INTERNATIONAL MONETARY FUND

IMF POLICY PAPER

CENTRAL BANK STRESS TESTING—GUIDANCE NOTE

December 2024

IMF staff regularly produces papers proposing new IMF policies, exploring options for reform, or reviewing existing IMF policies and operations. The following documents have been released and are included in this package:

- The Guidance Note, prepared by IMF staff and completed on December 5, 2024.

The IMF's transparency policy allows for the deletion of market-sensitive information and premature disclosure of the authorities' policy intentions in published staff reports and other documents.

Electronic copies of IMF Policy Papers
are available to the public from
http://www.imf.org/external/pp/ppindex.aspx

International Monetary Fund
Washington, D.C.

CENTRAL BANK STRESS TESTING—GUIDANCE NOTE

December 5, 2024

EXECUTIVE SUMMARY[1]

Many central banks expanded their balance sheets substantially in the wake of the Global Financial Crisis and again in response to the COVID-19 pandemic. While their actions were aimed at achieving their price and financial stability objectives—and were often spurred by limited conventional policy space—they exposed central banks to significant balance sheet risks. These risks have materialized into sizeable central bank losses in recent years as high inflation has induced central banks to tighten policy. These losses contrast with the period before the Global Financial Crisis, in which central banks maintained much smaller balance sheets and were almost always profitable.

This guidance note develops a tractable modeling framework—building on Hall and Reis (2015)—that can help gauge the balance sheet risks faced by central banks that arise from interest rate duration risk, foreign exchange risk, and credit risk. Specifically, our model allows for a quantitative assessment of how central bank equity would evolve under alternative scenarios. These scenarios can be used to capture how the balance sheet would evolve if macroeconomic conditions turn out differently than under the baseline—for example, if inflation runs higher than expected—as well as under specific policy actions such as a lower remuneration rate on excess reserves. This framework fills a gap as few central banks currently utilize full-fledged stress testing to guide their capital policy.

The illustrative model simulations provided in the note show that larger balance sheets tend to result in considerably more volatility in central bank equity—and a higher likelihood of sizeable losses—than when balance sheets are small and the main liability currency is circulation. Notably, the central bank not only tends to be "robustly" profitable in the latter case, but profits rise with higher interest rates. With larger balance sheets, the simulations highlight the importance of duration and foreign exchange risk for the path of profits and central bank equity.

This framework can be useful both in helping central banks assess the risks that policy actions pose for their capital position, as well as in informing decisions about how to retain central bank profits and build adequate financial buffers or about when a recapitalization may be needed. It should also be instrumental in improving central bank transparency and accountability for these policies, while keeping in mind that balance sheet actions have a range of macroeconomic, financial, and fiscal effects that must be considered in addition to their effects on central bank profits.

[1] The authors would like to thank Christopher Erceg, Romain Lafarguette, Robin Darbyshire, Yuan Gao Rollinson, Darryl King, and Rudy Wytenburg for their useful discussions and suggestions. The authors are also grateful for the comments garnered from staff in other IMF departments and the participants in the MCM policy forum.

Approved by
Tobias Adrian

Authored by the IMF staff, this document was developed by a team consisting of Romain Veyrune, Roger McLeod, Cecilia Melo Fernandes, Yuji Sakurai, Luyao Liu, Julie Vaselopulos, Evelyn Schimpf, and all members of the Monetary and Capital Markets Department.

CONTENTS

FIGURES

TABLES

APPENDICES

Glossary

AE	Advanced Economy
AIC	Akaike's Information Criterion
APF	Asset Purchase Facility
ARIMA	Autoregressive Integrated Moving Average
ARIMAX	Auto Regressive Integrated Moving Average with Exogenous Inputs
BoE	Bank of England
CB	Central Bank
CBST	Central Bank Stress Testing
CDS	Credit Default Swap
CiC	Currency in Circulation
CPI	Consumer Price Index
DCF	Discount Cash Flow
DNB	De Nederlandsche Bank
DSGE	Dynamic Stochastic General Equilibrium
EAD	Exposure at Default
ECB	European Central Bank
ECL	Expected Credit Loss
EM	Emerging Markets
ESG	Environmental–Social–Governance
Fed	Federal Reserve
FTPL	Fiscal Theory of Price Level
FX	Foreign Exchange
GDP	Gross Domestic Product
IFRS	International Financial Reporting Standards
LGD	Loss-Given Default
NFR	Net Foreign Reserves
NS	Nelson–Siegel Model
MAS	Monetary Authority of Singapore
MCM	Monetary and Capital Markets Department
OLS	Ordinary Least Squares
OMO	Open Market Operations
PD	Probability of Default
PLS	Partial Least Squares
QE	Quantitative Easing
QT	Quantitative Tightening
SNB	Swiss National Bank
SVAR	Structural Vector Autoregression
UIP	Uncovered Risk Parity
USD	United States Dollars
VAR	Vector
VIX	Volatility Index

INTRODUCTION

1. Central bank balance sheet exposure to risks has increased in the past decade. Following the Global Financial Crisis, a number of advanced economy (AE) central banks used asset purchase programs to provide monetary stimulus when the effective lower bound was binding and/or to stave off financial instability. In doing so, they incurred interest rate risk, which has recently materialized and resulted in sizeable balance sheet losses. Similarly, central banks in emerging markets (EMs) and some AEs were exposed to exchange rate risks due to large FX holdings and were affected by the recent global increase in interest rates. In addition, deteriorating fiscal balances in some EMs have increased the risk of sovereign default, significantly affecting expected credit losses (ECL). Furthermore, developmental activities, such as subsidized lending during the COVID-19 pandemic, have also contributed to inflate the balance sheets of some central banks.

2. What is central bank stress testing? Given these developments, it is useful to have a framework that can help quantify the risks to central bank balance sheets arising from various sources (including from interest rate, foreign exchange, and credit risks) and provide guidance on the procedural and analytical framework for determining adequate risk provisioning and capital. In this note, we overview such a central bank stress testing (CBST) model and use it to forecast central bank equity at a five-year horizon. It starts with the fundamental accounting identities of the balance sheet and income statement, which determine realized equity (the "core model"). Subsequently, CBST relies on satellite models to estimate risks and forecast each item of the core model (e.g., currency in circulation [CiC] and operational cost) based on the evolution of macroeconomic variables. The baseline capital path is derived from the combined forecasts. Finally, the baseline path is stress-tested using deterministic and at-risk macroeconomic scenarios. To illustrate the model's output, we analyze how the same macroeconomic shocks affect a central bank with different balance sheet characteristics: one with a lean balance sheet,[2] another with a large domestic asset portfolio (duration risk), a third with a large FX portfolio (exchange rate risk and duration risk), and, lastly, one with exposure to restructured assets (expected credit loss).

3. What differentiates stress testing for central banks from commercial banks? Existing stress testing frameworks are tailored to commercial banks. Both central and commercial banks encounter similar financial risks, including credit, interest rate, and exchange rate risks. However, unlike commercial banks, central banks do not seek profits, are not subject to financial solvency ratios,[3] and do not experience liquidity constraints. Finally, central banks have interest-free indefinite-duration liabilities that are not available to commercial banks (e.g., banknotes), which they can invest to earn revenue (seigniorage). However, central banks can be *policy* insolvent if they do

[2] A central bank has a lean balance sheet if its assets grow only as a result of the demand for CiC and the reserve requirement.

[3] Commercial banks can take excessive risk for profits, which is not a concern for central banks. However, excessive risk could also be taken for a given policy objective.

not have enough income to pursue their inflation objective without incurring persistent losses (Hall and Reis (2015) and Del Negro and Sims (2015)).

4. Why do central banks need a CBST? Most central banks must decide how profits should be distributed between general reserves (which, along with paid-in capital, belong to statutory capital) and its shareholders (most commonly, the government).[4] While central banks can apply other techniques to estimate potential losses, such as conditional VaR, the assessments of capital adequacy for financial institutions are usually based on stress tests to include a comprehensive set of risks and allow for scenario analysis. Regular stress tests, conducted by the risk unit, could inform the decisions of the boards of central banks on profit distribution and, possibly, on the need for recapitalization of the central bank by the government. The stress test scenarios can simulate the "extreme but plausible shocks" that additional capital may serve to buffer in a forward-looking and risk-based manner (also known as "dynamic" capital policy). The stress testing framework is more important in the context of a large balance sheet, where higher and more volatile risk exposures call for larger and more intricately calibrated capital buffers. This framework could help fill this gap, as few central banks have full-fledged stress testing, and most rely on rigid minimum capital and profit distribution rules.

5. Why do governments need a CBST? While it is straightforward that the CBST provides useful information for the risk management of the "agent," i.e., the central bank, it should, first and foremost, interest the "principal," i.e., the government, because it is the shareholder that pockets the benefits or ultimately must cover the losses. The fiscal authorities can use the simulated net income forecasts from the CBST as inputs to evaluate their claim on the profits and losses from their respective central bank, depending on institutional agreement between the central bank and the fiscal authority. This ensures that the central bank's actions are transparent and well-understood, which, in turn, helps to mitigate risks to the central bank's independence. While CB losses only partially capture the fiscal implications of respective CB actions (as they may have substantial macroeconomic benefits that improve the fiscal balance), it has become increasingly important for governments to anticipate transfers emanating from the prevailing arrangement with the CB.

6. The remainder of this guidance note is organized as follows. Section II reviews the academic literature. Section III reports on stylized facts. Section IV describes the core model. Section V explains modeling risks. Section VI show how to forecast balance sheet and financial accounting items. Section VII discusses scenario designs. Section VIII presents model outputs under stylized balance sheets.

[4] Current capital policies often rely on distributing a fixed percentage of the profit. In the best cases, the distribution is conditional to a minimum level of capital, which is usually defined as a percentage of monetary liabilities.

ACADEMIC LITERATURE REVIEW

A. General Framework

7. Stella (1997) pioneered the research on central bank capital adequacy. He examined the cases that central banks in several developing countries were recapitalized or incurred losses. Then, he discussed the implications of the central bank's losses for their independence. His studies are qualitative and empirical and do not formally develop a central bank balance sheet model.

8. To the best of our knowledge, Bindseil et al. (2004) is the first study to develop a quantitative model of the central bank balance sheet. They developed a simple model where the central bank balance sheet consists of four items: securities purchased under monetary policy operations, bank notes, capital, and other financial assets. They analyze under which conditions the central bank capital remains positive. However, their models are highly stylized and hence difficult to apply to actual balance sheet data and macroeconomic time series.

9. The next generation of the studies on the central bank balance sheet formulated general equilibrium models with the central bank's intertemporal budget constraint. The constraint is that the discounted present value of remittances to the fiscal authority must be equal to the central bank's equity (i.e., the market value of assets minus liabilities) plus the discounted present value of seigniorage, regardless of the remittance rule.[5] Buiter (2007) formulates a general equilibrium model in which the intertemporal budget constraint faced by central bank is explicitly represented and separately modeled from the Treasury's.[6] He emphasizes the importance of decomposing the consolidated government budget constraint into the central bank's constraint and Treasury's constraint because the decomposition allows us to analyze the financial constraint on the central bank to achieve the price stability. Del Negro and Sims (2015) construct a continuous-time dynamic general equilibrium model with the intertemporal budget constraint. They also calibrate the model for the US data. They show that the seigniorage dynamics under high inflation are important for determining recapitalization might be needed. Reis and Hall (2015) provide systematic analyses of the central bank's financial soundness. They examine how different specifications of the dividend rule to the Treasury affect the reserves. These studies provide solid macroeconomic foundations and motivation for our CBST framework, though the latter does not embed a general equilibrium model.

10. Relevant but theoretical studies are conducted under the Fiscal Theory of Price Level (FTPL). The basic FTPL emphasizes the role of fiscal policy for analyzing inflation based on the analysis of the consolidated government's intertemporal budget constraint.[7] These theoretical studies include but not limited to Park (2015) and Williamson (2018).

---

[5] The intertemporal budget constraint arises to rule out Ponzi scheme finance.

[6] Bassetto and Messer (2013) develop a stylized competitive equilibrium model and analyze under which conditions a central bank can ensure positive profits and transfers to the fiscal authority.

[7] The key implication of the FTPL is that the price adjusts itself to ensure the real value of the government debt should be equal to the real present value of fiscal surplus. In other words, the monetary policy is passive while fiscal policy is active in determining the price level. See Cochrane (2023).

B. Applications

11. There are several academic studies on the central bank balance sheet in specific countries. Fujiki and Tomura (2017) construct a model of the balance sheet of the Bank of Japan. Franta et al. (2022) develop the balance sheet model of the Czech National Bank. Buiter and Rahbari (2012) analyze the stylized balance sheet of the central bank and government from the perspective of the ECB. Carpenter et al. (2015) project selected balance sheet items and income and expenses of the Federal Reserve. Ize (2005) constructs a simple balance sheet model for studying central banks in Costa Rica, Chile, and Mozambique during their crisis periods. Manuelli and Vizcaino (2017) develop a simple model of the budget constraints of the central bank and the fiscal authority and study the recent experience of Argentina.[8]

12. There is also a strand of literature focusing on financial stress testing and how this should be developed specifically for central banks. Benjamin et al. (2022) present the capital framework that had been developed by the Bank of England (BoE) in 2018.[9] It assesses "potential losses in a set of severe but plausible events for activities that are backed by the BoE's capital" (that do not benefit from ex ante government indemnities) based on "a forward-looking, scenario-based approach." The framework of Benjamin et al. (2022) explains the analytical underpinnings of this BoE model and its institutional set up, then explores how central banks in general may apply this framework.

13. There are limited number of empirical studies on the central bank balance sheet and its implications. Perera et al. (2013) examine the empirical relationship between the financial strength of the central bank and inflation using data sets of selected advanced and emerging countries. They find that the relationship between central bank financial strength and inflation is negative with statistical significance. Goncharov et al. (2023) construct a data set of more than 150 countries and document that central banks tend to report slightly positive profits more often than slightly negative profits and this tendency is associated with political and market pressures and central banker's career concerns.

C. Other Relevant Studies

14. There are two strands of the literature which are less closely related to but still relevant to the CBST framework. First, there are emerging studies on quantifying the impact of normalizing a large balance sheet on the macroeconomy by explicitly modeling the central bank budget constraint. These studies include but are not limited to the following: Domínguez and Gomis-Porqueras (2023) examine the impact of reducing the size of the central bank's balance sheet and changing its composition toward short-term bonds on the macroeconomy. To do so, they develop a dynamic equilibrium model where fiscal policy is passive and monetary policy follows the Taylor rule

---

[8] Different from these studies, Bolt et al. (2023) adopt a game-theoretic model to analyze the limit of keeping the value of fiat money in the context of the failure of the Bank of Amsterdam in the early 19th century.

[9] For more details: [Exchange of letters between the Governor and Chancellor on the Bank's financial framework | Bank of England.]

and the central bank has short-term and long-term bonds. They show that the change in the balance sheet affects the long-run inflation and the real economy when short-term and long-term bond premia exist. This is because economic agents face imperfect asset substitutability. Arce et al. (2020) place an endogenous market for interbank loans into a standard new-Keynesian model to analyze the transmission of balance sheet policy. Yet, they do not study the implications of balance sheet policy for the intertemporal insolvency of the central bank. Benigno and Benigno (2022) extend a New-Keynesian model by incorporating reserves to study monetary policy normalization when existing a liquidity trap. They solve optimal combination of reserves and interest rate policy, but they do not discuss central bank balance sheet issues given the assumption that central bank is not subject to any solvency condition. Hayashi and Koeda (2019) formulate regime-switching structural vector autoregression to analyze the macroeconomic impacts of quantitative easing (QE) and quantitative tightening (QT). They apply it for Japan and find that higher reserves at the zero lower bound raises inflation and output and the effect of QT depends on the state of economy. They do not model the entire balance sheet of the Bank of Japan but estimate the excess reserves demand equation as a function of macro variables. The current CBST framework does not allow the feedback effects from reducing the size of the balance sheet to the macroeconomy because it complicates the analysis given the fact that we model all balance sheet items in a granular manner. Yet, capturing the feedback effects under the CBST framework is a potentially fruitful future research and development topic.

15. Second, there are only a few academic studies on the implications of the balance sheet concern of central bankers for optimal monetary policy. Jeanne and Svensson (2007) develop a simple model to analyze when an independent central bank can commit to a higher future price level in the context of escaping from a liquidity trap. They show that the optimal commitment policy can be achieved provided the balance sheet concern is sufficiently strong. Berriel and Bhattarai (2009) develop a model where a central bank minimizes not only deviation of inflation and output but also the real value of its own capital. They show that the concern about the real value of the capital and fiscal independence can act as a commitment device for the central bank given cost-push inflation. Although we do not adopt a general equilibrium framework that might allow us to consider optimal policy, the importance of the balance sheet concern documented in these studies provides additional justification for our CBST work.

16. There are several papers reviewing the practices employed by the central bank regarding the accounting framework and profit distributions such as Archer and Moser-Boehm (2013) and Bunea et al. (2016). Tanaka (2021) is a recent survey of the literature on the central bank capital.

STYLIZED FACTS

A. By Level of Economic Development

17. Balance sheet data reveals a few stylized facts. Large, advanced economies with a floating exchange rate traditionally had lean balance sheets with minimal risk exposures. They were robustly profitable as most of their liabilities consisted of interest-free CiC, which could be invested in income-earning assets. In contrast, emerging market central banks often faced financial issues due to the need for relatively large FX portfolios to support a commitment to a fixed (stable) exchange rate. This picture dramatically changed for AEs after the Global Financial Crisis, as AE central banks engaged in asset purchase programs that significantly increased the size of their balance sheets, and, thus, their risk exposure, surpassing those of other countries.

- Balance sheet size, measured as the ratio of monetary base to GDP, remained between 5 and 10 percent of GDP in 2005. However, by 2022, the distribution widened to between 15 and 45 percent. AE central balance sheets now appear to be statistically larger, while the difference between the two periods is less pronounced for other countries (Figure 1).

- Purchases of government securities have been the main source of balance sheet expansion for AE central banks. Portfolios of government securities contribute less to balance sheet expansion in other countries (Figure 2).

