Macroeconomic and Fiscal Consequences of Quantitative Easing*

Tobias Adrian
International Monetary Fund and CEPR

Christopher Erceg
International Monetary Fund

Marcin Kolasa
International Monetary Fund

Jesper Lindé
International Monetary Fund and CEPR

Pawel Zabczyk
International Monetary Fund

Preliminary and Incomplete

This Draft: September 9, 2024

Abstract

Central banks have come under increasing criticism for large balance sheet losses associated with quantitative easing (QE), and some observers have also argued that QE helped fuel the post-COVID inflation boom. In this paper, we emphasize that the merits of QE should be evaluated based on the macroeconomic stimulus it provides and its effects on the consolidated fiscal position relative to other means to provide stimulus. QE should not simply be evaluated on central bank profits or losses. Using a DSGE model with segmented asset markets, we show how QE can provide a sizeable boost to output and inflation in a deep recession, and improve the consolidated fiscal position – even if the central bank experiences considerable losses. From a consolidated government fiscal perspective, we also find that QE is likely a significantly more cost-effective tool to stimulate output and inflation than conventional fiscal stimulus.

JEL Classification: E1, E3, E5.

Keywords: Effective Lower Bound, Fiscal Policy, Forward Guidance Puzzle, Government Debt, Monetary Policy, New Keynesian Model, Term Premium.

*We are grateful for discussions with Andrew Levin and comments by conference participants at the April 10-11 Nederlandsche Bank conference and IMF staffers. This paper represents the view of the authors and not those of IMF Staff, IMF Management, or of the IMF Executive Board.

Contents

1. Introduction

The recovery from the COVID-19 pandemic and subsequent inflation surge have raised important questions about the tools and approaches policymakers should use to confront future recessions. On the fiscal side, the runup in public debt during the pandemic has further compressed fiscal space, compounding the pressures from unfavorable demographics. On the monetary side, many questions have been raised about the use of quantitative easing (QE) by central banks, including in the wake of sizeable central bank losses.

The use and implementation of QE merits particular attention in light of the post-pandemic inflation surge. Policymakers deployed QE in 2020 to support recovery, but with the view that there was little upside inflation risk given the perceived flatness of the Phillips Curve. Moreover, QE was coupled with forward guidance about the policy rate path that involved some degree of commitment to keep policy rates low even after asset purchases had ended. But evidence from the recent high inflation experience suggests potentially significant nonlinearities in price and wage-setting that may have interacted with QE to fuel some of the inflation runup. And central bank forward guidance about QE may have inhibited a timelier liftoff of policy rates (Eggertsson and Kohn, 2023, and Orphanides, 2023).

In this paper, we use a DSGE modeling framework to assess quantitatively the merits of using QE under different conditions and how the implementation of QE might be refined in light of recent experience (Gopinath, 2023). While QE was deployed initially by central banks during and after the Global Financial Crisis in the context of deep recessions, it was subsequently used to address "lowflation" problems even after unemployment had moved close to record lows. Accordingly, we consider the macroeconomic benefits of QE, and also its implications for the consolidated fiscal position and for central bank profits, under these alternative economic conditions. Moreover, we compare the implications of QE to standard fiscal policy actions – here an expansion in government spending – to provide commensurate economic stimulus.

Our DSGE modeling framework is well-suited to explore the benefits and costs of QE. We build on the literature incorporating bond market segmentation exemplified by Andres et al. (2004), Chen et al. (2012), Kiley (2014), and Kolasa and Wesolowski (2020) to allow QE to affect term premiums. Following Erceg et al. (2024), we make two key modifications of this framework to give QE an important role in a slump when inflation falls below the central banks target and the short-term policy rate becomes constrained by the effective lower bound (ELB). First, it embeds behavioral discounting as in Gabaix (2020) to mitigate the "forward guidance puzzle," so that the central bank can't simply rely on forward guidance about the policy rate to achieve its objectives when constrained by the ELB for a protracted period. Second, the model also incorporates a nonlinear Phillips Curve as in Harding, Linde, and Trabandt (2022, 2023) which endogenously lowers the sensitivity of inflation to policy actions in an economic slump and when inflation is low, but with the upshot that stimulative policies can potentially cause overheating when a combination of strong demand and adverse cost-push shocks causes inflation to surge. Finally, the model allows for price and wage stickiness – so that inflation has some intrinsic persistence – and also incorporates real rigidities to better match empirical evidence on monetary transmission. These features prove helpful in identifying potential risks from QE and ways of addressing these risks.

We begin by showing that QE is likely to have substantial macroeconomic benefits in a deep recession and liquidity trap, even while calibrating the model so that the effects of asset purchases on term premiums are fairly modest from the perspective of the empirical literature.[1] Moreover, QE tends to significantly improve the consolidated fiscal position of the government. In particular, the debt/GDP ratio falls as the faster output recovery boosts the primary balance, debt service costs fall due to lower interest payments, higher bond prices make issuance of new debt cheaper, and increases in the aggregate price level lower the real value of existing debt. The central bank also typically makes profits as it purchases long-term assets that pay a premium over their refinancing cost

By comparison, fiscal expansion – modeled as a rise in government spending — results in a much less favorable path for the consolidated fiscal position, even when the fiscal shock is scaled to generate nearly the same output path. As we show, aside from the direct impact of higher spending on the fiscal deficit, this reflects that fiscal impetus boosts wages and hence labor income tax revenue by less; leaves private consumption and hence sales tax revenue largely unchanged; and does not decrease the cost of servicing and issuing debt as its effect on inflation and bond prices is small.

Despite these seemingly very supportive arguments in favor of QE, there are some important caveats about how much reliance can be put on QE to boost the economy in a recession. Notably, QE works through term premiums, and there are likely limits to depressing them further when already low. Moreover, in practice central banks may worry about expanding their balance sheets enormously and incurring large balance sheet risks (though these political economy aspects are

[1] In a meta-analysis, Fabo et al. (2021) notes that the effectiveness of QE is found to be notably smaller by academics than central bank economists. We benchmark our model to their evidence excluding central banks, thus taking a conservative view on the effectiveness of QE.

not captured in our modeling framework). But our results do make a strong case for using QE aggressively in a deep liquidity trap at least as an important component of the overall policy response.

We then devote particular attention to the use of QE in a “shallow” liquidity trap, where interest rates are pinned at zero mainly because inflation and inflation expectations are running well below target, even while output is only modestly below potential. A number of countries faced such conditions before the pandemic. In particular, QE was used because the central banks were concerned that inflation would otherwise run below target for several years hence, posing downside risks to inflation expectations and ultimately to central bank credibility. They aimed to amplify the stimulus from QE by in effect promising not to lift rates until sometime after asset purchases had ended.

We show that the benefits of a given-sized QE program tend to be considerably smaller in this environment than in a deep liquidity trap. Moreover, even if there are benefits should the economy evolve roughly in line with the modal outlook, QE in a shallow trap can pose eventual overheating risks if the economy recovers faster than expected. In particular, the “commitment” aspect of QE can amplify overheating, especially in there are significant nonlinearities in the Phillips Curve and intrinsic inflation persistence – corroborating the concerns raised by Orphanides (2023) that QE can make it difficult to respond nimbly to overheating pressures. Of course, these issues can pertain with equal force to conventional fiscal actions that may be undertaken if the central bank abstains from QE.

However, while QE can increase the risks of overheating, we find that these risks can largely be contained by simply allowing the policy rate to adjust freely according to the normal reaction function (assumed to be a Taylor-style rule). While the expected benefits of QE are somewhat diminished by tempering the commitment-based forward guidance associated with it, QE still has considerable expected benefits. We illustrate this by using stochastic simulations in which the model’s underlying shocks are calibrated to match the 1960-2024 period. This sample period – which encompasses both the Great Inflation and COVID-inflation – in effect poses a severe test about whether QE poses overheating risks.

An interesting implication of the stochastic simulations is that QE against the shallow liquidity trap baseline often leads to large central bank losses – reflecting that the central bank incurs substantial duration risk, and then takes a big hit when forced to tighten to contain inflation in upside inflation scenarios. Even so, the consolidated fiscal position – that takes account not only of central bank losses but of the implications for overall government revenue and expenditure – tends to improve significantly with high probability.[2]

Overall, our results suggest that central banks should proceed cautiously with deploying QE in a shallow liquidity trap. Nevertheless, promising an aggressive response to inflation may be enough to largely contain overheating risks and still imply considerable benefits of using QE, especially when relating the fiscal costs of QE to conventional fiscal policy on the consolidated public sector financial position.

The remainder of the paper is organized as follows. Section 2 review existing empirical evidence of the transmission of QE and conventional fiscal stimulus on inflation, output and the term-premium, focusing on the U.S. and Euro area. Section 3 presents the quantitative macroeconomic model with real rigidities and gradual nominal price and wage adjustment. Section 4 discusses our results for QE and conventional fiscal policy in severe recessions when the shadow rate is well below the ELB, comparing the effects of QE with conventional fiscal stimulus (higher government spending), with a particular focus on the impact of the consolidated fiscal position for given boost to output by the two policy tools. In Section 5, we assess the limits of QE in a shallow trap when allowing for shock uncertainty, wheres Section 6 studies the sensitivity of the results to the initial size of the term premium and how the macroeconomic benefits and fiscal costs of QE varies with the size of the long-term asset purchases. Finally, Section 7 provides some concluding remarks.

2. Empirical Evidence on Monetary and Fiscal Policy

As we will subsequently compare the macroeconomic benefits and costs of QE with those of conventional fiscal policy, this section reviews the existing empirical evidence of Quantitative easing (QE) and government consumption (the fiscal tool we compare against). This evidence will guide the subsequent calibration of our model.

Erceg et al. (2024) studies the effects of QE for the US, and finds that a QE induced decline in the term premium on 10-year bonds with 10 basis points stimulates industrial production with 1/2 percent during the first year. This finding compares well with the results in the meta-analysis in Fabo et al. (2021), who review research by academics and central banks on the effects of balance sheet policies in the euro area, the United States and the United Kingdom. The academic studies considered by Fabo et al. implies that 10 percent of QE has a peak impact on output with 1.1

[2] However, one important condition for this conclusion to hold is that QE does not drive term premiums into negative territory, which could easily happen if premiums are already compressed, or asset purchases are very large. In this case the central bank could make losses even under the modal outlook, which an earlier recovery can turn into even bigger ones, and hence pose a larger burden for the treasury.

percent. The survey in Fabo et al. suggest the effects are greater in the United States than in the euro area and the United Kingdom. They also note that researchers at central banks report notably larger effects. But the estimated effects in various studies can differ due to different monetary policy frameworks, the duration of the effective lower bound on short-term policy rates, the pre-QE level of the term-premium, the way the financial markets function, the way the central banks have designed their purchase programmes, or simply by which method has been used to estimate the effect.[3] Di Casola (2021) argue that the seemingly large discrepancies between academics and central bank researchers reported in Fabo et al. (2021) can be reconciled by accounting for which method was used to calculate the effects and whether the study has been published in a scientific journal or not, after outliers are excluded.

On the fiscal side, the effects of government spending hikes have been studied empirically and through structural macroeconomic models. Starting with the former, prominent papers in the empirical literature on aggregate data are Blanchard and Perotti (2002), Galí, Lopez-Salido and Valles (2007), Auerbach and Gorodnichenko (2010), Ramey (2011), Ramey and Zubairy (2018), and Leeper, Traum and Walker (2017). While the papers obtain different estimates of the fiscal multiplier, defined as the change in output normalized by the impetus in government consumption during the same horizon, these papers and the body of literature they cite suggest a multiplier close to unity in normal times.

Notably fewer papers have assessed the impact of fiscal policy on inflation in both the US and the euro area, but this gap is bridged by an extensive literature examining fiscal policy in a liquidity trap with structural macroeconomic models. This literature shows that the spending multiplier is likely to be substantially larger than in normal times, e.g., Eggertsson (2011), Christiano, Eichenbaum, and Rebelo (2011), Woodford (2011), and Coenen et al. (2012). The higher multiplier reflects that the central bank does not raise nominal policy rates even though inflation rises, so that real interest rates fall and domestic demand is crowded in. These crowding in effects can be large if inflation is responsive to resource slack. For example, Christiano, Eichenbaum, and Rebelo (2011) showed that the peak multiplier exceeds 2 in a long-lived liquidity trap under their preferred model specification. Our model with discounting and real rigidities that lower senstivity of actual and

[3] For instance, the research of Engen, Laubach, and Reifschneider (2016) showed how the effects of a given-sized QE program on unemployment are much larger under a calibration of the Taylor rule implying a more gradual liftoff of policy rates than a standard Taylor rule. They argued that the Fed's communication after the Global Financial crisis evolved in a way consistent with this more accommodative form of the Taylor rule. Central banks generally perceived that some degree of commitment could enhance the stimulus from QE (Bernanke, 2017). Using evocative language from Greek mythology, the president of Federal Reserve Bank of Chicago Charles Evans described this as "Odyssean" forward guidance: just as Odysseus tied himself to the mast to resist bewitchment by Circe, central banks could promise to keep policy rate low—at least to a point—even if economic conditions called for raising interest rates under their normal reaction function.

expected inflation to shocks will moderate the multiplier relative to these papers, but will still imply an elevated fiscal spending multiplier in a protracted liquidity trap.

A number of empirical papers have corroborated the implication of a larger spending multiplier when monetary policy is constrained by an effective lower bound for a protracted period. Some of this analysis has focused on the Great Depression period given that monetary policy was arguably unreactive to fiscal stimulus during most of that time. Almunia et al. (2010) found a spending multiplier of over 2 using a panel VAR for major industrial economies that is estimated over the interwar period and uses the same identifying assumptions as in Blanchard and Perotti (2002). Gordon and Krenn (2010) estimated a spending multiplier of slightly under 2 for the United States in a narrow window preceding the U.S. entry into World War II. They argue that this is an ideal period for estimating the multiplier given that government spending rose massively (by 13 percent of U.S. GDP between 1940:Q2-1941:Q4), monetary policy was passive, resource slack still large, and tax rates weren't (yet) adjusted up. They also document a substantial crowding in of private demand.

Blanchard and Leigh (2014) focused on the recent experience of fiscal consolidation in the euro area during the 2010-2012 period. While some analysis suggested that deep spending cuts would exert only a modest drag on output – or possibly even raise output through confidence channels (Alesina and Ardagna 2010) – Blanchard and Leigh showed that fiscal multipliers in euro area countries turned out to be much larger than forecast ex ante, implying that fiscal cuts in the periphery had considerably more adverse effects than anticipated. Their estimates suggest a spending multiplier of around 1.5 for the euro area.

Both the theoretical and empirical literature has attempted to identify key factors influencing the size of the aggregate spending multiplier. In addition to the inflation response, the multiplier is larger in a longer-lived liquidity trap, if the bulk of spending occurs when the zero bound constraint is still binding (see the papers by CER and Woodford mentioned above), or if the economy is in a deep recession with substantial excess capacity (Auerbach and Gorodnichenko 2012 and Gordon and Krenn 2010). Moreover, as indicated by Uhlig (2010), Erceg and Linde (2014), and Drautzberg and Uhlig (2015), the tax reaction function can be quite consequential: the spending multiplier can be significantly lower if tax rates are adjusted quickly and if distortionary tax rates account for most of the adjustment. In our analysis, we assume that fiscal stimulus can be implemented fairly quickly, and that tax ratesadjust very slowly. The fiscal multipliers derived from our liqudity trap simulations would be lower under less favorable assumptions on these dimensions.

3. Quantitative Model

The quantitative model that we develop for this study builds on Erceg et al. (2024), casting it in a closed economy setup and extending in several directions. Compared to a standard DSGE model, our framework has the following features. First, we assume bond market segmentation similar to Andres et al. (2004), Chen et al. (2012), Kiley (2014), and Kolasa and Wesolowski (2020). This ensures that short- and long term bonds are imperfect substitutes and that, by affecting the term premiums, asset purchases (quantitative easing) by the central bank have real effects. Second, we allow for a moderate degree of cognitive discounting in the spirit of Gabaix (2020) to mitigate the so-called forward guidance puzzle (see Del Negro et al., 2012). This creates the need to complement interest rate policy with additional tools, such as QE or fiscal expansion, when the policy rate becomes constrained by the effective lower bound. Third, we incorporate strategic complementarities in price setting to allow for a lower sensitivity of inflation in recessions (the "missing deflation puzzle") as well as its surge during fast recoveries (see Harding, Lindé and Trabandt, 2022, 2023).

Relative to the model in Erceg et al. (2024), we make two additional modifications to feature empirically realistic transmission of both standard monetary (interest rate) and fiscal (government consumption) policy shocks. First, we allow for habit formation in consumer preferences to obtain a gradual and hump-shaped peak effect on output following an easing of the short-term interest rate. As the model is largely Ricardian and excludes endogenous capital formation, habit formation moderates the crowding out of private spending following an increase in government consumption and hence allows the model to better align with the empirical evidence on the effects of fiscal policy on output. Second, we allow for gradual nominal wage adjustment to moderate the effects on inflation of policies for an empiricially realistic degree of price adjustment and to obtain more realistic behavior of wages and profits (see Christiano, Eichenbaum and Evans, 2005, and Bilbiie and Trabandt, 2023). These two modifications are important to appropriately capture the response of fiscal revenue, which mainly relies on proportional taxation of consumption and labor income, to shocks and policies.

Below we provide an overview of the model. For any variable X_t: $x_t = X_t/P_t$ denotes its real value, where P_t is the aggregate domestic price level, and x denotes x_t's steady state. We will also occasionally use a bar to distinguish aggregate quantities that agents take as given from their individual choices. Naturally, in equilibrium we have $x_t = \bar{x}_t$.

