

Annex 2.1 Data Sources, Sample Coverage, and Variable Definitions

Data sources used in the chapter are listed in Annex Table 2.1.1. In general, the sample is the group of advanced economies as defined by the *World Economic Outlook* (WEO), for a total of 36 economies. The exact samples used varies with the analyses and exercises based on the time coverage and data available. See Annex Table 2.1.2 for the economies included, time coverage, and the analytical and statistical samples where they appear.

Data from the WEO database are extended backwards for key indicators using several sources as possible. The following indicators are extended backwards using their respective additional sources listed in Annex Table 2.1.1: gross public debt; nominal GDP; real GDP; short-term government interest rate; and long-term government interest rate. Construction of long-run historical data for short-term monetary policy rate varies by country, with the vast majority of countries' data coming from national sources.

Forecast errors are utilized in several analytical exercises—the analyses of contributions to the interest rate–growth differential ($r - g$) and debt dynamics (see Annex 2.2 for details) as well as in the analysis of fiscal multipliers (see Annex 2.3 for details). Forecast errors used in the analyses of contributions to $r - g$ and debt dynamics are calculated using forecasted annual data from World Economic Outlook database vintages beginning in 1990. Forecast errors used in the analysis of fiscal multipliers are calculated using forecasted annual data from OECD Economic Outlook database vintages beginning in 1985.

Annex Table 2.1.1. Data Sources

Indicator	Source(s)
Short-term policy rate	Bank for International Settlements; Global Data Source; Haver; International Financial Statistics; and national sources.
Primary fiscal balance to GDP	Mauro and others (2015); and World Bank.
Gross public debt to GDP	Jordà and others (2019); Mauro and others (2015); IMF Historical Public Debt Database; and World Economic Outlook database.
Central bank assets to GDP	European Central Bank; Haver; and Ferguson and others (2015).
Nominal GDP	Jordà and others (2019); and World Economic Outlook database.
Real GDP	Mauro and others (2015); Global Data Source; Maddison Project database; and World Economic Outlook database.
Short-term government interest rate	Global Data Source; Jordà and others (2019); Organisation for Economic Co-operation and Development; and World Economic Outlook database.
Long-term government interest rate	Global Data Source; Jordà and others (2019); Organisation for Economic Co-operation and Development; and World Economic Outlook database.
Primary deficit	World Economic Outlook database.
Population by age	United Nations.
Interest cost of reserves	Federal Reserve.
Long-run total factor productivity	Bergeaud, Cette, and Lecat (2016).
Real public consumption	OECD Economic Outlook database.
Exchange rate classification (flex vs. fixed)	Ilzetzki, Reinhart, and Rogoff (2019).
Systemic banking crisis classification	Laeven and Valencia (2018).

Source: IMF staff compilation.

Annex Table 2.1.2. Sample of Economies Included in Analytical Exercises

Economies	Years	Exercise ¹			
		I	II	III	IV
Australia; Austria; Belgium; Canada; Denmark; Finland; France; Germany; Greece; Iceland; Ireland; Italy; Japan; Luxembourg; Netherlands; New Zealand; Norway; Portugal; Spain; Sweden; Switzerland; United Kingdom; United States.	1960–90	X			
Australia; Austria; Belgium; Canada; Cyprus; Czech Republic; Denmark; Estonia; Finland; France; Germany; Greece; Iceland; Ireland; Israel; Italy; Japan; Korea; Latvia; Lithuania; Luxembourg; Malta; Netherlands; New Zealand; Norway; Portugal; Singapore; Slovak Republic; Slovenia; Spain; Sweden; Switzerland; Taiwan Province of China; United Kingdom; United States.	1991–2018	X			
Australia; Austria; Belgium; Canada; Cyprus; Czech Republic; Denmark; Finland; France; Germany; Greece; Iceland; Ireland; Israel; Italy; Japan; Korea; Luxembourg; Malta; Netherlands; New Zealand; Norway; Portugal; Slovak Republic; Slovenia; Spain; Sweden; Switzerland; United Kingdom; United States.	2016–19		X		
Australia; Austria; Belgium; Canada; Czech Republic; Denmark; Finland; France; Germany; Ireland; Italy; Japan; Korea; Netherlands; New Zealand; Norway; Portugal; Slovak Republic; Spain; Sweden; Switzerland; United Kingdom; United States.	1985–2018			X	
Belgium; Denmark; Finland; France; Germany; Italy; Japan; Netherlands; Norway; Portugal; Spain; Sweden; Switzerland; United Kingdom; United States.	1871–2019				X

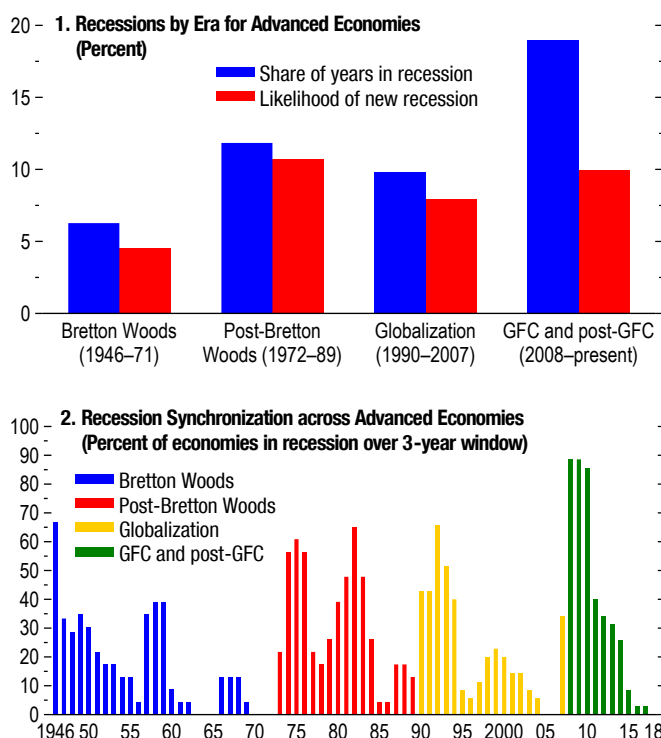
Source: IMF staff compilation.

¹Analytical exercises performed in the chapter. I = monetary and fiscal policy trends (Figures 2.1–2.3); II = public debt decomposition (Figures 2.4); III = fiscal multipliers (Figures 2.5 and 2.6); and IV = persistence and drivers of $r - g$ (Figures 2.2.1 and 2.2.2).

Additional Stylized Facts

Figure 2.1.1, panel 1 exhibits how recessions since 2008 have become longer and the likelihood of a recession has risen compared to the 1990s and early 2000s. Figure 2.1.1, panel 2 shows how the degree of synchronization of recessions across advanced economies has increased on average in the current era compared to earlier periods.

Annex Figure 2.1.1. Recessions in Advanced Economies

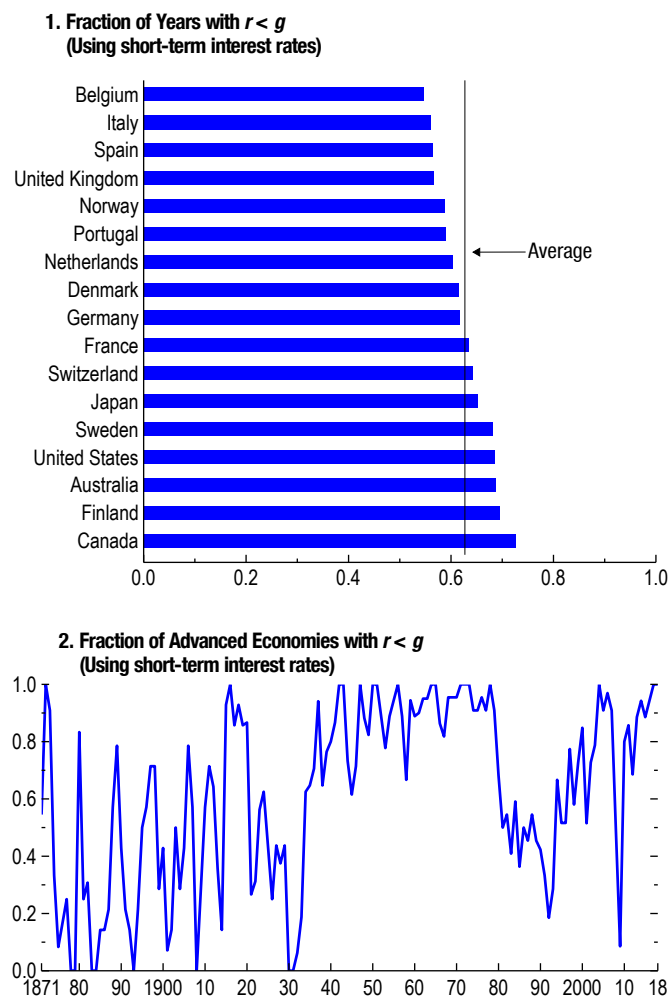


Sources: Maddison Project; Mauro and others (2015); and IMF staff calculations. Note: Recessions are defined to be years with negative output growth. The percent of advanced economies in a recession at a point in time t is calculated as the number of advanced economies in a recession in either years $(t-1)$, t , or $(t+1)$, divided by the total number of advanced economies. GFC = global financial crisis.

Annex 2.2. Interest Rate–Growth Differentials, Primary Deficits, and Contributions to Debt Dynamics

This section of the annex provides additional details on the backward-looking analysis of how unanticipated developments in countries' interest rates, nominal growth, and primary deficits affected the evolution of debt. It also presents more background information on interest rate–growth differentials ($r - g$) and the related analyses on the persistence of $r - g$ featured in Box 2.2. First, it provides some stylized facts about the evolution of the interest rate–growth differential over the long run, using a sample of 17 advanced economies and covering the period between 1871 and 2018. The following subsection outlines a framework for assessing the relationship between unanticipated changes in debt-to-GDP and unanticipated changes in key fiscal variables relative to expectations from a past date. This framework is used to generate Figure 2.4 in the main text. The subsequent subsection describes an alternative framework to conduct a purely backwards-looking accounting decomposition of outturns. Then, the statistical model that forecasts the path of the interest rate–growth differential underlying Figure 2.5 is described. The next subsection provides the background to construct Figure 2.6 and explains how to identify and predict the common international component of interest rate–growth differentials, and to understand its drivers. A final section concludes with a forward-looking estimate of the potential impact on debt-to-GDP arising from a further unexpected decline in $r - g$, conditional on the maturity structure of new debt.

Annex Figure 2.2.1. $r - g$ in Advanced Economies



Sources: Bank for International Settlements; Haver Analytics; IMF, *International Financial Statistics*; Jordà and others (2019); national sources; and IMF staff calculations.
Note: Sample includes 17 advanced economies.

