,t}$ and $CCP_{h,t}$ are, respectively, the mortgage, personal loan, and credit card payments of household h at time t .

Mortgage and personal loan payments $MP_{h,t}$ and $PP_{h,t}$ refer to households' scheduled payments.

Table 1. Measuring Core Variables Based on Available Information

Variable	Use	Best Indicator	Possible Proxy	Data Source
Income	To assess liquidity and solvency.	(Monthly, quarterly, or annual) disposable income.	(Monthly, quarterly, or annual) gross income.	Household income survey or credit registry.
Debt Payments	To assess liquidity.	(Monthly, quarterly, or annual) scheduled debt payments.	<ul style="list-style-type: none"> • First choice: (monthly, quarterly or annual) actual debt payment. • Second choice: estimates based on available micro- and/or macro-data sources. 	<ul style="list-style-type: none"> • If available, microdata source including information on scheduled (or actual) debt payments for individual households. • Alternatively, central bank information on average household DTI ratios, residual maturity and lending rates, complemented with information on indebtedness of different income groups (from authorities' data or Findex database).
Living Costs	To assess liquidity and solvency.	(Monthly, quarterly, or annual) expenses for necessary items.	National poverty line.	<ul style="list-style-type: none"> • If available, household survey. • Alternatively, authorities' estimates, and/or national accounts.
Liquid Assets	To assess liquidity and solvency.	Assets that can be readily converted into cash with minimal loss in value.	Information on deposits held by households.	<ul style="list-style-type: none"> • Household income and wealth survey. • Monetary survey complemented with information on distribution of deposits across income groups (from authorities' data or Findex).
Debt Levels	<p>To assess solvency</p> <p>To assess macro-financial impact.</p> <p>To compute debt payments, if these are not available in a microdata source.</p>	<p>Individual outstanding debt levels by debt category.</p> <p>Individual debt levels at origination.</p>	<p>Average household DTI ratios (possibly by debt category).</p> <p>N.A.</p>	<ul style="list-style-type: none"> • Household survey or another microdata source; • Central bank information on average household DTI ratios, complemented with information on indebtedness of different income groups (from authorities' data or Findex database).
Status of Debt Obligations	To calibrate and validate vulnerability estimates.	Individual information on status of debt payments (numeric or categorical variable).	Nation-wide exposures in arrears or in forbearance, or non-performing.	<ul style="list-style-type: none"> • If available, microdata source (i.e. household indebtedness survey, banks' loan books). • Alternatively, central bank information on aggregate NPLs or arrears.

While it is possible to proxy scheduled payments with actual payments, these may either overestimate or underestimate the actual future debt service burden in case the household repaid more principal than required (Bilston *et al.*, 2015), or if it holds a mortgage with atypical structure (such as interest only), respectively.

When information on individual mortgage (or personal loan) payments is not available, this can be derived via a credit-Foncier rule,¹⁶ which assumes that borrowers make constant payments over the life of the loan.¹⁷ In this case, if the interest rate at time t has not changed since origination, the scheduled mortgage payment $MP_{h,t}$ (or personal loan payment $PP_{h,t}$) is a function of the initial amount borrowed $M_{h,b}$, the per-period interest rate i_{h,t_0} and the total number of repayments T to be made over the life of a loan, as shown in equation (iii.):

$$MP_{h,t} = \frac{M_{h,t_0} \cdot i_{h,t_0} \cdot (1 + i_{h,t_0})^T}{(1 + i_{h,t_0})^T - 1} \quad (\text{ii.})$$

If the amount of the mortgage at origination is not available, the outstanding mortgage and the interest rates at time t ($MP_{h,t}$ and $i_{h,t}$) can be used, while the total number of payments over the life of the loan should be replaced by the *actual* number of remaining payments. Using the *scheduled* number of debt payments, rather than the actual number, would lead to an underestimation of the mortgage payments if the household paid more principal than scheduled in the past.¹⁸ This, in turn, may lead to an underestimation of the debt service burden of households with a higher income, that are more likely to repay their debt ahead of maturity.

¹⁶ Under a “credit Foncier” rule, the borrower must repay the principal and pay applicable interest on the outstanding balance over an agreed maximum term by making regular payments. This is the most-common repayment schedule for mortgage loans.

¹⁷ It must be noted that the use of a credit Foncier rule to estimate debt payments of “interest-only” mortgages will lead to an over-estimation of debt payments during the first period of the life of the loan, when the borrower only pays the interests on the mortgage, and to an under-estimation of payments during the remaining part of the loan, when the borrower begins paying off the principal of the loan, particularly if he/she must make a lump sum payment (balloon payment).

¹⁸ Based on standard re-payment rules for mortgages, accelerated payoffs shorten the life of the loan (i.e. reduce the number of remaining instalments) without modifying the amount of each payment, which is constant and set at origination. This implies that, if the borrower paid more principal than scheduled in the past, using the *scheduled* number of remaining payments in (iii.), rather than the *actual* number associated with the payoff, would lead to an underestimation of payments because the remaining balance would be spread over a longer period than actual. Scheduled and actual payments coincide for borrowers that have not paid more principal than scheduled in the past.

In absence of information on individual debt levels, the average household debt to income (DTI) ratio at the national level, complemented with information on indebtedness of different income groups, can be used and plugged in (iii.) to compute debt payments.^{19, 20}

Credit card payments $CCP_{h,t}$ at time t can, instead, be estimated as the reported outstanding balance.

- *Living Costs* $LC_{h,t}$ refer to (monthly, quarterly, or annual) basic consumption, which consists of expenses for essential items (food, transportation, health, education), that the household cannot cut in case of financial stress. In case of lack of individual information on expenditure items, living costs can be proxied with the national defined minimum consumption expenditures or with the national poverty line multiplied by the number of household members.
- *Liquid Assets* $LA_{h,t}$ refer to the stock of assets that can be readily converted into cash at time t with minimal loss on its value. The range of liquid assets available to households varies across countries and may include, in addition to cash reserves, checking and saving accounts (term deposits), money market accounts, and tradeable securities (bonds and stocks). Information on individual liquid assets may be derived from household wealth and income surveys, but these are frequently not available in developing countries. In those countries, finding an appropriate proxy for the total level of household liquid assets might be difficult, even at an aggregate level. Information on deposits may be derived from the monetary survey,²¹ complemented

¹⁹ For countries with limited information on household indebtedness the World Bank Global Findex Database can be used. This is a comprehensive database on financial inclusion, based on interviews with about 150,000 adults in over 140 countries. It provides data on how individuals save, borrow, make payments, and manage risks. It is collected in partnership with the Gallup World Poll and funded by the Bill & Melinda Gates Foundation.

²⁰ For example, if the shares of households with a mortgage in the lowest four income deciles and in the upper six income deciles are 20 percent and 40 percent, respectively (from Global FINDEX), assuming the same average DTI ratio in the two groups, equal to the national average of, say, 50 percent, the indebted households in the lowest 4 deciles have a DTI of 250 percent ($=0.5/0.2*100$) while those in the highest 6 deciles have a DTI ratio of 125 percent ($=0.5/0.4*100$). Outstanding debt levels for individual households in each of the two groups can then be derived by *i*) randomly selecting the indebted households (corresponding to 20 and 40 percent of the total households in the two groups) and, then, *ii*) multiplying the household income level for the DTI ratio of the respective group. Finally, monthly (quarterly, or annual) payments can be estimated using formula (iii.) and assuming a constant amortization schedule and using the average remaining maturity at the national level.

²¹ The information on household deposits from the monetary survey may be incomplete if the institutional coverage of the other depository institution sector covers only commercial banks, and excludes credit unions, savings institutions and money market mutual funds.

with information on the share of household deposits on total deposits and how this share is distributed among households.²² Information on the level of other liquid assets held by the household sector might be not available but holdings of financial assets different from deposits is frequently negligible in developing economies and can thus be ignored in those countries.

- Information on *debt levels* refers to the total households' outstanding debt, ideally split between the different debt types. Common debt categories for households include home mortgages, personal loans, and credit cards. Additional information on the level of loans at origination can be helpful to estimate debt payments if needed (see above). In absence of information on individual debt levels, the average household DTI ratio at the national level, complemented with information on indebtedness of different income groups, can be used as proxy.
- The *status of financial obligations* refers to their paid/unpaid status past due date and gives information on the *actual* financial distress of individual households. The corresponding variable should ideally be numeric, with values restricted to non-negative integers, measuring the days of arrears (if any). Another (less detailed) option is a categorical variable, indicating whether the financial obligation is performing/forborne/in arrears/in default. Information on the status of financial obligations allows to estimate how financial distress is associated with households' vulnerability measures and to validate vulnerability estimates. If individual data on the status of debt obligations are not available, average (country-level) information on arrears, forbearance, and non-performing loans remains useful to calibrate and validate vulnerability estimates (See Section III.B.2).^{23,24}

²² For instance, the World Bank Global Findex Database could be used.

²³ For debt-free households, information on the status of debt obligations should be replaced by the status of other financial obligations (i.e. overdue or not on non-debt-related payments), or, if not available, by other information on economic or financial difficulty (i.e. incapacity to cope with unexpected payments, subjective perception of financial vulnerability, data about loan rejection, etc.). If individual data are not available, information on the share of household below the national poverty line remains helpful to test the quality of vulnerability estimates and help the calibration of distress thresholds, although it may lead to an underestimation of the share of fragile households.

²⁴ If, in addition to the status of financial obligations, data on other aspects of financial distress are available (such as information on loan rejection, on subjective feeling of vulnerability, on incapacity to cope with unexpected payments, etc.), this information can be consolidated in a synthetic index through a dimensionality reduction process. For instance, Anderloni *et al.* (2011) used the Nonlinear Principal Components Analysis (NLPCA) methodology to obtain an optimal synthesis of observed variables of financial distress in a reduced space.

3. COMBINING INFORMATION FROM DIFFERENT MICRODATA SOURCES

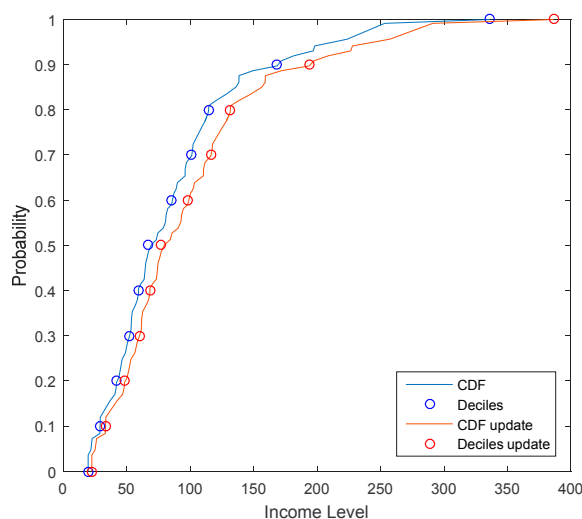
When relevant information is fragmented in several micro-data sources with different reference periods, data need to be consolidated in one time-consistent database. This entails: *i*) extrapolating data in microdata sources to a common reference period, and *ii*) merging the different microdata sources into a unified database.