3.1. Households

The two types of households are labeled “restricted" and “unrestricted", and indexed with $j \in \{r, u\}$, respectively, with $\omega_r \in (0,1)$ denoting the share of restricted households. The lifetime utility maximized by household of type j is given by

$$U_t^j = \mathbb{E}_t^j \sum_{s=0}^{\infty} \beta_j^s \exp\{\varepsilon_{t+s}^d\} \left[\exp\{\varepsilon_{t+s}^c\} \log(c_{t+s}^j - \varkappa \bar{c}_{t-1+s}^j) - \frac{(n_{t+s}^j)^{1+\varphi}}{1+\varphi} \right], \tag{1}$$

where $\beta_j \in [0,1)$ is the subjective discount factor, $\varkappa \in [0,1)$ is the external habit formation parameter, and $\varphi > 0$ is the (inverse) Frisch elasticity of labor supply.

Household preferences in consumption c_t^j (adjusted for habits that depend on aggregate consumption of the same type of agents $\bar{c}_t^j$) and labor n_t^j are perturbed by the discount factor shock ε_t^d and the consumption preference shock ε_t^c, both of which are assumed to follow a stationary AR(1) process. In the lifetime utility maximand (1), $\mathbb{E}_t^j$ indicates the expected value operator under the subjective expectations of type j households. We allow for deviations from rational expectations by following Gabaix (2020) by assuming that households can be myopic.[4]

There are two types of nominal assets in the model economy, short-and long-term bonds, and we indicate their holdings by agents of type j as B_t^j and $B_{L,t}^j$, respectively. Following Woodford (2001), we model long-term bonds as perpetuities paying an exponentially decaying coupon $1, \kappa, \kappa^2, \ldots$ starting in the period following issuance, where $\kappa \in (0,1]$. By absence of arbitrage, $P_{L-s,t}$, i.e., the current price of a long term bond issued s periods ago, then has to be related to the price of a newly issued perpetuity $P_{L,t}$ via $P_{L-s,t} = \kappa^s P_{L,t}$. A convenient implication is that we only need to keep track of the long term bond price issued contemporaneously, as prices of all past vintages can be easily recovered using the preceding formula. This structure also means that the yield to maturity on long-term bonds, which we will also refer to as the long-term rate, is simply $R_{L,t} = \kappa + 1/P_{L,t}$.

Unrestricted households can trade both types of bonds, but have to pay transaction costs ζ_t to hold position in long-term markets. These considerations translate into the following flow budget constraint

$$P_t\left(1+\tau_t^c\right) c_t^u + B_t^u + \left(1+\zeta_t\right) P_{L,t} B_{L,t}^u + T_t^u$$
$$= R_{t-1} B_{t-1}^u + \left(1+\kappa P_{L,t}\right) B_{L,t-1}^u + W_t\left(1-\tau_t^n\right) \bar{n}_t^u + D_t^u + \Xi_t^u,$$

[4] More specifically, when anticipating the future, households shrink their expectations toward the economy's steady state. Formally, for any variable X_t that households of type j take as given during optimization, their perceived law of motion is $X_{t+1} - X = m_j \mathbb{G}^X(\mathbf{X}_t^s - \mathbf{X}^s, \boldsymbol{\epsilon}_{t+1})$, where $\mathbf{X}_t^s$ is a vector of aggregate state variables, $\boldsymbol{\epsilon}_t$ is a vector of innovations to stochastic processes driving the economy, $\mathbb{G}^X$ is the equilibrium aggregate policy function for variable X_t, and $0 \leq m_j \leq 1$ is a cognitive discounting parameter for agent j, with $m_j = 1$ corresponding to the standard case of rational expectations.

where τ_t^c denotes the sales (VAT) tax rate, τ_t^n is the labor income tax rate, T_t stands for lump sum taxes, R_t is the short-term policy rate, and $W_t \bar{n}_t^u$ denotes pre-tax labor income, inclusive of net insurance payments insulating households from idiosyncratic income risk (see more details below). The bond holding adjustment costs are rebated lump sum through Ξ_t^u and they are assumed to satisfy

$$\frac{1+\zeta_t}{1+\zeta} = \left(\frac{P_{L,t} b_{L,t}^u}{P_L b_L^u}\right)^{\xi}, \tag{2}$$

with $\xi > 0$.

In contrast, restricted households trade only in long-term bonds and their transaction costs are negligible. Their budget constraint is then

$$P_t \left(1+\tau_t^c\right) c_t^r + P_{L,t} B_{L,t}^r + T_t^r = \left(1+\kappa P_{L,t}\right) B_{L,t-1}^r + W_t \left(1-\tau_t^n\right) \bar{n}_t^r + D_t^r. \tag{3}$$

3.2. Wage Setting

We assume that labor supplied by individual households is differentiated and aggregated by perfectly competitive labor unions according to a constant elasticity of substitution aggregation function controlled by parameter $\phi_w > 0$. The homogeneous labor services sold to firms are then given by the following formula

$$n_t = \left(\int_0^1 n_t(h)^{\frac{1}{1+\phi_w}} dh\right)^{1+\phi_w}, \tag{4}$$

where h indexes individual households.

Wages are set by labor unions in a Calvo-style staggered fashion. Each period, a randomly selected fraction $1-\theta_w$ of households get their nominal wage reset, while the remaining households mechanically index their wages to steady state inflation π. While resetting wages, labor unions do it on behalf of all households, taking into account aggregate rather than household-type specific preferences. This leads to the following optimisation problem

$$\max_{\tilde{W}_t} \mathbb{E}_t \sum_{s=0}^{\infty} \left(\beta\theta_w\right)^s \left[\Lambda_{t+s} \exp\{\varepsilon_{t+s}^w\} \Pi^s \frac{\tilde{W}_t}{P_{t+s}} - \frac{\exp\{\varepsilon_{t+s}^d\}}{1+\varphi} \left(\frac{\tilde{W}_t}{W_{t+s}}\right)^{\frac{\phi_w}{1-\phi_w}\varphi} n_{t+s}^{\varphi}\right] \left(\frac{\tilde{W}_t}{W_{t+s}}\right)^{\frac{\phi_w}{1-\phi_w}} n_{t+s}, \tag{5}$$

where $\tilde{W}_t$ is the newly set wage, Λ_t is a population-weighted average of marginal utility of consumption of restricted and unrestricted households, and ε_t^w is a wage cost-push shock that follows a stationary AR(1) process.

For tractability, we also assume the existence of perfect insurance schemes against idiosyncratic income risk associated with staggered wage setting. This ensures that all households of a given type make the same consumption and asset choices, and allows us to write their labor income net of insurance payments as $W_t \bar{n}_t^j$.

3.3. Firms

Perfectly competitive firms combine inputs into final goods according to

$$\int_0^1 G\left(\frac{y_t(i)}{y_t}\right) di = 1, \tag{6}$$

where we parameterize the Kimball aggregator as in Dotsey and King (2005) by assuming

$$G(x) \equiv \frac{\phi}{1+\psi}\left[(1+\psi)x - \psi\right]^{\frac{1}{\phi}} - \frac{\phi}{1+\psi} + 1, \tag{7}$$

with $\psi \leq 0$. This specification implies that the steady state (gross) markup μ equals $\frac{\phi}{(1-\phi)(1+\psi)+\phi}$, and it nests the standard Dixit-Stiglitz (1977) aggregator for $\psi = 0$.

Intermediate inputs are produced by monopolistically competitive firms indexed by i and operating a production function that is linear in labor

$$y_t(i) = \exp\{\varepsilon_t^z\} n_t(i) - f, \tag{8}$$

where ε_t^z denotes a productivity shock (driven by a stationary AR(1) process) and $f > 0$ is a fixed cost of production. Every period these firms face a fixed probability θ_p of price reoptimization, with non-resetting firms indexing prices to steady state inflation. We additionally assume that marginal costs faced by firms are distorted by an exogenous stationary AR(1) shock ε_t^p, which can be interpreted as reflecting stochastic variation in proportional taxes levied on firms that are rebated back to them in a lump sum fashion.

Assuming no myopia and local firm ownership – by "restricted" and "unrestricted" agents in proportion to their shares in the population – the problem of reoptimizing firms becomes

$$\max_{\tilde{P}_t} \mathbb{E}_t \sum_{s=0}^{\infty} (\theta_p)^s \frac{\Lambda_{t+s}}{P_{t+s}} \left(\tilde{P}_t \pi^s - \exp\{\varepsilon_{t+s}^p - \varepsilon_{t+s}^z\} W_{t+s}\right) y_{t+s}(i), \tag{9}$$

where $\tilde{P}_t$ is the newly reset price. The process for the price cost-push shock ε_t^p is given by

$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + u_{p,t}, \;\; u_{p,t} = i.i.d.\; N(0, \sigma_p). \tag{10}$$

3.4. Fiscal Authority

The fiscal authority operates subject to the following nominal flow budget constraint

$$B_t^f + P_{L,t} B_{L,t}^f + \mathcal{T}_t + \Phi_t^c = R_{t-1} B_{t-1}^f + (1 + \kappa P_{L,t}) B_{L,t-1}^f + P_t g_t, \tag{11}$$

i.e., it finances its expenditures (right-hand side in eq. 11) net of taxation $\mathcal{T}_t$ and profits made by the central bank on its asset portfolio Φ_t^c by issuing nominal short-term (B_t^f) and long-term ($B_{L,t}^f$) bonds.

As the government issues bonds of different maturity, measurement of consolidated government debt neccesitates taking a stand on the valuation of long-term debt. We adopt the spirit of the government debt statistics, which are based on face rather than market value of debt,[5] and value long-term government debt (perpetuities paying an exponentially decaying coupon $1, \kappa, \kappa^2, \ldots$ starting in the period following issuance) by discounting the outstanding stock of long-term bonds with $\frac{1}{1-\kappa}$, so that the consolidated government debt level as share of annualized trend GDP is defined as

$$GD_t^{con} = \frac{B_t^f + \frac{1}{1-\kappa} B_{L,t}^f}{4P_t Y}, \tag{12}$$

Importantly, our measure of the consolidated fiscal position GD_t^{con} is not mark-to-market, i.e. $B_t^f + P_{L,t} B_{L,t}^f$. This implies that we exclude direct revaluation effects and that purchases of long-term bonds on the secondary market by the central bank that changes the price of outstanding long-term debt has no direct impact on GD_t^{con}. Even so, we have checked that neither any qualitative nor significant quantitive longer-term aspects (say after 5 years) of the results for GD_t^{con} are notably affected by reasonable alternative definitions, including a mark-to-market based measure.[6]

Government consumption g_t is given by $g_t = g \exp\{\varepsilon_t^g\}$, where ε_t^g is assumed to follow an exogenous stationary AR(2) process:

$$\Delta\varepsilon_t^g = \rho_{g,1} \Delta\varepsilon_{t-1}^g - \rho_{g,2} \left(\varepsilon_{t-1}^g - 1\right) + u_{g,t}, \tag{13}$$

where Δ is the first difference operator, i.e. $\Delta x_t \equiv x_t - x_{t-1}$. We use an AR(2)-process for government consumption for two reasons. First, to assess the transmission to inflation for a commensurate boost to output as conventional interest rate policy changes and large scale asset purchases. Second, to capture implementation lags associated with conventional fiscal stimulus; i.e. although a spending bill has passed the parliament, it takes some time to fully boost the government consumption level.

On the revenue side, total nominal tax revenues $\mathcal{T}_t$ consists of proportional taxes levied on consumption and labor income and lumps sum taxes, i.e.

$$\mathcal{T}_t = \tau_t^c P_t c_t + \tau_t^n W_t n_t + T_t \tag{14}$$

[5] In particular, the debt definition in the Maastricht treaty is also based on the face value.

[6] We have studied two alternative measures. First, a market-to-market valuation of debt by defining the numerator of GD_t^{con} as $B_t^f + P_{L,t} B_{L,t}^f$. This measure is subject to near-term revaluation effects. A second measure is similar to our utilized concept in eq. (12) and discounts long-term debt with its steady state price, i.e. $B_t^f + P_L B_{L,t}^f$.

Consumption sales taxes are assumed to be constant ($\tau_t^c = \tau^c$), but labor incomes taxes vary gradually around its steady state level τ^n to stabilize the consolidated government debt position GD_t^{con} in the long-run according to:

$$\tau_t^n - \tau^n = \psi_\tau \left(\tau_{t-1}^n - \tau^n\right) + (1-\psi_\tau)\,\psi_b(GD_t^{con} - GD^{con}). \tag{15}$$

Setting ψ_τ near unity and ψ_b small ensures government debt sustainability in the long-run with very smooth and gradual adjustment of labor income taxes following the normative analysis in Bohn (1990). An additional key advantage of this setup is that short- and medium term debt dynamics is driven by other endogenous forces that we want to highlight.

As the treasury can issue both short- and long-term debt, we also need to make an assumption of the debt composition issuance strategy. To simplify the analysis, the fiscal authority is assumed to keep the composition of outstanding short- and long-term bonds (the latter evaluated at steady state prices) constant, i.e., $B_t^f/\ B_{L,t}^f = b/\ b_L^f$. Hence, when the central bank engages in QE and purchases long-term bonds issued by the treasury with central bank short-term assets, it shrinks the duration of outstanding government debt held by private agents.

When discussing the fiscal consequences of shocks and policies, it is instructive to define the fiscal deficit as

$$D_t^f = (R_{t-1}-1)B_{t-1}^f + B_{L,t-1}^f + P_t g_t - \mathcal{T}_t - \Phi_t^c. \tag{16}$$

The deficit hence is a sum of debt servicing costs, which includes (net) interest payments on short-term bonds and coupon payments on long-term bonds, and of government spending, less tax revenue and central bank profit. Note that, since debt is nominal, its servicing cost is predetermined in nominal terms and hence gets eroded by surprise inflation. Obviously, a deficit must be equal to the net debt issuance by the treasury, which we can show by combining equations eqs. (11) and (16), rewriting them in real terms

$$d_t^f = b_t^f - \frac{b_{t-1}^f}{\pi_t} + P_{L,t}\left(b_{L,t}^f - \kappa\frac{b_{L,t-1}^f}{\pi_t}\right). \tag{17}$$

Recall that our preferred measure of debt (eq. 12) relies on its face value, which in real terms (and before rescaling by steady state output) is $b_t^f + b_{L,t}^f/(1+\kappa)$. Then eq. 17 makes clear, that a change in this measure of fiscal position will be positively affected by fiscal deficits and negatively so by an increase in inflation and long-term bond prices, the latter simply reflecting the debt deflation channel – since debt is nominal, its real value is eroded by inflation. Higher bond prices will also typically contribute negatively to the fiscal position as they imply that the government needs to issue less bonds of a given face value if it can sell them at a higher price.

3.5. Monetary Authority

The monetary authority conducts "conventional" monetary policy according to a Taylor-type feedback rule for the gross nominal policy rate R_t subject to an effective lower bound (ELB henceforth, which we assume to be zero) constraint

$$R_t = \max\left\{1, \tilde{R}_t\right\}, \qquad \frac{\tilde{R}_t}{R_t^*} = \left(\frac{\tilde{R}_{t-1}}{R_{t-1}^*}\right)^{\gamma} \left[\left(\frac{\pi_t}{\pi}\right)^{\gamma_\pi} \left(\frac{y_t}{y}\right)^{\gamma_y}\right]^{1-\gamma} \exp\{\varepsilon_t^r\}. \tag{18}$$

$\tilde{R}_t$ is the unrestricted (shadow) rate that would prevail if monetary policy was not subject to an ELB, in which case $\gamma_r \in (0,1)$ controls the degree of interest rate smoothing, and γ_π and γ_y determine the strength of the long-term interest rate response to deviations of inflation and output from their steady state values, respectively. Since the model features variations in the effective discount factor in eq. (1) that can be neutralized by suitable adjustments of the short-term policy rate path, the rule (18) allows for a time-varying neutral gross policy rate R_t^*, defined as the steady state nominal gross policy rate adjusted for the expected change in the discount factor shock ε_t^d

$$R_t^* = R\mathrm{E}_t \exp\{\varepsilon_{t+1}^d / \varepsilon_t^d\}. \tag{19}$$

Since we want to study the role of QE when policy rates can be pinned at the ELB for an extended period, we assume the discount factor shock ε_t^d follows a persistent AR(1) process:

$$\varepsilon_t^d - 1 = \rho_d \left(\varepsilon_{t-1}^d - 1\right) + u_{d,t}. \tag{20}$$

With this specification we can trigger a hump-shaped persistent decline in R_t^* below the ELB. Finally, the policy rule (18) allows for a standard i.i.d. normally distributed short-term policy rate shock ε_t^r.

The central bank can also be active in the domestic bond market, i.e., it can take a position $B_{L,t}^c$ in long term bonds, financing it entirely by issuing one-period reserves B_t^c that pay interest R_t, and hence which – from the perspective of private agents – are indistinguishable from short-term government bonds. We shall refer to $QE_t \equiv P_{L,t} B_{L,t}^c = B_t^c$ as the size of LSAP and assume it obeys the following rule

$$QE_t \equiv \left(1 + (1-\varrho)\left(\kappa \frac{P_{L,t}}{P_{L,t-1}} - 1\right)\right) QE_{t-1} + \varepsilon_t^{QE}, \tag{21}$$

where $0 \leq \varrho < 1$ is a parameter controlling the reinvestment strategy. Hence, the cental banks QE portfolio closely resembles a standard AR(1) process with the exception that the persistence coefficient is partly endogenous.[7] ε_t^c denotes discretionary purchases of long-term assets by the central bank, and is assumed to follow an AR(1) process.