- FX reserves, which are a larger balance sheet items in economies other than in AEs, did not notably contribute to balance sheet growth (Figure 3). As a result, the share of local currency to FX portfolio increased in all countries, but the increase was markedly more for AE central banks.

- CiC used to be more than 80 percent of the monetary base in AEs. With the balance sheet expansion, bank reserves at the central bank, which are usually remunerated at the policy rate for proper monetary policy transmission, are now more than two times larger than CiC (Figure 4). While the balance sheet expansion generated sizeable profits for AE central banks when the term structure was upward sloping (as occurred in the wake of the Global Financial Crisis), many central banks have experienced losses in recent years as high inflation pushed central banks to significantly tighten monetary policy.

18. In the absence of international standards regarding central bank capital adequacy, practices vary from one country to the other.[10] The median capital is 2.1 percent when expressed as a percentage of GDP and 10.5 percent as a percentage of the monetary base. While capital increased in term of GDP from 2005 to 2022 (from 1.3 to 2.1 percent), it declined as a percentage of the monetary base from 14.7 to 10.5 percent, reflecting a capital accumulation that is lagging the increase in central bank balance sheets. The decline in capital vis-a-vis the monetary base concerns mainly AE central banks because their balance sheets expanded the most.

[10] The IMF model central bank law advocates for capital between 8 and 10 percent of "monetary liabilities." The latter are understood as liabilities to third parties, excluding its sovereign and the IMF. In other words, it mainly consists of currency in circulation and liabilities toward monetary policy counterparties (i.e., banks).

Figure 1. Monetary Base as a Percentage of Gross Domestic Product

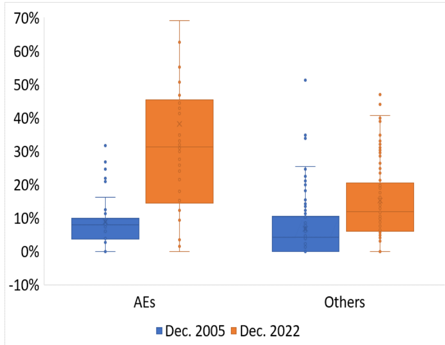


Figure 2. Government Securities Portfolio as a Percentage of Gross Domestic Product

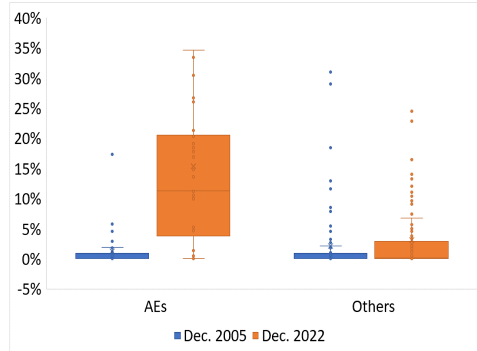
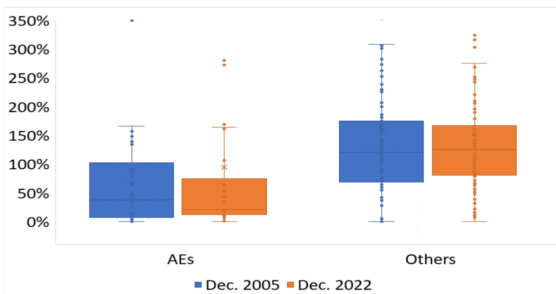


Figure 3. Gross Foreign Reserves as a Percentage of Monetary Base



Source: IMF's Monetary and Financial Statistics.

Figure 4. Currency in Circulation as a Percentage of Monetary Base

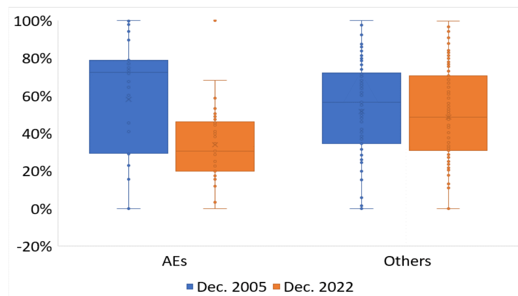
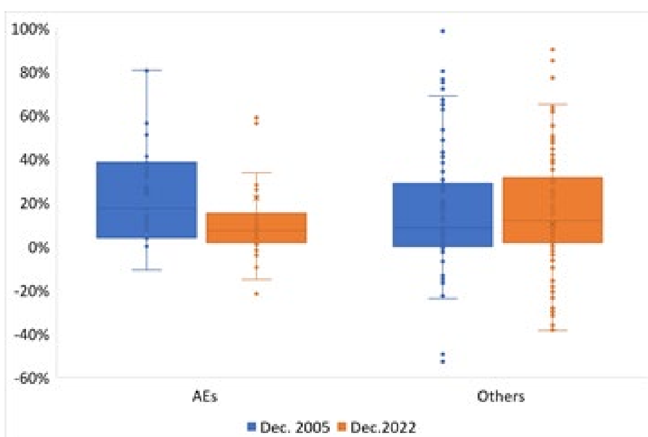


Figure 5. Capital Account as a Percentage of Monetary Base



Source: IMF's Monetary and Financial Statistics.

B. By Type of Exchange Rate Arrangement

19. Large central bank balance sheets were traditionally concentrated in fixed exchange rate arrangements. Fixed exchange rate economies typically had larger balance sheets than floating ones, mainly because they needed enough FX reserves to support an exchange rate commitment, which floating rate systems do not have. They were, thus, more exposed to exchange rate risk and duration risk. However, the scenario has evolved, and now the size of the balance sheets of central banks with a floating exchange rate often exceeds those with fixed exchange rates. The distribution of the balance sheet sizes among central banks with floating exchange rates is more varied, with the central banks of the largest economies coming across as outliers.

20. Countries with floating rate exchange systems are more likely to hold large portfolios of government securities. The large balance sheet expansions in AEs noted earlier typically reflects the effects of quantitative easing and shift to floor operating systems.

21. The erosion of seigniorage mainly impacts central banks with floating arrangements. FX reserve accumulation typically resulted in larger stocks of remunerated bank reserves at central banks with fixed exchange rates, making these central banks less profitable than those with floating systems. However, with the introduction of asset purchase programs, this trend has shifted, leading to a frequently lower share of CiC in base money in floating than in flexible exchange rate systems.

Figure 6. Monetary Base as a Percentage of Gross Domestic Product

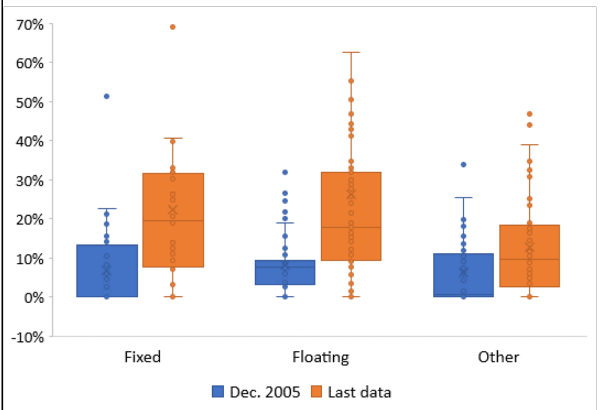


Figure 7. Government Securities Portfolio as a Percentage of Gross Domestic Product

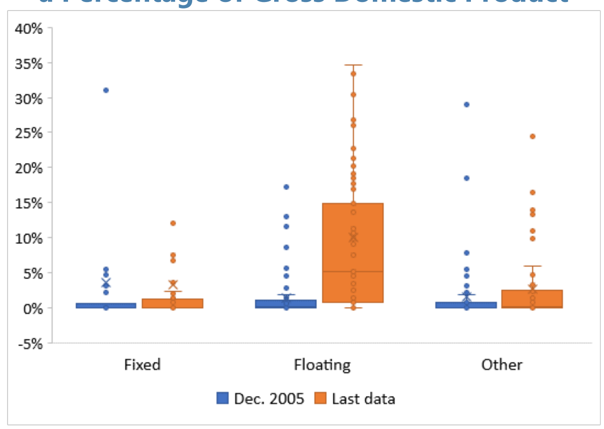


Figure 8. Gross Foreign Reserves as a Percentage of Monetary Base

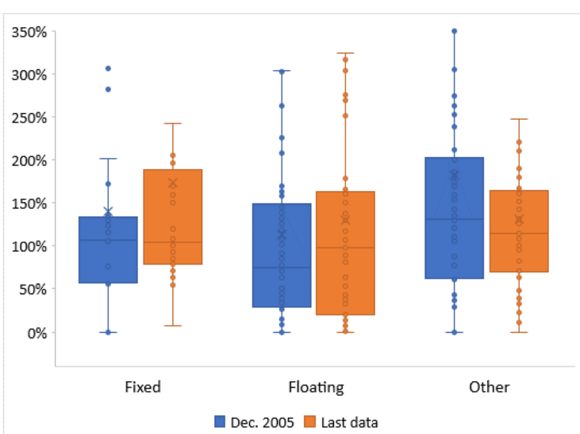
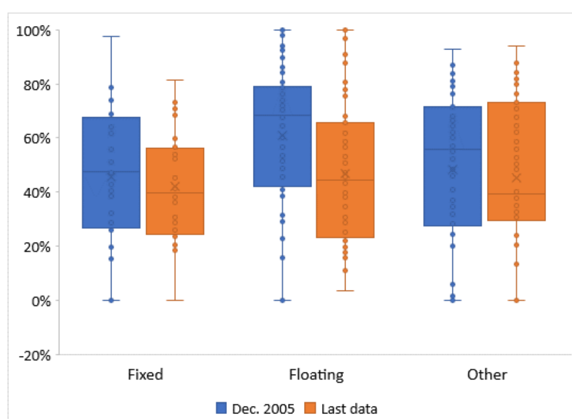


Figure 9. Currency in Circulation as a Percentage of Monetary Base



Source: IMF's Monetary and Financial Statistics.

22. Capitalization seems to have weakened in floating arrangements. Due to relatively higher risk exposure, central banks operating fixed exchange rates started with higher capital as a percentage of base money than floaters. However, despite the faster growth in floaters' balance sheets, which now exceeds those of fixers, capitalization did not keep pace. As a result, the capital of floating systems, as percentage of base money, has declined, while it slightly increased for fixed exchange rate systems.

23. Most central bank distributed dividends until 2022. Distribution rules vary depending on the central bank. However, most central banks in the sample, regardless of the size of their balance sheets, have distributed non-negligible dividends, as a percentage of GDP, to their governments through 2022 (Figure 10). In years during which those central banks made profit, they transferred most of it to the government, with few exceptions (Figure 11).

Figure 10. Central Bank Transfers to the Government as a Percentage of GDP

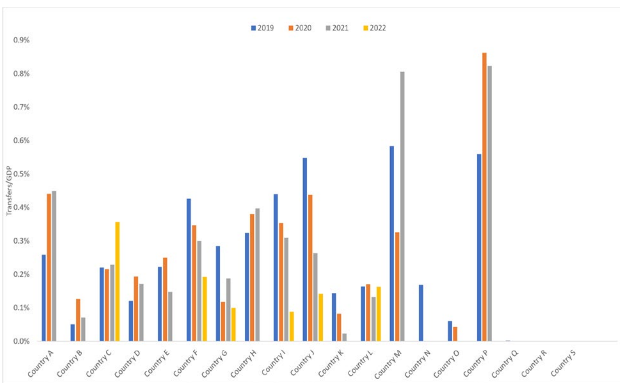
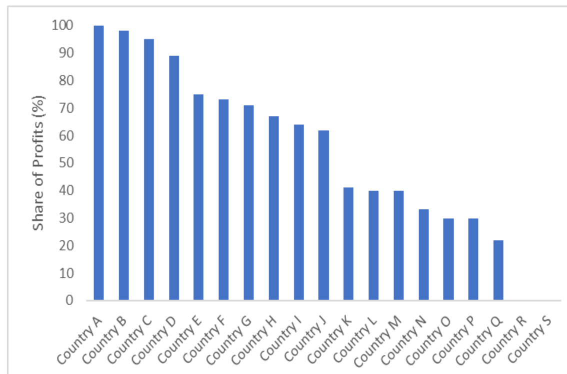


Figure 11. Share of Profits Distributed to the Government (Average Over 2019-22)



Note: The central banks of the following countries were included: Australia; Brazil; Canada; Denmark; France; Germany; Greece; Italy; Japan; Malta; Mexico; New Zealand; Norway; Poland; Portugal; Spain; Switzerland; United Kingdom; United States. Profits for all Euro area countries are calculated as distributable net profits plus risk provision.
Source: IMF's Monetary and Financial Statistics.

24. After 2022, central bank losses became a widespread issue and a focus of communication efforts. Losses were often large and broad-based across many central banks. Communications usually stated that these losses would not hinder the ability of the central bank to pursue policy objectives (Table 1). Some central banks explained how they would offset their losses, such as by using future profits or, in few cases, recapitalization. Additionally, the losses were cited in decisions to stop remunerating certain accounts held by central banks' counterparties, like banks or governments. The European Central Bank and the Swiss National Bank decided to stop remunerating the reserve requirement. In both cases, the communication of the central banks explicitly mentions cost savings among the motivations for the decision. The Deutsch Bundesbank stopped remunerating the government account.[11]

[11] See European Central Bank (2023), Swiss National Bank (2023), and Bundesbank (2023).

Table 1. International Experience on Central Bank Losses and Communication

Central Bank	Losses in bn USD*	Communication/Actions
Bank of Canada (BOC)	USD 3.3 billion (nine-month period ended in September 2023).	The Bank of Canada communicated that the losses do not affect the Bank's ability to conduct monetary policy or its operations and that it expects to return to a net income position.
Bank of England (BoE)	Losses projected to be USD 50 billion in 2023 and 2024.	Quantitative Easing at the BoE is carried out through the Asset Purchase Facility (APF). Under agreements in 2009 and 2012, the APF is indemnified by the Treasury, and any realized profits or losses generated by the APF are regularly reconciled with the Treasury, with transfers from or to the Treasury occurring each quarter. The BoE also publishes projections of these transfers each quarter for possible scenarios for the full run-down of the APF, which takes place over about 10 years. By May 2024, the total amount transferred from the APF to the Treasury was £63bn.
Bank of Ghana (BoG)	USD 5.3 billion (2022).	The Bank, in a press release, explained the reasons for these losses and stated that its present financial position has no immediate impact on its ability to pursue the policies it deems appropriate. A large part of the losses (USD 4.1 billion) was a direct result of the Government's domestic debt restructuring exercise. However, the Bank said its equity and its earning capacity should be high enough in the long run to ensure that it is sufficiently financially independent of the government.
Danmarks Nationalbank	USD 1.3 billion (2022).	There were no transfers to the central government for the 2022 financial year.
De Nederlandsche Bank (DNB)	USD 3.8 billion (2023).	The DNB communicated that they expect losses to continue until 2028. Nonetheless, buffers are expected to continue positive. Yet, should the buffers become too depleted or expected profits remain too low, additional measures may be necessary to restore its balance sheet to solidity, such as capital contribution from the Dutch State in extreme cases. USD 1.2 billion of 2023 losses were covered by reserves.
Deutsche Bundesbank	USD 0.00 (2023). A balanced financial result was obtained after releasing risk provisions and withdrawals from reserves.	The Bundesbank released its risk provisions in full (USD 21.1 billion), and USD 2.6 billion was drawn down from reserves to cover losses in 2023. The Bundesbank communicated that it expects the burdens to be considerable again in 2024, which are likely to exceed the remaining reserves. In this case, it will report a loss carryforward and then offset it through future profits. No profit has been transferred to the federal budget since 2020.
European Central Bank (ECB)	USD 1.41 billion (2023).	The ECB communicated in its February 2024 press release that it expects to incur further losses after fully releasing its remaining USD 7.1 billion (€6.6bn) general risk provision, but that any losses would also be carried forward against future profits, avoiding any need for a recapitalization.
Federal Reserve (Fed)	USD 114.3 billion (2023).	The Federal Reserve Act mandates that if earnings are insufficient to cover its costs, a deferred asset is recorded. This deferred asset represents the future excess earnings that the regional Reserve Banks need to accumulate before they can resume remitting earnings to the U.S. Treasury.
Monetary Authority of Singapore (MAS)	USD 22.8 billion (financial year ending in March 2023).	MAS communicated that it was the largest loss it has ever recorded, reflecting the effects of monetary policy tightening to bring down inflation. The tightening of monetary policy also led to the broad appreciation of the Singapore dollar against other currencies, including the US dollar. The losses were mostly attributed to negative currency translation effects resulting from this broad appreciation of the Singapore dollar, which did not affect MAS' ability to conduct monetary

Table 1. International Experience on Central Bank Losses and Communication (concluded)		
Central Bank	**Losses in bn USD***	**Communication/Actions**
		policy or support financial stability. The MAS also noted that it was unlikely to be able to contribute to the Consolidated Fund over the next few years due to the loss. To ensure that MAS remains well-capitalized relative to its assets, MAS doubled its issued and paid-up capital to SGD 50 billion (or USD 37 billion).
Reserve Bank of Australia (RBA)	USD 3.96 billion (fiscal year 2022/23).	The Bank suspended the transfers to the government and communicated that it is important that its equity is restored over time. The Board has also communicated its strong expectation to the government that future distributable earnings would be applied, in full, to offsetting the accumulated losses and restoring the balance of the Reserve Bank Reserve Fund to the Board's target. This measure was endorsed by the government. The Bank had a negative position of USD 11.6 billion by 30 June 2023.
Reserve Bank of New Zealand (RBNZ)	No surplus equity for distribution (fiscal year 2022/23).	The Bank has indemnity agreements with the Treasury. No dividend was paid to the Treasury in the years 2020, 2022, and 2023. In August 2023, the Minister of Finance agreed to provide USD 2 billion in new capital (on top of USD 806 million in July 2023). Furthermore, a new indemnity was established, covering losses up to USD 8 billion on bond purchase programs, should such losses occur in the future.
Riksbank	USD 7.94 billion (2022).	As a result of the loss, the Riksbank's equity was negative, USD 1.76 billion (SEK 18.2 billion). It communicated that a new Act contains provisions that its equity should have a target level of SEK 60 billion and a basic level of SEK 40 billion. The Executive Board of the Riksbank intends to submit a petition in March 2024 to the government that includes a request for capital injections and possibly includes proposals for additional earnings capacity.
Swiss National Bank (SNB)	USD 3.54 billion (2023).	SNB communicated that it would not make a payment to the Swiss central or local governments, nor pay a dividend to investors, as a result of the loss for 2023. In 2022, it had a record loss of USD 152 billion (CHF 133 billion).

