THE NATURAL RATE OF INTEREST: DRIVERS AND IMPLICATIONS FOR POLICY

ONLINE ANNEXES

Online Annexes: Data Sources, Sample Coverage, Variable Definitions, Methods and Chapter Figure Notes

Online Annex 2.1–2.4 to Chapter 2 of the April 2023 World Economic Outlook lay out the data sources, sample coverage, variable definitions and methodologies used in the main chapter. These online annexes follow the structure of the Chapter in describing the measurement of natural rates using the Kalman Filter approach first and then the structural macroeconomic modelling approach, before moving on to the quantification of fiscal implications.

Online Annex 2.1. Data Sources and Transformations for Figures 2.1 and 2.2

Figure 2.1. Panel 1: The real interest rates are computed as the difference between the US treasury rates for each maturity and the Cleveland Fed measure of inflation expectations over the same period. Data is from The Federal Reserve Economic Data (FRED). FRED mnemonics DSG[X] and EXPINF[X]YR where [X] stands for the respective maturity.

Figure 2.1. Panel 2: Real interest rates are the difference between the 3-month treasury rates and the realized annual inflation over the same horizon in each country. Japan's 3-month interbank rates are spliced with certificates of deposit from 1979 to 2002 to achieve a longer series. The realized inflation rate is calculated as the average of the annual inflation in the next three months.

$$real\ interest\ rates = i_t - \frac{1}{3}\left(\left(\frac{p_{t+1}}{p_{t-11}} - 1\right) + \left(\frac{p_{t+2}}{p_{t-10}} - 1\right) + \left(\frac{p_{t+3}}{p_{t-9}} - 1\right)\right) \times 100$$

where i_t is the nominal interest rates and p_t is the monthly CPI index. Data are also from the Federal Reserve Economic Data (FRED) and the following FRED mnemonics are used in the calculations:

FRED mnemonic	Series Description
IR3TIB01[XX]M156N	3-Month or 90-day Rates and Yields: Interbank Rates for [XX] where [XX] stands for the respective country's alphabetic ISO2-code, Percent, Monthly, Not Seasonally Adjusted
IR3TIB01JPM156N	3-Month or 90-day Rates and Yields: Interbank Rates for Japan, Percent, Monthly, Not Seasonally Adjusted
IR3TCD01JPM156N	3-Month or 90-day Rates and Yields: Certificates of Deposit for Japan, Percent, Monthly, Not Seasonally Adjusted

INDIR3TIB01STM	Interest Rates: 3-month or 90-day rates and yields: Interbank rates: Total for India, Percent, Monthly, Not Seasonally Adjusted
CPALTT01[XX]M659N	Consumer Price Index: Total All Items for [XX] where [XX] stands for the respective country's alphabetic ISO2-code, Growth rate same period previous year, Monthly, Not Seasonally Adjusted

Figure 2.2. uses real long term bond yield (WEO mnemonic: FIGB_R) from the WEO database. The sample includes 34 advanced economies (AUS, AUT, BEL, CAN, CHE, CYP, CZE, DEU, DNK, ESP, FIN, FRA, GBR, GRC, HKG, IRL, ISL, ISR, ITA, JPN, KOR, LUX, MAC, MLT, NLD, NOR, NZL, PRT, SGP, SVK, SVN, SWE, TWN, and USA) and 25 emerging market and developing economies (ARG, BIH, BLR, BRA, CHL, CHN, GHA, HRV, HUN, IDN, IND, MUS, MYS, NAM, PAN, PHL, POL, RUS, SLE, SYC, THA, TUR, TZA, UKR, and ZAF) and aggregated using market-exchange-rate-based GDP weight (WEO mnemonics: GDPWGT and NGDPD) as below:

$$GDPWGT_t = \frac{1}{3}(NGDPD_{t-1} + NGDPD_{t-2} + NGDPD_{t-3})$$

The maturity of the bonds is greater than 1 year.

Online Annex 2.2. Measuring the real natural rate

In measuring the real neutral rate, the chapter conducts three different empirical exercises based on a Kalman filter estimation of the natural rate inspired by Laubach and Williams (2003): (i) a country-by-country estimate of the natural rate, (ii) a real-time estimate of natural rates over the pandemic, that contrasts filtered and smoothed estimates of the natural rate to illustrate the real-time uncertainty monetary policy makers faced, and (iii) a two-region simultaneous estimation for the US and the rest-of-the world that quantifies international dependencies.

2.2.1. Country-by-Country Estimates of the real natural rate

The country-by-country estimates view data through the lens of a Kalman filter that is connected to a simple New Keynesian model of aggregate demand/investment-savings (AD/IS) and aggregate supply/Phillips Curve (AS/PC). The IS-curve links the percent deviation of real output from potential – the output gap $\widetilde{y_t}$ – to the corresponding policy stance measure, the real rate gap $(r_t - r_t^\star)$, in the IS-curve (A.2.2.1).

$$\widetilde{y_t} = \alpha_1 \widetilde{y_{t-1}} + \alpha_2 \widetilde{y_{t-2}} + \frac{a_r}{2} \sum_{i=1}^{2} (r_{t-i} - r_{t-i}^\star) + \varepsilon_{\tilde{y},t} \qquad \text{(A.2.2.1)}$$

where $\widetilde{y_t} = 100 * (y_t - y_t^\star)$ and y_t and $y_t^\star$ are the logarithms of real GDP and the unobserved potential output. r_t is the real short-term interest rate and $r_t^\star$ the real natural rate consistent with output being at potential. The PC is a standard relation between current inflation π_t and distributed lags of past inflation π_{t-1} , and responses to the output gap $\widetilde{y_{t-1}}$ as in (A.2.2.2).

$$\pi_t = B(L)\pi_{t-1} + + b_y \widetilde{y_{t-1}} + \varepsilon_{\pi,t} \qquad \text{(A.2.2.2)}$$

The law of motion for the natural rate of interest $r_t^\star$ is driven by trend growth g_t and other determinants z_t:[1]

$$r_t^\star = g_t + z_t$$

The "other" component z_t is similarly assumed to follow a random walk process.

$$z_t = z_{t-1} + \varepsilon_{z,t}$$

Log potential output $y_t^\star$ is a random walk with a stochastic drift g_t that is equally assumed to be a random walk.

$$y_t^\star = y_{t-1}^\star + g_{t-1} + \varepsilon_{y^\star,t}$$

$$g_t = g_{t-1} + \varepsilon_{g,t}$$

This system is cast as a Kalman filter of measurement and transition equations where the unobserved states includes both potential output and the real natural rate of interest. The observables are real output, inflation and the lagged ex-ante real interest rate and past output and inflation as predetermined variables. The output gap and thus real rate gap and natural rate, growth trend and stochastic growth drift as well as the innovation variances are estimated. The appendix of Wynne and Zhang (2018) is an excellent further reference for details on the empirical implementation.