As noted earlier, any profits or losses on the central banks' asset portfolio is fully backed by the government. The holding profits by the central bank – Φ_t^c in eq. (11) – associated with previous central bank asset purchases can be written as

$$\Phi_t^c \equiv R_{t-1} B_{H,t-1}^c + (1+\kappa P_{L,t}) B_{H,L,t-1}^c. \tag{22}$$

The first term on the rhs in eq. (22) is the gross cost of financing a given portfolio of long-term assets, because if there had been no LSAPs, $B_{H,t-1}^c$ would be nil. The second term is the gross value of the long-term assets the central bank has purchased until period $t-1$, including the current coupon payment. A purchase of long-term assets in period $t-1$ implies that $B_{L,t-1}^c$ is positive but that B_{t-1}^c is negative, hence the summation of the negative short- and positive long-positions forms a net profit for the central bank. As these profits are immediately transferred or financed by the treasury, we think about Φ_t^c as the period-by-period profit. Hence, the accumulated central bank profits in period $t+h$ on a QE portfolio purchased in period t is given by

$$CBPROF_{t+h}^{acc} = \Sigma_{s=0}^{h} \Phi_{t+s}^c. \tag{23}$$

3.6. Market Clearing Conditions

Equilibrium in the goods market requires

$$y_t = \omega_r c_t^r + (1-\omega_r) c_t^u + g_t, \tag{24}$$

and

$$y_t \Delta_t = \exp\{\varepsilon_t^z\} n_t - f, \tag{25}$$

where

$$\Delta_t \equiv \frac{1}{1+\psi} \left(\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{1}{1-\phi}} di \right)^{-\phi} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{\phi}{1-\phi}} di + \frac{\psi}{1+\psi}, \tag{26}$$

captures price dispersion arising on account of staggered price setting in the intermediate goods sector.

Complementing these, we also have market clearing conditions for bonds issued by the home economy's government, including central bank reserves

$$(1-\omega_r) B_t^u = B_t^f - B_t^c, \tag{27}$$

[7] See Appendix A.5 for detailed derivations and a comprehensive discussion of eq. (??).

and

$$\omega_r B^r_{L,t} + (1-\omega_r) B^u_{L,t} = B^f_{L,t} - B^c_{L,t}. \tag{28}$$

3.7. Calibration

Broadly speaking, we have three key considerations when selecting parameters in the model. First, we set parameters pertaining primarily to the dynamics of the model to match the empirical evidence reviewed in Section 2 regarding the transmission of short-term interest rates, QE, and government spending. To this end, we study changes in the policy tools which are implemented by shocks to the short-term interest rate (ε^r_t in eq. 18) , central banks asset purchases (ε^{QE}_t in eq. 21), and government consumption (ε^g_t in eq. 13). Second, following standard practice, we set other parameters to match key steady state and fiscal proportions observed in the data, or rely on an extant literature for parameters not pinned down by either dynamics or steady state considerations. Third and finally, we set persistence and standard deviations for the stochastic shocks used to construct uncertainty bands to match unconditional standard deviations and cross correlations with output for a key set of macroeconomic variables for the period 1960-2024. This is a long sample period, and implies that we include episodes when price and wage inflation, and nominal interest rates were volatile. The stochastic shocks include shocks to productivity (ε^z_t), cost-push impulses for firms (ε^p_t) and labor unions (ε^w_t), and private consumption demand shocks (ε^c_t). Table 1 shows the parameter values we adopted, and Table 2 the targeted steady state ratios, while Table 3 reports the selected stochastic moments we match in the data with the shocks in eqs (29). The time period throughout corresponds to a quarter.

An important part of our calibration concerns the structure of the bond market. We set the steady state share of sovereign bonds in annual GDP to 0.75, which is close to what was observed in many countries before the Covid-19 pandemic. We also assume that central bank holdings of government bonds were initially zero. The share of long-term bonds in total sovereign bond issuance is calibrated at 0.65 and their duration is set at 10 years. These choices imply that the effective duration of outstanding public debt is close to 7 years.

Another important group of parameters determines the degree of bond market segmentation. This is governed by the share of restricted households ω_r, which we set to 0.2, and the sensitivity of transaction costs to changes in bond holdings by unrestricted households ξ, which we calibrate at 0.02. These choices allow us to generate the response of the term premium and output to LSAP in the large economy that is consistent with empirical evidence for the US (see also Section 2). We use the US average levels of inflation, short-term rates and long-term rates (and hence also the term premiums) to pin down, respectively, the inflation target π at 1.005 (2% annualized), the discount factor of unrestricted households β_u at 0.99875, and that of restricted agents β_r at 0.99625.

Table 1: Model Parameter Values.

Parameter	Value	Description
		Households
ω_r	0.2	Share of restricted households
σ	1	Inv. elasticity of intertemporal substitution
φ	2	Inv. Frisch elasticity of labor supply
β_u	0.99875	Discount factor, unrestricted households
β_r	0.99625	Discount factor, restricted households
m_u	0.95	Cognitive discounting, unrestricted households
m_r	1	Cognitive discounting, restricted households
ϕ_w	0.5	Wage markup
θ_w	0.75	Calvo wage probability
ξ	0.02	Transaction cost on long-term bonds
		Firms
μ	1.15	Gross price markup
ψ	-12	Kimball parameter
θ_p	0.667	Calvo price probability
		Fiscal Policy
τ^c	0.15	Steady state Consumption Sales Tax
τ^n	0.35	Steady state Labor income Tax
ψ_τ	0.98	Tax Smoothing Coeff in eq. (15)
ψ_b	0.01	Gov't Debt Response Coeff in eq. (15)
$\rho_{g,1}$	0.7	First difference coeff in eq. (13)
$\rho_{g,2}$	0.03	Error correction coeff in eq. (13)
D	40	Long-term bond duration
		Monetary Policy
γ	0.9	Interest rate smoothing
γ_π	2.5	Interest rate response to inflation
γ_y	0.125	Interest rate response to output gap
ϱ	0	Reinvestment strategy

Notes: D and μ are composite parameters defined as $D = \pi\beta_r^{-1}/(\pi\beta_r^{-1} - \kappa)$ and $\mu = \phi/[(1-\phi)(1+\psi)+\phi]$.

Given the focus of our study, the crucial part of calibration concerns parameters governing inflation dynamics, especially in response to monetary policy actions. We allow for a modest degree of cognitive myopia by setting the corresponding discounting parameter of unrestricted households m_u to 0.95. Compared to papers that estimate this parameter within a DSGE framework – for instance Gust, Herbst and Lopez-Salido (2022) or Kolasa, Ravgotra and Zabczyk (2022) – our choice implies a rather small deviation from rational expectations, which, however, turns out to be sufficient to make the potency of forward guidance small when the economy is in a deep liquidity trap. To account for state-dependence in the slope of the Phillips curve, we follow Harding, Lindé and Trabandt (2022, 2023) and set the Kimball curvature parameter ψ to -12. The Calvo probability for prices θ_p is set to 0.667 and that for wages θ_w is calibrated at 0.75, consistent with the empirical evidence on average price and wage duration.

Table 2: Targeted Steady State Ratios.

Steady State	Value	Formulae
Government Consumption to GDP	0.2	$\frac{g}{y}$
Government Debt to Annual GDP	0.75	$\frac{b^f}{4y}$
Net inflation (Annualized)	2.0	$400(\pi - 1)$
Nominal Policy Rate (Annualized)	2.5	$400(R - 1)$
Term-premium	1.0	$400(R_L - R)$
Share of Long-term Bonds in Total Bonds	0.65	$\frac{P_L b_L^f}{b^f}$
Central Bank Assets	0	$P_L b_L^c = b^c$

Notes: The moments in the U.S. data are based on the period 1960Q2-2024Q1. Inflation π_t^{ann} is measured with annualized core PCE inflation. Policy rate R_t^{ann} is measured with the annualized net Federal Funds Rate. Real GDP growth $\Delta \ln y_t$ and non-farm business hours worked is scaled with working age population, and $\hat{n}_t$ is then calculated as $(n_t - n)/n$. Annualized nominal wage is measured with wages in the non-farm business sector. The model moments are based on a simulation of a long-sample of 100,000 observations. Finally, notice that we match all moments except for R_t^{ann}, which is used to validate our moment procedure.

The remaining parameters are relatively well-established in the literature. The steady state government spending is set to 20% of GDP, roughly in line with the long-run averages observed in the data. The elasticity of intertemporal substitution σ, the Frisch elasticity of labor supply φ, and price markups μ are all set to typical values used in New Keynesian models. The monetary policy rule coefficients γ, γ_π and γ_y also reflect typical values used in the DSGE literature.

Given these parameters, Figure 1 show the transmission of the key policy instruments we will study in paper around the steady state. We size the innovation to each policy instrument – conventional white noise short-term policy rate shocks ε_t^r in eq. (18) , an innovation to large scale asset purchases $u_{QE,t}$ in eq. (21), and government consumption $u_{g,t}$ in eq. (13) – so that each policy instrument moves output in the upper left panel at peak with the same magnitude. We normalize the output effect around a 100 basis cut in the short-term policy rate. The figure documents that the effects on output from short-term interest policy cuts (blue solid line) and government spending hikes (red dotted) are well aligned with the empirical evidence discussed in Section 2. A one percent policy rate cut drives up output with a little more than 1/2 percent after about one year, which is similar to the VAR evidence in the seminal paper by Christiano, Eichenbaum and Evans (2005).

Similarly, a government spending hike with 0.60 percent of baseline GDP results in an output increase with about 0.55 percent, i.e. a fiscal spending multiplier close but slightly below unity. This is also in the mid-range of existing empirical evidence, see for instance Ramey and Zubairy

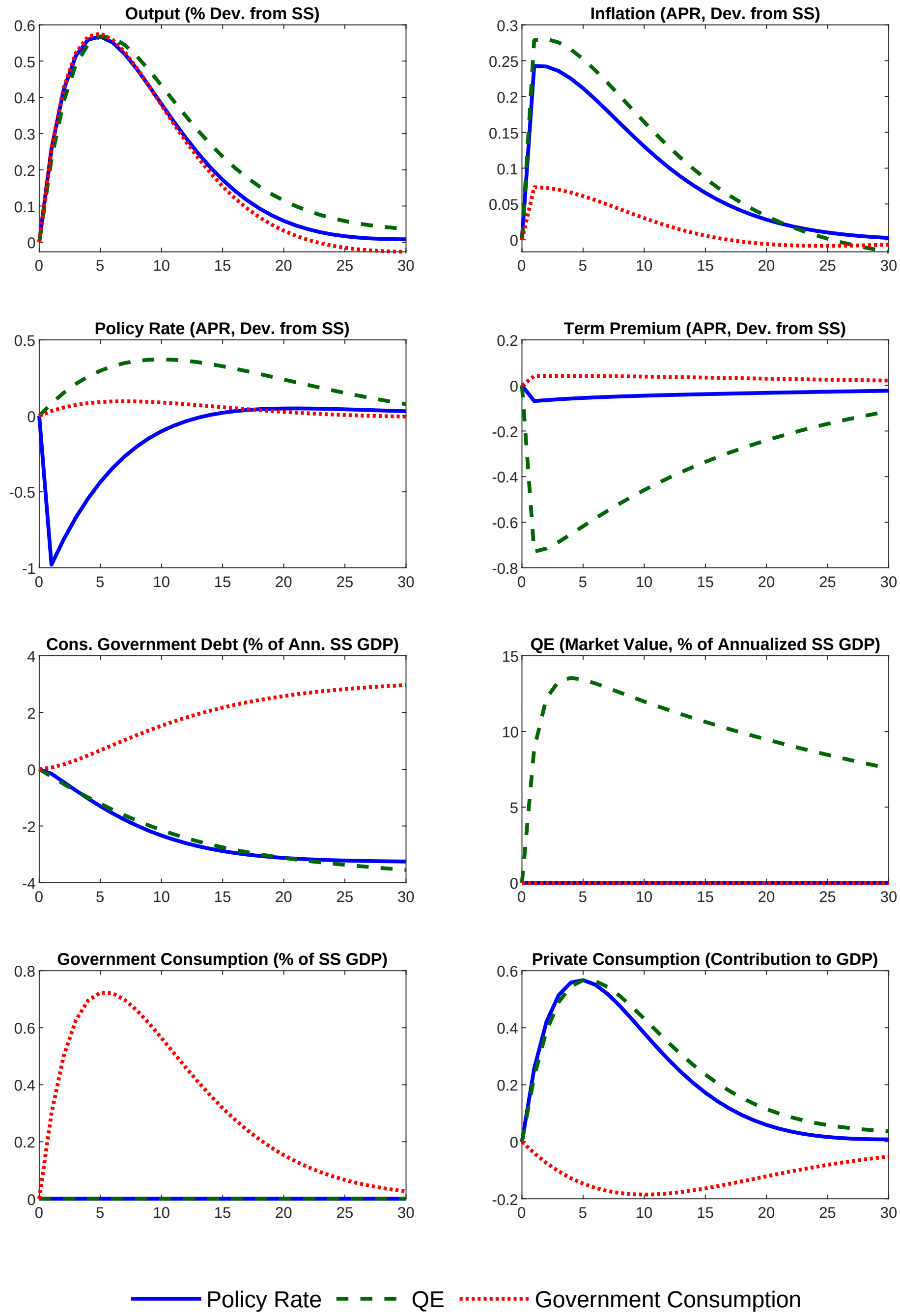

Figure 1: Transmission of Policy Instruments in Calibrated Model.

(2018) and Leeper, Traum and Walker (2017) and the references therein. Even so, while both instruments provide a similar output boost, the composition of the stimulus to output is rather different. The bottom right panel shows that higher government spending crowds out private consumption, whereas higher private consumption is the sole driver of the output expansion for a policy rate cut.

The figure also documents that it takes large scale asset purchases with 13 percent of baseline GDP to generate a commensurate output expansion as a 100 basis points policy rate cut when the short-term rate adjusts in response to QE purchases. This implies a peak output impulse with about 0.04 percent for QE with 1 percent of baseline GDP. This is a very conservative (i.e. small) estimate, even relative to the empirical evidence surveyed by Fabo et al (2021) that excludes central banks evidence on the transmission of QE. Now, the transmission of QE to output will be somewhat larger in a liqudity trap when nominal interest rates does not respond for some time, but since we address the FG puzzle with discounting in expectations, we err on the conservative side on the potency of QE to stimulate output and raise inflation. By implication, we are cautious in our assessment of QE's favorable impact on the consolidated fiscal position.

A final observation regards the transmission of the different instruments to inflation. Comparing QE with conventional short-term policy rate cuts, we see that the inflation transmission is very similar. Figure 1 shows slightly larger impact from QE than conventional policy due to the fact that the calibration of the model implies slightly more persistent effects of QE on output. Comparing the two monetary policy instruments with conventional fiscal policy, we find that the monetary policy tools are notably more effective in stimulating inflation for given boost to output. This difference is driven by the fact that monetary and fiscal poliy has very distinct differential transmission on labor supply. Higher government spending increases labor supply and reduce real wages which attenuates the upward pressures on output. Put differently, higher government spending drives up potential output and is hence associated with a smaller increase in the output gap than monetary policy when we normalize by the same stimulus to output for the different policy tools.

Since we aim to compare the transmission of alternative policy tools under uncertainty, we also need to parameterize the shock processes. To this end, we adopt a simple moments matching procedure and select AR(1) coefficients and standard deviations of the innovations to a parsimonous set of shocks that enables the model to match the unconditional standard deviations of output growth per capita, annualized core PCE inflation, nominal wage growth, and hours worked per capita (measured as deviation from its sample mean) and the correlation between output growth per capita and price inflation, nominal wage growth, hours worked, respectively. After striving to match these moments, we cross-check if our procedure implies a reasonable volatility of the nominal policy rate. The AR(1)-shock processes for productivity (ε_t^z), private consumption demand (ε_t^c), cost-push impulses for firms (ε_t^p) and labor unions (ε_t^w) that our simple matching procedure results in are as follows:

$$\begin{aligned} \varepsilon_t^z &= 0.90\varepsilon_{t-1}^z + u_{z,t},\ u_{z,t} \backsim i.i.d.N\,(0, 0.015) \\ \varepsilon_t^c &= 0.90\varepsilon_{t-1}^c + u_{c,t},\ u_{c,t} \backsim i.i.d.N\,(0, 0.048) \\ \varepsilon_t^p &= 0.85\varepsilon_{t-1}^p + u_{p,t},\ u_{p,t} \backsim i.i.d.N\,(0, 0.043) \\ \varepsilon_t^w &= 0.85\varepsilon_{t-1}^w + u_{w,t},\ u_{w,t} \backsim i.i.d.N\,(0, 0.140) \end{aligned} \tag{29}$$

where the numbers in the $N(.)$ parentheses are mean and standard deviations for each independent and normally distributed shock innovation.

Table 3: Targeted Stochastic Moments.

Moment	US Data	Model
Std($\Delta \ln y_t$)	0.81	0.85
Std(π_t^{ann})	2.17	2.19
Std($\pi_t^{w,ann}$)	3.45	3.71
Std($\hat{n}_t$)	4.99	4.83
Std(R_t^{ann})	3.65	3.12
Corr($\Delta \ln y_t$,π_t^{ann})	-0.18	-0.23
Corr($\Delta \ln y_t$,$\pi_t^{w,ann}$)	-0.12	-0.10
Corr($\Delta \ln y_t$,$\hat{n}_t$)	0.07	0.00

Notes: The moments in the U.S. data are based on the period 1960Q2-2024Q1. Inflation π_t^{ann} is measured with annualized core PCE inflation. Policy rate R_t^{ann} is measured with the annualized net Federal Funds Rate. Real GDP growth $\Delta \ln y_t$ and non-farm business hours worked is scaled with working age population, and $\hat{n}_t$ is then calculated as $(n_t - n)/n$. Annualized nominal wage is measured with wages in the non-farm business sector. The model moments are based on a simulation of a long-sample of 100,000 observations. Finally, notice that we match all moments except for R_t^{ann}, which is used to validate our moment procedure.