Extrapolating data in microdata sources

A variety of methods are available to extrapolate data in micro-databases to the period of interest. *Uprating* is the simplest approach, which entails updating monetary variables of individual households in line with developments of aggregate counterparts,²⁵ while non-monetary characteristics are assumed to remain constant. *Static aging* updates the monetary variables of individual households in line with other known information (usually macro-aggregate variables), as in the uprating approach, but entails also adjusting the weights of individual households to reflect the changes in the size of the group to which they belong over the relevant period. Finally, *dynamic ageing* ages individual households' characteristics by modelling processes such as fertility, household formation and dissolution as well as labor market behavior (using transition probabilities).²⁶

Most of the existing applications on household vulnerability assessment have employed the first approach (Ampudia et al., 2014; ECB, 2014; Hlaváč et al., 2014; IMF, 2012; Michelangeli and Pietruni, 2014). This approximation maintains significant heterogeneity in the data because of the cross-sectional structure of the initial database, which ensures household-specific variation (Ampudia et al., 2014). However, given that the “temporal adjustment factor” is identical for all households, it induces some distortion in distributions, by increasing the difference between the first and last deciles of monetary variables (Figure 3). In

Figure 3: Update of the Cumulative Distribution Function of Income Using Country-Level Growth Rates



Note: The figure portrays a proportional increase of 15 percent across the income distribution.

²⁵ For instance, income for each household can be adjusted in line with the observed changes in total income at the national level, while expenditure can be updated using inflation rates.

²⁶ See O'Donoghue and Loughrey (2014) for a survey of microdata extrapolating methods and corresponding properties.

addition, this extrapolation method ignores the fact that the structure of the population might have changed. This latter issue, however, is expected to be less relevant when the time gap between the period when the data are collected and the period of simulation is short.

A few factors should be taken into consideration to reduce sources of estimation bias when extrapolating microdata.

First, at this stage of the procedure, it is preferable to limit variables update to what is needed to obtain a time-consistent database, avoiding updating variables to the period when the analysis is conducted (if different from the reference period of the microdata sources) because this could introduce undue distortions in the estimation method. The update of variables to the period of interest can be performed in the third step of the framework after the estimation method has been validated and before modeling the baseline and alternative scenarios.²⁷

Second, typically it is convenient to project variables in other databases to the reference period of the most recent database. An exception might apply when the updating approach is used and the status of financial obligations is available in one data source. In this case, the status of financial obligations cannot be updated because the updating approach is only suitable for the update of monetary variables. Consequently, data on monetary variables in other micro-data sources should be extrapolated to the observation period of the database which includes information on the status of financial obligations, even if this is not the most recent period among available data sources. This would allow obtaining initial state vulnerability estimates (in the second step of the framework) without the need to update the status of financial obligations. Alternatively, static or dynamic aging should be used.²⁸

²⁷ For instance, if there are two micro-data sources with reference periods 2009 and 2014, respectively, and the period when the analysis is conducted is 2015, variables in the 2009 database should be extrapolated to 2014 to obtain a unified database. Then, vulnerability estimates as of 2014 should be derived and validated. Only in the third step, before crafting and modeling the baseline and alternative scenarios for the stress test, the database should be updated to 2015. It is worth noticing that this approach relies on the assumption that there are no structural changes in the underlying relation between vulnerability measures and household distress between 2014 and 2015 (for instance due to a change in the personal bankruptcy regime), and thus the estimation method validated with 2014 data is valid also for 2015.

²⁸ While static aging does not update non-monetary variables, it produces an updated distribution of households with overdue financial obligations by changing the weights of individual households. The preferable method, however, is dynamic aging as it allows modelling the status of household financial obligation endogenously through transition probabilities.

Merging microdata sources

Once variables in available databases are projected to the same reference period, data can be combined into a unified database. If the household identifier is not available in the different microdata sources, statistical methods can be used to merge the databases. Both parametric and non-parametric matching options are possible.

Parametric methods are available when the databases to be matched include common information on households' characteristics (e.g. region of residence, household head age group, employment status, education level, etc.). In this case, information can be transferred from one database to the other based on regression estimates, where the common variables serve as the independent variables in the regression equation. Non-parametric methods can be used when the databases to be combined do not include information on household characteristics or this information is limited. In this case, it is possible to use any variable available in both databases and/or a monotonic relation between two (or more) variables belonging to the different databases as matching keys. The mapping of information would then be performed by using the empirical distributions of households along the matching keys.

Further details on parametric and non-parametric methods to merge information from different databases can be found in Appendix I.

B. STEP 2: DEFINITION OF HOUSEHOLD DISTRESS AND VALIDATION OF VULNERABILITY ESTIMATES

1. CHOICE OF THE VULNERABILITY MEASURES AND DEFINITION OF DISTRESS

Once the database is built, the next step entails assessing whether a household is “at risk of distress” based on its vulnerability indicators.

First, there is need to assess whether vulnerability indexes derived from core variables [such as extended financial margins in (i.1) and (i.2)] should be complemented with additional measures of vulnerability to assess *household distress*. As highlighted in Section III.A.1., complementary variables can indeed be used to build additional vulnerability indexes that may enhance the capacity to discriminate distressed households.

At this stage, the selection of vulnerability measures can be guided by macro correlations and/or the institutional setting.²⁹ For example, in jurisdictions envisaging the possibility of personal bankruptcy, such as the United States or Australia, a measure of net wealth or leverage can be helpful to complement the assessment based on the core variables, because a

²⁹ The validation of variable and model selection will be illustrated in Section III.B.3.

household may choose to default on a loan despite being able to service it (strategic default). This can be the case with “underwater borrowers”, whose property value has dropped below the balance of the loan.

The selected vulnerability measures are then used to determine the likelihood that the household will run into financial distress. In this context, both non-parametric and parametric approaches are possible.

- i) Most of the studies on household vulnerability have adopted a *non-parametric binary approach* and have assumed that the household is at risk of financial distress when its measures of financial vulnerability cross a specified threshold. When more than one vulnerability measure is used, two options are possible.
1. The indicators are considered separately, each with its own reference threshold. In this case, the household is qualified as distressed when some or all the indicators fall below their reference threshold.³⁰

$$PD_{h,t} = \begin{cases} 1, & \text{if } \bigcup_{n=1}^N I_{n,t} < \tau_n \quad (\text{iii.1}) \\ 0, & \text{otherwise} \end{cases} \quad \text{or} \quad PD_{h,t} = \begin{cases} 1, & \text{if } \bigcap_{n=1}^N I_{n,t} < \tau_n \quad (\text{iv.2}), \\ 0, & \text{otherwise} \end{cases}$$

where $PD_{h,t}$ is a binary variable taking value 1 when indicators fall below the identified thresholds at time t and zero otherwise. $PD_{h,t}$ can be interpreted as the household’s probability of distress at time t .³¹ $I_{n,t}$ and τ_n are the vulnerability indicator n at time t and its reference threshold, respectively, and N is the total number of vulnerability indices. This approach corresponds to attributing the same weight (relevance) to each vulnerability measure, with the formulation in (iv.1) more conservative than the formulation in (iv.2) (i.e. the household is more likely to be declared vulnerable).

2. The indicators are combined in a synthetic index with the household considered at risk when the index falls below an identified threshold:

³⁰ In formulas (iv.1), (iv.2) and (v), the $<$ sign should be replaced by a $>$ sign for indicators that take higher values the higher is household financial vulnerability, such as DSTI ratios.

³¹ It must be noted that while the specifications in (iv.1), (iv.2) and (v) refer to the assessment of distress associated with the “liquidity” of financial positions at time t (see Section III.A.1), similar specifications can be used to assess the solvency of financial positions. For instance, a specification to assess distress associated with solvency could require that the present value of the sum of the household income-expenditure balances over the projection period is inferior to a certain threshold, where the threshold could be determined differently depending on whether the concept of solvency is associated with a declining level of net debt or with a level that is inferior to a certain benchmark (see Section III.A.1).

$$PD_{h,t} = \begin{cases} 1, & \text{if } f(I_{1,t}, \dots, I_{n,t}, \dots, I_{N,t}) < \tau \\ 0, & \text{otherwise} \end{cases} \quad (\text{iv.}),$$

where $f(\cdot)$ is the composite index and τ is its indicative threshold. When individual information on the status of debt obligations is available, the fitted values of a regression, estimating the association of vulnerability measures with household distress, could be used as synthetic composite index of vulnerability (Su and Liu, 1993; McIntosh and Pepe, 2002). Another option consists in deriving the weights of individual indicators non-parametrically through an algorithm aimed at maximizing the capacity to correctly predict distressed households (this method is used in Namibia's application, see Section IV.B.2 and Appendix II). The use of a synthetic index permits to reflect the relative importance of the composing indicators in explaining household vulnerability. A caveat of this approach is that from a policy perspective an ideal early warning indicator should be easy to interpret (Drehemann and Juselius, 2014), requirement that is not likely to be satisfied by a composite index.

In addition to the binary approach, non-parametric methods to extract information from multiple indicators include classification trees and other machine learning methods but, to the best of our knowledge, these methods have not yet been used for the assessment of household financial vulnerability.

- ii) Some applications have adopted *parametric approaches* to assess household vulnerability, by modelling the relation between household vulnerability measures (explanatory variables) and the status of financial obligations (dependent variable).³² These approaches allow to derive continuous probabilities of distress, lying in the interval $[0,1]$. Among the parametric estimation methods adopted in empirical literature there are binary regressions and artificial neural network models (Holló and Papp, 2007; Herrala and Kauko, 2007; Arins *et al.*, 2014).

The binary method in specifications (iv.1) and (iv.2) has the advantage to be simple and parsimonious. It does not require individual information on the status of financial obligations (if thresholds are not calibrated) and entails limited calculation. The binary method in specification (v.) is more compute and data intensive, as it encompasses the construction of the synthetic index and requires the calibration of the distress threshold. Both methods have the advantage of identifying potential non-linearities in the relation between household distress and vulnerability measures. In addition, they specify indicative thresholds for

³² Depending on the type of information available on the status of financial obligations and the purpose of the analysis, parametric methods may model the relation between household vulnerability measures (explanatory variables) and arrears or forbearances (dependent variable), rather than defaults.

vulnerability indicators (or synthetic indexes), thus providing policy makers with easy benchmarks for policy analysis and formulation. The limitation of binary approaches, however, is that they lead to a dichotomic classification of households (non-distressed/distressed), where “distressed” households are usually interpreted as “defaulted”, thus not accounting for the fact that financial distress has different degrees of severity.³³ Binary methods can be extended to obtain a multi-category classification of households, rather than binary, by calibrating different thresholds for different categories of distress (arrears, non-performing, forborne).