* Unless otherwise indicated.
Source: Central banks' communiqués.

THE "CORE" MODEL

25. The CBST projects the capital of central banks by combining macroeconomic forecasts and accounting techniques. All balance sheet items are modeled separately, then aggregated to provide a calculation for realized income, provisioning, and thus, statutory capital as well as revaluation income (Tables 1 to 4). Macroeconomic variable forecasts are usually sourced from the country's IMF macroeconomic framework. This typically consists of projections of real GDP, inflation, net foreign reserves, and the exchange rate. Other policy-determined macroeconomic variables, such as the forecast of the policy interest rate, are determined by satellite models. Other exogenously determined macroeconomic forecasts—such as foreign interest rates or credit default swap (CDS) spreads—are sourced from the *World Economic Outlook* and Bloomberg. Several techniques are used to forecast balance sheet items, with the choice of technique varying according to nature of the balance sheet items in question.

Table 2. The Analytical Balance Sheet

Assets	Liabilities
Net Foreign Assets	Currency in Circulation
Foreign Reserves (net)	Commercial Banks' Reserves
Monetary Gold	Required Reserves
Other Foreign Assets (net)	Excess Reserves
Domestic Assets	Government Deposit
Government Securities & Advances	Central Bank Securities
Domestic Securities	Other Liabilities
Loans to Financial Institutions	**Equity**
Other Assets	Statutory Equity
	Revaluation Reserves
Total Assets	***Total Equity and Liabilities***

Source: IMF staff.
Note: Statutory equity includes paid-in capital and general reserves (i.e., retained earnings).

Table 3. The Analytical Income Statement

Statement of Profit or Loss	
Interest Income from International Reserves	(1)
Interest Income From Monetary Policy Operations	(2)
Other Interest Income	(3)
Total Interest Income	(4)=(1)+(2)+(3)
Interest Expenses on Foreign Currency Liabilities	(5)
Interest Expenses From Monetary Policy Operations	(6)
Other Interest Income	(7)
Total Interest Expenses	(8)=(5)+(6)+(7)
Net Interest Income	(9)=(4)-(8)
Impairment Recovery or Charge	(10)
Net Interest Income After Impairment Provision	(11)=(9)+(10)
Net Gains or Losses from Foreign Currency Revaluation	(12)
Net Realized gain on Instruments Measured at FVOCI	(13)
Net Realized gain on Instruments Measured at FVTPL	(14)
Non-Interest Income	(15)=(12)+(13)+(14)
Non-Interest Expenses (Operational Costs)	(16)
Profit and Loss Before Comprehensive Income	(17)=(11)+(15)-(16)
Other Comprehensive Income	(18)
Total Comprehensive Income	(19)=(17)+(18)

Source: IMF staff.
Note: FVOCI = Fair Value through Other Comprehensive Income; FVTPL = Fair Value through Profit and Loss. Operational costs include compensation of central bank staff. In our simulation analysis, we also use the following definitions to disentangle domestic interest income or expenses from net foreign interest income: Interest income = (2)+(3); Interest expenses = (6)+(7); and Net foreign interest income = (1)-(5).

26. The model forecasts the comprehensive income, which is then distributed to the different equity items. Based on the exchange rate projections, part of the profit would be assigned to the revaluation reserve. The rest would be distributed between the general reserves and a transfer to the government (dividend payment). A central bank can have provisions against specific losses (e.g., against losses on securities transactions, foreign exchange, etc.) and/or general reserves to be used for all types of losses. For simplicity, we will assign all retained earnings to a general reserve. Losses will reduce the general reserve. Because the stress test is an input in the distribution decision, no distribution is assumed *ex ante*. The initial "paid-up" capital of the central bank plus the general reserves determines the statutory capital, which is the definition of capital that interests us. The statutory capital plus the revaluation reserve define equity.

Table 4. Arriving at Profits Available for Distribution

Total Comprehensive Income
Profits Transferred to Revaluation Reserve
Previous Unrealized Gains/Losses Recognized in Year
Profits Transferred to Retained Earnings
Profits Distributed to the Shareholder(s)

Source: IMF staff calculations.

27. The CBST has several core model equations, which are derived using standard accounting identities. They include the following:

- *Equity*. The total equity can be obtained in two ways, which together provide for a consistency check. FX revaluation gains or losses go to the revaluation account. Note that the notation, Δ Revaluation Account, in the equation below includes Net Gains or Losses from Foreign Currency Revaluation in Table 3 and other gains or losses which are not classified as realized earnings.

$$(1)\ \ \mathrm{Equity_t} = \mathrm{Total\ Assets_t} - \mathrm{Total\ Liabilities_t}$$

$$(2)\ \ \mathrm{Equity_t} = \mathrm{Equity_{t-1}} + Realized\ earnings_t + \Delta\ Revaluation\ Account_t$$

- *A clearing item*. Open Market Operations (OMO) are the clearing item that captures changes in assets and liabilities.[12] Typically, OMO can result in negative or positive outcomes, depending on the structural liquidity position. However, they generally generate income for central banks with lean balance sheets and incur costs for central banks with large balance sheets.[13]

$$\mathrm{OMO_t} = \mathrm{OMO_{t-1}} + [-\,\Delta\mathrm{FXInt_t} - \Delta\mathrm{DAH_t} + \Delta\ \mathrm{CiC_t} + \Delta\ \mathrm{RR_t} + \mathrm{IRIncome_t} - \mathrm{Op_t}],$$

Where $\mathrm{DAH_t}$ is domestic asset holdings, CiC_t is currency in circulation, $\mathrm{IRIncome_t}$ is the net interest rate income,[14] $\mathrm{RR_t}$ is required reserves, and $\mathrm{Op_t}$ is operational expenses.

- *Foreign reserve accumulation*. The dynamics for gross foreign reserve accumulation (denoted by $\mathrm{FXRes_t}$) is driven by three terms, namely:[15] (i) foreign currency interest income (denoted by

[12] In a floor system, OMO include recourse to a deposit facility or remunerated reserves.

[13] The structural position is the position of the monetary counterparties (i.e., banks) vis-à-vis the central bank. Typically, this position is short for banks (indicating a need to borrow from the central bank) under a lean balance sheet approach and long (where they place excesses with the central bank) under a large balance sheet approach.

[14] In general, this net interest income may include the interest income from repo and the interest expenses on the excess reserves and reverse repo, but the actual specification depends on each central bank.

[15] The equation is developing in "Exiting from an Exchange Rate Floor in a Small Open Economy: Balance Sheet Implications of the Czech National Bank's Exchange Rate Commitment" (Franta, Holub, and Saxa 2022).

$\mathrm{FIRIncome_t}$); (ii) FX revaluation gain or losses (denoted by $\mathrm{Revaluation_t^{FXRes}}$);[16] and (iii) net USD purchase for Net Foreign Reserves (NFR) accumulation (denoted by $\mathrm{\Delta FXInt_t}$).

$$\mathrm{FXRes_t = FXRes_{t-1} + FIRIncome_t + Revaluation_t^{FXRes} + \Delta FXInt_t}$$

28. The calculation of balance sheet items may differ across central banks. The first step in developing the CBST involves converting the balance sheet to an internationally acceptable accounting standard for comparability across regions.[17] On the asset side, foreign reserves denominated in cash are term deposits valued at amortized cost. In contrast, foreign currency securities, being longer-term investments, are measured using fair valuation. For domestic loans and securities, valuation is based on amortized cost or fair value, depending on the maturity. Typically, both local and foreign loans acquired by the central bank are recorded at amortized cost.

MODELING RISKS

A. Credit Risk

29. Credit risk is measured using ECL provisioning. A prudent risk management framework requires modeling expected losses on all assets, including FX and sovereign exposures.[18] In the CBST, ECL provisioning for sovereign debts is forecasted by conditioning the ECLs calculations on projected macroeconomic variables, which are derived from the country's macroeconomic framework. This approach allows one to incorporate a forward-looking component into the stress testing framework. The ECLs for corporate debts can also be calibrated and forecasted. In the model's benchmark setting, ECL provisioning for corporate debts is forecasted by assuming that the ECL provision, as a percentage of the outstanding debt, remains constant over for the forecast horizon.

30. ECL are computed using the standard formula, which incorporates the probability of default (PD), loss-given default (LGD), and exposure at default (EAD). This formula is as follows:[19]

$$ECL_t = PD_t \cdot LGD \cdot EAD_t,$$

[16] Note that ΔFX Revaluation Account in the equation below corresponds to Net Gains or Losses from Foreign Currency Revaluation in Table 3.

[17] For expository reasons, this guidance note discusses the valuation of balance sheet items in line with the International Financial Reporting Standards for classifying and measuring financial assets and liabilities (IFRS 9).

[18] FX Reserve managements should be subject to prudent risk management guidelines, but this has to be corroborated on a case-by-case basis. Eligible domestic assets are often limited to the sovereign (which itself could require ECL) but could also include less-liquid assets, especially for emergency liquidity support.

[19] More rigorously, we may use the lifetime ECL, defined as:

$$ECL_t = E_t\left[\int_t^T \exp\left(-r_s(s-t)\right) PD_s(X_s) \cdot LGD \cdot EAD(X_s)\,ds\right],$$

Where *r* is the discount rate, *t* is the expiry, and *X* is a vector of exogenous variables that impact the PD and EAD. For details, see "Applying IFRS Impairment of Financial Instruments under IFRS 9" (Ernst and Young 2018) and "IFRS 9 and CECL Credit Risk Modelling and Validation: A Practical Guide with Examples Worked in R and SAS" (Bellini 2019).

Where EAD_t is set equal to the estimated exposure at the time of the default t.[20] The LGD is derived from the concept of the recovery rate, R, where R=1-LGD. Recovery rates for corporate bonds can be easily obtained from sources such as Bloomberg. Conversely, the recovery rate for sovereign bonds can be difficult to calculate for individual sovereigns, due to the small number of sovereign defaults compared with corporate defaults. Nonetheless, several studies estimate regional LGDs, which can be used for corresponding countries.[21]

31. PDs can be calculated using three methods: (i) credit rating-based PD; (ii) CDS-implied PD; or (iii) discount-implied PD. Some central banks employ the credit rating-based PD method, which is generally applicable to standard sovereign and corporate bonds. Loans extended to local non-bank entities, which are often unrated, require judgment on the part of the central bank, with applications dependent upon data availability. The credit-rating based method is the preferred methodology out of the three options because it is widely utilized by central banks when developing financial statements. However, this method is data intensive and relies on an internal credit rating model to provide indicative ratings over the medium term. If the necessary data and corresponding modeling approach are not available, the CDS implied PD method can be utilized with important caveats.[22] If all other methods are unavailable, the discount-implied PD should be used. To reconcile potential differences between the calculated ECL from each option, the credit-rating based method should be used as a reference point.

Credit Rating-Based PD Method

32. The credit rating-based PD method is the benchmark setting employed in the model. In the case of sovereign bonds, the credit rating-based PD method is employed by forecasting credit ratings over the forecast horizon using an internal credit rating model (see Appendix III for technical details). The forecasts of the credit ratings over the forecast horizon incorporate three key components: (i) macroeconomic forecasts, including changes in GDP and its related components like GDP growth and volatility, economic diversification, credit growth, and per capita GDP; (ii) forecasts for fiscal variables from the latest Debt Sustainability Analysis of the respective countries, encompassing debt level and trajectory, fiscal and primary balances, and indicators for interest payments and the level of coverage to meet such payments; and (iii) assumptions regarding governance and susceptibility to event risks arising from political risk, climate risk, and Environmental-Social-Governance (ESG) considerations. Once the sovereign credit ratings is forecasted, this is used to determine the corresponding level of PD that should be applied over the forecast horizon.

[20] If the exposure changes over time, and depending on the market environment, we may need to model such a stochastic feature of the EAD. For example, please refer to Bellini (2019).

[21] See, for example, "Sovereign Default, Debt Restructuring, and Recovery Rates: Was the Argentinean 'Haircut' Excessive?" by Edward (2015), and "Sovereign Default and Recovery Rates, 1983–2010" (Moody's 2011). Edward (2015) shows how the historical average of haircut rates on sovereign bonds differs across countries. Moody's (2011) documents that the average historical recovery rate on sovereign bonds is 53 percent.

[22] An important caveat to note is that CDS implied PDs are risk neutral, and as such they tend to diverge somewhat from physical PDs, which are utilized in credit ratings. In addition to this, CDS implied PDs may suffer from risk endogeneity particularly in the case when the central bank has a large market footprint.

33. PDs are derived by way of a mapping methodology. Tables mapping the sovereign credit rating to the historical PD are available from major rating agencies. All sovereign credit ratings have a corresponding PD that may differ based on years to maturity. The standard approach of the CBST involves employing a one-year PD, which represents the probability of default within a year. Generally, this is lower than the probability of default over a longer period. On the other hand, a lifetime PD, which is the probability of default within the lifetime of the security up to its maturity, is employed for non-investment grade sovereigns with heightened credit risk.[23]

34. ECL provisioning are modelled to reflect scenario-based changes in macro conditions over the forecast horizon. ECL provisioning over the forecast horizon is derived from forecasted credit ratings, which reflect the underlying macroeconomic scenario. This ensures that ECL provisioning in the stress-testing model automatically responds to stress testing scenarios. For instance, simulating a significantly deteriorating macro environment will exert downward pressure on sovereign credit ratings, which increases the PD used in the derivation of ECL provision over the forecast horizon.

CDS-Implied PD Method

35. PDs can also be derived in the model using the CDS-implied PD method. This method is a particularly direct way to obtain market-implied PDs for sovereign debts because the payoff is directly linked to the event of underlying sovereign default. The sovereign CDS spread reflects the default probability of a sovereign and the corresponding recovery rate.[24] Approximately, sovereign CDS spread is given by:

$$spread_t = (1 - R_{CDS}) \cdot \lambda_t,$$

Where λ_t is the default intensity.[25] For example, $1 - \exp(-\lambda_t) \approx \lambda_t$ is the 1-year default probability.[26] The recovery rate for sovereign CDS, R_{CDS}, is often assumed to be equal to 25 percent among market participants.[27] Note that this assumption is a market convention and, thus, irrelevant to the historical average of realized recovery rates for sovereign debts.

[23] It should be noted that IFSR 9 allows for some amount of discretion and judgment to be applied in the ultimate determination of ECL provisioning, both in terms of the PD used and the ECL provisioning that is chosen.

[24] Precisely speaking, the PD implied from the sovereign CDS spread is the PD from the view of the risk-neutral investor. To obtain the PD under historical (physical) measure from the PD under risk-neutral measure, we need to specify the functional form of the market price of risk and estimate the relevant parameters. When it is difficult to estimate market price of risk due to the limited data availability, we use the risk-neutral PD as a proxy for the historical PD. The same argument holds for the LGD. This approach is consistent with market practice.

[25] For a definition of credit intensity, see "Credit Risk: Pricing, Measurement, and Management" (Duffie and Singleton 2003).

[26] The exact formula is given by $spread_t = \int_t^{t+T} ds\,(1-R)\lambda_s \exp(-(r_t+\lambda_t)s)\ /\int_t^{t+T} ds \exp(-(r_t+\lambda_t)s)$ in a continuous time framework. As this is computationally intensive, and our main goal is to reflect the macroeconomic environment to the default probability, we rely on the approximate formula.

[27] For a discussion, see "Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads" (Pan and Singleton 2008).

36. The CDS-implied PD method also enables ECL provisioning to be linked to macroeconomic variables for the purpose of stress testing. The CDS-implied PD method involves collecting a historical time series of CDS spreads, then conditioning this on macro variables such as inflation, real GDP growth, and debt as a percentage of GDP. By assuming a linear relationship between these three macroeconomic variables and the CDS spreads, an empirical link is estimated. Then, by using scenario-based values of the three macroeconomic variables, it becomes possible to derive the ECL provisioning for multiple scenarios over the forecast horizon.

37. The discount-implied PD method, or more formerly, the Discount Cash Flow (DCF) method can also be employed as a third option for deriving PDs. The DCF method includes a discount rate that reflects sovereign default risk. The difference between a risk-free discount rate and the default-risk-embedded discount rate (i.e., bond premium) can be considered as equivalent to ECL provisioning under the assumption that LGD=100 percent.[28] As done for the CDS spread method, the DCF method links the bond premium to macroeconomic variables. The bond premium is defined as the difference between the bond yield and the policy rate. As is the case for the CDS implied method, this bond premium would be conditioned on macroeconomic variables.

Exposure at Default

38. A yield curve model is crucial for determining the price and remuneration of the domestic assets bought by the central bank, as it is important in determining the EAD. The approach in the CBST model is to apply an accounting treatment consistent with International Financial Reporting Standards for classifying and measuring financial assets and liabilities (IFRS 9) to enable country comparability, but alternative accounting treatments that are internationally accepted standards can also apply for other central banks. Under the fair value hierarchy of IFRS 9, the preferred approach is to use exact market prices (Level 1). If there are no exact market equivalents, the next option is Level 2, which uses prices derived from market transactions of comparable securities. If there are no suitable comparable securities to derive a price, a Level 3 approach is adopted, using an internal model that discounts future cash flows of the bond using an estimated yield curve.

39. Yield curves are estimated using reduced form models, such as the Nelson-Siegel functional form. Following this, the future yield curve is projected based on the dynamic model developed in Nelson and Siegel (1987). The dynamic model, as discussed in Appendix II, projects yields with any maturity as a linear function of three factors: its level, slope, and curvature. The dynamic Nelson-Siegel model allows these three factors to be driven by exogenous regressors, such as real GDP growth, inflation, the exchange rate, and other variables. This model can be used to generate a stressed yield curve. The level factor in the model corresponds to the very long-term

[28] Consider the zero-coupon bond with T-year maturity. The fair value under a risk-free discount rate is $\exp(-rT)$, while the fair value under a default-risk-embedded discount rate is $\exp(-(r+\lambda)T)$. The diff is $diff = \exp(-rT)\,(1-\exp(-\lambda T))$. The second term is equal to the default probability from today to the maturity T. Given the assumptions of 100 percent LGD and EAD of $\exp(-rT)$, the difference is the ECL provision.

interest rate, while the slope factor captures the difference between the short-term yield and the long-term yield. The curvature factor describes to what extent the yield curve is hump shaped.