Table 3 reports the selected stochastic moments we match in the U.S. data for the period 1960-2024 along with the corresponding model moments. As can be seen, the model matches the selected moments well although we only allow for a small set of stochastic supply and demand shocks as underlying drivers of fluctuations in the model. Notice that the standard deviation for the policy rate is just slightly lower in the model (3.12) than in the data (3.65) as we do not include a monetary policy shock (ε_t^r) among the set of shocks. The reason for this is that we do not want deviations from normal policy behavior to be a source of uncertainty; we only want fundamental reasons – i.e. inflation and output developments – to be a source of earlier policy normalization and a source of central bank losses on their long-term asset portfolios.

It is important to note that the parameterization of the various stochastic shocks in eqs. (29) imply that the bulk (87 percent) of the unconditional volatility in output growth per capita is explained by the consumption demand shock, while only explaining a small part (1.5 percent) of inflation volatility. In line with estimated structural macroeconomic models, for instance the Smets and Wouters (2007) model, fluctuations in inflation are instead explained by the productivity innovations (24 percent), the price cost-push innovations (28 percent) and wage cost-push innovations (46 percent). As a result all four shocks contribute to the variation in the nominal policy rate; ε_t^z with 18 percent, ε_t^c with 31 percent, ε_t^p with 14 percent, and ε_t^w for the remaining 37 percent. None of these shocks are assumed to affect R_t^* in eq. (19), but they impact $\tilde{R}_t$ via their influence on π_t and y_t in the policy rule (18).

3.8. Solution

To preserve nonlinearities associated with the Kimball aggregator and the effective lower bound constraint, we do not linearize the model before solving it. To deal with behavioral discounting, which is not tractable in a fully non-linear setting, we proceed as follows. We first derive the linearized first-order conditions describing the decisions of behavioral agents. In an open economy setting these yield aggregate Euler conditions in which the forward looking terms are multiplied by the cognitive discounting parameter m, with an additional additive term that depends on agents' asset holdings (which, for reasons discussed in Kolasa, Ravgotra and Zabczyk, 2022, is very small). Guided by these considerations, we approximate the relevant first-order conditions in the non-linear model with formulas that, after linearization, yield equations that match the linear derivations, up to the above mentioned quantitatively small term. We solve the resulting model using the extended path approach of Fair and Taylor (1983) readily available in Dynare (Juillard, 1996).

4. Transmission of QE in Deep Liquidity Traps

It is useful to assess the effects of QE in a liquidity trap generated by economic conditions of varying severity and characteristics. We study two situations in this paper. In this section, we analyze the transmission of QE in a "deep" liquidity trap where the output gap is substantially negative, inflation is projected to be well below the central banks target for some time, and the short-term policy rate is constrained by the ELB for a prolonged period of time. Roughly speaking, this scenario would be similar to that prevailing in the aftermath of the global financial crisis (GFC), where financial conditions had improved but unemployment was running far above its long-run level. In Section 5, we analyze the transmission of QE in a "shallow" liquidity trap, in which economic slack is less negative, the inflation undershoot below target is projected to be smaller, and the policy rate is less constrained by the ELB. This is reasonably similar to the situation that a number of advanced economy central banks faced before the pandemic in which inflation seemed stuck well below target even as unemployment moved toward record lows.

4.1. A Baseline with Slow Recovery

Our first simulation considers QE in a very deep liquidity trap associated with a severe recession in which the output gap is deeply negative, and inflation well below target. The blue solid line in Figure 2 presents a baseline simulation in this vein. Starting from an initial position in period 0 where the output gap is closed and inflation is at its 2 percent target, a persistent increase in desired household savings, i.e. a positive innovation to the discount factor shock ε_t^d in period 1, drives the neutral net short-term policy rate ($R_t^* - 1$, see eq. 19) and the nominal net shadow rate $\tilde{R}_t - 1$ in the rule (18) below 0 for a protracted period (about 5 years) as shown in the lower left panel in Figure 2.[8] As a result, the actual short-term policy rate becomes pinned at the ELB for over 5 years absent any new shocks, and the undesired positive persistent gap between the actual and neutral policy rate triggers a sharp decline in output with nearly 4 percent. As the shocks leave potential output unaffected, they also trigger a commensurate negative output gap (not shown) along with significant persistent decline in inflation below the central bank 2 percent target.

Amid this outlook, in which forward guidance in the form of a lower-for-longer policy rate path is ineffective in boosting output and impoving the inflation outlook due to behavioral discounting and Kimball asymmetries in price setting, the cental bank engages in large scale asset purchases.[9] By allowing for some persistence in the AR(1) process for purchases of long term assets in eq. (21), QE is assumed to be implemented in an anticipated yet gradual fashion. Given the calibration of the central banks' reinvestment policy, we set ρ_{QE} and the purchase announcement innovation $u_{QE,t}$ in period 1 such that the stock of assets held by the central bank peaks at 10 percent of baseline GDP after one year as can be seen from the dashed orange line in the lower right panel in Figure 2. This sequence of anticipated purchases reduces the term premium initially by about 50 basis points.

In the deep recession scenario, QE clearly has sizeable macroeconomic benefits. As seen in Figure 2, QE boosts output (upper left panel) about 0.8 percent relative to baseline after six

[8] When simulating the modal baseline projection in Figure 2, we set the AR(1) coefficient in the process driving ε_t^d to 0.95 and its period 1 innovation to 0.20. The figure plots net annualized actual and shadow policy rates, i.e. $400(R_t - 1)$ and $400(\tilde{R}_t - 1)$ respectively. Finally, notice that the steady state annualized nominal policy rate $400(R-1)$ equals 2.5 percent, but the initial values for the actual and shadow rates in period 0 are lower (2 percent) driven by a below steady state neutral rate.

[9] See Erceg et al. (2024) for a detailed discussion why these two features of our model limit the effectiveness of forward guidance in a prolonged ELB episode.

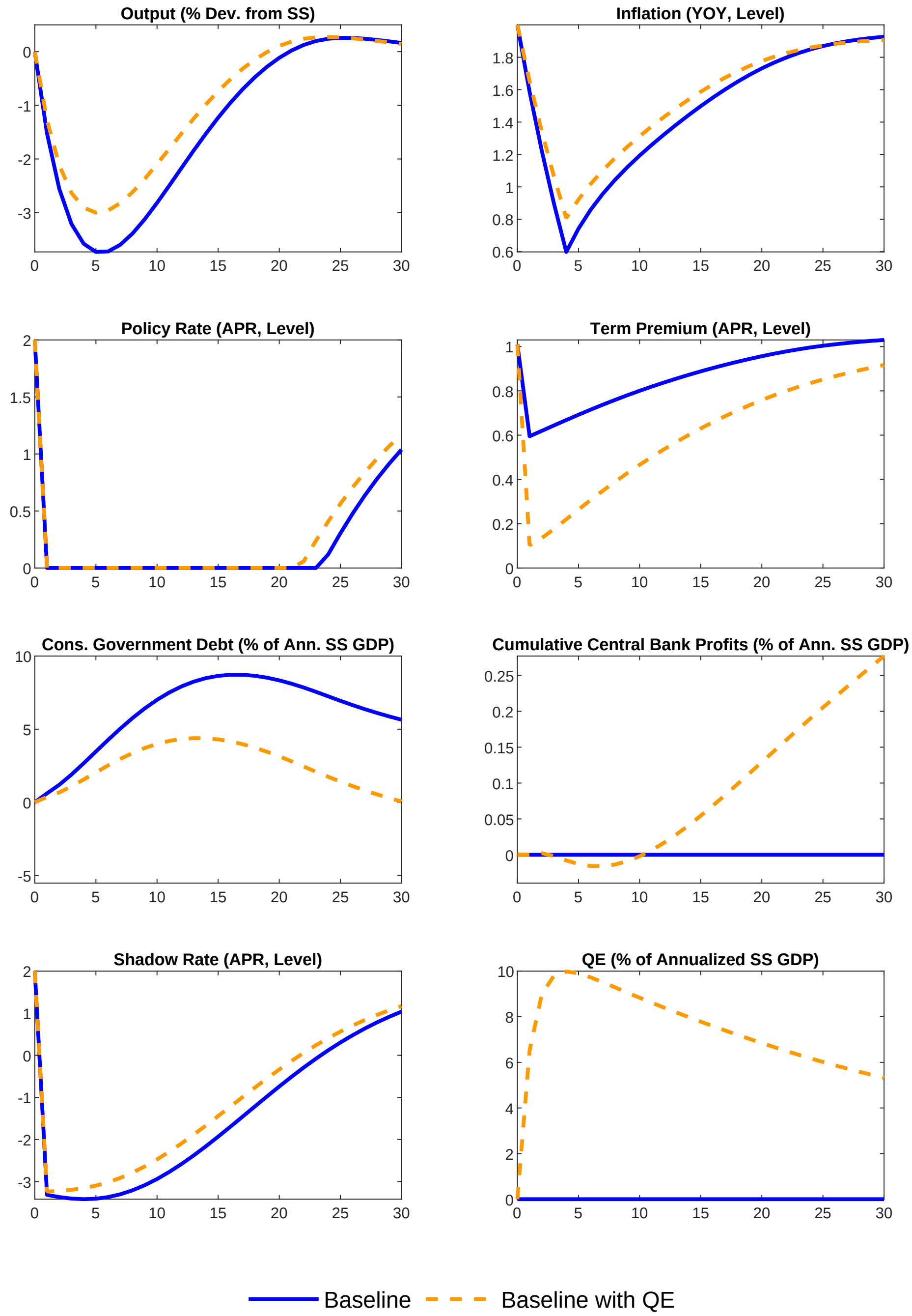

Figure 2: Deep Recession without and with QE.

quarters and core inflation (upper right panel) 0.2 percentage points. This stimulus reflects that markets expect the central bank to progressively expand its balance sheet through asset purchases, so that the term premium declines persistently (middle right panel). The macro stimulus induces a slightly earlier liftoff of the policy rate (middle left panel). Even so, this offset is relatively modest, so that most of the decline in the term premium passes through to longer-term yields.

On the fiscal side, the strong stimulus to output boosts the government's primary balance substantially, reduces the real value and cost of servicing past debt and the cost of issuing new debt (due to higher bond prices), and hence results in a large reduction in government debt. The central bank also makes profits, with the central bank's capital rising close to 0.3 percent after eight years (middle right panel). The central bank profits reflect that long-term yields – while declining substantially – remain well above the policy rate that is pinned at zero for several years and then rises only gradually to its long-run level. As a result, the consolidated fiscal position (middle left panel) improves substantially by about 6 percent of baseline GDP after five years (difference between blue solid line without QE and orange dashed line with QE), of which the bulk reflects increased tax revenue. The consolidated position includes central bank profits, which are assumed to be remitted immediately in our model. In effect, QE more than pays for itself insofar as the macro stimulus is accompanied by falling government debt.

This "holy trinity" of significant macroeconomic benefits, a boost to government revenues, and positive central bank profits seems a reasonable characterization of the central bank experience with QE in the aftermath of the GFC. Moreover, in a deep recession, the macro and fiscal benefits are likely to remain substantial even if there are shocks that occur after the implementation of a QE program that would call for faster policy rate hikes under normal conditions. In particular, given that the central bank would like to set a deeply negative policy rate if constrained by the ELB, modest-sized aggregate demand shocks – such as from fiscal expansion – would not push toward significantly earlier liftoff, and the benefits of QE from a macro perspective and consolidated fiscal perspective would be similar to those in Figure 2.

4.2. A Post-COVID Scenario with Faster Recovery

As suggested by recent experience, very large shocks could materially affect the benefits of a given QE program, especially if they induce a rapid recovery and inflation surge that pushes the central bank to exit a stimulative stance much earlier than in the modal outlook and raise policy rates well above its normal long-run level. Under these circumstances, the central bank could face sizeable losses as it gets stuck with a large stock of low-yielding assets but has to pay higher interest on reserves (see eq. 22). To illustrate this, Figure 3 shows the effects of QE in the baseline with a slow projected recovery from the recesssion, and in a scenario with an unexpected faster recovery.

The left column shows the effects in the baseline without QE, the modal "slow-recovery" projection with QE, and a third scenario with an unexpected faster recovery. The modal outlook trajectories without and with 10 percent QE were already shown in Figure 2, whereas the green dash-dotted line show a scenario with notably faster recovery precipitates inflation and long-term yields to rise notably faster than under the modal outlook in Figure 2.[10] The scenario with faster recovery features a mix of cost-push impulses for firms (ε_t^p in eq. 29) and stronger consumption demand (ε_t^c, in eq. 1, which does not impact R_t^* in eq. 19) that unexpectedly hit the economy in period 7 as indicated by the vertical black dashed lines (notice that quarter 0 is the initial period before any shock hits the economy). The right column shows the marginal effects of QE without and with the faster recovery. The impact of QE under the modal outlook without the (surprising) faster recovery is simply the difference between the orange-dashed and solid blue lines in Figure 2). The marginal impact of QE with the faster recovery is calculated as the difference betwen the trajectory with faster recovery in the left column in Figure 3 minus a counterfactual simulation without QE and faster recovery (not shown).

We see from the right column panels for output and inflation that accounting that the unexpected faster recovery reduces the "ex post" macroeconomic benefits of QE; earlier liftoff implies less stimulus to output, and the surge in inflation implies an uptilt in the senstivity of inflation to asset purchases, which is not desired when inflation is surging above the central bank's target. Moreover, as seen from the lower right panel, the associated fall in bond prices and higher funding costs of the asset purchases implies that the central bank makes significant losses on its QE portfolio (worth about 0.5 percent of baseline GDP) when the economy bounces back faster than previously envisioned.[11] The next to last bottom panel shows that the consolidated fiscal position improves until the surprising adverse shocks get full traction although the central bank makes sizeable losses (mark to market) directly as the prices of their purchased long-term bonds fall.

Overall, QE still provides substantial macro benefits, also from this ex post perspective – boosting output and inflation when economic slack is high, and the welfare benefits are presumably highest. While a possible eventual overheating and overshoot of the inflation target may require a sharp rise in the policy rate earlier than envisioned when QE was undertaken and hence pare back

[10] The size of the scenario is informed by the core inflation surge in the US during the post-COVID period, which is in line with the 99th percentile according to our stochastic simulations that we discuss later and which will be shown in Figure 7. Consequently, this scenario implies notably faster policy normalization than in the modal outlook (but still less faster than the 99th percentile under general uncertainty).

[11] Quantitatively, the losses are here smaller than observed following the COVID pandemic. This reflects our assumption that the liquidity trap is deep and the unexpected faster recovery materializes late after asset purchases have ended. In Figure 7, we will show that central bank losses can exceed 2 percent of baseline GDP for a QE program of the same size and when allowing for general shock uncertainty in a shallow liquidity trap.

(A) **Scenario Paths** (B) **Marginal Effects of QE**

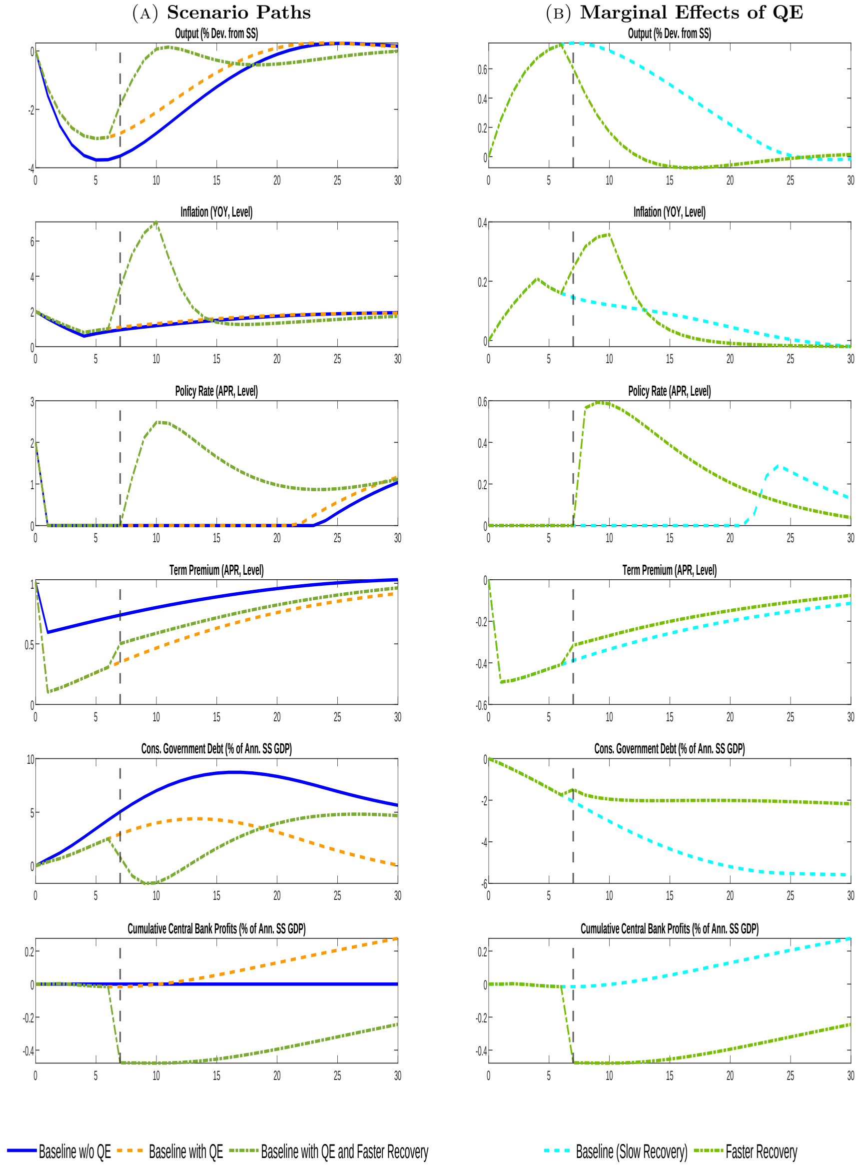

Figure 3: QE in a Deep Recession with Unexpected Faster Recovery.[7]

central bank profits significantly, the consolidated fiscal position still improves considerably. There is only a slim risk that QE worsens the overall fiscal position marginally in a deep liquidity trap although the central bank may possibly make some significant losses. These results underscore the robustness of QE in a deep trap, and the importance of looking at the consolidated fiscal position in assessing the fiscal consequences.