The model is estimated for six advanced economies: Canada, France, Germany, Japan, the United Kingdom and the United States. The observable inputs are log real GDP, inflation and the ex ante real policy rate, which are computed as the expectation of average inflation over the four quarters ahead from a univariate AR(3) model of inflation estimated over the 40 quarters prior to the data at which expectations are being formed. Specifically, Online Annex Table 2.2.1 shows the data sources for the exercise.

Online Annex Table 2.2.1. Country-by-Country Kalman Filter Data Sources

Country	Concept	Data
Canada	Real GDP	St Louis FRED mnemonic: NAEXKP01CAQ189S
Canada	Interest rates	St Louis FRED mnemonic: IRSTCB01CAM156N
Canada	Inflation	St Louis FRED mnemonic: CANCPICORMINMEI
France	Real GDP	ECB SDW mnemonic: Q.Y.FR.W2.S1.S1.B.B1GQ._Z._Z._Z.EUR.LR.N

[1] See the estimation in Laubach and Williams (2003) and the discussion thereof in Holston, Laubach and Williams (2017) of the (unit) coefficient on g_t.

France	Interest rates	St Louis FRED mnemonic: Q.U2.EUR.RT.MM.EURIBOR3MD_.HSTA St Louis FRED mnemonic: IRSTCI01FRM156N
France	Inflation	St Louis FRED mnemonic: FRACPICORMINMEI ECB SDW mnemonic: M.FR.N.XEF000.4.INX
Germany	Real GDP	St Louis FRED mnemonic: NAEXKP01DEQ661S
Germany	Interest rates	St Louis FRED mnemonic: IRSTCI01DEM156N
Germany	Inflation	St Louis FRED mnemonic: DEUCPICORMINMEI
Japan	Real GDP	Haver mnemonic: S158NGPC@G10 St Louis FRED mnemonic: RGDPNAJPA666NRUG
Japan	Interest rates	St Louis FRED mnemonic: IRSTCB01JPM156N
Japan	Inflation	St Louis FRED mnemonic: JPNCPICORMINMEI
UK	Real GDP	Office of National Statistics mnemonic: ABMI/UKEA
UK	Interest rates	St Louis FRED mnemonic: BOERUKM
UK	Inflation	OECD MEI mnemonic: CPGRLE01
USA	Real GDP	St Louis FRED mnemonic: GDPC1
USA	Interest rates	St Louis FRED mnemonic: FEDFUNDS
USA	Inflation	St Louis FRED mnemonic: PCEPILFE

Online Annex Table 2.2.2. Country-by-Country Kalman Filter Sample Periods

Country	Sample Start Date	Sample End Date
Canada	1962 Q4	2019 Q4
France	1971 Q4	2019 Q4
Germany	1971 Q4	2019 Q4
Japan	1961 Q4	2019 Q4
UK	1956 Q4	2019 Q4
USA	1960 Q4	2019 Q4

2.2.2. Two-Region Estimates

The open economy, two region version of the Laubach and Williams (2003) setup, first implemented developed by Wynne and Zhang (2018), brings the New Keynesian setting of aggregate demand and the Phillips curve to a two-region setting. Specifically, there's a home (A.2.2.3) and a foreign aggregate demand (A.2.2.3) relations as earlier in the country-by-country estimates:

$$\widetilde{y_t^h} = \alpha_1^h \widetilde{y_{t-1}^h} + \alpha_2^h \widetilde{y_{t-2}^h} + \frac{a_r^h}{2}\sum_{i=1}^{2}\left(r_{t-i}^h - r_{t-i}^{h,*}\right) + \varepsilon^h{}_{\tilde{y},t} \qquad \text{(A.2.2.3)}$$

$$\widetilde{y_t^f} = \alpha_1^f \widetilde{y_{t-1}^f} + \alpha_2^f \widetilde{y_{t-2}^f} + \frac{a_r^f}{2}\sum_{i=1}^{2}\left(r_{t-i}^f - r_{t-i}^{f,\star}\right) + \varepsilon^f{}_{\tilde{y},t} \qquad \text{(A.2.2.4)}$$

where h indexes the home economy and f indexed the foreign economy. Similarly, the Phillips Curve relationship (A.2.2.5 and A.2.2.6) is specified as

$$\pi_t^h = B^h(L)\,\pi_t^h + b_y^h\, \widetilde{y_{t-1}^h} + b_{\pi,I}^h\left(\pi_t^{I,h} - \pi_t^h\right) + b_{\pi,o}^h\left(\pi_{t-1}^{O,h} - \pi_{t-1}^h\right) + \varepsilon_{\pi,t}^h \qquad \text{(A.2.2.5)}$$

$$\pi_t^f = B^f(L)\,\pi_t^f + b_y^f\, \widetilde{y_{t-1}^f} + b_{\pi,I}^f\left(\pi_t^{I,f} - \pi_t^f\right) + b_{\pi,o}^f\left(\pi_{t-1}^{O,f} - \pi_{t-1}^f\right) + \varepsilon_{\pi,t}^f \qquad \text{(A.2.2.6)}$$

because of the open economy nature of the two-region model, the Phillips curve specification also includes import prices $\pi_t^{I,i}$ and oil prices inflation $\pi_{t-1}^{O,i}$ following Wynne and Zhang (2018) to account for the open economy aspects of inflation targeting subject to external shocks that drive both home and foreign inflation.

