TECHNICAL NOTES AND MANUALS

A Guide and Tool for Projecting Public Debt and Fiscal Adjustment Paths with Local- and Foreign-Currency Debt

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A Guide and Tool for Projecting Public Debt and Fiscal Adjustment Paths with Local- and Foreign-Currency Debt

Prepared by Santiago Acosta-Ormaechea and Leonardo Martinez

This technical note and manual (TNM) addresses the following questions:

- How can we project the evolution of public debt denominated in local and foreign currency?
- How can we project the effects of intra-year exchange rate fluctuations on public debt?
- Are public debt projections obtained with the government's borrowing requirements (overall balance plus other net debt-creating flows) consistent with public debt projections obtained with the government's net debt issuances?
- How can fiscal adjustment paths be computed to meet any debt target after any adjustment period?
- How can the simple Excel-based Public Debt Dynamics Tool (DDT) be used to project public debt levels (using the government's borrowing requirements) that are fully consistent with the baseline MAC DSA and LIC DSF projections (obtained using the government's net issuances)?
- How can the DDT be used to compute fiscal adjustment paths and debt projections in stress tests and alternative scenarios, including fan charts?

CONTENTS

GLOSSARY

EC . . . . European Commission

GFN . . . . Public gross financing needs

LIC DSF . . . . Low-income countries debt sustainability framework

MAC DSA . . Market-access countries debt sustainability analyses

DDT . . . . Public Debt Dynamics Tool

I. OVERVIEW

This guide presents the analytical underpinnings and a user manual for the Excel-based Public Debt Dynamics Tool (DDT). The DDT projects public debt as a percent of GDP, computes fiscal adjustment paths to achieve a user-defined debt target, performs predefined and customized stress tests, produces fan charts, and presents results in standardized output tables. This tool—developed as part of the capacity development work on macroeconomic frameworks for forecasting and policy analysis of the IMF Institute for Capacity Development—could easily be integrated into a country's macroeconomic framework. Only 10 macrofiscal variables, which are often publicly available, are needed to perform a rich analysis of public debt dynamics, making the DDT accessible for users without detailed information on debt issuances and amortizations.

II. INTRODUCTION

This guide explains how to project public debt and the fiscal adjustment paths necessary to achieve a public debt target using either the primary or the overall fiscal balance, while computing the associated interest expense. The note also explains step-by-step how this can be done using the Excel-based Public Debt Dynamics Tool (DDT). This tool computes fiscal adjustment paths for any user-defined debt target and adjustment period, performs pre-defined and customized stress tests, produces fan charts, and identifies key contributors to changes in public debt.

The debt dynamics equation presented here and used in the DDT differs from other often-used equations because it accounts for the effect of intra-year exchange rate fluctuations in debt projections, which allows us to properly compute the interest expense and the overall balance consistent with any debt path. This is also essential for reproducing—using the government's borrowing requirements (overall balance plus other net debt-creating flows)—the baseline public debt projections obtained using the government's net debt issuances with the IMF / World Bank debt sustainability frameworks for low-income countries (LIC DSF) and market-access countries (MAC DSA). The DDT's debt dynamics equation also enables users to identify how intra-year exchange rate fluctuations affect public debt, a debt driver often left unexplained in debt sustainability frameworks, and to compute fiscal adjustment paths that take into account all exchange rate valuation effects.[2]

The simplicity, precision, and functionality of the DDT allow users to easily understand the basic principles of projecting public debt and fiscal adjustment paths in baseline and alternative scenarios. This simplicity arises because the DDT does not depend on often-hard-to-find financing data,[3] making it ideal for multi-country analyses and for understanding and communicating public debt and fiscal adjustment projections under baseline and alternative scenarios (even in low-capacity environments). The DDT can thus be viewed as a steppingstone for the more complex LIC DSF and MAC DSA. The DDT also complements the MAC DSA and LIC DSF, which do not present the fiscal adjustment paths computed by the DDT. The DDT also includes fan charts with a longer projection horizon than those available in the MAC DSA (fan charts are not available in the LIC DSF), and it allows users to easily adjust the length of the projection period, offering more flexibility than the LIC DSF and the MAC DSA.

Compared with the LIC DSF and the MAC DSA, however, the DDT focuses only on the public-debt-to-GDP ratio. It does not present other solvency indicators (e.g., the external-public-debt to exports ratio), liquidity indicators (e.g., public gross financing needs), or indicators of the profile

2 Our approximation of intra-year exchange rate valuation effects assumes that all transactions affecting the debt stock occur at the average exchange rate, and that no relevant cross-exchange rate movements are present when countries have debt in multiple foreign currencies. This allows us to reproduce the debt projections of the LIC DSF and the MAC DSA. For further details on how intra-year exchange rate fluctuations affect public debt and their empirical relevance see Acosta-Ormaechea (2020).

3 The data used in the DDT can be retrieved from a country's macroeconomic framework, official government reports or the "Baseline debt" worksheet of either the LIC DSF or MAC DSA templates. Typically, the data required for working with the DDT are also available in the IMF WEO database and IMF staff reports.

of public debt (besides the share of foreign-currency debt).[4] The MAC DSA and LIC DSF also present thresholds for debt burden indicators that are used for constructing signals of the risk of debt distress, and a comprehensive set of tools not considered in the DDT for evaluating the realism of assumptions.

The rest of the guide is organized as follows. Section III describes how to project public debt. Section IV details the inputs needed to run the DDT. Section V explains how to compute fiscal adjustment paths to achieve user-defined debt targets. Section VI discusses how the DDT builds alternative scenarios, including through standardized stress tests, customized scenarios, and fan charts. Section VII presents the standardized outputs produced by the DDT. The Appendices discuss the equivalence of debt projections obtained with either the government's borrowing requirements (as in the DDT) or the government's net debt issuances (as in the LIC DSF and MAC DSA), and explain the derivation of the key formulas discussed in this guide and used in the DDT.

[4] A companion tool being developed, the DDT_GFN, projects fully consistent public debt and public gross financing needs (GFN) in baseline and alternative scenarios, including GFN fan charts.

III. BASIC DEBT DYNAMICS

A. The Public Debt Dynamics Equation

A standard public debt dynamics equation is obtained using the government's borrowing requirements, as given by the overall fiscal balance (the primary balance minus the interest expense) and other net debt-creating flows.[5,6] Thus, measuring all terms in nominal values, the evolution of public debt in local currency can be summarized as:

$$D_t = e_t^{eop} D_{t-1}^f + D_{t-1}^d + e_t^{avg} i_t^f D_{t-1}^f + i_t^d D_{t-1}^d - PB_t + OF_t + SF_t^{ier}, \tag{1}$$

where D_t ($\equiv e_t^{eop} D_t^f + D_t^d$) refers to the total stock of gross public debt measured in local currency at the end of period t; D_t^f and D_t^d are the end-of-period t stocks of public debt denominated in foreign and in domestic currency; e_t^{eop} and e_t^{avg} are the end-of-period and average nominal exchange rates for period t (local currency per unit of the foreign currency); i_t^f and i_t^d are the period-t nominal effective interest rates for foreign- and local-currency-denominated debt[7]; PB_t and OF_t refer to the primary balance (primary government revenues minus primary government expenditures) and other net debt-creating flows during period t[8]; and SF_t^{ier} is the stock-flow adjustment due to intra-period exchange rate fluctuations (Section III.E).

B. The Dynamics of the Debt-to-GDP Ratio

Because a country's GDP is an indicator of its capacity to generate resources to service its debt, looking at the debt-to-GDP ratio facilitates assessing the risk of debt distress. For example, if country A has more debt than country B, but also has higher GDP and a lower debt-to-GDP ratio, if everything else is equal, there is less reason to be concerned about country A's indebtedness.

5 The definition of debt presented in this note *excludes* uncalled guarantees and other types of contingent liabilities that do not yet represent liabilities for the sovereign. However, since some countries include such contingent liabilities in their definition of public debt, the DDT allows users to augment the debt dynamics equation to include the exogenous evolution of uncalled guarantees, which are reported in output tables as a stand-alone debt-creating flow.

6 The relation between the overall and the primary balance presented throughout the note assumes that the government's interest expense is net of any interest revenue. The interest revenue is often negligible in many countries.

7 The effective interest rate on foreign-currency debt is given by $i_t^f = \frac{I_t^f}{D_{t-1}^f}$, where I_t^f denotes the interest expense for foreign-currency debt during period t, expressed in foreign currency. For simplicity, it is assumed that all foreign-currency debt and its flow payments are in the same currency. Therefore, since both I_t^f and D_{t-1}^f are expressed in the *same* foreign currency, i_t^f is not affected by exchange rate fluctuations.