Some Stylized Facts on Interest Rate–Growth Differentials

Advanced economies in the sample experienced negative interest rate–growth differentials a majority of the time between 1871 to 2018 (Annex Figure 2.2.1). However, the median $r - g$ across these advanced economies at each point in time fluctuated markedly in periods prior to World War II (Annex Figure 2.2.2). Between World War II and throughout the 1970s, the

median $r - g$ was negative, while through the 1980s it varied near zero. During the 1990s, the median $r - g$ was positive, but then declining, with the overall trend mostly negative, apart from a brief positive period around the Global Financial Crisis (GFC).

A Simple Government Budget Constraint

As a preliminary, to derive the simplified budget constraint, one starts with nominal debt and deficit, labeled B_t and D_t , respectively. The budget constraint under the simplified assumption that all debt has one-period maturity is given by:

$$B_t = D_t + B_{t-1}(1 + r_{t-1}).$$

r is the nominal interest rate on debt. Denoting nominal output by Y_t and the debt and deficit ratios (to GDP) by $b_t = B_t/Y_t$ and $d_t = D_t/Y_t$ respectively the budget constraint can be rewritten as:

$$b_t = d_t + b_{t-1}(1 + r_{t-1}) \times \left(\frac{Y_{t-1}}{Y_t}\right)$$

And thus:

$$b_t = d_t + b_{t-1} \left(\frac{1+r_{t-1}}{1+g_t}\right),$$

where nominal growth is given by $g_t = Y_t/Y_{t-1} - 1$. Using a first order Taylor approximation, the gross interest rate–growth ratio is approximately the net difference:

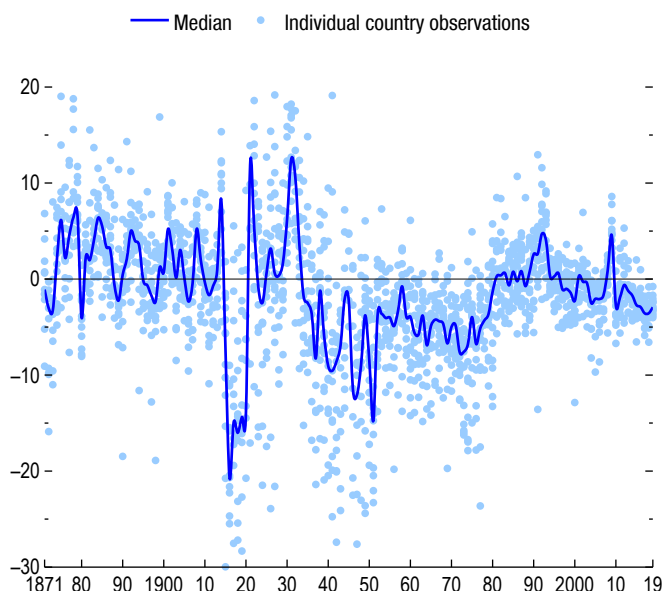
$$\left(\frac{1+r_{t-1}}{1+g_t}\right) \approx 1 + r_{t-1} - g_t.$$

Substituting in and rearranging yields a simplified version of the government’s budget constraint:

$$b_t - b_{t-1} = d_t + (r_{t-1} - g_t)b_{t-1}.$$

The importance of the interest rate–growth differential can be seen from this last equation. It says that the change in the government’s debt-to-GDP ratio b from period $t - 1$ to t is equal to the primary deficit d (current government spending minus current income, excluding interest payments, relative to GDP) in period t plus the difference between the nominal interest rate r at time $(t - 1)$ and nominal GDP growth g at time t , quantity times

Annex Figure 2.2.2. Interest Rate–Growth Differentials in Advanced Economies
(Percentage points)



Sources: Jordà and others (2019); Organisation for Economic Co-operation and Development; and IMF staff calculations.
Note: The sample includes 35 advanced economies. Time coverage is unbalanced across countries.

outstanding $(t - 1)$ debt to GDP.¹ As the interest rate–growth differential falls, the change in the debt-to-GDP ratio also comes down, all else equal.

Thus, this simplified version of the budget constraint illustrates that it is the difference between the nominal interest rate and nominal growth, known as the interest rate–growth differential or $r - g$ that is essential to understand public debt dynamics. Changes in interest rates affect the numerator of the debt-to-GDP ratio, while changes in nominal growth rates impact the denominator.²

A Framework for Decomposing Unanticipated Debt Changes into Unanticipated Changes in Fiscal Indicators

Next follows an outline of a framework which can trace through the impact of unanticipated changes to interest, growth, and primary deficits to the unanticipated change debt levels relative to past expectations. This is also built around a government budget constraint, but extends the simple case outlined in the preceding section.

Notation

Growth: Annualized nominal log output growth between periods t and $t+j$ is denoted by g_t^j . That is:

$$g_t^j = \frac{1}{j} (\log Y_{t+j} - \log Y_t)$$

Interest rates: The one-period policy rate at the central bank is denoted by r_t^{rf} (the risk-free rate). The government debt yield curve at time t is represented by $\{r_t^j\}_{j \geq 1}$, where j is the maturity of debt and r_t^j is the log yield on maturity j debt. That is, the government can sell a zero-coupon bond of maturity j for price $e^{-jr_t^j}$.

Term, risk and other premia may potentially drive the yield curve away from the expected future sequence of short rates. The bond premium at horizon j is therefore given by:

$$\tau_t^j = r_t^j - \frac{1}{j} \sum_{k=0}^{j-1} E_t r_{t+k}^{rf},$$

where E_t is the expectation operator conditional on information at time t .

Debt Structure

It is assumed that the government issues only long-term debt with exponentially decaying coupons (see Hatchondo and Martinez 2009). This means that for each unit of debt outstanding, the government pays a coupon c and makes a principal repayment λ . The remaining quantity of outstanding debt is then $1-\lambda$. Thus, a unit of debt issued in period t will

¹ In practice, the selection of the nominal interest rate in the analysis of debt dynamics will depend on the research question. For example, whether the average effective rate on outstanding debt or the yield to maturity on newly issued debt is appropriate depends on how payments are attributed to interest and principal. A further discussion of these choices and what is used in the subsequent exercises is provided further below.

² Nominal growth may also affect debt dynamics through its impact on the government's borrowing needs in a period. For example, tax revenues could rise with higher growth and incomes, reducing the primary deficit.

yield a stream of payments $(c + \lambda)$, $(1 - \lambda)(c + \lambda)$, $(1 - \lambda)^2(c + \lambda)$, and so forth in subsequent periods.

The price of the bond is therefore the price of this stream of future claims, priced from the yield curve:

$$q_t = (c + \lambda) \sum_{j=1}^{\infty} (1 - \lambda)^j e^{-jr_t^j}.$$

The yield-to-maturity, \bar{r}_t , is the constant yield which prices the bond:

$$\begin{aligned} q_t &= (c + \lambda) \sum_{j=1}^{\infty} (1 - \lambda)^j e^{-j\bar{r}_t} \\ &= \frac{c + \lambda}{e^{\bar{r}_t} - (1 - \lambda)}. \end{aligned}$$

These equations can be inverted to get an explicit formula for the yield-to-maturity:

$$\bar{r}_t = \log \left(1 - \lambda + \frac{1}{\sum_{j=1}^{\infty} (1 - \lambda)^j e^{-j\bar{r}_t}} \right). \quad (1)$$

Government Budget Constraint

The government issues new debt to cover its gross financing needs—the primary deficit plus coupon payments and amortizing debts. Total outstanding debt is the sum of non-maturing debt plus new debt issuance. Let b_t denote the debt-to-GDP ratio at the start of period t and s_t be the primary surplus ratio during period t . Then the debt-to-GDP ratio evolves according to:

$$b_{t+1} = (1 - \lambda)b_t e^{-g_t^1} + \frac{1}{q_t} ((c + \lambda)b_t - s_t) e^{-g_t^1}.$$

Substituting in for prices, ones gets the following:

$$b_{t+1} = (b_t - \theta_t s_t) e^{\bar{r}_t - g_t^1},$$

where

$$\theta_t = \left(\frac{1 - (1 - \lambda)e^{\bar{r}_t}}{c + \lambda} \right).$$

This is approximately one (to first order) if:

$$c = (1 - \lambda)\bar{r}_t.$$

Sample and Data for Backward and Forward Decomposition Exercises

The underlying data for the illustration depicted in the chapter comes from the IMF October 2015 WEO forecasts for 30 advanced economies for 2016–18. This vintage is selected since the projections from then incorporate the expected effects of the large-scale asset purchase programs undertaken prior to that date in advanced economies (including the ECB’s public sector purchase program). The 2018 end point is selected since that is when the latest final data on public debts and deficits are available. Each forecast vintage contains a contemporaneous measure of the ten-year government bond spread and the sequence of five-year forecasts of the policy rate. Assuming that these are valid measures for r_t^{10} and $E_t r_{t+k}^{rf}$ respectively (for $k=0, \dots, 5$), one can compute the ten-year bond spread from:

$$\tau_t^{10} = r_t^{10} - \frac{1}{10} \sum_{k=0}^4 E_t r_{t+k}^{rf} - \frac{1}{2} E_t r_{t+5}^{rf}. \quad (2)$$

This is equivalent to assuming that the five-year forward policy rate is an unbiased forecast of policy rates at the six- to ten-year horizon. Another assumption is that the term spread grows linearly with the yield curve horizon:

$$\tau_t^j = \frac{1}{j} \tau_t^{10}.$$

Robustness checks also use a variant where $\tau_t^j = \tau_t^{10}$ for all j .

These assumptions produce a full yield curve for government interest rates up to $j=10$. For $j>10$, the expectations hypothesis is supplemented in equation (2) with the assumption that the bond premium is constant. That is:

$$r_t^j = \frac{10}{j} r_t^{10} + \left(\frac{j-10}{j} \right) (E_t r_{t+5}^{rf} + \tau_t^{10}).$$

Using this yield curve, one can compute the yield-to-maturity \bar{r}_t using equation (1) for any given maturity λ . As λ is the inverse of the Macaulay duration of the bond, it is set to $\lambda = 1/8$ to match the average debt maturity of countries in the sample.

The government budget constraint only holds for net debt (financial liabilities minus financial assets of the general government), hence, it needs to be modified to:

$$b_{t+1} = (b_t - \theta_t s_t) e^{\bar{r}_t - g_t^1} + \eta_t,$$

where η_t is the period- t stock-flow adjustment, which is computed as a residual from the data.

Backward-Looking Decomposition

For each country, three alternative histories are computed, based on the 2015 forecasts for primary surplus ratios, nominal growth rates and nominal interest rates. Let \tilde{s}_t be the 2015 WEO forecast for primary surplus ratios in a given country. Then one can compute an alternative history for the evolution of the debt ratio using:

$$\begin{aligned} \tilde{b}_{2015} &= b_{2015} \\ \tilde{b}_{t+1} &= (\tilde{b}_t - \theta_t \tilde{s}_t) e^{\bar{r}_t - g_t^1} + \eta_t. \end{aligned}$$

Similarly, one can compute similar alternate histories for the debt stock under the assumptions that nominal growth and interest rates followed their 2015 forecasts. In the case of interest rates, this requires computing the yield curve and yield-to-maturity at each point in time. The cross-country distributions for the contributions of unanticipated changes in these components to the unanticipated change in the debt path are shown in Figure 2.4, panel 2.