Parametric approaches are more data and compute intensive than the binary approach in specifications (iv.1) and (iv.2) but allow a more granular modelling of household financial distress. Depending on the available information on the status of financial obligations and the purpose of the analysis, parametric methods may indeed model the relation between household vulnerability measures and different categories of distress. In addition, these approaches allow to derive continuous probabilities of distress for individual households, thus providing information on how likely is the household to fall into the analyzed category. The limitation of parametric methods is that test statistics are sensitive to how well data meet the underlying distributional assumptions, particularly in small samples.

It must be noted that non-parametric and parametric methods are not mutually exclusive. For instance, as illustrated above, parametric methods can be employed to derive synthetic indicators of financial vulnerability to be used in the binary method in specification (v.), thus entailing an overall semi-parametric approach.

2. ADDRESSING ENDOGENEITY ISSUES IN VULNERABILITY ESTIMATES

Endogeneity issues are frequent in household microdata—stemming from selection bias, measurement errors and omitted variables—and may lead to inconsistent vulnerability estimates.

This issue is particularly (but not exclusively) relevant in the binary approach, where the precision of household vulnerability estimates relies critically on the selection of the reference thresholds for each measure. The question then arises as to how thresholds should be determined to correct for potential sources of endogeneity bias.

To date, relevant literature has mainly used *ad hoc* thresholds for each measure, based on economic considerations or historical evidence on arrears/defaults. For instance, the reference threshold for financial margins has been universally set at zero, while that of DSTI

³³ For instance, in the case of debt obligations, banks categorize the severity of loan arrears based on a mix of qualitative and quantitative criteria (such as the number of days past due). This categorization is used to calculate provisions. Categorizations and provisioning requirements vary across jurisdictions (see Bank for International Settlements, 2016).

ratios has been set within a range of 20 and 40. These thresholds may be good approximations of the actual thresholds when the indicator used in the analysis faithfully reproduces the underlying vulnerability measure. However, when the most precise indicator is replaced with a proxy or with an indicator which is incomplete or subject to measurement error, the assessment based on standard thresholds can bias the results of the vulnerability assessment.

For instance, using gross income instead of disposable income, because of a lack of information on net current transfers and taxes, would lead to underestimating (overestimating) the income of poorer (wealthier) households. This entails that the estimated financial margins would be lower (higher) than actual for the poorer (wealthier):

$$\overline{FM}_{h,t}^p < FM_{h,t}^p \quad \text{and} \quad \overline{FM}_{k,t}^w > FM_{k,t}^w$$

where $\overline{FM}_{h,t}^p$ and $\overline{FM}_{k,t}^w$ are the estimated financial margins for a generic poor household h and for a generic wealthy household k , respectively, while $FM_{h,t}^p$ and $FM_{k,t}^w$ are the actual (unobserved) financial margins for the same households. This in turn implies that setting the distress threshold for both groups at zero would result in an overestimation of the number of distressed households among the poor and an underestimation of distressed households among the wealthy.

When the sources of endogeneity deriving from unobserved heterogeneity and/or measurement errors are assessed to be non-negligible, distress thresholds should be calibrated separately for each households group where the sources and size of estimation bias are expected to be uniform. For example, when disposable income is proxied with gross income, the estimation bias deriving from the lack of information on transfers, subsidies and taxes can be corrected by setting distress thresholds for financial margins below zero for the poor, which implies that they can fulfill their debt obligation even if their gross income is inferior to the sum of living costs and debt payments because of transfers and subsidies (unobserved variables). The distress threshold for the wealthy, instead, should be set higher than zero, implying that they can default even if their gross income is sufficient to cover the sum of living costs and debt payments because of taxes (unobserved variable).

Calibration

Calibration can help determine the thresholds for each vulnerability measure in each household group. For indicators that can be expressed either in monetary terms or as indexes (such as financial margins), calibration should be performed on the index-version to allow cross-sectional and time comparison. For instance, in the case of financial margins the formulation in (i.2) should be used for calibration.

In general, calibration entails (i) partitioning the observations in subsets where the sources of estimation bias are expected to be uniform, and (ii) estimating specific distress thresholds for each subset.

- i) *Partitioning*. The quality of the applied partition is critical to obtain unbiased vulnerability estimates. A successful partition should aim at suppressing (or, more realistically, significantly reducing) variation in the sources of estimation bias in each subset (i.e. each subset should no longer be affected by unobserved heterogeneity and measurement errors should be constant). This objective can be achieved by using observable variables that are correlated with the sources of endogeneity to perform the partition. For instance, when endogeneity is associated with the lack of information on net current transfers, households could be divided in subgroups based on their gross income, given that net current transfers depend on this variable: poor households receive positive transfers while rich households receive negative transfers. The more granular the partition the more likely is the suppression of endogeneity within each subset. Granularity of the partition comes however at the cost of a smaller number of observations in each group, which can complicate the estimation of thresholds.
- ii) *Estimating Thresholds for Each Subset*. The estimation method depends on data availability.
 - If individual information on the status of financial obligations is available, the simplest approach entails setting distress thresholds for any index in each subset equal to the average (or median) value of the index among defaulted households in the same subset. A preferable approach entails determining thresholds with statistical methods, for instance through a signaling detection approach, which uses non-parametric estimation to find the threshold striking the best balance between detecting the vulnerable households and sending too many false alarms. A caveat of the signaling detection approach is that it can be computationally demanding in case of a multivariate analysis with several vulnerability indicators (as in specifications iv.1 and iv.2).³⁴ In those cases, there are gains from combining information from different vulnerability measures into a synthetic index [as in specification (v.)] and then derive thresholds for this index.³⁵ Another option is to set a fixed threshold for

³⁴ In principle, the signaling approach could be used in a multivariate analysis. However, this would require high dimensionality grid search and the testing of many indicator combinations. To date, the method has been mainly applied to univariate analyses. Bivariate and three-variate applications are limited [Borio and Lowe, 2002; Alessi and Detken, 2011; Ferrari et al., 2015].

³⁵ As mentioned in Section III.B.1, computational tractability comes at the cost of easiness of interpretability when a synthetic index is used.

vulnerability indexes that are expected to be free from unobserved heterogeneity and measurement errors and calibrate only the thresholds of measures that are expected to be biased by data limitations.

- If individual information on the status of financial obligations is not available and the analysis employs only one vulnerability index, calibration can be performed using average data on the level of non-performing loans (or loan arrears).³⁶ In this case, the distress threshold for households in each subset can be set equal to the value of the vulnerability index that permits to reproduce the average value of non-performing loans (or loans in arrears).³⁷ In absence of more precise information, this procedure assumes that the level of non-performing loans in each household subset is the same as the national average. If the analysis employs more than one vulnerability index, one option entails setting a fixed threshold for vulnerability indexes that are expected to be free from unobserved heterogeneity and measurement errors and calibrate only the threshold of the indicator that is expected to be biased by endogeneity issues using average data on the level of non-performing loans (or loan arrears).³⁸

³⁶ For debt-free households, thresholds can be calibrated by using the share of population below the poverty line.

³⁷ The vulnerability index would be tested on a fine grid of possible thresholds with the goal of finding the threshold that permits to reproduce the observed non-performing loans (or arrears or other average measure of household distress). In detail, given the range $[a,b]$ of observed values of the vulnerability index I in subset s , simulated non performing-loans in the subset $NPL_s^{SIM}(x)$ are derived for any threshold $x \in [a,b]$ by computing the share of total debt in the subset held by households that cross the threshold x (i.e. that are classified as distressed, given the threshold x):

$$NPL_s^{SIM}(x) = \sum_{n=1}^{H_s} PD_{h,s}(x) \cdot D_{h,s} / D_s$$

where $PD_{h,s}(x)$ is the probability of distress of household h in subset s , given the threshold x , $D_{h,s} / D_s$ is household h 's share of total debt in subset s , and H_s is the total number of households in subset s . The optimal threshold x^* would then satisfy:

$$x^* : \left| NPL_s^{SIM}(x^*) - NPL \right| = \min \left| NPL_s^{SIM}(x) - NPL \right|, \text{ with } x \in [a,b],$$

where NPL is the observed non-performing loan ratio.

³⁸ Ampudia *et al.* (2014) and Meriküll and Rõõm (2017) used a similar approach by calibrating the threshold level of liquid assets, while setting the distress threshold for financial margins (excluding liquid assets) at zero. These works differ from the proposed approach in that they calibrate the threshold over the full sample rather than in individual household groups, which may entail some estimation bias in case of data limitations.

As mentioned in section III.B.1, calibration can also be used to obtain a multi-category classification of households, rather than binary, by producing different thresholds for different categories of distress (arrears, defaults, etc.).

Up to this point we have discussed how to correct for endogeneity issues in non-parametric methods. In theory, calibration can be employed also to correct for sources of estimation bias in parametric methods. This would entail (i) partitioning the observations in subsets where the sources of estimation bias are expected to be uniform—as in the binary case—and (ii) estimating specific model parameters for each subset. In practice, however, this procedure can result in a restricted number of observations in each subset making parametric inference potentially unreliable, particularly if data violate the underlying distribution assumptions. Other approaches are preferable in those cases. For instance, in the case of binary regression models several methods exist to deal with endogenous repressors, such as linear probability model estimators, maximum likelihood estimation, control function based estimation, and special regressor methods.

To conclude, it must be noted that while the focus of this Section has been on addressing endogeneity issues in the context of distress associated with the liquidity of financial positions, the method could be extended to address endogeneity issues in the context of distress associated with solvency (see Appendix III).

3. VALIDATION OF VULNERABILITY ESTIMATES

Before proceeding with the stress test, it is relevant to validate the initial-state vulnerability estimates. Validation entails assessing how accurately the model can replicate the underlying phenomenon (household distress in our case). Common validation techniques imply assessing the in-sample and/or out-sample predictive performance of the selected estimation method.

If individual information on the status of financial obligations is available, a simple approach to test *in-sample predictive performance* consists in computing, for each household subset, the fitting deviation (or estimation error) between the simulated share of distressed household and the actual share of households in distress. A more accurate validation option entails calculating for each subset the area under the Receiver Operating Characteristic (ROC) curve, which provides information on the capacity of the estimation method to correctly discriminate the event of interest (i.e. distressed households) (Holló and Papp, 2007; Arins et al., 2014). If individual information on the status of financial obligations is not available, validation can be performed by comparing the average simulated non-performing loans with their national-average counterpart (actual non-performing).