8 The primary and overall balance concepts used here are those considered in the related literature and in the MAC DSA and LIC DSF frameworks (e.g., Escolano 2010; IMF 2013; IMF 2018). These concepts belong to the 'old' *Government Finance Statistics Manual* (GFSM1986). Other net debt-creating flows include transactions that affect gross debt but are not captured in the overall balance, such as recognition of contingent liabilities, debt relief, and drawdown of assets. These transactions are sometimes referred to as stand-alone items (e.g., in IMF 2018). The recognition of contingent liabilities and debt relief are treated below-the-line in GFSM1986 but are treated above-the-line in GFSM2014, where they are included in the concepts that replace the overall balance (net lending/borrowing) or the primary balance (primary net lending/borrowing).

Following standard notation, lowercase letters are used to present variables as ratios to GDP (previously we used capital letters to present the same variables in nominal terms). Thus, it is relatively easy to re-write the nominal debt dynamics equation in (1) as a share of GDP (Appendix II):

$$d_t = \varphi_t d_{t-1} - pb_t + of_t + sf_t^{ier}, \tag{2}$$

where d_t is public debt; pb_t is the primary balance; of_t are other net debt-creating flows; and sf_t^{ier} is the stock-flow adjustment due to intra-period exchange rate fluctuations as a percent of GDP. The coefficient for automatic debt dynamics, $\varphi_t \equiv \frac{1+r_t^w}{1+g_t}$, measures how the previous-period debt ratio affects the current-period ratio, where g_t is the real GDP growth rate, $1 + r_t^w = \frac{\alpha_{t-1}(1+i_t^f e_t^{avg}/e_t^{eop})(1+\varepsilon_t^{eop})+(1-\alpha_{t-1})(1+i_t^d)}{1+\pi_t}$ is the (gross) real cost of debt that includes exchange rate valuation effects; π_t is domestic inflation measured with the GDP deflator; $\alpha_t = \frac{e_t^{eop} D_t^f}{D_t}$ is the share of foreign-currency debt in total debt, and $\varepsilon_t^{eop} = e_t^{eop}/e_{t-1}^{eop} - 1$ is the rate of change of the end-of-period exchange rate (or depreciation rate). From here on, we assume that a period is a calendar year.

C. Main Drivers of Changes in the Debt-to-GDP Ratio

To better understand the contributors to changes in the debt-to-GDP ratio, one can subtract d_{t-1} from both sides of (2). Although there are different ways in which changes in the debt ratio can be decomposed, an intuitive representation is given by (a similar decomposition can be found in IMF 2013):

$$\begin{aligned} d_t - d_{t-1} = {} & \frac{i_t - (1+g_t)\pi_t}{(1+g_t)(1+\pi_t)} d_{t-1} + \frac{\varepsilon_t^{eop} + i_t^f \,[(1+\varepsilon_t^{eop})\, e_t^{avg}/e_t^{eop} - 1]}{(1+g_t)(1+\pi_t)} \alpha_{t-1} d_{t-1} \\ & - \frac{g_t}{(1+g_t)(1+\pi_t)} d_{t-1} - pb_t + of_t + sf_t^{ier}, \end{aligned} \tag{3}$$

where $i_t = \alpha_{t-1}(1+i_t^f) + (1-\alpha_{t-1})(1+i_t^d) - 1$ denotes the nominal weighted average effective interest rate *without* exchange rate valuation effects.

In addition to the primary balance, other net debt-creating flows, and the stock-flow adjustment, sf_t^{ier} (the last three terms on the right-hand side of (3), there are three identifiable contributors that explain public debt changes: (i) the contribution of the real effective interest rate (first term on the right-hand side of (3)); (ii) the contribution of the exchange rate (second term); and (iii) the contribution of real GDP growth (third term).

The contributors to changes in the debt-to-GDP ratio can also be presented using the real interest and exchange rates:

$$d_t - d_{t-1} = \frac{\alpha_{t-1} r_t^f + (1-\alpha_{t-1}) r_t^d}{1+g_t} d_{t-1} - \frac{g_t}{1+g_t} d_{t-1} + \frac{\alpha_{t-1} z_t (1+r_t^f)}{1+g_t} d_{t-1} - pb_t + of_t + sf_t^{ier} \tag{4}$$

where $1+r_t^f = \frac{1+i_t^f e_t^{avg}/e_t^{eop}}{1+\pi_t^f}$ and $1+r_t^d = \frac{1+i_t^d}{1+\pi_t}$ are the (gross) real effective rates for foreign- and local-currency denominated debt; π_t^f is the foreign inflation rate (measured by the foreign GDP deflator); and $1+z_t = \frac{(1+\pi_t^f)(1+\varepsilon_t^{eop})}{1+\pi_t}$ is the (gross) rate of change of the real exchange rate. A similar decomposition can be found in IMF (2018).

Note that when looking at historical data for changes in the debt ratio and what contributed to these changes, (3) and (4) may not hold. This may occur because unidentified flows may arise due to cross-exchange rate valuation effects that are not properly accounted for (when there is debt denominated in multiple foreign currencies); intra-period exchange rate fluctuations not captured by the annual difference between the average and the end-of period exchange rate (e_t^{avg}/e_t^{eop}); differences between accrual and cash accounting that can introduce breaks between the government's borrowing requirements and the evolution of the stock of debt; and other accounting issues including countries using the market or face (instead of the nominal) value of public debt. In these situations, a residual term would appear, reflecting the differences between the observed changes in the debt ratio and the identified contributors to these changes.[9]

D. The Overall Balance

While (2) presents the most common representation of public debt dynamics using the primary balance, it is sometimes useful to evaluate public debt dynamics using the overall balance instead. For example, many countries have fiscal rules that limit the size of the government's overall balance, and one may want to know what these rules imply for debt dynamics.

As a percentage of GDP, the primary balance and the overall balance are linked as follows:

$$ob_t = pb_t - \left[\alpha_{t-1} i_t^f (1+\varepsilon_t^{eop})\, e_t^{avg}/e_t^{eop} + (1-\alpha_{t-1})\, i_t^d\right] \frac{d_{t-1}}{(1+g_t)(1+\pi_t)}, \tag{5}$$

where ob refers to the overall balance as a share of GDP.[10] Observe that the second term in the RHS of (5) is the total interest expense as a share of GDP, which also depends on intra-year exchange rate changes as reflected by the term e_t^{avg}/e_t^{eop}. Substituting (5) into (2) and rearranging terms, we obtain

$$d_t = \varphi_t^{ob} d_{t-1} - ob_t + of_t + sf_t^{ier}, \tag{6}$$

where the coefficient that measures how previous-period debt affects current-period debt is now defined as $\varphi_t^{ob} \equiv \frac{\alpha_{t-1}(1+\varepsilon_t^{eop})+1-\alpha_{t-1}}{(1+g_t)(1+\pi_t)}$.

9 For example, in the DDT "Output – Baseline" worksheet discussed in Section VI, a residual term res_t appears when accounting for the historical evolution of the debt-to-GDP ratio. Elements affecting the evolution of the stock of public debt that can be identified should be included in other net debt-creating flows.

10 Note that from fiscal statements one might be able to obtain the total interest expense for both local and foreign-currency debt converted into local currency (Int_t). The total stock of debt (i.e., D_t), is also likely to be reported in local currency. The nominal effective interest rate that can be derived from fiscal accounts should then be equal to $i_t^{fisc} = \frac{Int_t}{D_{t-1}} = \alpha_{t-1} i_t^f (1+\varepsilon_t^{eop}) e_t^{avg}/e_t^{eop} + (1-\alpha_{t-1}) i_t^d$, which is the nominal effective weighted interest rate presented in (5).

E. Stock-flow Adjustment due to Intra-Period Exchange Rate Fluctuations

To illustrate the need for the stock-flow adjustment due to intra-period exchange rate fluctuations, suppose that throughout the year, the government uses resources from its primary balance (a flow) to reduce the stock of foreign-currency debt. To that end, the government exchanges local currency from its primary balance to buy foreign currency to cancel its foreign-currency debt obligations. Thus, to obtain an accurate account of the effect of the primary balance on the stock of foreign-currency debt, it would be necessary to keep track of the exchange rate at which the government buys foreign currency to cancel its foreign-currency debt obligations.

For simplicity, we assume that these transactions occurred at the average exchange rate, e_t^{avg}. This allows the debt projections produced by the DDT with the borrowing-requirements approach to coincide with those produced by the MAC DSA and the LIC DSF with the net-debt-issuance approach.