4.3. A Comparison with Conventional Fiscal Stimulus

In this section, we compare QE with conventional fiscal stimulus. The fiscal instrument we consider is an increase in government consumption, for which there is ample empirical evidence as discussed in Section 2. We compare the effects of central bank QE with fiscal stimulus by normalising both policies to imply the same boost to aggregate output during the coming 7.5 years, i.e. $\Sigma_{t=1}^{30}\beta^t y_t$. Given the same output impulse, we can compare the effects on inflation and the consolidated government budget position (i.e. government debt). We keep our focus on the case of a deep liquidity trap, as this is the most realistic situation where both policies can be meaningfully deployed.[12] We discuss the transmission of both policies on the modal outlook (i.e. excludig future shock uncertainty), but next consider a situation with a surprising fast recovery from the recession. Finally, we decompose the impact of the two alternative means to provide stimulus on the consolidated government fiscal position (including the central banks' balance sheet).

The left column in Figure 4 compares the marginal impact of QE with 10 percent of baseline GDP to an anticipated increase in government spending ε_t^g that is sized to give the same discounted positive impulse in aggregate output under the modal outlook with a projected slow recovery from the recession.[13] Following the analysis in Figure 3, the right column compares the marginal impact of QE and conventional fiscal stimulus when the economy unexpectedly recover faster than expected from period 7 and onwards (as indicated by the dashed vertical line). In the simulation with the unexpected faster recovery, both QE and fiscal stimulus that was announced and implemented in period 1 is assumed to be fixed and the same as under the modal projection with a slow recovery.

The marginal impact of QE has already been extensively discussed, so in the following we focus on the transmission of conventional fiscal policy. Starting with our baseline results in the left column with a slow recovery from the recression, we see that fiscal policy stimulates inflation less than QE for given output boost. This feature was noted when we discussed Figure 1 and is driven by the fact that higher government spending increases potential output in our model, whereas QE leaves potential output largely unaffected. This implies that fiscal stimulus generates a smaller positive output gap when we normalize both policy interventions on a given sized increase in output, causing

[12] However, in Appendix TBA, we compare the tools in a situation when inflation is at target and the output gap is closed, so that the central bank is unconstrained to adjust its policy rate. We also provide a comparison of the two policies in a shallow liquidity trap.

[13] The marginal impact of QE is the same as in the right column of Figure 3. The marginal impact of higher government spending is calculated analogously.

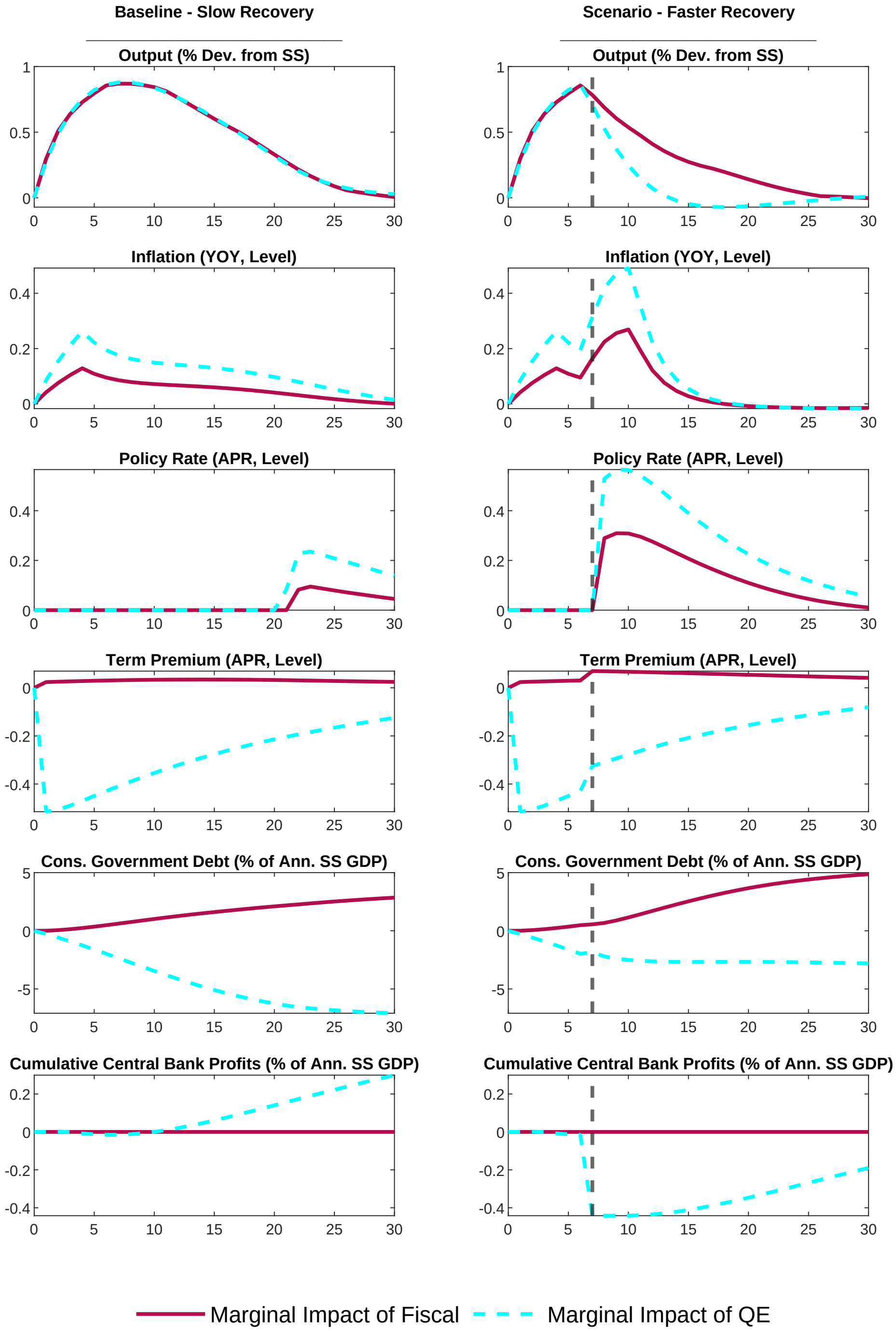

Figure 4: QE versus Fiscal Stimulus in a Deep Trap with Slow and Faster Recovery.

fiscal policy to induce a smaller increase in inflation although both policies transmit via increasing aggregate demand in our model. Another difference between QE and conventional fiscal stimulus is that the latter will actually nudge the term premium a little higher as can be seen from the

4th left panel. The increase in the term-premium is driven by larger issuance of long-term assets as the government debt increases notably under fiscal stimulus via higher government spending. Quantitatively, our model implies that the consolidated government debt position after 7.5 years worsen with about 3 percent of steady state GDP, whereas QE improves the fiscal position with about 7 percent.

The right column in Figure 4 presents the results when a mix of cost-push impulses for firms (ε_t^p) and stronger consumption demand (ε_t^c) trigger a faster recovery with stronger output and well above target inflation as considered in Section 4.2. The scenario with a faster recovery means that both policy tools feature notably stronger transmission to inflation, whereas the output stimulus is notably reduced for QE but remains similar for government spending (as the path for g_t is the same). As we have seen, a faster-than-expected recovery from the recession mutes the improvement in the consolidated fiscal position due to QE notably. The same as is true for conventional fiscal stimulus, but the change is smaller. Thus, the transmission of conventional fiscal policy is less sensitive to uncertainty than QE. Still, Figure 4 makes it crystal clear that QE in expectation should be associated with notably lower consolidated fiscal costs than conventional fiscal policy, although there is a risk that QE ends up exposing the central bank to significant capital losses on its portfolio of long-term bonds.

4.3.1. Drivers of the Consolidated Public Debt Position

In this section we discuss in detail the drivers behind the consolidated public debt position GD_t^{con} in eq. (12) depicted in Figure 4 for the alternative means to stimulate the economy in a deep liquidity trap. Accordingly, the drivers of GD_t^{con} in Figure 5 are splitted into the following categories: revenues from consumption and labor income taxes, government consumption expenditures, government debt service costs and central bank profits, and debt deflation and revaluation effects. Summing these drivers provides us with the overall fiscal position GD_t^{con} in equation (12) in periods $t = 1, 2, ..., T$, and the various bars depicted in Figure 5 show each of these categories contribution to GD_t^{con} for $t = 1, 2, ..., T$. The blue dashed (red solid) line in the left (right) panels is simply the same line as in Figure 4 (slow and faster recovery, respectively).

Panel A in Figure 5 shows the decompositions of government debt for QE and higher government spending in our baseline with slow recovery, i.e. the left column in Figure 4. As expected we see that increased labor income tax revenues – and to some extent also consumption income taxes – contributes notably to the improvement in the fiscal position over time. But also higher inflation,

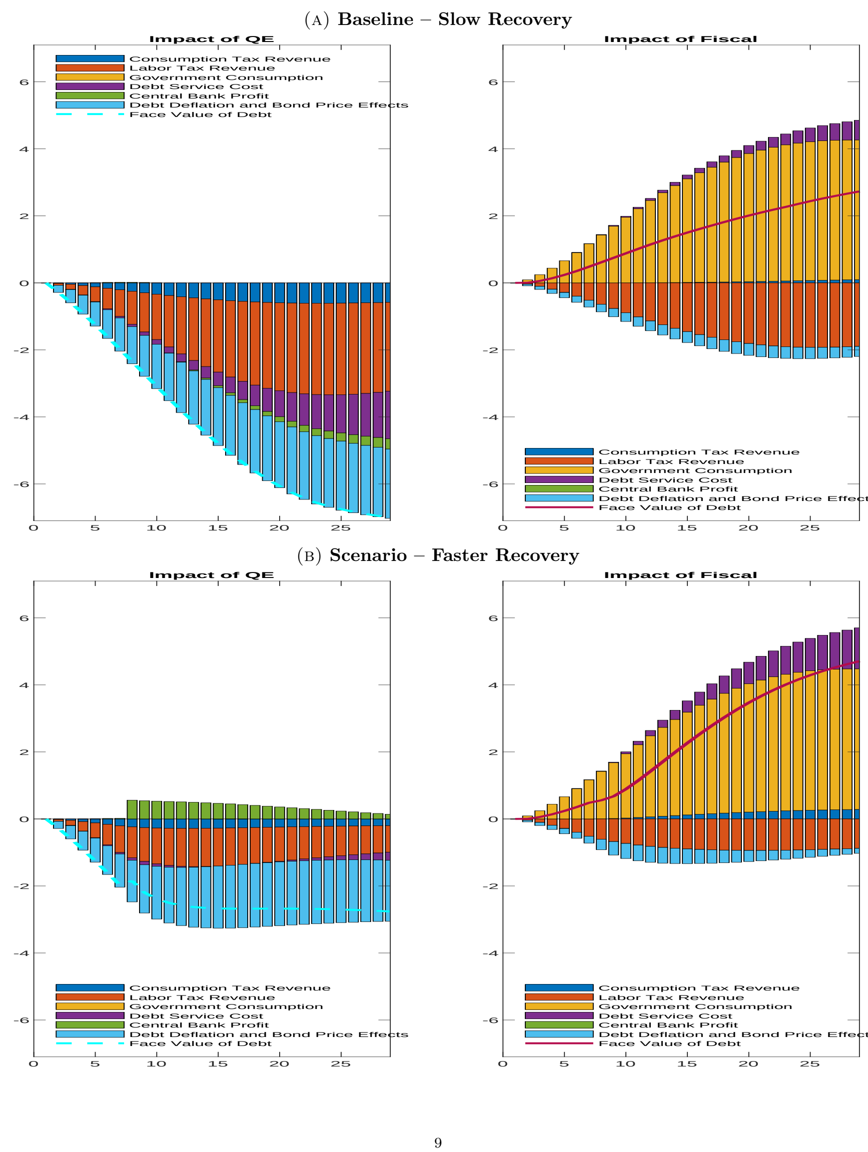

Figure 5: Decomposition of QE and Fiscal Stimulus to Consolidated Government Debt.

lower debt service costs and gains on the central banks' QE portfolio improves the consolidated public debt position. Stimulus via higher government spending, shown in the upper right panel, also improves labor income tax revenues, albeit less so than QE. This difference between QE and conventional fiscal stimulus may at first glance be surprising, since the output increase is by construction made (nearly) identical. As the model does not feature physical capital and technology is the same, the output expansion is completely accounted for by an identical expansion in hours worked. Even so, QE generates notably larger labor income tax revenues as real wages rises less under higher government spending. Moreover, because higher government spending eventually leads to some crowding out of private consumption, sales tax revenues do not contribute significantly to the change in the fiscal position, in constrast to QE. But the major difference in the consolidated fiscal position with higher government spending relative to QE is the costs of the fiscal stimulus: accumulating these costs over time is a key driver of the deterioration in the fiscal position. On top of this, by driving up government debt, conventional fiscal stimulus also induces higher debt service costs that over time is another source of relative worsening of the consolidated government debt position relative to stimulus via QE. Finally, when compared to government spending, QE leads to a sizable increase in bond prices and has a much stronger effect on inflation. This means that QE contributes much more to lower consolidated public debt by decreasing the quantity of bonds that need to be issued to finance the deficits and by decreasing the real value of outstanding bonds .

Panel B shows the corresponding decompositions for the scenario with a faster recovery, i.e. right column in Figure 4. While the drivers of debt dynamics for conventional fiscal stimulus are very similar to our baseline scenario with slow recovery (albeit quantitatively attenuated), there are some notable differences for the results with QE and conventional fiscal stimulus. In the left chart in Panel B we see that central bank losses is now a source of deterioration in the public debt position. Less labor income tax revenues is also key to smaller improvement in consolidated debt when the economy recovers notably faster than envisioned when QE is implemented. All told, the favorable debt implications of QE are more affected by uncertainty than for conventional fiscal stimulus. But despite this larger uncertainty associated with QE, we find that QE provides more effective output stimulus for given fiscal cost when accounting for stochastic shock uncertainty as in Figures **??** and 7. And QE has the additional advantage of stimulating inflation more than conventional fiscal stimulus. [Remains to be done: simulate fiscal stimulus for the 500 shock sequences considered in Figure 7. Report two histograms/kernel distributions with average/accumulated increase in output the first five years (left panel) and the consolidated debt level after 5 years in the right panel. Most likely it will show about the same output boost histogram, but consolidated government debt will be notably smaller. Possibly, we can add inflation as additional middle panel, taking the three year average in each simulation. We an merge this into panel c in the debt dynamics figure.]

5. QE in Shallow Liquidity Traps

We now turn to the case of a shallow liquidity trap with a smaller economic decline and below target inflation shown in Figure 6. Compared to the case of a deep trap, we generate the modal baseline projection (solid blue line) in the shallow trap with a smaller innovation to the discount factor shock (0.10) but assuming that it is slightly more persistent (AR(1) coefficient equal 0.96). As a result, we get a shadow rate that just slightly falls below the ELB.

Amid this modal projection, the orange-dashed lines in Figure 6 add identical purchases of long term assets as in Figure 2 (compare QE paths in the bottom right panels in Figures 2 and 6). QE still appears beneficial provided that the economy evolves reasonably in line with the modal (no-uncertainty) outlook (i.e., that it would stay close to the baseline projection absent QE). As illustrated in Figure 6, the baseline without QE has a slightly negative output gap of a little less than 0.5 percent, and inflation is about 0.8 percentage points below target. A modest-sized QE program is helpful in boosting inflation and in closing the output gap, and after some initial losses on its purchases the central bank experiences some eventual improvement in its capital position, as the yield curve remains upward sloping (with the policy rate only converging gradually to its long-run level). Higher tax revenues and lower debt service costs imply that the impact on the consolidated fiscal position remain favorable, although the improvement with 3 percent after 5-7 years is about twice as small as the improvement in the consolidated fiscal position in the deep trap in Figure 2.

However, there are several noteworthy differences between the effects of QE in a shallow liquidity trap compared to deep liquidity trap case considered earlier. First, a given-sized QE program has less "bang for the buck" in reducing long-term yields and hence in boosting output and inflation, reflecting that the stimulus from QE tends to cause policy rates to rise more quickly than in a deep liquidity trap. Thus, the increase in output is only about 2/3 as large in Figure 6 as in Figure 2.

Second, as suggested by recent experience, there is a greater risk that QE can be counterproductive in a shallow liquidity trap if upside inflation risks materialize. Such shocks could materially affect the benefits of a given QE program, especially if they induce a rapid economic recovery and inflation surge that pushes the central bank to exit a stimulative stance much earlier than in the modal outlook and raise policy rates well above its normal long-run level. Under these circumstances, the central bank could face large losses.