Emergency Liquidity Lending

40. Central banks are not expected to provide unsecured loans even when providing emergency lending. Collateralized lending provides two lines of defense for the central bank: (i) the solvency of the institutions that receive the loan; and (ii) the collateral itself. The risk faced by the central bank in the event of a counterparty default is the potential need to liquidate the collateral. To mitigate this risk, an appropriate haircut can be applied to the collateral value. If the demand for liquidity exceeds what the central bank could provide with full collateralization after the appropriate haircut, the government should provide a guarantee to the central bank to cover any shortfall.

41. In practice, unsecured lending or collateralization shortfalls are not a null probability during the crisis. Central banks are often hard pressed during a liquidity crisis to provide extensive funding, leading the central bank to deliberately increase its risk tolerance (lower haircut) in the pursuit of financial stability. In addition, haircuts are not always accurately calibrated for less liquid collateral in these urgent circumstances. Simultaneously, the sovereign might be under stress and, thus, in no position to provide a credible guarantee. After these episodes, the central bank's balance sheet could be durably damaged.

42. "Emergency liquidity" credit risk is modelled as an ECL with a contingent EAD. Emergency lending could be construed as a contingent credit line implicitly extended to predetermined counterparties. The PDs for these counterparties would be computed based on their credit ratings, including solvency ratios. To consider the contingent nature of the liquidity support, the EAD should reflect the likelihood of a large liquidity needs from the eligible counterparties. The ECL could also be applied or refined ex-post.

B. Interest Rate Risk

Inflation Risk and the Policy Rate

43. Central banks with large balance sheets are particularly exposed to an inverted yield curve. In normal circumstances, risk-free and perfectly liquid bank reserves at the central bank are the lowest-yielding assets in a jurisdiction. The central bank issued them to monetary counterparties (banks) in exchange for assets that have a higher yield. However, this situation can change if inflation forces the central bank to increase its policy rate. In such a scenario, bank reserves will be immediately repriced. The extent of the loss incurred depends on several factors: the size of the increase in the policy rate, the size of the asset portfolio held by the central bank, and the pace of re-pricing of this portfolio (i.e., its average duration). Accordingly, a portfolio with a shorter duration would reduce the risk exposure, as it would be quicker to adjust to the new interest rate environment. The policy rate is modelled as the reaction function the most consistent with the monetary policy framework of the central bank.

44. For inflation-targeting central banks, the policy rate is typically modelled as a function of inflation and possibly other variables. The most prominent example is the Taylor rule. However, the model can use any reaction function that the central bank implements. In this specification, the domestic policy rate, i_t^*, is determined by several factors: the target domestic policy interest rate $\bar{\iota}_t$, the long-run equilibrium interest rate r^*, the inflation differential $(\pi_t - \bar{\pi})$, and the output gap $(y_t - \bar{y})$, as follows:

$$i_t = \bar{\pi} + r^* + \gamma_\pi(\pi_t - \bar{\pi}) + \gamma_y(y_t - \bar{y}) + \gamma_i i_{t-1} + \varepsilon_t.$$

It is often assumed that central banks adjust their policy rates gradually toward the target rate.

45. For a fixed exchange rate regime, the policy rate should reflect the anchor currency policy rate plus an additional premium to account for the country specific risk. This approach ensures the balancing of capital flows and brings the interest rate towards its equilibrium value. The most difficult challenge is to determine the country risk premium. In the CBST model, country risk premium is derived from the uncovered risk parity (UIP) condition, as detailed in Appendix IV. Denote the domestic nominal interest rate per annum in period t by i_t, and the corresponding foreign interest rate by i_t^*, and the exchange rate in terms of domestic currency per reference foreign currency by s_t, then the UIP can be written as follows:

$$i_t - i_t^* = (s_{t+1}^e - s_t)/\, s_t,$$

Where $s_{t+1}^e$is the expectation of the exchange rate. Of course, this conventional UIP condition only applies when the domestic and foreign currencies are perfect substitutes. Therefore, a risk premium must be applied to the right-hand side of the equation, which becomes $i_t - i_t^* = (s_{t+1}^e - s_t)/\, s_t + \varphi$, where φ_t is the time-varying country risk premium. In addition to this, the change in the exchange rate is zero under a fixed exchange rate regime. Taken together, the equation becomes $\varphi_t = i_t - i_t^*$.

46. Theoretically, the country risk premium is affected by multiple factors. These include economic indicators (e.g., debt, currency, and political stability) as well as geopolitical factors, regulatory factors, liquidity and market access, perceptions of risk and global economic conditions, amongst other factors.

47. In the CBST, a model-determined risk premium is forecasted by conditioning the risk premium time series on the quantification of these factors. The selection of indicators used as explanatory variables would be based on their relevance to each specific country. After empirical estimation, the model-determined level for the risk premium is forecasted using the saved coefficients and the forecasts for the explanatory variables. The risk premium forecast is then denoted by φ_{t+1}^{e}, representing the expected risk premium for the next data point.[29] The interest rate reaction function, thus becomes:

$$i_t = i_t^* + \varphi_{t+1}^{e}.$$

See Appendix IV for further technical details.

Duration Risk

48. We explicitly model the duration of the domestic portfolio. We employ a satellite model in which the central bank rolls over the domestic bond portfolio. This satellite model allows us to capture the effect of the average yield of the bond portfolio, which is partially and gradually adjusted to the new path of the domestic policy rate. The following assumptions could be considered:

- *The maturing assets will not be replaced:* Reserves shrink as the portfolio matures, lowering the cost of monetary policy.
- *The maturing assets are replaced as they expire, and the average duration of the portfolio is kept unchanged:* The reserve stock remains unchanged, and the monetary policy expense is derived from the forecasted policy rate path. On the other hand, the average return on assets changes based on the forecasts of the policy rate and the yield curve.
- *The portfolio is partially replaced and the average maturity changes:* Several central banks are following this approach under quantitative tightening. In practice, this would be a combination of the previous two approaches.

In our simulation analyses below, we employ the second approaches so that we can analyze the impact of duration and yield curve changes on the path of the central bank's equity.

Mark-to-Market

49. Mark-to-market losses are primarily a concern for the FX portfolio of central banks with fixed exchange rate agreements or those in emerging markets.[30] These central banks might need to sell FX securities at any time to meet the objective of their FX intervention policy, facing the risk of making a loss depending on how the market price of the security has changed. The central banks that are the most exposed are those in emerging markets or with fixed exchange rate arrangements that tend to need more FX reserves to cover vulnerabilities. The losses could be

[29] When this country risk premium φ_{t+1}^{e} purely reflects the risk premium of sovereign default, the PD embedded in the risk premium φ_{t+1}^{e} and the PD embedded in the sovereign CDS spread discussed in the previous section should be consistent with each other. When running the CBST model, this theoretical consistency needs to be checked. This approach is consistent with market practice. Note that when φ_{t+1}^{e} reflects not only the sovereign default risk premium but also liquidity risk premium, there would be a gap between φ_{t+1}^{e} and the sovereign CDS spread.

[30] As noted in our description of exposure at default, the CBST discusses mark-to-market valuation of the FX portfolio consistent with IFRS9 to enable country comparability (given that many central banks are IFRS compliant), but alternative internationally accepted accounting treatments can also apply for other central banks.

particularly pronounced when major central banks engage in a sharp and prolonged tightening cycle after a long period of low interest rates, a period during which portfolio duration often increases as the result of the search for yields. Therefore, most (or for simplicity, all) of the FX portfolio is expected to be mark-to-market and would, thus, be exposed to changes in portfolio valuation due to changes in the policy rate of foreign central banks and the slopes of major currency yield curves.

50. **We apply the net present value approach for valuing the FX portfolio, given the foreign yield curves.** More specifically, we assume that the FX portfolio consists of an equal amount spread across maturities up to a predetermined longest maturity. Once the bond with shortest maturity expires, its principal amount is reinvested in the bond with longest maturity, based on the currently available yield.

51. **There is generally less need to mark-to-market for domestic securities portfolio.** Central banks are not liquidity constrained for the currency that they issue, which means that they do not have to sell domestic assets if they do not want to sell them. As the result, they can be placed in the banking book (held to maturity) and do not need to be marked-to-market.

C. Exchange Rate Risk

52. **The framework aligns exchange rate assumptions with those of the country team macroframework.** In the absence of such information, the stress testing framework would model the exchange rate as an exogenous process.

53. **Exchange rate risk depends on the composition of the FX reserves held by the central bank.** For instance, consider a central bank that has foreign currency assets in USD—cash deposits, FX securities held at fair value, and derivative financial instruments. These assets are respectively measured on a typical central bank balance sheet as follows:

$$Cash_{USD,t} = FX_{USD,t}/FX_{USD,t-1} \cdot \left(1 + i_{cash,USD,t}\right) \cdot Cash_{USD,t-1}$$

$$Fair\ Value_{USD\ ,t} = FX_{USD,t}/FX_{USD,t-1} \cdot \left(1 + i_{YTM,USD,t}\right) \cdot F_{USD,t}/(1 + i_{USD,t})^{\tau}$$

$$Income\ on\ Derivatives_{USD\ ,t} = FX_{USD,t}/FX_{USD,t-1} \cdot \left(i_{\ USD,t} - i_{\ USD,Derivative}\right).\left(D_{A,t} - D_{L,t}\right),$$

Where $FX_{USD,t}$ is the local currency-to-USD exchange rate, $i_{cash,USD,t}$ is the short-term (three months) foreign interest rate, $i_{YTM,USD,t}$ is the yield to maturity, $F_{USD,t}$ is the face value of USD investments, $i_{\ USD,Derivative}$ is the interest rate used to price derivatives at execution, and $\left(D_{A,t} - D_{L,t}\right)$ is the net position of the value of derivatives on the balance sheet.

FX Revaluation Gains and Losses

54. **A significant part of exchange rate gains or losses does not affect statutory capital.** Typically, central banks operate with a long FX position to provide some insurance to the market against exchange rate risk or FX liquidity risk (FX emergency liquidity assistance). Therefore, an exchange rate depreciation (appreciation) of the local currency against the U.S. dollar leads to a

revaluation gain (loss) for a central bank with a long FX position. The risk would be reversed in the less common case of a central bank operating with a short FX position. However, in both cases, these FX revaluation gains or losses are considered unrealized and, consequently, booked in the revaluation reserve account.[31] Contrary to realized gains and losses, these unrealized gains cannot be distributed.

55. An important consideration is the eventual distribution of some of the revaluation gains or the coverage of losses. The revaluation account could be construed as a capital buffer specifically dedicated to exchange rate risk. These gains are typically not distributed but are held to offset possible losses in the future. However, central banks with long standing positive revaluation gains have developed practices to realize a portion of these gains that exceed possible exchange rate losses with a high probability. On the other hand, a revaluation deficit would remain unrealized if exchange rate gains are expected to cover the losses with a high probability. If not, it would be deducted from statutory capital.

56. In the CBST, the conditional distribution of the exchange rate is forecasted to identify tail exchange rate risk. Based on this forecast, the revaluation account distribution rule could be calibrated. The forecast for the conditional distribution of the exchange rate employs an at-risk functional form using the 5th percentile to simulate exchange rate appreciation, and thereby revaluation losses. The equation for the general specification is as follows:

$$\Delta S_{t+k} = \alpha + \Delta S_t + \gamma Domestic\ Marco_t + \delta Balance\ of\ Payments_t + \theta Domestic\ Price_t + \sigma External\ Factors_t + \varepsilon_t,$$

Where ΔS_t is the log return of the exchange rate (percentage change). The log return of the exchange rate at time *t+k* is modelled with an autoregressive term, domestic macro, balance of payments, domestic price, and external factors at time *t*. Forecasting the distribution based on the 5 percent value at risk ensures that the top 5 percent of exchange rate appreciations are forecasted. For further technical details on at risk approach, see Appendix V.

57. Exchange rate risks are particularly relevant to cross-currency exposure. In some cases, a central bank might have "borrowed" one currency to invest in another one, thereby exposing it to cross-currency risk. Of course, for central banks in a dollarized economy, with the balance sheet indexed in U.S. dollars, the U.S. dollar depreciations against foreign currencies on the balance sheet leads to revaluation gains so long as the central bank holds a net asset position in those foreign currencies.

Exposure on Derivative

58. Some central banks are significantly involved in providing the market with FX derivatives. The most common are forwards, which are simply FX purchases or sales that settle at more than t+2, and FX swaps that correspond to lending or borrowing FX against local currency. The main risk for the central banks in these transactions arises from the liquidation of positions in the

[31] The revaluation account and statutory capital together constitute equity.

event of a counterparty default. For example, if the central bank lends local currency for U.S. dollars and the counterparty defaults, the central bank must sell the U.S. dollars in the market. This is a market risk which is usually mitigated by appropriate haircuts and regular marginal calls. The CBST would assess the exposure to market risk once mitigation measures have been factored in.

D. Commodity Risk

59. In some rare cases, central banks are exposed to changes in commodity prices. For instance, this concerns central banks that have purchased equities or gold. The framework models commodity-like items as a stochastic process.

E. FX Liquidity Risk

60. While central banks are not liquidity constrained regarding their local currency, they face constraints like commercial banks in currencies that they do not issue. Central banks in fully dollarized economies are subject to standard liquidity risk. In non-dollarized economies, central banks are not normally exposed to a run of their depositors because they normally have a long FX position. However, central banks can be prevented from aiding the market in a FX liquidity crisis due to the depletion of its FX reserves which is akin to *policy* insolvency as the central does not have the resources to achieve its financial stability mandate. This risk is not covered in this guidance note as a tool has already been proposed in Oura (2022).

FORECASTING BALANCE SHEET AND FINANCIAL ACCOUNT ITEMS

A. Currency in Circulation and the Reserve Requirement

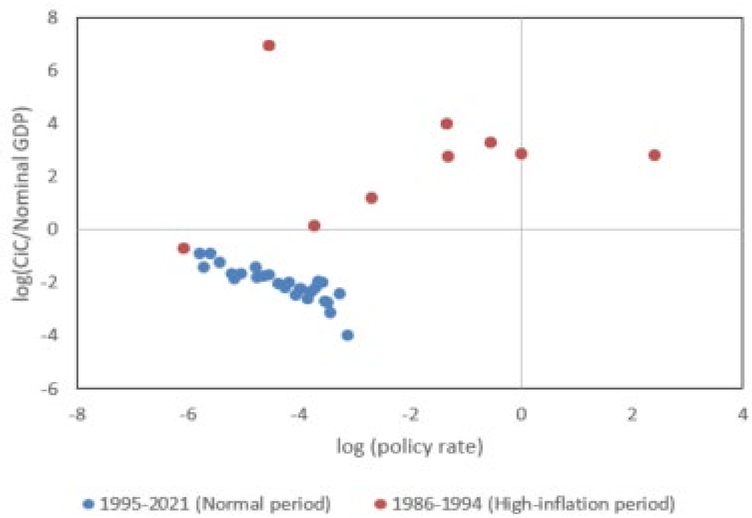
61. First, we propose a seasonal autoregressive integrated moving average (ARIMA) model to which nominal GDP is added as an exogenous variable. This exogenous variable capture both the inflation and real GDP impacts on CiC. Conditioning CiC on macroeconomic variables ensures that CiC dynamically responds to stress testing scenarios.

62. A second specification involves the addition of interest rate effects as follows:

$$\mathrm{CiC_t} = \eta_{\mathrm{CiC}}\, \mathrm{NGDP_t} \cdot \left(\frac{i_0}{i_t}\right)^{\gamma_{\mathrm{CiC}}},$$

Where, η_{CiC} is a scaling factor derived from the data, and $\left(\frac{i_0}{i_t}\right)^{\gamma_{\mathrm{CiC}}}$ captures the impact of policy rate changes to CiC, with γ_{CiC} measuring the level of sensitivity. The drop in the demand for money during episodes of high inflation could introduce nonlinearities in the relationship between CiC and prices that need to be modeled (see Figure 12).

Figure 12. Non-Linearities in the Demand for Currency



Source: IMF's International Financial Statistics and IMF staff calculations.

63. Assuming a constant ratio and definition of the base, the balances held at the central bank for the reserve requirement should depend on the demand for money, similar to cash in circulation. This is particularly important if the requirement is not remunerated, as its growth would change the interest paid or received, depending on whether banks have a deficit or a surplus with the central bank. This dynamic would not occur if the requirement was remunerated at the policy rate. The CBST could be used to simulate the impact on equity of remunerated versus unremunerated reserve requirements to test the strength of the central bank's balance sheet.

B. Operational Cost

64. Operational costs are modeled to increase log-linearly with nominal GDP as follows:

$$\text{Operational Cost}_t = \mu NGDP_t^{\alpha},$$

Where, μ is a scaling factor, while the coefficient α represents the growth path for operational costs relative to nominal GDP over the period. This is the benchmark setting in the CBST.

65. This specification allows the dynamic response of operational costs to stress testing scenarios. The CBST model can be used to simulate the impact on retained earnings and, thus, possible improvement in capital resulting from efforts in operational efficiency.

C. Relation with the Treasury

66. In most cases, the central bank is the designated banking agent of the government, in which case its position should be forecasted. Two situations could be encountered:

- *The Treasury has an account but does not borrow from the central bank*: The assumption is that the government, which is not a saving institution, keeps at the central bank the buffer necessary to safely cover its typical cash flow needs. Short-term cash flows do not matter here; thus, a constant balance is projected based on the historic average of the account as a percentage of GDP.

- *The Treasury has an account and does borrow from the central bank:* Projections would have to come from the government and central bank on the expected borrowing and reimbursement, eventually based on the provision in the law if central bank borrowing is regulated.

D. Asset Purchase Programs and Other Unconventional Operations

67. Asset purchase programs are designed to determine the size of the balance sheet. In that sense, they differ from OMO that act as a residual, absorbing the impact of other items. The trajectory of the asset portfolio of such programs would be derived from the policy announcement of the central bank, i.e., whether they intend to increase the balance sheet or to reduce it.