The relations for the evolution of the real natural rate of interest now allow for both foreign and domestic stochastic growth:

$$r_t^{h,*} = c_h^h g_t^h + c_f^h g_t^f + z_t^h$$

$$r_t^{f,*} = c_h^f g_t^h + c_f^f g_t^f + z_t^f$$

$$g_t^h = g_{t-1}^h + \varepsilon_t^h$$

$$g_t^f = g_{t-1}^f + \varepsilon_t^f$$

and the evolution of log natural output for the domestic and foreign economy is as before

$$y_t^{h,\star} = y_{t-1}^{h,\star} + g_{t-1}^h + \varepsilon_{y^\star,t}^h$$

$$y_t^{f,\star} = y_{t-1}^{f,\star} + g_{t-1}^f + \varepsilon_{y^\star,t}^f$$

The data used for estimation of the "home" economy are US data for log real GDP, core inflation, oil price inflation and relative import price inflation as well as real rates with ex ante inflation expectations computed as in the single country case.

The data construction follows Wynne and Zhang (2018b). The countries that comprise the rest-of-the world are Australia, Austria, Belgium, Brazil, Canada, China, Finland, France, Germany, Greece, India, Ireland, Italy, Japan, Korea, Netherlands, Norway, Portugal, Russia, South Africa, Spain, Sweden, Switzerland, United Kingdom with Brazil, Russia, India, China and South Africa (BRICS) representing the Emerging and Developing Economies (EMDEs). The sample starts in 1960 Q1, but coverage, especially for BRICS, is spottier in the early part of the sample. Overall, 90 percent of world GDP measured at PPP (IMF) is represented. At the end of the sample 2022 the subset of countries considered account for about 70 percent of IMF recorded world PPP GDP.

Online Annex Table 2.2.3. Two region Kalman Filter Data Sources

Country	Concept	Data
Australia, Austria, Belgium, Brazil, Canada, Finland, France, Germany, Greece, India, Ireland, Italy, Japan, Korea, Netherlands, Norway, Portugal, Russia, South Africa, Spain, Sweden, Switzerland, United Kingdom, United States	Real GDP	OCED National Quarterly Accounts databased mnemonic: [XYZ].B1_GE.VPVOBARSA.Q where [XYZ] stands for the following ISO3-country codes AUT, CAN, FRA, DEU, ITA, JPN, GBR, USA, AUS, BEL, FIN, GRC, IRL, NLD, NOR, PRT, KOR, ESP, SWE, CHE, BRA, IND, ZAF, RUS
China	Real GDP	For China, China WEO quarterly data is used is spliced and reindex to the purchasing power adjusted OECD NQA data.
Australia, Austria, Belgium, Canada, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States	Core Inflation	CED MEI database with mnemonics [XYZ].CPGRLE01.IXOB.Q for [XYZ] as ISO3-codes AUT, CAN, FRA, DEU, ITA, JPN, GBR, USA, AUS, BEL, FIN, GRC, IRL, NLD, NOR, PRT, KOR, ESP, SWE, and CHE.
All countries	Interest rates	For interest rates, we reply on Haver Analytics with a mixture of national sources, the IMF's International Financial Statistics dataset such the longest possible interest rate data series of the constituent countries can be spliced together with Haver mnemonics: FFED@DAILY, N023RTAR@G10, N112RTAR@G10, N193RTAR@G10, C134IM@IFS, C146IC@IFS, C158IM@IFS, C542IFC@IFS, C136IC@IFS,

		B111GDPC@DAILY, C122IC@IFS, C124IM@IFS, C172IFC@IFS, C174IC@IFS, C178IC@IFS, C138IC@IFS, C138FRIO@IFS, C142IC@IFS, C182IC@IFS, C184IC@IFS, that are spliced together to obtain the longest possible unbalanced panel of interest rates. Three-digit numbers here refer to countries IFS-codes. These rates are complemented by interest rates from the OECD Main Economic Indicators (MEI) with mnemonics 156IRSTCB01ST.Q (Canada), 132IRSTCI01ST.Q (France), 144IRSTCI01ST.Q (Sweden), 142IRSTCI01ST.Q (Norway), 138IR3TIB01ST.Q (Netherlands), 223IRSTFR01ST.Q (Brazil), 922IRSTCB01ST.Q (Russia).

Online Annex 2.3. Drivers of the natural rate in the structural model

2.3.1. Model's main features (Platzer and Peruffo, 2022)

An adapted version of the model presented in Platzer and Peruffo (PP, 2022) is used to quantify the contribution of various drivers to the change in the natural rate and simulate its future path. The model's main features include:

Demographic forces are modelled via an overlapping generations structure, which permits the model to match empirical population-age pyramids and, crucially, their changes over time. The different generations introduce an important life-cycle aspect to the model, allowing intergenerational income and wealth inequality: individuals work while young and save for their retirement. Once retired, households receive a social security transfer and rely on their savings to finance their consumption as well as out-of-pocket health expenditure shocks (e.g. long-run care). There is also *income inequality* within generations, and because the marginal propensity to consume varies across the income distribution, this leads to aggregate effects from changes in inequality. *Aggregate productivity* evolves according to an exogenous, deterministic process, chosen to match the data. Likewise, the inclusion of *monopolistic competition* introduces a mark-up that is varied exogenously to reflect changing market power. The *public sector* also plays an important role in the model: it issues debt and can thereby influence the balance of desired savings supply and demand – and thus natural interest rates. The government also runs a social security system, provides transfers to households, and finances its operation with various taxes. The PP model captures features of an open economy model in the sense that exogenous changes in net foreign asset position (NIIP), for example driven by a change in foreign demand for safe assets, affect the domestic natural rate. A drop in the NIIP, for example, lowers domestic natural rates as additional supply of foreign savings raises the total supply of savings available for domestic investment.

The adjusted PP model is calibrated separately to represent eight major economies: the United States, Japan, Germany, the UK, France, China, India, and Brazil. These are the five largest advanced and the three largest emerging and developing economies, covering some 70 percent of global GDP. For each country the model matches demographics, the age-earnings profile, the share of income going to the top ten percent, productivity trends, the retirement age, size of the pension system, labor share, government debt, government consumption, and the net foreign asset position.

2.3.2. Further details and important mechanisms

Demographic dynamics

The model features 74 overlapping generations. Households enter the model at biological age 26 and live at most until age 99. Households face uncertainty about the length of their life. In every period, new individuals enter the model at the youngest age ("newborns"), according to a fertility rate $\tilde{n}_t$. The growth rate of total population N_t is given by n_t such that $N_{t+1} = (1 + n_t)N_t$, and is determined by both the fertility rate and survival probabilities. Demographic change can be caused by a change in fertility, a change in survival probabilities, or both, where these values can also reflect migration flows.