Approaches to assess the *out-of-sample predictive performance* entail more data and compute-intensive statistical methods, such as cross-validation. This technique requires information on the status of financial obligations and involves splitting the data into

complementary subsets, performing the analysis on one subset (called the training set), and validating the analysis on the other subset (called the validation set or testing set). Given that the resulting test error can be highly variable, depending on the splitting method, multiple rounds of cross-validation are performed using different partitions, and the validation results are averaged over the rounds.

It must be noted that assessing the accuracy of estimates is relevant also when data quality is expected to be high. Indeed, if there is significant difference between the simulated and actual share of distressed households, this is a signal that some relevant variable has not been considered and/or measurement errors might be present, requiring to proceed with a new estimation which is based on additional (or alternative) vulnerability indices and/or employs techniques to correct for sources of endogeneity if these were not used in the initial assessment.

C. STEP 3: STRESS TESTING

The final step of the framework provides an assessment of household vulnerability under a baseline and/or adverse scenarios.

Prior to simulate the impact of alternative scenarios, variables in the unified database need to be extrapolated to the period when the analysis is conducted (if different from the reference period of the microdata sources) by employing one of the methods described in Section III.A.3. The framework is then suitable to conduct both one- and multi-period analyses. The former requires less computation and is apt to assess the liquidity of household financial positions, the latter is more compute-intensive and appropriate to assess solvency (Section III.A.1).

In the baseline, forward paths of vulnerability measures can be derived using simulations obtained from macroeconomic data (Djoudad, 2010, 2012; Gross and Población, 2017) or theoretical models (for instance DSGE). In case of multiperiod analyses, simulations may also include feedback loops between the household sector and the rest of the economy (Gross and Población, 2017).

Adverse scenarios consist in departures from the baseline of one or more variables. Variables typically shocked in household stress-tests are income, interest and exchange rates, and asset prices. Shocks may be applied uniformly to all households in the database or selectively to some subsets, depending on the purpose of the analysis. Several applications test also the impact of changes in unemployment rates—which affect households’ income—using Monte Carlo simulations (see, among the others, Albacete and Fessler, 2010; Arins et al., 2014; Fuenzalida and Ruiz-Tagle, 2011; Galuščák et al., 2016).³⁹

³⁹ Monte Carlo simulations are used to address the issue that changes in unemployment rates cannot be applied uniformly to the whole household sample since unemployment affects individual members of a household.

Vulnerability measures are then recomputed under both the baseline and adverse scenarios to assesses the changes in the share of distressed households.

It must be noted that the quality of household distress projections depends critically on baseline projections that, however, have a degree of uncertainty. To reflect this uncertainty, the vulnerability assessment under baseline could include, for each household group and each point in time along the simulation horizon, the lower and upper values of a confidence interval centered on baseline distress projections. This would entail computing, for each household group and each point in time along the simulation horizon, the joint distribution of vulnerability indicators⁴⁰ and, then, deriving the share of distressed household associated with the lower and upper values of the confidence interval.

⁴⁰ The distribution of vulnerability measures could be obtained through stochastic simulations of forward paths of macroeconomic data from VAR residuals.

IV. APPLICATION ON NAMIBIA

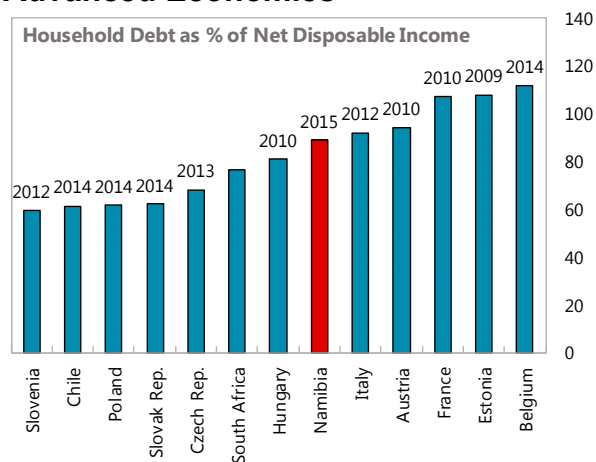
To date, the framework described in Section III has been used to assess household vulnerability in Namibia (IMF, 2016), Finland (IMF, 2017), and Luxemburg (IMF, 2017b).

In this paper, we will focus on Namibia’s application, for which we provide a step-by-step description of how the analysis was performed. This example will illustrate how the methodology described in the previous section permits to correct estimation bias associated with data limitations through the calibration of distress thresholds, allowing to obtain a meaningful assessment of household vulnerability also in a country with limited data availability and quality. The applications on Finland and Luxemburg, instead, used information from the ECB Housing Finance and Consumption Survey (HFCS), whose data quality and granularity is expected to be high, and didn’t require calibration of distress thresholds (Appendix IV summarizes the main features of the three applications).

The application on Namibia was prepared in the context of the 2016 Article IV consultation to analyze the financial stability implications of banks’ large exposure to the household sector. Consequently, the analysis restricts the focus on indebted households.

In Namibia, double-digit house price increases over a long period have left households with relatively high debt, estimated by Bank of Namibia (BON) at about 90 percent of disposable income at end-2015.⁴¹ While this level of debt is lower than in advanced economies, it is relatively high compared to emerging markets, including neighbor South Africa (Figure 4). In addition, the debt service burden, estimated at 16.1 percent of disposable income, is high compared to countries for which information is available⁴² and skewed toward interest payments, corresponding to roughly 10 percent of disposable income (Figure 5, Left Panel).

Figure 4. Namibia: Household Debt— Comparison with Emerging Markets and Advanced Economies

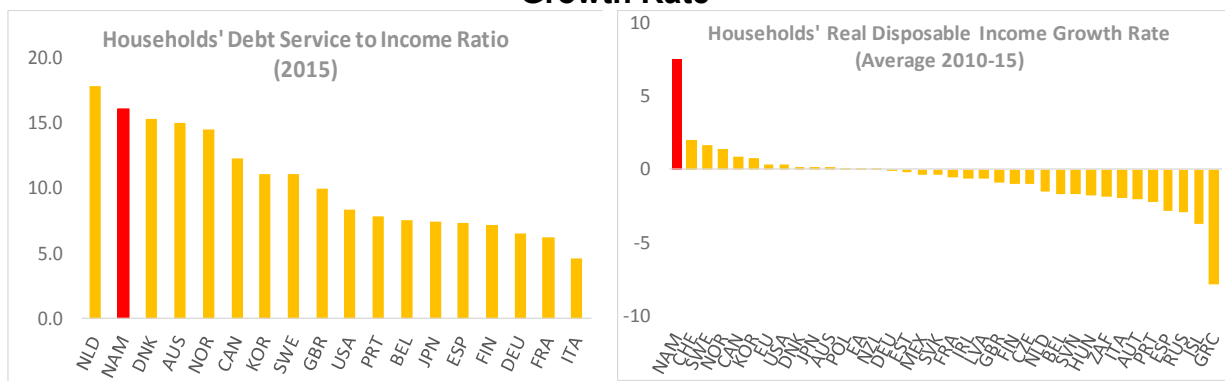


Source: OECD and BON.

⁴¹ Bank of Namibia estimates two country-wide indicators associated with household indebtedness in Namibia using data on total household credit by formal financial institutions as a proxy of household debt: the ratio of household debt to household disposable income and the debt servicing cost of household debt to household disposable income. See Appendix 2 in Bank of Namibia (2012).

⁴² BIS publishes debt service ratios for the household, the non-financial corporate and the total private non-financial sectors (PNFS) for 17 countries. Debt service ratios for the total PNFS are also available for 15 additional countries. See <http://www.bis.org/statistics/dsr.htm>.

Figure 5. Namibia: Households' Debt Service to Income Ratio and Real Income Growth Rate



Sources: BON, BIS, and OECD

Despite high debt and debt service burdens, impairment figures in Namibia's household loan books have remained low and broadly stable at about 1.5 percent of total loans and advances, owing to a benign economic environment and income growth rates superior to international standards (Figure 5, Right Panel), which have supported households' debt repayment capacity. Nevertheless, changes in economic and policy conditions may erode disposable income and/or increase the debt service burden (as the bulk of mortgage loans are at adjustable rates), potentially affecting the capability of indebted households to fulfill the undertaken financial liabilities timely and adequately.

The risks associated with a deterioration in economic and policy conditions are compounded by the limited diversification of the Namibian economy, which entails that households' sources of income are only a few and, thus, that defaults can be highly correlated. A reduction in the public wage bill, the bankruptcy of a large company, or the reversal of the current trend in house prices would impact the income of many households. This in turn could have financial stability implications, as commercial banks have a large exposure to the residential mortgage sector, corresponding to 38 percent of total loans and advances at end-2016.

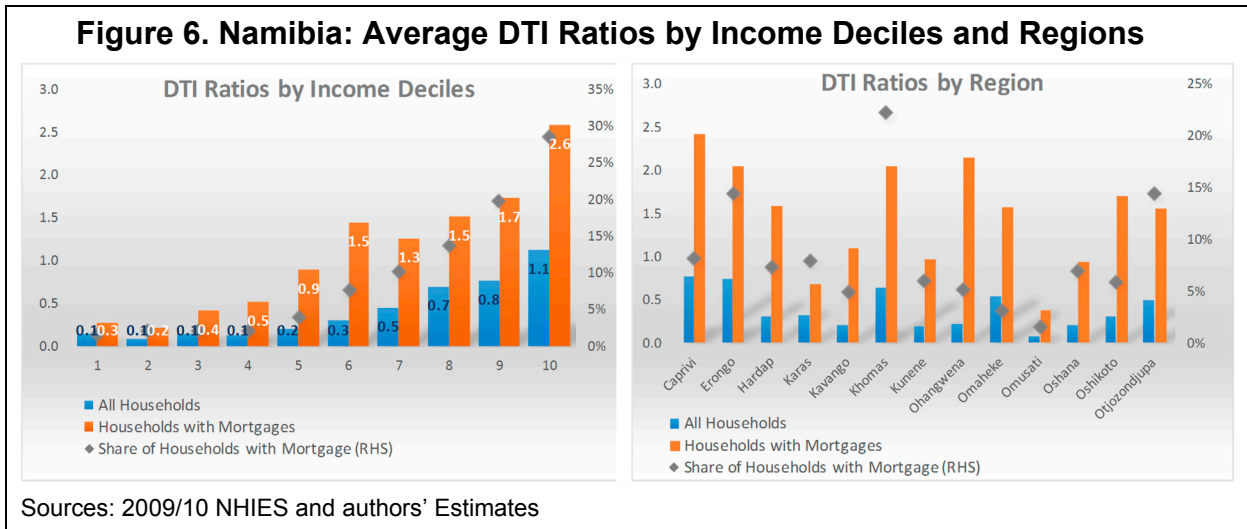
Prior to this application, the assessment of the credit risks associated with household indebtedness had been based on aggregate data, which has severe limitations because it masks the expected concentration of borrowers among selected groups, and may lead to underestimating risks. Among micro-data sources, no dataset is available for Namibia including all the core variables needed to build measures of vulnerability/strength for individual households. The methodology introduced in Section III can overcome this data limitation and provide a meaningful assessment of the risks connected to the household sector despite fragmented and incomplete data sources.