This is illustrated in Figure 7, which introduces uncertainty around the modal outlook with QE. To introduce uncertainy around the modal outlook with QE, we add stochastic shocks and

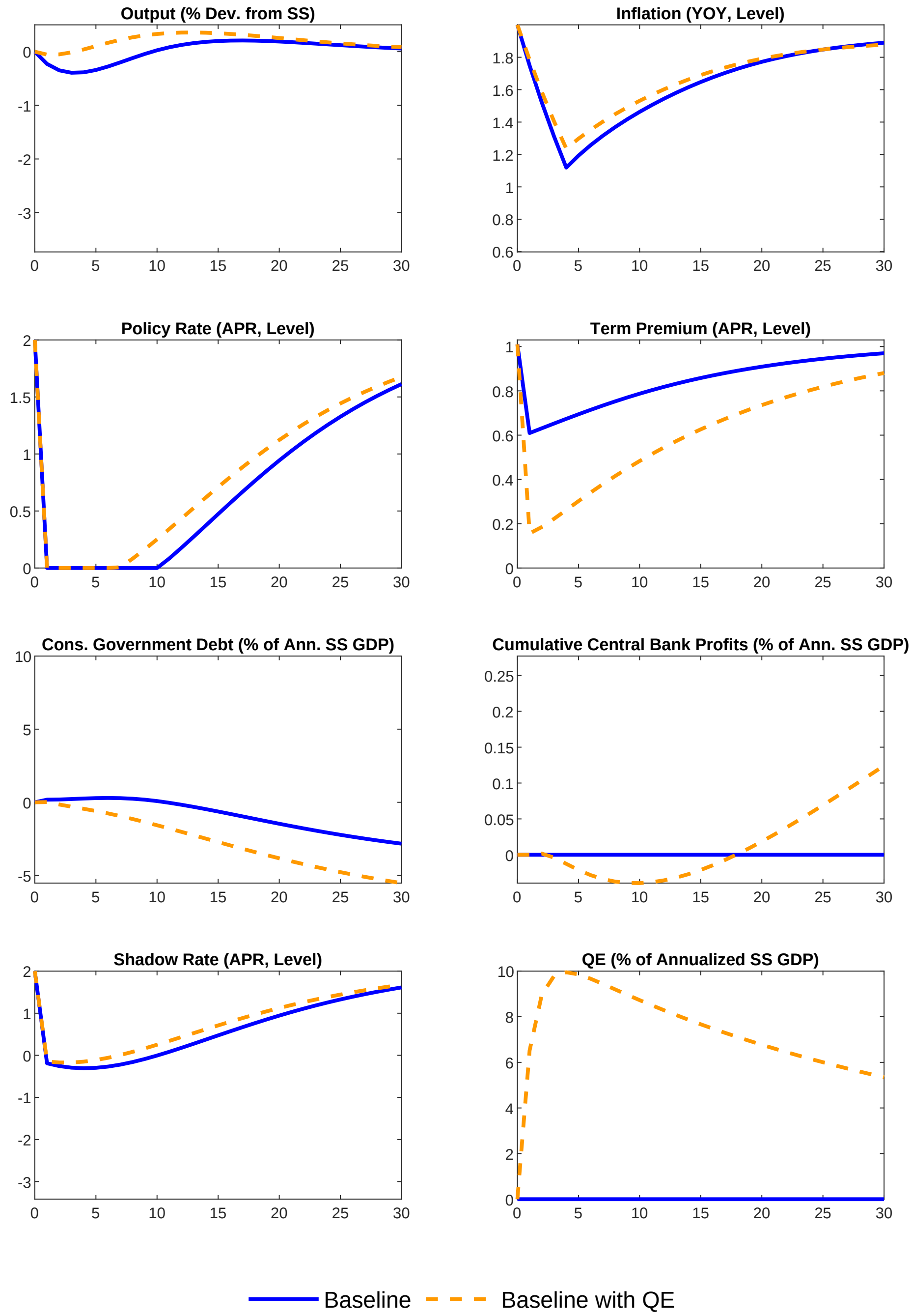

Figure 6: Shallow Recession without and with QE.

simulate 500 trajectories in periods $t = 1, 2, ..., T$. Each of the 500 trajectories allow for stochastic innovations to productivity (ε_t^z), consumption demand (ε_t^c), cost-push impulses for firms (ε_t^p), and cost-push impulses in wage setting (ε_t^w) where the innovations hitting the economy in period t are unexpected in period $t-1$. The processes for these stochastic shocks are given in eqs. (29) , and as noted earlier they enable the model to match the unconditional standard deviations of the key macrovariables in Table 3 for the period 1960-2024.

The left column in Figure 7 shows the modal (no-uncertainty) baseline projection with QE (i.e. the orange-dashed line in Figure 6) along with the mean and the 80^{th}, 90^{th} and 99^{th} percentiles from the distribution with 500 simulations. We see that the uncertainty is sizeable, the percentiles imply that the policy rate may lift-off from the ELB quickly when allowing for uncertainty. The rather high probability of an early lift-off from the ELB is primarily driven by three aspects of our model. First, we consider a rather large response coefficient (2.5) on annualized quarterly inflation in the policy rule (18). Second, the risks to inflation are asymmetric due to the real rigidities in firms price-setting behavior; the Kimball (1995) aggregator implies that there is more upward than downside inflation risk, which can be seen from the inflation fan chart in the second left panel. Third, the size of shocks we introduce in the stochastic simulations are choosen to be large so that they can match the volatility of inflation, output growth and policy rates for a long sample period (1960-2024), i.e. not just the "Great Moderation" but including a number of episodes with volatile and high inflation and policy rates. Had we focused on a sample with more stably macroeconomic conditions, the shadow rate would not have moved so much and precipated a notably erlier exit. All told, these three factors imply that the shadow rate can move substantially and long-term yields rise materially in the near-term. The possible combination of swift repricing of long-term assets and higher funding costs of long-term asset purchases due to rising short-term rates implies that a central bank that has purchased long-term assets via QE equal of 10% of baseline GDP can make significant losses on its portfolio as shown in the bottom left panel.

The right column in Figure 7 shows the marginal impact of QE, i.e. we calculate the difference between the 500 trajectories with QE and the same simulations without QE (not shown). By computing the difference with the trajectories with and without QE, we can parse out a distribution of 500 draws which captures the marginal impact of QE. And in the right column we plot the mean and the 80^{th}, 90^{th} and 99^{th} percentiles of this marginal distribution along with the no uncertainty modal path, which is simply the difference between the baseline with QE (organge-dashed lines) and the baseline without QE (blue solid lines) in Figure 6.

While allowing for uncertainty does not moderate the impact of QE on the output trajectory compared to the no-shock modal projection, the inflation impact is attenuated when considering stochastic shock uncertainty in a shallow liquidity trap. This implies a larger risk that QE in a

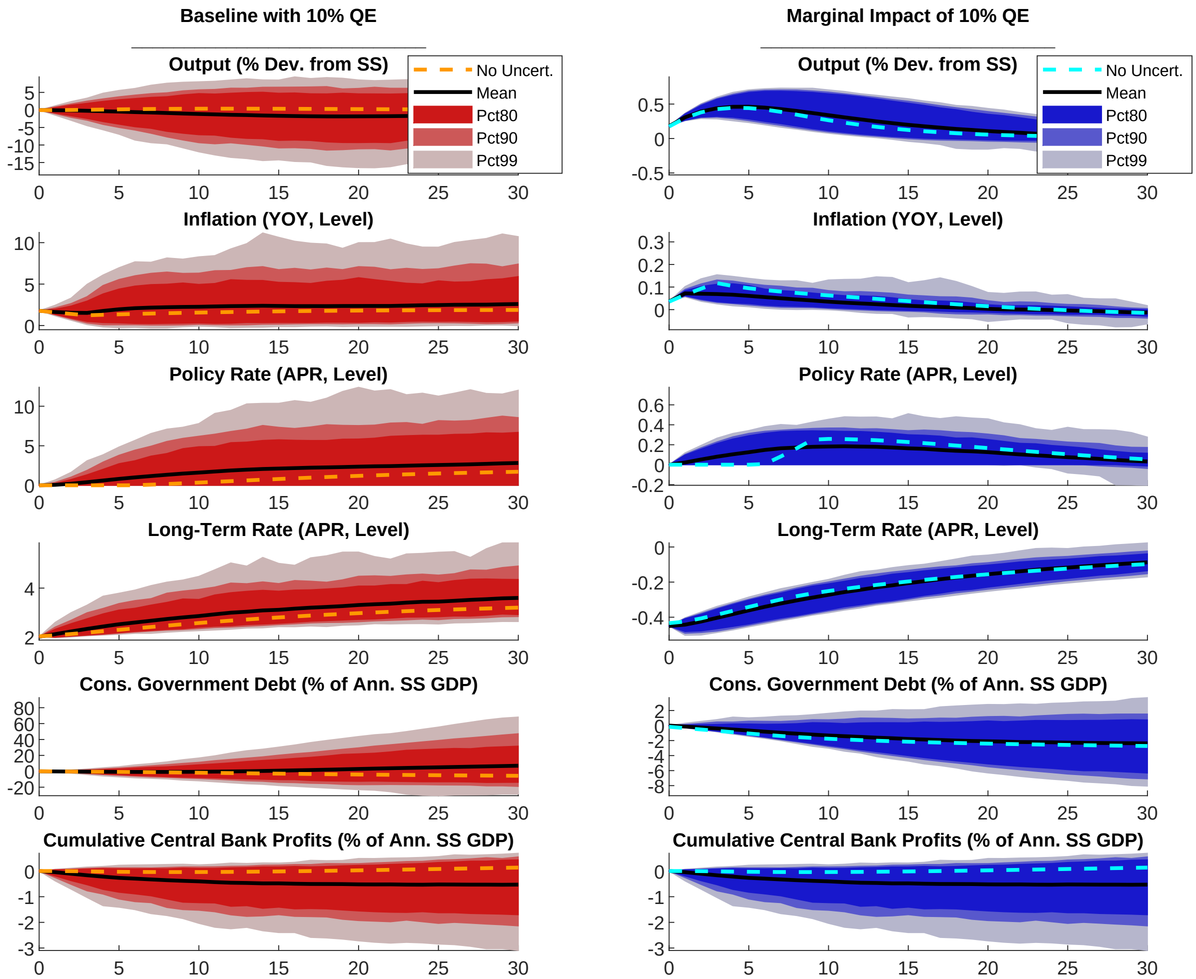

Figure 7: Impact of Uncertainty on QE in a Shallow Recession.

shallow trap contributes to an overheating than if QE is conducted in a deep trap, but the risk of contributing to inflation cycle should be contained. Moreover, the uncertainty bands in the bottom two panels – which are identical as we in both cases compare to scenarios without QE – show there is a big risk in a shallow trap that QE generates large losses for the central bank; the accumulated losses of the central bank may be as large as 3 percent of baseline GDP after 5 years. Even so, the next to last panel in the right column documents that the risk for sizeable consolidated fiscal losses are small. Thus, a striking feature conveyed by Figure 7 is that the adverse impact on the consolidated public debt position is likely very modest although the central bank is at risk of making outsized losses on its portfolio of long term assets. And it should be borne in mind that we have calibrated sizeable impact of shock uncertainty by forcing the model to match the standard deviation of output growth and inflation for 1960-2024 period, a sample that contains several episodes with inflation surges. Had we reduced shock uncertainty by matching standard deviations of inflation during the Great moderation period before he Covid pandemic (i.e. 1984-2019), the impact of shock uncertainty and scope for consolidated fiscal losses of QE would have been attenuated further.

However, there are some important arguments why QE can be less beneficial for macrostabilization and associated with larger fiscal risks than accounted for by our analysis sofar. If QE embodies a commitment on the short-term policy, it can additionally fuel the overheating and make the central bank less nimble in responding to the upside inflation surprise. As discussed previously, QE can involve guidance about how long it is likely to last, as well as a conditional promise to delay hiking rates until well-after QE ends. In effect, the central bank may feel "locked into" keeping policy rates low even when it would otherwise raise them quickly, which can trigger more overheating than shown in Figure 6. This case is shown in Figure 8, which shows a baseline scenario with faster recovery without QE, and compares this with a simulation with QE and faster recovery under the assumption that the central bank commits to keep the short-term policy rate unchanged without any QE and before the unanticipated shocks hit the economy (i.e. period 10 in Figure 6). In this case, we see that the commitment path leads to overheating in output and sizeable QE losses on the central bank's QE portfolio. In practice, central banks may also feel quite constrained by the guidance they give about the likely duration of QE program, including both from concerns about credibility and financial stability. These risks associated with the low nimbleness of QE are clearly higher when upside inflation risks are more pronounced, as would be the case if the Phillips Curve is more nonlinear than we assume and QE is initiated reasonably close to full employment.

On the possibility of larger adverse fiscal consequences of QE than accounted for in our analysis, we first recall that QE in a shallow trap is more likely to result in central bank losses (see accumulated central bank profits in Figure 7) given a greater likelihood that policy rates would eventually have to be raised notably faster and persistently, and thus that prices of long-term bonds fall (i.e. long-term yields rise). In our model, it still turns out that QE likely reduces government debt from a consolidated perspective (next to last lower right panel in Figure 7). However, the consolidated fiscal implications could be less favorable in the event the central bank felt it was necessary to shift its reaction function and act more forcefully. This would amplify central bank losses and could cause the consolidated fiscal position to deteriorate. Such an outcome would be more likely if the nonlinearities in the Phillips Curve were more pronounced—and lead to more persistent inflation

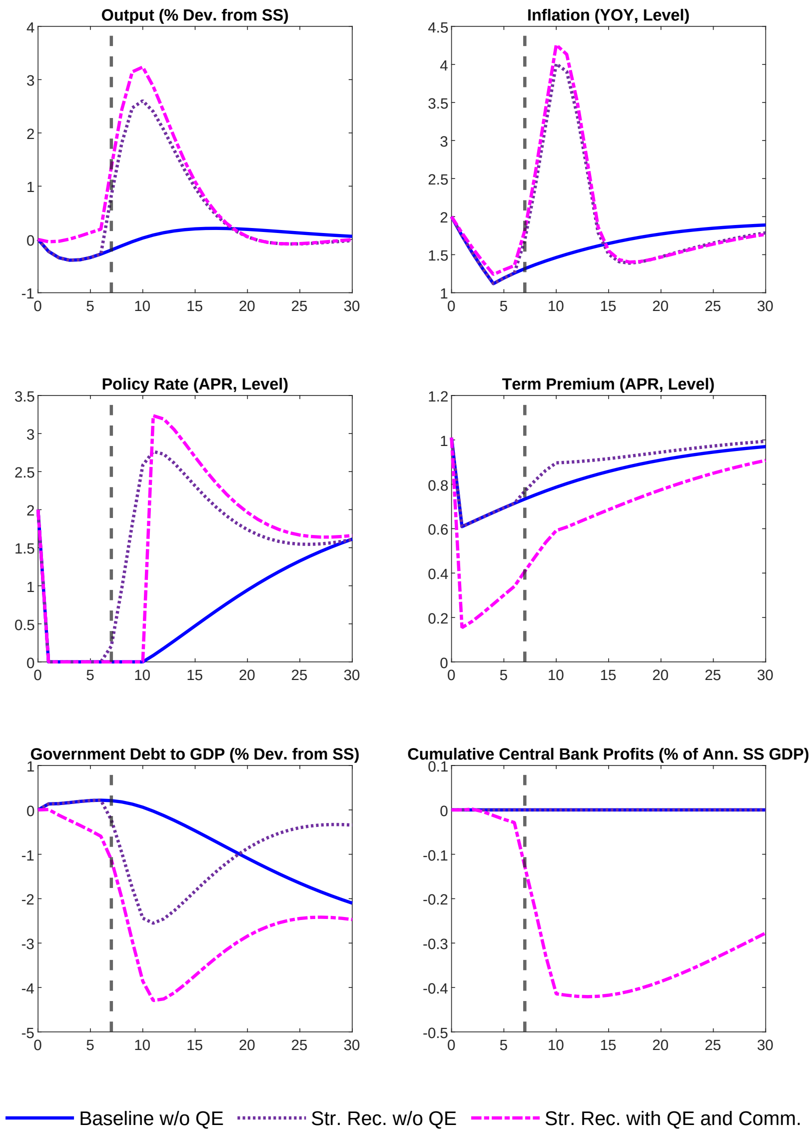

Figure 8: Impact of QE with Faster Recovery and Exit Commitment.

effects—than in our model.

Moreover, as we will explore in detail in the next section, the initial level of the term premium and how large purchases the central bank decides to undertake matters for the budgetary costs. The smaller the term premium before making purchases, and the larger the size of the purchase program, the higher the risk of significant losses on the central banks' QE portfolio, which will adversely impede the consolidated fiscal gains of QE.

Despite these caveats, the findings in Figures 6 and 7 suggest that QE may be a good complementary tool to ensure against adverse macroeconomic outcomes and disanchoring of inflation expectations when inflation is well below target and the policy rate constrained by the ELB although the outlook is highly uncertain and the economy may recovery much faster than envisioned when the central bank decides to announce purchase of long-term assets. Although it is conceivable that the central bank will end up making sizeable losses, the consolidated fiscal implications should notably be more favorable than conventional fiscal stimulus.

6. Additional Key Determinants of Fiscal Costs of QE

In this section, we highlight two additional key determinants of the fiscal costs and risk to the central banks balance sheet with QE; the initial level of the term premium and the size of asset purchases. These factors are on top of future shock uncertainty and the assessment of how much the unconstrained shadow rate is below the ELB.