E. Foreign Reserves

68. The impact of exchange reserve depends on the FX policy. Three cases could be considered:

- *Freely floating exchange rate*: FX reserves are not actively used. The stock, denominated in foreign currency, is assumed constant.

- *Managed float*: The central bank has an active FX intervention policy. The FX reserves trajectory would be determined based on the central bank's FX accumulation or decumulation plan.

- *Fixed exchange rate*: Here, FX reserves are unpredictable as the central bank responds on demand to the request of the market to buy or sell FX. However, if the interest rate rule correctly captures the risk premium, the FX reserves could be assumed as growing with the demand for money. This is because the need arising from CiC and the reserve requirement in the market would be satisfied by selling FX to the central bank. This would be equivalent to keeping FX reserves stable as percentage of nominal GDP.

SCENARIO DESIGN

69. The design of a baseline scenario must include macro-financial variables that are internally consistent. The risk-horizon of the CBST model spans five years because the accuracy of the forecasts declines with the forecast horizon. The macroeconomic assumptions and forecasts for the medium term could be derived from the official central bank forecasts, which is developed by the research (or equivalently entitled) department and subsequently provided to the Risk Unit. The macroeconomic assumptions associated with the stress testing scenarios can then be developed by the Risk Unit, with the forecasting results corresponding to these assumptions provided by the research department. Sufficient variation around the baseline scenario is necessary for risk assessment and serves as a robustness check. Scenario design could be based on (i) at-risk models; (ii) other models commonly used at the IMF, including SVAR (Structural Vector Autoregression) and

DSGE (Dynamic Stochastic General Equilibrium); and (iii) deterministic models. We prefer at-risk models because they provide "extreme but plausible" shocks by definition and are relatively easy to set up in different contexts. While the general equilibrium approach would be ideal because it accounts for endogeneity issues in central bank stress testing, it is also quantitatively difficult to estimate and would require a greater extent of resources to maintain such macro-models.

70. The central bank balance sheet projection follows a recursive method over the stressed horizon. The process is as follows:

- The scenarios on real GDP growth, inflation, foreign and domestic interest rates, and the exchange rate, determine the macro-financial environment for the period.
- The satellite models map the scenarios into the different items on the balance sheet and income statement.
- Cash flows, the net profit (or losses), and capital for the period are then computed. The balance sheet is projected without dividend distribution.
- The model is iterated to the next period.

A. At-risk Models

71. At-risk types of models are based on explanatory variables and on projections of future distributions conditionally to a given economic situation. Therefore, they are useful for designing "extreme but plausible" macroeconomic scenarios. The at-risk models project asymmetric distributions reflecting future risks relying on quantile regressions and on variables related to financial conditions and macro-financial vulnerabilities. Different from approaches purely based on a historical calibration, the at-risk projections capture structural changes within the economy and can quantify both upside and downside risks to the variables of interest (e.g., GDP growth, inflation rates, exchange rates, etc.) by using the most recent available observations. The advantage of this approach is that it captures nonlinear interaction between shocks, financial conditions, and economic outcomes predicted by theory (Adrian et al., 2019; Prasad et al., 2019).[32]

72. The at-risk methodology is mainly suitable for short horizons of time, ideally up to one year. After estimating and applying the shocks, various econometric techniques can be employed to construct a trajectory for macroeconomic variables for the remaining forecast horizons. Techniques such as VARs can be employed, which can be estimated using frequentist or Bayesian techniques (see Figure 13). These techniques should be combined with policy responses, ideally using the central bank's reaction functions.

73. Using a growth-at-risk model enables the stress testing of equity against deteriorating macro conditions. The "at-risk" model forecasts conditional distributions for changes in real GDP.

[32] While a macro model is the preferred approach for developing macroeconomic scenarios, it is not always available in the area departments for all countries, and thus cannot be the default method.

Four steps are required to obtain a full distribution of the variable of interest: (i) dimensionality reduction on explanatory variables to avoid overfitting and multicollinearity problems; (ii) conditional average prediction using the Theil-Sen model to accommodate small and "noisy" samples; (iii) skewness estimation of the distribution by the Firth model; and (iv) along with an assumption of Theil-Sen variance, the parametrization of an asymmetrical Gaussian distribution.[33] Growth-at-risk is transmitted through higher PDs and other balance sheet impacts from lower output changes, such as slower growth in CiC.

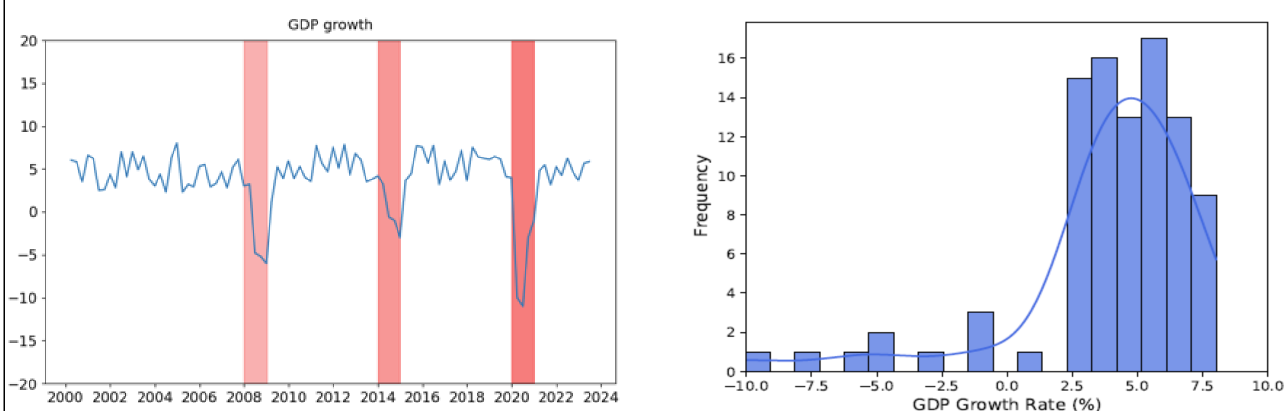
74. An inflation-at-risk model is employed to enable the stress testing of equity against extreme but plausible inflation levels over a short forecast horizon. Satellite models are also used to estimate the impact of shocks to inflation on other macroeconomic variables and on autonomously driven balance sheet items such as CiC, operational expenses, and the policy rate.

75. At-risk could be used to model other variables of interest. For instance, exchange-rate-at-risk could be used to determine how much of the revaluation reserve should be realized based on an assessment of the tail distribution of the exchange rate risk. The "at-risk" model forecasts conditional distributions for the bilateral exchange rate at different horizons.

Figure 13. Illustration of Growth-at-Risk Scenario

Historical data is employed to identify economic crisis that substantially impacted GDP, such as the Global Financial Crisis (2008–09), a hypothetical banking crisis in 2014, and the COVID-19 crisis (2020)…

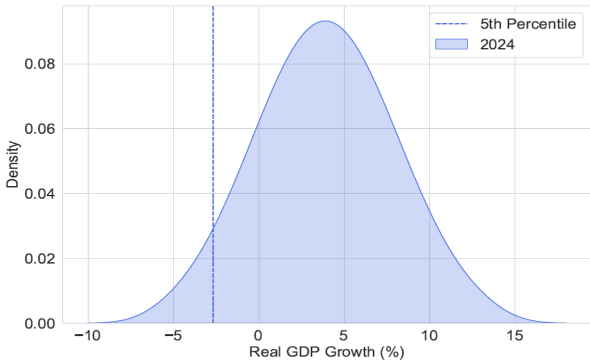
… which allows the derivation of a distribution for Real GDP growth…



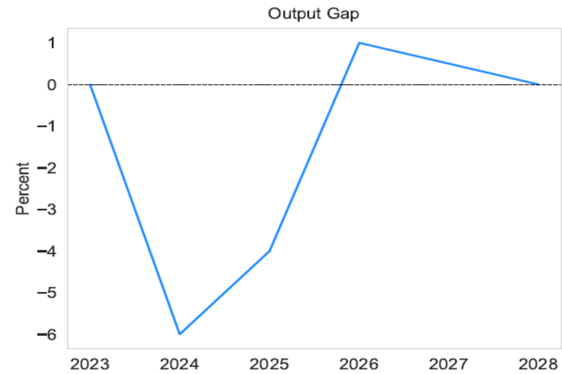
[33] See R. Lafarguette and Z. Chen, "West African Economic and Monetary Union: Technical Note on Stress Tests—Credit, Concentration, and Interest Rate Risks" (IMF 2022). See Appendix V for further technical details on at-risk models.

Figure 13. Illustration of Growth-at-Risk Scenario (concluded)

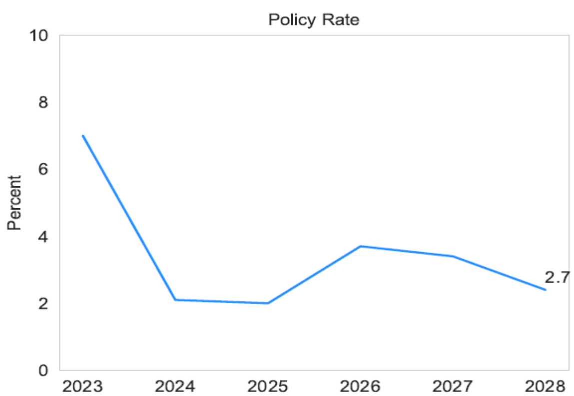
...allowing the estimation of a forecast for the 5th percentile for Real GDP for 2024...



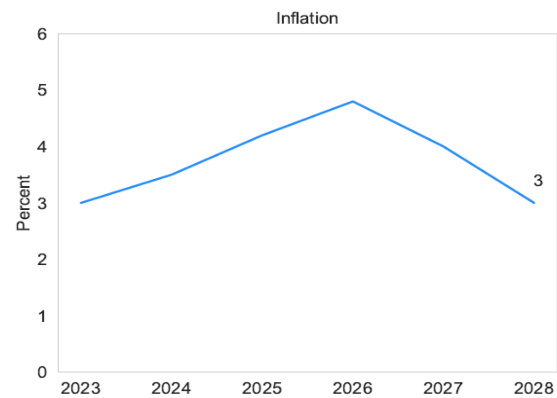
...which corresponds to a sharp fall in the output gap of -6.2 percent in 2024.



The central bank responds by decreasing the policy rate, which is again raised after 2026 when the economy is heating up.



Inflation follows the trend, going back to target.



Source: IMF staff calculations.
Note: Illustrative example using mock data.

B. Deterministic Models

76. Deterministic models focus on a fixed set of parameters to assess the impact of specific scenarios without the influence of random variables. These scenarios are typically utilized to assess the robustness of the equity path to costs that are mandated by discretionary central bank decisions, such as whether to remunerate required reserves. Deterministic scenarios can also be employed to assess policy changes by the central bank that can increase equity. As an example, scenarios could capture the positive impact on equity due to cost efficiency gains.

C. Other Models

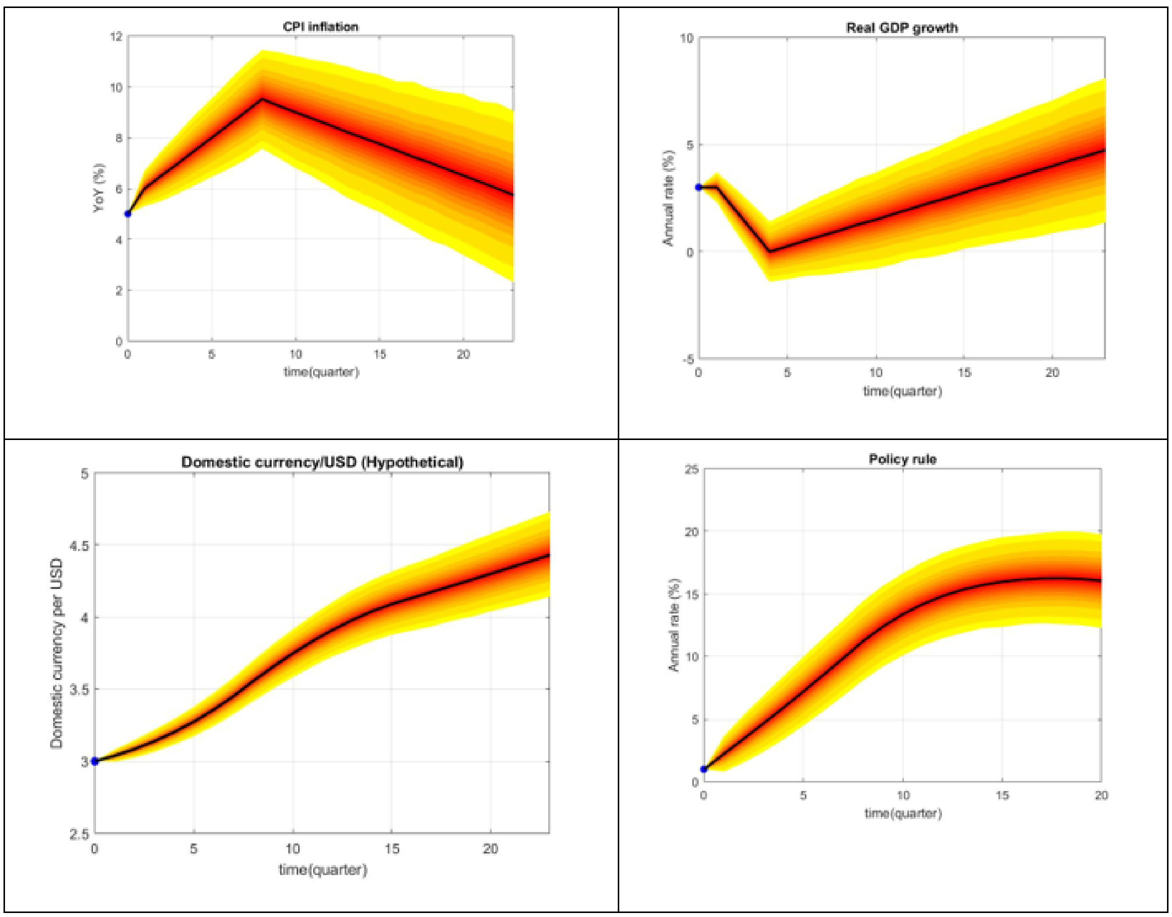
77. Other model could be considered, data availability permitting. SVAR models allow for the assessment of how an economy responds to changes in policy and external shocks by capturing the dynamic relationships between multiple time series variables. DSGE models, on the other hand, use microeconomic principles to forecast macroeconomic phenomena, making them useful for understanding the impact of shocks and policy changes over time.

MODEL OUTPUT USING STYLIZED BALANCE SHEETS

A. Macroeconomic Environment

78. In what follows, we simulate the impact of the macro-economic environment on central bank capital. These examples, while hypothetical, reflect the proportion of the key items in the balance sheet of central banks. They illustrate how the CBST model works under different balance sheet configurations. In these scenarios, macro-financial variables are the same across different cases, allowing us to see the impacts of the initial balance sheets on the capital path, independent of macro-financial factors.

Figure 14. Projection of Macroeconomic Variables



Source: IMF staff calculations.

79. The macroeconomic scenario describes an inflation shock. Inflation is high at the beginning of the simulation but declines over time back to the target level. This decrease is a result of the central bank increasing its policy rate in response to the inflation shocks (as derived from the reaction function of the model). The increase in the policy rate leads to a short recession, which is necessary for lowering inflation.[34] The exchange rate depreciates over time, but the speed of the depreciation becomes slower after two years.

80. We make several assumptions about returns for the claim on the government and the renumeration of the reserves from banks so that our simulation analyses are plausible. The key assumptions are summarized in the table below.

Table 5. Key Assumptions

Balance Sheet Type	Claim on Government	Claim from Banks
Lean	Policy Rate	Policy Rate
Large Domestic Portfolio	Function of Yield Curve	Policy Rate
Large Foreign Portfolio	Function of Yield Curve	Policy Rate
Debt Restructuring	Zero Return for Government Loan	Zero for Required Reserves

Source: IMF staff calculations.

B. Lean Balance Sheet Simulation

81. The expected macroeconomic environment is first applied to a lean balance sheet. Most of the central bank liabilities is CiC. Banks maintain what is necessary to fulfill the reserve requirement on their accounts at the central bank. As the demand for CiC increases, banks satisfy their demand for liquidity by borrowing from the central bank (OMO) at the policy rate.[35] The central bank operates a flexible exchange rate. It does not need ample FX reserves but keeps a portfolio to eventually intervene in the FX market and diversify its income stream. Net foreign assets represent 30 percent of total assets. Starting equity is 5 percent of total assets, and the initial operational costs are 0.5 percent.

Table 6. Lean Balance Sheet

Assets	Domestic Currencies (thousand)
Net Foreign Assets	396,000
Net Claim on Government	903,000
Total Assets	1,299,000

Total Equity and Liabilities	Domestic Currencies (thousand)
Currency in Circulation	907,000
Net Claim from Banks	303,000
Other Liabilities	20,000
Total Equity	69,000
Total Liabilities and Equity	1,299,000

Source: IMF staff calculations using hypothetical data.