A preliminary assessment can be derived by analyzing indebtedness of different household groups using data from the most recent available household survey, the 2009/10 National

Household Income and Expenditure Survey (NHIES) conducted by the Namibia Statistics Agency (NSA).⁴³ This preliminary analysis reveals that the median DTI ratio in the overall population of households is inferior to 0.01, thus suggesting that most households are debt-free. The average DTI ratio, scoring 0.44, is however significantly higher,⁴⁴ implying that the debt burden of the few indebted households is very high. Not surprisingly, Figure 6 illustrates that debt is largely concentrated among households with mortgages. Survey data show also that the average DTI ratio is significantly higher in richer (as measured by income) households. This is notably due to the higher access to mortgage credit in the highest deciles, including for investment purposes.

As regards the territorial distribution of household indebtedness, average DTI ratios of households with mortgage are equal or superior to one in all regions with the exception of Karas and Imusati. There is however stark difference in the share of people with mortgage loans among urban and rural areas, with the province of Khomas, including the capital Windhoek, having the highest value, which entails that the share of households with high debt burden is higher than in other regions.

Based on this preliminary analysis, the household vulnerability assessment which follows focuses only on the share of households with mortgages, as they hold the bulk of household debt.



⁴³ The most recent NHIES survey (2015/16) was completed in the first quarter of 2016 but data are not yet made available. Some caution is warranted in the interpretation of debt data from the NHIES, as this survey is not specifically designed to collect information on households' liabilities, focusing instead on poverty and social developments. The NSA, however, plans to introduce and pilot a Household Indebtedness and Financial Inclusion survey in the coming years.

⁴⁴ Because the analysis is based on survey data, DTI ratios do not correspond to the aggregate value as reported in Bank of Namibia's Financial Stability Report, which is based on data on total household credit by formal financial institutions (for debt) and national accounts (for income).

A. STEP 1: DATABASE PREPARATION

The information on Namibia's household vulnerability/strength used in this exercise is fragmented in two microdata sources: the NHIES and a large sample of banks' mortgage loans. The NHIES includes information on 9,600 individual households as of 2009-2010, including gross income, different expenditure items, outstanding debt levels, region of residence, and employment status. The mortgage database includes details on 22,000 individual mortgage loans at end-2014,⁴⁵ including the value of the loan at origination, outstanding balance, the monthly amortization payment, the monthly interest payment, the number of days the payment is overdue, the value of the property, additional guaranties, and the region where the property is located. Flow variables in the two databases (income, expenditures, amortizations and debt payments) are annualized.

As the reference period of the two databases is different, the information in the NHIES is updated to end-2014⁴⁶ by replicating the development of country-level variables (Section III.A.3): income of individual household is uniformly increased based on the national yearly growth rate reported in the Financial Stability Report of the Bank of Namibia, while expenditures are increased at the average annual CPI rate reported by NSA.

Then, relevant information of the household survey is mapped into the mortgage data source to create a unified database.⁴⁷ The mapping follows a non-parametric approach (Section III.A.3 and Appendix I) and it is based on two matching keys: the region (present in both data sources) and the positive relation between household income (in the NHIES) and amortization payment (in the mortgage database), supported by the fact that banks follow internal guidelines that put limits on DSTI ratios.⁴⁸

Initially, the two databases are partitioned into 130 cells based on the two matching keys. In detail, observations in the NHIES are distributed within a grid box according to their region

⁴⁵ This represents broadly one fourth of all outstanding mortgage loans.

⁴⁶ It must be noted that the update is performed by extrapolating variables in the NHIES to the observation period of the mortgage database, which includes information on the status of financial obligations, and not vice versa. As illustrated in Section III.A.3, this approach allows to derive the pre-shock vulnerability estimates without the need to update the status of financial obligations which could create distortions in estimates.

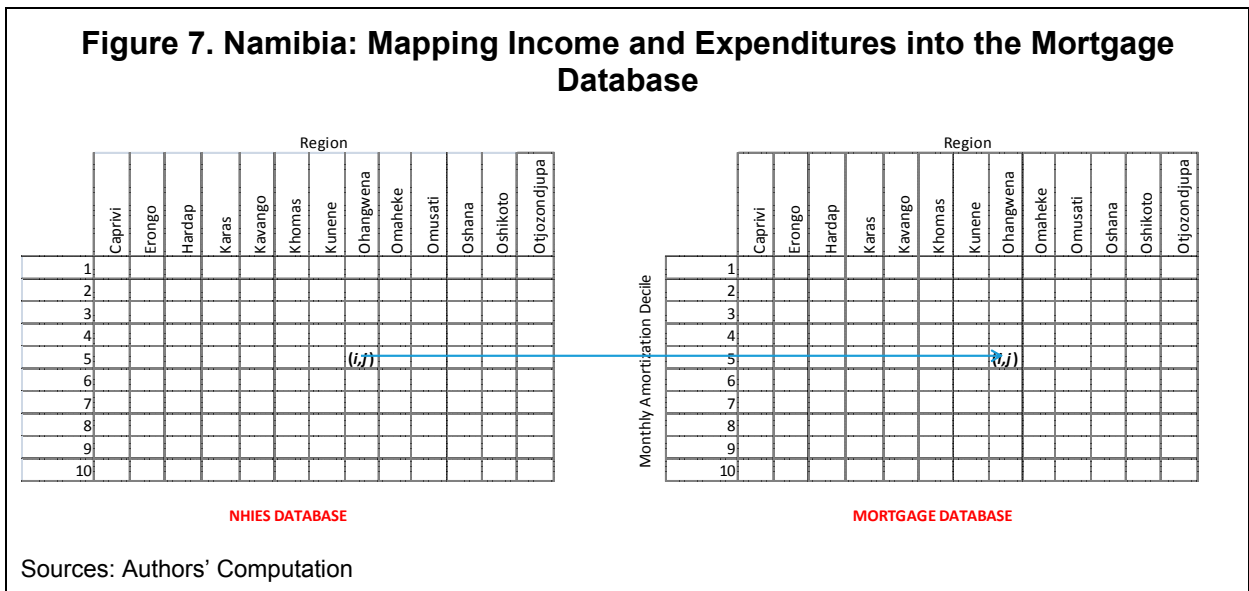
⁴⁷ By merging the two databases, we implicitly assume that each loan corresponds to a different household. This assumption is a simplification as some of the loans in the mortgage database may belong to the same household, of which we do not have information. This leads to an underestimation of the debt-service for households with more than one mortgage loans. The calibration of distress thresholds in the next step aims at reducing the estimation bias stemming from this and other sources of unobserved heterogeneity.

⁴⁸ Internal guidelines of most banks impose a 30-35 percent limit on the debt service to gross income ratio. Guidelines allow however for some flexibility for high net worth individuals.

of residence and gross income decile, and observations in the mortgage data source are distributed within a grid box according to the region where the property is located and the amortization payment decile. Then, information on income and expenditure from the NHIES is mapped into corresponding cells of the mortgage book database (Figure 7) based on the loan performance status:

- i. Each performing loan⁴⁹ belonging to cell (i, j) in the loan book grid box is paired with the average values of gross income and living costs⁵⁰ of cell (i, j) in the NHIES grid box.
- ii. Non-performing loans in cell (i, j) in the loan book grid box are paired with weighted averages of the minimum and average values of gross income and expenditures in cell (i, j) in the NHIES grid box, where the weights are derived through a numerical procedure aimed at maximizing the predictive performance of household vulnerability for cell (i, j) (see following Section IV.B.2 for more details).

The mapping is applied differently to performing and non-performing loans to enhance the variability of financial margins in each cell and enhance predictive performance. The economic rationale behind the use of a weighted average of the median and the minimum income of cell (i, j) for households with non-performing loans is that these households are likely to have a lower income than households with loans that are performing.



⁴⁹ In the exercise, we assume that a loan is non-performing if the monthly payment has an overdue superior to 30 days, implying that the household is at least one installment behind.

⁵⁰ We define “living costs” the sum of “basic” expenditure items (food, transport, health, education). We exclude from living costs “non-necessary” expenditure items (travel, luxury goods, sport, entertainment, etc.).

The unified database includes individual information on all the core variables listed in Section III.A.1., except for liquid assets, that, however, are not expected to be sizable in Namibia, allowing to build individual relative financial margins at end-2014, as follows:

$$FM_{h,2014}^R = \frac{Mean(Y_{2014,i,j}) - Mean(LC_{2014,i,j}) - DP_{h,2014}}{Mean(Y_{2014,i,j})}$$

if household h 's loan is performing, and

$$FM_{h,2014} = \frac{(1 - \alpha_{i,j}) \cdot Mean(Y_{2014,i,j}) + \alpha_{i,j} \cdot Min(Y_{2014,i,j}) - (1 - \beta_{i,j}) \cdot Mean(LC_{2014,i,j}) - \beta_{i,j} \cdot Min(LC_{2014,i,j}) - DP_{h,2014}}{Mean(Y_{2014,i,j})}$$

if household h 's loan is non-performing, where $Mean(Y_{2014,i,j})$, $Min(Y_{2014,i,j})$, $Mean(LC_{2014,i,j})$, and $Min(LC_{2014,i,j})$ are, respectively, households' average and minimum gross income and living costs in cell (i, j) at end-2014 (mapped from the updated household survey), while $DP_{h,2014}$ is the debt payment of household h at end-2014 (from the mortgage book). Weights $\alpha_{i,j}$ and $\beta_{i,j}$ will be derived through calibration in the next step.

Based on this matching procedure, debt servicing costs are different for each household h belonging to the unified database, while gross income and living costs are equal for each household belonging to cell (i, j) with the same loan performance status (performing/non-performing).

In addition to financial margins, information in the merged database permits to derive other individual measures of household strength/vulnerability. In particular, the "outstanding loan-to-value ratio" ($LTV_{h,2014}$) is a measure of the household's leverage⁵¹ and is expected to be positively correlated with the probability of default. The "outstanding loan-to-loan-at-origination ratio" ($OTO_{h,2014}$) is also expected to be positively correlated with defaults as it is likely that the household would do its best to repay the loan if only a small portion of it is left. Both variables are different for each household h belonging to the unified database.

⁵¹ Information on household assets is not available except for the property value. In the case of Namibia, however, household financial assets are likely negligible and thus the property value is a good proxy of a household's gross wealth. Nevertheless, this measure remains incomplete because it provides only information on the value of the property connected to the mortgage loan and not on all the household's properties.