6.1. Initial size of the Term-Premium

Figure 9 shows the sensitivity of our results to the size of the term premium in the steady state. Our baseline calibration features a steady state term-premium of 100 basis points, and we show results for two alternative calibrations; when the term premium is markedly lower (25 basis points) and higher (200 basis points). The left column shows the marginal impact of QE with 10 percent of baseline in a deep liquidity trap, whereas the right column shows results in the shallow liquidity trap. This implies the marginal impact in the baseline (light-blue dashed line) 100 basis points term-premium is the difference between the orange-dashed and blue solid lines in Figures 2 and 6, respectively.

As can be seen from the figure, the macroeconomic transmission of QE to output, inflation and policy rates are largely invariant to the calibration of the initial level of term premium. Still, although the difference in macro transmission is modest, it is evident that the larger the size of the initial term-premium, the larger the transmission of QE is to output and inflation at the margin. This is driven by a larger compression of the term-premium when the initial (steady state) term-premium is higher as can be seen from the middle panels.

However, even if the macroeconomic transmission is very similar, the bottom row in Figure 9 shows that the effects on the central bank profits are quite different. There is a notably larger risk for central bank losses when QE is done when the term-premium is low than when its high. The adverse impact on central bank losses when the term-premium is low initially implies that the improvement in the consolidated government fiscal position is paired back notably in such a situation. Out of

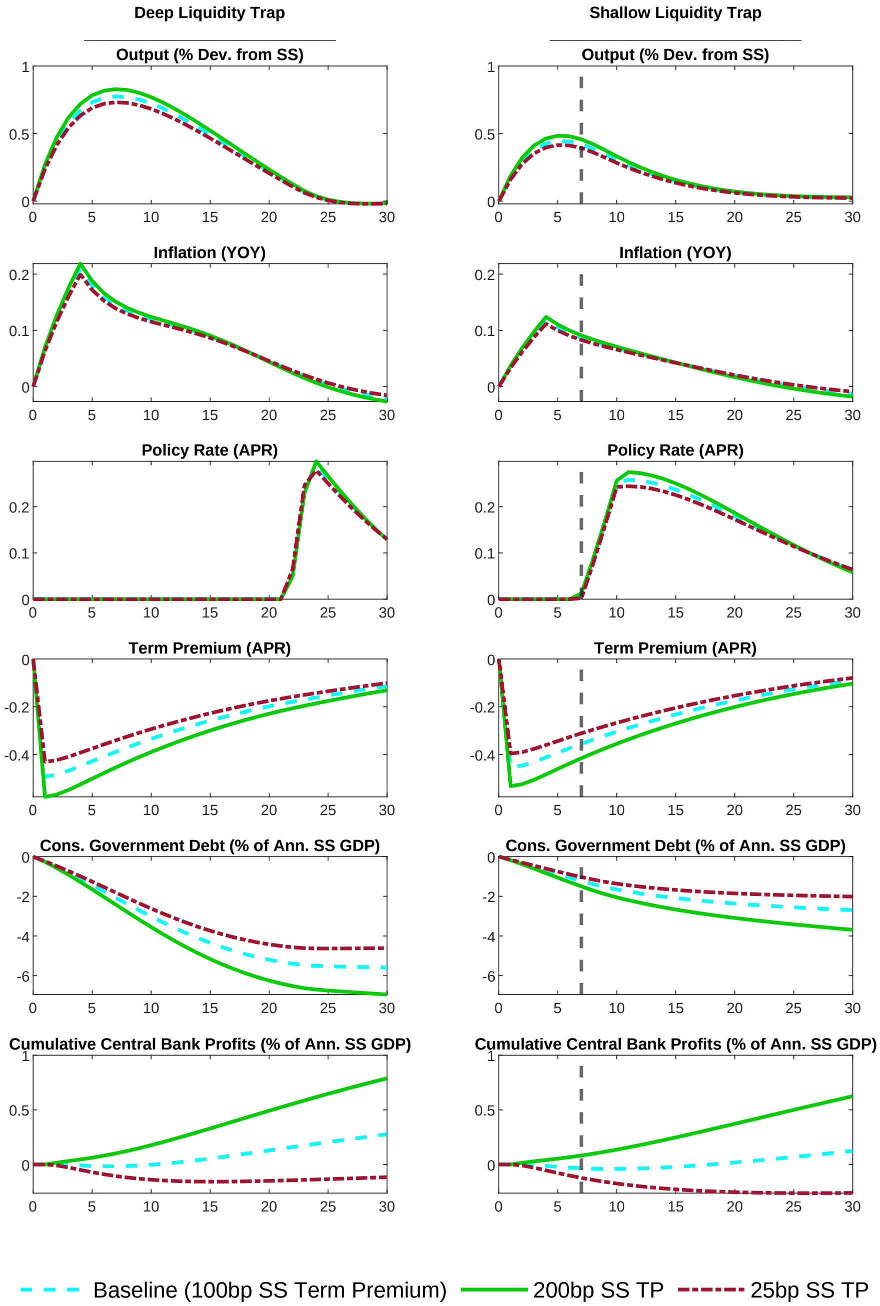

Figure 9: Sensitivity of QE transmission to Alternative Steady State Term Premium.

the 2 percent of baseline GDP less favorable impact on the consolidated fiscal position after 5 years under the high (200 basis points) and low (25 basis points) initial term-premium levels, about half is explained by the adverse impact of accumulated central bank profits and the other half by smaller accumulated tax revenues and reduction in debt service costs. Finally, by comparing the left and right columns in Figure 9, we see that the conclusions are invariant whether we consider a deep or shallow liquidity trap in the modal outlook.

6.2. Size of Long-term Asset Purchases

Figure 10 shows the sensitivty of our results when increasing the size of the QE intervention from 10 (baseline) to 20, and finally to 40 percent of pre-shock GDP. The results in Figure 10 are normalized so that they show the equivalent of a 10 percent of baseline GDP LSAP (i.e. the 20 and 40 percent QE purchases are divided by 2 and 4, respectively). As in Figure 9, we report the marginal impact of QE so that the results in the left (right) column for 10 percent QE equals the line for the trajectories in Figure 2 (Figure 6).

The findings for alternative (normalized) QE sizes are very similar to those for alternative term premium sizes. Although the transmission to output and inflation is close to linearly related to QE size, larger purchases have a significant adverse impact on cumulated central bank profits. This is because larger QE means a larger compression of the term premium, and hence a higher price the central bank needs to pay for bonds. If the premium is driven sufficiently low, the profit made by the central bank on its asset portfolio becomes negative even under the modal outlook. The scope for larger central bank losses per unit of QE for larger purchases reduces the improvement in the consolidated fiscal position when the central bank implements a larger QE intervention. The worsening of the consolidated fiscal position per 10 percent of QE is quantitatively similar as when we varied the initial term-premium in the previous section, but a difference here is that a larger share of this worsening (about 3/4 instead of about 1/2 in Figure 9) is now driven by the adverse impact on the cumulated central banks' profits. Finally, we see by comparing the left and right columns in Figure 10 that the conclusions are invariant whether we consider a deep or shallow liquidity trap.

7. Concluding Remarks

This paper aims at providing a preliminary assessment of the implications of the recent inflation surge for use of quantitative easing and for central bank capital policies (including distributions to the government). We emphasize four key points.

The first is that QE policies are likely to have substantial benefits in a deep recession in which policy rates are expected to be constrained by the ELB for a protracted period. QE boosts output and inflation and improves the consolidated fiscal position of the government. Thus, QE is likely to be a very useful tool in the event that the ELB again becomes severely binding.

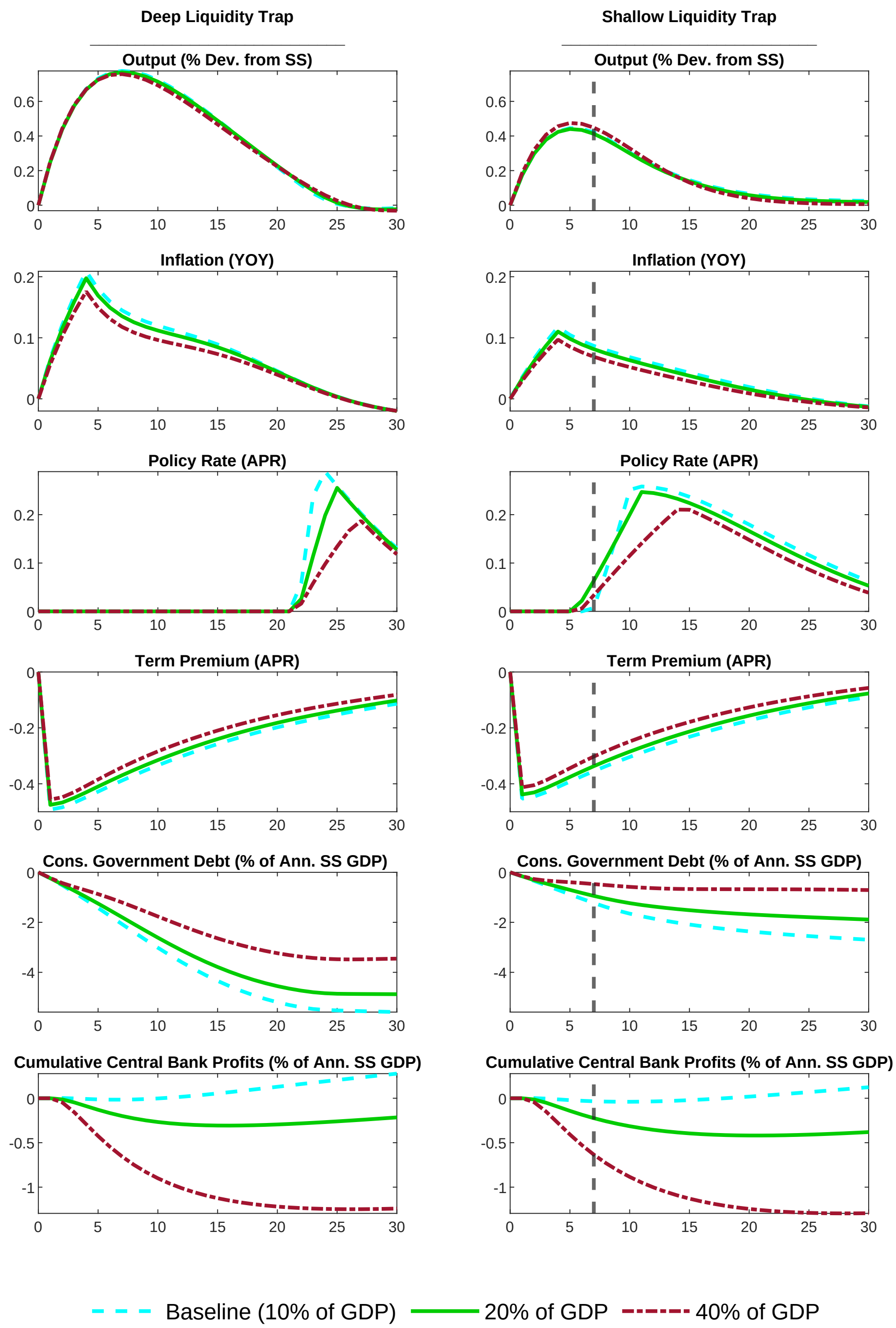

Figure 10: Sensitivity of QE Transmission to Alternative QE Size (Normalized).

The second is that—in light of the recent experience of high inflation – more caution is warranted in using QE in a shallow liquidity trap in which the central bank mainly faces a low inflation problem. While QE may appear beneficial ex ante, there is considerable risk that QE may cause overheating ex post given important nonlinearities in the Phillips Curve, the potential for an outsized easing of financial conditions, and that other inflation-raising shocks may hit after the deployment of QE. Negative interest rates may be preferable in these circumstances.

The third is that the duration risk that central banks take on with QE has the intended outcome of fueling risk-taking, which compresses term premia and eases financial conditions more broadly. But a side effect is to make central banks highly exposed to losses if interest rates rise enough. We argue that the benefits of QE are often significantly positive even when the central bank experiences losses, as can occur if the recovery from recession is unexpectedly fast and the yield curve inverts. Even so, the losses still represent a potentially first order headwind for central bank credibility and may ultimately weaken its independence, at least in some cases.

The fourth is that relative to conventional fiscal stimulus, QE is likely to be associated with significantly lower fiscal costs to boost the economy in an economic slump. QE lower debt service costs and boost tax revenues more that conventional fiscal policy for a given stimulus to output. This finding holds up even if the economy rebounds notably faster than envisioned at the time QE is implemented, but a problem for the central bank is that it may visible economic losses, whereas the gains for the treasury are less visible.

All told, our analysis suggests that merits of QE should be measured on the basis of its impact on the consolidated government finances, and how it can help stabilize the business cycle and inflation relative to other policy instruments. Doing so is critical to formulate good policy.

References

Acharya, Viral, S. Chauhan, Raghuram Rajan, and Sasha Steffen (2023). "Liquidity Dependence and the Waxing and Waning of Central Bank Balance Sheets." NBER Working Paper No. 31050.

Adrian, Tobias, Christopher Erceg, Marcin Kolasa, Jesper Lindé, and Pawel Zabczyk (2021): "A Quantitative Microfounded Model for the Integrated Policy Framework," IMF Working Papers 21/292, International Monetary Fund.

Adrian, Tobias, Ashraf Khan, and Lev Menand (2024). "A New Measure of Central Bank Independence." IMF Working paper Series No. 24/35.

Alesina, Alberto and Silvia Ardagna (2010). "Large Changes in Fiscal Policy: Taxes vs. Spending." In *Tax Policy in the Economy*, edited by Jeffrey Brown, NBER: University of Chicago Press, 35-68.

Almunia, Miguel, Augustin Bénétrix, Barry Eichengreen, Kevin O'Rourke, Gisela Rua, Silvana Tenreyro, and Fabrizio Perri (2010). "From Great Depression to Great Credit Crisis: Similarities, Differences, and Lessons," in *Economic Policy* 62 (eds G. De Ménil, R. Portes and H.-W. Sinn), John Wiley & Sons, Ltd, Chichester, UK.

Andres, Javier, J. David Lopez-Salido, and Edward Nelson (2004). "Tobin's imperfect asset substitution in optimizing general equilibrium," Journal of Money, Credit and Banking 36, 665–90.

Auerbach, Alan J. and Yuriy Gorodnichenko (2012). "Measuring the Output Responses to Fiscal Policy," *American Economic Journal: Economic Policy,* 4(2), 1-27.

Bilbiie, Florin and Mathias Trabandt (2023), "Sticky Prices or Sticky Wages? An Equivalence Result", mimeo, www.mathiastrabandt.com/.

Bernanke, Ben S. (2017). "Monetary Policy in a New Era." Paper prepared for a conference on Rethinking Macroeconomic Policy at the Peterson Institute of International Economics on October 2.

Blanchard, Olivier and Roberto Perotti (2002). "An Empirical Characterization of The Dynamic Effects of Changes in Government Spending and Taxes on Output," *The Quarterly Journal of Economics* 117(4), 1329-1368.

Blanchard, Olivier and Daniel Leigh (2014). "Learning about Fiscal Multipliers from Growth Forecast Errors," *IMF Economic Review* 62(2), 179-212.

Brzoza-Brzezina, M., P. Galinski, and K. Makarski (2022): "Monetary policy in a two-country model with behavioral expectations," Narodowy Bank Polski Working Papers 353.

Chen, Han, Vasco Cúrdia, and Andrea Ferrero (2012). "The Macroeconomic Effects of Large-scale Asset Purchase Programmes," Economic Journal 122, 289-315.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", Journal of Political Economy 113(1), 1–45.

Christiano, Lawrence J., Martin Eichenbaum, and Sergio Rebelo (2011). "When is the Government Spending Multiplier Large?," *Journal of Political Economy*, 119(1), 78-121.

Coenen, Günter, Christopher J. Erceg, Charles Freedman, Davide Furceri, Michael Kumhof, René Lalonde, Douglas Laxton, Jesper Lindé, Annabelle Mourougane, Dirk Muir, Susanna Mursula, Carlos de Resende, John Roberts, Werner Roeger, Stephen Snudden, Mathias Trabandt, and Jan in't Veld (2012). "Effects of Fiscal Stimulus in Structural Models," *American Economic Journal: Macroeconomics* 4(1), 22–68.

Del Negro, Marco, Marc Giannoni, and Christina Patterson (2012), “The forward guidance puzzle,” Staff Reports 574, Federal Reserve Bank of New York.

Di Casola, Paola (2021), “What does research say about the effects of central bank balance sheet policies?,” Sveriges Riksbank Economic Commentary No. 2.

Dixit, A. K. and J. E. Stiglitz (1977). “Monopolistic Competition and Optimum Product Diversity,” American Economic Review 67, 297–308.

Dotsey, M. and R. G. King (2005). “Implications of state-dependent pricing for dynamic macroeconomic models,” Journal of Monetary Economics 52, 213–242.

Drautzburg, Thorsten and Harald Uhlig, 2015. "Fiscal Stimulus and Distortionary Taxation,” *Review of Economic Dynamics* 18(4), 894-920.

Du, Wenxin, Kristin Forbes, and Matthew Luzzetti (2024). “Quantitative Tightening around the Globe: What Have We Learned?” Paper prepared for the U.S. Monetary Forum on February 28.

Erceg, Christopher J., Marcin Kolasa, Jesper Lindé, Haroon Mumtaz, and Pawel Zabczyk (2024). “Central Bank Exit Strategies Domestic Transmission and International Spillovers,” IMF Working Paper No. 2024-73.

Eggertsson, Gauti (2011). “What Fiscal Policy is Effective at Zero Interest Rates?” *NBER Macroeconomics Annual 2010, Volume 25,* NBER Books, National Bureau of Economic Research, 59-112.