Formally, since the stock of foreign-currency debt is valued in local currency using the end-of-period exchange rate, e_t^{eop}, to properly capture the effect of the primary balance and other flow variables on the *change* in the stock of foreign-currency debt, this *change* must be corrected by the intra-period fluctuation of the exchange rate. Thus, the stock-flow adjustment due to intra-period exchange rate changes in (1) is endogenized as $SF_t^{ier} = (e_t^{eop} - e_t^{avg})(D_t^f - D_{t-1}^f)$, which is different from zero only if the end-of-period and the average exchange rates differ and the stock of foreign-currency debt changes.[11]

As a percent of GDP, the stock-flow adjustment due to intra-period fluctuations of the exchange rate is given by:

$$sf_t^{ier} = \frac{e_t^{eop} - e_t^{avg}}{e_t^{eop}} \left[\alpha_t d_t - \alpha_{t-1} d_{t-1} \frac{1 + \varepsilon_t^{eop}}{(1+g_t)(1+\pi_t)} \right]. \tag{7}$$

Note that in (7), sf_t^{ier} depends on the level of debt d_t. Therefore, (2) does not present d_t as an explicit function of d_{t-1}. However, we can rewrite (2) as a closed-form debt dynamics equation:

$$d_t = \hat{\varphi}_t d_{t-1} - \mu_t pb_t + \mu_t of_t, \tag{8}$$

where $\hat{\varphi}_t \equiv \frac{1+\hat{r}_t^w}{1+g_t}$, $1 + \hat{r}_t^w$ is the (gross) real cost of debt that fully incorporates the effects of intra-period exchange rate fluctuations, and $\mu_t = [1 - \alpha_t(1 - e_t^{avg}/e_t^{eop})]^{-1}$ is the stock-flow coefficient that adjusts for intra-period exchange rate fluctuations.[12] Equation (8) is the main equation used for projecting the public debt ratio in the DDT.

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[11] To see this intuitively, suppose that a country only has foreign-currency debt, and that interest rates and other net debt-creating flows are zero. Then, the *flow* nominal budget constraint of the government expressed in local currency is given by: $e_t^{avg}(D_t^f - D_{t-1}^f) = -PB_t$ Since the stock of foreign-currency debt is measured using the end-of-period exchange rate, this *flow* budget constraint can equivalently be written as $e_t^{eop}(D_t^f - D_{t-1}^f) = -PB_t + (e_t^{eop} - e_t^{avg})(D_t^f - D_{t-1}^f)$ which is consistent with (1) with $SF_t^{ier} = (e_t^{eop} - e_t^{avg})(D_t^f - D_{t-1}^f)$.

[12] Formally, $1 + \hat{r}_t^w \equiv \mu_t \frac{\alpha_{t-1}(1+i_t^f)(1+\varepsilon_t^{eop})e_t^{avg}/e_t^{eop} + (1-\alpha_{t-1})(1+i_t^d)}{1+\pi_t}$. The (gross) nominal valuation-adjusted cost of debt reported in the DDT is defined as $1 + \hat{\imath}_t^w \equiv \mu_t[\alpha_{t-1}(1 + i_t^f)(1 + \varepsilon_t^{eop})e_t^{avg}/e_t^{eop} + (1 - \alpha_{t-1})(1 + i_t^d)]$.

In (8), the stock-flow coefficient μ_t distributes the stock-flow adjustment term sf_t^{ier} presented in (2) among the flow variables that affect the evolution of debt. Note that μ_t is positive. If $e_t^{avg}/e_t^{eop} = 1$ (there are no intra-period exchange rate fluctuations), μ_t is equal to one. If $e_t^{avg}/e_t^{eop} < 1$ (the currency depreciates during the year) and there is foreign-currency debt, μ_t is larger than one. If $e_t^{avg}/e_t^{eop} > 1$ (the currency appreciates during the year) and there is foreign-currency debt, μ_t is smaller than one.

Coming back to the previous example, if the currency depreciates after the primary balance was used to pay back foreign-currency debt, the effect of the primary balance on the change in the stock of public debt converted to local currency using the end-of-period exchange rate e_t^{eop} would be *larger* than the size of the primary balance itself. This occurs because the primary balance was used to repay foreign-currency debt at the average exchange rate, e_t^{avg}, which is lower than the end-of-period exchange rate, e_t^{eop}, used to estimate the stock of debt. This explains why the effect of the primary balance in the stock of debt needs to be adjusted by μ_t, as shown in (8). The same occurs with other flow variables.

The overall balance: Recall that to obtain the debt dynamics equation with the overall balance and the stock-flow adjustment term sf_t^{ier} in (6), we used the relation between the primary and the overall balance in (5), and the debt dynamics equation (2). We can now use (6) and (7) to derive the debt dynamics equation with the overall balance and the stock-flow coefficient μ_t:

$$d_t = \hat{\varphi}_t^{ob} d_{t-1} - \mu_t ob_t + \mu_t of_t, \tag{9}$$

where $\hat{\varphi}_t^{ob} = \mu_t \frac{\alpha_{t-1}(1+\varepsilon_t^{eop})e_t^{avg}/e_t^{eop}+1-\alpha_{t-1}}{(1+g_t)(1+\pi_t)}$ Because equation (9) closely resembles (8), it is very easy to switch from the primary to the overall balance as needed to assess public debt dynamics.

Local and foreign-currency real interest rates and the real exchange rate: Equation (8) can be re-arranged to highlight the role of the real exchange rate, instead of the nominal exchange, and the real cost of local- and foreign-currency debt, instead of the nominal cost. This is essentially what was done in (4) to understand debt drivers, while presenting the stock-flow adjustment sf_t^{ier} as a separate term. Equivalently, we can rearrange the debt dynamics equation (8), to be explicit about the role of local- and foreign-currency real interest rates, and the real exchange rate:

$$d_t = \frac{\alpha_{t-1}(1+\hat{r}_t^f)(1+z_t) + (1-\alpha_{t-1})(1+\hat{r}_t^d)}{1+g_t} d_{t-1} - \mu_t pb_t + \mu_t of_t, \tag{10}$$

where $1+\hat{r}_t^f \equiv \mu_t \frac{(1+i_t^f)e_t^{avg}/e_t^{eop}}{1+\pi_t^f}$ and $1+\hat{r}_t^d \equiv \mu_t \frac{1+i_t^d}{1+\pi_t}$ represent the (gross) real effective foreign- and local-currency cost of debt. The DDT uses (10) to build fan charts, as discussed in Section VI.

F. Uncalled Guarantees

We have focused on the dynamics of public debt, which comprises financial claims on the government that require payment of principal, interest, or both. Thus, contingent liabilities are excluded from the analysis.

However, governments often report uncalled guarantees and other purely contingent liabilities as part of their total debt, even though these do not constitute a contractual obligation for the government until they are called upon. To the extent there is a way to inform how such liabilities evolve over time, their evolution can be added to the debt projections obtained with the formulas presented previously. For instance, if for a particular country the reported stock of debt includes uncalled guarantees, one could subtract them to project debt using (8), and then add the stock of uncalled guarantees back in the next period, including any exogenously-driven variation of this stock. The DDT allows users to do this.

IV. DDT INPUTS

The first step in using the DDT is to populate country-specific and macrofiscal data. Information and relevant data should only be entered in *yellow-shaded* cells, which are in worksheets with *yellow* tabs. Light *yellow-shaded* cells are used to indicate that the user can select the input from a drop-down menu. *White* cells contain formulas that users should not override. Five different "blocks" of inputs are described next.

A. Basic Inputs

The user needs to go through the pull-down menus in the Readme worksheet under the "Basic Inputs" header, to select:

1. country of interest;
2. the first year of projections;
3. the "Year for medium-term indicators," which is the year for which debt in the historical and constant-primary-balance scenarios, and the medium-term debt-stabilizing primary balance are presented in the Output - Baseline worksheet;
4. coverage of the public sector; and
5. whether uncalled guarantees (and similar contingent liabilities) are included in the definition of public debt. If guarantees are included in total debt, a menu option pops up to specify the type of guarantee, as in the picture below. In such a case, the user should indicate which type of guarantees are included in total debt, an information that is automatically reflected in footnote "1/" of the main output table in the Output - Baseline worksheet.