B. STEP 2: DEFINITION OF HOUSEHOLD DISTRESS AND VALIDATION OF VULNERABILITY ESTIMATES

In the second step of the exercise we derive and validate initial-state vulnerability estimates. First, we assess whether all the three vulnerability measures ($FM_{h,2014}$, $LTV_{h,2014}$, $OTO_{h,2014}$) are good indicators of household distress in our sample. For this purpose, we estimate a binary logistic regression to model the relationship between arrears⁵² and the vulnerability indices.⁵³

Table 2 reports the results of the regression. As expected, (relative) financial margins have a negative impact on the probability to accumulate arrears and are a significant variable in the regression. Loan-to-value (LTV) ratios seem to have a significant impact on the probability to incur arrears but the sign is not expected. The negative sign is likely to reflect reverse causality: households' probability to incur arrears explains LTV ratios rather than the other way around. This is associated with the fact that banks set LTV ratios based on households' repayment capacity, with the best borrowers getting the highest LTV ratios. Finally, the outstanding loan-to-loan-at-origination ratio is not significant in the regression.

Table 2: Binary Logit Regression

Variable	Coefficient	SE	p-value
FM	-0.041	0.019	0.0296
LTV	-0.681	0.167	0.000
OTO	0.0000	0.003	0.958
Constant	-3.726	0.087	0.000

Source: 2009/10 NHIES, commercial banks' data, BON, and authors' computations.

Overall, results appear consistent with previous studies that find that indebtedness is only weakly correlated with household financial stress (Worthington, 2006; Costa and Farinha, 2012; Dietsch and Welter-Nicol, 2014), and reflects the fact that in Namibia, as in most jurisdictions, mortgage loans are full recourse, which implies that borrowers cannot

⁵² In our exercise, we assume that households are in distress when their mortgage payment is overdue for more than 30 days. This likely overestimates the number of indebted households at risk, which would be more correctly associated to an overdue of 90 days (standard non-performing loan classification). However, using the standard definition would not allow to conduct the vulnerability assessment. The number of loans with an overdue of more than 90 days is indeed very small in our database and would allow the computation of vulnerability thresholds only for a limited number of cells, while the precision of estimates would be likely inferior.

⁵³ In this preliminary analysis, we assume that $\alpha_{i,j}$ and $\beta_{i,j}$ are equal to zero for any cell (i,j) .

strategically default when the value of the property plunges or if the debt burden is considered too high.⁵⁴

Based on the regression results,⁵⁵ we use only financial margins to derive household vulnerability estimates through a non-parametric binary approach (Section III.B.1). This entails defining the household as vulnerable when its financial margin crosses a specified threshold. In the exercise, we use individual information on arrears to calibrate specific distress thresholds for each cell (i,j) of the unified database. As discussed in Section III.B.2, this approach permits to account for measurement errors and unobserved heterogeneity, based on the assumption that these sources of estimation bias are constant within each cell.

This methodology fits well Namibia where data quality is weak and there are important omitted variables. In detail, information on net current transfers is not available, hampering the possibility to derive estimates of disposable income. This is an important source of unobserved heterogeneity because in Namibia the tax system is highly progressive and the social system is very generous, implying that disposable income is higher than gross income in the lowest deciles and lower in the highest deciles. In addition, the World Bank audit of the NHIES has assessed that gross income is under-reported in the lowest deciles, which is a source of measurement error. On the debt repayment side, we assume that each loan in the mortgage database belongs to a different household, thus underestimating the debt-service for households with more than one mortgage loan, who are expected to be concentrated in the highest income deciles. Finally, as mentioned before, information on liquid assets is not available and this is likely to lead to an overestimation of the vulnerability of the wealthy households, which are more likely to hold financial assets.

For the estimation of distress thresholds, we use a signal detection approach. According to this methodology, when the indicator (in our case the financial margin) takes a value that is below a certain threshold, this is a signal that the event of interest (in our case a household's arrears) materializes. Comparing the signal with the actual realization of the event permits to assess the predictive performance of the indicator for a given threshold. For each cell, financial margins are tested in this way on a grid of different thresholds with the goal of finding the "critical" threshold that optimizes the balance between missing a distressed household (type I error) and producing too many false alarms (type II errors), i.e. the threshold that minimizes the noise-to-signal ratio, defined as the ratio of type II errors to one

⁵⁴ Only in the high-end part of the property market there is anecdotal evidence of a larger use of "closed corporations" arrangements, which entail limited liability for debt and could give rise to strategic defaults.

⁵⁵ It must be noted that the estimation results of the logistic regression are only used to help with the selection of vulnerability indexes and will not be employed in the rest of the application. Despite consistency with other empirical findings, these results are indeed likely to suffer from endogeneity issues associated with data limitations.

minus type I errors. For each cell (i, j) , the computation of the optimal threshold is nested in an algorithm aimed at selecting the weights $\alpha_{i,j}$ and $\beta_{i,j}$ that maximize the capacity to correctly predict arrears, as measured by the area under the Receiver Operating Characteristic (ROC) curve (see Appendix II).^{56,57}

The average estimated distress thresholds by income deciles are reported in Table 3 (first row in bold). As expected, thresholds are significantly lower for the lowest deciles, which corrects for the underestimation of disposable income associated with the use of gross income. Households in the first to the fourth deciles, in particular, have negative distress thresholds, implying that households in these deciles can fulfill debt obligations also when their gross income is lower than the sum of basic living costs and debt payments because they are supported by high transfers and subsidies (that are not observed). On the other side of the spectrum, households in the last two deciles tend to be vulnerable at relatively high financial margins, which reflects the fact that disposable income is lower than gross income and that for some households the debt servicing payment is higher than observed since they have more than one outstanding loan.

Table 3. Namibia: Distress Thresholds by Income Deciles

	1	2	3	4	5	6	7	8	9	10
Calibration Using Signal Detection Approach	-4.02	-1.18	-0.40	-0.01	0.14	0.39	0.54	0.59	0.63	0.66
<i>Calibration Using Average Share of Distressed Households</i>	<i>-5.11</i>	<i>-1.34</i>	<i>-0.51</i>	<i>-0.13</i>	<i>0.07</i>	<i>0.37</i>	<i>0.50</i>	<i>0.57</i>	<i>0.60</i>	<i>0.63</i>

Source: 2009/10 NHIES, commercial banks' data, BON, and authors' computations.

For validation, we compare the in-sample predictive performance of this model with those of two alternative models:

1. The first alternative model uses the average share of distressed households in the overall sample to calibrate vulnerability thresholds for each income decile, as illustrated in footnote 37. The thresholds estimated with this method are also reported in Table 3 (second row in italic).
2. The second alternative model corresponds to the predominant approach in literature, employing a fixed threshold equal to zero for all households.

⁵⁶ We use the area under the curve (AUC) metric to measure the signaling performance of financial margins in each cell. The AUC corresponds to the area under the ROC curve and synthesizes the signaling power of the indicator. An AUC of 0.5 indicates that the indicator is not informative, as for any positive signal the probability that the event of interest will materialize in the forecast horizon is equal to the probability of a false alarm. This case corresponds to a ROC curve coincident with the 45-degree diagonal. The higher is the distance of the AUC from 0.5 and the closer to 1 the more informative is the indicator.

⁵⁷ The database includes loans with overdue payments in excess of 30 days only in 74 cells. In the remaining 56 cells the procedure selects the lowest financial margin in the cell as critical threshold.

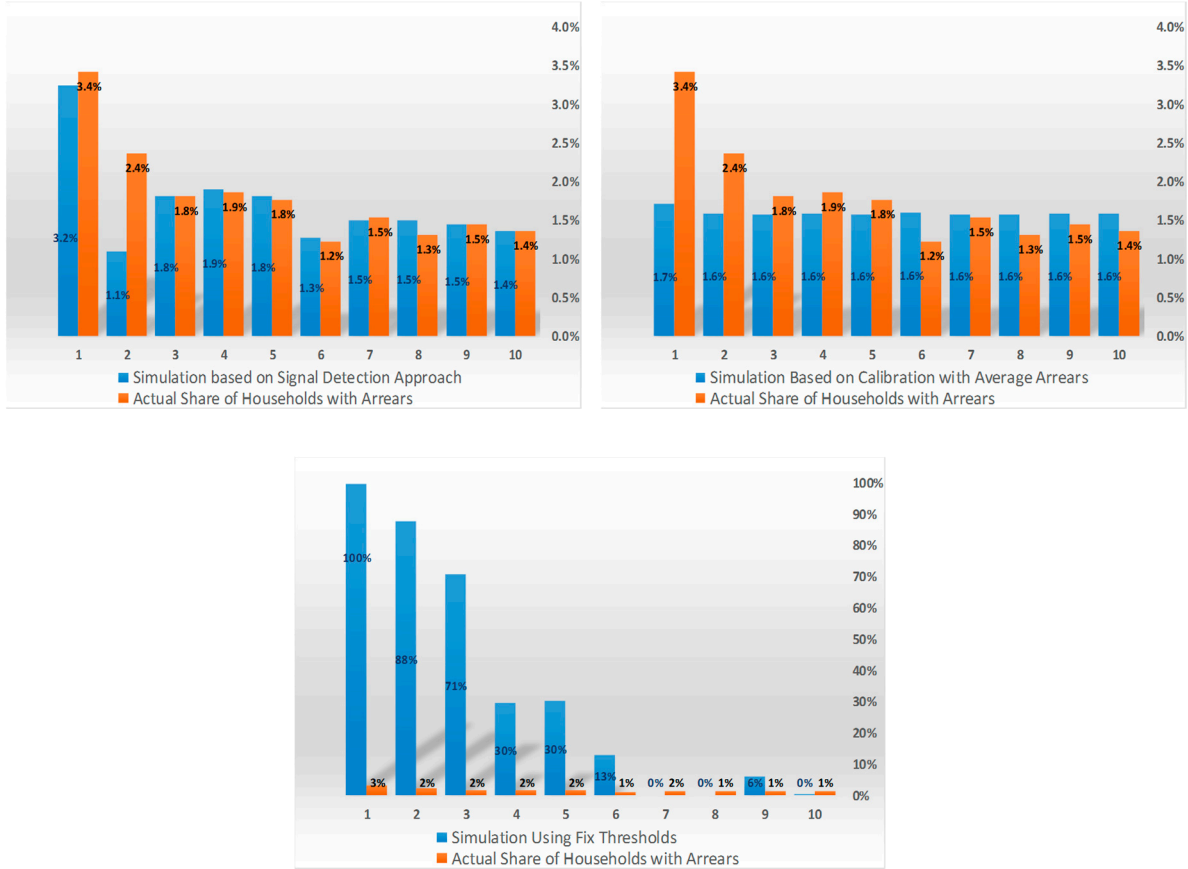
We use the thresholds of each estimation method to simulate the corresponding share of distressed households in any income decile. Figure 8 compares the simulated figures for each model (in blue) with the actual number of distressed households in the sample (in orange).