Households

The individual (or agent) in the model is a household. Mortality rates and other household-specific variables are not gender specific. Agents in this economy move through two stages of life: work and retirement. Working individuals exogenously supply their labor to firms in exchange for wages. There is an exogenous retirement age. Retired individuals do not work, and they obtain earnings from their savings and from a social security system. There are two assets in this economy. Households can save by holding government bonds and capital, earning a return r. A no-arbitrage condition ensures that net return on both is the same.[2] Households face a no-borrowing constraint and can also receive dividend income. Our frictionless model implies that output is always at potential, meaning that the return on government bonds coincides with the natural rate in this economy.

[2] We will amend this assumption in an extension of the model, see subsection "convenience yield" below.

Labor productivity is the product of three terms: permanent productivity, a transitory productivity shock, and an age-dependent productivity term. Permanent productivity can take on a high or a low value, and the assignment of the type to a household does not change over its lifetime. We can interpret this term as differences in ability or (exogenous) educational attainment. Transitory productivity follows an autoregressive process of order one and introduces idiosyncratic income risk into the model. Intra-generational inequality in the model can increase due to larger differences in the permanent productivity term or a larger variance of the transitory productivity process. The age-dependent term of productivity is hump shaped in age and accounts for the fact that real wage profiles are hump shaped in the data.

Preferences

Consumption preferences are of the addilog form (Houthakker, 1960, Straub, 2019). The elasticity of intertemporal substitution is age-dependent and assumed to decline in age. As a result, households with larger permanent productivity, and consequently income, have higher saving rates. We choose this utility function in order to be able to match empirically estimated elasticities of consumption out of permanent income. This introduces the feature, in line with empirical evidence (by Carroll, 2000, Straub, 2019) that richer households tend to have higher saving rates and explains why inequality matters for the natural interest rate in our model.

Production

The production function is of the Cobb-Douglas form, with labor augmenting technological progress. The production side features monopolistic competition that gives rise to a markup.

Government

The government has four roles in our model economy. First, public debt issuance, by directly impacting the demand for savings, affects the equilibrium interest rate; more debt issuance tends to increase the natural rate. Second, it establishes a social security system; the more generous the social security system the less need for individual saving and the higher the natural rate of interest. Third, there is exogenous government consumption G, which only enters the model as a resource cost, the more of it the, the higher the natural rate. Finally, it runs a system of taxes and transfers. We include a progressive income tax system following Heathcote et al (2017), consumption tax, capital income tax, and profit tax. The more progressive the tax system, the higher the natural rate of interest since the tax burden disproportionally falls on households with high savings rates. Transfers are a means-tested transfer that ensures a consumption floor in old age.

Capital Flows

In order to assess the potential impact of capital flows on the natural rate within our framework, we introduce an exogenous stock of net savings in the economy. This term is meant to capture accumulated net flows of capital into an economy and is calibrated to match the net foreign asset position of a country (NIIP). The natural rate in the model balances the supply and demand of savings. A change in the net foreign asset position can thus have a direct effect on interest rates. The market clearing condition for the stock of assets/savings is:

$$\mathrm{K} + \mathrm{B} = \sum_i a_i + A^* \quad \text{(A.2.3.1)}$$

Here, K denotes domestic capital used in production, B denotes borrowing by the government, a_i denotes savings supply by household i, so that the sum term stands for aggregate savings from the household sector. A^* denotes the net savings stock owned by foreigners, the NIIP.

Convenience Yield

To simulate the effect of a decline in the willingness to hold – safe and liquid – government bonds, we introduce a wedge between the return on government bonds and the net return on capital. We adjust the no-arbitrage condition for assets to:

$$1 + \mathrm{r} = 1 + r^k(1 - \omega) - \delta \quad \text{(A.2.3.2)}$$

Here, ω can be seen as a tax on the gross return on capital, and δ is the depreciation rate. This will have the effect that in equilibrium, the pre-wedge net return on capital, $r^k - \delta$, will be higher than the return on a government bond:

$$(r^k - \delta) - \mathrm{r} = \omega r^k \equiv \text{ wedge} \quad \text{(A.2.3.3)}$$

We rebate the proceeds from introducing the wedge back to households as a lump-sum transfer in proportion to their capital holdings. On a transition path, we adjust ω to hold the wedge at a desired level.[3]

Online Annex Table 2.3.1 depicts the drivers of the natural rate featured in the model and their respective effect on demand and supply of savings, and the natural rate. *Higher TFP growth* leads to more investment in physical capital by firms, increasing demand for savings. At the same time, higher TFP growth implies larger expected wage growth for workers. Since workers expect to make more money in the future, they need to save less today to smooth their overall consumption path. Higher demand, and lower supply implies a higher interest rate in equilibrium. *Demographic change* can take the form of an increase in life expectancy. A higher life expectancy, at a constant retirement age, increase the expected duration in retirement and induces households to save more while they are still in the labor force. Another implication of demographic change is a shift of population mass to older age, or population aging. Savings and effective labor supply profiles over the lifecycle are non-monotonous, so the impact of this change on the supply and demand of savings could be positive or negative in theory. Lastly, a lower population growth rate implies a lower output growth rate as well, having a negative effect

[3] Returns on capital can exceed the return on government bonds for several reasons: risk and liquidity premia, real intermediation costs, and regulatory requirements in the banking system. Our modeling approach is thought to capture this wedge in reduced-form, following Mehrotra (2017).

on investment and consequently savings demand. The implied impact on the natural rate coming from demographic change is the combination of all these effects and therefore ambiguous. An *increase in inequality* in our model implies that a larger share of output goes to high-saving households, increasing the supply of savings and depressing the natural rate. An expansion of *public debt* directly increases the demand for savings, raising natural rates. *Government consumption* has to be financed by issuance of public debt or taxation. Our assumption is that all the burden of adjustment ultimately falls on the latter, in the form of higher income taxes. This will depress the supply of savings and increase the natural rate. A rise in *market power* depresses production and demand for savings. At the same time, profits disproportionately go to the elderly population, reducing the supply of savings. The sign of the change in the natural rate is therefore ambiguous. *Net international capital flows* increase the available savings in the domestic economy, lowering the natural rate.