Basic Inputs	
Select country	Macondo
First year of projection	2021
Year for medium-term indicators	2027
Definition of the public sector	central government
Includes public guarantees?	Yes
Specify or define the guarantees	outstanding amount of loans guaranteed by the central government

The remainder inputs of the Readme worksheet are for fiscal adjustment paths and stress tests. Before populating them, it is recommended that the user fills out the input sheet Input - Data, where key macrofiscal data are included; Input – Data is used to define key variables for computing fiscal adjustment paths.

B. Input – Data

The purpose of the Input - Data worksheet is to provide the template with 10 years of historical annual data and projections for the key macrofiscal inputs that are necessary to project debt-to-GDP ratios. While baseline projections require only data for the last year before the projection period, the inclusion of 10 years of historical data is desirable to use all functions of the DDT (for example, the historical scenario). The projection period is pre-specified to be 12 years, but the user can easily adjust it as needed, though that would have to be done manually by extending the projection horizon in all relevant worksheets. Once these data are entered, the DDT does all necessary calculations by transferring inputs to several interconnected worksheets.

Required DDT inputs are those needed to project public debt using (8), plus uncalled guarantees if these are included in the debt stock, and foreign inflation to build fan charts using (10). Thus, the required annual data include:

i. *Debt-related data:*
   a. stock of total gross public debt, up to the year before the projection period starts (row 9, historical data only);
   b. uncalled guarantees as percent of GDP in local and foreign currency (only needed if these are included in the definition of reported debt, rows 10-11);
   c. share of public debt in foreign currency as percent of total debt (excluding uncalled guarantees, row 12). The template automatically computes the share of local-currency debt (excluding uncalled guarantees) as percent of total debt.

ii. *Macrofiscal indicators:*
   a. average and end-of-period nominal exchange rates (local currency per unit of foreign currency, rows 13-14);
   b. effective nominal interest rates on local- and foreign-currency debt (rows 15-16);
   c. GDP deflator inflation (row 17);
   d. real GDP growth (row 18);
   e. primary balance (row 19);
   f. other net debt-creating flows (row 20); and
   g. foreign GDP deflator inflation (e.g., that of the US) to be used only for fan charts (row 21).

C. Adjustment Paths Inputs

Note that it is advisable to complete this section after the Input - Data worksheet is populated. Under the "Adjustment Paths Data Inputs" header in the Readme worksheet, the user needs to specify the "Public debt-to-GDP target" and the years in which "Adjustment path" starts and ends. For the example, in the figure below, the DDT would compute adjustment paths to achieve a debt ratio of 60 percent of GDP by 2030, with fiscal adjustments that start in 2022.

Adjustment Paths Inputs	
Debt target and adjustment period	
To be completed after "Input - Data" Sheet is populated	
Public debt-to-GDP target (e.g., for 60 percent of GDP write 60)	60.0
Adjustment path starts	2022
Adjustment path ends	2030

For the fiscal adjustment scenarios, the DDT allows users to choose values of key macrofiscal inputs of the debt dynamics equation that are different from the baseline values in Input – Data. For example, users may understand that because of the fiscal adjustment needed to achieve the debt target, GDP growth may be lower than in the baseline, reflecting assumptions about fiscal multipliers.[13] The user can input its preferred assumptions into the DDT using either constant or "time-varying values" for the key variables presented in the figure below.

Values of Key Variables for Adjustment Paths (for time-varying values please review cells F29:S38)			
		Manually-set constant values	Drop-down menu constant values
α = Share of foreign currency debt	time-varying values		
ε = Depreciation rate	time-varying values		
i^d = Local-currency effective interest rate	time-varying values		
i^f = Foreign-currency effective interest rate	time-varying values		
π = Inflation rate	time-varying values		
g = Real GDP growth rate	10-year projection average		3.81
of = Other net debt-creating flows	10-year projection average		0.90

Choosing constant values for fiscal adjustment paths implies that key variables would take the same value throughout the adjustment period. Pre-defined options for constant values include selections that reflect either historical data or projection assumptions (10-year or 5-year historical averages; 5-year or 10-year projection averages; or latest year before projections start), or to "Set constant value manually" in the *yellow-shaded* cells in column C, rows 31-37. As illustrated in the figure above, if one of the historical or projected values is chosen, column D, rows 31-37, automatically reproduces the relevant value for each variable. In the figure, the 10-year projection average is selected for real GDP growth and other net debt-creating flows, and the associated values are shown in column D, rows 36-37 (this information may help users find their preferred assumptions).

Under the "time-varying values" option, the DDT computes fiscal adjustment paths using the baseline assumptions in Input – Data plus the "Additions to Baseline Assumptions for Fiscal Adjustment Paths" chosen by the user. For example, a user who believes the adjustment scenario would imply a GDP growth rate that is 1 percentage point lower than the baseline rate must enter "–1" for the corresponding years in row 37, columns H-S.

Baseline Assumptions From "Input - Data" Sheet, plus Additions to Baseline for Fiscal Adjustment Paths	2020	2021	2022	2023	2024	2025
α_t (share excl. guarantees): share of foreign currency denominated debt in total debt, percent of total debt	60.45	63.47	63.47	60.30	57.28	57.28
e_t *(LCU/FCU, avg)*: nominal average exchange rate, local currency per unit of foreign currency	24.58	25.02	25.48	25.97	26.44	26.89
e_t *(LCU/FCU, eop)*: nominal end of period exchange rate, local currency per unit of foreign currency	24.11	24.30	24.47	24.96	25.46	25.97
i_t^d : nominal effective interest rate on local currency denominated debt, percent	10.39	9.87	9.38	9.84	10.34	10.34
i_t^f : nominal effective interest rate on foreign currency denominated debt, percent	3.60	3.78	3.78	3.59	3.41	3.41
π_t : GDP deflator inflation, percent	4.01	4.04	3.65	3.76	3.74	3.72
g_t : Real GDP growth, percent	-8.99	4.23	3.75	3.41	3.66	3.84
of_t *(other net debt-creating flows)*: Other net debt creating flows, percent of GDP	0.00	0.90	0.90	0.90	0.90	0.90

Additions to Baseline Assumptions for Fiscal Adjustment Paths (for time-varying values)	2020	2021	2022	2023	2024	2025
α_t (share excl. guarantees): share of foreign currency denominated debt in total debt, percent of total debt						
e_t *(LCU/FCU, avg)*: nominal average exchange rate, local currency per unit of foreign currency						
e_t *(LCU/FCU, eop)*: nominal end of period exchange rate, local currency per unit of foreign currency						
i_t^d : nominal effective interest rate on local currency denominated debt, percent						
i_t^f : nominal effective interest rate on foreign currency denominated debt, percent						
π_t : GDP deflator inflation, percent						
g_t : Real GDP growth, percent						
of_t *(other net debt-creating flows)*: Other net debt creating flows, percent of GDP						

[13] Of course, this can be an iterative process. The DDT computes fiscal adjustments of a given set of assumptions. If the user believes the implied adjustment would lead to a lower growth rate, this lower growth rate would in turn necessitate a larger adjustment. Assumptions can be updated until the user is satisfied with the implied scenario.

As illustrated in the figure above, for the user's convenience, the resulting values of key variables are reported above "Additions" in the Readme worksheet. Choosing "Additions" equal to zero implies that fiscal adjustments are computed using baseline values.

D. Stress Tests Inputs

The final set of inputs in the Readme worksheet are those needed for the stress test scenarios, which appear under the header "Stress Tests Data Inputs." They include:

1. The size of the shock to each relevant variable: the primary balance, GDP growth, the interest rate, and the exchange rate depreciation. The size of the shock is measured as a scalar factor that multiplies the historical standard deviation of each variable considered (column B, rows 43-46).[14] For instance, in the figure below, –1 reflects that the shocks to the primary balance and GDP growth are calibrated as a reduction in the values of these two variables by 1 historical standard deviation relative to the baseline.

Stress Tests Inputs

Stress testing scenario design: size of shocks	**Size of shocks (number of historical standard deviations)**
Primary balance shock	-1
GDP growth shock	-1
Interest rate shock	1
Exchange rate depreciation shock	1
Stress testing scenario design: duration of shocks	
Primary balance shock starts	2022
Primary balance shock ends	2023
GDP growth shock starts	2022
GDP growth shock ends	2023
Interest rate shock starts	2022
Interest rate shock ends	2023
Exchange rate depreciation shock starts	2022
Exchange rate depreciation shock ends	2023

2. The years in which each shock starts and finishes.

In the DDT, only one variable is shocked at a time. This allows for more transparency about the sensitivity of debt projections to changes in each of the key variables that affect debt dynamics (while keeping everything else as in the baseline).