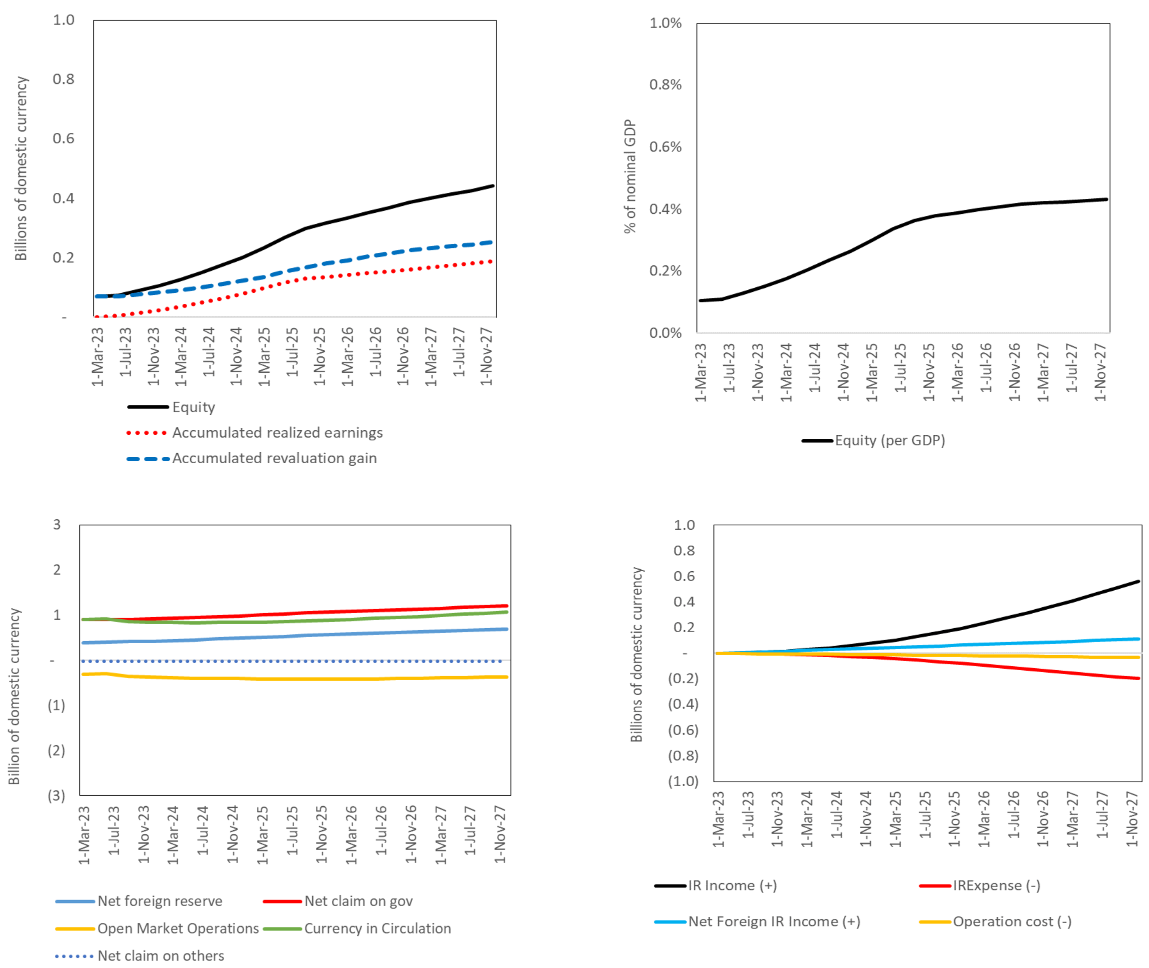
82. The inflation shock is positive for the equity of the lean central bank. Assuming no distribution, higher inflation supports capital via two channels: (i) increased interest income due to

[34] While the 15-percentage point rise in the policy rate is a significant shock, this has been seen in a few emerging markets following the COVID-19 pandemic, whereby the policy rate was raised by over 10 percentage points to combat inflation.

[35] Alternatively, banks sell high-quality assets to the central bank, such as government securities, for reserves.

the higher policy rate; and (ii) larger OMO stocks due to faster growth in CiC, which is itself a consequence of higher inflation.[36] Equity also increases as a percentage of GDP when the real interest rate turns positive (and, thus, realized earnings grow faster than nominal GDP). In this scenario, the central bank's capital exceeds the minimum required to break even over the medium term. The decision on whether to retain profits in general reserves for adequate capitalization or distribute them to the government would require stressing the macroeconomic framework.

Figure 15. Macro Impact Simulation on a Lean Balance Sheet



Source: IMF staff calculations.

[36] It should be noted that the exchange rate is held fixed for this scenario. If the exchange rate is allowed to dynamically respond, then some of the gains in the accumulation of capital would be marginally tempered by the appreciation of the exchange rate (which in turn causes losses on the FX holdings because these balances would be expressed in domestic currency on the balance sheet) in response to higher domestic interest rates. The impact, however, would be limited to only a marginal effect because the lean balance sheet would contain only a small FX portfolio relative to the size of the balance sheet, therefore rendering the result quite close to the illustration in Figure 15.

C. Large Domestic Portfolio

83. The central bank holds a large portfolio of domestic securities. Reflecting actual trends, its total assets increased 7.6 times compared with the lean balance sheet. The increase is entirely due to domestic asset purchases. The starting point in nominal terms remains the same for CiC (whose share drops to 9 percent of total assets, consistent with recent ratios in AE balance sheets), equity, and operational costs. The domestic policy rate path follows the pattern of the lean balance sheet scenario, where the policy rate increases over time from 1 percent to 16 percent. We also assume a flat yield curve over time for simplicity.

84. We explicitly model the duration of the domestic bond portfolio. Specifically, we developed a satellite model wherein an equal amount of the bond is held across various maturities up to the pre-fixed longest maturity. In each quarter, a portion of the bond portfolio reaches maturity, and the principal amount is re-invested into the bond with the longest maturity. In our simulation, we consider two cases for the average maturity: (i) one year; and (ii) five years. The valuation of the domestic bond portfolio is based on the amortization method (the portfolio is held to maturity) and, thus, there is no mark-to-market loss arising from an increase in the domestic yield curve.

Table 7. Balance Sheet with a Large Domestic Portfolio

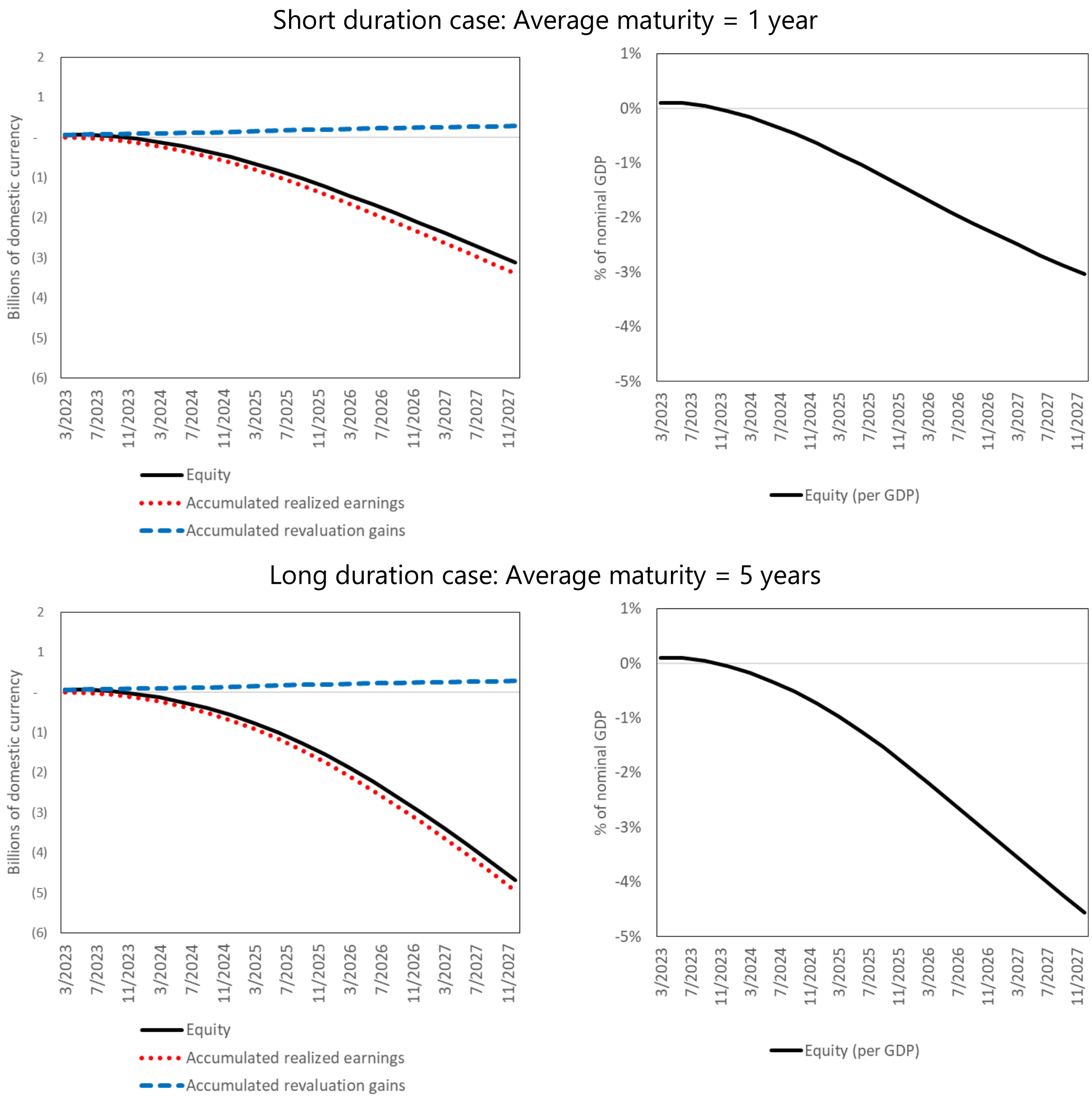
Assets	Domestic Currencies (thousand)
Net Foreign Assets	396,000
Net Claim on Government	9,500,000
Total Assets	9,896,000

Total Equity and Liabilities	Domestic Currencies (thousand)
Currency in Circulation	907,000
Net Claim from Banks	8,900,000
Other Liabilities	20,000
Total Equity	69,000
Total Liabilities and Equity	9,896,000

Source: IMF staff calculations using hypothetical data.

85. The central bank capital declines over time because the interest income from the domestic portfolio is smaller than the interest expenses on excess reserves. Our results show that longer durations exacerbate the decline in capital due to the slower adjustment of bond portfolio returns. To see this, let us compare two cases: the short duration case vs. the long duration case. In the first case, the capital ends at -3 percent of the nominal GDP, while it ends at -5 percent in the second case. Hence, the longer duration makes the equity path worse due to the slower adjustment of bond portfolio return.

Figure 16. Macro Impact Simulation on a Balance Sheet with a Large Domestic Portfolio



Source: IMF staff calculations.

D. Large Foreign Portfolio

86. In this case, the central bank has a large portfolio of foreign securities within a fixed exchange regime. Total assets have increased 4.7 times compared to the lean balance sheet (Table 8). The increase is entirely due to the portfolio of foreign government bonds. Assuming the reaction function of a fixed exchange rate, the domestic policy rate follows the U.S. policy rate plus a spread, assumed to be 25 basis points. We also consider two cases for the term spread dynamics of the U.S. yield curve. In the first case, the U.S. yield curve remains flat over time. In the second case, the U.S. yield curve steepens in the first two years before gradually flattening after December 2025 (Figure 17).

Table 8. Balance Sheet with a Large Foreign Portfolio

Assets	Domestic Currencies (thousand)
Net Foreign Assets	5,170,000
Net Claim on Government	903,000
Total Assets	6,073,000

Total Equity and Liabilities	Domestic Currencies (thousand)
Currency in Circulation	907,000
Net Claim from Banks	5,077,000
Other Liabilities	20,000
Total Equity	69,000
Total Liabilities and Equity	6,073,000

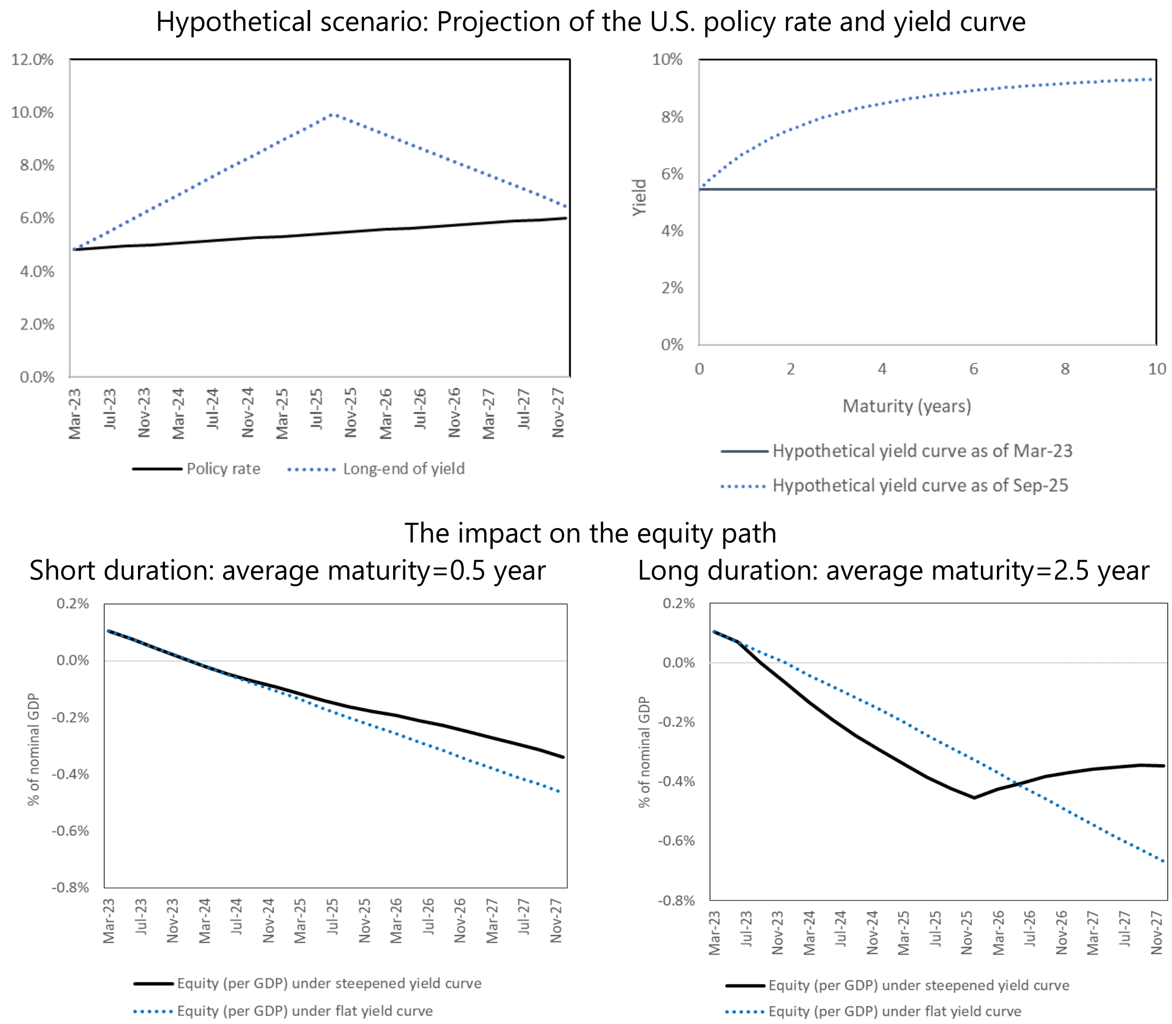
Source: IMF staff calculations using hypothetical data.

87. We explicitly model the duration of the foreign portfolio in a similar manner to the large domestic bond portfolio case, but with mark-to-market valuation. We employ a satellite model where the central bank rolls over the foreign bond portfolio, valuated under mark-to-market accounting. The satellite model allows us to capture the effect of the average yield of the bond portfolio, which only partially adjusts to the foreign policy rate, proxied by the USD policy rate. For illustrative purpose, we assume the U.S. policy rate increases over time from 4.8 percent to 6 percent in the next five years. We consider two cases: (i) the average duration is 0.5 years; and (ii) 2.5 years. Prudent FX risk management requires a short average duration. A longer duration might represent a scenario where a central bank, holding substantial FX reserves, pursued yields by investing in longer maturity assets during a period characterized by low interest rates in reserve currencies.

88. The central bank's capital declines over time because the interest income from the foreign portfolio is smaller than the domestic interest expenses on excess reserves. Recall that the domestic policy rate is set as the U.S. policy rate plus the spread. The average yield of the foreign bond portfolio gradually adjusts through the rollover, eventually catching up to the U.S. policy rate. To see this effect, consider the flat yield curve case, illustrated by the blue-dotted line in Figure 17. It demonstrates equity paths under two assumptions: the short duration (bottom left) and the long duration (bottom right). Under the short duration, the capital ends at -0.5 percent of nominal GDP, while it ends at -0.7 percent under the long duration. Hence, the results show that a longer duration leads to poorer capital outcomes due to the slower adjustment of the bond portfolio return, echoing the trend seen in the large domestic portfolio case.

89. The mark-to-market loss impacts the equity path, especially when the duration is longer. By comparing the black solid lines in the bottom two panels of Figure 17, it is evident that equity declines up to December of 2025 and then begins to recover under the long duration assumption (the right bottom panel). This pattern occurs because the bond portfolio with a longer duration is more severely impacted by an increase at the long end of the yield curve. The stronger recovery observed after 2025 can be attributed to two factors: (i) the long-end of the yield curve declines after 2025, and thus produces mark-to-market gains; and (ii) the returns for long-term bonds are higher in the scenario with a steepened yield curve than in the flat yield curve case.

Figure 17. Macro Impact Simulation on a Balance Sheet with a Large Foreign Portfolio



Source: IMF staff calculations.

E. Sovereign Credit Risk due to Possibility of Debt Restructuring

90. In the final hypothetical case, the central bank holds a large portfolio of domestic government securities, which is impacted by heightened credit risk. Total assets increased 7.6 times compared to the lean balance sheet. The initial balance sheet mirrors that of the large domestic bond portfolio case. However, a key distinction in this scenario is Expected Credit Losses (ECL) on the domestic government bond portfolio, which arises due to the heightened sovereign credit risk associated with possibility of a debt restructuring process. Another distinction is that we assume no return from the domestic bond portfolio and zero renumeration rate on the excess reserves, therefore no duration risk.[37]

[37] The assumption of zero remuneration rate is applied only for expository reasons to remove the duration impact, thereby directing the focus of the analysis on the ECL impact.