Online Annex Table 2.3.1

Increase in	Demand for Savings	Supply of Savings	R*
TFP growth	+	-	+
Life expectancy		+	-
Population aging	+/-	+/-	+/-
Inequality		+	-
Public debt	+		+
Government consumption		-	+
Market power	-	-	+/-
Net international capital flows		+	-

2.3.3. Calibration

We can distinguish between two types of parameters. Type I parameters are calibrated by relying on estimates from the literature or setting them equal to an empirical counterpart like for example, population distribution, growth rate or mortality rates, government spending or public debt ratios. Parameters for the earnings and medical-expense processes are unobserved, structural parameters for which we rely on estimates from the literature. Type II parameters are estimated by minimizing the distance between appropriate moments from the data and corresponding quantities in the model.

We match the population distribution (share of population in each age group) from UN data. The population growth rate refers to the growth rate of 26-to-99-years old. Since UN data are total-head counts, this means our data implicitly includes net migration flows. Migrants are identical in all regards to domestic population, including their asset holdings. Mortality rates are taken from UN data as well. In the steady state, we match the population-age distribution of the respective year and the population growth rate. The discrepancy between steady state mortality rates and the change in size of population groups of consecutive ages can be interpreted as steady state migration flows. The parameters that are country specific, together with the sources, are listed in Online Annex Table 2.3.2. Relative to PP (2022), we implement the following changes: we match the population distribution and mortality rates for each point in time, including the steady state years. As an additional target in the calibration, we add the semi-elasticity of savings supply to a change in the interest rate. This should be interpreted as a long run elasticity. We add the median coefficient of risk aversion to the calibration and target a semi-elasticity of savings supply of 15, following sources cited in Auclert et al (2021). For TFP growth, we calculate the Solow residual as described below in section 2.3.4. We do not include a bequest motive, leaving all bequests to be accidental. The distribution of bequests is according to the bequest distribution in SCF. The distribution of profits is still from Piketty et al (2018), but updated relative to PP (2022). Labor supply is inelastic, and we hold out-of-pocket health expenditures constant over time. Some parameters are different (depreciation rate $\delta = 0.5$, capital income tax $\tau_k = 0.3$), and we choose a more conservative target for the elasticity of consumption out of permanent income, $\phi_{PI} = 0.8$, deviating from the baseline in Straub (2019) of 0.7.[4] When we change the difference between permanent productivity terms, we hold the average constant. For the inequality driver, we only change permanent productivity, and hold the variance of the innovation in the transitory productivity term constant. Since data for the top 10 percent labor income share is not available for all countries, we use the top 10 percent income share as a proxy of the former. Finally, some of the parameters and targeted data moments are from a different source in order to provide consistency with the seven additional countries, other than the U.S., included in the chapter.

2.3.4. Simulations

Figure 2.7: baseline simulation

We run two main experiments in the chapter. The *first* is a comparison between hypothetical steady states. Here we compare a 2015-2019 steady-state with a 1975-1979 steady-state (Figure 2.7 in the main text).[5] Type I and II parameters are calibrated in reference to the period 2015-2019. When simulating the 1975-1979 steady-state, we only change the parameter values that correspond to our candidate drivers of the decline of the natural rate.[6] We then solve again for

[4] We are not aware of many estimates for this elasticity. Fagereng et al (2019) find limited evidence for savings rates that differ by income, using Norwegian data.

[5] The 1970s are a common reference point in the theoretical literature on the natural rate (Gagnon, Johannsen, Lopez-Salido 2021, Eggertsson, Mehrotra, Robbins, 2019, Rachel and Summers, 2019) and coincides with the approximate timing of important macroeconomic developments in the United States: entrance of the baby boom generation into the labor force, a slowdown in productivity growth, a rise in income inequality, and others.

[6] The parameters are the following (with corresponding driver in parenthesis): population growth, mortality rates, and population distribution by age (Life expectancy, Population aging), growth rate of TFP (TFP growth), difference in permanent productivity between high and low type

the equilibrium of the model, which will give us the model-implied value for the natural rate in the 1975-1979 period. To calculate the contribution of an individual driver, we only change the corresponding parameter value to the 1975-1979 value, leaving all others at the 2015-2019 values. Since the model is not linear, the individual contributions do not have to exactly sum to the total change in the natural rate resulting from changing all drivers simultaneously, reflecting possible interactions between the drivers

We target the natural rate at one point in time, the 2015-2019 steady-state. We use our own Kalman filter estimates as targets for advanced economies. For Emerging Markets, we rely on literature estimates of the natural rate (Guofeng and Rees, 2021 (China), Pattanaik et al, 2022 (India), Ruch, 2021 (Brazil)).

We deviate from the standard calibration procedure in the following cases: According to data from Penn World Tables, India experienced a sharp decline in the labor share of about 21 percentage points between the mid-1970s and 2019, by far the largest of our eight countries in the sample. Using our standard procedure, this would imply a negative mark-up in the 1975-1979 steady-state. We decide instead to impose a minimum markup of zero and therefore do not fully match the change in the labor share for India.[7] For China, the data implies a very large capital-to-output ratio. Given the equilibrium conditions of the model and our baseline parametrization, this would imply a negative mark-up in the 2015-2019 steady state. We set the markup to 5% and use the inferred capital-to-output ratio of 5.4.[8]

Collecting the data or estimating the values for the respective parameters and targets for all eight countries goes beyond the scope of this chapter. Therefore, all other parameters and calibration targets are held constant across countries at the respective 2015-2019 value for the United States. This is the case for the following: bend-points of social security system, minimum consumption floor in retirement, productivity process for transitory productivity shock, medical expense process, tax rates, including parameter determining progressivity of income tax, scaling factor o in utility function, distribution of bequest received by households, distribution of profits received by households, elasticity of consumption out of permanent income.

For TFP growth we calculate a Solow residual based on the growth rate of the capital stock and labor share from the PWT as well as the growth rate of the effective labor force implied by

(Inequality), public debt as share of output (Public debt), government consumption as share of output (Government consumption), NIIP (Net international capital flows), labor share of income (Market power). For inequality, we change the permanent-productivity realization to match the change in the top ten percent labor income share using a numerical optimization routine.

[7] This implies a labor share in 1975-1979 of 61.3% vs the 73% in the data. The labor share in the 2015-2019 steady state is 52%. India still exhibits the largest decline in the labor share between 1975-1979 and 2015-2019 of our eight countries given this assumption. Given the large share of informal labor in India, estimating the labor share is a daunting undertaking.