All selections and the data inputted in the Readme and Input - Data worksheets are automatically transferred into the relevant worksheets of the template. Worksheets with green tabs present debt projections under the baseline and alternative scenarios.

[14] Historical standard deviations consider 10 years of data, except for exchange rate depreciation, for which the nine years of data available are used.

V. FISCAL PATHS TO ACHIEVE A PUBLIC DEBT TARGET

Details about the underlying equations are left to Appendix III. It should be emphasized that all calculations in green worksheets are automatic; the user does not need to input any information in these worksheets but may choose to revise them to look at additional details (all the needed information is entered in yellow worksheets, with the adjustment years, the debt target, and the value of key variables chosen by the user in Readme). Key results from the calculations in green worksheets are presented in the Output - Baseline worksheet.

A. Debt-stabilizing Primary and Overall Balance

The worksheets DebtStabPrimBalance and DebtStabOverallBalance automatically compute the primary and overall balance that would be required to stabilize debt at the level of the previous year, in either the first projection year or the "Year for medium-term indicators" defined in Readme. Results are reported in the Output - Baseline worksheet. See Appendix IV.A for the underlying equation.

B. Constant Primary and Overall Balance to Reach a Debt Target

The worksheets DebtTargetConstant_pb and DebtTargetConstant_ob automatically compute the primary and overall balance levels that if implemented every adjustment year would achieve the debt target. The Output - Baseline worksheet reports the resulting primary balance. See Appendix IV.B for the equation used.

C. Constant Yearly Adjustment in the Primary and Overall Balance to Reach a Debt Target

The constant level of the fiscal balance that allows to achieve the debt target typically implies a significant adjustment (fiscal balance increase) in the first adjustment year, and no adjustment (a constant fiscal balance) thereafter. One may also want to consider a smoother adjustment path that achieves the same debt target. The worksheets DebtTargetConstantAdjustment_pb and DebtTargetConstantAdjustment_ob present the constant fiscal *yearly adjustment* (change in the primary and overall balances) that would achieve the debt target. This adjustment is reported in Output - Baseline. See Appendix IV.C for the relevant equations.

D. Primary and Overall Balance to Reach a Debt Target through Constant Yearly Debt Reductions

Alternatively, fiscal policy can aim at constant annual debt-ratio reductions until the debt target is reached. The worksheets DebtTargetConstantDebtChange_pb and DebtTargetConstantDebtChange_ob present the primary and overall balance paths that would achieve this objective. Since this exercise implies a different fiscal balance for each adjustment year, a summary statistic is not presented in the standardized output. See Appendix IV.D for the relevant equations.[15]

[15] Note that the constant yearly debt reductions are for the stock of debt *excluding* uncalled guarantees. But, as in the other scenarios, the debt target includes uncalled guarantees, if applicable.

E. Customized Adjustment Scenario

To illustrate the possibility of different adjustment modalities, the DDT also allows users to combine a first period with a constant yearly adjustment (as in Subsection C) and a second period with a constant primary balance (as in Subsection B), such that the debt target is reached at end of the second period. To do this, additional inputs are needed in the worksheet CustomizedAdj. The user can set the years in which the first period of adjustment starts and ends in cells B51 and B52; the year in which the debt target is reached in cell D52; and the constant primary balance for the second period in cell B54. The debt target and the value of key variables are as set in Readme. This customized adjustment scenario is set for the primary balance, but a similar exercise could be done for the overall balance.

VI. REALISM, STRESS TESTS, AND FAN CHARTS

A. Historical Scenario

The historical scenario consists of setting key macrofiscal inputs (nominal effective interest rates in local and in foreign currency, growth, inflation, the nominal exchange rate depreciation, the primary balance, and other net debt-creating flows) at their historical averages throughout the projection period. To fully capture historical contributions to debt changes, the average residual res_t is added to other net-debt creating flows.[16] Thus, this scenario tells us what would happen with public debt if key variables behave as in the past. Therefore, the historical scenario can be a useful realism check: a historical scenario with debt levels higher than in the baseline may indicate that baseline assumptions are too optimistic; lower debt levels may indicate baseline assumptions that are too pessimistic. Of course, baseline assumptions that differ from historical averages can often be justified. Assumptions of the historical scenario affect debt dynamics starting in the second projection year. The historical scenario is automatically created in the worksheet HistoricalSc. The debt level in the historical scenario for the "Year for medium-term indicators" (defined in Readme) is reported in the Output - Baseline worksheet (cell W23).

B. Constant Primary Balance Scenario

Throughout the projection period, this scenario sets all key macro fiscal inputs as in the baseline, except for the primary balance that, starting in the second projection year, is set at the level of the first projection year. The scenario is automatically created in worksheet ConstanPBSc, and helps users assess what would happen with public debt if the government does not fulfill the baseline fiscal plan. The debt level in the constant primary balance scenario for the "Year for medium-term indicators" (defined in Readme) is reported in the Output - Baseline worksheet (cell Y23).

C. Shock Scenarios

The worksheets Sh-pb, Sh-g, Sh-int, and Sh-er present shock scenarios for the primary balance, GDP growth, the effective nominal interest rates in local and foreign currencies, and the average and end-of-period nominal exchange rates, respectively. For each of these variables, between the starting and end years for the shock (defined in the Readme worksheet), the shocked variable is equal to its baseline value plus a multiple (also defined in the Readme worksheet) of its historical standard deviations.[17] All other macrofiscal variables take their baseline values.

D. Customized Scenario

The DDT also allows users to create a fully customized scenario in the worksheet CustomizedSc, where any value can be inputted (in the *yellow-shaded* cells) to define the variables that determine the evolution of debt. Possible events modeled in this scenario include natural disasters, failures of public-private partnerships (PPPs), and bailouts of state-owned enterprises (SOEs) and subnational governments.

[16] Historical averages consider 10 years of data, except for the exchange rate depreciation and the residual, for which the 9 years of available data are used.

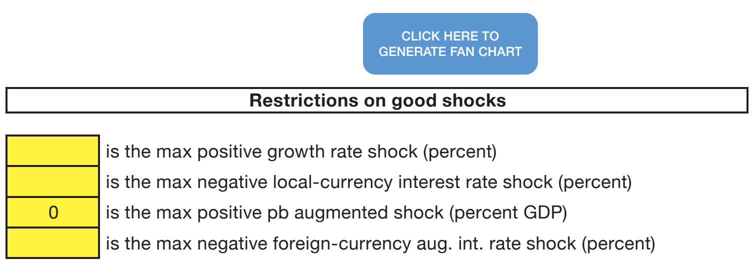
[17] For the exchange rates, the shock is to the depreciation rate.

E. Fan Charts

To generate stochastic simulations, the fan charts included in the DDT follow the template of the 2013 MAC DSA (currently being reviewed). These fan charts summarize the possible evolution of the debt-to-GDP ratio over the medium term by incorporating shocks to relevant macrofiscal variables and ordering the resulting debt paths through confidence intervals. For simplicity, shocks are drawn considering only contemporaneous correlations among variables; intertemporal correlations and autocorrelations of variables are ignored.[18] Fan charts rely on historical annual data of four key variables that determine debt dynamics. These data are used to generate the sample means and the variance-covariance matrix that define the joint normal distribution of shocks.

The four variables that are shocked for building the fan charts are $\hat{r}_t^d$, g_t, the augmented primary balance $pb_t^{aug} = \ \mu_t(pb_t - of_t) - res_t$, where res_t is the residual in period t, and the augmented real foreign effective interest rate $\hat{r}_t^{f,aug}$, which is such that $1 + \hat{r}_t^{f,aug} \equiv (1 + \hat{r}_t^f)(1 + z_t)$.[19] The behavior of these variables can be summarized as $Y_t = \theta + \varepsilon_t$, where $Y_t = \left(\hat{r}_t^d, g_t, pb_t^{aug}, \hat{r}_t^{f,aug}\right)$ the vector θ contains the baseline mean value of the four variables; and $\varepsilon_t \sim N(0,\Omega)$ is a vector of normally distributed shocks, with Ω being the variance co-variance matrix that characterizes the joint statistical properties of the historical contemporaneous interrelations among the variables. Using 1,000 shocks for the key variables, the DDT adds those shocks to the baseline assumptions to compute 1,000 debt paths using (10). These 1,000 debt paths are used to build the fan charts.