Correlations underlying the changes in Public debt

As a supplement to Figure 2.4, Table 2.2.1 presents key moments of the cross-sectional distributions of components of changes in debt ratios since 2015 for WEO forecast vintages prior to 2015. This table summarizes effect of each component on the evolution of the debt ratio 2015-2018 by: their average, cross-country standard deviations, and correlations. Numbers

cited in the chapter come from the rightmost column of this table. The difference in the median impact on debt ratios of $r - g$ compared to primary deficits is typically small (around 1-3pp) relative to the cross-country standard deviation within each component (around 3-8pp). In other words, the average variation across components is a relatively small determinant of changes in the debt ratio relative to the variation across countries within components.

Annex Table 2.2.1. Summary Statistics for Unexpected Changes of Fiscal Variables on Debt Ratio, 2015–18, by WEO Forecast Vintage

Vintage	2013	2014	2015
<i>Median impact on debt ratio</i>			
News about $r - g$	-2.94	-2.79	-1.47
News about primary deficits	-1.00	-0.05	-2.35
<i>Mean impact on debt ratio</i>			
News about $r - g$	-5.70	-3.81	-2.20
News about primary deficits	-0.06	-0.20	-2.30
<i>Cross-country standard deviation of impact on debt ratio</i>			
News about $r - g$	8.27	7.05	3.27
News about primary deficits	5.99	5.35	4.23
<i>Relative contributions to cross-country debt ratio differences</i>			
News about $r - g$	58%	57%	44%
News about primary deficits	42%	43%	56%
<i>Correlation of news about $r - g$ and primary deficits</i>			
Point estimate	-0.14	0.08	0.11
p-value	0.48	0.67	0.58

Source: IMF staff calculations.

Forward-Looking Decomposition

The forward-looking decomposition holds fixed the 2024 WEO debt level and then iterates back to 2019 by inverting the budget constraint. For example, for an alternate sequence of future nominal growth rates \tilde{g}_t , the start-2019 debt level can be computed from:

$$\tilde{b}_{2024} = b_{2024}$$

$$\tilde{b}_t = \theta_t \tilde{s}_t + (\tilde{b}_{t+1} - \eta_t) e^{-\tilde{r}_t + \tilde{g}_t^1}$$

For an alternate sequence of future short-term policy rates, a new yield curve and yield-to-maturity are computed at each point in time, and the same methodology applied to generate a counterfactual debt level in 2019. Annex Figure 2.2.3 shows by how much advanced economies could hypothetically increase their borrowing while keeping debt stable at its 2024 projected level if interest rate–growth differentials were to drop by a further 100 basis points. This experiment also measures the impact of past changes in $r - g$ on borrowing capacity so long as the persistence of those changes was also in line with the data (see the subsequent section on the common component of $r - g$ and its persistence for the estimates used).

If such a decline were to happen through an increase in nominal growth, then the increase in borrowing capacity is moderate, averaging about 3 percentage points of GDP across advanced economies.

If instead the decline in $r - g$ were to occur due to short-term policy rates, however, depends on how that change is that transmit through the yield curve; the impact on government borrowing costs depends on the maturity of debt. At the average 8-year maturity of debt in advanced economies, the impact is small, averaging only a fraction of 1 percent of GDP. For

declines in short-term interest rates to have a similar impact to declines in short-term growth rates, all debt would need to be short-term (i.e. one-year maturity).

If the shock is permanent, the impact of a given change in the interest-growth differential is much larger. Moreover, the sensitivity to debt maturity goes away when the decline in short-term interest rates is permanent. In this case, there is no difference between the impact if debt is one period or infinitely long-lived.

In general, countries with larger debts are more sensitive to changes in interest rate-growth differentials, as $r - g$ determines the growth rate of the debt ratio. A given change in $r - g$ therefore leads to a larger change in borrowing capacity when the debt ratio is higher. This applies equally to negative shocks. The magnitude sensitivities computed here are not a function of the sign of the shock and are still valid for increases in $r - g$ (albeit with a negative sign). And so higher-debt countries are more exposed to increases in interest rate-growth differentials.

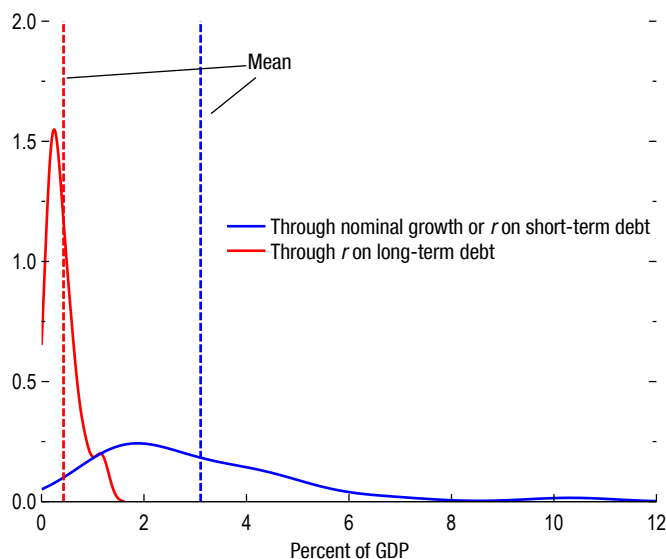
Comparison to an Accounting Decomposition

The backward-looking exercise described here explains current debt levels as a result of the changes in the economic environment relative to what was expected. As shocks can be persistent and yield curves are forward looking, it is only through a comparison relative to expected outcomes that the timing of the impact of past shocks can be correctly identified.

Nevertheless, it can also be instructive to compare the results of the foregoing exercise (presented in Figures 2.4) to the results of a purely accounting decomposition. This explains movements in debt by the realized components of the budget constraint: primary deficits, interest and growth rates, inflation, and a stock-flow adjustment.³

Annex Figure 2.2.3. Additional Borrowing Capacity if $r - g$ Falls 100 Basis Points
(Kernel density)

If the interest rate-growth differential were to fall further, the average additional borrowing capacity consistent with debt stability would be about 3 percent at most, depending on how it would be financed. In general, the savings gained scales with the size of debt outstanding.



Sources: Bank for International Settlements; Haver Analytics; IMF, *International Financial Statistics*; Jordà and others (2019); national sources; and IMF staff calculations.

Note: Chart shows the cross-country distribution of the increase in the debt ratio consistent with achieving the forecast debt levels in 2024 (as of the January 2020 WEO vintage) if $r - g$ were to fall by 100 basis points in 2019. Specifically, the thought experiment assumes an unexpected 100 basis point drop in the common component of $r - g$ across countries, which then evolves according to the statistical model used in Figure 2.2.1. The persistence of this decline is assumed to match the persistence of the estimated common factor for $r - g$ under alternative assumptions about debt maturity.

³ This last component is required because the overall deficit is equal to the change in net debt, whereas the final decomposition is presented in terms of changes to gross debt. This difference principally arises from treatment of public acquisition of financial assets. This has a net zero impact on government wealth and so is excluded from the primary deficit. However, funding for the acquisition of such assets creates a gross financing need, increasing gross debt (all else equal).

Such an approach has an obvious disadvantage relative to the exercise described in the preceding section; the timing of the impact of changes will not be properly identified. For example, changes in yield curves in a given year will affect realized interest payments for many subsequent years. A comparison to the anticipated yield curve will correctly assign the impact of these changes to the date when the yield curve changes, whereas the realized decomposition will attribute them to the horizon at which interest is paid. These could be very different, particularly for long-maturity bonds.

Yet his method does have an advantage: clarity. There are fewer assumptions required about yield curves and the like – it is entirely an accounting decomposition.

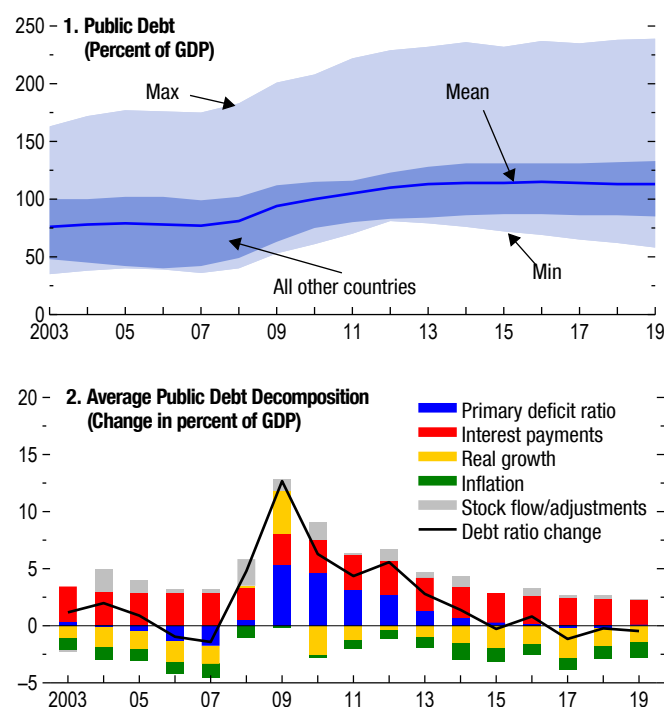
Annex Figure 2.2.4 shows this decomposition averaged across the G7 plus Spain. The top panel shows the evolution of gross debt since 2003. The bottom panel explains the year-to-year change in average debt ratios as a function of the accounting terms. The black line in the lower panel thus equals the slope of the blue line in the upper panel.

During the global financial crisis, debt levels rose sharply across advanced economies. This was principally due to a sharp increase in primary deficits from 2009 onwards (blue bars) plus a sharp decline in real growth and inflation in 2008–9. Primary deficits fell until 2015, slowing the rate at which the debt ratio grew.

Since 2015, primary deficits and interest payments have been very close to pre-crisis levels. Yet debt ratios are stable at much higher levels. This implies that higher primary deficits during the crisis broadly offset the gains from lower interest-growth differentials.

More recently, debt ratios have started to drift down (since around 2016). Mechanically, this has been due largely to higher nominal growth. Yet, as the analysis in the main text highlights, this underplays the role of fiscal consolidation in recent years. Fiscal policy was expected to be mildly expansionary over these years but has instead been broadly balanced (and very heterogeneous across advanced economies).

Annex Figure 2.2.4. Relative Average Public Debt to GDP and Decomposition, G7 + Spain



Source: IMF staff calculations.
 Note: Public debt is measured by general government gross debt. Average decomposition across advanced economies is shown for the bottom panel. G7 = Group of Seven (Canada, France, Germany, Italy, Japan, United Kingdom, United States).

Forecasting $r - g$

This section outlines the methodology used in Box 2.2 and Figure 2.2.1, describing how to isolate the common international component of $r - g$ and how to construct its forecast.