Table 9. Balance Sheet Under Debt Restructuring

Assets	Domestic Currencies (thousand)
Net Foreign Assets	396,000
Net Claim on Government	10,500,000
ECL on Government Bonds	(1,000,000)
Total Assets	9,896,000

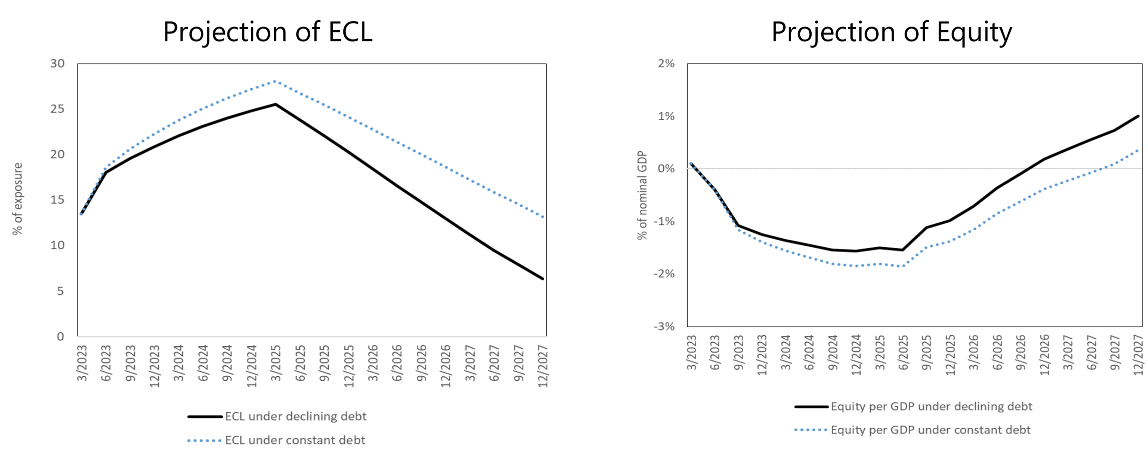
Total Equity and Liabilities	Domestic Currencies (thousand)
Currency in Circulation	907,000
Net Claim from Banks	8,900,000
Other Liabilities	20,000
Total Equity	69,000
Total Liabilities and Equity	9,896,000

Source: IMF staff calculations using hypothetical data.

91. In this scenario, the ECL on government debt, which is impacted by heightened credit risk due to the increased possibility of debt restructuring is simulated. We allow the ECL to be driven by the macroeconomic environment. To do so, we employ a satellite model where the probability of default (PD) is inferred from changes in sovereign CDS spreads, varying in response to three macroeconomic variables: real GDP growth, inflation, and public debt as a percentage of GDP. For the first two, we use the same scenario as shown above. For the third one, we consider two cases: (i) debt as percentage of GDP starts from 90 percent and declines by 1 percent in each quarter; and (ii) debt as a percentage of GDP holds at 90 percent over time.

92. The path of public debt impacts the central bank capital through the ECL. In both scenarios outlined above, the capital of the central bank declines for the first two years, and then increases over time. However, there is a stark difference between the two cases**.** In the case of a declining debt path, capital reaches 1 percent of nominal GDP at the end of the simulation because ECL declines quickly after the first two years. In contrast, capital reaches 0.4 percent in the constant debt path because ECL declines more slowly. This disparity arises from the differing dynamics of ECL in the two scenarios.

Figure 18. Macro Impact Simulation on a Large Domestic Portfolio Under Debt Restructuring



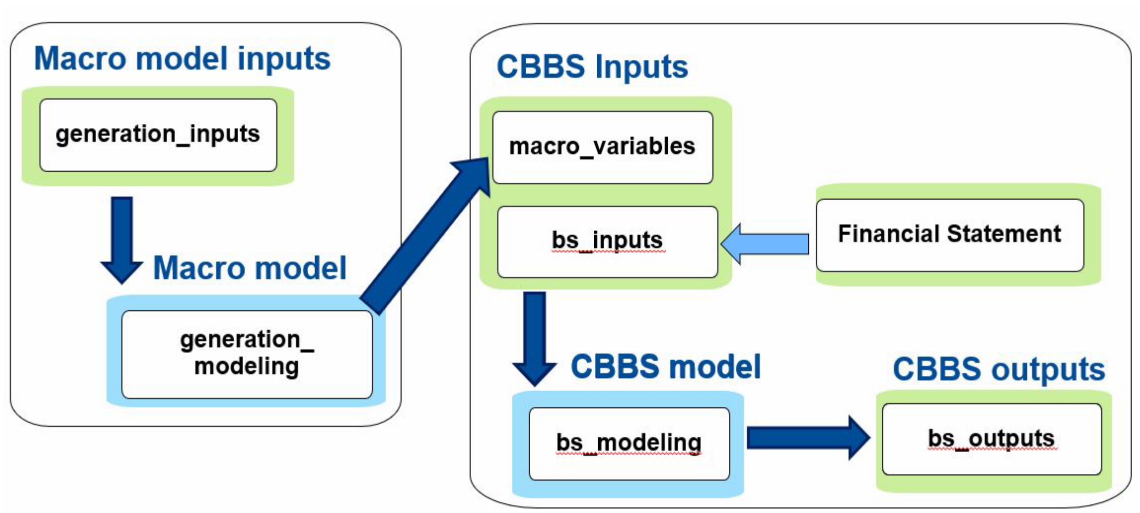
Source: IMF staff.

Appendix I. Structure of Codes for CBST

1. **The codes for CBST are structured to ensure readability and modularity.** There are multiple components in modeling central bank balance sheets. Therefore, CBST codes are structured in a modular way to enhance code readability. In the figure below, the blue box is a folder of codes. The green box is a folder of data files.

2. **The macroeconomic component is separated from the main component for CBST.** For example, one can change how macroeconomic variables are generated by replacing exiting macro model codes with new macro model codes in the folder named "generation_modeling." One can also directly overwrite generated macroeconomic variables in files located in the folder named "macro_variables."

Figure I.1. Structure of CBST Codes

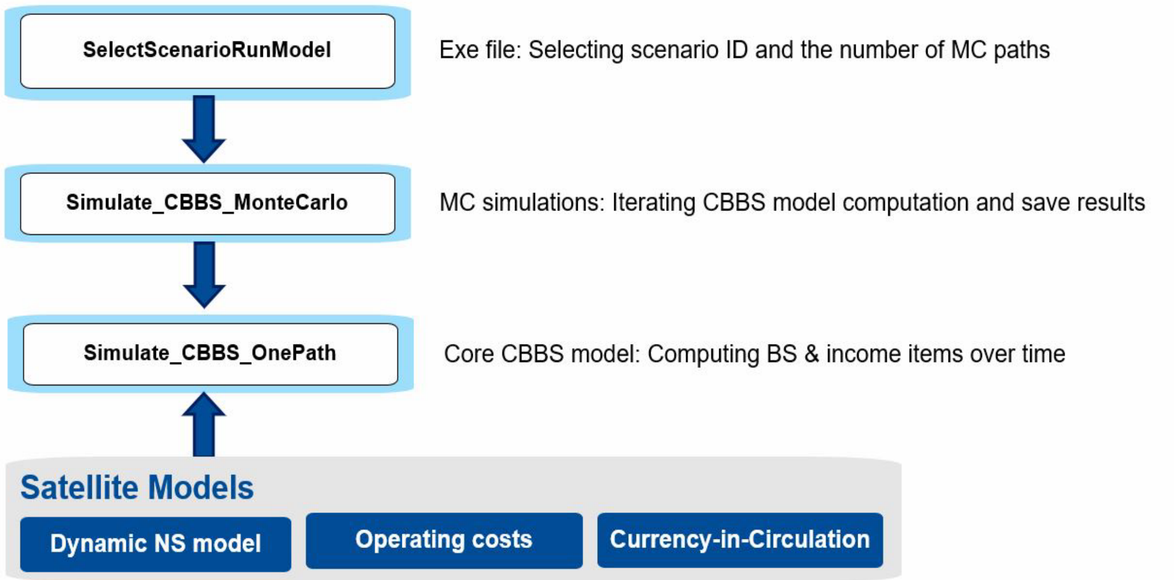


Source: IMF calculations

3. **The codes in the main component are organized in a user-friendly manner.** Specifically, there are three executions codes. The first code, named SelectScenarioRunModel, allows users to choose a scenario identification number and specify the number of Monte-Carlo simulation paths. The second code, Simulate_CBBS_MonteCarlo, is utilized for generating projections of central bank balance sheet items and income statements. This code repeatedly processes these projections using the same inputs of balance sheet and fundamental parameters.

4. **The third execution code represents the core of central balance sheet model, where all balance sheet items and income statement items are computed over time, taking into account the macroeconomic inputs, the initial balance sheet, and fundamental parameters.**

Figure I.2. Layer of Main Exe Codes



Source: IMF calculations

Appendix II. Dynamic Nelson-Siegel Model

1. **The Dynamic Nelson-Siegel model extends the Nelson-Siegel model by explicitly modeling factor dynamics.**[1] Let us first explain the original Nelson-Siegel (NS) model.

2. **The NS model is a widely used functional form to interpolate yields with different maturities.** It is a parsimonious functional form to describe various types of yields curves because it summarizes the shape of yield curves with three factors and one parameter. Specifically, the NS functional form is represented as:

$$y_{model,t}(\tau, \lambda) = L_t + \frac{1-\exp(-\lambda\tau)}{\lambda\tau} S_t + \left(\frac{1-\exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau)\right) C_t,$$

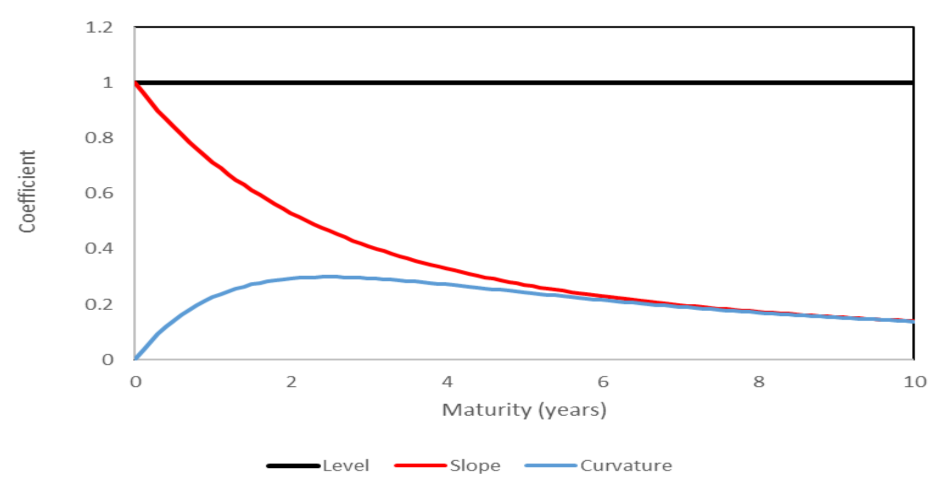
3. **Where L_t, S_t, and C_t are called Level, Slope, and Curvature factors, respectively. τ is the maturity.** λ is a constant parameter, calibrated using the historical data of yield curves. Specifically, λ is obtained as a solution of this nonlinear optimization problem:

$$\min_{\lambda} \sum_{t=1}^{T} \sum_{m=1}^{M} \left(y_{data,t}(\tau_m) - y_{model,t}(\tau_m, \lambda)\right)^2 ,$$

4. **Where $y_{data,t}(\tau_m)$ is the actual yield with maturity τ_m and $y_{model,t}(\tau_m, \lambda)$ is the Nelson-Siegel-model-based yield that is computed with three NS factors.** These three NS factors are obtained by an ordinary least square (OLS) regression of three yields on three NS factors at each point in time.

5. **The following figure shows each coefficient as a function of maturity.**

Figure II.1. Level, Slope, and Curvature



Source: IMF staff.

[1] This extension was proposed by Diebold and Li (2006) and Diebold, Rudebusch, and Aruoba (2006).

6. When the maturity goes to zero, we obtain the short-end of the yield as

$$y_t(0) = L_t + S_t.$$

7. We interpret this short-end of the yield curve as the policy rate $\boldsymbol{i_t = y_t(0)(= L_t + S_t)}$.

8. Let us represent the Nelson-Siegel model in matrix form. To do so, we introduce the Nelson-Siegel coefficient vector $\beta(\tau) = \left(1, \frac{1-\exp(-\lambda\tau)}{\lambda\tau}, \frac{1-\exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau)\right)$. Let us denote a vector of the three factors with $X_t = (L_t, S_t, C_t)'$.

9. For yields with $\boldsymbol{M}$ number of maturities, $\boldsymbol{y_t = (y_t(\tau_1), y_t(\tau_2), y_t(\tau_3) \ldots y_t(\tau_M))}$, the Nelson-Siegel model is re-written as the matrix form:

$$y_t = \beta \cdot X_t \,,$$

10. Where the $\boldsymbol{3 \times M}$ coefficient matrix is given by

$$\beta = \begin{pmatrix} \beta(\tau_1) \\ \vdots \\ \beta(\tau_M) \end{pmatrix}.$$

11. Under the dynamic Nelson-Sigel model, we assume that these three factors evolve over time following the vector autoregressive (VAR) model. The VAR is specified as:

$$X_t = \Phi\, X_{t-1} + \mu + \epsilon_t,$$

12. Where the shock $\boldsymbol{\epsilon_t}$ is sampled from a three-dimensional multivariate normal distribution with zero mean and the covariance matrix $\boldsymbol{\Sigma}$.

13. It is also noteworthy that the unconditional mean of the vector of the factors $\boldsymbol{X_t}$ is given by $\boldsymbol{\bar{X} = (I - \Phi)^{-1}\mu}$.

14. The dynamic NS model generates dynamics of long-term yield proxied by $\boldsymbol{L_t}$ and policy rate $\boldsymbol{i_t(= L_t + S_t)}$ separately. Therefore, one can capture potential divergence between them. This is important because it impacts central bank profitability. Suppose that a central bank holds and rolls over its domestic government bond portfolio while paying policy rates on its reserves. The net interest income is lower if policy rates increase faster than the long-term yield.

Appendix III. Technical Details of Internal Credit Rating Model

1. **In order to forecast ECLs, forecasted values of the PD, LGD, and EAD are all needed.** The internal credit rating model is used to derive a forecast for the PD, as noted in section IV of this guidance note. This is a key component of the calculation of the ECL provision in CBST models. The credit rating-based approach for computing ECLs is widely employed in central banks' financial statements. However, to the best of our knowledge, central banks do not forecast ECL as a part of stress testing their balance sheet. Employing the credit rating model enables the macro framework to be linked to ECL provisioning. The credit rating model can also be back tested to ensure accuracy. Its structure is applicable to all countries, accommodating varying MP regimes and economic structures.

2. **The credit rating model has four blocks and an ESG overlay.** The blocks measure a country's economic strength, institutional strength, fiscal strength, and susceptibility to event risks such as political turmoil, banking sector crisis, and natural disasters.[1]

Table III.1. Indicators Used in the Credit Rating Model

Economic Strength	Institutional Strength	Fiscal Strength	Vulnerability Indicators
GDP Growth	Inflation Level	Debt Trajectory	Political Governance Measures
GDP Volaility	Inflational Volatility	Domestic Debt/GDP	Banking Sector Risk Indicators
GDP Per Capita	FATF Compliance	Domestic Debt/Revenue	Natural Disaster Risk Indicators
Credit Growth	Default Record	Foreign Debt/Domestic Debt	Government Liquidity Risk Indicators
Economic Diversity	Governance Index	Interest Payments/GDP	External Vulnerability Indicators
		Interest Payments/Revenue	
		Other Public Debt	

Source: IMF staff

3. **Each block and sub-block are weighted based on historical calibration.** ESG considerations and other remaining risk factors may require qualitative adjustments based on vulnerability models and/or judgment. All blocks, except for the event risk block, uses a portion of the variables as adjustment factors for the weighted sum of all other variables, the choice of which is also based on historical calibration.[2]

4. **Prior to employing the internal credit rating model to derive PDs, a back testing exercise is completed.** This back testing involves using the credit rating model to derive a country's credit ratings over the past five to ten years. The results are then compared to the actual credit ratings from rating agencies over the same period to determine the model's accuracy. The accuracy is evaluated to ensure the internal credit rating model is appropriately specified.

[1] The ESG overlay is applied only in exceptional circumstances to adjust model-determined credit ratings when necessary. The ESG overlay primarily focuses on climate risk, utilizing publicly available global indices that measure the risk for countries.

[2] The final model is not empirically estimated but is rather a credit rating scorecard, with its sub-weightings calibrated based on historical data. The model is back tested internally by producing indicative credit ratings of the past and comparing these to actual historical credit ratings.

5. Once the accuracy is deemed satisfactory, the credit ratings are forecasted for the respective country using internal macroeconomic forecasts and debt sustainability analysis results. Governance measures are assumed to stay constant unless alternative assumptions are deemed more appropriate. The forecasted credit ratings are then converted to PDs. Tables mapping the sovereign credit rating to the historical PD are available from major rating agencies. All sovereign credit ratings have a corresponding PD that may differ based on the years to maturity. The standard approach is to employ a one-year PD. This is the probability of default within one year, which in the general case is lower than the probability of default within a higher number of years. The lifetime PD—which is the probability of default within the lifetime of the security up to its maturity—is employed for non-investment grade sovereigns with heightened credit risk. After making an appropriate choice for the PD in line with acceptable international accounting standards, the ECL is calculated using estimates for the LGD and the EAD.

Appendix IV. Estimating Country Risk Premium

1. **In its basic form, the uncovered risk parity (UIP) posits that the interest rate differential between two countries should align with the corresponding exchange rate change.** Deviations from this relationship are crucial in fixed exchange rate regimes, as any divergence in interest rates from the UIP condition inevitably applies pressure on the peg. The risk premium of a country can be affected by multiple factors, such as macroeconomic conditions, fiscal strength, political stability, geopolitical factors, regulatory framework, liquidity and market access, perceptions of risk, amongst other secondary factors.

2. **A model-based assessment, such as the CBST model, can be used to forecast country risk premium in a fixed exchange rate regime.** The model is set to operate at a monthly frequency to exclude the short-term volatility that may occur in higher frequency data. This approach improves the accuracy of medium to long-term forecasts. The target variable is designed as a spread shown below:

$$Spread_t = I_t - I_{0t},$$

3. **Where, I_t refers to the three-month interbank offered rate in the country and I_0 uses the three-month LIBOR rate, which can also be replaced by other international benchmarks in the future if needed.** This equation captures the excess risk over the international benchmark.