Fan charts are built in the worksheet Fan chart. Nine years of historical data and projections for the four variables are automatically populated using baseline assumptions (cells AS7:BA10). Additional years of historical data can be included (cells C7:AR10). Stochastic simulations produce fan charts for a 12-year projection horizon.[20] To generate the fan charts with the relevant data, users must activate an embedded Excel macro by clicking the blue button:



Users can choose to deactivate shocks to any of the four variables (cells B42:45). Note that the symmetric distribution of shocks implies a symmetric fan chart (where debt levels higher than in the baseline scenario are as likely as debt levels lower than in the baseline scenario). But users can also build an asymmetric fan chart by restricting the value of shocks (cells D42:45), as illustrated in the figure. For example, there could be reasons for believing that negative shocks to the primary balance are more likely than positive shocks making an asymmetric fan chart relevant.

[18] For a discussion of alternative methodologies for building fan charts, see Celasun, Debrun, and Ostry (2006) and the IMF Public Debt Dynamics Under Uncertainty online course (DDUx) at edX.org.

[19] Following (10), debt projections for the fan charts are computed using $d_t = \frac{\alpha_{t-1}(1+\hat{r}_t^{f,aug}) + (1-\alpha_{t-1})(1+\hat{r}_t^d)}{1+g_t} d_{t-1} - pb_t^{aug}$, where pb_t^{aug} includes res_t that is zero in the baseline projections but not in the history

[20] This departs from the 2013 MAC DSA which only considers a six-year projection horizon for fan charts.

VII. STANDARDIZED OUTPUTS

Once the user populates all necessary inputs in the yellow worksheets, the DDT automatically produces the baseline and alternative scenarios, and output tables and charts. Following the user's choices, standardized outputs present the name of the country, the coverage of public sector debt, the first year of projections and whether uncalled guarantees are included. The main results are presented in the Output - Baseline and Output - Shocks blue worksheets.

Output - Baseline reports the evolution of public debt in the baseline scenario, based on the public debt dynamics equation (8). Using (3), it also presents a standard decomposition of contributions to debt changes in a table and a chart, reporting the stock-flow adjustment due to intra-period fluctuations of the exchange rate as a stand-alone item.[21] The next figure presents an example of the Output - Baseline worksheet in the DDT. Observe that Output - Baseline includes a "Realism and Fiscal Adjustments" blue box with the following indicators:

i. debt levels over the medium term ("Year for medium-term indicators" in Readme) under the historical and primary balance scenarios;

ii. the debt-stabilizing primary balance in the first year of projections and over the medium term ("Year for medium-term indicators" in Readme);

iii. the level and yearly change in the primary balance needed to reach a debt target after the adjustment period (under parameters chosen by the user in Readme); and

iv. an assessment of the probability of the debt level being below that of the last historical year and below the user-defined debt target (Readme) in the year in which the adjustment path ends (as defined by the user in Readme).

[21] The effective nominal interest rate in the upper panel of the table is computed as in footnote 7. If the user inputted uncalled guarantees, the output table will include those guarantees when reporting public debt dynamics.

Macondo Public Sector Debt Dynamics - Baseline Scenario

(in percent of GDP unless otherwise indicated)

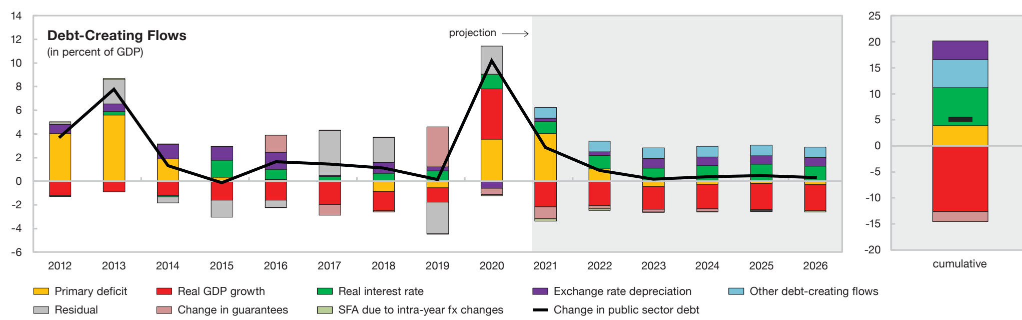
Debt and Economic Indicators[1/]

	Actual		Est.	Projections											
	2012-2018[2/]	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
Nominal gross public debt	44.2	48.7	58.9	61.7	62.6	62.8	63.2	63.7	64.0	64.4	64.7	65.0	65.4	65.7	66.0
Of which: guarantees (uncalled)[3/]	0.3	3.8	3.3	2.3	2.0	1.8	1.5	1.4	1.4	1.5	1.4	1.4	1.5	1.4	1.4
Real GDP growth (in percent)	3.8	2.7	-9.0	4.2	3.7	3.4	3.7	3.8	3.8	3.8	3.8	3.8	3.8	3.8	3.8
Inflation (GDP deflator, in percent)	4.4	4.1	4.0	4.0	3.7	3.8	3.7	3.7	4.0	4.0	4.0	4.0	4.0	4.0	4.0
Nominal GDP growth (in percent)	8.3	6.9	-5.3	8.4	7.5	7.3	7.5	7.7	8.0	8.0	8.0	8.0	8.0	8.0	8.0
Effective interest rate (in percent)[4/]	5.8	6.2	6.3	6.3	5.9	6.0	6.3	6.5	6.5	6.6	6.3	6.5	6.6	6.3	6.5

Contribution to Changes in Public Debt

	Actual		Est.	Projections						
	2012-2018	2019	2020	2021	2022	2023	2024	2025	2026	cumulative
Change in gross public sector debt	2.4	0.1	10.2	2.9	0.9	0.2	0.4	0.5	0.3	5.1
Identified debt-creating flows	1.6	2.8	7.8	2.9	0.9	0.2	0.4	0.5	0.3	5.1
Primary deficit	1.6	-0.6	3.5	4.0	1.1	-0.5	-0.3	-0.2	-0.3	3.9
Automatic debt dynamics	-0.1	0.0	4.9	-0.9	-0.6	0.0	0.0	0.0	-0.2	-1.8
Interest rate/growth differential	-1.0	-0.3	5.5	-1.2	-0.9	-0.8	-0.8	-0.8	-0.9	-5.4
Of which: real interest rate	0.5	0.9	1.2	1.0	1.1	1.1	1.3	1.4	1.3	7.3
Of which: real GDP growth	-1.5	-1.2	4.3	-2.2	-2.1	-1.9	-2.1	-2.2	-2.2	-12.7
Exchange rate depreciation	0.9	0.3	-0.6	0.3	0.3	0.8	0.8	0.7	0.7	3.6
Other identified debt-creating flows	0.0	0.0	0.0	0.9	0.9	0.9	0.9	0.9	0.9	5.4
SFA due to intra-year exchange rate changes[5/]	0.0	0.0	-0.1	-0.2	-0.1	0.0	0.0	-0.1	-0.1	-0.5
Change in guarantees (uncalled)	0.1	3.4	-0.5	-1.0	-0.3	-0.2	-0.2	-0.1	0.0	-1.8
Residual[6/]	0.8	-2.7	2.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Realism and Fiscal Adjustments	
debt levels in 2027: historical and constant pb scenarios	
77.2	88.3
debt-stabilizing primary balance in 2021 and in 2027	
-0.1	0.6
level and yearly change in primary balance to yield 60 percent debt in 2030[7/]	
0.7	0.9
probability of debt below: end-2020 and 60 percent in 2030	
31.3	34.0



Source: Ministry of Finance

1/ Public sector is defined as central government and includes public guarantees, defined as outstanding amount of loans guaranteed by the central government .

2/ Based on available data.

3/ Only includes uncalled guarantees (contingent liabilities) that might be included in the total stock of debt but do not generate interest payments, and have an exogenously-driven evolution.

4/ Defined as interest payments divided by debt stock (excluding guarantees) at the end of previous year.

5/ Calculated as the effect of intra-year exchange rate fluctuations on total debt.

6/ For the history it could include residual changes in gross debt due to accounting issues and cross-exchange rate variations.

7/ Measures the constant annual primary balance and the yearly change in the primary balance (in ppts of GDP) necessary to reach the pre-determined debt target. Adjustment starts in 2022 and ends in 2030.

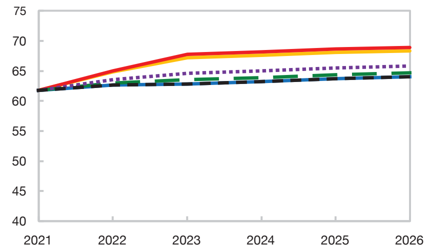
Output - Shocks presents debt dynamics for alternative scenarios, the underlying assumptions for these scenarios, and fan charts. The figure below shows an example of Output - Shocks.