The model that calibrates thresholds using a signal detection approach outperforms the other models. The goodness of fit is very high in all deciles except for the second, where, despite calibration, financial margins are a noisy indicator. Excluding this decile, the mean absolute percentage error is 3 percent. This high in-sample predictive performance derives by construction as the algorithm selects the weights $\alpha_{i,j}$ and $\beta_{i,j}$ that maximize the area under the ROC curve. In the second decile, the procedure, aimed at minimizing the noise-to-signal ratio given the weights $\alpha_{i,j}$ and $\beta_{i,j}$, selects thresholds that are relatively high to limit the number of false alarms. An alternative method to select thresholds, aimed at minimizing the noise-to-signal ratio conditional to at least 2/3 of arrears are captured, would result in a large overestimation of arrears in this decile and we have thus rejected it.

The model calibrating thresholds with the average share of distressed households has a good performance in deciles where the share of distressed households resembles the population's share. Overall, the average mean absolute percentage error is 20 percent. While the model has a weaker performance compared to the method based on the signal detection approach, it remains a valid alternative when financial distress is uniformly distributed among households.

Finally, the method using a fixed threshold equal to zero has an extremely poor performance, particularly among the lowest deciles, because it does not correct for the sources of endogeneity.

**Figure 8. Namibia: Share of Household in Arrears
(In-Sample Predictive Performance Using Different Calibration Methods)**



Sources: Authors' Computation

C. STEP 3: STRESS TESTING

Before proceeding with the stress-test, the unified database is updated to end-2016 (last available data point for macro variables) by uprating the monetary variables in line with the development of country-level counterparties (Section III.A.3).

In the exercise, we perform one-period ahead analysis. Projections for the baseline scenario are derived as follows:

- Individual household income is projected to grow in line with non-mining GDP per capita (used to proxy nation-wide income), where non-mining GDP projections are derived from the April 2017 IMF WEO database and population growth is obtained with a linear projection model based on historical population estimates from the WB development indicators.
- Individual household debt is assumed to grow in line with non-mining GDP per capita (i.e. we assume that the average household DTI ratio remains constant). This is consistent with the fact that the credit to disposable income ratio has remained flat over the past two years (Bank of Namibia, 2017).
- Individual household expenditures are projected to grow in line with inflation projections, as derived from the April 2017 IMF WEO database.
- Lending rate increases are proxied by the Jibar forward rates (because of the peg with the South African rand).

The adverse scenarios simulate the impact of an increase in interest rate by 200bp and 300bp under different assumptions on nominal gross income growth. Shocks are assumed to be uniform across income deciles. Household indebtedness is projected to grow in line with income, while inflation projections are as in baseline in all the scenarios.

Financial margins in both baseline and adverse scenarios are recomputed to calculate how many households fall below the thresholds identified in the previous step.

The stress test shows that the share of distressed households would rise significantly under the 300bp shocks, ranging between 19.5 and 23 percent (Table 4), depending on the income growth assumptions.⁵⁸ The analysis shows also that the most vulnerable households are those in middle- and middle/upper-income deciles (Figure 9). Households in the lowest deciles are less impacted by lower income growth as they are heavily dependent on the generous social system of subsidies and transfers (not included in the income definition). Households in the

⁵⁸ This version of Namibia's household stress test uses a more negative (and plausible) scenario compared to the version in IMF (2016). The previous version assumed a constant level of household debt as of 2014, thus implying that the average household leverage ratio, measured by the DTI ratio, declined over time. This explains the larger share of distressed household in the current update.

highest decile, instead, are less vulnerable, despite higher indebtedness, because of larger buffers. It is worth noticing that, while results in the second decile appear consistent with bordering deciles, the share of distressed households in this income group might be higher than projected because vulnerability thresholds in the decile are set at a relatively high level to limit false alarms (see Section IV.B).

Based on the distribution of mortgage loans across income deciles (yellow lines in Figure 9), banks' arrears on mortgages are projected to increase up to 19.6 percent of total mortgage loans in the most adverse scenario.

It relevant to highlight that the accuracy of the vulnerability estimates under baseline and adverse scenarios relies on the assumption that there are no structural changes in the underlying relation between financial margins and household distress between 2014 and 2017 (for instance due to a change in the social benefits or tax rates), and thus that the distress thresholds validated with 2014 data are valid also for 2017.

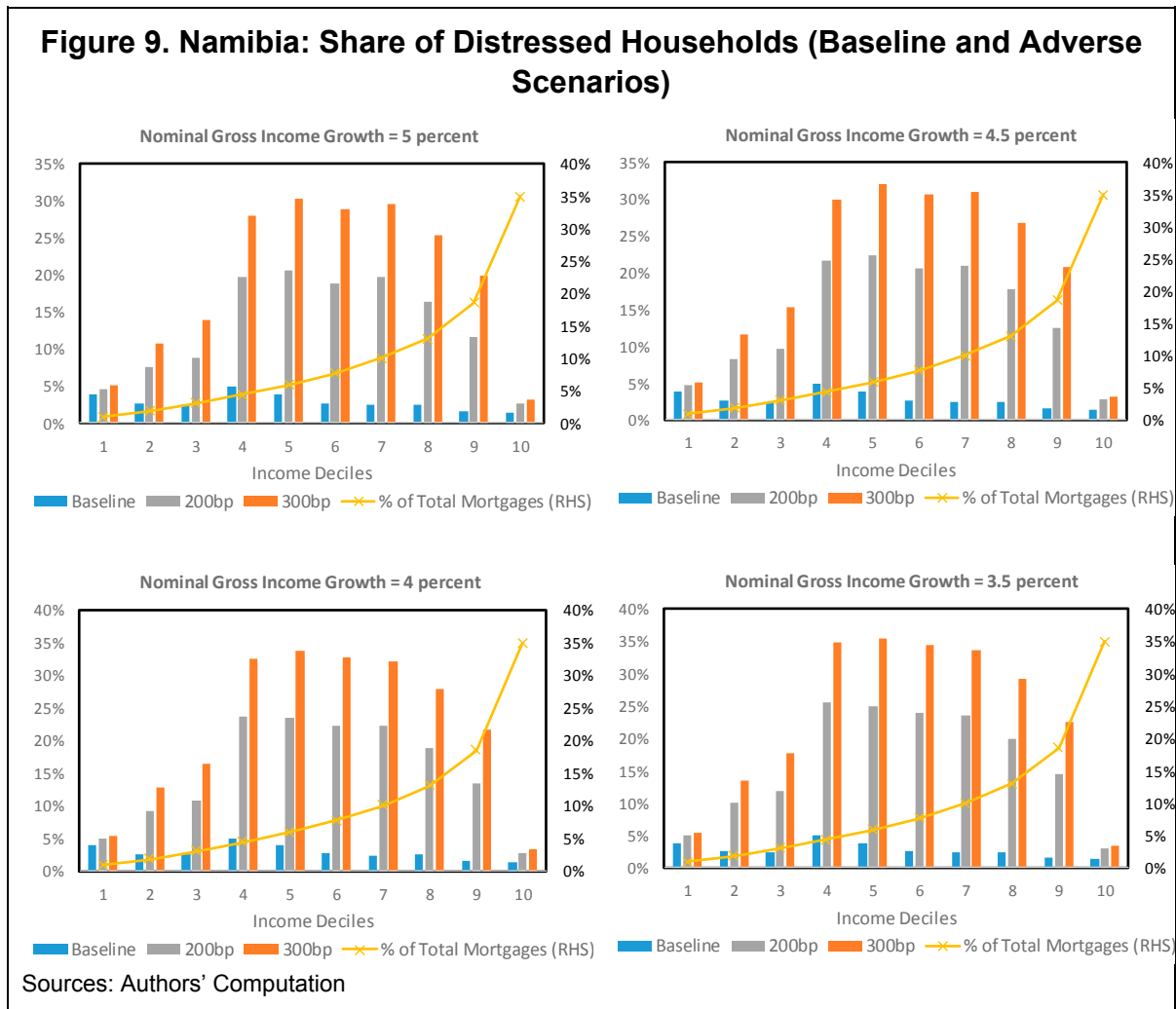
Table 4. Namibia: Share of Distressed Households (Baseline and Alternative Scenarios)

	Nominal Gross Income Growth Rate (Percent)				
	Baseline	Adverse Scenarios			
	5.2*	5.0	4.5	4.0	3.5
Interest Rate Increase (bps)					
None	2.8				
200		13.0	14.1	15.2	16.3
300		19.5	20.6	21.8	23.0

Source: Authors' Computations.

* Nominal Gross Income Growth Rate is proxied by the growth rate in non-mining GDP per capita.

Figure 9. Namibia: Share of Distressed Households (Baseline and Adverse Scenarios)



V. CONCLUSIONS

In this paper, we reviewed and integrated in a coherent fashion best practices emergent in the literature on household vulnerability analysis. We also proposed several analytical and estimation extensions aimed at improving the quality of estimates and allowing implementation in presence of data limitations. The result of this effort is a comprehensive framework, that has wide applicability to both advanced and developing economies.

The methodology described in the paper offers a helpful tool to support policy analysis and formulation. First, it may provide inputs for banks' stress testing, as it offers an alternative to macroeconomic credit risk models for the computation of household average probability of default (see, among the others, Holló and Papp, 2007, Ampudia et al., 2014, and Bilston et al., 2015). The approach allows capturing potential non-linear responses to shocks by taking into account the distributional aspects of the ability-to-pay of households. This addresses a key weakness of traditional macro stress testing frameworks, which are unable to capture the non-linear effects associated with default correlations (Ampudia et al., 2014), particularly when the country under analysis has never experienced a crisis.

Second, household vulnerability estimates can be used to calibrate macroprudential instruments. In most jurisdictions, the calibration of tools such as LTV and DSTI limits has been based on average historical observation of these ratios and qualitative assessment. This may lead to an incorrect assessment of the risks associated with the household sector, particularly if the country has never experienced a crisis, with the risk of either imposing unneeded credit restrictions on some household segments (type II error), or, on the opposite side of the spectrum, leaving systemic risk buildup undetected (type I error). A micro-based calibration, instead, may allow a more precise assessment and permits to consider both the benefits and costs associated with specific limits (Bańbuła et al., 2016).