[8] In the data the capital-to-output ratio for China is 6.3, implying a markup of -1.5%. Our assumption of a capital-to-output ratio of 5.4 is still the largest capital-to-output ratio of all eight countries in our sample.

our model. When calibrating the age-specific productivity term, we match the profile of average labor earnings from the Luxembourg Income Study Database.[9] When targeting a change in the labor share, we adjust the markup. The interpretation is that a fall in the labor share goes hand-in-hand with an increase in the profit share. This approach is consistent with findings in Barkai (2020).

Figure 2.8: alternative scenarios

The *second* experiment is a transition-path analysis. The starting date is 1950, and the terminal date is set to 2200, which ensures that the model has sufficiently converged. The calibration of Type II parameters relies on an iterative procedure, with a total of P iterations. The algorithm starts ($p = 1$) with a calibration of a 2015-2019 steady state to a set of targeted moments (m_p^t). Here we choose the same targets and respective values as in the first experiment, which are also the ultimate targets of interest (denoted $\bar{m}^t$). Along the transition, we allow the drivers of the natural rate to change, described in more detail below. We simulated the full transition and derive model-implied moments (m_p^m). Since 2015-2019 are not a steady state in the transition-path exercise, some model-implied moments, in particular the average level of the natural rate in this period, do not have to overlap with the targeted values in the steady state. Therefore, we derive $\Delta(\mathrm{r})_p = \bar{m}(r)^t - m(r)_p^m$, i.e. the difference of the targeted natural rate to the model-implied natural rate in the 2015-2019 period in iteration step p. If this difference is above a tolerance, we move to the next iteration step, which starts with a steady-state calibration with targeted moments $\mathrm{m(r)}_{p+1}^t = m(r)_p^t + \Delta(\mathrm{r})_p$.[10] The iteration continues until the tolerance is met.[11]

Regarding the drivers of the natural rate along the transition, we do the following: in the baseline (Figure 2.8, panel 1 in the main text), we let demographic drivers (population distribution, mortality rates, population growth rate) change from 1950 to 2100, and from 2100 on we hold the values fixed at the 2100 level. Demographic variables are the only ones we start in 1950 in order to avoid a jump in 2020. All other drivers start to change earliest at 2019. We take the debt trajectory and forecast until 2028 from WEO (as of February 28, 2023), and hold public debt constant from there on. For advanced economies, we hold TFP growth constant at the 2015-2019 average value. For emerging markets, we assume a linear convergence to the advanced economies' average value over 2015 to 2019 by 2050.

[9] We use the same quadratic function as in PP (2022) and choose the parameters of the quadratic function manually to minimize the distance to the empirical profile. A difference to PP (2022) is that we use the same profile for both permanent productivity types.

[10] We only adjust the target for the natural rate in the iterative procedure.

[11] In practice, we stop after $p = 2$. The procedure ensures that the average natural rate in 2015 to 2019 on the transition path is close to the data.

When we simulate the scenario of *higher government borrowing costs* we introduce a wedge between the return on physical capital and on government bonds (see Equation A.2.3.3). We calibrate the steady-state wedge in 2015-2019 to the five-year average of the spread between a 10 years' government bond and investment grade corporate bond yields.[12] In addition, we rerun the calibration routine to match the same targets in the 2015-2019 steady-state as in the baseline model. In all other simulations, this wedge is assumed to be zero. When we run the scenario, we feed in a change in the wedge, assuming a linear transition by 2050 to the new level. The 2050 level is calculated as a decline in the 2015-2019 average wedge by 44 basis points. This change is calculated as the difference between the United States corporate bond spread in 2015-2015 and pre-2000 (1970-1999). The model does not capture the endogenous response of capital flows to a change in the convenience yield, but this effect could be sizeable for safe asset providers like the United States. For this reason, we change the net savings stock owned by foreigners, A^* (see Equation A.2.3.1), simultaneously in this same scenario, only for the United States. We feed in a linear change by 79 percent of GDP by 2050, which amounts to the difference of gross foreign portfolio investments in the United States between the 2015-2019 average and the pre-2000 average (1976-1999), according to the Bureau of Economic Analysis. Consequently, in this scenario the natural rate increases by around 100 basis points by 2050 in the United States.

Online Annex Table 2.3.2

Indicator	Sources
Mortality rates	UN World Population Prospects, 2022 Revision
Population size by age	UN World Population Prospects, 2022 Revision
TFP growth	Penn World Table PWT 10.0
Government consumption	Penn World Table 10.0
Public debt	IMF Global Debt Database (GDD), Mauro et al (2015)
Asset-to-income ratio	wid.world
Net International Investment Position	IMF
Labor income profiles	LIS
Labor share of income	PWT 10.0, WEO
Top 10% income share	wid.world
Natural rate of interest	Own calculations, Guofeng and Rees (2021), Pattanaik et al (2022), Ruch (2021)

[12] We use Moody's Seasoned Baa Corporate Bond Yield for the U.S. and S&P Investment Grade Corporate Bond Index for U.K. and Japan, and Eurozone S&P Investment Grade Corporate Bond Index for France and Germany.

Out-of-pocket health spending	OECD
Social Security Spending	OECD, Cuevas et al (2017), Fang and Feng (2018), Narayana (2015)
Retirement age	OECD

Online Annex 2.4. Fiscal policy implications

This section presents further details on the Mian, Straub and Sufi (2021c) framework for assessing debt sustainability. The model assumes that saving households draw convenience benefits from holding government debt. These convenience benefits – principally associated with safety and liquidity properties of government bonds – are reflected in a "convenience yield" which reduces the cost of borrowing for the government (Krishnamurthy and Vissing-Jorgensen 2012; Caballero, Farhi and Gourinchas 2017b).

In this framework, it is assumed that the utility savers derive from holding government debt decreases with the debt level—the convenience yield gets eroded. For r and b denoting the interest rate on government debt and the debt-to-GDP ratio, respectively, the model-derived interest rate on government debt, $r(b)$, depends on the level of debt, amongst other factors.