Macondo Public Sector Debt Dynamics - Stress Tests and Alternative Scenarios

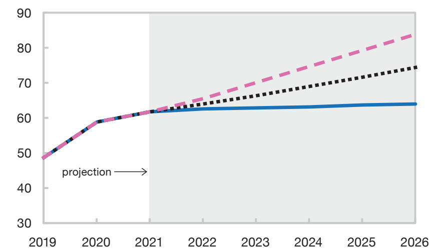
Macro-Fiscal Stress Tests and Alternative Scenarios

Baseline — Primary Balance Shock — Real Interest Rate Shock
Real GDP Growth Shock — Exchange Rate Shock — Customized Scenario
Constant Primary Balance — Historical Scenario

Gross Nominal Public Debt: Stress Tests
(in percent of GDP)



Gross Nominal Public Debt: Alternative Scenarios
(in percent of GDP)



Underlying Assumptions
(in percent)

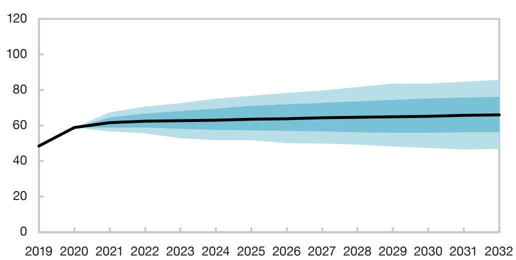
	2021	2022	2023	2024	2025	2026
Baseline						
Real GDP growth	4.2	3.7	3.4	3.7	3.8	3.8
Inflation	4.0	3.7	3.8	3.7	3.7	4.0
Primary balance	-4.0	-1.1	0.5	0.3	0.2	0.3
Effective interest rate	6.3	5.9	6.0	6.3	6.5	6.5
Real Interest Rate Shock						
Real GDP growth	4.2	3.7	3.4	3.7	3.8	3.8
Inflation	4.0	3.7	3.8	3.7	3.7	4.0
Primary balance	-4.0	-1.1	0.5	0.3	0.2	0.3
Effective interest rate	6.3	6.6	6.7	6.3	6.5	6.5
Exchange Rate Shock						
Real GDP growth	4.2	3.7	3.4	3.7	3.8	3.8
Inflation	4.0	3.7	3.8	3.7	3.7	4.0
Primary balance	-4.0	-1.1	0.5	0.3	0.2	0.3
Effective interest rate	6.3	6.0	6.1	6.3	6.5	6.5
Constant Primary Balance Scenario						
Real GDP growth	4.2	3.7	3.4	3.7	3.8	3.8
Inflation	4.0	3.7	3.8	3.7	3.7	4.0
Primary balance	-4.0	-4.0	-4.0	-4.0	-4.0	-4.0
Effective interest rate	6.3	5.9	6.0	6.3	6.5	6.5

	2021	2022	2023	2024	2025	2026
Primary Balance Shock						
Real GDP growth	4.2	3.7	3.4	3.7	3.8	3.8
Inflation	4.0	3.7	3.8	3.7	3.7	4.0
Primary balance	-4.0	-3.3	-1.8	0.3	0.2	0.3
Effective interest rate	6.3	5.9	6.0	6.3	6.5	6.5
Real GDP Growth Shock						
Real GDP growth	4.2	-0.3	-0.6	3.7	3.8	3.8
Inflation	4.0	3.7	3.8	3.7	3.7	4.0
Primary balance	-4.0	-1.1	0.5	0.3	0.2	0.3
Effective interest rate	6.3	5.9	6.0	6.3	6.5	6.5
Customized Scenario						
Real GDP growth	4.2	3.7	3.4	3.7	3.8	3.8
Inflation	4.0	3.7	3.8	3.7	3.7	4.0
Primary balance	-4.0	-1.1	0.5	0.3	0.2	0.3
Effective interest rate	6.3	5.9	6.0	6.3	6.5	6.5
Historical Scenario						
Real GDP growth	4.2	2.4	2.4	2.4	2.4	2.4
Inflation	4.0	4.4	4.4	4.4	4.4	4.4
Primary balance	-4.0	-1.7	-1.7	-1.7	-1.7	-1.7
Effective interest rate	6.3	5.9	5.9	6.1	6.3	6.3

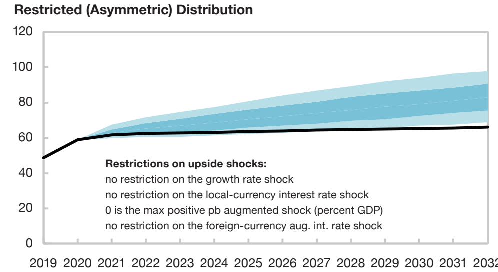
Evolution of Predictive Densities of Gross Nominal Public Debt
(in percent of GDP)

Baseline — Percentiles: 10th-25th — 25th-50th — 75th-90th

Symmetric Distribution



Restricted (Asymmetric) Distribution



Source: Ministry of Finance.

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APPENDICES

Appendix I: Equivalence between the Borrowing-Requirements Approach and Net-Debt-Issuances Approach for Projecting Public Debt

Recall that (1) presents how the evolution of public debt can be computed using the government's borrowing requirements determined by the fiscal balance and other net debt-creating flows. The evolution of public debt can alternatively be derived considering government net debt issuances as

$$D_t = e_t^{eop} D_{t-1}^f + D_{t-1}^d + e_t^{avg}\left(Iss_t^f - Amort_t^f\right) + Iss_t^d - Amort_t^d + SF_t^{ier}, \quad (11)$$

where $Iss_t^f - Amort_t^f$ refers to net debt issuances (i.e., issuances minus amortizations) of foreign-currency debt (converted to local currency using the average exchange rate) and $Iss_t^d - Amort_t^d$ indicates net debt issuances of local-currency debt. In both (1) and (11), the stock-flow adjustment term SF_t^{ier} is included to revalue foreign-currency flows from the average to the end-of-period exchange rate.

To see the consistency between (1) and (11), let us define the government's gross financing needs (GFN) as the sum of the resources the government needs to finance its period-t overall deficit (primary deficit plus the interest expense) and its period-t debt amortizations. Note also that all GFN not financed by other net debt-creating flows (e.g., use of financial assets) must be covered by gross debt issuances:

$$GFN_t \equiv e_t^{avg} i_t^f D_{t-1}^f + i_t^d D_{t-1}^d - PB_t + e_t^{avg} Amort_t^f + Amort_t^d = e_t^{avg} Iss_t^f + Iss_t^d - OF_t. \quad (12)$$

Rearranging terms in (12), we can see that the government's overall deficit plus other net debt-creating flows must be equal to the net debt issuances. Therefore, starting from (11) and using (12) it is possible to recover (1).

Whether (1) or (11) is used for projecting public debt depends on a number of considerations, such as the availability of information to undertake projections, and the type of questions to be addressed (e.g., identifying government financing needs or the key drivers of public debt changes). Given the prominent role that assessing liquidity risks in member countries has for the IMF, understanding the evolution of financing needs has become a priority for both the MAC DSA and the LIC DSF, which thus project debt levels with (11). Note from (12) that to project GFN, one would need projections for amortizations.

Appendix II: Debt-to-GDP Ratio

Dividing (1) by nominal GDP at time $t(=P_tY_t)$ and rearranging terms gives:

$$\frac{D_t}{P_tY_t} = \left(\frac{e_t^{eop}}{e_{t-1}^{eop}} + \frac{e_t^{avg}}{e_t^{eop}}\frac{e_t^{eop}}{e_{t-1}^{eop}} i_t^f\right)\frac{e_{t-1}^{eop}D_{t-1}^f}{P_{t-1}Y_{t-1}}\frac{P_{t-1}Y_{t-1}}{P_tY_t} + \left(1+i_t^d\right)\frac{D_{t-1}^d}{P_{t-1}Y_{t-1}}\frac{P_{t-1}Y_{t-1}}{P_tY_t} - \frac{PB_t}{P_tY_t} + \frac{OF_t}{P_tY_t} + \frac{SF_t^{ier}}{P_tY_t}.$$