Lastly, the framework may be used to inform macroeconomic policy decisions, as it allows to assess the potential impact of changes in policy variables on household financial positions and, consequently, on the economy. Recent empirical research has shown indeed that the distribution of income, assets and debt across households play a critical role in determining the economy's overall response to policy changes and shocks (see, among the others, Bertrand and Morse, 2013; Bricker et al., 2014; and Mian and Sufi, 2014a and 2014b; and Devlin-Foltz and Sabelhaus, 2015). The methodology can also be used to assess the impact of policy decisions and shocks on household welfare, including the effects on poverty and income distribution (Tiongson et al., 2010), which is of interest in developing countries.

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APPENDIX I — AGGREGATION METHODS FOR HOUSEHOLD MICRO-DATA SOURCES

When information on relevant variables belongs to different micro-data sources, data can be combined in a unified database using statistical methods, if the household identifier is not available in the different microdata sources.

Two matching options are possible:

- i. *Parametric methods.* This option is available when the databases to be matched include common information on households' characteristics (e.g. region of residence, household head age group, employment status, education level, etc.). In this case, information can be transferred from one database to the other based on regression estimates, where the common variables serve as the independent variables in the regression equation. For instance, if database 1 includes variable X and database 2 variable Y and both include information on household characteristics A , B , and C , then information on Y can be mapped into database 1 by using the coefficients of a regression of Y against A , B , and C estimated using data from database 2:

$$Y = \alpha \cdot A + \beta \cdot B + \gamma \cdot C + \text{residual}$$

This mapping method allows any household in database 1 to have a different value for variable Y , based on its characteristics. The validity of this imputation method would depend on how well the variable which is being imputed (Y) is explained by the variables (i.e. household characteristics) which are in common. This matching method has already been used in some empirical applications, for instance ECB (2014).

- ii. *Non-parametric methods.* This option can be used when the databases to be combined do not include information on households' characteristics or this is limited. In this case, it is possible to use any variable available in both databases and/or a monotonic relation between two (or more) variables belonging to the different databases as matching keys. The mapping of information would then be performed by using the empirical distributions of households along the matching keys. For instance, consider a situation where database 1 includes variable X and database 2 includes variables Y and Z and there is a theoretical (or empirical) relation between X and Y (for example, income and expenditure), such that:

$$Y = f(X) \quad \text{with } X \in [a, b]$$

where $f(\cdot)$ is a strictly monotone function.⁵⁹ In this case, the distribution of households over the support of Y is strictly related to the distribution of households over the support of X ⁶⁰ and it is possible to: i) compute the empirical distribution of households along X and Y , using data from databases 1 and 2, respectively, ii) partition the two distributions into an equal number of ordered subsets of observations (or cells), in a way that the relative frequency of each cell in the distribution of X is equal to the frequency of the

⁵⁹ It must be noted that, rather than a transformation of X , Y is likely to be a function of X plus an error term: $Y=f(X) + \varepsilon$.

⁶⁰ In terms of theoretical distributions, let X be a continuous random variable with generic probability density function $p_X(x)$ defined over the support $a < x < b$ and $Y=f(X)$, with $f(\cdot)$ strictly monotone with inverse $X=f^{-1}(Y)$, then the probability density function of Y is:

$$p_Y(y) = p_X(f^{-1}(y)) \cdot |f^{-1}'(y)|$$

defined over the support $f^{-1}(a) < y < f^{-1}(b)$.

corresponding cell in the distribution of Y ,⁶¹ and *iii*) map statistics of Y and Z of each cell of database 2 into the corresponding cell of database 1. With this matching method, all households in a cell of database 1 would have the same value of variable Y , corresponding to the selected statistic (mean, median, minimum, maximum or other summary statistics) value of variable Y in the corresponding cell of database 2. The more numerous are the matching keys and the finer is the partition for each of the matching keys, the larger would be the set of possible values for the variables mapped from a database to the other. An alternative to the discrete estimate is a smooth estimate of the distribution function of X and Y (e.g. through a kernel estimate). In this case, if the derived empirical cumulative distribution functions (edcf) are strictly monotone, it is possible to perform a one-to-one matching of values of X and Y (and Z) by using the inverse distribution function. In detail, let F_X and F_Y be the edcf for X and Y , respectively, then for each $p \in [0,1]$, $F_X^{-1}(p)$ matches $F_Y^{-1}(p)$. The validity of this matching method depends critically on the existence of a monotonic relation between X and Y .

⁶¹ For example, the partition could be based on quantiles of the empirical distributions.

APPENDIX II—ALGORITHM TO DETERMINE WEIGHTS AND OPTIMAL THRESHOLDS

Weights and optimal thresholds are determined in MATLAB using the following algorithm for each cell (i, j) of the merged database.

<i>for</i> $\alpha_{i,j} = 0:0.01:1$	
<i>for</i> $\beta_{i,j} = 0:0.01:1$	
$\text{for } \tau = \text{Min}(FM_{2014,i,j} \alpha_{i,j}, \beta_{i,j}) : \frac{\text{Max}(FM_{2014,i,j} \alpha_{i,j}, \beta_{i,j}) - \text{Min}(FM_{2014,i,j} \alpha_{i,j}, \beta_{i,j})}{100} : \text{Max}(FM_{2014,i,j} \alpha_{i,j}, \beta_{i,j})$ $nts_{\tau \alpha_{i,j}, \beta_{i,j}} = \frac{\text{type II error}_{\tau \alpha_{i,j}, \beta_{i,j}}}{1 - \text{type I error}_{\tau \alpha_{i,j}, \beta_{i,j}}};$ <i>end;</i>	<i>Computes the noise-to-signal ratio for 100 thresholds (τ) evenly spaced in the interval defined by the minimum and maximum observed financial margins in cell (i, j), given the weights $\alpha_{i,j}$ and $\beta_{i,j}$</i>
$AUC_{\alpha_{i,j}, \beta_{i,j}} = \text{trapez}(\text{false positive}_{\alpha_{i,j}, \beta_{i,j}}, \text{true positive}_{\alpha_{i,j}, \beta_{i,j}});$ <i>end;</i> <i>end;</i>	<i>Computes a vector of AUC for each value of $\alpha_{i,j}$ and $\beta_{i,j}$ via trapezoidal approximation</i>
$(\alpha_{i,j}^*, \beta_{i,j}^*) = \arg \max (AUC_{\alpha_{i,j}, \beta_{i,j}});$	<i>Select $\alpha_{i,j}$ and $\beta_{i,j}$ that maximize the AUC:</i>
$\tau_{i,j}^* = \arg \min \left(nts_{\tau \alpha_{i,j}^*, \beta_{i,j}^*} \right);$	<i>Select τ that minimizes nts given the optimal weights $(\alpha_{i,j}^*, \beta_{i,j}^*)$.</i>

APPENDIX III—HOW TO ADDRESS ENDOGENEITY ISSUES IN THE CONTEXT OF SOLVENCY

The “operational” definition of solvency introduced in Section III.A.1 requires that, at the end of the forecast period, the projected net-debt either stabilizes or declines or does not exceed a benchmark level considered “risky”. Based on the truncated intertemporal budget constraint, this solvency condition implies that the present discounted value of the truncated sum of income-expenditure balances over the projection period needs to be not lower than a certain benchmark. For instance, if solvency is associated with an end-period debt level that is not larger than the initial debt level, then the truncated intertemporal budget constraint implies that the present discounted value of the truncated sum of income-expenditure balances needs to be not lower than the initial debt level times one minus the relevant discount factor. This can be easily derived starting from the household debt accumulation formula:

$$ND_{h,t+1} = (1 + r_t) \cdot ND_{h,t} - Y_{h,t} + LC_{h,t} \quad (1)$$

where $ND_{h,t}$ and r_t are, respectively, household h 's net debt and the interest rate at time t .⁶² Solving forward recursively gives:

$$ND_{h,t} = \sum_{i=0}^{T-1} \prod_{j=0}^i \frac{(Y_{h,t+i} - LC_{h,t+i})}{(1 + r_{t+j})} + \prod_{j=0}^{T-1} \frac{1}{(1 + r_{t+j})} \cdot ND_{h,t+T}$$

Imposing the terminal condition that net debt at the at the end of the projection period needs to be not higher than the initial level of net debt (i.e. $ND_{h,t+T} \leq ND_{h,t}$) leads to:

$$ND_{h,t} \cdot \left(1 - \prod_{j=0}^{T-1} \frac{1}{(1 + r_{t+j})} \right) \leq \sum_{i=0}^{T-1} \prod_{j=0}^i \frac{(Y_{h,t+i} - LC_{h,t+i})}{(1 + r_{t+j})} \quad (2)$$

Were all variables in (2) precisely measured at time t , condition (2) could be easily tested empirically by using the initial level of net debt at time t and projections of income, living costs and interest rates. However, in presence of data limitations at time t , calibration might be needed. For instance, if disposable income is not available and replaced with gross income (both at time t and in the projections), condition (2) would not hold anymore and the condition that ensures that the end-period level of debt is not larger than the initial debt level needs to be found through calibration.

In the example above (where disposable income is replaced by gross income), calibration would entails estimating how debt accumulation relates to gross income in each household subset s where the sources of unobserved heterogeneity are expected to be constant. This implies using historical data to estimate α_s in the following “modified debt accumulation formula”:

$$ND_{h,t+1} = (1 + r_t) \cdot ND_{h,t} - \alpha_s \cdot Y_{h,t} + LC_{h,t} + \varepsilon_{h,t+1} \quad (3)$$

where $\varepsilon_{h,t+1}$ is an error term with zero mean.

Plugging the estimated parameter $\hat{\alpha}_s$ into (3), solving forward recursively, and imposing the terminal condition that the end-period level of debt is not larger than the initial debt level would lead to the following modified solvency condition for households in subset s :

$$ND_{h,t} \cdot \left(1 - \prod_{j=0}^{T-1} \frac{1}{(1 + r_{t+j})} \right) \leq \sum_{i=0}^{T-1} \prod_{j=0}^i \frac{(\hat{\alpha}_s \cdot Y_{h,t+i} - LC_{h,t+i})}{(1 + r_{t+j})} \quad (4)$$

This can be tested empirically by using the initial level of net debt at time t and projections of *gross* income, living costs and interest rates.

⁶² To simplify notation, we assume that the interest rate on assets and liabilities is the same.

APPENDIX IV—APPLICATIONS OF THE FRAMEWORK

Country	Data Sources	Extrapolation Method	Merging of Databases Method	Data Quality	Estimation Method	Thresholds	Reference
Namibia	NHIES and sample of mortgage loans	Uprating	Non-Parametric Approach	Low	Binary non-parametric with one indicators	Calibrated Through Signaling Detection Approach	IMF, 2016
Finland	HFCS and Finland Survey on Income and living conditions	Uprating	Non-Parametric Approach	High	Binary non-parametric with two indicators	Fixed	IMF, 2017
Luxemburg	HFCS	Uprating	N.A. (Only one microdata source used)	High	Binary non-parametric with two indicators	Fixed	IMF, 2017b