Data

The interest rate–growth differential used for forecasting is the difference between the annual average short-term policy rate and the annual nominal growth rate.

This choice of interest rate has several advantages. First, it strips out variation in risk premia, which are often endogenous to fiscal policy. It is also clear that one can compare the annual short-term rate to the annual nominal growth rate (with longer maturity bonds the comparison is to equivalently longer periods of growth).

Nevertheless, this approach does suppress other sources of variation not due to short-term policy rates, including term premia. Yet this is not overly costly when studying the long-term variation in interest rates. The long decline in government interest rates since the early 90s has been driven overwhelmingly by reductions in future expected policy rates, not term premia.

This source of variation in $r - g$ is interpreted as the primitive shock driving realized interest rate–growth differentials. For example, the mapping of changes in policy rates into debt levels in the preceding subsection is done so through a yield curve which includes various premia. But the fluctuations in that yield curve are only due to current and future expected short-term policy rates.

Theoretical Basis

The Euler equation in the Ramsey-Cass-Koopmans and associated models relates asset prices to investors' valuation of their payoffs (Galí 2008). One application of this to nominal risk-free interest rates (when investors' preferences have logarithmic preferences over consumption) yields:

$$r_t = E_t g_{t+1}^c + E_t \pi_{t+1} + \text{constant},$$

where g_{t+1}^c is investor consumption growth, π_{t+1} inflation and E_t the time t expectation operator. Then if the expected share of consumption in output is constant, $E_t g_{t+1}^c = E_t g_{t+1}^y$ where g_{t+1}^y represents real output growth. Substituting this into the expression for the interest rate and using that nominal growth is the sum of real growth and inflation, one gets that:

$$r_t - g_{t+1} = (E_t g_{t+1}^y - g_{t+1}^y) + (E_t \pi_{t+1} - \pi_{t+1}) + \text{constant}.$$

Thus, the variation in the realized interest-growth differential is driven by the sum of forecast errors on inflation and real growth.

Annex Figure 2.2.5 performs this decomposition for a sample of advanced economies using WEO forecasts for expectations. Errors to growth and inflation explain much of the short-term variation in the interest-growth differential, including during the global financial crisis. Yet the residual (in blue) declines slowly throughout the period, suggesting that slow-moving common forces drive deviations of realized $r - g$ from the theoretical predictions.

Empirical Specification

The following equation is estimated using an unbalanced sample of 15 countries starting between 1871 (11 countries) and 1914 and ending in 2019,

$$(r_{i,t} - g_{i,t+1}) = \alpha_i + \delta_t + \beta^\pi \pi_{i,t+1}^* + \beta^g g_{i,t+1}^* + \epsilon_{i,t}$$

where $\pi_{i,t+1}^*$ and $g_{i,t+1}^*$ are expectations of inflation and real growth respectively, α and δ are country and time fixed effects respectively, and ϵ is a mean zero error term. The inflation and real growth expectations are constructed in two steps.

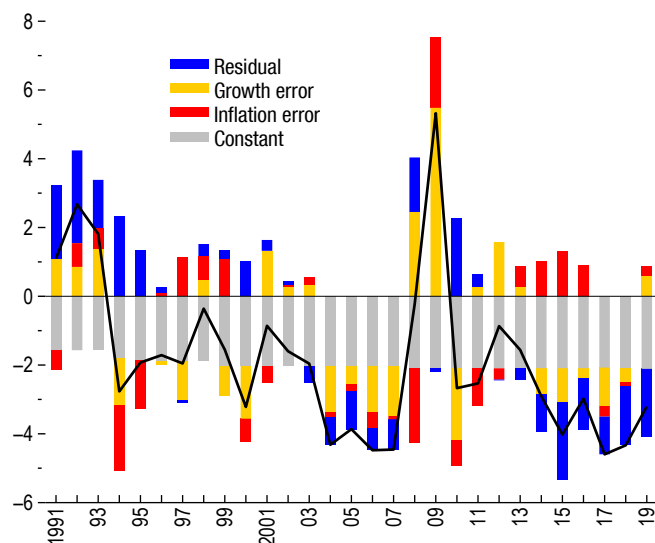
First, averages are taken over plausibly distinct monetary eras: Pre-WW1 (1871–1913), WW1 (1914–1918), Interwar (1919–1938), WW2 (1939–1945), Bretton Woods (1946–1971), post-Bretton Woods (1972–1990), Global Financial Integration (1991–2007), Global Financial Crisis and aftermath (2008–2019). Second, these era averages are filtered using an equally-weighted five-year moving average to prevent sudden jumps in the expectation series. Forecast errors for inflation and growth are then the difference between the expectation and the realized outcome.

The time fixed effects therefore capture the extent to which the simplest Euler equation fails to explain the data, sometimes termed “wedges” between theory and reality. Given that the Euler equation framework has strong implications for predictability (specifically, that interest rate–growth differentials are unpredictable white noise), the time fixed effects therefore measure the component of the data which has information about future international trends in interest rate–growth differentials. Moreover, the drivers of these time fixed effects must be factors which are not captured by the simple Euler equation relationship between growth and interest rates. This point motivates the choice of candidate drivers described in Box 2.2 and underlying Figure 2.2.2.

Regression Results

Annex Table 2.2.2 shows the results of regressing nominal interest rate–growth differentials on forecast errors for inflation and real growth. Several points are worthy of note. First, that the magnitude of coefficients on the forecast errors are statistically indistinguishable from unity in the first column when there are no additional explanatory variables included. The simple Euler equation cannot be rejected in the text. Further below, more explanation is provided on the additional columns and their interpretation.

Annex Figure 2.2.5. Average Contributions of Inflation and Growth Surprises to $r - g$
(Percentage points)



Source: IMF staff calculations.
Note: Expectations for inflation and growth calculated from one-year ahead WEO forecasts. The average decomposition across advanced economies is shown.

The remaining specifications include country-specific potential drivers of interest rate–growth differentials. As time fixed effects capture international determinants of the interest rate–growth differential, these should be interpreted as factors which might drive interest rate–growth differentials in a given country in ways distinct from the forces shaping international trends.

These other factors are mostly insignificant, suggesting that country-specific trends in, for example, productivity or demographics, have little impact on individual countries’ interest rate–growth differentials. Instead, global trends are likely a more important determinant. These non-results therefore are an extra motive for focusing on global drivers of interest rate–growth differentials.

Table 2.2.2 also includes details on the share of variance explained by each of the regressors, following the absolute average deviation measure of Sterck (2019). In the baseline case, time fixed effects explain around 20 percent of the total variation. This is quite important, since by construction this is the only component with predictable and cross-country predictive power. It is also very persistent, suggesting that international factors producing unusually low (or high) $r - g$ take many years to dissipate. About 60 percent of the variation come from forecast errors (inflation and growth surprises) and from country fixed effects, but those are transitory and unpredictable, and thus, have no predictive power. Thus, the common component represents almost half of the variability in $r - g$ that can be predicted. That this share increases in the other specifications reflects a negative correlation between the time fixed effect estimated in specification (1) and the time pattern of the additional variables. In contrast, inflation and growth surprises explain around 40 percent of the variation, suggesting that these are an important channel of fluctuations in interest rate–growth differentials.

Annex Table 2.2.2. Interest Rate–Growth Differentials and Forecast Errors

	15 countries: 1871–2019				
	Interest rate-growth differential				
	(1)	(2)	(3)	(4)	(5)
Inflation surprise	–1.022*** (0.117)	–0.368** (0.179)	–0.297** (0.135)	–0.367** (0.179)	–0.364** (0.177)
Growth surprise	–1.082*** (0.156)	–0.742*** (0.198)	–0.792*** (0.164)	–0.750*** (0.202)	–0.776*** (0.175)
Fraction 40–64		0.067 (0.225)	0.025 (0.234)		0.057 (0.224)
Dependency ratio				0.035 (0.133)	
UIP debt gain		–0.098*** (0.036)		–0.098*** (0.036)	–0.098*** (0.036)
UIP error			0.120*** (0.036)		
TFP growth		–0.012 (0.166)	0.015 (0.156)	–0.004 (0.17)	
Labor productivity growth					0.043 (0.137)
NFA-GDP ratio		0.0002 (0.005)	–0.001 (0.004)	0.001 (0.005)	0.0004 (0.005)
Share of variation					
<i>Of which,</i>					
Growth surprise	0.14	0.16	0.17	0.16	0.17
Inflation surprise	0.28	0.11	0.09	0.11	0.11
Country Fixed Effects	0.11	0.09	0.08	0.09	0.09
Time Fixed Effects	0.19	0.32	0.32	0.32	0.32
Residuals	0.28	0.26	0.26	0.26	0.25
Other variables	0.00	0.06	0.09	0.05	0.07
Residual autocorrelation p-value	0	0	0	0	0
Mean within-country residual persistence	0.67 (0.06)	0.35 (0.07)	0.33 (0.09)	0.33 (0.09)	0.33 (0.09)
Observations	2125	1466	1511	1466	1466
R ²	0.848	0.299	0.288	0.299	0.3
Adjusted R ²	0.835	0.223	0.213	0.223	0.224

Source: IMF staff calculations.

Note: Double-clustered robust standard errors in parentheses. Share of variation is computed using the average absolute deviation measure by Sterck (2019).

*p<0.1; **p<0.05; ***p<0.01.

Notable exceptions are the variables related to the uncovered interest rate parity condition (UIP). The UIP condition, states that the difference in safe returns in two countries should be equal on average to the appreciation of the bilateral nominal exchange rate. Systematic violation of this condition suggests that capital cannot flow freely to take advantage of differential returns. Thus, violations of UIP can be interpreted as evidence of financial repression. This motivates both the inclusion of the UIP error (defined as the exchange rate differential relative to the US less bilateral exchange rate depreciation versus the dollar) and the product of the UIP error with the outstanding debt stock (which measures the fiscal gains from UIP violation). That the coefficients on these terms are negative and significant suggests that country-specific $r - g$ is low when returns on domestic safe currencies are low relative to foreign-currency alternatives. Financial repression is an important part of the story linking this to low $r - g$; without some sort of quantity restriction on capital, low interest rates mean lower saving and growth in future. Empirically, these findings give further weight to the results in Mauro and Zhou (forthcoming).

Note that the coefficient estimates on forecast errors vary as extra controls are added (columns 2–5). This does not invalidate the earlier conclusion that the simplified Euler equation cannot be rejected. Instead, it simply means that country- and time-varying additional controls are absorbing the predictive capacity of the forecast errors, since they are likely to be correlated with them. Some of these country-specific forces captured by these explanatory variables operate *through* realized inflation or growth, impacting interest rate–growth differentials consistent with the Euler equation. Indeed, to the extent that such channels have an unpredictable component, projecting them onto inflation and growth forecast errors acts to strip them out of forecasts for $r - g$.