Using $\alpha_t = e_t^{eop}D_t^f/D_t$, $\varepsilon_t^{eop} = e_t^{eop}/e_{t-1}^{eop} - 1$, $g_t = \frac{Y_t}{Y_{t-1}} - 1$, $\pi_t = \frac{P_t}{P_{t-1}} - 1$, and lower-case variables to denote shares of GDP gives:

$$d_t = \alpha_{t-1}(1+\varepsilon_t^{eop})\left(1 + \frac{e_t^{avg}}{e_t^{eop}} i_t^f\right)\frac{d_{t-1}}{(1+g_t)(1+\pi_t)} + (1-\alpha_{t-1})\left(1+i_t^d\right)\frac{d_{t-1}}{(1+g_t)(1+\pi_t)} - pb_t + of_t + sf_t^{ier}.$$

Rearranging terms and defining $\varphi_t \equiv \frac{1+r_t^w}{1+g_t}$, where $1 + r_t^w = \frac{\alpha_{t-1}(1+i_t^f e_t^{avg}/e_t^{eop})(1+\varepsilon_t^{eop})+(1-\alpha_{t-1})(1+i_t^d)}{1+\pi_t}$ gives (2). Considering that $sf_t = \frac{(e_t^{eop}-e_t^{avg})}{e_t^{eop}}[\alpha_t d_t - \alpha_{t-1}d_{t-1}\frac{(1+\varepsilon_t^{eop})}{(1+g_t)(1+\pi_t)}]$, and defining $\hat{\varphi}_t \equiv \frac{1+\hat{r}_t^w}{1+g_t}$ where $1 + \hat{r}_t^w \equiv \frac{(1+i_t^f)(1+\varepsilon_t^{eop})e_t^{avg}/e_t^{eop}\alpha_{t-1}+(1+i_t^d)(1-\alpha_{t-1})}{(1+\pi_t)}\mu_t$ and $\mu_t = [1 - \alpha_t\left(1 - \frac{e_t^{avg}}{e_t^{eop}}\right)]^{-1}$ gives (8).

Appendix III: Solution of the Public Debt Dynamics Equation

The solution of the linear first-order non-homogeneous difference equation given by (8) takes the form:

$$d_{t-1} = \frac{d_T}{\prod_{v=t}^{T} \hat{\varphi}_v} + \sum_{s=t}^{T} \frac{\mu_s (pb_s - of_s)}{\prod_{v=t}^{s} \hat{\varphi}_v}. \tag{13}$$

A similar solution can be obtained with (9) using the overall balance. Thus, while all the analyses presented here are carried out using the primary balance as in (8), they could easily be modified to consider instead the overall balance as in (9).

Appendix IV: Derivations of Fiscal Adjustment Path Rules

A. Debt-stabilizing Primary Balance

The primary balance needed to stabilize the debt ratio ($d_t - d_{t-1} = 0$) can easily be computed from (8):

$$pb_t^{ds} = d_{t-1}(\hat{\varphi}_t - 1)/\mu_t + of_t\,. \tag{14}$$

Since $\mu_t > 0$, if $\hat{\varphi}_t > 1$, everything else being equal, a larger stock of debt in period *t-1* requires a larger debt-stabilizing primary balance in period t.

B. Constant Primary Balance to Reach a Specific Public Debt Target

Equation (13) implies that the constant primary balance between years t and T that allows the government to reach debt target d_t in period T is given by:

$$pb^{constant} = \frac{d_{t-1}\prod_{v=t}^{T}\hat{\varphi}_v - d_T + \sum_{s=t}^{T}\prod_{v=s+1}^{T}\hat{\varphi}_v\,\mu_s of_s}{\sum_{s=t}^{T}\prod_{v=s+1}^{T}\hat{\varphi}_v\,\mu_s}. \tag{15}$$

To better understand this expression, suppose that the discount factor $\hat{\varphi}_t \equiv \frac{1+\hat{r}_t^w}{1+g_t}$, other net debt-creating flows, the gap between the average and the end-of-period exchange rates, and the share of foreign-currency debt in total debt (and thus μ) are constant. Since $\prod_{v=s+1}^{T}\hat{\varphi} = \hat{\varphi}^{T+1-(s+1)}$ and $\sum_{s=t}^{T}\hat{\varphi}^{T+1-(s+1)} = \frac{1-\hat{\varphi}^{T+1-t}}{1-\hat{\varphi}}$,

$$pb^{constant} = d_{t-1}(\hat{\varphi} - 1)/\mu + of + (d_{t-1} - d_T)\frac{\hat{\varphi}-1}{\hat{\varphi}^{(T+1-t)}-1}/\mu\,. \tag{16}$$

That is, the constant primary balance needed to meet the debt target d_T after $T+1-t$ years is given by the combination of two elements. First, the primary balance needed for stabilizing debt at the period *t-1* level, represented by the first two terms of (16) (recall equation 14). Second, the additional primary balance needed to reduce the debt ratio from the initial d_{t-1} to the desired d_T, represented in the last term of (16). Observe that when the initial debt level is the debt target ($d_{t-1} = d_T$), (16) is equivalent to (14).

C. Constant Adjustment of the Primary Balance to Reach a Specific Debt Target

With an adjustment, adj that would take place every period from t to T with the objective of meeting the debt target d_T at a time T, the primary balance evolves according to

$$pb_t = pb_{t-1} + adj\,, \tag{17}$$

which implies that for any period $s \geq t$ the primary balance is given by

$$pb_s = pb_{t-1} + adj(s + 1 - t).$$

Substituting this into (13) implies that the needed constant adjustment between years t and T that allows the government to reach debt target d_T in period T is given by:

$$adj = \frac{d_{t-1}\prod_{v=t}^{T}\hat{\varphi}_v - d_T + \sum_{s=t}^{T}\prod_{v=s+1}^{T}\hat{\varphi}_v\,\mu_s of_s - pb_{t-1}\sum_{s=t}^{T}\prod_{v=s+1}^{T}\hat{\varphi}_v\,\mu_s}{\sum_{s=t}^{T}\prod_{v=s+1}^{T}\hat{\varphi}_v\,\mu_s(s+1-t)}. \tag{18}$$

To better understand this expression, suppose that the discount factor $\hat{\varphi}_t \equiv \frac{1+\hat{r}_t^w}{1+g_t}$, other net debt-creating flows, and μ are constant. Since $\sum_{s=t}^{T}(s+1-t)\hat{\varphi}^{T+1-(s+1)} = \frac{1-\hat{\varphi}^{T+1-t}}{1-\hat{\varphi}}\kappa$ with $\kappa \equiv \frac{T+1-t}{1-\hat{\varphi}^{T+1-t}} - \frac{\hat{\varphi}}{1-\hat{\varphi}}$, we can obtain

$$adj = \frac{d_{t-1}\dfrac{\hat{\varphi}-1}{\mu} + of - pb_{t-1} + (d_{t-1} - d_T)\dfrac{\hat{\varphi}-1}{\mu[\hat{\varphi}^{(T+1-t)} - 1]}}{\kappa}. \tag{19}$$

Equation (19) underscores that the yearly adjustment needed to achieve the debt target is the sum of two components divided by κ, which depends on the number of periods envisaged for the fiscal consolidation, $T+1-t$. First, the gap between the debt-stabilizing primary balance for the current debt level d_t and the primary balance pb_{t-1} (first three terms in the numerator). Second, the adjustment needed for bringing debt down from d_{t-1} to d_T (last term in the numerator). Note that EC (2019) referred to a similar expression as the S1 indicator.

D. Primary Balance to Reach a Debt Target through Constant Debt Reductions

From (8), the change in the debt ratio is given by

$$d_t - d_{t-1} = d_{t-1}(\hat{\varphi}_t - 1) - \mu_t pb_t + \mu_t of_t. \tag{20}$$

Then, the constant debt reduction to achieve the debt target d_T in $T+1-t$ periods is given by[22]

$$\frac{d_T - d_{t-1}}{T+1-t} = d_{t-1}(\hat{\varphi}_t - 1) - \mu_t pb_t + \mu_t of_t,$$

which solving for the primary balance gives

$$pb_t^{reduction} = d_{t-1}(\hat{\varphi}_t - 1)/\mu_t + of_t + \frac{d_{t-1} - d_T}{T+1-t}/\mu_t. \tag{21}$$

Equation (21) shows that the primary balance required to achieve the debt target d_T in $T+1-t$ periods, starting from d_{t-1}, is the combination of two elements. First, the primary balance required to stabilize debt at the t-1 level (first two terms). Second, the additional primary balance needed to bring debt down from d_{t-1} to d_T by a fixed amount every period divided by μ_t (to take into account intra-year exchange rate changes).

22 To see this, consider solving the first-order linear difference equation, $d_t - d_{t-1} = K$, which has a solution for any initial period t-1 and terminal period T, $d_{t-1} = d_T - K(T+1-t)$.

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