Solving the model yields the following key equation (A.2.4.1) capturing debt dynamics:

$$\dot{b} = \underbrace{z}_{direct\ effect} + \underbrace{(r(b) - g)b}_{indirect\ effect} \qquad \text{(A.2.4.1)}$$

where $\dot{b}$ is the change in the debt-to-GDP ratio, z denotes the primary deficit and g is the economy's growth rate. A primary deficit directly increases debt-to-GDP over time, while the second term is the indirect effect coming from the difference between the cost of borrowing and the rate of output growth, a term widely known as $r - g$.

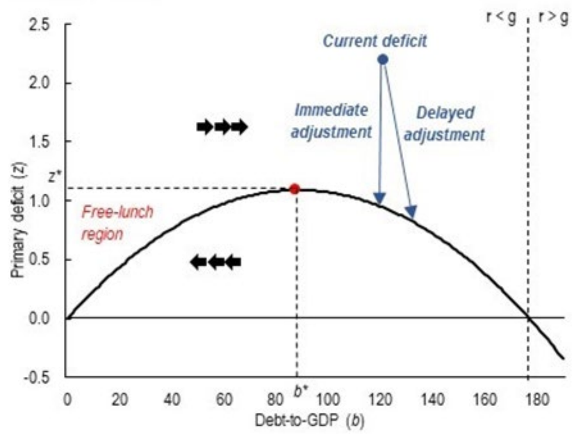
As is well-known, when $r > g$, primary deficits are unsustainable as debt-to-GDP will be on an explosive path. Governments will have to run a primary surplus to stabilize the debt-to-GDP ratio. When $r < g$, however, and if interest rates are not endogenous to the debt level, the debt dynamic equation (A.2.4.1) suggests that any level of primary deficit can be entertained. To each level of primary deficit will correspond a *stable* debt-to-GDP ratio (see Blanchard 2019); fiscal space appears infinite. [13]

The Mian, Sufi and Straub framework used in this chapter introduces continuity between these two polar worlds by setting r as endogenous to the level of debt-to-GDP b, or $r(b)$. Increasing debt will lead to higher rates, pinning down a unique level of primary deficit z that stabilizes b

[13] Blanchard (2023) cautions against an interpretation of the result as indicating unconditional infinite fiscal space given the endogeneity of r^*, and accordingly of actual rates, to fiscal policy. In addition, future forecasts of $r - g$ are subject to uncertainty, and the probability of sign reversals should be taken into account when assessing debt sustainability.

when $r(b) < g$. This can be seen easily by setting $\dot{b} = 0$ in the previous equation to obtain: $z = -(r(b) - g)b$. This is a key relationship as it identifies, for any level of debt, the unique primary deficit that is necessary to stabilize debt-to-GDP. These debt-stabilizing (b, z) combinations are plotted in the phase diagram in Online Annex Figure 2.4.1. Any point above the solid arc means $\dot{b} > 0$ and debt will rise forever, with the converse also true for points below the arc. With $r - g < 0$, z can be positive indicating that the government can run a primary deficit, given the favorable dynamics. At a low debt level, the government can even increase the primary deficit and still converge to the stable arc. This region is indeed a "free lunch" for the government as a higher deficit is possible while still maintaining stable debt dynamics. This is available to the government for low debt levels, those lower than b^*, which is at about 90 percent of GDP in the hypothetical economy used in the chart for illustration. At any point beyond b^*, the government must reduce its deficit to ensure debt stabilization. Beyond the vertical dashed line that corresponds to a debt of 180 percent of GDP, $r(b)$ becomes larger than g and the government must run a primary surplus to ensure debt sustainability.

Online Annex Figure 2.4.1. Illustrative Example of Stability in Debt Dynamics



Source: IMF staff calculations.

Suppose that the economy has a current primary deficit that is above the stability arc, and a debt level that exceed b^*, like the blue dot marking the *current deficit* in the figure. Here, deficit reduction is needed to stabilize debt-to-GDP at the current level. The reduction could take place in the near term, or over the medium term. In the latter case, debt-to-GDP will increase and the deficit will need to be reduced even more to stabilize b as indicated by the blue arrows. The stabilization would occur at a higher debt-to-GDP ratio and also at a higher cost of borrowing for the government.

Because the critical element in debt sustainability analysis is $r - g$ and its future trajectory, we use the projections for r^* and and the corresponding growth rate – which underlie the "Outlook for the Natural Rate" section – to construct the debt-stabilizing arc for each economy, identify the point implied by current policies (primary deficit in 2022) and compute the necessary deficit reduction to place public debt on a sustainable trajectory – on the long-run stabilizing arc. For the US and China, this is displayed in Table 2.1 in the Chapter. This section of the Online Annex includes the analysis for Brazil, France, Germany, India, Japan, and the UK.

Online Annex Table 2.4.1 presents the deficit reduction required in each of the six economies. As shown in the top panel, the required near-term reductions range from about 1.5 pp in France to 7.6 pp in Japan. The $r - g$ trajectories for France and Germany are rather similar, but a little less favorable for the UK. This explains the variation in the required fiscal adjustment among them despite starting from comparable primary deficits in 2022 at around 2.7 percent of GDP.

The large reduction for Japan reflects a sizable primary deficit in 2022, at about 7.4 percent of GDP, and a targeted primary surplus at 0.2 percent of GDP. For Brazil and India, the needed adjustments are 3.2 pp and 1.8 pp, respectively.

Under the higher debt scenario, more adjustment is needed given a worsening $r - g$ profile for all economies. With higher debt, demand for savings is higher, which increases the path of the natural rate. Along the transition, growth will also be lower due to the crowding out effect on productive capital from the increase in the natural rate. The 1970s labor share scenario changes $r - g$ differentially across countries. Labor share changes are modeled as a change in the markup and have an ambiguous effect on the natural rate: a decline in markups implies more production, raising the demand for savings. But since profits disproportionately go to the elderly, savings supply increases with a lower markup.[14] The results indicate that France, the UK, and India would need more deficit reduction, while Brazil, Germany, and Japan require less consolidation.

The middle and bottom panels of Online Annex Table 2.4.1 show the additional required deficit reduction if a longer adjustment is deemed more palatable. It is mostly Japan and Brazil where this cost appears larger relative to the rest given a large current primary deficit as well as a sizable debt level at 261 percent of GDP for Japan, and worsening $r - g$ dynamics for Brazil. Germany and the UK would experience comparable costs for delayed adjustment given a similar $r - g$ trajectory over the medium term. France and India have relatively more favorable medium-term dynamics for $r - g$, which lowers the cost of delayed fiscal adjustment relative to the other economies.