Forecasting the Common International Component of $r - g$

To forecast the international component of the interest-growth differential, the time fixed effects are isolated from the estimated form of specification (1). The resulting time series is then fitted with an autoregressive integrated moving average (ARIMA) model. The lag structure is chosen using an Akaike Information Criterion. This selects a model with three autoregressive lags and two moving average terms and no unit root. The long-term persistence is estimated to be 0.87, suggesting a half-life of five years for a unit shock.

Drivers of $r - g$

Data

The exercise on the long-run drivers of $r - g$ uses linear regression with the common international component of $r - g$ as a dependent variable. The choice of explanatory variables is based on the literature—see in particular, Andrade and others (2019), Gordon (2015), Eggertsson, Mehrotra, and Robbins (2019), and Chapter 3 of the 2014 WEO, among others. The sources for these variables are the following:

- *Long-run total factor productivity (TFP) data.* These come from the long-run productivity database v2.3 (see Bergeaud, Cetto, and Lecat 2016 for details). A subset of the series is matched to the 15 advanced economies used in the $r - g$ forecasting exercise and aggregated. Robustness checks were conducted using measures of labor productivity in the same dataset.

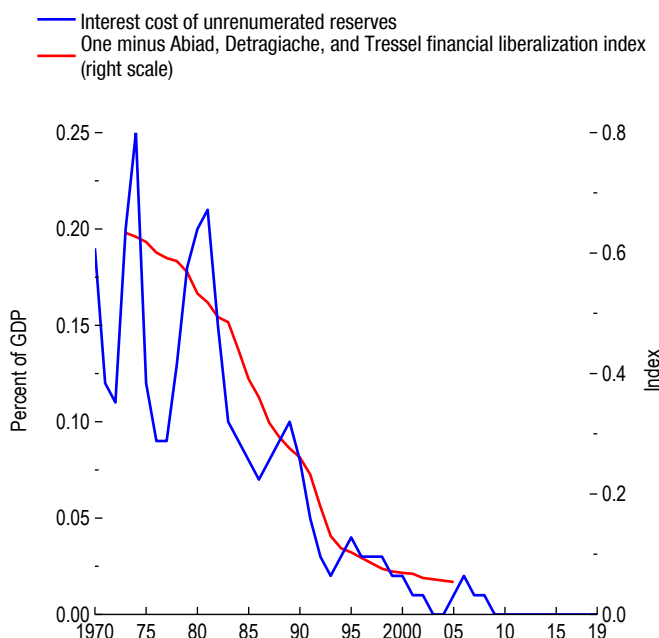
- *Global share of middle-aged.* Global population age shares come from the UN’s 2019 Revision of World Population Prospects. Advanced economy shares are calculated by aggregating country-specific age shares from the Human Mortality Database.
- *Share of emerging market and developing economies in the global economy.* World Economic Outlook (WEO) data, are spliced with data from the Maddison Project 2018. Robustness checks also use cumulated current account deficits as a proxy for emerging market and developing economy asset holdings in advanced economies, computed using data from the Jordà-Schularick-Taylor (2017) Macrohistory Database.
- *Opportunity cost of required reserves in the United States.* This is calculated from:

$$cost_t = (Overnight\ rate_t - interest\ on\ reserves_t) \times \frac{Required\ reserves_t}{GDP_t}$$

This is therefore the savings to the Federal Reserve from paying interest on reserves below the market overnight rate, expressed as a fraction of GDP. Since 2009 the Federal Reserve has paid interest on reserve equal to the federal funds rate, so this cost has been zero. Prior to 1955, the overnight rate is measured as the Federal Reserve Bank of New York’s discount rate. Subsequently, the federal funds rate is used. In robustness checks the required reserve ratio is also used. An alternative natural measure of financial repression is the UIP differential—the difference in bilateral rates of return between two countries after adjusting for nominal exchange rate depreciation.⁴ However, this only measures *relative* financial repression

between two countries and cannot be used as a global measure; the average UIP differential is always zero. Instead, the cost of unremunerated reserves is best compared to the financial liberalization index of Abiad, Detragiache, and Tressel (2008). This index uses a variety of measures to compute a single number between 0 and 1 measuring financial liberalization during 1973-2005. As financial repression is simply the opposite of financial liberalization,

Annex Figure 2.2.6. Global Measures of Financial Repression



Sources: Abiad, Detragiache, and Tressel (2008); and IMF staff calculations. Note: The Abiad, Detragiache, and Tressel (2008) index combines eight measures of financial liberalization, shown here as the average for the sample of 15 advanced economies used in Figure 2.2.1.

⁴ If trade in financial and exchange rate markets is free, this will be zero on average. If financial repression acts to depress interest rates without allowing a corresponding depreciation of the exchange rate, this will produce a persistently negative differential. For example, Mauro and Zhou (2020) thus use the UIP differential as a proxy for financial repression.

Annex Figure 2.2.6 displays 1 minus the global annual average of this measure for the overlapping samples. The correlation between the two measures is high, at 0.87, and the timing—increasingly fast liberalization in the 1980s before easing in the 1990s—matches that of the interest cost of unremunerated reserves almost exactly.

Share of Variance

Full results are reported in Annex Table 2.2.3. Note that there are no country fixed effects as this is a single time series and not a panel. Shares discussed in Box 2.2 are computed in line with Sterck (2019) using an absolute deviation metric. Moreover, the shares capture the economic significance of explanatory variables rather than statistical significance. Coefficients on TFP growth and the global fraction of middle age are consistently statistically significant. The emerging market and developing economy share is less reliably statistically significant in the robustness checks, but the variation in the share is large so it still explains a relatively large share of the variance of the dependent variable. In contrast, the proxy of the global level of financial repression is not reliably significant. This is in line with the cross-country regressions, which suggest that financial repression in a country can affect its relative interest rate–growth differential, but not the cross-country average.

Annex Table 2.2.3. International Drivers of Interest Rate–Growth Differentials

	15 countries: 1950–2018							
	Interest Rate–Growth Differentials, common component							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
TFP growth	–0.660** (0.259)		–0.879** (0.350)	–0.331 (0.208)	–0.874*** (0.339)	–0.456** (0.184)	–1.026*** (0.336)	–0.396** (0.175)
Labor productivity growth		–0.750*** (0.233)						
Global fraction 40–64	–0.889*** (0.288)	–0.883*** (0.310)				–0.265 (0.195)	–1.247*** (0.348)	–1.371* (0.737)
Global dependency ratio			0.124 (0.308)					
AE dependency ratio				–1.188*** (0.234)				
AE fraction 40–64					–0.285 (0.322)			
Interest cost of reserves (GDP ratio)	–12.740 (9.447)	–9.429 (9.720)	–2.512 (14.110)	24.277*** (7.695)	–4.385 (10.638)		–8.677 (10.591)	6.593 (7.367)
Unremunerated reserves (GDP ratio)						–1.484*** (0.334)		
EMDE GDP share	0.290*** (0.057)	0.257*** (0.063)	0.143 (0.090)	0.126*** (0.042)	0.164* (0.091)	–0.038 (0.079)		–0.053 (0.077)
AE cumulated CA deficit							–0.212*** (0.059)	
Constant	3.884 (5.593)	6.026 (5.657)	–14.536 (16.648)	39.029*** (10.453)	–0.972 (9.824)	7.791** (3.307)	25.575*** (7.942)	29.405* (15.762)
Quadratic time trend	No	No	No	No	No	No	No	Yes
Observations	69	69	69	69	69	69	67	69
R ²	0.620	0.641	0.468	0.687	0.476	0.741	0.535	0.768
Adjusted R ²	0.596	0.619	0.435	0.668	0.443	0.725	0.505	0.746

Source: IMF staff calculations.

Note: Newey–West standard errors in parentheses. AE = advanced economy. EMDE = emerging market and developing economy.

TFP = total factor productivity. CA = current account.

*p<0.1; **p<0.05; ***p<0.01.

The importance of the differing drivers of interest rate–growth differentials (Figure 2.2.2) varies over the sample period. For example, the importance of global aging is largely a function of large changes in the last few decades. The significance of these results is also subject to change depending on the sample period (for example, restricting the estimation sample to pre-2007). This is largely a function of the relatively small sample—standard errors increase but point estimates (and, more importantly, their signs) remain broadly stable.

Annex 2.3 Fiscal Multipliers

This section provides details about the estimation of fiscal multipliers during periods of slack and when monetary policy is accommodative. It also presents additional exercises that investigate the channels that might explain why multipliers are higher when the effective lower bound on interest rates is binding and other dimensions of multiplier heterogeneity.

Empirical Model for Government Consumption Multipliers

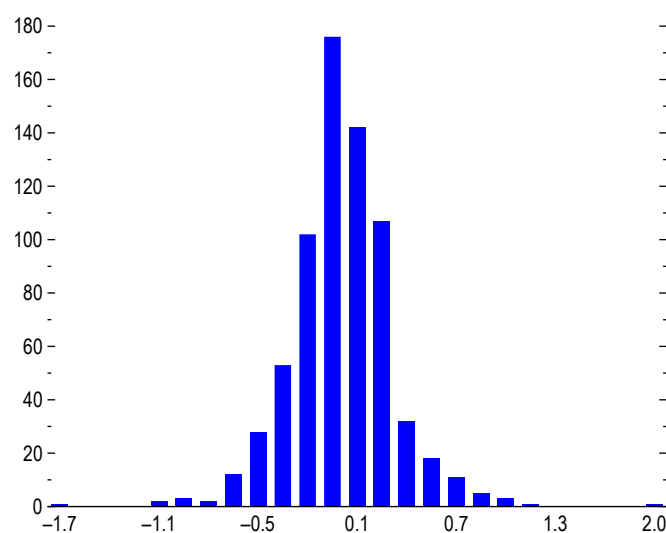
To compute government consumption multipliers, the analysis follows the methodology developed by Auerbach and Gorodnichenko (2012a, 2012b, 2013, 2017) that exploits forecast errors to proxy for the unexpected and exogenous movement in government spending. The *real time* forecast errors FE_{it} of government consumption for country i and time t are defined as:

$$FE_{it} = \% \Delta GC_{it} - E_{t-1}[\% \Delta GC_{it}],$$

where $\% \Delta GC_{it}$ represents the percentage change in actual government consumption in country i and year t (measured in real time using data realized in year $t+1$), and $E_{t-1}[\% \Delta GC_{it}]$ is the forecast for public consumption growth for year t projected in $t-1$.¹

Forecast errors have desirable properties as shocks since they are serially uncorrelated and unanticipated. They also address the problem of fiscal foresight and the importance of anticipation for estimating the effects of government spending shocks (Ramey 2011). To ensure that any remaining predictable component is purged from the estimation, the model also includes a broad set of macroeconomic variables together with other components of spending (for example, transfers) and total revenues. We focus on government consumption, rather than investment, as different components of spending might have different multipliers (Kraay 2012).²

Annex Figure 2.3.1. Distribution of Public Consumption Shocks (Frequency)



Sources: OECD Economic Outlook; and IMF staff calculations.