Eggertsson, Gauti and Don Kohn (2023), “The Inflation Surge of the 2020s: The Role of Monetary Policy”, Hutchins Center Working Paper No. 87, Brookings.

Erceg, Christopher and Jesper Lindé (2014). “Is there a Fiscal Free Lunch in a Liquidity Trap?,” *Journal of the European Economic Association* 12(1), 73-107.

Fabo, Brian, Elisabeth Kempf, Martina Janoková, and Luboš Pástor (2021). ”Fifty Shades of QE: Comparing Findings of Central Banks and Academics”, Journal of Monetary Economics 120, 1-20.

Fair, Ray C. and John B. Taylor (1983), “Solution and Maximum Likelihood Estimation of Dynamic Rational Expectations Models”, Econometrica 51(4), 1169-1185.

Gabaix, Xavier (2020). “A Behavioral New Keynesian Model.” American Economic Review 110(8), 2271-2327.

Galí, Jordi, J. David López-Salido, and Javier Vallés (2007) “Understanding the Effects of Government Spending on Consumption,” Journal of the European Economic Association vol. 5(1), 227-270.

Gordon, Robert, and Robert Krenn (2010). “The End of the Great Depression 1939-41: Policy Contributions and Fiscal Multipliers,” NBER Working paper 16380.

Gopinath, Gita (2022). “How Will the Pandemic and War Shape Future Monetary Policy?” presentation prepared for the Jackson Hole Symposium on August 26.

Gopinath, Gita (2023). “Three Uncomfortable Truths for Monetary Policy.” Remarks prepared for the European Central Bank Forum on Central Bank on June 28.

Gopinath, Gita, Emine Boz, Camila Casas, Federico J. Diez, Pierre-Olivier Gourinchas, and Mikael Plagborg-Moller (2020): “Dominant Currency Paradigm,” American Economic Review 110, 677–719.

Gust, Christopher, Ed Herbst, and J. David Lopez-Salido (2022): “Short-Term Planning, Monetary Policy, and Macroeconomic Persistence,” American Economic Journal: Macroeconomics, 14, 174–209.

Hall, Robert and Ricardo Reis (2015). “Maintaining Central Bank Financial Stability under New-Style Central Banking.” NBER Working Paper No. 21173.

Harding Martin, Jesper Linde, and Mathias Trabandt (2022). “Resolving the Missing Deflation Puzzle.” Journal of Monetary Economics 126C, 15-34.

Harding Martin, Jesper Linde, and Mathias Trabandt (2023). “Understanding Post-COVID Inflation Dynamics” Journal of Monetary Economics, forthcoming.

Juillard, Michel (1996). “Dynare : a program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm,” CEPREMAP Working Papers (Couverture Orange) 9602.

Kiley, M. T. (2014). “The Aggregate Demand Effects of Short- and Long-Term Interest Rates,” International Journal of Central Banking 10, 69–104.

Kimball, Miles S. (1995), “The Quantitative Analytics of the Basic Neomonetarist Model,” Journal of Money, Credit, and Banking 27(4), 1241–1277.

Kolasa, Marcin and Grzegorz Wesolowski (2020). “International Spillovers of Quantitative Easing.” Journal of International Economics 126(C).

Kolasa, Marcin, Sahil Ravgotra, and Pawel Zabczyk (2022): “Monetary Policy and Exchange Rate Dynamics in a Behavioral Open Economy Model,” IMF Working Papers 2022/112.

Leeper, Eric M., Nora Traum, and Todd B. Walker (2017): “Clearing Up the Fiscal Multiplier Morass.” American Economic Review 107(8), 2409–54.

Orphanides, Athanasios (2023). “The Forward Guidance Trap.” Institute for Monetary and Economic Studies, Bank of Japan Discussion Paper No. 20234-E-6.

Ramey, Valerie A. (2011). “Identifying Government Spending Shocks: It’s All in the Timing,” *Quarterly Journal of Economics*, 126(1), 1-50.

Ramey, Valerie A., and Sarah Zubairy (2018), “Government Spending Multipliers in Good Times and in Bad: Evidence from U.S. Historical Data.” Journal of Political Economy 126(2), 850-901

Stella, Peter (2008). “Central Bank Balance Sheet Strength, Policy Constraints, and Inflation.” IMF Working Paper 08/49.

Tucker, Paul (2018). “Unelected Power.” Princeton, NJ: Princeton University Press.

Uhlig, Harald (2010). “Some Fiscal Calculus.” *American Economic Review Papers and Proceedings* 100(2), 30-34.

Woodford, M. (2001). “Fiscal Requirements for Price Stability,” Journal of Money, Credit and Banking 33, 669–728.

Woodford, Michael (2011). “Simple Analytics of the Government Expenditure Multiplier,” *American Economic Journal: Macroeconomics* 3(1), 1-35.

Appendix A. Additional Model Details

This appendix contains additional details on the model.

A.1. Term Premium and Duration

Typically, the 10Y term premium denotes the difference between the yield on a 10 year bond and the expected yield on a series of short term bonds. In a world with no uncertainty or adjustment costs, there would be no term premium, meaning that the yield on long term bonds would have to equal the expected yield of investing short term. Since the expected yield on the hypothetical long-term bond would equal the expected return on short term bonds, therefore we can define the term premium as

$$TP_t = R_{L,t} - R_{L,t}^{EH},$$

where $R_{L,t}^{EH}$ is the counterfactual yield to maturity on a longer-term bond in the absence of transaction costs.

Duration, in turn, is, by definition, equal to

$$D \equiv \frac{\sum_{s=1}^{+\infty} \frac{\kappa^{s-1} s}{R^s}}{P}$$

where R is the yield to maturity, and where, for a consol $R - \kappa = \frac{1}{P}$. Substituting the former into the numerator we end up with

$$D \equiv (R - \kappa) \sum_{s=1}^{+\infty} \frac{\kappa^{s-1} s}{R^s}.$$

Since

$$\sum_{s=1}^{+\infty} \frac{\kappa^{s-1} s}{R^s} = \frac{1}{R} \sum_{s=1}^{+\infty} s \left(\frac{\kappa}{R}\right)^{s-1},$$

therefore, letting $x \equiv \kappa/R$, we obtain

$$\frac{1}{R} \sum_{s=1}^{+\infty} s x^{s-1} = \frac{1}{R} \sum_{s=1}^{+\infty} \frac{d}{dx} x^s = \frac{1}{R} \frac{d}{dx} \sum_{s=1}^{+\infty} x^s = \frac{1}{R} \frac{d}{dx} \frac{x}{1-x} = \frac{1}{R} \frac{1}{(1-x)^2}.$$

Substituting back in for x we arrive at the formula used in the paper, i.e.,

$$D \equiv (R - \kappa) \left(\frac{1}{R} \frac{1}{\left(1 - \frac{\kappa}{R}\right)^2} \right) = (R - \kappa) \frac{R^2}{R (R - \kappa)^2} = \frac{R}{R - \kappa}.$$

A.2. The Government and Central Bank Balance Sheets

The CB balance sheet can be written as in Table Appendix A.1.

Table Appendix A.1: Central Bank Balance Sheet.

Assets	Liabilities
$\tilde{B}_t$	$\tilde{B}_t^{cp}$
$P_{L,t}B_{L,t}$	

This is because the central bank is assumed to be active in the asset market, i.e., it can take positions $\tilde{B}_t^c$ in short term bonds, and $B_{L,t}^c$ in long term bonds. These positions are entirely financed by issuing central bank commercial paper $\tilde{B}_t^{cp}$, which constitutes 100% of central bank liabilities, and which – from the perspective of private agents – is indistinguishable from short term government bonds.

Because of the perfect substitutability between central bank commercial paper and short term treasuries, purchases of the latter only affect the size of the CB balance sheet but not the quantity of short term assets outstanding.[A.1] For that reason we will only focus on purchases of long term assets and will "net out" shorter term assets, defining

$$B_t^c \equiv \tilde{B}_t^c - \tilde{B}_t^{cp}.$$

We can thus simplify the CB balance sheet in Table Appendix A.1 as in Table Appendix A.2.

Table Appendix A.2: Consolidated Central Bank Balance Sheet.

Assets	Liabilities
$P_{L,t}B_{L,t}^c$	B_t^c

This balance sheet also accounts for simplified definitions of government debt outstanding

$$B_F^g \equiv B_t^f - B_t^c,\ B_{L,t}^g \equiv B_{L,t}^f - B_{L,t}^c.$$

[A.1] The total quantity of short and long term government securities (i.e., B_t^g and $B_{L,t}^g$, respectively) equals

$$\begin{aligned} B_t^g &\equiv B_t^f - \tilde{B}_t^c + \tilde{B}_t^{cp} \\ B_{L,t}^g &\equiv B_{L,t}^f - B_{L,t}^c. \end{aligned}$$

The first of these equations immediately implies that purchases of short term assets $\tilde{B}_t^c$ financed by issuing commercial paper $\tilde{B}_t^{cp}$ leave government supplied short-term bonds $B_t^{g,\star}$ unchanged.

Importantly, because $B_t^c = P_{L,t}B_{L,t}^c$ therefore we shall refer to $P_{L,t}B_{L,t}^c$ as the size of QE.[A.2] Relatedly, the holding period profits associated with unconventional monetary policy equal

$$\Phi_t^c \equiv R_{t-1}B_{t-1}^c + P_{L,t}R_{L,t}B_{L,t-1}^c.$$

To keep matters simple, we assume that any QE "carry costs" are fully rebated to the treasury and that losses are rebated lump sum as well. As a consequence, the central bank budget constraint becomes

$$P_{L,t}B_{L,t}^c + B_t^c = R_{t-1}B_{t-1}^c + P_{L,t}R_{L,t}B_{L,t-1}^c - \Phi_t^c,$$

or alternatively

$$P_{L,t}B_{L,t}^c + B_t^c = 0,$$

which states that the central bank starts every period with a "clean" balance sheet.

Runoff, Revaluation and Reinvestment. We now formally define runoff and reinvestment, the first or which describes the mechanical phenomenon of assets maturing and leaving the central bank balance sheet, while the second essentially pins down the baseline investment strategy of the central bank that we shall analyze deviations from.

In our model, if the central bank purchased $P_{L,t-1}B_{L,t-1}^c$ worth of consols, then at the start of the following period it would have a coupon worth $B_{L,t-1}^c$ and a stock of assets with a market value of $\kappa P_{L,t}B_{L,t-1}^c$.[A.3] Mechanically, the change in value of long term assets Ψ_t thus equals

$$\Psi_t \equiv \kappa P_{L,t}B_{L,t-1}^c - P_{L,t-1}B_{L,t-1}^c,$$

i.e., it can be written as a combination of runoff and revaluation, as follows

$$\underbrace{\Psi_t}_{\text{“passive” change in portfolio value}} = \underbrace{-B_{L,t-1}^c}_{\text{runoff}} + \underbrace{\left(\kappa P_{L,t}B_{L,t-1}^c - P_{L,t-1}B_{L,t-1}^c + B_{L,t-1}^c\right)}_{\text{revaluation}},$$

where runoff is defined as being negative, as it tends to decrease the, typically positive, value of long term bonds held by the central bank . Exploiting $R_{L,t} \equiv (P_{L,t})^{-1} + \kappa$, we can then simplify

[A.2] Expressed alternatively, the model will be mute on the maturity composition of CB assets, because, in principle, we could have netted out any quantity of short term treasuries. In addition, since the central bank is assumed to purchase long-term treasuries when intervening, therefore the size and maturity composition of government debt determines the maximum size of unconventional stimulus. While intriguing, the potential implications of such considerations for optimal issuance policies are beyond the scope of our paper.

[A.3] A potentially helpful way of thinking about the consol is as a portfolio of zero coupon bonds with exponentially decaying nominal face value. Under that interpretation, it becomes clear that run-off would simply be equal to the face value of the first coupon, or $-B_{L,t-1}^c$.

the expression for the revaluation component as

$$
\begin{aligned}
(1+\kappa P_{L,t})\, B^c_{L,t-1} - P_{L,t-1} B^c_{L,t-1} &= P_{L,t}\left(\frac{1}{P_{L,t}}+\kappa\right) B^c_{L,t-1} - P_{L,t-1}B^c_{L,t-1} \\
&= (P_{L,t}R_{L,t} - P_{L,t-1})\, B^c_{L,t-1} = (\Pi_{L,t}R_{L,t}-1)\, P_{L,t-1}B^c_{L,t-1},
\end{aligned}
$$

which shows that positive inflation and yield to maturity will translate into positive nominal revaluation. Similarly, we can also express run-off in terms of the original value of the long term bond portfolio to arrive at

$$
\underbrace{\Psi_t}_{\text{“passive” change in LT portfolio value}} = \left(\underbrace{-\frac{1}{P_{L,t-1}}}_{\text{runoff}} + \underbrace{\Pi_{L,t}R_{L,t} - 1}_{\text{revaluation}} \right) P_{L,t-1}B^c_{L,t-1}.
$$

Of course, typically, the central bank will also have a reinvestment strategy in place to counterbalance run-off and revaluation, and it may occasionally wish to deviate from that strategy. To capture such considerations, yet still keep the analysis tractable, we assume that the passive reinvestment strategy is expressed as a share of runoff and revaluation, and that it is governed by parameter ϱ, i.e., that total reinvestment Θ_t is given by

$$
\underbrace{\Theta_t}_{\text{total reinvestment}} \equiv \underbrace{\varrho\left(\frac{1}{P_{L,t-1}} - \Pi_{L,t}R_{L,t} + 1\right) P_{L,t-1}B^c_{L,t-1}}_{\text{passive reinvestment}} + \underbrace{\epsilon^c_t}_{\text{active reinvestment}}.
$$

Collecting terms, the expression for the evolution of the value of the central bank's portfolio becomes

$$
\begin{aligned}
P_{L,t}B^c_{L,t} &= \underbrace{P_{L,t-1}B^c_{L,t-1}}_{\text{previous value}} + \underbrace{\Psi_t}_{\text{mechanical change in value of QE portfolio}} + \underbrace{\Theta_t}_{\text{passive and active reinvestment}} \\
&= \left(1 + (1-\varrho)\left(-\frac{1}{P_{L,t-1}} + \Pi_{L,t}R_{L,t} - 1\right)\right) P_{L,t-1}B^c_{L,t-1} + \epsilon^c_t,
\end{aligned}
$$

which confirms that with ϱ set to one, and absent active reinvestment $\epsilon^c_t = 0$, the nominal value of the long term bond portfolio would stay constant.[A.4] We conclude by presenting a real equivalent of the above expression, which we obtain by dividing through by P_t and simplifying to arrive at

$$
P_{L,t}b^c_{L,t} = \frac{\left(1 + (1-\varrho)\left(-\frac{1}{P_{L,t-1}} + \Pi_{L,t}R_{L,t} - 1\right)\right)}{\Pi_t} P_{L,t-1}b^c_{L,t-1} + \frac{\epsilon^c_t}{P_t}.
$$

[A.4] Conversely, with ϱ set to zero, corresponding to no reinvestment, the value of the portfolio would decrease at its fastest possible rate (barring active asset sales).

This relationship confirms that positive inflation acts to erode the real value of the central banks' portfolio, even if the nominal value of debt outstanding is held fixed (i.e., even if ϱ is equal to one).[A.5]

The fiscal authority operates subject to the following constraint

$$B_t^f + P_{L,t}B_{L,t}^f + T_t + \Phi_t^c = R_{t-1}B_{t-1}^f + P_{L,t}R_{L,t}B_{L,t-1}^f + P_t g_t,$$

i.e., it finances its expenditures by issuing long-term $B_{L,t}^f$, and short-term B_t^f bonds, as well as through lump sum taxation T_t and any holding-period profits made by the central bank on its asset portfolio Φ_t^c, since we assume these are transferred back to the treasury. As the expression makes clear, the total amount requiring financing is a sum of the value of maturing obligations $R_{t-1}B_{t-1}^f + P_{L,t}R_{L,t}B_{L,t-1}^f$ as well as government expenditures $P_t g_t$, with the total market value of outstanding government debt given by

$$\mathcal{B}_t^f \equiv B_t^f + P_{L,t}B_{L,t}^f.$$

Government expenditures on final goods are assumed to follow $g_t \equiv g\exp\{\varepsilon_t^g\}$, where ε_t^g is the government spending shock. Taxes per capita are set equal across the two household types, which implies that they are levied in proportion to the restricted and unrestricted households' population shares ω_r and $1-\omega_r$, respectively. In addition, in the baseline version of our model, the fiscal authority of the large country is assumed to keep the composition of government debt evaluated at steady state prices $\left(P_L B_{L,t}^f\right) / \left(P_L B_{L,t}^f + B_t^f\right)$ constant.

[A.5] See also Appendices Appendix A.3 and Appendix A.4 for a more detailed discussion of the underlying considerations.

A.3. Results Under Lower QE Macrotransmission

The empirical evidence discussed in Section 2 does not rule out the possibility of a very modest impact of QE, especially when QE is done for boosting inflation faster to target outside of a financial stress episode (initial term premium is small or possibly even negative) and the ELB is not expected to be binding for an extended period. Although the baseline calibration of the model embodies a conservative view on the macrotransmission of QE, we here consider an calibration with even smaller macroeconomic benefits of QE, to examine the implications for the central banks' balance sheet and the consolidated governments' fiscal position. The results are reported in Figure TBA

[**Considering adding a figure with half the macrotransmission of QE to examine the robustness of conclusions. The figure is distinguished from Figure 9 by featuring the same effect on the Term-premium but a smaller elasticity between the term-premium and the macroeconomy (output and inflation).**]