4. **This model uses time series dynamics on an ARIMAX formulation for estimation and forecast.** ARIMAX refers to an auto-regressive integrated moving average with exogenous variables, which incorporates exogenous variables alongside the autoregressive and moving average components. This method helps mitigate the risk of spurious connections between input factors and the risk premium. Table AIV.1. below presents a benchmark list of variables for use with this model. However, this list can be augmented with other relevant variables in specific cases. For example, in an oil export-oriented economy, variables like oil prices, government revenue from oil, and non-oil government revenue allocations can be included to refine the model’s estimation, given their impact on the risk premium expectations. In addition, other fiscal or banking sector indicators can be incorporated. All the input variables are differenced and normalized to become stationary and to facilitate model estimation.

Table IV.1. Factors Considered in the Risk Premium ARIMAX

	Factor
1	Government revenue
2	CDS 1 year
3	CDS 5 years
4	CDS 10 years
5	GDP
6	Stock market index
7	Stock market turnover
8	Unemployment (international)
9	Unemployment (domestic)
10	VIX
11	CPI

Source: IMF staff

5. **To identify the most relevant factors among possible combinations of input variables and identify the best group of regressors,** this model uses Akaike's Information Criterion (AICc), corrected for sample size, to test and rank these combinations**.** The ARIMA orders within the model are selected based on their AIC rank. All combinations of input variables are considered, each of which with optimal ARIMA dynamics.

Appendix V. Technical Details of the At-Risk Model[1]

1. Synthetic variables in the at-risk models are obtained through data reduction from a set of variables. We extract the common trends of several variables under the same "theme" (see the complete list in Tables V.1., V.2., and V.3.) using partial least squares (PLS) regression.[2] The PLS estimator models the covariance between two datasets, named Y and X, based on the latent structure of the underlying data. The latent structure is obtained by projecting both the Y and X matrices on a vectorial lower-dimension subspace, such that the covariance between the projections of Y and X in this new subspace is maximized. The PLS method is useful for analyzing data with numerous multicollinear variables that are potentially noisy and may even have incomplete observations. PLS excels in data reduction, particularly when aggregating a large number of collinear variables (X) with the aim of maximizing their correlation with a supervisory variable (Y).

Table V.1. Examples of Synthetic Regressors and Underlying Variables for Growth at Risk Model

Domestic Demand	External Demand	Domestic Financial Conditions
Real GDP growth lag Government consumption (percentage change) Household consumption (percentage change)	Oil price (percentage change) Food price (percentage change) China real GDP growth U.S. real GDP growth	Capital adequacy ratio (first diff.) Non-performing loans ratio (first diff.) Deposit rate (first diff.) Lending rate (first diff.) Market repo rate (first diff.)

*First difference and percentage change are calculated on a year-on-year basis unless otherwise indicated.

Sources: Bloomberg, Financial Sector Indicators, Haver, IMF Country Desk, IMF staff calculations, and World Economic Outlook.

[1] Extracted from R. Lafarguette and Z. Chen, "West African Economic and Monetary Union: Technical Note on Stress Tests—Credit, Concentration, and Interest Rate Risks," (IMF 2022).

[2] See Wold, Sjostrom, and Eriksson (2001).

Table V.2. Examples of Synthetic Regressors and Underlying Variables for Inflation at Risk Model

Domestic Macro	External Demand	Monetary Factor
Real GDP growth	Local currency to GBP FX rate (first diff.)	Capital adequacy ratio (first diff.)
Public consumption (percentage change)	Local currency to USD FX rate (first diff.)	Non-performing loans ratio (first diff.)
Private consumption (percentage change)	China inflation	Deposit rate (first diff.)
Import (percentage change)	U.S. inflation	Lending rate (first diff.)
Export (percentage change)	REER (percentage change)	Market repo rate (first diff.)
		M2 (percentage change)

*First difference and percentage change are of year-on-year basis unless otherwise indicated.

Possible Sources: Bloomberg, FSI, Haver, IMF Country Desk, IMF staff calculations, and World Economic Outlook.

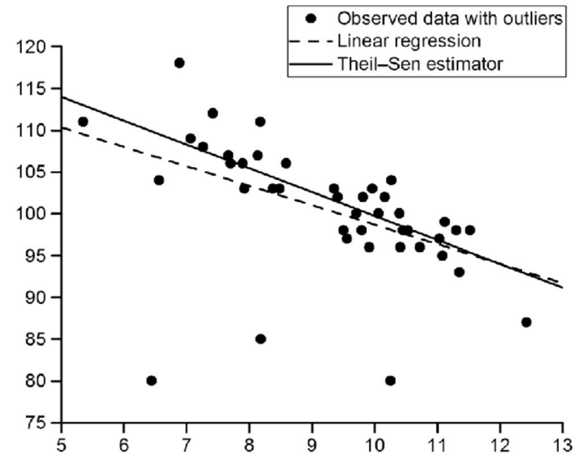
Table V.3. Examples of Synthetic Regressors and Underlying Variables for Exchange Rate at Risk Model

Domestic Macro	Inflation and Interest Rate	Balance of Payment
Real GDP growth	Deposit rate (first diff.)	Current account balance (percentage change)
Public consumption (percentage change)	Lending rate (first diff.)	Import (percentage change)
Private consumption (percentage change)	Market repo rate (first diff.)	Export (percentage change)

*First difference and percentage change are of year-on-year basis unless otherwise indicated.

Possible Sources: Bloomberg, FSI, Haver, IMF Country Desk, IMF staff calculations, and World Economic Outlook.

2. The conditional mean is estimated using a Theil-Sen model (Theil 1950; Sen 1968), which is a regression model improving the OLS estimator to make it more accurate for estimation on small samples and, particularly, robust to outliers. A "jackknife" Theil-Sen estimator—one that systemically removes one observation at a time from the initial sample—is constructed. For example, if the sample contains 20 observations, it creates 20 subsamples of 19 observations, with a different observation removed from the original sample at each iteration. It then estimates a classic ordinary least squares (OLS) regression on each subsample, thus obtaining 20 values for each coefficient. The Theil-Sen estimator is the average of these 20 values, i.e., the average of the OLS coefficients estimated on each of the subsamples. It is thus highly robust to outliers, to the extent that the impact of such observations is diluted in the estimators for each subsample. Taking the median makes it possible to reduce the impact of coefficients that are too extreme. The specification of the Theil-Sen model is like the one of an OLS:



Source: Scikit-Learn documentation.

$$y_{t+h} = \alpha + \beta^{TS} X_t + \epsilon_t^{TS},$$

3. Where y_{t+h} is the real GDP growth in t+h, X_t is a vector of conditional variables, α is the intercept, and ϵ_t^{TS} is the residuals of the Theil-Sen regression.

4. The Firth model, a logistic regression model with penalized likelihood, is employed to estimate the asymmetry around the average projection of growth, inflation, or exchange rate. Like the classical logistic model, the Firth model estimates the binary probability of an event. This probability may be coded as a binary 0/1 indicator, taking 1 if the event occurs and 0 if it does not. The event is coded as being dependent variable y_t, higher than a given value $\bar{\bar{y}}$. Thus, the specification of the Firth model is written as a classic logistic model.

$$\mathbb{P}[y_{t+h} > \bar{\bar{y}} | X_t] = \alpha + \beta^{LR} X_t + \epsilon_t^{LR},$$

5. Where y_{t+h} is the real GDP growth in t+h, $\bar{\bar{y}}$ is a given growth threshold, X_t is a vector of conditional variables, α is the intercept, and ϵ_t^{LR} is the residuals of the logistic regression.

6. Firth's innovation relies on the estimation method. For small and noisy samples, or samples with a weak degree of separation (a lot of 1s and few 0s, for example), the classic logistic estimator is biased. Firth shows that by modifying the likelihood function (the logistic models are estimated based on maximum likelihood) and by introducing a penalizing term, it is possible to eliminate the estimation bias.[3]

[3] See "Bias Reduction of Maximum Likelihood Estimates" (Firth 1993).

7. The Theil-Sen/Firth dual model thus estimates two moments in the conditional distribution of the variable of interest, and the third moment is obtained based on a parametric assumption. The first statistic is the conditional expectation estimated by the Theil-Sen model $\mathbb{E}[y_{t0+h}|X_{t0}] = \hat{\alpha} + \hat{\beta}^{TS} X_{t0}$, while the second is the asymmetry of the distribution, obtained as the cumulative density estimated at the conditional mean[4] $\mathrm{F}(y_{t0}|X_t) = \hat{\alpha} + \hat{\beta}^{LR} X_{t0}$. These two statistics are not sufficient to parametrize a distribution, as the second order moment is missing, i.e., the variance. Estimating the conditional variance on a limited sample is discouraged, as the estimators of conditional variance need a lot of information to estimate heteroskedasticity (as in the case of an ARCH/GARCH model, for example). Thus, the at-risk model makes the simplifying but realistic assumption that the variance is unconditional and equal to the residual variance of the Theil-Sen estimation (i.e., heteroskedasticity is assumed to be constant over the course of time). This approach also addresses a recurring problem of projection models, i.e., that the variance of the projection tends to increase with the projection's horizon. With constant heteroskedasticity, there is no inflation in the variance. Thus, under this assumption, the at-risk model obtains three conditional moments: the expectation (Theil-Sen projection), the variance (constant heteroskedasticity, derived from Theil-Sen), and the skewness (obtained from the Firth logistic model).

8. The team parametrizes an asymmetrical Gaussian distribution from the three estimated moments. The at-risk model further stabilizes the projection by using an over-parametrized fit, where the distribution is assumed to follow an asymmetric Gaussian process. This assumption is realistic, insofar as an asymmetric Gaussian distribution naturally encompasses both the standard normal distributions and the asymmetric ones. This approach retains a high degree of generality while conserving simplicity. It presents the most interesting metrics for economists (i.e., central tendency, interquartile range, and balance of risks). The choice of an asymmetrical Gaussian rather than another asymmetrical distribution is constrained by the number of moments. To estimate an asymmetric Student distribution, four moments are needed (including the kurtosis), which, due to the limited size of the sample, is unfeasible. Another approach consists of using nonparametric distributions, like kernels, but again, the limited size of the samples makes this approach unsuitable. Finally, a major advantage of the asymmetric Gaussian distribution is that it provides simple analytical relationships between moments, cumulative density, and parameters. This property greatly simplifies the distribution fit on conditional moments, as the parameters are derived manually in closed algebraic form and not through optimized approximation.

[4] This quantity is not directly a measure of asymmetry. However, in the case of an asymmetrical Gaussian distribution, it is possible to infer the asymmetry coefficient from 〖F(y〗_t0 |X_t) via a simple bijective transformation.

References

Adrian, Tobias, Nina Boyarchenko, and Domenico Giannone. 2019. "Vulnerable Growth." American Economic Review, 109 (4): pp. 1263-89.

Bellini, T. 2019. *IFRS 9 and CECL Credit Risk Modelling and Validation: A Practical Guide with Examples Worked in R and SAS.* Amsterdam: Elsevier Inc.

Benigno, P., 2020. "A Central Bank Theory of Price Level," *American Economic Journal: Macroeconomics* 12(3). 253-283.

Benjamin, N., Haddow, A., and Jacobs, D. 2022. Stress Testing a Central Bank's Own Balance Sheet. In J. Doyne Farmer, A. M. Kleinnijenhuis, & T. Schuermann (Eds.). *Handbook of Financial Stress Testing* (pp.635-658). Cambridge, England: Cambridge University Press.

Bundesbank. 2023. "Bundesbank Adjusts Remuneration of Domestic Government Deposits. Press release, August 4, 2023. Available at https://www.bundesbank.de/en/press/press-releases/bundesbank-adjusts-remuneration-of-domestic-government-deposits-914060.

Bunea, D., P. Karakitsos, N. Merriman, and W. Studener. 2016. "Profit Distribution and Loss Coverage Rules for Central Banks." Occasional ECB Paper 169, European Central Bank, Frankfurt, Germany.

Carpenter, S., Ihrig, J., Klee, E., Quinn, D., Boote, A., 2015. "The Federal Reserve's Balance Sheet and Earnings: a Primer and Projections," *International Journal of Central Banking* 11(2). 199-236.

Cochrane, J., 2023. The Fiscal Theory of the Price Level, Princeton University Press.

Del Negro, M., and C. Sims. 2015. "When Does a Central Bank's Balance Sheet Require Fiscal Support?" *Journal of Monetary Economics* 73: 1-19.

Diebold, F. X., and C. Li. 2006. "Forecasting the Term Structure of Government Bond Yields." *Journal of Econometrics* 130 (2): 337-364.

Diebold, F. X., G. Rudebusch, and S. B. Aruoba. 2006. "The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach." *Journal of Econometrics* 131 (1-2): 309-338.

Domínguez, B, Gomis-Porqueras, P., 2023. "Normalizing the Central Bank's Balance Sheet: Implications for Inflation and Debt Dynamics," *Journal of Money, Credit and Banking* 55(4). 945-974.

Duffie, D., and K. J. Singleton. 2003. *Credit Risk: Pricing, Measurement and Management*. Princeton, NJ: Princeton University Press.

Edwards, S. 2015. "Sovereign Default, Debt Restructuring, and Recovery Rates: Was the Argentinean 'Haircut' Excessive?" NBER Working Paper 20964, National Bureau of Economic Research, Cambridge, MA.

Ernst & Young. 2018. *Applying IFRS Impairment of Financial Instruments under IFRS 9*. London: EYGM Limited.

European Central Bank. 2023. "ECB Adjusts Renumeration of Minimum Reserves." Press release, July 27, 2023. Available at https://www.ecb.europa.eu/press/pr/date/2023/html/ecb.pr230727~7206e9aa48.en.html.

Firth, D. 1993. "Bias Reduction of Maximum Likelihood Estimates." *Biometrika* 80 (1): 27-38.

Franta, M., T. Holub, and B. Saxa. 2022. "Exiting from an Exchange Rate Floor in a Small Open Economy: Balance Sheet Implications of the Czech National Bank's Exchange Rate Commitment." *International Journal of Central Banking* 18 (2): 51-105.

Fujiki, H., Tomura, H. 2017. "Fiscal Cost to Exit Quantitative Easing: the Case of Japan," *Japan and the World Economy* 42.1-11.

Goncharov, I., Ioannidou, V., Schmalz, M.C., 2023. "(Why) Do Central Banks Care about Their Profits?," *Journal of Finance* 78(5). 2991-3045.

Hall, R., and R. Reis (2015). Maintaining Central-Bank Financial Stability Under New-Style Central Banking, National Bureau of Economic Research, Working Paper 21173.

Hayashi, F, Koeda, J., 2019. "Exiting from Quantitative Easing," *Quantitative Economics* 10(3). 1069-1107.

IMF (International Monetary Fund). 2016. "IMF Updates Central Bank Legislation Database." https://www.imf.org/en/News/Articles/2016/08/15/NA081516-IMF-updates-central-bank-legislation-database.

Ize, A. 2005. "Capitalizing Central Banks: A Net Worth Approach," IMF Working Paper.

Jeanne, O, Svensson, L., 2007. "Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank," *American Economic Review* 97(1). 474-490.

Lafarguette, R., Z. Chen. 2022. "West African Economic and Monetary Union: Technical Note on Stress Tests—Credit, Concentration, and Interest Rate Risks." Country Report 22/279, International Monetary Fund, Washington, DC.

Manuelli, R., Vizcaino, J., 2017. "Monetary Policy with Declining Deficits: Theory and an Application to Recent Argentine Monetary Policy," Federal Reserve Bank of St. Louis Review.

Moody's. 2011. "Sovereign Default and Recovery Rates, 1983–2010." Available at https://www.moodys.com/Pages/Sovereign-Default-Research.aspx.

National Audit Office. 2018. "Managing the Bank of England's Central Services." Report by the Comptroller and Auditor General, Bank of England.

Nelson, C. R., and A. F. Siegel. 1987. "Parsimonious Modeling of Yield Curves." *Journal of Business* 60 (4): 473–89.

Oura, H. 2022. "Systemwide Liquidity Stress Testing Tool." IMF Working Paper 22/252, International Monetary Fund, Washington, DC.

Pan, J., and K. J. Singleton. 2008. "Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads." *The Journal of Finance* 63 (5): 2345-84.

Park, S.G, 2015. "Central Banks' Quasi-Fiscal Policies and Inflation," *International Journal of Central Banking* 11(2). 199-236.

Perera, A., Ralston, D., Wickramanayake, J., 2013. "Central Bank Financial Strength and Inflation: Is There a Robust Link?," *Journal of Financial Stability* 9. 399-414.

Prasad, A., Elekdag, S., Jeasakul, P., Lafarguette, R., Alter, A., Fend, A. X., Wang, C., 2019. "Growth at Risk: Concept and Application in IMF Country Surveillance." IMF Working Paper 19/36, International Monetary Fund, Washington, DC.

Sen, P. K. 1968. "Estimates of the Regression Coefficient Based on Kendall's Tau." *Journal of the American Statistical Association* 63 (324): 1379-89.

Stella, P. 1997. "Do Central Banks Need Capital?," IMF Working Paper.

Swiss National Bank. 2023. "SNB Adjusts Remuneration of Sight Deposits." Press release, October 30, 2023. Available at https://www.snb.ch/en/publications/communication/press-releases/2023/pre_20231030.

Tanaka, A., 2021. "Central Bank Capital and Credibility: A Literature Survey," *Comparative Economic Studies* 63(2). 249-262.

Theil, H. 1950. "A Rank-invariant Method of Linear and Polynomial Regression Analysis." *Proceedings of the Royal Netherlands Academy of Sciences*, Series A Mathematical Sciences (53): 386-92, 521-5, 1397-412.

Veyrune, R., and S. Zerbo. 2023. "Estimation and Determinants of Cost Efficiency: Evidence from Central Banks Operational Expenses." IMF Working Paper 23/195, International Monetary Fund, Washington, DC.

Williamson, S.D., 2018. "Can the Fiscal Authority Constrain the Central Bank?," Journal of Economic Dynamics and Control 89. 154-172.

Wold, S., M. Sjostrom, and L. Eriksson. 2001. "PLS-Regression: A Basic Tool of Chemometrics." *Chemometrics and Intelligent Systems* 58 (2): 109-30.