¹ Comparing forecasts to contemporaneous measures of real time data is important to take account of subsequent data revisions (see Auerbach and Gorodnichenko 2012b)

² As discussed in Boehm (2019), if the actual stimulus measures have a different expenditure composition than the one underlying the estimation for total purchases, the resulting multiplier estimates could provide erroneous guidance for policymakers. The empirical evidence on investment multipliers varies markedly. Chapter 3 of the October 2014 *World Economic Outlook* reports public investment multipliers, at around 1.5. In contrast, Boehm (2019) finds that fiscal stimulus packages with large investment components are less effective, with large falls in private investment after government investment shocks due to crowding out. In an earlier contribution, Perotti (2004) estimates no difference in the effectiveness of public investment versus public consumption in boosting GDP.

The baseline shock definition is based on the OECD Spring forecasts, but the results are robust to using the Fall vintages (see Table 1). The forecast errors FE_{it} are winsorized excluding the bottom 1st and top 99th percentiles to eliminate extreme observations and normalized to transform the shocks into percent of GDP using each country sample average of government consumption as a share of GDP (following Auerbach and Gorodnichenko 2017; Hall 2009; Barro and Redlick 2011). This is useful to capture actual multipliers instead of elasticities (Ramey 2019).

$$share_i^g = \left(\frac{G_{it}}{GDP_{it}} \right)$$

$$Shock_{it} = FE_{it} * share_i^g$$

Annex Figure 2.3.1 reports the distribution of the government consumption shocks scaled with real GDP.³ The distribution of the shocks is centered around zero and varies in a range between 2 and -2 percent of GDP. The sample includes 23 advanced OECD countries.⁴

The baseline specification is the following:

$$\frac{y_{it+h} - y_{it-1}}{y_{it-1}} = \alpha_i^h + \delta_t^h + \beta^h \hat{G}_{it} + \sum_{k=0}^2 \gamma_k^h \mathbf{X}_{it-k} + \varepsilon_{it+h},$$

where y_{it} is the real GDP of country i in year t . The vector of controls \mathbf{X}_{it} includes two lags of the following variables: percentage change in real GDP, the shock itself (to control for any serial correlation, see Stock and Watson 2018 for a discussion), the first difference of government revenues, and of government transfers (proxied by security benefits paid by general government). These controls are all scaled by real GDP lagged by one year. The short-term policy rate, the level of unemployment, the degree of trade openness (measured as imports plus exports divided by GDP), a dummy variable for fixed exchange rates regimes, a linear trend and country and year fixed effects are also included as controls. The baseline estimation employs standard errors clustered at the country level that are heteroscedasticity and autocorrelation consistent. Results are robust to the use of Driscoll-Kraay standard errors that account for cross-sectional dependence (see Table 1). The cumulative multiplier is computed adopting the methodology proposed by Ramey and Zubairy (2018) that uses the shocks as instruments to jointly estimate the response of government consumption and real GDP. \hat{G}_{it} is therefore the predicted government consumption obtained from a first-stage regression where the shock is used as an instrumental variable.⁵ This procedure has the advantage to provide consistent standard errors and to underline the properties of the shocks through the first stage F-statistics.⁶

³ As an alternative, the share is computed with respect to potential output and the resulting distribution is unchanged.

⁴ The countries in the sample are: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Korea, Netherlands, New Zealand, Norway, Portugal, Slovak Republic, Spain, Sweden, Switzerland, the United Kingdom and the United States.

⁵ The two stages are estimated jointly.

⁶ Alternative definitions of multipliers are often used in the literature, such as the *peak to impact* multipliers (Blanchard and Perotti 2002) and *present value multipliers* (Mountford and Uhlig 2009).

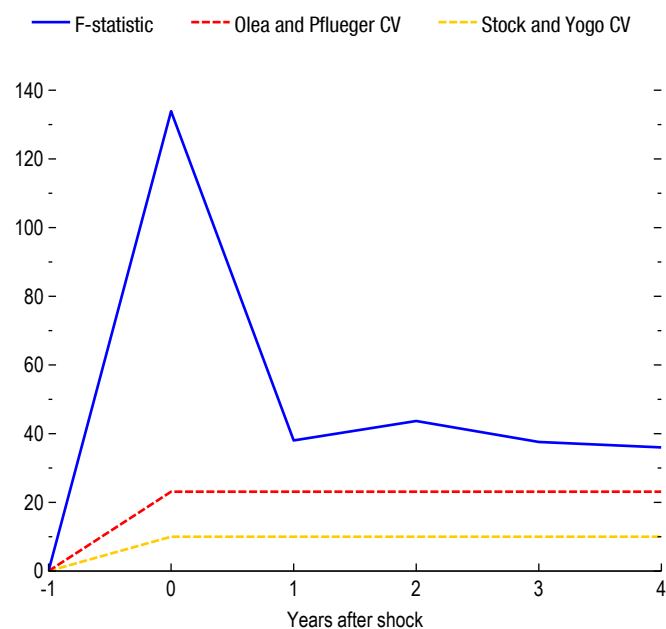
Baseline Results

In line with the literature (see, for instance, Gechert and Rannenberg 2018 and Ramey 2019), the baseline results of the linear model suggest a public consumption multiplier of about 1 throughout the estimation horizon (up to 4 years after the shock hits). The multiplier is different from zero at the 90 percent level in the first 3 periods (see Figure 2.9 in the main text). The first stage F-statistics are above the Stock and Yogo (2005) rule of thumb of 10 throughout the horizon and above the Olea and Pflueger (2013) critical value, which is robust to weak instruments (Annex Figure 2.3.2). This suggests that the forecasts errors are a relevant instrument to predict the endogenous public consumption changes, providing reassurance against weak instruments concerns.

Annex Table 2.3.1 presents a set of robustness tests for the baseline estimation. Column 1 reports the baseline one-year multiplier as a reference. The multiplier excluding the years associated with the global financial crisis (from 2008 to 2010 inclusive) is slightly smaller and equal to 0.95 (Column 2). Column 3

excludes the 1980s, as several countries in the sample start reporting only at the beginning of the 1990s and the results are broadly unchanged. The level of statistical significance is unaffected when Driscoll-Kraay standard errors are computed (Column 4). The one-year multiplier is slightly higher when using the forecasts errors computed with the Fall vintages.

Annex Figure 2.3.2. Instrumental Variable First Stage Tests
(F-statistic on excluded instruments)



Sources: OECD Economic Outlook; and IMF staff calculations.
Note: CV = critical value; dashed lines denote critical values at the 10 percent.

Annex Table 2.3.1. Linear Multiplier Robustness Exercises

	1-year multiplier				
	(1)	(2)	(3)	(4)	(5)
Multiplier	1.240** (0.600)	0.950*** (0.365)	1.360** (0.647)	1.240** (0.613)	1.640** (0.802)
Observations	614	545	599	614	631
R ²	0.631	0.527	0.631	0.631	0.316
Number of countries	23	23	23	23	23
Country Fixed Effect	Yes	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes	Yes
Sample	All	No GFC	No 80s	All	Full
Controls	Baseline	Baseline	Baseline	Baseline	Baseline
Shock	Spring	Spring	Spring	Spring	Fall
Standard errors	Cluster country	Cluster country	Cluster country	Dkraay	Cluster country
F-stat	54.719	48.324	50.260	36.865	38.306

Source: IMF staff calculations.
*p<0.1; **p<0.05; ***p<0.01.

Multipliers During Slack and When Monetary Policy is Accommodative

To estimate state-dependent fiscal multipliers, the procedure proposed by Ramey and Zubairy (2018) is adopted and the baseline estimation is augmented with interaction terms to proxy for different states of the economies captured by an indicator variable I_{it} .

$$\frac{y_{it+h} - y_{it-1}}{y_{it-1}} = \alpha_i^h + \delta_t^h + \beta^h \hat{G}_{it} * I_{it-1} + \beta \lambda^h \hat{G}_{it} * (1 - I_{it-1}) + \sum_{k=0}^2 \gamma_k^h X_{it-k} + \varepsilon_{it+h}$$

Annex Table 2.3.2
Column (1) presents the results with the indicator variables taking the value of 1 when unemployment is above the country-specific median, whereas in Column (2) an alternative definition is adopted using the country specific mean. In Column (3), instead, recessions are defined following the Harding and Pagan business cycle algorithm (see also Figure 2.9 in the main text). The first stage F-statistics exceed the rule of thumb value for instrument strength of 10.

The results reported in the first two columns indicate that multipliers are above one during periods of slack, however only when slack is defined using the country-specific mean the point estimates under the regimes are statistically different from each other (see last row of Annex Table 2.3.2, where a t-test on the coefficient difference is reported). The findings are not suggestive of higher multipliers during recessions (Column 3). Overall, the adopted specification in a cross-country setting with yearly data is relatively demanding and this might explain the lack of statistical significance in some of the results. The literature also finds mixed results on the role played by slack. Ramey and Zubairy (2018) do not find evidence of higher multipliers in periods of slack in the United States. In contrast, Auerbach and Gorodnichenko (2012b) report multipliers of 2.2 in recessions and -0.3 in expansions.

Annex Table 2.3.2. One-Year Multipliers During Different Business Cycle Phase:

	1-year multiplier		
	(1)	(2)	(3)
Unemployment below country-specific median	1.000* (0.604)		
Unemployment above country-specific median	1.410* (0.717)		
Unemployment below country-specific mean		0.540 (0.689)	
Unemployment above country-specific mean		1.710** (0.728)	
Expansion			0.900 (0.726)
Recession			0.790 (0.934)
Observations	614	614	614
R ²	0.634	0.639	0.666
Number of countries	23	23	23
Country Fixed Effect	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	No
Sample	Full	Full	Full
Shock	Spring	Spring	Spring
Standard errors	Cluster country	Cluster country	Cluster country
F-stat	23.974	31.467	21.722
P-value of difference	0.385	0.026	0.918

Source: IMF staff calculations.

*p<0.1; **p<0.05; ***p<0.01.

The same empirical strategy is used to tease out the role of monetary policy accommodation in determining the success of discretionary fiscal actions.⁷ In this case the indicator variable takes the value of one when the short-term policy rate is below 0.75 to proxy for the effective lower bound (see Boehm, 2019). Column (1) in Annex Table 2.3.3 indicates a multiplier of above two when monetary policy is constrained by the effective lower bound (ELB) on interest rates. It is important to note that

Annex Table 2.3.3. One-Year Multipliers when Monetary Policy is Accommodative

	1-year multiplier		
	(1)	(2)	(3)
No ELB	0.530 (0.579)		
ELB	2.590** (1.320)		
Flexible exchange rates		0.090 (0.315)	
Fixed exchange rates		2.050** (1.042)	
Pre-GFC			-0.280 (0.687)
Post-GFC			2.920* (1.604)
Observations	614	614	614
R ²	0.657	0.650	0.520
Number of countries	23	23	23
Country Fixed Effect	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	No
Sample	Full	Full	Full
Shock	Spring	Spring	Spring
Standard errors	Cluster country	Cluster country	Cluster country
F-stat	22.754	19.085	24.991
P-value of difference	0.062	0.048	0.004

Source: IMF staff calculations.