Online Annex Table 2.4.1. Required Fiscal Adjustment under Different Scenarios

(Changes in primary deficit; percentage points of GDP)

		Scenarios	
	Baseline	Higher debt	1970s labor share
Near-term adjustment			
France	-1.46	-1.57	-1.56
Germany	-1.81	-1.91	-1.77
Japan	-7.64	-8.22	-7.58
United Kingdom	-1.99	-2.12	-2.03
Brazil	-3.22	-3.48	-3.12
India	-1.77	-1.82	-2.02
Additional consolidation needed for medium-term adjustment (3 years)			
France	-0.03	-0.03	-0.02
Germany	-0.05	-0.06	-0.05
Japan	-0.28	-0.32	-0.26
United Kingdom	-0.05	-0.06	-0.04
Brazil	-0.19	-0.21	-0.19
India	0.00	0.00	0.00
Additional consolidation needed for medium-term adjustment (5 years)			
France	-0.04	-0.05	-0.04
Germany	-0.10	-0.12	-0.09
Japan	-0.49	-0.54	-0.46
United Kingdom	-0.09	-0.10	-0.09
Brazil	-0.31	-0.33	-0.34
India	-0.01	-0.01	-0.01

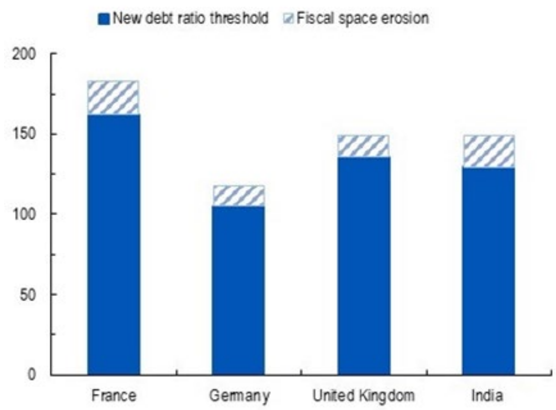
Source: IMF staff calculations.
Note: The required fiscal adjustment is the gap from the long-run debt-stabilization level, calculated as the difference between the 2022 primary deficit from the *World Economic Outlook* database and the model-based estimate of the primary deficit that stabilizes debt-to-GDP at the long run given projections for the natural rate of interest and growth.

Online Annex Figure 2.4.2 shows the sensitivity of the results to a key parameter in this exercise, which is the elasticity of interest rates with repect to the debt level. This parameter is denoted by φ. The objective is to trace the impact of an increase in φ on the threshold debt level that, once crossed, the sign of $r - g$ reverses

[14] The size of each effect depends on the parametrization of the model, which is determined by individual country calibrations and can therefore differ among countries.

from negative to positive— see the vertical dashed line in Online Annex Figure 2.4.1. This requires shifting from a primary deficit to surplus to maintain a stable debt-to-GDP ratio, and thus can be interpreted as some measure of fiscal space: how much additional debt can a country accumulate before primary surpluses are necessary for debt stabilization. An increase in φ, indicating that a given increase in debt now causes a larger increase in borrowing rates for the government, results in a relatively large erosion of fiscal space as indicated by the dashed portion of the bars. The results show varied threshold debt levels at the baseline estimate of φ, with France enjoying the largest threshold at around 183 percent of GDP, followed by India and the UK at roughly similar levels, then followed by Germany at 118 percent of GDP. An increase in φ leads to an erosion of the fiscal space as indicated by the dashed portions. The loss in fiscal space is non-negligible ranging from around 13 pp in Germany to 21 pp in France. This would be more impactful for countries that are already nearing their respective threshold levels, especially if currently running large primary deficits.[15]

Online Annex Figure 2.4.2. Impact of Higher Interest Elasticity on Debt Threshold
(Percentage points of GDP)



Source: IMF staff calculations.
Note: The full bar height (solid plus dashed area) is the threshold level of debt beyond which a primary surplus would be required to stabilize debt-to-GDP. This is calculated using the baseline value of 0.017 for the elasticity parameter. Increasing the elasticity parameter leads to a higher borrowing cost for the government at all debt levels, which lowers the debt threshold. The reduced debt threshold is given by the solid bars and uses an elasticity estimate of 0.022, which is the upper bound for estimates identified from the literature; see Mian, Straub and Sufi (2022) for details. The dashed portion of the bars is the lost fiscal space due to a higher elasticity estimate.

Online Annex 2.5. Geopolitical Fragmentation

Box 2.2 on geo-economic fragmentation and natural rates defines three country blocs. The blocs were determined primarily by the geopolitical alignment between countries using the "ideal point distance" (IPD) proposed by Bailey and others (2017), which is based on the similarity of voting patterns at the United Nations General Assembly (UNGA). The IPD is used in WEO Chapter 4 on spillovers, and explained a bit further in the Technical Annex for the chapter. The China Bloc are those regions with strong tendencies towards China; the US bloc towards the United States. The nonaligned are weakly aligned with both China and the United States under the IPD measure.

Here are the blocs and regions decomposed in detail. The US Bloc is the United States, European Union Plus, and Other Advanced economies, which are:

[15] The sensitivity analysis uses the baseline projections for r^*. Brazil, China, Japan and the US are excluded from the analysis given that their current debt levels already exceed this threshold. For these economies, a primary surplus is needed to stabilize the debt-to-GDP ratio in the long run.

1. United States
2. European Union Plus (EU+) = EU and Switzerland
3. Other Advanced Economies = Australia, Canada, Iceland, Israel, Japan, Korea, New Zealand, Norway, and the United Kingdom

The China Bloc is China, Southeast Asia, and the Remaining Countries, which are:

1. China = China and Hong Kong SAR
2. Southeast Asia = Brunei, Cambodia, Lao PDR, Malaysia, Myanmar, Philippines, Singapore, Thailand, and Vietnam
3. Remaining Countries = Russia, South Africa, and Turkey plus the regions of Africa, the Caribbean, Central Asia, other Latin America, the Middle East, and Oceania, plus any other EMDEs not accounted for elsewhere

The non-aligned regions are India and Indonesia and Latin America, which are:

1. India and Indonesia = India, Indonesia
2. Latin America = Argentina, Brazil, Chile, Colombia, Costa Rica, Mexico, and Peru

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