Note: GFC = global financial crisis; and ELB = effective lower bound on interest rates.

*p<0.1; **p<0.05; ***p<0.01.

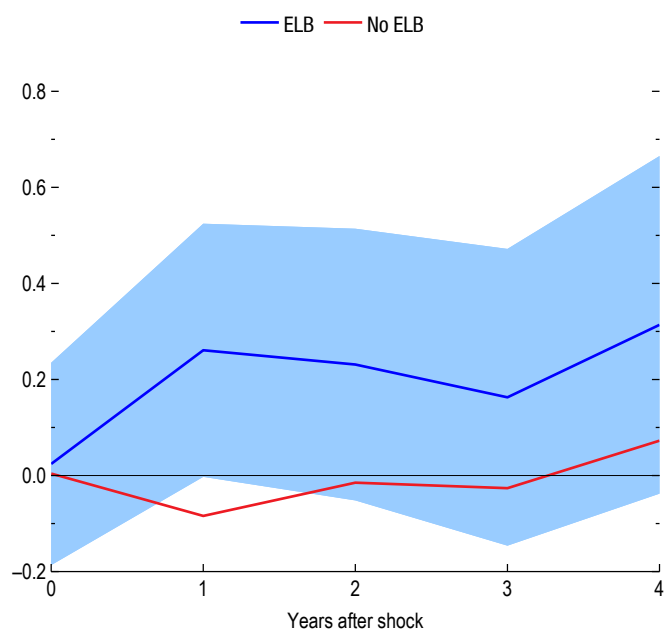
statistical significance is affected by the error structure and the inclusion of the lags of the shock itself. The economic magnitude of the effect, however, is in the range of theoretical estimates of fiscal multipliers at the ELB. A series of theoretical papers reports multipliers substantially higher than one at the ELB, in the range of 2 to 5 (Christiano, Eichenbaum, and Rebelo 2011; Coenen and others 2012; Eggertsson 2011; Woodford 2011). The empirical evidence is still relatively limited given lack of data. Estimates for the US, where time series are longer and include more instances of interest rates at the zero lower bound suggest a multiplier of 1.5 (Ramey and Zubairy 2018). In the case of Japan, Miyamoto, Nguyen, and Sergeyev (2018) report results similar in magnitude. The results of Amendola and others (2019) who exploit shadow rates in the Euro area are suggestive of multipliers between 1.6 and 2.8 at the ELB.

Excluding Japan from the sample does not change the results (the effect at the ELB is equal to 2.25). Given the relatively few observations at the ELB (about one-third of the country-year panel) and given the challenges in disentangling episodes of ELB from recessions, an alternative exercise considers fixed exchange regimes in which monetary policy actions are also constrained. The point estimate in Column (2) exhibits a higher multiplier under fixed exchange rates (for similar results see Ilzetzki, Mendoza, and Végh, 2013). Finally, the evidence on multipliers after the global financial crisis (GFC; Column 2) is also suggestive of higher potency of fiscal stimulus when interest rates are low.

⁷ In this specification year fixed effects are omitted, and recessions—identified using the Harding and Pagan (2002) algorithm where negative growth in a year is a recession—and systemic banking crisis are explicitly controlled for. The specification that includes year fixed effects and the baseline controls provides a multiplier at the effective lower bound of around 3.

Why would fiscal policy be more potent when monetary policy is at the ELB? First, when the economy is at the ELB, interest rates do not rise in response to fiscal stimulus and hence the normal crowding-out channel does not operate. Second, the increase in inflation expectations at the ELB permanently reduces real rates and may therefore have a permanent positive effect on aggregate demand (Christiano, Eichenbaum, and Rebelo 2011). Some supportive evidence of this channel is presented in Annex Figure 2.3.3 that reports the impact of the fiscal shocks on one-year ahead inflation expectations from Consensus Economics. In line with the findings of Miyamoto, Nguyen, and Sergeyev (2018) a 1 percent increase in government consumption increases inflation expectations by 0.3 after one year, whereas inflation expectations move around zero in normal times. While the point estimates confirm the role played by inflation expectations, the effect is not statistically significant.

Annex Figure 2.3.3. Response of One-Year Ahead Inflation Expectations (Units)



Sources: Consensus Economics; Organisation for Economic Co-operation and Development; and IMF staff calculations.
 Note: The blue solid line plots the response of inflation expectations to a public consumption shock equal to one percent of GDP under ELB. The red line corresponds to the response during no-ELB periods. The shaded area corresponds to 90 percent confidence interval. ELB = effective lower bound.

Alternative State-dependent Multipliers

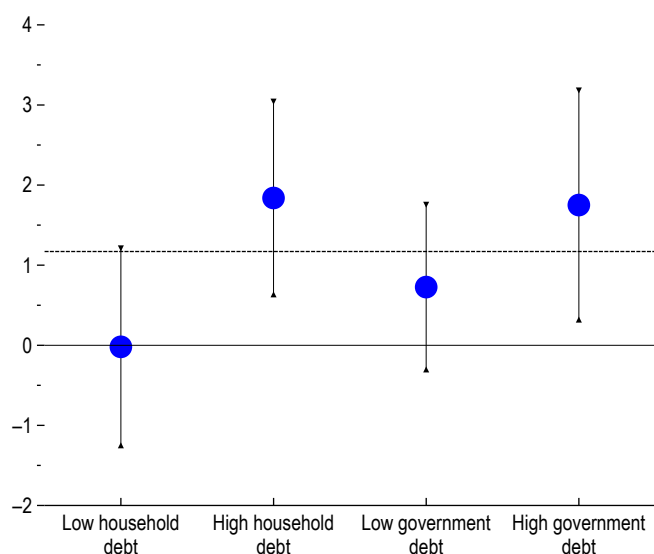
A final empirical exercise computes fiscal multiplier during period of high household and government debt. In line with the findings of Bernardini and Peersman (2017) and Klein (2017) multipliers are higher when household debt is large (Annex Figure 2.3.4).⁸ This confirms the idea that expansionary fiscal policy is more potent when consumer deleveraging is high due to a higher propensity to spend (Eggertsson and Krugman 2012).⁹ A series of paper have also been asking whether public debt might play a role in determining the size of the multiplier. Discretionary measures in an economy characterized by high debt might trigger fiscal sustainability concerns that in turn raise the cost of borrowing and hence fail to boost output. Ilzetki, Mendoza and Vegh (2013) find that a stimulus may be less effective in economies with a public debt overhang in a sample of 44 advanced and emerging economies. Corsetti, Meier and Müller (2012), in a sample of OECD countries between 1975 and 2008, find some evidence that

⁸ For evidence at the subnational level, see Bernardini and Peersman (2017) and Demyanyak, Loutschina, and Murphy (2019).

⁹ Sahm, Shapiro, and Slemrod (2015), on the contrary, argue that deterioration of balance sheets may reduce the effectiveness of fiscal stimulus to boost consumer spending.

high public debt (above 100% of GDP) reduces fiscal multipliers, however the difference in the two sets of multipliers is not statistically significant.¹⁰ Auerbach and Gorodnichenko (2017) also find little variation in the size of multipliers across different public debt states for OECD economies, whereas interest rates and CDS spreads tend to increase more in economies with high debt.¹¹ In line with the literature, the variation in the estimated multipliers across high and low-debt states is too limited to achieve definitive conclusions. (Annex Figure 2.3.4).¹²

Annex Figure 2.3.4. Fiscal Multipliers during Times of High Debt
(Real output effect)



Sources: Consensus Economics; Organisation for Economic Co-operation and Development; and IMF staff calculations.

Note: The black dashed line corresponds to the baseline linear multiplier. The blue dots plot the point estimates and the black whiskers are 90 percent confidence intervals. High and low debt states correspond to above and below the country-specific median.

¹⁰ Corsetti, Kuester, Meier, and Muller (2013) study the sovereign risk channels in a theoretical model and find that high debt has an effect on fiscal transmission only if monetary policy is constrained.

¹¹ The point estimates in the output regressions are larger under high-debt states than under low-debt ones. Other papers investigating the link between the state of public finances and the size of multipliers are Nickel and Tudyka (2013) for 17 European countries from 1970 to 2010, and Huidron, Kose, Lim and Ohnsorge (2019) in a sample of 19 advanced and 15 developing economies. Broner, Clancy, Erce, and Martin (2018) study both theoretically and empirically the link between foreign debt holdings and multipliers.

¹² High-debt states are defined when the debt is above the country-specific median.

Annex 2.4 Model-Based Analysis of Rules-Based Fiscal Stimulus

The analysis of the rules-based fiscal stimulus is carried out using the IMF's G20MOD. The model is one of the modules in the Flexible System of Global Models (FSGM), described in detail in Andrieu and others (2015a).

In the model, output is produced using capital and labor, with labor provided by households. Investment is driven by decisions of profit-maximizing and forward-looking firms, subject to investment adjustment costs, resulting in a version of the Tobin's Q model. The cost of borrowing of firms is affected by an endogenous risk premium, which increases in a downturn (financial accelerator). Labor is provided by households at the market wage, with households choosing the rate of labor-force participation.

Private consumption in G20MOD is driven by two types of households: (i) optimizing overlapping-generations (OLG) households with access to financial markets and (ii) liquidity-constrained households. Liquidity-constrained households consume their full disposable income every period. Their disposable income is formed by their after-tax labor income, transfers from the government, and received remittances, if applicable.

Imports of goods and services are driven by the relative prices of domestic and foreign goods and import requirements of domestic consumption, investment, government expenditures, and exports. Exports of goods and services are given by trading partners demand for imports.

There is a fully specified fiscal sector with multiple policy instruments. There is a full stock-flow accounting of fiscal policy with a fiscal balance that accumulates to a stock of debt and reflects the interest rate costs. Fiscal policy stabilizes debt as a percent of GDP in the long run. On the revenue side, households are subject to a labor-income tax, an ad-valorem consumption tax, and lump-sum taxes. Firms pay capital income taxes. On the expenditure side, the model features government consumption, productive government investment, transfers to households, and the interest cost of the outstanding government debt.

Public investment cumulates into public capital stock, which acts as a positive private-sector productivity spillover. Government can differentiate between general transfers to households or transfers only to liquidity-constrained households. In the model, public consumption does not enter utility of households and has no productivity spillovers.

Monetary Policy

Central bank operates under the inflation-forecast targeting regime, responding to an expected deviation of inflation from its inflation target and to an estimate of the output gap. The monetary policy rate is set to level corresponding to the interest rates implied by the monetary policy rule, $INTP_{RULE}$, unless it breaches a pre-specified effective lower bound (ELB), specified by the value of the interest rate floor, $INTMP_{FLOOR}$. The formulation for the monetary policy is thus:

$$INTMP_t = \max (INTMP_{FLOOR}, INTP_{RULE})$$

In the baseline simulations, when the ELB is reached, no other form of monetary policy easing is considered (i.e. no quantitative easing, etc.) to keep the analysis tractable. While this is a

useful assumption to keep the analysis tractable, the effects of the binding ELB cannot be directly compared to historical experience of economies under the ELB, since quantitative easing and forward guidance have been used. Such policies can be understood in terms of the “shadow” interest rate that would breach the ELB.

Fiscal Policy

To meet its long-run fiscal goals in terms of exogenous debt-to-GDP target, one or a combination of fiscal instruments always needs to adjust or be expected to adjust. The adjustment keeps the government solvent in the long run. The default fiscal rule uses general lump-sum transfers as the adjusting instrument. Over the course of the business-cycle, the exogenous deficit-to-GDP target is met the debt-to-GDP ratio is stabilized in the long run.

The rule explicitly reflects the position in the business cycle (the output gap, \widehat{Y}_t) in the size of adjustment of the deficit-to-GDP, $GDEF_RAT_t$, to its target value:

$$GDEF_RAT_t = GDEF_RAT_t^* + \alpha \widehat{Y}_t.$$

The parameter α reflects automatic stabilizing response of the government and is calibrated using the median estimates from Girouard and Andre (2005) and Price, Dang, and Botev (2015).

Rules-based Fiscal Stimulus

The rules-based fiscal stimulus with explicit macro triggers is implemented on top of the baseline fiscal rule (on top of baseline automatic stabilizers). The fiscal impulse, as a share of output, $IMPULSE_RAT_t$, increases only when the unemployment rate, UR_t , increases above the benchmark value, UR_t^* :

$$IMPULSE_RAT_t = \phi \max(0, UR_t - UR_t^*).$$

The calibration of the cyclical rule draws on, but is not identical to, a proposal by Sahm (2019). When the unemployment rate increases above the benchmark by half of a percentage point, the impulse of 0.7 percent of GDP is automatically triggered. This corresponds to $\phi = 1.4$, which is used for all fiscal instruments to ease comparisons.

The augmented fiscal rule is defined as follows:

$$GDEF_RAT_t = GDEF_RAT_t^* + \alpha \widehat{Y}_t + \phi \max(0, UR_t - UR_t^*),$$

which creates the space for temporary financing by government debt. The other part of the rule specification consists of choosing the fiscal instrument, or a combination of instruments, that are used to stabilize the public finances.

The default instrument in the model is general transfers to all households. Other instruments can be used and each of the used instrument is then augmented by the effect of the cyclical impulse, e.g. for transfers:

$$TR_t = TR_t^* + IMPULSE_RAT_t \times GDP.,$$

where TR_t^* is an exogenous path of the instrument that would prevail without the use of the rules-based fiscal stimulus.

Simulations of Alternative Fiscal and Monetary Policy

To evaluate the response of the economy to changes in rules-based fiscal stimulus, both deterministic and stochastic simulations are carried out. Given the severe non-linearity introduced by the ELB, both the size of the shock and the initial distance of the interest rates from the ELB matter. Should the shock be small enough that the ELB is not hit, it wouldn't be representative. In the same way, the shock shouldn't be unrealistically large. Stochastic simulations help to fully understand the implications of the rule, with an empirically-motivated range of structural shocks.

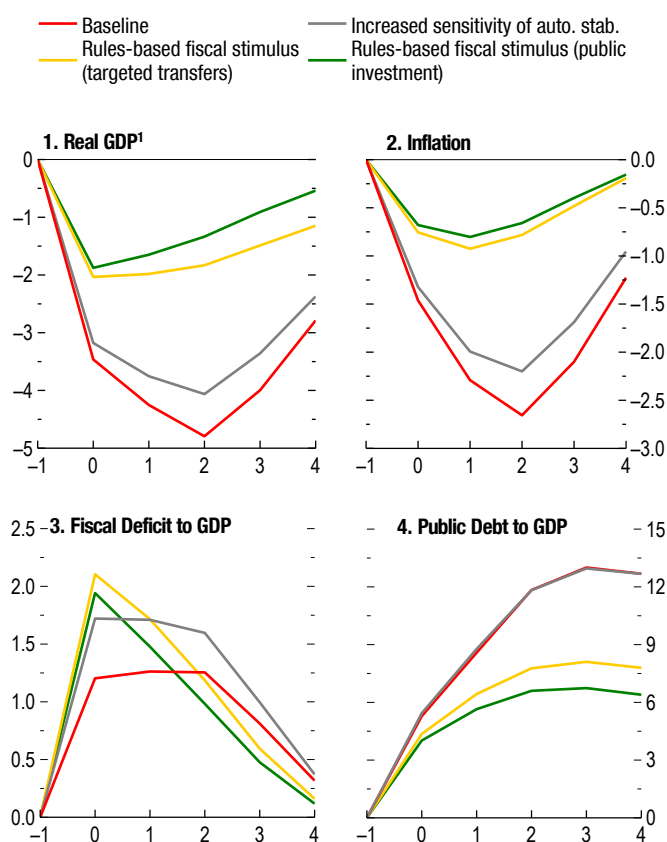
All simulations use a global non-linear solution technique, “stacked-time algorithm”, suitable for simulating large-scale dynamic general equilibrium (DSGE) models with many state variables, see Juillard (1996) for intuition or Hollinger (2008) for the algorithm used.

Simulations (Impulse-Response Function)

The economy is exposed to an adverse demand shock. The shock is transitory but persistent “demand shock”. It is shock to consumption and investment preferences of households and firms, resulting in a typically-observed response of key macroeconomic variables over the business cycle. Consumption and investment both decline, inflation decelerates, and unemployment increases, for instance. While there are no long-run effects of the demand shock, the potential output of the economy temporarily declines with respect to the baseline growth path, due slower capital formation and a lower level of capital available for production.

When the monetary policy is unconstrained, the central bank cuts the interest rates to counter the adverse economic development. When the monetary policy reaches the ELB, it can no longer accommodate the adverse output shock and the downturn of the economy is much steeper.

Annex Figure 2.4.1. Response to Negative Demand Shock at the Effective Lower Bound
(Percentage point deviation from baseline, unless otherwise noted)



Source: IMF staff calculations.
Note: Targeted transfers go to liquidity-constrained households. X-axis represents the number of years after the shock.
¹Percent deviation from baseline.

The “fiscal multipliers” in the model, for given coefficient values, depend on multiple things—the choice of the fiscal instrument, if the shock is transitory or permanent, or if monetary policy is or is not accommodative, for instance—see Andrieu and others (2015b) for details. For the calibration used, a two-year, debt-financed fiscal expansion of 1 percent of GDP using transfers to liquidity constrained household results in the average increase of real output by 0.57 percent in the first two years. When monetary policy is fully accommodative and keeps monetary policy rate unchanged for two years, the fiscal expansion results in an increase of 1.1 and 1.2 percent in the first and second year, respectively. See Figure 2.4.1 for an illustration of how the choice of fiscal instrument affects the dynamic responses of output and other variables. The rules-based fiscal stimulus helps stabilize real output and also avoid a significant decline in inflation from an adverse shock. Together, the improved paths of real output and the price level contribute to more favorable dynamics of the debt-to-GDP ratio (since nominal GDP is higher).

While the effects of the rules-based fiscal stimulus are intimately linked to the effectiveness of the fiscal instrument and the distance of the economy from the ELB, the effects are not identical. On top of standard analysis of fiscal multipliers an implementation of a widely-understood fiscal rule induces an “expectation effect”, where households and firms know about the fiscal rule and they react to the adverse demand shock to a smaller degree, thus lowering the need for the fiscal action itself.

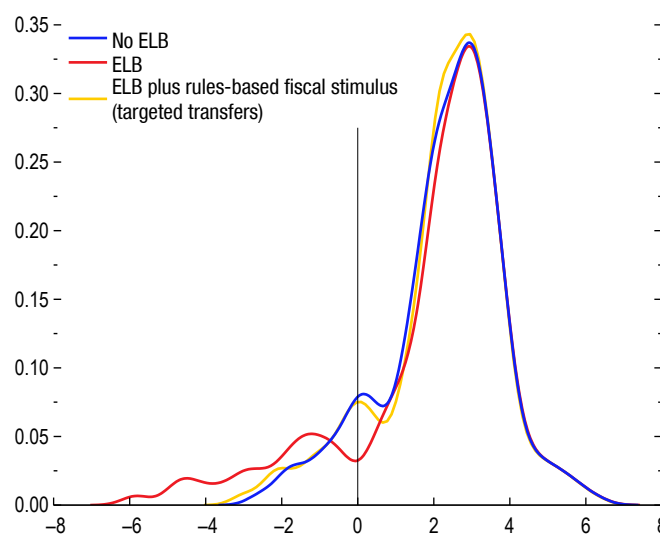
Stochastic Simulations

To extend the impulse-response analysis, the effects of the changes in fiscal and monetary policy framework on the variance and distribution of key macroeconomic variables over the business cycle. This is relevant also due to the inherently non-linear and asymmetric nature of the effective lower bound (ELB) on interest rates and the asymmetric rules-based fiscal stimulus.

The stochastic simulations proceed as follows:

- Using the historical data from 1965—2018 the model is used to estimate the structural shocks. The shocks are estimated by inverting the model and solving for a sequence of unexpected shocks that replicates the observed data. The estimated stochastic shocks also display significant deviations from the Gaussian distribution.

Annex Figure 2.4.2. Distribution of GDP Growth under Alternative Rules-Based Fiscal Stimulus Actions, One Year Ahead (Percent)



Source: IMF staff calculations.
 Note: Targeted transfers go to liquidity-constrained households. Stochastic simulations are used to generate the distribution of output under alternative scenarios. Underlying demand shocks for the stochastic simulations are drawn from the empirical distribution calibrated to the empirical variance and centered at the baseline growth projection. ELB = effective lower bound on interest rates.

- Sampling the shocks from their distribution and simulating the model, the distribution of other variables is obtained. Two variants of stochastic simulations are used: (i) the shocks are assumed to be Gaussian with the standard deviation estimate from the data, and (ii) the shocks are drawn from the kernel density estimate of their unknown distribution function.

The stochastic simulations use $N=500$ draws, each draw consisting of $T=5Y$ periods, around the baseline projection. For all policy alternatives, instrument choices, and ELB or not, it is important to keep the stochastic shocks identical. The change in the resulting distributions and variance of macroeconomic variables are thus purely deterministic reaction to the change in the policy assumptions. See figure 2.4.2 for an illustration of how the distribution of real output growth varies across policy scenarios given the empirical distribution of underlying demand shocks.

The motivation and the methodology for the stochastic simulations are fully detailed in Andrieu and Hunt (2020), with emphasis on estimating structural shocks with large-scale non-linear models, and on the importance of using non-Gaussian, empirical distribution function of shocks for realistic risk assessment.